Linear Regression models

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Credit Card Dataset

Income	Limit	Rating	Cards	Age	Balance
14.891	3606	283	2	34	333
106.025	6645	483	3	82	903
104.593	7075	514	4	71	580
148.924	9504	681	3	36	964
55.882	4897	357	2	68	331
80.18	8047	569	4	77	1151
20.996	3388	259	2	37	203
71.408	7114	512	2	87	872
15.125	3300	266	5	66	279
71.061	6819	491	3	41	1350
63.095	8117	589	4	30	1407

Questions set

- *How to predict card balance using the Income of customer ?*
- *How to choose class of function for predicting the balance using the income ?*
- *What happens when we work with complex class of functions ?*
- *How to control the complexity of a given class of function for accurate prediction ?*

Task

Income (hundred thousand dollar) x	Balance (thousand dollar) y
0.550798	5.651202
0.708148	7.321263
0.290905	5.167304
0.510828	5.609367
0.892947	9.406379
0.896293	9.379439
0.125585	2.734997
0.207243	4.876649
0.051467	3.584138
0.44081	5.437239

Training data

Task

Income (hundred thousand dollar)	Balance (thousand dollar)
0.550798	5.651202
0.708148	7.321263
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0.207243	4.876649
0.051467	3.584138
0.44081	5.437239

Training data

Income (x) (hundred thousand dollar)	Balance (y) (thousand dollar)
0.96703	9.675083
0.547232	6.293266
0.972684	9.730614
0.714816	7.474346
0.697729	7.342933
0.216089	4.619033
0.976274	9.765597
0.00623	4.012784
0.252982	4.762698
0.434792	5.626166
0.779383	7.989045
0.197685	4.552625
0.862993	8.705537
0.983401	9.835217
0.163842	4.43522
0.597334	6.622444
0.008986	4.01882
0.386571	5.37153
0.04416	4.096522
0.956653	9.574695

Testing Data

Task

Income (hundred thousand dollar)	Balance (thousand dollar)
x_1= 0.550798	y_1= 5.651202
x_2 =0.708148	y_2= 7.321263
x_3 =0.290905	y_3= 5.167304
x_4 =0.510828	y_4= 5.609367
x_5 =0.892947	y_5= 9.406379
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x_7 =0.125585	y_7= 2.734997
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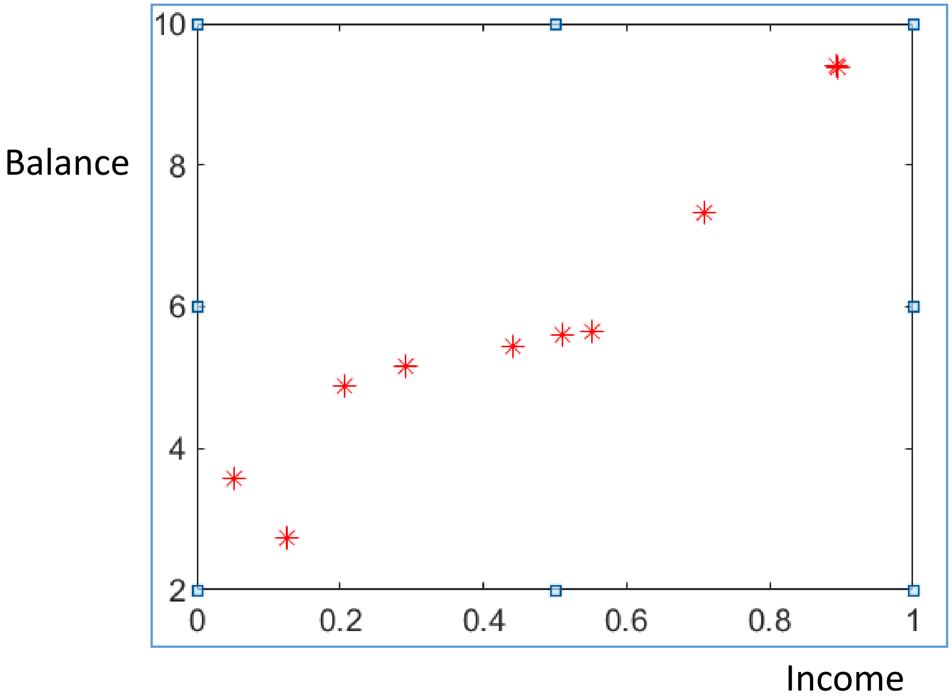
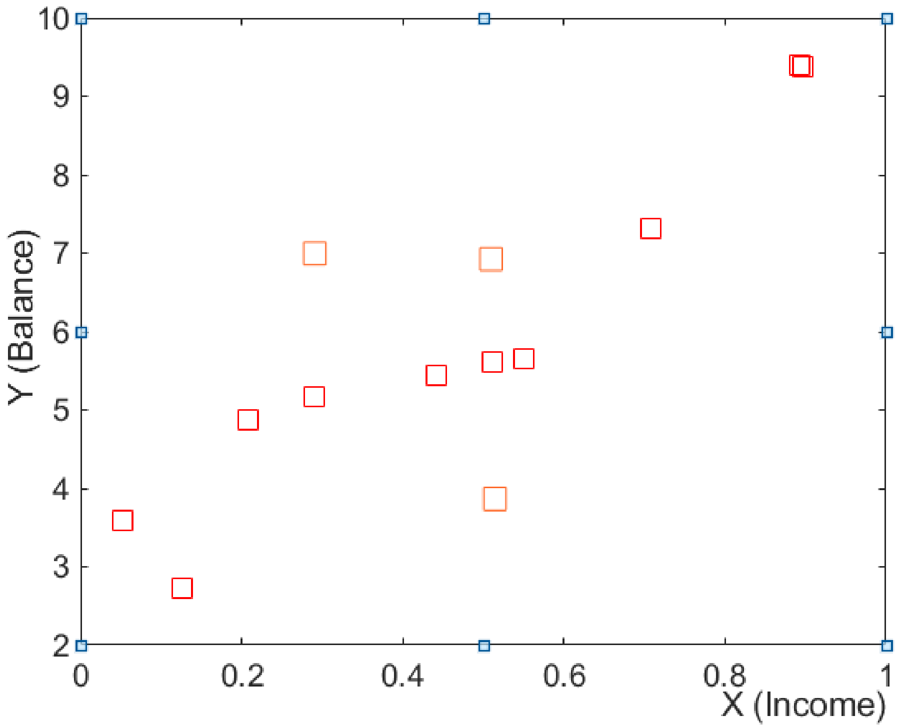
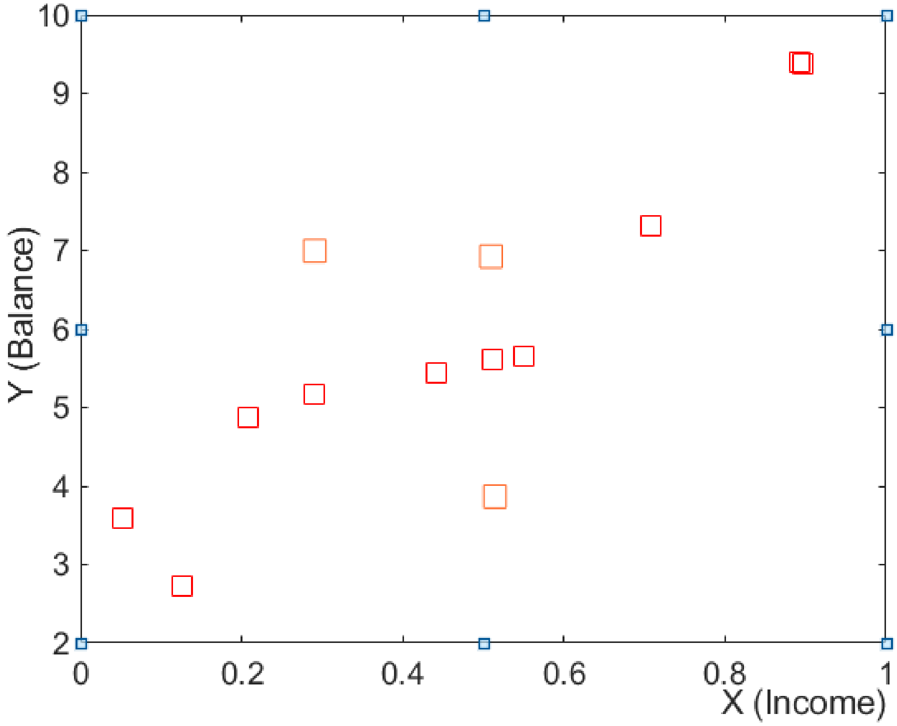
x_9 =0.051467	y_9= 3.584138
x_{10} =0.44081	y_{10}= 5.437239



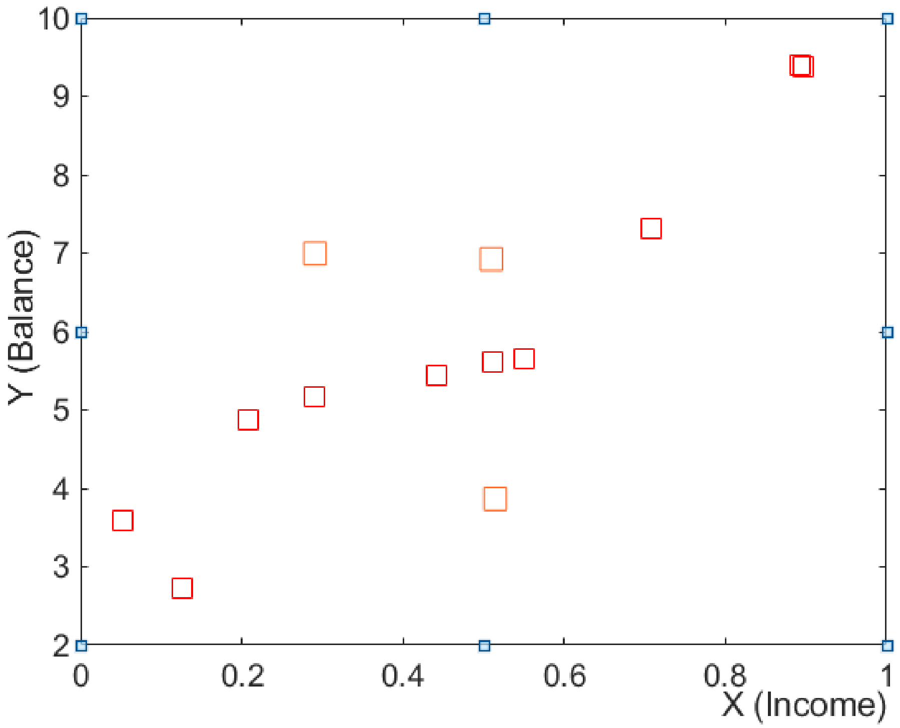
Random Relation



$$y_i = \quad E(y_i | x) + \epsilon_i$$

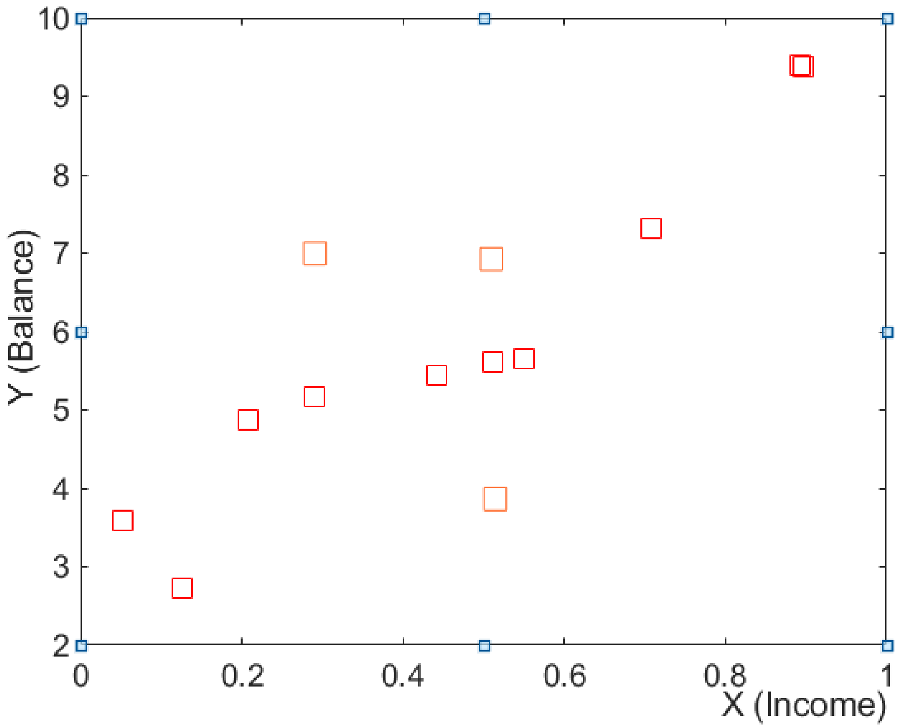


$$y_i = \underbrace{E(y_i|x)}_{f(x)} + \epsilon_i$$



$$y_i = \underset{\overset{\uparrow}{f(x)}}{E(y_i|x)} + \epsilon_i$$

$$E(\epsilon_i) = 0$$



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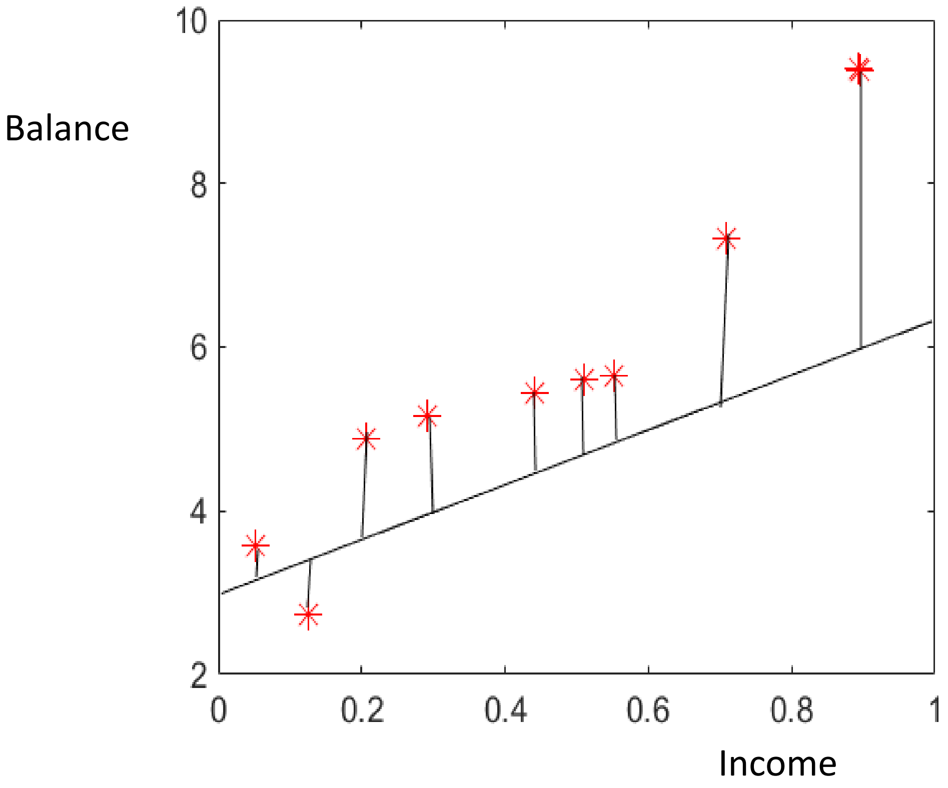

$f(x) = \beta_0 + \beta_1 x$

Income (hundred thousand dollar)	Balance (thousand dollar)
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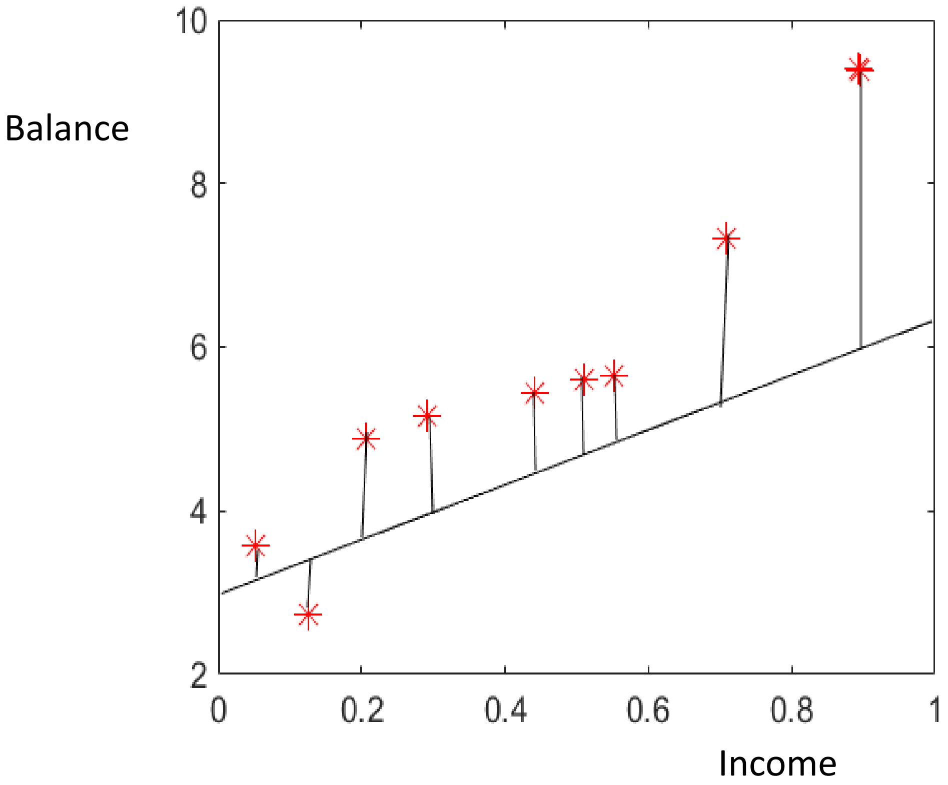
Error (thousand dollar)
$(y_1-(\beta_0 + \beta_1 x_1))^2$
$(y_2-(\beta_0 + \beta_1 x_2))^2$
$(y_3-(\beta_0 + \beta_1 x_3))^2$
$(y_4-(\beta_0 + \beta_1 x_4))^2$
$(y_5-(\beta_0 + \beta_1 x_5))^2$
$(y_6-(\beta_0 + \beta_1 x_6))^2$
$(y_7-(\beta_0 + \beta_1 x_7))^2$
$(y_8-(\beta_0 + \beta_1 x_8))^2$
$(y_9-(\beta_0 + \beta_1 x_9))^2$
$(y_{10}-(\beta_0 + \beta_1 x_{10}))^2$



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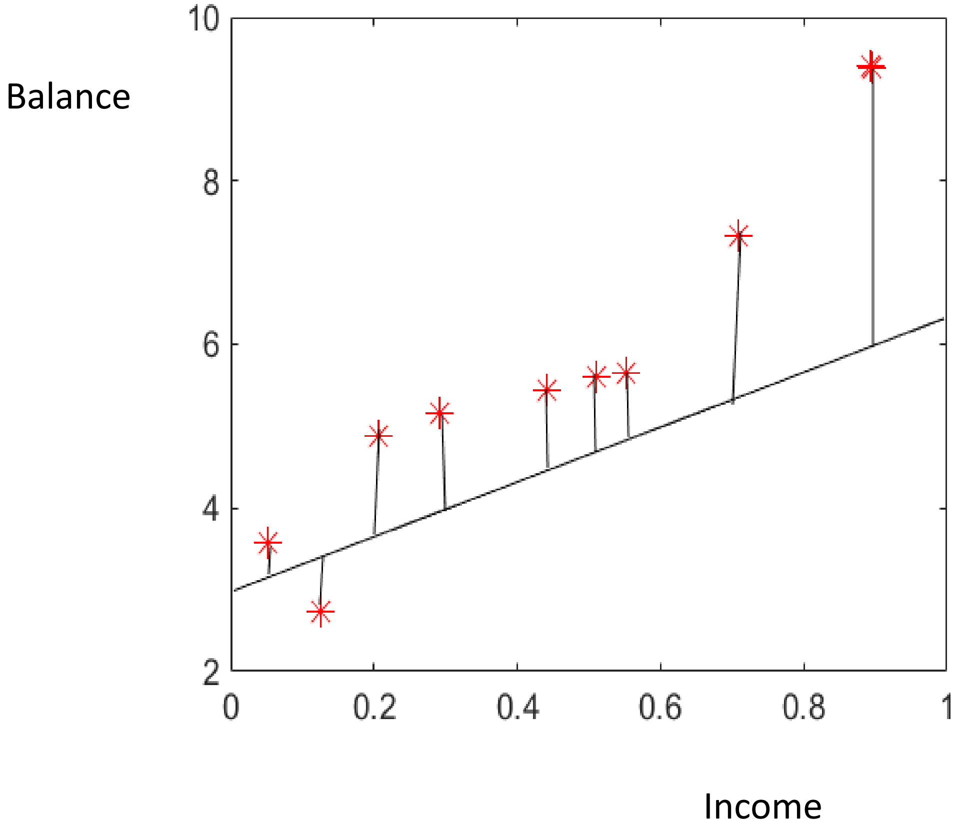
$(y_2-(\beta_0 + \beta_1 x_2))^2$
$(y_3-(\beta_0 + \beta_1 x_3))^2$
$(y_4-(\beta_0 + \beta_1 x_4))^2$
$(y_5-(\beta_0 + \beta_1 x_5))^2$
$(y_6-(\beta_0 + \beta_1 x_6))^2$
$(y_7-(\beta_0 + \beta_1 x_7))^2$
$(y_8-(\beta_0 + \beta_1 x_8))^2$
$(y_9-(\beta_0 + \beta_1 x_9))^2$
$(y_{10}-(\beta_0 + \beta_1 x_{10}))^2$

$$\frac{1}{10}\sum_{i=1}^{10}(y_i-(\beta_0 + \beta_1 x_i))^2$$



$f(x) = \beta_0 + \beta_1 x$

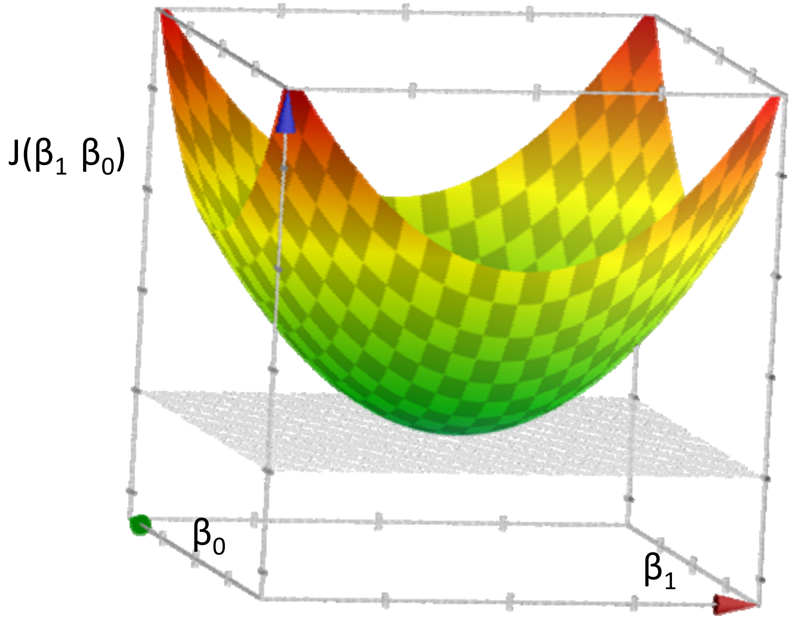
$x_i \in R$, $y_i \in R$

For given Training Set T = { (x_1,y_1), (x_2,y_2),….., (x_n,y_n)}, we need to solve

$$\underline{Min_{(\beta_1,\beta_0)}} \quad J(\beta_1,\beta_0) = \boxed{\sum_{i=1}^{n} (yi-(\beta_0 + \beta_1 x_i))^2}$$

$$\min_{(\beta_1,\beta_0)} \sum_{i=1}^{n} \left(y_i - \underline{(\beta_0 + \beta_1 x_i)}\right)^2$$

[1]



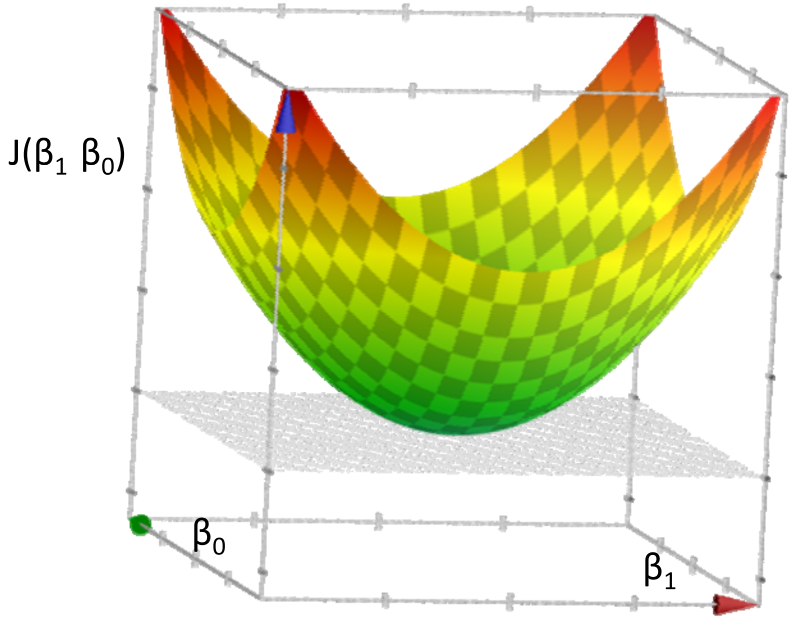
[1] https://www.google.com/search?q=plot+z%3Dx%5E2%2By%5E2&oq=plot+z%3D&aqs=chrome.1.69i57j0i512l4j0i22i30l5.6542j0j4&sourceid=chrome&ie=UTF-8

$(Y - Au)^T (Y - Au)$

For given Training Set T = { (x_1,y_1), (x_2,y_2),….., (x_n,y_n)}, we need to solve

$$Min_{(\beta_1,\beta_0)} \quad J(\beta_1,\beta_0) = \sum_{i=1}^{n} (y_i - (\beta_0 + \beta_1 x_i))^2$$

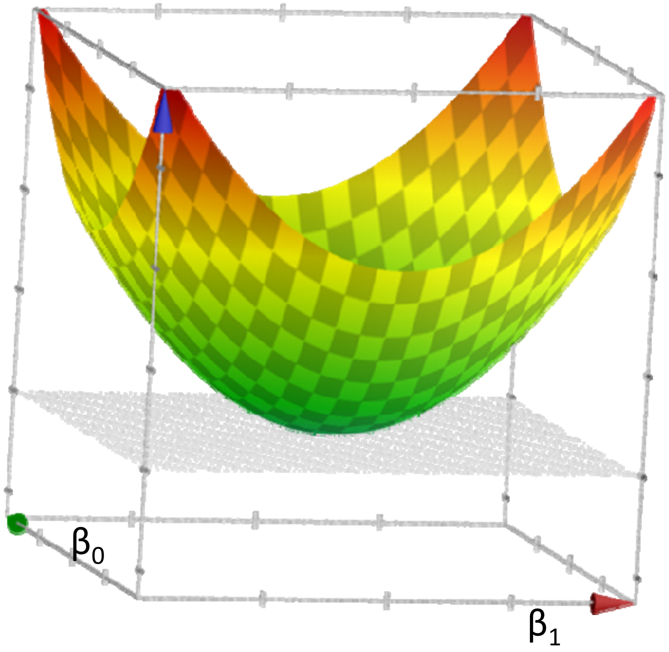
$$u = \begin{bmatrix} \beta_1 \\ \beta_0 \end{bmatrix} \qquad A = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \\ x_4 & 1 \\ x_5 & 1 \\ x_6 & 1 \\ x_7 & 1 \\ & \\ x_n & 1 \end{bmatrix} \qquad Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ \\ y_n \end{bmatrix}$$



For given Training Set T = { (x_1,y_1), (x_2,y_2),....., (x_n,y_n)}, we need to solve

$$Min_{(\beta_1,\beta_0)} \quad J(\beta_1,\beta_0) = \sum_{i=1}^{n} (yi-(\beta_0+\beta_1 x_i))^2 \qquad .....(1)$$

$$u = \begin{bmatrix} \beta_1 \\ \beta_0 \end{bmatrix} \qquad A = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \\ x_4 & 1 \\ x_5 & 1 \\ x_6 & 1 \\ x_7 & 1 \\ & \\ x_n & 1 \end{bmatrix} \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ \\ y_n \end{bmatrix}$$



The Least Square problem reduces to

$$\boxed{Min_{(u)} \quad J(u) = (Y - Au)^T (Y - Au)}$$

$x \in R^n$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix} \qquad x_i \in R^k$$

$$\nabla_x \; x^T x = \nabla_x \boxed{\sum_{i=1}^{k} x_i^2} = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \\ \vdots \\ \frac{\partial}{\partial x_k} \end{bmatrix} = 2\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix}$$

$m \times k$ $\qquad$ $k \times 1$

$\nabla_x \; Ax$ $\qquad$ A^T

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{1k} \\ a_{21} & a_{22} & a_{2k} \\ a_{m1} & a_{m2} & a_{mk} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_k \end{bmatrix}$$

$$\begin{bmatrix} \boxed{a_{11}x_1 + a_{12}x_2 \ldots} \\ a_{21}x_1 \ldots \\ a_{m1}x_1 \ldots \end{bmatrix}$$

$$= 2x$$

$$\begin{bmatrix} a_{11} & a_{21} & \ldots \\ a_{12} & a_{22} & \ldots \\ a_{1k} & a_{2k} & \end{bmatrix}$$

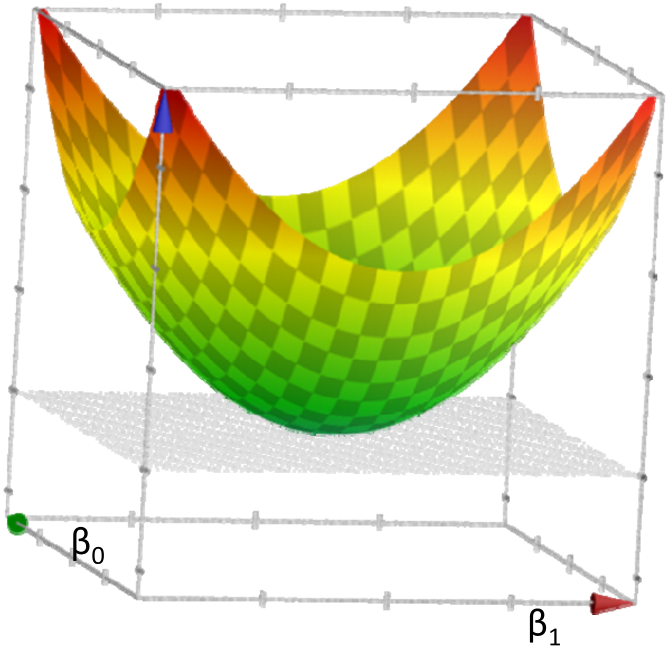
$J(u) = z^T z$

$z = (y - Au)$

For given Training Set $T = \{ (x_1,y_1), (x_2,y_2),\ldots, (x_n,y_n)\}$, we need to solve

$$\min_{(\beta_1,\beta_0)} J(\beta_1,\beta_0) = \sum_{i=1}^{n} (y_i-(\beta_0 + \beta_1 x_i))^2 \quad \ldots..(1)$$

$z = (y - Au)$



The Least Square problem reduces to

$$\min_{(u)} J(u) = \boxed{(Y - Au)^T (Y - Au)}$$

$2(y - Au)$

Setting the gradient for J(u) = 0

$$\nabla_u J(u) = 0$$

$$A^T (Y - Au) = 0$$

$$\boxed{u = \begin{bmatrix}\beta_1 \\ \beta_0\end{bmatrix} = (A^T A)^{-1} A^T Y}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{1k} \\ a_{21} & a_{22} & a_{2k} \\ a_{m1} & a_{m2} & a_{mk} \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_k \end{bmatrix}$$

$R^n \rightarrow R$

$$Ax = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \ldots a_{1k}x_k \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_2 \\ a_{m1}x_1 + a_{m2}x_2 + \cdots a_{mk}x_k \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{21} & , \ \ldots \\ a_{12} & a_{22} & \\ a_{1k} & a_{2k} & \end{bmatrix}$$

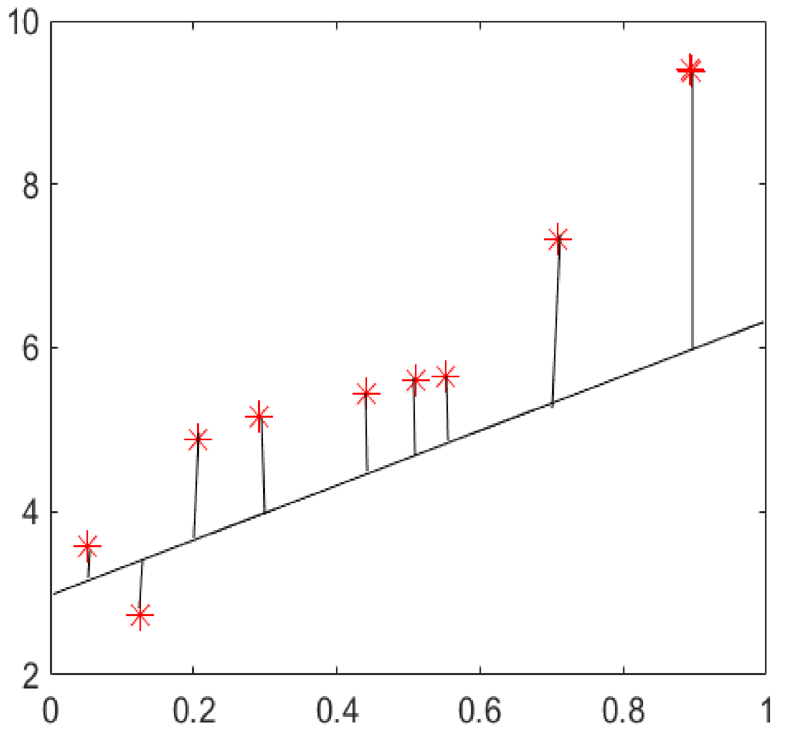
$$\nabla_{\underline{(y-Au)}} \boxed{(y-Au)^T(y-Au)} \cdot \nabla_u (y-Au)$$

$$2(y-Au)$$

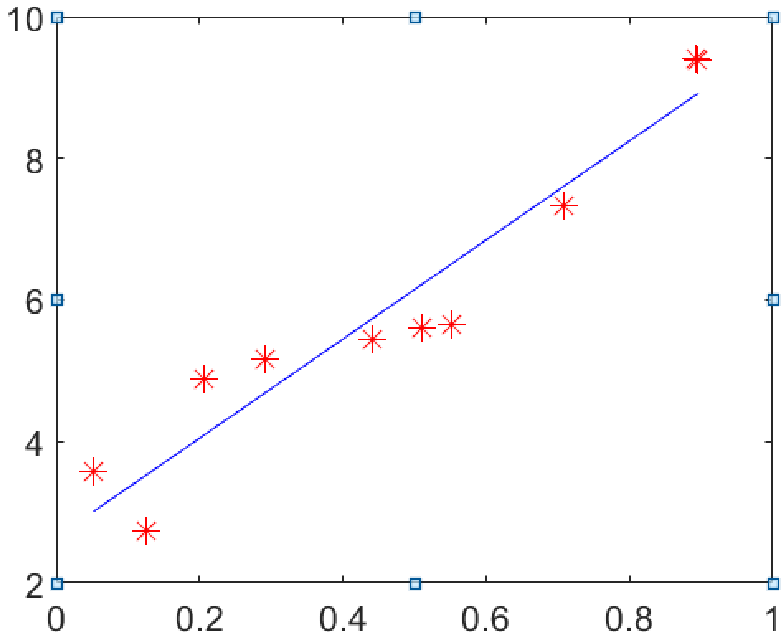
$$-2A^T(y-Au)=0$$

Best Fitting Line

y
Balance



x
Income



Best Fitting Line

f(x) = 2.6 + 6.9 x

Best Fitting Line

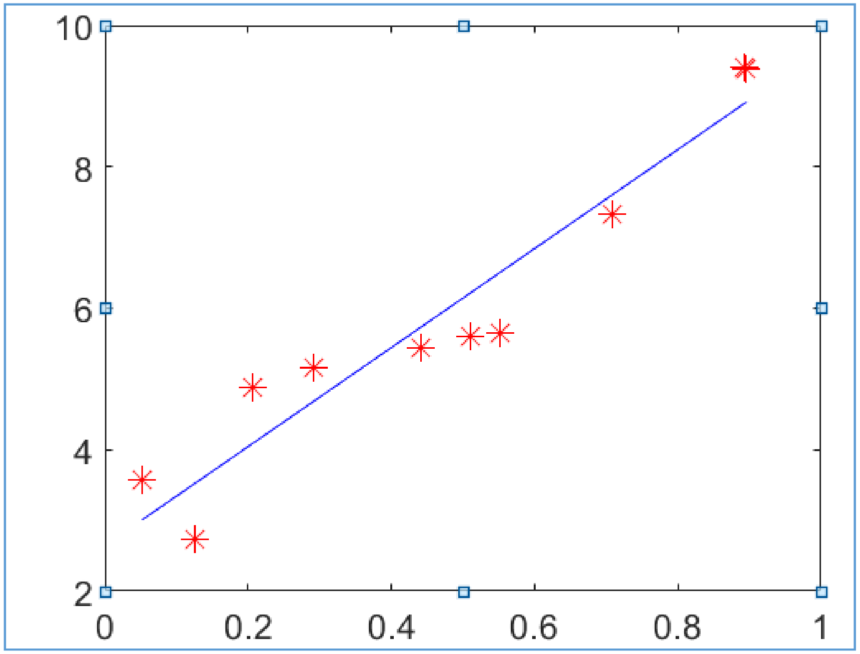
Training Error:-

$$\frac{1}{10}\sum_{i=1}^{10}(yi-(\beta_0+\beta_1 x_i))^2 = 0.3537$$

Training RMSE =

$$\sqrt{\frac{1}{10}\sum_{i=1}^{10}(yi-(\beta_0+\beta_1 x_i))^2}$$

= 0.5947



Best Fitting Line

$f(x) = 2.6 + 6.9\,x$

Income (x) (thousand dollar)	Balance (y) (thousand dollar)	Estimated f(x) (thousand dollar)
0.96703	9.675083	9.41205399
0.547232	6.293266	6.47467789
0.972684	9.730614	9.45161938
0.714816	7.474346	7.64728226
0.697729	7.342933	7.5277212
0.216089	4.619033	4.15763074
0.976274	9.765597	9.47673972
0.00623	4.012784	2.68921946
0.252982	4.762698	4.41577473
0.434792	5.626166	5.68791617
0.779383	7.989045	8.09906511
0.197685	4.552625	4.02885271
0.862993	8.705537	8.6840969
0.983401	9.835217	9.52660279
0.163842	4.43522	3.79205019
0.597334	6.622444	6.8252457
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Test $RMSE = \sqrt{\frac{1}{k}\sum_{i=1}^{k}(yi - f(xi))^2} = 0.9426$

Training $RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{k}(yi - f(xi))^2} = 0.5947$

Testing $RMSE = \sqrt{\frac{1}{k}\sum_{i=1}^{k}(yi - f(xi))^2} =$ 0.9426

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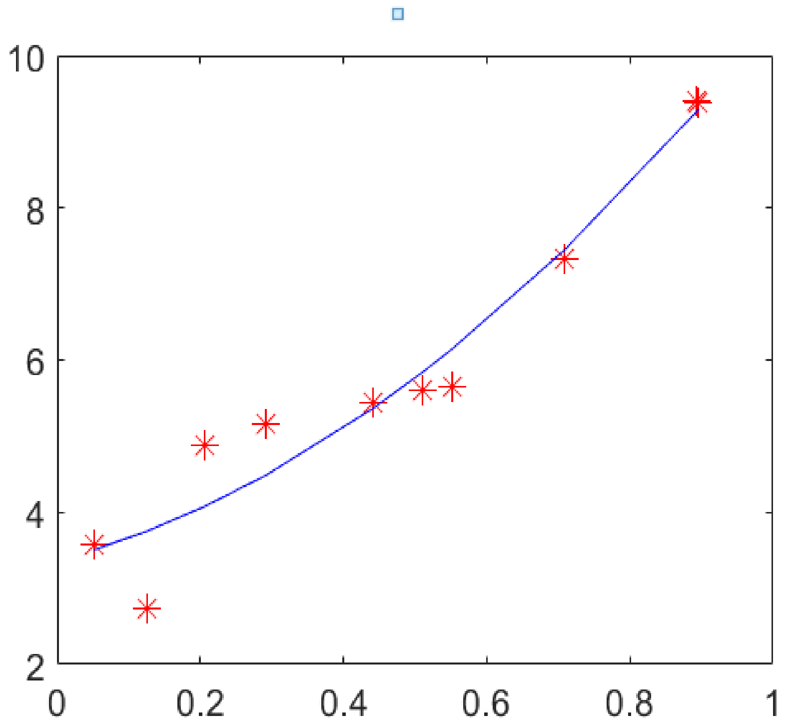
Quadratic Fitting



$f(x) = \beta_0 + \beta_1 x + \beta_2 x^2$

Quadratic Fitting

For given Training Set T = { (x_1,y_1), (x_2,y_2),....., (x_n,y_n)}, we need to solve

$$Min_{(\beta_2,\beta_1,\beta_0)} \quad J(\beta_2,\beta_1,\beta_0) = \quad \sum_{i=1}^{n} (yi-(\beta_0+\beta_1 x_i+\beta_2 x_i^2)^2 \qquad .....(2)$$

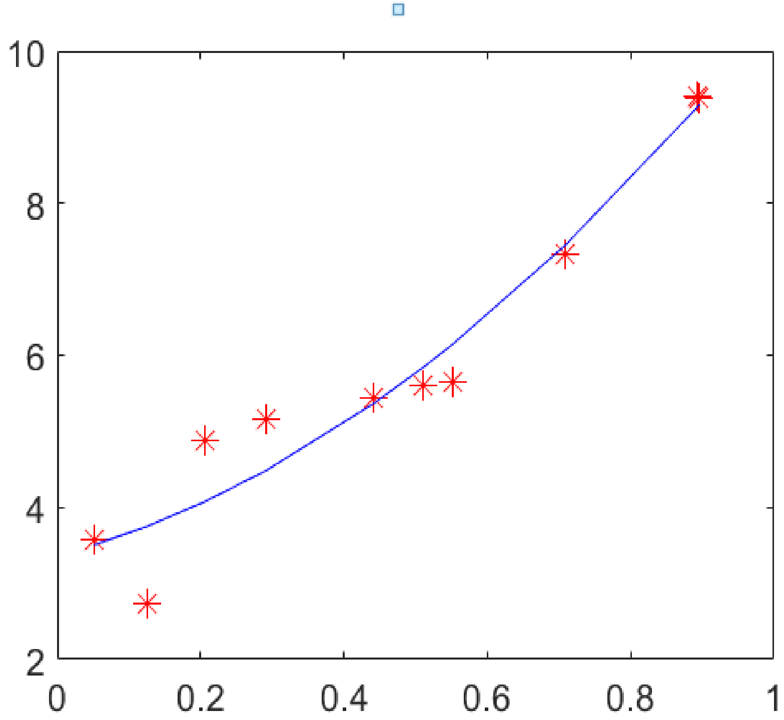
$$u=\begin{bmatrix}\beta_2\\ \beta_1\\ \beta_0\end{bmatrix} \qquad A = \begin{bmatrix}(x_1)^2 & x_1 & 1\\ (x_2)^2 & x_2 & 1\\ (x_3)^2 & x_3 & 1\\ (x_4)^2 & x_4 & 1\\ (x_5)^2 & x_5 & 1\\ (x_6)^2 & x_6 & 1\\ & & \\ & & \\ (x_n)^2 & x_n & 1\end{bmatrix} \qquad Y = \begin{bmatrix}y_1\\ y_2\\ y_3\\ y_4\\ y_5\\ y_6\\ \\ \\ y_n\end{bmatrix}$$

The Least Square problem (2) reduces to

$$Min_{(u)} \quad J(u) = \quad (Y - Au)^T\,(Y - Au)$$

$$u= \begin{bmatrix}\beta_2\\ \beta_1\\ \beta_0\end{bmatrix} = (A^T A)^{-1} \; A^T Y$$

Quadratic Fitting

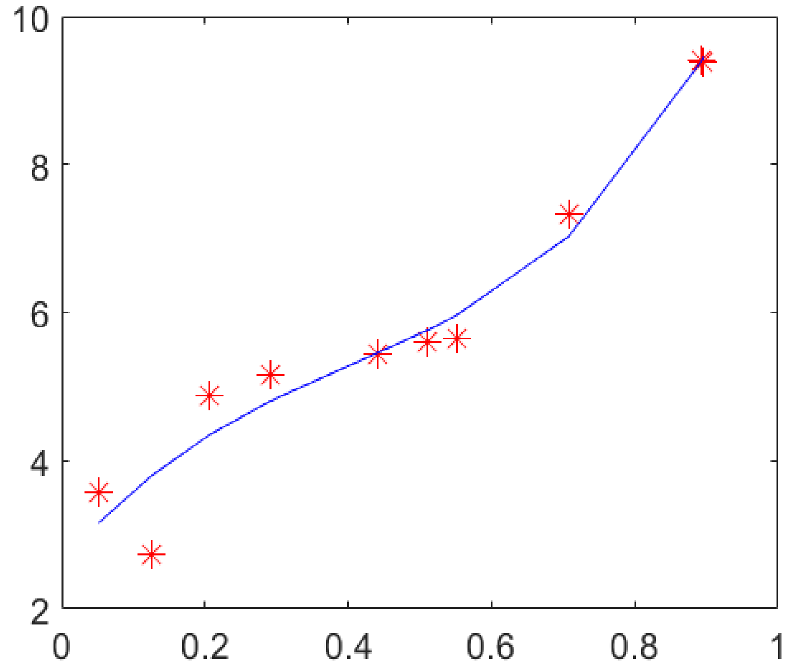


$f(x) = \beta_0 + \beta_1 x + \beta_2 x^2$

$= 3.36 + 2.51\, x \; + 4.58x^2$

Training $RMSE =\ \ 0.4980$

Fitting with fourth order polynomial



$f(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4$

For given Training Set T = { (x_1,y_1), (x_2,y_2),….., (x_n,y_n)}, we need to solve

$$Min_{(\beta_4,\beta_3,\beta_2,\beta_1,\beta_0)} \quad J(\beta_4,\beta_3,\beta_2,\beta_1,\beta_0) = \sum_{i=1}^{n} (yi - (\beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i^3 + \beta_4 x_i^4))^2 \quad \text{…(3)}$$

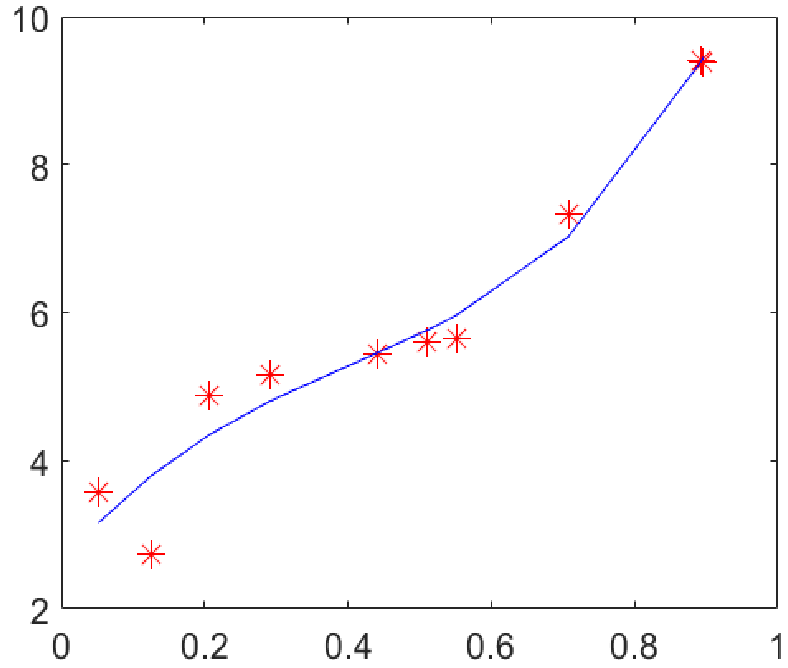
$$u= \begin{bmatrix} \beta_4 \\ \beta_3 \\ \beta_2 \\ \beta_1 \\ \beta_0 \end{bmatrix} \qquad A = \begin{bmatrix} (x_1)^4 & (x_1)^3 & (x_1)^2 & x_1 & 1 \\ (x_2)^4 & (x_2)^3 & (x_2)^2 & x_2 & 1 \\ (x_3)^4 & (x_3)^3 & (x_3)^2 & x_3 & 1 \\ & & & & \\ & & & & \\ (x_n)^4 & (x_n)^3 & (x_n)^2 & x_n & 1 \end{bmatrix} \qquad Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ \\ y_n \end{bmatrix}$$

The Least Square problem (3) reduces to

$$Min_{(u)} \; J(\mathrm{u}) = \; (Y - Au)^T \, (Y - Au)$$

$$u= \begin{bmatrix} \beta_4 \\ \beta_3 \\ \beta_2 \\ \beta_1 \\ \beta_0 \end{bmatrix} = (A^T A)^{-1} \; A^T Y$$

Fitting with fourth order polynomial

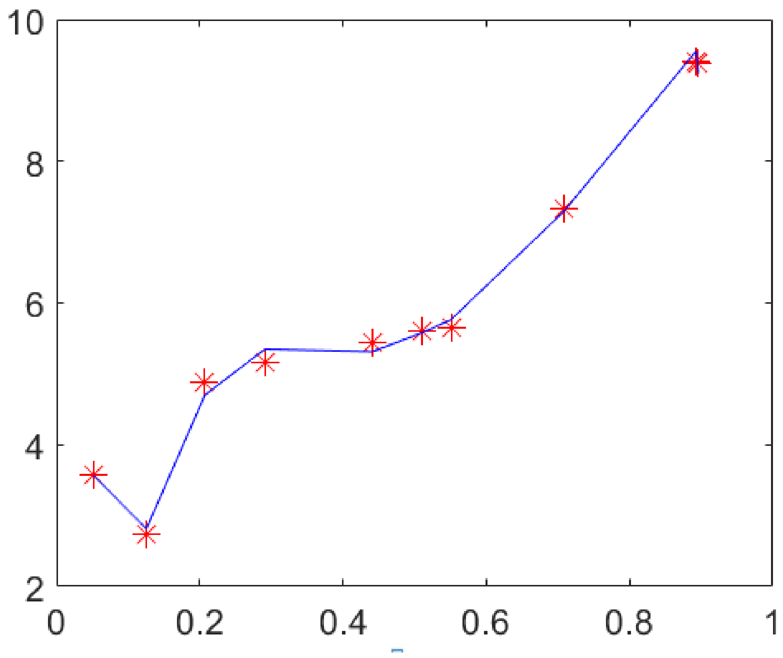


$f(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4$

$= 2.62 + 11.14\,x \; - 15.58\,x^2 \; + 9.17\,x^3 \; + 4.26 x^4$

Training $RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{k}(yi - f(xi))2} = \; 0.4387$

Fitting with seven order polynomial

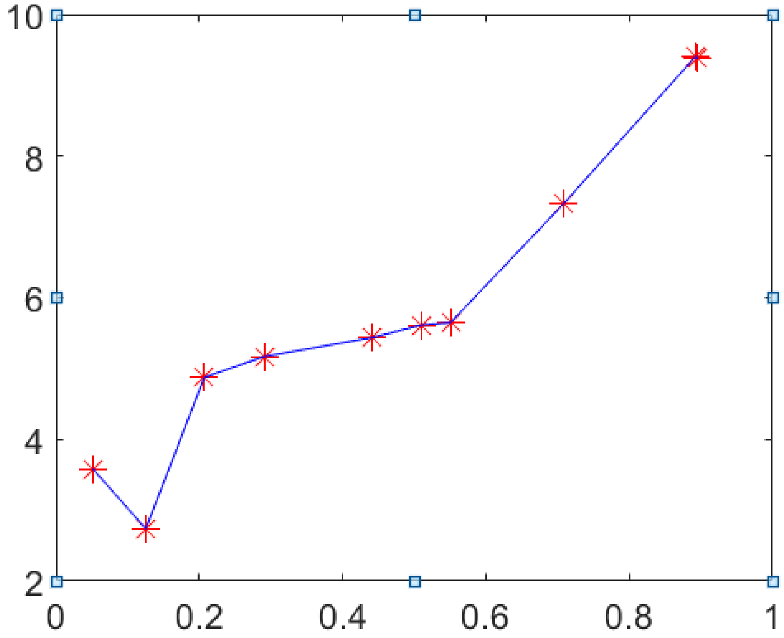


$f(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4 + \beta_5 x^5 + \beta_6 x^6 + \beta_7 x^7$

$= 11.89 - 283.034\, x + 3015\, x^2 - 14643.7 x^3 + 38006.62 x^4 - 54565.9 x^5 + 40844.5 x^6 - 12458.5 x^7$

Training $RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(yi - f(xi))2} = 0.1186$

Fitting with eight order polynomial

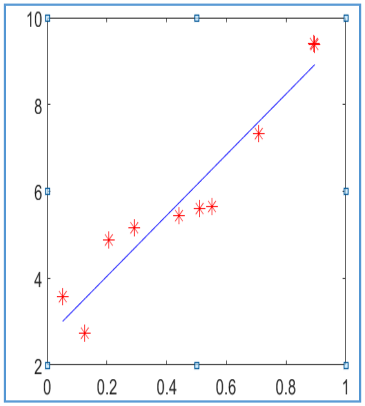


$f(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4 + \beta_5 x^5 + \beta_6 x^6 + \beta_7 x^7 + \beta_8 x^8$

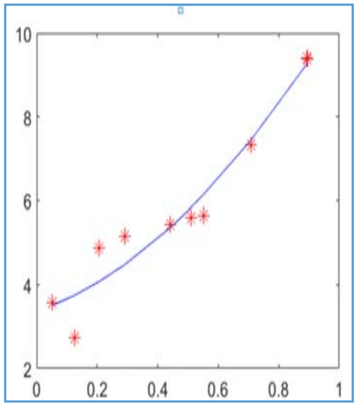
$= 18.14 - 527.837\,x + 6379\,x^2 - 37080.8x^3 + 120518.8x^4 - 230990x^5 + 256860.2x^6 - 154208x^7 + 38542x^8$

Training $RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{k}(yi - f(xi))2} = \ \ 0.0026$

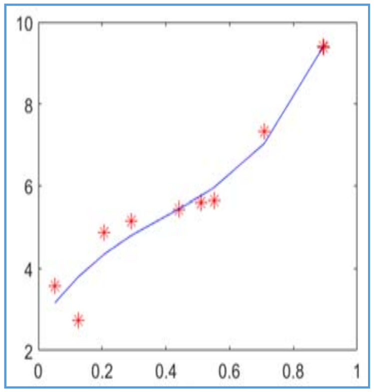
M = 1


10
8
6
4
2
0
0.2
0.4
0.6
0.8
1

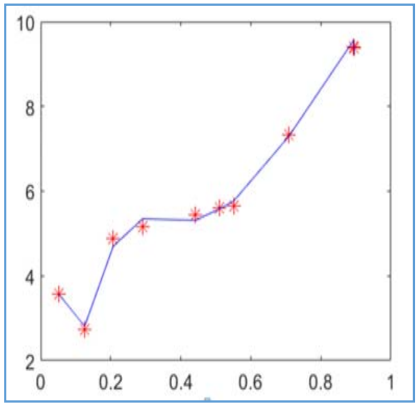
M = 2


10
8
6
4
2
0
0.2
0.4
0.6
0.8
1

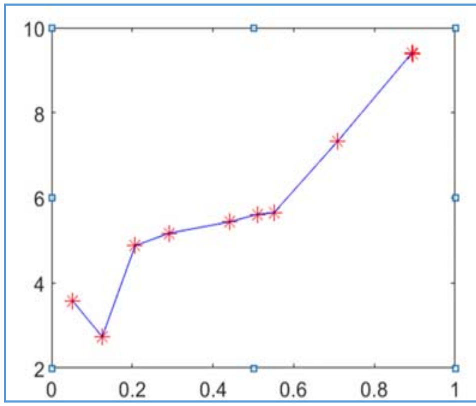
M = 4


10
8
6
4
2
0
0.2
0.4
0.6
0.8
1

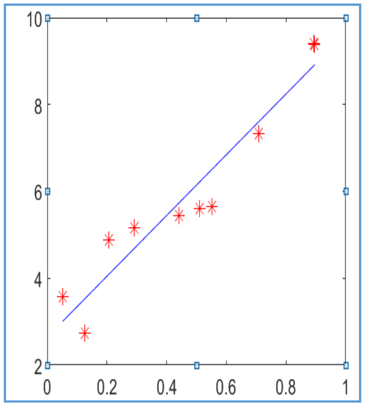
M = 7


10
8
6
4
2
0
0.2
0.4
0.6
0.8
1

M = 8

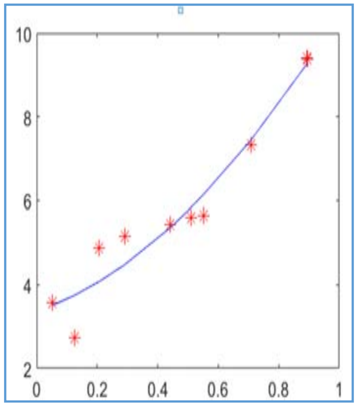

10
8
6
4
2
0
0.2
0.4
0.6
0.8
1

M = 1



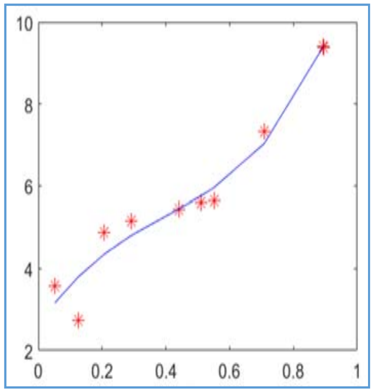
Train RMSE
= 0.5947

M = 2



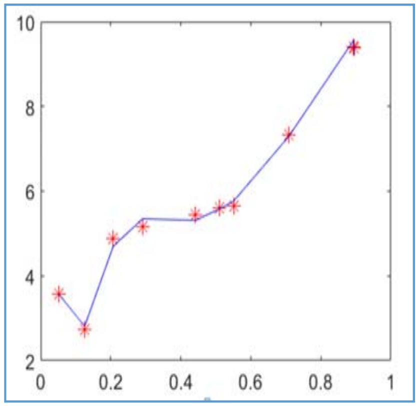
Train RMSE
= 0.4980

M = 4



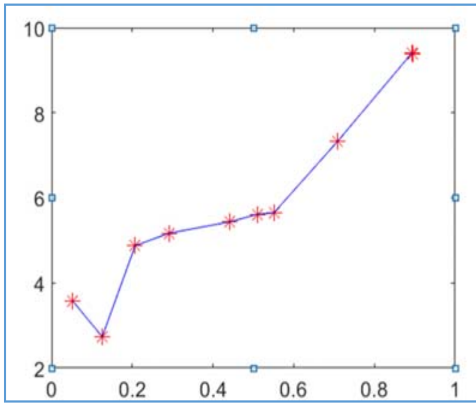
Train RMSE
= 0.4387

M = 7



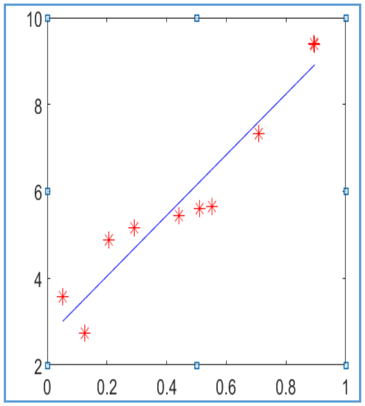
Train RMSE
= 0.1186

M = 8



Train RMSE
= 0.0026

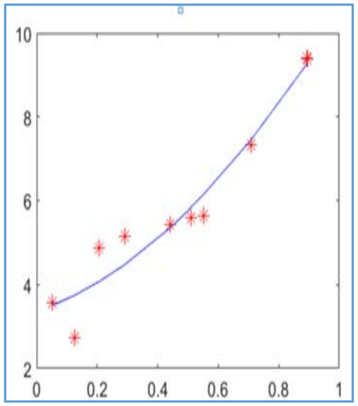
M = 1



Train RMSE
= 0.5947

Test RMSE
= 0.9426

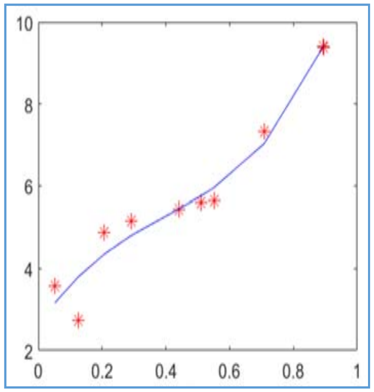
M = 2



Train RMSE
= 0.4980

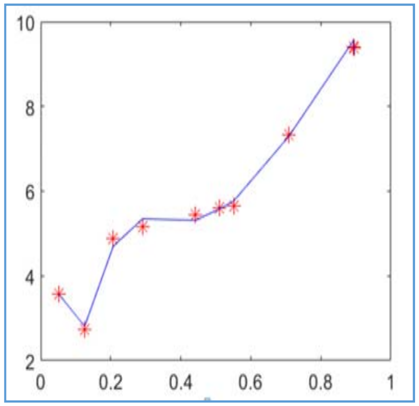
Test RMSE
= 0.7711

M = 4



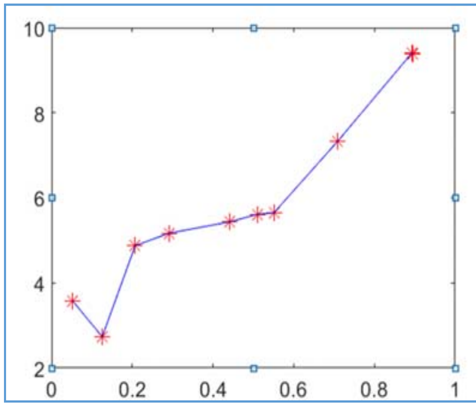
Train RMSE
= 0.4387

M = 7



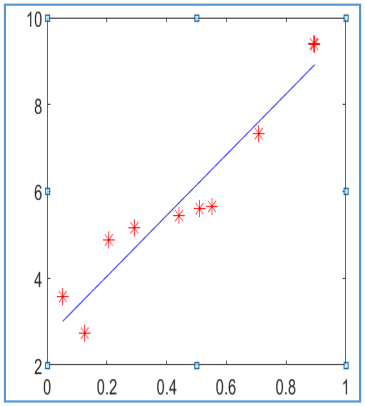
Train RMSE
= 0.1186

M = 8



Train RMSE
= 0.0026

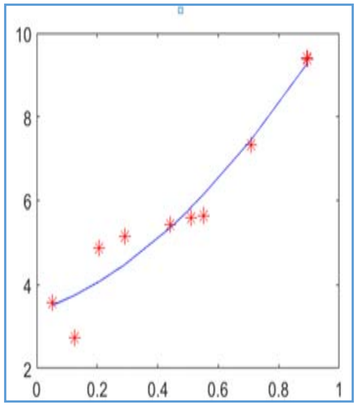
M = 1



Train RMSE
= 0.5947

Test RMSE
= 0.9426

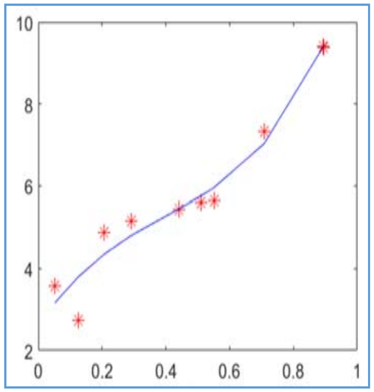
M = 2



Train RMSE
= 0.4980

Test RMSE
= 0.7711

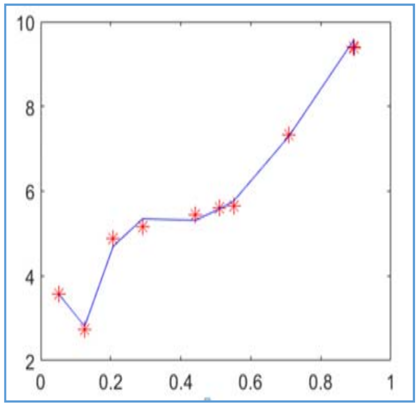
M = 4



Train RMSE
= 0.4387

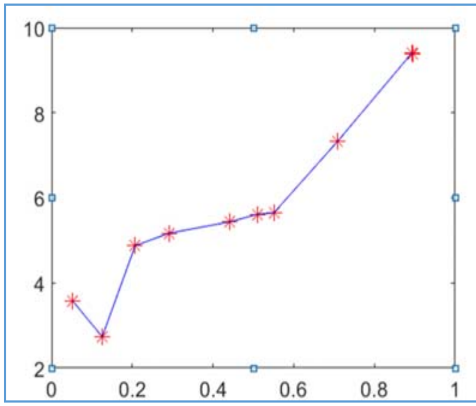
Test RMSE
= 0.9811

M = 7



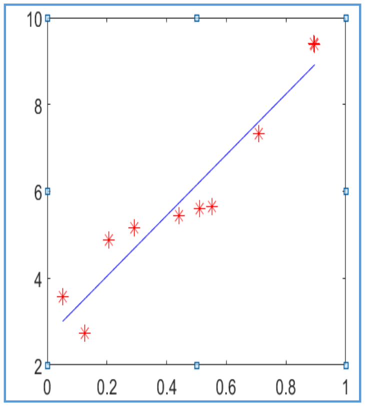
Train RMSE
= 0.1186

M = 8



Train RMSE
= 0.0026

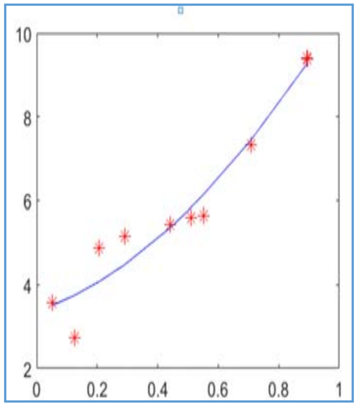
M = 1



Train RMSE
= 0.5947

Test RMSE
= 0.9426

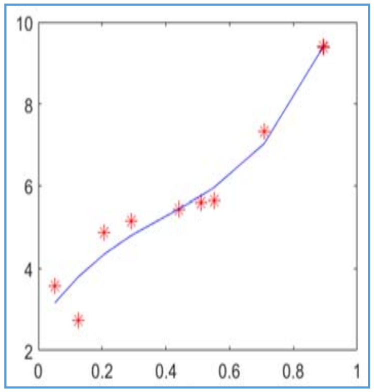
M = 2



Train RMSE
= 0.4980

Test RMSE
= 0.7711

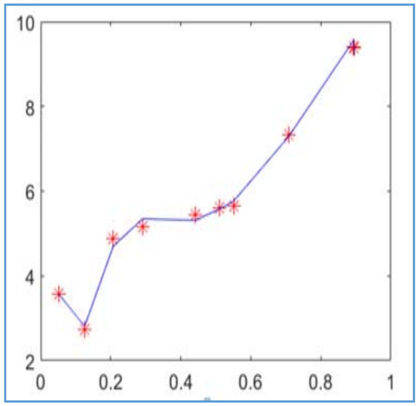
M = 4



Train RMSE
= 0.4387

Test RMSE
= 0.9811

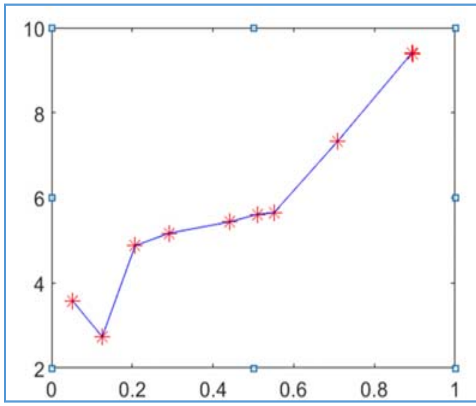
M = 7



Train RMSE
= 0.1186

Test RMSE
=1.179

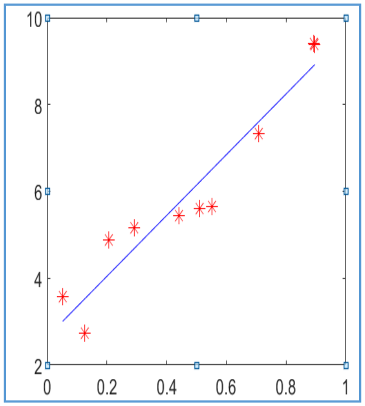
M = 8



Train RMSE
= 0.0026

Test RMSE
= 3.90

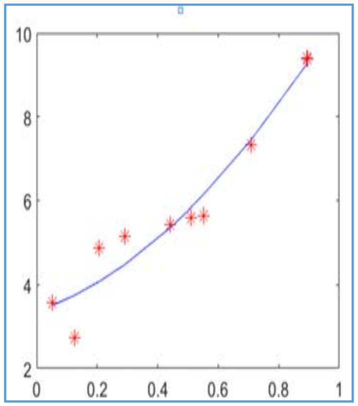
M = 1



Train RMSE
= 0.5947

Test RMSE
= 0.9426

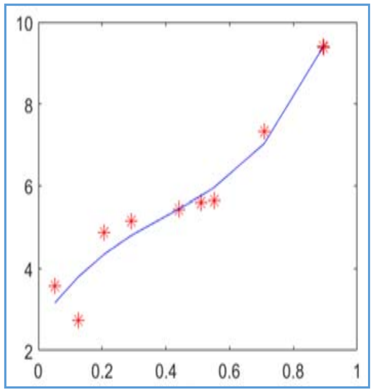
M = 2



Train RMSE
= 0. 4980

Test RMSE
= 0.7711

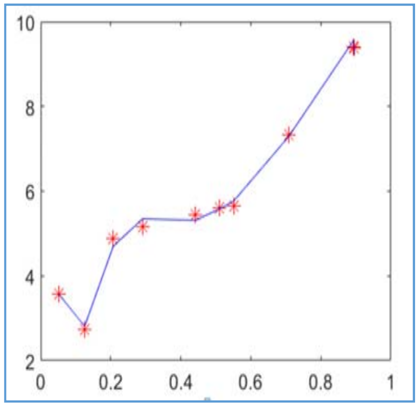
M = 4



Train RMSE
= 0. 4387

Test RMSE
= 0.9811

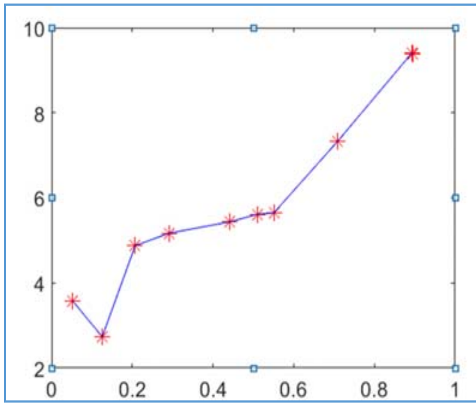
M = 7



Train RMSE
= 0. 1186

Test RMSE
=1.179

M = 8

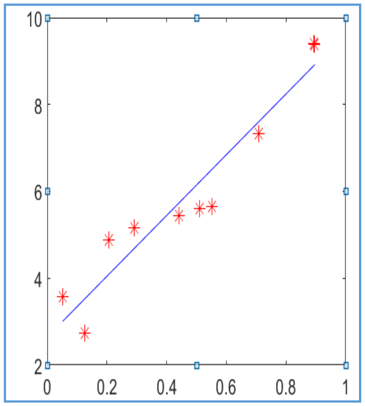


Train RMSE
= 0. 0026

Test RMSE
= 3.90

https://colab.research.google.com/drive/1APfTBXi3U_1ADX1PmLkJ8DyhVCQBegtE?usp=sharing

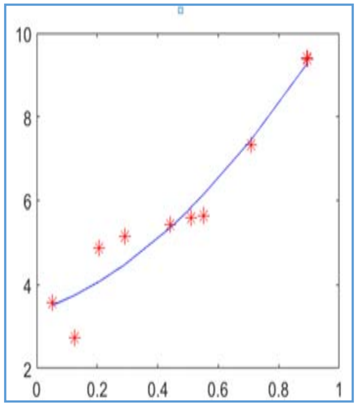
M = 1



Train RMSE
= 0.5947

Test RMSE
= 0.9426

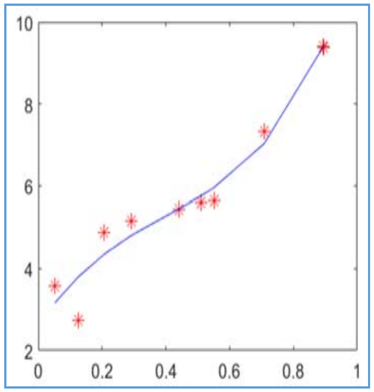
M = 2



Train RMSE
= 0.4980

Test RMSE
= 0.7711

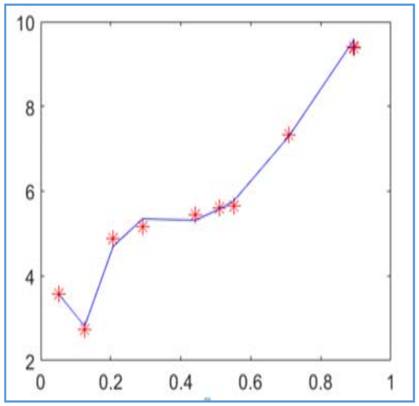
M = 4



Train RMSE
= 0.4387

Test RMSE
= 0.9811

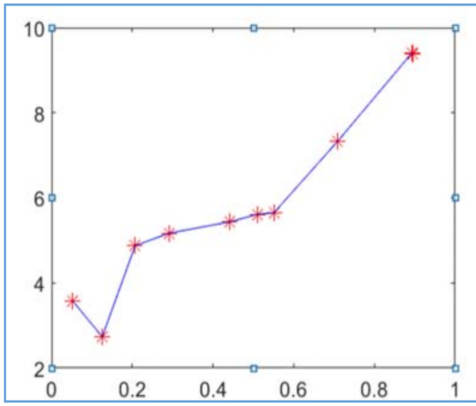
M = 7



Train RMSE
= 0.1186

Test RMSE
=1.179

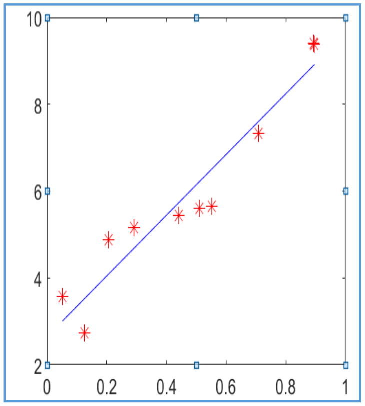
M = 8



Train RMSE
= 0.0026

Test RMSE
= 3.90

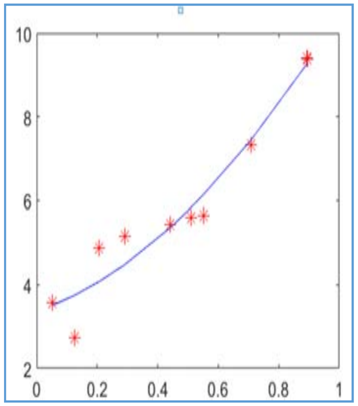
M = 1



Test RMSE
= 0.9426

$$\begin{bmatrix}\beta_0\\ \beta_1\end{bmatrix} = \begin{bmatrix}2.6\\ 6.9\end{bmatrix}$$

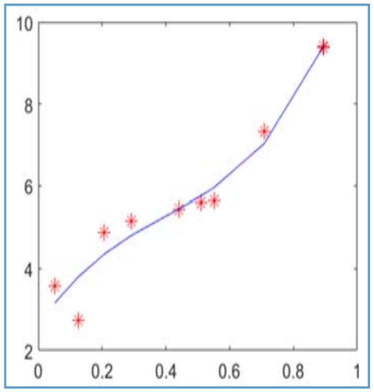
M = 2



Test RMSE
= 0.7711

$$\begin{bmatrix}\beta_0\\ \beta_1\\ \beta_2\end{bmatrix} = \begin{bmatrix}3.36\\ 2.51\\ 4.58\end{bmatrix}$$

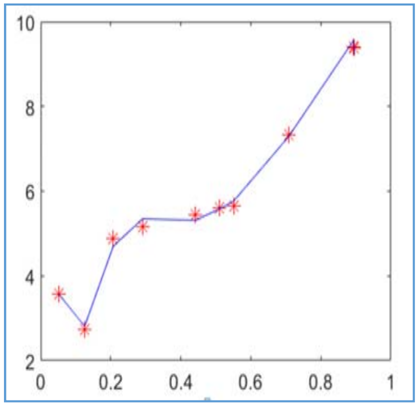
M = 4



Test RMSE
= 0.9811

$$\begin{bmatrix}\beta_0\\ \beta_1\\ \beta_2\\ \beta_3\\ \beta_4\end{bmatrix} = \begin{bmatrix}2.62\\ 11.14\\ 15.58\\ 9.17\\ 4.26\end{bmatrix}$$

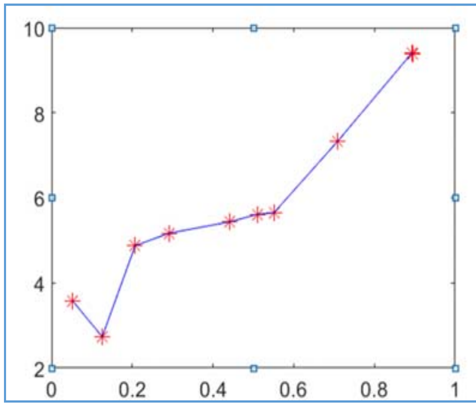
M = 7



Test RMSE
=1.179

$$\begin{bmatrix}\beta_0\\ \beta_1\\ \beta_2\\ \beta_3\\ \beta_4\\ \beta_5\\ \beta_6\\ \beta_7\end{bmatrix} = \begin{bmatrix}11.89\\ -283.034\\ 3015.61\\ -14643.7\\ 38006.62\\ -54565.9\\ 40844.45\\ -12458.5\end{bmatrix}$$

M = 8



Test RMSE
= 3.90

$$\begin{bmatrix}\beta_0\\ \beta_1\\ \beta_2\\ \beta_3\\ \beta_4\\ \beta_5\\ \beta_6\\ \beta_7\\ \beta_8\end{bmatrix} = \begin{bmatrix}18.14\\ -527.83\\ 6379.38\\ 37080\\ 120518.80\\ -230390\\ 256860.2\\ 154208\\ 38542.75\end{bmatrix}$$

Improving the prediction for M=7

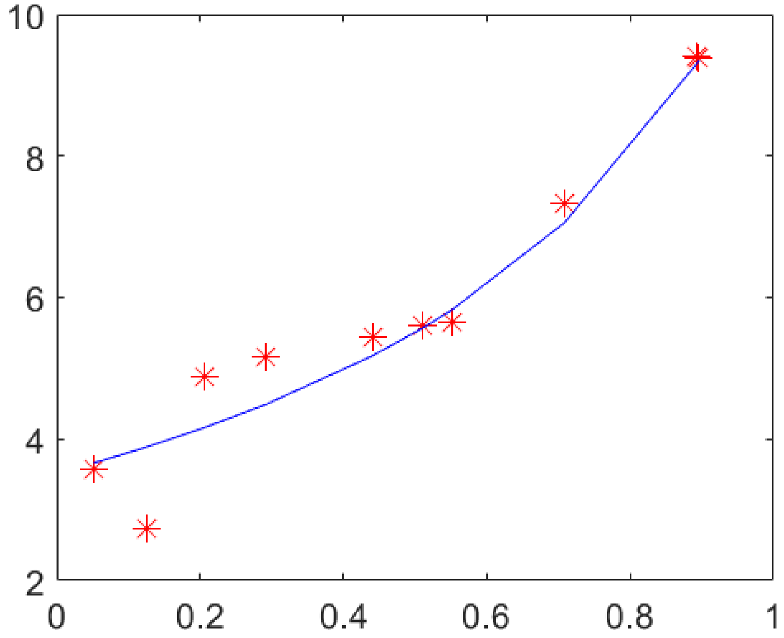
$$Min\left\{ \frac{1}{n}\sum_{i=1}^{n}(yi-(\beta_0 + \beta_1 x_i+\beta_2 x_i+\beta_3 x_i+\beta_4 x_i+\beta_5 x_i+\beta_6 x_i+\beta_7 x_i))^2 \right\}$$

$$\left\{ + \ \frac{\lambda}{2}(\beta_0{}^2+\beta_1{}^2+\beta_2{}^2+\beta_3{}^2+\beta_4{}^2+\beta_5{}^2+\beta_6{}^2+\beta_7{}^2) \right\}$$

User defined
parameter

Estimation with regularization

$$\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_7 \end{bmatrix} = \begin{bmatrix} 3.51 \\ 2.75 \\ 1.60 \\ 1.05 \\ 0.78 \\ 0.61 \\ 0.50 \\ 0.41 \end{bmatrix}$$



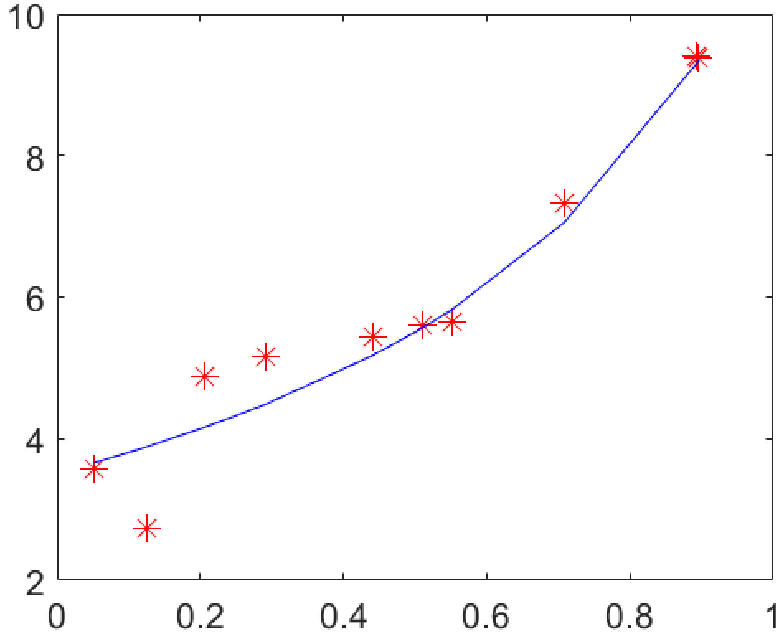
M = 7 order polynomial estimation

Estimation with regularization

Train RMSE
= 0.4989

Test RMSE
= 0.8646

$$\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_7 \end{bmatrix} = \begin{bmatrix} 3.51 \\ 2.75 \\ 1.60 \\ 1.05 \\ 0.78 \\ 0.61 \\ 0.50 \\ 0.41 \end{bmatrix}$$



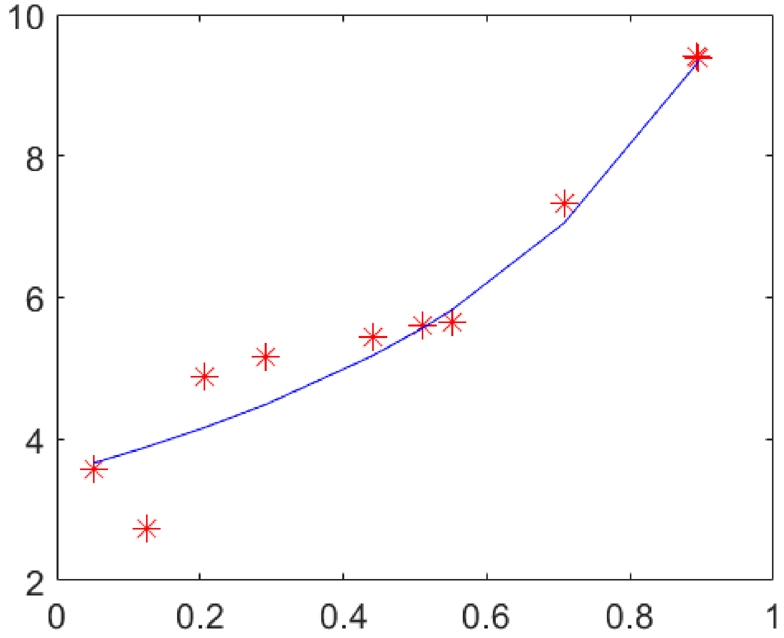
M = 7 order polynomial estimation

Estimation with regularization

Train RMSE
$= 0.4989$

Test RMSE
$= 0.8646$
which was 3.90

$$\begin{bmatrix}\beta_0\\ \beta_1\\ \beta_2\\ \beta_3\\ \beta_4\\ \beta_5\\ \beta_6\\ \beta_7\end{bmatrix} = \begin{bmatrix}3.51\\ 2.75\\ 1.60\\ 1.05\\ 0.78\\ 0.61\\ 0.50\\ 0.41\end{bmatrix}$$



M = 7 order polynomial estimation
with λ = 0.25

Age	Income (hundred thousand dollar)	Balance (thousand dollar)
32	0.550798	5.651202
22	0.708148	7.321263
45	0.290905	5.167304
78	0.510828	5.609367
54	0.892947	9.406379
39	0.896293	9.379439
42	0.125585	2.734997
51	0.207243	4.876649
21	0.051467	3.584138
19	0.44081	5.437239

Credit Card Dataset

Income	Limit	Rating	Cards	Age	Balance
14.891	3606	283	2	34	333
106.025	6645	483	3	82	903
104.593	7075	514	4	71	580
148.924	9504	681	3	36	964
55.882	4897	357	2	68	331
80.18	8047	569	4	77	1151
20.996	3388	259	2	37	203
71.408	7114	512	2	87	872
15.125	3300	266	5	66	279
71.061	6819	491	3	41	1350
63.095	8117	589	4	30	1407