HWB6B Due on **Friday** February 22

6.4 Non-Polynomial Models ("Transforming" the Data)

x_i	y_i
2	6
3	5
4	3
5	1

The four data points at right are measurements of the amount of mustard (y_i) left behind in a container as a function of the force (x_i) you squeezed the bottle. You want to calibrate an analytic model $y = f(x)$ for future mustard-container analysis. You're considering the following five models for $f(x)$:

i. $f(x) = a + be^x$ for two unknown coefficients ***a*** and ***b***.

ii. $f(x) = \frac{1}{2} P x^Q e^{Sx}$ for three unknown coefficients ***P***, ***Q*** and ***S***.

iii. $f(x) = ux^2 + wx + 1$ for two unknown coefficients ***u*** and ***w***.

iv. $f(x) = \dfrac{a}{1+kx}$ for two unknown coefficients ***a*** and ***k***.

v. $f(x) = X_0 + V_0 x + \frac{1}{2}(9.8)x^2$ for two unknown coefficients $\boldsymbol{X_0}$ and $\boldsymbol{V_0}$.

For EACH model (i) through (v), complete both tasks below:

(**A**) Write the problem out in matrix form $r = y - Ac$. Be sure to **explicitly** …

- show me how you're analytically "transforming" the model (if necessary),
- define the vector ***c*** in terms of the unknown(s) in the model,
- write out the values of **all the elements** in the vector ***y*** and matrix ***A*** using the provided data.

(**B**) Use MATLAB to evaluate the *least-squares* best-fit *c* using the command `c = A \ y` …

- evaluate the **vector** *c*, and all the **unknown coefficients** in the model (write each value to at least 4 significant figures),
- substitute the coefficients back into the general form to write out the **final equation** for the best-fit model $f(x)$.

6.5 Non-Polynomial Models (using `polyfit`)

For EACH of the five models above, show me how (<u>IF POSSIBLE</u>) you can get the unknown coefficients directly from using the command `P = polyfit(X,Y,n)` (thereby skipping all the steps of creating the matrix form). That is,

- Write out explicitly what you use for vectors `X` and `Y`, and your choice for the value of `n`.
- Also show explicitly how you use the output `P` to give you the required unknown(s).

If you don't think you can use `polyfit` for that particular model, then say so explicitly.

Everything above is to be written out *on paper*. You're using MATLAB to solve for $c = A \setminus y$, but just write values out. Don't submit your answers on Carmen.

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6.6 Trying to correlate a random data set!

Load the ***y*** and ***t*** data provided in the file HW6data.mat into MATLAB (type: `load HW6data`). Make a quick plot of ***y*** versus ***t*** (*i.e.* `plot(t,y,'o')`) to see what it looks like, then write a code called `HW6_6.m` with commands that do ALL of the steps (a) through (f) below. That is, I should be able to run your code and it would automatically create all the best-fit models, evaluate the requested values, and make all the plots.

a) Determine the **1st-order** polynomial, $f_1(t) = a_1\, t + a_0$, best-fit of the data.

b) Calculate the R^2 value of the least-squares best fit model $f_1(t)$. (*Hint*: you might find the built-in command `mean(Y)` helpful).

c) Determine the **2nd-order** polynomial, $f_2(t) = a_2\, t^2 + a_1\, t + a_0$, best-fit of the data.

d) Calculate the R^2 value of the 2nd-order least-squares best fit $f_2(t)$.

e) Plot the original (t,y) data as circles, *i.e.* `plot(t,y,'o')`, overlaid with lines of the 1st-order best-fit model $f_1(t)$ <u>and</u> the 2nd-order model $f_2(t)$ on the same plot. Make the line for $f_1(t)$ in **blue** and the line for $f_2(t)$ in **red**. Save this plot as `PLOT6A.pdf`.

f) Plot the residual errors, *i.e.* `plot(t,r,'o')`, for the residual error vector of the 2nd-order model, $\underline{r} = \underline{y} - f_2(\underline{t})$. Save this plot as `PLOT6B.pdf`.

Do **not** hand in anything on paper for 6.6. Everything below must be submitted on **Carmen**:

- Upload your <u>*documented*</u> code `HW6_6.m`, and both plots `PLOT6A.pdf` and `PLOT6B.pdf`.
- Write "1st-order fit", followed by the coefficients a_1, a_0 and the R^2 value of the best-fit model $f_1(t)$ in the comment box.
- Write "2nd-order fit", followed by the coefficients a_2, a_1, a_0 and the R^2 value of the best-fit model $f_2(t)$ in the comment box.
- Use the results from (b) and (d) – (f) to comment explicitly on the "goodness" of the final fit $f_2(t)$ in the comment box. What **exactly** does the shape & distribution of residuals, and both of the R^2 values tell you? (Be succinct, specific and mathematical in your write-up using ideas from class, not a vague essay. Think about how this problem brings together the "engineering step" ideas from class.)