RISK MANAGEMENT

GROUP WORK PROJECT: 2

Forward Algorithm

Pseudocode

the pseudocode versions for the forward algorithm , shown below:

Input: $\boldsymbol{A},\ \psi_{1:N},\ \boldsymbol{\pi}$

$[\boldsymbol{\alpha}_1,\ C_1] = \text{normalize}(\psi_1 \odot \boldsymbol{\pi})$;

for $t = 2 : N$ **do**

$\quad [\boldsymbol{\alpha}_t,\ C_t] = \text{normalize}(\psi_t \odot (\boldsymbol{A}^\top \boldsymbol{\alpha}_{t-1}))$;

Return $\boldsymbol{\alpha}_{1:N}$ and $\log P(X_{1:N}) = \sum_t \log C_t$

Sub: $[\alpha,\ C] = \text{normalize}(\boldsymbol{u})$: $C = \sum_j u_j$; $\alpha_j = u_j / C$;

Figure 2 - Forward Algorithm Pseudocode (Version 1)
Source: Adapted from Alperen Degirmenci [16]

Toy Example

Following is a toy example based on the hmms library (forward algorith) [19][20]:

In [2]:

```
pip install hmms
```

```
Collecting hmms
  Downloading hmms-0.2.3.tar.gz (524 kB)
     ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 524.8/524.8 kB 12.4 MB/s eta 0:00:00
  Preparing metadata (setup.py) ... done
Requirement already satisfied: Cython in /usr/local/lib/python3.10/dist-packages (from hm
ms) (0.29.36)
Requirement already satisfied: NumPy in /usr/local/lib/python3.10/dist-packages (from hmm
s) (1.23.5)
Requirement already satisfied: ipython in /usr/local/lib/python3.10/dist-packages (from h
mms) (7.34.0)
Requirement already satisfied: matplotlib in /usr/local/lib/python3.10/dist-packages (fro
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ms) (1.5.3)
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s) (1.10.1)
Requirement already satisfied: setuptools>=18.5 in /usr/local/lib/python3.10/dist-package
s (from ipython->hmms) (67.7.2)
Collecting jedi>=0.16 (from ipython->hmms)
  Downloading jedi-0.19.0-py2.py3-none-any.whl (1.6 MB)
     ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 1.6/1.6 MB 57.5 MB/s eta 0:00:00
Requirement already satisfied: decorator in /usr/local/lib/python3.10/dist-packages (from
ipython->hmms) (4.4.2)
Requirement already satisfied: pickleshare in /usr/local/lib/python3.10/dist-packages (fr
om ipython->hmms) (0.7.5)
Requirement already satisfied: traitlets>=4.2 in /usr/local/lib/python3.10/dist-packages
(from ipython->hmms) (5.7.1)
Requirement already satisfied: prompt-toolkit!=3.0.0,!=3.0.1,<3.1.0,>=2.0.0 in /usr/local
```

```
/lib/python3.10/dist-packages (from ipython->hmms) (3.0.39)
Requirement already satisfied: pygments in /usr/local/lib/python3.10/dist-packages (from
ipython->hmms) (2.16.1)
Requirement already satisfied: backcall in /usr/local/lib/python3.10/dist-packages (from
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Requirement already satisfied: matplotlib-inline in /usr/local/lib/python3.10/dist-packag
es (from ipython->hmms) (0.1.6)
Requirement already satisfied: pexpect>4.3 in /usr/local/lib/python3.10/dist-packages (fr
om ipython->hmms) (4.8.0)
Requirement already satisfied: contourpy>=1.0.1 in /usr/local/lib/python3.10/dist-package
s (from matplotlib->hmms) (1.1.0)
Requirement already satisfied: cycler>=0.10 in /usr/local/lib/python3.10/dist-packages (f
rom matplotlib->hmms) (0.11.0)
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es (from matplotlib->hmms) (4.42.1)
Requirement already satisfied: kiwisolver>=1.0.1 in /usr/local/lib/python3.10/dist-packag
es (from matplotlib->hmms) (1.4.5)
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Requirement already satisfied: pillow>=6.2.0 in /usr/local/lib/python3.10/dist-packages (
from matplotlib->hmms) (9.4.0)
Requirement already satisfied: pyparsing>=2.3.1 in /usr/local/lib/python3.10/dist-package
s (from matplotlib->hmms) (3.1.1)
Requirement already satisfied: python-dateutil>=2.7 in /usr/local/lib/python3.10/dist-pac
kages (from matplotlib->hmms) (2.8.2)
Requirement already satisfied: pytz>=2020.1 in /usr/local/lib/python3.10/dist-packages (f
rom pandas->hmms) (2023.3.post1)
Requirement already satisfied: parso<0.9.0,>=0.8.3 in /usr/local/lib/python3.10/dist-pack
ages (from jedi>=0.16->ipython->hmms) (0.8.3)
Requirement already satisfied: ptyprocess>=0.5 in /usr/local/lib/python3.10/dist-packages
(from pexpect>4.3->ipython->hmms) (0.7.0)
Requirement already satisfied: wcwidth in /usr/local/lib/python3.10/dist-packages (from p
rompt-toolkit!=3.0.0,!=3.0.1,<3.1.0,>=2.0.0->ipython->hmms) (0.2.6)
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.10/dist-packages (from
python-dateutil>=2.7->matplotlib->hmms) (1.16.0)
Building wheels for collected packages: hmms
  Building wheel for hmms (setup.py) ... done
  Created wheel for hmms: filename=hmms-0.2.3-cp310-cp310-linux_x86_64.whl size=1936113 s
ha256=6d50a9aaf56ecf4cb8dadd703b2c5d17dd0f0cf6ca7951789bda81c6d1a62d14
  Stored in directory: /root/.cache/pip/wheels/aa/6f/a4/1dbae244341f24881dce9465aa533729d
2ae870cff3866070f
Successfully built hmms
Installing collected packages: jedi, hmms
Successfully installed hmms-0.2.3 jedi-0.19.0
```

In [3]:

```python
import numpy as np
import hmms

# Define the model parameters
A = np.array([[0.9, 0.1], [0.4, 0.6]])
B = np.array([[0.9, 0.1], [0.2, 0.8]])
pi = np.array([0.8, 0.2])
emission_sequence = np.array([0, 1])

# Create the Hidden Markov Model
hmm_model = hmms.DtHMM(A, B, pi)

# Compute the forward probabilities
forward_probabilities = hmm_model.forward(emission_sequence)
```

In [7]:

```python
# Define the model parameters
A = np.array([[0.72, 0.04], [0.0664, 0.0768]])
B = np.array([[0.9, 0.1], [0.2, 0.8]])
pi = np.array([0.8, 0.2])
emission_sequence = np.array([0, 1])

# Create the Hidden Markov Model
```

```python
dhmm = hmms.DtHMM(A, B, pi)

# Compute and exponentiate the forward probabilities
forward_probabilities = np.exp(dhmm.forward(emission_sequence))

# Print the results
print("A:\n", A)
print("X:\n", forward_probabilities)
```

```
A:
 [[0.72   0.04  ]
 [0.0664 0.0768]]
X:
 [[0.72      0.04      ]
 [0.0521056 0.0254976]]
```

In [8]:

```python
# Display the forward algorithm parameters
hmms.print_parameters(dhmm);
```

```
Initial probabilities (π) :
```

	0
0	0.8
1	0.2

```
Transition probabilities matrix (A):
```

	0	1
0	0.7200	0.0400
1	0.0664	0.0768

```
Emission probabilities matrix (B):
```

	0	1
0	0.9	0.1
1	0.2	0.8

Backward Algorithm

Pseudocode

The pseudocode (backward algorithm) from Alperen Degirmenci shown :

Input: $\boldsymbol{A}$, $\psi_{1:N}$, $\boldsymbol{\alpha}$
$\beta_N = 1$;
for $t = N-1 : 1$ **do**
  $\beta_t = \text{normalize}(\boldsymbol{A}(\psi_{t+1} \odot \beta_{t+1})$;
$\gamma = \text{normalize}(\boldsymbol{\alpha} \odot \boldsymbol{\beta},\ 1)$
Return $\gamma_{1:N}$

Figure 7 - Backward Algorithm Pseudocode
Source: Adapted from Alperen Degirmenci [16]

Toy Example

toy example (backward algorithm) using hmms library [19][20]:

In [9]:

```
import numpy as np
import hmms

# Define the Hidden Markov Model parameters
transition_matrix = np.array([[0.9, 0.1], [0.4, 0.6]])
emission_matrix = np.array([[0.9, 0.1], [0.2, 0.8]])
initial_state_probabilities = np.array([0.8, 0.2])
emission_sequence = np.array([0, 1])

# Create a Hidden Markov Model using the specified parameters
hmm_model = hmms.DtHMM(transition_matrix, emission_matrix, initial_state_probabilities)

# Compute the backward probabilities using the backward algorithm
backward_probabilities = hmm_model.backward(emission_sequence)
```

In [10]:

```
import numpy as np

# Define the expected transition matrix A for validation
expected_transition_matrix = np.array([[0.17, 0.52], [1, 1]])

# Compute and exponentiate the backward probabilities using the Hidden Markov Model
backward_probabilities = np.exp(dhmm.backward(emission_sequence))

# Print the expected transition matrix and computed backward probabilities
print("Expected Transition Matrix (A):\n", expected_transition_matrix)
print("Computed Backward Probabilities (X):\n", backward_probabilities)
```

```
Expected Transition Matrix (A):
 [[0.17 0.52]
 [1.   1.  ]]
Computed Backward Probabilities (X):
 [[0.104   0.06808]
 [1.      1.     ]]
```

In [11]:

```
# Display the backward algorithm parameters
hmms.print_parameters(dhmm);
```

```
Initial probabilities (π) :
```

	0
0	0.8
1	0.2

```
Transition probabilities matrix (A):
```

	0	1
0	0.7200	0.0400
1	0.0664	0.0768

```
Emission probabilities matrix (B):
```

	0	1
0	0.9	0.1
1	0.2	0.8

Baum Welch Algorithm

Baum Welch Algorithm

Toy Example

toy example (Baum Welch algorithm) from hmms library [19][20]:

In [12]:

```
# Install required packages
!pip install "git+https://github.com/lopatovsky/HMMs"
!pip install fredapi

# Import necessary libraries
import numpy as np
import pandas as pd
from fredapi import Fred
import hmms

# Get oil price data from the Federal Reserve Economic Data (FRED) API
# Create data frames for current and forecasted spot crude oil prices
fred = Fred(api_key='87769799aa7b2dc41a0590ed8a688283')
current = pd.DataFrame(fred.get_series('WTISPLC'), columns=['WTISPLC'])
current.index.names = ['Date']
forecast = pd.DataFrame(fred.get_series('WTISPLC').shift(-1), columns=['Forecast'])
forecast.index.names = ['Date']

# Filter the data to a specific date range
current = current.drop((current[(current.index < '2003-01-01') | (current.index > '2023-
12-01')]).index, axis=0)

# Compute the difference (returns) and create a binary sequence
price = current['WTISPLC']
price_diff = price.diff()[1:]
e_seq = np.array(price_diff.apply(lambda x: 1 if x > 0 else 0).values)
```

```
Collecting git+https://github.com/lopatovsky/HMMs
  Cloning https://github.com/lopatovsky/HMMs to /tmp/pip-req-build-qiph5_8h
  Running command git clone --filter=blob:none --quiet https://github.com/lopatovsky/HMMs
/tmp/pip-req-build-qiph5_8h
  Resolved https://github.com/lopatovsky/HMMs to commit c7ef1a72d49de388ed62632137566586c
d06812c
  Preparing metadata (setup.py) ... done
Requirement already satisfied: Cython in /usr/local/lib/python3.10/dist-packages (from hm
ms==0.2.3) (0.29.36)
Requirement already satisfied: NumPy in /usr/local/lib/python3.10/dist-packages (from hmm
s==0.2.3) (1.23.5)
Requirement already satisfied: ipython in /usr/local/lib/python3.10/dist-packages (from h
mms==0.2.3) (7.34.0)
Requirement already satisfied: matplotlib in /usr/local/lib/python3.10/dist-packages (fro
m hmms==0.2.3) (3.7.1)
Requirement already satisfied: pandas in /usr/local/lib/python3.10/dist-packages (from hm
ms==0.2.3) (1.5.3)
Requirement already satisfied: scipy in /usr/local/lib/python3.10/dist-packages (from hmm
s==0.2.3) (1.10.1)
Requirement already satisfied: setuptools>=18.5 in /usr/local/lib/python3.10/dist-package
s (from ipython->hmms==0.2.3) (67.7.2)
Requirement already satisfied: jedi>=0.16 in /usr/local/lib/python3.10/dist-packages (fro
m ipython->hmms==0.2.3) (0.19.0)
Requirement already satisfied: decorator in /usr/local/lib/python3.10/dist-packages (from
ipython->hmms==0.2.3) (4.4.2)
Requirement already satisfied: pickleshare in /usr/local/lib/python3.10/dist-packages (fr
om ipython->hmms==0.2.3) (0.7.5)
Requirement already satisfied: traitlets>=4.2 in /usr/local/lib/python3.10/dist-packages
(from ipython->hmms==0.2.3) (5.7.1)
Requirement already satisfied: prompt-toolkit!=3.0.0,!=3.0.1,<3.1.0,>=2.0.0 in /usr/local
/lib/python3.10/dist-packages (from ipython->hmms==0.2.3) (3.0.39)
Requirement already satisfied: pygments in /usr/local/lib/python3.10/dist-packages (from
ipython->hmms==0.2.3) (2.16.1)
Requirement already satisfied: backcall in /usr/local/lib/python3.10/dist-packages (from
ipython->hmms==0.2.3) (0.2.0)
Requirement already satisfied: matplotlib-inline in /usr/local/lib/python3.10/dist-packag
es (from ipython->hmms==0.2.3) (0.1.6)
```

```
es (from ipython->hmms==0.2.3) (0.1.6)
Requirement already satisfied: pexpect>4.3 in /usr/local/lib/python3.10/dist-packages (fr
om ipython->hmms==0.2.3) (4.8.0)
Requirement already satisfied: contourpy>=1.0.1 in /usr/local/lib/python3.10/dist-package
s (from matplotlib->hmms==0.2.3) (1.1.0)
Requirement already satisfied: cycler>=0.10 in /usr/local/lib/python3.10/dist-packages (f
rom matplotlib->hmms==0.2.3) (0.11.0)
Requirement already satisfied: fonttools>=4.22.0 in /usr/local/lib/python3.10/dist-packag
es (from matplotlib->hmms==0.2.3) (4.42.1)
Requirement already satisfied: kiwisolver>=1.0.1 in /usr/local/lib/python3.10/dist-packag
es (from matplotlib->hmms==0.2.3) (1.4.5)
Requirement already satisfied: packaging>=20.0 in /usr/local/lib/python3.10/dist-packages
(from matplotlib->hmms==0.2.3) (23.1)
Requirement already satisfied: pillow>=6.2.0 in /usr/local/lib/python3.10/dist-packages (
from matplotlib->hmms==0.2.3) (9.4.0)
Requirement already satisfied: pyparsing>=2.3.1 in /usr/local/lib/python3.10/dist-package
s (from matplotlib->hmms==0.2.3) (3.1.1)
Requirement already satisfied: python-dateutil>=2.7 in /usr/local/lib/python3.10/dist-pac
kages (from matplotlib->hmms==0.2.3) (2.8.2)
Requirement already satisfied: pytz>=2020.1 in /usr/local/lib/python3.10/dist-packages (f
rom pandas->hmms==0.2.3) (2023.3.post1)
Requirement already satisfied: parso<0.9.0,>=0.8.3 in /usr/local/lib/python3.10/dist-pack
ages (from jedi>=0.16->ipython->hmms==0.2.3) (0.8.3)
Requirement already satisfied: ptyprocess>=0.5 in /usr/local/lib/python3.10/dist-packages
(from pexpect>4.3->ipython->hmms==0.2.3) (0.7.0)
Requirement already satisfied: wcwidth in /usr/local/lib/python3.10/dist-packages (from p
rompt-toolkit!=3.0.0,!=3.0.1,<3.1.0,>=2.0.0->ipython->hmms==0.2.3) (0.2.6)
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.10/dist-packages (from
python-dateutil>=2.7->matplotlib->hmms==0.2.3) (1.16.0)
Collecting fredapi
  Downloading fredapi-0.5.1-py3-none-any.whl (11 kB)
Requirement already satisfied: pandas in /usr/local/lib/python3.10/dist-packages (from fr
edapi) (1.5.3)
Requirement already satisfied: python-dateutil>=2.8.1 in /usr/local/lib/python3.10/dist-p
ackages (from pandas->fredapi) (2.8.2)
Requirement already satisfied: pytz>=2020.1 in /usr/local/lib/python3.10/dist-packages (f
rom pandas->fredapi) (2023.3.post1)
Requirement already satisfied: numpy>=1.21.0 in /usr/local/lib/python3.10/dist-packages (
from pandas->fredapi) (1.23.5)
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.10/dist-packages (from
python-dateutil>=2.8.1->pandas->fredapi) (1.16.0)
Installing collected packages: fredapi
Successfully installed fredapi-0.5.1
```

Subsequently, we will generate a model with initial random parameters. The training of this model will culminate in aligning it with the data, representing three (3) hidden states within a discrete-time HMM: "bull," "bear," or "stagnant," and two (2) outcome variables indicating either a rise or decline.

In [13]:

```python
import numpy as np
import hmms

# Create a random Hidden Markov Model with 3 hidden states and 2 outcome variables
dhmm_r = hmms.DtHMM.random(3, 2)

# Split the emission sequence into arrays of length 32 or less
e_seq = np.array_split(e_seq, 32)

# Learn the model parameters using the Baum-Welch Algorithm
dhmm_r.baum_welch(e_seq, iterations=100)

# Display the learned parameters of the HMM
hmms.print_parameters(dhmm_r)
```

```
iteration  1 / 100
iteration  2 / 100
iteration  3 / 100
iteration  4 / 100
iteration  5 / 100
iteration  6 / 100
iteration  7 / 100
```

```
iteration  7 / 100
iteration  8 / 100
iteration  9 / 100
iteration  10 / 100
iteration  11 / 100
iteration  12 / 100
iteration  13 / 100
iteration  14 / 100
iteration  15 / 100
iteration  16 / 100
iteration  17 / 100
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iteration  78 / 100
iteration  79 / 100
```

```
iteration  79 / 100
iteration  80 / 100
iteration  81 / 100
iteration  82 / 100
iteration  83 / 100
iteration  84 / 100
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iteration  86 / 100
iteration  87 / 100
iteration  88 / 100
iteration  89 / 100
iteration  90 / 100
iteration  91 / 100
iteration  92 / 100
iteration  93 / 100
iteration  94 / 100
iteration  95 / 100
iteration  96 / 100
iteration  97 / 100
iteration  98 / 100
iteration  99 / 100
iteration  100 / 100
Initial probabilities (π) :
```

	0
0	0.515617
1	0.077432
2	0.406952

```
Transition probabilities matrix (A):
```

	0	1	2
0	0.192016	0.077708	0.730276
1	0.634010	0.101115	0.264875
2	0.493125	0.238583	0.268292

```
Emission probabilities matrix (B):
```

	0	1
0	0.365635	0.634365
1	0.505559	0.494441
2	0.381168	0.618832

Viterbi Algorithm

Toy Example

toy example (Viterbi algorithm) from hmms library [19][20]:

In [15]:

```python
import numpy as np
import warnings
warnings.filterwarnings('ignore')

def create_small_hmm():
    A = np.array([[0.9, 0.1], [0.4, 0.6]])
    B = np.array([[0.9, 0.1], [0.2, 0.8]])
    pi = np.array([0.8, 0.2])
    return hmms.DtHMM(A, B, pi)
```

```python
def create_medium_emission():
    hmm = create_small_hmm()
    em = np.array([0, 1, 0, 1, 1])
    return hmm, em

def test_viterbi_p():
    hmm, em = create_medium_emission()
    p, seq = hmm.viterbi(em)
    out_p = 0.0020155392
    print(np.exp(p), out_p)

def test_viterbi_seq():
    hmm, em = create_medium_emission()
    p, seq = hmm.viterbi(em)
    out_seq = np.array([0, 0, 0, 1, 1])
    print(seq, out_seq)

test_viterbi_p()
test_viterbi_seq()

def create_small_random_hmm():
    return hmms.DtHMM.random(2, 2)

def create_hmm_small_out():
    A = np.array([[0, 1], [0, 1]])
    B = np.array([[1, 0], [0, 1]])
    pi = np.array([1, 0])
    return hmms.DtHMM(A, B, pi)

def create_long_emission():
    hmm, em = create_small_hmm(), np.array([0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0
, 0, 1, 1, 0, 0, 0, 1, 0, 1, 1, 1, 1, 0, 0, 1, 1, 1])
    return hmm, em

def test_viterbi_long_seq():
    hmm, em = create_long_emission()
    p, seq = hmm.viterbi(em)
    out_seq = np.array([0, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0,
0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 1])
    print(seq, out_seq)

test_viterbi_long_seq()
```

```
0.0020155392000000001 0.0020155392
[0 0 0 1 1] [0 0 0 1 1]
[0 0 0 1 1 1 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 1 1 1 1 0 0 1 1 1] [0 0 0 1 1 1 1 1 0 0
0 0 0 0 0 0 1 1 0 0 0 0 0 1 1 1 1 0 0 1 1 1]
```

Regime Detection

Regime detection involves identifying recurring periods of volatility within a time series, which can encompass hidden states such as bull, bear, or stagnant phases, as well as high or low volatility intervals. These periods are often categorized based on quantitative attributes. Since these volatility patterns are latent and observable only through returns (referred to as emissions), Hidden Markov Models (HMMs) are well-suited for solving the common volatility detection problem.

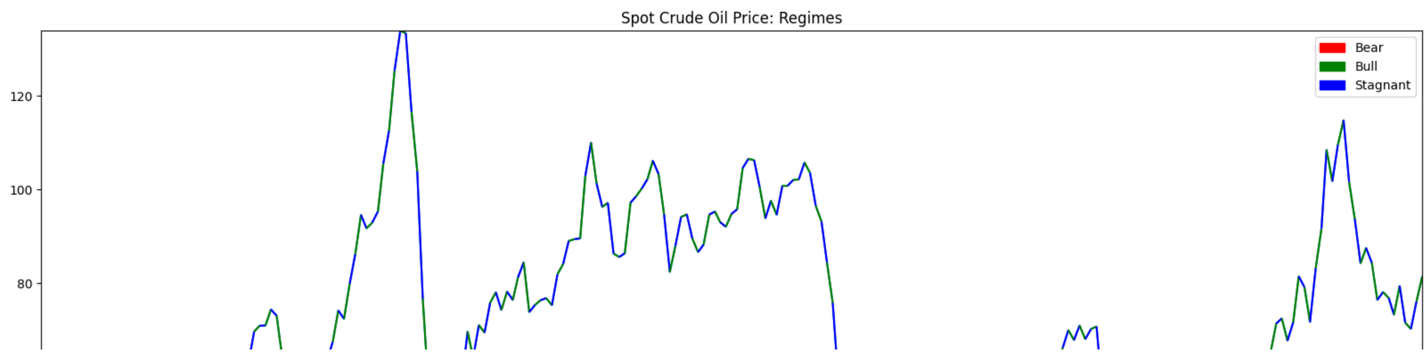
In the context of a discrete-time Hidden Markov Model (HMM), the goal is to detect three primary hidden states: bull, bear, and stagnant states, as described in the subsections below. Utilizing graphical representations of these hidden states enables visualizing and identifying the regimes detected by the HMM.

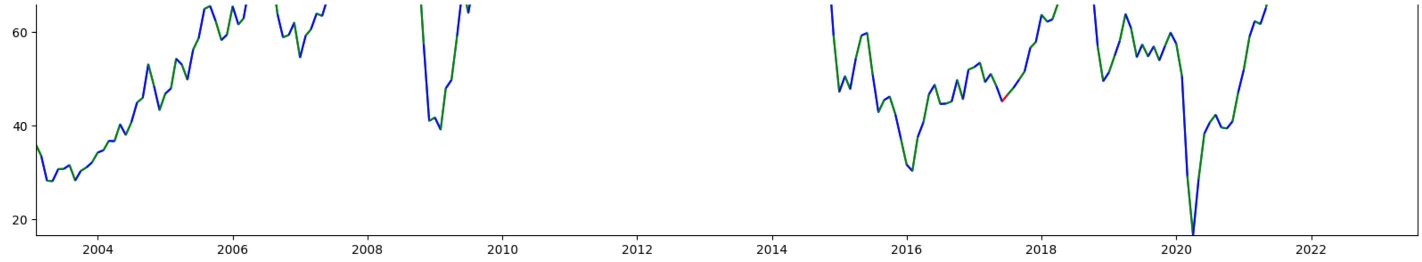
Below is the code for plotting regime detection/identification.

In [17]:

```python
import numpy as np
import warnings
import matplotlib.pyplot as plt  # Import plt module
import matplotlib.dates as mdates
```

```python
import matplotlib.patches as mpatches
from matplotlib.collections import LineCollection
from matplotlib.colors import ListedColormap, BoundaryNorm
import pandas as pd
import hmms

warnings.filterwarnings('ignore')

# Assuming you have defined the variables and functions here:

# Using Viterbi Algorithm to identify market regimes
log_prob, s_seq = dhmm_r.viterbi(np.concatenate(e_seq).ravel())

# Create a DataFrame with price, regime, and price difference
price_plot = pd.DataFrame(price[1:], index=price[1:].index, columns=['WTISPLC'])
price_plot['Regime'] = s_seq
price_plot['diff'] = price_diff

# Get means of all assigned states and map them
means = price_plot.groupby(['Regime'])['diff'].mean()
map_regimes = {k: v for v, k in enumerate(means.sort_values().index)}

# Map regime values and display the DataFrame
price_plot['Regime'] = price_plot['Regime'].map(map_regimes)
price_plot.head()

# Count the occurrences of each regime
regime_counts = price_plot['Regime'].value_counts()

# Plot the regime detection
fig, ax = plt.subplots(figsize=(20, 8))
plt.title("Spot Crude Oil Price: Regimes")
plt.plot(price_plot.index, price_plot['WTISPLC'])

# Define colormap for regimes
cmap = ListedColormap(['r', 'b', 'g'], 'indexed')
norm = BoundaryNorm(range(3 + 1), cmap.N)

# Create line segments for plotting regimes
inxval = mdates.date2num(price_plot.index.to_pydatetime())
points = np.array([inxval, price_plot['WTISPLC']]).T.reshape(-1, 1, 2)
segments = np.concatenate([points[:-1], points[1:]], axis=1)
lc = LineCollection(segments, cmap=cmap, norm=norm)
lc.set_array(price_plot['Regime'])

# Add line collection to the plot
plt.gca().add_collection(lc)
plt.xlim(price_plot.index.min(), price_plot.index.max())
plt.ylim(price_plot['WTISPLC'].min(), price_plot['WTISPLC'].max())

# Create legend
legend_patches = [
    mpatches.Patch(color='red', label='Bear'),
    mpatches.Patch(color='green', label='Bull'),
    mpatches.Patch(color='blue', label='Stagnant')
]
plt.legend(handles=legend_patches)

# Show the plot
plt.show()
```





Bull Regimes

The bull regime, often associated with a bull market, draws its name from a bull's upward charge and is marked by rising equity (stock) prices [23]. Bull regimes typically involve substantial positive price changes [1] and exhibit the following traits [23]:

Ascending stock (equity) prices, often indicated by a significant increase of approximately 20% or more over two or more months, as observed in broad market indices like the Dow Jones Industrial Average (DJIA) or the S&P 500.

Typically, high levels of stockholder confidence.

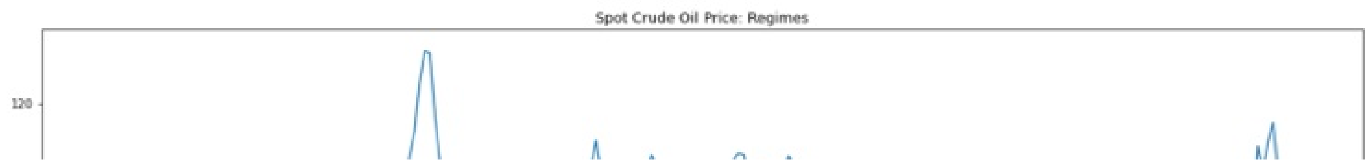
Often coincides with a robust national economy.

Figure 8 - Regime Graph - Bull Identification and Illustration
Source: Own Adaptation

The hidden state 0 corresponds to the "Bear" regime, represented in red, while hidden state 1 represents the "Stagnant" regime in blue. Hidden state 2 signifies the "Bull" regime, depicted in green. In the graph, the green section corresponds to bull regimes, which are periods when crude oil prices consistently rise after a period of decline or stagnation.

Bear Regimes

To identify bear regimes, we applied the Viterbi Algorithm. As illustrated in the graph below, the periods marked with red lines represent the bear regimes detected by the algorithm. While there are some occasional false alarms, the algorithm consistently detects bear regimes accurately. Notably, it successfully identified major financial events such as the 2008 financial crisis and the market crash during the pandemic. Additionally, it detected an oil price crash in 2014-15 attributed to oversupply and unfavorable OPEC policies.



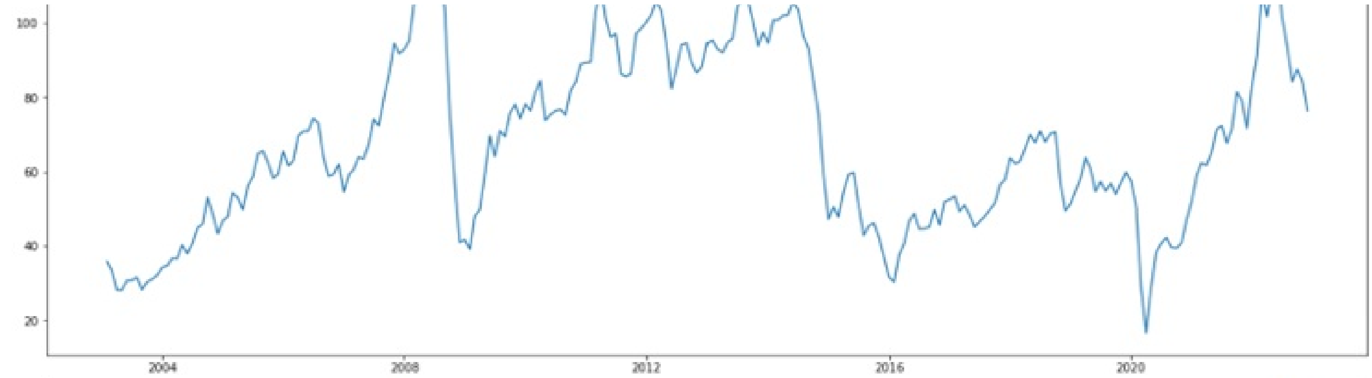


Figure 9 - Regime Graph: Bear Identification and Illustration

Source: Own Adaptation

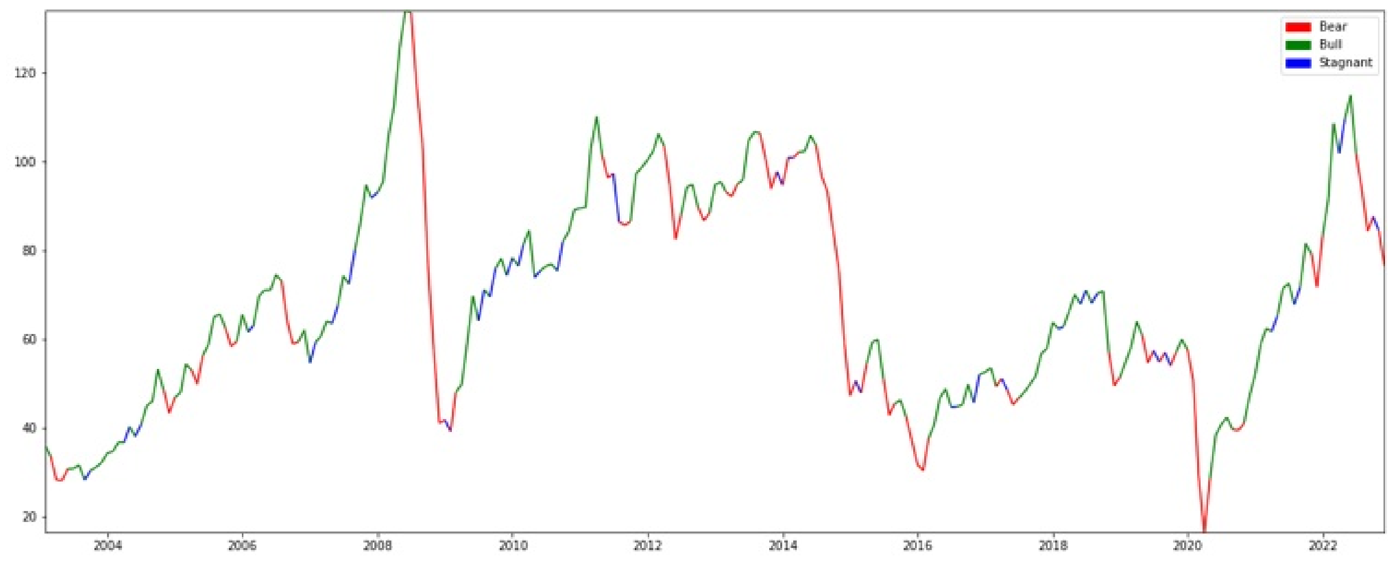
Stagnant Regimes

We will utilize the Viterbi Algorithm to discern the most probable state-transition path, representing market regimes, in the time series data obtained in the preceding step. The different regimes have been visualized using the algorithm discussed earlier. In the stagnant regime, there is minimal fluctuation in returns, as indicated by the blue line.

Model Development

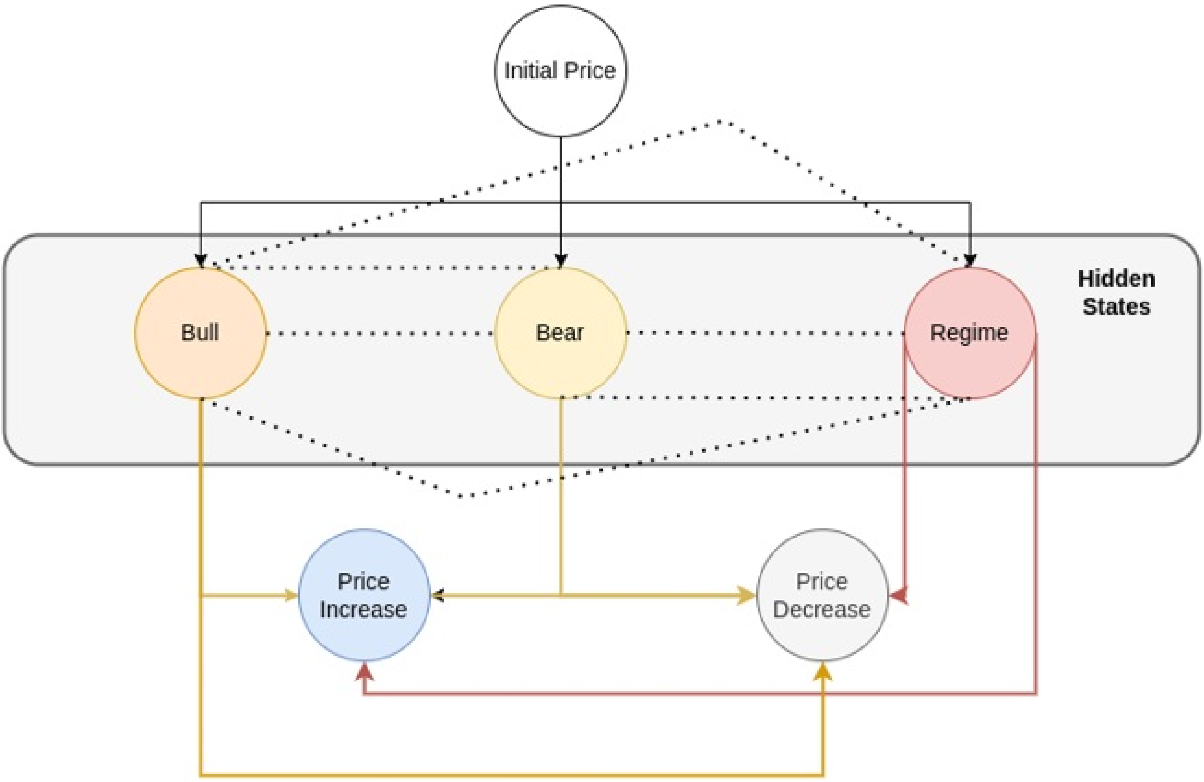
Hidden States Sequence

In our scenario, the hidden sequence corresponds to distinct market regimes, namely 'bull,' 'bear,' and 'stagnant.' These distinctive regimes are visually represented in the image below.



Additionally, we possess a variety of probabilities, including the initial probabilities associated with each state, denoted as follows: 0 (Bear) - represented by red, 1 (Stagnant) - depicted in blue, and 2 (Bull) - illustrated in green. The transition probability signifies the likelihood of transitioning from one state to another. In our case, for instance, Class 0 has a transition probability of 0.79 to remain in Class 0 (self-transition), and it has a transition probability of approximately 0.207 to transition to Class 1. These sequential transitions accumulate over time, resulting in an evolving sequence of states.

It's important to note that the hidden states and their respective transition probabilities ultimately determine the identification of the final states in the sequence.



Regime Process, Model Training, Validation and Testing

Minimal Working Example of Hill Climb Search

example of the Hill Climb Search is shown below

In [18]:

```
!pip install pgmpy
```

```
Collecting pgmpy
  Downloading pgmpy-0.1.23-py3-none-any.whl (1.9 MB)
     ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 1.9/1.9 MB 33.8 MB/s eta 0:00:00
Requirement already satisfied: networkx in /usr/local/lib/python3.10/dist-packages (from
pgmpy) (3.1)
Requirement already satisfied: numpy in /usr/local/lib/python3.10/dist-packages (from pgm
py) (1.23.5)
Requirement already satisfied: scipy in /usr/local/lib/python3.10/dist-packages (from pgm
py) (1.10.1)
Requirement already satisfied: scikit-learn in /usr/local/lib/python3.10/dist-packages (f
rom pgmpy) (1.2.2)
Requirement already satisfied: pandas in /usr/local/lib/python3.10/dist-packages (from pg
mpy) (1.5.3)
Requirement already satisfied: pyparsing in /usr/local/lib/python3.10/dist-packages (from
pgmpy) (3.1.1)
Requirement already satisfied: torch in /usr/local/lib/python3.10/dist-packages (from pgm
```

```
py) (2.0.1+cu118)
Requirement already satisfied: statsmodels in /usr/local/lib/python3.10/dist-packages (fr
om pgmpy) (0.14.0)
Requirement already satisfied: tqdm in /usr/local/lib/python3.10/dist-packages (from pgmp
y) (4.66.1)
Requirement already satisfied: joblib in /usr/local/lib/python3.10/dist-packages (from pg
mpy) (1.3.2)
Requirement already satisfied: opt-einsum in /usr/local/lib/python3.10/dist-packages (fro
m pgmpy) (3.3.0)
Requirement already satisfied: python-dateutil>=2.8.1 in /usr/local/lib/python3.10/dist-p
ackages (from pandas->pgmpy) (2.8.2)
Requirement already satisfied: pytz>=2020.1 in /usr/local/lib/python3.10/dist-packages (f
rom pandas->pgmpy) (2023.3.post1)
Requirement already satisfied: threadpoolctl>=2.0.0 in /usr/local/lib/python3.10/dist-pac
kages (from scikit-learn->pgmpy) (3.2.0)
Requirement already satisfied: patsy>=0.5.2 in /usr/local/lib/python3.10/dist-packages (f
rom statsmodels->pgmpy) (0.5.3)
Requirement already satisfied: packaging>=21.3 in /usr/local/lib/python3.10/dist-packages
(from statsmodels->pgmpy) (23.1)
Requirement already satisfied: filelock in /usr/local/lib/python3.10/dist-packages (from
torch->pgmpy) (3.12.2)
Requirement already satisfied: typing-extensions in /usr/local/lib/python3.10/dist-packag
es (from torch->pgmpy) (4.5.0)
Requirement already satisfied: sympy in /usr/local/lib/python3.10/dist-packages (from tor
ch->pgmpy) (1.12)
Requirement already satisfied: jinja2 in /usr/local/lib/python3.10/dist-packages (from to
rch->pgmpy) (3.1.2)
Requirement already satisfied: triton==2.0.0 in /usr/local/lib/python3.10/dist-packages (
from torch->pgmpy) (2.0.0)
Requirement already satisfied: cmake in /usr/local/lib/python3.10/dist-packages (from tri
ton==2.0.0->torch->pgmpy) (3.27.4.1)
Requirement already satisfied: lit in /usr/local/lib/python3.10/dist-packages (from trito
n==2.0.0->torch->pgmpy) (16.0.6)
Requirement already satisfied: six in /usr/local/lib/python3.10/dist-packages (from patsy
>=0.5.2->statsmodels->pgmpy) (1.16.0)
Requirement already satisfied: MarkupSafe>=2.0 in /usr/local/lib/python3.10/dist-packages
(from jinja2->torch->pgmpy) (2.1.3)
Requirement already satisfied: mpmath>=0.19 in /usr/local/lib/python3.10/dist-packages (f
rom sympy->torch->pgmpy) (1.3.0)
Installing collected packages: pgmpy
Successfully installed pgmpy-0.1.23
```

In [19]:

```python
import numpy as np
import pandas as pd
from pgmpy.models import BayesianModel
from pgmpy.estimators import HillClimbSearch, BicScore

# Create a dataset with dependencies
data = pd.DataFrame(np.random.randint(0, 3, size=(2500, 5)), columns=list('ABCGH'))
data['A'] += data['B'] + data['C']
data['H'] = data['G'] - data['A']

# Perform a Hill Climb search to learn the Bayesian Network structure
hc = HillClimbSearch(data)
best_model = hc.estimate(scoring_method=BicScore(data))

# Print the edges of the best Bayesian Network model
print(best_model.edges())
```

```
[('A', 'B'), ('A', 'C'), ('C', 'B'), ('G', 'A'), ('G', 'H'), ('H', 'A')]
```

In [20]:

```python
# Limit the maximum indegree to 1 and print the edges of the estimated model
estimated_model = hc.estimate(max_indegree=1)
print(estimated_model.edges())
```

```
[('A', 'B'), ('A', 'C'), ('H', 'A'), ('H', 'G')]
```

In [ ]:

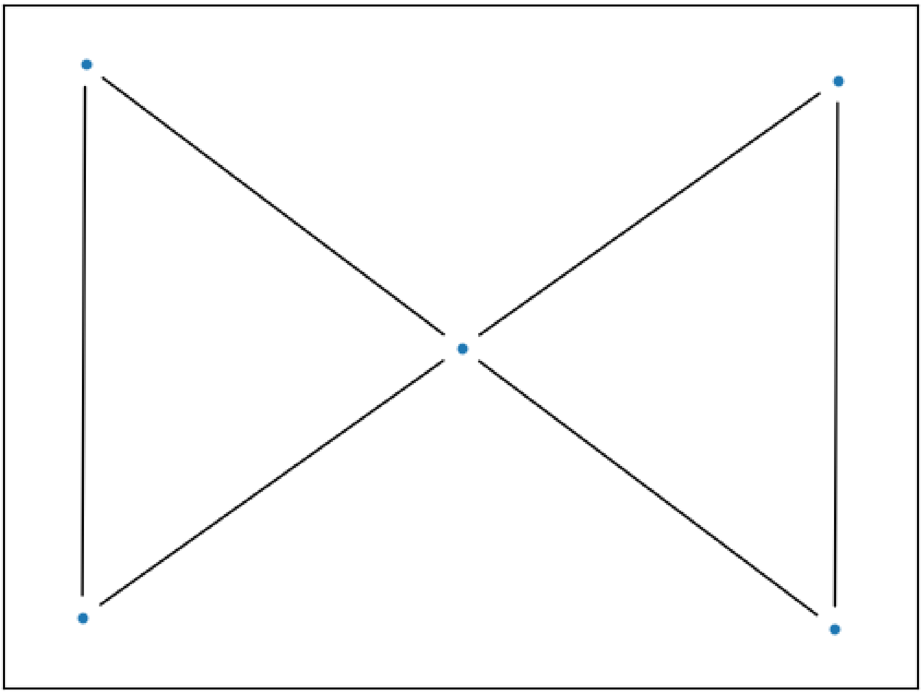
```

```

In [21]:

```
import networkx as nx
import pylab as plt

# Create a graph
G = nx.Graph()
G.add_edges_from(best_model.edges())

# Define the layout of the graph
pos = nx.spring_layout(G)

# Draw nodes
nx.draw_networkx_nodes(G, pos, node_size=10)

# Draw edges with arrows
nx.draw_networkx_edges(G, pos, arrows=True)

# Create and set the figure size
plt.figure(figsize=(20, 10))
```

Out[21]:

```
<Figure size 2000x1000 with 0 Axes>
```



```
<Figure size 2000x1000 with 0 Axes>
```

We can see the edges being represented in the form of network where Node A is connected to B and C and similarly Node H is connected to G and A.

In [22]:

```
import numpy as np
import pandas as pd
from pgmpy.models import BayesianNetwork

# Split the data into training, validation, and prediction sets
train_data = data[:1800]
val_data = data[1800:2000]
predict_data = data[2000:]

# Create a Bayesian Network model based on the best_model's edges
```

```python
model = BayesianNetwork(best_model.edges())

# Fit the model to the training data
model.fit(train_data)

# Prepare the validation data for prediction
y_val = val_data['H'].tolist()
val_data.drop('H', axis=1, inplace=True)

# Perform predictions on the validation data
y_val_pred = model.predict(val_data)
```

In [23]:

```python
# Evaluate the performance on validation data
error = np.mean(y_val != np.roll(y_val_pred, 1))
print("\nError on Validation Data: {:.2f} %".format(error * 100))
```

```
Error on Validation Data: 82.50 %
```

In [26]:

```python
# Minor validation tweaking
val_data = data[1800:2000]
from pgmpy.estimators import BayesianEstimator

model.fit(
    data=train_data,
    estimator=BayesianEstimator,
    prior_type="BDeu",
    equivalent_sample_size=1000,
    complete_samples_only=False
)
y_val = val_data['H'].tolist()
val_data.drop('H', axis=1, inplace=True)
y_val_pred = model.predict(val_data)

# Check the performance on validation data
error = np.mean(y_val != np.roll(y_val_pred, 1))
print("\nError on Validation Data: {:.2f} %".format(error * 100))
```

```
Error on Validation Data: 82.50 %
```

In [27]:

```python
# Making predictions
y_test = predict_data['H'].tolist()
predict_data = predict_data.drop('H', axis=1)  # Drop 'H' column
y_pred = model.predict(predict_data)
```

In [28]:

```python
# Calculate the prediction error
error = np.mean(y_test != np.roll(y_pred, 1))
# Print the error as a percentage
print("\nError on Testing Data: {:.2f} %".format(error * 100))
```

```
Error on Testing Data: 83.11 %
```

We can see that our validation and testing errors are 82% and 83% respectively.

In [29]:

```python
# Printing a CPD with it's state names defined.
print(model.get_cpds('H'))
```

```
+-------+----------------------+----------------------+----------------------+
| G     | G(0)                 | G(1)                 | G(2)                 |
+-------+----------------------+----------------------+----------------------+
```

H(-6)	0.06901701843336083	0.040819658747652865	0.038972680151213995
H(-5)	0.11889321227842627	0.06947505918850518	0.038972680151213995
H(-4)	0.17407538419211568	0.11796881378071677	0.058965665068786775
H(-3)	0.193176905239162	0.1664625683729284	0.10105615963209788
H(-2)	0.19105451401171242	0.223773369254633	0.1989165594917962
H(-1)	0.11570962543725188	0.17307535309004815	0.21680501968120344
H(0)	0.059466257909837665	0.11245815984978363	0.17366226275380955
H(1)	0.039303541249066536	0.05514735896807902	0.10736973381659455
H(2)	0.039303541249066536	0.040819658747652865	0.06527923925328344

Since the model performance is not so good, let's do the discretization of the data.

In [31]:

```
import numpy as np
import pandas as pd

# Number of tiers for data grouping
TIERS_NUM = 2

def boundary_str(start, end, tier):
    """Generate a string representation of a tier's boundary range."""
    return f'{tier}: {start:+0,.2f} to {end:+0,.2f}'

def relabel(v, boundaries):
    """Assign a label to a value based on its position in tier boundaries."""
    for tier, (start, end) in enumerate(boundaries, start=65):  # Use ASCII 'A' as the s
tarting integer
        if start <= v <= end:
            return boundary_str(start, end, chr(tier))  # Convert tier to ASCII characte
r
    return np.nan

def get_boundaries(tiers):
    """Calculate tier boundaries based on data distribution."""
    boundaries = [(tiers[0][0], tiers[0][-1])]
    for tier in tiers[1:]:
        boundaries.append((boundaries[-1][-1], tier[-1]))
    return boundaries

new_columns = {}
for label, series in data.items():
    # Sort and filter out NaN values
    values = np.sort(series.dropna().values.astype(float))

    if len(values) < TIERS_NUM:
        print(f'Error: There are not enough data points for label {label}')
        break

    # Split data into tiers
    tiers = np.array_split(values, TIERS_NUM)
    boundaries = get_boundaries(tiers)

    # Relabel the data based on tier boundaries
    new_columns[label] = [relabel(value, boundaries) for value in series]

# Create a new DataFrame with relabeled columns
df = pd.DataFrame(data=new_columns, columns=data.columns)
df.head(10)
```

Out[31]:

	A	B	C	G	H
0	B: +3.00 to +6.00	B: +1.00 to +2.00	A: +0.00 to +1.00	A: +0.00 to +1.00	A: -6.00 to -2.00
1	A: +0.00 to +3.00	A: +0.00 to +1.00	B: +1.00 to +2.00	B: +1.00 to +2.00	B: -2.00 to +2.00
2	A: +0.00 to +3.00	A: +0.00 to +1.00	A: +0.00 to +1.00	A: +0.00 to +1.00	A: -6.00 to -2.00
3	A: +0.00 to +3.00	A: +0.00 to +1.00	A: +0.00 to +1.00	B: +1.00 to +2.00	B: -2.00 to +2.00
4	A: +0.00 to +3.00	B: +1.00 to +2.00	A: +0.00 to +1.00	B: +1.00 to +2.00	B: -2.00 to +2.00
5	B: +3.00 to +6.00	B: +1.00 to +2.00	B: +1.00 to +2.00	A: +0.00 to +1.00	A: -6.00 to -2.00
6	A: +0.00 to +3.00	A: +0.00 to +1.00	B: +1.00 to +2.00	A: +0.00 to +1.00	A: -6.00 to -2.00
7	A: +0.00 to +3.00	A: +0.00 to +1.00	A: +0.00 to +1.00	A: +0.00 to +1.00	A: -6.00 to -2.00
8	A: +0.00 to +3.00	A: +0.00 to +1.00	A: +0.00 to +1.00	B: +1.00 to +2.00	B: -2.00 to +2.00
9	A: +0.00 to +3.00	A: +0.00 to +1.00	A: +0.00 to +1.00	B: +1.00 to +2.00	B: -2.00 to +2.00

In [32]:

```
# Splitting the DataFrame into training, validation, and prediction datasets
train_data = df.iloc[:1800]
val_data = df.iloc[1800:2000]
predict_data = df.iloc[2000:]
```

In [33]:

```
# Fit the Bayesian Network model to the training data
model.fit(data=train_data, estimator=BayesianEstimator, prior_type="BDeu", equivalent_sam
ple_size=1000, complete_samples_only=False)

# Extract the target variable 'H' for validation
y_val = val_data['H'].tolist()

# Remove the target variable 'H' from the validation data
val_data.drop('H', axis=1, inplace=True)

# Predict the target variable 'H' on the validation data
y_val_pred = model.predict(val_data)

# Calculate the prediction error on the validation data
error = np.mean(y_val != np.roll(y_val_pred, 1))

# Print the validation error
print("\nError on Validation Data: {:.2f} %".format(error * 100))
```

```
Error on Validation Data: 44.87 %
```

In [34]:

```
# Extract the target variable 'H' for testing
y_test = predict_data['H'].tolist()

# Create a copy of the predict_data DataFrame to avoid modifying the original data
predict_data_copy = predict_data.copy()

# Remove the target variable 'H' from the predict_data
predict_data_copy.drop('H', axis=1, inplace=True)

# Predict the target variable 'H' on the testing data
y_pred = model.predict(predict_data_copy)

# Calculate the prediction error on the testing data
error = np.mean(y_test != np.roll(y_pred, 1))
```

```
# Print the testing error
print("\nError on Testing Data: {:.2f} %".format(error * 100))
```

```
Error on Testing Data: 45.47 %
```

In [35]:

```
# Print the Conditional Probability Distribution (CPD) for the variable 'H' with state na
mes defined.
cpd_h = model.get_cpds('H')
print(cpd_h)
```

```
+----------------------+----------------------+----------------------+
| G                    | G(A: +0.00 to +1.00) | G(B: +1.00 to +2.00) |
+----------------------+----------------------+----------------------+
| H(A: -6.00 to -2.00) | 0.6797385620915033   | 0.4297224709042077   |
+----------------------+----------------------+----------------------+
| H(B: -2.00 to +2.00) | 0.3202614379084967   | 0.5702775290957923   |
+----------------------+----------------------+----------------------+
```

We can see performance improvement with discretization.

5.2.2 Learning Bayesian Network Using 'pgmpy' Hill Climbing

We will perform data preprocessing on our real macro dataset to prepare it for training and validation. The dataset has already undergone cleaning and null value handling. Our next step involves utilizing the Hill Climbing algorithm from the pgmpy library.

In [9]:

```
# First, let's install the fredapi package using pip
!pip install fredapi

# Now, let's import the necessary libraries for our data retrieval and processing
import numpy as np
import pandas as pd
from fredapi import Fred

# The fredapi library is a Python wrapper for the Federal Reserve Economic Data (FRED) AP
I.
# It allows us to easily access and retrieve economic data provided by the Federal Reserv
e.

# With this library, we can programmatically fetch economic data for analysis and researc
h.
```

```
Requirement already satisfied: fredapi in /usr/local/lib/python3.10/dist-packages (0.5.1)
Requirement already satisfied: pandas in /usr/local/lib/python3.10/dist-packages (from fr
edapi) (1.5.3)
Requirement already satisfied: python-dateutil>=2.8.1 in /usr/local/lib/python3.10/dist-p
ackages (from pandas->fredapi) (2.8.2)
Requirement already satisfied: pytz>=2020.1 in /usr/local/lib/python3.10/dist-packages (f
rom pandas->fredapi) (2023.3.post1)
Requirement already satisfied: numpy>=1.21.0 in /usr/local/lib/python3.10/dist-packages (
from pandas->fredapi) (1.23.5)
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.10/dist-packages (from
python-dateutil>=2.8.1->pandas->fredapi) (1.16.0)
```

In [10]:

```
# Import the necessary libraries
import pandas as pd
from fredapi import Fred

# Define the list of data series IDs from the FRED (Federal Reserve Economic Data)
datasets_fred = [
    'FEDFUNDS',          # Federal Funds Rate
    'CPIENGSL',          # Consumer Price Index for All Urban Consumers (CPI-U)
```

```python
    'CUSR0000SEHE',      # Civilian Unemployment Rate
    'PCU211211',         # Producer Price Index for Industrial Commodities
    'PCU213111213111',   # Producer Price Index for Drilling Oil and Gas Wells
    'PCU213111213111P',  # Producer Price Index for Drilling Oil and Gas Wells (PPI)
    'WPS601103',         # Employment-Population Ratio
    'PCU324191324191S',  # Producer Price Index for Petroleum and Coal Products (Seasonall
y Adjusted)
    'PCU3241913241910',  # Producer Price Index for Petroleum and Coal Products (Not Seaso
nally Adjusted)
    'PCU324110324110J',  # Producer Price Index for Petroleum Lubricants and Oils (Jet Fue
l)
    'CAPG211S',          # Capacity Utilization: Total Industry
    'CAPUTLG211S',       # Capacity Utilization: Total Industry, Capacity Utilization Rate
    'IPG211S',           # Industrial Production: Total Index
    'IPG211111CN',       # Industrial Production: Crude Oil Production
    'INDPRO',            # Industrial Production Index
    'IPN213111N',        # Industrial Production: Drilling Oil and Gas Wells
    'IPMINE',            # Industrial Production: Mining
    'IR10000'            # Interest Rates: 10-Year Treasury Constant Maturity Rate
]

# Define a method to retrieve data from FRED
def retrieveFREDData():
    # Specify the FRED API key
    fred_key = "87769799aa7b2dc41a0590ed8a688283"

    # Initialize a session with the FRED API
    fred = Fred(api_key=fred_key)

    # Create an empty DataFrame to store the data
    data_fred = pd.DataFrame()

    # Retrieve data for each series ID and merge it into the DataFrame
    for series_id in datasets_fred:
        series_data = pd.DataFrame(fred.get_series(series_id), columns=[series_id])
        series_data.index.names = ['Date']

        if data_fred.empty:
            data_fred = series_data
        else:
            data_fred = pd.merge(data_fred, series_data, how="inner", left_on='Date', ri
ght_on='Date')

    return data_fred

# Call the method to retrieve FRED data
df_fred = retrieveFREDData()

# Filter the data to include records between January 2003 and December 2023
df_fred = df_fred[(df_fred.index >= '2003-01-01') & (df_fred.index <= '2023-12-01')]

# Display the top 5 records of the filtered data
df_fred.head()

# Create data frames for the current and forecasted spot crude oil prices
fred = Fred(api_key='87769799aa7b2dc41a0590ed8a688283')
current = pd.DataFrame(fred.get_series('WTISPLC'), columns=['WTISPLC'])
current.index.names = ['Date']
forecast = pd.DataFrame(fred.get_series('WTISPLC').shift(-1), columns=['Forecast'])
forecast.index.names = ['Date']

# Merge the spot crude oil prices (current and forecasted) with the macro data frame
df_macro = pd.merge(
    df_fred,
    current,
    how="inner",
    left_on='Date',
    right_on='Date')
```

In [11]:

```python
# Fill missing values in the macro dataset using the backward filling (bfill) and forward
```

```
filling (ffill) techniques
# Backward filling: Fill missing values with the next available value in the column
df_macro.fillna(method='bfill', inplace=True)

# Forward filling: Fill any remaining missing values with the previous available value in
the column
df_macro.fillna(method='ffill', inplace=True)

# Check if there are any remaining missing values in the dataset
# Uncomment the following line to see the count of missing values for each column
# df_macro.isnull().sum()
```

In [12]:

```
# Install the pgmpy library if not already installed
!pip install pgmpy

# Import necessary libraries
import numpy as np
import pandas as pd
from pgmpy.models import BayesianModel  # Import BayesianModel class for creating Bayesia
n Networks
from pgmpy.models import BayesianNetwork  # Import BayesianNetwork class for Bayesian Net
work structure
from pgmpy.estimators import BayesianEstimator  # Import BayesianEstimator for parameter
estimation
from pgmpy.estimators import HillClimbSearch  # Import HillClimbSearch for Bayesian Netwo
rk structure learning
from pgmpy.estimators import BicScore  # Import BicScore for scoring Bayesian Network str
uctures
```

```
Requirement already satisfied: pgmpy in /usr/local/lib/python3.10/dist-packages (0.1.23)
Requirement already satisfied: networkx in /usr/local/lib/python3.10/dist-packages (from
pgmpy) (3.1)
Requirement already satisfied: numpy in /usr/local/lib/python3.10/dist-packages (from pgm
py) (1.23.5)
Requirement already satisfied: scipy in /usr/local/lib/python3.10/dist-packages (from pgm
py) (1.10.1)
Requirement already satisfied: scikit-learn in /usr/local/lib/python3.10/dist-packages (f
rom pgmpy) (1.2.2)
Requirement already satisfied: pandas in /usr/local/lib/python3.10/dist-packages (from pg
mpy) (1.5.3)
Requirement already satisfied: pyparsing in /usr/local/lib/python3.10/dist-packages (from
pgmpy) (3.1.1)
Requirement already satisfied: torch in /usr/local/lib/python3.10/dist-packages (from pgm
py) (2.0.1+cu118)
Requirement already satisfied: statsmodels in /usr/local/lib/python3.10/dist-packages (fr
om pgmpy) (0.14.0)
Requirement already satisfied: tqdm in /usr/local/lib/python3.10/dist-packages (from pgmp
y) (4.66.1)
Requirement already satisfied: joblib in /usr/local/lib/python3.10/dist-packages (from pg
mpy) (1.3.2)
Requirement already satisfied: opt-einsum in /usr/local/lib/python3.10/dist-packages (fro
m pgmpy) (3.3.0)
Requirement already satisfied: python-dateutil>=2.8.1 in /usr/local/lib/python3.10/dist-p
ackages (from pandas->pgmpy) (2.8.2)
Requirement already satisfied: pytz>=2020.1 in /usr/local/lib/python3.10/dist-packages (f
rom pandas->pgmpy) (2023.3.post1)
Requirement already satisfied: threadpoolctl>=2.0.0 in /usr/local/lib/python3.10/dist-pac
kages (from scikit-learn->pgmpy) (3.2.0)
Requirement already satisfied: patsy>=0.5.2 in /usr/local/lib/python3.10/dist-packages (f
rom statsmodels->pgmpy) (0.5.3)
Requirement already satisfied: packaging>=21.3 in /usr/local/lib/python3.10/dist-packages
(from statsmodels->pgmpy) (23.1)
Requirement already satisfied: filelock in /usr/local/lib/python3.10/dist-packages (from
torch->pgmpy) (3.12.2)
Requirement already satisfied: typing-extensions in /usr/local/lib/python3.10/dist-packag
es (from torch->pgmpy) (4.5.0)
Requirement already satisfied: sympy in /usr/local/lib/python3.10/dist-packages (from tor
ch->pgmpy) (1.12)
Requirement already satisfied: jinja2 in /usr/local/lib/python3.10/dist-packages (from to
rch->pgmpy) (3.1.2)
```

```
Requirement already satisfied: triton==2.0.0 in /usr/local/lib/python3.10/dist-packages (from torch->pgmpy) (2.0.0)
Requirement already satisfied: cmake in /usr/local/lib/python3.10/dist-packages (from triton==2.0.0->torch->pgmpy) (3.27.4.1)
Requirement already satisfied: lit in /usr/local/lib/python3.10/dist-packages (from triton==2.0.0->torch->pgmpy) (16.0.6)
Requirement already satisfied: six in /usr/local/lib/python3.10/dist-packages (from patsy>=0.5.2->statsmodels->pgmpy) (1.16.0)
Requirement already satisfied: MarkupSafe>=2.0 in /usr/local/lib/python3.10/dist-packages (from jinja2->torch->pgmpy) (2.1.3)
Requirement already satisfied: mpmath>=0.19 in /usr/local/lib/python3.10/dist-packages (from sympy->torch->pgmpy) (1.3.0)
```

In [13]:

```
# Reset the index of the DataFrame and drop the old index
df_macro.reset_index(drop=True, inplace=True)
```

In [14]:

```
# Create a HillClimbSearch object using the df_macro DataFrame
hc = HillClimbSearch(df_macro)

# Perform the Hill Climbing search for the best model with a maximum of 10 iterations
best_model = hc.estimate(max_iter=10)

# Print the edges of the best model found
print(best_model.edges())
```

```
[('CPIENGSL', 'CUSR0000SEHE'), ('CUSR0000SEHE', 'CAPUTLG211S'), ('PCU324191324191S', 'CPIENGSL'), ('CAPUTLG211S', 'IPG211S'), ('IPG211S', 'INDPRO'), ('INDPRO', 'IPN213111N'), ('INDPRO', 'IPMINE'), ('INDPRO', 'IPG211111CN'), ('INDPRO', 'WTISPLC'), ('IPN213111N', 'CAPG211S')]
```

In [15]:

```
# Split the df_macro DataFrame into training, validation, and prediction datasets
train_data = df_macro[:180]
val_data = df_macro[180:200]
predict_data = df_macro[200:]
```

In [16]:

```
# Create a Bayesian Network model using the best_model's edges
model = BayesianNetwork(best_model.edges())

# Fit the model with the training data
model.fit(train_data)
```

In [17]:

```
model.nodes
```

Out[17]:

```
NodeView(('CPIENGSL', 'CUSR0000SEHE', 'CAPUTLG211S', 'PCU324191324191S', 'IPG211S', 'INDPRO', 'IPN213111N', 'IPMINE', 'IPG211111CN', 'WTISPLC', 'CAPG211S'))
```

In [20]:

```
# Select the desired columns from the DataFrame
selected_columns = ['CPIENGSL', 'WTISPLC', 'PCU324191324191S', 'CAPG211S', 'IPN213111N',
'CAPUTLG211S', 'IPMINE', 'IPG211111CN', 'INDPRO', 'CUSR0000SEHE', 'IPG211S']
data = df_macro.loc[:, selected_columns]

# Assign the entire dataset to the training data
train_data = data

# Slice the validation data from row 180 to 199
val_data = data[180:200]
```

```python
# Slice the prediction data from row 200 onwards
predict_data = data[200:]

# Fit the Bayesian Network model with the training data
model.fit(train_data)

# Extract the 'WTISPLC' column values from the validation data
y_val = val_data['WTISPLC'].tolist()

# Drop the 'WTISPLC' column from the validation data
val_data.drop('WTISPLC', axis=1, inplace=True)

# Predict using the Bayesian Network model on the validation data
model.predict(val_data)

# Extract the 'WTISPLC' column values from the training data
y_train = train_data['WTISPLC'].tolist()

# Drop the 'WTISPLC' column from the training data
train_data.drop('WTISPLC', axis=1, inplace=True)

# Predict using the Bayesian Network model on the training data
model.predict(val_data)

# Extract the 'WTISPLC' column values from the prediction data
y_test = predict_data['WTISPLC'].tolist()

# Create a copy of the prediction data and drop the 'WTISPLC' column
predict_data = predict_data.copy()
predict_data.drop('WTISPLC', axis=1, inplace=True)

# Predict using the Bayesian Network model on the prediction data
y_pred = model.predict(predict_data)
```

```
<ipython-input-20-350caa87d98e>:21: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user_g
uide/indexing.html#returning-a-view-versus-a-copy
  val_data.drop('WTISPLC', axis=1, inplace=True)
```