HW11 All problems due (on paper) **FRIDAY** April 5

11.1 Application: Finite Difference Rules

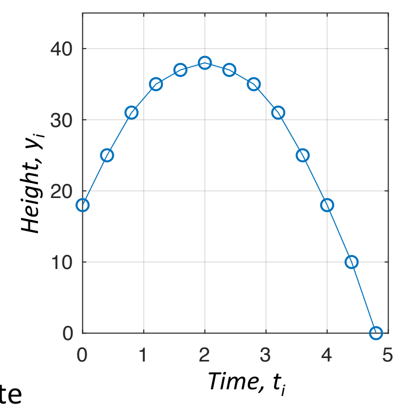
2 pts This is the equation for the Taylor Series expansion of $f(x+\delta)$, as a function of $f(x)$ and all its derivatives at x, that will be given to you on your midterm and exam cheat-sheets:

$$f(x+\delta) = f(x) + \delta f'(x) + \frac{\delta^2}{2} f''(x) + \frac{\delta^3}{3!} f'''(x) + \frac{\delta^4}{4!} f''''(x) + \cdots$$

Start with that equation and use it to <u>derive</u> the expression for $\boldsymbol{f(x_i - 3\Delta x)}$ (*i.e.* $f(x)$ evaluated three nodes to the left of x_i) as a function of $f(x_i)$ and all the derivatives at x_i up to the **SIXTH** derivative.

11.2 Application: Finite Difference Rules

7 pts At right are 13 measurements of height y_i (m) of a rocket as a function of time t_i (sec). Assume all the points are accurate and don't need any "smoothing" first.



t_i	y_i
0	18
0.4	25
0.8	31
1.2	35
1.6	37
2	38
2.4	37
2.8	35
3.2	31
3.6	25
4	18
4.4	10
4.8	0

a) Use the order $(\Delta t)^2$ *forward* difference rule to calculate the initial velocity (dy/dt) of the rocket at $t = 0$.

b) Use the order $(\Delta t)^2$ *central* difference rule to calculate the velocity (dy/dt) of the rocket at $t = 4$ seconds.

c) Use the order $(\Delta t)^2$ *backward* difference rule to calculate the velocity (dy/dt) of the rocket when it hits the ground (at $t = 4.8$ seconds).

d) Use the order $(\Delta t)^2$ *central* difference rule to calculate the acceleration (d^2y/dt^2) of the rocket at $t = 4$ seconds.

e) Knowing from physics that the *exact* acceleration of the rocket must be $g = -9.8$ m/s^2, what is the *error* in your approximation from (d)?

f) IF you only had measurements every **1** second (instead of 0.4 seconds, like in the table of data), and used *that* hypothetical data instead to re-calculate (d), quantify how much you expect your error in calculating the acceleration to **change**.

11.3 Concepts: Error Order and Precision

7 pts The following is a 5-point backward difference scheme, over equally-spaced x_i, for df/dx at $x = x_i$:

$$f'(x_i) \approx \frac{1}{12\Delta x}\left(25 f_i - 48 f_{i-1} + 36 f_{i-2} - 16 f_{i-3} + 3 f_{i-4}\right)$$

Write out Taylor Series expressions for each of the four f_{i-4}, f_{i-3}, f_{i-2}, f_{i-1} to the **SIXTH** derivative, like you did in 11.1, and then combine them using the difference scheme above to …

a) Calculate the discretization **error order** (*i.e.* write the error = $O(\Delta x^p)$ for some integer p).

b) Calculate the **precision** of the scheme.

Hint: **DON'T** write a Taylor Series for f_i. f_i is just itself (or just written as $f(x_i)$). There's nothing else you can do with it. Only ever create Taylor Series expansions for points *other* than f_i.

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11.4 Application: Finite Difference Rules

3 pts a) Apply the 5-point differencing scheme from 11.3, using nodes with spacing Δx = 0.5, to calculate the derivative (f') of the following function at x = 2:

$$f(x) = e^{(x^2)}$$

Be sure to write out all your nodes, and carry all calculations to at least ***4*** *decimals.*

b) Calculate the percent error in your calculation from (a). (Note that you'll have to first *analytically* evaluate $f'(1)$.)

11.5 DERIVING difference formula for ANY order derivative to ANY error order

7 pts Use Taylor series expansions to <u>derive</u> a *central* difference scheme, over equally-spaced points, using any or all of f_{i-2}, f_{i-1}, f_i, f_{i+1} and f_{i+2} that approximates $f''''(x_i)$ (**4th** derivative at x_i) to order (Δx^2) discretization error.