

GR8

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**SCM 651 Fall 2018 Group Assignment 2**

**Due Date: Midnight, Tuesday, 10/16/2018, Total Points = 40**

38½  
40

**Please use the programming language R to complete this assignment. Copy and paste relevant parts of the R output and/or screen shots into a Word file to prepare the answers.**

1.(20 points) Please use the data set 651F18 Orange Juice Homework 2.csv to do all parts of question 1.

This data set provides, for a random sample of 5780 cases drawn from the Dominicks data base, made available by University of Chicago, Kilts Center:

- MOVE: Number of units sold for three brands of orange juice: Florida's Natural Home-squeezed (FLNAT), Tree Fresh (TF), and Tropicana Grove Stand (TROPICANA), at a store in a given week.
- PRICE: Unit price of the brand.
- logMOVE: Natural logarithm of MOVE.
- logPRICE: Natural logarithm of price.
- BRAND
- Season
- Feat (1 if product is on sale, 0 if not)
- Demographic variables at the store location: AGE9, AGE60, EDUC, ETHNIC, INCOME, HSIZEAV, HH3PLUS, HH4PLUS, HHSINGLE, HHLARGE, HVAL150, HVAL200, MORTGAGE, NOCAR, NWHITE, SINGLE, POVERTY, RETIRED, SINGLE, UNEMP, WORKWOM

1(a)(4+3+3 = 10 points) Fit a regression model with dependent variable logMOVE and the following independent variables:

- BRAND
- logPRICE
- Interaction between BRAND and logprice
- Feat
- Season
- Demographic variables given in the data set.

1(a)(i) From the estimated parameters, what are the price elasticities of demand of the three brands?

Call:

```
lm(formula = logMOVE ~ BRAND + logPRICE + BRAND * logPRICE +
    Feat + Season + AGE9 + AGE60 + EDUC + ETHNIC + INCOME + HSIZEAVG +
    HH3PLUS + HH4PLUS + HHSINGLE + HHLARGE + HVAL150 + HVAL200 +
    MORTGAGE + NOCAR + NWHITE + SINGLE + POVERTY + RETIRED +
    UNEMP + WORKWOM, data = oj)
```

Residuals:

Min	1Q	Median	3Q	Max
-4.8048	-0.4487	0.0203	0.4549	3.3468

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-10.83734	4.36791	-2.481	0.013125 *
BRAND[T.IF]	-0.54003	0.13661	-3.953	7.81e-05 ***
BRAND[T.TROPICANA]	1.06738	0.16127	6.619	3.95e-11 ***
logPRICE	-3.31164	0.13166	-25.153	< 2e-16 ***
Feat	0.23030	0.02409	9.559	< 2e-16 ***
Season[T.Spring]	-0.02445	0.02885	-0.847	0.396757
Season[T.Summer]	-0.02978	0.02849	-1.045	0.295895
Season[T.Winter]	0.04221	0.02909	1.451	0.146865
AGE9	10.67244	2.23239	4.781	1.79e-06 ***
AGE60	2.39892	1.45809	1.645	0.099972 .
EDUC	-0.40258	0.34949	-1.152	0.249411
ETHNIC	-1.50328	0.39754	-3.782	0.000157 ***
INCOME	0.06580	0.19917	0.330	0.741123
HSIZEAVG	8.52585	2.13839	3.987	6.77e-05 ***
HH3PLUS	-4.68900	3.52432	-1.330	0.183417
HH4PLUS	-11.28363	3.63435	-3.105	0.001914 **
HHSINGLE	3.96397	2.69468	1.471	0.141337
HHLARGE	-32.06710	6.78987	-4.723	2.38e-06 ***
HVAL150	0.94689	0.17724	5.343	9.52e-08 ***
HVAL200	-0.08945	0.22792	-0.392	0.694721
MORTGAGE	0.15890	0.20936	0.759	0.447917
NOCAR	1.70522	0.45465	3.751	0.000178 ***
NWHITE	0.71943	0.39578	1.818	0.069153 .
SINGLE	6.52714	0.91567	7.128	1.14e-12 ***
POVERTY	-1.87986	1.63822	-1.148	0.251221
RETIRED	-1.41060	1.82679	-0.772	0.440043
UNEMP	-3.95899	2.67228	-1.482	0.138527
WORKWOM	-4.82137	1.44372	-3.340	0.000845 ***
BRAND[T.IF]:logPRICE	0.40820	0.14540	2.807	0.005010 **
BRAND[T.TROPICANA]:logPRICE	-0.33128	0.16793	-1.973	0.048577 *

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.768 on 5750 degrees of freedom

Multiple R-squared: 0.5283, Adjusted R-squared: 0.526

F-statistic: 222.1 on 29 and 5750 DF, p-value: < 2.2e-16

FLNAT: -3.31164

TF:  $-3.31164 - 0.54003 = -3.85167$

you added coeffs of  
TF & TROPICANA.

Need to add coeffs  
of  $\log \text{Price} + \text{TF}$

2 Log Price \* TROPICANA  
(-1/2)



1(a)(iii) Starting from the full model estimated, test the following hypothesis at a 99% level of confidence:

```
> local({
+   .Hypothesis <- matrix(c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,-1), 1, 30, byrow=TRUE)
+   .RHS <- c(0)
+   linearHypothesis(LinModel.4, .Hypothesis, rhs=.RHS)
+ })
Linear hypothesis test

Hypothesis:
BRAND[T.TF]:logPRICE - BRAND[T.TROPICANA]:logPRICE = 0 ✓

Model 1: restricted model
Model 2: logMOVE ~ BRAND + logPRICE + BRAND * logPRICE + Feat + Season +
AGE9 + AGE60 + EDUC + ETHNIC + INCOME + HSIZEAVG + HH3PLUS +
HH4PLUS + HHSINGLE + HHLARGE + HVAL150 + HVAL200 + MORTGAGE +
NOCAR + NWHITE + SINGLE + POVERTY + RETIRED + UNEMP + WORKWOM

Res.Df    RSS Df Sum of Sq      F       Pr(>F)
1     5751 3409.3
2     5750 3391.7    1      17.616 29.865 0.000000004828 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



Since the P value is **much smaller than 0.01**, so REJECT the null hypothesis, thus the price elasticity is not same for TF and TROPICANA at 99% level of confidence.

1(b)(5 points) From the output from the full model in 1(a)(i), identify the demographic variables that are not significant at a 90% level of confidence. At a 99% level of confidence, test the null hypothesis that none of these variables is significant. ✓

At 90% level of confidence, the following demographic variables that are not significant: EDUC, INCOME, HH3PLUS, HHSINGLE, HVAL200, MORTGAGE, POVERTY, RETIRED, UNEMP

Linear hypothesis test

Hypothesis:

EDUC = 0

INCOME = 0

HH3PLUS = 0

HHSINGLE = 0

HVAL200 = 0

MORTGAGE = 0

POVERTY = 0

RETIRED = 0

UNEMP = 0

Model 1: restricted model

Model 2:  $\log\text{MOVE} \sim \text{BRAND} + \log\text{PRICE} + \text{BRAND} * \log\text{PRICE} + \text{Feat} + \text{Season} + \text{AGE9} + \text{AGE60} + \text{EDUC} + \text{ETHNIC} + \text{INCOME} + \text{HSIZEAVG} + \text{HH3PLUS} + \text{HH4PLUS} + \text{HHSINGLE} + \text{HHLARGE} + \text{HVAL150} + \text{HVAL200} + \text{MORTGAGE} + \text{NOCAR} + \text{NWHITE} + \text{SINGLE} + \text{POVERTY} + \text{RETIRED} + \text{UNEMP} + \text{WORKWOM}$

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	5759	3398.5				
2	5750	3391.7	9	6.8022	1.2813	0.2413

 ✓

Since the P value is **much greater than 0.01**, so CANNOT REJECT the null hypothesis. We cannot conclude none of the variables is significant.

1(c)(5 points) Based on the result of the hypothesis test in question 1(b), keep the full model or the restricted model as appropriate as the next model. Based on variance inflation factor (VIF), is there any problem with multi-collinearity with this model? If yes, then modify the model to reduce the effect of multi-collinearity. (To do this, identify groups of demographic variables that are strongly correlated with one another. Then, keep one variable from each group.) Fit the model and check if the problem of multi-collinearity is reduced.

VIF for the full model shows below:



	GVDF	DF	GVDF*(1/(2*DF))
BRAND	908.396783	2	5.4389956
logPRICE	8.441930	1	2.912875
Feat	1.294573	1	1.187798
SEASON	1.033472	3	1.005502
AGE9	29.103878	1	5.394301
AGE60	79.711189	1	8.928112
ETHNIC	15.116887	1	3.888044
ETHNIC	55.784658	1	7.468913
INCOME	32.555378	1	5.7095732
HSEAEVG	2849.879032	1	53.384258
HH3PLUS	774.413378	1	27.382834
HH4PLUS	470.9978588	1	21.7092041
HH5SNGLE	455.442956	1	21.3411109
HH4LARGE	394.934248	1	19.8724499
HVAL150	18.362552	1	4.2851555
HVAL200	17.246656	1	4.1529099
MORTGAGE	8.813916	1	2.968824
NOCAR	33.896115	1	5.822037
NWHITE	55.788803	1	7.4691909
SNGLE	35.205826	1	5.9334509
POVERTY	52.275853	1	7.230204
RETIRED	82.343848	1	9.074351
UNEMP	36.907010	1	6.075114
WORKWOM	56.334651	1	7.505641
BRAND: logPRICE	678.412724	2	5.103563

Since we have multiple variables P value >0.05 and strongly correlated, we reduce the full model to model B.

```
lm(formula = logMOVE ~ BRAND + logPRICE + Feat + AGE9 + ETHNIC +
    HSEAEVG + HH4PLUS + HHLARGE + HVAL150 + NOCAR + SINGLE +
    WORKWOM, data = oj)
```

Residuals:

Min	1Q	Median	3Q	Max
-5.0138	-0.4480	0.0208	0.4617	3.3884

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-3.60416	0.95798	-3.762	0.000170 ***
BRANDTF	-0.20027	0.03001	-6.673	2.74e-11 ***
BRANDTROPICANA	0.77765	0.02523	30.818	< 2e-16 ***
logPRICE	-3.11603	0.06038	-51.608	< 2e-16 ***
Feat	0.25643	0.02361	10.863	< 2e-16 ***
AGE9	3.70177	1.06801	3.466	0.000532 ***
ETHNIC	-0.88561	0.12676	-6.986	3.14e-12 ***
HSEAEVG	5.46294	0.50107	10.903	< 2e-16 ***
HH4PLUS	-15.46418	1.67914	-9.210	< 2e-16 ***
HHLARGE	-16.81392	1.89758	-8.861	< 2e-16 ***
HVAL150	0.91361	0.06010	15.202	< 2e-16 ***
NOCAR	1.80118	0.24657	7.305	3.15e-13 ***
SNGLE	3.63382	0.55522	6.545	6.47e-11 ***
WORKWOM	-2.37155	0.35095	-6.758	1.54e-11 ***

---  
 signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.7719 on 5766 degrees of freedom  
 Multiple R-squared: 0.5223, Adjusted R-squared: 0.5212  
 F-statistic: 484.9 on 13 and 5766 DF, p-value: < 2.2e-16

VIF for the model B shows below:

	GVDF	DF	GVDF*(1/(2*DF))
BRAND	1.1518678	2	1.103196
logPRICE	1.766118	1	1.328954
FEAT	1.230449	1	1.103256
AGE9	6.595235	1	2.568119
ETHNIC	5.615731	1	2.369753
HSIZEAVG	154.925761	1	12.446918
HH4PLUS	99.538993	1	9.976923
HHLARGE	80.538334	1	5.526200
HVAL150	2.030401	1	1.445822
NOCAR	9.870323	1	3.141707
SINGLE	12.815321	1	3.579849
WORKWOM	3.295816	1	1.815438

Although all p value is smaller than 0.05, meaning significant in model B, we have some variables, such as HSIZEAVG, HH4PLUS and HHLARGE are strongly correlated. So we reduce the model again to model C.

Residuals:

Min	1Q	Median	3Q	Max
-5.2380	-0.4534	0.0214	0.4722	3.4361

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	7.33526	0.29206	25.116	< 2e-16	***
BRANDTF	-0.20316	0.03036	-6.691	2.42e-11	***
BRANDTROPICANA	0.77747	0.02554	30.442	< 2e-16	***
logPRICE	-3.11392	0.06098	-51.067	< 2e-16	***
Feat	0.25776	0.02386	10.803	< 2e-16	***
AGE9	-1.57308	0.91317	-1.723	0.085005	.
ETHNIC	0.02048	0.09934	0.206	0.836644	
HSIZEAVG	-0.37539	0.10692	-3.511	0.000450	***
HVAL150	1.06644	0.05310	20.083	< 2e-16	***
NOCAR	0.83080	0.22943	3.621	0.000296	***
SINGLE	-0.44401	0.44288	-1.003	0.316123	
WORKWOM	-1.75765	0.34393	-5.110	3.32e-07	***

---

signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.7814 on 5768 degrees of freedom  
Multiple R-squared: 0.5102, Adjusted R-squared: 0.5093  
F-statistic: 546.3 on 11 and 5768 DF, p-value: < 2.2e-16

VIF for the model C shows below:

	Q <sup>2</sup> OF 10F	Q <sup>2</sup> OF A(1/(2*10F))	
BRAND	1.511612	2	1.100417
logPRICE	1.757564	1	1.825711
FEAT	1.226671	1	1.100752
AGE	4.704531	1	2.168993
ETHNIC	3.365364	1	1.834493
HSCZEAUG	6.883462	1	2.623635
PAVALLSD	1.582418	1	1.261910
NOCAR	8.33412	1	2.887631
SINGLE	7.956414	1	2.820712
WORKING	3.088553	1	1.757428

In this case, both model B and model C are better than full model, because they have less correlation between variables. However, by reducing the # of variables in the model, the Rsquare and adjust Rsquare become smaller.

2.(5 points) Please use the data set 651F18 Orange Juice Homework 2.csv to answer this question.

Fit a regression model with dependent variable log of move, and independent variables BRAND, Feat, logPRICE and BRAND\*logPRICE. For the six cases given below, use R to construct 99% prediction intervals for logMOVE.

Case	BRAND	logPRICE	Feat
1	FLNAT	.9	1
2	FLNAT	1.0	0
3	TF	.55	1
4	TF	.75	0
5	TROPICANA	.80	1
6	TROPICANA	.95	0

**Solution:**

**r-CODE:**

```
lm2a<-lm(logMOVE~BRAND+Feat+logPRICE+BRAND*logPRICE,data=oj1)
summary(lm2a)
predict(lm2a,interval="prediction",level=.99,newdata=Book1)
```

## Output:

```
> lm2a<-lm(logMOVE~BRAND+Feat+logPRICE+BRAND*logPRICE,data=oj1)
> summary(lm2a)

Call:
lm(formula = logMOVE ~ BRAND + Feat + logPRICE + BRAND * logPRICE,
    data = oj1)

Residuals:
    Min       1Q   Median       3Q      Max
-5.3859 -0.4938  0.0086  0.5080  3.2308

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)    5.55183    0.14210   39.071 < 2e-16 ***
BRANDTF        -0.19352    0.14776   -1.310  0.1904
BRANDTROPICANA  1.17171    0.17540    6.680 2.61e-11 ***
Feat            0.30155    0.02593   11.629 < 2e-16 ***
logPRICE       -2.83936    0.14140  -20.080 < 2e-16 ***
BRANDTF:logPRICE  0.09249    0.15748    0.587  0.5570
BRANDTROPICANA:logPRICE -0.42307    0.18265   -2.316  0.0206 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8377 on 5773 degrees of freedom
Multiple R-squared:  0.4366, Adjusted R-squared:  0.4361
F-statistic: 745.7 on 6 and 5773 DF, p-value: < 2.2e-16

> predict(lm2a,interval="prediction",level=.99,newdata=Book1)
      fit      lwr      upr
1 3.297963 1.1384684 5.457457
2 2.712472 0.5533087 4.871636
3 4.149094 1.9895548 6.308634
4 3.298166 1.1389990 5.457333
5 4.415150 2.2555732 6.574726
6 3.624230 1.4650688 5.783391
```

3.(a)(4 points) Please perform logit analysis using Personal Loan as the dependent variable, and all the remaining variables as independent variables. (For education, include the two dummy variables GRAD and PROF.) Include **only main effects** in your model. Which variables are significant at a 90% level of confidence? Copy and paste screen shots from R analysis to support your answers.



## Solution

```
GLM3a<-glm(PersonalLoan~Age+Experience+Income+Family+CCAvg+GRAD+PROF+Mortgage+SecuritiesAccount+
            CDAccount+Online+CreditCard,data=Bank)
```

```
summary(GLM3a)
```

### Deviance Residuals:

Min	1Q	Median	3Q	Max
-0.79039	-0.13533	-0.03435	0.07122	1.05807

### Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-0.17725205	0.06969237	-2.543	0.0110 *
Age	-0.00534073	0.00273640	-1.952	0.0510 .
Experience	0.00588222	0.00273375	2.152	0.0315 *
Income	0.00306523	0.00009623	31.854	< 2e-16 ***
Family	0.03017912	0.00289139	10.438	< 2e-16 ***
CCAvg	0.01215732	0.00243823	4.986	0.0000006369 ***
GRAD	0.14539853	0.00817964	17.776	< 2e-16 ***
PROF	0.15441144	0.00816394	18.914	< 2e-16 ***
Mortgage	0.00006759	0.00003267	2.069	0.0386 *
SecuritiesAccount	-0.05983812	0.01130047	-5.295	0.0000001240 ***
CDAccount	0.32609565	0.01568961	20.784	< 2e-16 ***
Online	-0.02752139	0.00673604	-4.086	0.0000446353 ***
CreditCard	-0.04380917	0.00748760	-5.851	0.0000000052 ***

---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for gaussian family taken to be 0.05249746)

Null deviance: 433.92 on 4999 degrees of freedom  
Residual deviance: 261.80 on 4987 degrees of freedom  
AIC: -530.58

Number of Fisher Scoring iterations: 2

```
confint(GLM3a,level=0.9,type="LR")
```

```
> confint (GLM3a, level=0.9, type="LR")
```

	5 %	95 %
(Intercept)	-0.29188579497	-0.0626183074
Age	-0.00984171512	-0.0008397473
Experience	0.00138559564	0.0103788462
Income	0.00290695137	0.0032235099
Family	0.02542321646	0.0349350305
CCAvg	0.00814678649	0.0161678612
GRAD	0.13194422027	0.1588528371
PROF	0.14098294150	0.1678399301
Mortgage	0.00001385491	0.0001213223
SecuritiesAccount	-0.07842574124	-0.0412504974
CDAccount	0.30028854550	0.3519027636
Online	-0.03860118748	-0.0164415890
CreditCard	-0.05612516982	-0.0314931612

Variables significant at a 90% level of confidence according to the R Analysis in the screenshots above are:

Income, Family, CCAvg, GRAD, PROF, CDAccount, SecuritiesAccount, Online and CreditCard

3(b)(4 points) From your answer to question 3(a), list the variables that are not significant at a 90% level of confidence. Using test linear hypothesis, test the null hypothesis that none of these variables is significant using a 99% level of confidence. Copy and paste R screenshots to support your answer and clearly state your conclusion.

**Solution:**

The variables that are not significant at a 90% level of confidence according to the results from question 3(a) are:

Age, Experience, Mortgage

Testing the null hypothesis that none of the above mentioned variables are significant:



Loan. Identify which one among the 1/0 variables has the highest effect on the probability of personal loan.

(Hint: For a continuous predictor, compare how much the indicator function  $I$  changes if that predictor changes by one standard deviation. For a 1/0 variable, find how much  $I$  changes if that variable changes from 0 to 1.)

**Solution:**

```
GLM3b<-glm(PersonalLoan~Income+Family+CCAvg+GRAD+PROF+SecuritiesAccount+
           CDAccount+Online+CreditCard,data=Bank)
```

```
summary(GLM3b)
```

```
Call:
glm(formula = PersonalLoan ~ Income + Family + CCAvg + GRAD +
    PROF + SecuritiesAccount + CDAccount + Online + CreditCard,
    data = Bank)

Deviance Residuals: |
    Min       1Q   Median       3Q      Max
-0.79372  -0.13612  -0.03349   0.07087   1.05748

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  -0.29586747  0.01194597 -24.767 < 2e-16 ***
Income         0.00310278  0.00009479  32.732 < 2e-16 ***
Family        0.02969212  0.00288685  10.285 < 2e-16 ***
CCAvg         0.01160466  0.00243490   4.766 0.00000193287 ***
GRAD          0.14321729  0.00813260  17.610 < 2e-16 ***
PROF          0.15046947  0.00790261  19.040 < 2e-16 ***
SecuritiesAccount -0.06101605  0.01130446  -5.398 0.00000007069 ***
CDAccount      0.32978272  0.01565696  21.063 < 2e-16 ***
Online       -0.02780050  0.00673977  -4.125 0.00003770269 ***
CreditCard   -0.04420399  0.00749134  -5.901 0.00000000386 ***
```



Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for gaussian family taken to be 0.05259411)

Null deviance: 433.92 on 4999 degrees of freedom  
Residual deviance: 262.44 on 4990 degrees of freedom  
AIC: -524.38

Number of Fisher Scoring iterations: 2

The variables that have a positive significant affect on Perosnal Loan are :

Income, Family, CCAvg, GRAD, PROF, CDAccount

The variables that have a negative significant affect on Personal Loan are:

SecuritiesAccount, Online and CreditCard

Running l/0 model

Deviance Residuals:

Min	1Q	Median	3Q	Max
-1.4818	-0.4786	-0.3296	-0.2544	2.8958

Coefficients:

	Estimate	Std. Error	z value	Pr(> z )
(Intercept)	-2.8856	0.1307	-22.074	< 2e-16 ***
PROF	1.3057	0.1391	9.384	< 2e-16 ***
GRAD	1.2458	0.1422	8.760	< 2e-16 ***
SecuritiesAccount	-1.2920	0.2055	-6.286	3.25e-10 ***
CDAccount	3.9494	0.2127	18.568	< 2e-16 ***
CreditCard	-1.1481	0.1553	-7.393	1.44e-13 ***
Online	-0.5292	0.1127	-4.698	2.63e-06 ***

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 3162.0 on 4999 degrees of freedom  
Residual deviance: 2616.1 on 4993 degrees of freedom  
AIC: 2630.1

-1/2

Need to tell  
which variables  
have highest  
effects on  
probability.



The variables affecting Personal Loan positively are:

PROF, GRAD, CDAccount

The variables affecting Personal Loan Negatively are:

SecuritiesAccount, CreditCard and Online

**3(d)(5 points)** Please perform a logit analysis with Personal Loan as the dependent variable and the following independent variables:

CCAvg, CDAccount, CreditCard, ED, Family, Income, Online, SecuritiesAccount, and the interaction of Income with each of ED, CD Account, Credit Card, Online and Securities Account.

(In this case, we will be able to check if the effect of income is “moderated,” that is, affected by the level of ED, DC Account, etc. Here, ED, CD Account, etc., are called moderating variables.)

Focus on the interaction terms and identify the interactions that are not significant at a 90% level of confidence. At a 99% level of confidence, test the null hypothesis that none of these interaction terms is significant. Based on the result, keep the original model or the reduced model as appropriate. Briefly discuss how the effect of income is affected by the moderating variable.

**Solution:**

```
GLM3d <- glm(PersonalLoan ~ Income + Family + CCAvg + ED + SecuritiesAccount + CDAccount + Online + CreditCard +
Income*ED + Income*CDAccount +
```

```
Income*CreditCard + Income*Online + Income*SecuritiesAccount, family=binomial(logit), data=Bank)
```

```
summary(GLM3d)
```

```
Deviance Residuals:
    Min       1Q   Median       3Q      Max
-2.1408  -0.1591  -0.0514  -0.0035   3.4862

Coefficients:
              Estimate Std. Error z value Pr(>|z|)
(Intercept)  -7.8648725   0.6173288  -12.740  < 2e-16 ***
Income         0.0228901   0.0041195   5.557 0.0000000275 ***
Family        0.8169216   0.0942480   8.668  < 2e-16 ***
CCAvg         0.2091331   0.0517929   4.038 0.0000539382 ***
ED           -10.6892661   1.1041484  -9.681  < 2e-16 ***
SecuritiesAccount -1.8852091   1.0529051  -1.790   0.0734 .
CDAccount     4.8925547   1.1567438   4.230 0.0000234115 ***
Online       -0.8347757   0.6479559  -1.288   0.1976 .
CreditCard   -1.6638308   0.8750384  -1.901   0.0572 .
Income:ED     0.1259426   0.0103406   12.179  < 2e-16 ***
Income:CDAccount -0.0080032   0.0091572  -0.874   0.3821 .
Income:CreditCard 0.0037484   0.0069832   0.537   0.5914 .
Income:Online  -0.0007825   0.0051085  -0.153   0.8783 .
Income:SecuritiesAccount 0.0078930   0.0085382   0.924   0.3553 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

According to the results above, the following interactions are not significant at 90% level of confidence:

Following is the result of running null hypothesis test for the above insignificant interaction terms:

## Running the reduced model

```
summary(GLM.6)
```

# Deviance Residuals:

Min	1Q	Median	3Q	Max
-2.1320	-0.1618	-0.0537	-0.0037	3.4936

## Coefficients:

	Estimate	Std. Error	z value	Pr(> z )	
(Intercept)	-7.822625	0.515658	-15.170	< 2e-16	***
Income	0.022601	0.003089	7.316	2.56e-13	***
Family	0.810191	0.093781	8.639	< 2e-16	***
CCAvg	0.212874	0.051715	4.116	3.85e-05	***
ED	-10.691213	1.102557	-9.697	< 2e-16	***
SecuritiesAccount	-0.962788	0.339646	-2.835	0.00459	**
CDAccount	3.942669	0.384271	10.260	< 2e-16	***
Online	-0.922362	0.204460	-4.511	6.45e-06	***
CreditCard	-1.214731	0.259601	-4.679	2.88e-06	***
Income:ED	0.125879	0.010327	12.189	< 2e-16	***

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

(Dispersion parameter for binomial family taken to be 1)

Hall deviance: 3162.04 on 4999 degrees of freedom  
Residual deviance: 257.67 on 4990 degrees of freedom  
AIC: 270.67

Number of Fisher Scoring iterations: 9

```
> exp(coef(GLM.6)) # Exponentiated coefficients ("odds ratios")
              (Intercept)      Income      Family      CCAvg      ED SecuritiesAccount      CDAccount      Online      CreditCard
0.00040056591      1.02265884828      2.24933793263      1.23722920950      0.00022274392      0.39182689915      51.55601917839      0.39757896434      0.29678991819
Income:ED
1.13414515440
```

When running the reduced model with splitting ED into GRAD and PROF, we see the effect of both these variables are similarly significant.

Need to tell how the effect of  
income is moderated by  
income  
education.

$-1/2$

Deviance Residuals:

Min	1Q	Median	3Q	Max
-1.4505	-0.2200	-0.1137	-0.0088	3.6024

Coefficients:

	Estimate	Std. Error	z value	Pr(> z )	
(Intercept)	-5.474462	0.332276	-16.476	< 2e-16	***
Income	0.021789	0.002353	9.261	< 2e-16	***
GRAD	-9.412213	1.341865	-7.014	2.31e-12	***
PROF	-9.666831	1.364517	-7.084	1.40e-12	***
Income:GRAD	0.113072	0.012616	8.963	< 2e-16	***
Income:PROF	0.113433	0.012569	9.024	< 2e-16	***

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

{Dispersion parameter for binomial family taken to be 1}

Null deviance: 3162.0 on 4999 degrees of freedom  
Residual deviance: 1093.3 on 4994 degrees of freedom  
AIC: 1105.3

Number of Fisher Scoring iterations: 9

