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In this paper we will discuss about Exotic Options. Given the nature of Exotic Options they are not OTC traded rather than in Exchange. Exotic products come into existence because of several reasons, tax, accounting, legal or to reflect a view on potential future movements in particular market variables. We will look at three flavours of exotic options. Look back, Barrier and Compound Options .

I. Lookback Options

An exotic option gives the buyer of the option to exercise the option optimally depending on maximum or minimum price attained by the underlying asset during the option's lifetime. The payoff depends on the optimal (maximum or minimum) underlying asset's price occurring during the life of the option. The option allows the holder to "look back" over time to determine the payoff. However, the lookback option's strong reduction to risk exposure comes with a sufficiently high premium.

Unpredictability is therefore the prime feature of an asset that would drive demand and usage for a lookback option based upon that asset. Lookback options benefit the holder given greater volatility in such a market. Thus, it is an effective instrument for hedging against large price movements and reducing the risk of destabilising events that may cause markets to either rise or fall. These include political election outcomes or other unforeseen chance occurrences such as market anomalies. Also among its potential uses is to hedge exposure in cryptocurrency markets, where extreme volatility has increasingly become a menace as investors look for ways to mitigate such risk.In 1982, the Mocatta Metals Corporation issued one of the first “lookbacks,” that allowed a trader to buy gold at the lowest price attained within a period.

There exist two kinds of lookback options: with floating strike and with fixed strike.

Lookback with floating Strike

A floating strike lookback option with its strike price set equal to the optimal value that is achieved by the underlying asset over the option´s life. In the case of a call, that optimal value is the lowest value achieved by the underlier during the life of the option, so it pays off the difference between the final value of the underlying asset and that lowest value. In the case of a put, the option pays off the difference between the highest value achieved and the value of the underlying asset at expiration.

The valuation of a European lookback call at time zero is

$$S_0 e^{-qT} N(a_1) - S_0 e^{-qT} \frac{\sigma^2}{2(r-q)} N(-a_1) - S_{\min} e^{-rT} \left(N(a_2) - \frac{\sigma^2}{2(r-q)} e^{Y_1} N(-a_3) \right)$$

$$a_1 = \frac{\ln(S_0/S_{\min}) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}},$$
$$a_2 = a_1 - \sigma\sqrt{T}$$
$$a_3 = \frac{\ln(S_0/S_{\min}) + (-r + q + \sigma^2/2)T}{\sigma\sqrt{T}}$$
$$Y_1 = -\frac{2(r - q - \sigma^2/2)\ln(S_0/S_{\min})}{\sigma^2}$$

here
S_{min} is the minimum asset price achieved to date.

The value of the European lookback put is

$$S_{\max}e^{-rT}\left(N(b_1) - \frac{\sigma^2}{2(r-q)}e^{Y_2}N(-b_3)\right) + S_0e^{-qT}\frac{\sigma^2}{2(r-q)}N(-b_2) - S_0e^{-qT}N(b_2)$$

where

$$b_1 = \frac{\ln(S_{\max}/S_0) + (-r + q + \sigma^2/2)T}{\sigma\sqrt{T}}$$
$$b_2 = b_1 - \sigma\sqrt{T}$$
$$b_3 = \frac{\ln(S_{\max}/S_0) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}}$$
$$Y_2 = \frac{2(r - q - \sigma^2/2)\ln(S_{\max}/S_0)}{\sigma^2}$$

S_{max} is the maximum asset price achieved till date.

The option's strike price is floating and determined at maturity.

Lookback with Fixed Strike.

As for the standard European options, the option's strike price is fixed. The difference is that the option is not exercised at the price at maturity.A fixed strike lookback option has a strike price set in advance. Its exercise depends on the optimal value achieved by the underlying asset during the life of the option. In the case of a call, the optimal value is the highest value the underlier achieves, so the call pays off the difference between that value and the strike price, if positive, and zero otherwise. In the case of a put, the optimal value is the lowest value achieved by the underlying asset and the put pays off the difference between the strike price and that lowest value, if positive, and zero otherwise.

II. Barrier Options

Barrier options are the options where the payoff depends on whether the underlying asset's price reaches a certain level, known as a barrier(K) during a certain period of time. also considered a type of path-dependent option because their value fluctuates as the underlying value changes during the option's contract term. In other words, a barrier option's payoff is based on the underlying asset's price path. Barrier Options have two trigger prices, strike and knock out levels. The option becomes worthless or may be activated upon the crossing of a price point barrier. Barrier options are typically classified as either knock-in or knock-out.

Barrier Options are appealing to some market participants as they are less expensive compared to regular options.

Knock in Barrier Option - A knock in option comes into existence only when the underlying asset reaches a certain barrier. Knock-in barrier options are further classified into **up-and-in or down-and-in** options. In an up-and-in barrier option, the option contract starts only when the price of the underlying asset exceeds the predetermined price barrier. Conversely, if it is a down-and-in barrier option, it turns valid as the underlying asset value drops below the initially set barrier price.

Knock Out Barrier Option - A knock Out option ceases to exist when the underlying asset reaches a certain barrier. Knock-out barrier options can also be further decomposed into up-and-out or down-and-out options.

An **up-and-out** option stops existing when the underlying security moves above the barrier that was set above the initial price of the underlying security.A **down-and-out** option stops existing when the underlying security moves below the barrier that was set below the initial price of the underlying security. If an asset underlying the barrier option strikes the barrier anytime during the option's life, the option is terminated or knocked out.

If H is greater than or equal to the strike price K, the value of a down and out call is given by

$$c_{do} = S_0 N(x_1) e^{-qT} - K e^{-rT} N(x_1 - \sigma\sqrt{T}) - S_0 e^{-qT} (H/S_0)^{2\lambda} N(y_1) + K e^{-rT} (H/S_0)^{2\lambda-2} N(y_1 - \sigma\sqrt{T})$$

$$c_{di} = c - c_{do}$$

$$x_1 = \frac{\ln(S_0/H)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}, \qquad y_1 = \frac{\ln(H/S_0)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

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III. Compound Options.

Compound Options are options on options. Compund Options are exotoc options for which underlying security is an option. The inherent idea is, the purchaser of a compound option acquires the right, but not the obligation to buy or sell underlying option for a predetermined premium. They can have four variations involving the vanilla call and put options as underlying instruments.

- Call on a call.

A call on a call is a type of exotic option that gives the holder the right to buy a call option on the same underlying. Essentially, a call on a call option is an option to buy an option. It will have two strike prices(K1 and K2) and two expiration/exercise dates(T1,T2). One is for the compound call option, and the other is for the underlying vanilla option. A call on a call can be beneficial to an investor if it is offered at an optimal price. The holder of the secondary call has the right but not the obligation to buy a plain vanilla call.

- Put on a call

If the option owner exercises the put option they will be short a call option, which is an option that gives the owner the right but not the obligation to buy a specific asset at a set price within a defined time period.

- Call on a put.

If the option owner exercises the call option, they receive a put option, which is an option that gives the owner the right but not the obligation to sell a specific asset at a set price within a defined time period.

- Put on a put.

A put on a put is an options contract that gives the holder the right to sell an underlying put options contract. The put on a put trade is one of four types of compound options.Essentially, a put on a put option is an option to sell an option. The underlying asset of the put on a put option is the original option. Put on a put options are more common on European exchanges than in the United States.

Compound Options are good for covering risks that might not materialise in future. It is common for a company to tender for an export contract with the price quoted in a foreigm currency, the company can expect foreign currency receipts and can therefore hedge this contingent exposure with the forward contract. However, if the bid is unsuccessful, a commitment to forward contracts could prove expensive if the foreighn currency subsequently rises in value. Currency Options may be alternate but need significant upfront payments to cover risks that might not materialise.

Pricing of a compound Option

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European Call on Call Option

$$S_0 e^{-qT_2} M(a_1, b_1; \sqrt{T_1/T_2}) - K_2 e^{-rT_2} M(a_2, b_2; \sqrt{T_1/T_2}) - e^{-rT_1} K_1 N(a_2)$$

$$a_1 = \frac{\ln(S_0/S^*) + (r - q + \sigma^2/2)T_1}{\sigma\sqrt{T_1}}, \qquad a_2 = a_1 - \sigma\sqrt{T_1}$$

$$b_1 = \frac{\ln(S_0/K_2) + (r - q + \sigma^2/2)T_2}{\sigma\sqrt{T_2}}, \qquad b_2 = b_1 - \sigma\sqrt{T_2}$$

M is a cumulative bivariate normal distribution function. S* is asset price at time T1 for which the option price at time T1 is K1

European Call on put

$$K_2 e^{-rT_2} M(-a_2, b_2; -\sqrt{T_1/T_2}) - S_0 e^{-qT_2} M(-a_1, b_1; -\sqrt{T_1/T_2}) + e^{-rT_1} K_1 N(-a_2)$$

European call on put

$$K_2 e^{-rT_2} M(-a_2, -b_2; \sqrt{T_1/T_2}) - S_0 e^{-qT_2} M(-a_1, -b_1; \sqrt{T_1/T_2}) - e^{-rT_1} K_1 N(-a_2)$$

European put on put.

$$S_0 e^{-qT_2} M(a_1, -b_1; -\sqrt{T_1/T_2}) - K_2 e^{-rT_2} M(a_2, -b_2; -\sqrt{T_1/T_2}) + e^{-rT_1} K_1 N(a_2)$$

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Part B

Hindsight Option : Formulas covered in section 1 alongside the explanation.

LookBack Option with Floating Strike.

Time	0.08
Risk Free Rate	0.73% 10 Yr Treasury as of Jun 30 , 2019
Vol	12.5%
VIX as on Jun 30 2019	30.43
Min VIX	11.54
Call	**18.89**
Put	**19.41**

Lookback Option with Fixed Strike.

Time	0.08
Risk Free Rate	0.73% 10 Yr Treasury as of Jun 30 , 2019
Vol	12.5%
Spot VIX as on Jun 30 2019	30.43
Min VIX	11.54
Max VIX	82.69
Call	**52.23**
Put	**18.88**

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