$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \end{array}\right]$$

Upto here it's Gauss-Jordan elimination method.

$\therefore$ $x_1 = 2$ //

$x_2 = -1$ //

<u>Set</u> : A well-defined collection of objects

binary operation

$S \times S \rightarrow S$

$a, b \in S$

$a+b = c$

P1) <u>Closure law</u>:

$+ : N \times N \rightarrow N$

$2, 3 \in N$ $(N, +)$

$2+3 = 5$

eg: $(N, -)$ is not ~~to~~ closed as $1-2 = -1 \notin N$

$(I, -)$ is closed

$(N, *)$ is defined under closure law

P2) <u>Associative law</u>: let $a, b, c \in S$

$a + (b+c) = (a+b) + c$

eg i) $(N, +)$ ✓

ii) $(I, -)$ ✗ eg: $1-3-4$

iii) $(N, *)$ ✓

P3) <u>Identity</u>: if $\exists$ an element $e \ni$
$a+e = e+a = a$, $\forall a \in S$

eg. i) $(I, +)$ ✓
ii) $(N, *)$ ✓
iii) $(N, +)$ ✗ ($\because$ 0 is not natural)

P4) <u>Inverse</u>: for each $a \in S$ $\exists$ $b \ni$
$a+b = b+a = e$
$a*b = b*a = e$

eg: i) $(N, *)$ ✗
ii) $(R-\{0\}, *)$ ✓
iii) $(Q, +)$ ✓
iv) $(R, +)$ ✓
v) $(R-\{0\}, *)$ ✓

P5) <u>Commutative law</u>:

$a+b = b+a$
$a*b = b*a$

eg: i) $(N, +)$ ✓
ii) $(N, *)$ ✓
iii) $(R, +)$ ✓
iv) $(Q, +)$ ✓
v) $(R, *)$ ✓
vi) $(N, -)$ ✗

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Group: A structure $(S,+)$ satisfying P1, P2, P3, P4

eg: $(R,+)$

Commutative group: group satisfying P1, P2, P3, P4, P5

Field:

1) $(S,+)$ Commutative group

2) $(S-\{0\}, *)$ commutative group

3) ~~Distributive~~ law

P6) Distributive law: $a(b+c) = ab + ac$

eg: 1) $(R,+)$ group? Ans: Yes, commutative group

$(R-\{0\}, *)$? Ans: "

Hence R is a field.

2) $(C, +, *)$ ✓ (complex)

3) $(Q, +, *)$ ✓

4) Which of the following is ~~not~~ a field:

a) $(R, +, *)$

b) $(C, +, *)$

c) $(I, +, *)$ ✓

d) $(Q, +, *)$

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Vector Space: (set of all possible functions)

Let $<F, +, \cdot>$ be a Field and V is a non-empty set with two operations.

Vector addition: This assigns to any $\alpha, \beta \in V$ to $\alpha + \beta \in V$

Scalar multiplication: This assigns to any $\alpha \in V$, $a \in F$ to a product ~~ad~~ $a\alpha \in V$

Then V is said to be vector space over the Field if the following hold :-

[A1] $(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$ – Associative

[A2] There is a vector 0 in V.
Such that $\alpha + 0 = 0 + \alpha = \alpha$, $\forall \alpha \in V$
0 is said to be the identity of V.

[A3] For each $\alpha \in V$ there exist $-\alpha \in V \ni$
$\alpha + (-\alpha) = (-\alpha) + \alpha = 0$ – Inverse

[A4] $\alpha + \beta = \beta + \alpha$ (Commutative)

[M1] $a(\alpha + \beta) = a\alpha + a\beta$, $\forall a, b \in F$

[M2] $(a+b)\alpha = a\alpha + b\alpha$

[M3] $(ab)\alpha = a(b\alpha)$

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[M 4] ~~for~~ $1 \cdot \alpha = \alpha$, $\forall \alpha$, $1 \in F$

NOTE: The elements of Field (F) are said to be scalar and elements of V are said to be vectors

Q. 1 Show that V is a vector over R.
Is V(R) a vector space ??

Sol: 1: 1) Addition is closed:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix} \in V$$

2) ///ly $A+(B+C) = (A+B)+C$, where A, B, C are 3 different matrices

3) $I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ identity of V

$$A+I = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = A$$

///ly $I+A = A$

4) $A+B = B+A$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix} = \begin{bmatrix} b_{11}+a_{11} & b_{12}+a_{12} \\ b_{21}+a_{21} & b_{22}+a_{22} \end{bmatrix}$$

$$= \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = B+A$$

5) Inverse: $-A = \begin{bmatrix} -a_{11} & -a_{12} \\ -a_{21} & -a_{22} \end{bmatrix}$

$A+(-A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, |||ly $(-A)+A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

|||ly prove scalar multiplication and M_1, M_2, M_3, M_4.

Q2. Is Set of all polynomials of degree 2 is a vector space over R?

Sol: $V = \{ a_0 + a_1 x + a_2 x^2 : a_0, a_1, a_2 \in R, a_2 \neq 0 \}$

~~Sol:~~ $\alpha = 1 + x + 2x^2$, $\beta = 1 + 3x - 2x^2$

$\alpha+\beta = (1+x+2x^2) + (1+3x-2x^2)$

$= 2+4x$ ($\therefore$ not a vector space) ✓

Also identity doesn't exist

Q3. Same as above → just degree <3 or ≤ 3

Sol: Yes, vector space ✓

Q.4 Set of all polynomials is a vector space over R. T/F.

Sol: vector space ✓.

eg: let $V = \{(a,b) : a,b \in R\}$ be a non-empty set with two operations:

~~(a_1, b_1)~~

Vector addition $(a_1, b_1) + (a_2, b_2) = (a_1 + a_2, b_1 + b_2)$

Scalar multiplication: $k(a_1, b_1) = (ka_1, 0)$, $k \in R$

eg: $1 \cdot (a_1, b_1) = (1 \cdot a_1, 0) = (a_1, 0) \neq (a_1, b_1)$

$1 \cdot (2,3) = 1(2,0) = (2,0) \neq (2,3)$. Not V space

eg: $\begin{cases} (a_1, b_1) + (a_2, b_2) = (a_1 + b_1, a_2 + b_2) \\ k(a,b) = (ka, kb) \end{cases}$

→ Not V-space ✓

eg: $(a_1, b_1) = (2,3)$

$(2,3) + (0,0) = (5,0)$ (Idendity element doesn't exist)

eg: Another operation

$\begin{cases} (a_1, b_1) + (a_2, b_2) = (a_1, b_1) \\ k(a_1, b_1) = (ka_1, kb_1) \end{cases}$

→ Not V-space ✓

eg: Another operation defined as

$(a_1,b_1) + (a_2,b_2) = (a_1+a_2, b_1+b_2)$

$k(a,b) = (ka, kb)$

Check whether V is a vector space over R.

Sol^n: Vector addn: $(a_1,b_1) + (a_2,b_2) = (a_1+a_2, b_1+b_2) \in V$

Scalar multip... : $k(a,b) = (ka, kb) \in V$

[A1] Commutative : $(a_1,b_1) + (a_2,b_2) = (a_1+a_2, b_1+b_2)$

$= (a_2+a_1, b_2+b_1) = (a_2,b_2) + (a_1,b_1)$

[A2] Ily Associative

[A3] Identity element $(0,0)$: $(a_1,b_1) + (0,0) = (a_1+0, b_1+0)$

$= (a_1,b_1) = (0+a_1, 0+b_1)$

$= (0,0) + (a_1,b_1)$

[A4] Inverse : $(a_1,b_1) + (-a_1,-b_1) = (a_1-a_1, b_1-b_1)$

$= (0,0) = (-a_1+a_1, -b_1+b_1)$

$= (-a_1,-b_1) + (a_1,b_1)$

[M1] $k[(a_1,b_1) + (a_2,b_2)] = k(a_1,b_1) + k(a_2,b_2)$

(prove it).

[M2] $(k_1+k_2)[(a,b)] = ((k_1+k_2)a, (k_1+k_2)b)$

(prove it.)

[M3] $k_1k_2(a,b) = k_1[k_2a, k_2b]$

[M4] $1 \cdot (a,b) = (1 \cdot a, 1 \cdot b) = (a,b)$.

$\because$ all the 10 axioms are satisfied by V. Hence V is V-space.

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Subspace :- ~~of~~

1. V (F)
2. $W \subseteq V$
3. W is itself a vector space under the given operation

Theorem: V is a vector space, & W is non-empty ~~set~~ subset of V. Then W is a subspace of V if and only if

$$a\alpha + \beta \in W \quad \forall\ \alpha, \beta \in W$$
$$a \in F$$

eg: Let V be a set of all real valued continuous functions over R, and W be a set of all solution of valued continuous function such that

$$\boxed{2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 5y = 0}$$

$$W = \left\{ y : 2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 5y = 0 \right\} \subseteq V$$

Sol^n: let $y_1, y_2 \in W$

$$\Rightarrow 2\frac{d^2y_1}{dx^2} + 3\frac{dy_1}{dx} + 5y_1 = 0 \quad -(1)$$

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and $2\frac{d^2y_2}{dx^2} + \frac{3dy_2}{dx} + 5y_2 = 0$ — (2)

satisfying ay_1+y_2 in place of y we get

$$2\frac{d^2(ay_1+y_2)}{dx^2} + \frac{3d(ay_1+y_2)}{dx} + 5(ay_1+y_2)$$

$$= \cancel{2d^2}\ 2a\frac{d^2y_1}{dx^2} + 2\frac{d^2y_2}{dx^2} + 3a\frac{dy_1}{dx} + 3\frac{dy_2}{dx} + 5ay_1 + 5y_2$$

$$= a\left[2\frac{d^2y_1}{dx^2} + 3\frac{dy_1}{dx} + 5y_1\right] + 0 \quad \text{(from (2))}$$

$$= a(0) + 0 \quad \text{(from (1))}$$

$$= 0$$

$\therefore$ $ay_1+y_2 \in W$ (proved) //

$\therefore$ W is subspace of V. ✓

Assignm

Ex: $V = \{(a,b,c) : a,b,c \in R\}$

$W_1 = \{(a,b,c) : a+b+c = 1, a,b,c \in R\}$
definidely $W \subseteq V$ — Not sub-space

$W_2 = \{(a,b,c) : 2a+3b-c = 0, a,b,c \in R\}$
— subspace

$W_3 = \{(a,b,c) : 2a = b ; a,b,c \in R\}$ — Subspace

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