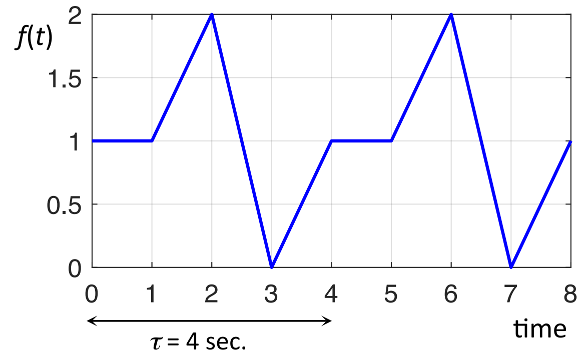
HW8B (8.4 – 8.7) due Friday March 8 – On paper *and* on-line.

8.4 Fundamentals: POINTS to use in a Discrete Fourier Analysis, and COEFFICIENTS to calculate

5 pts Remember: when picking points you need to pick an **EVEN** number of points, starting at $\boldsymbol{t = 0}$, which **end ONE Δt** before the period. Other than that, there's really no other rules (in this course).

Look at the function at right, with the smallest period t = 4 seconds, and consider picking **8 points** to do a Fourier analysis.



a) Just write out the **values** of all the times t_j and function f_j you would use in the Fourier Analysis.

b) What are the values of $\boldsymbol{\Delta t}$ and $\boldsymbol{N}$ for this problem?

c) List the **names** of all the non-trivial Fourier coefficients (*A*s & *B*s) that you would evaluate.

- *Don't* actually do the Fourier analysis or calculate values for any coefficients;
  I just want a <u>list</u> of coefficient names (like "B_0") you would *plan* on calculating "later".
- *Don't* include "trivial" coefficients which you already know are always zero (and wouldn't even attempt calculations for).

Due **on paper**: **Everything**.
Due on-line: Nothing.

Get off the railroad tracks!

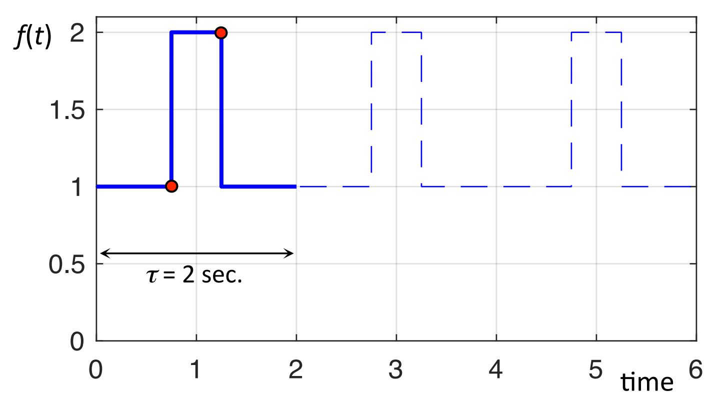
True story – I once flew into Calgary with friends and got a car rental at midnight. New to the city, we got a little lost, and with no traffic downtown I accidentally turned down a ramp and ended up driving below the city roads on their light-rail transit line. Ah! The things you do with rental cars.

Clearly, it's important we study the impact of driving on bumpy railway ties.

The following 3 HW questions walk you through an investigation of the sinusoids that make up the motion of driving over railway ties. For $f(t)$ we'll use one period ($\tau = 2$) of the "pulse" function drawn below

An important feature are the discontinuities at $t = ¾$ and 1¼ seconds. For all your subsequent analyses assume that $f(0.75)$ = **1** (not 2) and $f(1.25)$ = **2** (not 1), like the red dots and the formula below indicate.

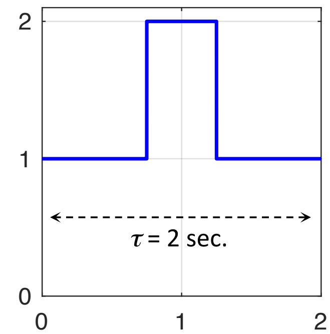
$$f(t) = \begin{cases} 1 & 0 \le t \le 0.75 \\ 2 & 0.75 < t \le 1.25 \\ 1 & 1.25 < t \le 2 \end{cases}$$



Use this function for the next three problems:

- 8.5 gets you to visualize the *analytic* Fourier coefficients (given to you).
- 8.6 gets you to do a 4-point *discrete* Fourier transform of this function, *by-hand*.
- 8.7 gets you to do a DFT with a *large* number of points using MATLAB's built-in `fft` command.

HW8B (8.4 – 8.7) due Friday March 8 – On paper *and* on-line.



8.5 Analytic Fourier Decomposition - VISUALIZATION.

3 pts The Analytic **Fourier series** of $f(t)$ over one period ($\tau = 2$) is given by:

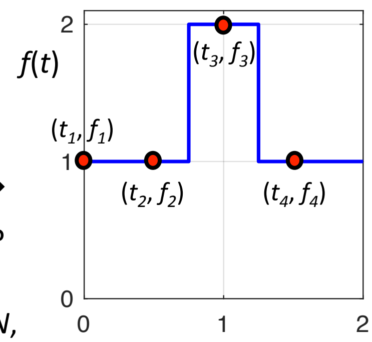
$$f(t) = B_0 + \sum_{k=1}^{\infty}\left[A_k \sin\left(\frac{k\pi t}{L}\right) + B_k \cos\left(\frac{k\pi t}{L}\right)\right] \quad \text{where}$$

B_0 = *something*, $B_k = -\dfrac{2}{k\pi}\sin\left(\dfrac{3k\pi}{4}\right)$ and $A_k = 0$ for all $k \geq 1$.

a) Analytically determine the value of B_0.

b) Use MATLAB to compare the **exact** pulse $f(t)$ with an **approximation** of $f(t)$ over one period using only the Fourier terms up to k = 20. Use a finely-spaced time vector of 6,000 points between 0 and 2 seconds (*i.e.* `t=linspace(0,tau,6000)`). Plot your exact $f(t)$ in **solid black**, your final approximated $f(t)$ in **dashed-red**, and save your plot as HW8_5.pdf with your name in the title at the top (*e.g.* use a command like `title('HW8.5 Dirk Pitt')`).

Due **on-line**: The plot **HW8_5.pdf**, and your answer to "**B_0 = …**" in the comment box. (No script.)
Due on paper: Nothing.



8.6 Discrete Fourier Decomposition – *By Hand*

7 pts You now sample the same pulse $f(t)$ at 2-Hz (i.e. Δt = 0.5), starting with $f_1 = 1$ at $t_1 = 0$, and getting four discrete points (t_j, f_j) over one period →

a) Write out the **values** (t_j, f_j) of these four points. What's ***N*** and **2*N***?

b) Use the four points in (a) to calculate (by-hand) the DFT for $f(t)$:
   i. Calculate all the Fourier coefficients A_k and B_k, for $0 \leq k \leq N$,
   ii. Substitute the values of the coefficients into the discrete Fourier Series representation of the pulse function $f(t_j) = B_0 + \Sigma$(… etc. …), and express in a simplified form.

That's it! You've done the by-hand analysis! Now let's check that against MATLAB:

c) Use the built-in MATLAB function `fft` to re-calculate the Discrete Fourier coefficients for the same four points. Write out on paper exactly how you used `fft`:
   i. what vectors you created as input to the `fft` function,
   ii. how you called `fft`,
   iii. exactly what your MATLAB output from `fft` was (I want you to write out **all** four complex values from MATLAB),
   iv. and finally, exactly how you used that output to get all the relevant A_k and B_k.

Hint: There should be ***NO*** difference between the coefficients calculated in (b) and (c). If there are, you're either having difficulty calculating the coefficients correctly by-hand, or else misinterpreting the output of `fft`. Go back and fix the problem. A good idea to confirm you're using `fft` correctly is to maybe first try it on the known 4-pt example from class.

The whole point of part (c) is to make sure you fully understand how to use and interpret the built-in `fft` command before moving on to the last (more complicated) question, where you don't know the "by-hand" coefficients.

Due **on paper**: **All parts** (a, b, c).
Due on-line: Nothing.

HW8B (**8.4 – 8.7**) due Friday March 8 – On paper *and* on-line.

8.7 Discrete Fourier Decomposition – *using MATLAB*.

7 pts Go back to the "railway track" pulse $f(t)$ in 8.5, except this time sample it at a higher frequency of 100 Hz (*i.e.* $\Delta t = 1/100$), over $0 \le t < 2$ s. This is too much to do DFT calculations "by-hand", so you now want to use MATLAB `fft` to do a discrete Fourier transformation of $f(t_j)$ over one period.

Before you start, consider the concepts: what is N and $2N$ for this problem? Exactly how many points (t_j, f_j) should you include in your discrete Fourier analysis? Are these meeting the rules/ assumptions from class? You should probably sketch out on paper what you're trying to do (I'm not asking that to be handed in though).

Develop a *documented* MATLAB script **HW8_7.m** that does the following:

a) Creates vectors of the appropriate data points (t_j, f_j) for $1 \le j \le 2N$. Call these vectors `tj` and `fj`.

b) Appropriately calls the `fft` command.

c) Uses the output from (b) to create two vectors:

  i. `Ak`, with length N, containing all the A_k coefficients for $1 \le k \le N$,

  ii. `Bk`, with length N+1, containing all the B_k coefficients for $0 \le k \le N$.

d) Creates a "stem" plot of the coefficients, similar to the bottom of our "Class22_SquareWave" handout, with all the A_k to the left, and B_k to the right. Save this plot as **HW8_7stem.pdf**.

e) Reconstructs an approximation of $f(t)$ over one period using all these calculated A_k and B_k Fourier terms (*i.e.* does an "inverse transform"). The trick is that I want you to use the finely-spaced time vector, like in 8.5 (*i.e.* `t=linspace(0,tau,6000)`).

f) Makes a plot, called **HW8_7ift.pdf**, similar to the top of our "Class22_SquareWave" handout, comparing this approximation of $f(t)$ with the exact force $f(t)$ over one period. Plot your exact $f(t)$ in solid black, your approximated $f(t)$ in dashed-red, and all your discrete points as red dots, say, with a command like:

```
plot(t,f_exact,'-k', t,f_approx,'--r', tj,fj,'or')
```

This plot is a great check to see you created and interpreted all the Fourier coefficients properly because the two traces should be somewhat similar, and your inverse transform (`f_approx`) should exactly pass through all your discrete (t_j, f_j) points.

g) Look at your results for B_k and get the value for B_0. Notice that you've now calculated B_0 for this same motion $f(t)$ *three* different ways: (i) analytically in 8.5, (ii) by-hand using 4 points (sampled at 2-Hz) in 8.6, and now (iii) with MATLAB using many more points sampled at 100-Hz. In the Carmen comment field, **enter your values for B_0** from 8.6 **and** 8.7 (to 4 decimals), and **comment** as to which one is more accurate (compared to the analytic value).

Due **on-line**: your *documented* script **HW8_7.m**, <u>two</u> plots **HW8_7stem.pdf** and **HW9_7ift.pdf**, and your answer to part (g) "**B_0 = ... (8.6) and ... (8.7)**" and comment on their accuracy in the comment box.