WORLDQUANT UNIVERSITY

Group Work Project

MScFE 622

Continuous-time Stochastic Processes

Revision Date: 04/27/2021

Overview

This document describes the requirements for the three Group Work Project assignments which must be submitted at the end of week 3, 5, and 7 respectively. Within a week of each submission, your group will receive feedback from the WQU Instructional Team, enabling you to use the feedback to revise your assignment ahead of the second and third submissions. You will use the Group Work Forum to communicate with your peers throughout the course.

Please make use of the **LIRN Library** located on the left pane of your screen as the primary resource for your research.

Your research should favor authoritative, scholarly sources, and you must cite all sources where relevant. The task is not to reproduce the research of others, but instead to develop your own systematic narrative that addresses the research topic and is informed by the research of others. Not only are you required to cite accurate and relevant facts, but you must also present your own clear logic when linking and contextualizing these facts.

Visit the **Student Resource Center (SRC)** where you can find resources on **how to conduct research**, how to use different sources of information, how to **cite references to** a**void plagiarism**, and how to use the **MLA citation style.**

Note: All Group Work Project assignments must be submitted via **Turnitin**, the anti-plagiarism software.

Group Work Project Objectives

Submission 1: Understand continuous-time stochastic terminology by exploring the history of the development of stochastic integration in the 20th century

Submission 2: Apply the Black-Scholes model to price classes of exotic options

Submission 3: Compare and contrast the Two-Factor Hull-White Model with other interest short rate models

Submission 1: Stochastic Calculus in the 20th Century

Submit a paper illustrating the early history of stochastic processes and the development of stochastic integration in the 20th century. Your discussion must address the following:

a) The role played by Louis Bachelier in mathematical finance and how his ideas were improved upon by Paul Samuelson

b) The contribution of Kiyosi Itō and other Japanese mathematicians

c) The contribution of Paul-André Meyer's 'Strasbourg School' of probability

Required length for each part (a, b, and c): 2 pages or about 1,000 words, for a total of ~3,000 words.

d) From Parts a and b, show how Brownian motion (assumed by Bachelier) and Ito's lemma are the steps in deriving Black-Scholes.

e) Write down the Black-Scholes partial differential equation in Greek form.

f) Assuming the option buyer and seller are delta-hedging, show that theta and gamma have opposite signs. Interpret the results.

Required length for parts d, e, and f: 2 pages total.

Submission Requirements

To develop your assignment, use LaTeX editor (https://www.overleaf.com), which significantly simplifies the writing of complex mathematical formulae. You can replicate in LaTeX the "**MScFE_622_CTSP_REPORT_TEMPLATE**" provided in the course room along with this Group Work Project Requirements. Then, convert the assignment to PDF and submit.

Note: If you submit multiple files, the PDF file with your report must be uploaded **separately** from the zipped folder that includes your other files. This allows Turnitin to generate the similarity report.

Submission 2: Pricing Exotic Options

Part A

For each of the following three types of options:

a) Lookback options

b) Barrier options

c) Compound options

write a brief one-page (500 words) covering the following:

- Explain each option along with its variations (i.e. sub-types e.g. up-and-out, fixed vs float, etc.). Be sure to list the valuation/pricing equation for each type.
- Give a real-world use case for when the option has been used to hedge against what type of risk. Or describe a potential profiting strategy for each of the options.

Part B

You are a quant at a bank and a senior trader comes to you explaining that he barely understands the Black Scholes equation. All he knows is that for one asset class S_t and the riskless bank account B satisfying the following stochastic differential equation:

$$dS_t = S_t(\mu dt + \sigma dW_t),$$

$$dB_t = rB_t dt$$

He has requested you to price one of the following options along with the justification for how you came up with the price starting from the basic Black Scholes equation given above:

Use the above equation and the ***"20200701_Index Price & Volume Data"*** Excel spreadsheet provided in the course room to answer **only one** of the following questions:

I. A proprietary trading firm expects a spike in volatility and would like to take a leveraged position on VIX. The trader recommended options on VIX and suggested hindsight option in particular to maximize the returns. The proprietary trading firm decided to use 1-month long option and agreed to present a proposal with prices for both fixed and floating strike hindsight option.

II. With the recent volatility in the market due to COVID-19, a hedge fund wants to buy a barrier option on Nasdaq Index. However, they wanted the opinion of the trader if they should purchase up-and-in at a level of 10,000 or down-and-out at a level of 9,000. Price both the options for the client justifying how you came up with the price.

III. A pension fund bought OTC long expiry put options on S&P 500 index at a strike of 3000 expiring in December 2020. However, they are unsure about the position and were thinking of either selling a Put or a Call on their position. Price both the options for them and make a recommendation based on the price you derive.

Again, you need to answer **only <u>ONE</u>** of the above. For the question you pick, you must derive the *pricing* and the *payoff* function starting with the Black Scholes equation. Then using the pricing function to derive the price of the options stated in the question.

For this submission, your instructors will use the following grading rubric:

Group #	**Points**	**Out of**
<u>Part A:</u>		**21**
Lookback Options		7
Explanation		4
Real World use case		3
Barrier Options		7
Explanation		4
Real World use case		3
Compound Options		7
Explanation		4
Real World use case		3
<u>Part B:</u>		**14**
Derive the Pricing and Payoff function from Black Scholes Equation		7
Pricing the actual options from the example stated		7
Writing, formatting & Spelling		**5**
References		**5**
Total		**45**

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Submission 3: Interest Rate Models

Read the following paper: The Two-Factor Hull-White Model: Pricing and Calibration of Interest Rates Derivatives by Arnaud Blanchard.

Find more references on the two-Factor Hull-White Model on your own.

For this submission complete the following tasks:

a) Describe, in mathematical terms, the Two-Factor Hull-White Model

b) Compare and contrast this approach with the short rate models discussed in Module 7 and provide a critical assessment

c) Show how to price an interest rate cap using the two-factor Hull-White model.

   a. Identify the target audience of an interest-rate cap.
   b. Define each input.
   c. Use recent market values, not made-up values, for as many inputs as you can.
   d. Apply the analytical approach.
   e. Apply the Monte Carlo approach.
   f. Compare your answers to parts d and e

Show all steps in your calculations and state all external references you used.

Submission Requirements

Submit a PDF report including a response of about **1,500 words for each part**, for a total of 4,500 words.

To develop your assignment, use LaTeX editor (https://www.overleaf.com), which significantly simplifies the writing of complex mathematical formulae. You can replicate in LaTeX the "**MScFE_622_CTSP_REPORT_TEMPLATE**" provided in the course room along with this Group Work Project Requirements. Then, convert the assignment to PDF and submit.

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