HW4 Due **Friday** February 8 – **All** on paper "by hand"

4.1 "Physics" – Making M equations for M unknowns, and setting up a problem in *Ax* = *b* form.

3 pts Your business plan uses a 3-D printer (that uses laser sintering of steel powder) to create four products: bolts, nuts, hinges, and door handles. To calculate the weight of powder required for each individual item, you have the following four bulk measurements:

- 120 bolts and 200 nuts require a total of 5160 grams,
- 40 bolts, 90 nuts, 16 hinges and 4 door handles require 4458 grams,
- 36 nuts, 4 hinges and 20 door handles require a total of 3084 grams,
- 1 bolt, 1 nut, 3 hinges and 2 door handles require a total of 651 grams.

Define the weight of powder (in grams) required for a single item as ***b*** (for 1 bolt), ***n*** (for 1 nut), ***h*** (for 1 hinge) and ***d*** (for 1 door handle).

a) Write each of the four bulk measurements above as an equation relating *b*, *n*, *h* and *d.* You should now have four equations for the four unknowns.

b) Write the equations from (a) in *matrix* form ***A x*** = ***b***. Remember: there's more than one way to define the ***x*** vector with the 4 unknowns, so clearly show me how you chose to define it.

** Do **NOT** solve for any of the unknowns! You're just writing out the equations from the problem statement, ready to solve "later".

4.2 "Physics" – Making M equations for M unknowns, and setting up a problem in *Ax* = *b* form.

3 pts When operating your 3D printing business, you find that the time (**T**, in minutes) to produce **N** number of hinges seems to match the following formula for unknowns ***x***, ***y*** and ***z***:

$$T = \left(x+z\right)N + \frac{\left(y-1\right)}{N} + z\,N^2$$

To determine the values of *x*, *y* and *z*, you ran three "time trials" with your printer and found:

- It takes 2 minutes to make 2 hinges,
- It takes 5 minutes to make 4 hinges,
- It takes 20 minutes to make 10 hinges.

a) Express your time trial data as 3 <u>linear</u> equations with the 3 unknowns *x*, *y* and *z*.

b) Write the equations from (a) in matrix form ***A x*** = ***b*** using the vector of unknowns ***x*** = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

c) Now *rewrite* the equations from (b) in matrix form using a different vector of unknowns ***x*** = $\begin{bmatrix} z \\ y \\ x \end{bmatrix}$ (*i.e.* with the unknowns *x* and *z* swapped).
How did the *A* matrix and *b* vector change from your answer in (b) (if at all)?

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4.3 "Order" of a process.

4 pts For each of the following processes (i) write the "order" of *n* with respect to *M* (*i.e.* write your answer as *n* = *O*(*… something involving M*)), and (ii) **use that order approximation** to estimate the requested times.

Process 1: $n = 40M + 2M^2$	If it takes 1 minute to finish a process with *M* = 100, estimate the time it takes to finish a process with *M* = 500.
Process 2: $n = 3\left(M^2+2\right)^2 + 7M^3$	If it takes 8 hours to finish a process with *M* = 1000, estimate the time it takes to finish a process with *M* = 500.
Process 3: $n = \log_{10} M + 0.1M$	If it takes 4 days to finish a process with *M* = 50, estimate the time it takes to finish a process with *M* = 500.

4.4 Establishing the "Order" of your own process.

3 pts Consider the process of multiplying two matrices *C* = *A B*, where *A* and *B* are general *M* x *M* matrices.

a) Derive the equation for the number of individual element-to-element multiplications *n* it takes to complete the process.

b) Use the "order" of the equation above (*i.e. n* as the "order" of *M* to some power) to answer the following question: given it takes 0.002 seconds to multiply two [100 x 100] matrices, how long should it take to multiply two [800 x 800] matrices?

4.5 "Math" (Gaussian Elimination with Partial Pivoting)

8 pts Solve the ***A x*** = ***b*** problem at right **by hand** using Gaussian Elimination with partial pivoting. *Show your work!!* Divide the process into (1) forward elimination and (2) backward substitution to solve for *j*, *a* and *x*. Be sure to follow the *exact* G-E steps from class (using pivoting, the multipliers, etc.) and write out all your decimals *exactly* (don't round).

Hint: Catch possible mistakes by comparing your answer to `x = A\b` in MATLAB!

$$\begin{bmatrix} 2 & 2.2 & -3.6 \\ 10 & 1 & -3 \\ -1 & -8.1 & 4.3 \end{bmatrix} \begin{pmatrix} j \\ a \\ x \end{pmatrix} = \begin{pmatrix} 10 \\ 15 \\ -21.5 \end{pmatrix}$$

4.6 P-L-U Factorization Method, and re-solving the problem for a new *b* vector

11 pts a) Use the results of your G-E process in 4.5 to manually construct the ***P - L - U*** matrices for the *A* matrix in the problem above. Show your work! You're being graded on demonstrating your

+4 understanding between the G-E process and the P-L-U matrices, but you can catch mistakes by comparing your work to the output from the MATLAB "**lu**" command, and/or checking *PA* = *LU*.

+7 b) Use your *P-L-U* factorization of *A* from (a) to "efficiently" solve (by-hand!) the *new* problem at right:

$$\begin{bmatrix} 2 & 2.2 & -3.6 \\ 10 & 1 & -3 \\ -1 & -8.1 & 4.3 \end{bmatrix} \begin{pmatrix} c \\ u \\ d \end{pmatrix} = \begin{pmatrix} 4 \\ -10 \\ -39 \end{pmatrix}$$

The point of (b) is to recognize the new problem has the same *A* matrix, so you must NOT go through the entire G-E process again. Just use the P-L-U s from part (a) to more quickly solve *A x* = *b* for the new *b* vector.