

bayes_optimal

September 18, 2017

1 Bayes optimal classifier

1.1 Generating 10 centroids for inputs (x's)

In [124]: %matplotlib notebook

```
import matplotlib.pyplot as plt
import numpy as np, pickle
from sklearn import linear_model
from sklearn.metrics import confusion_matrix

# Generating 10 centroids for inputs (x's)
x_centroids_class1 = np.random.multivariate_normal(mean = [1, 0], cov = np.identity(2))
x_centroids_class2 = np.random.multivariate_normal(mean = [0, 1], cov = np.identity(2))
x_centroids = np.append(x_centroids_class1, x_centroids_class2, axis = 0) #All centroids

#plotting
ax = plt.subplots()[1]
ax.plot(x_centroids_class1[:, 0], x_centroids_class1[:, 1], marker = '^', linestyle = 'none')
ax.plot(x_centroids_class2[:, 0], x_centroids_class2[:, 1], marker = '^', linestyle = 'none')
```

<IPython.core.display.Javascript object>

<IPython.core.display.HTML object>

Out[124]: [<matplotlib.lines.Line2D at 0x29a7b7f0>]

1.2 Generating training data around centroids

In [125]: # Generating data around centroids

```
x_train, y_train = [], []

for _ in range(100):
    centroid_idx = np.random.choice(10) #sample i for m_i from 1 to 10
    x_train.append(np.random.multivariate_normal(mean = x_centroids[centroid_idx], cov = np.identity(2)))
    if centroid_idx < 5: y_train.append(0) #if i in {0, ..4}, y = 0
```

```

        else: y_train.append(1) #if i in {5, ..9}, y = 1

x_train, y_train = np.array(x_train), np.array(y_train) #lists to arrays
#plotting
ax = plt.subplots()[1]
ax.plot(x_train[y_train == 0, 0], x_train[y_train == 0, 1], linestyle = '', marker =
ax.plot(x_train[y_train == 1, 0], x_train[y_train == 1, 1], linestyle = '', marker =

<IPython.core.display.Javascript object>

```

```

<IPython.core.display.HTML object>

```

```

Out[125]: [matplotlib.lines.Line2D at 0x29c5c668>]

```

1.3 Boundary plotting function

```

In [126]: def plot_boundary(ax, clf, x, y, **params):
           """Plot the decision boundaries for a classifier.

           Parameters
           -----
           ax: matplotlib axes object
           clf: a classifier
           x: data to base x-axis meshgrid on
           y: data to base y-axis meshgrid on
           params: dictionary of params to pass to contourf, optional
           """

           def make_meshgrid(x, y):
               grid_res = 1000
               x_min, x_max = x.min(), x.max()
               y_min, y_max = y.min(), y.max()
               xx, yy = np.meshgrid(np.arange(x_min, x_max, (x_max - x_min)*1./grid_res)[:g
                                   np.arange(y_min, y_max, (y_max - y_min)*1./grid_res)[:g

               return xx, yy

           xx, yy = make_meshgrid(x, y)
           Z = clf(np.c_[xx.ravel(), yy.ravel()])
           Z = Z.reshape(xx.shape)
           ax.contourf(xx, yy, Z, **params)
           ax.set_xlim(xx.min(), xx.max())
           ax.set_ylim(yy.min(), yy.max())

```

1.4 Testing a linear model

We fit a linear model to the training data and draw the decision boundary (subspace where $y \approx 0.5$). We highlight the false positive and false negatives in the model predictions and print the rates of both.

```

In [127]: #train regression model
model = linear_model.LinearRegression()
model.fit(X = x_train, y = y_train)

#plotting
ax = pickle.loads(pickle.dumps(ax))
plot_boundary(ax, lambda x: np.floor(model.predict(x) + 0.5), x_train[:, 0], x_train[:, 1])

# False positives and false negatives for training data
y_pred = np.vectorize(lambda y: 0 if y < 0.5 else 1)(model.predict(X = x_train)) #rounded
x_fp, x_fn = x_train[np.logical_and(y_pred == 1, y_train == 0)], x_train[np.logical_and(y_pred == 0, y_train == 0)]
#plotting
ax.plot(x_fp[:, 0], x_fp[:, 1], linestyle = '', marker = 's', color = 'red', alpha = 0.5)
ax.plot(x_fn[:, 0], x_fn[:, 1], linestyle = '', marker = 's', color = 'blue', alpha = 0.5)

#fp and fn rates
tn, fp, fn, tp = confusion_matrix(y_train, y_pred).ravel()
fp_rate, fn_rate = (fp + 0.)/(fp + tn)*100., (fn + 0.)/(fn + tp)*100.

print 'false positive rate = ' + str(fp_rate) + '% , false negative rate = ' + str(fn_rate) + '%'

```

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false positive rate = 38.8888888889% , false negative rate = 30.4347826087%

In []: