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Group Number: 02

Group Members: 3

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Statement of integrity: By typing the names of all group members in the text box below, you confirm that the assignment submitted is original work produced by the group (*excluding any non-contributing members identified with an "X" above*).

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Use the box below to explain any attempts to reach out to a non-contributing member. Type (N/A) if all members contributed.

N/A

** Note, you may be required to provide proof of your outreach to non-contributing members upon request.*

1(a) American Call Option Pricing

Stock price upper movement (u) is set as follows:

u = (1.10 + Group Number /100) = 1.10 + 2/100 = 1.12

Stock price downward movement (d) is d = 1/u and the initial stock price is S_0 = 95. The strike price is K=90.

Upward movement risk-neutral probability is given by:

$$\mathbb{P}^* = \frac{1-d}{u-d} = \frac{25}{53} = p^*$$

Downward movement risk-neutral probability is given by:

$$q = 1 - p^* = \frac{28}{53}$$

Binomial tree generated using R code.

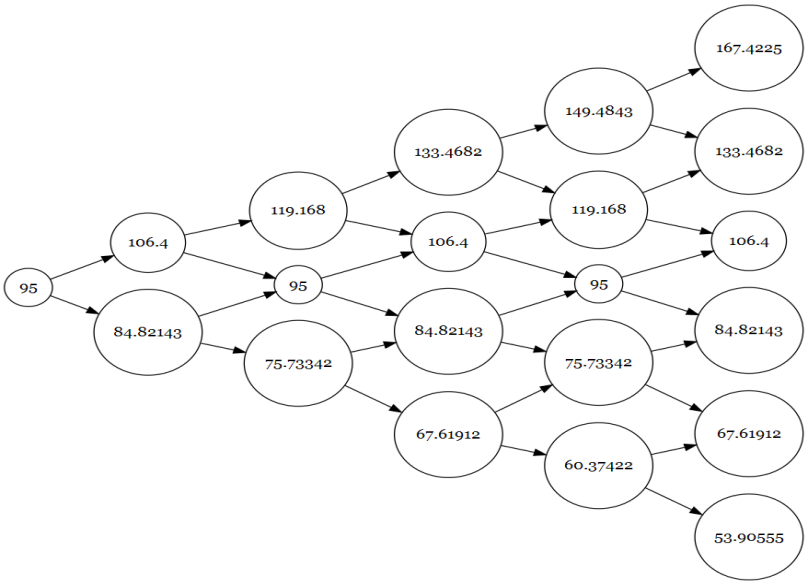

Figure 1.1: Stock Price Evolution

Value of the derivative at each node

Below is a plot generated using R code and it shows the derivative price for all sample paths.

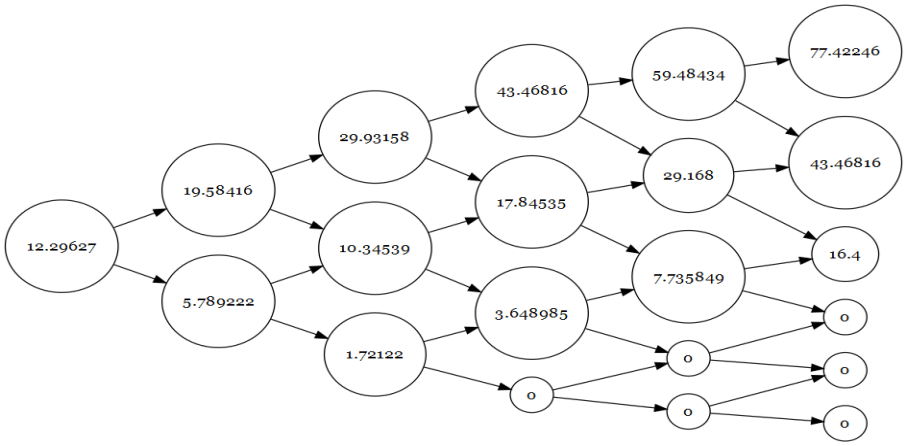

Figure 1.2: American Call option Price Values

1(b) Benefit from American Call Option early exercise

For non-dividend paying stock $\mathbf{S_t}$, an early exercise of an American Call Option is never optimal, hence intuitively it is not beneficial to have an early exercise of an American Call Option. Exercising an American Call Option requires the holder to pay a strike price **K**. Holding some cash **K** until expiration of the option at time **T** will result on some interest being realized on the cash amount **K.**

We consider two portfolios:

Portfolio **A**: One American call option (**C**) and some cast $\boldsymbol{Ke^{-r(T-t)}}$

Portfolio **B**: One share $\boldsymbol{S_t}$

Let **r** be the interest rate and **τ** the exercise time.

If **τ < T** then we have

$$\boldsymbol{A = (S_t - K) + Ke^{-r(T-\tau)} < S_t = B}$$

If **τ = T** then we have

$$\boldsymbol{A = max\{S_T - K, 0\} + K = max\{S_T, K\} \geq S_T = B}$$

Thus, for all times it is not ideal make an early exercise. As the price of the stock rises so is the premium and the call price as indicated in the call price evolution diagram.

1(c) It is not always the case because a significantly large upward jump in the stock price before expiration of the option will present a beneficial early exercise opportunity.

2(a) American Put Option Pricing

Value of the derivative at each node

Below is a plot generated using R code and it shows the derivative price for all sample paths.

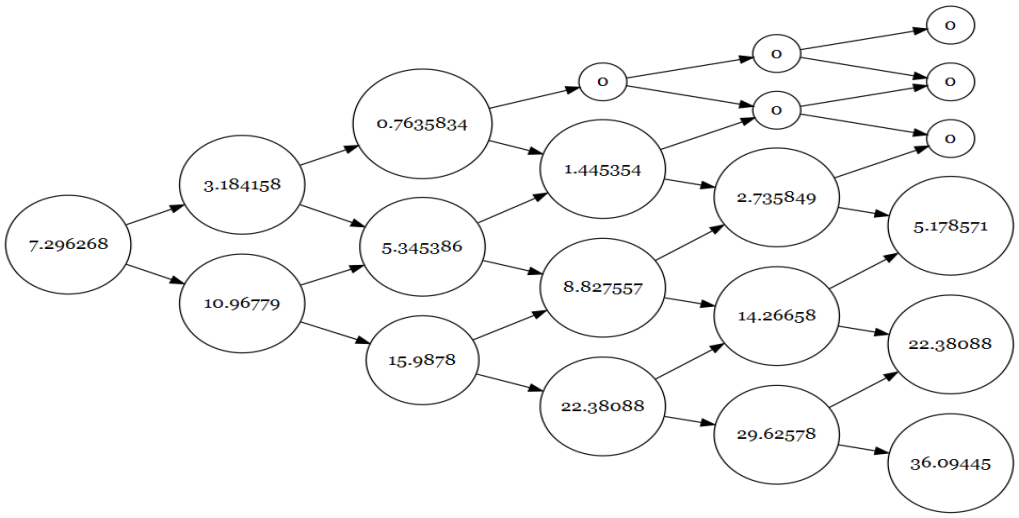

Figure 2.1: American Put option Price Values

3. Pricing Exotic Options

3(a) Pricing an Up-and-Out (UAO) European Call Option

Initial Stock price S_0 = \$100; Up-and-out barrier L = \$130

From R code we have:

Up-and-Out (UAO) European Call Option = \$ 3.222177

3(b) From the R code we have:

European Call price = \$ 9.894281

UAO European call price = \$ 3.222177

Thus, the European Call is more expensive.

3(c) An advantage of the up-and-out European call option is that it provides the investor with additional protection or leverage. A rebate is added to the contract specification, which is a coupon paid to the holder of the option in case the barrier is breached.

3(d) In-Out Parity for barrier options

Knock-In (K, T, B) + Knock-out (K, T, B) = Vanilla (T, B)

Thus, Knock-In (K, T, B) = Vanilla (T, B) - Knock-out (K, T, B)

UAI European call price = European Call price - UAO European call price = 9.894281 - 3.222177

UAI European call price = $ 6.672104

4. Market completeness

4(a) Pricing an American Call Option or an American Put Option using a binomial tree result in a complete market in the sense that the price path for all states of the world is known and available information in the market is regarded as perfect information. Hence the market will be complete if the condition of the put-call parity is satisfied.

4(b) Pricing a European UAO call using a binomial tree result in a complete market in the sense that the price path for all states of the world is known and available information in the market is regarded as perfect information.

4(c) Consider the system of equations: $Ax = b$

A is the payoff matrix of the basis assets with A_{ij} being the payoff of the j^{th} asset in the i^{th} state.

x is a column vector of basis portfolios where x_j is the amount held in the j^{th} portfolio.

b denotes payoff of a focus asset with b_j being the payoff of the j^{th} focus asset.

$j = 1,2, \ldots \ldots, n$ and $i = 1,2, \ldots \ldots m$

Thus, we have **n** assets in the portfolio and **m** states of the world. Markets are complete if $rank(A) = m$

4(d) A portfolio of assets denotes a complete market if a price is available for each asset under all possible states of the world under negligible transactional costs.

References

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