Submission Number: **1**

Group Number: **02**

Group Members: 3

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Statement of integrity: By typing the names of all group members in the text box below, you confirm that the assignment submitted is original work produced by the group (*excluding any non-contributing members identified with an "X" above*).

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Use the box below to explain any attempts to reach out to a non-contributing member. Type (N/A) if all members contributed.

N/A

** Note, you may be required to provide proof of your outreach to non-contributing members upon request.*

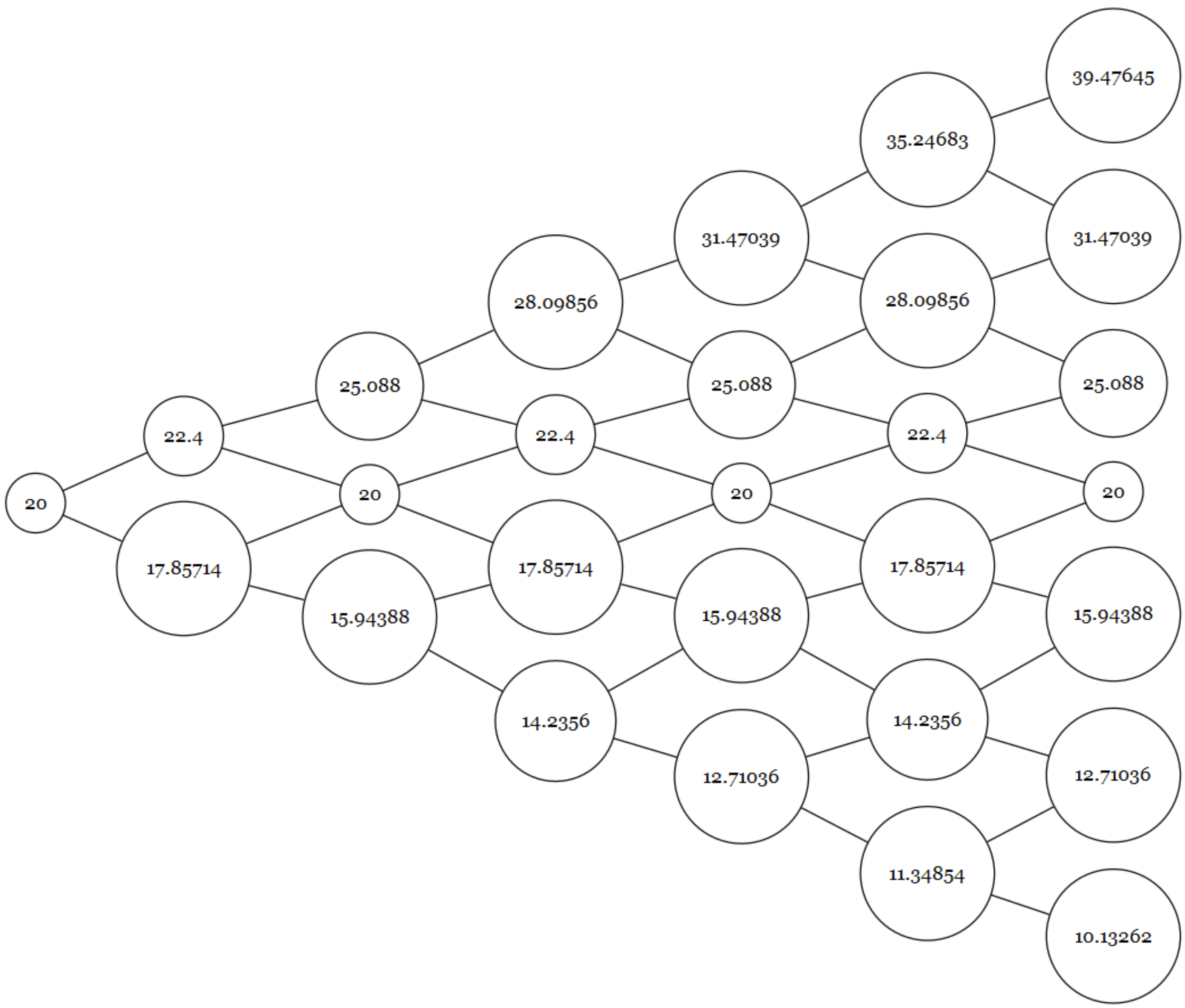
1(a) Binomial tree for N=6

Below is a six step (N = 6) binomial tree for stock price evolution. Upper movement of the price (u) is set based on our group number as follows:

u = (1.10 + Group Number /100) = 1.10 + 2/100 = 1.12

Downward movement (d) of the stock price evolution is d = 1/u and the initial stock price is S_0 = 20.

Below is the binomial tree generated using R code.



MScFE 620 Discrete-time Stochastic Processes

Group #: ___02__ Submission #: ___1__

1(b) The terminal values of each path and associated Natural Filtrations

Let $\Omega = \{\omega\colon \omega\ is\ a\ six\ step\ random\ walk\}$, where u=upward movement and d=downward movement.

Let (Ω, Ŧ, 𝔽, ℙ) be a filtered probability space with a natural filtration given by:

$$\mathbb{F} = (\mathbb{F}_{\mathbf{0}}^{\boldsymbol{X}}, \mathbb{F}_{\mathbf{1}}^{\boldsymbol{X}}, \mathbb{F}_{\mathbf{2}}^{\boldsymbol{X}}, \mathbb{F}_{\mathbf{3}}^{\boldsymbol{X}}, \mathbb{F}_{\mathbf{4}}^{\boldsymbol{X}}, \mathbb{F}_{\mathbf{5}}^{\boldsymbol{X}}, \mathbb{F}_{\mathbf{6}}^{\boldsymbol{X}})$$

Let $\mathbb{I} = \{0, 1, 2, 3, 4, 5, 6\}$ be the index set

X: Ω → ℝ

A stochastic process $X = \{X_t\colon\ t \in \mathbb{I}\}$ is adapted to the filtration 𝔽 where:

$$\mathbb{F}_0^X = (\emptyset, \Omega)$$

$$\mathbb{F}_1^X = \sigma(\{\{u\}, \{d\}\})$$

$$\mathbb{F}_2^X = \sigma(\{\{uu\}, \{du, up\}, \{dd\}\})$$

$$\mathbb{F}_3^X = \sigma(\{\{uuu\}, \{udu, uud, duu\}, \{udd, dud, ddu\}, \{ddd\}\})$$

$$\mathbb{F}_4^X = \sigma(\{\{uuuu\}, \{uuud, uudu, uduu, duuu\}, \{uudd, udud, uddu, dudu, dduu\}, \{uddd, dudd, ddud, dddu\}, \{dddd\}\})$$

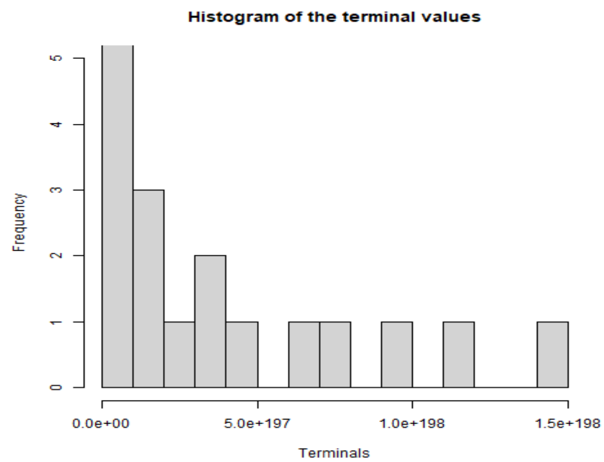
$$\begin{aligned}\mathbb{F}_5^X = \sigma(\{\{uuuuu\}, &\{uuuud, uuudu, uuduu, uduuu, duuuu\},\\ &\{uuudd, uudud, uuddu, uduud, ududu, udduu, duduu, dduuu, duudu, duuud\},\\ &\{uuddd, ududd, uddud, udddu, duudd, dudud, duddu, ddudu, dduud, ddduu\},\\ &\{udddd, duddd, ddudd, dddud, ddddu\}, \{ddddd\}\})\end{aligned}$$

$$\mathbb{F}_6^X = \sigma(2^{\Omega})$$

The terminal values of each path = {39.47645, 31.47039, 25.088, 15.94388, 12.71036, 10.13262}

2. **Binomial tree for N=4000**

a. Terminal prices produced by the model plot



b. The terminal prices resemble a Fisher–Snedecor distribution (F-Distribution) or a Chi-Square Distribution.

c. $Return = \ln\left(\frac{price_t}{price_{t-1}}\right)$ where $price_t$ is the prices of the stock at time t.

$Return$ follows a normal distribution with a mean μ and variance σ^2 i.e., $Return \sim N(\mu, \sigma^2)$

Let Ω = ℝ be a sample space defined on a real number line and (Ω, 𝔽, ℙ) be a probability space.

𝔽 be a σ-algebra define on Ω and ℙ is a probability measure.

Let 𝔹(ℝ) be a Borel σ-algebra generated by intervals ℝ where the elements are called Borel sets.

X = $Return$

X: (Ω, 𝔽) → (ℝ, 𝔹(ℝ)) is a random variable.

The random variable X has a normal distribution and is absolutely continuous with a density given by:

$\frac{d\mathbb{P}_X}{d\lambda_1}(x) = \frac{1}{\sqrt{2\Pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$, $x \in \mathbb{R}$ and λ_1 is a Lebesgue measure.

3. Market completeness

a. There are two fundamental securities in the market namely:
   i. Debt Securities and
   ii. Equity Securities

   Other forms of securities are derived from the fundamental securities.

b. At any given node there are only two possible future states in a binomial tree. One of the states is a known fixed upward price movement and the other is a some known fixed downward price movement.

c. A complete market is one in which there is availability of perfect information. Prices for all assets (Debt Securities and Equity Securities) are known for all possible future states. Thus, in a complete market there are no surprises in price movements and the assets are of basic form without any complex methodology in their pricing.

d. In a complete market there are no surprises in price movements. Jumps in prices would mean non availability of perfect information in the market and there will not be a price for all future states of assets. Thus, with jumps in prices the market will not be complete.

References

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