Rotational Dynamics

- Circular Motion
- Rotational Motion

➢ Angular Displacement

Related Formula

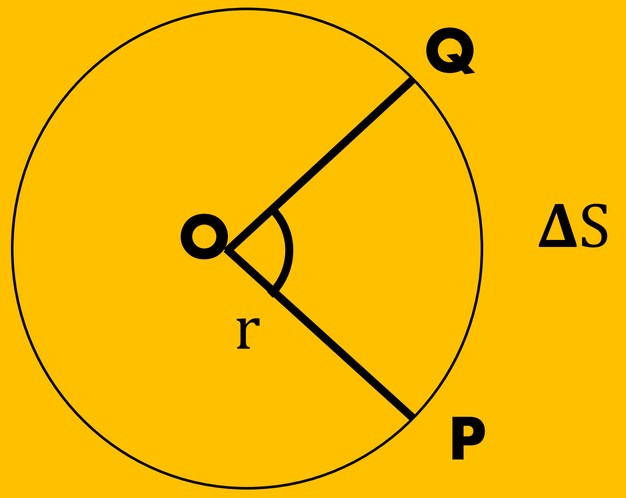
$$\theta = \omega t$$

$$\theta = 2\pi n t$$

$$\theta = \frac{2\pi t}{T}$$

$$\text{ANGLE} = \frac{Arc}{Radius}$$

$$\theta = \frac{s}{r}$$



Angular Velocity

The rate of change of the angular displacement of the body undergoing circular motion

$$\omega = \frac{d\theta}{dt}$$

Related Formula

$$\omega = \frac{v}{r} \qquad \omega = 2\pi n$$

$$\omega = \frac{\theta}{t} \qquad \omega = \frac{2\pi}{T}$$

UNIT : Radian Per Second

➢ Linear Velocity

"The rate of change of displacement with respect to time when the object moves along a straight path."

Related Formula

$\mathbf{v} = \mathbf{r}\omega$ $\mathbf{v} = 2\pi nr$

➢ Linear Acceleration

➢ Angular Acceleration

The rate of change of angular velocity of a body

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Related Formulae

$$\alpha = \frac{\omega_2 - \omega_1}{t}$$

$$\alpha = \frac{2\pi}{t} \left(n_2 - n_1\right)$$

Unit : (rad/s^2)

Dimension : $[M^1T^{-2}]$

Circular Motion

Uniform Circular Motion

1. Constant Speed
2. Acceleration is radial and always directed towards the centre

Centripetal Force

$$\vec{F} = \frac{-mv^2}{r}$$

Centrifugal Force

$$\vec{F} = \frac{mv^2}{r}$$

Non -Uniform Circular Motion

1. No Constant Speed
2. Acceleration of the Particle is Radial →
3. Acceleration of the Particle is Tangential

Rate of change in Speed - $a_r = \frac{d\,|v|}{dt}$

Centripetal Force

The force acting on an object in curvilinear motion directed towards the axis of rotation or center of curvature

Related Formula

$$F_{CP} = \frac{mv^2}{r}$$

$$F_{CP} = mr\omega^2$$

$$F_{CP} = mr4\pi^2n^2$$

$$F_{CP} = \frac{4\pi^2 mr}{T^2}$$

Centrifugal Force

The tendency of an object moving in a circle to travel away from the center of the circle

Related Formula

$$F_{CP} = -F_{CP}$$

BANKING OF ROAD

BANKING OF ROAD

Horizontal Plain Curved Road

$$v_s = \sqrt{\mu r g}$$

Banked Road

1.**ABSENCE OF FRICTION** $v = \sqrt{rg\ tan\theta}$

2.**Banking Angle** $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

3.**Speed Limits**

$$v_{min} = \sqrt{rg\left(\frac{tan\theta - \mu_s}{1+\mu_s\, tan\theta}\right)}$$

$$v_{max} = \sqrt{rg\left(\frac{tan\theta + \mu_s}{1-\mu_s\, tan\theta}\right)}$$

Kinematic Equations for Circular Motion

$\omega = \boldsymbol{\omega_0} + \alpha t$

$\boldsymbol{\Theta = \omega_0 t + ½}\ \alpha t^2$

$\omega 2 = \boldsymbol{\omega_0^2 + 2}\ \alpha\boldsymbol{\Theta}$

WELL OF DEATH (WALL OF DEATH)

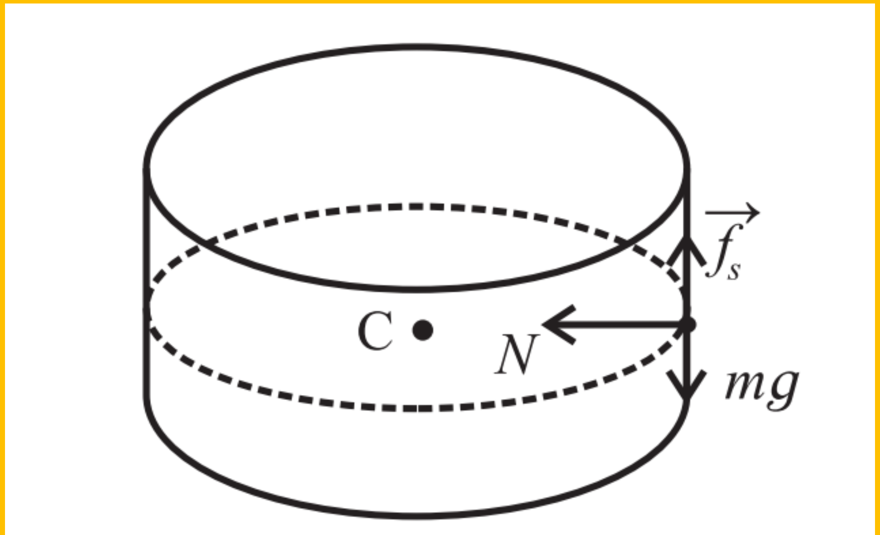
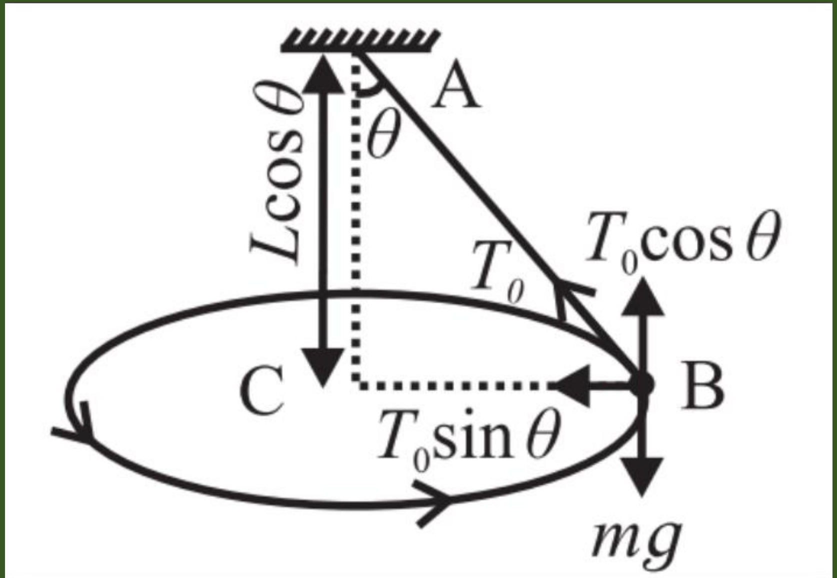


Fig. 1.5: Well of death.

$$v_{min} = \sqrt{\frac{rg}{\mu_s}}$$

Conical Pendulum



$$T = 2\pi\sqrt{\frac{l\cos\theta}{g}} = 2\pi\sqrt{\left(\frac{h}{g}\right)}$$

Vertical Circular Motion

Highest Point

Tension = (minimum)

Velocity = $\sqrt{rg}$ (minimum)

Middle Point

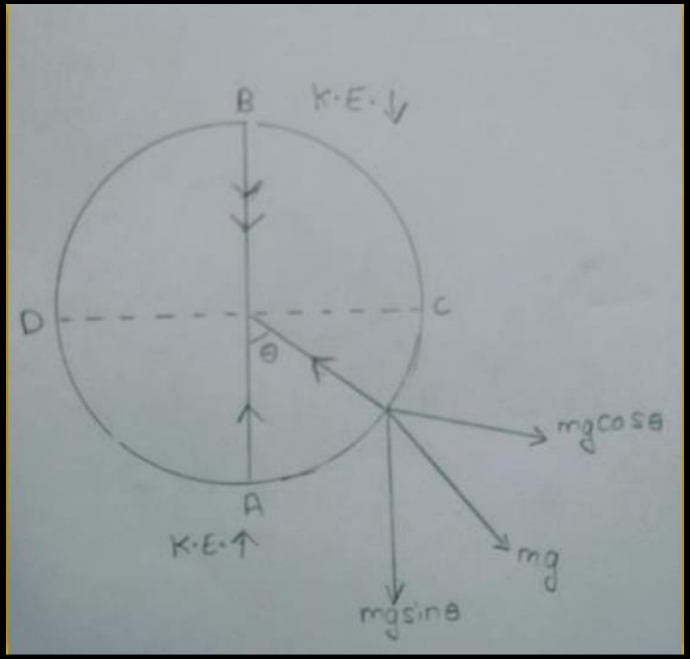
Tension = (intermediate)

Velocity = $\sqrt{3rg}$

Lowest Point

Tension = (maximum)

Velocity = $\sqrt{5rg}$



TENSION AT HIGHEST POINT : $T_H = \frac{mv_H^2}{r} - mg$

TENSION AT Midway POINT : $T_M = \frac{mv_m^2}{r}$

Tension at Lowest POINT : $T_L = \frac{mv_L^2}{r} + mg$

Difference between tension at lower most and uppermost point : $T_L - T_H = 6mg$

Rotational Motion

1. **Moment of Inertia :** $I = mr^2$

2. **Kinetic Energy of Rotation**

$$K_R = \frac{1}{2} I\omega^2 \qquad K_{Transitional} = \frac{1}{2} mv^2$$

$$K.E_{ROLLING} = \frac{1}{2}\,[Mv^2 + I\omega^2]$$

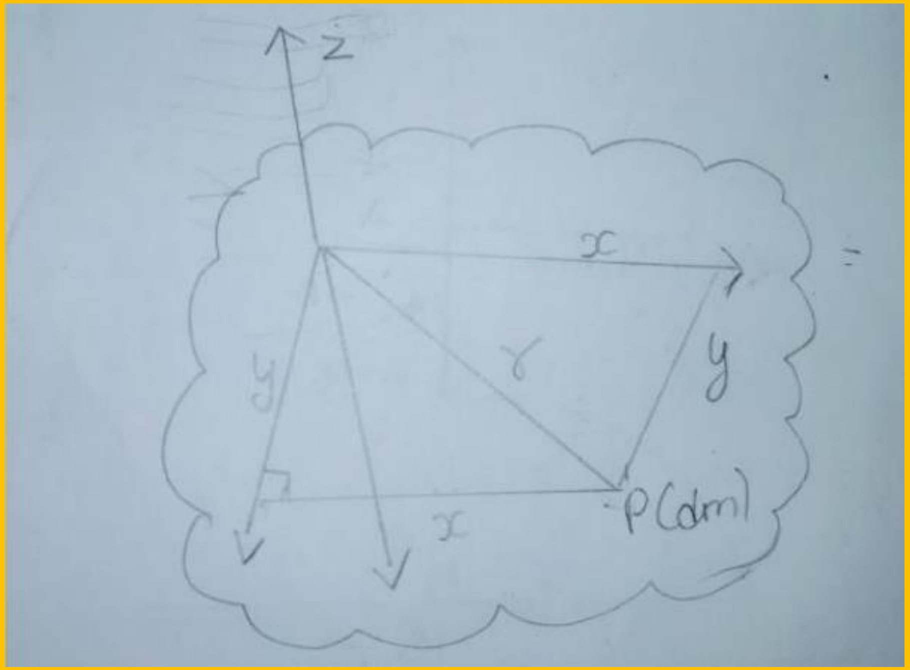
3. **Torque :** $\tau = rF$ $\quad \tau = I\alpha = \frac{dL}{dt} \quad \tau = I\alpha = \frac{dL}{dt} = 2\pi I\left(\frac{n_2 - n}{t}\right)$

4. **Angular Momentum** $L = I\omega = I(2\pi n)$

5. **Radius of Gyration** =

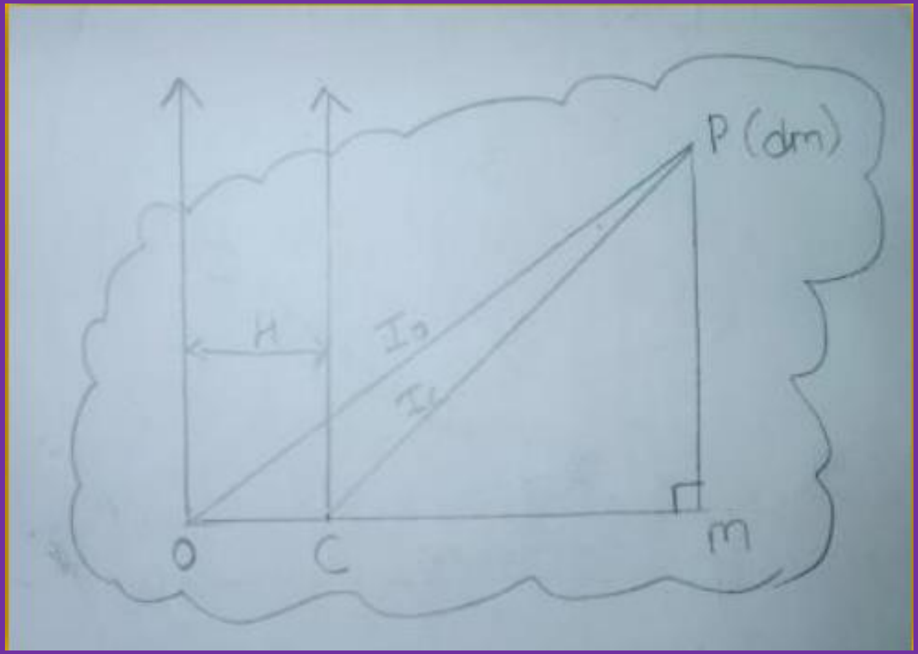
Perpendicular Axis Theorem

$$I_Z = I_x + I_y$$



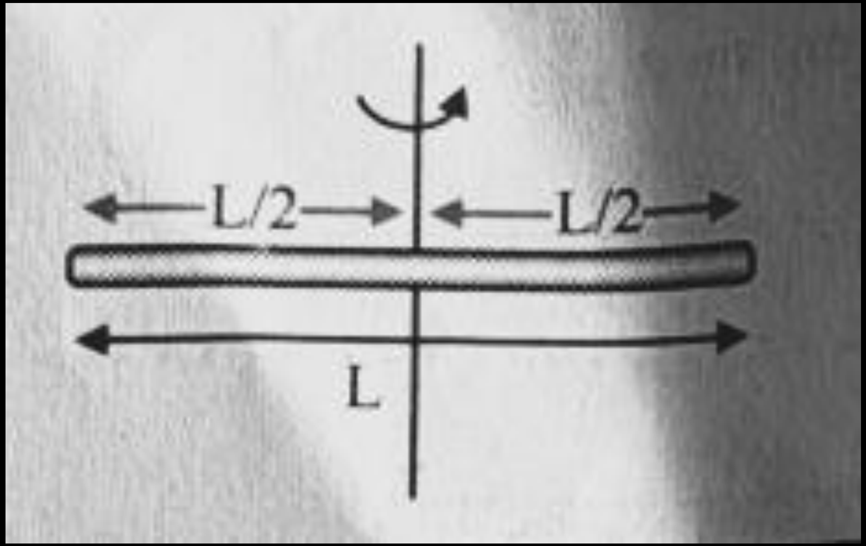
Parallel Axis Theorem

$$I = I_c + Mh^2$$

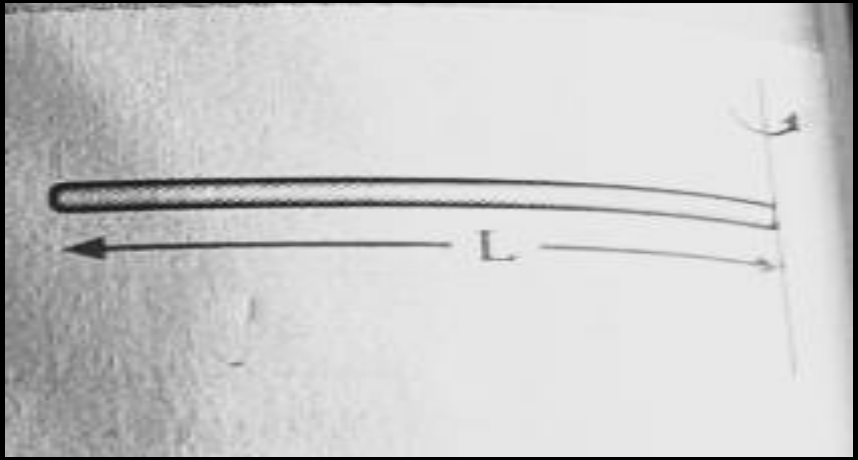


M.I of a thin rod of mass M and length L about

An axis passing through its centre and perpendicular to its length $\frac{ML\,2}{12}$

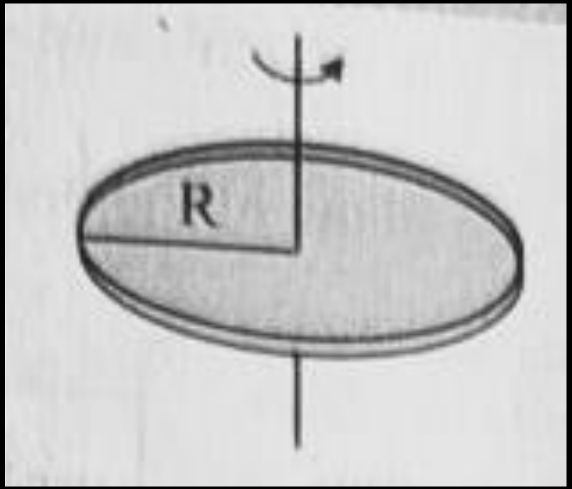


An axis passing through its one end and perpendicular to its length $\frac{ML\,2}{3}$

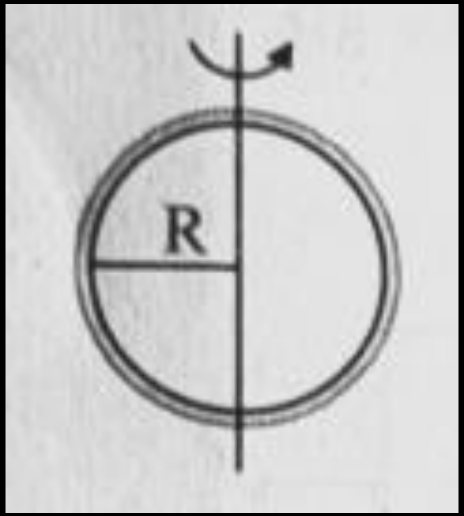


M.I of a circular ring of mass M and radius R about

An axis passing through its centre and perpendicular to the plane of the ring MR^2

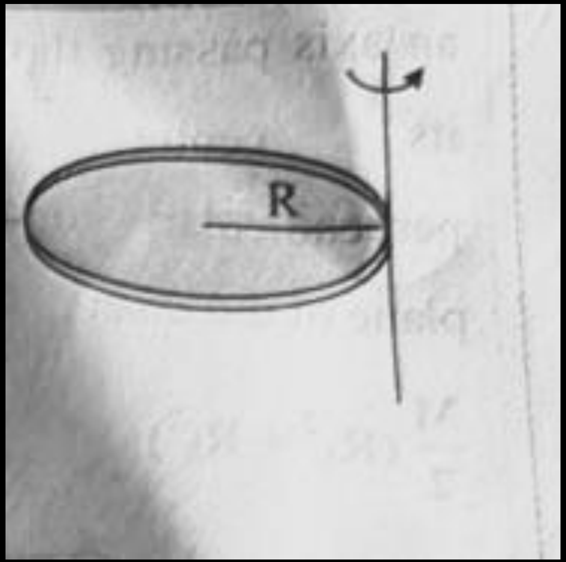


An axis passing through its diameter : $\frac{1}{2}MR^2$

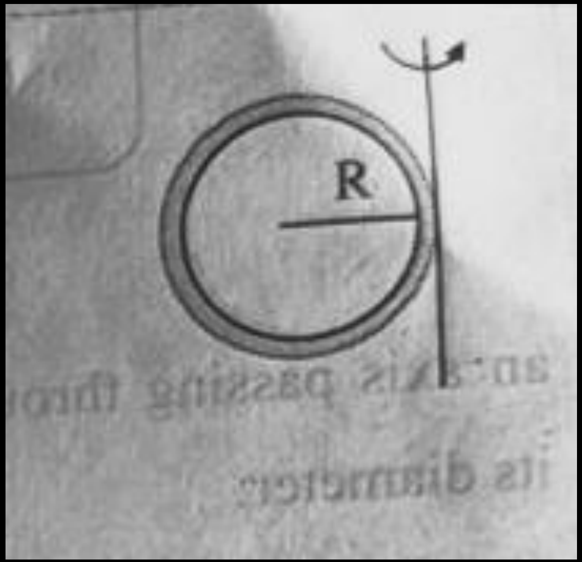


M.I of a circular ring of mass M and radius R about

A tangent , and perpendicular to the plane of the ring 2 MR^2

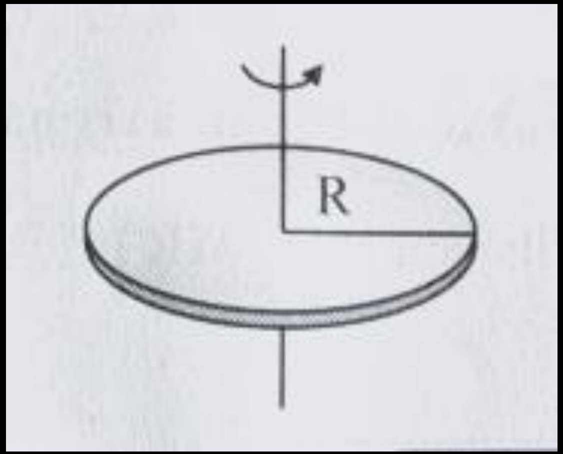


A tangent , and in the plane of the ring $\frac{3}{2}$ MR^2

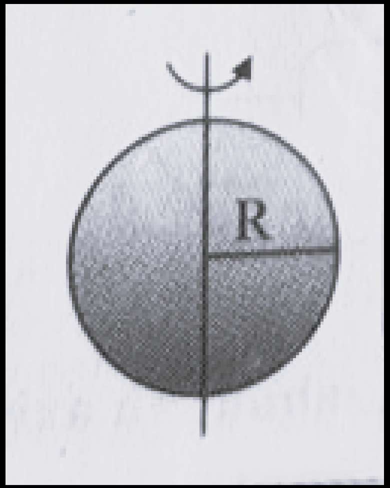


M.I of a circular disc of mass M and radius R about

An axis passing through its centre and perpendicular to the plane of the disc $\frac{1}{2}$ MR²

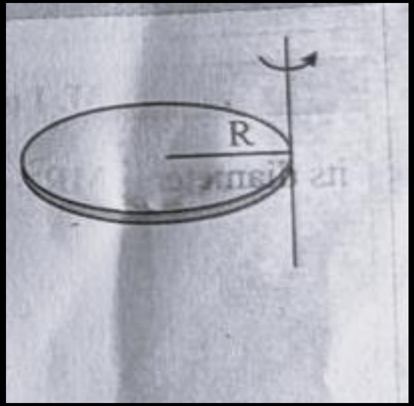


An axis passing through its diameter $\frac{1}{4}$ MR²



M.I of a circular disc of mass M and radius R about

A tangent , and perpendicular to *the plane of the disc* $\frac{3}{2}$ $\mathbf{MR^2}$



A tangent , and in the plane of the disc $\frac{5}{4}$ $\mathbf{MR^2}$