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Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an "X" above).	
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Use the box below to explain any attempts to reach out to a non-contributing member. Type (N/A) if all members contributed.
Note: You may be required to provide proof of your outreach to non-contributing members upon request.

N/A

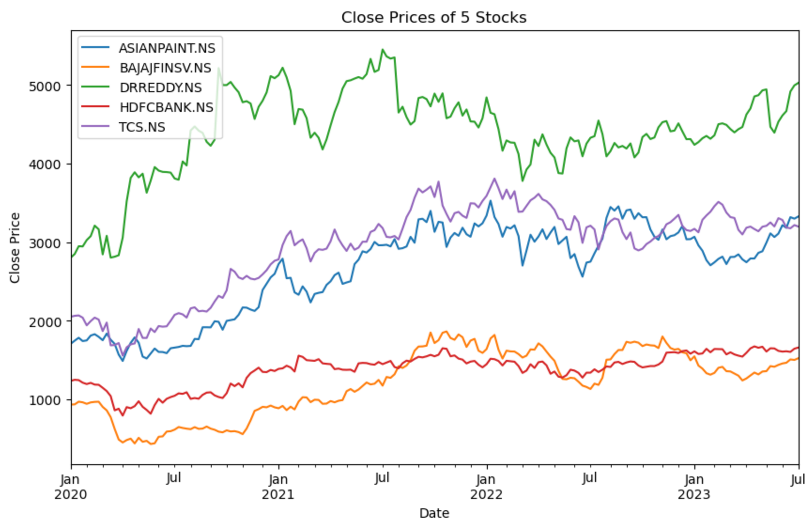
TASK 1:

a. Select a portfolio with a minimum of 5 securities. (Note that you should choose stocks for which you can find recent news, headlines, analyst reports, etc.)

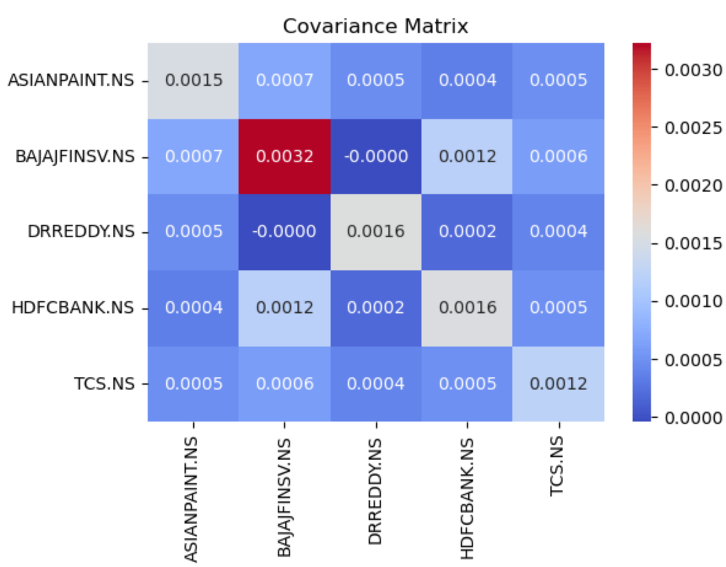
We selected 5 stocks from the Nifty50 HDFCBANK (Banking), ASIANPAINTS (Building Materials), BAJAJ Finance (NBFC), DR REDDY (Pharmaceuticals), TCS (Information Technology). All these companies were recently in news after their quarterly results.

b. Download daily or weekly returns for a period that allows you to have at least 100 data points (about 6 months daily returns, or 2 years of weekly returns). Assume you cannot short any securities.

We have used 2.5 years of weekly data for analysis.

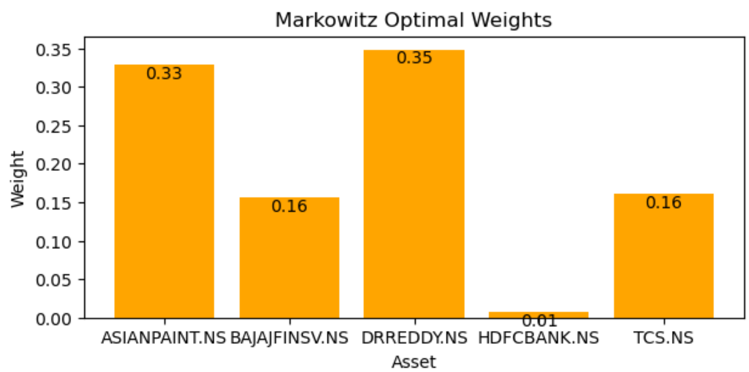


c. Compute the covariance matrix from the observations.



- ASIANPAINT.NS: Relatively low covariance with other stocks in the portfolio, can provide diversification.
- BAJAJFINSV.NS: Higher compared to ASIANPAINT.NS. potential to higher co-movements in prices.
- DRREDDY.NS: Moderate covariance with other stocks but negative with BAJAJFINSV. Might give diversification chances.
- HDFCBANK.NS: Moderate to high covariance. Being a bank the alignment is logical.
- TCS.NS: Moderate covariance.

d. Run a classical Markowitz portfolio optimization. Be sure to print / display / graph the weights of each security in the portfolio optimization results.




```
Average Daily Return
ASIANPAINT.NS    0.004438
BAJAJFINSV.NS    0.004296
DRREDDY.NS       0.003993
HDFCBANK.NS      0.002392
TCS.NS           0.003060
```

In terms of returns HDFCBANK has the lowest of the daily returns and hence we see the lowest weight given to it. BAJAJFINSV is a very high Beta Stock (1.96) therefore lower weight despite high return si also a logical outcome.

TASK 2: Black-Litterman Model (BL model)

Theory

The BL model allows us to utilize the Bayesian approach to get better optimal weights through their calculation being dependent on both the historical data and the portfolio manager's point of view. This portfolio manager's view is quantitative in nature, but it can even be derived from the real-world events which are qualitative, like the news, if the manager is willing/able to define the event in terms to quantitative change in return. Bayes Theorem helps us link the probability of the effect being due to event to the probability of the event, probability of the effect and the probability of the event given the effect is observed. This is achieved by using the technique of reverse optimization, which is basically to calculate the expected value of posterior returns(π) based on market capitalization weights, assuming that market portfolio is our optimal portfolio, in conjunction with the risk averse factor(d), which come out to be market portfolio's Sharpe ratio, and the covariance of the assets in consideration. We then calculate the error corresponding to the calculation of π, denoted as Δ. Therefore, the combined error/standard deviation for the expected posterior returns comes from both asset historical covariance and this Δ.

Implementation

Step 1 – Calculation of all variables and definition of all constants

```
#all necessary variables
mkt_sharpe_ratio = 1.67
mkt_volatility=0.75
rf_rate=0.02
r=1/(data.shape[0]-data.shape[1])
port_eq_wts=mkt_top_50.loc[['ASIANPAINT','BAJAJFINSV','DRREDDY','HDFCBANK','TCS'],"eq_mkt_wt"]
port_eq_wts.index=tickers
# port_eq_wts=port_eq_wts/port_eq_wts.sum()
port_eq_wts
```

```
ASIANPAINT.NS     0.008642
BAJAJFINSV.NS     0.013161
DRREDDY.NS        0.005902
HDFCBANK.NS       0.126520
TCS.NS            0.019522
Name: eq_mkt_wt, dtype: float64
```

Here, we've utilized NIFTY 50's Sharpe ratio as 'd' risk aversion constant.

$$\pi = d\Sigma w_m$$

Where $w_m = c_i / c_m$

c_m = NIFTY 50 market capitalization = INR 246,991,688,592.80

c_i is the individual assets capitalization.

Step 2 – Portfolio manager's views are input using matrices P and Q. where Q defines the different opinion's threshold and P defines the respective number of opinions based on the number of rows it contains, that is each row has one opinion. There are two types of opinion:

1. Absolute – then the row sum corresponding to absolute opinion is 1.
2. Relative – then the row sum corresponding to absolute opinion is 0

Here we have considered three opinions, as follows:

```
In [130]: #manager views
P=np.array([[1,0,0,0,0],[0,-1,0,1,0],[0,0,1,0,0]])
Q=np.array([-0.05,0.025,0.1])
```

View 1 - Asian Paints is negative (Year End was good, but in Jan they acquired a company hence cash is low) so any activity from other firms in the same industry could impact it. And the expected decrement is 5 %
View 2- HDFC Bank will outperform BajajFinsv by 2.5%, HDFC Bank positive, but the HDFC and HDFC BANK merger is on so there are structural uncertainties.
View 3- DRReddy will perform well with increased expected decrement by 10%.

1. **Step 3- Calculation of expected posterior return and covariance based on below formulas.**
   [Walters, Jay. "The Black-Litterman Model in Detail." 2014. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1314585]

Formula 1

$$\mu_{BL} := E[r] = \pi + \tau\Sigma P^T[(P\tau\Sigma P^T) + \Omega]^{-1}[q - P\pi].$$

Formula 2

$$\Sigma_{BL} := \tilde{\Sigma} = \Sigma + M = \Sigma + ((\tau\Sigma)^{-1} + P^T\Omega^{-1}P)^{-1}.$$

```
In [131]: #BL implementation
#return calculation
returns = data.pct_change()
returns = returns.dropna()
#cov_matrix
covar=returns.cov()
#Step-2 calculation of pie-posterior estimate of expected returns
pie=mkt_sharpe_ratio*np.matmul(covar,port_eq_wts)
delta=r*covar
omega= np.matmul(np.matmul(P,delta),P.T)
```

```
In [132]: #expected posterior return
exp_post_ret=pie+np.matmul(np.matmul(np.matmul(delta,P.T),np.linalg.inv(omega+np.matmul(P,np.matmul(delta,P.T)))
                             ),(Q-np.matmul(P,pie)))
exp_post_ret

Out[132]: ASIANPAINT.NS   -0.024982
BAJAJFINSV.NS   -0.014694
DRREDDY.NS       0.050009
HDFCBANK.NS     -0.002189
TCS.NS           0.003040
dtype: float64
```

```
In [133]: exp_post_covar=covar+np.linalg.inv(np.linalg.inv(delta)+np.matmul(P.T,np.matmul(np.linalg.inv(omega),P)))
exp_post_covar
```

Out[133]:

	ASIANPAINT.NS	BAJAJFINSV.NS	DRREDDY.NS	HDFCBANK.NS	TCS.NS
ASIANPAINT.NS	0.000321	0.000172	0.000084	0.000109	0.000098
BAJAJFINSV.NS	0.000172	0.000639	0.000066	0.000255	0.000144
DRREDDY.NS	0.000084	0.000066	0.000300	0.000050	0.000079
HDFCBANK.NS	0.000109	0.000255	0.000050	0.000351	0.000111
TCS.NS	0.000098	0.000144	0.000079	0.000111	0.000268

Step 4- Optimizing weights by maximizing below:

[Walters, Jay. "The Black-Litterman Model in Detail." 2014. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1314585]

$$\max_w w^T \mu_{BL} - \frac{1}{2} w^T \Sigma_{BL} w$$

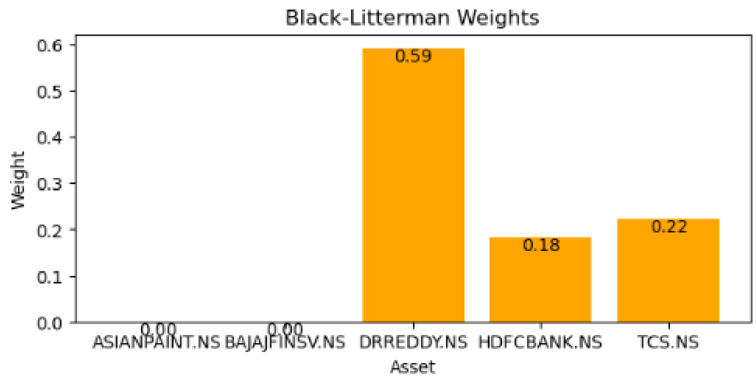
Constrained to all the weights adding to 1.

In [98]:
```
# Define function to maximise maximising function
def max_function(weights, mean_ret, cov, rf):
    a=np.matmul(weights.T,mean_ret)
    b=0.5*np.sqrt(np.dot(weights.T, np.dot(cov, weights))) * np.sqrt(252)
    op_fun = a-b
    return -op_fun

def max_sharpe_ratio(mean_ret, cov, rf):
    num_assets = len(mean_ret)
    args = (mean_ret, cov, rf)
    constraints = ({'type': 'eq', 'fun': lambda x: np.sum(x) - 1})
    bound = (0.0,1.0)
    bounds = tuple(bound for asset in range(num_assets))
    result = sco.minimize(max_function, num_assets*[1./num_assets,], args=args,
                          method='SLSQP', bounds=bounds, constraints=constraints)
    return result
```

In [99]:
```
optimal_portfolio_sharpe = max_sharpe_ratio(exp_post_ret, exp_post_covar, rf_rate)
opt_wts=optimal_portfolio_sharpe['x']
opt_wts=opt_wts/opt_wts.sum()
```

In [100…
```
plot_weights(opt_wts, 'Black-Litterman Weights')
```

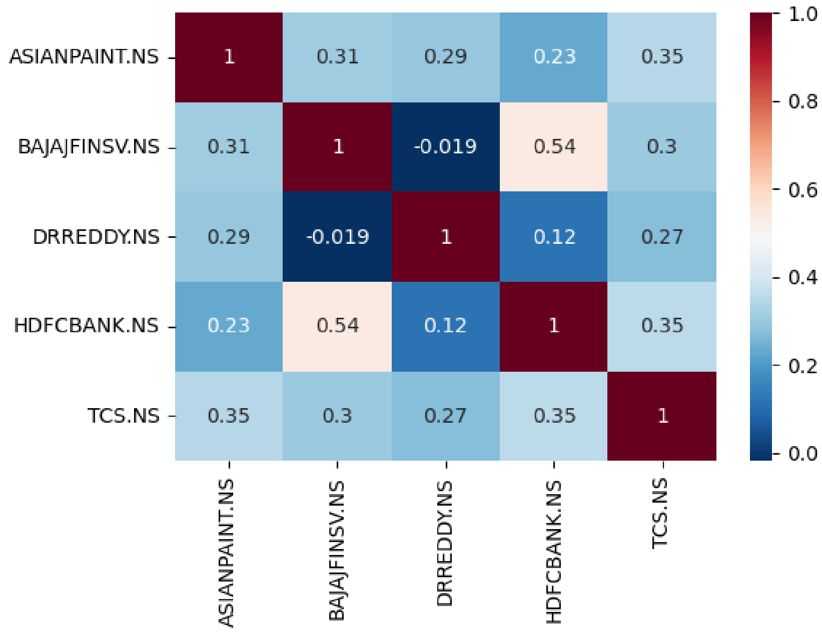


We conclude that the portfolio manager's opinions about the different assets had profound impact post optimization.

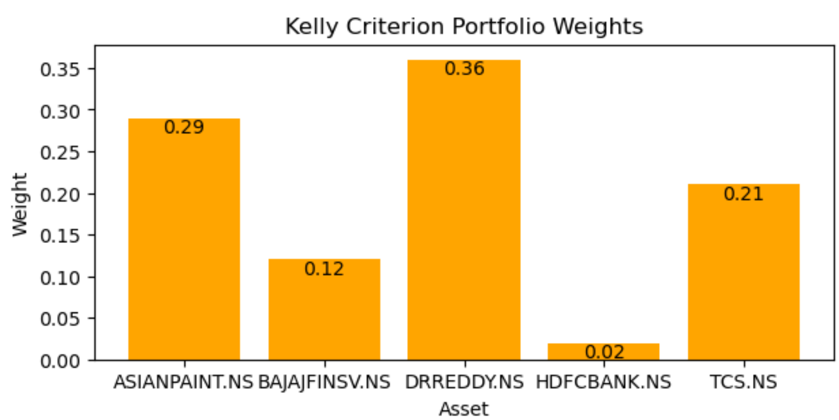
TASK 3:

a. Perform back-testing using the Kelly criterion for each security in the portfolio to size the allocation to that security.

The stocks selected have a weak to moderate positive correlation. Hence we have proceeded with correlated analysis for Kelly criterion



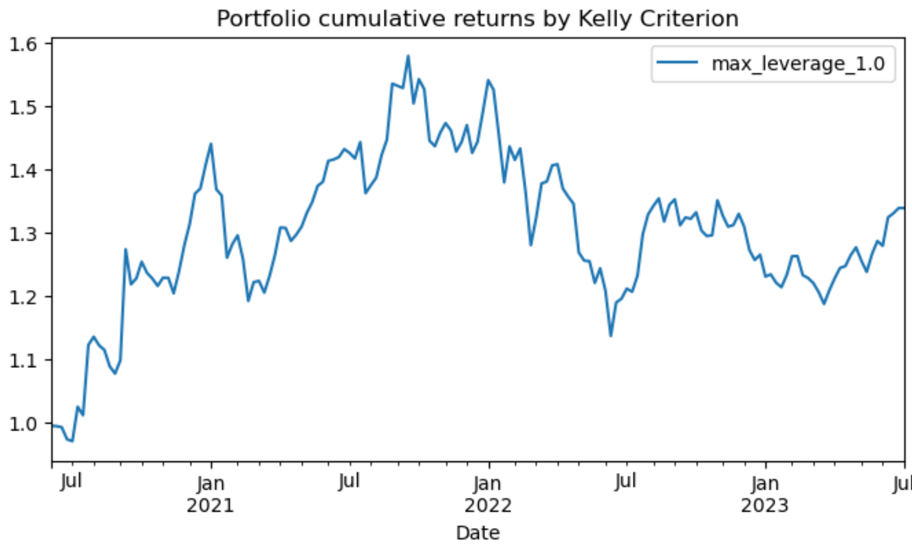
Kelly Criterion weight is on similar lines to that of the Markowitz Optimal weight.

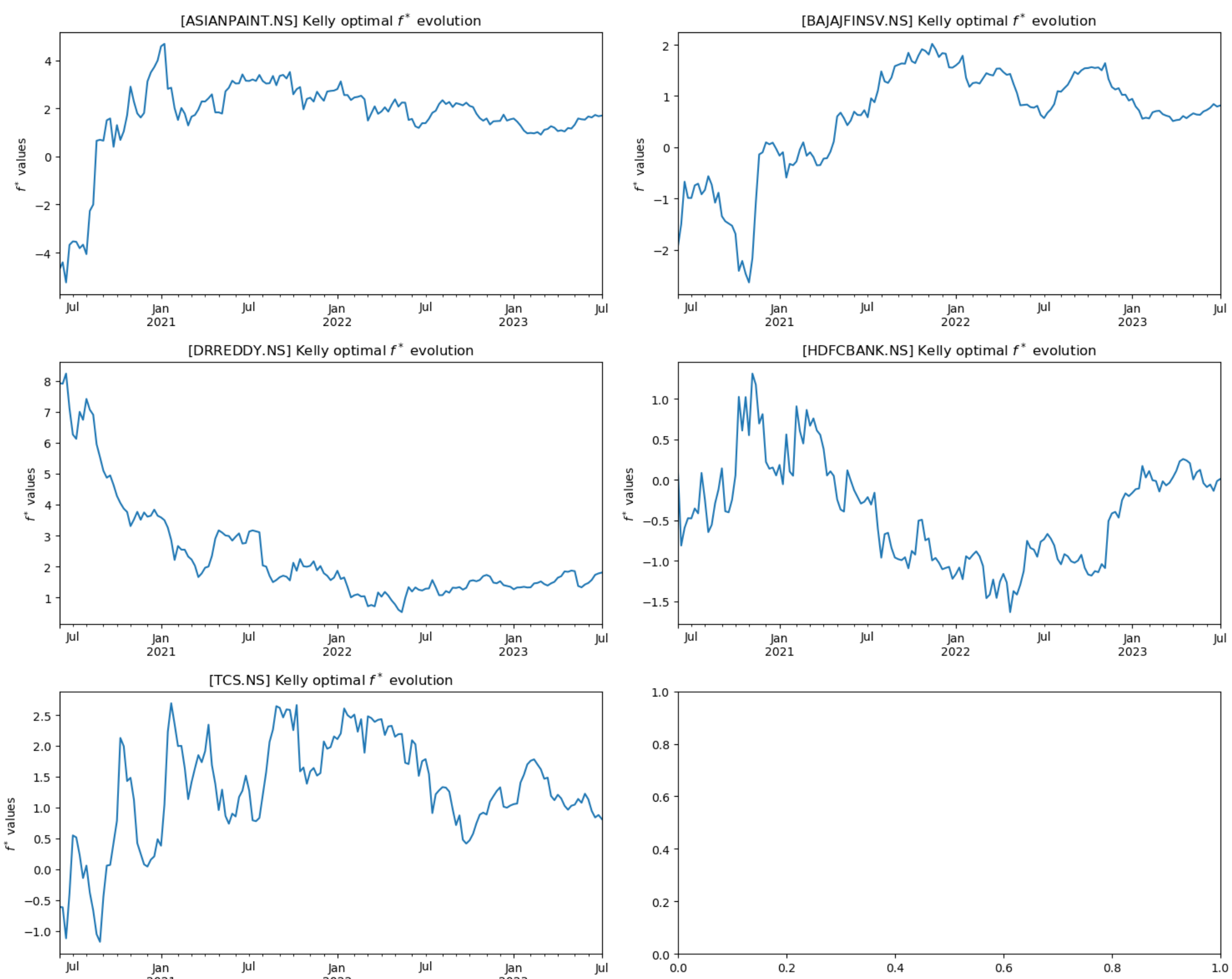


b. Perform a series of historical backtests to see how the combined portfolio performs.

Kelly criterion allocation to the stocks delivers a healthy return of ~56% with relatively how std dev and a healthy risk-adjusted return as evident by the Sharpe ration of 1.1808.

```
Kelly Returns: 55.91%
Kelly Std. Dev.:  0.0288
Kelly Sharpe Ratio:  1.1808
```


Portfolio cumulative returns by Kelly Criterion
max_leverage_1.0
1.6
1.5
1.4
1.3
1.2
1.1
1.0
Jul
Jan
2021
Jul
Jan
2022
Jul
Jan
2023
Jul
Date


[ASIANPAINT.NS] Kelly optimal f^* evolution
[BAJAJFINSV.NS] Kelly optimal f^* evolution
[DRREDDY.NS] Kelly optimal f^* evolution
[HDFCBANK.NS] Kelly optimal f^* evolution
[TCS.NS] Kelly optimal f^* evolution
f^* values
Jul
Jan
2021
Jul
Jan
2022
Jul
Jan
2023
Jul

c. Write 1-2 pages of background information. The background information provides equations, explains the terms in the equation, distinguishes inputs and outputs, explains how the parameters are estimated and calibrated, gives any interpretation to what the parameters mean, etc. The background is a technical summary of how the model works using equations, graphs, figures, tables, charts, and other illustrations, along with written explanations and interpretations.

The criteria is used to find out one possible allocation strategy of investments into assets for long term appreciation in a portfolio of investible assets.

Suppose that some random variable X represent some assets return that

$$P[X = \mu + \sigma] = \frac{1}{2} = P[X = \mu - \sigma]$$

Assume the capital is V0, r is risk free rate and f is the capital allocated to risky assets.

V1(f) = V0(1+(1-f)r +fX) = V0(1 + r + f(X – r))

Calculating the expected log return V1(f)/V0, will be

$$G(f) = E\left[ln\frac{V_1(f)}{V_0}\right] = \frac{1}{2}\ln\left(1 + r + f(\mu + \sigma - r)\right) + \frac{1}{2}\ln\left(1 + r + f(\mu - \sigma - r)\right)$$

The time interval is divided in n equal independent steps and the mean and variance is kept constant. Therefore instead of single return X, we have to consider n different returns, Xi, i=1,2,….,n where

$$P[X_i = \frac{\mu}{n} + \frac{\sigma}{\sqrt{n}}] = \frac{1}{2} = P[X_i = \frac{\mu}{n} - \frac{\sigma}{\sqrt{n}}]$$

Then

$$\left(\frac{V_n(f)}{V_0}\right) = \prod_{i=1}^{n}\left(1 + \frac{r}{n} + f\left(X_i - \frac{r}{n}\right)\right)$$

and

$$ln\left(\frac{V_n(f)}{V_0}\right) = \sum_{i=1}^{n} ln\left(1 + \frac{r}{n} + f\left(X_i - \frac{r}{n}\right)\right)$$

Simplifying using Taylor series,

$$\ln(1 + z) = z - \frac{z^2}{2} + \frac{z^3}{3} + \cdots + \frac{(-1)^{n+1}z^n}{n}$$

in our case

$$z = \frac{r}{n} + f\left(X_i \pm \frac{r}{n}\right)$$

which will give

$$z = \frac{r^2}{n^2} + f^2\left(X_i - \frac{r}{n}\right)^2 + 2\frac{r}{n}f\left(X_i - \frac{r}{n}\right)$$

Some terms will go to zero as n approached infinity. We will not require terms that are smaller than O($n^{-3/2}$).Now when we take the expected value of z2 the relevant term will be

$$E\lfloor f^2 X_i^2 \rfloor = f^2 E\lfloor x_i^2 \rfloor = f^2\left(\frac{\sigma^2}{n} + \frac{\mu^2}{n^2}\right) \sim f^2 \frac{\sigma^2}{n}$$

Therefore,

$$G_i(f) = \frac{r}{n} + \frac{f}{n}(\mu - r) + \frac{f^2\sigma^2}{2n} + O(n^{-3/2})$$

Finally,

$$G(f) = \lim_{n\to\infty} \sum_{i=1}^{n} G_i(f) = r + f(\mu - r) + \frac{f^2\sigma^2}{2} + \lim_{n\to\infty} O\left(n^{-\frac{1}{2}}\right) = r + f(\mu - r) + \frac{f^2\sigma^2}{2}$$

G(f) can be maximised by

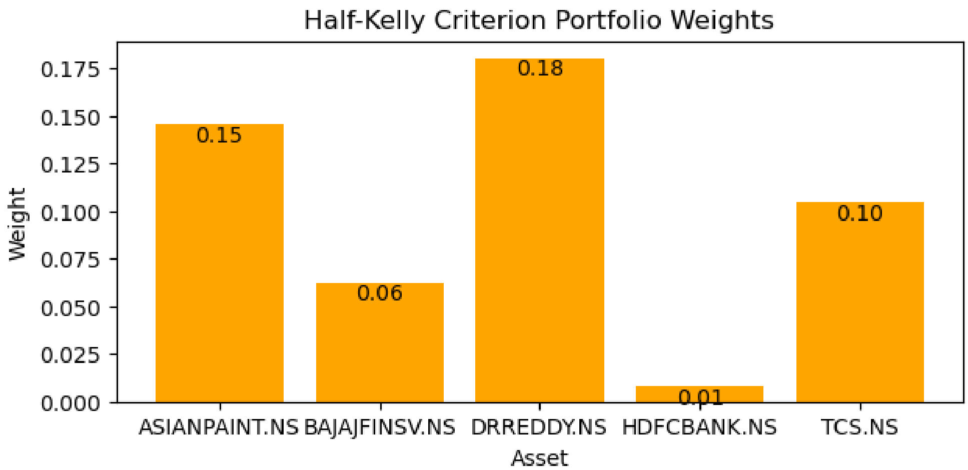
$$f^* = \frac{\mu - r}{\sigma^2}$$

Which is the optimal fraction of investible amount to be invested in a given security.
As can be seen from the equation that the required input for the criterion are easy to find pertaining to any stock or asset where μ is the mean return, σ is the standard deviation and r is the risk free rate.

TASK 4:

a. Apply a leverage constraint smaller than Kelly (such as half-Kelly).

The sum of weights comes to 0.50 as required for a Half Kelly. The same distribution across the stocks from earlier 2 models is reflected

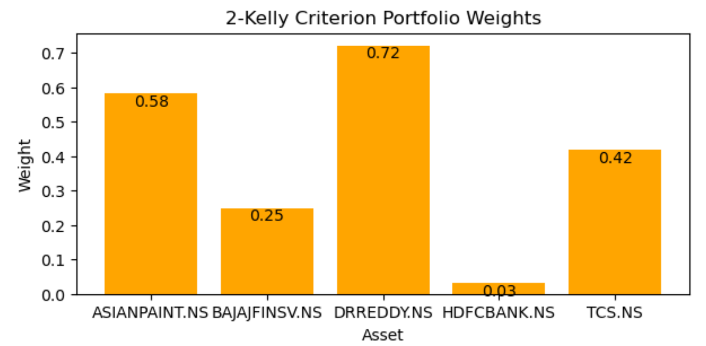


Returns from the portfolio is also almost half of what we encountered in the Kelly Criterion

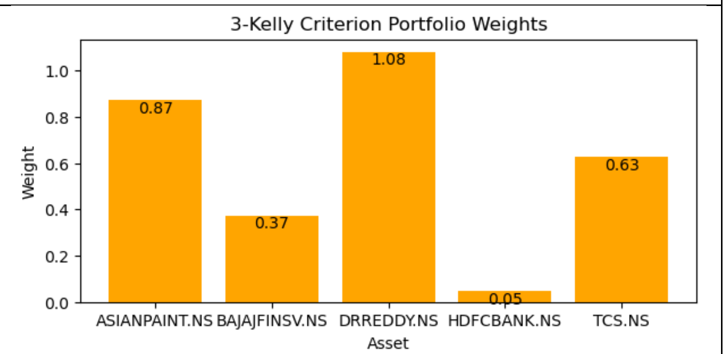
```
HALF-KELLY:
Kelly Returns      : 27.95%
Kelly Std. Dev.    :  0.0144
Kelly Sharpe Ratio :  1.137
```

b. Apply a leverage constraint larger than Kelly (such as two to five times Kelly).

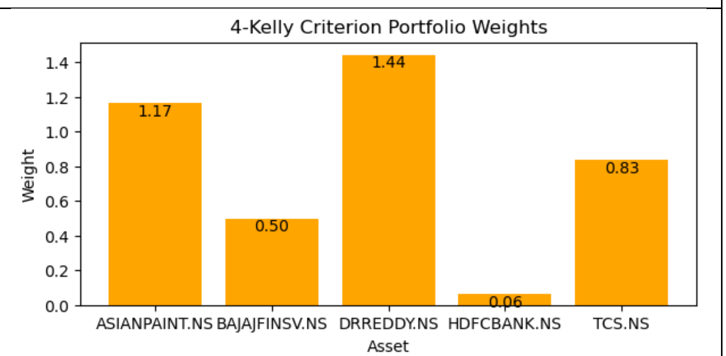
```
2-KELLY:
Kelly Returns       : 111.82%
Kelly Std. Dev.     :  0.0575
Kelly Sharpe Ratio  :  1.2027
```



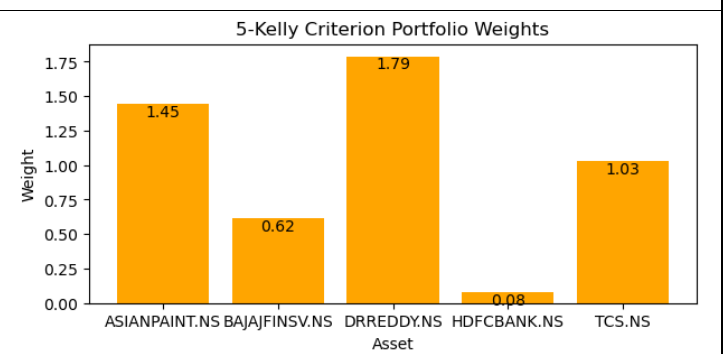

```
3-KELLY:
Kelly Returns       : 167.72%
Kelly Std. Dev.     :  0.0863
Kelly Sharpe Ratio  :  1.21
```




```
4-KELLY:
Kelly Returns       : 223.63%
Kelly Std. Dev.     :  0.115
Kelly Sharpe Ratio  :  1.2137
```




```
5-KELLY:
Kelly Returns: 274.38%
Kelly Std. Dev.    :  0.0288
Kelly Sharpe Ratio:  1.1808
```



As evident by the sum of weights > 1 which implies recommending of leverage (borrowing at risk free rate to invest). Leverage magnifies both gain and loss.

The table below provide the comparative view of the weights calculated for all the different models.

	ASIANPAINT.NS	BAJAJFINSV.NS	DRREDDY.NS	HDFCBANK.NS	TCS.NS
Original	0.33	0.16	0.35	0.01	0.16
Kelly	0.29	0.12	0.36	0.02	0.21
Half Kelly	0.15	0.06	0.18	0.01	0.10
2 Kelly	0.58	0.25	0.72	0.03	0.42
3 Kelly	0.87	0.37	1.08	0.05	0.63
4 Kelly	1.17	0.50	1.44	0.06	0.83
5 Kelly	1.45	0.62	1.79	0.08	1.03

c. Write 1-2 pages of background information. The background information provides equations, explains the terms in the equation, distinguishes inputs and outputs, explains how the parameters are estimated and calibrated, gives any interpretation to what the parameters mean, etc. The background is a technical summary of how the model works using equations, graphs, figures, tables, charts, and other illustrations, along with written explanations and interpretations.

Example of Double Kelly:

Let

$G_p := E_p - \frac{1}{2} V_p$ where E_p , V_p and G_p are the portfolio expected return, variance and expected log.

As per CAPM,

$$E_p = r + (E_m - r)f, V_p = \sigma_M^2 f^2$$

Where E_m is the market return, and r the risk free rate and f the portfolio weight. This will be simplified to

$$f^* = \frac{E_M - r}{\sigma_M^2}$$

This is the optimal Kelly bet with optimal growth given by

$$g^* = r + \frac{(E_M - r)^2}{\sigma_M^2} - \frac{1}{2}\frac{(E_M - r)^{\frac{2}{\sigma_M^2}}}{\sigma_M^4}$$

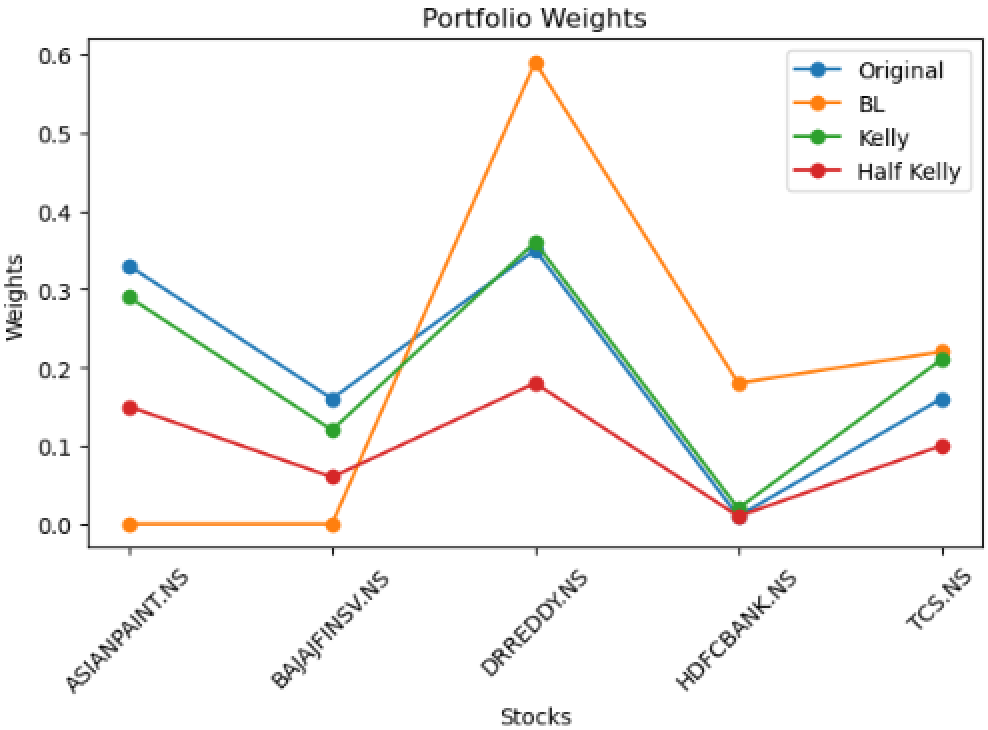
$$= r + \frac{E_M - r}{2\sigma_M^2}$$

When we use a double strategy, which is we bet f’ = 2f, and replace f with f’ we get

$$G(p) = r + \frac{2(E_M - r)^2}{\sigma_M^2} - \frac{4(E_M - r)^2}{2\sigma_M^2} = r$$

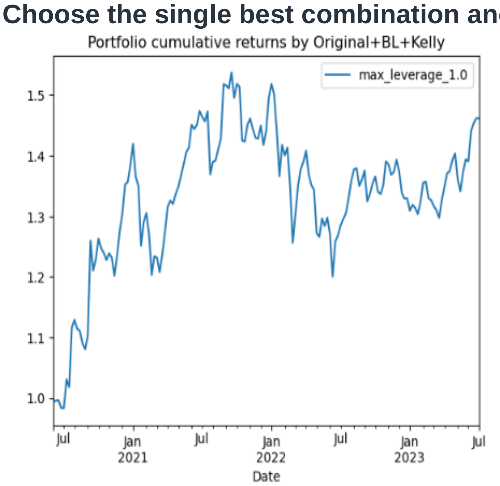
TASK 5:

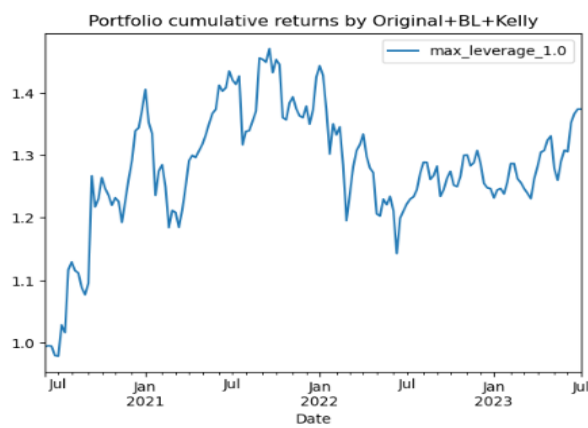
a. **Compare the original portfolio allocation, BL allocation, Kelly allocation, and Leverage Constraint.**



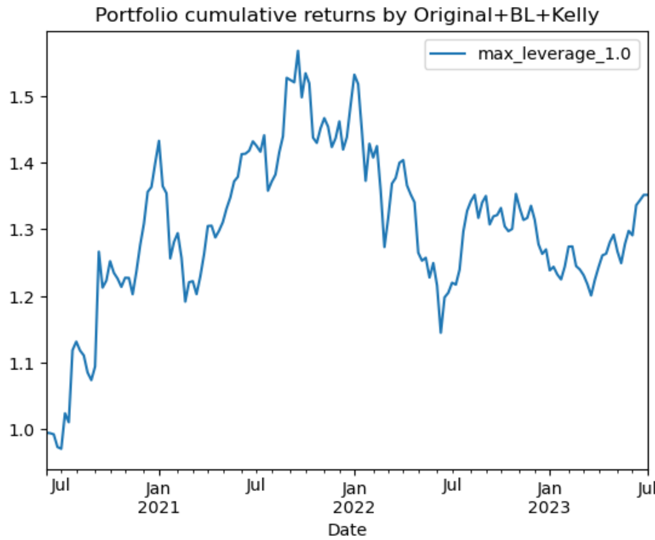
We see that estimated weights for all methods are in agreement with each other at least in terms of ranking except BL model, which is due to the manager's own opinion, hence his opinion was considered. Except Asian Paints and Bajaj, BL model is also in line with all other models.

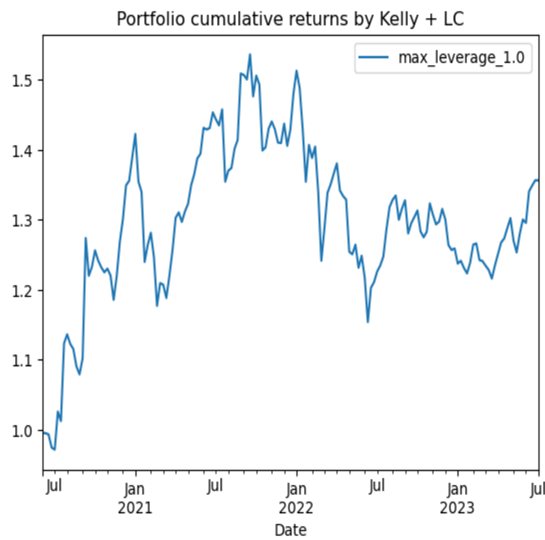
b. **Show how different combinations (all 3, BL & Kelly, BL & LC, Kelly & LC, etc.) compare in performance using metrics that you choose.**
c. **Choose the single best combination and explain why it is optimal.**





d.





The ALL 3 method has the highest Return (68.40%) and the best Sharpe Ratio (1.561) among the methods. It has the superior risk-adjusted return. It combines the Kelly criterion, Black-Litterman model, and leverage constraint to optimize the allocation of wealth and achieve a superior risk-adjusted return. The BL + KELLY method has a lower Return (59.00%) and Sharpe Ratio (1.3116) compared to the ALL 3 method. The BL + LC method has the lowest Return (57.06%) and Sharpe Ratio (1.2261) among the methods considered. The LC limits the allocations and reduces both the potential return and risk. The KELLY + LC method has almost a similar Return (57.68%) and Sharpe Ratio (1.2379) compared to the BL + LC method.

Therefore, based on these comparisons, the ALL 3 method demonstrates the highest risk-adjusted return and can be considered the most optimal method among the ones evaluated.