



GRAPH THEORY USE IN TRANSPORTATION PROBLEMS AND RAILWAY NETWORKS

SC-205 PROJECT





□ ABSTRACT

- This model gives an idea about how graph theory used in optimization and reduction of cost and time in transportation in railway systems.
 - We discussed below, there are types of graphs and information that are used specifically for railroad systems in graph theory and problems in railway system and some applications.
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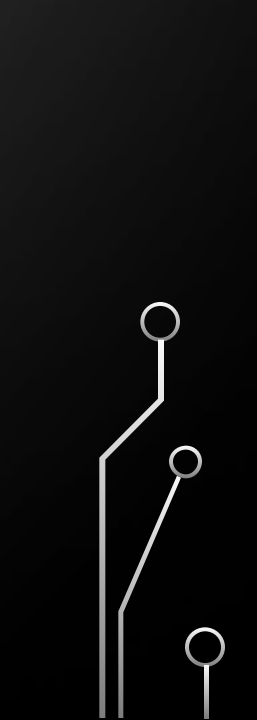


□ INTRODUCTION

- We use Graph Theory in many researches . It helps researchers to research of provable techniques in discrete mathematics .
- The graph theory is explored in many applications in computing, industry, natural science, and social science .
- In the history of transportation and telecommunication, graph theory networks have played a significant role. Additionally, there are specific graphs and information types utilized in graph theory for railway systems . In application there are three London Underground train lines are converted into graphs, and each is then examined in relation to one of three design and safety issues with railroad systems (the blocking problem, the yard location problem, and the train schedule problem).





□ TYPES OF GRAPHS

- Three different forms of graphs—weighted graphs, direct graphs, and double vertex graphs—will be covered in this section.
 - A) Weighted Graphs
 - B) Direct Graphs
 - C) Double Vertex Graphs
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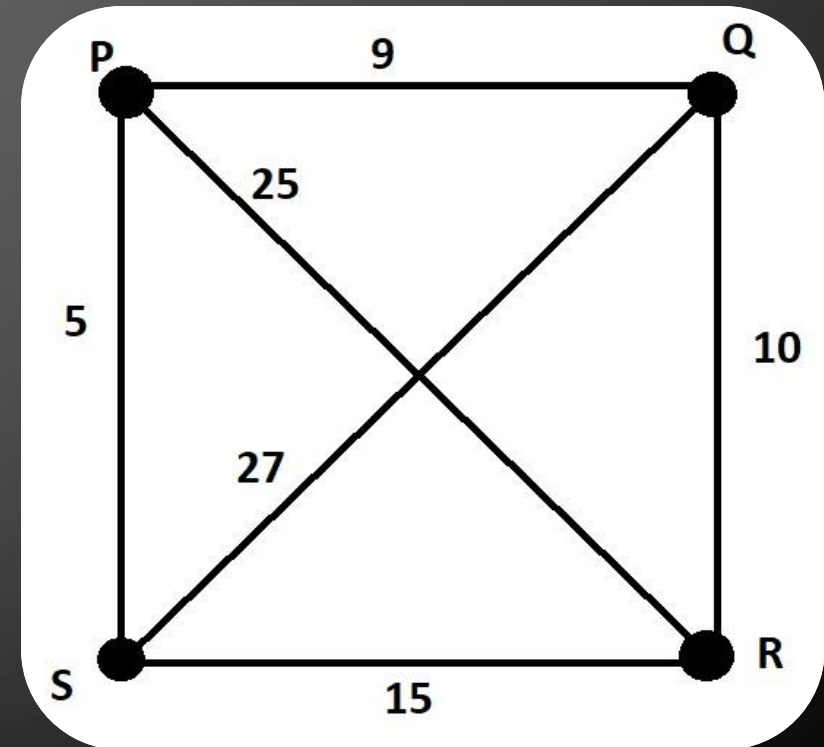


A)WEIGHTED GRAPHS

- Each edge, denoted by "t", has a "weight," such as the separation between two vertices.
 - The shortest and most effective path to take can be found using weighted graphs.
 - Two well-known examples of this are the Chinese Postman Problem and the Travelling Salesman Problem.
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EXAMPLE

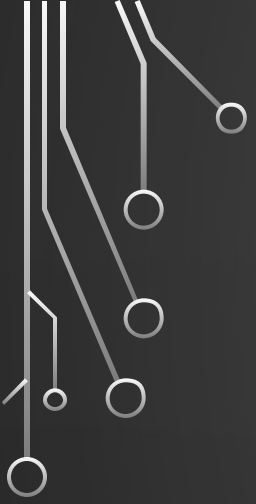
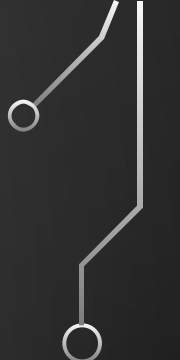


- This example will be a version of the Traveling Salesman Problem.
- Consider a graph G with the following vertices and edges (that have weights).
- Imagine that G represents a route, where P , Q , R , and S are different cities.
- Each edge connecting to cities represents the total length (in kilometers) between the two cities.



- For example, between City P and City Q the travel between the two cities is 9 kilometers.
- A person has to travel from City P, visit every other city only once, and return back to City P.
- What is the shortest route this person can possibly take?

• The

ROUTE	TOTAL DISTANCE OF ROUTE(KM)
PQRSP	39
PQSRP	76
PRQSP	67
PRSQP	77
PSQRP	67
PSRQP	39

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- From the table, we can see that the shortest routes to take are either PQRSP or PSRQP, which are both 39 kilometers.
 - Additionally the circuits, PQRSP and PSRQP would also be considered Hamiltonian circuits.
 - We can build railway systems using data and graphs like this, but there are more effective graphs that consider every aspect of a railway system.
 - Note that this version of the problem was inspired by two examples The Chinese Postman Problem and The Traveling Salesman Problem.
 - . As a result, we can observe that some graphs have edges that can be traversed in both ways, indicating that the edges do not have a defined direction.
 - An edge that can be travel upon in both directions is denoted pictorially as a single line in a graph.

B)DIRECTED GRAPHS

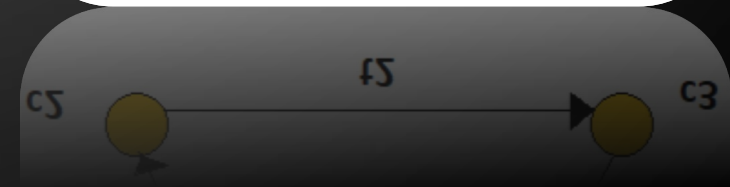
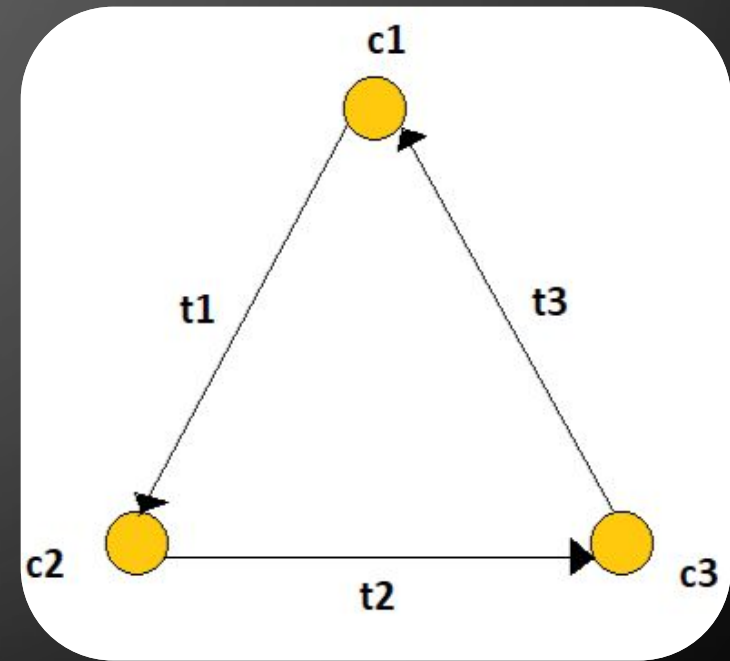
- A directed graph D is a finite, nonempty set of vertices , together with some directed edges joining pairs of vertices. It can be said that the edge-endpoint function associates each edge, t to an ordered pair of vertices, (v, w) , where v is the initial vertex (“starting” vertex) of edge t and w is the terminal vertex (“ending” vertex) of edge t . Directed graphs are held to additional restriction: that the initial and terminal vertices cannot be the same.
- A single line with an arrow drawn as an edge that must only be traversed in one direction.

EXAMPLE

- Consider graph H beside. Graph H is a directed graph.
- In order to complete a circuit that starts and ends at c1, we can only perform the following circuit:

c1t1c2t2c3t3c1

- This is the only circuit, starting and ending at c1 because of the directed edges.

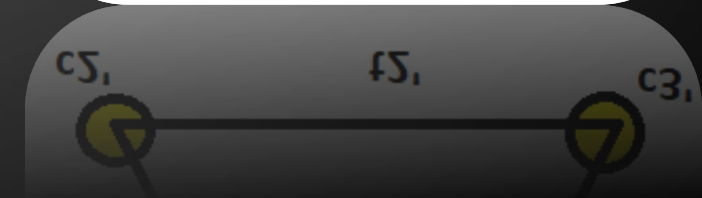
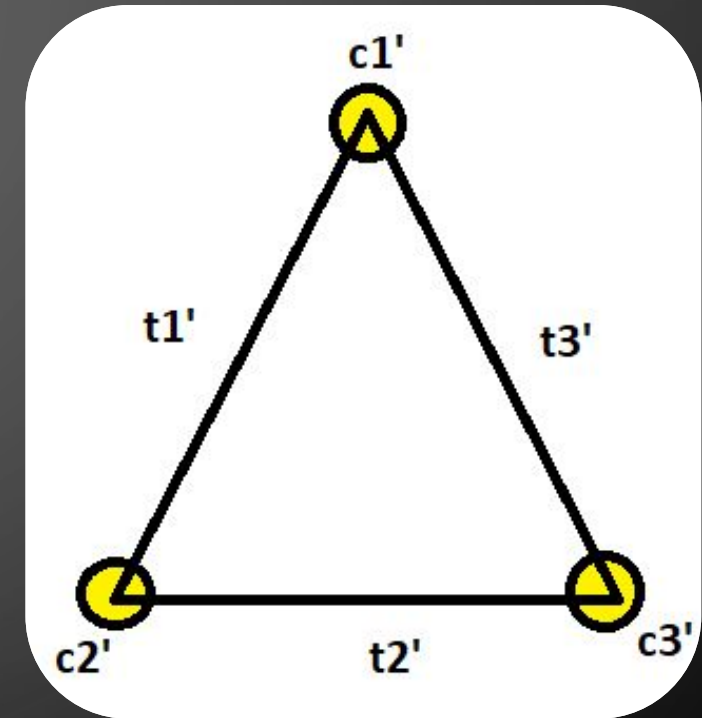


- Consider beside graph.
- If we want to complete a circuit, with it starting and ending at $c1'$, we will have the following two circuits:

Circuit 1 : $c1't1'c2't2'c3't3'c1'$

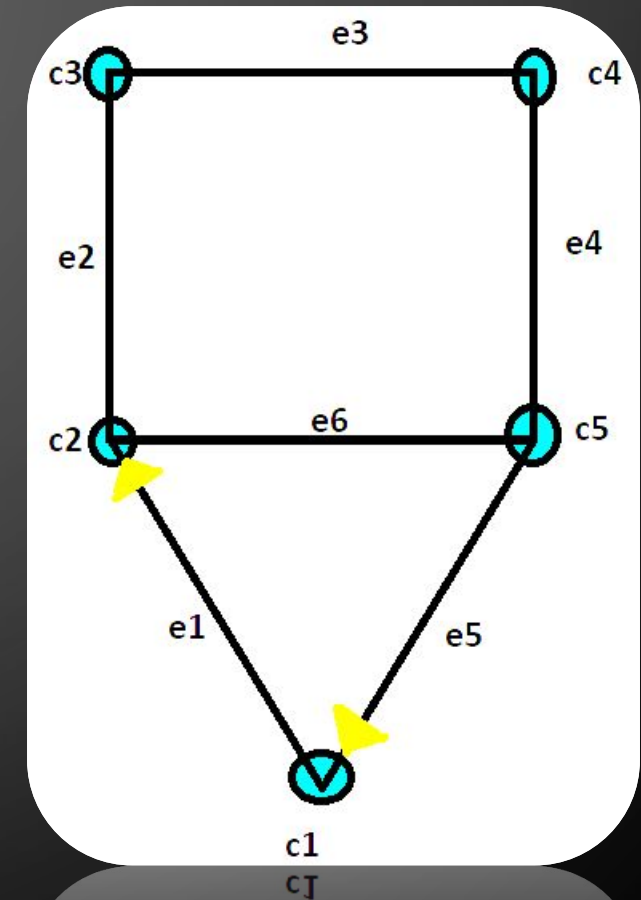
Circuit 2 : $c1't3'c3't2'c2't1'c1'$

- Therefore, since there are no directed edges, we can make a circuit by starting from $c1'$ to $t1'$ or from $c1'$ to $t3'$.
- Notably, there are some edges that are directed and some that are not in some versions of directed graphs. The edges that can only be traversed in one direction are always indicated by an arrow. Undirected edges can be presumed to exist on edges that are not indicated by an arrow.



EXAMPLE

- Consider this graph.
- We can see that there are edges that can be traveled upon in both directions (e2, e3, e4 and e6) and edges that can only be traveled upon in one direction (e1 and e5).
- Therefore, when applied to the railroad system, we can have train tracks that are only traveled upon in one direction, while there are other tracks that can be traveled upon in both directions.



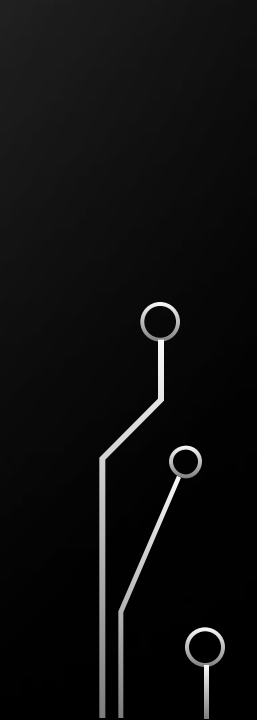


C) DOUBLE VERTEX GRAPH

- In a railway system, we must take into account more than just the track's length or the direction that a train goes in order to make decisions.
- For example, we must consider the speed of a train on a given track.
- A graph, weighted graph, or directed graph of the railway system alone would not contain the specific, physical information that a double vertex graph of the railway system does.



DEFINITION


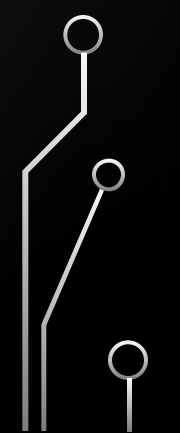
- Let a double vertex graph T be a finite, nonempty set of vertices, $V(G)$, and there be a finite set of edges, $E(G)$, between the vertices.
 - Where none of the edges are loops or that there are multiple edges between a pair of vertices.
 - Additionally, these graphs are specifically for railroad systems and every edge holds different pieces of information for the railroad, such as track length, maximum speed, etc.
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□ DESIGN AND SAFETY PROBLEMS IN THE RAILROAD SYSTEM

- The design of railway systems must take into consideration of a number of safety and design issues.
- Because certain trains can only run in one direction but use a two-way line, railway systems need to be properly built.
- Therefore, it is important to prevent train accidents because, if nothing is done to consider for the fact that two trains will be travelling on the same rail at the same time, it is possible that they may collide.
- We will be focusing on two:
 - 1)Chinese Postman's Problem
 - 2)The Yard Location Problem

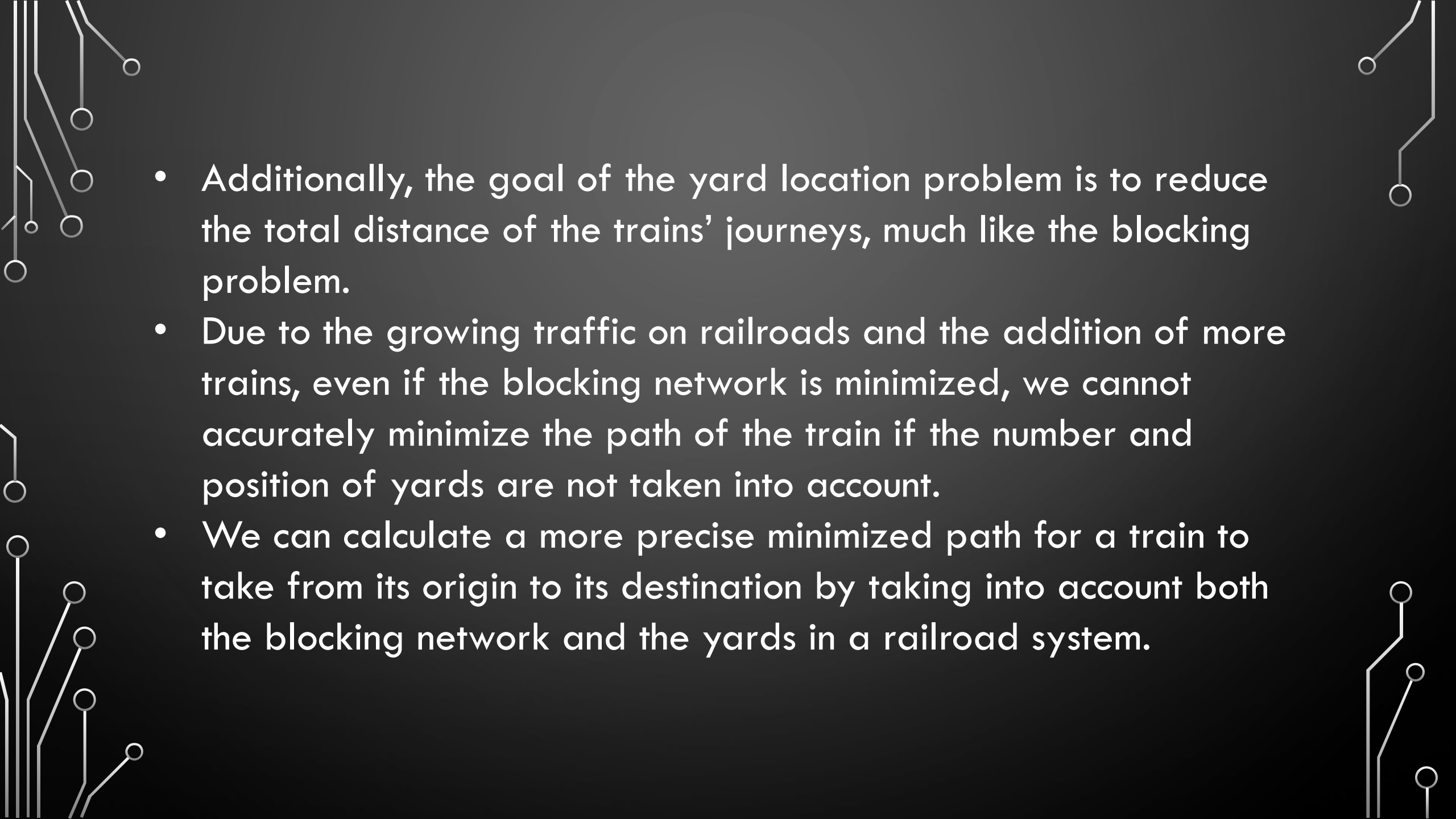


1) CHINESE POSTMAN'S PROBLEM

- In 1962, A Chinese mathematician called Kuan Mei-ko was interested in a postman delivering mail to a number of streets.
 - Such that the total distance walked by the postman was as short possible.
 - How could the postman ensure that the distance walked was minimum.
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2) YARD LOCATION PROBLEM

- We try to find the best network configuration in terms of number and location of yards where cars can be reclassified into new blocks and switch trains
- The train travels to a "yard" on its way from its departure location, which we shall refer to as an origin, to its final destination, as was previously stated. Trains from other train lines in the system are also classed in yards before being sent to different locations.
- Local yards, system yards, and regional yards are the three different categories of yards. Local yards are used when trains are arriving from stations that are divided into blocks but are not in yards. System yards and regional yards are regarded as "hub yards," as they both handle larger quantities of trains and commodities. Compared to system yards and regional yards, local yards are smaller.

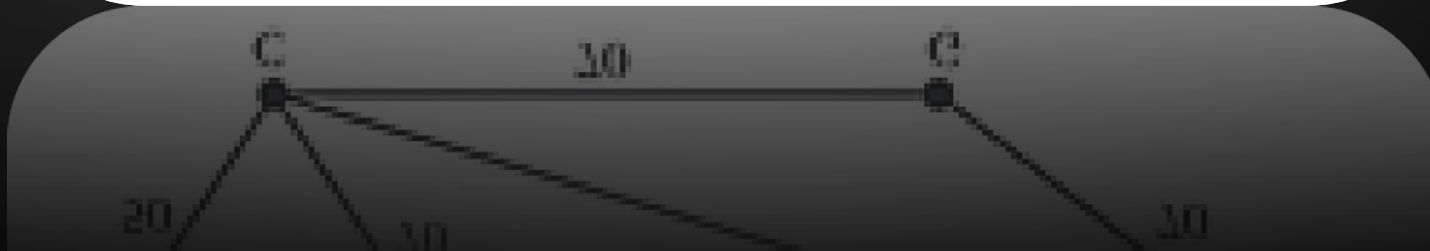
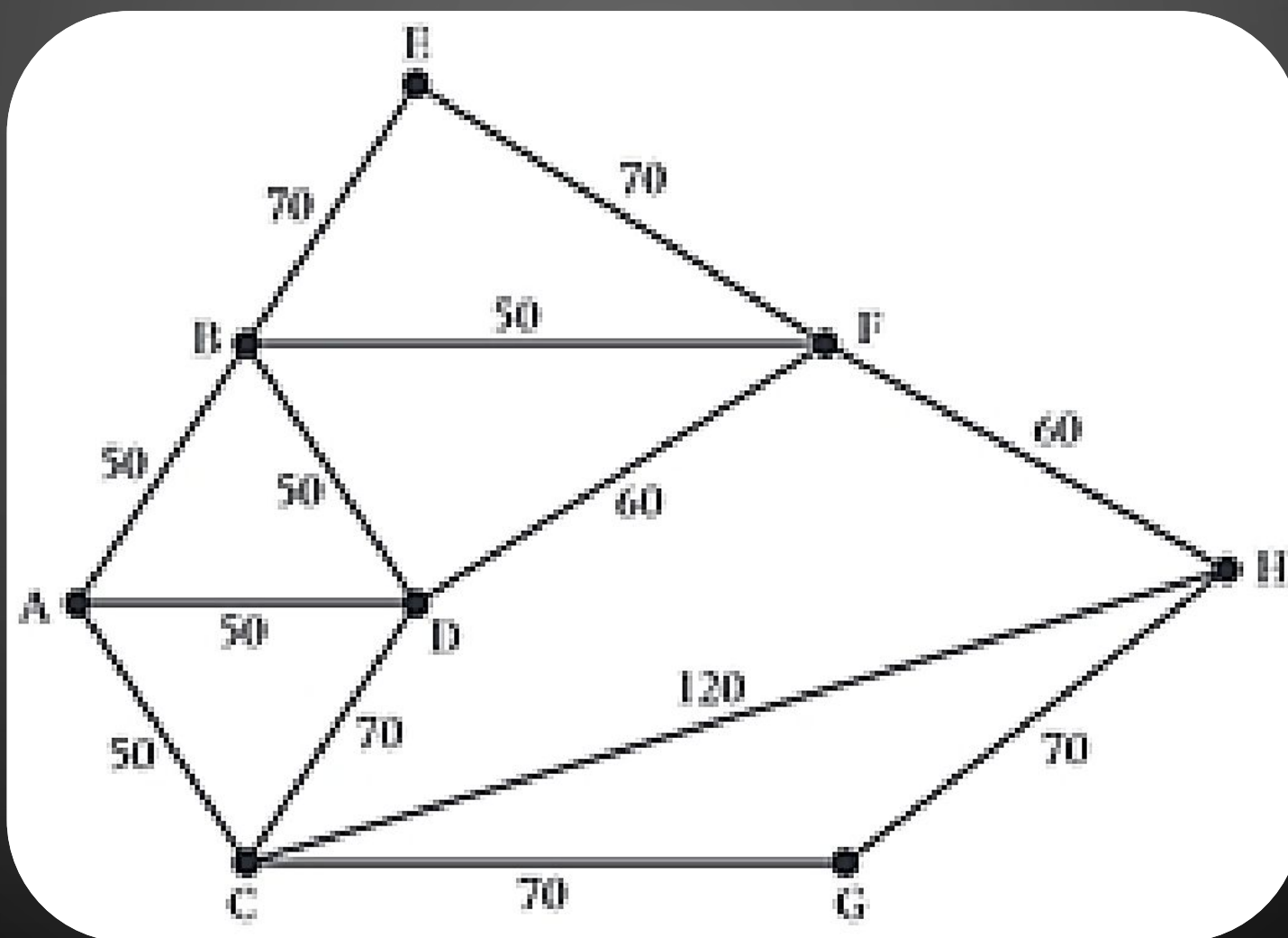
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- Additionally, the goal of the yard location problem is to reduce the total distance of the trains' journeys, much like the blocking problem.
 - Due to the growing traffic on railroads and the addition of more trains, even if the blocking network is minimized, we cannot accurately minimize the path of the train if the number and position of yards are not taken into account.
 - We can calculate a more precise minimized path for a train to take from its origin to its destination by taking into account both the blocking network and the yards in a railroad system.



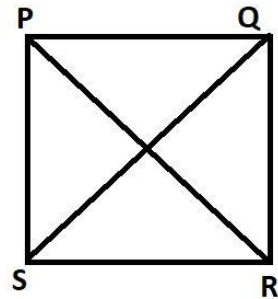
APPLICATIONS

1) CHINESE POSTMAN SOLUTION

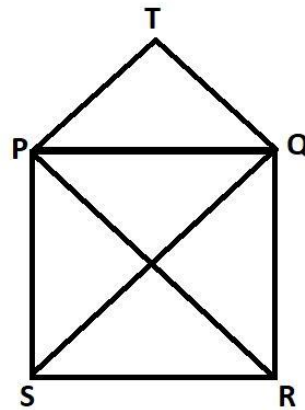
- A postman has to start at A, walk along all 13 streets and return to A.
- The numbers on each edge represent the Length, in meters, of each street.
- The problem is to find a train that uses all the edges of a graph with minimum Length.



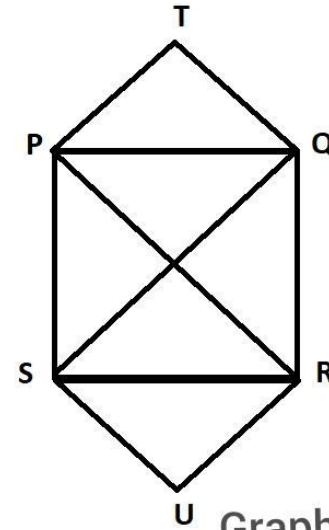
- ❖ We'll look at making different graphs first, and then we'll get back to solving the actual problem. The graphs below can be used to navigate the Chinese postman.



Graph 1



Graph 2



Graph 3

- Drawing Graph 1 requires either removing the pen off the page or retracing an edge.
- We can draw graph 2, but only by starting at either A or D-in each case the path will end at the other vertex of D or A.
- Graph 3 can be drawn regardless of the starting position and you will always return to the start vertex.

Vertex	Order
A	3
B	3
C	3
D	3

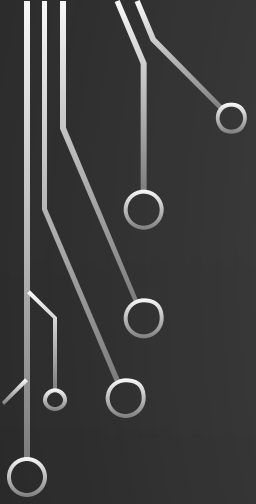
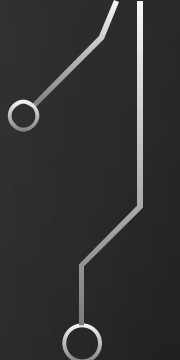

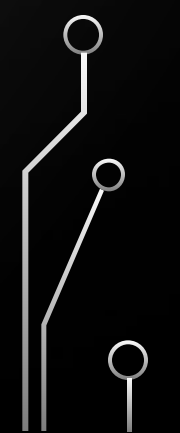
Graph 01

Vertex	Order
A	3
B	4
C	4
D	3
E	2

Graph 02

Vertex	Order
A	4
B	4
C	4
D	4
E	2
F	2

Graph 03

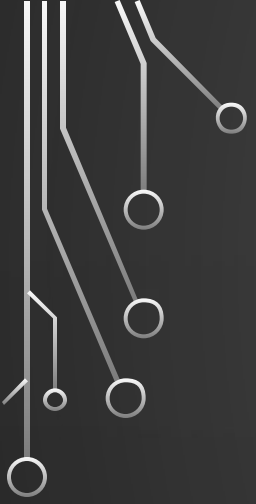
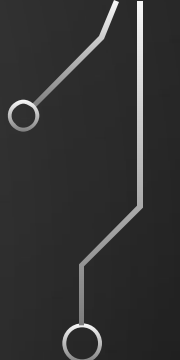

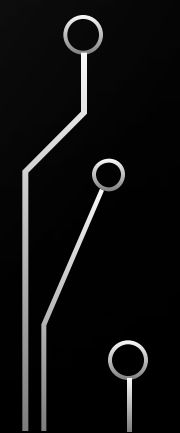
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- The graph is Traversable if all of the vertices are arranged in even numbers.
 - We can draw the graph when there are two odd vertices, but the start and end vertices are different.
 - The graph cannot be drawn when there are four odd vertices without repeating an edge.



CHINESE POSTMAN ALGORITHM

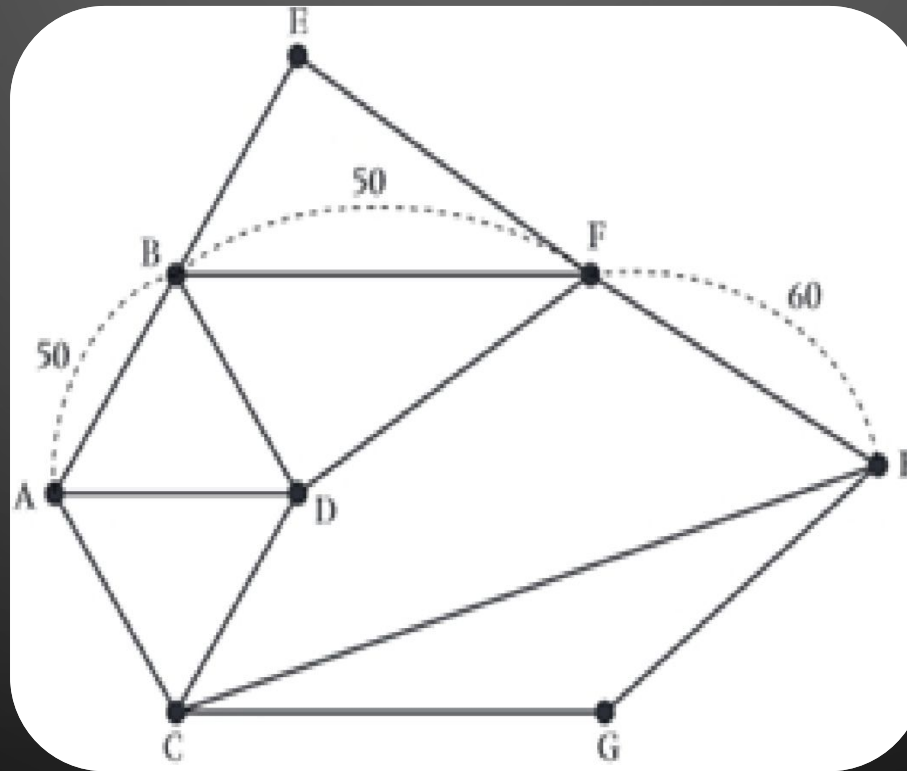
An algorithm for finding an optimal Chinese postman route is.

- Step 1 :- List all odd vertices.
- Step 2 :- List all possible pairing of odd vertices.
- Step 3 :- For each pairing find the edges that connect the vertices with the minimum weight.
- Step 4 :- Find the pairing such that the sum of the weights is minimized.

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- Step 5 :- On the original graph add the edges that have been found in step 4.
 - Step 6 :- The length of an optimal Chinese postman route is the sum of all the edges added to the total found in step 4.
 - Step 7 :- A route corresponding to this minimum weight can then be easily found.



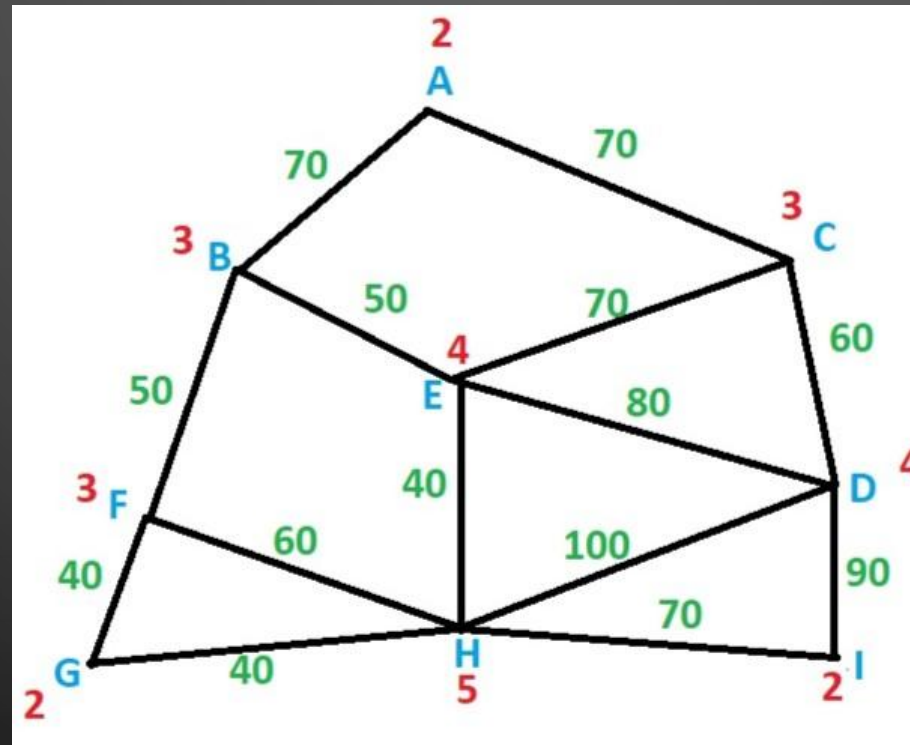
APPLICATION OF THE ALGORITHM IN FOLLOWING FIGURE



- Step 1:- The odd vertices are A and H.
- Step 2:- There is only one way of pairing these odd vertices namely AH.
- Step 3:- The shortest way of joining A to H is using the path AB, BF, FH a total length of 160.
- Step 4:- These edges on to the original network in this fig.
- Step 5:- The length of the optimal Chinese postman route is the sum of all the edges in the original network. Which is 840 meter Plus the answer found in step 4, which is 160 meter, Hence the length of the optimal Chinese postman route is 1000 meter.
- Step 6:- One possible route corresponding to this length is ADCGH CABDFBEHFBA, but many other possible routes of the sum minimum length can be found.

□ EXAMPLE :-

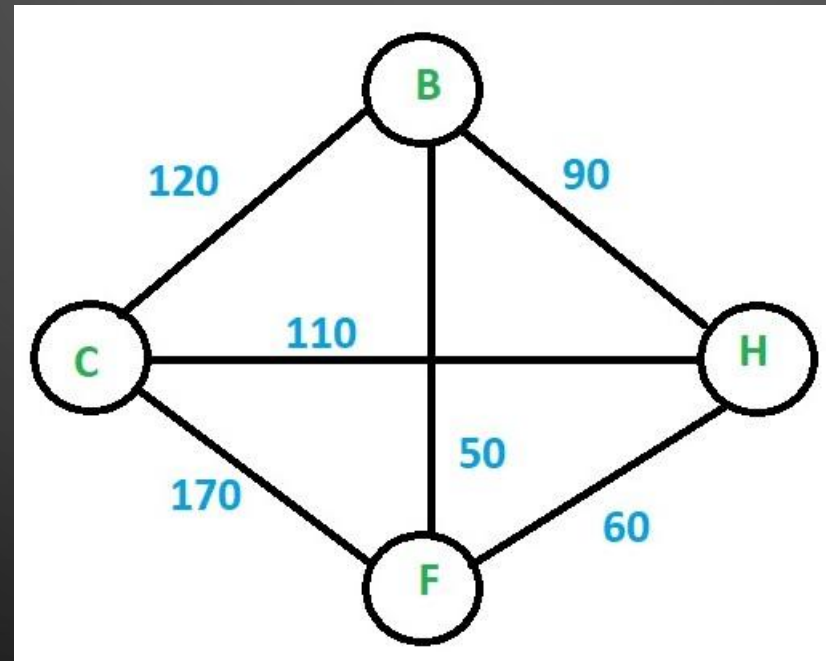
FIND THE CHINESE POSTMAN OPTIMAL ROUTE.



SOLUTION :-

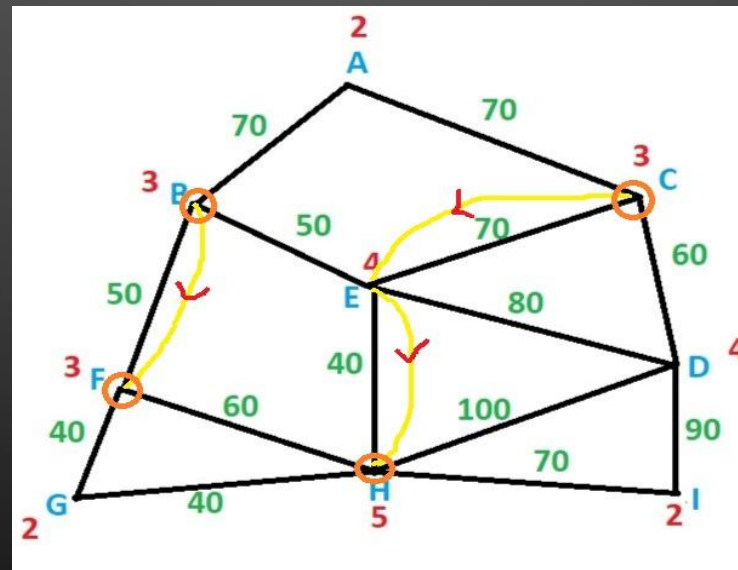
[1] ODD VERTICES ARE B,C,F AND H.

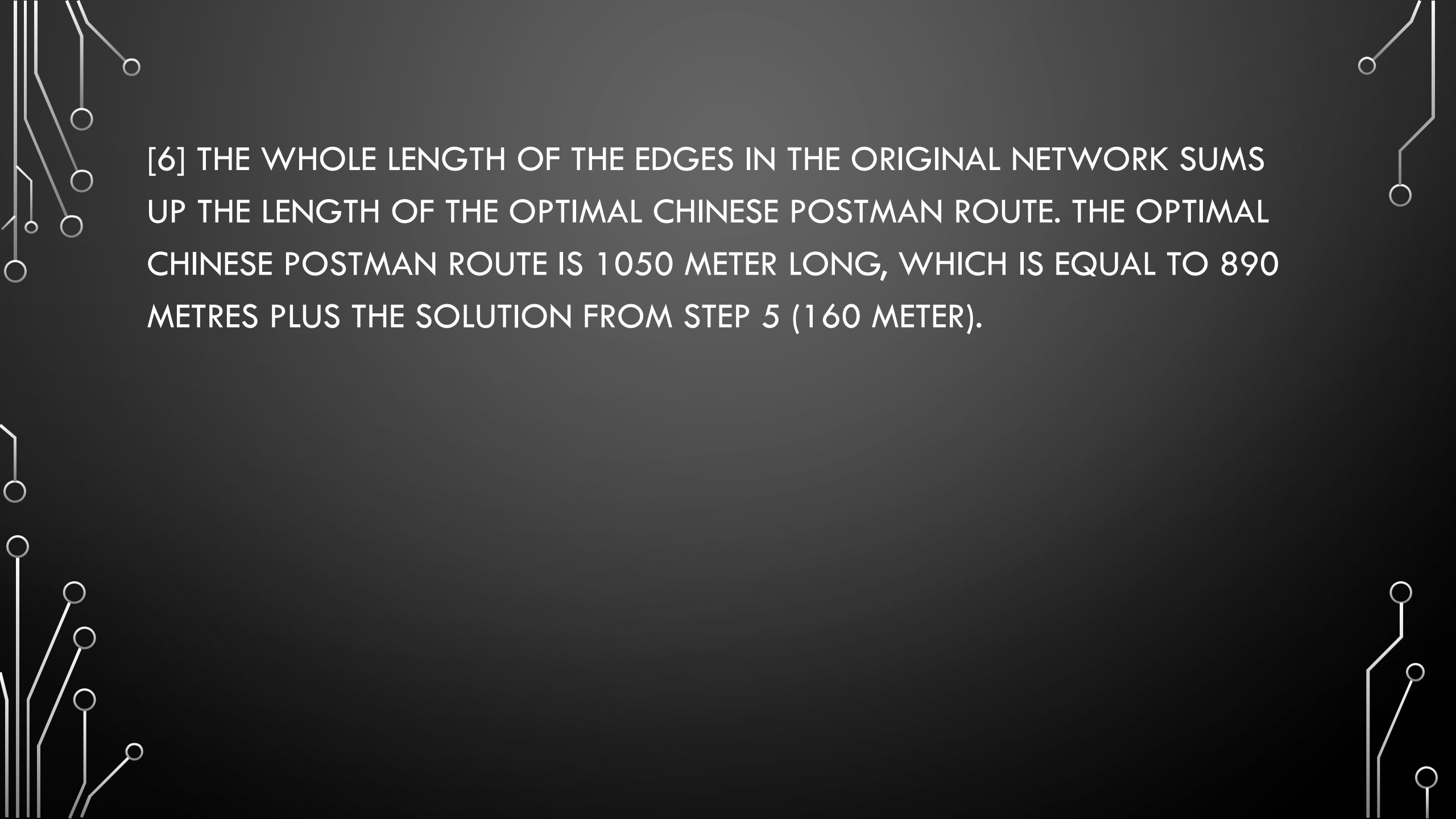
[2] FROM STEP2, STEP3 AND STEP 4 AS ABOVE MENTION ALGORITHM ,WE HAVE DRAW ONE FIGURE SUCH THAT SHOW HOW MANY PATHS ARE POSSIBLE.



- PATH 1 \rightarrow BF \rightarrow 50 CH \rightarrow 110 TOTAL=160
- PATH 2 \rightarrow BC \rightarrow 120 FH \rightarrow 60 TOTAL=180
- PATH 3 \rightarrow BH \rightarrow 90 CF \rightarrow 170 TOTAL=260

[5] WE CAN SAY THAT OPTIMAL DISTANCE IS FROM PATH 1 AND THESE DISTANCE IS 160. THESE EDGES ON TO THE ORIGINAL NETWORK IN THIS FIGURE.



The background is a dark gray gradient. In the four corners, there are decorative white line art elements that resemble circuit boards or network diagrams. These elements consist of thin lines connecting small circles, some of which are larger than others, creating a stylized, abstract pattern.

[6] THE WHOLE LENGTH OF THE EDGES IN THE ORIGINAL NETWORK SUMS UP THE LENGTH OF THE OPTIMAL CHINESE POSTMAN ROUTE. THE OPTIMAL CHINESE POSTMAN ROUTE IS 1050 METER LONG, WHICH IS EQUAL TO 890 METRES PLUS THE SOLUTION FROM STEP 5 (160 METER).





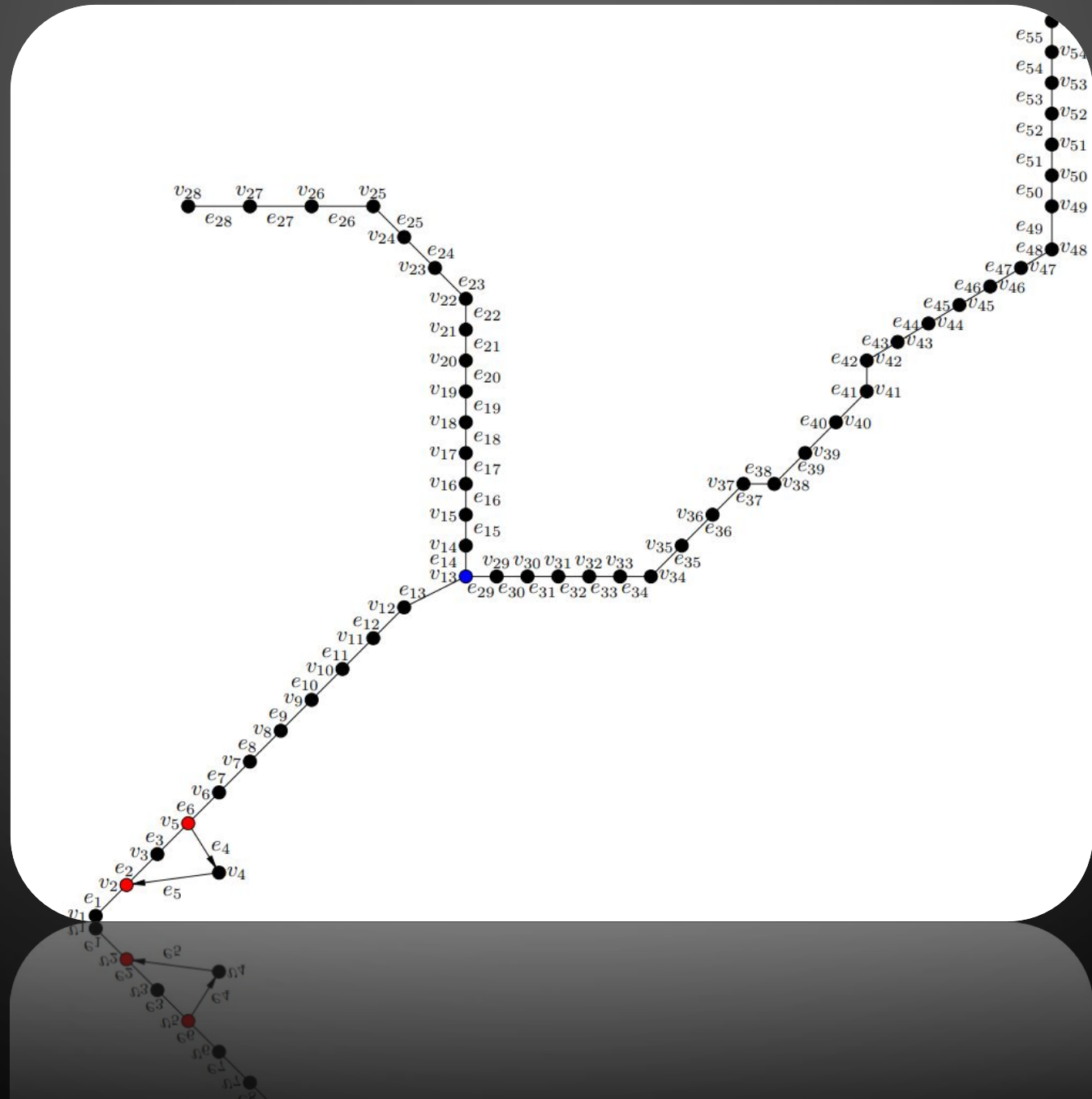
LONDON UNDERGROUND SYSTEM

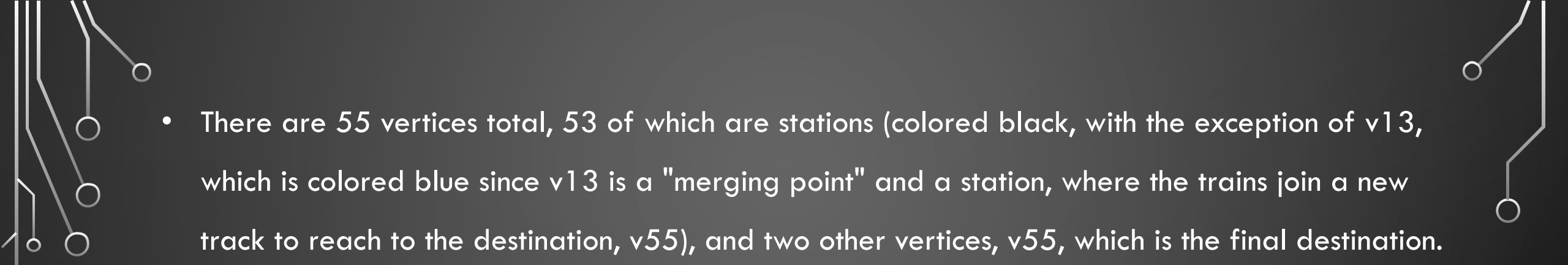
- We will use the London Underground, sometimes known as "The Tube," to apply graph theory to the design and safety elements of railway systems.
- There are eleven train lines in the London Underground , which spans 250 miles.
- The Central Line, the Piccadilly Line, and the Circle Line will be specifically examined, and each line will be related to one of three design and safety issues with the railway system: the blocking problem, the yard location problem, and the train route problem.



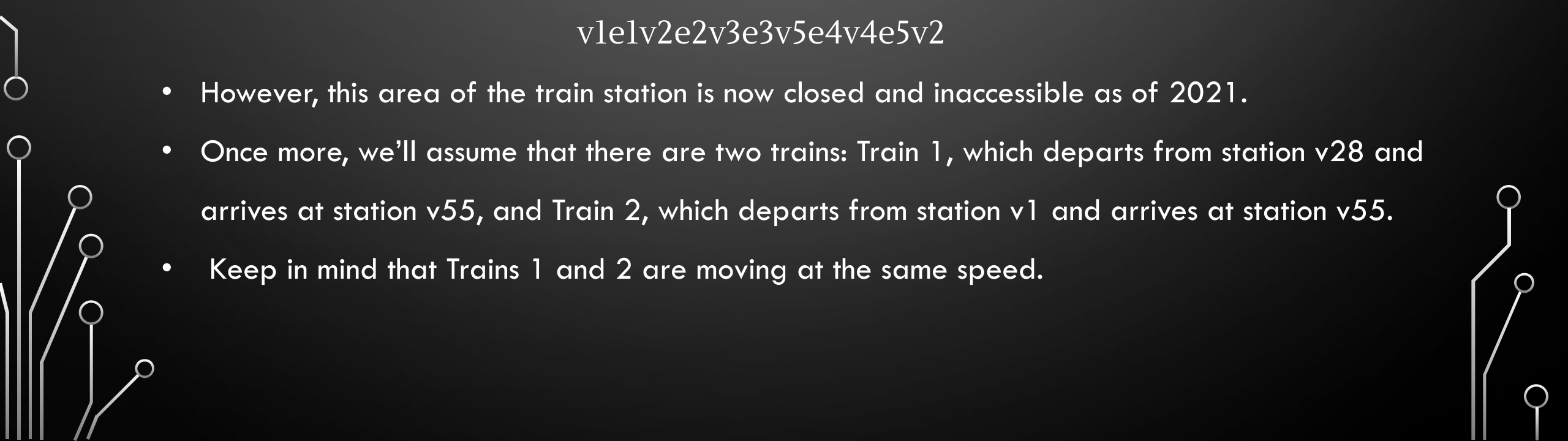
❖ PICCADILLY LINE

- We'll now talk about the Piccadilly Line and the issue with the yard location problem.
 - The Piccadilly Line has 53 stations and travels a distance of around 44 miles.
 - For a visual illustration of the Piccadilly Line, see following Figure.
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- There are 55 vertices total, 53 of which are stations (colored black, with the exception of v13, which is colored blue since v13 is a "merging point" and a station, where the trains join a new track to reach to the destination, v55), and two other vertices, v55, which is the final destination.
 - There are two "merging points" (in red). Keep in mind that the graph's overall directed graph component is created by these two merging points. Consequently, there is a walk for the directed graph portion:

v1e1v2e2v3e3v5e4v4e5v2

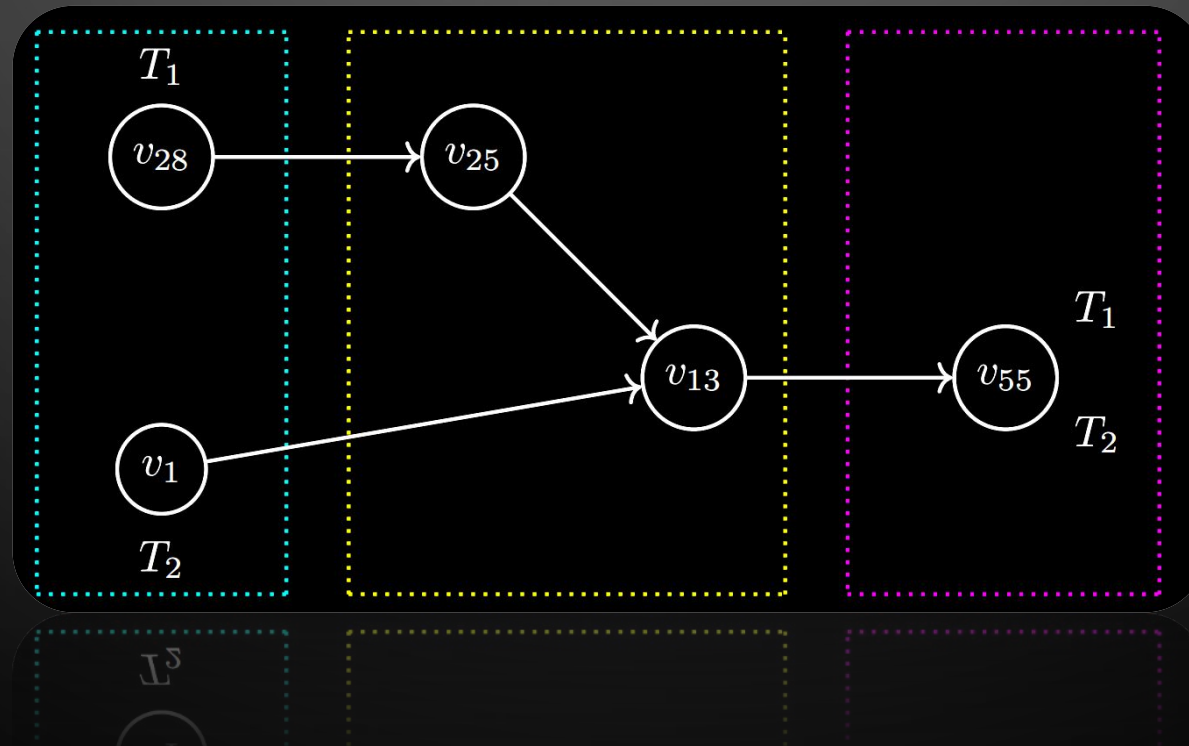
- However, this area of the train station is now closed and inaccessible as of 2021.
 - Once more, we'll assume that there are two trains: Train 1, which departs from station v28 and arrives at station v55, and Train 2, which departs from station v1 and arrives at station v55.
 - Keep in mind that Trains 1 and 2 are moving at the same speed.
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- Consequently, we can determine the subsequent walks:

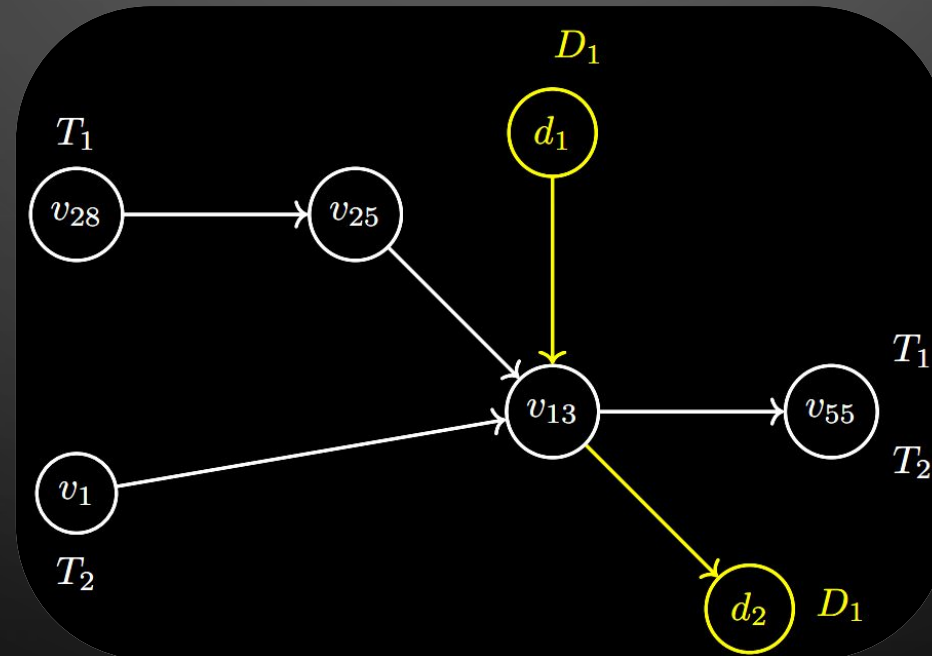
Train 1 : $v_{28}e_{28}v_{27}e_{27} \dots v_{13}e_{29} \dots v_{53}e_{54}v_{54}e_{55}v_{55}$

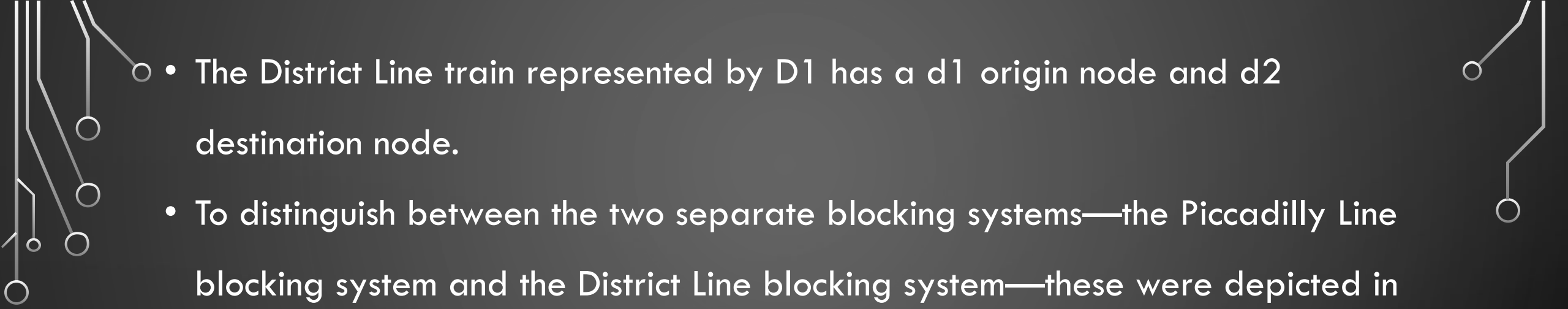
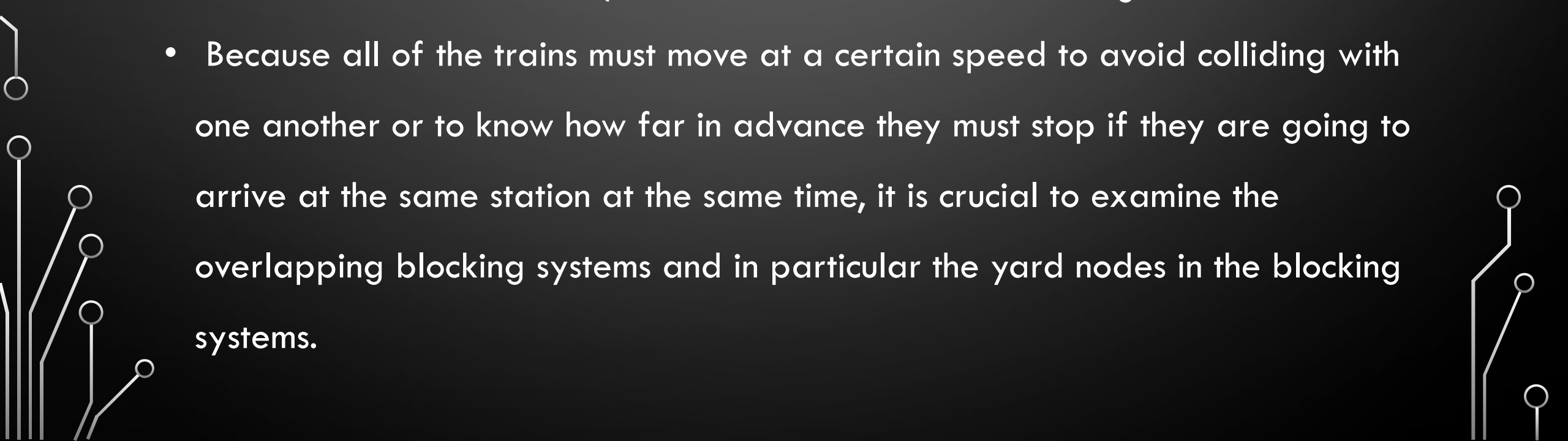
Train 2 : $v_1e_1v_2e_2v_3e_3v_5e_6 \dots v_{13}e_{29} \dots v_{53}e_{54}v_{54}e_{55}v_{55}$

- Moreover, to see the Piccadilly Line as a network system that blocks traffic.
- The Piccadilly Line's blocking network system is shown in following figure.



- The blue dotted box highlights the origin nodes (v_{28} and v_1), the yellow dotted box the yard nodes (v_{25} and v_{13}) that both trains pass through before arriving at their final destination or the station where Train 1 turns (v_{25}), and the purple dotted box the destination nodes of both trains. Overall, it depicts a local yard.. If we were to consider the whole London Underground, that would be a system yard.
- Note that another train line in the London Underground, the District Line, also visits v_{13} .
- If we were to refer back to Figure 11 and add that connection from the District Line, it would look like following Figure.


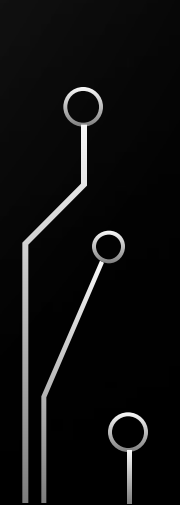


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- The District Line train represented by D1 has a d1 origin node and d2 destination node.
 - To distinguish between the two separate blocking systems—the Piccadilly Line blocking system and the District Line blocking system—these were depicted in blue.
 - Three trains travel via v13, as can be seen from the blocking network.
 - Because all of the trains must move at a certain speed to avoid colliding with one another or to know how far in advance they must stop if they are going to arrive at the same station at the same time, it is crucial to examine the overlapping blocking systems and in particular the yard nodes in the blocking systems.
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- In order for Train 1 (T1) to travel from v28 to v55, it takes approximately 1 hour and 41 minutes, and specifically it takes 37 minutes to travel from v28 to v13.
- In order for Train 2 (T2) travel from v1 to v55, it takes 1 hour and 33 minutes, and specifically it takes 29 minutes to travel from v1 to v13. Lastly, it takes D1 6 minutes to travel from d1 to v13.
- By just looking graphically at Piccadilly line , We can only view the Piccadilly Line and the Piccadilly Line blocking network by simply glancing at Figure.
- We can see overall that it is unlikely that the three trains would collide because there is less space and stations between d1 and v13 when we combine some of the District Line blocking system with the Piccadilly Line blocking system (particularly its yard nodes) together with time information.
- Despite the relative accuracy of arrival and departure schedules, trains might become stopped or break down at a station.
- As a result, in addition to looking at the yards of the other train lines in the blocking network that could be impacted, we need to analyze the blocking networks of many train lines.



❖ CONCLUSIONS

- When designing a railway system, especially in terms of safety, using graph theory is an effective technique.
 - We see Chinese postman problem and it's solution to find most optimal Chinese postman route. We created a graph of train line in the London Underground, specifically the Piccadilly Line utilizing the fundamental characteristics of graph theory.
 - In these line, we were then able to apply one of the design and safety problem (Yard location problem).Overall , the basic and complex properties in graph theory can be utilized and analyzed for design and safety purposes in the railroad system.
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