Submission Number: **2**

Group Number: **02**

Group Members: 3

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Statement of integrity: By typing the names of all group members in the text box below, you confirm that the assignment submitted is original work produced by the group (*excluding any non-contributing members identified with an "X" above*).

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Use the box below to explain any attempts to reach out to a non-contributing member. Type (N/A) if all members contributed.

N/A

** Note, you may be required to provide proof of your outreach to non-contributing members upon request.*

1(a) European Call Option Pricing

Upper movement of the price (u) is set as follows:

u = (1.10 + Group Number /100) = 1.10 + 2/100 = 1.12

Downward movement (d) of the stock price evolution is d = 1/u and the initial stock price is S_0 = 95. The strike price is K=105.

Risk-Neutral upward movement probability is given by:

$$\mathbb{P}^* = \frac{1-d}{u-d} = \frac{25}{53} = p^*$$

Risk-Neutral downward movement probability is given by:

$$q = 1 - p^* = \frac{28}{53}$$

The payoff function **H** of the European Call option is given by:

$$H = \max\{S_T - K, 0\}$$

The price of the European Call option is given by:

$$\pi_C = \pi(H) = \mathbb{E}^*(H) = \sum_{y=0}^{T} (S_0 u^y d^{T-y} - K)^+ \binom{T}{y} p^{*-y} (1-p^*)^{T-y} = 3.481498$$

Binomial tree generated using R code.

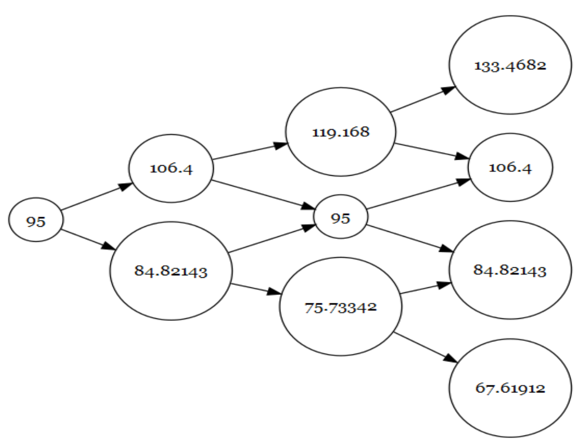

Figure 1.1: Stock Price Evolution

1(b) Value of the derivative H(ω)

Below is a plot generated using R code and it shows the derivative price for all sample paths.

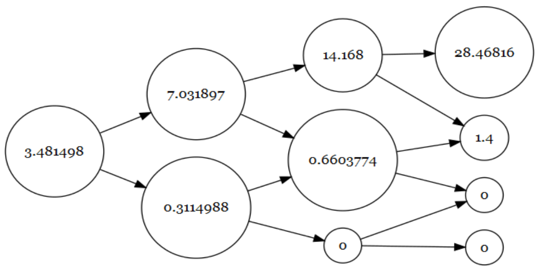

Figure 1.2: Derivative Price Values

2(a) European Put Option Pricing

The payoff function **H** of the European Put option is given by:

$$H = \max\{K - S_T, 0\}$$

The price of the European Put option is given by:

$$\pi_P = \pi(H) = \mathbb{E}^*(H) = \sum_{y=0}^{T} (K - S_0 u^y d^{T-y})^+ \binom{T}{y} p^{*-y} (1 - p^*)^{T-y} = 13.15237$$

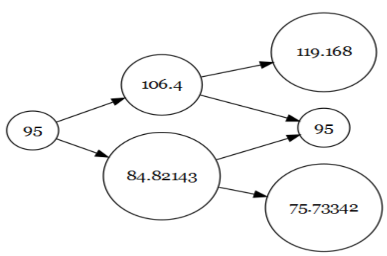

Figure 2.1: Stock Price Evolution

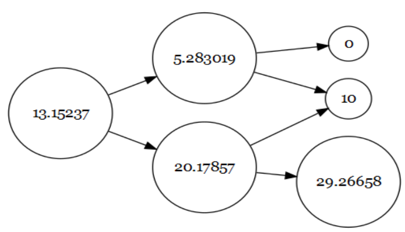

Figure 2.2: Derivative Price Values

2(b) Hedging

Below is a table generated using R code and it shows stock price evolution and the derivative price for all sample paths.

Table 2.1: European Put Option Hedging

ω	$X_0(\omega)$	$X_1(\omega)$	$X_2(\omega)$	$H(\omega)$	$V_0^H(\omega)$	$V_1^H(\omega)$	$V_2^H(\omega)$	$\psi_1^H(\omega)$	$\psi_2^H(\omega)$
(u,u)	95	106.40000	119.16800	0.00000	13.15237	5.283019	0.00000	-0.6902937	-0.4137703
(u,d)	95	106.40000	95.00000	10.00000	13.15237	5.283019	10.00000	-0.6902937	-0.4137703
(d,u)	95	84.82143	95.00000	10.00000	13.15237	20.178571	10.00000	-0.6902937	-1.0000000
(d,d)	95	84.82143	75.73342	29.26658	13.15237	20.178571	29.26658	-0.6902937	-1.0000000

3. Market completeness

We pick the node at time **t=1** for the stock evolution and the resulting matrix A is given by:

$$A = \begin{pmatrix} 1 & 106.4 \\ 1 & 84.82143 \end{pmatrix}$$

First column represents the Bond values and second column the stock values.

Matrix **b** is the payoff of the European Put Option at time **t=1** and is given by:

$$b = \begin{pmatrix} 5.283019 \\ 20.17857 \end{pmatrix}$$

First row represents the payoff values when the stock price goes up and the second row represents the payoff when the stock price goes down.

The no arbitrage equation is given by:

$Ax = b$ and x is a column matrix of the hedging strategy.

We solve for x by using the R code $x = solve(A, b)$

$$x = \begin{pmatrix} \psi_1^0 \\ \psi_1^1 \end{pmatrix} = \begin{pmatrix} 78.73027 \\ -0.6902937 \end{pmatrix}$$

It must be observed that the solution matches the binomial tree solutions presented in table 2.1.

4. Put Call Parity

We have a two-step binomial model, where the stock price can rise by a fixed factor **u = 1.12** or fall by a fixed factor **d = 1/u**. Time horizon is **T=1** and the price of the stock at time **t =0** is **S_0 = 95**. The constant risk-free rate is **r=0.05**. The strike price is **K=95**.

The risk neutral probability for an upward stock price movement is given by:

$$p = \frac{1 + r - d}{u - d}$$

The price of a European Call option at time **t=T=1** is given by:

$$C_T = \max\{S_T - K, 0\} = 24.168$$

The price of a European Put option at time **t=T=1** is given by:

$$P_T = \max\{K - S_T, 0\} = 19.26658$$

The price of a European Call option at time **t=0** is given by:

$$C_0 = \mathbb{E}_p\left[\frac{C_T}{(1+r)^T}\right] = 11.5673$$

The price of a European Put option at time **t=0** is given by:

$$P_0 = \mathbb{E}_p\left[\frac{P_T}{(1+r)^T}\right] = 1.829795$$

4(a)

- **i.** **Portfolio A**
  The value of the portfolio is $P_0 - C_0 = -9.7375$
- **ii.** **Portfolio B**
  The value of the portfolio is $K - S_0 = 0$

4(b)

From 4(a) we see that the strike price (**K**) and the initial stock price (**S_0**) are equal, but the call option price (**C_0**) is not equal to the put option price (**P_0**) hence the current state of the values does not satisfy the put-call parity. To satisfy the put-call parity portfolio A and portfolio B must be equal.

Failure to satisfy the put-call parity can be attributed to the choice of the strike price and the value of the risk-free interest rate. Appropriate values would give the same portfolio values with respect to portfolio A and portfolio B.

References

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