Eighth Edition

# GATE

## ELECTRONICS & COMMUNICATION

## Analog Circuits

Vol 5 of 10

**RK Kanodia**
**Ashish Murolia**

NODIA & COMPANY

GATE Electronics & Communication Vol 5, 8e
Analog Circuits
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MRP 660.00

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Printed by Nodia and Company, Jaipur

*To Our Parents*

# *Preface to the Series*

For almost a decade, we have been receiving tremendous responses from GATE aspirants for our earlier books: GATE Multiple Choice Questions, GATE Guide, and the GATE Cloud series. Our first book, GATE Multiple Choice Questions (MCQ), was a compilation of objective questions and solutions for all subjects of GATE Electronics & Communication Engineering in one book. The idea behind the book was that Gate aspirants who had just completed or about to finish their last semester to achieve his or her B.E/B.Tech need only to practice answering questions to crack GATE. The solutions in the book were presented in such a manner that a student needs to know fundamental concepts to understand them. We assumed that students have learned enough of the fundamentals by his or her graduation. The book was a great success, but still there were a large ratio of aspirants who needed more preparatory materials beyond just problems and solutions. This large ratio mainly included average students.

Later, we perceived that many aspirants couldn't develop a good problem solving approach in their B.E/B.Tech. Some of them lacked the fundamentals of a subject and had difficulty understanding simple solutions. Now, we have an idea to enhance our content and present two separate books for each subject: one for theory, which contains brief theory, problem solving methods, fundamental concepts, and points-to-remember. The second book is about problems, including a vast collection of problems with descriptive and step-by-step solutions that can be understood by an average student. This was the origin of *GATE Guide* (the theory book) and *GATE Cloud* (the problem bank) series: two books for each subject. *GATE Guide* and *GATE Cloud* were published in three subjects only.

Thereafter we received an immense number of emails from our readers looking for a complete study package for all subjects and a book that combines both *GATE Guide* and *GATE Cloud*. This encouraged us to present GATE Study Package (a set of 10 books: one for each subject) for GATE Electronic and Communication Engineering. Each book in this package is adequate for the purpose of qualifying GATE for an average student. Each book contains brief theory, fundamental concepts, problem solving methodology, summary of formulae, and a solved question bank. The question bank has three exercises for each chapter: 1) Theoretical MCQs, 2) Numerical MCQs, and 3) Numerical Type Questions (based on the new GATE pattern). Solutions are presented in a descriptive and step-by-step manner, which are easy to understand for all aspirants.

We believe that each book of GATE Study Package helps a student learn fundamental concepts and develop problem solving skills for a subject, which are key essentials to crack GATE. Although we have put a vigorous effort in preparing this book, some errors may have crept in. We shall appreciate and greatly acknowledge all constructive comments, criticisms, and suggestions from the users of this book. You may write to us at rajkumar.kanodia@gmail.com and ashish.murolia@gmail.com.

**Acknowledgements**

We would like to express our sincere thanks to all the co-authors, editors, and reviewers for their efforts in making this project successful. We would also like to thank Team NODIA for providing professional support for this project through all phases of its development. At last, we express our gratitude to God and our Family for providing moral support and motivation.

We wish you good luck !
R. K. Kanodia
Ashish Murolia

# SYLLABUS

## GATE Electronics & Communications

Small Signal Equivalent circuits of diodes, BJTs, MOSFETs and analog CMOS. Simple diode circuits, clipping, clamping, rectifier. Biasing and bias stability of transistor and FET amplifiers. Amplifiers: single-and multi-stage, differential and operational, feedback, and power. Frequency response of amplifiers. Simple op-amp circuits. Filters. Sinusoidal oscillators; criterion for oscillation; single-transistor and op-amp configurations. Function generators and wave-shaping circuits, 555 Timers. Power supplies.

## IES Electronics & Telecommunication

Transistor biasing and stabilization. Small signal analysis. Power amplifiers. Frequency response. Wide banding techniques. Feedback amplifiers. Tuned amplifiers. Oscillators. Rectifiers and power supplies. Op Amp, PLL, other linear integrated circuits and applications. Pulse shaping circuits and waveform generators.

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# CONTENTS

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# CHAPTER 1

## DIODE CIRCUITS

### 1.1 INTRODUCTION

A general goal of this chapter is to develop the ability to use the piece wise linear model and approximation techniques in the hand analysis and design of various diode circuits. The chapter includes the following topics:

- Introduction to diode
- AC and DC analysis of diode.
- Application of diodes to perform signal processing functions: rectification, clipping and clamping.
- Zener diode, which operates in the reverse breakdown region
- Application of Zener diode in voltage regulators

### 1.2 DIODE

Diode is a two terminal device with nonlinear $i$-$v$ (current-voltage) characteristics. Figure 1.1 shows the circuit symbol of a diode. In the diode symbol, the triangular head denoting the allowable direction of current flow and the vertical bar representing the blocking behaviour for currents in the opposite direction. The corresponding terminals are called the anode (or $p$ -terminal) and the cathode (or $n$-terminal) respectively.

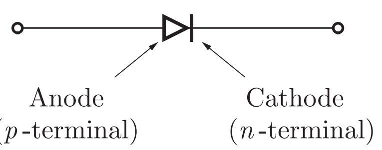



Figure 1.1: Diode Circuit Symbol

#### 1.2.1 Operating Modes of a Diode

A diode operates in the following two modes:

1. Forward bias
2. Reverse bias

**Forward Bias**

If the $p$-terminal of a diode is at higher voltage level than the $n$-terminal (i.e. positive voltage applied across diode), a positive current flows through the diode. The diode, operating in this mode, is said to be turned ON or *forward biased*. Mathematically, we define the condition for a forward biased diode as

$$\left.\begin{aligned} V_{\text{anode}} &> V_{\text{cathode}} \\ V_p &> V_n \\ V_D &= V_p - V_n > 0 \end{aligned}\right\} \text{Forward bias}$$

**Reverse Bias**

If the $p$-terminal of an ideal diode is at lower voltage level than the $n$ -terminal (i.e. negative voltage applied across diode), then there is no current across the diode. The diode operating in this mode is said to be turned OFF or *reverse biased*. Mathematically, we define the condition for a reverse biased diode as

$$\left.\begin{aligned} V_{\text{anode}} &< V_{\text{cathode}} \\ V_p &< V_n \\ V_D &= V_p - V_n < 0 \end{aligned}\right\} \text{Reverse bias}$$

### 1.2.2 Current-Voltage Characteristics of a Diode

The theoretical relationship between the voltage ($V_D$) and current ($i_D$) in the $pn$ junction is given by

$$i_D = I_S[\exp(\tfrac{V_D}{\eta V_T}) - 1] \qquad ...(1.1)$$

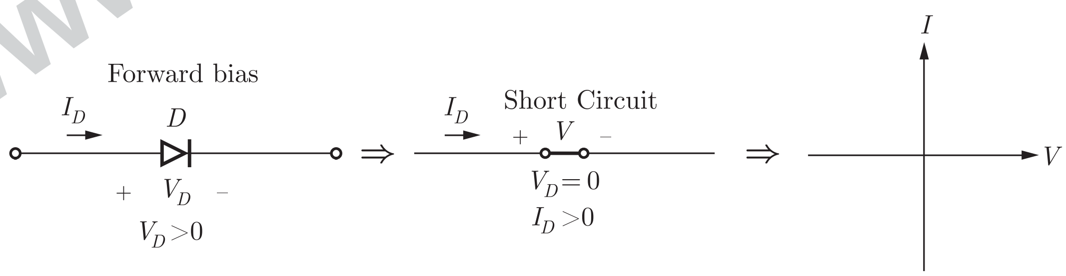
where the parameter $I_S$ is the reverse saturation current, $V_T$ is thermal voltage, and $\eta$ is the emission coefficient or ideality factor.

NOTE :
Thermal voltage at room temperature is $V_T = 0.026$ V. The value of ideality factor is in the range $1 \le \eta \le 2$. For germanium, $\eta = 1$, and for silicon, $\eta = 2$.

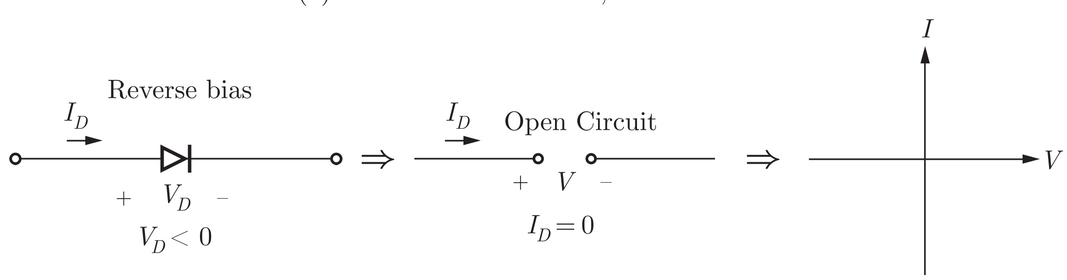


### 1.2.3 Current-Voltage Characteristics of an Ideal Diode

The ideal diode may be considered as the most fundamental non-linear circuit element. Figure 1.2 (a) and (b) shows the current-voltage characteristics of an ideal diode in the forward bias and reverse bias regions, respectively.




(a) Forward Biased Diode,




(b) Reverse Biased Diode

Figure 1.2: Current-Voltage Characteristic of (a) Forward Biased Diode, (b) Reverse Biased Diode

Combining the two graphs, we get the overall current-voltage characteristic of an ideal diode (as opposed to the $i$-$v$ characteristic of a diode given in equation (1.1)) as shown in Figure 1.3.

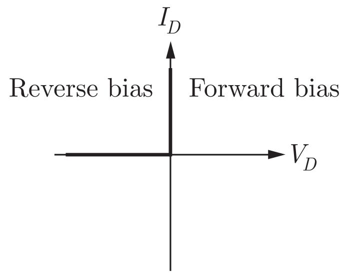



Figure 1.3: Current-Voltage Characteristic of an Ideal Diode

## 1.3 LOAD LINE ANALYSIS

The applied load to a device normally have an important impact on the point or region of operation of the device. *Load line analysis* is the graphical approach to analyse the operation of a circuit. Consider the network shown in Figure 1.4 (a). Applying KVL in the circuit,

$$E - V_D - V_R = 0$$

$$E = V_D + I_D R \qquad \text{...(1.2)}$$

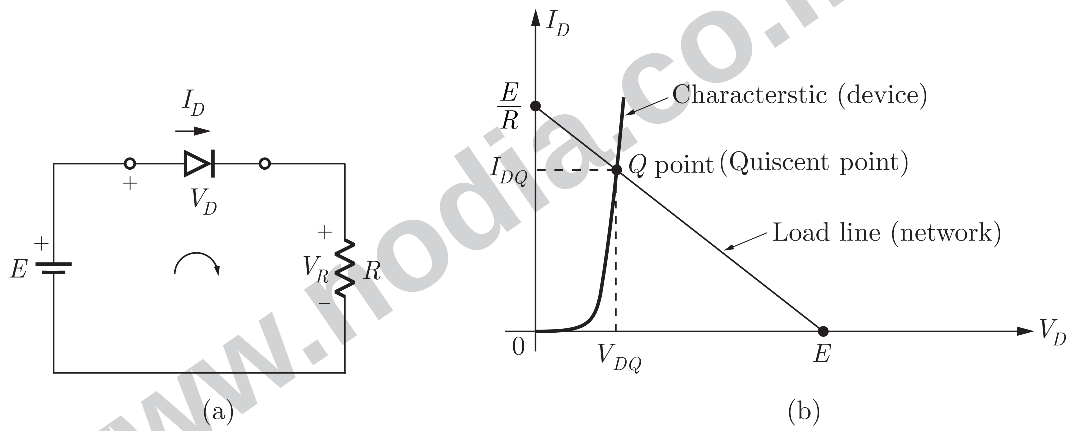



Figure 1.4: (a) A Simple Diode Circuit, (b) Load Line Characteristic for the Diode Circuit



Substituting $V_D = 0$ in equation (1.2), we have

$$I_D = \frac{E}{R}\Big|_{V_D=0} \qquad \text{...(1.3)}$$

Again, substituting $I_D = 0$ in equation (1.2), we get

$$V_D = E\Big|_{I_D=0}$$

The two variables ($V_D$, $I_D$) are the same as the diode axis variable, so we draw the load line and determine the point of intersection, as shown in Figure 1.4 (b).

## 1.4 PIECEWISE LINEAR MODEL

Figure 1.5 shows the piecewise linear model of a diode forward characteristic. The exponential curve is approximated by two straight lines, line $A$ with zero slope and line $B$ with a slope of $r_d$. The straight line model or piecewise linear model can be described as

$$I_D = 0, \qquad V_D \leq V_\gamma$$

$$I_D = \frac{V_D - V_\gamma}{r_d}, \quad V_D \geq V_\gamma$$

where $\quad r_d$ = Diode forward resistance

$V_\gamma$ = cut in voltage of diode

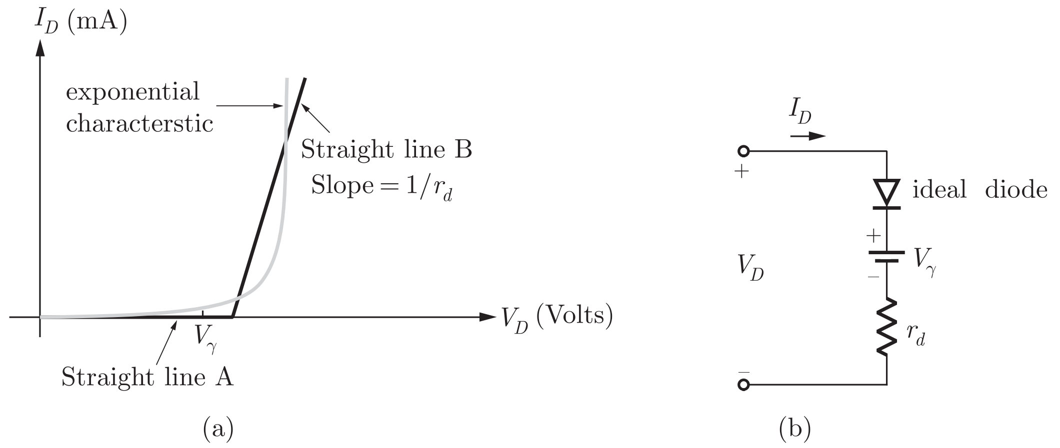



Figure 1.5: (a) Piecewise Linear Model of the Diode Forward Characteristics, and (b) its Equivalent Circuit Representation

## 1.5 SMALL SIGNAL MODEL

When *pn* junction diode is used in a linear amplifier circuit, the time varying or ac characteristics of the diode becomes important. For these circuits, we define the small signal model of diode. For the small signal model analysis, assume that the ac signal is small compared to the dc component, so that a linear ac model can be developed from the non linear diode. The relationship between the diode current and voltage can be written as

$$i_D \approx I_S e^{\frac{V_D}{V_T}} = I_S e^{\frac{V_{DQ}+v_d}{V_T}}$$

where $V_{DQ}$ is the dc quiescent voltage and $v_d$ is the ac component. So,

$$i_D = I_S e^{\frac{V_{DQ}}{V_T}} e^{\frac{v_d}{V_T}} \qquad \text{...(1.4)}$$

Since, the ac signal is small, i.e

$$v_d << V_T$$

So,

$$e^{v_d/V_T} \approx 1 + \frac{v_d}{V_T} \qquad \text{...(1.5)}$$

Substituting the above expression to equation (1.4), we get

$$i_D = \left(I_S e^{\frac{V_{DQ}}{V_T}}\right)\left(1 + \frac{v_d}{V_T}\right) = I_{DQ}\left(1 + \frac{v_d}{V_T}\right)$$

$$= \underbrace{I_{DQ}}_{\substack{\text{Quiescent diode current} \\ \text{(dc bias current)}}} + \underbrace{I_{DQ}\frac{v_d}{V_T}}_{\substack{\text{AC current} \\ \text{(signal current)}}}$$

where $I_{DQ} \approx I_S e^{\frac{V_{DQ}}{V_T}}$ is the quiescent diode current. Thus, the relationship between the ac components of the diode voltage and current is

$$i_d = I_{DQ}\frac{v_d}{V_T} \qquad \text{...(1.6)}$$

### 1.5.1 Small Signal Resistance

The small signal incremental resistance of a diode is defined as

$$r_d = \frac{v_d}{i_d} = \frac{V_T}{I_{DQ}} \qquad \text{[see equation (1.6)]}$$

where $V_T$ is thermal voltage and $I_{DQ}$ is the quiescent diode current.

### 1.5.2 AC and DC Equivalent Model

In the above sections, we have already discussed the diode characteristics for ac and dc supply. Figure 1.4 (a) and (b) illustrate the equivalent dc model (bias model) and ac model (small signal model) of a diode.

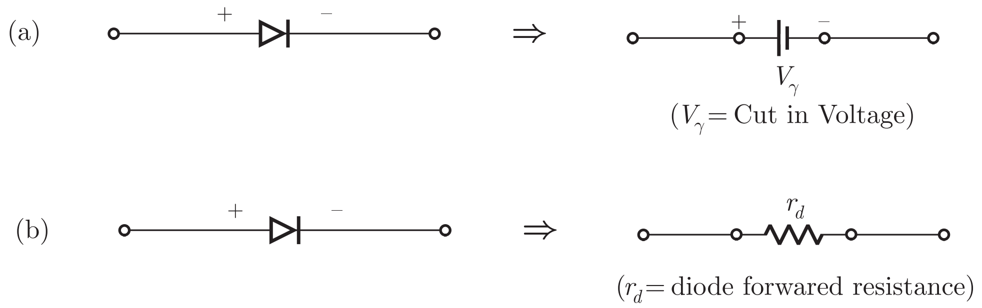



Figure 1.4 (a): Bias Model and (b): Small Signal Model of a Diode

### METHODOLOGY: ANALYSIS OF SINGLE DIODE CIRCUIT

In using the piecewise linear model to the diode, the first objective is to determine the Linear region (forward bias or reverse bias) in which the diode is operating To do this, we can :

Step 1: Find the input voltage conditions such that a diode is 'ON' (forward bias) then find the output signal for this condition.

Step 2: Find the input voltage condition such that a diode is OFF (Reverse bias) then find the output signal for this condition.

NOTE :
Step 2 can be performed before step 1.

### METHODOLOGY: ANALYSIS OF MULTIPLE DIODE CIRCUIT

Analyzing multidiode circuit requires determining if the individual diodes are ON or OFF. In many cases, the choice is not obvious so we must initially guess the state of each diode, then analyse the circuit to determine if we have a solution consistent with our initial guess. To do this, we may follow the steps given below.

Step 1: Assume the state of a diode. If a diode is assumed ON the voltage across the diode is assumed to be $V_\gamma$. If a diode is assumed to be OFF the current through the diode is assumed to be zero.

Step 2: Analyse the linear circuit with the assumed states.

Step 3: Evaluate the resulting state of each diode. If the initial assumption were that a diode is OFF and the analysis shows that $I_D = 0$ or $V_D \le V_\gamma$ then the assumption is correct if, however, the analysis actually shows that $I_D > 0$ or $V_D > V_\gamma$, then the initial assumption is incorrect. Similarly, if the initial assumption were that a diode is ON and the analysis shows that $I_D \ge 0$ or $V_D \ge V_\gamma$, then the initial assumption is correct. If, however, the analysis shows that $I_D < 0$ or $V_D < V_f$, then the initial assumption is incorrect.

Step 4: If any initial assumption is proven incorrect then a new assumption must be made and the new linear circuit must be analysed. Step 3 must than be repeated.

NOTE :
For simplification at the initial step always assume that all the diodes are OFF then find the voltage across diodes $V_D$ then follow step 3.

## 1.6 CLIPPER AND CLAMPER CIRCUITS

Diode can be used in wave shaping circuits that either limit or clip portion of a signal, or shift the dc voltage level. The circuits are called clippers and clampers, respectively.

### 1.6.1 Clippers

Clipper circuits, also called limiter circuits are used to eliminate portion of a signal that are above or below a specified level without distorting the remaining part of the alternating waveform. The simple form of diode clipper-one resistor and diode depending on the orientation of the diode the positive or negative region of the input signal is clipped OFF. There are two general categories of clippers:

1. Series clipper
2. Parallel clipper.

**1. Series Clipper**

The series configuration is defined as one where the diode is in series with the load. Table 1.1 summarizes the output waveform of various biased and unbiased series clipper circuits for the input waveform shown in Figure 1.5.

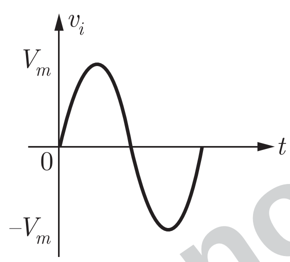
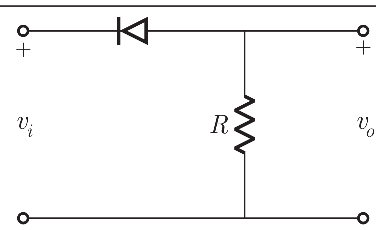
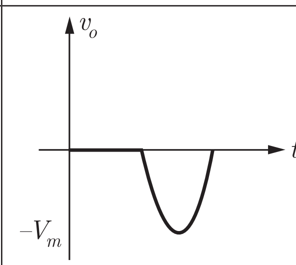
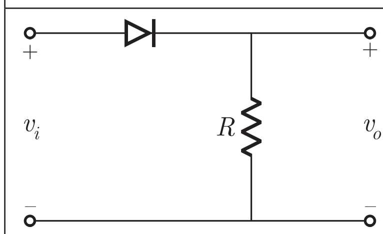
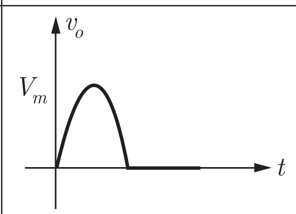
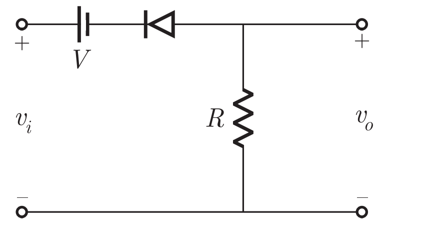
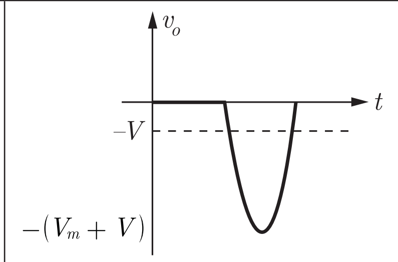
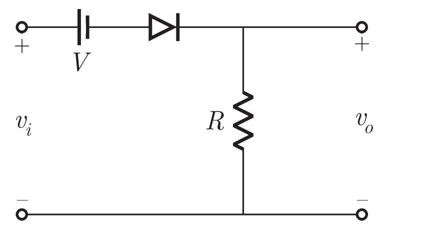
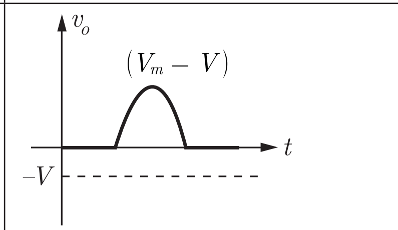
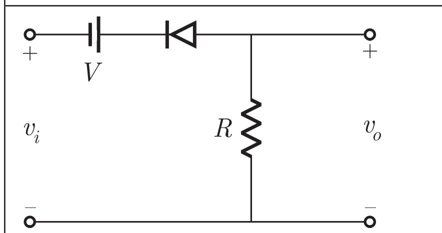
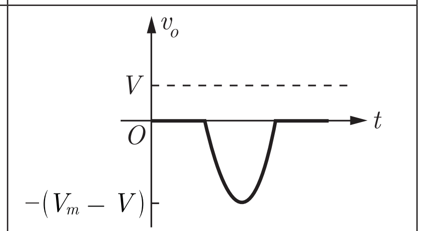
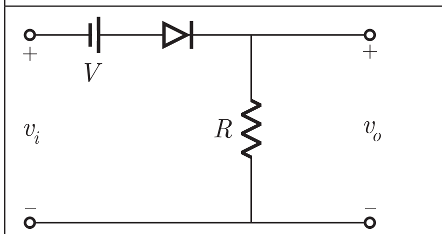
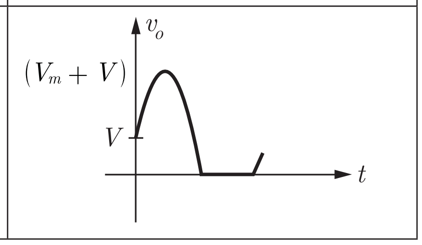



Figure 1.5: Sinusoidal Input Waveform



Table 1.1: Series Clipper Circuits and its Output Waveform

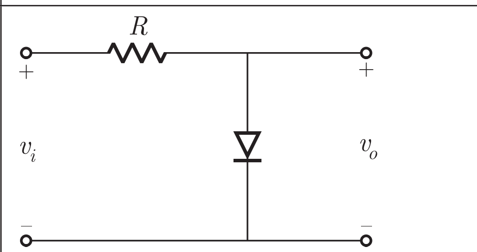
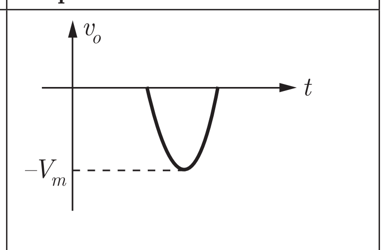
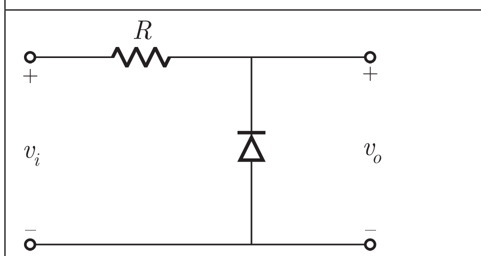
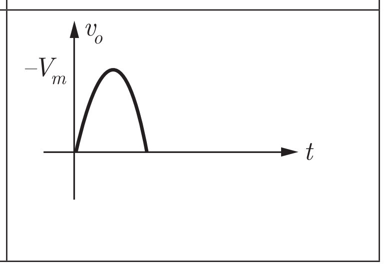
| Series Clippers | Clipper Circuits | Output Waveforms |
|---|---|---|
| Unbiased Series Clipper Using Ideal Diode |   |   |
| |   |   |

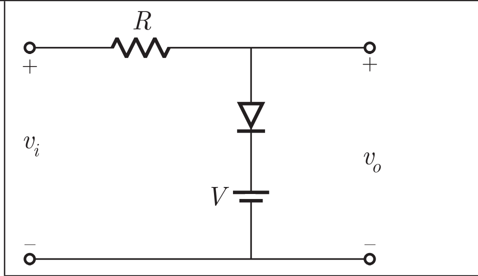
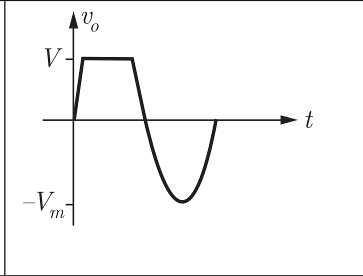
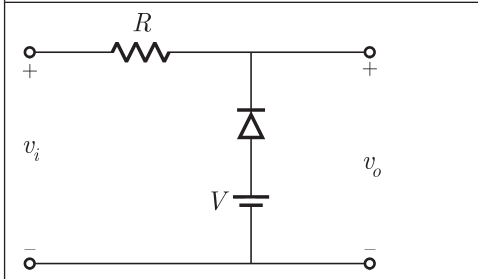
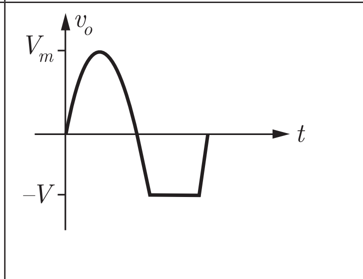
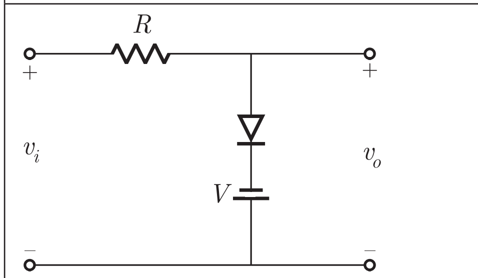
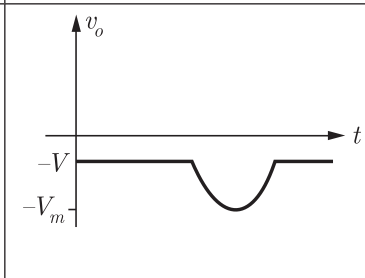
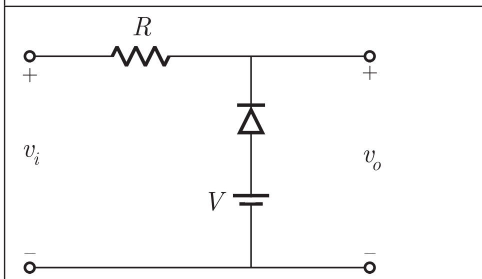
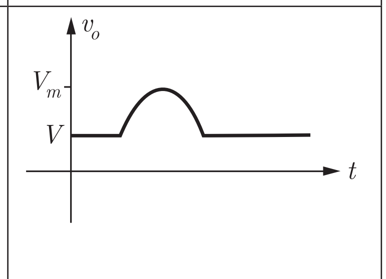
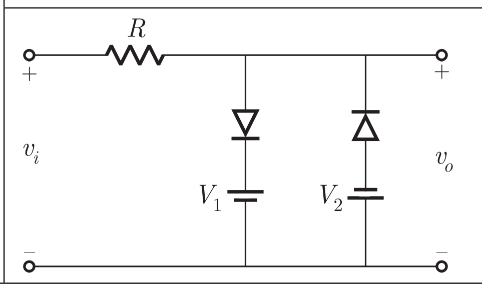
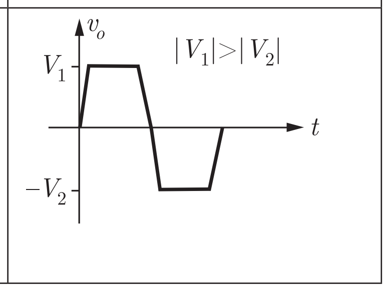
| Series Clippers | Clipper Circuits | Output Waveforms |
|---|---|---|
| Biased Series Clipper Using Ideal Diode |   |   |
| |   |   |
| |   |   |
| |   |   |

## 2. Parallel Clipper

The parallel clipper (or shunt clipper) has the diode in a branch parallel to the load. Table 1.2 summarizes the output waveform of various biased and unbiased parallel clipper circuits for the input waveform shown in Figure 1.5.

Table 1.2: Parallel Clipper Circuits and its Output Waveform

| Parallel Clippers | Clipper Circuits | Output Waveforms |
|---|---|---|
| Unbiased Parallel Clippers Using Ideal Diode |   |   |
| |   |   |

| Parallel Clippers | Clipper Circuits | Output Waveforms |
|---|---|---|
| Biased Parallel Clippers Using Ideal Diode |   |   |
| |   |   |
| |   |   |
| |   |   |
| |   |   |

Following are some important points that must be remembered while analysing a parallel clipper circuit:

> PROBLEM SOLVING TECHNIQUES IN PARALLEL CLIPPER
>
> 1. When the diode is in downward direction the signal will be transmitted below the reference voltage.
> 2. When the diode is in upward direction the signal will be transmitted above the reference voltage.
> 3. Under any circumstances - dc, ac, instantaneous values, pulses, and so on; Kirchoff's voltage law must be satisfied.

### 1.6.2 Clampers

Clamping shifts the entire signal voltage by a dc level. In a steady state, the output waveform is an exact replica of the input waveform, but the output signal is shifted by a dc value that depends on the circuit. The network must have a capacitor, a diode, and a resistive elements but it can also employ an independent dc supply to introduce an additional shift. Figure 1.6 shows a clamper circuit and a sinusoidal input waveform.

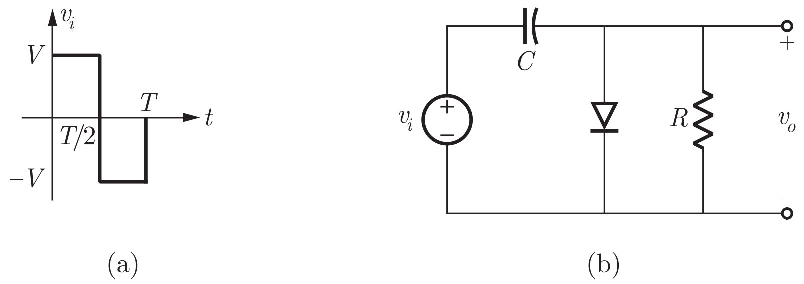



Figure 1.6: (a) Input Sinusoidal Waveform, (b) Clamper Circuit

For the positive half cycle of input waveform, diode is short and the output is zero, as shown in Figure 1.7 (a). The capacitor is charged in the positive half cycle. In the negative cycle, the diode is open and the output is

$$v_o = -V - V = -2V$$

Thus, we get the output waveform of the clamper as shown in Figure 1.7 (c).

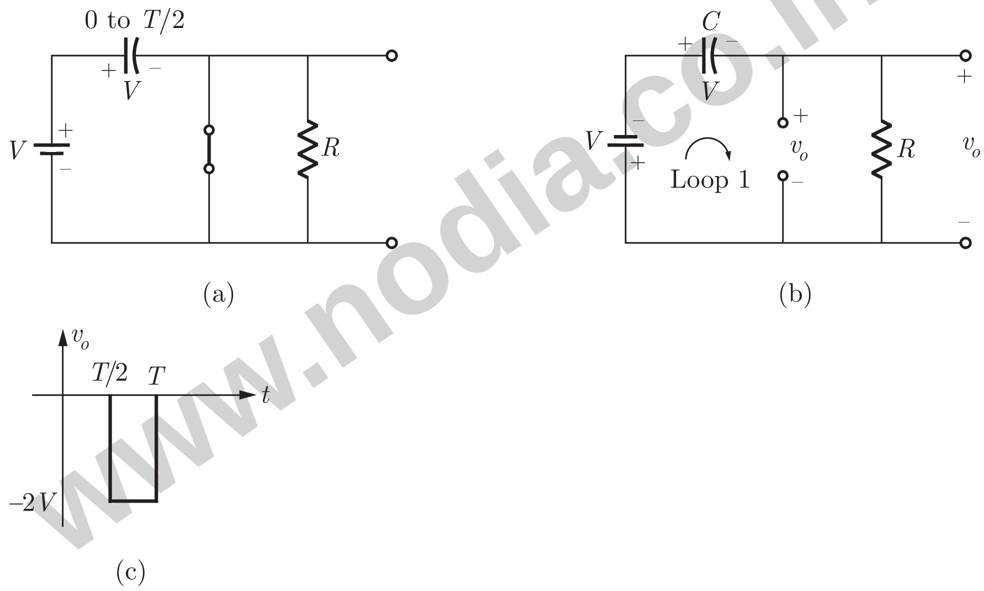



Figure 1.7: Equivalent Clamping Circuit for (a) Positive Half Cycle, (b) Negative Half Cycle of Input Waveform, (c) Output Waveform of Clamper Circuit

Following are some important points that must be remembered while solving a clamper circuit:

PROBLEM SOLVING TECHNIQUES IN CLAMPER CIRCUIT

1. When the diode is in downward direction the total signal will be clamped below the reference voltage.
2. When the diode is in upward direction the total signal will be clamped above the reference voltage.

## 1.7 VOLTAGE MULTIPLIER CIRCUIT

Voltage multiplier Circuits provide a means by which multiple dc voltages can be generated from a single ac source and power transformer. Here, we will discuss some typical voltage multiplier circuits.

### 1.7.1 Voltage Doubler

Voltage doubler circuits provide twice the peak voltages of the transformer secondary. Figure 1.8 shows the half wave voltage doubler circuit.

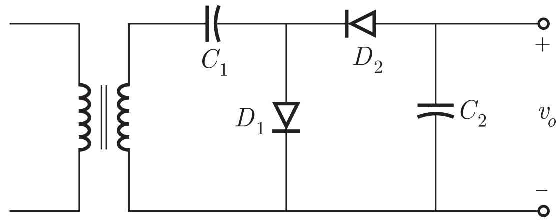
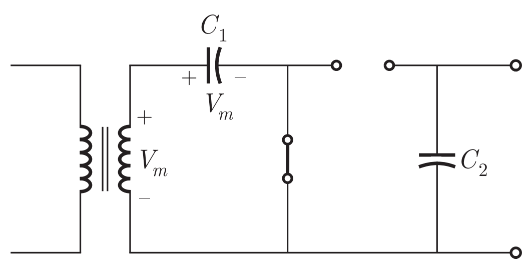



Figure 1.8: Half Wave Voltage Doubler Circuit

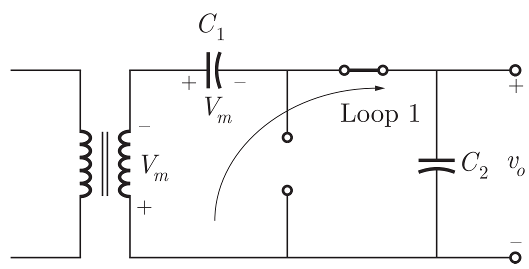
The output waveform of the voltage doubler can be determined by following two methods:

**METHODOLOGY 1: TO ANALYSE OUTPUT OF VOLTAGE DOUBLER**

Step 1: For positive voltage half cycle across the transformer secondary, diode $D_1$ conducts so capacitor $C_1$ will be charged.




Step 2: For negative voltage half cycle across the transformer secondary, diode $D_2$ will conduct and capacitor $C_2$ will be charged.



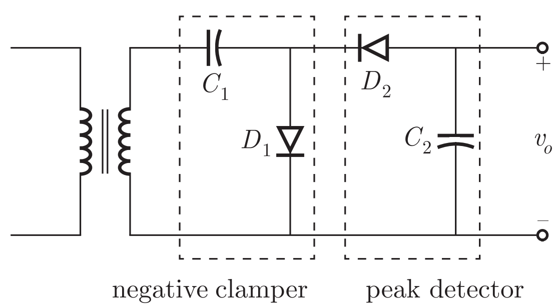

Step 3: Applying KVL in loop 1, we get

$$-V_m - V_m - v_o = 0$$

$$v_o = -2V_m$$

**METHODOLOGY 2: TO ANALYSE OUTPUT OF VOLTAGE DOUBLER**

Step 1: For the given voltage doubler circuit, we deduce that it includes a negative clamper and a peak detector.



Step 2: For the negative clamper (diode is in downward direction), the total signal will be clamped below the reference voltage, i.e.

$$(v_o)_{\text{clamper}} = -V_m - v_i$$

Step 3: Since, the peak detector provides the peak voltage at output, so we get

$$v_o = -2V_m$$

NOTE :
If a circuit includes a diode and a capacitor, then the circuit is either a clamper or a peak detector. For a clamper, output is always across diode; whereas for a peak detector, output is across capacitor.

### 1.7.2 Voltage Tripler and Quadrupler

Figure 1.9 shows the voltage tripler and quadrupler. It is an extension of the half wave voltage doubler, which develops three and four times the peak input voltage. For positive half cycle $D_1$ and $D_3$ will conduct so capacitor $C_1$, $C_3$, $C_2$ will be charged. For negative half cycle $D_2$ and $D_4$ will conduct.

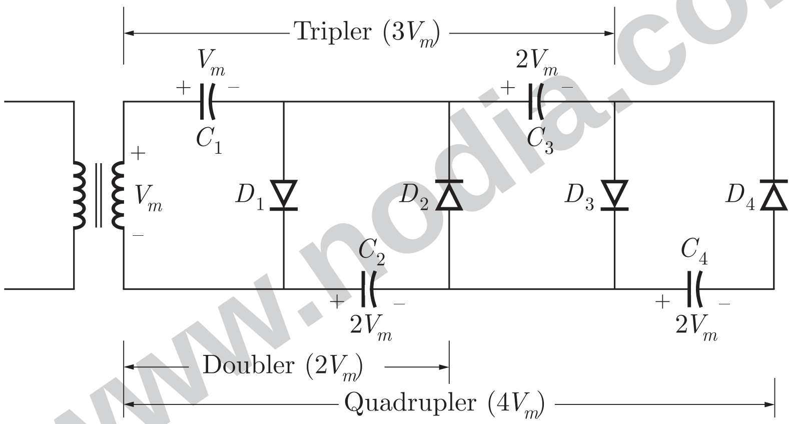



Figure 1.9: Votage Tripler and Quadrupler

## 1.8 RECTIFIER CIRCUIT

Rectification is the process of converting an alternating voltage into one that is limited to one polarity. The diode is useful for this function because of its nonlinear characteristic that is current exists for one voltage polarity, but is essentially zero for the opposite polarity. The block diagram of a rectifier circuit is shown in Figure 1.10.

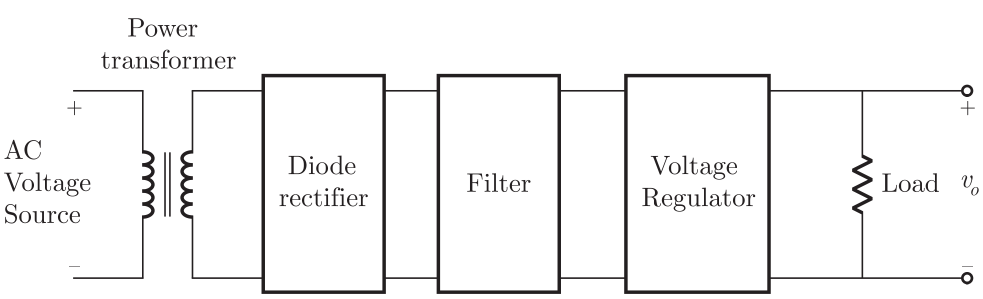



Figure 1.10: Block diagram of an electronic power supply

### 1.8.1 Parameters of Rectifier Circuit

Some important parameters for rectifier are described in the following texts:

1. **DC load current:** The dc load current of a rectifier is given by

$$I_{dc} = \frac{\text{area under curve } of \text{ output waveform}}{2\pi}$$
$$= \frac{1}{2\pi}\int_0^{2\pi} I\, d(\omega t)$$

2. **DC load voltage:** If the rectifier is terminated to load $R_L$, then the dc load voltage of the rectifier is given by

$$V_{dc} = I_{dc} R_L$$

3. **RMS load current:** The rms load current of a rectifier is defined as

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} I^2\, d(\omega t)}$$

4. **RMS load voltage:** The rms load voltage of a rectifier is given by

$$V_{rms} = I_{rms} R_L$$

5. **Ripple:** The time dependent component present in the output of the rectifier filter is known as ripple.

6. **Ripple factor:** The ripple factor of a rectifier is obtained as

$$r.f. = \frac{\text{rms value of alternating component in output current}}{\text{averge value of output current}}$$
$$= \frac{\sqrt{(I_{rms})^2 - (I_{dc})^2}}{I_{dc}}$$
$$= \sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1}$$

7. **Voltage regulation:** The percentage voltage regulation for a rectifier is defined as

$$\%\text{VR} = \frac{(V_{dc})_{\text{NL}} - (V_{dc})_{\text{FL}}}{(V_{dc})_{\text{FL}}}$$

where $(V_{dc})_{\text{FL}}$ is the full load dc voltage and $(V_{dc})_{\text{NL}}$ is the null load voltage. Ideally VR should be zero.

8. **Rectifier efficiency:** The rectifier efficiency is defined as

$$\eta = \frac{\text{DC power delivered to the load}}{\text{AC input power}} = \frac{P_{dc}}{P_{ac}}$$

9. **Peak inverse voltage:** It is the maximum reverse bias voltage across the diode where the diode withstand.

### 1.8.2 Classification of Rectifiers

Rectifier circuits can be classified as

1. Half wave rectifier
2. Full wave rectifier

In the following sections, we will discuss some typical classes of rectifiers.

## 1.9 HALF WAVE RECTIFIERS

For a half wave rectifier, the output voltage appears only during the half cycle of the input signal. Figure 1.11 shows a half wave rectifier, its input waveform, and the corresponding output waveform.

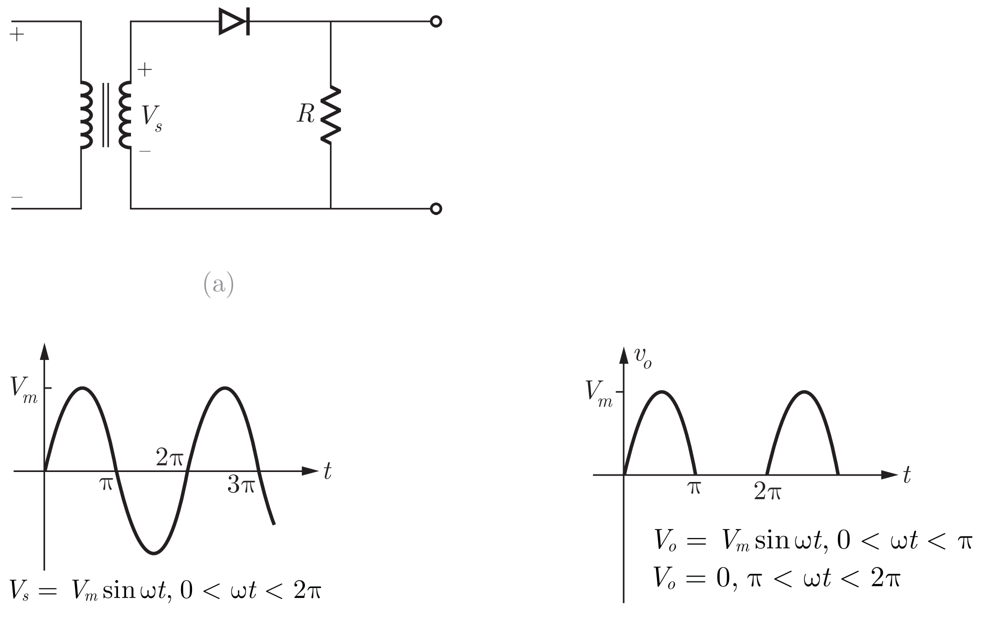



Figure 1.11: (a) Half Wave Rectifier Circuit, (b) Input Waveform, and (c) Output Waveform

Some important parameters for a half wave rectifier are described in the following table.

Table 1.3: Parameters of a Half Wave Rectifier Circuit

| S.N. | Parameters | Expression |
|---|---|---|
| 1. | DC load current | $I_{dc} = \dfrac{I_m}{\pi}$ |
| 2. | DC load voltage | $V_{dc} = \dfrac{V_m}{\pi\left(\dfrac{R_f + R_S}{R_L} + 1\right)}$ where $R_L$ is load resistance, $R_S$ is second transformer resistance, and $R_f$ is forward resistance of diode. |
| 3. | Null load dc voltage | $(V_{dc})_{\text{NL}} = \dfrac{V_m}{\pi}$ if $R_L = \infty$, i.e. load terminal is open |
| 4. | RMS load current | $I_{rms} = \dfrac{I_m}{2}$ |
| 5. | RMS load voltage | $V_{rms} = \dfrac{V_m}{2}$ |
| 6. | Ripple factor | $r.f. = 1.21$ |
| 7. | Voltage regulation | Ideally VR should be zero, i.e. %VR $= 0$ |
| 8. | Rectifier efficiency | $\eta \approx 40.6$ |
| 9. | Peak inverse voltage | In half wave rectifier, the PIV rating of the diode must equal or exceed the peak value of the applied voltage, i.e. PIV $\geq V_m$ |
| 10. | Output frequency | $f$, i.e. same as input frequency |

## 1.10 FULL WAVE RECTIFIERS

The full wave rectifier inverts the negative portions of the sine wave so that a unipolar output signal is generated during both halves of the input sinusoidal. Following are the full wave rectification methods:

1. Centre tap full wave rectifier
2. The Bridge rectifier

### 1.10.1 Centre Taped Full wave Rectifier

One possible implementation of full wave rectifier is centre taped full wave rectifier. Figure 1.13 shows a centre taped full wave rectifier, input waveform, and the corresponding output waveform. The current through the two diodes in the circuit is given by

$$I_1 = I_m \sin \omega t, \quad 0 < \omega t < \pi$$
$$I_2 = I_m \sin \omega t, \quad \pi < \omega t < 2\pi$$

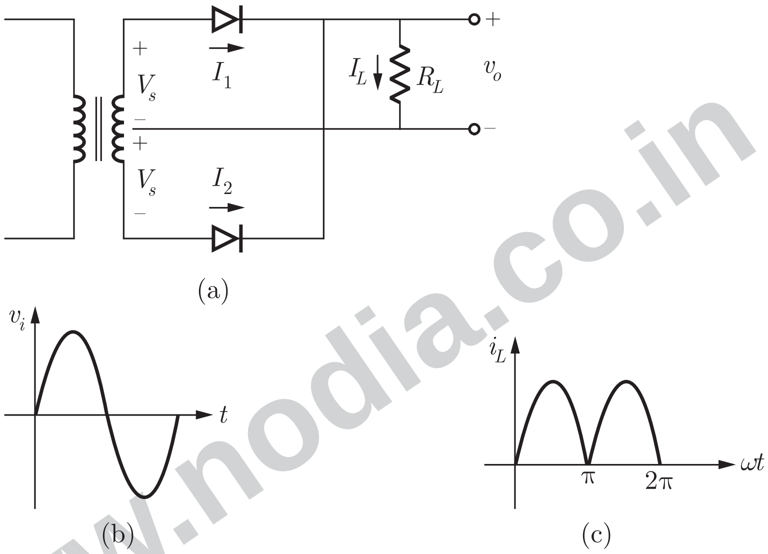



Figure 1.13: (a) Full Wave Rectifier, (b) Input Waveform, and (c) Output Waveform



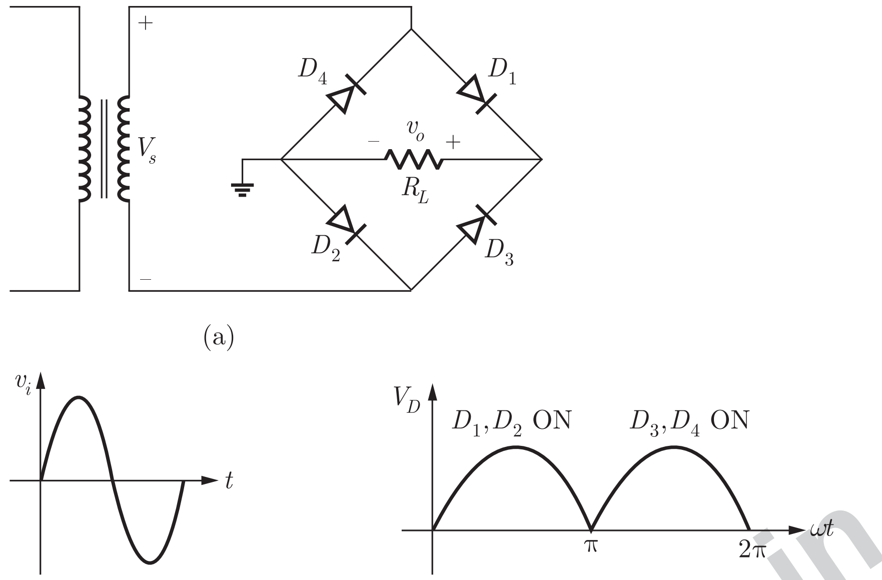
Some important parameters for a centre taped full wave rectifier are summarized in the following table.

Table 1.4: Parameters of Centre Taped Full Wave Rectifier

| S.N. | Parameters | Expression |
|---|---|---|
| 1. | DC load current | $I_{dc} = \frac{2I_m}{\pi}$ |
| 3. | Null load dc voltage | $(V_{dc})_{\text{NL}} = \frac{2V_m}{\pi}$ if $R_L = \infty$, i.e. load terminal is open |
| 4. | RMS load current | $I_{rms} = \frac{I_m}{\sqrt{2}}$ |
| 5. | RMS load voltage | $V_{rms} = \frac{V_m}{\sqrt{2}}$ |
| 6. | Ripple factor | $r.f. = 0.48$ |
| 8. | Rectifier efficiency | $\eta \approx 81.2\%$ |
| 9. | Peak inverse voltage | PIV $= 2V_m$ |
| 10. | Output frequency | $2f$, where $f$ is the input frequency. |

### 1.10.2 Bridge Rectifier

An alternative implementation of the full wave rectifier is bridge rectifier. Figure 1.14 shows a bridge rectifier, input waveform, and the corresponding output waveform.




(b) (c)

Figure 1.14: (a) Bridge Rectifier, (b) Input Waveform, (c) Output Waveform

Some important parameters for a bridge rectifier are summarized in the table below.

| S.N. | Parameters | Expression |
|---|---|---|
| 1. | DC load current | $I_{dc} = \dfrac{2I_m}{\pi}$ |
| 3. | Null load dc voltage | $(V_{dc})_{\text{NL}} = \dfrac{2V_m}{\pi}$<br>if $R_L = \infty$, i.e. load terminal is open |
| 4. | RMS load current | $I_{rms} = \dfrac{I_m}{\sqrt{2}}$ |
| 5. | RMS load voltage | $V_{rms} = \dfrac{V_m}{\sqrt{2}}$ |
| 6. | Ripple factor | $r.f. = 0.48$ |
| 8. | Rectifier efficiency | $\eta \approx 81.2\,\%$ |
| 9. | Peak inverse voltage | $\text{PIV} = V_m$ |
| 10. | Output frequency | $2f$, where $f$ is the input frequency. |

## 1.11 FILTERS

A capacitor is added in parallel with the load resistor of a half wave rectifier to form a simple filter circuit as shown in fig 1.15.

### Conduction Interval

The diode only conducts for a short time $\Delta T$ during each cycle. This time is called the conduction interval.

### Output Voltage

During the discharge period the voltage across the capacitor is described by

$$v_o(t) = V_m e^{-t/RC}$$

At the end of discharge interval, we have the output

$$V_L = V_m e^{-T'/RC}$$

where $T'$ is the discharge time.

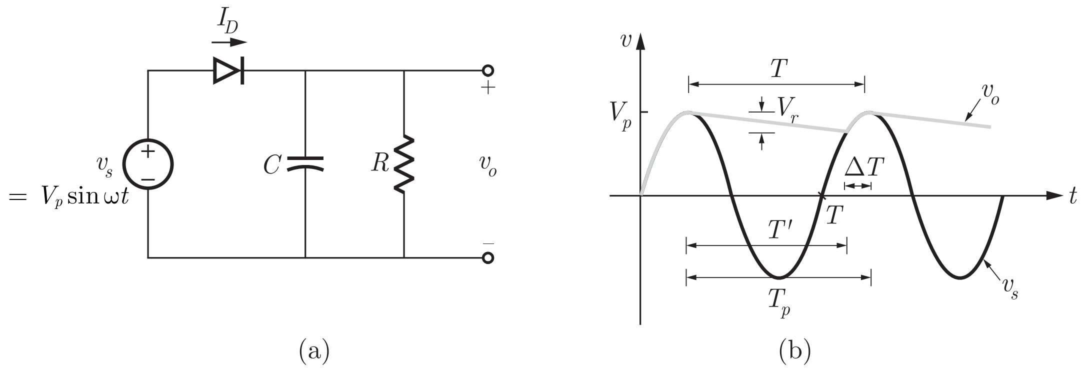



Figure 1.15: (a) Capacitor Filter Circuit, (b) Input and Output Waveform for the Filter

### Ripple Voltage

The output voltage is no longer constant as in the ideal peak detector circuit but has ripple voltage ($V_r$). The ripple voltage is defined as the difference between $V_m$ and $V_L$, i.e.

$$V_r = V_m - V_L = V_m(1 - e^{-T'/RC})$$

Since, $RC >> T'$, so we may write

$$V_r \approx V_m\left(\frac{T'}{RC}\right)$$

or $$V_r \approx V_m \frac{T}{RC} \qquad (T' \approx T)$$

or $$V_r = \frac{V_m}{fRC} \qquad (f = 1/T)$$

## 1.12 ZENER DIODE

Zener diodes are designed to provide a specified breakdown voltage. The breakdown voltage of the zener diode is nearly constant over a wide range of reverse bias currents. This makes the zener diode useful in a voltage regulator, or a constant voltage reference circuit. Figure 1.16 shows the zener diode and its equivalent circuit models for ON and OFF states.

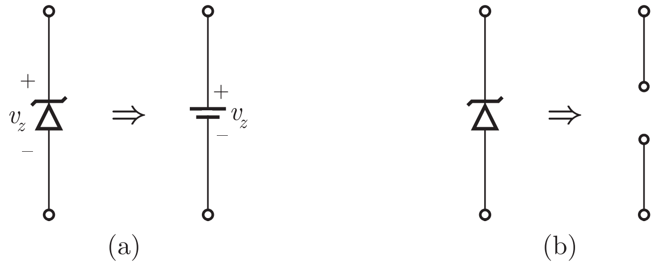



Figure 1.16: Equivalent Circuit Model for a Zener Diode for (a) ON State and (b) OFF State

## 1.13 VOLTAGE REGULATORS

A Voltage regulator is a device or combination of devices designed to maintain the output voltage of a power supply as nearly constant as possible. One of the simplest discrete Regulator consists of only a resistor and a zener diode, as shown in Figure 1.17.

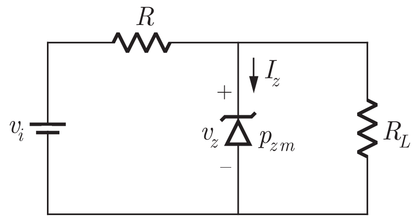
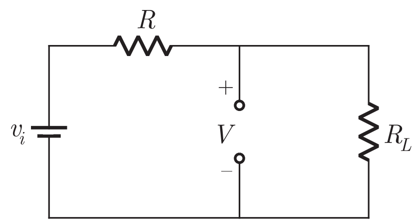



Figure 1.17: Basic Zener Regulator

METHODOLOGY: TO ANALYSE ZENER REGULATOR CIRCUIT

The applied dc voltage is fixed, as is the load resistor. The zener regulator can be analysed in the following steps.

Step 1: Remove the zener diode from the network.




Step 2: Calculate the voltage across the resulting open circuit.

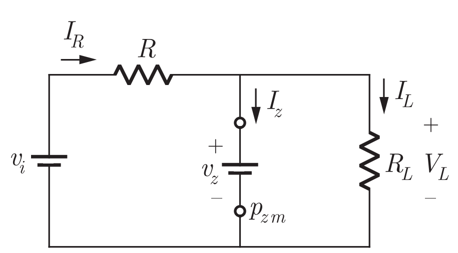
$$V = V_L = \frac{R_L}{R + R_L} v_i$$

Step 3: Determine the state of the zener diode by checking the obtained value of voltage $V$ for the following conditions

$$\text{If } V \geq V_z, \text{ the zener diode is ON}$$
$$\text{If } V < V_z, \text{ the diode is OFF}$$

Step 4: Substitute the appropriate equivalent circuit for the resulting state of zener diode. For ON state, equivalent mode of Figure 1.16(a) can be substituted, while for OFF state the open circuit equivalence of Figure 1.16(b) is substituted.

Step 5: Solve the resulting equivalent circuit for the desired unknowns. For example, assume that the equivalent circuit for the zener regulator is as shown below.




So, we get the various parameters for the circuit as

Load voltage, $V_L = V_z$

Load current, $I_L = \dfrac{V_L}{R}$

Current through resistance $R$, $I_R = \dfrac{V_R}{R} = \dfrac{V_i - V_L}{R}$

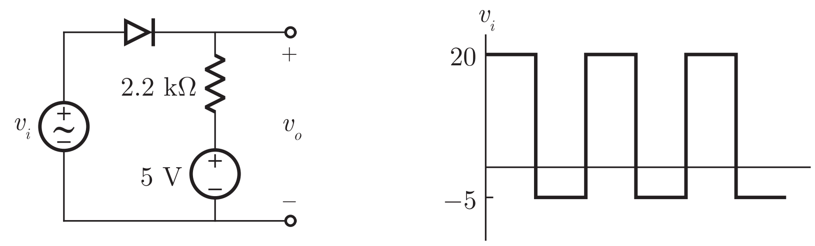
Zener current, $I_z = I_R - I_L$

Power dissipated by Zener diode, $P_z = V_z I_z$

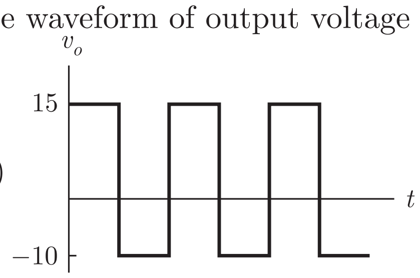
***********

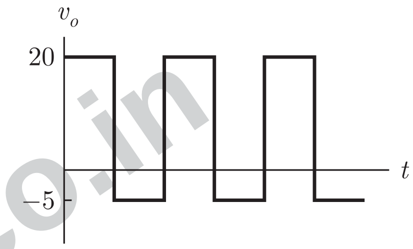
# EXERCISE 1.1

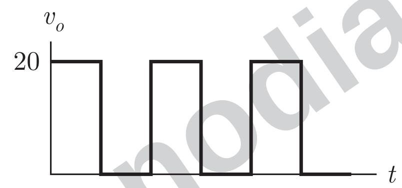
MCQ 1.1.1 Consider the given circuit and a waveform for the input voltage, shown in figure below. The diode in circuit has cutin voltage $V_\gamma = 0$.

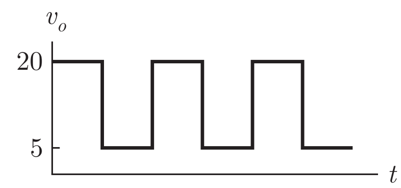



The waveform of output voltage $v_o$ is

(A)


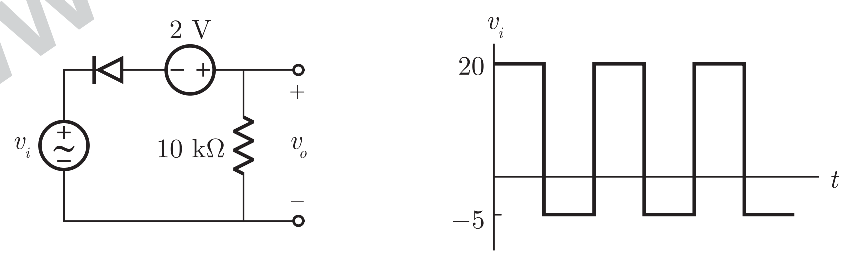

(B)



(C)


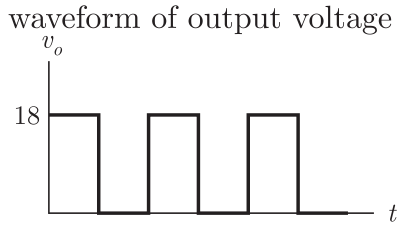

(D)


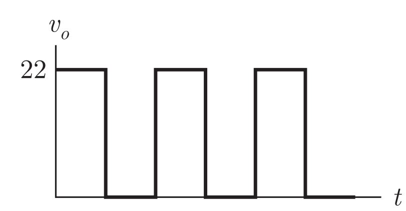


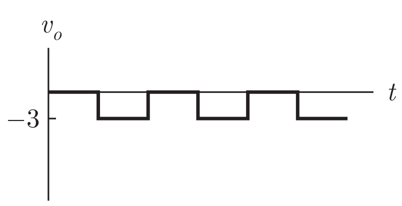

MCQ 1.1.2 Consider the given circuit and a waveform for the input voltage, shown in figure below. The diode in circuit has cutin voltage $V_\gamma = 0$.

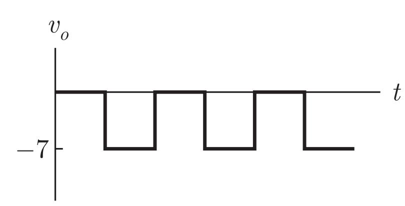



The waveform of output voltage $v_o$ is

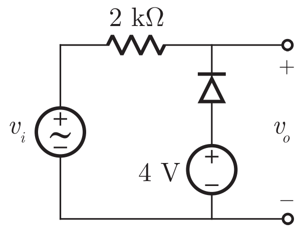
(A)


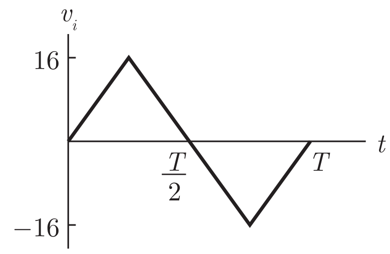

(B)



(C)


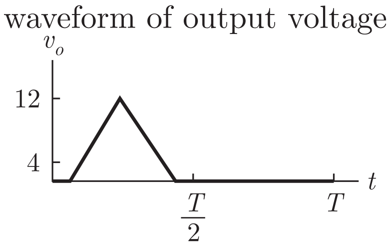

(D)


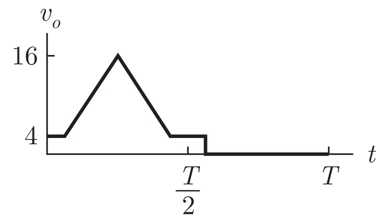

MCQ 1.1.3 Consider the given circuit and a waveform for the input voltage, shown in figure below. The diode in circuit has cutin voltage $V_\gamma = 0$.

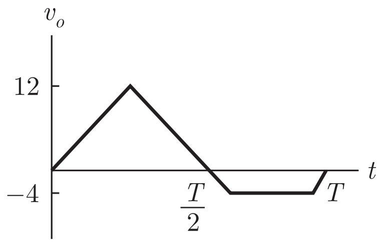


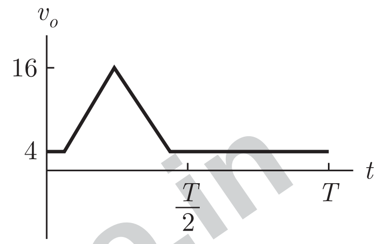



The waveform of output voltage $v_o$ is

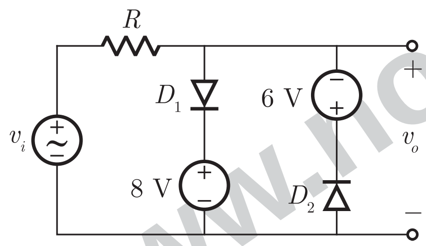
(A)


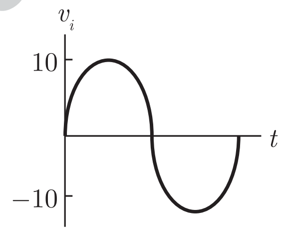
(B)


(C)


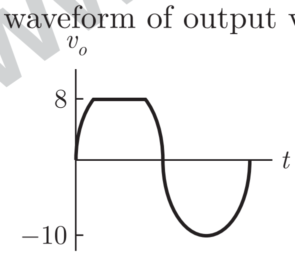
(D)


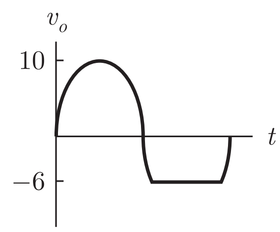
MCQ 1.1.4 Consider the given circuit and a waveform for the input voltage. The diode in circuit has cutin voltage $V_\gamma = 0$.

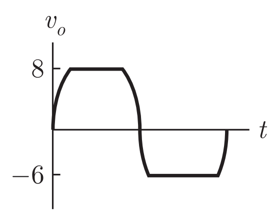


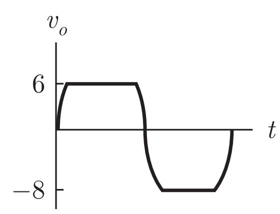



The waveform of output voltage $v_o$ is

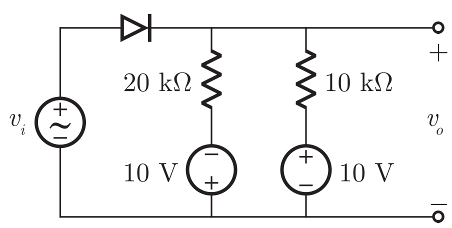
(A)


(B)


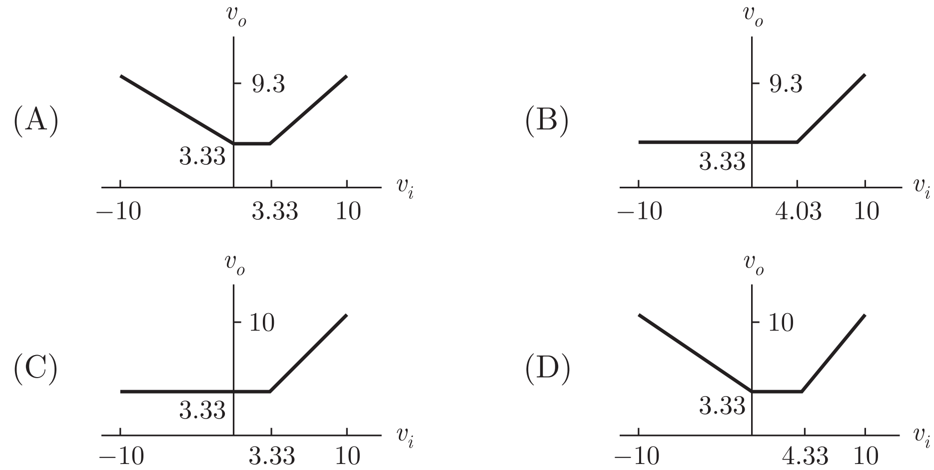
(C)


(D)


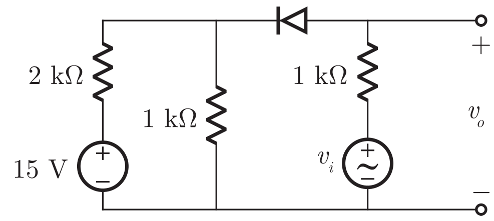
MCQ 1.1.5 For the circuit shown below, let cut in voltage $V_\gamma = 0.7$ V.



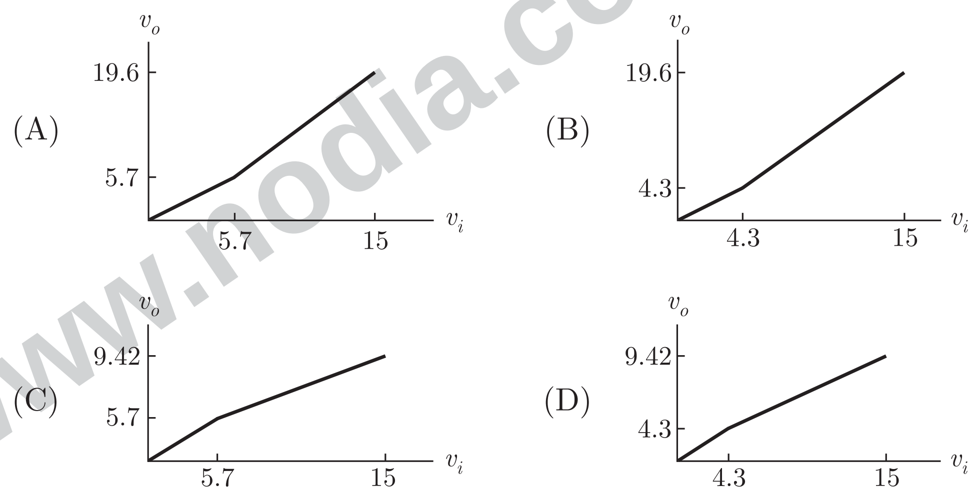

The plot of $v_o$ verses $v_i$ for $-10 \le v_i \le 10$ V is




MCQ 1.1.6 For the circuit shown below the cutin voltage of diode is $V_\gamma = 0.7$ V.

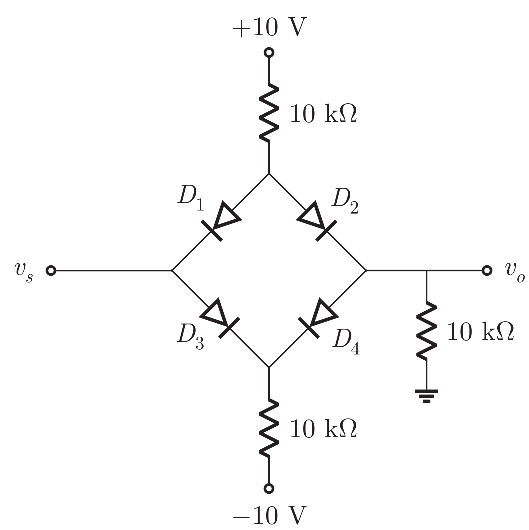



The plot of $v_o$ versus $v_i$ is

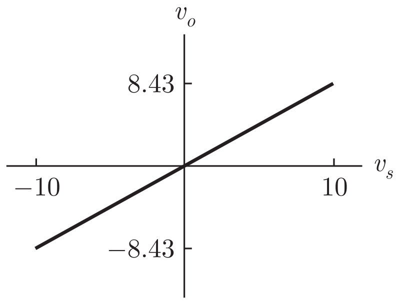


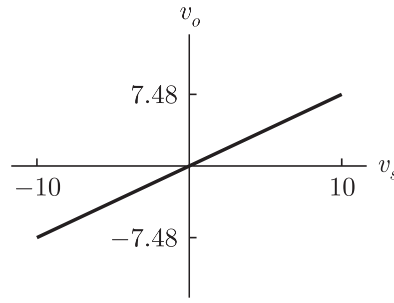


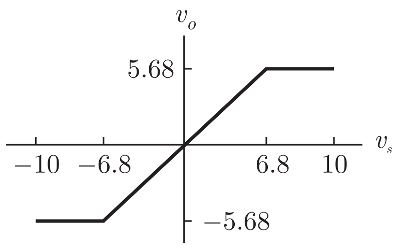

MCQ 1.1.7 For the circuit shown below each diode has $V_\gamma = 0.7$ V.

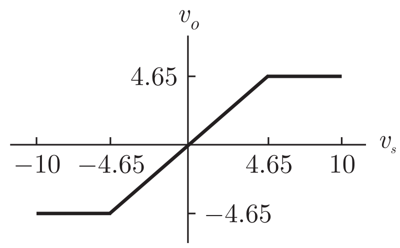



The $v_o$ for $-10 \le v_s \le 10$ V is

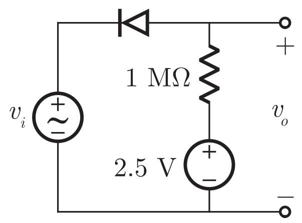
(A)



(B)


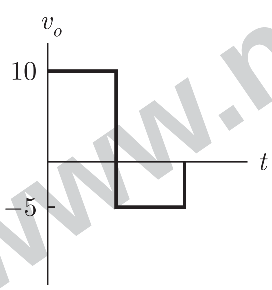

(C)


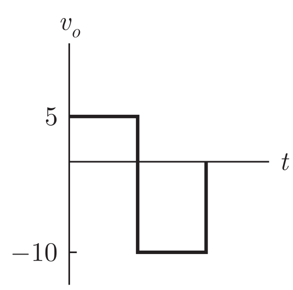

(D)


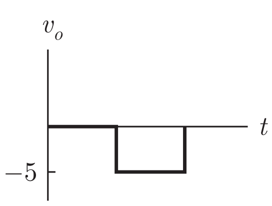

MCQ 1.1.8 A symmetrical 5 kHz square wave whose output varies between $+10$ V and $-10$ V is impressed upon the clipping shown in figure below

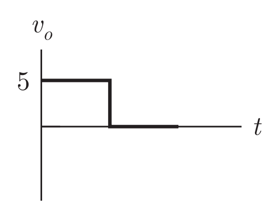



If diode has $r_f = 0$ and $r_r = 2$ MΩ and $V_\gamma = 0$, the output waveform is

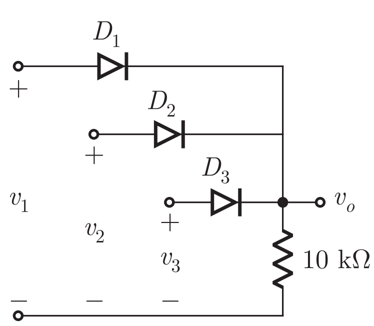
(A)


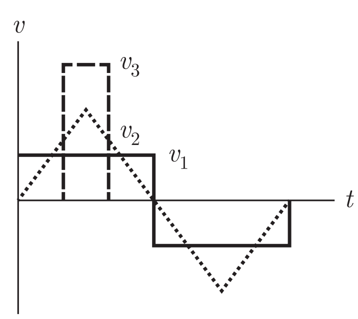

(B)



(C)


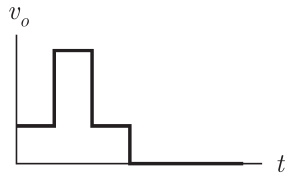

(D)


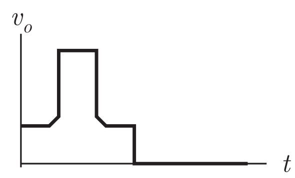

MCQ 1.1.9 In the circuit shown below, the three signals of fig are impressed on the input terminals.

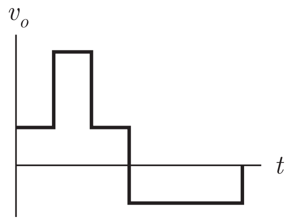


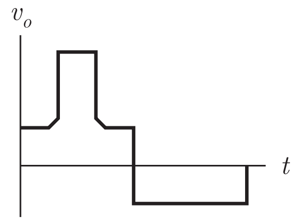




If diode are ideal then the voltage $v_o$ is

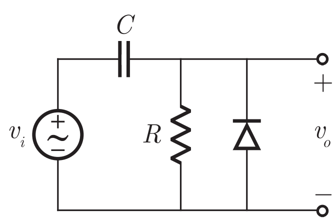
(A)


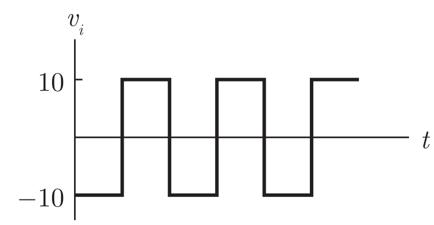

(B)



(C)


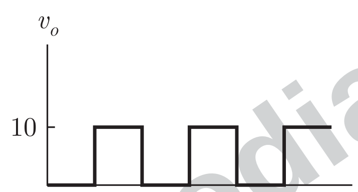

(D)


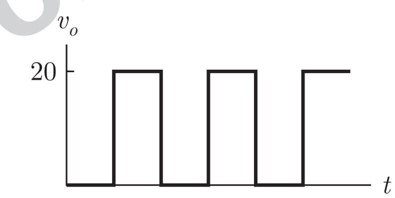

MCQ 1.1.10 For the circuit shown below the input voltage $v_i$ is as shown in figure.

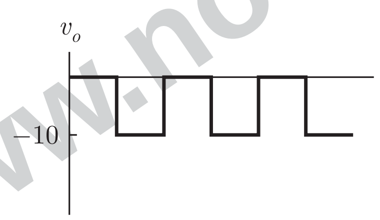


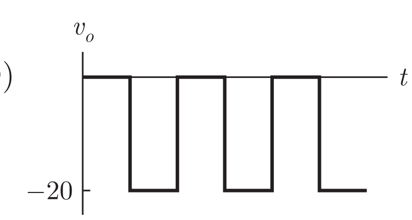




Assume the $RC$ time constant large and cutin voltage of diode $V_\gamma = 0$. The output voltage $v_o$ is

(A)


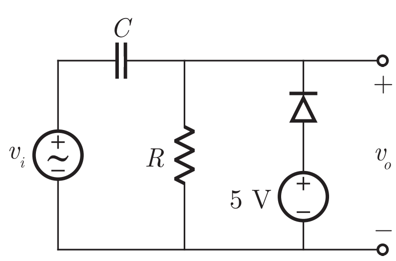

(B)


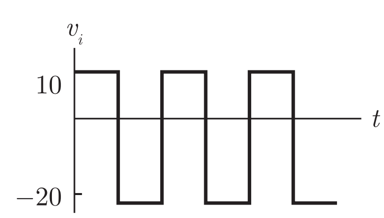

(C)



(D)


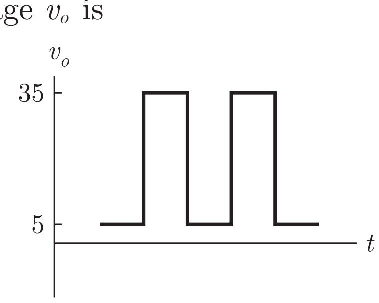


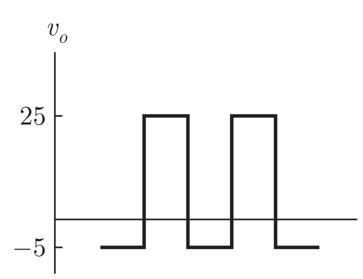

MCQ 1.1.11 For the circuit shown below, the input voltage $v_i$ is as shown in figure.

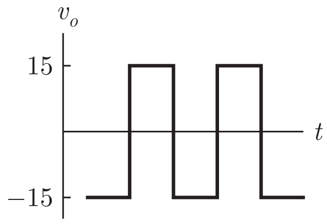



Assume the $RC$ time constant large and cutin voltage $V_\gamma = 0$. The output voltage $v_o$ is

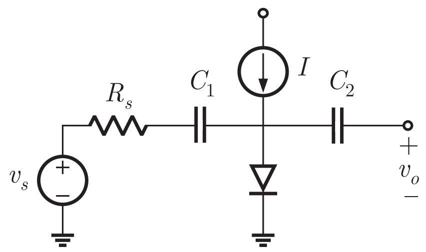
(A)



(B)


(C) 


(D) None of above

MCQ 1.1.12 In the circuit $I$ is DC current and capacitors are very large. Using small signal model which of following is correct ?



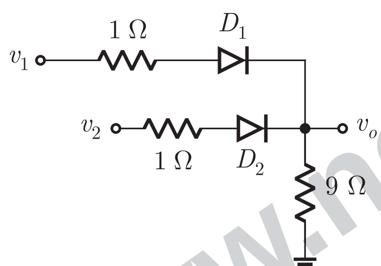

(A) $v_o = v_s \dfrac{\eta V_T}{\eta V_T + IR_s}$ (B) $v_o = v_s$

(C) $v_o = \dfrac{v_s}{\eta V_T + IR_S}$ (D) $v_o = 0$



**Common Data For Q. 13 to 15 :**

Consider the circuit shown below. Assume diodes are ideal.




MCQ 1.1.13 If $v_1 = 10$ V and $v_2 = 5$ V, then output voltage $v_o$ is

(A) 9 V (B) 9.474 V

(C) 0 V (D) 8.943 V

MCQ 1.1.14 If $v_1 = v_2 = 10$ V, then output voltage $v_o$ is

(A) 9 V (B) 9.474 V

(C) 4 V (D) 8.943 V

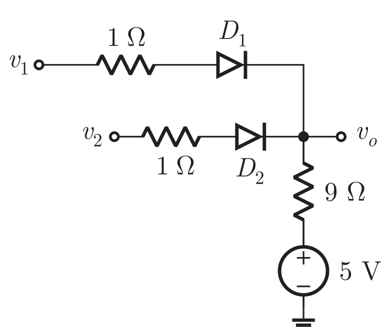
MCQ 1.1.15 If $v_1 = -5$ V and $v_2 = 5$ V then $v_o$ is

(A) 9.474 V (B) 8.943 V

(C) 4.5 V (D) 9 V

**Common Data For Q.16 and 17 :**

Consider the circuit shown below. Assume diodes are ideal.



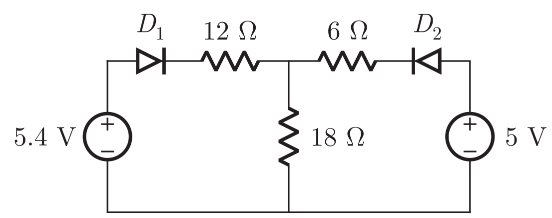
MCQ 1.1.16 If $v_1 = v_2 = 10$ V, then output voltage $v_o$ is

(A) 0 V (B) 9.737 V
(B) 9 V (D) 9.5 V

MCQ 1.1.17 If $v_1 = -5$ V and $v_2 = 10$ V, the output voltage $v_o$ is

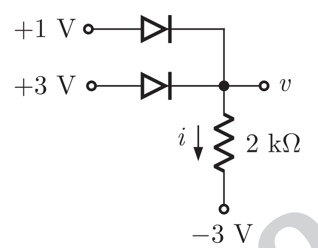
(A) 9 V (B) 9.737 V
(C) 9.5 V (D) 4.5 V

MCQ 1.1.18 In the circuit shown below diodes has cutin voltage of 0.6 V. The diode in ON state are



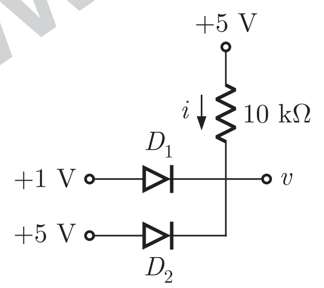

(A) only $D_1$ (B) only $D_2$
(C) both $D_1$ and $D_2$ (D) None of these

MCQ 1.1.19 For the circuit shown below cutin voltage of diode is $V_\gamma = 0.7$. What is the value of $v$ and $i$ ?



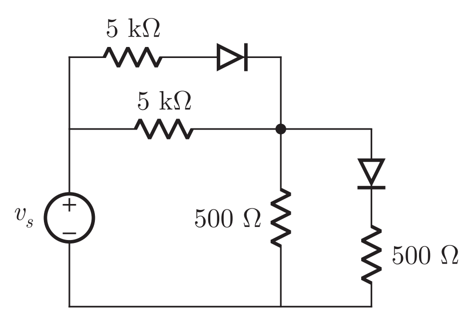

(A) 2.3 V, 2.65 mA (B) 2.65 V, 2.3 mA
(C) 2 V, 0 mA (D) 0 V, 2.3 mA

MCQ 1.1.20 For the circuit shown below the value of $v$ and $i$ are (if the diode is ideal)




(A) +5 V, 0 mA (B) +1 V, 0.6 mA
(C) +5 V, 0.4 mA (D) +1 V, 0.4 mA

MCQ 1.1.21 For the circuit shown below each diode has $V_\gamma = 0.6$ V and $r_f = 0$. Both diode will be ON if

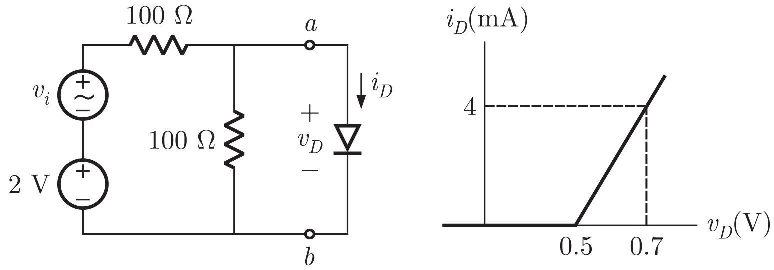


(A) $v_s > 3.9$ V (B) $v_s > 4.9$ V

(C) $v_s > 6.3$ V (D) $v_s > 5.3$ V

**Common Data For Q. 22 and 23 :**

The diode In the circuit shown below has the non linear terminal characteristic as shown in figure. Let the voltage be $v_i = \cos\omega t$ V.




MCQ 1.1.22 The current $i_D$ is

(A) $2.5(1+\cos\omega t)$ mA (B) $5(0.5+\cos\omega t)$ mA

(C) $5(1+\cos\omega t)$ mA (D) $5(1+0.5\cos\omega t)$ mA

MCQ 1.1.23 The voltage $v_D$ is

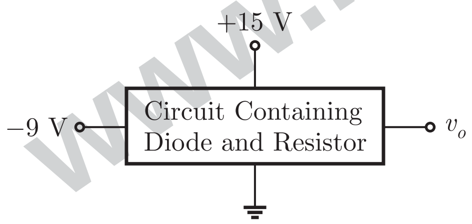
(A) $0.25(3+\cos\omega t)$ V

(B) $0.25(1+3\cos\omega t)$ V

(C) $0.5(3+1\cos\omega t)$ V

(D) $0.5(2+3\cos\omega t)$ V

MCQ 1.1.24 The circuit inside the box in figure shown below contains only resistor and diodes. The terminal voltage $v_o$ is connected to some point in the circuit inside the box.




The largest and smallest possible value of $v_o$ most nearly to is respectively

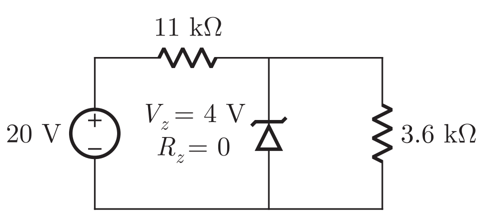
(A) 15 V, 6 V

(B) 24 V, 0 V

(C) 24 V, 6 V

(D) 15 V, −9 V

MCQ 1.1.25 The $Q$-point for the Zener diode shown below is




(A) (0.34 mA, 4 V)

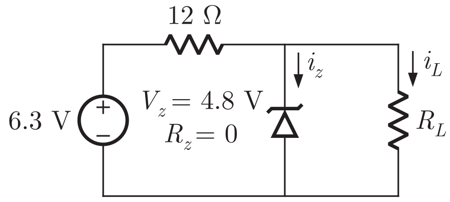
(B) (0.34 mA, 4.93 V)

(C) (0.94 mA, 4 V)

(D) (0.94 mA, 4.93 V)

**Common Data For Q. 26 to 28 :**

In the voltage regulator circuit shown below the zener diode current is to be limited to the range $5 \le i_z \le 100$ mA.




MCQ 1.1.26 The range of possible load current is

(A) $5 \le i_L \le 130$ mA

(B) $25 \le i_L \le 120$ mA

(C) $10 \le i_L \le 110$ mA

(D) None of the above

MCQ 1.1.27 The range of possible load resistance is

(A) $60 \le R_L \le 372\,\Omega$

(B) $60 \le R_L \le 200\,\Omega$

(C) $40 \le R_L \le 192\,\Omega$

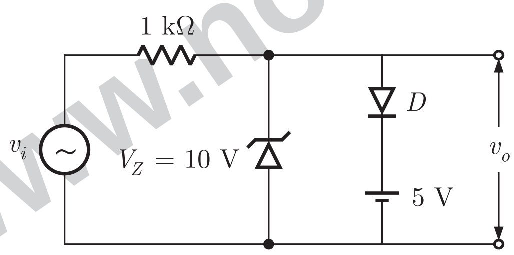
(D) $40 \le R_L \le 360\,\Omega$

MCQ 1.1.28 The power rating required for the load resistor is

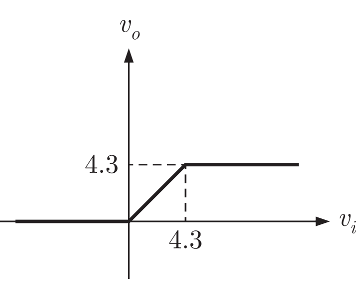
(A) 576 mW (B) 360 μW

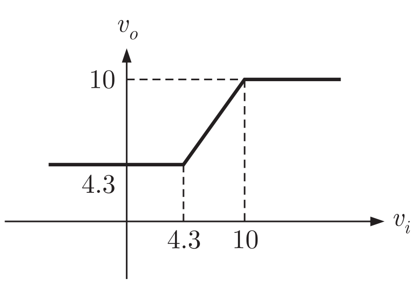
(C) 480 mW (D) 75 μW

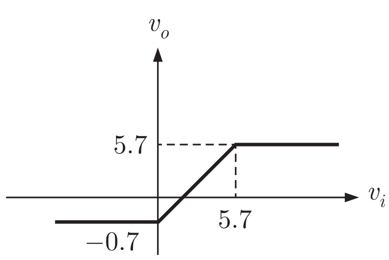
MCQ 1.1.29 A clipper circuit is shown below.

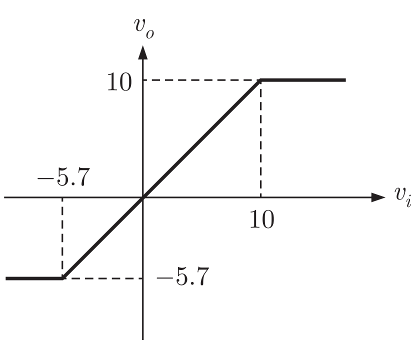



Assuming forward voltage drops of the diodes to be 0.7 V, the input-output transfer characteristics of the circuit is

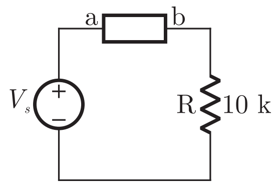
(A)


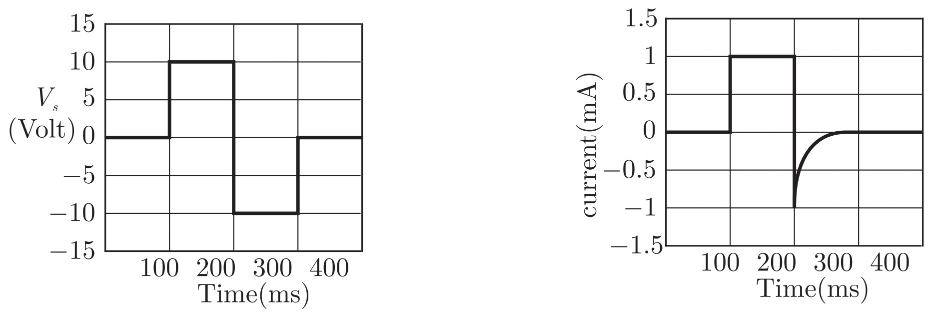

(B)



(C)



(D)


MCQ 1.1.30 The following circuit has a source voltage $V_s$ as shown in the graph. The current through the circuit is also shown.




The element connected between $a$ and $b$ could be

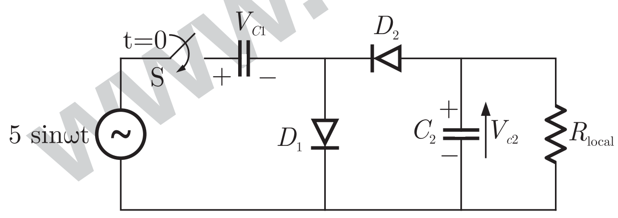
(A)


(B)


(C)

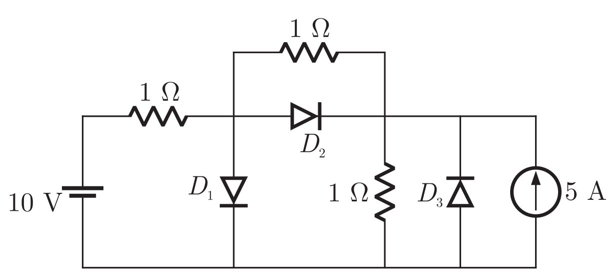

(D)


MCQ 1.1.31 In the voltage doubler circuit shown in the figure, the switch '$S$' is closed at $t = 0$. Assuming diodes $D_1$ and $D_2$ to be ideal, load resistance to be infinite and initial capacitor voltages to be zero. The steady state voltage across capacitor $C_1$ and $C_2$ will be



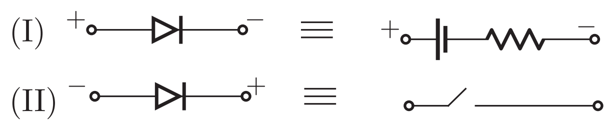

(A) $V_{c1} = 10$ V, $V_{c2} = 5$ V
(B) $V_{c1} = 10$ V, $V_{c2} = -5$ V
(C) $V_{c1} = 5$ V, $V_{c2} = 10$ V
(D) $V_{c1} = 5$ V, $V_{c2} = -10$ V

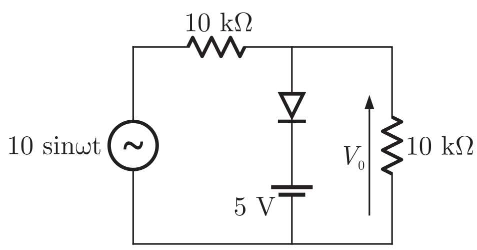
MCQ 1.1.32 What are the states of the three ideal diodes of the circuit shown in figure ?




(A) $D_1$ ON, $D_2$ OFF, $D_3$ OFF
(B) $D_1$ OFF, $D_2$ ON, $D_3$ OFF
(C) $D_1$ ON, $D_2$ OFF, $D_3$ ON
(D) $D_1$ OFF, $D_2$ ON, $D_3$ ON

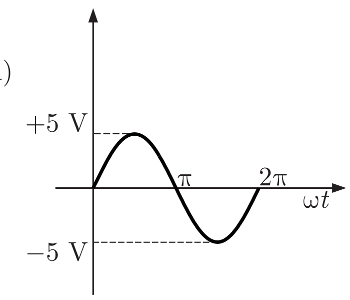
MCQ 1.1.33 The equivalent circuit of a diode, during forward biased and reverse biased conditions, are shown in the figure.



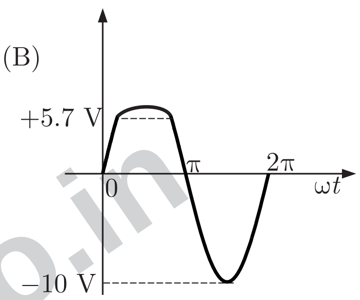




If such a diode is used in clipper circuit of figure given above, the output voltage $V_o$ of the circuit will be

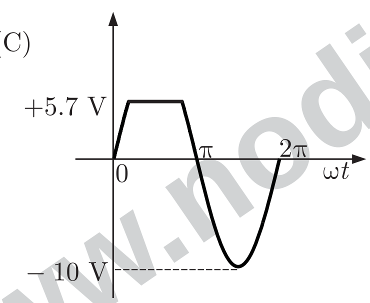
(A)



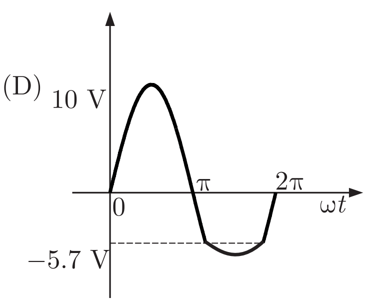

(B)



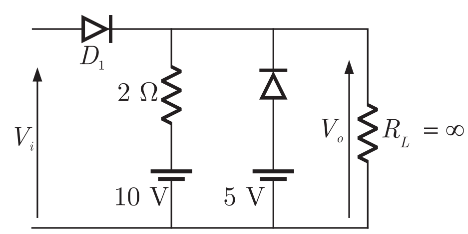

(C)



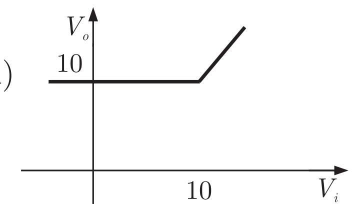

(D)



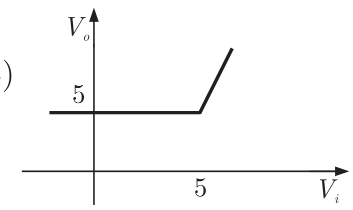

MCQ 1.1.34 Assuming the diodes $D_1$ and $D_2$ of the circuit shown in figure to be ideal ones, the transfer characteristics of the circuit will be



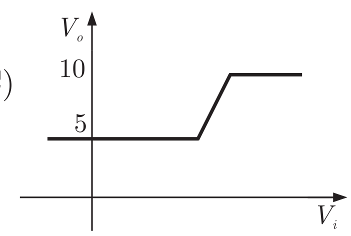

(A)



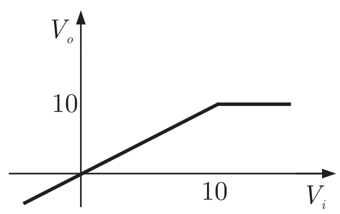

(B)



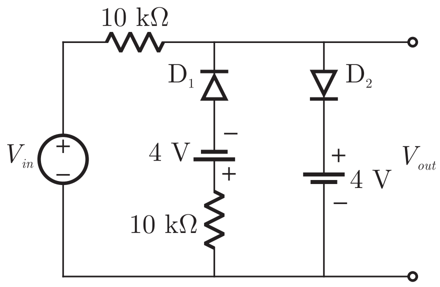

(C)




(D)

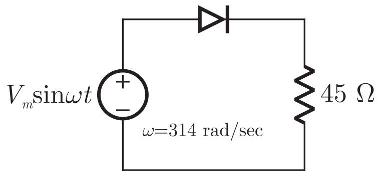


MCQ 1.1.35 A voltage signal $10\sin\omega t$ is applied to the circuit with ideal diodes, as shown in figure, The maximum, and minimum values of the output waveform $V_{out}$ of the circuit are respectively




(A) $+10$ V and $-10$ V
(B) $+4$ V and $-4$ V
(C) $+7$ V and $-4$ V
(D) $+4$ V and $-7$ V

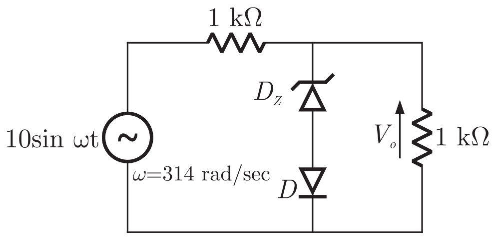
MCQ 1.1.36 The forward resistance of the diode shown in Figure is $5\,\Omega$ and the remaining parameters are same at those of an ideal diode. The dc component of the source current is




(A) $\frac{V_m}{50\pi}$ (B) $\frac{V_m}{50\pi\sqrt{2}}$
(C) $\frac{V_m}{100\pi\sqrt{2}}$ (D) $\frac{2V_m}{50\pi}$



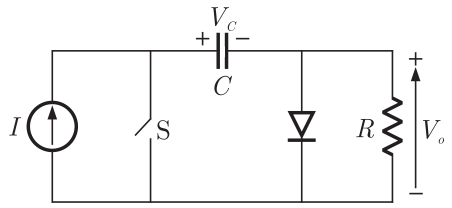
MCQ 1.1.37 The cut-in voltage of both zener diode $D_Z$ and diode $D$ shown in Figure is 0.7 V, while break-down voltage of $D_Z$ is 3.3 V and reverse break-down voltage of $D$ is 50 V. The other parameters can be assumed to be the same as those of an ideal diode. The values of the peak output voltage ($V_o$) are

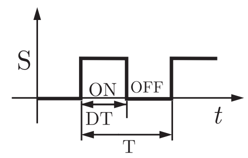



(A) 3.3 V in the positive half cycle and 1.4 V in the negative half cycle.
(B) 4 V in the positive half cycle and 5 V in the negative half cycle.
(C) 3.3 V in both positive and negative half cycles.
(D) 4 V in both positive and negative half cycle

**Common Data For Q. 38 and 39**

In the circuit shown in Figure, the source $I$ is a dc current source. The switch $S$ is operated with a time period $T$ and a duty ratio $D$. You may assume that the capacitance $C$ has a finite value which is large enough so that the voltage. $V_C$ has negligible ripple, calculate the following under steady state conditions, in terms of $D$, $I$ and $R$




MCQ 1.1.38 The voltage $V_c$, with the polarity shown in Figure,

(A) $\frac{I}{C}$ (B) $\frac{I}{C}(1-DT)$

(C) $\frac{I}{C}(1-D)T$ (D) $-\frac{I}{C}T$

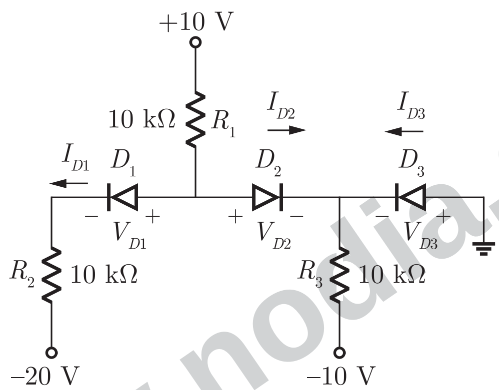
MCQ 1.1.39 The average output voltage $V_o$, with the polarity shown in figure

(A) $-\frac{I}{C}T$ (B) $-\frac{I}{2C}D^2T$

(C) $\frac{I}{2C}(1-DT)$ (D) $\frac{I}{2C}(1-D)T$

**Common Data For Q. 40 and 41:**

Consider the given circuit and the cut in voltage of each diode is $V_\gamma = 0.6$ V.






MCQ 1.1.40 What is the value of $I_{D1}$, $I_{D2}$, $I_{D3}$ respectively?

(A) 1.94 mA, −0.94 mA, 1.86 mA (B) 1.47 mA, 0.94 mA, 0 mA

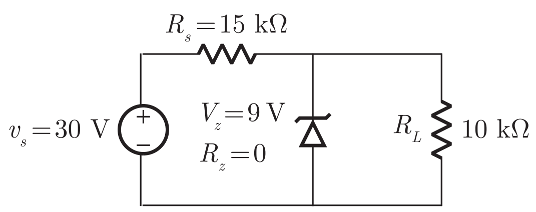
(C) 1.47 mA, 0 mA, 0.94 mA (D) 1.94 mA, 0 mA, 0.94 mA

MCQ 1.1.41 The value of $V_{D1}$, $V_{D2}$, $V_{D3}$ respectively is

(A) 0.6, −4.10, 0.6 (B) 0.6, −8.8, 0.6

(C) 0.6, 0.6, 0.6 (D) 0.6, 0.6, −4.10

MCQ 1.1.42 The voltage regulator shown below, what are the nominal and worst case values of zener diode current if the power supply voltage, zener break down voltage and registor all have 5 % tolerance?

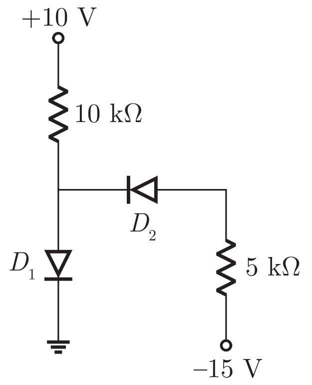



| | $I_z^{\text{nom}}$ | $I_z^{\text{nom}}$ | $I_{z(\min)}^{\text{worst}}$ |
|---|---|---|---|
| (A) | 1.2 mA | 0 mA | 0 mA |
| (B) | 0.5 mA | 0.70 mA | 0.103 mA |
| (C) | 0.5 mA | 0.60 mA | 0.346 mA |
| (D) | 0.5 mA | 0.796 mA | 0.215 mA |

MCQ 1.1.43 What are the nominal and worst case values of zener power dissipation?

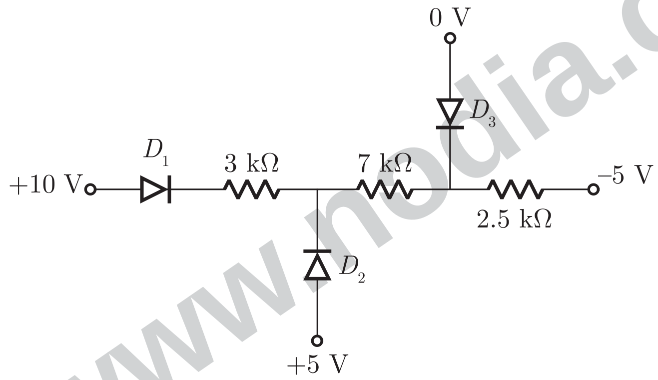
(A) 10.8 mW, 0 mW, 0 mW
(B) 4.5 mW, 7.52 nW, 1.80 mW
(C) 4.5 mW, 6.81 mW, 2.03 mW
(D) 4.5 mW, 5.13 mW, 5.67 mW

MCQ 1.1.44 The diode circuit in figure shown below the biasing of the diode $D_1$, $D_2$ is



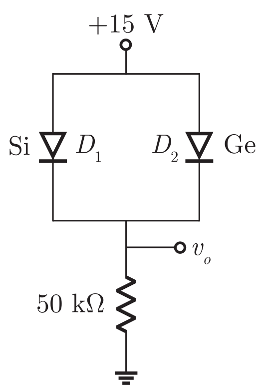

(A) ON, ON
(B) ON, OFF
(C) OFF, ON
(D) OFF, OFF

MCQ 1.1.45 The diode circuit shown in figure. Assume that diode is ideal, what will be the biasing modes of diode $D_1$, $D_2$ and $D_3$? (FB → forward biased, RB → reverse biased)



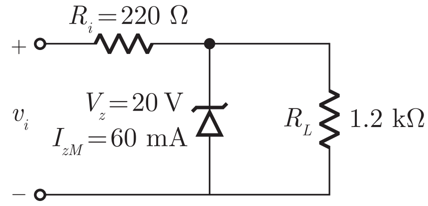

| | $D_1$ | $D_2$ | $D_3$ |
|---|---|---|---|
| (A) | FB | FB | FB |
| (B) | FB | FB | RB |
| (C) | FB | RB | RB |
| (D) | FB | RB | FB |

MCQ 1.1.46 The diode in the circuit shown below have linear parameter of $V_\gamma = 0.7$ (for Si), $V_\gamma = 0.3$ (for Ge) and $r_f = 0$ for both the diode. What is the biasing condition of diode $D_1$ and $D_2$?




(A) $D_1$ – ON, $D_2$ – ON
(B) $D_1$ – ON, $D_2$ – OFF
(C) $D_1$ – OFF, $D_2$ – ON
(D) $D_1$ – OFF, $D_2$ – OFF

MCQ 1.1.47 In the voltage regulator circuit below rating of zener diode is given, then the range of values of $v_i$ that will maintain the zener diode in the 'ON' state is



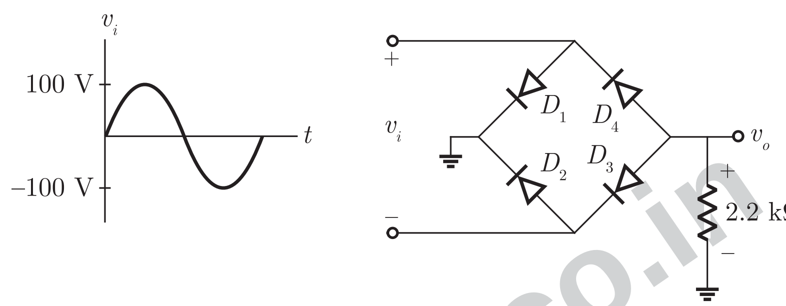

(A) $16.87 < v_i < 36.87$ V

(B) $16.87 < v_i < 23.67$ V

(C) $23.67 < v_i < 36.87$ V

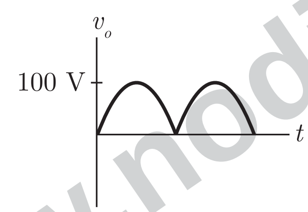
(D) None of the above

MCQ 1.1.48 Consider the given a circuit and a waveform for the input voltage.

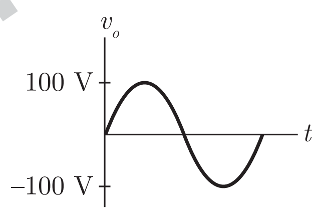



If the diode has cut in voltage $V_\gamma = 0$, the output waveform of the circuit is

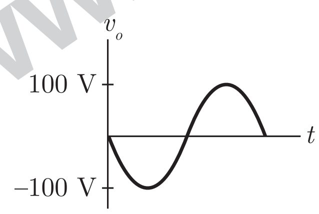
(A)



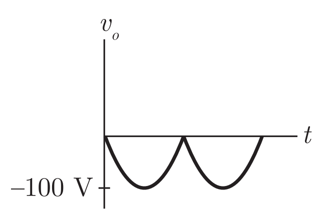

(B)




(C)

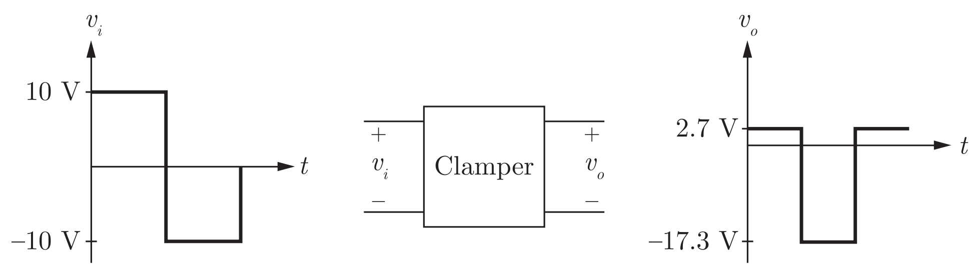



(D)

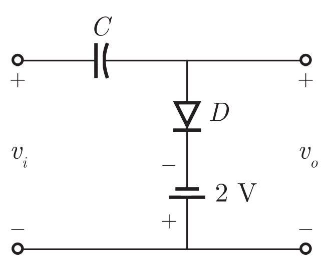




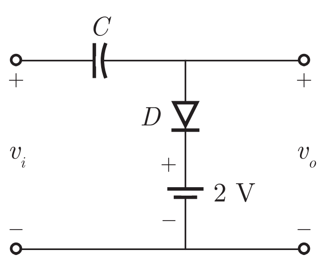

MCQ 1.1.49 Assume that the diode cut in voltage for the circuit shown below is $V_\gamma = 0.7$V . Which of clamper circuit perform the function shown below ?



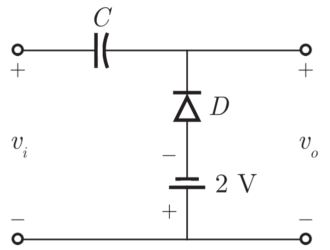

(A)



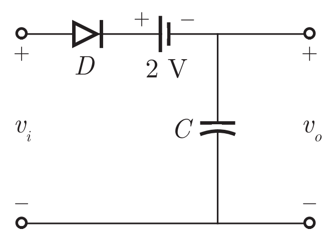

(B)



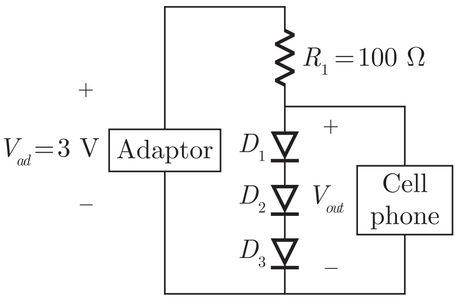
(C)




(D)




MCQ 1.1.50 Having lost his 2.4 V cellphone charger, an electrical engineering student tries several stores but does not find adaptors with outputs less than 3 V. He then decides to put his knowledge of electronics to work and constructs the circuit shown in figure where three identical diodes in forward bias produce a total voltage of $V_{out} = 3V_D \simeq 2.4$ V and resistor $R_1$ sustain remaining voltage. Neglect the current drawn by the cellphone and assume $V_{Thermal} = 26$ mV. The reverse saturation current $I_S$ for the diode is

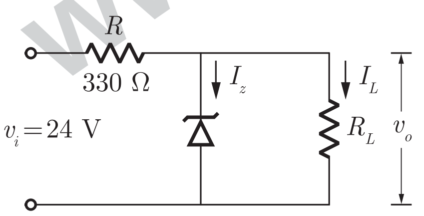



(A) 6 mA
(B) $2.602 \times 10^{-16}$ A
(C) $8.672 \times 10^{-14}$ A
(D) $7.598 \times 10^{-14}$ A



**Common Data For Q. 25 to 27:**

The circuit diagram of a zener regulator is shown in figure below. The data sheet specification for zener IN4742A provides following values
$V_z = 12$ V at $I_{zT} = 21$ mA, $R_z = 9$ (assume constant) $I_{zM} = 76$ mA, $I_{zk} = 1$ mA
.




MCQ 1.1.51 The values of zener diode voltages $(V_z)_{max}$, $(V_z)_{min}$ respectively are

(A) 11.82 V, 12.5 V
(B) 12.5 V, 11.82 V
(C) 12.18 V, 11.82 V
(D) 12.5 V, 11.5 V

MCQ 1.1.52 The maximum value of load current over which the zener diode is in ON state.

(A) 36.88 mA
(B) 35.88 mA
(C) 36.36 mA
(D) 35.36 mA

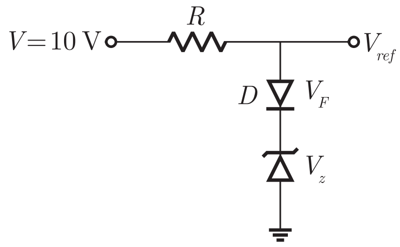
MCQ 1.1.53 The value of $R_L$ (Load resistance) corresponding to maximum load current is

(A) $329.43\ \Omega$
(B) $334.44\ \Omega$
(C) $325.37\ \Omega$
(D) $320.49\ \Omega$

**Common Data For Q. 54 and 55 :**

A breakdown diode has $V_z = 6.2\text{ V}$ at $25^\circ\text{C}$ and $\alpha_z = 0.02\%/^\circ\text{C}$. A silicon diode with $V_F = 0.7\text{ V}$ and a temperature coefficient of $-1.8\text{ mV}/^\circ\text{C}$ is connected in series with the breakdown diode.

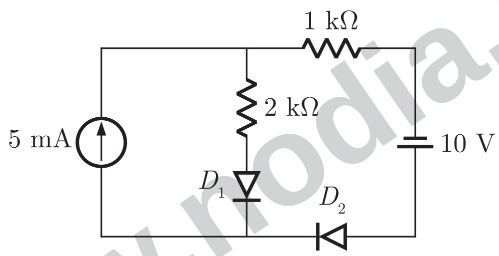
MCQ 1.1.54 The new value of reference voltage and the temperature coefficient of the series combination of diode $D$ and zener diode.




(A) 6.9 V, $-0.008\,\%/^\circ\text{C}$ (B) 6.9 V, $-0.0289\,\%/^\circ\text{C}$
(C) 6.9 V, $-0.056\,\%/^\circ\text{C}$ (D) 6.2 V, $-0.056\,\%/^\circ\text{C}$

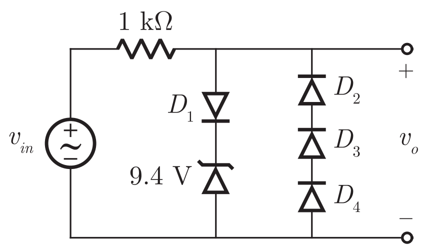
MCQ 1.1.55 The new value of $V_{ref}$ at a temperature of $50^\circ\text{C}$ is
(A) 6.186 (B) 6.886 V
(C) 6.914 V (D) 6.700 V

MCQ 1.1.56 The diode circuit given below. Assume diode is ideal. The operating states of diodes $D_1$, $D_2$ are



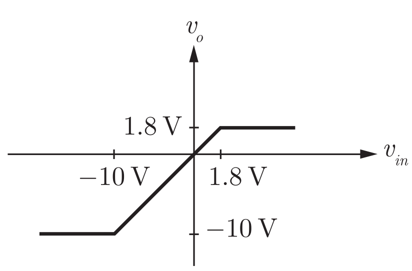

| | $D_1$ | $D_2$ |
|---|---|---|
| (A) | ON, | ON |
| (B) | ON, | OFF |
| (C) | OFF, | ON |
| (D) | OFF, | OFF |

MCQ 1.1.57 Consider the given circuit. The diode in circuit has cut in voltage $V_\gamma = 0.6\text{ V}$ and zener diode voltage $V_z = 9.4\text{ V}$.

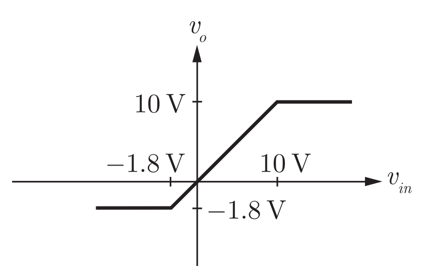



Plot $v_o$ versus $v_i$ is

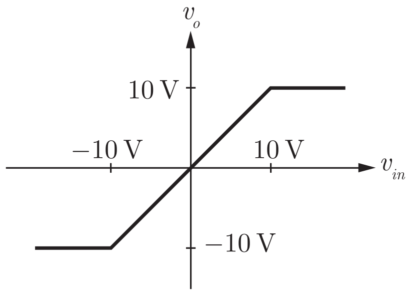
(A)



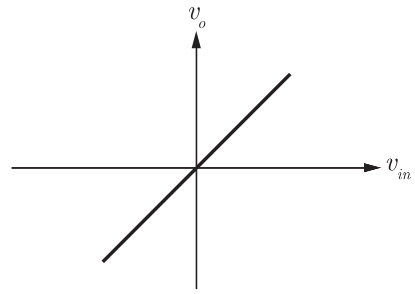

(B)



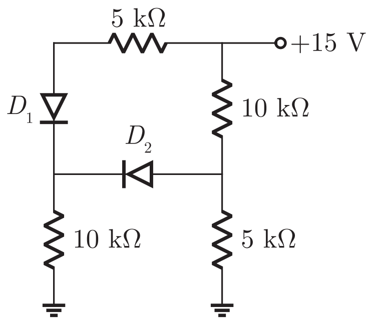
(C)




(D)

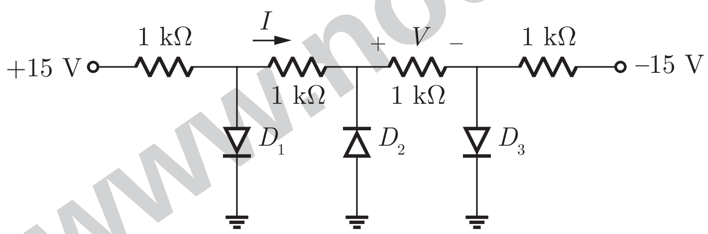



MCQ 1.1.58 Assume that the diodes in the circuit are ideal. What are the operating states of diodes ?




| | $D_1$ | $D_2$ |
|---|---|---|
| (A) | ON | ON |
| (B) | ON | OFF |
| (C) | OFF | ON |
| (D) | OFF | OFF |

MCQ 1.1.59 Assuming that the diodes are ideal in the given circuit.




Status of the diodes $D_1$, $D_2$ and $D_3$ are respectively

(A) ON, OFF, OFF

(B) ON, OFF, ON

(C) ON, ON, OFF

(D) ON, ON, ON

MCQ 1.1.60 What is the value of voltage V and current $I$ respectively ?

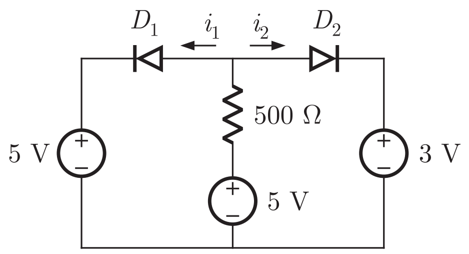
(A) 0 V, 0 Amp (B) 7.5 V, 7.5 mAmp

(C) 7.5 V, 0 Amp (D) 0 V, 7.5 mAmp

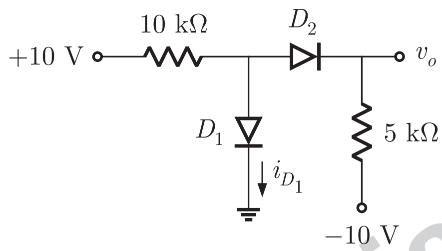
**********

# EXERCISE 1.2

QUES 1.2.1 In the circuit shown below, $D_1$ and $D_2$ are ideal diodes. The current $i_2$ is _____ mA

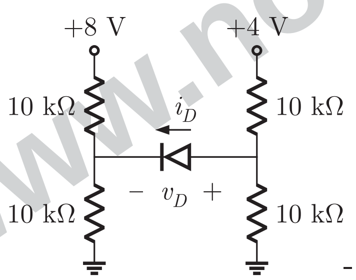



QUES 1.2.2 Let cutin voltage $V_\gamma = 0.7$ V for each diode in the circuit shown below.

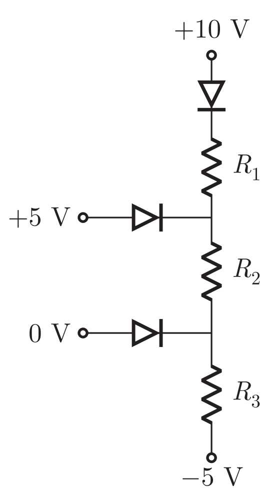



The voltage $v_o$ is _____ volts

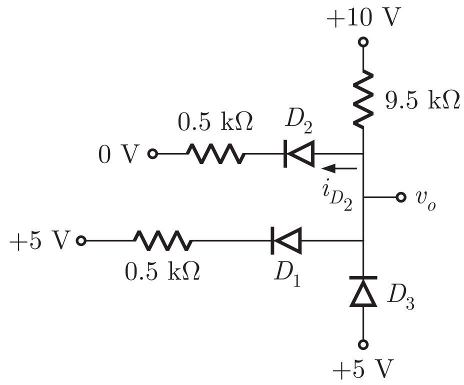
QUES 1.2.3 For the circuit in the figure below. The value of $v_D$ is _____volts.




QUES 1.2.4 The cutin voltage for each diode in circuit shown below is $V_\gamma = 0.6$ V. Each diode current is 0.5 mA. The value of $R_3$ will be _____ kΩ.

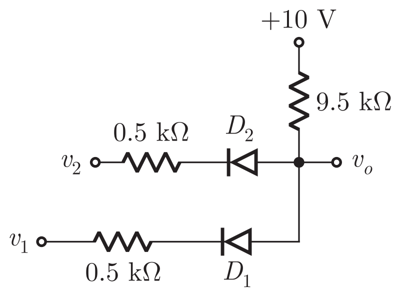



QUES 1.2.5 The diodes in the circuit shown below has parameters $V_\gamma = 0.6$ V and $r_f = 0$ . The current $i_{D_2}$ is ____ mA.




**Common Data For Q. 6 to 8 :**

The diodes in the given circuit have linear parameter of $V_\gamma = 0.6$ V and $r_f = 0$.



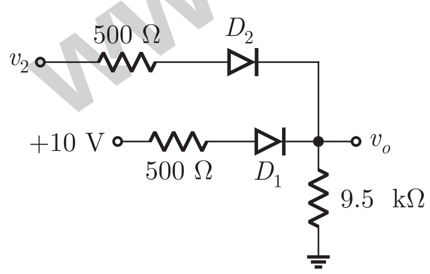

QUES 1.2.6 If $v_1 = 10$ V and $v_2 = 0$ V, then $v_o$ is _____ volts.

QUES 1.2.7 If $v_1 = 10$ V and $v_2 = 5$ V, then $v_o$ is_____ volts.

QUES 1.2.8 If $v_1 = v_2 = 0$, then output voltage $v_o$ is _____ volts.

**Common Data For Q. 9 to 11 :**

The diodes in the circuit shown below have linear parameters of $V_\gamma = 0.6$ V and $r_f = 0$.



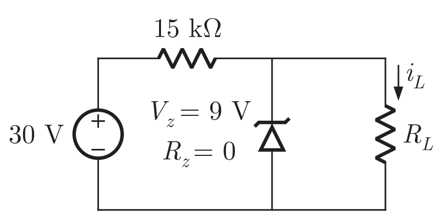

QUES 1.2.9 If $v_2 = 0$, then output voltage $v_o$ is_____ volts.

QUES 1.2.10 If $v_2 = 5$ V, then $v_o$ is____volts.

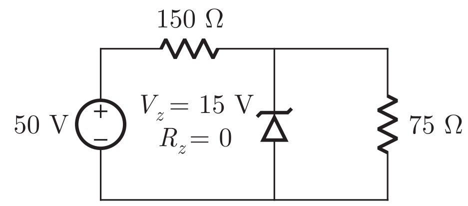
QUES 1.2.11 If $v_2 = 10$ V, then $v_o$ is_____ volts.

QUES 1.2.12 Ten diodes, each of them provides 0.7 V drop when the current through it is 20 mA, connected in parallel operate at a total current of 0.1 A. What current flows in each diode (in Amp) ?

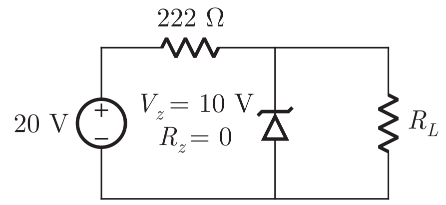
QUES 1.2.13 In the voltage regulator circuit shown below the maximum load current $i_L$ that can be drawn is _____ mA.



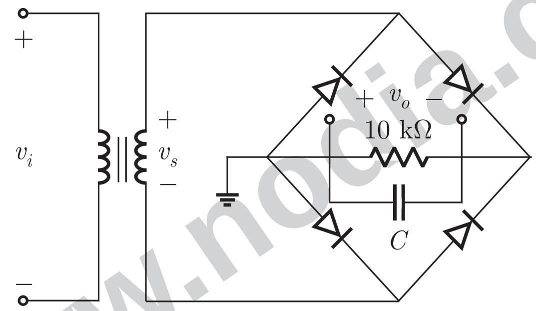
QUES 1.2.14 In the voltage regulator shown below the power dissipation in the Zener diode is _____ watts.



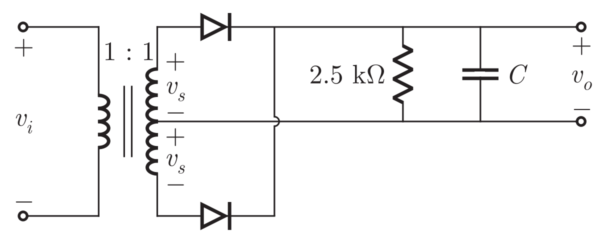

QUES 1.2.15 In the voltage regulator circuit shown below the power rating of Zener diode is 400 mW. The value of $R_L$ that will establish maximum power in Zener diode is _____ kΩ.



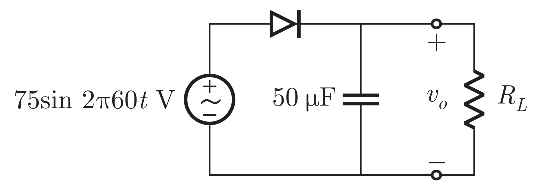

QUES 1.2.16 The secondary transformer voltage of the rectifier circuit shown below is $v_s = 60\sin 2\pi 60t$ V. Each diode has a cut in voltage of $V_\gamma = 0.7$ V. The ripple voltage is to be no more than $v_{rip} = 2$ V. The value of filter capacitor will be _____ μF.



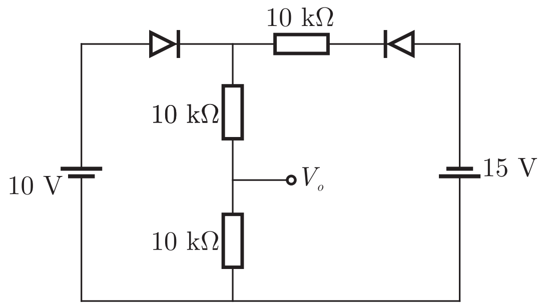

QUES 1.2.17 The input to full-wave rectifier shown below is $v_i = 120\sin 2\pi 60t$ V. The diode cutin voltage is 0.7 V. If the output voltage cannot drop below 100 V, the required value of the capacitor is _____ μF.



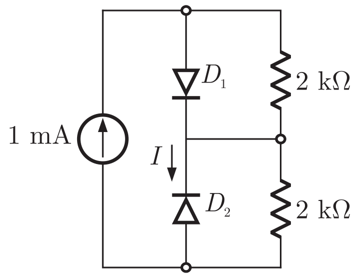

QUES 1.2.18 For the circuit shown below diode cutin voltage is $v_{in} = 0$. The ripple voltage is to be no more than $v_{rip} = 4$ V. The minimum load resistance, that can be connected to the output is _____ kΩ.



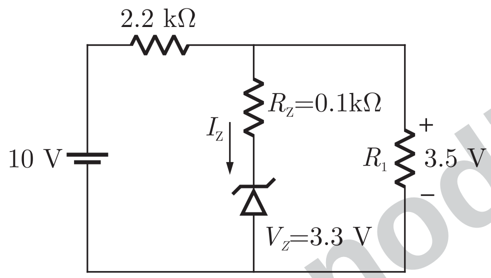

QUES 1.2.19 Assuming that the diodes in the given circuit are ideal, the voltage $V_o$ is_____ volts.



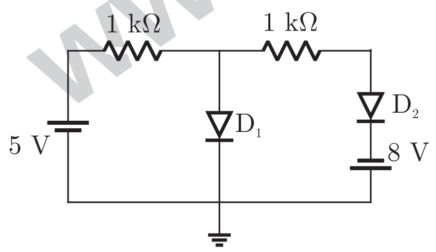

QUES 1.2.20 Assume that $D_1$ and $D_2$ in given figure are ideal diodes. The value of current is _____ mA.




QUES 1.2.21 The current through the Zener diode in figure is _____ mA.



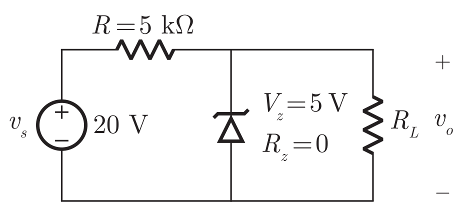

QUES 1.2.22 Assuming that in given circuit the diodes are ideal. The current in diode $D_1$ is ____ mA.




**Common Data For Q. 23 and 24 :**

In the zener diode voltage regulator circuit shown in the figure below, the zener diode has the following parameter.

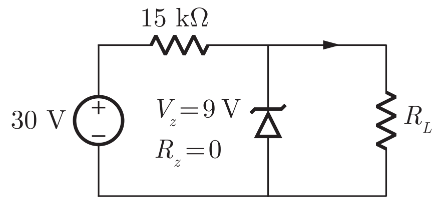
$$V_z = 5\,\text{V}, \quad R_z = 0\,\Omega$$




QUES 1.2.23 What is the value of $R_{L\,\min}$ (Minimum load resistance) for zener voltage regulator circuit (in kΩ) ?

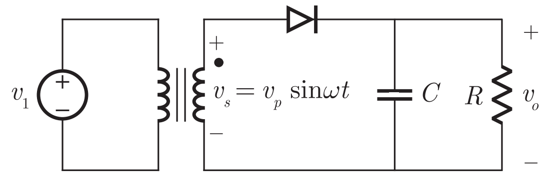
QUES 1.2.24 The value of output voltage $v_o$ for $R_L = 1\,\text{k}\Omega$ is ____ volts.

QUES 1.2.25 If $R_L = \infty$, the value of power dissipation in the zener diode is ____ mW.




**Common Data For Q. 26 and 27 :**

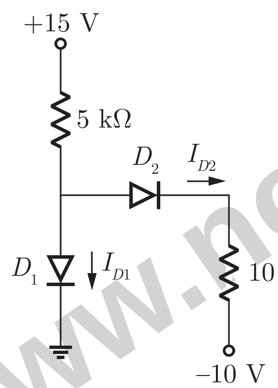
Given that a half-wave rectifier circuit driven from a transformer having a secondary voltage of $12.6\,V_{\text{rms}}$, $f = 60$ Hz with $R = 15\,\Omega$ and $C = 25$ mF. Assume the diode on voltage $V_{on} = 1$ V.



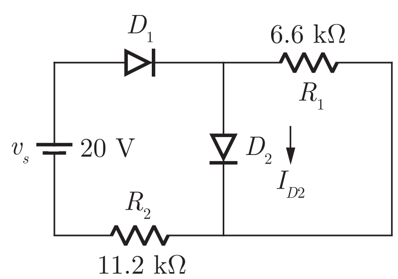

QUES 1.2.26 The value of the DC output voltage ($V_{o,\text{dc}}$) is ______ volts.

QUES 1.2.27 The value of ripple voltage is ____ volts.

QUES 1.2.28 Assume that in the given circuit diodes are ideal. The value of $I_{D2}$ is __mA.




QUES 1.2.29 The diode in the circuit shown below have linear parameter of $V_\gamma = 0.7$ V and $r_f = 0\,\Omega$. The value of the current ($I_{D2}$) in diode $D_2$ is ____ mA.




QUES 1.2.30 Given that $V_{\text{Thermal}} = 26$ mV. A diode is biased with a current of 1 mA. What is the value of the current change (in μA), if $V_D$ (Diode voltage) changes by 1 mV?

QUES 1.2.31 A transformer convert the 110 V, 60 Hz line voltage to a peak to peak swing of 9 V. A half wave rectifier follows the transformer to supply the power to the laptop computer of $R_L = 0.436\,\Omega$. What is the minimum value of the filter capacitor (in Farad) that maintains the ripple below 0.1 V ? (Assume $V_{D,on} = 0.8$ V)

QUES 1.2.32 A full wave rectifier is driven by a sinusoidal input $V_{in} = V_o \cos\omega t$, where $V_o = 3$ V and $\omega = 2\pi(60\text{ Hz})$. Assuming $V_{D,on} = 800$ mV, what is the value of the ripple amplitude (in volt) with a $1000\,\mu$F smoothing capacitor and a load resistance of $30\,\Omega$.

**Common Data For Q. 33 and 34 :**

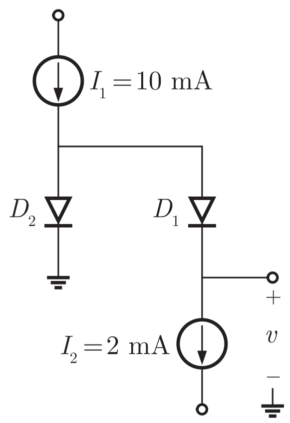
A full wave rectifier circuit is powered by ac mains. Power transformer has a center-tapped secondary. RMS secondary voltage across each half of secondary is 20 V. The dc winding resistance of each half of secondary is $5\,\Omega$. Forward resistance of diode is $2\,\Omega$. If the equivalent load resistance is $50\,\Omega$,

QUES 1.2.33 The value of dc power delivered to load is ____ watt.

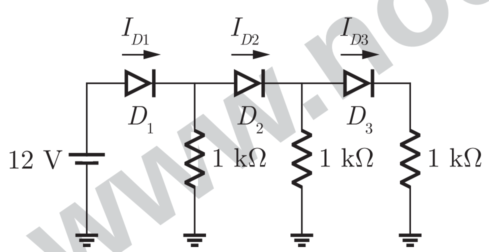
QUES 1.2.34 The percentage load regulation is _____.

QUES 1.2.35 The efficiency of rectification is ____

QUES 1.2.36 In the circuit shown in figure, both diodes have $\eta = 1$, but $D_1$ has 10 times the junction area of $D_2$. The value of voltage $V$ is ____ mV.



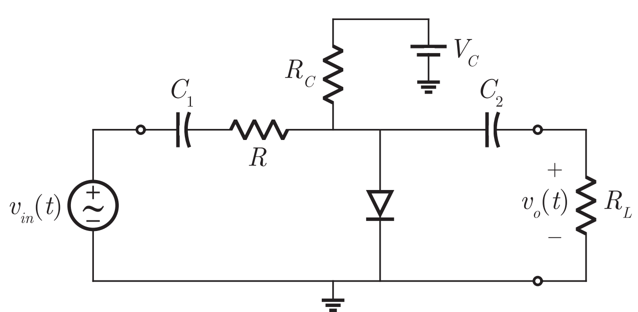

QUES 1.2.37 Assume that the voltage drop across each of the diodes in the circuit shown below is 0.7 V. The value of current through diode $D_1$, is $I_{D1} =$ _____ mA






**Common Data For Q. 38 and 39 :**

The circuit shown below has $R = 100\,\Omega$, $R_C = 2\,\text{k}\Omega$, $R_L = 2\,\text{k}\Omega$. Also, assume a constant diode voltage of 0.6 V and capacitors are very large using the small signal model for $V_C = 1.6\,\text{V}$.




QUES 1.2.38 What is the Q-point value of the diode current (in mA) ?

QUES 1.2.39 The value of $\dfrac{v_o(t)}{v_{in}(t)}$ is _____

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# EXERCISE 1.3

MCQ 1.3.1 Which of the following filter types is suitable only for large values of load resistance ?

(A) Capacitor filter (B) Inductor filter

(C) Choke-input filter (D) $\pi$-type CLC filter

MCQ 1.3.2 Which of the following filter types is suitable only for small values of load resistance ?

(A) Capacitor filter (B) Inductor filter

(C) Choke-input filter (D) $\pi$-type CLC filter

MCQ 1.3.3 A rectifier is used to

(A) convert a.c. voltage to d.c. voltage

(B) convert d.c. voltage to a.c. voltage

(C) both (a) and (b)

(D) convert voltage to current

MCQ 1.3.4 The ripple factor of a inductor filter is

(A) $\frac{R_L w}{\sqrt{2}\sqrt{3}L}$ (B) $\frac{R_L}{\sqrt{2}\sqrt{3}L}$

(C) $\frac{R_L}{3\sqrt{2}\,wL}$ (D) $\frac{R_L}{\sqrt{3}\,2wL}$



MCQ 1.3.5 A bleeded resistor is used in a d.c. power supply because it

(A) keeps the supply OFF (B) keeps the supply ON

(C) improves filtering action (D) improves voltage regulation

MCQ 1.3.6 A half wave diode rectifier and full wave diode bridge rectifier both have an input frequency of 50 Hz. The frequencies of outputs respectively are

(A) 100 Hz and 50 Hz (B) 50 Hz and 100 Hz

(C) 100 Hz each (D) 50 Hz each

MCQ 1.3.7 A full wave bridge diode rectifier uses diodes having forward resistance of 50 ohms each. The load resistance is also 50 ohms. The voltage regulations is

(A) 20% (B) 50%

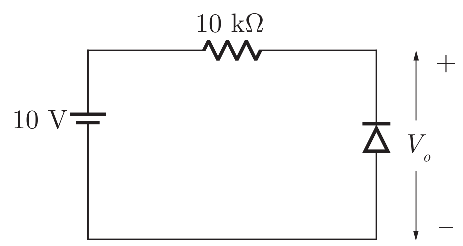
(C) 100% (D) 200%

MCQ 1.3.8 Which of the following components is essential for a voltage multiplier circuit

(A) resistor (B) inductor

(C) capacitor (D) both (A) and (C)

MCQ 1.3.9 In the circuit of Fig. the approximate value of $V_o$ across the germanium diode is



(A) 10 V (B) 5 V
(C) 2.5 V (D) 0.3 V

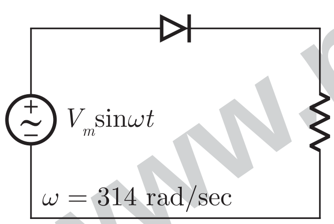
MCQ 1.3.10 When used in a circuit, a zener diode is always
(A) forward-biased (B) reverse-biased
(C) connected in series (D) at a temperature below 0°C

MCQ 1.3.11 For ideal Rectifier and filter circuits, % regulations must be
(A) 1% (B) 0.1%
(C) 5% (D) 0%

MCQ 1.3.12 A particular green LED emits light of wavelength 5490°A. The energy bandgap of the semiconductor material used there is (Planck's constant $= 6.626 \times 10^{-34}$ J-s)
(A) 2.26 eV (B) 1.98 eV
(C) 1.17 eV (D) 0.74 eV

MCQ 1.3.13 The function of bleeder resistor in a rectifier with $LC$ filter is to
(A) maintain the minimum current through $C$
(B) maintain the minimum current necessary for optimum inductor operation
(C) maintain maximum current through $L$
(D) Charge capacitor $C$ to maximum value

MCQ 1.3.14 The forward resistance of the diode shown in figure is $5\,\Omega$ and the remaining parameters are same as those of an ideal diode. The dc component of the source current is




(A) $\frac{V_m}{50\pi}$ (B) $\frac{V_m}{50\pi\sqrt{2}}$
(C) $\frac{V_m}{100\pi\sqrt{2}}$ (D) $\frac{2V_m}{50\pi}$

MCQ 1.3.15 The relation between diode current, voltage and temperature is given by
(A) $I = I_o\left(1 - e^{\frac{V}{\eta V_T}}\right)$ (B) $I = I_o\left(e^{\frac{V}{\eta V_T}} - 1\right)$
(C) $I = I_o\left(e^{\frac{V}{\eta V_T}} + 1\right)$ (D) $I = I_o e^{\frac{V}{\eta V_T}}$

MCQ 1.3.16 The Transformer Utilization Factor (TUF) is defined as
(A) $\frac{P_{dc}}{P_{ac\,\mathrm{rated}}}$ (B) $\frac{P_{dc}}{P_{ac}}$
(C) $\frac{I_{rms}}{I_{dc}}$ (D) $\frac{I_m}{I_{dc}}$

MCQ 1.3.17 The Peak inverse voltage (PIV) of an half wave rectifier and full-wave rectifier are
(A) $V_m$, $2V_m$ respectively (B) $2V_m$, $V_m$ respectively
(C) $\frac{V_m}{2}$, $V_m$ respectively (D) $V_m$, $\frac{V_m}{2}$ respectively

MCQ 1.3.18 Inductor filter should be used when
(A) load current is high (B) load current is low
(C) high load resistance $R_L$ (D) none of the above

MCQ 1.3.19 The value of critical inductance in an $LC$ filter is

(A) $L_c = 3\omega R_L$ (B) $L_c = \frac{R_L}{3\omega}$

(C) $L_c = \frac{3\omega}{R_L}$ (D) None of the above

MCQ 1.3.20 Transformer utilization factor is more for

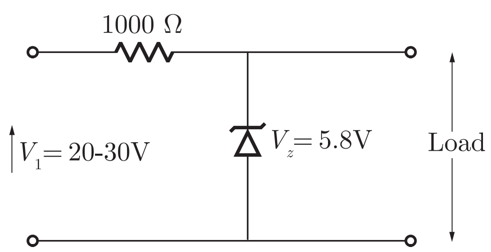
(A) half wave rectifier

(B) center tapped full wave rectifier

(C) bridge rectifier

(D) all of the above

MCQ 1.3.21 The zener diode in the regulator circuit shown in figure has a Zener voltage of 5.8 Volts and a Zener knee current of 0.5 mA. The maximum load current drawn from this circuit ensuring proper functioning over the input voltage range between 20 and 30 Volts, is




(A) 23.7 mA (B) 14.2 mA

(C) 13.7 mA (D) 24.2 mA

MCQ 1.3.22 The circuit which converts undirectional flow to D.C. is called

(A) Rectifier circuit

(B) Converter circuit

(C) filter circuit

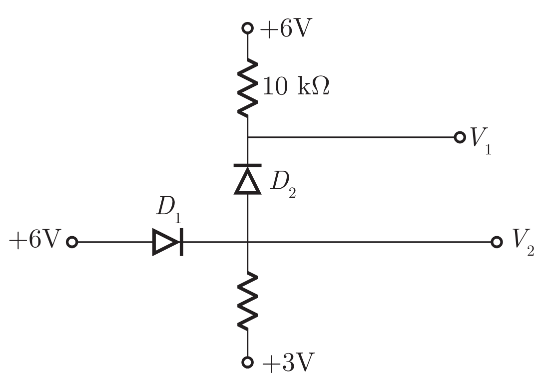
(D) Eliminator

MCQ 1.3.23 The value of current that flows through $R_L$ in a '$\pi$' section filter circuit at no load is

(A) $\infty$ (B) 0.1 mA

(C) 0 (D) few mA

MCQ 1.3.24 The voltage at $V_1$ and $V_2$ of the arrangement shown in Fig. will be respectively. (Assume diode cut in voltage $V_\gamma = 0.6$ V)



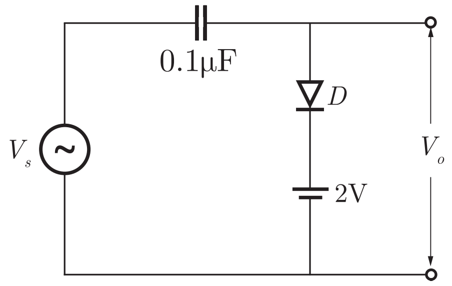

(A) 6 V and 5.4 V

(B) 5.4 V and 6 V

(C) 3 V and 5.4 V

(D) 6 V and 5 V

MCQ 1.3.25 For an input $V_S = 5\sin\omega t$ (assume ideal diode), the circuit shown in Fig. will be behave as a




(A) clipper, sine wave clipped at $-2$ V

(B) clamper, sine wave clamped at $-2$ V

(C) clamper, sine wave clamped at zero volt

(D) clipped, sine wave clipped at 2 V

MCQ 1.3.26 A clipper circuit always

(A) needs a dc source

(B) clips both half cycles of input signal

(C) clips upper portion of the signal

(D) clips some part of the input signal

MCQ 1.3.27 The primary function of a clamper circuit is to

(A) suppress variations in signal voltage

(B) raise positive half-cycle of the signal

(C) lower negative half-cycle of the signal

(D) introduce a dc level into an ac signal

MCQ 1.3.28 A zener diode has a dc power dissipation rating of 500 mW and a zener voltage rating of 6.8 V. The value of $I_{ZM}$ is

(A) 70 mA

(B) 72 mA

(C) 73.5 mA

(D) 75 mA

MCQ 1.3.29 When the reverse current in a zener diode increases from 20 mA to 30 mA, the zener voltage changes from 5.6 V to 5.65 V. The zener resistance is

(A) $2\,\Omega$ (B) $3\,\Omega$

(C) $4\,\Omega$ (D) $5\,\Omega$

MCQ 1.3.30 A 4.7 V zener has a resistance of $15\,\Omega$. When a current 20 mA passes through it, then the terminal voltage is

(A) 5 V

(B) 10 V

(C) 15 V

(D) 4.7 V

MCQ 1.3.31 In a full wave rectifier, the current in each of the diodes flows for

(A) the complete cycle of the input signal

(B) half cycle of the input signal

(C) for zero time

(D) more than half cycle of input signal

MCQ 1.3.32 The ripple factor of power supply is a measure of

(A) its filter efficiency

(B) diode rating

(C) its voltage regulation

(D) purity of power output

MCQ 1.3.33 Larger the value of filter capacitor

(A) larger the $p$-$p$ value of ripple voltage

(B) larger the peak current in the rectifying diode

(C) longer the time that current pulse flows through the diode

(D) smaller the d.c. voltage across the load

MCQ 1.3.34 In a centre-tap full-wave rectifier, if $V_m$ is the peak voltage between the centre tap and one end of the secondary, the maximum voltage coming across the reverse-biased diode is

(A) $V_m$ (B) $2V_m$

(C) $V_m/2$ (D) $V_m/\sqrt{2}$

MCQ 1.3.35 In a bridge type full wave rectifier, if $V_m$ is the peak voltage across the secondary of the transformer, the maximum voltage coming across each reverse-biased diode is

(A) $V_m$ (B) $2V_m$

(C) $V_m/2$ (D) $V_m/\sqrt{2}$

MCQ 1.3.36 In a half wave rectifier, the peak value of the ac. voltage across the secondary of the transformer is $20\sqrt{2}$ V. If no filter circuit is used, the maximum d.c. voltage across the load will be

(A) 28.28 V (B) 25 V

(C) 14.14 V (D) 9 V

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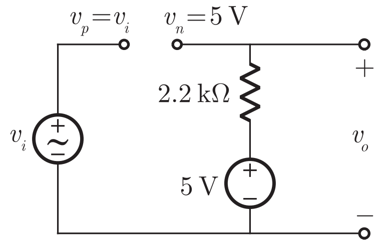
# SOLUTIONS 1.1



SOL 1.1.1 Option (D) is correct.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the diode is operating, and then obtain the output.

Step 1: Assume that the diode is OFF, and replace it by open circuit. So, equivalent circuit is,




Step 2: The voltage across the diode terminal is obtained as

$$v_p = v_i \qquad \text{at the } p\text{-terminal}$$

$$v_n = 5\text{ V} \qquad \text{at the } n\text{- terminal}$$

Step 3: Now, we have the condition

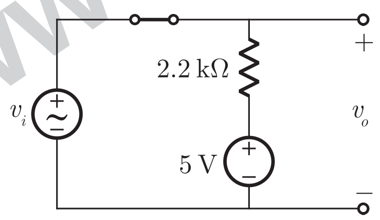
$$v_p > v_n \qquad \text{diode is ON}$$

$$v_p < v_n \qquad \text{diode is OFF}$$

Applying these conditions, we determine the output voltage.

CASE I:

If $v_i > 5$ V, then diode is ON. So the equivalent circuit is

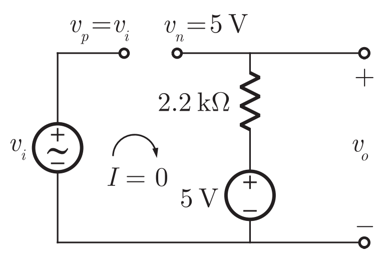



So, the output voltage is

$$v_o = v_i$$

CASE II:

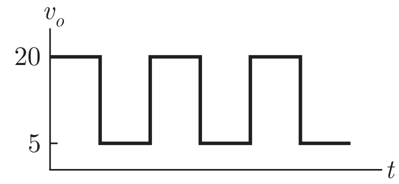
If $v_i < 5$ V, then diode is OFF. So equivalent circuit is




Since no current flows in the circuit (i.e. $i = 0$), so we get

$$v_o = 5\text{ V}$$

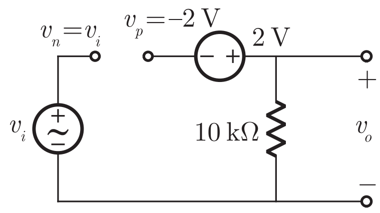
Step 4: From the two results obtained in the above steps, we sketch the output waveform as shown below.




SOL 1.1.2 Option (C) is correct.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the diode is operating, and then obtain the output.

Step 1: Assume that the diode is OFF, and replace it by open circuit. So, equivalent circuit is,




Step 2: The voltage across the diode terminal is obtained as

$$v_p = -2\text{ V} \qquad \text{at the } p\text{-terminal}$$

$$v_n = v_i \qquad \text{at the } n\text{- terminal}$$

Step 3: Now, we have the condition

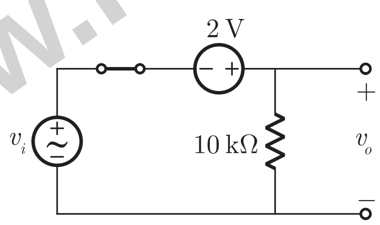
$$v_p > v_n \qquad \text{diode is ON}$$

$$v_p < v_n \qquad \text{diode is OFF}$$

Applying these conditions, we determine the output voltage.

CASE I:

If $v_i < -2$ V, then diode is ON. So the equivalent circuit is,

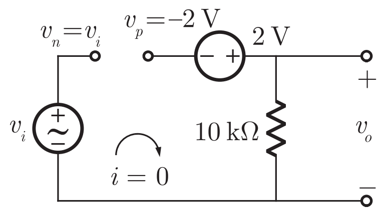



So, the output voltage is obtained as

$$v_o = v_i + 2$$

CASE II:

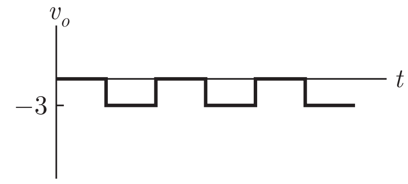
If $v_i > -2$ V, then diode is OFF. So, the equivalent circuit is,




Since no current flows in the circuit (i.e. $i = 0$), so we get

$$v_o = 0\text{ V}$$

Step 4: From the two results obtained in the above steps, we sketch the output waveform as shown below.

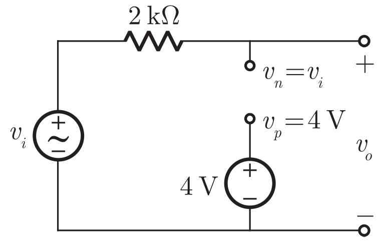



SOL 1.1.3 Option (D) is correct.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the diode is operating, and then obtain the output.

Step 1: Assume that the diode is OFF, and replace it by open circuit.

So, equivalent circuit is,




Step 2: The voltage across the diode terminal is obtained as

$$v_p = 4\text{ V} \qquad \text{at the } p\text{-terminal}$$
$$V_n = v_i \qquad \text{at the } n\text{- terminal}$$

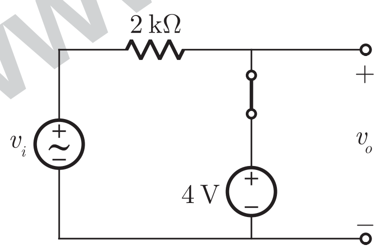
Step 3: Now, we have the condition

$$v_p > v_n \qquad \text{diode is ON}$$
$$v_p < v_n \qquad \text{diode is OFF}$$

Applying these conditions, we determine the output voltage.

CASE I:

If $v_i < 4$ V, then diode is ON. So the equivalent circuit is

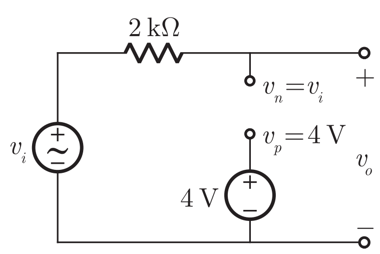



So, the output voltage is,

$$v_o = 4\text{ V}$$

CASE II:

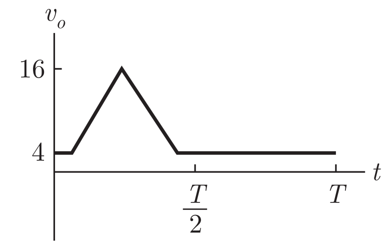
If $v_i > 4$ V, then diode is OFF. So equivalent circuit is




So, the output voltage is,

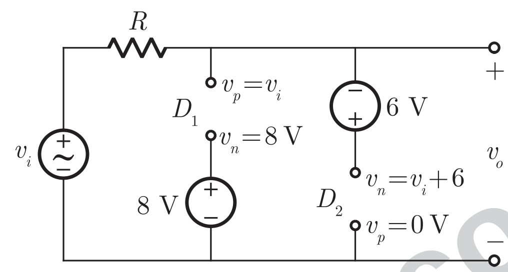
$$v_o = v_i$$

Step 4: From the two results obtained in the above steps, we sketch the output waveform as shown below.




SOL 1.1.4 Option (C) is correct.
For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the two diodes are operating, and then obtain the output.

Step 1: Assume that both the diodes are OFF, and replace it by open circuit. So, equivalent circuit is,




Step 2: For the assumption, the voltage across diode $D_1$ is obtained as

$$v_{p1} = v_i \qquad \text{at the } p\text{-terminal}$$
$$v_{n1} = 8\text{ V} \qquad \text{at the } n\text{- terminal}$$

Step 3: Voltage across the diode $D_2$ is obtained as

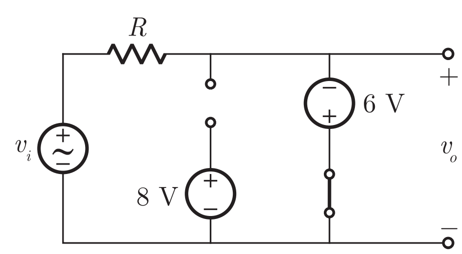
$$v_{p2} = 0\text{ V} \qquad \text{at the } p\text{-terminal}$$
$$v_{n1} = v_i + 6 \qquad \text{at the } n\text{- terminal}$$

Step 4: Now, we have the condition for both the diodes

$$v_p > v_n \qquad \text{diode is ON}$$
$$v_p < v_n \qquad \text{diode is OFF}$$

Applying these conditions, we determine the output voltage.

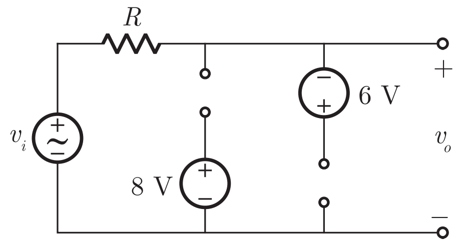
CASE I:
If $v_i + 6 < 0$ or $v_i < -6$ diode $D_1$ is OFF and diode $D_2$ is ON. So, the equivalent circuit is




So, the output voltage is,

$$v_o = -6\text{ V}$$

CASE II:
If $-6\text{ V} < v_i < 8\text{ V}$, then both diodes $D_1$ and $D_2$ are OFF. So, the equivalent circuit is

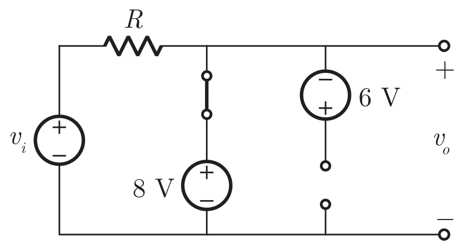



So, the output voltage is

$$v_o = v_i$$

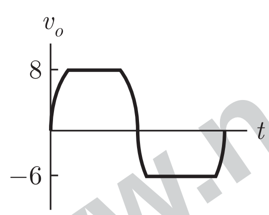
CASE III:

When $v_i > 8\,\text{V}$, then diode $D_1$ is ON and $D_2$ is OFF. So, the equivalent circuit is,




$$v_o = 8\,\text{V}$$

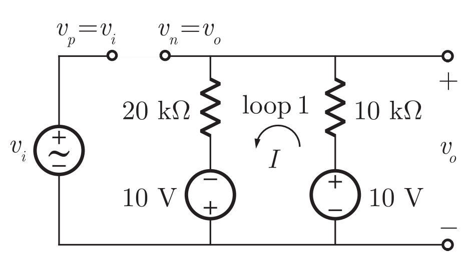
Step 5: From the results obtained in the above step, we sketch the output waveform as shown below.




SOL 1.1.5 Option (B) is correct.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the diode is operating, and then obtain the output.

Step 1: Assume that the diode is OFF, and replace it by open circuit. So, the equivalent circuit is




Step 2: Apply KVL in loop 1,

$$10 - 10\text{k}I - 20\text{k}I + 10 = 0$$

$$I = \frac{20}{30}\,\text{mA} = \frac{2}{3}\,\text{mA}$$

$$v_o = 10 - 10 \times \frac{2}{3} = \frac{10}{3} = 3.333\,\text{V}$$

Step 3: Now, we have the condition

$$v_p > v_n \qquad \text{diode is ON}$$

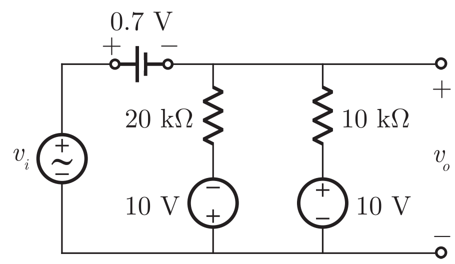
$$v_p < v_n \qquad \text{diode is OFF}$$

Using these conditions, we determine the output voltage.

CASE I

$$v_i \geq 3.333 + 0.7 \text{ or } v_i > 4.03$$

Diode is ON. So, the equivalent circuit is



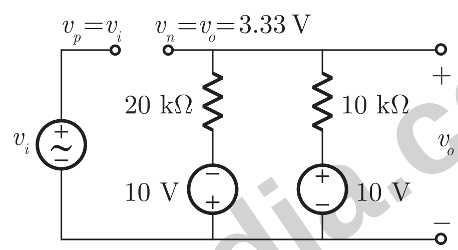

So, the output voltage is

$$v_o = v_i - 0.7$$

CASE II

$$v_i \leq 4.03$$

Diode is OFF. So, the equivalent circuit is

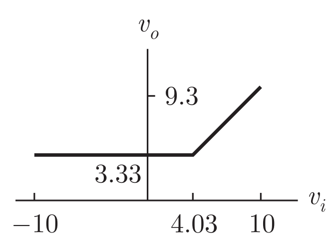





So, the output voltage is

$$v_o = 3.33$$

Step 4: From the results obtained in the above step, we sketch the ransfer characteristic is as shown below.

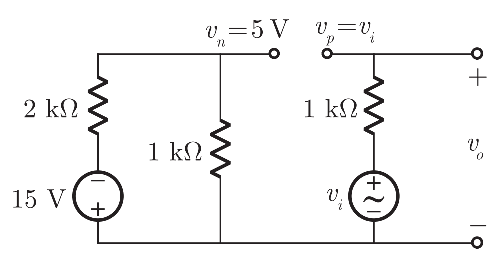



SOL 1.1.6

Option (C) is correct.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the diode is operating, and then obtain the output.

Step 1: Assume that the diode is OFF, and replace it by open circuit. So, the equivalent circuit is,




Step 2: Now, we have the condition

$$v_p > v_n \qquad \text{diode is ON}$$

$$v_p < v_n \qquad \text{diode is OFF}$$

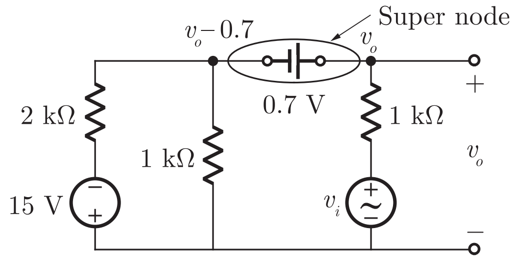
Applying these conditions, we determine the output voltage. By using voltage divider rule,

$$v_n = \frac{1\text{k}}{1\text{k} + 2\text{k}} \times 15 = 5\text{ V}$$

CASE I :
If $v_i \le 5.7$ V then Diode is OFF, we have the same circuit as shown above. So, the output voltage is,

$$v_o = v_i$$

CASE II.
If $v_i \ge 5.7$ then Diode is ON and So, the equivalent circuit is,



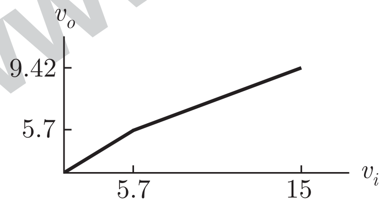

By using super node technique

$$\frac{v_o - 0.7 - 15}{2\text{k}} + \frac{v_o - 0.7}{1\text{k}} + \frac{v_o - v_i}{1\text{k}} = 0$$

So, the output voltage is

$$v_o = 0.4 v_i + 3.42$$

Step 3 From the two results obtained in the above steps, we sketch the Transfer characteristic is as shown below. So, the transfer characteristic is




SOL 1.1.7 Option (D) is correct.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the diodes are operating, and then obtain the output.

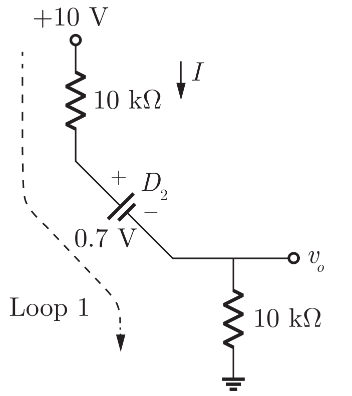
Step 1: Assume that the diodes are OFF, and replace it by open circuit

Step 2: Now, we have the condition

$$v_p > v_n \qquad \text{diode is ON}$$
$$v_p < v_n \qquad \text{diode is OFF}$$

Applying these conditions, we determine the output voltage

CASE I:
For positive part of $v_s$ $(0 \le v_s \le 10\text{ V})$: For $v_s > 0$ V, when $D_1$ is OFF current through $D_2$ is ($D_2$ is ON). So, the equivalent circuit stage is

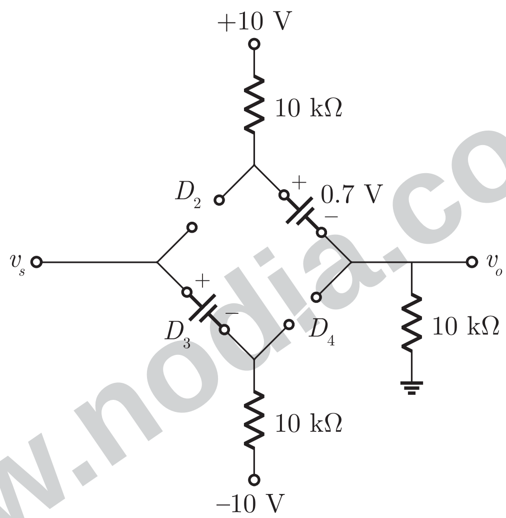



$$I = \frac{10 - 0.7}{10 + 10} = 0.465\,\text{mA}$$

So, the output voltage is

$$v_o = 10\text{k} \times 0.465\,\text{mA} = 4.65\text{ V}$$

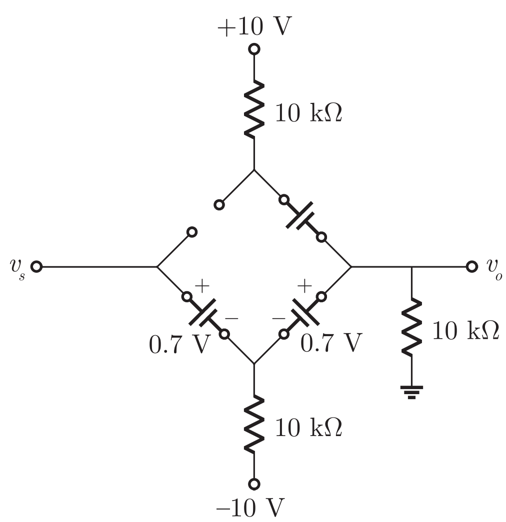
If $v_s > 4.65$ then diode $D_4$ is always OFF. So, the equivalent circuit is




So, the output voltage is

$$v_o = 4.65\text{ V}$$

If $0 < v_s < 4.65$, diode $D_3$ and $D_4$ is ON. So, the equivalent circuit is,

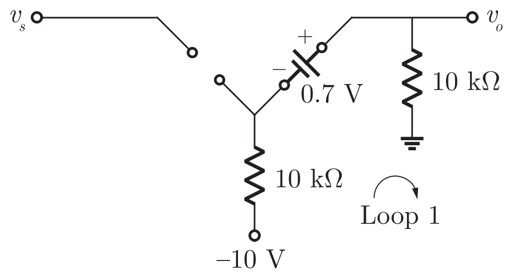



So, the output voltage is

$$v_o = v_s - 0.7 + 0.7 = v_s$$

CASE II:

For negative part of $v_s$ $(-10 \le v_s \le 0)$: For $v_s < 0\,\text{V}$, when $D_3$ is OFF and $D_4$ is ON. So, the equivalent circuit is



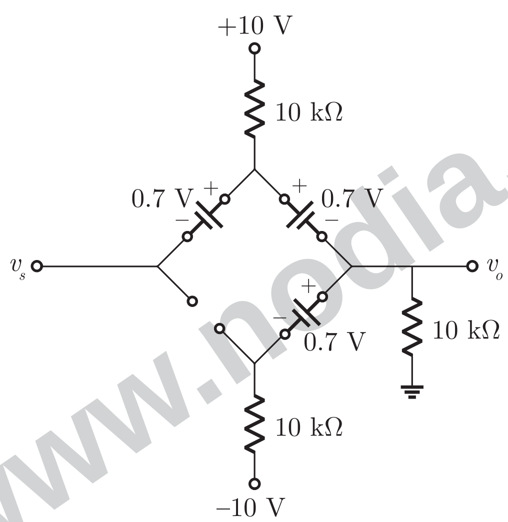

Current through $D_4$ is

$$I = \frac{-0.7-(-10)}{10+10} = 0.465\,\text{mA}$$

So, the output voltage is

$$v_o = -(10\text{k} \times 0.465) = -4.65$$

If $v_s > -4.65$ then $D_1$ & $D_2$ is ON. So, the equivalent circuit is,

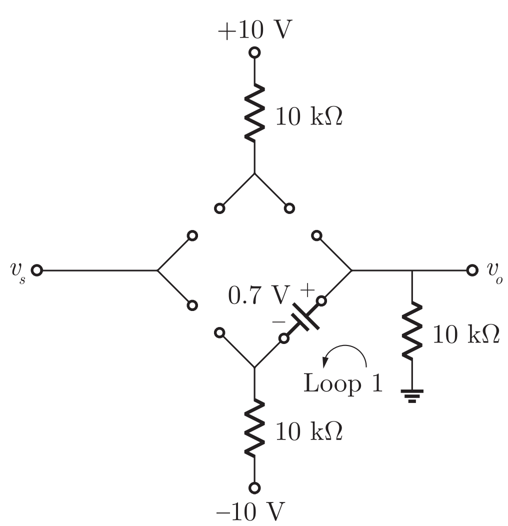





So, the output voltage is

$$v_o = v_s$$

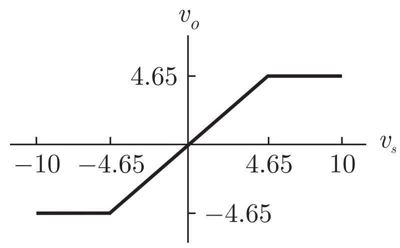
If $-10 < v_s < -4.65$ then $D_2$ is OFF. So, the equivalent circuit is




So, the output voltage is,

$$v_o = -4.65\,\text{V}$$

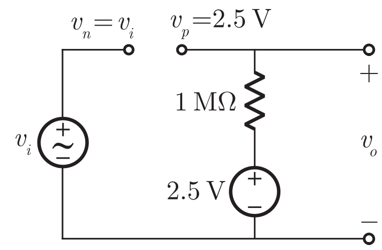
Step 3: From the results obtained in the above steps, we sketch the Transfer characteristic is as shown below.




SOL 1.1.8 Option (B) is correct.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the two diodes are operating, and then obtain the output.

Step 1: Assume that the diodes is OFF, and replace it by open circuit. So, the equivalent circuit is



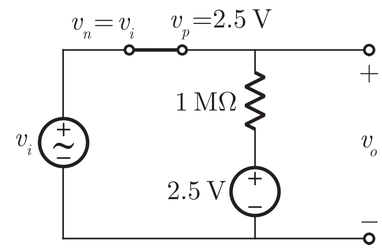

Step 2: Now, we have the condition

$$v_p > v_n \qquad \text{diode is ON}$$
$$v_p < v_n \qquad \text{diode is OFF}$$

Applying these conditions, we determine the output voltage.

CASE I:

If $v_i < 2.5$ V then diode is ON. So, the equivalent circuit is,

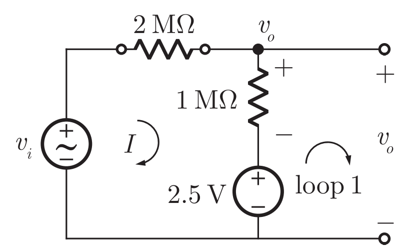



So, the output voltage is

$$v_o = v_i \Rightarrow v_o = -10 \text{ V}$$

CASE II.

If $v_i > 2.5$ V,then diode is OFF, $r_r = 2\,\text{M}\Omega$. So, the equivalent circuit is




For positive wave waveform,

$$v_i = 10\text{ V}$$

Nodal analysis at Node $v_o$,

$$\frac{v_o - v_i}{2M} + \frac{v_o - 2.5}{1M} = 0$$

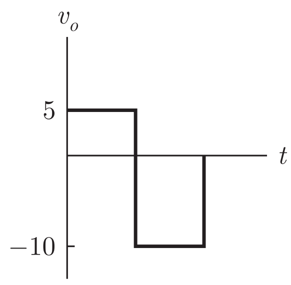
$$3v_o = v_i + 5$$

$$3v_o = 15$$

So, the output voltage is

$$v_o = 5\text{ V}$$

Step 3: From the two results obtained in the above steps, we sketch the output waveform is as shown below.




SOL 1.1.9 Option (B) is correct.

For this circuit KVL gives,

$$v_1 - v_2 = v_{D_1} - v_{D_2},$$

$$v_1 - v_3 = v_{D_1} - v_{D_3}$$

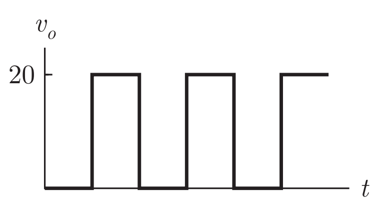
Suppose that $v_1$ is positive and exceeds $v_2$ and $v_3$. Then $D_1$ must be forward-biased with $v_{D_1} = 0$ and consequently $v_{D_2} < 0$ and $v_{D_3} < 0$. Hence $D_2$ and $D_3$ block, while $v_1$ is passed as $v_o$. This is so in general. The largest positive input signal is passed as $v_o$, while the remainder of the input signals are blocked. If all input signals are negative, $v_o = 0$.

SOL 1.1.10 Option (B) is correct.

This is a clamper circuit. Clamper circuit consists capacitor and a diode. The capacitor is in series and the diode in shunt. In clamper circuit,

1. When the diode is in downward direction the total signal will be clamp below the reference voltage.
2. When the diode is in upward direction the total signal will be clamp above the reference voltage.

In the given circuit, the diode is in upward direction and the reference voltage is zero then the total signal will be clamp above the 0 V. So, the output voltage is



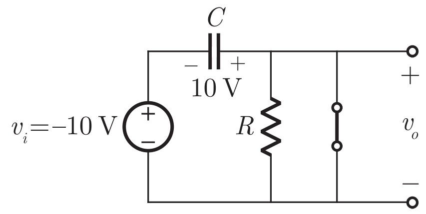

ALTERNATIVE METHOD :

For the given circuit, we consider the following two cases:

CASE I:

In negative cycle diode will be ON and Capacitor will be charged

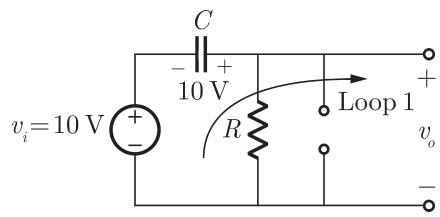
So, the equivalent circuit is,




$$v_o = 0$$

CASE II:

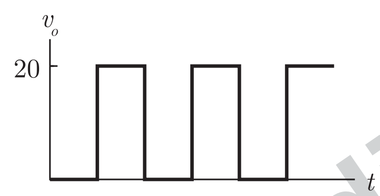
In positive half cycle diode will be OFF. So, the equivalent circuit is




By using KVL in loop 1, we have

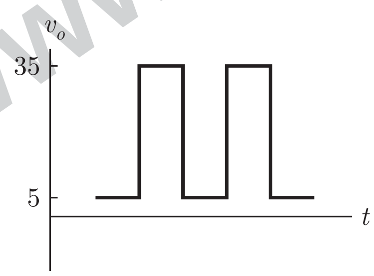
$$v_o = 10 + 10 = 20\text{ V}$$

From the two results obtained in the above cases, we sketch the output voltage as shown below.




SOL 1.1.11 Option (A) is correct.

This is a clamper circuit. In this circuit, the diode is in upward direction. So, total signal will be clamp above the $+5$ V. From the results obtained in the above steps, we sketch the output waveform is as shown below.



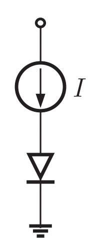

ALTERNATIVE METHOD :

During $-20$ V cycle of $v_i$, diode is ON and capacitor will charge up instantaneously to 25 V. Output is $+5$ V during this cycle.

During the $+10$ V of $v_i$, diode is OFF and capacitor will hold on this voltage level, giving total output voltage $+35$ V.

SOL 1.1.12 Option (A) is correct.

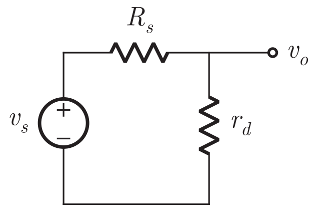
In DC equivalent circuit capacitor is open. So, the equivalent circuit is



Current through diode is $I$. So, we get the small signal resistance

$$r_d = \frac{\eta V_T}{I_{dc}} = \frac{\eta V_T}{I}$$

The small signal equivalent circuit is as follows




For small signal response, open $dc$ current source, short capacitor $C_1$ and $C_2$, and replace diode with it small signal resistance $r_d$. So, the output voltage is

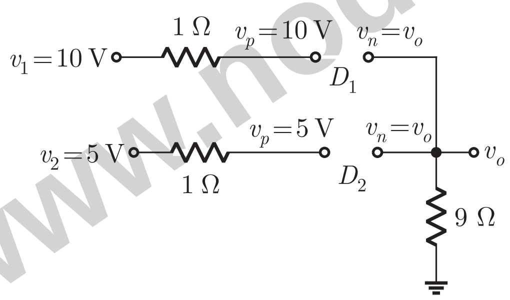
$$v_o = \frac{v_s r_d}{r_d + R_s} = v_s \frac{\frac{\eta V_T}{I}}{\frac{\eta V_T}{I} + R_s}$$

$$= v_s \frac{\eta V_T}{\eta V_T + R_s} = v_s \frac{\eta V_T}{\eta V_T + IR_s}$$

SOL 1.1.13 Option (A) is correct.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the two diodes are operating, and then obtain the output.

Step 1: Assume that the two diodes are OFF, and replace it by open circuit. So, the equivalent circuit is



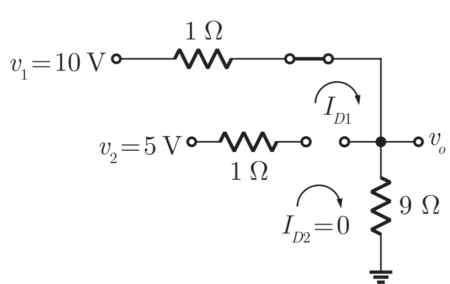

Step 2: Now, we have the condition for both the diodes

$$v_p > v_n \qquad \text{diode is ON}$$
$$v_p < v_n \qquad \text{diode is OFF}$$

Applying these conditions, we determine the output voltage. So,

$$v_{pn1} = 10 - 0 = 10 \text{ V}$$
$$v_{pn2} = 5 - 0 = 5 \text{ V}$$

Step 3: Assume diode is $D_1$ is ON and diode $D_2$ is OFF.




Current through diode $D_1$,

$$i_{D1} = \frac{v_1 - 0}{1 + 9} = \frac{10 - 0}{1 + 9} = 1 \text{ Amp}$$

So output voltage,

$$v_o = 1 \times 9 = 9\text{ V}$$

Voltage across diode $D_2$

$$v_{pn2} = v_{p2} - v_{n2}$$
$$= 5 - 9 = -4$$

So diode $D_2$ is in OFF state.

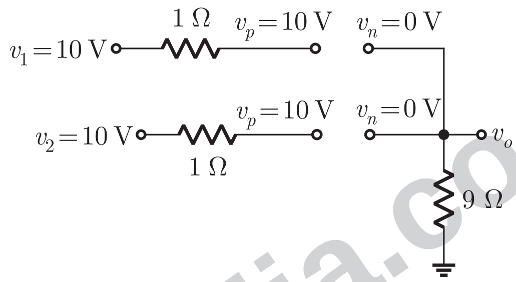
Step 4: From the results obtained in the above steps, the output voltage is,

$$V_o = 9\text{ V}$$

SOL 1.1.14 Option (B) is correct.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the two diodes are operating, and then obtain the output.

Step 1:Assume that the two diodes are OFF, and replace it by open circuit. Since, the equivalent circuit is

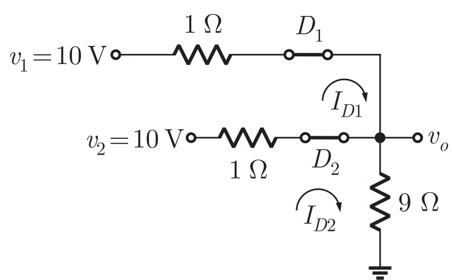



Step 2: Now, we have the condition for both the diodes

$$v_p > v_n \qquad \text{diode is ON}$$
$$v_p < v_n \qquad \text{diode is OFF}$$

Applying these conditions, we determine the output voltage.

Step 3: Assume that diode $D_1$ and $D_2$ is in ON state. So, the equivalent circuit is




Nodal analysis

$$\frac{v_0 - 10}{1} + \frac{v_o - 10}{1} + \frac{v_o}{9} = 0$$

Step 4: From the two results obtained in the above steps, the output voltage is,

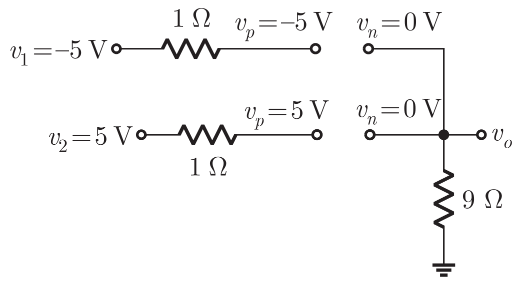
$$v_o = \frac{20 \times 9}{19} = 9.474\text{ V}$$

In this case $i_{D1}$ and $i_{D2}$ both are positive. So, $D_1$ and $D_2$ are ON (assumption is correct)

SOL 1.1.15 Option (C) is correct.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the two diodes are operating, and then obtain the output.

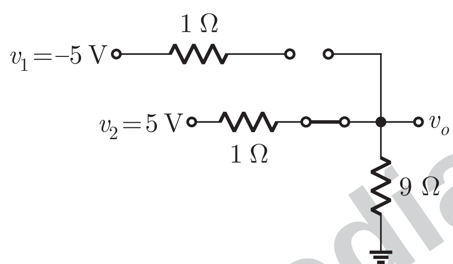
Step 1: Assume that the two diodes are OFF, and replace it by open circuit So, the equivalent circuit is




Step 2: Now, we have the condition for both the diodes

$$v_p > v_n \qquad \text{diode is ON}$$
$$v_p < v_n \qquad \text{diode is OFF}$$

Applying these conditions, we determine the output voltage.So the equivalent circuit is,



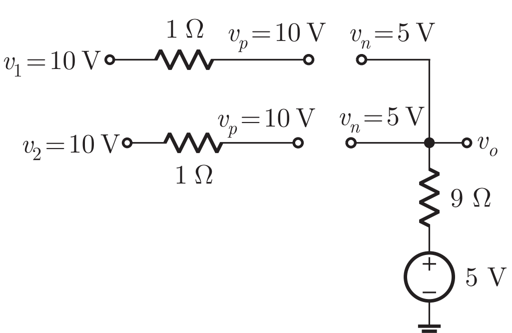

Step 3: From the results obtained in the above steps, the output voltage is,

$$v_o = \frac{5}{10} \times 9 = 4.5 \text{ V}$$

SOL 1.1.16 Option (B) is correct.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the two diode are operating, and then obtain the output.

Step 1: Assume that the two diodes are OFF, and replace it by open circuit. So, the equivalent circuit is

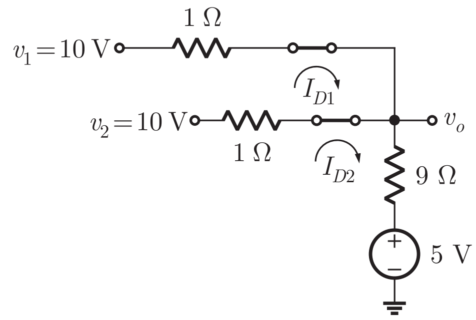



Step 2: Now, we have the condition for both the diodes

$$v_p > v_n \qquad \text{diode is ON}$$
$$v_p < v_n \qquad \text{diode is OFF}$$

Applying these conditions, we determine the output voltage

Step 3: Diode $D_1$ and $D_2$ is ON. So, the equivalent circuit is




Using Nodal analysis

$$\frac{v_o - 10}{1} + \frac{v_o - 10}{1} + \frac{v_o - 5}{9} = 0$$

So, the output voltage is

$$v_o = 9.737 \text{ V}$$

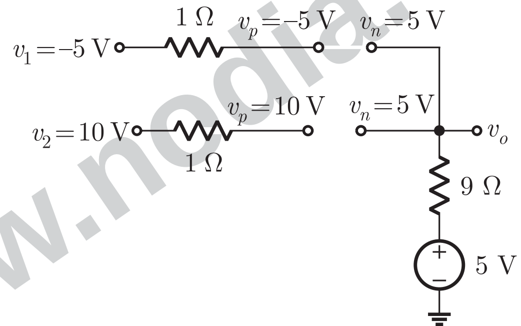
For this voltage, $i_{D1}$ and $i_{D2}$ are positive, so $D_1$ and $D_2$ are ON (assumption is correct).

SOL 1.1.17 Option (C) is correct.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the two diode are operating, and then obtain the output.

Step 1: Assume that the two diodes are OFF, and replace it by open circuit

So, the equivalent circuit is,



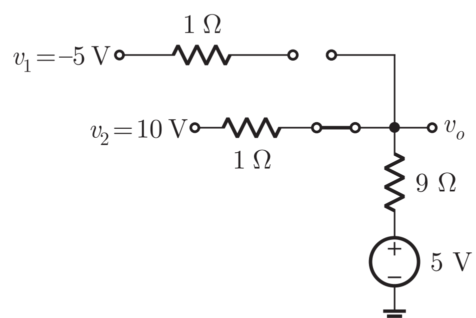



Step 2: Now, we have the condition for both the diodes

$$v_p > v_n \qquad \text{diode is ON}$$
$$v_p < v_n \qquad \text{diode is OFF}$$

Applying these conditions, we determine the output voltage.

Step 3: Diode $D_1$ is OFF and $D_2$ is ON. So, the equivalent circuit is




Node analysis at node $v_o$

$$\frac{v_o - 10}{1\,\Omega} + \frac{v_o - 5 \text{ V}}{9\,\Omega} = 0$$

$$10v_o = 95$$

So, the output voltage is

$$v_o = 9.5\text{ V}$$

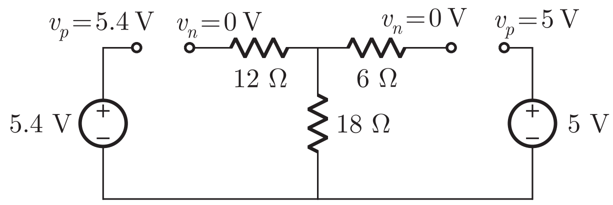
In this case, $i_{D1}$ is negative, $i_{D2}$ is positive, so $D_1$ is OFF and $D_2$ is ON (i.e. assumption is correct).

SOL 1.1.18 Option (C) is correct.

For the given circuit, we determine the linear region (forward bias or reverse bias) in which the two diodes are operating.

Step 1: Assume that the two diodes are OFF, and replace it by open circuit

So, the equivalent circuit is

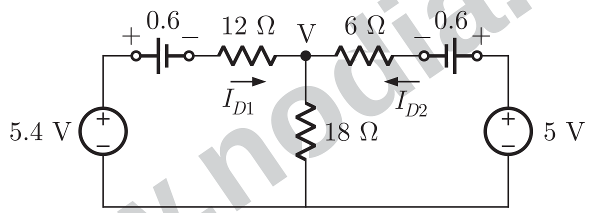



Step 2: Now, we have the condition for both the diodes

$$v_p > v_n \quad \text{diode is ON}$$
$$v_p < v_n \quad \text{diode is OFF}$$

Applying these conditions, we analyse the circuit.

Step 3: Both diodes are ON. So, the equivalent circuit is




Using Nodal analysis at Node 1,

$$\frac{V-4.8}{12} + \frac{V}{18} + \frac{V-4.4}{6} = 0$$

$$V = 3.71\text{ V}$$

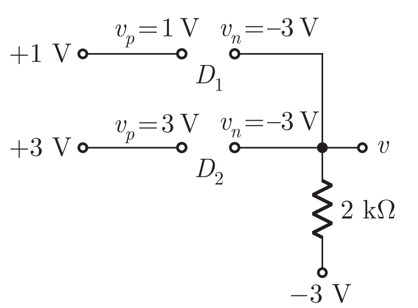
In this case, $i_{D1}$ and $i_{D2}$ are positive, so $D_1$ and $D_2$ are ON (assumption is correct).

SOL 1.1.19 Option (A) is correct.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the two diodes are operating, and then obtain the output.

Step 1: Assume that the two diodes are OFF, and replace it by open circuit

So, the equivalent circuit is,

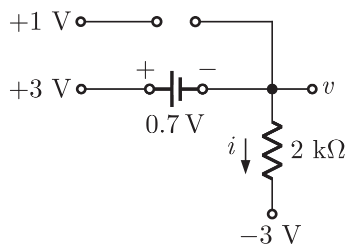



Step 2: Now, we have the condition for both the diodes

$$v_p > v_n \quad \text{diode is ON}$$
$$v_p < v_n \quad \text{diode is OFF}$$

Applying these conditions, we obtain the required unknowns.

Step 3: When diode $D_2$ is ON. So, the equivalent circuit is




$$v = 3 - 0.7 = 2.3\text{ V}$$

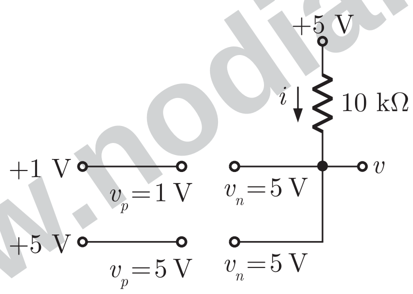
This diode $D_1$ is not conducting because of diode $D_2$ is ON. (highly forward biased).

$$i = \frac{2.3 - (-3)}{2} = 2.65\text{ mA}$$

SOL 1.1.20 Option (A) is correct.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the two diode are operating, and then obtain the output.

Step 1: Assume that the two diodes are OFF, and replace it by open circuit So, the equivalent circuit is,

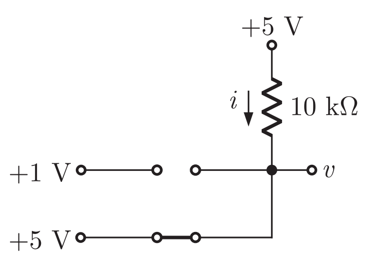



Step 2: Now, we have the condition for both the diodes

$$v_p > v_n \quad \text{diode is ON}$$
$$v_p < v_n \quad \text{diode is OFF}$$

Applying these conditions, we obtain the required unknowns

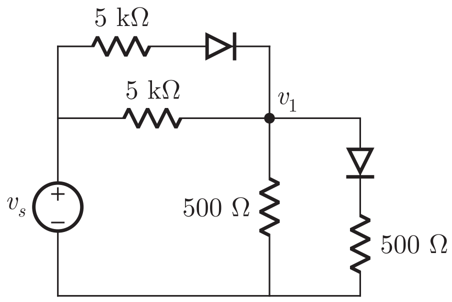
Step 3: Diode $D_1$ is OFF & $D_2$ is ON. Since the equivalent circuit is




$$v = 5\text{ V}$$
$$i = \frac{5 - v}{10\text{ k}\Omega}$$
$$= \frac{5 - 5}{10\text{k}} = 0\text{ mA}$$

SOL 1.1.21 Option (A) is correct.

The given circuit is,






$$v_s - v_1 > 0.6$$

For both diode is ON when $v_1 > 0.6$. Using nodal analysis at node $v_1$,

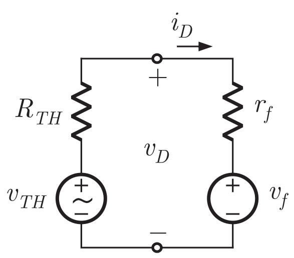
$$\frac{v_1 + 0.6 - v_s}{5\text{k}} + \frac{v_1 - v_s}{5\text{k}} + \frac{v_1}{500} + \frac{v_1 - 0.6}{500} = 0$$

$$v_1 = \frac{2v_s + 5.4}{22} > 0.6$$

$$v_s > 3.9 \text{ V}$$

SOL 1.1.22 Option (C) is correct.

The thevenin equivalent circuit for the network to the left of terminal $ab$ is shown below.




$$v_{TH} = \frac{100}{200}(2 + \cos\omega t) = 1 + 0.5\cos\omega t \text{ V}$$

$$R_{TH} = \frac{100 \times 100}{100 + 100} = 50\,\Omega$$

The diode can be modeled with $v_f = 0.5$ V and

$$r_f = \frac{0.7 - 0.5}{0.004} = 50\,\Omega,$$

$$i_D = \frac{v_{TH} - v_f}{R_{TH} + r_f} = \frac{1 + 0.5\cos\omega t - 0.5}{50 + 50}$$

$$= 5(1 + \cos\omega t) \text{ mA}$$

SOL 1.1.23 Option (A) is correct.

The voltage, $v_D = r_f i_D + v_f$

$$= 50 \times 5(1 + \cos\omega t) \times 10^{-3} + 0.5$$

$$= 0.75 + 0.25\cos\omega t = 0.25(3 + \cos\omega t) \text{ V}$$

SOL 1.1.24 Option (D) is correct.

The output voltage cannot exceed the positive power supply voltage and cannot be lower than the negative power supply voltage.

SOL 1.1.25 Option (B) is correct.

For Zener diode

$|V_{pn}| > V_z$ Zener diode is ON

$|V_{pn}| < V_z$ Zener diode is OFF

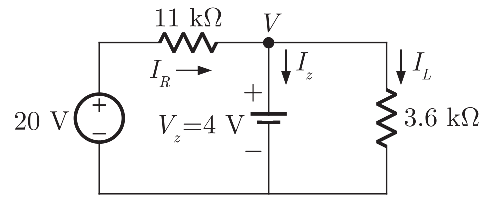
Applying these conditions, we determine the Q-point of the zener diode.

If we assume that the Zener diode is off, then the voltage across resistor (the diode) is given by

$$V = \frac{3.6}{11+3.6} \times 20$$
$$= 4.93$$

So, $V > v_z$

Therefore, the zener diode is in ON State. In ON condition, the diode circuit is




So, the current through the diode is

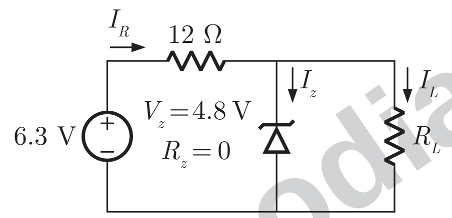
$$I_z = i_R - i_C = \frac{20-4}{11\text{k}} - \frac{4}{3.6}$$
$$= 0.343 \text{ mA}$$

SOL 1.1.26 Option (B) is correct.

Given the current through the Zener diode,

$$5 \le i_z \le 100 \text{ mA}$$

The given circuit is




From the circuit, we have

$$i_L = i_R - i_z$$

So, $$i_{L(\max)} = i_R - i_{z(\min)} = \frac{6.3-4.8}{12} - 5 = 125 - 5$$
$$= 120 \text{ mA}$$

and $$i_{L(\min)} = i_R - i_{z(\max)} = 125 - 100$$
$$= 25 \text{ mA}$$

Thus, $25 \text{ m} \le i_L \le 120 \text{ mA}$

SOL 1.1.27 Option (C) is correct.

In Previous Question, the range of $i_L$ is

$$25 \le i_L \le 120 \text{ mA}$$

So, $$25 \text{ m} \le \frac{4.8}{R_L} \le 120 \text{ mA}$$

or $$40 \le R_L \le 192\,\Omega$$

SOL 1.1.28 Option (A) is correct.

The power rating required for the load resistor is

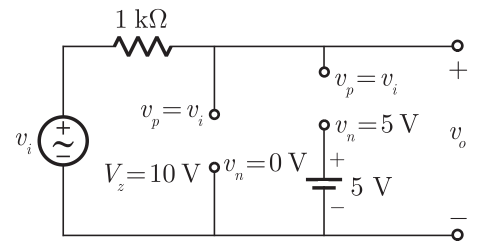
$$p_L = i_{L(\max)} v_z = (120)(4.8) = 576 \text{ mW}$$

SOL 1.1.29 Option (C) is correct.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the two diodes are operating, and then obtain the output.

Step 1: Assume that the two diodes are OFF, and replace it by open circuit.

So, the equivalent circuit is,




Step 2: Now, we have the condition for both the diodes are

For normal diode,

$$v_p > v_n \qquad \text{diode is ON}$$
$$v_p < v_n \qquad \text{diode is OFF}$$

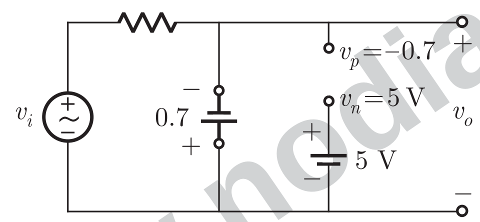
For Zener diode,

$$|V_{pn}| > V_z \qquad \text{Zener diode is ON}$$
$$|V_{pn}| < V_z \qquad \text{Zener diode is OFF}$$

Applying these conditions, we obtain the required unknowns.

CASE I:

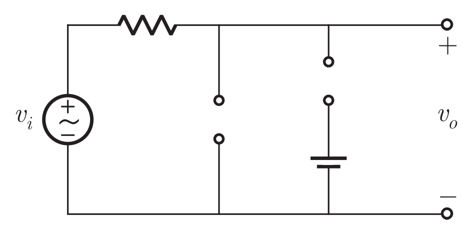
If $v_i < -0.7$, $pn$ diode are OFF and zener diode behave as forward bias. So, the equivalent circuit is




$$v_o = -0.7$$

CASE II:

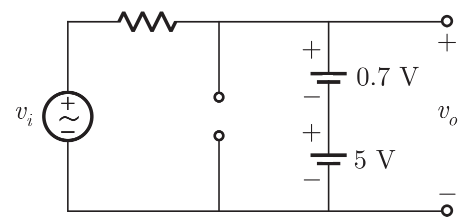
$-0.7 \le v_i \le 5.7$ both zener and diode $D$ will be off. So, the equivalent circuit is,




$$v_o = v_i$$

CASE III:

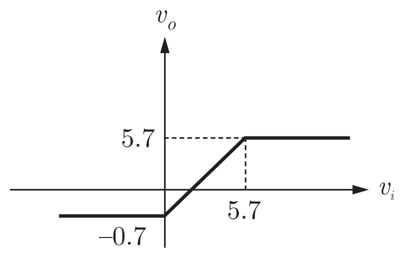
$5.7 < v_i < 10$ Diode $D$ will be ON and zener will be OFF. So, the equivalent circuit is




So, the output voltage is

$$v_o = 5 + 0.7 = 5.7 \text{ V}$$

Step 3: From the results obtained in the above steps, the transfer characteristics is

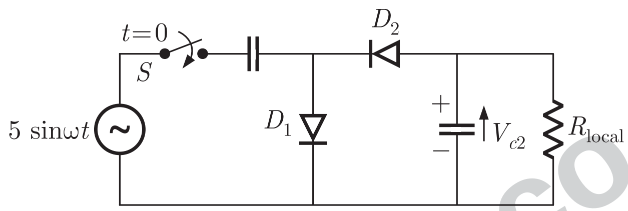



SOL 1.1.30 Option (A) is correct.

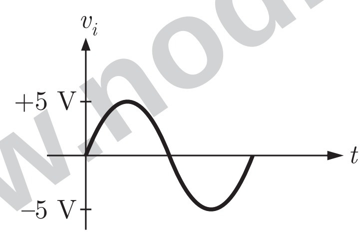
From the characteristics of voltage and current, we observe that only positive current flowing through the element and negative cycle is blocked. Therefore, given element is forward diode.

SOL 1.1.31 Option (D) is correct.

The given circuit is

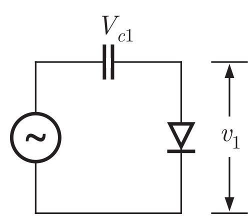



Step 1: We have the input waveform, $v_i = 5\sin\omega t$. So, we draw the waveform as

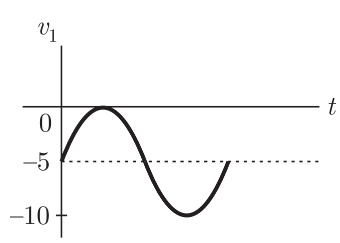



Step 2: For half part of the circuit. When positive half cycle of input is applied, diode $D_1$ is ON and $D_2$ is OFF. So, capacitor $C_1$ will charge upto $+5$ Volt

$$V_{C_1} = +5 \text{ Volt}$$

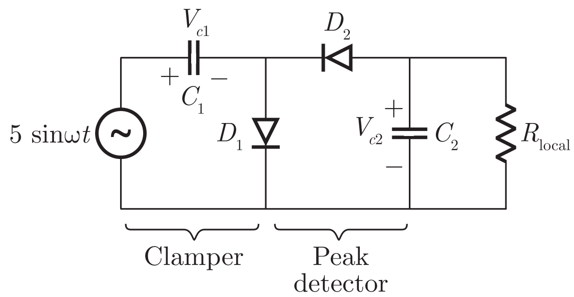



This is a clamper circuit. So, output of the circuit is




In this clamper, diode is in downward position. So, it is negative clamper.

Step 3: Second part of the circuit is peak detector as shown below.



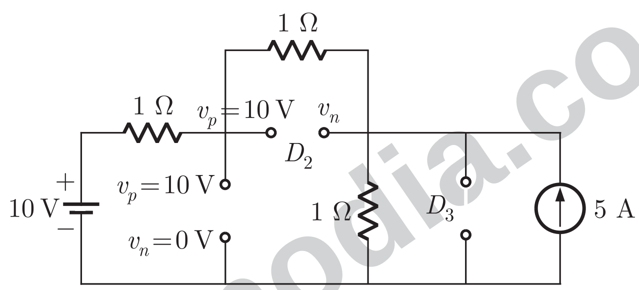

So, it allows only peaks at the output. Thus, from the results obtained in the above step, the output voltage is

$$V_{C_2} = -10 \text{ Volt}$$

SOL 1.1.32 Option (A) is correct.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the diode are operating, and then obtain the output.

Step 1: Assume that the diodes are OFF, and replace it by open circuit So, the equivalent circuit is,



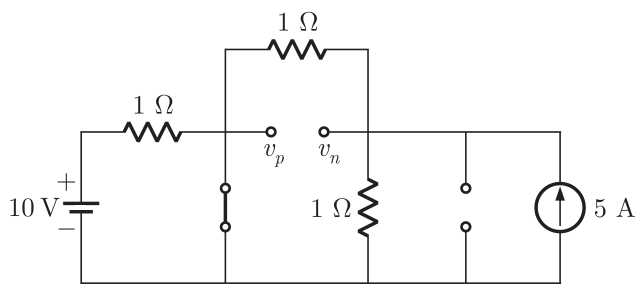

Step 2: Now, we have the condition for both the diodes

$$v_p > v_n \qquad \text{diode is ON}$$
$$v_p < v_n \qquad \text{diode is OFF}$$

Applying these conditions, we obtain the required unknowns

Step 3: In the given circuit, $D_3$ is always OFF because it is connected to 5 A current source and $D_1$ is always ON because it is connected to 10 V battery source. So, voltage drop across $D_1$ is always positive.

Step 4: Now, we check for $D_2$ diode.




$$v_p = 0 \text{ V}, \quad v_n = 2.5 \text{ V} \text{ (current divider rule)}$$

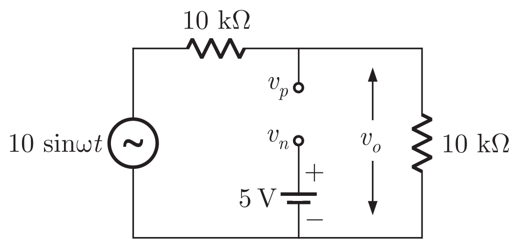
Since, diode current and voltage is negative. So, $D_2$ is OFF.

Step 5: Thus, from results obtained in above case, we conclude that $D_1$ is ON, $D_2$ is OFF, and $D_3$ is OFF.

SOL 1.1.33 Option (A) is correct.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the diode is operating, and then obtain the output.

Step 1: Assume that the diode is OFF, and replace it by open circuit. The equivalent circuit is

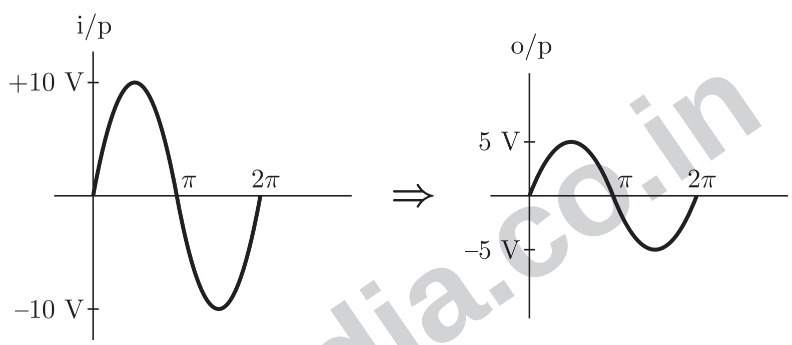



According to voltage divide rule, the output voltage is

$$v_o = \frac{10\text{k}}{10\text{k} + 10\text{k}} \times 10\sin\omega t = 5\sin\omega t$$

$$v_p < v_n$$

So diode is always OFF. Thus, we get the output voltage as shown below.

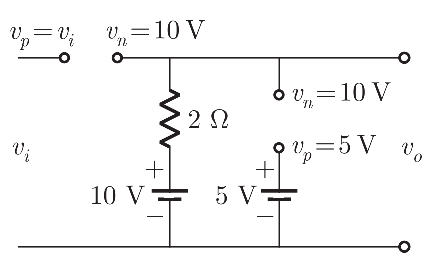





SOL 1.1.34 Option (A) is correct.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the two diodes are operating, and then obtain the output.

Step 1: Assume that the two diodes are OFF, and replace it by open circuit. The equivalent circuit is




Step 2: Now, we have the condition for both the diodes

$$v_p > v_n \qquad \text{diode is ON}$$

$$v_p < v_n \qquad \text{diode is OFF}$$

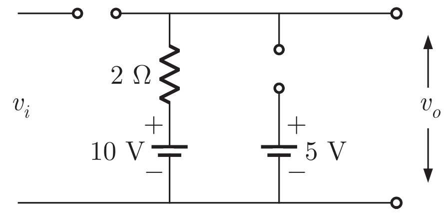
Applying these conditions, we obtain the required unknowns.

CASE I:

If $v_i < 10$ V $D_1$ is OFF. Since,

$$v_o = 10 \text{ V}$$

So, $D_2$ is OFF. Since the equivalent circuit is



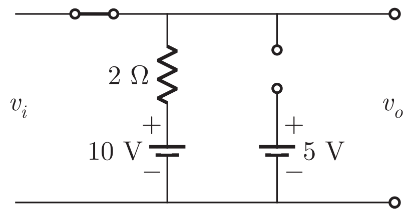

The output voltage is

$$v_o = 10 \text{ V}$$

CASE II:
If $v_i > 10$ Volt, $D_1$ is ON, i.e.

$$v_o = v_i$$

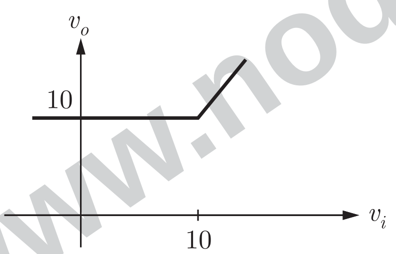
So, $D_2$ is OFF. Since the equivalent circuit is




The output voltage is

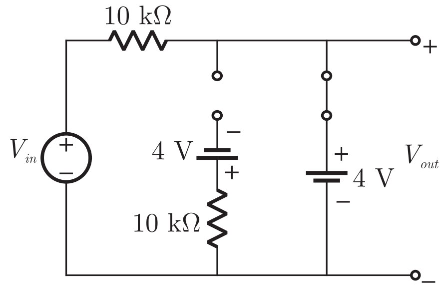
$$v_o = v_i$$

Step 3: In both cases diode $D_2$ is always OFF. From the results obtained in above steps, we get the transfer characteristic as




SOL 1.1.35 Option (D) is correct.
For the given input signal, we obtain the output in following steps:

Step 1: In the positive half cycle (when $V_{in} > 4$ V), diode $D_2$ conducts and $D_1$ will be off. So, the equivalent circuit is

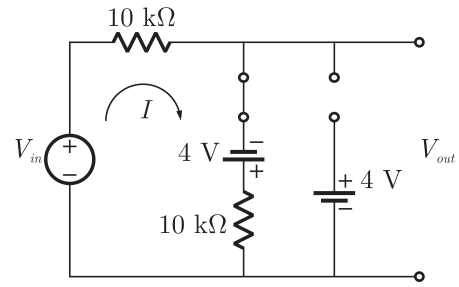



So, the output voltage is

$$V_{out} = +4 \text{ Volt}$$

This is the maximum output voltage.

Step 2: In the negative half cycle diode $D_1$ conducts and $D_2$ will be off so the circuit is




Applying KVL,

$$V_{in} - 10\text{k}I + 4 - 10\text{k}I = 0$$

$$\frac{V_{in} + 4}{20\text{k}} = I$$

Since, the minimum input voltage is $V_{in} = -10$ V, so we obtain the corresponding current as

$$I = \frac{-10 + 4}{20\text{k}} = -\frac{3}{10} \text{ mA}$$

Thus, the minimum output voltage is obtained as

$$\frac{V_{in} - V_{out}}{10} = I$$

$$\frac{-10 - V_{out}}{10\text{k}} = -\frac{3}{10}\text{m}$$

$$V_{out} = -(10 - 3)$$

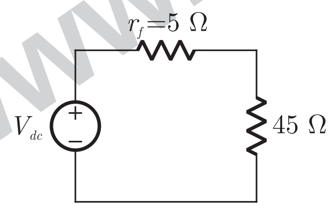
$$= -7 \text{ volt}$$

SOL 1.1.36 Option (A) is correct.

This is a half-wave rectifier circuit. So, the dc voltage is given by,

$$V_{dc} = \frac{V_m}{\pi}$$

Equivalent circuit with forward resistance is




So, the dc current in the circuit is

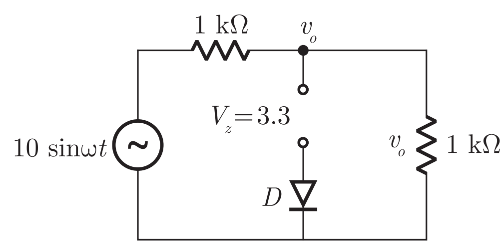
$$I_{dc} = \frac{\frac{V_m}{\pi}}{r_f + R} = \frac{(V_m/\pi)}{(5+45)} = \frac{V_m}{50\pi}$$

SOL 1.1.37 Option (B) is correct.

For the given input voltage, we consider the following two cases.

CASE I:

For positive half cycle ($v_o < 4$ V), the equivalent circuit is

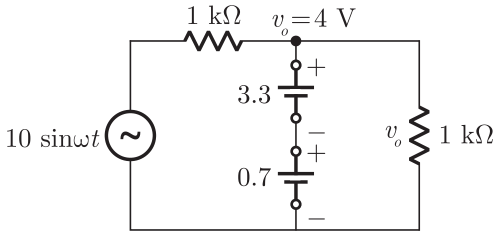



By using voltage divider rule,

$$v_o = \frac{1\text{k}}{1\text{k}+1\text{k}} 10\sin\omega t$$
$$v_o = 5\sin\omega t$$

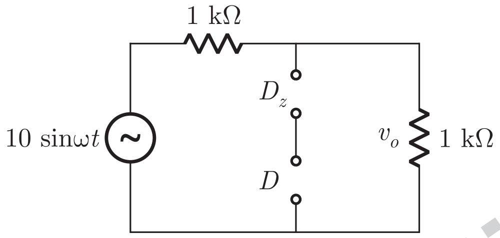
For peak positive value of input zener diode will be ON state So,

$$v_o = 3.3 + 0.7$$
$$v_o = 4\text{ V}$$




CASE II:

For negative value of input, the zener diode will be in OFF state. So, the equivalent circuit is




So, we get the output voltage as

$$v_o = -\frac{1\text{k}}{1\text{k}+1\text{k}} \times 10\sin\omega t$$
$$= -5\sin\omega t$$

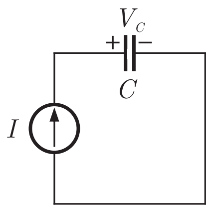
Thus, we have the peak value

$$v_o = -5$$



SOL 1.1.38 Option (C) is correct.

When the switch is opened, current flows through capacitor and diode is ON in this condition. So, the equivalent circuit during $T_{\text{OFF}}$ is




$$I = C\frac{dV_c}{dt}$$

$\Rightarrow$ $$V_c = \frac{I}{C}t + V_c(0)$$

Initially, $$V_c(0) = 0$$
$$V_c = \frac{I}{C}t$$

At $t = \text{T}_{\text{off}}$, $$V_c = \frac{I}{C}T_{\text{off}}$$

So, the duty cycle is

$$D = \frac{T_{\text{ON}}}{T_{\text{ON}}+T_{\text{OFF}}} = \frac{T_{\text{ON}}}{T}$$
$$\text{T}_{\text{ON}} = DT$$

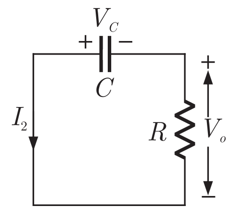
$$T_{OFF} = T - T_{ON} = T - DT$$

So,
$$V_c = \frac{I}{C}(T - DT)$$
$$= \frac{I}{C}(1 - D)\,T$$

During $T_{OFF}$, output voltage is
$$V_o = 0 \text{ volt.}$$

SOL 1.1.39 Option (B) is correct.

When the switch is closed, diode is off and the circuit is,



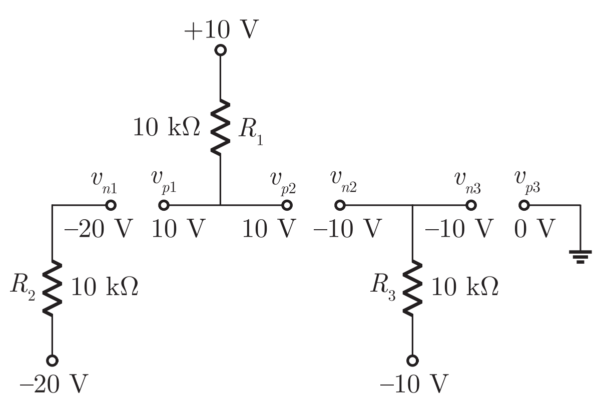

In steady state condition,
$$C\,\frac{dV_c}{dt} = I_2$$
$$I_2 = C\frac{I}{C} \qquad \because \frac{dV_c}{dt} = \frac{I}{C}$$
$$V_o = -V_c = \frac{-I}{C}t$$

Average output voltage,
$$V_o = \frac{1}{T}\left[\int_0^{DT=T_{ON}}\left(-\frac{I}{C}t\right)dt + \int_0^{T_{OFF}} 0\,dt\right]$$
$$= -\frac{1}{T}\cdot\frac{I}{C}\left[\frac{t^2}{2}\right]_0^{DT}$$
$$= -\frac{1}{T}\cdot\frac{I}{C}\cdot\frac{D^2T^2}{2} = -\frac{I}{C}\frac{D^2}{2}.T$$

SOL 1.1.40 Option (C) is correct.

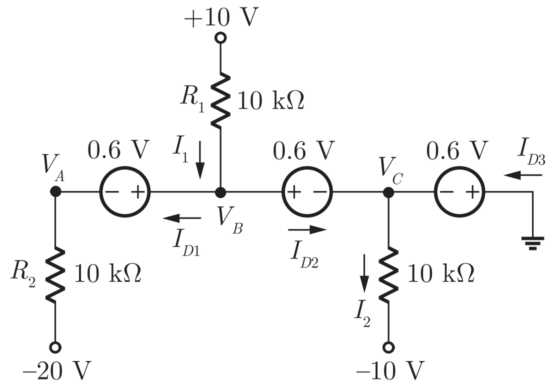
For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the three diodes are operating, and then obtain the output.

Step 1: Assume that the three diodes are OFF, and replace it by open circuit. So, equivalent circuit is




As seen by biasing of diode, for each diode the $p$-terminal of diode is greater than the $n$-terminal. So, our assumption is incorrect.

Step 2: For first iteration, assume that all three diode are ON. So, equivalent circuit is




$$V_C = 0 - 0.6 = -0.6 \text{ V}$$
$$V_B = -0.6 + 0.6 = 0 \text{ V}$$
$$V_A = V_B - 0.6 = 0 - 0.6 = -0.6 \text{ V}$$

So, current $I_1$ is

$$I_1 = \frac{10 - V_B}{10 \text{ k}\Omega} = \frac{10 - 0}{10\text{k}} = 1 \text{ mA}$$

Current through diode $D_1$ is

$$I_{D1} = \frac{V_A - (-20)}{10 \text{ k}\Omega} = \frac{-0.6 + 20}{10\text{k}} = 1.94 \text{ mA}$$

Also, current $I_2$ is

$$I_2 = \frac{V_C - (-10)}{10\text{k}} = \frac{-0.6 + 10}{10\text{k}} = 0.94 \text{ mA}$$

Applying KCL at node $V_B$,

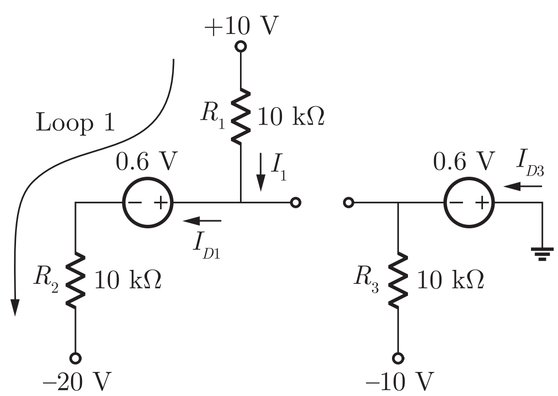
$$I_1 = I_{D1} + I_{D2}$$
$$1 \text{ mA} = 1.94 \text{ mA} + I_{D2}$$
$$I_{D2} = -0.94 \text{ mA} < 0$$

Applying KCL at node $V_C$,

$$I_{D2} + I_{D3} = I_2$$
$$-0.94 \text{ mA} + I_{D3} = 0.94 \text{ mA}$$
$$I_{D3} = 1.86 \text{ mA} > 0$$

From the result $I_{D1}$ and $I_{D3}$ are greater than zero and $I_{D2}$ which is less than zero, represent a contradiction. So, diode $D_3$ will be OFF.

Step 3: For second iteration let us assume $D_1$ and $D_3$ are ON and $D_2$ is OFF so




Applying KVL in Loop 1,

$$10 - 10\text{k}I_1 - 0.6 - 10I_{D1} - (-20) = 0$$

In Loop 1, we have $I_1 = I_{D1}$. Substituting it in above expression,

$$I_{D1} = \frac{29.4}{20\text{k}} = 1.47\text{ mA} > 0$$

Current through diode $D_3$ is

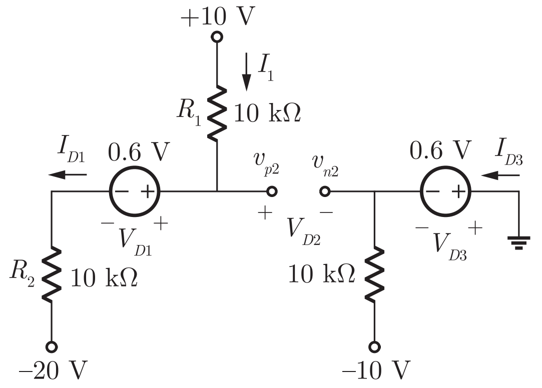
$$I_{D3} = \frac{-0.6-(-10)}{10\text{k}} = 0.94\text{ mA} > 0$$

Current through diode $D_2$ is

$$I_{D2} = 0 \text{ (because } D_2 \text{ is OFF)}$$

SOL 1.1.41 Option (A) is correct.

From previous solution the equivalent circuit is




So voltage drop at diode $D_1$ is

$$V_{D1} = 0.6\text{ V}$$

Voltage drop across diode $D_3$ is

$$V_{D3} = 0.6\text{ V}$$

and voltage across the diode $D_2$ is

$$V_{D2} = V_{P2} - V_{n2}$$

where $V_{n2} = 0 - 0.6 = -0.6\text{ V}$

and $V_{P2} = 10 - I_1 R_1$ (Put $I_1 = 1.47\text{ m}$)

$$= 10 - 1.47\text{ m} \times 10\text{ k}\Omega = -4.70\text{ V}$$

So, $V_{D2} = V_{P2} - V_{n2}$

$$= -4.70 - (-0.6) = -4.10\text{ V}$$

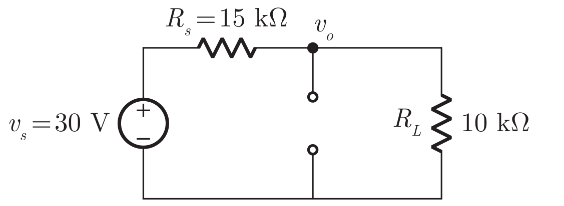


SOL 1.1.42 Option (D) is correct.

Let us consider the given circuit for nominal and worst cases.

CASE I: For Nominal Values

Assume that Zener diode is OFF and redraw the given circuit,

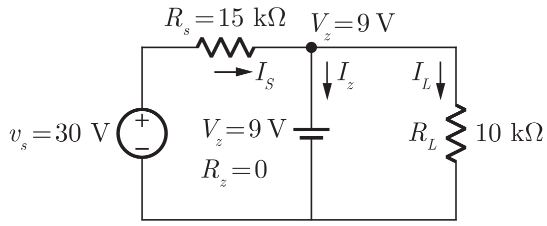



By using voltage divider rule,

$$v_o = \frac{10\text{k}}{10\text{k}+15\text{k}} \times 30 = 12\text{ V}$$

$$v_o > v_z$$

So, zener diode is in ON state. Therefore, the equivalent circuit is






Applying KCL at $v_z$ node,

$$I_S = I_z + I_L$$

or $$\frac{30-9}{15\text{ k}\Omega} = I_z + \frac{v_z - 0}{10\text{k}}$$

or $$I_z = \frac{21}{15\text{k}} - \frac{9}{10\text{k}} = 0.5\text{ mA}$$

So, nominal value of zener diode current is

$$I_z^{nom} = 0.5\text{ mA}$$

CASE II: For Worst Values

For 5 % tolerance (worst case), we have

$$V_{S\max} = 30(1.05) = 31.5$$
$$V_{S\min} = 30(0.95) = 28.5$$
$$R_{S\max} = 15\text{k}(1.05) = 15.75\text{ k}\Omega$$
$$R_{S\min} = 15(0.95) = 14.25\text{ k}\Omega$$
$$R_{L\max} = 10\text{k}(1.05) = 10.5\text{ k}\Omega$$
$$R_{L\min} = 10(0.95) = 9.5\text{ k}\Omega$$
$$V_{z\max} = 9(1.05) = 9.45\text{ V}$$
$$V_{z\max} = 9(0.95) = 8.55\text{ V}$$
$$I_z = I_S - I_L$$
$$I_z = \frac{V_S - V_z}{R_S} - \frac{V_z - 0}{R_L}$$

For $I_{z\max}$ $$I_{z\max}^{\text{worst}} = \frac{V_{S\max} - V_{z\min}}{R_{S\min}} - \frac{V_{z\min}}{R_{L\max}}$$

$$I_{z\max}^{\text{worst}} = \frac{31.5 - 8.55}{14.25\text{k}} - \frac{8.55}{10.5}$$

$$= 1.610\text{ mA} - 0.814\text{ m} = 0.796\text{ mA}$$

For $I_{z\min}$ $$I_{z\min}^{\text{worst}} = \frac{V_{S\min} - V_{z\max}}{R_{S\max}} - \frac{V_{z\max}}{R_{L\min}}$$

$$I_{z\min}^{\text{worst}} = \frac{28.50 - 9.45}{15.75\text{k}} - \frac{9.45}{9.5\text{k}}$$

$$= 1.209\text{ mA} - 0.99\text{ mA} = 0.215\text{ mA}$$

SOL 1.1.43 Option (C) is correct.

From the solution obtained in previous problem,

$$I_z^{nom} = 0.50\text{ mA}$$

So, $$P_z^{nom} = V_z^{nom}(I_z)^{nom}$$
$$= 9 \times 0.5 = 4.5\text{ mW}$$

In calculation of $I_z^{\max}$ we use $V_z^{\min}$. So, $P_z^{\max}$ is also calculated for the same condition. Therefore, the maximum power dissipation is

$$P_z^{\max} = V_z^{\min} I_z^{\max}$$
$$= 8.55 \times 0.796\text{ m} = 6.81\text{ mW}$$

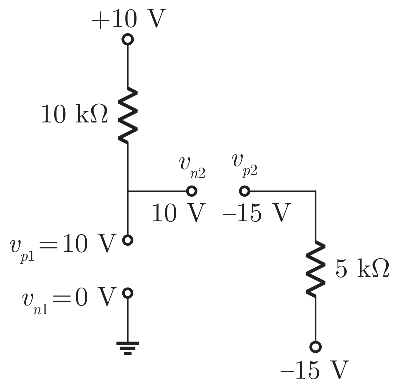
Similarly, $$P_z^{\min} = V_z^{\max} I_z^{\min}$$

$$= 9.45 \times 0.215 \text{ m} = 2.03 \text{ mW}$$

SOL 1.1.44 Option (B) is correct.

For the given circuit, we determine the linear region (forward bias or reverse bias) in which the two diode are operating.

Step 1: Assume that the two diodes are OFF, and replace it by open circuit. So, equivalent circuit is

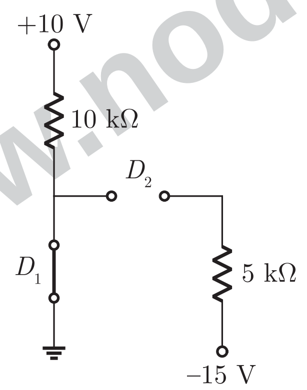



Step 2: Now, we have the condition for both the diodes

$$v_p > v_n \quad \text{diode is ON}$$
$$v_p < v_n \quad \text{diode is OFF}$$

For these condition, we conclude from the above circuit that diode $D_1$ is ON and diode $D_2$ is OFF.

Step 3: For the result obtained in above step, we draw the equivalent circuit as

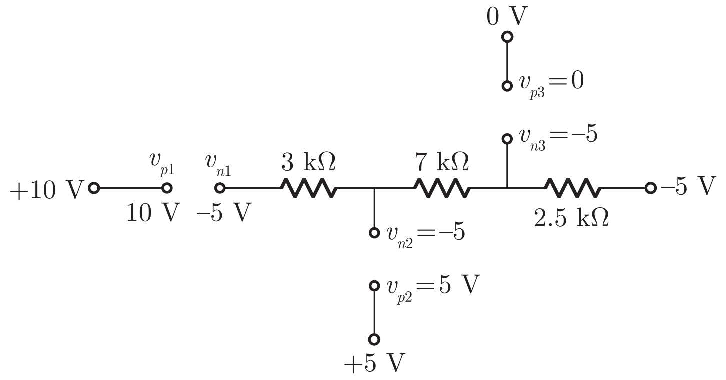



Thus, $D_1$ is ON and $D_2$ is OFF is correct option.

SOL 1.1.45 Option (D) is correct.

For the given circuit, we determine the linear region (forward bias or reverse bias) in which the diode are operating.

Step 1: Assume that the diodes are OFF, and replace it by open circuit. So, equivalent circuit is

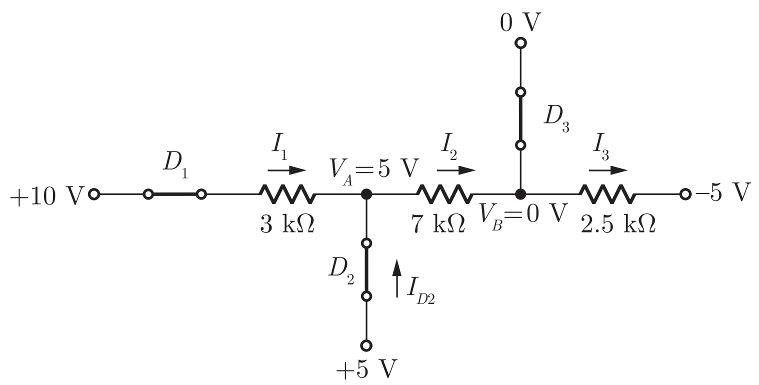


Step 2: Now, we have the condition for both the diodes

$$v_p > v_n \qquad \text{diode is ON}$$
$$v_p < v_n \qquad \text{diode is OFF}$$

Applying these conditions to above circuit, we conclude that $D_1$, $D_2$, $D_3$ are ON.

Step 3: If $D_1$, $D_2$, $D_3$ are ON, then equivalent circuit is




$$\text{Current } I_1 = \frac{10-5}{3\text{k}} = 1.667 \text{ mA}$$

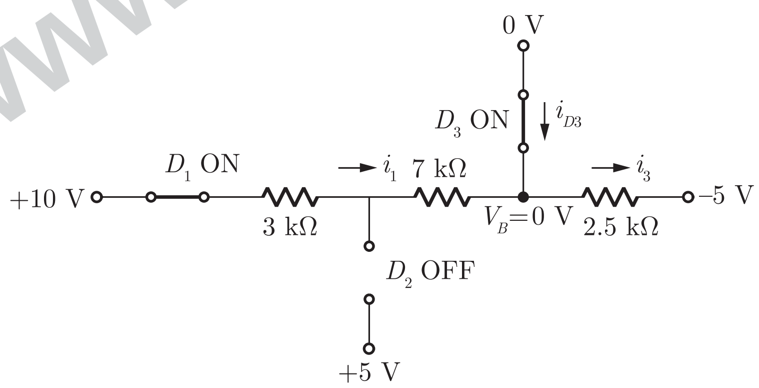
$$\text{Current } I_2 = \frac{5-0}{7\text{k}} = 0.714 \text{ mA}$$

Applying KCL at $v_A$ node,

$$i_1 + i_{D2} = I_2$$
$$i_{D2} = 0.714 \text{ mA} - 1.667 \text{ m}$$
$$= -0.9527 \text{ mA}$$

So, diode $D_2$ is OFF because $i_{D2} < 0$

Step 4: For diode $D_2$ to be OFF, the modified circuit is




$$\text{Current } i_1 = \frac{10-0}{3\text{k}+7\text{k}} = 1 \text{ mA}$$

$$\text{Current } i_3 = \frac{0-(-5)}{2.5\text{k}} = 2 \text{ mA}$$

Applying KCL at Node $V_B$,

$$i_1 + i_{D3} = i_3$$
$$i_{D3} = 2 \text{ m} - 1 \text{ mA} = 1 \text{ mA}$$
$$I_{D3} > 0$$

So, diode $D_3$ is ON.

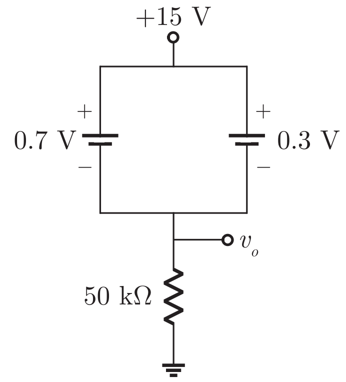
Step 5: Thus, from the above results, we have

| $D_1$ | $D_2$ | $D_3$ |
|---|---|---|
| FB | RB | FB |

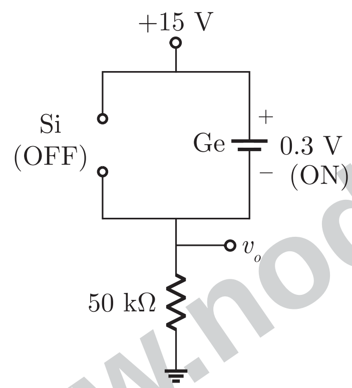
SOL 1.1.46

Option (C) is correct.

Since, 15 V voltage source is used for biasing the diodes. So, the applied voltage will turn both diodes ON. Therefore, we get the equivalent circuit as




In this case, the voltage across the parallel arm is not same. So it does not follow the KVL rule. When the supply is turned ON, it will increase from 0 to 15 V within milliseconds. During the rise in voltage across the diodes, when 0.3 V is established across the germanium diode it will turn 'ON' and maintain a level of 0.3 V. So, silicon diode always remains OFF. Therefore, we get the equivalent circuit as



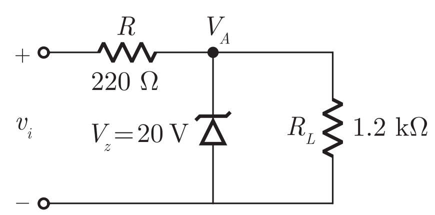



SOL 1.1.47

Option (C) is correct.

For the given circuit, we first determine the region (ON or OFF region) in which the zener diode is operating, and then obtain the output.

Step 1: Assume that the zener diode is OFF, and replace it by open circuit.




Condition for ON state of zener diode is

$$V_A \geq v_z$$

Since, zener diode will be ON. So, we have

$$V_{A\,\min} = 20 \text{ V}$$

By using voltage divider rule,

$$V_A = \frac{R_L}{R + R_L} \times v_L$$

$$V_{A\,\min} = \frac{R_L}{R + R_L} \times v_{i\,\min}$$

$$v_{i\,\min} = \frac{(R + R_L)}{R_L} \times v_{A\,\min}$$

$$= \frac{1200 + 220}{1200} \times 20$$

$$= 23.67 \text{ V}$$

So, the load current is

$$i_L = \frac{V_z - 0}{1.2\text{ k}} = \frac{20}{1.2\text{ k}} = 16.67 \text{ mA}$$

Applying KCL at node $V_A$,

$$I_R = I_z + I_L$$

$$I_{R\max} = I_{z\max} + I_L$$

$$\frac{v_{i\max} - v_z}{R} = 60\text{m} + 16.67 \text{ mA}$$

$$v_{i\max} = 76.67\text{m} \times 220 + v_z$$

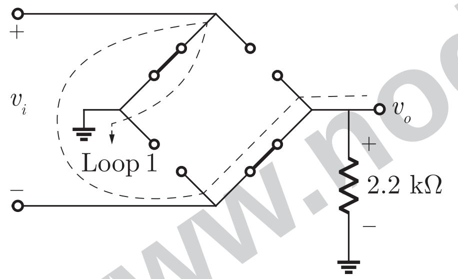
$$= 16.87 \text{ V} + 20 \text{ V} = 36.87 \text{ V}$$

SOL 1.1.48 Option (D) is correct.

For the given input waveform, we obtain the output voltage waveform considering the positive and negative half cycles.

CASE I:

For positive half wave of the input, the diode $D_1$ and $D_3$ will be forward biased and diode $D_2$, $D_4$ will be reverse diode. So, equivalent circuit diagram is



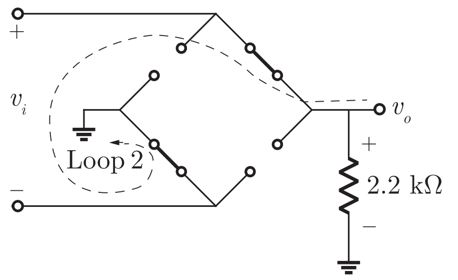

Applying KVL in Loop 1,

$$v_o + v_i = 0$$

$$v_o = - v_i \quad ...(1)$$

CASE II:

For negative half cycle of the input, the diode $D_4$ and $D_2$ will be ON, and $D_1$ and $D_3$ will be OFF. So, the equivalent circuit is

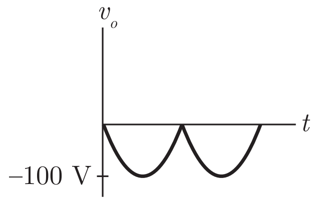



Applying KVL in Loop 2,

$$v_o - v_i + 0 = 0$$

$$v_o = v_i$$

Thus, from the obtained results in above two cases, we get the complete output waveform as




SOL 1.1.49 Option (B) is correct.

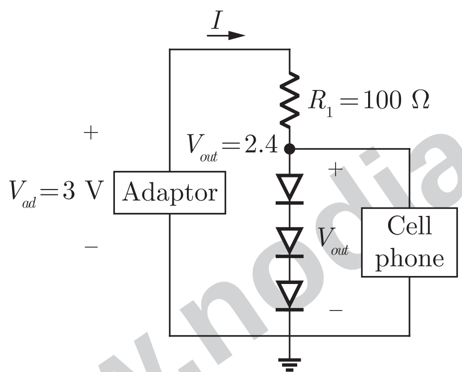
For the given problem, we analyze the clamper circuit as

Step 1: In clamper circuit, the capacitor in series and diode in shunt. In peak detector diode is in series and capacitor in shunt. So, we conclude that circuit given in options (a), (b), and (c) are clamper circuit, but the circuit given in option (d) is peak detector.

Step 2: Again, we observe that the output waveform is clamped downward the reference voltage, it means the direction of diode must be in downward direction and the reference voltage is $V = 2\text{ V}$. Thus, option (b) follows the condition.

SOL 1.1.50 Option (B) is correct.

Given the circuit,




So, the current flowing through resistor $R_1$ is given by

$$I = \frac{V_{ad} - V_{out}}{R_1}$$

$$= \frac{3 - 2.4}{100} = 6\text{ mA}$$

Since, no current is drawn by cellphone, so this current $I$ flows through each diode and hence diode current equation is

$$I = I_S e^{\frac{V_D}{V_T}}$$

or $$6\text{ m} = I_S e^{\frac{800\text{m}}{26\text{m}}}$$

or $$I_S = 2.602 \times 10^{-16}\text{ A}$$

SOL 1.1.51 Option (B) is correct.

When there is no load $(R_L = \infty)$, $I_L = 0$. Under this condition, maximum current flows through the zener diode. So, maximum current passing through zener is $I_{zM}$, when load current is zero. This current is more than $I_{zT}$, so voltage across zener diode changes due to small zener resistance. Therefore, the maximum zener voltage is

$$(V_z)_{\max} = V_z + (I_{z\max} - I_{zT})R_z$$

$$= 12 + (76\text{m} - 21\text{m})9$$

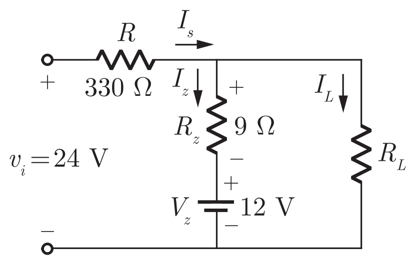
$$= 12 + 0.495 = 12.5\text{ V}$$

Similarly, we obtain the minimum Zener voltage as

$$\begin{aligned}(V_z)_{\min} &= V_z - (I_{zT} - I_{z\,\min})R_z \\ &= V_z - (I_{zT} - I_{zK})R_z \\ &= 12 - (21 - 1)9\text{ m} = 11.82\text{ V}\end{aligned}$$

SOL 1.1.52 Option (B) is correct.

Consider the given circuit,




Applying KCL at node $A$,

$$\begin{aligned}I_L &= I_S - I_z \\ (I_L)_{\max} &= (I_S)_{\max} - I_{z\,\min} \\ &= \frac{V_i - V_{z\,\min} - I_{zk}R_z}{R} - I_{zk} \\ &= \frac{24 - 11.82 - 1\times 9}{330} - 1\text{m} \\ &= 36.88\text{m} - 1\text{m} = 35.88\text{ mA}\end{aligned}$$

SOL 1.1.53 Option (A) is correct.

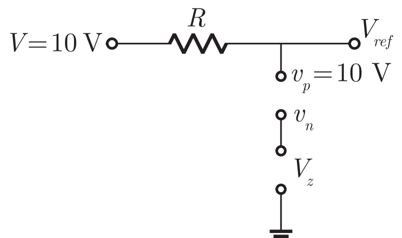
When load current is maximum, the load resistance will be

$$\begin{aligned}R_L &= \frac{V_{z\,\min}}{I_{L\,\max}} \\ &= \frac{11.82}{35.88\text{m}} = 329.43\,\Omega\end{aligned}$$

SOL 1.1.54 Option (A) is correct.

For the given circuit, we first determine the region (ON or OFF region) in which the zener diode is operating, and the normal diode is forward or reverse then obtain the output.

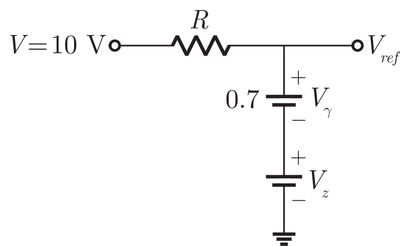
Step 1: Assume that the zener diode is OFF, and replace it by open circuit




Zener diode operates in ON state, if

$$V_{ref} \geq v_z$$

Step 2: From the biasing, we observe that the diode is in forward bias and zener diode is in ON state So, equivalent circuit is



The reference voltage for the circuit is

$$\begin{aligned} V_{ref} &= V_z + V_\gamma \\ &= 6.2 + 0.7 = 6.9 \text{ V} \end{aligned}$$

Step 3: Thus, the temperature coefficient is obtained as

$$\begin{aligned} \frac{V_z\alpha_z + V_f\alpha_f}{V_z + V_f} &= \frac{6.2 \times \frac{0.02}{100} + (-1.8\text{m})}{6.9} \\ &= \frac{1.24\text{m} - 1.8\text{m}}{6.9} \\ &= -\frac{0.56\text{m}}{6.9}/^\circ\text{C} \\ &= -0.0811\,\text{m}/^\circ\text{C} \\ &= -0.008\,\%/^\circ C \end{aligned}$$

SOL 1.1.55 Option (B) is correct.

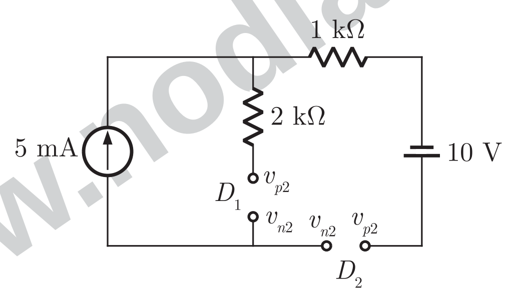
The new value of $V_{ref}$ at $50^\circ$C $= V_{ref} + \Delta T \times$ temperature coefficient

$$\begin{aligned} &= 6.9 - 0.56\text{m} \times (50 - 25) \\ &= 6.9 - 0.014 = 6.886 \text{ V} \end{aligned}$$

SOL 1.1.56 Option (C) is correct.

For the given circuit, we determine the linear region (forward bias or reverse bias) in which the two diodes are operating.

Step 1: Assume that the two diodes are OFF, and replace it by open circuit. So, the equivalent circuit is

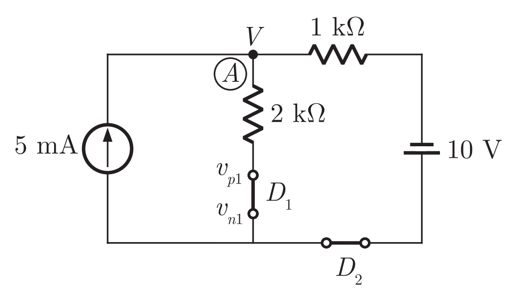



Step 2: Now, we have the condition for both the diodes

$$\begin{aligned} v_p &> v_n \quad \text{diode is ON} \\ v_p &< v_n \quad \text{diode is OFF} \end{aligned}$$

Applying these conditions, we conclude that the diodes $D_1, D_2$ are ON.

Step 3: When both diodes are operating in ON state, we have the equivalent network as




By nodal analysis, we obtain the voltage at node $A$ as

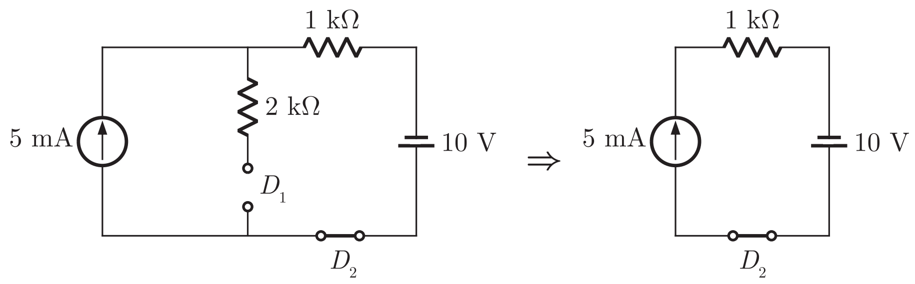
$$\frac{V - 0}{2\text{k}} + \frac{V - (-10)}{1\text{k}} = 5\text{m}$$

$$V + 2V + 20 = 5\text{m} \times 2\text{k}$$
$$3V = -10$$
$$V = -\frac{10}{3}$$

Step 4: At node $A$ voltage is negative, so we get

$$V_{Pn1} < 0\ ,\ V_{Pn2} > 0$$

Therefore, diode $D_1$ is OFF and diode $D_2$ is ON. So, equivalent circuit is as shown below.




From the above circuit, $I_{D2} > 0$. Thus, diode $D_1$ is OFF and Diode $D_2$ is ON.

SOL 1.1.57 Option (B) is correct.

We have the condition for both the diodes are

For normal diode,

$v_p > v_n$ diode is ON

$v_p < v_n$ diode is OFF

For Zener diode,

$|V_{pn}| > V_z$ Zener diode is ON

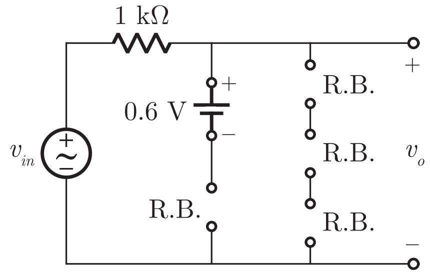
$|V_{pn}| < V_z$ Zener diode is OFF

Using these conditions, we analyze the given circuit.

Step 1: For positive half cycle, the diode $D_2$, $D_3$, $D_4$ are operating in OFF state and diode $D_1$ is in forward biased for $V_i > 0.6$. Now, consider the following two cases for positive half cycle.

CASE I:

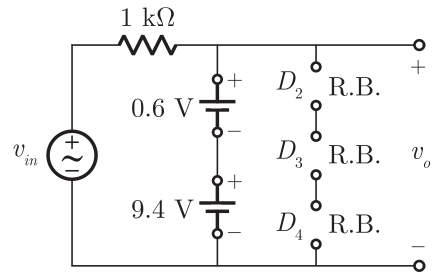
If $0 < v_i < 10\,\text{V}$, then zener diode is in OFF state. So, it operates as normal diode. In this case, the voltage across zener diode $V_{Pn} < 0$. So, it operates in reverse bias.




So, output voltage $v_o = v_{in}$

CASE II:

If $v_i > 10\,\text{V}$, then voltage across zener diode is $V > v_z$. So, zener diode is in ON state.

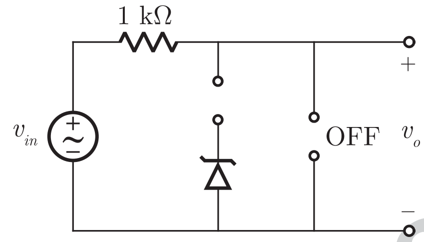



So, the output voltage is

$$v_o = 9.4 + 0.6 = 10 \text{ V}$$

Step 2: For negative half cycle, the diode $D_1$ is in reverse bias. So, no more use of zener diode and diode $D_2$, $D_3$, $D_4$ are ON if $v_i < -1.8$ V other wise it is OFF. Consider the following cases for negative half cycle.

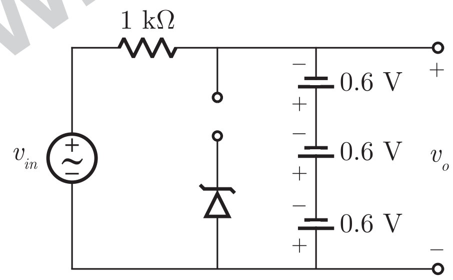
CASE I:
When $-1.8 < v_{in} < 0$, the equivalent circuit is




So, the output voltage is

$$v_o = v_{in}$$

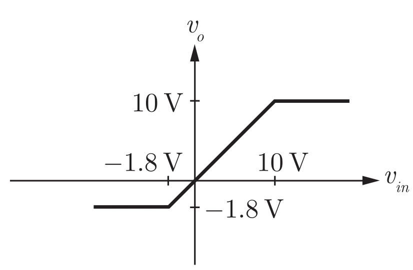
CASE II:
If $v_{in} < -1.8$ V, then diode $D_2$, $D_3$ and $D_4$ is in ON state. So, equivalent circuit is




Therefore, the output voltage is

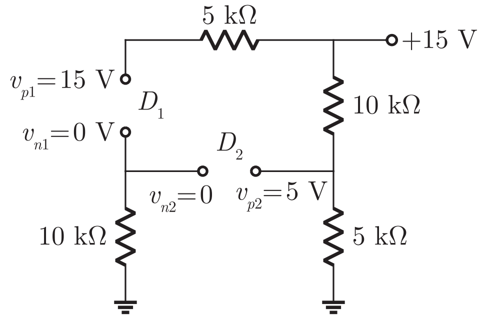
$$\begin{aligned} v_o &= -0.6 - 0.6 - 0.6 \\ &= -1.8 \text{ V} \end{aligned}$$

Step 3: From the above results, we sketch the plot between $v_{in}$ and $v_o$ as




SOL 1.1.58 Option (B) is correct.
For the given circuit, we determine the linear region (forward bias or reverse bias) in which the two diodes are operating.

Step 1: Assume that the two diodes are OFF, and replace it by open circuit. So, the equivalent circuit is




Step 2: By using voltage divide rule, we have

$$v_{P2} = \frac{5\text{k}}{5\text{k}+10\text{k}} \times 15 = 5\text{ V}$$

So, voltage across diodes $D_1$ and $D_2$ are

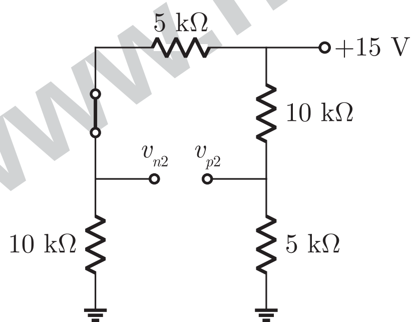
$$v_{Pn1} = 15\text{ V} \quad \text{and} \quad v_{Pn2} = 5\text{ V}$$

Step 3: Now, we have the operating conditions for the diode

$$v_p > v_n \qquad \text{diode is ON}$$
$$v_p < v_n \qquad \text{diode is OFF}$$

Applying these conditions, we conclude that both diodes are operating in ON state. But, the $n$-terminals of diode are commonly connected, so the result is incorrect.

Step 4: For the obtained voltage, we conclude that diode $D_1$ approaches the ON state first. So, we check for $D_1$ ON and $D_2$ OFF. The equivalent circuit is




Step 5: By using voltage divider rule, we have

$$v_{P2} = \frac{5\text{k}}{5\text{k}+10\text{k}} \times 15 = 5\text{ V}$$

$$v_{n2} = \frac{10\text{k}}{5\text{k}+10\text{k}} \times 15 = 10\text{ V}$$

So, voltage across diode $D_2$ is

$$v_{Pn2} = 5 - 10$$
$$v_{Pn2} = -5$$
$$v_{Pn2} < 0$$

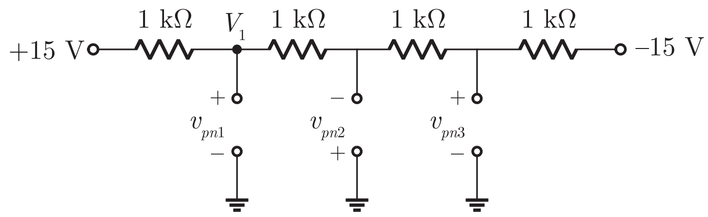
i.e. diode $D_2$ is OFF (assumption is correct). Thus, diode $D_1$ is ON and $D_2$ is OFF.

SOL 1.1.59 Option (C) is correct.

For the given circuit, we determine the linear region (forward bias or reverse bias) in which the diodes are operating.

Step 1: Assume that all the diodes are OFF, and replace it by open circuit.

So, the equivalent circuit is




Step 2: By using voltage divider rule,

$$V_1 = \frac{1\text{k}}{1\text{k}+1\text{k}+1\text{k}+1\text{k}} \times (15-(-15))$$

$$V_1 = \frac{1}{4} \times 30 = 7.5\text{ V}$$

Voltage across diode $D_1$ is

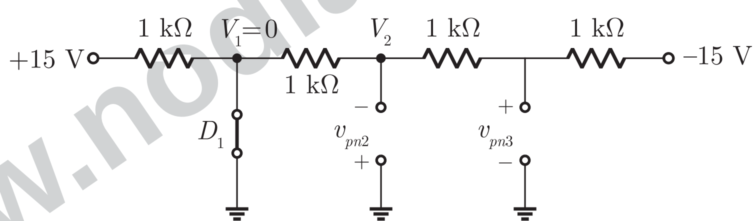
$$v_{Pn1} = v_{P1} - v_{n1}$$
$$= V_1 - 0 = 7.5\text{ V}$$
$$v_{Pn1} > 0$$

Step 3: Now, we have the condition for the diode

$$v_p > v_n \qquad \text{diode is ON}$$
$$v_p < v_n \qquad \text{diode is OFF}$$

For these conditions, we conclude that diode $D_1$ is ON.

Step 4: Now, we draw the equivalent circuit (for the diode $D_1$ ON) for checking the operating regions of diodes $D_2$ and $D_3$.

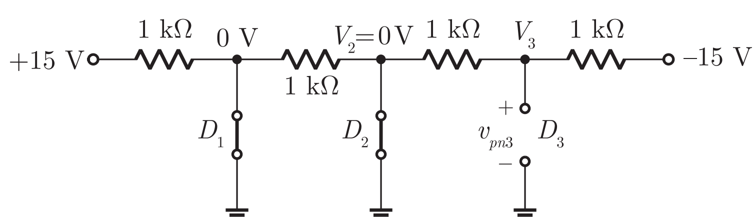



Step 5: Using voltage divider rule,

$$V_2 = \frac{1\text{k}}{1\text{k}+1\text{k}+1\text{k}}(-15\text{ V})$$
$$= -5\text{ V}$$
$$v_{Pn2} = v_{P2} - v_{n2}$$
$$= 0-(-5) = 5$$

So, diode $D_2$ is also ON.

Step 6: Again, we redraw the equivalent circuit (for the diodes $D_1$ and $D_3$ ON) for checking the operating region of diode $D_3$.




By using voltage divider rule,

$$V_3 = \frac{1\text{ k}\Omega}{1\text{k}+1\text{k}}(-15) = -7.5\text{ V}$$

$$v_{Pn3} = V_{P3} - V_{n3}$$

$$= -7.5 - 0 = -7.5\text{ V}$$

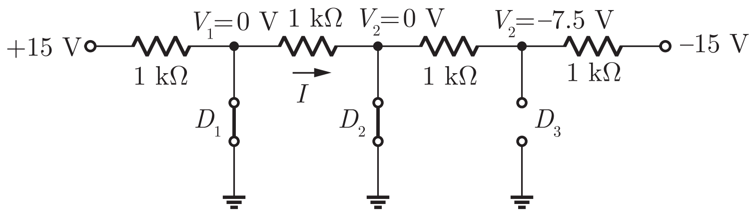
$$v_{Pn3} < 0$$

So, diode $D_3$ is OFF.

Step 7: Thus, combining the all results obtained in above cases, we conclude that diode $D_1$ is ON, $D_2$ is ON, and diode $D_3$ is OFF.

SOL 1.1.60 Option (C) is correct.

From the results obtained in previous problem, we have the equivalent circuit as




Thus, we get the voltage,

$$V = V_2 - V_3$$
$$= 0 - (-7.5\text{ V})$$
$$= 7.5\text{ V}$$

and the current,

$$I = \frac{V_1 - V_2}{1\text{ k}\Omega}$$
$$= \frac{0-0}{1\text{k}} = 0\text{ Amp}$$

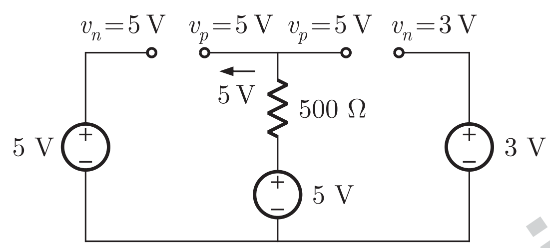
**********

# SOLUTIONS 1.2

SOL 1.2.1 Correct answer is 4.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the diodes are operating, and then obtain the required unknowns.

Step 1: Assume that the diodes are OFF, and replace it by open circuit. So, equivalent circuit is,

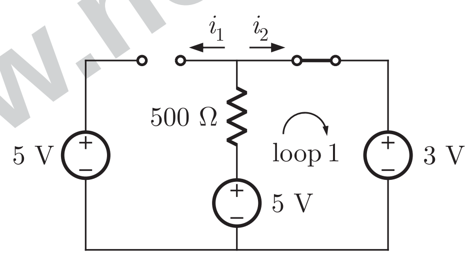



Step 2: Now, we have the condition

$$v_p > v_n \qquad \text{diode is ON}$$
$$v_p < v_n \qquad \text{diode is OFF}$$

Applying these conditions, we conclude that diode $D_1$ is OFF and $D_2$ is ON.

Step 3: For the result obtained in the above case, the equivalent circuit is

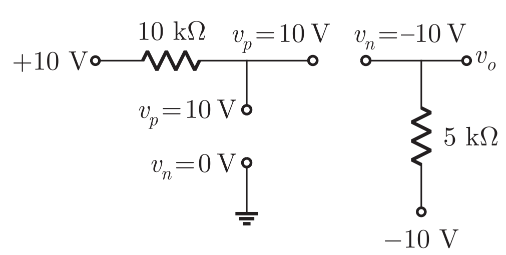



$$i_1 = 0\,\text{mA}, \quad i_2 = \frac{5-3}{500} = 4\,\text{mA}$$

SOL 1.2.2 Correct answer is $-3.57$.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the diodes are operating, and then obtain the output.

Step 1: Assume that the diodes are OFF, and replace it by open circuit. So, equivalent circuit is

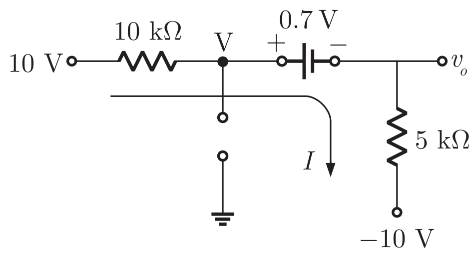



Step 2: Now, we have the condition

$$v_p > v_n \qquad \text{diode is ON}$$
$$v_p < v_n \qquad \text{diode is OFF}$$

Applying these conditions, we conclude that both diodes are ON. But, the $p$-terminals of the diodes are commonly connected, so it violates the result.

Step 3: When diode $D_1$ is OFF, and diode $D_2$ is ON, then equivalent circuit is




$$I = \frac{10 - 0.7 - (-10)}{15\text{k}} = 1.287 \text{ mA}$$

$$V = 10 - (1.287 \text{ m}) \times 10\text{k} = -2.87 \text{ V}$$

So, diode $D_1$ is OFF (our assumption is correct).

Step 4: For the diode $D_1$ OFF, we have

$$i_{D1} = 0 \text{ mA}$$

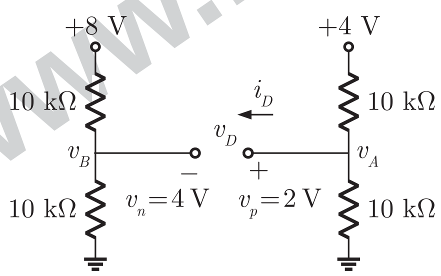
So,

$$v_o = -10 + (5\text{k})(1.287 \text{ m}) = -3.57 \text{ V}$$

SOL 1.2.3 Correct answer is $-2$.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the diode is operating, and then obtain the output.

Step 1: Assume that the diode is OFF, and replace it by open circuit. So, equivalent circuit is




Step 2: By using voltage divider rule,

$$v_A = \frac{10\text{k}}{10\text{k} + 10\text{k}} \times 4 = 2 \text{ V}$$

$$v_B = \frac{10\text{k}}{10\text{k} + 10\text{k}} \times 8 = 4 \text{ V}$$

Step 3: Now, we have the condition

$$v_p > v_n \qquad \text{diode is ON}$$

$$v_p < v_n \qquad \text{diode is OFF}$$

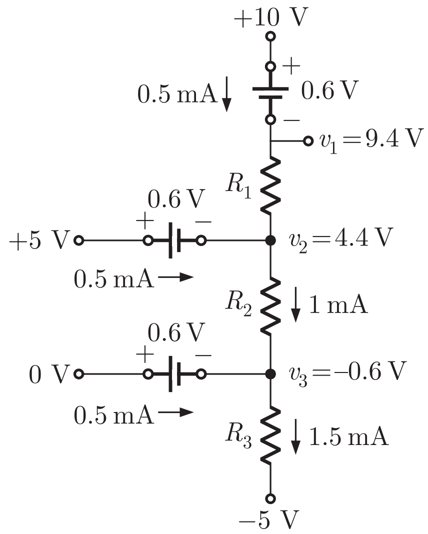
Applying these conditions, we conclude that diode is OFF.

Step 4: So, the voltage across diode is

$$v_D = v_A - v_B = 2 - 4 = -2 \text{ V}$$

SOL 1.2.4 Correct answer is 2.93.

Each diode having a forward bias current or $i_D > 0$. So, each diode is operating in ON state. Therefore, the equivalent circuit is




From the circuit,we obtain the resistance

$$R_1 = \frac{9.4 - 4.4}{0.5}$$

$$= 10\text{ k}\Omega$$

$$R_2 = \frac{4.4 - (-0.6)}{1}$$

$$= 5\text{ k}\Omega$$

$$R_3 = \frac{-0.6 - (-5)}{1.5}$$

$$= 2.93\text{ k}\Omega$$

SOL 1.2.5 Correct answer is 7.6 .

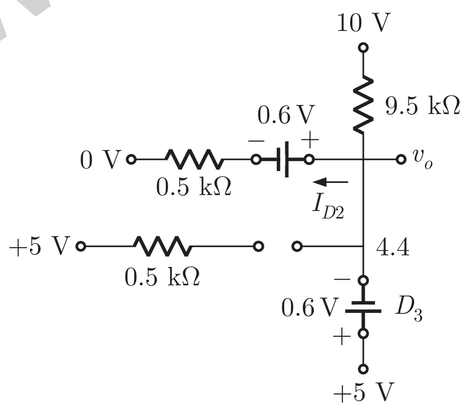
If $D_3$ is ON, then

$$v_o = 5 - 0.6 = 4.4$$

In this case,

$$i_{D1} < 0,\ i_{D2} > 0$$

So, diode $D_1$ is OFF and diode $D_2$ is ON. Therefore, the equivalent circuit is




Thus, the current through the diode $D_2$ is

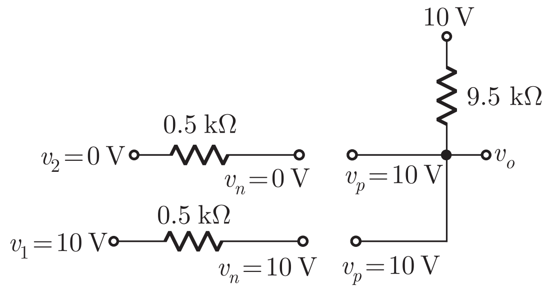
$$i_{D2} = \frac{4.4 - 0.6}{0.5\text{k}}$$

$$= 7.6\text{ mA}$$

SOL 1.2.6 Correct answer is 1.07 .

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the diodes are operating, and then obtain the output.

Step 1: Assume that the diodes are OFF, and replace it by open circuit. So, equivalent circuit is

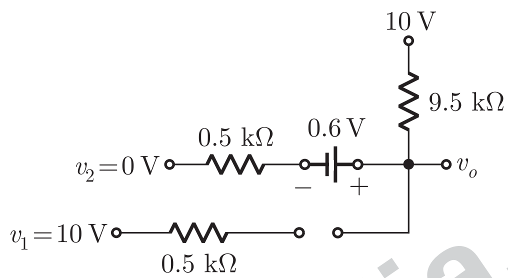



Step 2: Now, we have the condition

$$v_p > v_n \qquad \text{diode is ON}$$
$$v_p < v_n \qquad \text{diode is OFF}$$

Applying these conditions, we conclude that diode $D_1$ is OFF and $D_2$ is ON.

Step 3: So, the equivalent circuit is



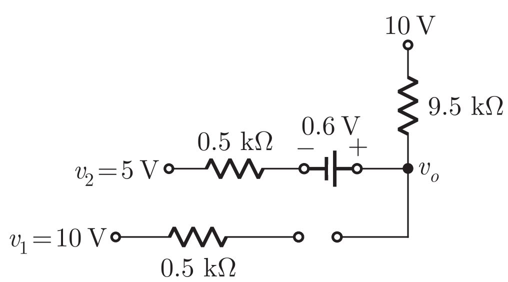

Using Nodal analysis, we obtain the output voltage as

$$\frac{v_o - 10}{9.5} + \frac{(v_o - 0.6) - 0}{0.5\text{k}} = 0$$

$$v_0 = 1.07 \text{ V}$$

SOL 1.2.7 Correct answer is 5.82 .

If $v_1 = 10$ V, $v_2 = 5$ V, then $D_2$ is OFF and $D_2$ is ON. In that case, the equivalent circuit is




Using nodal analysis at node $v_o$,

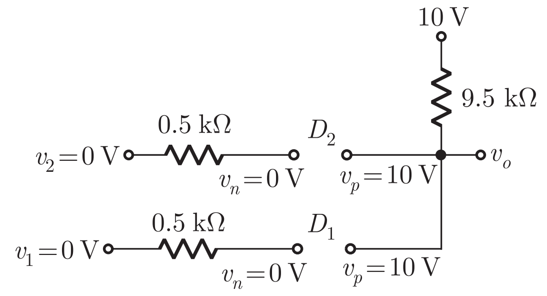
$$\frac{v_o - 10}{9.5} + \frac{v_o - 0.6 - 5}{0.5} = 0$$

$$v_0 = 5.82 \text{ V}$$

SOL 1.2.8 Correct answer is 0.842.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the diodes are operating, and then obtain the output.

Step 1: Assume that the diodes are OFF, and replace it by open circuit. So, the equivalent circuit is (for $v_1 = v_2 = 0$)

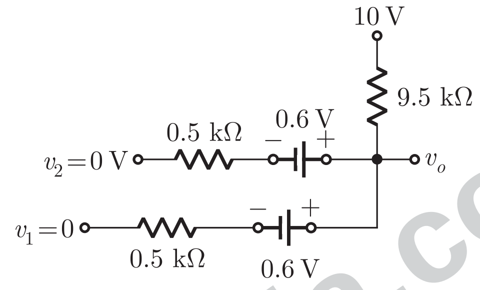



Step 2: Now, we have the condition

$$v_p > v_n \quad \text{diode is ON}$$
$$v_p < v_n \quad \text{diode is OFF}$$

Applying these conditions, we conclude that both diodes will be ON.

Step 3: For the both diodes operating in ON condition, we have the equivalent circuit as




By using nodal analysis at node $v_o$,

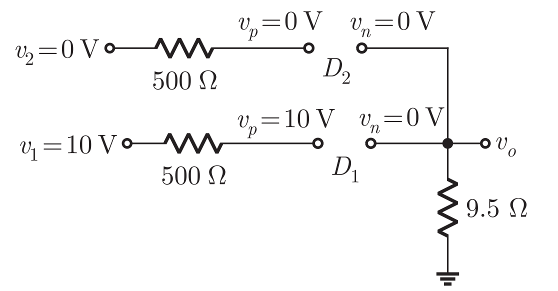
$$\frac{v_o - 10}{9.5\text{k}} + \frac{(v_o - 0.6) - 0}{0.5\text{k}} + \frac{(v_o - 0.6) - 0}{0.5\text{k}} = 0$$

$$v_o = 0.842 \text{ V}$$

SOL 1.2.9 Correct answer is 8.93 .

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the diodes are operating, and then obtain the output.

Step 1: Assume that the diodes are OFF, and replace it by open circuit. So, the equivalent circuit is

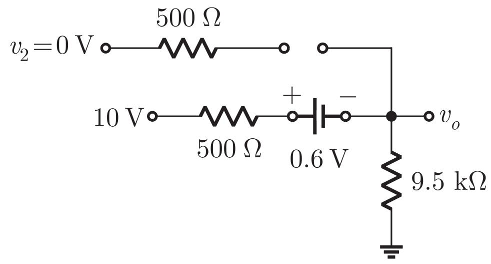



Step 2: Now, we have the condition

$$v_p > v_n \quad \text{diode is ON}$$
$$v_p < v_n \quad \text{diode is OFF}$$

Applying these conditions, we conclude that diode $D_2$ is OFF and $D_1$ is ON.

Step 3: For the obtained operating state, the equivalent circuit is






Using Nodal analysis, we get the output voltage as

$$\frac{v_o - 0}{9.5\text{k}} + \frac{(v_o + 0.6) - 10}{500} = 0$$

$$v_o = 8.93 \text{ V}$$

SOL 1.2.10 Correct answer is 8.93.

In previous problem, we have obtained the output voltage

$$v_o = 8.93 \text{ V}$$

This is the voltage at $n$-terminal of $D_2$. So, the required voltage $v_2$ to turn ON the diode $D_2$ is

$$v_2 = 8.93 + 0.60 = 9.53 \text{ V}$$

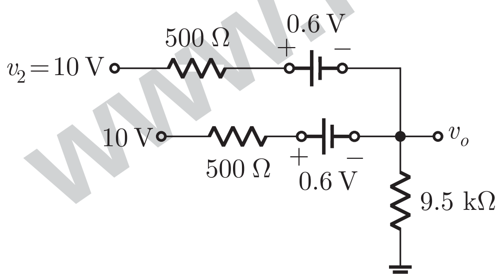
Since, for the given problem we have $v_2 = 5$ V. So, the output in this case is same as in previous.

SOL 1.2.11 Correct answer is 9.16.

Now, we have

$$v_2 = 10 \text{ V}$$

which is greater than the required voltage to turn ON the diode $D_2$. So, the diodes $D_1$ and $D_2$ both are ON for the given voltage. Therefore, the equivalent circuit is




Applying KCL at node $v_o$,

$$\frac{v_o - 0}{9.5\text{k}} + \frac{(v_o + 0.6) - 10}{.5\text{k}} + \frac{(v_o + 0.6) - 10}{.5\text{k}} = 0$$

or $$v_o = 9.16 \text{ V}$$

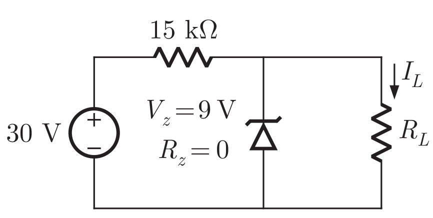
SOL 1.2.12 Correct answer is 0.01.

Since the diodes are connected in parallel, so each diode has the current

$$i_D = \frac{0.1}{10} \text{ A} = 0.01 \text{ A}$$

SOL 1.2.13 Correct answer is 1.4.

The given circuit is

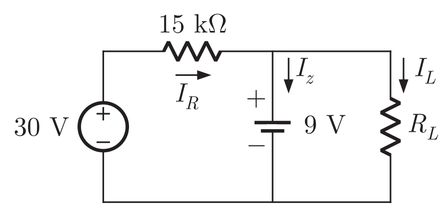


For Zener diode,

$$v_o > v_z \qquad \text{Zener diode is ON}$$

, $$v_o < v_z \qquad \text{Zener diode is OFF}$$

Zener diode in ON state.
So, the equivalent circuit is,




According to KCL

$$i_R = i_z + i_L$$

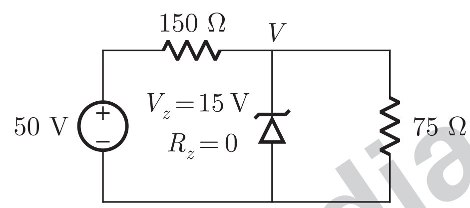
For $$i_{L(\max)} \Rightarrow i_{z(\min)} = 0$$

$$i_R = i_{L(\max)}$$

$$i_{L(\max)} = \frac{30 - 9}{15\ \text{k}\Omega} = 1.4\ \text{mA}$$

SOL 1.2.14 Correct answer is 0.5.

We have the given circuit,






Step 1: From the circuit, the voltage across Zener diode is obtained as

$$V = \frac{75}{225} \times 50 = 16.66\ \text{V}$$

Step 2: For Zener diode, we have the condition

$$V > v_z \qquad \text{Zener diode is ON}$$

$$V < v_z \qquad \text{Zener diode is OFF}$$

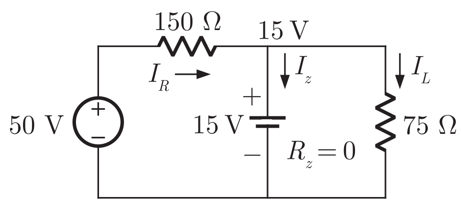
Checking this condition for the obtained value $V$, we conclude that

$$V > V_z$$

$$16.66 > 15\ \text{V}$$

i.e. the Zener diode is ON.

Step 3: For the Zener diode operating in ON condition, the equivalent circuit is




Step 4: From the equivalent circuit, we obtain the current through Zener diode as

$$i_R = i_z + i_L$$

$$i_z = i_R - i_L = \frac{50 - 15}{150} - \frac{15}{75}$$

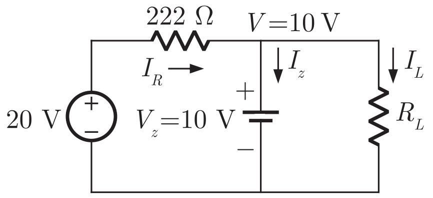
$$i_z = 33.33\ \text{mA}$$

Step 5: So, the power dissipation in Zener diode is

$$p_z = v_z \times i_z = 15 \times 33.33 = 0.5 \text{ Watt}$$

SOL 1.2.15 Correct answer is 2.

Zener diode is operating in ON state, so equivalent circuit is




Now, we obtain the required value of unknown in following steps.

Step 1: Since, the maximum power (power rating) in zener diode is

$$p_z = 400 \text{ mW}$$

So, the maximum current through the zener diode is

$$i_z = \frac{p_z}{v_z} = \frac{400}{10} = 40 \text{ mA}$$

Step 2: Again, from the circuit we have the current through the resistance 222 Ω as

$$i_R = \frac{20 - 10}{222} = 45 \text{ mA}$$

Step 3: For maximum power in zener diode, the value of $R_L$ is obtained as

$$i_{L(\min)} = i_R - i_{z(\max)}$$

$$\frac{10}{R_L} = 45 - 40$$

$$R_L = \frac{10}{5} = 2 \text{ k}\Omega$$



SOL 1.2.16 Correct answer is 24.4.

Given the secondary transformer voltage,

$$v_s = 60 \sin 2\pi 60t \text{ V}$$

So, the maximum voltage across capacitor is given by

$$V_{\max} = (v_s)_{\max} - 2 \times V_{D,\text{on}}$$

$$= 60 - 1.4 = 58.6 \text{ V}$$

Thus, the capacitance is given by

$$C = \frac{V_{\max}}{2fRv_{rip}} = \frac{58.6}{2(60)\,10 \times 10^3 \times 2} = 24.4 \ \mu\text{F}$$

SOL 1.2.17 Correct answer is 20.6.

Given the input to full-wave rectifier,

$$v_i = 120 \sin 2\pi 60t \text{ V}$$

So, we have the voltage across both the secondary transformer as

$$v_s = 120 \sin 2\pi 60t \text{ V}$$

Therefore, the maximum voltage across the capacitor is

$$V_{\max} = (v_s)_{\max} - V_{D,\text{on}}$$

$$= 120 - 0.7 = 119.3 \text{ V}$$

Since, the input cannot drop below 100 V, so we have

$$v_{rip} = 119.3 - 100 = 19.3 \text{ V}$$

Thus, the capacitance is obtained as

$$C = \frac{V_{\max}}{2fRv_{rip}} = \frac{119.3}{2(60)\,2.5 \times 10^3 \times 14.4} = 20.6 \,\mu\text{F}$$

SOL 1.2.18 Correct answer is 6.25.

Since, we know

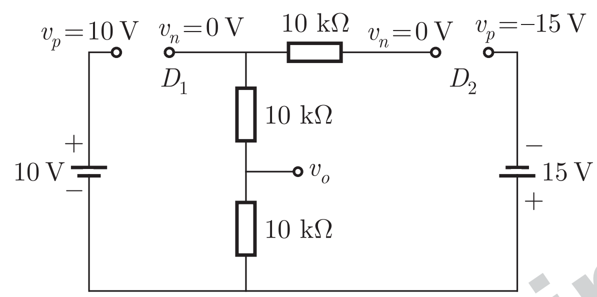
$$v_{rip} = \frac{v_{\max}}{fR_L C}$$

So,
$$R_L = \frac{v_{\max}}{fCv_{rip}} = \frac{75}{60 \times 50 \times 10^{-5} \times 4} = 6.25 \text{ k}\Omega$$

SOL 1.2.19 Correct answer is 5.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the diode is operating, and then obtain the output.

Step 1: Assume that the diode is OFF, and replace it by open circuit. So, equivalent circuit is

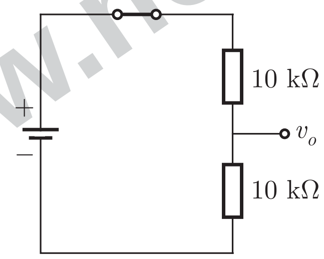



Step 2: Now, we have the condition

$$v_p > v_n \quad \text{diode is ON}$$
$$v_p < v_n \quad \text{diode is OFF}$$

Checking the given circuit for these conditions, we conclude that $D_2$ is always OFF and $D_1$ is ON.

Step 3: So, we draw the equivalent circuit for the obtained result as



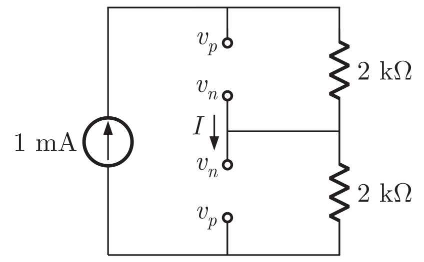

Step 4: By to voltage divider rule, we obtain

$$v_o = \frac{10\text{k}}{10\text{k} + 10\text{k}} \times 10$$
$$= 5 \text{ V}$$

SOL 1.2.20 Correct answer is 0.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the diodes are operating, and then obtain the output.

Step 1: Assume that the diodes are OFF, and replace it by open circuit. So, equivalent circuit is



Step 2: Now, we have the condition

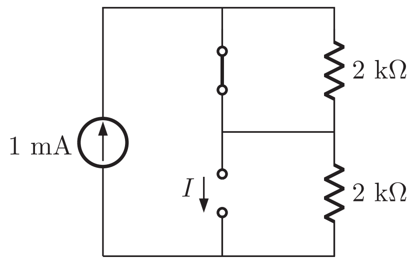
$$v_p > v_n \qquad \text{diode is ON}$$
$$v_p < v_n \qquad \text{diode is OFF}$$

Checking the given circuit for the conditions,

Diode $D_1 \to v_{pn} = 2 = +\,\text{ve}$. So $D_1$ is ON.

Diode $D_2 \to v_{pn} = -2 = -\,\text{ve}$. So $D_2$ is OFF.

Step 3: Thus, for the obtained result, we draw the equivalent circuit as



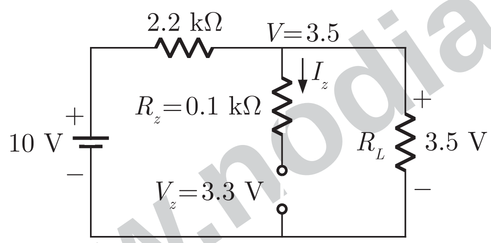

Step 4: From the equivalent circuit, we get

$$I = 0$$

SOL 1.2.21 Correct answer is 2.

We analyze the given circuit in following steps.

Step 1: Assume that Zener diode is OFF, and draw the equivalent circuit as




Step 2: For Zener diode, the operating regions are defined as

$$V > V_z \qquad \text{Zener diode is ON}$$
$$V < V_z \qquad \text{Zener diode is OFF}$$

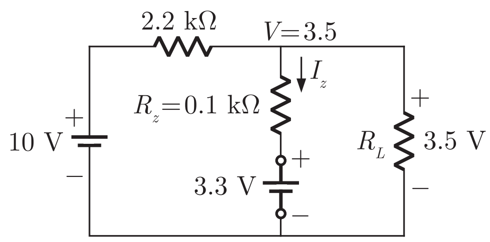
,

Checking the given circuit for the conditions, we have

$$V > v_z$$

i.e. zener diode is ON and work as a voltage regulation.

Step 3: For the obtained result, we draw the equivalent circuit as



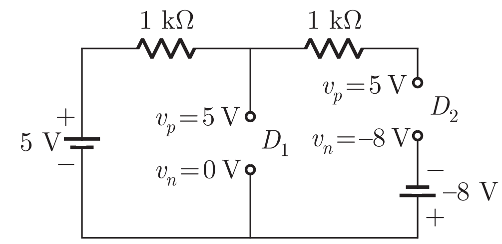

Step 4: From the circuit, we obtain

$$I_z = \frac{3.5 - 3.3}{0.1\text{k}} = 2\,\text{mA}$$

SOL 1.2.22 Correct answer is 0.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the two diodes are operating, and then obtain the output.

Step 1: Assume that the two diodes are OFF, and replace its by open circuit. So, the equivalent circuit is

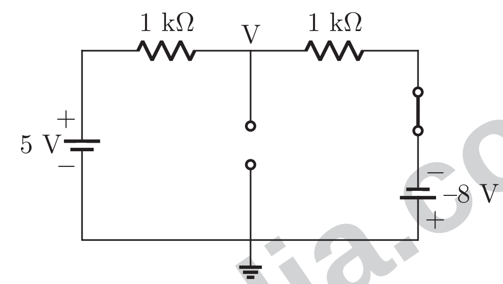



Step 2: Now, we have the condition for both the diodes

$$v_p > v_n \qquad \text{diode is ON}$$
$$v_p < v_n \qquad \text{diode is OFF}$$

Checking the given circuit for the conditions, we conclude that $V_{pn}$ of diode $D_2$ is highly positive respect to $D_1$. So, $D_1$ is OFF and $D_2$ is ON.

Step 3: For the obtained result, we draw the equivalent circuit as




From the circuit, we have

$$\text{Current, } I = \frac{5-(-8)}{2\text{k}} = 6.5\,\text{mA}$$

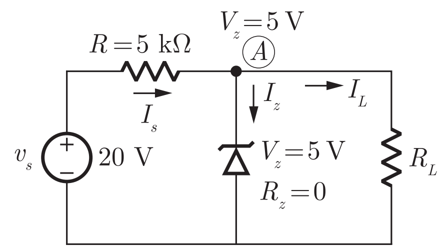
$$\text{Voltage, } V = 5 - 6.5 \times 1 = -1.5\,\text{V}$$

So, diode $D_1$ is OFF (assumption is correct). Therefore,

$$I_D = 0$$

SOL 1.2.23 Correct answer is 1.66.

We have the regulator circuit as




For $R_{L\,\text{min}}$ the load current will be maximum. Since, by applying KCL at Node $A$, we have

$$I_S = I_L + I_z$$

So, for $I_{L\,\text{max}}$ the value of $I_z$ has to be minimum, i.e.

$$I_{z\,\text{min}} = 0\,\text{mA}$$

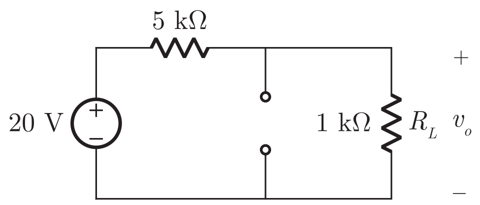
or $$\frac{V_S - V_z}{R} = 0 + I_{L\,\text{max}}$$

or $$\frac{V_z}{R_{L\,\text{min}}} = \frac{20-5}{5\text{k}} \qquad \left[\text{Put } I_{L,\text{max}} = \frac{V_z}{R_{L\,\text{min}}}\right]$$

Thus, $$R_{L\text{min}} = \frac{5}{3\text{m}} = 1.66\,\text{k}\Omega$$

SOL 1.2.24 Correct answer is 3.33.

Assume that zener diode is OFF and draw the equivalent circuit




By using voltage divider rule,

$$v_o = \frac{1\text{k}}{5\text{k}+1\text{k}} \times 20$$

$$3.33 \text{ V}$$

Across the zener diode the voltage is $v_o = 3.33$ V, and the Zener voltage is $v_z = 5$ V. So, we have

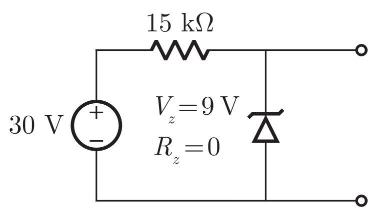
$$v_o < v_z$$

i.e. zener diode operates in OFF state (assumption is correct).Thus,

$$v_o = 3.33 \text{ V}$$

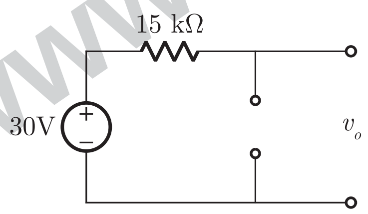
SOL 1.2.25 Correct answer is 12.6.

For $R_L = \infty$, the equivalent circuit is




Now, we analyse the circuit in following steps.

Step 1: Assume zener diode in OFF state and draw the equivalent circuit.




Step 2: From the circuit, we have

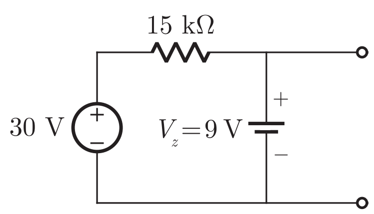
$$v_o = 30 \text{ V}$$

Since $v_z = 9$ V, so we conclude that

$$v_o > v_z$$

i.e. Zener diode is operating in ON state (assumption is incorrect).

Step 3: For the obtained result, we draw the equivalent circuit as




Step 4: So, the current flowing through zener diode is

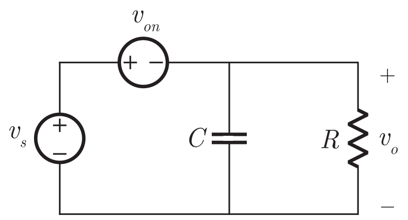
$$I_z = \frac{30-9}{15\text{k}} = 1.4 \text{ mA}$$

Step 5: Thus, the power dissipation in Zener diode is

$$P_z = v_z I_z$$
$$= 9 \times 1.4\text{m} = 12.6 \text{ mW}$$

SOL 1.2.26 Correct answer is 16.82.

For the diode operating in ON state, simplified circuit is




The ideal dc output voltage in the absence of the ripple is given by

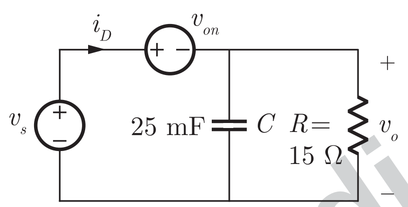
$$(\mathrm{V_{dc}})_{\text{output}} = (\mathrm{V_{dc}})_{\text{input}} - V_{on}$$

Since, $$(\mathrm{V_{dc}})_{\text{input}} = (\mathrm{V_{rms}})_{\text{input}} \times \sqrt{2}$$
$$= 12.6 \times \sqrt{2} = 17.82 \text{ V}$$

So, $$(\mathrm{V_{dc}})_{\text{output}} = 17.82 - 1 = 16.82 \text{ V}$$

SOL 1.2.27 Correct answer is 0.747.

Again, for the diode operating in ON state, we have the simplified circuit as




From the circuit, the ripple voltage is

$$V_{rip} \approx \frac{V_m - V_o}{R}\frac{T}{C}$$
$$= I_{dc}\frac{T}{C} \qquad \left[\because I_{dc} = \frac{V_m - V_o}{R}\right]$$

Given that $f = 60$ Hz. So,

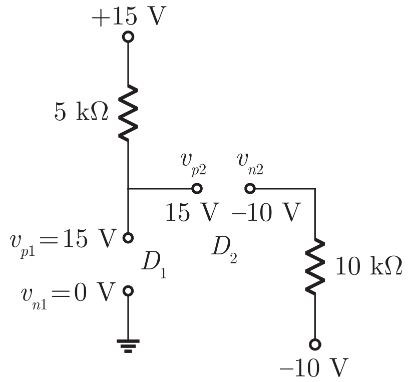
$$T = \frac{1}{f} = \frac{1}{60} \text{ sec}$$

Therefore, $$V_{rip} = \frac{16.8}{15} \times \frac{1/60}{25 \times 10^{-3}}$$
$$= 0.747 \text{ V}$$

SOL 1.2.28 Correct answer is 0.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the diodes are operating, and then obtain the output.

Step 1: Assume that the diodes are OFF, and replace it by open circuit. So, equivalent circuit is

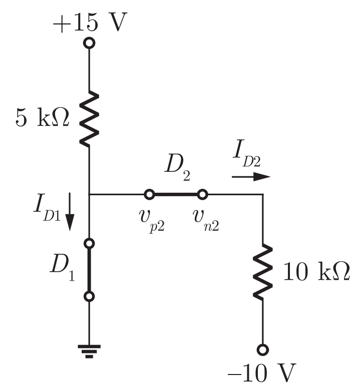


Step 2: Now, we have the condition

$$v_p > v_n \quad \text{diode is ON}$$
$$v_p < v_n \quad \text{diode is OFF}$$

Checking the given circuit for these conditions, we conclude that the diodes $D_1$ and $D_2$ will be ON.

Step 3: For the obtained result, we get the equivalent circuit as




From the circuit, we obtain

$$I_{D2} = \frac{15-(-10)}{5\text{k}+10\text{k}} = 1.66\text{ mA}$$

$$V_{n2} = 10\text{k} \times 1.66\text{m} - 10$$
$$= 6.6\text{ V}$$

Step 4: Since, we have assumed diode $D_1$ ON, so

$$V_{p2} = 0\text{ V}$$

Therefore, the voltage across diode $D_2$ is

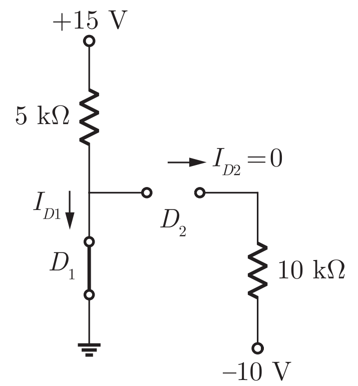
$$V_{pn2} < 0$$

i.e. $D_2$ will be OFF due to $D_1$ ON (assumption is incorrect).

Step 5: Therefore, the total current from 15 V supply will go through diode $D_1$ and no current flow through diode $D_2$, i.e.

$$I_{D2} = 0$$

Step 6: Thus, the actual equivalent circuit is




Current through diodes $D_1$ and $D_2$ are

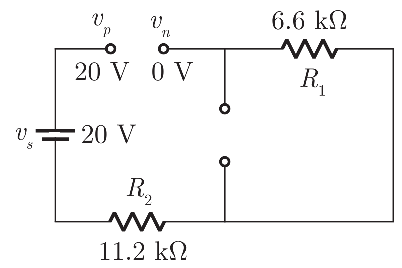
$$I_{D1} = \frac{15-0}{5\text{k}\Omega} = 3\text{ mA}$$

$$I_{D2} = 0$$

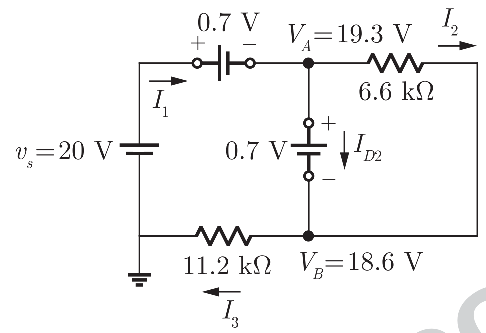
SOL 1.2.29 Correct answer is 1.55.

For the given circuit, we first determine the linear region (forward bias or reverse bias) in which the diodes are operating, and then obtain the output.

Step 1: Assume that the diodes are OFF, and replace it by open circuit. So, equivalent circuit is




Step 2: For the equivalent circuit, we may conclude that the applied voltage forward biases the diodes. So, both the diodes are operating in ON state. So equivalent Circuit is




Step 3: For the equivalent circuit, we obtain

$$I_2 = \frac{0.7}{6.6\,\text{k}\Omega} = 0.11\,\text{mA}$$

$$I_3 = \frac{V_B - 0}{R_2}$$

$$= \frac{18.6 - 0}{11.2\text{k}} = 1.66\,\text{mA}$$

Step 4: Again, applying KCL at node $V_B$,

$$I_3 = I_{D2} + I_2$$

$$1.66\,\text{m} = I_{D2} + 0.11\,\text{mA}$$

$$I_{D2} = 1.55\,\text{mA}$$

SOL 1.2.30 Correct answer is 38.4.

The diode current equation is

$$I_D = I_S e^{\frac{V_D}{V_T}} \qquad ...(1)$$

Differentiating with respect to $V_D$,

$$\frac{dI_D}{dV_D} = I_S e^{\frac{V_D}{V_T}} \frac{1}{V_T} \qquad ...(2)$$

So we may write

$$\frac{\Delta I_D}{\Delta V_D} = \frac{I_D}{V_T}$$

Thus, $$\Delta I_D = \frac{I_D}{V_T} \times \Delta V_D$$

$$= \frac{1\text{m}}{26\text{m}} \times 1\text{m} = 38.4\,\mu\text{A}$$

ALTERNATIVE METHOD :

We have

DC bias current, $I_D = 1\,\text{mA}$

Small signal resistance, $r_d = \frac{V_T}{I_D} = \frac{26\text{m}}{1\text{m}} = 26\,\Omega$

By ac analysis, we get small change in current as

$$\Delta I_D = \frac{\Delta V_D}{r_d} = \frac{1\text{m}}{26} = 38.4\,\mu\text{A}$$

SOL 1.2.31 Correct answer is 1.417.

The ripple voltage is given by

$$V_{rip} = \frac{V_m - V_{D,on}}{R_L}\frac{T}{C} \qquad \text{where } V_m = \text{peak voltage}$$

or $$V_{rip} = I_{dc}\frac{T}{C}$$

Given that peak to peak swing is

$$V_{\text{peak}-\text{peak}} = 9\text{ V}$$

So, the peak voltage is obtained as

$$V_m = \frac{V_{\text{peak}-\text{peak}}}{2} = \frac{9}{2} = 4.5\text{ V}$$

Also, we have $$V_{D,on} = 0.8\text{ V}$$

$$R_L = 0.436\,\Omega$$

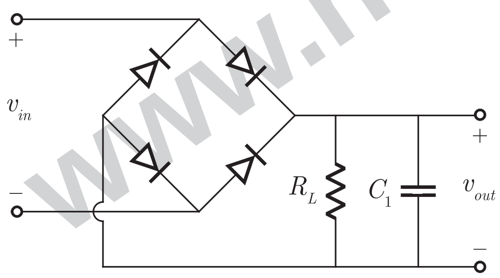
$$T = \frac{1}{f} = \frac{1}{60} = 16.67\text{ ms}$$

Thus, the capacitance is obtained as

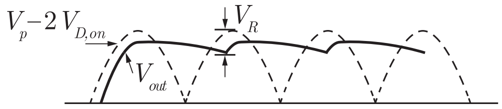
$$C = \frac{4.5 - 0.8}{0.436\,\Omega} \times \frac{16.67\text{ ms}}{0.1} = 1.417\text{ F}$$

SOL 1.2.32 Correct answer is 0.389.

Full wave rectifier with smoothing capacitor and load resistance $R_L$ is




Ripple in full wave rectifier is




The output remains equal to zero for

$$|V_{in}| < 2V_{D,on}$$

So output voltage is

$$V_{out} = V_P - 2V_{D,on}$$

The capacitor is discharged for half of the input cycle (approx.), so the ripple is approximately equal to half of that in half wave rectifier. Therefore, ripple voltage in full wave rectifier is

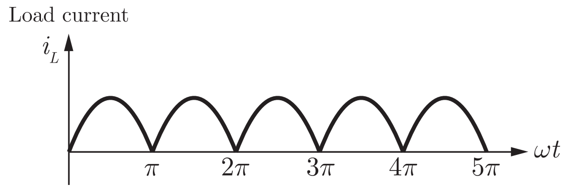
$$V_{rip} = \frac{1}{2}\frac{V_m - 2V_{D,on}}{R_1 C_1}T$$

$$= \frac{1}{2}\frac{3 - 2\times 0.8}{30\times 1000\times 10^{-6}\times 60}$$

$$= 0.389\text{ V}$$

SOL 1.2.33 Correct answer is 4.98.

Load current waveform for full wave rectifier is




By definition, average or dc current is given by

$$I_{dc} = \frac{1}{T}\int_0^T i_L(t)\,d(\omega t)$$
$$= \frac{1}{2\pi}\int_0^{2\pi} i_L\,d(\omega t)$$

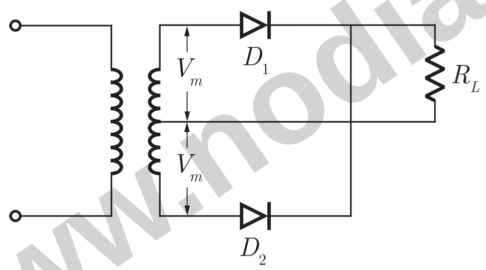
The load current is given by

$$i_L = \begin{cases} I_m \sin\omega t & 0 \le \omega t \le \pi \\ -I_m \sin\omega t & \pi \le \omega t \le 2\pi \end{cases}$$

So, the average dc current is obtained as

$$I_{dc} = \frac{1}{2\pi}\left[\int_0^{\pi} I_m \sin\omega t\,d(\omega t) + \int_{\pi}^{2\pi} - I_m \sin\omega t\,d(\omega t)\right]$$
$$= \frac{I_m}{2\pi}\left[(-\cos\omega t)_0^{\pi} + (\cos\omega t)_{\pi}^{2\pi}\right]$$
$$= \frac{2I_m}{\pi} \qquad \text{...(1)}$$

Now, the rectifier circuit is




From the circuit, we have

$$I_m = \frac{V_m}{R_S + R_f + R_L} \qquad \text{...(2)}$$

Where $R_S \rightarrow$ Secondary winding resistance
$R_f \rightarrow$ Diode forward resistance
$R_L \rightarrow$ Load resistance

Substituting equation (2) in equation (1), we get

$$I_{dc} = \frac{2I_m}{\pi} = \frac{2V_m}{\pi(R_S + R_f + R_L)}$$

Again, for positive half cycle diode $D_1$ is ON and $D_2$ is OFF, and for negative half cycle diode $D_1$ is OFF and $D_2$ is ON. Average or dc voltage $V_{dc}$ is load voltage produced by $I_{dc}$ flowing through the load, i.e.

$$V_{dc} = I_{dc}R_L = \frac{2I_m R_L}{\pi}$$

Substituting equation (2) in above expression, we have

$$V_{dc} = \frac{2V_m R_L}{\pi(R_S + R_f + R_L)}$$

Since $R_S + R_f << R_L$, so

$$V_{dc} = \frac{2V_m}{\pi}$$

Therefor, dc power delivered to the load is given by

$$P_{dc} = V_{dc} I_{dc} = I_{dc}^2 R_L$$
$$= \left(\frac{2I_m}{\pi}\right)^2 R_L = \left[\frac{2V_m}{\pi(R_S + R_f + R_L)}\right]^2 R_L \qquad ...(3)$$

Given that, $V_{S(rms)} = 20\text{ V}$

$$R_S = 5\,\Omega,\ R_f = 2\,\Omega,\ R_L = 50\,\Omega$$
$$V_m = \sqrt{2}\, V_{S(rms)}$$
$$= \sqrt{2} \times 20 = 28.28\text{ V}$$

Substituting these values in equation (3), we get

$$P_{dc} = \left[\frac{2 \times 28.28}{3.14(5+2+50)}\right]^2 \times 50$$
$$= 4.98\text{ watt}$$

SOL 1.2.34 Correct answer is 14.

In previous solution, we have obtained

$$V_{dc} = \frac{2V_m R_L}{\pi(R_S + R_f + R_L)}$$

For calculation of Null load voltage ($V_{NL}$), we put $R_L \to \infty$ in above expression, i.e.

$$V_{NL} = \lim_{R_L \to \infty} \frac{2V_m}{\pi\left(1 + \frac{R_S + R_f}{R_L}\right)}$$
$$= \frac{2V_m}{\pi} = 18.01\text{ V}$$

Also, the full load voltage is obtained as

$$V_{FL} = I_{dc} R_L = \frac{2V_m R_L}{\pi(R_S + R_f + R_L)}$$
$$= \frac{2 \times 28.28 \times 50}{3.14(5+2+50)}$$
$$= 15.80\text{ V}$$

Thus, the percentage load regulation is

$$\%\text{ Load regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\,\%$$
$$= \frac{18.01 - 15.80}{15.80} \times 100 = 14\,\%$$

SOL 1.2.35 Correct answer is 0.711.

The rectifier efficiency is defined as

$$\eta = \frac{\text{Output dc power}}{\text{Input ac power}}$$

Now, we have the output dc power,

$$(P_{dc})_o = V_{dc} I_{dc} = I_{dc}^2 R_L$$
$$= \left(\frac{2I_m}{\pi}\right)^2 R_L$$

The input ac power is

$$(P_{ac})_{in} = (V_{rms})_i (I_{rms})_i = \left(\frac{V_m}{\sqrt{2}}\right)_i \left(\frac{V_m}{\sqrt{2}}\right)_i$$
$$= \frac{(V_m)_i (I_m)_i}{2}$$
$$= \frac{I_m^2}{2(R_S + R_f + R_L)}$$

Thus, we get

$$\eta = \frac{\frac{4}{\pi^2} I_m^2 R_L}{\frac{I_m^2}{2(R_S + R_f + R_L)}}$$

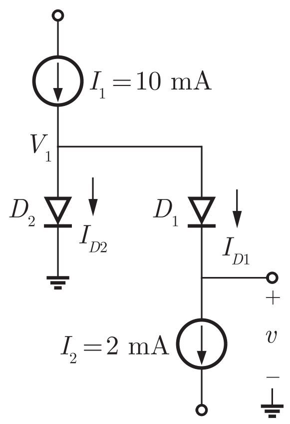
$$= \frac{8}{\pi^2} \frac{R_L}{(R_S + R_f + R_L)}$$

$$= \frac{8}{\pi^2} \frac{50}{5 + 2 + 50}$$

$$= 0.711$$

SOL 1.2.36 Correct answer is 92.22.

We redraw the given the diode circuit as




From circuit, we have the current through $D_2$ as

$$I_{D2} = I_{S2} e^{\frac{V_{D2}}{\eta V_T}} = I_{S2} e^{\frac{V_1}{\eta V_T}} \qquad ...(1)$$

Also, we have

$$V_{D1} = V_1 - V$$

So, the current through diode $D_1$ is

$$I_{D1} = I_{S1} e^{\frac{V_1 - V}{\eta V_T}} \qquad ...(2)$$

Now, the reverse saturation current is directly proportional to junction area, i.e.

$$I_S \propto A$$

where $A$ denotes the junction area. Since, $D_1$ has 10 times the junction area of $D_2$. So, we get

$$\frac{I_{S1}}{I_{S2}} = 10$$

Applying KCL at node $V_1$,

$$I_1 = I_{D1} + I_{D2}$$

or $$10\text{m} = I_{D2} + 2\text{m}$$

or $$I_{D2} = 8\text{ mA}$$

Dividing equation (2) by (1), we have

$$\frac{I_{D1}}{I_{D2}} = \frac{I_{S1} e^{\frac{V_1 - V}{V_T}}}{I_{S2} e^{\frac{V_1}{V_T}}}$$

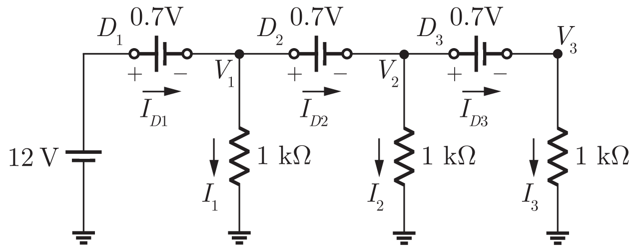
or $$\frac{2\text{m}}{8\text{m}} = 10 e^{-\frac{V}{V_T}}$$

Thus, $$V = V_T \ln(40)$$

$$= 92.22\text{ mV}$$

SOL 1.2.37 Correct answer is 31.8.

All the diodes are forward biased due to 12 V supply voltage. So, diode $D_1$, $D_2$, $D_3$ are ON.




From the circuit, we have

$$\text{Voltage, } V_1 = 12 - 0.7 = 11.3$$
$$\text{Voltage, } V_2 = 12 - 0.7 - 0.7 = 10.6 \text{ V}$$
$$\text{Voltage, } V_3 = 12 - 0.7 - 0.7 - 0.7 = 9.9 \text{ V}$$
$$\text{Current, } I_3 = \frac{V_3 - 0}{1 \text{ k}\Omega} = \frac{9.9 - 0}{1 \text{ k}\Omega} = 9.9 \text{ mA}$$

So, the current through diode $D_3$ is

$$I_{D3} = I_3 = 9.9 \text{ mA}$$

Again, we obtain

$$I_2 = \frac{V_2 - 0}{1 \text{ k}\Omega}$$
$$= \frac{10.6 - 0}{1 \text{ k}\Omega} = 10.6 \text{ mA}$$

Applying KCL at node $V_2$, we get current through diode $D_2$ as

$$I_{D2} = I_2 + I_{D3}$$
$$= 10.6\text{m} + 9.9\text{m}$$
$$= 20.5 \text{ mA}$$

From the circuit, we have the current

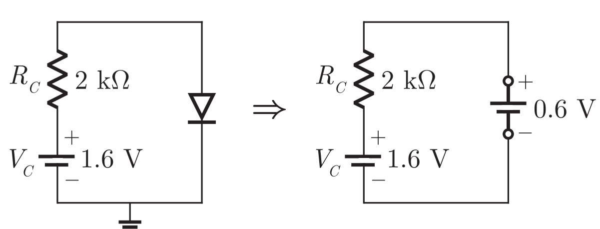
$$I_1 = \frac{V_1 - 0}{1 \text{ k}\Omega} = \frac{11.3}{1\text{k}} = 11.3 \text{ mA}$$

Thus, applying KCL at node $V_1$ we obtain the current through diode $D_1$ as

$$I_{D1} = I_1 + I_{D2}$$
$$= 11.3 \text{ mA} + 20.5\text{m} = 31.8 \text{ mA}$$

SOL 1.2.38 Correct answer is 0.5.

For DC analysis, coupling capacitor $C_1$ and $C_2$ is open circuited. So, equivalent circuit is




Since, diode is forward biased, so current through the diode is

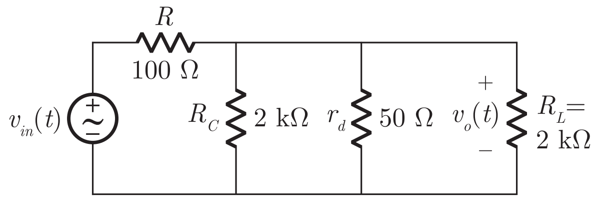
$$I_{DQ} = \frac{V_C - 0.6}{R_C}$$
$$= \frac{1.6 - 0.6}{2 \text{ k}\Omega}$$
$$= 0.5 \text{ mA}$$

SOL 1.2.39 Correct answer is 0.33.

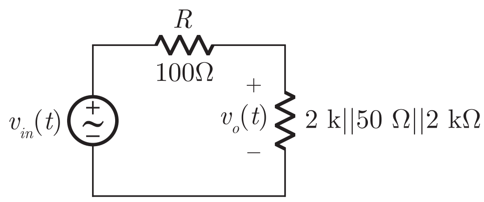
For ac analysis, we have the diode resistance

$$r_d = \frac{\eta V_T}{I_{DQ}} = \frac{1 \times 25\text{mV}}{0.5\text{m}} = 50\,\Omega$$

In ac analysis, capacitor is short circuit and dc voltage source acts as a short circuit. So, we get the small signal ac equivalent circuit is




The above circuit can be redrawn in simplified form as




Thus, by using voltage divider rule

$$v_o(t) = \frac{2\text{k}\,||\,50\,||\,2\text{k}}{100 + 2\text{k}\,||\,50\,||\,2\text{k}}\,v_i(t)$$

or

$$\frac{v_o(t)}{v_i(t)} = \frac{47.61}{100 + 47.61} = 0.33$$

***********

# SOLUTIONS 1.3



SOL 1.3.1 Option (A) is correct.
In capacitor filter,

$$\text{Ripple factor, } r = \frac{1}{4\sqrt{3}\,fCR_L}$$

For large value of $R_L$ (load resistance) the ripple factor should be small. So it is suitable for large value of load resistance.

SOL 1.3.2 Option (B) is correct.
In inductor filter,

$$\text{Ripple factor, } r = \frac{R_L}{3\sqrt{2}\,(\omega L)}$$

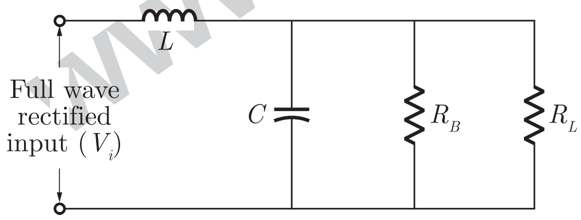
For small value of load resistance $(R_L)$ the ripple factor should be small. So inductor filter is suitable for small values of load-resistance.

SOL 1.3.3 Option (A) is correct.
The rectifier circuit is to convert a.c. voltage to d.c. voltage.

SOL 1.3.4 Option (C) is correct.
The ripple factor of Inductor filter is,

$$r = \frac{R_L}{3\sqrt{2}\,(\omega L)}$$

SOL 1.3.5 Option (D) is correct.
A bleeder resistance $R_B$, is connected in parallel with the load resistance as shown in figure




It improves voltage regulation of the supply by acting as the preload on the supply.

SOL 1.3.6 Option (B) is correct.
For half wave rectifier,
The output frequency $f_o = f_i$
Where, $f_i$ = input frequency
$f_o = 50$ Hz
For full wave bridge rectifier

$$\begin{aligned} f_o &= 2f_i \\ &= 2 \times 50 \\ &= 100 \text{ Hz} \end{aligned}$$

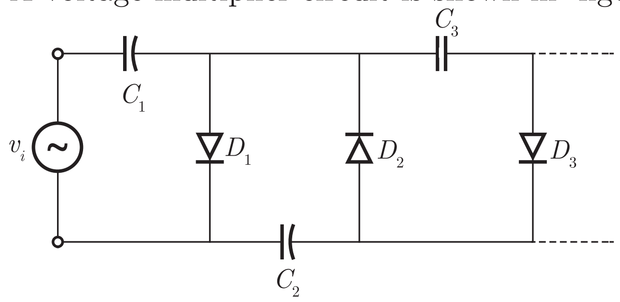
SOL 1.3.7 Option (D) is correct.
The full wave bridge diode rectifier,

$$\%\text{ Voltage regulation} = \frac{2R_f}{R_L} \times 100\%$$

$$= \frac{2 \times 50}{50} \times 100\%$$
$$= 200\%$$

SOL 1.3.8 Option (C) is correct.

A voltage multiplier circuit is shown in figure

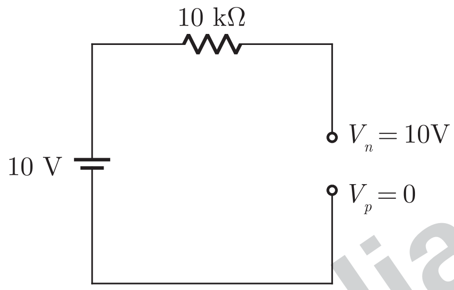



So essential component is capacitor.

SOL 1.3.9 Option (A) is correct.

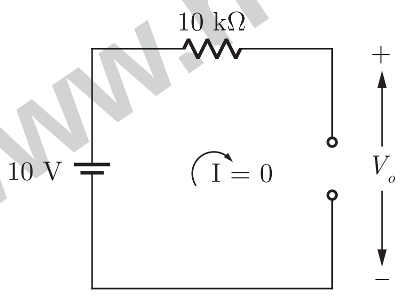
Let us assume diode is OFF.

So the circuit is



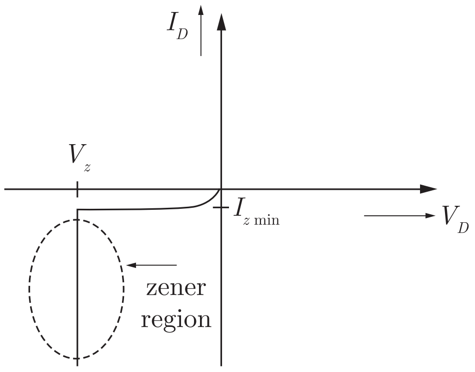

$V_{pn} < 0$, So diode is actually OFF.

So the equivalent circuit is




$$V_o = 10 \text{ V}$$

SOL 1.3.10 Option (B) is correct.




Zener diode can work in the breakdown region without getting damaged. So a Zener diode is always used in Reverse biased.

SOL 1.3.11 Option (D) is correct.

Voltage regulation – it is defined as the % change in regulated output voltage for a change in load current from minimum to maximum value.

$$\% VR = \frac{V_{NL} - V_{FL}}{V_{NL}} \times 100\%$$

So it should be as small as possible for ideal rectifier and filter circuit,

$$\% \text{ Regulation} = 0\%$$

SOL 1.3.12 Option (A) is correct.

From Planck's equation,

$$E = h\nu$$

$$E_g = \frac{hC}{\lambda}$$

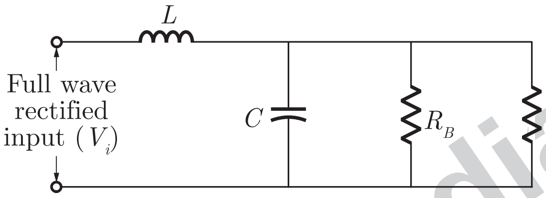
$$E_g = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{5490 \times 10^{-10}}$$

$$= 3.62 \times 10^{-19}\text{ J}$$

$$= 2.26\text{ eV}$$

SOL 1.3.13 Option (B) is correct.

When the load resistance is infinity the value of the inductance will also tend to be infinity to over come this problem, a bleeder resistor $R_B$, is connected in parallel with the load resistance as shown in Fig.




Therefore, a minimum current will always be present for optimum operation of the inductor.

SOL 1.3.14 Option (A) is correct.

The given circuit is half wave rectifier circuit so

$$\text{DC current, } I_{dc} = \frac{I_m}{\pi} = \frac{V_m}{(R_L + R_F)\pi}$$

$$= \frac{V_m}{(5+45)\pi} = \frac{V_m}{50\pi}$$

SOL 1.3.15 Option (B) is correct.

The relation between diode current voltage and temperature is given by

$$I = I_o(e^{V/\eta V_T} - 1)$$

where $I_o$ = reverse saturation current

$V$ = voltage across diode

$V_T$ = temperature equivalent voltage

$$= \frac{KT}{q}$$

SOL 1.3.16 Option (A) is correct.

The transformer utilization factor (TUF) is defined as,

$$\text{TUF} = \frac{\text{dc power delivered to the load}}{\text{ac rating of the transformer secondary}}$$

$$= \frac{P_{dc}}{R_{ac\text{ rated}}}$$

SOL 1.3.17 Option (A) is correct.

Peak inverse voltage (PIV) of an half wave rectifier is $V_m$.

PIV of an full wave rectifier with centre tapped is $2V_m$.

SOL 1.3.18 Option (A) is correct.

For inductor filter,

$$\text{Ripple factor, } r = \frac{R_L}{3\sqrt{2}(\omega L)}$$

So the ripple factor value is low for low value of load resistance and low value of load resistance means for heavy load currents.

SOL 1.3.19 Option (B) is correct.

The minimum value of inductance required to make the current to flow through the circuit at all times is known as critical inductance. It depends on load resistance and supply frequency. It is given by,

$$\text{The critical inductance} \quad L_c = \frac{R_L}{3\omega}$$

SOL 1.3.20 Option (C) is correct.

Transformer utilization factor,

For Half wave rectifier, $\text{TUF} = 0.287$

For Full wave centre tapped rectifier,

$$\text{TUF} = 0.693$$

For Full wave bridge rectifier,

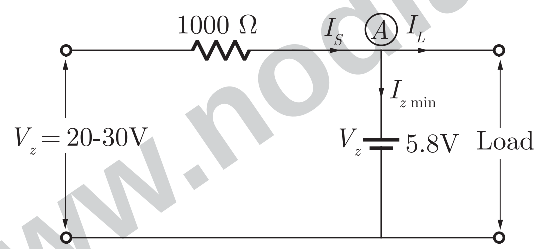
$$\text{TUF} = 0.812$$

So, TUF is more in bridge rectifier.

SOL 1.3.21 Option (A) is correct.

Zener diode is in OFF state. So, it work as voltage regulator.

So, equivalent circuit is,






KCL at node $A$

$$I_S = I_L + I_z$$

For maximum load current,

$$I_{S(\max)} = I_{L(\min)} + I_{z(\min)}$$

$$\frac{V_{i(\max)} - V_z}{R_S} = I_{L(\max)} + 0.5\text{m}$$

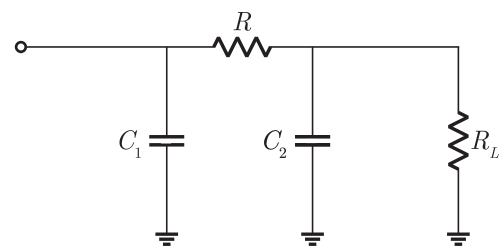
$$I_{L(\max)} = \frac{30 - 5.8}{1000} - 0.5\text{m}$$

$$= 24.2\text{m} - 0.5\text{m} = 23.7 \text{ mAmp}$$

SOL 1.3.22 Option (C) is correct.

SOL 1.3.23 Option (C) is correct.

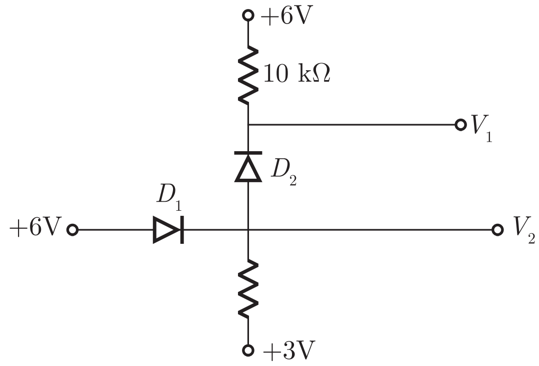
The $\pi$-section filter circuit is,



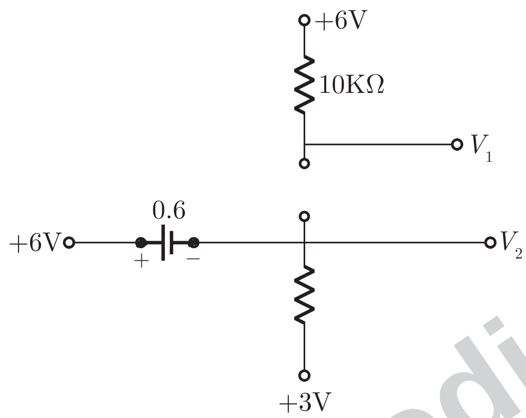

$$\text{At no load,} \quad R_L = \infty$$

So current flow through is zero.

SOL 1.3.24 Option (A) is correct.




The Diode $D_1$ is ON due to 6 V and due to $D_1$ is F.B. $D_2$ is always in R.B. So equivalent circuit is,

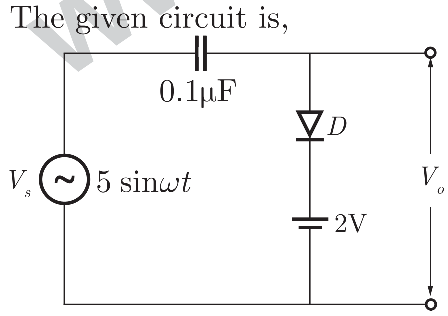



So,

$$V_2 = 6 - 0.6$$
$$= 5.4 \text{ V}$$
$$V_1 = 6 \text{ V}$$

SOL 1.3.25 Option (B) is correct.

The given circuit is,




This is a clamper circuit and the diode is in downward direction the total signal will be clamp below the reference voltage.

SOL 1.3.26 Option (D) is correct.

Clipper circuits, also called limiter circuits are used to eliminate portion of a signal that are above or below a specified level without distorting the remaining part of the alternating waveform.

SOL 1.3.27 Option (D) is correct.

Clamper circuits shifts the entire signal voltage by a dc level.

SOL 1.3.28 Option (C) is correct.

$$I_{ZM} = \frac{P_{ZM}}{V_Z} = \frac{500 \times 10^{-3}}{6.8}$$
$$= 73.5 \times 10^{-3} \text{ A} = 73.5 \text{ mA}$$

SOL 1.3.29 Option (D) is correct.

$$r_Z = \frac{\Delta V_Z}{\Delta I_Z} = \frac{0.05}{10 \times 10^{-3}} = 5\,\Omega$$

SOL 1.3.30 Option (A) is correct.

$$V'_Z = V_Z + I_Z r_Z = 4.7 + (20 \times 10^{-3}) \times 15 = 5\text{ V}$$

SOL 1.3.31 Option (B) is correct.

SOL 1.3.32 Option (D) is correct.

The output of a rectifier consists of d.c. component as well as a.c. component. The a.c. component in the output is called as ripple. So the ripple factor $(\gamma)$ is the ratio of ripple voltage or effective value of a.c. component voltage to the average or d.c. voltage, so it defined the purity of output power.

$$\gamma = \frac{\text{rms value of a.c. component}}{\text{d.c. value of wave}}$$

SOL 1.3.33 Option (B) is correct.

SOL 1.3.34 Option (B) is correct.

SOL 1.3.35 Option (A) is correct.

SOL 1.3.36 Option (D) is correct.

$$V_m = 20\sqrt{2}\text{ V}$$

$$V_{dc} = \frac{V_m}{\pi} = \frac{20\sqrt{2}}{\pi} = 9\text{ V}$$

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Eighth Edition

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MRP 490.00

NODIA & COMPANY
B – 8, Dhanshree Ist, Central Spine, Vidyadhar Nagar, Jaipur – 302039
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Printed by Nodia and Company, Jaipur

# *Preface to the Series*

For almost a decade, we have been receiving tremendous responses from GATE aspirants for our earlier books: GATE Multiple Choice Questions, GATE Guide, and the GATE Cloud series. Our first book, GATE Multiple Choice Questions (MCQ), was a compilation of objective questions and solutions for all subjects of GATE Electronics & Communication Engineering in one book. The idea behind the book was that Gate aspirants who had just completed or about to finish their last semester to achieve his or her B.E/B.Tech need only to practice answering questions to crack GATE. The solutions in the book were presented in such a manner that a student needs to know fundamental concepts to understand them. We assumed that students have learned enough of the fundamentals by his or her graduation. The book was a great success, but still there were a large ratio of aspirants who needed more preparatory materials beyond just problems and solutions. This large ratio mainly included average students.

Later, we perceived that many aspirants couldn't develop a good problem solving approach in their B.E/B.Tech. Some of them lacked the fundamentals of a subject and had difficulty understanding simple solutions. Now, we have an idea to enhance our content and present two separate books for each subject: one for theory, which contains brief theory, problem solving methods, fundamental concepts, and points-to-remember. The second book is about problems, including a vast collection of problems with descriptive and step-by-step solutions that can be understood by an average student. This was the origin of *GATE Guide* (the theory book) and *GATE Cloud* (the problem bank) series: two books for each subject. *GATE Guide* and *GATE Cloud* were published in three subjects only.

Thereafter we received an immense number of emails from our readers looking for a complete study package for all subjects and a book that combines both *GATE Guide* and *GATE Cloud*. This encouraged us to present GATE Study Package (a set of 10 books: one for each subject) for GATE Electronic and Communication Engineering. Each book in this package is adequate for the purpose of qualifying GATE for an average student. Each book contains brief theory, fundamental concepts, problem solving methodology, summary of formulae, and a solved question bank. The question bank has three exercises for each chapter: 1) Theoretical MCQs, 2) Numerical MCQs, and 3) Numerical Type Questions (based on the new GATE pattern). Solutions are presented in a descriptive and step-by-step manner, which are easy to understand for all aspirants.

We believe that each book of GATE Study Package helps a student learn fundamental concepts and develop problem solving skills for a subject, which are key essentials to crack GATE. Although we have put a vigorous effort in preparing this book, some errors may have crept in. We shall appreciate and greatly acknowledge all constructive comments, criticisms, and suggestions from the users of this book. You may write to us at rajkumar.kanodia@gmail.com and ashish.murolia@gmail.com.

Acknowledgements

We would like to express our sincere thanks to all the co-authors, editors, and reviewers for their efforts in making this project successful. We would also like to thank Team NODIA for providing professional support for this project through all phases of its development. At last, we express our gratitude to God and our Family for providing moral support and motivation.

We wish you good luck !
R. K. Kanodia
Ashish Murolia

# SYLLABUS

## GATE Electronics & Communications:

Random signals and noise: probability, random variables, probability density function, autocorrelation, power spectral density. Analog communication systems: amplitude and angle modulation and demodulation systems, spectral analysis of these operations, superheterodyne receivers; elements of hardware, realizations of analog communication systems; signal-to-noise ratio (SNR) calculations for amplitude modulation (AM) and frequency modulation (FM) for low noise conditions. Fundamentals of information theory and channel capacity theorem. Digital communication systems: pulse code modulation (PCM), differential pulse code modulation (DPCM), digital modulation schemes: amplitude, phase and frequency shift keying schemes (ASK, PSK, FSK), matched filter receivers, bandwidth consideration and probability of error calculations for these schemes. Basics of TDMA, FDMA and CDMA and GSM.

## GATE Instrumentation

Signals, Systems and Communications: Periodic and aperiodic signals. Impulse response, transfer function and frequency response of first- and second order systems. Convolution, correlation and characteristics of linear time invariant systems. Discrete time system, impulse and frequency response. Pulse transfer function. IIR and FIR filters. Amplitude and frequency modulation and demodulation. Sampling theorem, pulse code modulation. Frequency and time division multiplexing. Amplitude shift keying, frequency shift keying and pulse shift keying for digital modulation.

## IES Electronics & Telecommunication

Communication Systems: Basic information theory; Modulation and detection in analogue and digital systems; Sampling and data reconstructions; Quantization & coding; Time division and frequency division multiplexing; Equalization; Optical Communication: In free space & fiber optic; Propagation of signals at HF, VHF, UHF and microwave frequency; Satellite Communication.

## IES Electrical

Communication Systems Types of modulation; AM, FM and PM. Demodulators. Noise and bandwidth considerations. Digital communication systems. Pulse code modulation and demodulation. Elements of sound and vision broadcasting. Carrier communication. Frequency division and time division multiplexing, Telemetry system in power engineering.

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## CHAPTER 2 RANDOM PROCESS

## CHAPTER 6 DIGITAL TRANSMISSION

## CHAPTER 9 SPREAD SPECTRUM

# CHAPTER 6

## DIGITAL TRANSMISSION

### 6.1 INTRODUCTION

In this chapter, we will consider the digital transmission of analog messages via pulse modulation. This chapter covers the following topics:

- Sampling, which is basic to digital signal processing and digital communications.
- Pulse amplitude modulation (PAM), which is the simplest form of pulse modulation.
- Quantization, which represents an analog signal in discrete form in both amplitude and time.
- Pulse code modulation (PCM): standard method for the transmission of an analog message signal by digital means.
- Delta modulation (DM), which provides a staircase approximation to the oversampled version of the message signal.
- Digital signaling, which provides the mathematical representation for a digital signal waveform.
- Multiplexing, which enables the joint utilization of common channel by a plurality of independent message sources without mutual interference among them.

### 6.2 SAMPLING PROCESS

The sampling process is usually described in the time domain. In this process, an analog signal is converted into a corresponding sequence of samples that are usually spaced uniformly in time. Consider an arbitrary signal $x(t)$ of finite energy, which is specified for all time as shown in figure 6.1(a).

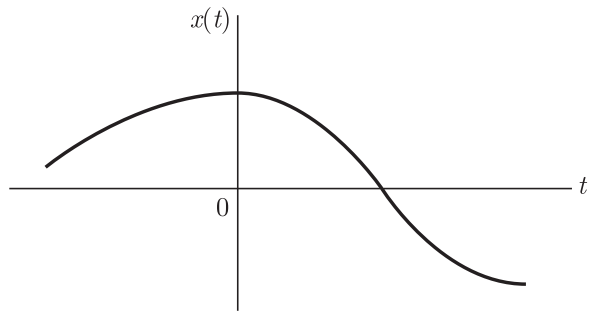
Suppose that we sample the signal $x(t)$ instantaneously and at a uniform rate, once every $T_s$ seconds, as shown in figure 6.1(b). Consequently, we obtain an infinite sequence of samples spaced $T_s$ seconds apart and denoted by $\{x(nT_s)\}$, where $n$ takes on all possible integer values.

Thus, we define the following terms:

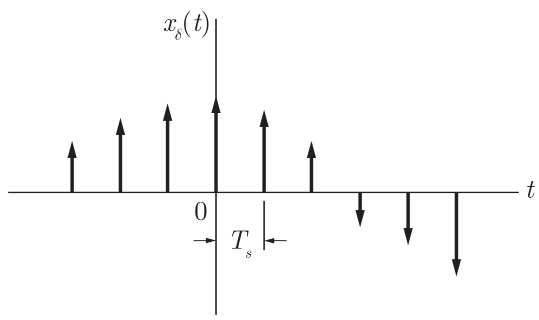
1. **Sampling Period:** The time interval between two consecutive samples is referred as sampling period. In figure 6.1(b), $T_s$ is the sampling period.

2. **Sampling Rate:** The reciprocal of sampling period is referred as sampling rate, i.e.

$$f_s = 1/T_s$$




(a)




(b)

Figure 6.1: Illustration of Sampling Process: (a) Message Signal, (b) Sampled Signal



### 6.2.1 Sampling Theorem

Sampling theorem provides both a method of reconstruction of the original signal from the sampled values and also gives a precise upper bound on the sampling interval required for distortionless reconstruction. It states that

1. A band-limited signal of finite energy, which has no frequency components higher than $W$ Hertz, is completely described by specifying the values of the signal at instants of time separated by $1/2\,\mathrm{W}$ seconds.
2. A band-limited signal of finite energy, which has no frequency components higher that $W$ Hertz, may be completely recovered from a knowledge of its samples taken at the rate of $2\,\mathrm{W}$ samples per second.

### 6.2.2 Explanation of Sampling Theorem

Consider a message signal $m(t)$ bandlimited to $W$, i.e.

$$M(f) = 0 \qquad \text{for } |f| \geq W$$

Then, the sampling Frequency $f_s$, required to reconstruct the bandlimited waveform without any error, is given by

$$f_s \geq 2W$$

### 6.2.3 Nyquist Rate

Nyquist rate is defined as the minimum sampling frequency allowed to reconstruct a bandlimited waveform without error, i.e.

$$f_N = \min\{f_s\} = 2W$$

where $W$ is the message signal bandwidth, and $f_s$ is the sampling frequency.

### 6.2.4 Nyquist Interval

The reciprocal of Nyquist rate is called the Nyquist interval (measured in seconds), i.e.

$$T_N = \frac{1}{f_N} = \frac{1}{2W}$$



where $f_N$ is the Nyquist rate, and $W$ is the message signal bandwidth.

## 6.3 PULSE MODULATION

Pulse modulation is the process of changing a binary pulse signal to represent the information to be transmitted. Pulse modulation can be either analog or digital.

### 6.3.1 Analog Pulse Modulation

Analog pulse modulation results when some attribute of a pulse varies continuously in one-to-one correspondence with a sample value. In analog pulse modulation systems, the amplitude, width, or position of a pulse can vary over a continuous range in accordance with the message amplitude at the sampling instant, as shown in Figure 6.2. These lead to the following three types of pulse modulation:

1. Pulse Amplitude Modulation (PAM)
2. Pulse Width Modulation (PWM)
3. Pulse Position Modulation (PPM)

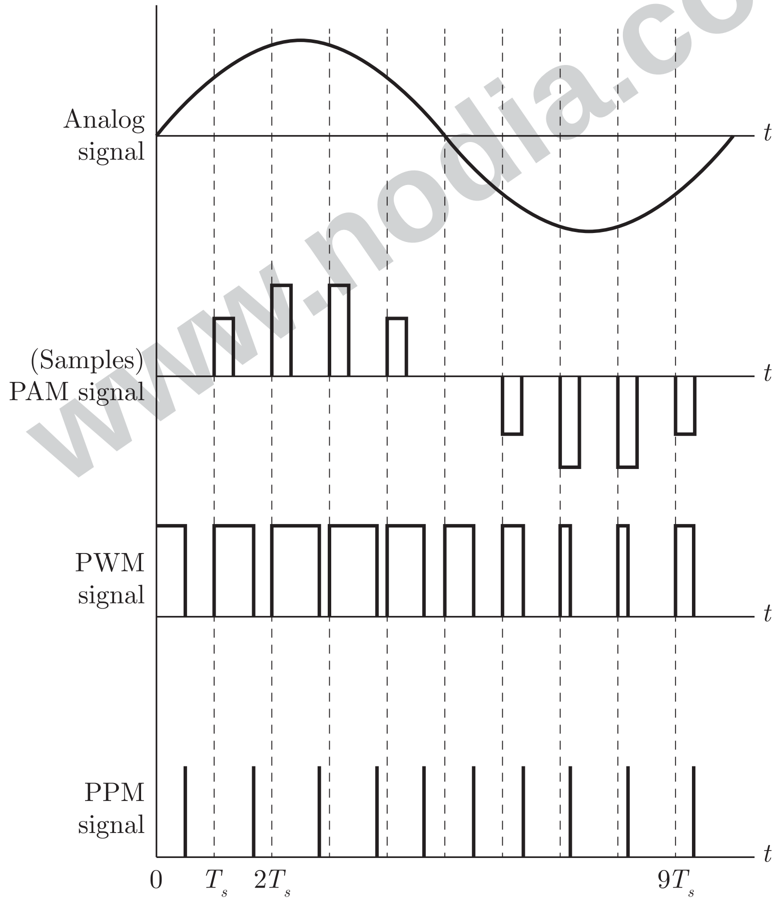



Figure 6.2: Representation of Various Analog Pulse Modulation

### 6.3.2 Digital Pulse Modulation

In systems utilizing digital pulse modulation, the transmitted samples take on only discrete values. Two important types of digital pulse modulation are:

1. Delta Modulation (DM)
2. Pulse Code Modulation (PCM)

In the following sections, we will discuss the various types of pulse modulation in some more detail.

## 6.4 PULSE AMPLITUDE MODULATION

Pulse amplitude modulation (PAM) is the conversion of the analog signal to a pulse-type signal in which the amplitude of the pulse denotes the analog information. PAM system utilizes two types of sampling: (1) Natural sampling and (2) Flat-top sampling.

### 6.4.1 Natural Sampling (Gating)

Consider an analog waveform $m(t)$ bandlimited to $W$ hertz, as shown in Figure 6.3(a). The PAM signal that uses natural sampling (gating) is defined as

$$m_s(t) = m(t)s(t)$$

where $s(t)$ is the pulse waveform shown in Figure 6.3(b), and $m_s(t)$ is the resulting PAM signal shown in Figure 6.3(c)

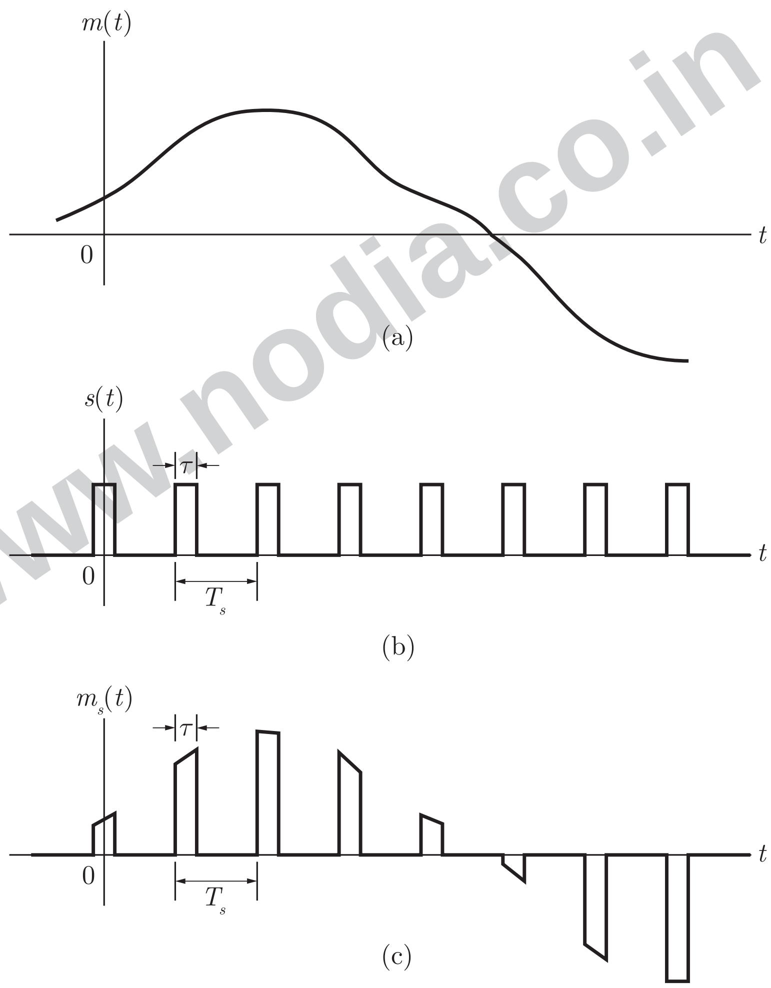





Figure 6.3: Illustration of Nautural Sampling Pulse Amplitude Modulation: (a) Message Signal, (b) Pulse Waveform, (c) Resulting PAM Signal

### 6.4.2 Instantaneous Sampling (Flat-Top PAM)

Analog waveforms may also be converted to pulse signalling by the use of flat-top signalling with instantaneous sampling, as shown in Figure 6.4. If $m(t)$ is an analog waveform bandlimited to $W$ hertz, the instantaneous sampled PAM signal is given by

$$m_s(t) = \sum_{k=-\infty}^{\infty} m(kT_s)h(t-kT_s)$$

where $h(t)$ denotes the sampling-pulse shape shown in Figure 6.4(b), and $m_s(t)$ is the resulting flat top PAM signal shown in Figure 6.4(c)

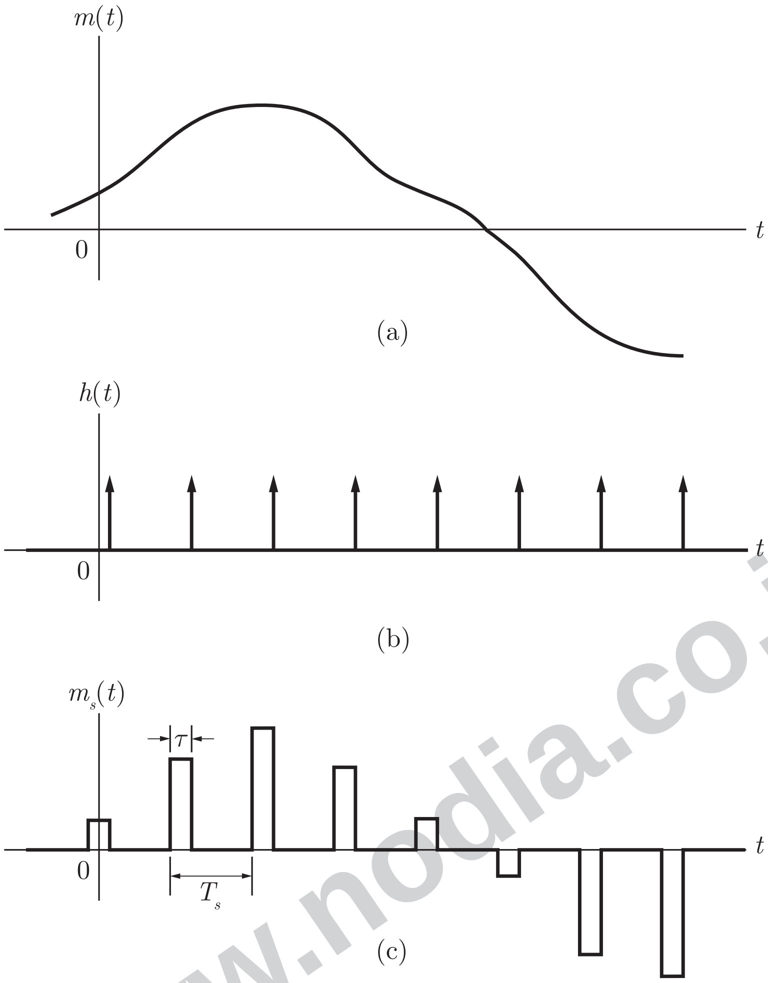





Figure 6.3: Illustration of Flat-top Sampling Pulse Amplitude Modulation: (a) Message Signal, (b) Sampling Pulse, (c) Resulting PAM Signal

POINT TO REMEMBER

The analog-to-PAM conversion process is the first step in converting an analog waveform to a PCM (digital) signal.

## 6.5 PULSE CODE MODULATION

Pulse code modulation (PCM) is essentially analog-to-digital conversion of a special type where the information contained in the instantaneous samples of an analog signal is represented by digital words in a serial bit stream. Figure 6.5 shows the basic elements of a PCM system. The PCM signal is generated by carrying out the following three basic operations:

1. Sampling
2. Quantizing
3. Encoding

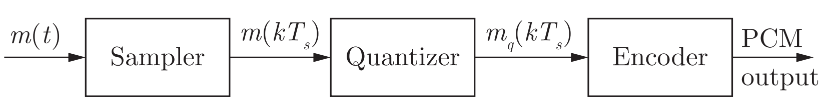



Figure 6.4 : Block Diagram Representation of PCM System

### 6.5.1 Sampling

The incoming message signal $m(t)$ is sampled with a train of narrow rectangular pulses so as to closely approximate the instantaneous sampling process. To ensure perfect reconstruction of the message signal at the receiver, the sampling rate must be greater than twice the highest frequency component $W$ of the message signal in accordance with the sampling theorem. The resulting sampled waveform $m(kT_s)$ is discrete in time.

**APPLICATION OF SAMPLING**

The application of sampling permits the reduction of the continuously varying message signal (of some finite duration) to a limited number of discrete values per second.

### 6.5.2 Quantization

A quantizer rounds off the sample values to the nearest discrete value in a set of $q$ quantum levels. The resulting quantized samples $m_q(kT_s)$ are discrete in time (by virtue of sampling) and discrete in amplitude (by virtue of quantizing). Basically, quantizers can be of a uniform or nonuniform type.

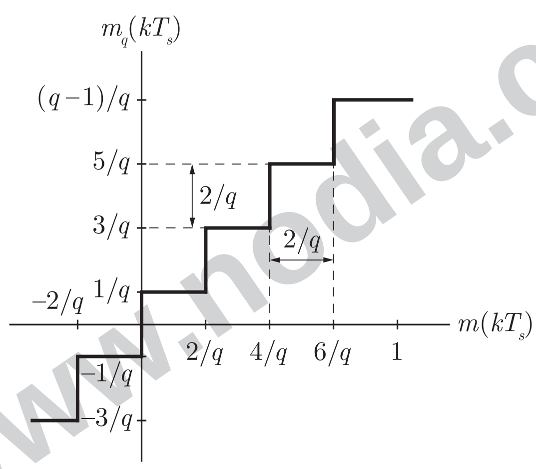



Figure 6.6: Representation of Relationship Between Sampled and Quantized Signal

#### Uniform Quantizer

A quantizer is called as a uniform quantizer if the step size remains constant throughout the input range. To display the relationship between $m(kT_s)$ and $m_q(kT_s)$, let the analog message be a voltage waveform normalized such that $|m(t)| \le 1\,V$. Uniform quantization subdivides the 2 V peak-to-peak range into $q$ equal steps of height $2/q$, as shown in Figure 6.6. The quantum levels are then taken to be at $\pm 1/q$, $\pm 3/q$,..., $\pm(q-1)/q$ in the usual case when $q$ is an even integer. A quantized value such as $m_q(kT_s) = 5/q$ corresponds to any sample value in the range $4/q < x(xT_s) < 6/q$.

#### Nonuniform Quantizer

Nonuniform quantization is required to be implemented to improve the signal to quantization noise ratio of weak signals. It is equivalent to passing the baseband signal through a compressor and then applying the compressed signal to a uniform quantizer. A particular form of compression law that is used in practice is the so called $\mu$-law, which is defined by

$$|m_q| = \frac{\ln(1+\mu|m|)}{\ln(1+\mu)}$$

where $m$ and $m_q$ are the normalized input and output voltages, and $\mu$ is a positive constant.

### 6.5.3 Encoding

An encoder translates the quantized samples into digital code words. The encoder works with $M$-ary digits and produces for each sample a code word consisting of $n$ digits in parallel. Since, there are $M^n$ possible $M$-ary codewords with $n$ digits per word, unique encoding of the $q$ different quantum levels requires that

$$M^n \geq q$$

The parameters $M$, $n$, and $q$ should be chosen to satisfy the equality, so that

$$q = M^n \quad \text{or} \quad n = \log_M q$$

#### Encoding in Binary PCM

For binary PCM, each digit may be either of two distinct values 0 or 1, i.e.

$$M = 2$$

If the code word of binary PCM consists of $n$ digits, then number of quantization levels is defined as

$$q = 2^n$$

or $$n = \log_2 q$$

In general, we must remember the following characteristics of a PCM system:

CHARACTERISTICS OF PCM SYSTEM

1. A sampled waveform is quantized into $q$ quantization levels; where $q$ is an integer.
2. If the message signal is defined in the range $(-m_p, m_p)$, then the step size of quantizer is
$$\delta = \frac{2m_p}{q}$$
3. For a binary PCM system with $n$ digit codes, the number of quantization level is defined as
$$q = 2^n$$
4. If the message signal is sampled at the sampling rate $f_s$, and encoded to $n$ number of bits per sample; then the bit rate (bits/sec) of the PCM is defined as
$$R_b = nf_s$$

METHODOLOGY 1 : TO EVALUATE BIT RATE FOR PCM SYSTEM

If the number of quantization levels $q$ and message signal frequency $f_m$ for a PCM signal is given, then bit rate for the PCM system is obtained in the following steps:

Step 1: Obtain the sampling frequency for the PCM signal. According to Nyquist criterion, the minimum sampling frequency is given by
$$f_s = 2f_m$$

Step 2: Deduce the number of bits per sample using the expression

$$n = \log_2 q$$

Step 3: Evaluate bit rate (bits/sec) for the PCM system by substituting the obtained values in the expression

$$R_b = nf_s$$

## 6.6 TRANSMISSION BANDWIDTH IN A PCM SYSTEM

The bandwidth of (serial) binary PCM waveforms depends on the bit rate and the waveform pulse shape used to represent the data. The dimensionality theorem shows that the bandwidth of the binary encoded PCM waveform is bounded by

$$B_{PCM} \geq \frac{1}{2}R_b = \frac{1}{2}nf_s$$

where $R_b$ is the bit rate, $n$ is the number of bits in PCM word, and $f_s$ is the sampling rate. Since, the required sampling rate for no aliasing is

$$f_s \geq 2W$$

where $W$ is the bandwidth of the message signal (that is to be converted to the PCM signal). Thus, the bandwidth of the PCM signal has a lower bound given by

$$B_{PCM} \geq nW$$

**POINT TO REMEMBER**

The minimum bandwidth of $\frac{1}{2}R = \frac{1}{2}nf_s$ is obtained only when $(\sin x)/x$ type pulse shape is used to generate the PCM waveform. However, usually a more rectangular type of pulse shape is used, and consequently, the bandwidth of the binary-encoded PCM waveform will be larger than this minimum. Thus, for rectangular pulses, the first null bandwidth is

$$B_{PCM} = R = nf_s \text{ (first null bandwidth)}$$

## 6.7 NOISE CONSIDERATION IN PCM

In PCM (pulse code modulation), there are two error sources:

1. Quantization noise
2. Channel noise

### 6.7.1 Quantization Noise

For a PCM system, the $k$th sample of quantized message signal is represented by

$$m_q(kT_s) = m(kT_s) + \varepsilon(kT_s)$$

where $m(kT_s)$ is the sampled waveform, and $\varepsilon(kT_s)$ is the quantization error. Let the quantization levels having uniform step size $\delta$. Then, we have

$$-\frac{\delta}{2} \leq \varepsilon \leq \frac{\delta}{2}$$

So, the mean-square error due to quantization is

$$\overline{\varepsilon^2} = \frac{1}{\delta}\int_{-\delta/2}^{\delta/2}\varepsilon^2 d\varepsilon = \frac{\delta^2}{12} \qquad \text{...(6.1)}$$

METHODOLOGY 2 : TO EVALUATE BIT RATE FOR PCM SYSTEM

For a PCM system, consider the message signal having frequency $f_m$ and peak to peak amplitude $2m_p$. If the accuracy of the PCM system is given as $\pm x$ % of full scale value, then the bit rate is obtained in the following steps:

Step 1 : Obtain the sampling frequency for the PCM signal. According to Nyquist criterion, the minimum sampling frequency is given by

$$f_s = 2f_m$$

Step 2 : Obtain the maximum quantization error for the PCM system using the expression

$$|\text{error}| = \left|\frac{\delta}{2}\right| = \left|\frac{2m_p}{2q}\right| = \left|\frac{m_p}{q}\right| = \left|\frac{m_p}{2^n}\right|$$

Step 3 : Apply the given condition of accuracy as

$$|\text{error}| \le x\% \text{ of full scale value}$$

Step 4 : Solve the above condition for the minimum value of number of bits per second $(n)$.

Step 5 : Obtain the bit rate by substituting the approximated integer value of $n$ in the expression

$$R_b = nf_s$$



### 6.7.2 Signal to Quantization Noise Ratio

For PCM system, we have the message signal $m(t)$, and quantization error $\varepsilon$. So, we define the signal to quantization noise ratio as

$$(\text{SNR})_Q = \frac{\overline{m^2(t)}}{\overline{\varepsilon^2}} = \frac{\overline{m^2(t)}}{\delta^2/12} \qquad ...(6.2)$$

where $\delta$ is the step size of the quantized signal defined as

$$\delta = \frac{2m_p}{q} \qquad ...(6.3)$$

Substituting equation (6.3) in equation (6.2), we get the expression for signal to quantization noise ratio as

$$(\text{SNR})_Q = 12\frac{\overline{m^2(t)}}{(2m_p/q)^2}$$

$$(\text{SNR})_Q = 3q^2\frac{\overline{m^2(t)}}{m_p^2} \qquad ...(6.4)$$

where $m_p$ is the peak amplitude of message signal $m(t)$, and $q$ is the number of quantization level. Let us obtain the more generalized form of SNR for the following two cases:

Case I :

When $m(t)$ is a sinusoidal signal, we have its mean square value

$$\overline{m^2(t)} = \frac{1}{2}$$

and the peak amplitude of sinusoidal message signal is

$$m_p = 1$$

So, by substituting these values in equation (6.4), we get the signal to quantization noise ratio for sinusoidal message signal as

$$(\text{SNR})_Q = 3q^2\frac{1/2}{(1)^2} = \frac{3q^2}{2}$$

Case II :

When $m(t)$ is uniformly distributed in the range $(-m_p, m_p)$, then we obtain

$$\overline{m^2(t)} = \frac{m_p^2}{3}$$

Substituting this value in equation (6.4), we get the signal to quantization noise ratio as

$$(\text{SNR})_Q = 3q^2\frac{m_p^2/3}{m_p^2} = q^2$$

Case III:

For any arbitrary message signal $m(t)$, the peak signal to quantization noise ratio is defined as

$$(\text{SNR})_{\text{peak}} = 3q^2\frac{m_p^2}{m_p^2} = 3q^2$$

### 6.7.3 Channel Noise

If a PCM signal is composed of the data that are transmitted over the channel having bit error rate $P_e$, then peak signal to average quantization noise ratio is defined as

$$(\text{SNR})_{\text{peak}} = \frac{3q^2}{1+4(q^2-1)P_e}$$

Similarly, for the channel with bit error probability $P_e$, the average signal to average quantization noise ratio is defined as

$$(\text{SNR})_{\text{ave}} = \frac{q^2}{1+4(q^2-1)P_e}$$



POINT TO REMEMBER

If the additive noise in the channel is so small that errors can be neglected, quantization is the only error source in PCM system.

### 6.7.4 Companding

Companding is nonuniform quantization. It is required to be implemented to improve the signal to quantization noise ratio of weak signals. The signal to quantization noise ratio for $\mu$-law companding is approximated as

$$(\text{SNR})_Q = \frac{3q^2}{[\ln(1+\mu)]^2}$$

where $q$ is the number of quantization level, and $\mu$ is a positive constant.

## 6.8 ADVANTAGES OF PCM SYSTEM

PCM is very popular because of the many advantages it offers, including the following:

1. Relatively inexpensive digital circuitry may be used extensively in the system.
2. PCM signals derived from all types of analog sources (audio, video, etc.) may be merged with data signals (e.g., from digital computers) and transmitted over a common high-speed digital communication system.

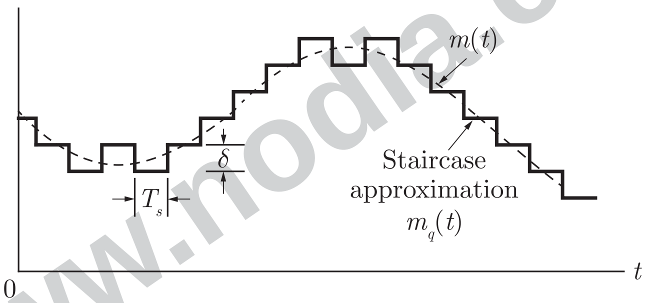
3. In long-distance digital telephone systems requiring repeaters, a clean PCM waveform can be regenerated at the output of each repeater, where the input consists of a noisy PCM waveform.
4. The noise performance of a digital system can be superior to that of an analog system.

POINT TO REMEMBER

The advantages of PCM usually outweigh the main disadvantage of PCM: a much wider bandwidth than that of the corresponding analog signal.

## 6.9 DELTA MODULATION

Delta modulation provides a staircase approximation to the oversampled version of the message signal, as illustrated in Figure 6.7. Let $m(t)$ denote the input (message) signal, and $m_q(t)$ denote its staircase approximation. The difference between the input and the approximation is quantized into only two levels, namely, $\pm\delta$, corresponding to positive and negative differences. Thus if the approximation falls below the signal at any sampling approach, it is increased by $\delta$. If on the other hand, the approximation lies above the signal, it is diminished by $\delta$.




Binary sequence at modulator output  0 0 1 0 1 1 1 1 1 0 1 0 0 0 0 0 0

Figure 6.7: Staircase Approximation in Delta Modulation

Following are some key points related to delta modulation.

POINTS TO REMEMBER

1. In delta modulation (DM), an incoming message signal is oversampled (i.e. at a rate much higher than the Nyquist rate) to purposely increase the correlation between adjacent samples of the signal
2. The staircase approximation remains within $\pm\delta$ of the input signal provided that the signal does not change too rapidly from sample to sample.

### 6.9.1 Noise Consideration in Delta Modulation

The quantizing noise error in delta modulation can be classified into two types of noise:

1. Slope Overload Noise
2. Granular Noise

1. Slope Overload Noise

Slope overload noise occurs when the step size $\delta$ is too small for the accumulator output to follow quick changes in the input waveform. The maximum slope that can be generated by the accumulator output is

$$\frac{\delta}{T_s} = \delta f_s$$

where $T_s$ is sampling interval, and $f_s$ is the sampling rate. To prevent the slope overload noise, the maximum slope of the message signal must be less than maximum slope generated by accumulator. Thus, we have the required condition to avoid slope overload as

$$\max\left|\frac{dm(t)}{dt}\right| \le \delta f_s$$

where $m(t)$ is the message signal, $\delta$ is the step size of quantized signal, and $f_s$ is the sampling rate.

2. Granular Noise

The granular noise in a DM system is similar to the granular noise in a PCM system. From equation (6.1), we have the total quantizing noise for PCM system,

$$(\overline{\varepsilon^2})_{\text{PCM}} = \frac{1}{\delta}\int_{-\delta/2}^{\delta/2}\varepsilon^2 d\varepsilon = \frac{\delta^2}{12} = \frac{(\delta/2)^2}{3}$$

Replacing $\delta/2$ of PCM by $\delta$ for DM, we obtain the total granular quantizing noise as

$$(\overline{\varepsilon^2})_{\text{DM}} = \frac{\delta^2}{3}$$

Thus, the power spectral density for granular noise in delta modulation system is obtained as

$$S_N(f) = \frac{\delta^2/3}{2f_s} = \frac{\delta^2}{6f_s}$$

where $\delta$ is the step size, and $f_s$ is the sampling frequency.



POINT TO REMEMBER

Granular noise occurs for any step size but is smaller for a small step size. Thus we would like to have $\delta$ as small as possible to minimize the granular noise.

METHODOLOGY : MINIMUM STEP SIZE IN DELTA MODULATION

Following are the steps involved in determination of minimum step size to avoid slope overload in delta modulation:

Step 1: Obtain the sampling frequency for the modulation. According to Nyquist criterion, the minimum sampling frequency is given by

$$f_s = 2f_m$$

Step 2: Obtain the maximum slope of message signal using the expression

$$\max\left|\frac{dm(t)}{dt}\right| = 2\pi f_m A_m$$

where $f_m$ is the message signal frequency, and $A_m$ is amplitude of the message signal.

Step 3: Apply the required condition to avoid slope overload as

$$\delta f_s \geq \max\left|\frac{dm(t)}{dt}\right|$$

Step 4: Evaluate the minimum value of step size $\delta$ by solving the above condition.

## 6.10 MULTILEVEL SIGNALING

In a multilevel signaling scheme, the information source emits a sequence of symbols from an alphabet that consists of $M$ symbols (levels). Let us understand some important terms used in multilevel signaling.

### 6.10.1 Baud

Let a multilevel signaling scheme having the symbol duration $T_s$ seconds. So, we define the symbols per second transmitted for the system as

$$D = \frac{1}{T_s}$$

where $D$ is the symbol rate which is also called *baud*.

### 6.10.2 Bits per Symbol

For a multilevel signaling scheme with $M$ number of symbols (levels), we define the bits per symbol as

$$k = \log_2 M$$

### 6.10.3 Relation Between Baud and Bit Rate

For a multilevel signaling scheme, the bit rate and baud (symbols per second) are related as

$$R_b = kD = D\log_2 M \quad \text{...(6.5)}$$

where $R_b$ is the bit rate, $k = \log_2 M$ is the bits per symbol, and $D$ is the baud (symbols per second).

### 6.10.4 Relation Between Bit Duration and Symbol Duration

For a multilevel signaling scheme, the bit duration is given by

$$T_b = \frac{1}{R_b}$$

where $R_b$ is the bit rate. Also, we have the symbol duration

$$T_s = \frac{1}{D}$$

where $D$ is the symbol rate. Thus, by substituting these expressions in equation (6.5), we get the relation

$$T_s = kT_b = T_b\log_2 M$$

where $k = \log_2 M$ is the bits per symbol.

### 6.10.5 Transmission Bandwidth

The null to null transmission bandwidth of the rectangular pulse multilevel waveform is defined as

$$B_T = D \text{ symbols/sec}$$

The absolute transmission bandwidth for $\frac{\sin x}{x}$ pulse multilevel waveform is defined as

$$B_T = \frac{D}{2} \text{ symbols/sec}$$

## 6.11 MULTIPLEXING

In many applications, a large number of data sources are located at a common point, and it is desirable to transmit these signals simultaneously using a single communication channel. This is accomplished using multiplexing. Let us study the two important types of multiplexing: FDM and TDM.

### 6.11.1 Frequency-Division Multiplexing (FDM)

Frequency-division multiplexing (FDM) is a technique whereby several message signals are translated, using modulation, to different spectral locations and added to form a baseband signal. The carriers used to form the baseband are usually referred to as subcarriers. Then, if desired, the baseband signal can be transmitted over a single channel using a single modulation process.

#### Bandwidth of FDM Baseband Signal

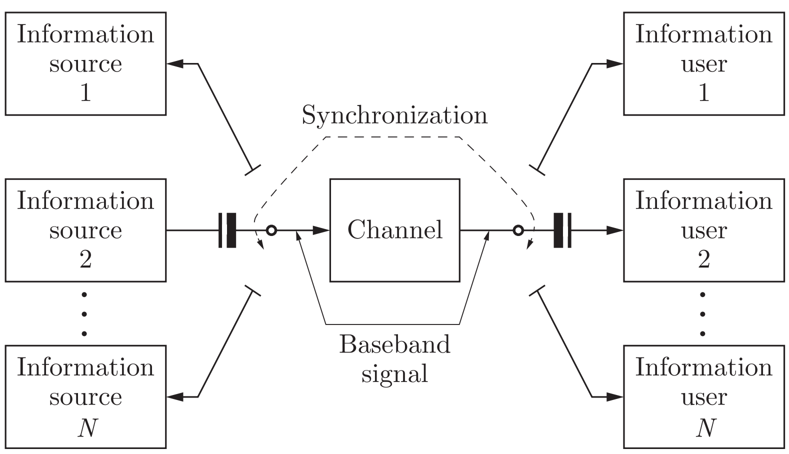
The bandwidth of FDM baseband is equal to the sum of the bandwidths of the modulated signals plus the sum of the guardbands, the empty spectral bands between the channels necessary for filtering. This bandwidth is lower-bounded by the sum of the bandwidths of the message signals, i.e.

$$B = \sum_{i=1}^{N} W_i$$

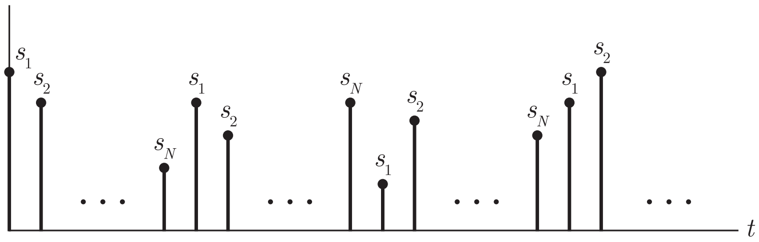
where $W_i$ is the bandwidth of $m_i(t)$, This bandwidth is achieved when all baseband modulators are SSB and all guardbands have zero width.

### 6.11.2 Time Division Multiplexing (TDM)

Time-division multiplexing provides the time sharing of a common channel by a large number of users. Figure 6.8(a) illustrates a TDM system. The data sources are assumed to have been sampled at the Nyquist rate or higher. The commutator then interlaces the samples to form the baseband signal shown in Figure 6.8(b). At the channel output, the baseband signal is demultiplexed by using a second commutator as illustrated. Proper operation of this system depends on proper synchronization between the two commutators.




(a)



(b)

Figure 6.8: (a) Time Division Multiplexing System, (b) Resulting Baseband Signal

In a TDM system, the samples are transmitted depending on the message signal bandwidth. For example, let us consider the following two cases:

1. If all message signals have equal bandwidth, then the samples are transmitted sequentially, as shown in Figure 6.8(b).
2. If the sampled data signals have unequal bandwidths, more samples must be transmitted per unit time from the wideband channels. This is easily accomplished if the bandwidths are harmonically related. For example, assume that a TDM system has four data sources $s_1(t)$, $s_2(t)$, $s_3(t)$, and $s_4(t)$ having the bandwidths respectively as $W, W, 2W, 4W$. Then, it is easy to show that a permissible sequence of baseband samples is a periodic sequence, one period of which is $...s_1s_4s_3s_4s_2s_4....$

### Bandwidth of TDM Baseband Signal

The minimum sampling bandwidth of a TDM baseband signal is defined as

$$B = \sum_{i=1}^{N} W_i$$

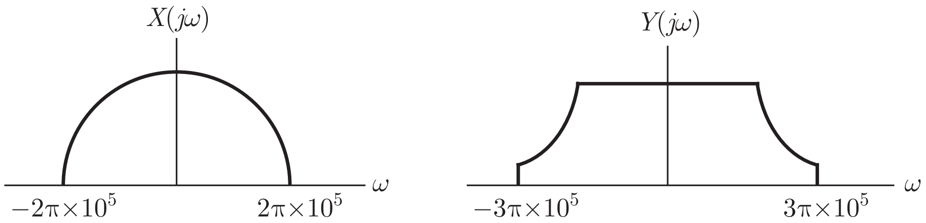
where $W_i$ is the bandwidth of the $i$th channel.

***********

# EXERCISE 6.1

## Common Data For Q. 1 to 5 :

Figure given below shows Fourier spectra of signal $x(t)$ and $y(t)$.




MCQ 6.1.1 The Nyquist sampling rate for $x(t)$ is

(A) 100 kHz (B) 200 kHz

(C) 300 kHz (D) 50 kHz

MCQ 6.1.2 The Nyquist sampling rate for $y(t)$ is

(A) 50 kHz (B) 75 kHz

(C) 150 kHz (D) 300 kHz

MCQ 6.1.3 The Nyquist sampling rate for $x^2(t)$ is

(A) 100 kHz (B) 150 kHz

(C) 250 kHz (D) 400 kHz

MCQ 6.1.4 The Nyquist sampling rate for $y^3(t)$ is

(A) 100 kHz (B) 300 kHz

(C) 900 kHz (D) 120 kHz

MCQ 6.1.5 The Nyquist sampling rate for $x(t)y(t)$ is

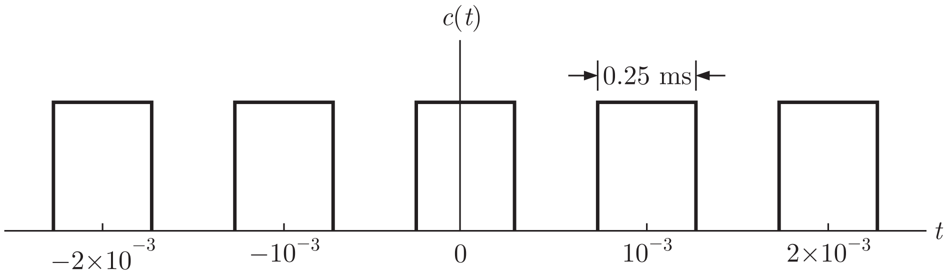
(A) 250 kHz (B) 500 kHz

(C) 50 kHz (D) 100 kHz



## Common Data For Q. 6 and 7

A signal $x(t)$ is multiplied by rectangular pulse train $c(t)$ shown in figure.




MCQ 6.1.6 $x(t)$ would be recovered from the product $x(t)c(t)$ by using an ideal LPF if $X(j\omega)=0$ for

(A) $\omega > 2000\pi$ (B) $\omega > 1000\pi$
(C) $\omega < 1000\pi$ (D) $\omega < 2000\pi$

MCQ 6.1.7 If $X(j\omega)$ satisfies the constrains required, then the pass band gain $A$ of the ideal lowpass filter needed to recover $x(t)$ from $c(t)x(t)$ is
(A) 1 (B) 2
(C) 4 (D) 8

## Common Data For Q. 8 and 9 :

Ten telemetry signals, each of bandwidth 2 kHz, are to be transmitted simultaneously by binary PCM. The maximum tolerable error in sample amplitudes is 0.2% of the peak signal amplitude. The signals must be sampled at least 20% above the Nyquist rate. Framing and synchronizing requires an additional 1% extra bits.

MCQ 6.1.8 The minimum possible data rate must be
(A) 272.64 k bits/sec
(B) 436.32 k bits/sec
(C) 936.32 k bits/sec
(D) None of the above

MCQ 6.1.9 The minimum transmission bandwidth is
(A) 218.16 kHz (B) 468.32 kHz
(C) 136.32 kHz (D) None of the above

MCQ 6.1.10 A binary channel with capacity 36 k bits/sec is available for PCM voice transmission. If signal is band limited to 3.2 kHz, then the appropriate values of quantizing level and the sampling frequency will be, respectively
(A) 32, 3.6 kHz (B) 64, 7.2 kHz
(C) 64, 3.6 kHz (D) 32, 7.2 kHz

## Common Data For Q. 11 to 13 :

Consider a linear DM system designed to accommodate analog message signals limited to bandwidth of 3.5 kHz. A sinusoidal test signal of amplitude $A_m = 1$ V and frequency $f_m = 800$ Hz is applied to system. The sampling rate of the system is 64 kHz.

MCQ 6.1.11 The minimum value of the step size to avoid slope overload is
(A) 240 mV (B) 120 mV
(C) 670 mV (D) 78.5 mV

MCQ 6.1.12 The granular-noise power would be
(A) $1.68 \times 10^{-3}$ W (B) $2.86 \times 10^{-4}$ W
(C) $2.48 \times 10^{-3}$ W (D) $1.12 \times 10^{-4}$ W

MCQ 6.1.13 The SNR will be
(A) 298 (B) $1.75 \times 10^{-3}$
(C) $4.46 \times 10^{3}$ (D) 201

## Common Data For Q. 14 and 15 :

A Signal has a bandwidth of 1 MHz. It is sampled at a rate 50% higher than the Nyquist rate and quantized into 256 level using a $\mu$-law quantizer with $\mu = 255$.

MCQ 6.1.14 The signal-to-quantization-noise ratio is

(A) 34.91 dB (B) 38.06 dB

(C) 42.05 dB (D) 48.76 dB

MCQ 6.1.15 It was found that a sampling rate 20% above the rate would be adequate. So the maximum SNR, that can be realized without increasing the transmission bandwidth, would be

(A) 64.4 dB (B) 70.3 dB

(C) 50.1 dB (D) None of the above

MCQ 6.1.16 The input to a linear delta modulator having step-size $\delta = 0.628$ is a sine wave with frequency $f_m$ and peak amplitude $A_m$. If the sampling frequency $f_s = 40$ kHz, the combination of the sine wave frequency and the peak amplitude, where slope overload will take place is

| | $A_m$ | $f_m$ |
|---|---|---|
| (A) | 0.3 V | 8 kHz |
| (B) | 1.5 V | 4 kHz |
| (C) | 1.5 V | 2 kHz |
| (D) | 3.0 V | 1 kHz |

MCQ 6.1.17 A sinusoidal signal is sampled at 8 kHz and is quantized using 8 bit uniform quantizer. If $\text{SNR}_q$ be the quantization signal to noise ratio and $R$ be the bit rate of PCM signal, which of the following provides the correct values of $\text{SNR}_q$ and $R$ ?

(A) $R = 32$ kbps, $\text{SNR}_q = 25.8$ dB

(B) $R = 64$ kbps, $\text{SNR}_q = 49.8$ dB

(C) $R = 64$ kbps, $\text{SNR}_q = 55.8$ dB

(D) $R = 32$ kbps, $\text{SNR}_q = 49.8$ dB

MCQ 6.1.18 A 1.0 kHz signal is flat-top sampled at the rate of 1800 samples/sec and the samples are applied to an ideal rectangular LPF with cut-off frequency of 1100 Hz. The output of the filter contains

(A) only 800 Hz component

(B) 800 and 900 Hz component

(C) 800 Hz and 1000 Hz components

(D) 800 Hz, 900 and 1000 Hz components

MCQ 6.1.19 A signal $x(t) = 100\cos(24\pi \times 10^3)t$ is ideally sampled with a sampling period of 50 $\mu$sec and then passed through an ideal lowpass filter with cutoff frequency of 15 kHz. Which of the following frequencies is/are present at the filter output

(A) 12 kHz only (B) 8 kHz only

(C) 12 kHz and 9 kHz (D) 12 kHz and 8 kHz

MCQ 6.1.20 The minimum sampling frequency (in samples/sec) required to reconstruct the following signal form its samples without distortion would be

$$x(t) = 5\left(\frac{\sin 2\pi 1000t}{\pi t}\right)^3 + 7\left(\frac{\sin 2\pi 1000t}{\pi t}\right)^2$$

(A) $2 \times 10^3$ Hz (B) $4 \times 10^3$ Hz
(C) $6 \times 10^3$ Hz (D) $8 \times 10^3$ Hz

MCQ 6.1.21 The minimum step-size required for a Delta-Modulator operating at 32 K samples/sec to track the signal (here $u(t)$ is the unit function)

$$m(t)$$
$$= 125t[u(t) - u(t-1)] + (250 - 125t)[u(t-1) - u(t-2)]$$

so that slope overload is avoided, would be
(A) $2^{-10}$ (B) $2^{-8}$
(C) $2^{-6}$ (D) $2^{-4}$

MCQ 6.1.22 If the number of bits in a PCM system is increased from $n$ to $n+1$, the signal-to-quantization noise ratio will increase by a factor.
(A) $\frac{(n+1)}{n}$ (B) $\frac{(n+1)^2}{n^2}$
(C) 2 (D) 4

MCQ 6.1.23 In PCM system, if the quantization levels are increased form 2 to 8, the relative bandwidth requirement will.
(A) remain same
(B) be doubled
(C) be tripled
(D) become four times



Common Data For Q. 24 to 26 :

A singer's performance is to be recorded by sampling and storing the sample values. Assume that the highest frequency tone to be recorded is 15800 hertz.

MCQ 6.1.24 What is the minimum sampling frequency that can be used ?
(A) $3.16 \times 10^4$ Hz (B) $15.8 \times 10^4$ Hz
(C) $7.9 \times 10^3$ Hz (D) $6.32 \times 10^4$ Hz

MCQ 6.1.25 How many samples would be required to store a three minutes performance ?
(A) $1.053 \times 10^4$ samples
(B) $5.688 \times 10^6$ samples
(C) $1.756 \times 10^6$ samples
(D) $9.48 \times 10^4$ samples

MCQ 6.1.26 If each sample is quantized into 128 levels, how many binary digits (bits) would be required to store the three minutes performance ?
(A) $8.125 \times 10^5$ bits (B) $7.28 \times 10^8$ bits
(C) $3.98 \times 10^7$ bits (D) $5.688 \times 10^6$ bits

**Common Data For Q. 27 and 28 :**

A voltage of 1 volt rms is applied at the sending end of a telephone cable of length 1000 meters. The attenuation in the cable is 1 dB/m.

MCQ 6.1.27 What will be the rms voltage at receiving end of the cable ?

(A) $10^{50}$ volt  (B) $10^{-50}$ volt

(C) $10^{5}$ volt  (D) $10^{-5}$ volt

MCQ 6.1.28 It is desired to obtain a 1 volt rms signal at the receiving end. This can be accomplished by using repeaters. If the repeaters yield a maximum output of 1 volt and have a voltage gain of 100, then the number of repeaters needed is

(A) 100  (B) 25

(C) 5  (D) 40

**Common Data For Q. 29 to 31 :**

The probability density of a signal $X(t)$ is given by

$$f_{X(t)}(x) = \begin{cases} ke^{-|x|} & \text{for } |X| \le 4 \text{ volt} \\ 0 & \text{otherwise} \end{cases}$$

MCQ 6.1.29 If the signal $X(t)$ is quantized into 4 levels, what will be the step size of each level ?

(A) 2 volt  (B) 8 volt

(C) 32 volt  (D) 1 volt

MCQ 6.1.30 What will be the value of constant $k$ ?

(A) $9.16 \times 10^{-3}$  (B) 0.4908

(C) 27.29  (D) 0.5093

MCQ 6.1.31 If the probability density function is not constant over each level, what will be the variance of the quantization error ?

(A) 0.1869  (B) 0.5093

(C) 0.7341  (D) 0.3739

MCQ 6.1.32 Consider the two message signals defined as

$$m_1(t) = \operatorname{sinc}(100t),\ m_2(t) = \operatorname{sinc}^2(100t)$$

what are the values of Nyquist rates $f_{N1}$ and $f_{N2}$ of the corresponding signals ?

| | $f_{N1}$ | $f_{N2}$ |
|---|---|---|
| (A) | 100 | 200 |
| (B) | 200 | 100 |
| (C) | 50 | 100 |
| (D) | 100 | 50 |

MCQ 6.1.33 In a binary system, the quantizing noise is not to exceed $\pm x$ % of the peak-to-peak analog level. The minimum number of bits in each PCM words needs to be

(A) $x/50$  (B) $50/x$

(C) $\log_2\left(\frac{50}{x}\right)$  (D) $\log_2\left(\frac{x}{50}\right)$

Common Data For Q. 34 and 35 :

The information in an analog voltage waveform is to be transmitted over a PCM system with a $\pm 0.1\%$ accuracy. The analog waveform has an absolute bandwidth of 100 Hz and an amplitude range of $-10$ V to $+10$ V.

MCQ 6.1.34 What is the minimum sampling rate needed for the waveform ?

(A) 102 Hz (B) 100 Hz
(C) 200 Hz (D) 50 Hz

MCQ 6.1.35 What is the minimum absolute channel bandwidth required for transmission of this PCM ?

(A) 1800 Hz (B) 900 Hz
(C) 450 Hz (D) 3600 Hz

MCQ 6.1.36 An 850 Mbyte hard disk is used to store PCM data. Suppose that a voice frequency signal is sampled at 8 k samples/sec and the encoded PCM is to have an average SNR of at least 30 dB. How many minutes of voice frequency conversation can be stored on the hard disk ?

(A) 2.833 minute (B) $170 \times 10^3$ minute
(C) 2833 minute (D) 1700 minute

Common Data For Q. 37 and 38 :

An analog signal with a bandwidth of 4.2 MHz is to be converted into binary PCM and transmitted over a channel. The peak signal quantizing noise ratio must be at least 55 dB.

MCQ 6.1.37 If there is no bit error and no inter symbol interferrence, what will be the number of quantizing steps needed ?

(A) 512 (B) 9
(C) 4 (D) 256

MCQ 6.1.38 What will be the channel bandwidth required if rectangular pulse shapes are used ?

(A) 75.6 MHz (B) 8.4 MHz
(C) 933.3 kHz (D) 17.4 MHz

Common Data For Q. 39 and 40 :

Compact disk (CD) players use 16 bit PCM, including one parity bit with 8 times oversampling of the analog signal. The analog signal bandwidth is 20 kHz.

MCQ 6.1.39 What is the null bandwidth of this PCM signal ?

(A) 640 MHz (B) 320 kHz
(C) 512 MHz (D) 512 MHz

MCQ 6.1.40 What will be the peak SNR for the signal ?

(A) 9.50 dB (B) 10.1 dB
(C) 95.08 dB (D) 101.1 dB

### Common Data For Q. 41 and 42 :

In a PCM system, the bit error rate due to channel noise is $10^{-4}$. Assume that the peak signal to noise ratio on the recovered analog signal needs to be at least 30 dB.

MCQ 6.1.41 What is the minimum number of quantizing steps that can be used to encode the analog signal into a PCM signal ?

(A) 32 (B) 16
(C) 64 (D) 8

MCQ 6.1.42 If the original analog signal had an absolute bandwidth of 2.7 kHz, what is the null bandwidth of the PCM signal ?

(A) 27 kHz (B) 13.5 kHz
(C) 10.4 kHz (D) 5.4 kHz

### Common Data For Q. 43 to 45 :

A multi level digital communication system sends one of 16 possible levels over the channel every 0.8 ms.

MCQ 6.1.43 What is the number of bits corresponding to each level ?

(A) 16 bits/level
(B) 2 bits/level
(C) 4 bits/level
(D) 8 bits/level

MCQ 6.1.44 What is the baud rate ?

(A) 8 baud
(B) 8000 baud
(C) 1250 baud
(D) 1.25 baud

MCQ 6.1.45 What is the bit rate for the signal ?

(A) 5 k bits/sec (B) 1.25 k bits/sec
(C) 0.31 k bits/sec (D) 4 k bits/sec

### Common Data For Q. 46 to 48 :

The information in an analog waveform is first encoded into binary PCM and then converted to a multilevel signal for transmission over the channel. The number of multilevels is eight. Assume that analog signal has a bandwidth of 2700 Hz and is to be reproduced at the receiver output with an accuracy of $\pm 1\%$ (full scale)

MCQ 6.1.46 What is the minimum bit rate of the PCM signal ?

(A) 900 bits/sec
(B) 32.4 k bits/sec
(C) 6 k bits/sec
(D) 1.11 k bits/sec

MCQ 6.1.47 The minimum baud rate of the multilevel signal is
(A) 97.2 k symbols/sec
(B) 32.4 k symbols/sec
(C) 4.05 k symbols/sec
(D) 10.8 k symbols/sec

MCQ 6.1.48 What is the minimum absolute channel bandwidth required for transmission of the PCM signal ?
(A) 10.8 kHz
(B) 5.4 kHz
(C) 12.8 kHz
(D) 21.6 kHz

### Common Data For Q. 49 and 50 :

A binary waveform of 9600 bits/sec is converted into an octal (multilevel) waveform that is passed through a channel with a raised cosine roll of Nyquist filter characteristic. The channel has a conditioned phase response out to 2.4 kHz.

MCQ 6.1.49 What is the baud rate of the multilevel signal ?
(A) 3 k symbol/sec
(B) 28.8 k symbol/sec
(C) 3.2 k symbol/sec
(D) 9.6 k symbol/sec

MCQ 6.1.50 What is the roll off factor of the filter characteristic ?
(A) 2
(B) 0.5
(C) 3
(D) 1

### Common Data For Q. 51 and 52 :

An analog signal is to be converted into PCM signal that is a binary polar NRZ line code. The signal is transmitted over a channel that is absolutely bandlimited to 4 kHz. Assume that the PCM quantizer has 16 steps and that the overall equivalent system transfer function is of the raised cosine roll off type with roll off factor $\alpha = 0.5$.

MCQ 6.1.51 What is the maximum PCM bit rate that can be supported by this system without introducing ISI (intersymbol interference) ?
(A) 5.33 k bits/sec
(B) 8 k bits/sec
(C) 12 k bits/sec
(D) 16 k bits/sec

MCQ 6.1.52 The maximum bandwidth that can be permitted for the analog signal is
(A) 666 Hz
(B) 2.22 kHz
(C) 333 Hz
(D) 1.33 kHz

### Common Data For Q. 53 and 54 :

Multilevel data with an equivalent bit rate of 2400 bits/sec is sent over a channel using a four level line code that has a rectangular pulse shape at the output of the transmitter. The overall transmission system (i.e., the transmitter, channel and receiver) has an $\alpha = 0.5$ raised cosine roll off Nyquist filter characteristic.

MCQ 6.1.53 What is the 6 dB bandwidth for this transmission system ?

(A) 2400 Hz (B) 1200 Hz

(C) 300 Hz (D) 600 Hz

MCQ 6.1.54 What is the absolute bandwidth for the system ?

(A) 900 Hz (B) 1800 Hz

(C) 450 Hz (D) 600 Hz

### Common Data For Q. 55 to 57 :

A DM (Delta modulation) system is tested with a 10 kHz sinusoidal signal, 1 volt peak-to-peak, at the input. The signal is sampled at 10 times the Nyquist rate.

MCQ 6.1.55 What is the minimum step size required to prevent slope overload ?

(A) 0.157 volt (B) 0.314 volt

(C) 0.05 volt (D) 0.1 volt

MCQ 6.1.56 What is the power spectral density for the granular noise ?

(A) $1.23 \times 10^{-7}$ $V^2/Hz$ (B) $2.47 \times 10^{-2}$ $V^2/Hz$

(C) $7.39 \times 10^{-7}$ $V^2/Hz$ (D) $2.06 \times 10^{-8}$ $V^2/Hz$

MCQ 6.1.57 If the receiver input is band limited to 200 kHz, what is the average signal quantizing noise power ratio ?

(A) 11.8 dB (B) 1.18 dB

(C) 14.8 dB (D) 1.48 dB

MCQ 6.1.58 Five messages bandlimited to $W$, $W$, $2W$, $4W$ and $4W$ Hz, respectively are to be time division multiplexed. What is the minimum transmission bandwidth required for this TDM signal ?

(A) $24W$ (B) $W$

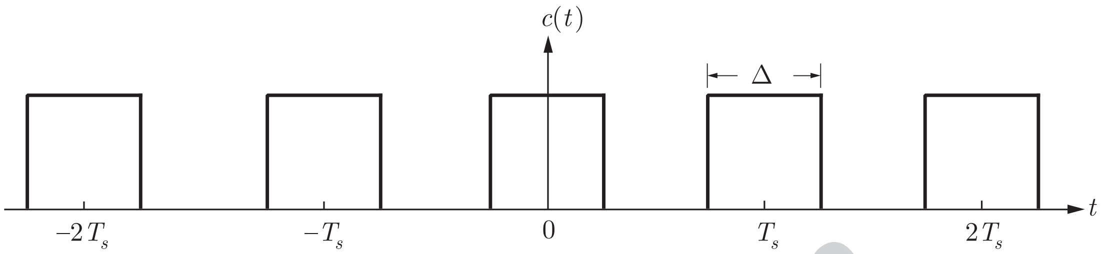
(C) $4W$ (D) $12W$

***********

# EXERCISE 6.2



QUES 6.2.1 Consider a set of 10 signals $x_i(t)$, $i = 1, 2, 3, ...10$. Each signal is band limited to 1 kHz. All 10 signals are to be time-division multiplexed after each is multiplied by a carrier $c(t)$ shown in figure.




If the period $T_s$ of $c(t)$ is chosen that have the maximum allowable value, the largest value of $\triangle$ would be _______ μs.

QUES 6.2.2 A compact disc recording system samples a signal with a 16-bit analog-to-digital convertor at 44.1 kHz. The CD can record an hour worth of music. The approximate capacity of CD is _______ Mbytes.

QUES 6.2.3 An analog signal is sampled at 36 kHz and quantized into 256 levels. The time duration of a bit of the binary coded signal is _______ μs.

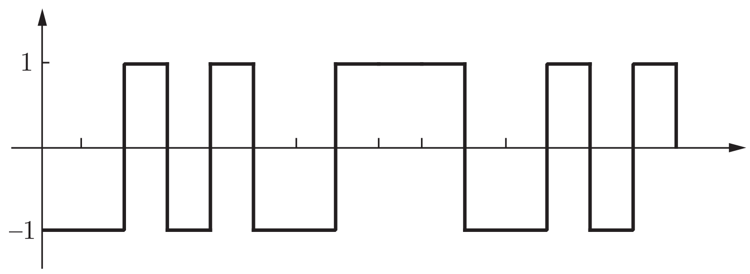
QUES 6.2.4 An analog signal is quantized and transmitted using a PCM system. The tolerable error in sample amplitude is 0.5% of the peak-to-peak full scale value. The minimum binary digits required to encode a sample is _______

QUES 6.2.5 A Television signal is sampled at a rate of 20% above the Nyquist rate. The signal has a bandwidth of 6 MHz. The samples are quantized into 1024 levels. The minimum bandwidth required to transmit this signal would be _______ Mbps.

QUES 6.2.6 A CD record audio signals digitally using PCM. The audio signal bandwidth is 15 kHz. The Nyquist samples are quantized into 32678 levels and then binary coded. What is the minimum bit rate (in kbps) required to encode the audio signal ?

QUES 6.2.7 The American Standard Code for information interchange has 128 characters, which are binary coded. If a certain computer generates 1,000,000 character per second, the minimum bandwidth required to transmit this signal will be _______ Mbps.

QUES 6.2.8 Figure given below shows a PCM signal in which amplitude level of + 1 volt and $-1$ volt are used to represent binary symbol 1 and 0 respectively. The code word used consists of three bits. The sampled version of analog signal from which this PCM signal is derived is _______




QUES 6.2.9 A PCM system uses a uniform quantizer followed by a 8-bit encoder. The bit rate of the system is equal to $10^8$ bits/s. The maximum message bandwidth for which the system operates satisfactorily is _______ MHz.

QUES 6.2.10 Twenty-four voice signals are sampled uniformly at a rate of 8 kHz and then time-division multiplexed. The sampling process uses flat-top samples with 1 $\mu$s duration. The multiplexing operation includes provision for synchronization by adding an extra pulse of 1 $\mu$s duration. What is the spacing (in $\mu$s) between successive pulses of the multiplexed signal ?

QUES 6.2.11 A linear delta modulator is designed to operate on speech signals limited to 3.4 kHz. The sampling rate is 10 times the Nyquist rate of the speech signal. The step size $\delta$ is 100 mV. The modulator is tested with a 1kHz sinusoidal test signal. The maximum amplitude of the test signal required to avoid slope overload is _______ mV.

QUES 6.2.12 The output signal-to-quantization-noise ratio of a 10-bit PCM was found to be 30 dB. The desired SNR is 42 dB. It can be increased by increasing the number of quantization level. In this way the percentage increase in the transmission bandwidth would be _______ %. (Assume $\log_2 10 = 0.3$)

QUES 6.2.13 For a PCM signal the compression parameter $\mu = 100$ and the minimum signal to quantization-noise ratio is 50 dB. The number of bits per sample would be _______

QUES 6.2.14 A sinusoid massage signal $m(t)$ is transmitted by binary PCM without compression. If the signal to-quantization-noise ratio is required to be at least 48 dB, the minimum number of bits per sample will be _______

QUES 6.2.15 A speech signal has a total duration of 20 sec. It is sampled at the rate of 8 kHz and then PCM encoded. The signal-to-quantization noise ratio is required to be 40 dB. The minimum storage capacity needed to accommodate this signal is _______ kbytes.

QUES 6.2.16 A sinusoidal signal with peak-to-peak amplitude of 1.536 V is quantized into 128 levels using a mid-rise uniform quantizer. The quantization-noise power is _______ $\times 10^{-6}$ V$^2$.

QUES 6.2.17 The Nyquist sampling interval, for the signal $\sin c(700t) + \sin c(500t)$ is _______ second.

QUES 6.2.18 In a PCM system, if the code word length is increased form 6 to 8 bits, then by what factor signal to quantization noise ratio improves ?

QUES 6.2.19 Four signals $g_1(t), g_2(t), g_3(t)$ and $g_4(t)$ are to be time division multiplexed and transmitted. $g_1(t)$ and $g_4(t)$ have a bandwidth of 4 kHz, and the remaining two signals have bandwidth of 8 kHz. Each sample requires 8 bits for encoding. What is the minimum transmission bit rate (kbps) of the system ?

QUES 6.2.20 Three analog signals, having bandwidths 1200 Hz, 600 Hz and 600 Hz, are sampled at their respective Nyquist rates, encoded with 12 bit words, and time division multiplexed. What is the bit rate (in kbps) for the multiplexed signal ?

QUES 6.2.21 Four signals each band limited to 5 kHz are sampled at twice the Nyquist rate. The resulting PAM samples are transmitted over a single channel after time division multiplexing. The theoretical minimum transmission bandwidth of the channel should be equal to _______ kHz.

QUES 6.2.22 Four independent messages have bandwidths of 100 Hz, 100 Hz, 200 Hz and 400 Hz respectively. Each is sampled at the Nyquist rate, time division multiplexed and transmitted. The transmitted sample rate in Hz, is given by _______

QUES 6.2.23 The Nyquist sampling rate for the signal $g(t) = 10\cos(50\pi t)\cos^2(150\pi t)$ where '$t$' is in seconds, is _______ samples per second.

QUES 6.2.24 A TDM link has 20 signal channels and each channel is sampled 8000 times/sec. Each sample is represented by seven binary bits and contains an additional bit for synchronization. The total bit rate for the TDM link is _______ kbps.

QUES 6.2.25 The Nyquist sampling interval for the signal $s(t) = \sin c(350t) + \sin c(250t)$ is _______ second.

QUES 6.2.26 In a CD player, the sampling rate is 44.1 kHz and the samples are quantized using a 16-bit/sample quantizer. The resulting number of Mega-bits for a piece of music with a duration of 50 minutes is _______

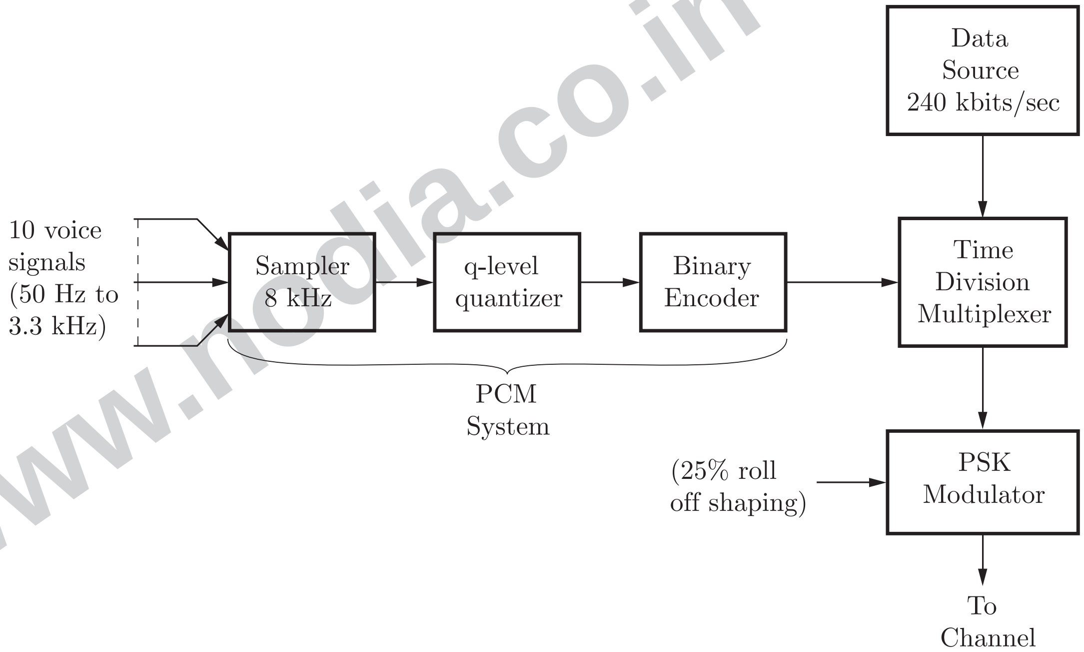
QUES 6.2.27 Four voice signals, each limited to 4 kHz and sampled at Nyquist rate are converted into binary PCM signal using 256 quantization levels. The bit transmission rate for the time-division multiplexed signal will be _______ kbps.

QUES 6.2.28 Analog data having highest harmonic at 30 kHz generated by a sensor has been digitized using 6 level PCM. What will be the rate (in kbps) of digital signal generated ?

QUES 6.2.29 In a PCM system, the number of quantization levels is 16 and the maximum signal frequency is 4 kHz; the bit transmission rate is _______ kbps.

QUES 6.2.30 A speech signal occupying the bandwidth of 300 Hz to 3 kHz is converted into PCM format for use in digital communication. If the sampling frequency is 8 kHz and each sample is quantized into 256 levels, then the output bit rate will be _______ kbps.

QUES 6.2.31 A PCM system is time division multiplexed with a data source, as shown in figure below. The output of the time division multiplexer is fed into a PSK modulator after which the signal is transmitted over a channel of bandwidth $B_T = 1\,\text{MHz}$ with center frequency at 100 MHz. What is the maximum number of quantization levels required in this system ?




QUES 6.2.32 A time division multiplexing system using PCM is used to multiplex telephone conversations over a single communication channel. It is found that a minima interval of $1\,\mu\text{s}$ must be allowed for reliable identification of bits at the receiving end. If the allowable voice bandwidth for telephone line transmission is 3 kHz and the quantization level is given as 16, what is the approximate number of voice signals that can be multiplexed ?

## Common Data For Q. 33 and 34 :

Consider a PCM multiplexing system using a 256 level signal quantizer for the transmission of three signals $m_1$, $m_2$ and $m_3$, bandlimited to 5 kHz, 10 kHz, and 5 kHz, respectively. Assume that each signal is sampled at Nyquist rate and 8 bits transmitted simultaneously.

QUES 6.2.33 What is the maximum bit duration (in μs) for the transmitted signal ?

QUES 6.2.34 The channel bandwidth required to pass the PCM signal is _______ kHz.

QUES 6.2.35 An information signal to be transmitted digitally is a rectangular wave with a period of 71.4 μs. It has been determined that the wave will be adequately passed if the bandwidth includes the fourth harmonic. The minimum sampling frequency required for the signal is _______ kHz.

QUES 6.2.36 The voltage range of an A/D converter that uses 14 bit numbers is $-6$ to $+6$ V. What will be the resolution (in μV) of digitization ?

QUES 6.2.37 The input voltage of a compander with a maximum voltage range of 1 volt and $\mu = 255$ is 0.25 volt. What will be the voltage gain of compander ?

QUES 6.2.38 A multilevel digital communication system is to operate at a data rate of 9600 bits/sec. If 4 bit words are encoded into each level for transmission over the channel, what is the minimum required bandwidth (in Hz) for the channel ?

QUES 6.2.39 An analog signal is to be converted into a PCM signal that is multilevel polar NRZ line code with four number of levels. The signal is transmitted over a channel that is absolutely bandlimited to 4 kHz. Assume that the PCM quantizer has 16 steps and that the overall equivalent system transfer function is of the raised cosine roll off type with roll off factor $\alpha = 0.5$. What is the maximum bandwidth (in kHz) that can be permitted for the analog signal ?

QUES 6.2.40 Assume that a PCM type system is to be designed such that an audio signal can be delivered at the receiver output. The audio signal is to have a bandwidth of 3400 Hz and an SNR of at least 40 dB. What is the bit rate requirement (in kbps) for a design that uses $\mu = 255$ companded PCM signaling ?

QUES 6.2.41 A continuous data signal is quantized and transmitted using a PCM signal. If each data sample at the receiving end of the system must be known to within $\pm 0.5\%$ of the peak-to-peak full scale value, how many binary symbols must each transmitted digital word contain ?

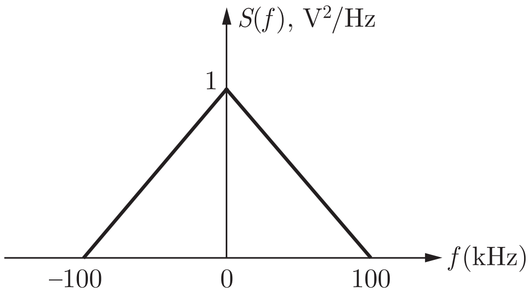
QUES 6.2.42 A delta modulator has the message signal,

$$m(t) = 4\sin 2\pi(10)t + 5\sin 2\pi(20)t$$

If the step size is $\delta = 0.05\pi$, what is the minimum sampling frequency (in Hz) required to prevent slope overload ?

QUES 6.2.43 A message signal has the following frequency components :
First component : A signal tone sine wave of 500 Hz.
Second component : A sound of frequency components with lowest value of 750 Hz and highest value of 1800 Hz.
What should be the minimum sampling frequency (in Hz) to sense the information present in this signal according to the sampling theorem ?

QUES 6.2.44 Consider a signal with power spectral density shown in figure below.




If the signal is sampled at 90% of its Nyquist rate and reconstruction filter has an ideal rectangular amplitude response, what will be signal to distortion ratio in decibel ?

***********

# EXERCISE 6.3



MCQ 6.3.1 A signal that is mistakenly sampled when the sampling frequency is less than twice the input frequency is a/an
(A) harmonic
(B) pseudo signal
(C) alias
(D) false wave

MCQ 6.3.2 The smallest increment of voltage that the D/A converter produces over its output range is called the
(A) step
(B) resolution
(C) pixel
(D) bit

MCQ 6.3.3 A circuit that converts an instantaneous value of analog voltage into a binary number is the
(A) multiplexer
(B) half-adder
(C) D/A converter
(D) A/D converter

MCQ 6.3.4 In what type of multiplexing does each signal occupy the entire bandwidth of the channel?
(A) frequency-division multiplexing
(B) time-division multiplexing
(C) pulse-width multiplexing
(D) phase-shift multiplexing

MCQ 6.3.5 Which of the following is a very popular form of multiplexing where multiple channels of digital data are transmitted in serial form?
(A) frequency-division multiplexing
(B) phase-shift multiplexing
(C) pulse-amplitude modulated multiplexing
(D) pulse-code modulated multiplexing

MCQ 6.3.6 Which of the following is not a common type of media used in data communication?
(A) wire cable (B) radio
(C) wave guide (D) coaxial cable

MCQ 6.3.7 The main advantage of TDM over FDM is that it
(A) needs less power (B) needs less bandwidth

(C) needs simple circuitry
(D) gives better S/N ratio

MCQ 6.3.8 The PWM needs
(A) more power than PPM
(B) more samples per second than PPM
(C) more bandwidth than PPM
(D) none of the above

MCQ 6.3.9 The PAM signal can be detected by
(A) bandpass filter
(B) bandstop filter
(C) high pass filter
(D) low pass filter

MCQ 6.3.10 Flat-top sampling leads to
(A) an aperture effect
(B) aliasing
(C) loss of signal
(D) none

MCQ 6.3.11 In PCM, the quantization noise depends on
(A) sampling rate
(B) number of quantization levels
(C) signal power
(D) none of the above

MCQ 6.3.12 Which of the following modulation is digital in nature
(A) PAM
(B) PPM
(C) DM
(D) none of the above

MCQ 6.3.13 Which of the following modulation is analog in nature
(A) PCM
(B) DPCM
(C) DM
(D) none of these

MCQ 6.3.14 Quantization noise occurs in
(A) PAM
(B) PWM
(C) DM
(D) none of the above

MCQ 6.3.15 Companding is used in PCM to
(A) reduce bandwidth
(B) reduce power
(C) increase S/N ratio
(D) get almost uniform S/N ratio

MCQ 6.3.16 The main advantage of PCM is
(A) less bandwidth
(B) less power
(C) better performance in presence of noise
(D) possibility of multiplexing

MCQ 6.3.17 The main disadvantage of PCM is
(A) large bandwidth
(B) large power
(C) complex circuitry
(D) quantization noise

MCQ 6.3.18 The main advantage of DM over PCM is
(A) less bandwidth
(B) less power
(C) better S/N ratio
(D) simple circuitry

MCQ 6.3.19 In pulse analog modulation, with respect to message signal, the modulation is achieved by varying
(A) pulse amplitude (B) pulse width
(C) pulse position (D) all the pulse parameters

MCQ 6.3.20 Pulse amplitude modulation involves
(A) varying amplitude of message signal according to amplitude of pulse train
(B) performing amplitude modulation and then multiplying the result with pulse train
(C) varying amplitude of pulse train according to instantaneous variations of message signal
(D) performing multiplcation of pulse train with message and then subjecting the result to amplitude modulation.

MCQ 6.3.21 Pulse width modulation involves
(A) varying duration of message signal according to width of pulse train
(B) varying width of pulses in the pulse train according to instantaneous variations of message signal
(C) performing duration modification of message signal and then multiplying the result width pulse train
(D) performing width modification of pulse train with message and then subjecting the result to width modulation

MCQ 6.3.22 Pulse position modulation involves
(A) varying position of message signal components according to the position of pulses in the pulse train
(B) varying position of pulses in the pulse train according to the instantaneous variations in the message signal
(C) varying position of pulses in the pulse train according to the message components position
(D) performing position modification of pulse train with message and then subjecting the result to position modification

MCQ 6.3.23 Pulse code modulation involves
(A) PAM followed by quantization
(B) Direct encoding using binary words
(C) PAM followed by quantization and encoding using binary words
(D) PAM followed by encoding using binary words

MCQ 6.3.24 Delta modulation involves
(A) PAM followed by encoding using one-bit binary words
(B) PAM followed by quantization and encoding using one-bit binary words
(C) PAM followed by one-bit quantization
(D) direct encoding using one-bit quantization

MCQ 6.3.25 Sampling is the process which convert :
(A) discrete signal to continuous signal
(B) analog signal to digital signal

(C) continuous signal to discrete signal

(D) digital signal to analog signal

MCQ 6.3.26 Which is correct for Nyquist rate

if $f_s$ = frequency of sampling

$f_m$ = highest frequency of band limited signal

(A) $f_s = f_m$ (B) $f_s > 2f_m$

(C) $f_m > 2f_s$ (D) $f_s < 2f_m$

MCQ 6.3.27 Amplitude of pulse train varies according to instantaneous value of modulating signal is called :

(A) PLM (B) PDM

(C) PAM (D) PWM

MCQ 6.3.28 Quantization noise occurs in :

(A) TDM (B) PCM

(C) FDM (D) None of the above

MCQ 6.3.29 Several signals can be time division multiplexed (TDM) because

(A) these signals cannot be sampled

(B) be ideally sampled

(C) be sampled with any arbitrary sample shape

(D) none of the above

MCQ 6.3.30 Aliasing occurs due to

(A) over sampling

(B) under sampling

(C) attenuation or amplification of signals

(D) none of the above

MCQ 6.3.31 A PAM signal is recovered by using

(A) low pass filter (B) high pass filter

(C) band pass filter (D) none of the above

MCQ 6.3.32 Pulse modulation is used in

(A) Radio navigation (B) Automatic landing system

(C) Data Communications (D) All of a above

MCQ 6.3.33 PAM signal can be demodulated by using

(A) a low pass filter (B) a band pass filter

(C) a high pass filter (D) None of the above

MCQ 6.3.34 In a pulse position modulation system, the transmitted pulse have

(A) Constant amplitudes but varying widths

(B) Constant amplitudes constant widths

(C) Constant width by varying amplitudes

(D) None of the above

MCQ 6.3.35 A “PWM” signal can be generated by

(A) a monostable multivibrator

(B) an astable multivibrator

(C) Integrating the PPM signal

(D) Differenting the PPM signal



MCQ 6.3.36 Time division multiplexing

(A) Can be used only with PCM

(B) Combines five groups into a super-group

(C) Stacks 24 channels in adjacent frequency slots

(D) Interleaves pulse belonging to different transmission

MCQ 6.3.37 If the transmission bandwidth is $W$ and available channel bandwidth is $W_{\text{Channel}}$, what should be the condition that will allow fruitful reception?

(A) $W = W_{\text{channel}}$ (B) $W < W_{\text{channel}}$

(C) $W > W_{\text{channel}}$ (D) All of the above

MCQ 6.3.38 In pulse modulation systems, the number of samples required to ensure no loss of information is given by

(A) Fourier transform (B) Nyquist theorem

(C) Parseval's theorem (D) Shannon's Theorem

MCQ 6.3.39 A PCM receiver can see

(A) Quantization nosie (B) Channel noise

(C) Interference noise (D) All of the above

MCQ 6.3.40 In PCM for $q$ quantizing levels, the number of pulses $p$ in a code group is given by

(A) $\log_{10} q$ (B) $\log_2 q$

(C) $\ln q$ (D) $2\log_2 q$

MCQ 6.3.41 The principal merit of PCM system is its

(A) Lower bandwidth

(B) Lower noise

(C) Lower power requirement

(D) Lower cost

MCQ 6.3.42 A compander is used in communication systems to

(A) Compress the bandwidth

(B) Improve the frequency response

(C) Reduce the channel noise

(D) Improve singal-to-noise ratio

MCQ 6.3.43 Which of the following systmes is digital?

(A) PPM (B) PWM

(C) PCM (D) PFM

MCQ 6.3.44 Which of the following is unlikely to happen when the quantizing noise is decreased in PCM?

(A) Increase in the bandwidth

(B) Increase in the number of standard levels

(C) Increase in the channel noise

(D) All of the above

MCQ 6.3.45 Which of the following systems is not digital?

(A) Differential PCM (B) DM

(C) ADPCM (D) PAM

MCQ 6.3.46 In which of the following methods of source coding, maximum compression is achieved?

(A) $A$-law PCM (B) $\mu$-law PCM

(C) DM (D) ADPCM

MCQ 6.3.47 A sine wave carrier cannot be modified by the intelligence signal through which of the following?

(A) amplitude modulation

(B) pulse modulation

(C) frequency modulation

(D) phase modulation

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# SOLUTIONS 6.1



SOL 6.1.1 Option (B) is correct.

From the Fourier spectra of signal $x(t)$, we observe that the bandwidth of $x(t)$ is

$$\omega_x = 2\pi \times 10^5$$

or, $$2\pi f_x = 2\pi \times 10^5$$

or, $$f_x = 10^5 \text{ Hz}$$

So, the Nyquist sampling rate for $x(t)$ is given by

$$\begin{aligned} f_s &= 2f_x \\ &= 2 \times 10^5 \text{ Hz} \\ &= 200 \text{ kHz} \end{aligned}$$

SOL 6.1.2 Option (D) is correct.

From the Fourier spectrum of signal $y(t)$, we obtain the bandwidth of $y(t)$ as

$$\omega_y = 3\pi \times 10^5$$

$$2\pi f_y = 3\pi \times 10^5$$

or, $$\begin{aligned} f_y &= 1.5 \times 10^5 \text{ Hz} \\ &= 150 \text{ kHz} \end{aligned}$$

So, the Nyquist sampling rate for $y(t)$ is

$$\begin{aligned} f_s &= 2f_y \\ &= 2 \times 150 \\ &= 300 \text{ kHz} \end{aligned}$$

SOL 6.1.3 Option (D) is correct.

Again, we have the bandwidth the signal $x^2(t)$ as

$$\begin{aligned} f_{x^2} &= 2f_x \\ &= 2 \times 10^5 \text{ Hz} \end{aligned}$$

So, the Nyquist sampling rate of $x^2(t)$ is given by

$$\begin{aligned} f_s &= 2f_{x^2} \\ &= 2 \times (2 \times 10^5) \\ &= 4 \times 10^5 \text{ Hz} \\ &= 400 \text{ kHz} \end{aligned}$$

SOL 6.1.4 Option (C) is correct.

The bandwidth of signal $y^3(t)$ will be thrice that of signal $y(t)$. So, we have the bandwidth of $y^3(t)$ as

$$\begin{aligned} f_{y^3} &= 3f_y \\ &= 3 \times 1.5 \times 10^5 \\ &= 4.5 \times 10^5 \end{aligned}$$

So, the Nyquist sampling rate of $y^3(t)$ is given by

$$f_s = 2f_{y^3} = 2 \times (4.5 \times 10^5)$$

$$= 9 \times 10^5 \text{ Hz}$$
$$= 900 \text{ kHz}$$

SOL 6.1.5 Option (B) is correct.

The bandwidth of signal $x(t)y(t)$ is obtained as the sum of the bandwidth of $x(t)$ and $y(t)$. i.e.

$$f_{xy} = f_x + f_y$$
$$= 10^5 + 1.5 \times 10^5$$
$$= 2.5 \times 10^5 \text{ Hz}$$

So, the Nyquist sampling rate of $x(t)y(t)$ is given by

$$f_s = 2f_{xy}$$
$$= 2 \times (2.5 \times 10^5)$$
$$= 5 \times 10^5 \text{ Hz}$$
$$= 500 \text{ kHz}$$

SOL 6.1.6 Option (B) is correct.

From the shown pulse waveform of carrier signal $c(t)$, we have the sampling frequency

$$f_s = \frac{1}{T_s} = \frac{1}{10^{-3}} = 10^3 \text{ Hz.}$$

This sampling frequency must satisfy

The Nyquist critesion for sampling the signal $x(t)$ i.e.,

$$f_s > 2f_x$$

or $$2\pi f_s > 2\pi(2f_x)$$

$$2\pi \times 1000 > 2\omega$$

or $$\omega < 1000\pi$$

So, the signal $X(j\omega)$ must be defined for the region $\omega < 1000\pi$. Therefore, we have $X(j\omega) = 0$ for $\omega > 1000\pi$



SOL 6.1.7 Option (C) is correct.

From the shown rectangular pulse train, we have the time period of $c(t)$,

$$T_s = 10^{-3} \text{ sec}$$

ON Time duration of pulse train is

$$\triangle = 0.25 \text{ ms} = 0.25 \times 10^{-3} \text{ sec}$$

So, the duty cycle of pulse-train is given by

$$D = \frac{\triangle}{T_s} = \frac{0.25 \times 10^{-3}}{10^{-3}} = 0.25$$

Now, the gain of LPF is $A$. To recover the signal $x(t)$, the overall gain of the system must be 1. So, we have

$$D \times A = 1$$
$$\frac{\triangle}{T_s} A = 1$$
$$A = \frac{T_s}{\triangle} = \frac{1}{0.25} = 4$$

SOL 6.1.8 Option (B) is correct.

Given the bandwidth of each telemetry signal, $f_m = 2$ kHz.

Let the peak signal amplitude be $m_p$. So, we have the maximum tolerable error

$$\text{tolerable error} = 0.2\% \text{ of peak signal amplitude}$$

$$= \frac{0.2}{100} \times m_p$$

$$= \frac{m_p}{500} \qquad \text{...(1)}$$

Now, the Nyquist sampling rate for each signal is

$$f_N = 2f_m = 2 \times 2 \times 1000$$
$$= 4000 \text{ Hz}$$

Since, the sampling must be 20% above the Nyquist rate so, we have the actual sampling rate as

$$f_s = 1.2 \times f_N = 1.2 \times 4000$$
$$= 4800 \text{ Hz}$$

Again, let the number of quantization level of the sampled signal be $q$. So, we have the maximum quantization error in the sampled signal as

$$\text{quantization error} = \frac{\frac{2m_p}{q}}{2} = \frac{m_p}{q} \qquad \text{...(2)}$$

This error must be less than the maximum tolerable error. So, from equations (1) and (2), we have

$$\frac{m_p}{q} \le \frac{m_p}{500}$$

or, $q \ge 500$

or $2^n \ge 500$

where $n$ is the number of bits used to encode the quantized signal. For satisfying the above condition, the minimum number of bits is $n = 9$ bits. Therefore, the bit rate of the sampled signal is given by,

$$R_b = nf_s$$
$$= 9 \times 4800$$
$$= 4.32 \times 10^4 \text{ bits/sec}$$

since, ten signals are multiplexed, and framing and synchronising requires an additional 1% extra bits. Therefore, the total data rate is given by

$$\text{Data rate} = 1.01 \times [10 \times R_b]$$
$$= 1.01 \times 10 \times (4.32 \times 10^4)$$
$$= 4.3632 \times 10^5 \text{ bits/sec}$$
$$= 436.32 \text{ kbits/sec}$$

SOL 6.1.9 Option (A) is correct.

We have just obtained the minimum possible data rate,

$$\text{Data rate} = 436.32 \text{ kbits/sec}$$

So, the minimum transmission bandwidth of the multiplexed signal is given by

$$B_T = \frac{\text{Data rate}}{2}$$
$$= \frac{436.32}{2}$$
$$= 218.16 \text{ kHz}$$

NOTE :

As we can transmit error free at most two peices of information per second per Hz bandwidth, here we have dividided the data rate by 2 bits (information) to get the minimum transmission bandwidth.

SOL 6.1.10 Option (D) is correct.

Given the transmission bandwidth capacity of binary channel,

$$B_{ch} = 36 \text{ kbits/sec}$$

Bandwidth of message signal,

$$f_m = 3.2 \text{ kHz}$$

So, the sampling frequency of signal must be

$$f_s \geq 2f_m$$

or,
$$f_s \geq 2 \times (3.2) = 6.4 \text{ kHz} \quad \text{...(1)}$$

Now, let the quantization level be $q$. So, we have $q = 2^n$, where $n$ is the number of bits required to encode the sample. So, we have the bit rate for the signal,

$$R_b = nf_s$$

Therefore, the maximum transmission bandwidth for the signal is given by

$$B_T = R_b = nf_s$$

This transmision bandwidth must be less than the bandwidth capacity of binary channel i.e.

$$R_b \leq B_{ch}$$

or,
$$nf_s \leq 36 \quad \text{...(2)}$$

Now, checking the options for inequality (1), we have the sampling frequency $f_s = 7.2$ kHz.

Substituting it in equation (2), we get

$$7.2n \leq 36$$

or
$$n \leq 5$$

or
$$2^n \leq 2^5$$

or
$$q \leq 32$$

Thus, the appropriate values of quantizing level and sampling frequency are $q = 32$ and $f_s = 7.2$ kHz

SOL 6.1.11 Option (D) is correct.

Given the bandwidth of message signal for which delta modulator is designed is

$$B = 3.5 \text{ kHz}$$

the amplitude of test signal, $A_m = 1$ volt

frequency of test signal, $f_m = 800$ Hz

sampling frequency of the system, $f_s = 64$ kHz

So, we have time duration of a sample as

$$T_s = \frac{1}{64 \times 10^3} = 1.56 \times 10^{-5} \text{ sec}$$

Let the step size of the delta modulated signal be $\delta$. So, the condition to avoid slope overload is

$$\frac{\delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right|$$

or
$$\frac{\delta}{1.56 \times 10^{-5}} \geq A_m(2\pi f_m)$$

or,
$$\delta \geq (1.56 \times 10^{-5}) \times 1 \times (2\pi \times 800)$$

or,
$$\delta \geq 7.84 \times 10^{-2} \text{ volt}$$

Thus, the minimum value of step size to avoid slope overload is

$$\delta = 78.5 \text{ mV}$$

SOL 6.1.12 Option (D) is correct.
Again, we have the analog signal band for which delta modulator is designed as $B = 3.5\text{ kHz}$.
Sampling frequency of the system, $f_s = 64\text{ kHz}$.
The step size, we have just obtained as $\delta = 78.5\text{ mV} = 78.5 \times 10^{-3}$
So, the granular noise power in the analog signal band is given by

$$N = \frac{\delta^2 B}{3f_s}$$
$$= \frac{(78.5 \times 10^{-3})^2 \times (3.5 \times 10^3)}{3 \times (64 \times 10^3)}$$
$$= 1.123 \times 10^{-4}\text{ watt}$$

SOL 6.1.13 Option (C) is correct.
We have just obtained the granular noise power as

$$N = 1.12 \times 10^{-4}\text{ watt}$$

Also, we have the amplitude of message signal (sinusoidal test signal)

$$A_m = 1\text{ volt}$$

So, the signal power is given by

$$S = \frac{A_m^2}{2} = \frac{1}{2} = 0.5\text{ watt}$$

Therefore, SNR is given by

$$\text{SNR} = \frac{S}{N} = \frac{0.5}{1.12 \times 10^{-4}} = 4.46 \times 10^3$$

SOL 6.1.14 Option (B) is correct.
Given, the number of quantization level $q = 256$
For the $\mu$-law quantizer, we have $\mu = 225$
So, according to $\mu$-law companding, signal to quantization noise ratio is given by

$$\frac{S_o}{N_o} = \frac{3q^2}{[\ln(\mu+1)]^2}$$
$$= \frac{3 \times (256)^2}{[\ln(255+1)]^2}$$
$$= 6.39 \times 10^3$$

In decibel, the SNR is

$$\left(\frac{S_o}{N_o}\right)_{\text{db}} = 10\log_{10}(6.39 \times 10^3)$$
$$= 38.06\text{ dB}$$

SOL 6.1.15 Option (C) is correct.
Given, the bandwidth of message signal, $W_m = 1\text{ MHz}$
So, the Nyquist rate for the signal is $f_N = 2W_m = 2\text{ MHz}$
Since, the signal to be sampled 50% higher than the Nyquist rate, we have

$$f_{s1} = 1.5 \times f_N = 3\text{ MHz}$$

Also, we have the number of quantization level for modulating the signal,

$$q_1 = 256$$

or $$2^{n_1} = 256$$

or $$n_1 = 8\text{ bits}$$

This is the number of bits required to sample the signal in first case. So, the transmission bandwidth for this case is

$$B_{T1} = n_1 f_{s1} = 8 \times 3 = 24\text{ Mbits/sec}$$

Now signal to be sampled at a rate 20% above the Nyquist rate (adequate rate). So ,we have the sampling frequency for the second case as

$$f_{s2} = 1.2 f_N = 2.4 \text{ MHz}$$

Again, the transmission bandwidth must be same as in previous case. So, we have

$$B_{T2} = B_{T1} = 24 \text{ M bits/sec.}$$

or, $$n_2 f_{s2} = 24$$

or, $$n_2 = \frac{24}{2.4} = 10 \text{ bits}$$

This is the number of bits required to encode the signal in second case. So, we have the number of quantization level in the second case as

$$q_2 = 2^{n_2} = 2^{10}$$

Thus, the SNR in this case would be

$$\frac{S_o}{N_o} = \frac{3q_2^2}{[\ln(1+\mu)]^2}$$
$$= \frac{3 \times (2^{10})^2}{[\ln(1+255)]^2} = \frac{3 \times 2^{20}}{[\ln(256)]^2}$$
$$= 1.023 \times 10^5$$

In decibel, the SNR is

$$\left(\frac{S_o}{N_o}\right)_{\text{dB}} = 10\log_{10}(1.023 \times 10^5)$$
$$= 50.1 \text{ dB}$$

SOL 6.1.16 Option (B) is correct.

Given the step size of linear delta modulator, $\delta = 0.628$

The sampling frequency, $f_s = 40 \text{ kHz} = 40 \times 10^3$ Hz.

The condition for slope overload in delta modulation is

$$\frac{\delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right|$$

or, $$\delta f_s \geq \max\left|\frac{dm(t)}{dt}\right|$$

or, $$\max\left|\frac{dm(t)}{dt}\right| \leq (0.628) \times 40 \times 10^3 = 2.512 \times 10^4 \qquad ...(1)$$

Since, the input to delta modulator is sine wave with frequency $f_m$ and peak amplitude $A_m$. So, we have

$$\max\left|\frac{dm(t)}{dt}\right| = A_m(2\pi f_m) \qquad ...(2)$$

Therefore, from equations (1) and (2), we have

$$A_m(2\pi f_m) \leq 2.512 \times 10^4$$

or, $$A_m f_m \leq \frac{2.512}{2\pi} \times 10^4 = 3.998 \times 10^3$$

This is the condition to avoid slope overload.

In the given options, (A), (C) and (D) satisfy this condition, where as option (B) does not satisfy the condition. Therefore, the slope overload will take place at

$$A_m = 1.5 \text{ V}, \; f_m = 4 \text{ kHz}$$

SOL 6.1.17 Option (B) is correct.

Given, the sampling frequency, $f_s = 8$ kHz

Number of bits required for quantization, $n = 8$ bits

So, the bit rate of the PCM signal is given by

$$\begin{aligned} R_b &= nf_s \\ &= 8\times 8 \text{ kHz} \\ &= 64 \text{ kbits/sec} \end{aligned}$$

Also, the SNR for quantization of sinusoidal signal obtained as

$$\begin{aligned} \mathrm{SNR}_q &= \frac{S_o}{N_o} = 3q^2\frac{\overline{m^2(t)}}{m_p} \\ &= 3q^2\frac{1/2}{1} = \frac{3(2^n)^2}{2} = \frac{3(2^8)^2}{2} \\ &= 9.83\times 10^4 \end{aligned}$$

In decidel, we have

$$\mathrm{SNR}_q = 10\log_{10}(9.83\times 10^4) = 49.93\text{ dB}$$

SOL 6.1.18 Option (B) is correct.

Given the bandwidth of message signal

$$W = 1\text{ kHz} = 1000\text{ Hz}$$

Sampling frequency

$$\begin{aligned} f_s &= 1800 \text{ sample/sec} \\ &= 1800 \text{ Hz} \end{aligned}$$

Since, from the Nyquist criterion, we know that the sampling rate must be greater than the Nyquist rate given as

$$f_s \ge f_N = 2f_m$$

So, for the sampling frequency $f_s = 1800\text{ Hz}$, the message signal recovered has the maximum frequency,

$$f_m \le \frac{f_s}{2} = \frac{1800}{2} = 900 \text{ Hz}$$

Thus, for the applied message signal of bandwidth $W = 1\text{ kHz} = 1000\text{ Hz}$, the recovered message signals are of the frequency less or equal to 900 Hz. i.e. the output of the filter contains 800 Hz and 900 Hz components.

SOL 6.1.19 Option (D) is correct.

Given the message signal

$$x(t) = 100\cos(24\pi\times 10^3 t)$$

Sampling period, $T_s = 50\,\mu\text{sec}$

Cut off frequency of the low pass filter $f_c = 15\text{ kHz}$

So, we have the sampling frequency

$$f_s = \frac{1}{T_s} = \frac{1}{50\times 10^{-6}} = 20 \text{ kHz}$$

Also, the frequency of the message signal

$$f_m = \frac{24\pi\times 10^3}{2\pi} = 12 \text{ kHz}$$

Now, for a message signal of frequency $f_m$ and sampling frequency $f_s$, the output consists the frequency components as $nf_s \pm f_m$, where $n = 0, 1, 2...$

So, for the given signal, we have frequency components of sampled output as

$$\begin{aligned} f_m &= 12 \text{ kHz} \\ f_s + f_m &= 20 + 12 = 22 \text{ kHz} \\ f_s - f_m &= 20 - 12 = 8\text{ kHz} \\ 2f_s + f_m &= 40 + 12 = 52 \text{ kHz} \\ 2f_s - f_m &= 40 - 12 = 28 \text{ kHz and so on} \end{aligned}$$

When these signal components are passed through low pass filter of cut-off frequency $f_c = 15$ kHz, we get the output with frequency components $f_m = 12$ kHz, $f_s - f_m = 8$ kHz

**SOL 6.1.20** Option (C) is correct.

Given the message signal

$$x(t) = 5\left(\frac{\sin 2\pi 1000t}{\pi t}\right)^3 + 7\left(\frac{\sin 2\pi 1000t}{\pi t}\right)^2$$

For this signal, we have the maximum frequency

$$f_m = 3 \times 1000 = 3000 \text{ Hz}$$

So, the minimum sampling frequency (Nyquist frequency) required to reconstruct the signal is

$$\begin{aligned} f_s &= 2f_m \\ &= 2 \times 3000 = 6000 \text{ Hz} \\ &= 6 \times 10^3 \text{ Hz} \end{aligned}$$

**SOL 6.1.21** Option (B) is correct.

Given the message signal,

$$m(t) = 12s[u(t) - u(t-1)] + (250 - 125t)[u(t-1) - u(t-2)]$$

Sampling frequency,

$$\begin{aligned} f_s &= 32 \text{ k samples/sec} \\ &= 32 \text{ kHz} \end{aligned}$$

Now, condition to avoid the slope overload is given as

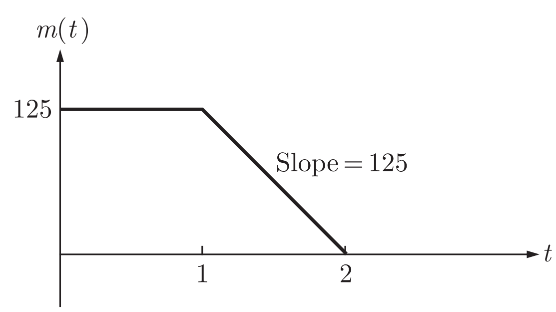
$$\frac{\delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right|$$

where $\delta$ is the step size of the delta modulator and $T_s$ is the sampling interval.

For the given message signal, we obtain

$$\begin{aligned} m(t) &= 125[u(t) - u(t-1)] + (250 - 125t)[u(t-1) - u(t-2)] \\ &= 125u(t) - 125(t-1)u(t-1) + 125(t-2)u(t-2) \end{aligned}$$

The waveform for this signal is shown below




So, from the waveform it is observed that

$$\max\left|\frac{dm(t)}{dt}\right| = 125$$

Therefore, the condition to avoid slope overload becomes

$$\frac{\delta}{T_s} \geq 125$$

or $$\delta \geq \frac{125}{f_s} = \frac{125}{32 \times 10^3} = \frac{1}{256}$$

So, the minimum value of step size is

$$\delta = \frac{1}{256} = 2^{-8}$$

SOL 6.1.22 Option (D) is correct.

The signal to quantization noise ratio for a PCM system is given by

$$\text{SNR} = \frac{S_o}{N_o} = 3q^2\frac{\overline{m^2(t)}}{m_p}$$

where $q$ is the number of quantization levels, $\overline{m^2(t)}$ is the power in the message signal $m(t)$ and $m_p$ is the peak amplitude of $m(t)$. Since, the number of bits in PCM system is increased from $n$ to $n+1$, so we have the number of quantization levels in the two cases as

$$q_1 = 2^n$$

$$q_2 = 2^{n+1}$$

Therefore, The factor by which the new SNR increase is given by

$$\frac{(\text{SNR})_2}{(\text{SNR})_1} = \frac{q_2^2}{q_1^2}$$

$$= \frac{(2^{n+1})^2}{(2^n)^2} = 4 \times \frac{2^{2n}}{2^{2n}} = 4$$

SOL 6.1.23 Option (C) is correct.

The transmission bandwidth of a channel for PCM system is given by

$$B_T = R_b = nf_s \qquad \text{...(1)}$$

where $R_b$ is the bit rate, $n$ is the number of bits per sample, and $f_s$ is the sampling frequency. Also, the quantization level are defined as

$$q = 2^n$$

So, we have

$$n = \log_2 q$$

Substituting it in equation (1), we have

$$B_T = n\log_2 q$$

So, for change in the quantization level from $q_1 = 2$ to $q_2 = 8$, the change in bandwidth is obtained as

$$\frac{B_{T_1}}{B_{T_2}} = \frac{n\log_2 q_1}{n\log_2 q_2} = \frac{n\log_2 2}{n\log_2 8}\backslash$$

$$3BT_1 = B_{T_2}$$

i.e., the bandwidth requirement will be tripled.

SOL 6.1.24 Option (A) is correct.

Given, the highest frequency tone to be recorded,

$$f_m = 15800 \text{ Hz}$$

So, according Nyquists sampling criterion, the minimum sampling frequency required for the signal is

$$f_s = f_{\text{Nyquist}} = 2f_m$$

$$= 2 \times 15800$$

$$= 31600 \text{ Hz}$$

$$= 3.16 \times 10^4 \text{ Hz}$$

SOL 6.1.25 Option (B) is correct.

We have the minimum sampling frequency for the recording,

$$f_s = 3.16 \times 10^4 \text{ Hz}$$

$$= 3.16 \times 10^4 \text{ samples/ sec}$$

So, the number of sample required to store a three minutes performance is

$$\text{Number of sample} = f_s \times \text{Time duration}$$

$$= (3.16 \times 10^4)(3 \times 60)$$
$$= 5.688 \times 10^6 \text{ samples}$$

SOL 6.1.26 Option (C) is correct.

Given, the number of quantized levels,

$$q = 128 \text{ levels.}$$

Also, we have the number of samples required to store the three minutes performance

$$\text{no. of samples} = 5.688 \times 10^6 \text{ samples}$$

For 128 quantized levels, required number of bits is obtained as

$$2^n = 128$$
$$n = 7 \text{ bits}$$

Thus, 7 bits per sample are used to quantify the signal. Therefore, we have the number of bits required to store the three minutes performance as

$$\text{number of bits} = (\text{number of samples}) \times (\text{number of bits per sample})$$
$$= (5.688 \times 10^6) \times 7$$
$$= 3.98 \times 10^7 \text{ bits}$$

SOL 6.1.27 Option (B) is correct.

Given the rms voltage at sending end, $v_i = 1$ volt rms

Length of the cable, $l = 1000$ m

Attenuation in the cable, $\alpha = 1$ dB/m

So, we have the total attenuation in the cable as

$$\alpha l = 1000 \text{ dB}$$

Thus, the rms voltage at receiving end is obtained as

$$20\log_{10}\left(\frac{v_o}{v_i}\right) = -1000 \text{ dB}$$
$$\log_{10}\frac{v_o}{v_i} = -50$$
$$v_o = v_i 10^{-50} = 1 \times 10^{-50} = 10^{-50} \text{ volt}$$

SOL 6.1.28 Option (B) is correct.

Given, the voltage gain of repeater

$$G = 100 \text{ V/V}$$

So, the gain in decide is obtained as

$$G = 20\log_{10}100$$
$$= 40 \text{ dB}$$

Since, the cable provides the attenuation of 1 dB/m. So, we need a repeater for every 40 m of cable. Thus, the total number of repeater required along the cable is

$$\text{no. of repeater} = \frac{\text{length of cable}}{40}$$
$$= \frac{1000}{40} = 25$$

SOL 6.1.29 Option (A) is correct.

Given, number of quantization levels, $q = 4$. Since, the random signal ranges between

$$-4 \le X(t) \le 4 \text{ volt}$$

So, we have the peak-to-peak amplitude of the signal as

$$2m_p = 4-(-4) = 8 \text{ volt}$$



Therefore, the step size of each level is

$$\delta = \frac{2m_p}{q} = \frac{8}{4} = 2 \text{ volt}$$

SOL 6.1.30 Option (D) is correct.

Given, the probability density of signal $X(t)$,

$$f_{X(t)}(x) = \begin{cases} ke^{-|x|}, & |X| \le 4 \text{ volt} \\ 0 \quad , & \text{otherwise} \end{cases}$$

From the property of probability density function, we know that

$$\int_{-\infty}^{\infty} f_{X(t)}(x)dx = 1$$

So, we solve the above equation for value of $k$ as

$$\int_{-4}^{4} ke^{-|x|}dx = 1$$

or $$2k\int_0^4 e^{-x}dx = 1$$

or $$2k\left[\frac{e^{-x}}{-1}\right]_0^4 = 1$$

or $$2k[-e^{-4}+1] = 1$$

Thus, $$k = \frac{0.5}{1-e^{-4}} = 0.5093$$

SOL 6.1.31 Option (D) is correct.

We have the four quantization levels in the range $-4 \le X \le 4$ given by

$$x_1 = -3 \text{ volt}$$
$$x_2 = -1 \text{ volt}$$
$$x_3 = 1 \text{ volt}$$
$$x_4 = 3 \text{ volt}$$

So, for any value between $-2$ to $-4$, the quantized value is $-3$. So for any value $x$ in the range $-2 < x < 4$, the quantization error is

$$e(X) = (-3-x)$$

Similar discussions can be made for other quantization levels. So, we get the variance of quantization error is

$$\text{var}[e(X)] = \int_{-4}^{-2}(-3-x)^2 ke^{-|x|}dx + \int_{-2}^{0}(-1-x)^2 ke^{|-x|}dx + \int_0^2 (1-x)^2 ke^{-|x|}dx + \int_2^4 (3-x)^2 ke^{-|x|}dx$$

$$= 2k\int_0^2 (1-x)^2 e^{-x}dx + 2k\int_2^4 (3-x)^2 e^{-x}dx$$

$$= 2k\left[(1-x)^2(-e^{-x}) - \{-2(1-x)\}e^{-x} + 2(-e^{-x})\right]_0^2 + 2k\left[(3-x)^2(-e^{-x}) - \{-2(3-x)\}e^{-x} + 2(-e^{-x})\right]_2^4$$

$$= 2k[e^{-x}(-1-x^2+2x+2-2x-2)]_0^2 + 2k[e^{-x}(-9-x^2+6x+6-2x-2)]_0^4$$

$$= 2k[e^{-x}(-1-x^2)]_0^2 + 2k[e^{-x}(-5-x^2+4x)]_2^4$$

$$= 2k[e^{-2}(-5)-(-1)] + 2k\begin{bmatrix} e^{-4}(-5-16+16) \\ \qquad -e^{-2}(-5-4+8) \end{bmatrix}$$

$$= 2\times 0.5093[1-5e^{-2}+e^{-2}-5e^{-4}]$$

$$= 0.3739$$

SOL 6.1.32 Option (A) is correct.

Given, the message signals

$$m_1(t) = \operatorname{sinc}(100t) = \frac{\sin(100\pi t)}{100\pi t}$$

$$m_2(t) = \operatorname{sinc}^2(100t) = \frac{\sin^2(100\pi t)}{(100\pi t)^2}$$

$$= \frac{1}{(100\pi t)^2}\left[\frac{1-\cos(200\pi t)}{2}\right]$$

So, we have the corresponding maximum frequency for these signals as

$$f_{m1} = \frac{100\pi}{2\pi} = 50 \text{ Hz}$$

$$f_{m2} = \frac{200\pi}{2\pi} = 100 \text{ Hz}$$

Therefore, the corresponding Nyquist rates are obtained as

$$f_{N_1} = 2f_{m1} = 2\times 50\,Hz = 100 \text{ Hz}$$

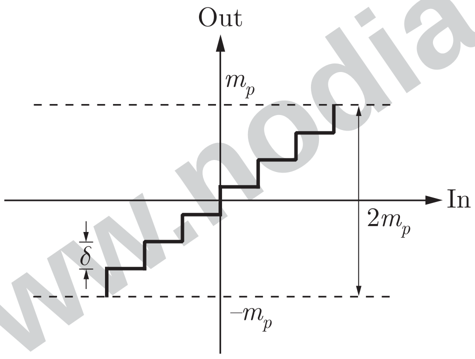
$$f_{N_2} = 2f_{m2} = 2\times 100 = 200 \text{ Hz}$$

SOL 6.1.33 Option (C) is correct.

Let the Binary PCM consists $n$ bits per sample. So, we have the quantization level,

$$q = 2^n \text{ Levels}$$

A quantized signal with peak-to-peak value $2m_p$ is shown in figure below.






From the figure, we have the step size

$$\delta = \frac{2m_p}{q}$$

So, the maximum quantization error is given by

$$|\text{error}| = \frac{\delta}{2} = \frac{m_p}{q}$$

Since, the quantizing noise is not to exceed $\pm x$ % of the peak-to-peak analog level. So, we have

$$|\text{error}| \le \frac{x}{100}(2m_p)$$

or $$\frac{m_p}{q} \le \frac{x}{100}(2m_p)$$

or $$\frac{1}{q} \le \frac{x}{50}$$

or $$q = 2^n \ge \frac{50}{x}$$

Thus, the required number of bits in each PCM word is

$$n \ge \log_2\left(\frac{50}{x}\right)$$

SOL 6.1.34 Option (C) is correct.
Given the absolute bandwidth of analog waveform,
$$W = 100 \text{ Hz}$$
So, the minimum sampling rate (Nyquist rate) for the waveform is obtained as
$$f_N = 2W = 2 \times 100 = 200 \text{ Hz}$$

SOL 6.1.35 Option (B) is correct.
We have the Nyquist rate for the waveform,
$$f_N = 200 \text{ Hz}$$
The peak-to-peak amplitude of message signal,
$$2m_p = 10 - (-10) = 20 \text{ V}$$
So, we have the maximum quantization error for the system,
$$|\text{error}| = \frac{\delta}{2} = \frac{2m_p}{2q} = \frac{m_p}{q}$$
Since, the accuracy for PCM system is $\pm 0.1\%$. So, we have
$$|\text{error}| \le 0.1\% \text{ of } (2m_p)$$
or $$\frac{m_p}{q} \le \frac{0.1}{100} \times 2m_p$$
or $$q \ge 500$$
or $$2^n \ge 500$$
or $$n \ge \log_2 500 = 8.96$$
i.e. the minimum number of bits per sample is 8.96. Approximating the value, we consider
$$n = 9 \text{ bits/samples}$$
Thus, the minimum bit rate for the transmitted signal is
$$\begin{aligned} R_b &= nf_s \\ &= (9 \text{ bits/sample}) \times (200 \text{ samples/ sec}) \\ &= 1800 \text{ bits/ sec} \end{aligned}$$
Therefore, the minimum absolute channel bandwidth required for transmission of the PCM is
$$\begin{aligned} B &= \frac{R_b}{2} = \frac{1800 \text{ bits/ sec}}{2} \\ &= 900 \text{ Hz} \end{aligned}$$

SOL 6.1.36 Option (C) is correct.
Given, the sampling frequency of signal,
$$f_s = 8 \text{ k samples/sec} = 8 \text{ kHz}$$
Average SNR (signal to noise ratio) in the PCM system,
$$\left(\frac{S}{N}\right)_{\text{ave}} = 30 \text{ dB} = 10^3 = 1000 \qquad ...(1)$$
In PCM system, we consider the noise due to quantization only. So, we have the average signal to quantization noise ratio defined as
$$\left(\frac{S}{N}\right)_{\text{ave}} = q^2 \qquad ...(2)$$
From equations (1) and (2), we get
$$q^2 = 1000$$
or $$q = \sqrt{1000} = 31.6 \approx 32$$
This is the number of quantization level. So, we obtain the number of bits per sample as

$$n = \log_2 q = \log_2 32 = 5 \text{ bits/sample}$$

Therefore, the bit rate of the voice frequency signal is given by

$$R_b = nf_s = (5 \text{ bits/sample}) \times (8 \text{ ksamples/ sec})$$
$$= 5 \times 8 = 40 \text{ k bits/ sec}$$

Converting it into bytes/sec, we have

$$R_b = \frac{40}{8} \text{ k bytes/ sec} = 5 \text{ k bytes/ sec}$$

Since, an 820 Mbyte hard disk is used to store the PCM data, so the duration of stored voice frequency conversation is

$$T = \frac{850 \text{ Mbyte}}{5 \text{ k bytes/ sec}}$$
$$= \frac{850 \times 10^6}{5 \times 10^3} \text{ sec}$$
$$= 170 \times 10^3 \times \frac{1}{60} \text{ minute}$$
$$= 2.833 \times 10^3 \text{ minute}$$
$$= 2833 \text{ minute}$$

SOL 6.1.37 Option (A) is correct.

Given, the peak signal to quantizing noise ratio,

$$\left(\frac{S}{N}\right)_{\text{peak}} = 55 \text{ dB} = 10^{5.5}$$
$$3q^2 = 10^{5.5}$$
$$q = \sqrt{\frac{10^{5.5}}{3}} = 3.247 \times 10^2$$
$$n = \log_2 q = \log_2 (3.247 \times 10^2)$$
$$= 8.34$$

Since, we are solving for the least value of quantizing noise ratio, so we use

$$n = 9 \text{ bits}$$

Therefore, the required number of quantizing steps is

$$q = 2^n = 2^9 = 512$$

SOL 6.1.38 Option (A) is correct.

We have the required number of bits per sample,

$$n = 9 \text{ bits/sample}$$

Also, the bandwidth of analog signal,

$$W = 4.2 \text{ MHz}$$

So, we get the sampling frequency of the signal,

$$f_s = 2W = 2 \times (4.2) = 8.4 \text{ MHz}$$

Therefore, the bit rate of the signal is

$$R_b = nf_s$$
$$= (9 \text{ bits/sample}) \times (8.4 M \text{ samples/ sec})$$
$$= 75.6 M \text{ bit/ sec}$$

For rectangular pulse shape, the null channel bandwidth is obtained as

$$B_{\text{null}} = R_b = 75.6 \text{ MHz}$$

SOL 6.1.39 Option (C) is correct.

Given, the analog signal bandwidth

$$W = 20 \text{ kHz} = 20 \times 10^3 \text{ Hz}$$

Number of bits per sample,

$$n = 16 \text{ bits/sample}$$

Since, PCM uses 8 times oversampling of the analog signal, so we get the sampling frequency

$$f_s = 8f_N = 8 \times (2W)$$
$$= 8 \times (2 \times 20 \times 10^3)$$
$$= 320 \text{ k samples/ sec}$$

Thus, the null bandwidth of this PCM signal is given by

$$B_{\text{null}} = R_b = nf_s$$
$$= (16 \text{ bits/sample}) \times (320 \text{ k samples/ sec})$$
$$= 512 \text{ MHz}$$

SOL 6.1.40 Option (C) is correct.

We have the number of bits per sample,

$$n = 16 \text{ bit}$$

Since, the PCM includes one parity bit, so we get the number of quantization level for the PCM as

$$q = 2^{n-1} = 2^{16-1} = 2^{15}$$

Therefore, the peak SNR for the signal is given by

$$\left(\frac{S}{N}\right)_{\text{peak}} = 3q^2$$
$$= 3(2^{15})^2 = 3 \times 2^{30}$$

In decibel, we get

$$\left(\frac{S}{N}\right)_{\text{peak}} = 10\log_{10}(3 \times 2^{30})$$
$$= 95.08 \text{ dB}$$

SOL 6.1.41 Option (A) is correct.

Given, the bit error rate due to channel noise (probability of bit error),

$$P_e = 10^{-4}$$

The peak signal to noise ratio on the recovered analog signal,

$$\left(\frac{S}{N}\right)_{\text{peak}} \geq 30 \text{ dB} = 10^3 = 1000 \qquad ...(1)$$

Again, for a PCM signal we have the peak signal to noise ratio,

$$\left(\frac{S}{N}\right)_{\text{peak}} = \frac{3q^2}{1 + 4(q^2 - 1)P_e}$$
$$= \frac{3 \times 2^{2n}}{1 + 4(2^{2n} - 1)10^{-4}} \qquad ...(2)$$

where $n$ is number of bits per sample. Equations (1) and (2) can be solved for the value of $n$ by hit and trial method. Firstly, we put $n = 4$ in equation (2)

$$\left(\frac{S}{N}\right)_{\text{peak}} = \frac{3 \times 2^{2 \times 4}}{1 + 4(2^{2 \times 4} - 1)10^{-4}}$$
$$= 6.969 \times 10^2$$
$$= 10\log_{10}(6.969 \times 10^2) \text{ dB}$$
$$= 28.4 \text{ dB}$$

Again, for $n = 5$ in equation (2) we obtain

$$\left(\frac{S}{N}\right)_{\text{peak}} = \frac{3 \times 2^{2 \times 5}}{1 + 4(2^{2 \times 5} - 1)10^{-4}}$$

$$= 2.1799 \times 10^3$$
$$= 10\log_{10}(2.1799 \times 10^3)\text{ dB}$$
$$= 33.4\text{ dB}$$

Thus, $n = 5$ satisfies the required condition. Therefore, the minimum number of quantizing steps is

$$q = 2^n = 2^5 = 32$$

SOL 6.1.42 Option (A) is correct.

Given, the bandwidth of the original analog signal,

$$W = 2.7\text{ kHz}$$

So, we get the sampling frequency of the PCM signal,

$$f_s = 2W = 2 \times 2.7\text{ kHz}$$
$$= 5.4\text{ k samples/ sec}$$

Also, we have number of bits per sample, $n = 5$ bits/sample.

Therefore, the null bandwidth of the PCM signal is

$$B = R_b = nf_s$$
$$= 5 \times (5.4\text{ k samples/ sec})$$
$$= 27\text{ kHz}$$

SOL 6.1.43 Option (C) is correct.

Given, the number of possible levels,

$$M = 16\text{ levels}$$

So, the number of bits corresponding to each level is obtained as

$$k = \log_2 M = \log_2 16$$
$$= 4\text{ bits/level}$$

SOL 6.1.44 Option (C) is correct.

Given that the digital communication system sends one level over the channel every 0.8 ms. so, we get the baud rate (symbol rate) of the signal as

$$D = \frac{1}{T_s} = \frac{1}{0.8 \times 10^{-3}}$$
$$= 1.25 \times 10^3$$
$$= 1250\text{ baud}$$
$$= 1250\text{ levels/ sec}$$

SOL 6.1.45 Option (A) is correct.

We have the baud rate of symbol,

$$D = 1250\text{ levels/ sec}$$

Number of bits per level,

$$k = 4\text{ bits/level}$$

So, we get the bit rate for the signal as

$$R_b = kD$$
$$= (4\text{ bits/level}) \times (1250\text{ levels/ sec})$$
$$= 5000\text{ bits/ sec}$$
$$= 5\text{ k bits/ sec}$$

SOL 6.1.46 Option (B) is correct.

Given, the bandwidth of analog signal

$$W = 2700\text{ Hz}$$

So, we have the sampling frequency,

$$f_s = 2W = 2 \times 2700$$
$$= 5400 \text{ Hz} = 5.4 \text{ kHz}$$

Now, accuracy at the output is $\pm 1\%$ of full scale $V_{PP}$. So, we have the step size of quantization.

$$\delta \le 2 \times \frac{1}{100} V_{PP}$$
$$\frac{V_{PP}}{2^n} \le 2 \times \frac{1}{100} V_{PP}$$
$$n \ge \log_2 \frac{100}{2} = 5.64$$

Therefore, we use $n = 6$. Thus, the minimum bit rate of the PCM signal is obtained as

$$R_b = nf_s = 6 \times 5.4$$
$$= 32.4 \text{ k bits/ sec}$$

SOL 6.1.47 Option (D) is correct.

We have the minimum bit rate of PCM signal as

$$R_b = 32.4 \text{ k bits/ sec}$$

This signal is converted to $M = 8$ levels signals. So, we have the number of bits per level (symbol) as

$$k = \log_2 M = \log_2 8 = 3 \text{ bits/symbol}$$

Therefore, we obtain the baud rate (symbol/sec) as

$$D = \frac{R_b}{k}$$
$$= \frac{32.4 \text{ k bits/ sec}}{3 \text{ bits/symbol}}$$
$$= 10.8 \text{ k symbol/ sec}$$

SOL 6.1.48 Option (B) is correct.

We have the baud rate of signal,

$$D = 10.8 \text{ k symbols/ sec}$$

So, we obtain the minimum absolute channel bandwidth required for transmission of PCM signal as

$$B = \frac{D}{2} = \frac{10.8}{2}$$
$$= 5.4 \text{ kHz}$$

SOL 6.1.49 Option (C) is correct.

Given, the bit rate of binary waveform,

$$R_b = 9600 \text{ bits/ sec}$$

This waveform is converted into octal waveform, so we have the number of levels,

$$M = 8$$

Therefore, the number of bits per level is obtained as

$$k = \log_2 8 = 3 \text{ bits/symbol}$$

Thus, the baud rate of the multilevel signal is obtained as

$$D = \frac{R_b}{k} = \frac{9600 \text{ bits/ sec}}{3 \text{ bits/symbol}}$$
$$= 3.2 \text{ k symbol/ sec}$$

SOL 6.1.50 Option (B) is correct.

We have the absolute bandwidth of the system,

$$B = 2.4\text{ kHz}$$

Also, the baud rate is

$$D = 3.2\text{ k symbols/ sec}$$

Since, the baud rate for a system is defined as

$$D = \frac{2B}{1+\alpha}$$

where $\alpha$ is the roll off factor. Substituting the given values, we obtain the roll off factor of the filter characteristic as

$$3.2 \times 10^3 = \frac{2 \times 2.4 \times 10^3}{1+\alpha}$$

or $$\alpha = \frac{1}{2} = 0.5$$

SOL 6.1.51 Option (A) is correct.

Given, the channel bandwidth

$$B = 4\text{ kHz}$$

Since the PCM signal is a binary polar NRZ line code $(M = 2)$, so we have number of bits per symbol.

$$k = 1\text{ bits/symbol}$$

Therefore, the maximum PCM bit rate is obtained as

$$R_b = kD = D = \frac{2B}{1+\alpha}$$

where $D$ is the baud rate (symbols/sec) and $\alpha$ is the roll off factor. So, we get

$$R_b = \frac{2 \times 4\text{ kHz}}{1+0.5} = 5.33\text{ k bits/ sec}$$

This is the maximum PCM bit rate that can be supported by this system without introducing ISI.

SOL 6.1.52 Option (A) is correct.

We have the maximum PCM bit rate, $R_b = 5.33\text{ k bits/ sec}$

Number of quantization level, $q = 16$

So, we get the number of bits per sample for the PCM signal as

$$n = \log_2 q = \log_2 16 = 4$$

Therefore, the sampling frequency for the signal is given by

$$f_s = \frac{R_b}{n}$$

$$= \frac{5.33 \times 10^3}{4} = 1.33\text{ kHz}$$

Thus, the maximum bandwidth of analog signal is

$$W = \frac{f_s}{2} = \frac{1.33}{2} = 0.666\text{ kHz} = 666\text{ Hz}$$

SOL 6.1.53 Option (D) is correct.

Given, the bit rate of data,

$$R_b = 2400\text{ bits/ sec}$$

This data is transmitted over a channel using $M = 4$ level line code. So, we get the number of bits per level,

$$k = \log_2 M = \log_2 4 = 2\text{ bits/symbol}$$

Therefore, the baud rate is given by

$$\begin{aligned} D &= \frac{R_b}{k} \\ &= \frac{2400 \text{ bits/sec}}{2 \text{ bits/symbol}} \\ &= 1200 \text{ symbols/sec} \end{aligned}$$

Thus, the 6 dB bandwidth of the channel is obtained as

$$\begin{aligned} B_{6\,\text{dB}} &= \frac{D}{2} = \frac{1200 \text{ symbols/sec}}{2} \\ &= 600 \text{ Hz} \end{aligned}$$

SOL 6.1.54 Option (A) is correct.

We have the baud rate (symbol rate) for the system as

$$D = 1200 \text{ symbols/sec}$$

Also, the roll off factor for the Nyquist characteristic is

$$\alpha = 0.5$$

So, we get the absolute bandwidth for the system as

$$\begin{aligned} B_{\text{absolute}} &= \frac{1}{2}(1+\alpha)D \\ &= \frac{1}{2}(1+0.5) \times 1200 \\ &= 900 \text{ Hz} \end{aligned}$$

SOL 6.1.55 Option (A) is correct.

Given, the frequency of message signal, $f_m = 10 \text{ kHz}$

Peak-to-peak amplitude of the test signal, $2A_m = 1 \text{ volt}$

So, we have the Nyquist rate for the signal, $f_N = 2f_m = 2 \times 10 = 20 \text{ kHz}$

Since, the signal is sampled at 10 times the Nyquist rate, so we get the sampling frequency

$$f_s = 10 f_N = 10 \times 20 = 200 \text{ kHz}$$

To avoid the slope overload, we have

$$\begin{aligned} \delta f_s &\geq \max\left|\frac{dm(t)}{dt}\right| \\ \delta \times 200 \text{ kHz} &\geq 2\pi f_m A_m \\ \delta &\geq \frac{2\pi \times (10 \text{ kHz}) \times \left(\frac{1}{2}\right)}{200 \text{ kHz}} \\ \delta &\geq 0.157 \text{ volt} \end{aligned}$$

SOL 6.1.56 Option (D) is correct.

For a delta modulation system, the power spectral density of granular noise is obtained as

$$\begin{aligned} S_N(f) &= \frac{\delta^2}{6f_s} = \frac{(0.157)^2}{6 \times 200 \text{ kHz}} \\ &= 2.06 \times 10^{-8} \text{ V}^2/\text{Hz} \end{aligned}$$

SOL 6.1.57 Option (A) is correct.

We have the power spectral density, of granular noise

$$S_N(f) = 2.06 \times 10^{-8} \text{ V}^2/\text{Hz}$$

Bandwidth of sampled signal,

$$B = f_s = 200 \text{ kHz}$$

So, we have the total noise at receiver output as

$$\begin{aligned} P_{no} &= S_N(f) \times 2B \\ &= (2.06 \times 10^{-8}) \times (2 \times 200 \times 10^3) \\ &= 8.24 \times 10^{-3} \text{ W} \end{aligned}$$

Also, we have the maximum amplitude of test signal,

$$A_m = \frac{1}{2} \text{ volt}$$

Therefore, the signal power at the receiver output is

$$P_{so} = \frac{A_m^2}{2} = 0.125 \text{ W}$$

Thus, the signal to quantization noise ratio at receiver output is

$$\begin{aligned} \left(\frac{S}{N}\right)_0 &= \frac{P_{so}}{P_{no}} \\ &= \frac{0.125}{8.24 \times 10^{-3}} = 15.2 \text{ W/W} \end{aligned}$$

In decibel, we get

$$\begin{aligned} \left(\frac{S}{N}\right)_{\text{dB}} &= 10\log(15.2) \\ &= 11.81 \text{ dB} \end{aligned}$$

SOL 6.1.58 Option (D) is correct.

Given, the bandwidth of five message signals, respectively as

$$W, W, 2W, 4W, 4W$$

Since, the bandwidths of message signals are harmonically related, so we have the minimum transmission bandwidth for TDM signal as

$$\begin{aligned} B &= W + W + 2W + 4W + 4W \\ &= 12W \end{aligned}$$

***********

# SOLUTIONS 6.2



SOL 6.2.1 Correct answer is 50.

Given, the bandwidth of each signal, $f = 1\,\text{kHz}$

From the Nyquist criterion the sampling rate must be such that $f_s \geq 2f$

Here, the period $T_s$ of $c(t)$ is chosen for the maximum allowable value. So, we have the sampling frequency (frequency of pulse train)

$$f_s = 2f = 2 \times 10^3 \text{ Hz}$$

Therefore, the time period of pulse train is given by

$$T_s = \frac{1}{f_s} = \frac{1}{2 \times 10^3} = 0.5 \times 10^{-3}\,\text{sec}$$

After sampling of the multiplexed signals (10 signals), each of the signal will appear for time duration $\triangle$. So, for the largest value of $\triangle$, we must have

$$10\,\triangle = T_s$$

$$\triangle = \frac{T_s}{10} = \frac{0.5 \times 10^{-3}}{10} = 5 \times 10^{-5}\,\text{s} = 50\,\mu\text{s}$$

SOL 6.2.2 Correct answer is 317.5.

Given the sampling rate of a signal,

$$f_s = 44.1\,\text{kHz} = 44.1 \times 10^3 \text{ samples/sec}$$

Since, recording system samples the signal with 16 bit analog to digital converter. Therefore, the rate of recording in bits/second is given by

$$f_s = (44.1 \times 10^3) \times 16 \text{ bits/sec}$$
$$= 7.056 \times 10^5 \text{ bits/sec}$$

As the CD can record an hour worth of music, the approximate capacity of CD is given by

$$\text{Memory capacity} = f_s \times \text{Time duration}$$
$$= (7.056 \times 10^5) \times (3600\,\text{sec})$$
$$= 2.54 \times 10^9 \text{ bits}$$

Converting it into bytes (1 byte = 8 bites), we get

$$\text{Memory capacity} = \frac{2.54 \times 10^9}{8} \text{ bytes}$$
$$= 3.175 \times 10^8 \text{ bytes}$$
$$= 317.5 \text{ M bytes}$$

SOL 6.2.3 Correct answer is 3.47.

Given, the sampling frequency $f_s = 36\,\text{kHz} = 36 \times 10^3\,\text{Hz}$

Number of quantization levels, $q = 256$

Let these quantization levels be encoded in binary by $n$ bits. So, we have

$$2^n = q = 256$$

or $$n = 8$$

So, there are 8 bits to represent a sample. The time duration of a sample is given by

$$T_s = \frac{1}{f_s} = \frac{1}{36 \times 10^3} = 2.78 \times 10^{-5} \text{ sec}$$

Therefore, the time duration of a bit of the binary coded signal is obtained as

$$T_b = \frac{T_s}{8} = \frac{2.78 \times 10^{-5}}{8}$$
$$= 3.47 \times 10^{-6} \text{ sec}$$
$$= 3.47 \ \mu\text{s}$$

SOL 6.2.4 Correct answer is 7.

Given, the tolerable error in sample amplitude $= 0.5\%$ of peak-peak full scale.

Let the analog signal lie in the internal $(-m_p, m_p)$; where $m_p$ is the peak value of the signal. If $q$ levels are used to quantize the signal, then the step size of each level is

$$\delta = \frac{2m_p}{q}$$

So, the maximum quantization error in the signal is obtained as

$$\text{error} = \frac{\delta}{2} = \frac{2m_p}{2q} = \frac{m_p}{q} \qquad ...(1)$$

Also, we have the tolerable error in sample amplitude

$$\text{error} = 0.5\% \text{ of peak-peak full scale}$$
$$= \frac{0.5}{100} \times (2m_p)$$
$$= 0.01\, m_p \qquad ...(2)$$

As the maximum quantization error of the signal must be less than the tolerable error, so from equations (1) and (2), we have

$$\frac{m_p}{q} \le 0.01\, m_p$$

or, $$q \ge 100$$

Let the minimum number of binary digits required to encode the sample be $n$.

Then, we have

$$2^n \ge 100$$

The minimum value of $n$ that satisfies the above condition is

$$n = 7$$

SOL 6.2.5 Correct answer is 144.

Given the bandwidth of signal, $f_m = 6\,\text{MHz} = 6 \times 10^6\,\text{Hz}$

Number of quantization levels, $q = 1024$

So, we have the Nyquist rate for the signal as

$$f_N = 2f_m$$
$$= 2 \times 6 \times 10^6$$
$$= 12 \times 10^6 \text{ Hz.}$$

Since, the sampling rate must be 20% above the Nyquist rate therefore, we get the sampling rate as

$$f_s = 1.2 \times f_N$$
$$= 1.2 \times (12 \times 10^6)$$
$$= 1.44 \times 10^7 \text{ Hz.}$$

Again, let the number of bits for quantization be $n$. So, we have

$$q = 2^n = 1024$$

or, $$n = 10 \text{ bits}$$

So, the bit rate of the sampled signal is given by

$$R_b = nf_s$$
$$= 10 \times (1.44 \times 10^7)$$
$$= 1.44 \times 10^8 \text{ bits/sec}$$
$$= 144 \text{ Mbits/sec}$$

Therefore, the maximum transmission bandwidth of the signal is

$$B_T = R_b = 144 \text{ Mbits/sec.}$$

This is the minimum channel bandwidth required to transmit the signal.

NOTE :

The maximum transmission bandwidth is defined for the case when one bit information is transmitted in a specific instant so, we have the maximum transmission bandwidth

$$B_T = R_b$$

SOL 6.2.6 Correct answer is 450.

Given, the audio signal bandwidth, $f_m = 15$ kHz

Number of quantization levels, $q = 32678$

So, we have the Nyquist sampling rate for the analog signal,

$$f_s = 2f_m$$
$$= 2 \times 15 = 30 \text{ kHz}$$

Let $n$ number of bits encode each sample, then we have

$$q = 2^n = 32678$$

or, $$n = 15 \text{ bits}$$

Therefore, the minimum bit rate for the analog signal is

$$R_b = nf_s$$
$$= 15 \times 30$$
$$= 450 \text{ kbits/s}$$

SOL 6.2.7 Correct answer is 7.

Given the rate of generation of characters

$$R_c = 1,000,000 \text{ characters/sec}$$

Total number of characters in the American standard code

$$q = 128 \text{ characters}$$

Let the minimum no. of bits required to encode these characters be $n$.

So, we have

$$q = 2^n = 128$$

or, $$n = 7 \text{ bits}$$

So, 7 bits are required to encode each character. Therefore, we have the minimum bit rate for the signal,

$$R_b = nR_c$$
$$= 7 \times 1,000,000$$
$$= 7,000,000 \text{ bits/second}$$
$$= 7 \text{ Mbits/sec}$$

So, the transmission bandwidth of the signal is

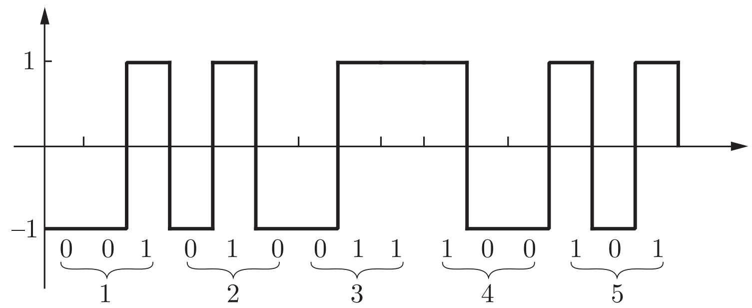
$$B_T = R_b = 7 \text{ Mbits/sec.}$$

This is the minimum channel bandwidth required to transmit the signal.

SOL 6.2.8

Correct answer is 12345.

Observing the PCM signal, we obtain the transmitted bits as shown below




In the figure, we have put 0 for level of $-1$ volt and 1 for level of $+1$ volt. Since, the code word used consists of 3 bits, so, we write the words for the transmitted bits as

1st word $\rightarrow$ 001 $\rightarrow$ 1

2nd word $\rightarrow$ 010 $\rightarrow$ 2

3rd word $\rightarrow$ 011 $\rightarrow$ 3

4th word $\rightarrow$ 100 $\rightarrow$ 4

5th word $\rightarrow$ 101 $\rightarrow$ 5

Thus, the sampled version of analog signal from which this PCM signal is driven is 12345



SOL 6.2.9

Correct answer is 6.25.

Given the number of bits of encoder, $n = 8$ bits.

Bit rate of the system, $R_b = 10^8$ bits/sec

Let the maximum message bandwidth be $W_m$. So, we have the sampling frequency for this signal as $f_s = 2W_m$.

Therefore, the bit rate for the signal is obtained as

$$\begin{aligned} R_b &= nf_s \\ 10^8 &= 8 \times (2W_m) \\ W_m &= \frac{10^8}{16} \\ &= 6.25 \times 10^6 \text{ Hz} \\ &= 6.25 \text{ MHz} \end{aligned}$$

SOL 6.2.10

Correct answer is 4.

Given, the sampling rate, $f_s = 8\,\text{kHz}$

ON time duration of the flat top samples, $\triangle = 1\,\mu\text{s}$

The time duration for synchronization, $T_{syn} = 1\,\mu\text{s}$

Since, each of the 24 voice signals are sampled for the time duration $\triangle$. So, the total time width for the 24 voice signal pulses is given by

$$\begin{aligned} T_{voice} &= 240 \\ &= 24 \times 1 = 24\,\mu\text{s} \end{aligned}$$

Therefore, the total time width required for voice signal pulses along with its synchronization pulse is

$$\begin{aligned} T_{pulse} &= T_{voice} + T_{syn} \\ &= 24 + 1 \\ &= 25\,\mu\text{s} \end{aligned}$$

Again, we have the time duration of each sample as

$$T_s = \frac{1}{f_s} = \frac{1}{8 \times 10^3} = 1.25 \times 10^{-4} \text{ sec}$$

$$= 125\,\mu\text{s}$$

Therefore, the total OFF duration in a sample is

$$T_{\text{OFF}} = T_s - T_{\text{pulse}}$$
$$= 125 - 25$$
$$= 100\,\mu\text{s}$$

Since, there are 25 pulses (24 voice signal pulses and 1 synchronization pulse) in a sample. So, the spacing between successive pulses is given by

$$T_{\text{spacing}} = \frac{T_{\text{OFF}}}{25}$$
$$= \frac{100}{25} = 4\,\mu\text{s}$$

ALTERNATIVE METHOD :

Given, the sampling frequency $f_s = 8\,\text{kHz}$

So, we have the time duration of a sample,

$$T_s = \frac{1}{f_s}$$
$$= \frac{1}{8\times 10^3} = 1.25\times 10^{-4}\,\text{sec}$$
$$= 125\,\mu\text{s}$$

Now, we have the 24 voice signals each sampled for $1\,\mu\text{s}$ duration and one synchronization pulse of $1\,\mu\text{s}$ duration. So, the total number of pulses in a sample is 25. Therefore, the alloted time duration to each of the pulse is

$$T_{s,\text{pulse}} = \frac{T_s}{25} = \frac{125}{25} = 5\,\mu\text{s}$$

Since the width of each pulse is $1\,\mu\text{s}$. Therefore, the spacing between the successive pulses is

$$T_{\text{spacing}} = T_{s,\text{ pulse}} - 1\,\mu\text{s}$$
$$= 5 - 1$$
$$= 4\,\mu\text{s}$$



SOL 6.2.11 Correct answer is 1.0823.

Given, the bandwidth of speech signal, $W_m = 3.4\,\text{kHz}$

So, the Nyquist rate of speech signal is $f_N = 2W_m = 2\times 3.4 = 6.8\,\text{kHz}$

Since, the sampling rate is 10 times the Nyquist rate so, we have

$$f_s = 10f_N = 10\times 6.8 = 6.8\ \text{kHz}.$$

Also, we have the step size for the delta modulated signal $\delta = 100\,\text{mV}$

To avoid the slope overload for this delta modulator, we have the condition

$$\frac{\delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right|$$

or, $$\max\left|\frac{dm(t)}{dt}\right| \leq \frac{100\times 10^{-3}}{1/(68\times 10^3)}$$

or, $$\max\left|\frac{dm(t)}{dt}\right| \leq 68\times 10^2$$

Now, we have the test signal with 1 kHz frequency and let the maximum amplitude of this test signal be $A_m$. therefore, substituting it in above condition, we get

$$A_m(2\pi f_m) \leq 68\times 10^2$$

or $$A_m \leq \frac{68\times 10^2}{2\pi\times 10^3} = 1.0823$$

Hence, $A_m = 1.0823$ is the maximum amplitude of test signal.

SOL 6.2.12 Correct answer is 20%.

Given the output signal to quantization noise ratio of $n_1 = 10$ bit PCM,

$$(\text{SNR})_1 = 30dB = 10^{30/10} = 10^3$$

After increasing the number of quantization level, the desired SNR

$$(\text{SNR})_2 = 42 \text{ dB} = 10^{42/10} = 10^{4.2}$$

Since, SNR for a PCM signal is given by

$$\frac{S_o}{N_o} = 3q^2 \frac{\overline{m^2(t)}}{m_p^2}$$

where $\overline{m^2(t)}$ is the power of message signal and $m_p$ is its peak value. So, we can write

$$\frac{S_o}{N_o} \propto q^2$$

or, $$\text{SNR} = kq^2$$

where $k$ is a constant for the given message signal. Here, the message signal for both the cases are same, so the value of $k$ will be constant for both the cases, whereas the value of $q$ varies.Therefore, the ratio of SNR is

$$\frac{(\text{SNR})_1}{(\text{SNR})_2} = \frac{q_1^2}{q_2^2} \qquad ...(1)$$

Initially, the quantization level is $q_1 = 2^{n_1} = 2^{10}$

For the desired value of SNR, let the system require $n_2$ bits. So, we have

$$q_2 = 2^{n_2}$$

Substituting all the values in equation (1), we get

$$\frac{10^3}{10^{4.2}} = \left(\frac{2^{10}}{2^{n_2}}\right)^2$$

$$10^{\frac{4.2-3}{2}} = 2^{n_2 - 10}$$

$$n_2 - 10 = \log_2 10^{0.6}$$

$$n_2 = 10 + 0.6\log_2 10$$

Thus, the increase in bits of PCM is

$$n_2 - n_1 = (10 + 0.6\log_2 10) - 10$$

$$= 0.6\log_2 10$$

Therefore, the percentage increase in the transmission bandwidth is given by

$$\%B_T = \frac{n_2 - n_1}{n_1} \times 100\%$$

$$= \frac{0.6\log_2 10}{10} \times 100\%$$

$$= 20\%$$

NOTE :

Since, the message signal (sampling frequency) for both the cases are same so, the percentage increase in transmission bandwidth is same as the percentage increase in number of encoding bits.

SOL 6.2.13 Correct answer is 10.

Given, the compression parameter of PCM signal, $\mu = 100$

Minimum signal to quantization noise ratio,

$$(\text{SNR})_{\min} = 50 \text{ dB} = 10^5$$

For a PCM system, the output SNR (for $\mu$-law companding) is given by

$$\frac{S_o}{N_o} = \frac{3q^2}{[\ln(1+\mu)]^2}$$

where $q$ is the number of quantization level defined as

$$q = 2^n$$

where $n$ is the number of bits required to encode the sample. So, we have

$$\frac{S_o}{N_o} = \frac{3(2^n)^2}{[\ln(1+100)]^2} > 10^5$$

$$\frac{3 \times 2^{2n}}{[\ln(101)]^2} > 10^5$$

$$2^{2n} > 7.1 \times 10^5$$

$$n > \frac{1}{2}\log_2(7.1 \times 10^5)$$

$$n > 9.72$$

Thus, the minimum number of bits per sample is

$$n = 10$$

SOL 6.2.14 Correct answer is 8.

Given, the signal to quantization ratio for the PCM,

$$\frac{S_o}{N_o} = 48\text{ dB} = 10^{4.8}$$

Now, the signal to quantization noise ratio for a PCM system is defined as

$$\frac{S_o}{N_o} = 3q^2\frac{\overline{m^2(t)}}{m_p^2} \qquad ...(1)$$

where $\overline{m^2(t)}$ is the power in the message signal $m(t)$ and $m_p$ is its peak value. Here, the message signal is sinusoid, so we have

$$\overline{m^2(t)} = \frac{1}{2}$$

and $$m_p = 1$$

Substituting it in equation (1), we have

$$\frac{S_o}{N_o} = 3q^2\frac{\frac{1}{2}}{1}$$

$$10^{4.8} = \frac{3q^2}{2}$$

$$q^2 = \frac{2 \times 10^{4.8}}{3} \qquad ...(2)$$

Now, if the number of bits per sample of PCM be $n$, then we must have

$$q < 2^n$$

Substituting it in equation (2), we get

$$(2^n)^2 > \frac{2 \times 10^{4.8}}{3}$$

or, $$n > \frac{1}{2}\log_2\left(\frac{2 \times 10^{4.8}}{3}\right)$$

or, $$n > 7.7$$

Thus, the minimum number of bits per sample is $n = 8$

SOL 6.2.15 Correct answer is 140.

Given the duration of speech signal, $T = 20$ sec.

Sampling frequency of the signal, $f_s = 8\text{ kHz}$

Signal to quantization noise ratio,

$$\frac{S_o}{N_o} = 40\text{ dB} = 10^4$$

Now, for a speech signal (sinusoid signal) we have the signal to quantization noise ratio

$$\frac{S_o}{N_o} = 3q^2\frac{\overline{m^2(t)}}{m_p}$$
$$= 3q^2\frac{1/2}{1} = \frac{3q^2}{2}$$

If $n$ be the minimum number of bits per sample so, we have

$$\frac{3q^2}{2} > 10^4$$

or
$$\frac{3(2^{2n})^2}{2} > 10^4$$

or,
$$n > \frac{1}{2}\log_2\left(\frac{2\times 10^4}{3}\right) = 6.35$$

So, we have

$$n = 7$$

Therefore, the data rate (bit rate) of the speech signal is given by

$$R_b = nf_s$$
$$= 7\times 8 \text{ kHz}$$
$$= 56 \text{ kbits/sec}$$

So, the minimum storage capacity needed to accommodate the 20 sec duration speech signal is

$$M = R_b \times T$$
$$= (56 \text{ kbits/ sec})\times(20 \text{ sec})$$
$$= 1120 \text{ kbits}$$

Converting it into bytes (1 byte = 8 bits), we get

$$M = \frac{1120}{8} = 140 \text{ k bytes.}$$

SOL 6.2.16 Correct answer is 12.

Given, the peak to peak amplitude of the message signal $m(t)$,

$$2m_p = 1.536 \text{ V or } m_p = 0.768 \text{ V}$$

number of quantization level, $q = 128$

So, the quantization noise power for the uniform mid rise quantized signal $m(t)$ is given by

$$N_o = \frac{m_p^2}{3q^2}$$
$$= \frac{(0.768)^2}{3\times(128)^2}$$
$$= 1.2\times 10^{-5}\text{ V}^2 = 12\times 10^{-6}\text{ V}^2$$

SOL 6.2.17 Correct answer is 0.0014.

Given the message signal,

$$m(t) = \sin c(700t) + \sin c(500t)$$
$$= \frac{\sin(700\pi t)}{\pi t} + \frac{\sin(500\pi t)}{\pi t}$$
$$= \frac{1}{\pi t}[\sin(700\pi t) + \sin(500\pi t)]$$

So, we have the frequency components of the signal as

$$f_1 = \frac{700\pi}{2\pi} = 350 \text{ Hz}$$

$$f_2 = \frac{500\pi}{2\pi} = 250 \text{ Hz}$$

Therefore, the signal is band limited to $f_m = 350$ Hz.

So, the Nyquist sampling rate for the signal is given by

$$f_N = 2f_m = 2 \times (350) = 700 \text{ Hz.}$$

Therefore, the Nyquist sampling interval is obtained as

$$T_N = \frac{1}{f_N} = \frac{1}{700} = 0.0014 \text{ sec.}$$

SOL 6.2.18 Correct answer is 16.

For a PCM system, the signal to quantization noise ratio is defined as

$$\text{SNR} = 3q^2 \frac{\overline{m^2(t)}}{m_p^2} = 3 \times (2)^{2n} \frac{\overline{m^2(t)}}{m_p^2}$$

where $n$ is the number of bits per sample, $\overline{m^2(t)}$ is the power in message signal, and $m_p$ is the peak amplitude of $m(t)$.

When the code word length $(n)$ is changed from $n_1 = 6$ to $n_2 = 8$ bits the signal to quantization noise ratio improves by the factor given as

$$\frac{(\text{SNR})_2}{(\text{SNR})_1} = \frac{3 \times (2)^{2n_2}}{3 \times (2)^{2n_1}} = \frac{3 \times (2)^{2 \times 8}}{3 \times (2)^{2 \times 6}} = 16$$

SOL 6.2.19 Correct answer is 384.

Given, the bandwidth of signals $g_1(t)$ and $g_4(t)$

$$f_1 = f_4 = 4 \text{ kHz}$$

Bandwidth of signals $g_2(t)$ and $g_3(t)$.

$$f_2 = f_3 = 8 \text{ kHz}$$

Required number of bits for each sample, $n = 8$ bits

For the given signals, we obtain the minimum sampling frequency (Nyquist sampling rate) as

$$f_{s1} = f_{s4} = 2 \times 4 = 8 \text{ kHz}$$

and $$f_{s2} = f_{s3} = 2 \times 8 = 16 \text{ kHz}$$

So, total number of pulses (frequency) per second for the multiplexed signal is

$$\begin{aligned} f_s &= f_{s1} + f_{s2} + f_{s3} + f_{s4} \\ &= 8 + 16 + 16 + 8 \\ &= 48 \text{ kHz} \end{aligned}$$

Therefore, the minimum transmission bit rate for the system is obtained as

$$\begin{aligned} R_b &= nf_s \\ &= 8 \times 48 \\ &= 384 \text{ k bits/sec} \end{aligned}$$

SOL 6.2.20 Correct answer is 57.6.

Given, the bandwidths of three analog signals,

$$f_1 = 1200 \text{ Hz}$$
$$f_2 = 600 \text{ Hz}$$
$$f_3 = 600 \text{ Hz}$$

Number of bits per sample, $n = 12$ bits

Since, the analog signals are sample at their Nyquist rate. So, we have their sampling frequencies as

$$f_{s_1} = 2f_1 = 2 \times 1200 = 2400 \text{ Hz}$$
$$f_{s_2} = 2f_2 = 2 \times 600 = 1200 \text{ Hz}$$
$$f_{s_3} = 2f_3 = 2 \times 600 = 1200 \text{ Hz}$$

So, we get the frequency (total number of pulses per second) for the multiplexed signal as

$$f_s = f_{s_1} + f_{s_2} + f_{s_3}$$
$$= 2400 + 1200 + 1200$$
$$= 4800 \text{ Hz}$$

Therefore, the bit rate of the multiplexed signal is

$$R_b = nf_s$$
$$= 12 \times 4800$$
$$= 57.6 \text{ kbits/sec}$$

SOL 6.2.21 Correct answer is 80.

Given, the bandwidth of each of the four signals,

$$f_m = 5 \text{ kHz}$$

Since, the signals to be sampled at twice the Nyquist rate. So, we have the sampling frequency of the signals as

$$f_s = 2f_N = 2(2f_m)$$
$$= 2 \times 2 \times 5 = 20 \text{ kHz}$$

Now, these signals are time division multiplexed. So, the total pulses per second (frequency) of the multiplexed signal is

$$f_T = 4 \times f_s = 4 \times 20 = 80 \text{ kHz}$$

This is the theoretical transmission bandwidth of the channel.



SOL 6.2.22 Correct answer is 1600.

Given, the bandwidths of four independent message signals.

$$f_1 = 100 \text{ Hz}$$
$$f_2 = 100 \text{ Hz}$$
$$f_3 = 200 \text{ Hz}$$
$$f_4 = 400 \text{ Hz}$$

Since, these signals are sampled at their Nyquist rate so, we have the sampling frequencies of the four signals as

$$f_{s1} = 2f_1 = 2 \times 100 = 200 \text{ Hz}$$
$$f_{s2} = 2f_2 = 2 \times 100 = 200 \text{ Hz}$$
$$f_{s3} = 2f_3 = 2 \times 200 = 400 \text{ Hz}$$
$$f_{s4} = 2f_4 = 2 \times 400 = 800 \text{ Hz}$$

Now, these signals are time division multiplexed. So, the total pulses per second (frequency) of the multiplexed signal is

$$f_s = f_{s1} + f_{s2} + f_{s3} + f_{s4}$$
$$= 200 + 200 + 400 + 800 = 1600 \text{ Hz}$$

This is the transmitted sample rate.

SOL 6.2.23 Correct answer is 350.

Given the message signal

$$g(t) = 10\cos(50\pi t)\cos^2(150\pi t)$$
$$= 10\cos(50\pi t)\left[\frac{1 + \cos(2 \times 150\pi t)}{2}\right]$$
$$= \frac{10}{2}\cos(50\pi t)[1 + \cos(300\pi t)]$$
$$= 5\cos(50\pi t) + 5\cos(50\pi t)\cos(300\pi t)$$
$$= 5\cos(50\pi t) + \frac{5}{2}[\cos(250\pi t) + \cos(350\pi t)]$$

So, the message signal is band limited to

$$f_m = \frac{350\pi}{2\pi} = 175 \text{ Hz}$$

Therefore, the Nyquist sampling rate for the signal is

$$f_N = 2f_m$$
$$= 2 \times 175 = 350 \text{ Hz}$$

SOL 6.2.24 Correct answer is 1280.

Given the sampling rate of one channel

$$f_{\text{ch}} = 8000 \text{ times/sec}$$
$$= 8000 \text{ Hz}$$

Since, 20 such channels are timedivision multiplexed so, we get the total pulse per second (frequency) of the multiplexed signal as

$$f_s = 20 f_{\text{ch}}$$
$$= 20 \times 8000$$
$$= 160 \text{ kHz}$$

Now, each of the sample is represented by 7 bits and contains an additional bit for synchronization. So, the total number of bits per sample is obtained as

$$n = 7 + 1 = 8 \text{ bits}$$

Therefore, the total bit rate of TDM link is given by

$$R_b = nf_s$$
$$= 8 \times 160 \text{ kHz}$$
$$= 1280 \text{ kbits/sec}$$

SOL 6.2.25 Correct answer is 0.0029.

Given, the signal

$$s(t) = \sin c(350t) + \sin c(250t)$$
$$= \frac{\sin(350\pi t)}{\pi t} + \frac{\sin(250\pi t)}{\pi t}$$
$$= \frac{1}{\pi t}[\sin(350\pi t) + \sin(250\pi t)]$$

So, the signal is band limited to

$$f_m = \frac{350\pi}{2\pi} = 175 \text{ Hz}$$

Therefore, the Nyquist sampling rate for the signal is

$$f_N = 2f_m = 2 \times 175 = 350 \text{ Hz}$$

So, the Nyquist sampling internal is given by

$$T_N = \frac{1}{f_N} = \frac{1}{350} = 0.0029 \text{ sec}$$

SOL 6.2.26 Correct answer is 4233.6.

Given the sampling frequency, $f_s = 44.1$ kHz

number of bits per sample, $n = 16$ bits

So, the resulting bit rate of the sampled signal is

$$R_b = nf_s$$
$$= 16 \times 44.1 \text{ kHz}$$
$$= 705.6 \text{ kbits/sec}$$

So, the resulting number of bits for the music with duration 50 minutes is

$$\text{Number of bits} = (50 \times 60) \times R_b$$
$$= 3000 \times 705.6 \times 10^3$$

$$= 2.1168 \times 10^9 \text{ bits}$$

This is the resulting number of bits per channel (left and right). Since, the two audio signals from the left (L) and right (R) microphones in a recording studio or a concert hall are sampled and digitized, so the overall number of bits is obtained as

$$\text{Number of bits} = 2 \times 2.1168 \times 10^9$$
$$= 4.2336 \times 10^9 \text{ bits}$$
$$= 4233.6 \text{ Mbits}$$

SOL 6.2.27 Correct answer is 256.

Given, the bandwidth of each of the four voice signals,

$$f_{m_o} = 4 \text{ kHz}$$

Number of quantization levels, $q = 256$

Since, the voice signals are sampled at Nyquist rate, so we have the sampling frequency for each of the signal as

$$f_{s0} = 2f_m = 2 \times 4 \text{ kHz}$$
$$= 8 \text{ kHz}$$

Therefore, the number of pulses per second (frequency) of the multiplexed signal is

$$f_s = 4 \times f_{s0}$$
$$= 4 \times 8 = 32 \text{ kHz}$$

So, the bit transmission rate of the time division multiplexed signal is

$$R_b = nf_s \qquad ...(1)$$

where $n$ is the number of bits per sample defined as

$$q = 2^n$$

So, $n = \log_2 256 = 8$

Substituting it in equation (1), we get the bit rate as

$$R_b = 8 \times 32 \text{ kHz}$$
$$= 256 \text{ kbits/sec}$$

SOL 6.2.28 Correct answer is 180.

Given, the maximum frequency of the analog data, $f_m = 30$ kHz

The quantization level (digitization level), $q = 6$

So, the required number of bits per sample for digitization is given by

$$2^n \geq 6$$

or, $n = 3$ bits

Now, the sampling frequency for the analog data is given by Nyquist rate as

$$f_s = 2f_m$$
$$= 2 \times 30$$
$$= 60 \text{ kHz}$$

Therefore, the rate of generated digital signal is

$$R_b = nf_s$$
$$= 3 \times 60 \text{ kHz}$$
$$= 180 \text{ kbits/sec}$$

SOL 6.2.29 Correct answer is 32.

Given, the number of quantization levels, $q = 16$

maximum signal frequency, $f_m = 4$ kHz

So, the sampling frequency for the message signal is given by its Nyquist rate as

$$f_s = 2f_m$$
$$= 2 \times 4 = 8 \text{ kHz}$$

Again, the number of bits per sample is obtained as

$$2^n \geq q$$

or, $$2^n \geq 16$$

or $$n = 4$$

Therefore, the bit transmission rate is given by

$$R_b = nf_s$$
$$= 4 \times 8$$
$$= 32 \text{ kbits/sec}$$

SOL 6.2.30 Correct answer is 64.

Given the frequency range of the speech signal

$$300 \text{ Hz} < f < 3 \text{ kHz}$$

So, the speech signal is band limited to $f_m = 3$ kHz

The sampling frequency of the signal must be greater than Nyquist rate

$$f_s \geq 2f_m = 2 \times 3 = 6 \text{ kHz}$$

Here, the sampling frequency is

$$f_s = 8 \text{ kHz}$$

This satisfies the Nyquist criterion. Since, the number of quantization levels is

$$q = 256$$

So, the number of bits per sample is

$$n = \log_2 256 = 8$$

Therefore, the output bit rate is given by

$$R_b = nf_s = 8 \times 8 = 64 \text{ kbps}$$



SOL 6.2.31 Correct answer is 128.

Given, the transmission bandwidth over the channel

$$B_T = 1 \text{ MHz}$$

This is the bandwidth at output of PSK modulator having roll factor

$$\alpha = 25\% = 0.25$$

So, we obtain the overall system bandwidth at time division multiplexer output as

$$B_{overall} = \frac{B_T}{1+\alpha}$$
$$= \frac{10^6}{1+0.25} = 8 \times 10^5 \text{ Hz}$$
$$= 800 \text{ kbits/ sec}$$

Since, the number of pulses from data source is 240 k bits/sec. So, we obtain the bit rate at the output of binary encoder as

$$R_b = 800 \text{ k bits/s} - 240 \text{ k bits/ sec}$$
$$= 560 \text{ kbits/ sec}$$

Now, sampler has the sampling frequency of 8 kHz. Since, there are 10 voice signals applied to the sampler. So, we have the overall sampling frequency,

$$f_s = 10 \times 8$$

$$= 80\text{ kHz}$$
$$= 8\times 10^4\text{ samples/second}$$

Therefore, the number of bits per sample is given by

$$n = \frac{R_b}{f_s} = \frac{560\text{ k bits/ sec}}{8\times 10^4\text{ samples/ sec}}$$
$$= 7\text{ bits/sample}$$

Thus, number of quantization level is given by

$$q = 2^n = 2^7 = 128\text{ levels}$$

SOL 6.2.32 Correct answer is 83.

The allowable voice bandwidth for telephone line transmission,

$$W = 3\text{ kHz} = 3000\text{ samples/sec}$$

The number of quantization level,

$$q = 16$$

Minimum interval allowed for reliable identification of a bit,

$$T_{\text{bit}} = 1\,\mu\text{s}$$

So, we have the maximum allowable bit rate as

$$R_{b,\max} = \frac{1}{T_{\text{bit}}} = \frac{1}{1\,\mu\text{s}} = 10^6\text{ bits/ sec}$$

Also, we get the number of bits per sample,

$$n = \log_2 q = \log_2 16 = 4\text{ bits/sample}$$

Now, let the number of voice signals that can be multiplexed be $M$ so, we have over all sampling frequency for the communication channel as

$$f_s = MW = M\times 3000$$
$$= 3000\,N\text{ samples/sec}$$

Therefore, the bit rate at the receiving end is given by

$$R_b = nf_s$$
$$= (4\text{ bits/sample})\times(3000M\text{ samples/ sec})$$
$$= 12000M\text{ bits/ sec}$$

This bit rate must be less than the maximum bit rate, i.e.

$$R_b < R_{b,\max}$$
$$12000M < 10^6$$

or $$M < \frac{10^6}{12000} = 83.33$$

or $$M \approx 83\text{ signals}$$

SOL 6.2.33 Correct answer is 3.125.

Given, the number of quantization level, $q = 256$

Bandwidth of three input signals,

$$W_1 = 5\text{ kHz},\ W_2 = 10\text{ kHz},\ W_3 = 5\text{ kHz}$$

So, we have the number of bits per sample as

$$n = \log_2 q = \log_2 256 = 8$$

Since, the bandwidth of signals are harmonically related, we have the overall bandwidth of multiplexed signals as

$$W = W_1 + W_2 + W_3$$
$$= 5 + 10 + 5 = 20\text{ kHz}$$

So, the Nyquist rate for multiplexed signal is given by

$$f_N = 2W = 2\times 20 = 40\text{ kHz}$$

This is the sampling frequency for the transmitted signal. Therefore, the bit rate of the transmitted signal is obtained as

$$\begin{aligned} R_b &= nf_s \\ &= 8 \text{ bits/sample} \times 40 \times 10^3 \text{ samples/ sec} \\ &= 320 \text{ k bits/ sec} \end{aligned}$$

Thus, the maximum bit duration for the transmitted signal is

$$\begin{aligned} T_{\text{bit}} &= \frac{1}{R_b} \\ &= \frac{1}{320 \times 10^3} \\ &= 3.125 \times 10^{-6} \text{ sec} = 3.125 \,\mu\text{sec} \end{aligned}$$

SOL 6.2.34 Correct answer is 160.

We have the bit rate of transmitted signal,

$$R_b = 320 \text{ k bits/ sec}$$

So, the minimum channel bandwidth required to pass the PCM signal is given by

$$\begin{aligned} B &= \frac{R_b}{2} \\ &= \frac{320}{2} = 160 \text{ kHz} \end{aligned}$$

SOL 6.2.35 Correct answer is 112.

Given, the time period of transmitting signal,

$$T = 71.4 \,\mu\text{s} = 71.4 \times 10^{-6} \text{ sec}$$

So, we have frequency of the signal as

$$f = \frac{1}{T} = \frac{1}{71.4 \times 10^{-6}} = 1.4 \times 10^4 \text{ Hz}$$

Therefore, the 4th harmonic of the signal is given by

$$\begin{aligned} f_{4^{\text{th}}\text{ harmonic}} &= 4 \times 1.4 \times 10^4 \\ &= 5.6 \times 10^4 \text{ Hz} \end{aligned}$$

This must be the bandwidth of the signal, i.e.

$$W = f_{4^{\text{th}}\text{harmonic}} = 5.6 \times 10^4 \text{ Hz}$$

Thus, the minimum sampling rate (Nyquist rate) for the signal is

$$\begin{aligned} f_s &= 2W = 2 \times (5.6 \times 10^4) \\ &= 11.2 \times 10^4 \text{ Hz} = 112 \text{ kHz} \end{aligned}$$

SOL 6.2.36 Correct answer is 732.5.

Given, the number of bits representing analog signal

$$n = 14 \text{ bits}$$

Peak to peak voltage range,

$$2m_p = 6 - (-6) = 12 \text{ V}$$

So, we have the number of digitzation (quantization) level,

$$q = 2^n = 2^{14} = 16384$$

Thus, the resolution of digitization is given by

$$\begin{aligned} \text{Resolution} &= \frac{12}{q-1} \\ &= \frac{12}{16384 - 1} = 7.3246 \times 10^{-4} \text{ V} \\ &= 732.5 \,\mu\text{V} \end{aligned}$$

SOL 6.2.37 Correct answer is 3.

Given, the maximum voltage range $V_{max} = 1$ volt

Input voltage of compander, $V_{in} = 0.25$ volt

For $\mu$-law compander, $\mu = 255$

So, the output voltage of compander is given by

$$V_{out} = \frac{V_{max}\ln\left[1+\frac{\mu V_{in}}{V_{max}}\right]}{\ln[1+\mu]}$$

$$= \frac{1\ln\left[1+\frac{255\times 0.25}{1}\right]}{\ln[1+255]}$$

$$= 0.752 \text{ volt}$$

Therefore, the gain of compander is obtained as

$$G = \frac{V_{out}}{V_{in}} = \frac{0.752}{0.25} = 3$$

SOL 6.2.38 Correct answer is 1200.

Given, the data rate of communication system

$$R_b = 9600 \text{ bits/ sec}$$

Number of bits encoded into each level,

$$k = 4 \text{ bits/level}$$

So, we obtain the baud rate for the system,

$$D = \frac{R_b}{k} = \frac{9600 \text{ bits/ sec}}{4 \text{ bits/level}}$$

$$= 2400 \text{ levels/ sec}$$

Therefore, the minimum required bandwidth for the channel is

$$B = \frac{D}{2} = \frac{2400}{2} = 1200 \text{ Hz}$$

SOL 6.2.39 Correct answer is 1.33.

Given, the number of levels for polar NRZ line code, $M = 4$

The bandwidth of channel, $B = 4$ kHz

Roll off factor, $\alpha = 0.5$

So, we get the number of bits per level,

$$k = \log_2 M = \log_2 4 = 2 \text{ bits/symbol}$$

Therefore, the bit rate of PCM signal is obtained as

$$R_b = kD$$

$$= (2 \text{ bits/symbol}) \times \left(\frac{2B}{1+\alpha}\right)$$

$$= 2 \times \frac{2\times 4 \text{ kHz}}{1+0.5}$$

$$= 10.67 \text{ kHz}$$

Again, we have the number of quantization level for PCM signal as $q = 16$.

So, the number of bits per sample for the PCM signal is given by

$$n = \log_2 q = \log_2 16 = 4$$

Therefore, the sampling frequency of the signal is given by

$$f_s = \frac{R_b}{n}$$

$$= \frac{10.67 \text{ kHz}}{4} = 2.67 \text{ k samples/ sec}$$

Thus, the maximum bandwidth analog signal is

$$W = \frac{f_s}{2}$$
$$= \frac{2.67\text{ kHz}}{2} = 1.33\text{ kHz}$$

SOL 6.2.40 Correct answer is 61.2.

Given, the bandwidth of audio signal, $W = 3400$ Hz

For compander, we have $\mu = 255$

For the $\mu$-law compander, we have signal to noise ratio

$$\left(\frac{S}{N}\right)_{\text{dB}} = 6.02n + 4.77 - 20\log[\ln(1+\mu)]$$

This value must be at least 40 dB, so we have

$$40 \le 6.02n + 4.77 - 20\log[\ln(1+255)]$$
$$n \ge \frac{40 - 4.77 + 20\log[\ln(256)]}{6.02}$$
$$= 8.32\text{ bits}$$

So, we use $n = 9$. Therefore bit rate is obtained as

$$R_b = nf_s$$
$$= 9 \times (2W)$$
$$= 9 \times 2 \times 3400$$
$$= 6.12 \times 10^4\text{ bits/sec}$$
$$= 61.2\text{ k bits/sec}$$

SOL 6.2.41 Correct answer is 7.

Given the error at receiving end,

$$|\text{error}| \le 0.5\% \text{ of peak-to-peak full scale value}$$

So, we have

$$\frac{1}{2}\left(\frac{2m_p}{2^n}\right) \le \frac{0.5}{100} \times 2m_p$$

or, $$2^n \ge 100$$

or, $$n \ge \log_2 100 = 6.65$$

Therefore, the minimum number of bits per sample (word) is

$$n = 7\text{ bits/word}$$

SOL 6.2.42 Correct answer is 5600.

Given the message signal,

$$m(t) = 4\sin 2\pi(10)t + 5\sin 2\pi(20)t$$

Also, we have the step size,

$$\delta = 0.05\pi$$

To avoid the slope overload, we have the required condition

$$\delta f_s \ge \max\left|\frac{dm(t)}{dt}\right|$$
$$0.05\pi f_s \ge \max\{80\pi\cos 2\pi(10t) + 200\pi\cos 2\pi(20)t\}$$
$$0.05\pi f_s \ge 280\pi$$
$$f_s \ge \frac{280\pi}{0.05\pi} = 5600\text{ Hz}$$

This is the minimum required sampling frequency.

SOL 6.2.43 Correct answer is 3600.

We have the frequency components in the message signal as

$$f_1 = 500 \text{ Hz}$$
$$f_2 = 750 \text{ Hz}$$
$$f_3 = 1800 \text{ Hz}$$

So, the message signal is bandlimited to

$$W = f_3 = 1800 \text{ Hz}$$

Therefore, the sampling frequency of the signal is obtained as

$$f_s \geq 2W = 2 \times 1800$$
$$= 3600 \text{ Hz}$$

SOL 6.2.44 Correct answer is 20.

Since, the message signal is bandlimited to

$$W = 100 \text{ kHz}$$

So, we have the Nyquist rate for the signal as

$$f_N = 2W = 2 \times 100 = 200 \text{ kHz}$$

Since, the signal is sampled at 90% of its Nyquist rate, so we have the sampling frequency of the signal as

$$f_s = \frac{90}{100} f_N$$
$$= 0.9 \times 200 = 180 \text{ kHz}$$

Therefore, the signal to distortion ratio (SDR) is obtained as

$$\text{SDR} = \frac{\int_0^{f_s/2} S(f)df}{\int_{f_s/2}^{W} S(f)df} = \frac{\int_0^{90\times10^3} (1-10^{-5}f)df}{\int_{90\times10^3}^{10^5} (1-10^{-5}f)df}$$

$$= \frac{\left[f - \frac{10^{-5}f^2}{2}\right]_0^{90\times10^3}}{\left[f - \frac{10^{-5}f^2}{2}\right]_{90\times10^3}^{10^5}}$$

$$= \frac{90\times10^3 - \frac{10^{-5}\times(90\times10^3)^2}{2}}{10^5 - \frac{10^{-5}\times(10^5)^2}{2} - 90\times10^3 + \frac{10^{-5}(90\times10^3)^2}{2}}$$

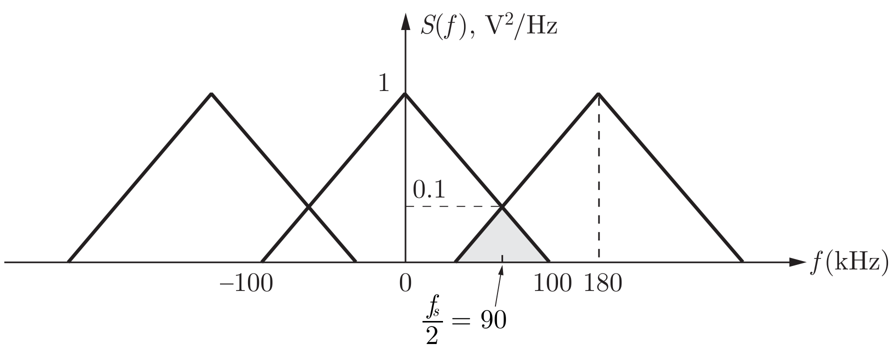
$$= 99 \text{ or } 20 \text{ dB}$$

ALTERNATIVE METHOD :

We have the sampling frequency,

$$f_s = 180 \text{ kHz}$$

If the message signal is sampled at this frequency, we get the frequency spectrum of sampled waveform as shown below.




Consider the sampled signal in the range

$$0 < f < 180 \text{ kHz}$$

For this region, we have the undistorted message signal power as

$$\begin{aligned} P_{so} &= \text{Area of unshaded region} \\ &= 2\times\left[\frac{1}{2}\times 100\times 1\right]-\frac{1}{2}\times 20\times 0.1 \\ &= 100-1=99 \end{aligned}$$

Also, the power in distorted signal is given by

$$\begin{aligned} P_{do} &= \text{Area of shaded region} \\ &= \frac{1}{2}\times 20\times 0.1=1 \end{aligned}$$

So, we get the signal to distortion ratio as

$$\text{SDR}=\frac{P_{so}}{P_{do}}=\frac{99}{1}=99 \text{ or } 20 \text{ dB}$$

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# SOLUTIONS 6.3

SOL 6.3.1 Option (C) is correct.
The required sampling rate for no aliasing in a PCM system is given by

$$f_s \geq 2f_m$$

where $f_m$ is the input message frequency. So, aliasing occurs when a signal is mistakenly sampled at less than twice the input frequency.

SOL 6.3.2 Option (B) is correct.

SOL 6.3.3 Option (D) is correct.

SOL 6.3.4 Option (B) is correct.

SOL 6.3.5 Option (D) is correct.

SOL 6.3.6 Option (C) is correct.

SOL 6.3.7 Option (C) is correct.

SOL 6.3.8 Option (A) is correct.

SOL 6.3.9 Option (D) is correct.

SOL 6.3.10 Option (A) is correct.
In flat-top sampling, the aperture effect occurs due to application of rectangular pulse. It causes an amplitude distortion and time delay.

SOL 6.3.11 Option (B) is correct.
In PCM, the quantization noise power is defined as

$$\overline{\varepsilon^2} = \frac{\delta^2}{12}$$

where $\delta$ is the step size of the quantization given by

$$\delta = \frac{2m_p}{q}$$

where $2m_p$ is the peak to peak amplitude and $q$ is the number of quantization level. Thus, the quantization noise depends on number of quantization levels.

SOL 6.3.12 Option (C) is correct.
Pulse modulation can be either analog or digital. Some typical analog and digital pulse modulations are categorized below.
Analog pulse modulation :
(i) Pulse amplitude modulation (PAM)
(ii) Pulse width modulation (PWM)
(iii) Pulse position modulation (PPM)
Digital pulse modulation :
(i) Delta modulation (DM)
(ii) Pulse code modulation (PCM)
(iii) Differential pulse code modulation (DPCM)

SOL 6.3.13 Option (D) is correct.

SOL 6.3.14 Option (C) is correct.

SOL 6.3.15 Option (D) is correct.

SOL 6.3.16 Option (C) is correct.

SOL 6.3.17 Option (A) is correct.

SOL 6.3.18 Option (D) is correct.

SOL 6.3.19 Option (D) is correct.
Analog pulse modulation results when some attributes of a pulse varies continuously in one-to-one correspondence with a sample value.
The attributes may be the amplitude, width, or position which lead to the following modulations:
(i) Pulse amplitude modulation (PAM)
(ii) Pulse width modulation (PWM)
(iii) Pulse position modulation (PPM)



SOL 6.3.20 Option (C) is correct.

SOL 6.3.21 Option (B) is correct.

SOL 6.3.22 Option (B) is correct.

SOL 6.3.23 Option (C) is correct.

SOL 6.3.24 Option (B) is correct.

SOL 6.3.25 Option (C) is correct.

SOL 6.3.26 Option (B) is correct.

SOL 6.3.27 Option (C) is correct.

SOL 6.3.28 Option (B) is correct.

SOL 6.3.29 Option (C) is correct.

SOL 6.3.30 Option (C) is correct.

SOL 6.3.31 Option (C) is correct.

SOL 6.3.32 Option (D) is correct.

SOL 6.3.33 Option (A) is correct.

SOL 6.3.34 Option (A) is correct.

SOL 6.3.35 Option (C) is correct.

SOL 6.3.36 Option (A) is correct.

SOL 6.3.37 Option (B) is correct.
The transmission bandwidth must have a value such that it passes through the channel without any attenuation.

Thus, we have

$$W \le W_{\text{channel}}$$

where $W$ is transmission bandwidth and $W_{\text{channel}}$ is channel bandwidth.

SOL 6.3.38 Option (B) is correct.
According to Nyquist theorem, the minimum sampling frequency required to ensure no loss in information is given by

$$f_s = 2f_m$$

SOL 6.3.39 Option (D) is correct.

SOL 6.3.40 Option (B) is correct.
The number of bits per sample in a PCM system is given by

$$n = \log_2 q$$

In PCM, each of $n$ bits is represented by a pulse. So, the number of pulses in a code group is given by

$$p = \log_2 q$$

SOL 6.3.41 Option (B) is correct.

SOL 6.3.42 Option (D) is correct.

SOL 6.3.43 Option (C) is correct.

SOL 6.3.44 Option (C) is correct.
In a PCM system there are two error sources:
(i) Quantization noise
(ii) Channel noise
Channel noise in the system is independent of quantization noise. So, with increase in quantization noise, channel noise will remain unaffected.

SOL 6.3.45 Option (D) is correct.

SOL 6.3.46 Option (C) is correct.

SOL 6.3.47 Option (B) is correct.

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# GATE

ELECTRONICS & COMMUNICATION

Vol 8 of 10

Eighth Edition

# GATE

## ELECTRONICS & COMMUNICATION

## Control Systems

Vol 8 of 10


**RK Kanodia**
**Ashish Murolia**



NODIA & COMPANY

GATE Electronics & Communication Vol 8, 8e
Control Systems
RK Kanodia & Ashish Murolia





MRP 540.00

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Printed by Nodia and Company, Jaipur

*To Our Parents*

## *Preface to the Series*

For almost a decade, we have been receiving tremendous responses from GATE aspirants for our earlier books: GATE Multiple Choice Questions, GATE Guide, and the GATE Cloud series. Our first book, GATE Multiple Choice Questions (MCQ), was a compilation of objective questions and solutions for all subjects of GATE Electronics & Communication Engineering in one book. The idea behind the book was that Gate aspirants who had just completed or about to finish their last semester to achieve his or her B.E/B.Tech need only to practice answering questions to crack GATE. The solutions in the book were presented in such a manner that a student needs to know fundamental concepts to understand them. We assumed that students have learned enough of the fundamentals by his or her graduation. The book was a great success, but still there were a large ratio of aspirants who needed more preparatory materials beyond just problems and solutions. This large ratio mainly included average students.

Later, we perceived that many aspirants couldn't develop a good problem solving approach in their B.E/B.Tech. Some of them lacked the fundamentals of a subject and had difficulty understanding simple solutions. Now, we have an idea to enhance our content and present two separate books for each subject: one for theory, which contains brief theory, problem solving methods, fundamental concepts, and points-to-remember. The second book is about problems, including a vast collection of problems with descriptive and step-by-step solutions that can be understood by an average student. This was the origin of *GATE Guide* (the theory book) and *GATE Cloud* (the problem bank) series: two books for each subject. *GATE Guide* and *GATE Cloud* were published in three subjects only.

Thereafter we received an immense number of emails from our readers looking for a complete study package for all subjects and a book that combines both *GATE Guide* and *GATE Cloud*. This encouraged us to present GATE Study Package (a set of 10 books: one for each subject) for GATE Electronic and Communication Engineering. Each book in this package is adequate for the purpose of qualifying GATE for an average student. Each book contains brief theory, fundamental concepts, problem solving methodology, summary of formulae, and a solved question bank. The question bank has three exercises for each chapter: 1) Theoretical MCQs, 2) Numerical MCQs, and 3) Numerical Type Questions (based on the new GATE pattern). Solutions are presented in a descriptive and step-by-step manner, which are easy to understand for all aspirants.

We believe that each book of GATE Study Package helps a student learn fundamental concepts and develop problem solving skills for a subject, which are key essentials to crack GATE. Although we have put a vigorous effort in preparing this book, some errors may have crept in. We shall appreciate and greatly acknowledge all constructive comments, criticisms, and suggestions from the users of this book. You may write to us at rajkumar.kanodia@gmail.com and ashish.murolia@gmail.com.

**Acknowledgements**

We would like to express our sincere thanks to all the co-authors, editors, and reviewers for their efforts in making this project successful. We would also like to thank Team NODIA for providing professional support for this project through all phases of its development. At last, we express our gratitude to God and our Family for providing moral support and motivation.

We wish you good luck !
R. K. Kanodia
Ashish Murolia

# SYLLABUS

## GENERAL ABILITY

**Verbal Ability :** English grammar, sentence completion, verbal analogies, word groups, instructions, critical reasoning and verbal deduction.

**Numerical Ability :** Numerical computation, numerical estimation, numerical reasoning and data interpretation.

## Engineering Mathematics

**Linear Algebra :** Matrix Algebra, Systems of linear equations, Eigen values and eigen vectors.

**Calculus :** Mean value theorems, Theorems of integral calculus, Evaluation of definite and improper integrals, Partial Derivatives, Maxima and minima, Multiple integrals, Fourier series. Vector identities, Directional derivatives, Line, Surface and Volume integrals, Stokes, Gauss and Green's theorems.

**Differential equations :** First order equation (linear and nonlinear), Higher order linear differential equations with constant coefficients, Method of variation of parameters, Cauchy's and Euler's equations, Initial and boundary value problems, Partial Differential Equations and variable separable method.

**Complex variables :** Analytic functions, Cauchy's integral theorem and integral formula, Taylor's and Laurent' series, Residue theorem, solution integrals.

**Probability and Statistics :** Sampling theorems, Conditional probability, Mean, median, mode and standard deviation, Random variables, Discrete and continuous distributions, Poisson, Normal and Binomial distribution, Correlation and regression analysis.

**Numerical Methods :** Solutions of non-linear algebraic equations, single and multi-step methods for differential equations.

**Transform Theory :** Fourier transform, Laplace transform, Z-transform.

## Electronics and Communication Engineering

**Networks :** Network graphs: matrices associated with graphs; incidence, fundamental cut set and fundamental circuit matrices. Solution methods: nodal and mesh analysis. Network theorems: superposition, Thevenin and Norton's maximum power transfer, Wye-Delta transformation. Steady state sinusoidal analysis using phasors. Linear constant coefficient differential equations; time domain analysis of simple RLC circuits, Solution of network equations using Laplace transform: frequency domain analysis of RLC circuits. 2-port network parameters: driving point and transfer functions. State equations for networks.

**Electronic Devices :** Energy bands in silicon, intrinsic and extrinsic silicon. Carrier transport in silicon: diffusion current, drift current, mobility, and resistivity. Generation and recombination of carriers. p-n junction diode, Zener diode, tunnel diode, BJT, JFET, MOS capacitor, MOSFET, LED, p-I-n and avalanche photo diode, Basics of LASERs. Device technology: integrated circuits fabrication process, oxidation, diffusion, ion implantation, photolithography, n-tub, p-tub and twin-tub CMOS process.

**Analog Circuits :** Small Signal Equivalent circuits of diodes, BJTs, MOSFETs and analog CMOS. Simple diode circuits, clipping, clamping, rectifier. Biasing and bias stability of transistor and FET amplifiers. Amplifiers: single- and multi-stage, differential and operational, feedback, and power. Frequency response of amplifiers. Simple op-amp circuits. Filters. Sinusoidal oscillators; criterion for oscillation; single-transistor and op-amp configurations. Function generators and wave-shaping circuits, 555 Timers. Power supplies.

**Digital circuits :** Boolean algebra, minimization of Boolean functions; logic gates; digital IC families (DTL, TTL, ECL, MOS, CMOS). Combinatorial circuits: arithmetic circuits, code converters, multiplexers, decoders, PROMs and PLAs. Sequential circuits: latches and flip-flops, counters and shift-registers. Sample and hold circuits, ADCs, DACs. Semiconductor memories. Microprocessor(8085): architecture, programming, memory and I/O interfacing.

**Signals and Systems :** Definitions and properties of Laplace transform, continuous-time and discrete-time Fourier series, continuous-time and discrete-time Fourier Transform, DFT and FFT, z-transform. Sampling theorem. Linear Time-Invariant (LTI) Systems: definitions and properties; causality, stability, impulse response, convolution, poles and zeros, parallel and cascade structure, frequency response, group delay, phase delay. Signal transmission through LTI systems.

**Control Systems :** Basic control system components; block diagrammatic description, reduction of block diagrams. Open loop and closed loop (feedback) systems and stability analysis of these systems. Signal flow graphs and their use in determining transfer functions of systems; transient and steady state analysis of LTI control systems and frequency response. Tools and techniques for LTI control system analysis: root loci, Routh-Hurwitz criterion, Bode and Nyquist plots. Control system compensators: elements of lead and lag compensation, elements of Proportional-Integral-Derivative (PID) control. State variable representation and solution of state equation of LTI control systems.

**Communications :** Random signals and noise: probability, random variables, probability density function, autocorrelation, power spectral density. Analog communication systems: amplitude and angle modulation and demodulation systems, spectral analysis of these operations, superheterodyne receivers; elements of hardware, realizations of analog communication systems; signal-to-noise ratio (SNR) calculations for amplitude modulation (AM) and frequency modulation (FM) for low noise conditions. Fundamentals of information theory and channel capacity theorem. Digital communication systems: pulse code modulation (PCM), differential pulse code modulation (DPCM), digital modulation schemes: amplitude, phase and frequency shift keying schemes (ASK, PSK, FSK), matched filter receivers, bandwidth consideration and probability of error calculations for these schemes. Basics of TDMA, FDMA and CDMA and GSM.

**Electromagnetics :** Elements of vector calculus: divergence and curl; Gauss' and Stokes' theorems, Maxwell's equations: differential and integral forms. Wave equation, Poynting vector. Plane waves: propagation through various media; reflection and refraction; phase and group velocity; skin depth. Transmission lines: characteristic impedance; impedance transformation; Smith chart; impedance matching; S parameters, pulse excitation. Waveguides: modes in rectangular waveguides; boundary conditions; cut-off frequencies; dispersion relations. Basics of propagation in dielectric waveguide and optical fibers. Basics of Antennas: Dipole antennas; radiation pattern; antenna gain.

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# SYLLABUS

### GATE Electronics & Communications Control Systems

Basic control system components; block diagrammatic description, reduction of block diagrams. Open loop and closed loop (feedback) systems and stability analysis of these systems. Signal flow graphs and their use in determining transfer functions of systems; transient and steady state analysis of LTI control systems and frequency response. Tools and techniques for LTI control system analysis: root loci, Routh-Hurwitz criterion, Bode and Nyquist plots. Control system compensators: elements of lead and lag compensation, elements of Proportional-Integral-Derivative (PID) control. State variable representation and solution of state equation of LTI control systems.

### IES Electronics & Telecommunication Control Systems

Transient and steady state response of control systems; Effect of feedback on stability and sensitivity; Root locus techniques; Frequency response analysis. Concepts of gain and phase margins: Constant-M and Constant-N Nichol's Chart; Approximation of transient response from Constant-N Nichol's Chart; Approximation of transient response from closed loop frequency response; Design of Control Systems, Compensators; Industrial controllers.

### IAS Electrical Engineering Control Systems

Elements of control systems; block-diagram representations; open-loop & closed-loop systems; principles and applications of feed-back. LTI systems : time domain and transform domain analysis. Stability : Routh Hurwitz criterion, root-loci, Nyquist's criterion. Bode-plots, Design of lead-lag compensators; Proportional, PI, PID controllers.

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# CONTENTS

## CHAPTER 7 STATE VARIABLE ANALYSIS

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# CHAPTER 1

## ROOT LOCUS TECHNIQUE

### 1.1 INTRODUCTION

The performance of a feedback system can be described in terms of the location of the roots of the characteristic equation, graphically, in the $s$ -plane. This qualitative nature of the solution will be examined in this chapter using the root-locus analysis. Following topics are included in the chapter:

- Basic concept of the root locus method
- Useful rules for constructing the root loci
- Effect of adding poles and zeros to $G(s)H(s)$
- Root sensitivity

### 1.2 ROOT LOCUS

A graph showing how the roots of the characteristic equation move around the $s$-plane as a single parameter varies is known as a *root locus plot*.

#### 1.2.1 The Root-Locus Concept

The roots of the characteristic equation of a system provide a valuable concerning the response of the system. To understand the root locus concept, consider the characteristics equation

$$q(s) = 1 + G(s)H(s) = 0$$

Now, we rearrange the equation so that the parameter of interest, $K$, appears as the multiplying factor in the form,

$$1 + KP(s) = 0$$

For determining the locus of roots as $K$ varies from 0 to $\infty$, consider the polynomial in the form of poles and zeros as

$$1 + \frac{K\prod_{i}^{m}(s+Z_i)}{\prod_{j}^{n}(s+P_i)} = 0$$

or $$\prod_{j}^{n}(s+P_j) + K\prod_{i}^{m}(s+Z_i) = 0$$

when $K = 0$, the roots of the characteristic equation are the poles of $P(s)$.

i.e. $$\prod_{j}^{n}(s+P_j) = 0$$

when $K = \infty$, the roots of the characteristic equation are the zeros of $P(s)$.

i.e. $$\prod_{i}^{m}(s+Z_i) = 0$$

Hence, we noted that the locus of the roots of the characteristic equation $1 + KP(s) = 0$ begins at the poles of $P(s)$ and ends at the zeros of $P(s)$ as $K$ increases from zero to infinity.

### 1.2.2 Properties of Root Locus

To examine the properties of root locus, we consider the characteristic equation as

$$1 + G(s)H(s) = 0$$

or $$1 + KG_1(s)H_1(s) = 0$$

where $G_1(s)H_1(s)$ does not contain the variable parameter $K$. So, we get

$$G_1(s)H_1(s) = -\frac{1}{K}$$

From above equation, we conclude the following result:

1. For any value of $s$ on the root locus, we have the magnitude

$$|G_1(s)H_1(s)| = \frac{1}{|K|}; \qquad -\infty < K < \infty$$

$$|G_1(s)H_1(s)| = \frac{\prod_{i=1}^{m}(s+Z_i)}{\prod_{j=1}^{n}(s+P_j)} = \frac{1}{|K|}; \quad -\infty < K < \infty$$

2. For any value of $s$ on the root locus, we have

$$\angle G_1(s)H_1(s) = (2k+1)\pi; \quad \text{where } k = 0, \pm 1, \pm 2, \ldots\ldots$$
$$= \text{odd multiple of } 180^\circ$$
$$\text{for } 0 \le K < \infty$$

$$\angle G_1(s)H_1(s) = 2k\pi; \quad \text{where } k = 0, \pm 1, \pm 2, \ldots\ldots$$
$$= \text{even multiple of } 180^\circ$$
$$\text{for } -\infty < K \le 0$$

3. Once the root locus are constructed, the values of $K$ along the loci can be determined by

$$K = \frac{\prod_{j=1}^{n}|(s+P_j)|}{\prod_{i=1}^{m}|(s+Z_i)|}$$

The value of $K$ at any point $s_1$ on the root locus is obtained from above equation by substituting value of $s_1$. Graphically, we write

$$K = \frac{\text{Product of vector lenghts drawn from the poles of } G(s)H(s) \text{ to } s_1}{\text{Product of vector lengths drawn from the zeros of } G(s)H(s) \text{ to } s_1}$$

**POINTS TO REMEMBER**

1. Root loci are trajectories of roots of characteristic equation when a system parameter varies.
2. In general, this method can be applied to study the behaviour of roots of any algebraic equation with one or more variable parameters.
3. Root loci of multiple variable parameters can be treated by varying one parameter at a time. The resultant loci are called the root contours.
4. Root-Loci refers to the entire root loci for $-\infty < K < \infty$,
5. In general, the values of $K$ are positive $(0 < K < \infty)$. Under unusual conditions when a system has positive feedback or the loop gain is negative, then we have $K$ as negative.

## 1.3 RULES FOR SKETCHING ROOT LOCUS



Some important rules are given in the following texts that are useful for sketching the root loci.

**Rule 1: Symmetry of Root Locus**

Root locus are symmetrical with respect to the real axis of the $s$-plane. In general, the root locus are symmetrical with respect to the axes of symmetry of the pole-zero configuration of $G(s)H(s)$.

**Rule 2: Poles and Zeros on the Root Locus**

To locate the poles and zeros on root locus, we note the following points.

1. The $K=0$ points on the root loci are at the poles of $G(s)H(s)$.
2. The $K=\pm\infty$ points on the root loci are at zeros of $G(s)H(s)$.

**Rule 3: Number of Branches of Root Locus**

The number of branches of the root locus equals to the order of the characteristic polynomial.

**Rule 4: Root Loci on the Real axis**

While sketching the root locus on real axis, we must note following points:

1. The entire real axis of the $s$-plane is occupied by the root locus for all values of $K$.
2. Root locus for $K\geq 0$ are found in the section only if the total number of poles and zeros of $G(s)H(s)$ to the right of the section is odd. The remaining sections of the real axis are occupied by the root locus for $K\leq 0$.
3. Complex poles and zeros of $G(s)H(s)$ do not affect the type of root locus found on the real axis.

**Rule 5: Angle of Asymptotes of the Root Locus**

When $n$ is the number of finite poles and $m$ is the number of finite zeros of $G(s)H(s)$, respectively. Then $|n-m|$ branches of the root locus approaches the infinity along straight line asymptotes whose angles are given by

$$\theta_a=\pm\frac{(2q+1)\pi}{n-m};\quad \text{for } K\geq 0$$

and

$$\theta_a=\frac{\pm(2q)\pi}{n-m};\quad \text{for } K\leq 0$$

where $q=0,1,2,.........(n-m-1)$

**Rule 6: Determination of Centroid**

The asymptotes cross the real axis at a point known as centroid, which is given by

$$\sigma_A=\frac{\sum\text{Real parts of poles of } G(s)H(s)-\sum\text{Real parts of zeros of } G(s)H(s)}{n-m}$$

**Rule 7: Angle of Departure**

The angle of departure from an open loop pole is given by (for $K\geq 0$)

$$\phi_D=\pm[(2q+1)\pi+\phi];\quad q=0,1,2,........$$

where, $\phi$ is the net angle contribution at this pole, of all other open loop poles and zeros. For example, consider the plot shown in figure below.

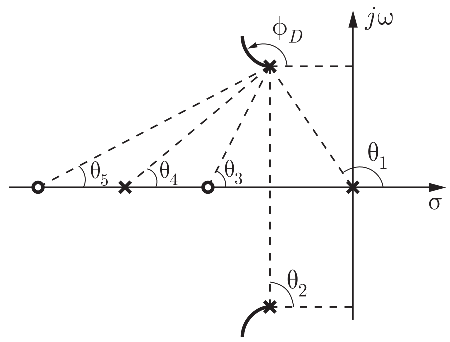



Figure 4.1: Illustration of Angle of Departure

From the figure, we obtain

$$\phi = \theta_3 + \theta_5 - (\theta_1 + \theta_2 + \theta_4)$$

and $$\phi_D = \pm[(2q+1)\pi + \phi]; \qquad q = 0,\ 1,\ 2, .......$$

**Rule 8: Angle of Arrival**

The angle of arrival at an open loop zero is given by (for $K \geq 0$)

$$\phi_a = \pm[(2q+1)\pi - \phi]; \qquad q = 0,\ 1,\ 2, .....$$

where $\phi =$ net angle contribution at this zero, of all other open loop poles and zeros. For example, consider the plot shown in figure below.

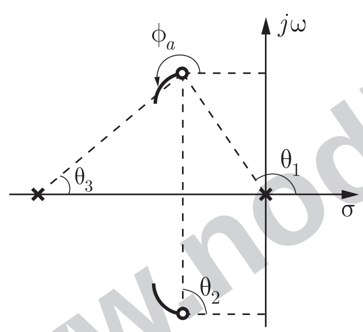



Figure 4.2: Illustration of Angle of Arrival

From the figure, we obtain

$$\phi = \theta_2 - (\theta_1 + \theta_3)$$

and $$\phi_a = \pm[(2q+1)\pi - \phi]; \qquad q = 0,\ 1,\ 2, .......$$

NOTE :

For $K \leq 0$, departure and arrival angles are given by

$$\phi_D = \mp[(2q+1)\pi + \phi]$$

and $$\phi_a = \mp[(2q+1)\pi - \phi]$$

**Rule 9: Break-away and Break-in Points**

To determine the break-away and break-in points on the root locus, we consider the following points:

1. A root locus plot may have more than one breakaway points.
2. Break away points may be complex conjugates in the $s$-plane.
3. At the break away or break-in point, the branches of the root locus form an angle of $\frac{180^\circ}{n}$ with the real axis, where $n$ is the number of closed loop poles arriving at or departing from the single breakaway or break-in point on the real axis.
4. The breakaway and break-in points of the root locus are the solution of

$$\frac{dK}{ds} = 0$$

i.e. breakaway and break in points are determined by finding maximum and minimum points of the gain $K$ as a function of $s$ with $s$ restricted to real values.

**Rule 10: Intersects of Root Locus on Imaginary Axis**

Routh-Hurwitz criterion may be used to find the intersects of the root locus on the imaginary axis.

## 1.4 EFFECT OF ADDITION OF POLES AND ZEROS TO G(S)H(S)

In this section, the effect of adding poles and zeros to $G(s)H(s)$ are described.

**Addition of Poles to $G(s)H(s)$**

Due to addition of poles to $G(s)H(s)$, the root locus is affected in following manner:

1. Adding a pole to $G(s)H(s)$ has the effect of pushing the root loci towards the right half.
2. The complex path of the root loci bends to the right.
3. Angle of asymptotes reduces and centroid is shifted to the left.
4. The system stability will be reduced.

**Addition of Zeros to $G(s)H(s)$**

Due to addition of zeros to $G(s)H(s)$, the root locus is affected in following manner:

1. Adding left half plane zero to the function $G(s)H(s)$, generally has the effect of moving or pushing the root loci towards the left half.
2. The complex path of the root loci bends to the left.
3. Centroid shifted to the right.
4. The relative stability of the system is improved.



## 1.5 ROOT SENSITIVITY

The sensitivity of the roots of the characteristic equation when $K$ varies is termed as the root sensitivity. Mathematically, the root sensitivity is defined as the ratio of the fractional change in a closed-loop pole to the fractional change in a system parameter, such as gain and is given by

$$S_K^S = \frac{K}{s}\frac{ds}{dK}$$

where $s$ is the current pole location, and $K$ is the current system gain. Converting the partials to finite increments, the actual change in closed-loop poles can be approximated as

$$\Delta s = s(S_K^S)\frac{\Delta K}{K}$$

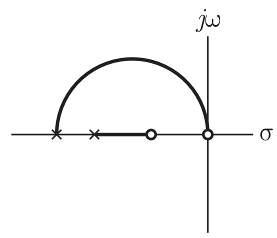
where, $\Delta s$ is the change in pole location and $\Delta K/K$ is the fractional change in the gain, $K$.

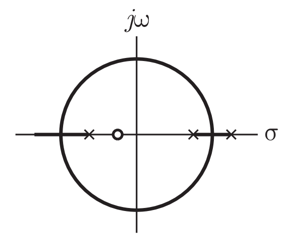
NOTE :
The root sensitivity at the breakaway points is infinite, because break away points are given by $\frac{dK}{ds} = 0$.

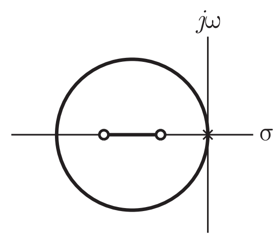
***********

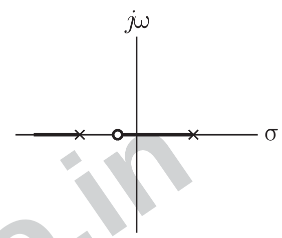
# EXERCISE 1.1

MCQ 1.1.1 Form the given sketch the root locus can be

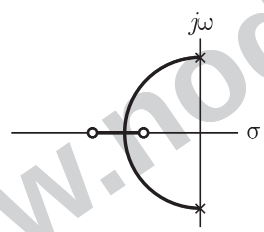
(A) 

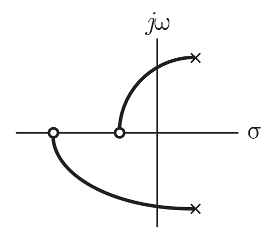

(B) 

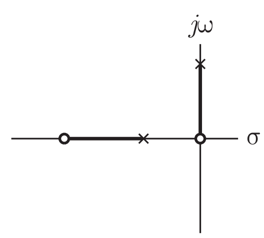

(C) 

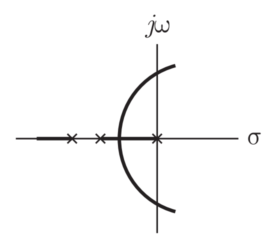

(D) 


MCQ 1.1.2 Consider the sketch shown below.

(1) 


(2) 


(3) 

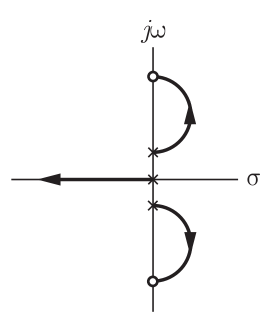

(4) 

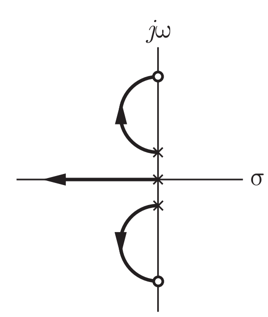

The root locus can be

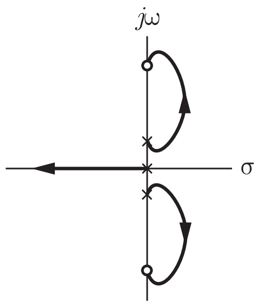
(A) (1) and (3)  (B) (2) and (3)

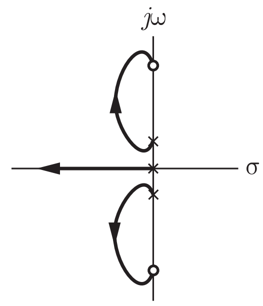
(C) (2) and (4)  (D) (1) and (4)

MCQ 1.1.3 The valid root locus diagram is

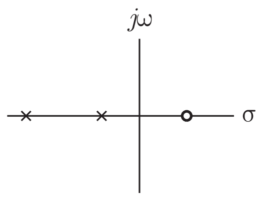
(A) 


(B) 

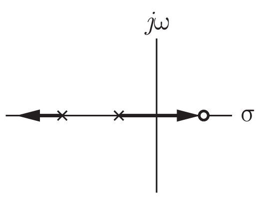
(C) 

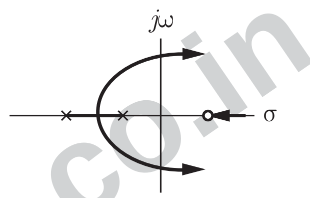

(D) 

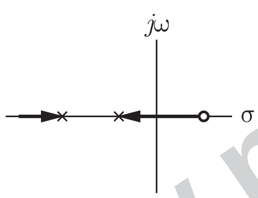

MCQ 1.1.4 An open-loop pole-zero plot is shown below

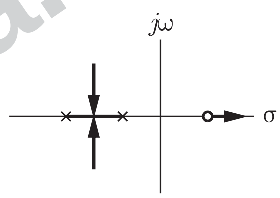



The general shape of the root locus is

(A) 

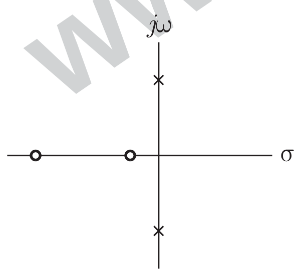

(B) 


(C) 

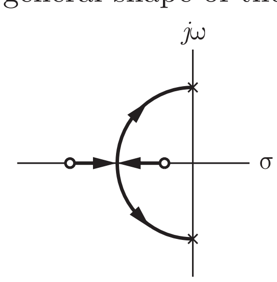

(D) 

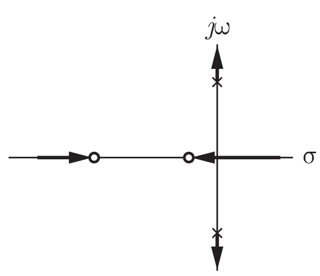


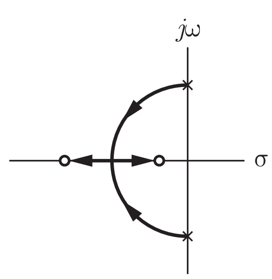

MCQ 1.1.5 An open-loop pole-zero plot is shown below

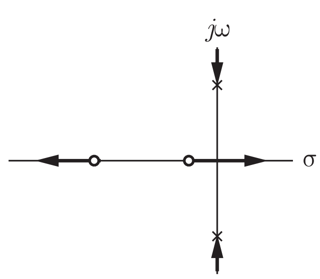



The general shape of the root locus is

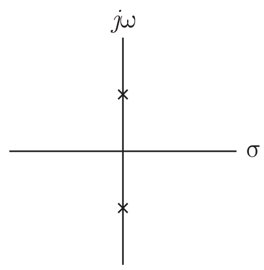
(A) 


(B) 

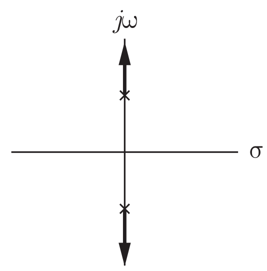

(C) 

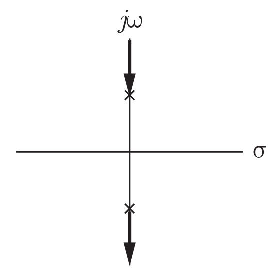

(D) 

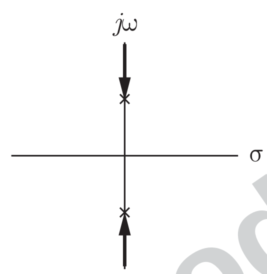
MCQ 1.1.6 An open-loop pole-zero plot is shown below.

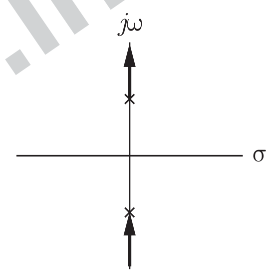



The general shape of the root locus is

(A)


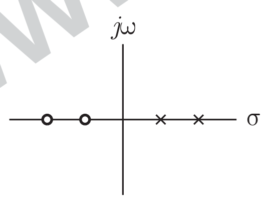

(B)



(C)


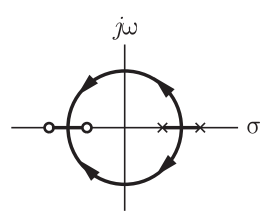

(D)


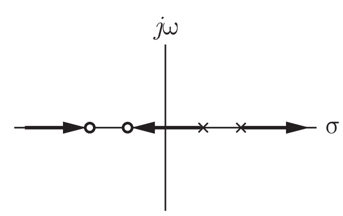


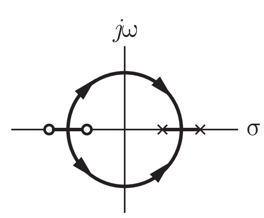

MCQ 1.1.7 An open-loop pole-zero plot is shown below.

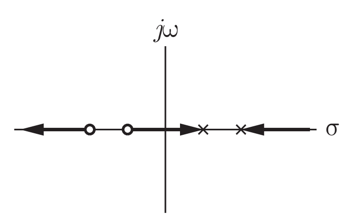



The general shape of the root locus is

(A)



(B)


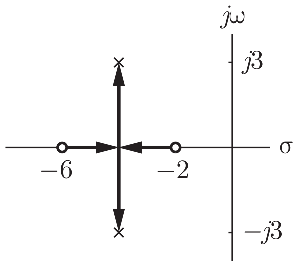

(C)


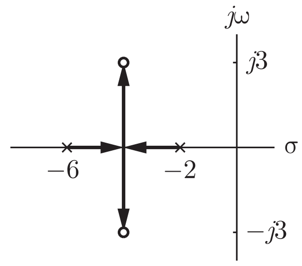

(D)


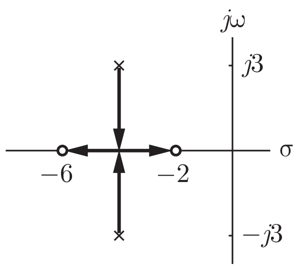
MCQ 1.1.8 The forward-path open-loop transfer function of a *ufb* system is

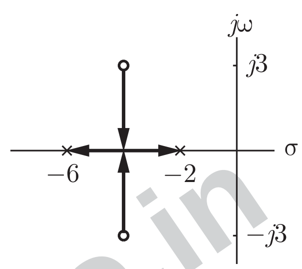
$$G(s) = \frac{K(s+2)(s+6)}{s^2+8s+25}$$

The root locus for this system is

(A)



(B)


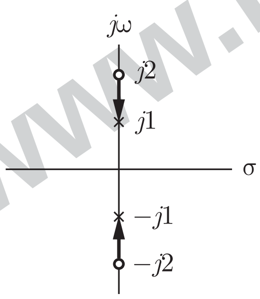

(C)


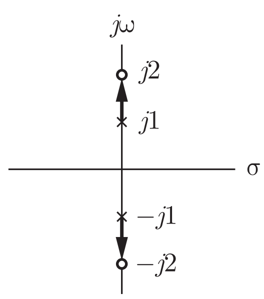

(D)


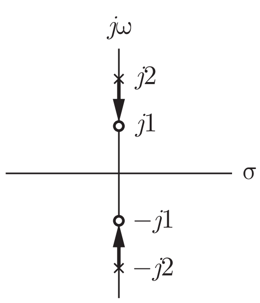

MCQ 1.1.9 The forward-path open-loop transfer function of a *ufb* system is

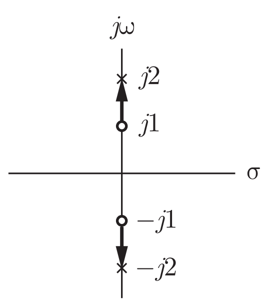
$$G(s) = \frac{K(s^2+4)}{(s^2+1)}$$

For this system, root locus is

(A)



(B)


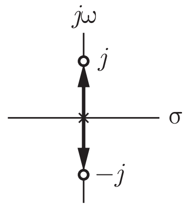

(C)


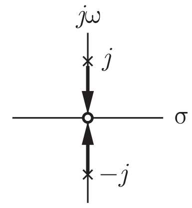

(D)


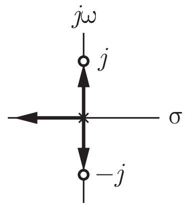

MCQ 1.1.10 The forward-path open-loop transfer function of a *ufb* system is

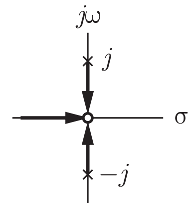
$$G(s) = \frac{K(s^2+1)}{s^2}$$

The root locus of this system is

(A)


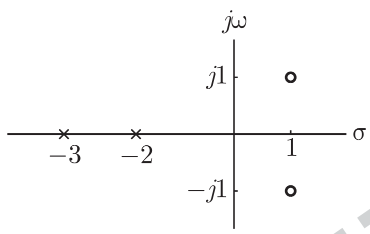

(B)



(C)



(D)



**Common Data For Q. 11 and 12**

An open-loop pole-zero plot is shown below.






MCQ 1.1.11 The transfer function of this system is

(A) $\dfrac{K(s^2+2s+2)}{(s+3)(s+2)}$ (B) $\dfrac{K(s+3)(s+2)}{(s^3+2s+2)}$

(C) $\dfrac{K(s^2-2s+2)}{(s+3)(s+2)}$ (D) $\dfrac{K(s+3)(s+2)}{(s^2-2s+2)}$

MCQ 1.1.12 The break point is

(A) break away at $\sigma=-1.29$

(B) break in at $\sigma=-2.43$

(C) break away at $\sigma=-2.43$

(D) break in at $\sigma=-1.29$

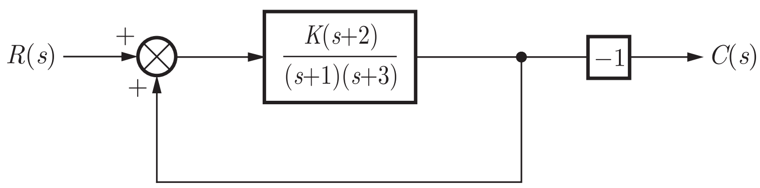
MCQ 1.1.13 The forward-path transfer function of a *ufb* system is

$$G(s)=\frac{K(s+1)(s+2)}{(s+5)(s+6)}$$

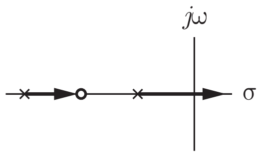
So, the break points are

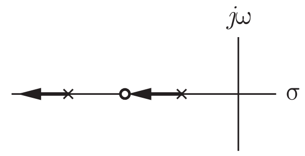
| | Break-in | Break-away |
|---|---|---|
| (A) | $-1.563$ | $-5.437$ |
| (B) | $-5.437$ | $-1.563$ |
| (C) | $-1.216$ | $-5.743$ |
| (D) | $-5.473$ | $-1.216$ |

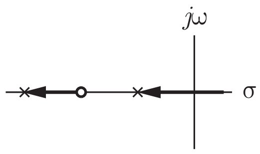
MCQ 1.1.14 Consider the feedback system shown below.

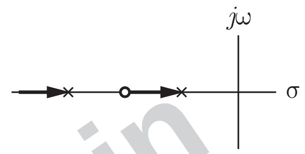



For this system, the root locus is

(A)



(B)



(C)



(D)



MCQ 1.1.15 For a *ufb* system, forward-path transfer function is

$$G(s) = \frac{K(s+6)}{(s+3)(s+5)}$$

The breakaway point and break-in points are located respectively at

(A) $\infty, 4.27$ (B) $7.73, 4.27$

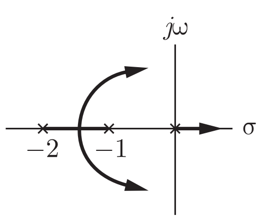
(C) $4.27, \infty$ (D) $4.27, 7.73$

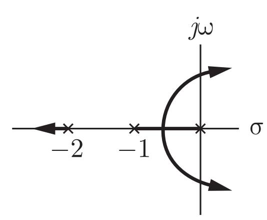


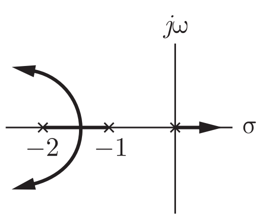
MCQ 1.1.16 The open loop transfer function of a system is given by

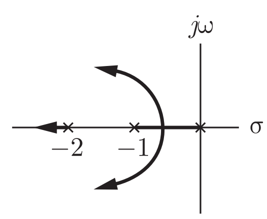
$$G(s)\,H(s) = \frac{K}{s(s+1)(s+2)}$$

The root locus plot of above system is

(A)



(B)


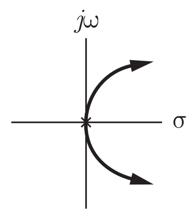

(C)


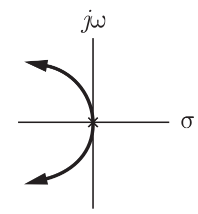

(D)


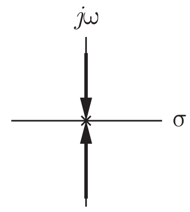

MCQ 1.1.17 A *ufb* system has forward-path transfer function,

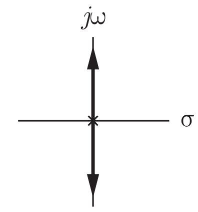
$$G(s) = \frac{K}{s^2}$$

The root locus plot is

(A) 

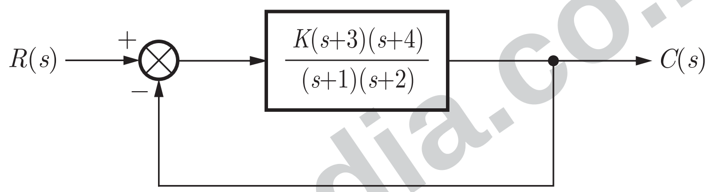

(B) 


(C) 


(D) 


MCQ 1.1.18 For the *ufb* system shown below, consider two points

$$s_1 = -2 + j3 \text{ and } s_2 = -2 + j\frac{1}{\sqrt{2}}$$




Which of the above points lie on root locus ?

(A) Both $s_1$ and $s_2$

(B) $s_1$ but not $s_2$

(C) $s_2$ but not $s_1$

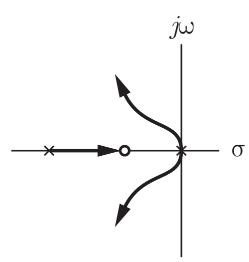
(D) neither $s_1$ nor $s_2$

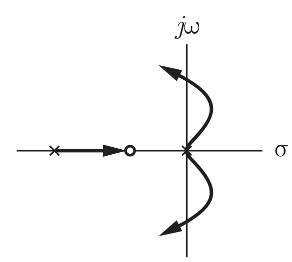


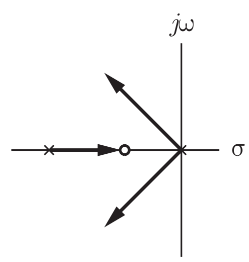
MCQ 1.1.19 A *ufb* system has open-loop transfer function,

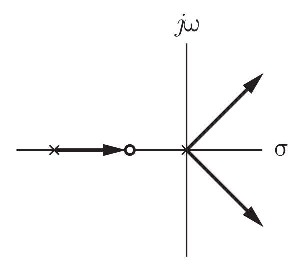
$$G(s) = \frac{K(s+\alpha)}{s^2(s+\beta)}, \beta > \alpha > 0$$

The valid root-loci for this system is

(A) 


(B) 


(C) 


(D) 

MCQ 1.1.20 The characteristic equation of a feedback control system is given by

$$(s^2+4s+4)(s^2+11s+30)+Ks^2+4K=0$$

where $K>0$. In the root locus of this system, the asymptotes meet in $s$-plane at

(A) $(-9.5, 0)$

(B) $(-5.5, 0)$

(C) $(-7.5, 0)$

(D) None of the above

MCQ 1.1.21 The root locus of the system having the loop transfer function,

$$G(s)H(s)=\frac{K}{s(s+4)(s^2+4s+5)} \text{ has}$$

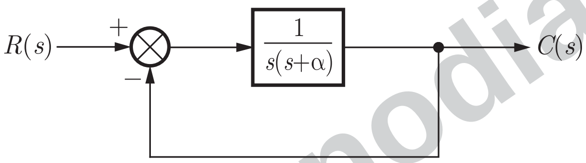
(A) 3 break-away points

(B) 3 break-in points

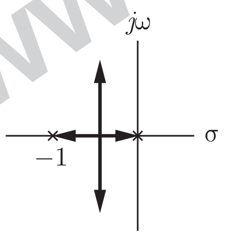
(C) 2 break-in and 1 break-away point

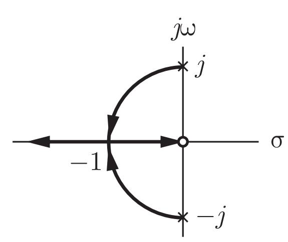
(D) 2 break-away and 1 break-in point

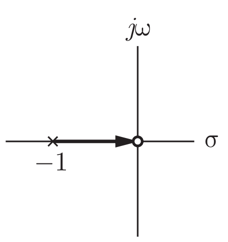
MCQ 1.1.22 Consider the *ufb* system shown below.

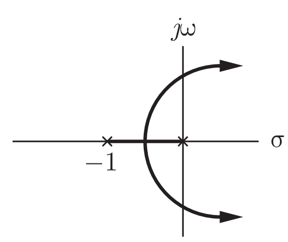



The root-loci, as α is varied, will be

(A) 


(B) 


(C) 

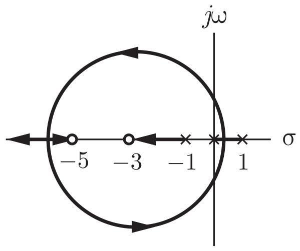

(D) 

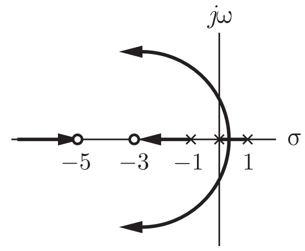

**Common Data For Q. 23 and 24**

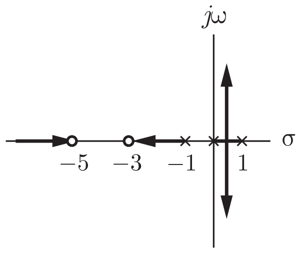
The forward-path transfer function of a *ufb* system is

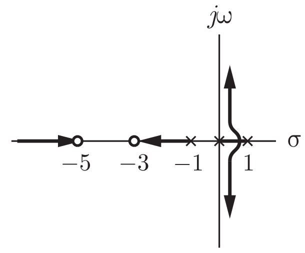
$$G(s)=\frac{K(s+\alpha)(s+3)}{s(s^2-1)}$$

MCQ 1.1.23 The root-loci for $K > 0$ with $\alpha = 5$ is

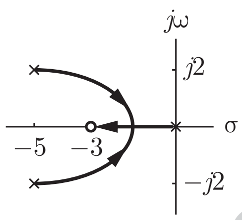
(A)


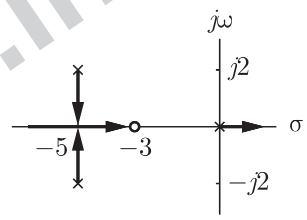

(B)


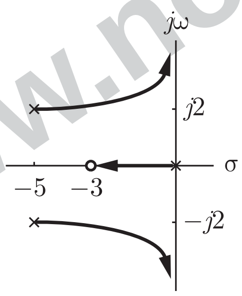

(C)


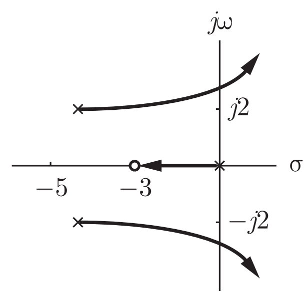

(D)



MCQ 1.1.24 The root-loci for $\alpha > 0$ with $K = 10$ is

(A)



(B)



(C)



(D)





MCQ 1.1.25 For the system $G(s)\,H(s) = \dfrac{K(s+6)}{(s+2)(s+4)}$, consider the following characteristic of the root locus :

1. It has one asymptote.
2. It has intersection with $j\omega$-axis.
3. It has two real axis intersections.
4. It has two zeros at infinity.

The root locus have characteristics

(A) 1 and 2 (B) 1 and 3

(C) 3 and 4 (D) 2 and 4

MCQ 1.1.26 The forward path transfer function of a *ufb* system is

$$G(s) = \frac{K(s+3)}{s(s+1)(s+2)(s+4)}$$

The angles of asymptotes are

(A) $0, \frac{\pi}{2}, \pi$

(B) $0, \frac{2\pi}{3}, \frac{4\pi}{3}$

(C) $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

(D) None of the above

MCQ 1.1.27 Match List-I with List-II in respect of the open loop transfer function

$$G(s)H(s) = \frac{K(s+10)(s^2+20s+500)}{s(s+20)(s+50)(s^2+4s+5)}$$

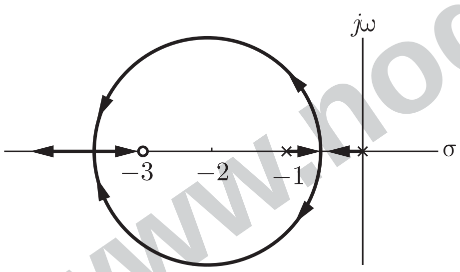
and choose the correct option.

| | List I (Types of Loci) | List II (Numbers) |
|---|---|---|
| P. | Separate Loci | 1. One |
| Q. | Loci on the real axis | 2. Two |
| R. | Asymptotes | 3. Three |
| S. | Break away points | 4. Five |

| | P | Q | R | S |
|---|---|---|---|---|
| (A) | 4 | 3 | 1 | 1 |
| (B) | 4 | 3 | 2 | 1 |
| (C) | 3 | 4 | 1 | 1 |
| (D) | 3 | 4 | 1 | 2 |



MCQ 1.1.28 The root-locus of a *ufb* system is shown below.




The open loop transfer function is

(A) $\frac{K}{s(s+1)(s+3)}$

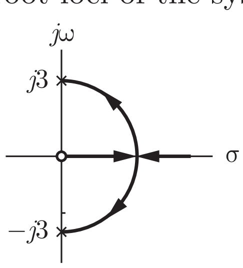
(B) $\frac{K(s+3)}{s(s+1)}$

(C) $\frac{K(s+1)}{s(s+3)}$

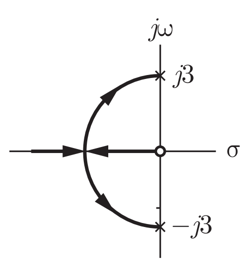
(D) $\frac{Ks}{(s+1)(s+3)}$

MCQ 1.1.29 The characteristic equation of a linear control system is $s^2 + 5Ks + 9 = 0$. The root loci of the system is

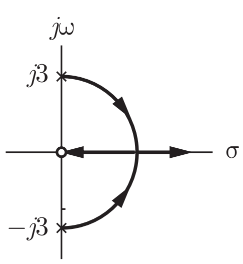
(A)



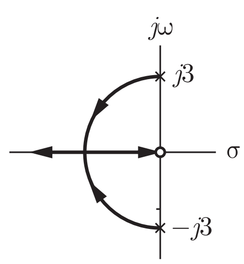

(B)



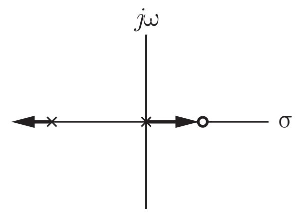

(C)

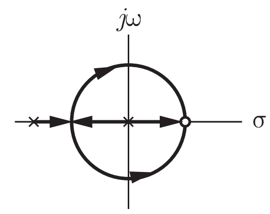


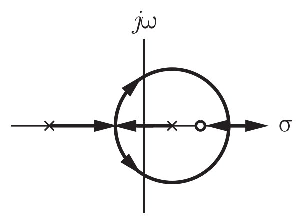

(D)

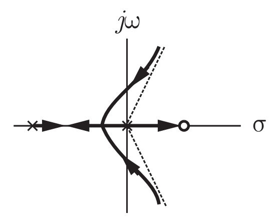


MCQ 1.1.30 An unity feedback system is given as $G(s) = \frac{K(1-s)}{s(s+3)}$. Which of the following is the correct root locus diagram ?

(A)



(B)



(C)



(D)



MCQ 1.1.31 The open loop transfer function $G(s)$ of a *ufb* system is given as

$$G(s) = \frac{K(s+\frac{2}{3})}{s^2(s+2)}$$

From the root locus, it can be inferred that when $K$ tends to positive infinity,

(A) three roots with nearly equal real parts exist on the left half of the $s$-plane

(B) one real root is found on the right half of the $s$-plane

(C) the root loci cross the $j\omega$ axis for a finite value of $K$; $K \neq 0$

(D) three real roots are found on the right half of the $s$-plane



MCQ 1.1.32 The characteristic equation of a closed-loop system is

$$s(s+1)(s+3) + K(s+2) = 0, \quad K > 0$$

Which of the following statements is true ?

(A) Its roots are always real

(B) It cannot have a breakaway point in the range $-1 < \mathrm{Re}[s] < 0$

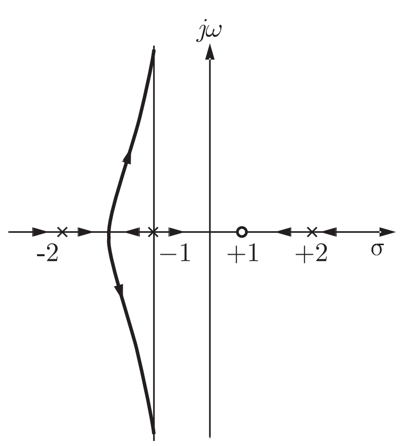
(C) Two of its roots tend to infinity along the asymptotes $\mathrm{Re}[s] = -1$

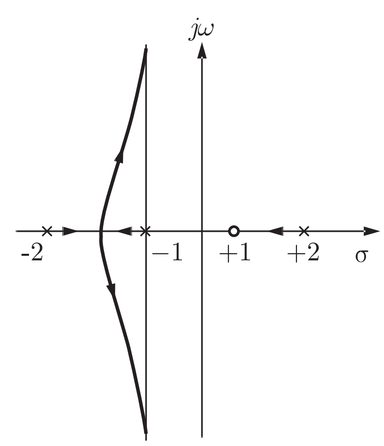
(D) It may have complex roots in the right half plane.

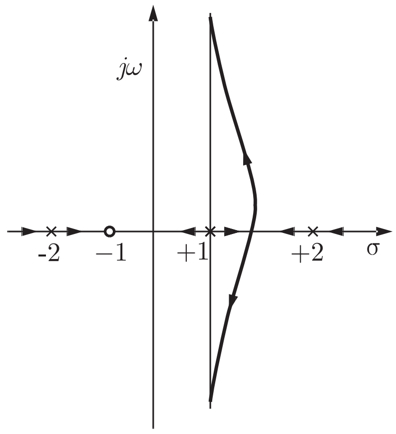
MCQ 1.1.33 A closed-loop system has the characteristic function,

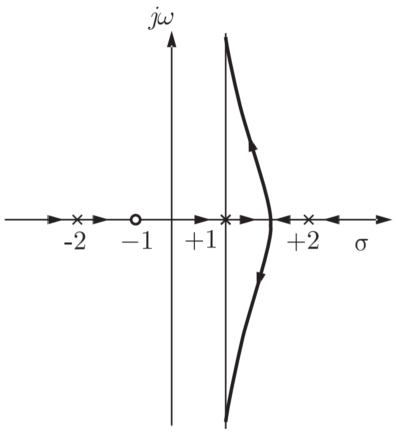
$$(s^2-4)(s+1) + K(s-1) = 0$$

Its root locus plot against $K$ is

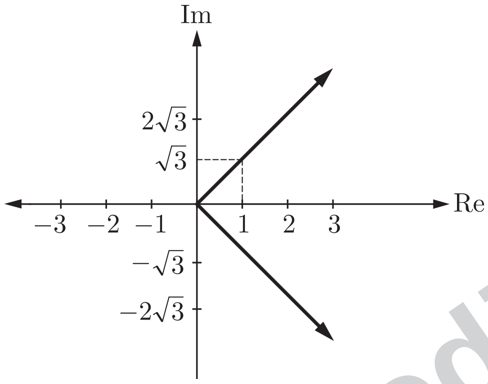
(A)



(B)


(C) 


(D) 


MCQ 1.1.34 Figure shows the root locus plot (location of poles not given) of a third order system whose open loop transfer function is




(A) $\frac{K}{s^3}$

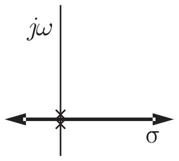
(B) $\frac{K}{s^2(s+1)}$

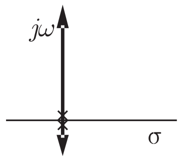
(C) $\frac{K}{s(s^2+1)}$

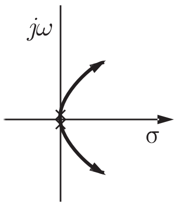
(D) $\frac{K}{s(s^2-1)}$

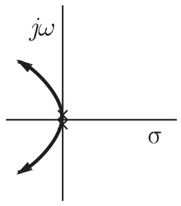


MCQ 1.1.35 A unity feedback system has an open loop transfer function, $G(s) = \frac{K}{s^2}$. The root locus plot is

(A) 


(B) 


(C) 


(D) 


**Common Data For Q. 36 to 38**

The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{2(s+\alpha)}{s(s+2)(s+10)};\ \alpha > 0$$

MCQ 1.1.36 Angles of asymptotes are

(A) $60^\circ, 120^\circ, 300^\circ$

(B) $60^\circ, 180^\circ, 300^\circ$

(C) $90^\circ, 270^\circ, 360^\circ$

(D) $90^\circ, 180^\circ, 270^\circ$

MCQ 1.1.37 Intercepts of asymptotes at the real axis is

(A) $-6$ (B) $-\frac{10}{3}$

(C) $-4$ (D) $-8$

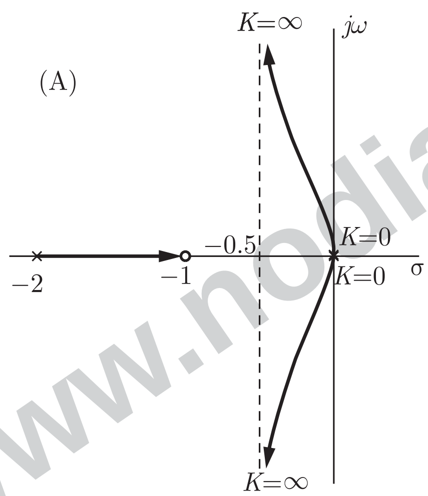
MCQ 1.1.38 Break away points are

(A) $-1.056, -3.471$ (B) $-2.112, -6.943$

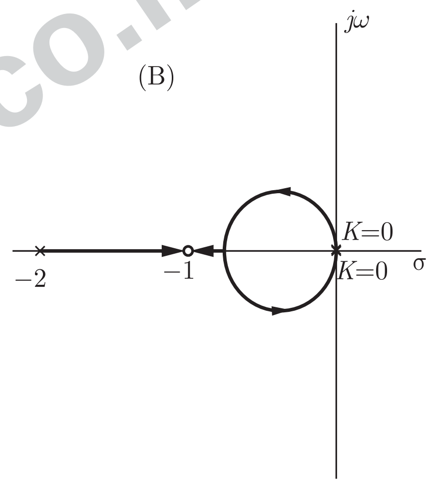
(C) $-1.056, -6.943$ (D) $1.056, -6.943$

MCQ 1.1.39 For the characteristic equation $s^3 + 2s^2 + Ks + K = 0$, the root locus of the system as $K$ varies from zero to infinity is

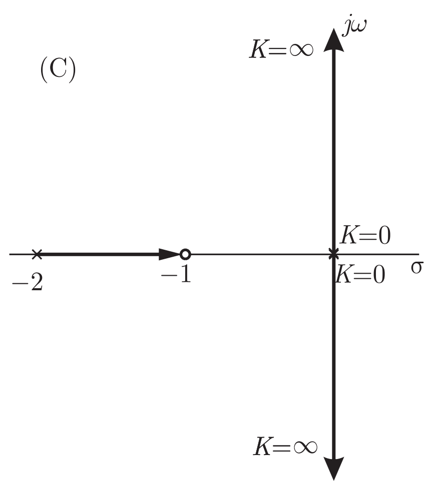
(A)




(B)



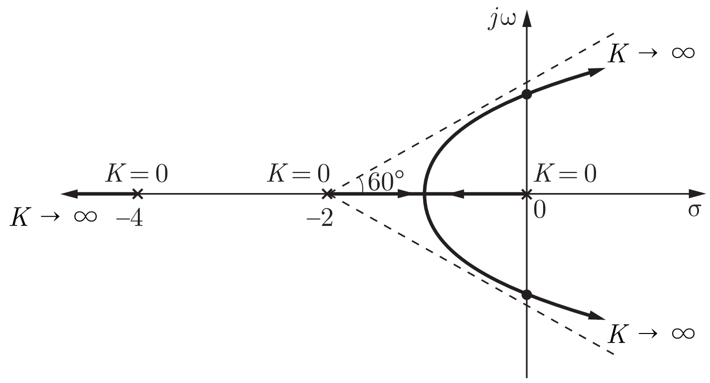

(C)




(D) none of these

MCQ 1.1.40 The open loop transfer function of a *ufb* system and root locus plot for the system is shown below.

$$G(s)H(s) = \frac{K}{s(s+2)(s+4)}$$

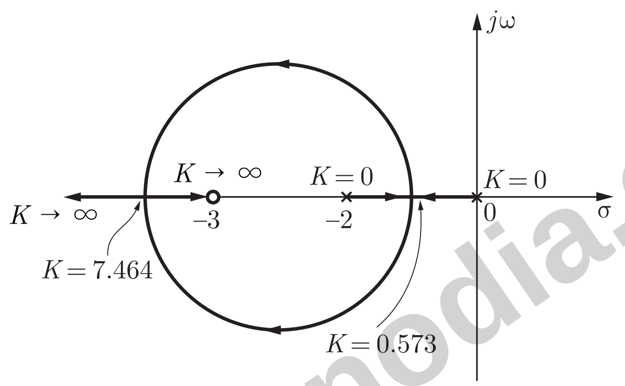



The range of $K$ for which the system has damped oscillatory response is

(A) $0 < K < 48$ (B) $K > 3.08$

(C) $3.08 < K < 48$ (D) $K > 48$

MCQ 1.1.41 The root locus plot for a control system is shown below.




Consider the following statements regarding the system.

1. System is stable for all positive value of $K$.
2. System has real and repeated poles for $0.573 < K < 7.464$.
3. System has damped oscillatory response for all values of $K$ greater than 0.573.
4. System is overdamped for $0 < K < 0.573$ and $K > 7.464$.

Which of the following is correct ?

(A) 1, 2 and 3 (B) 1 and 4

(C) 2 and 3 (D) all

MCQ 1.1.42 The open loop transfer function of a control system is

$$G(s)H(s) = \frac{Ke^{-s}}{s(s+2)}$$

For low frequencies, consider the following statements regarding the system.

1. $s = 2.73$ is break-away point.
2. $s = -0.73$ is break-away point.
3. $s = -0.73$ is break-in point.
4. $s = 2.73$ is break-in point.

Which of the following is correct ?

(A) 1 and 2 (B) 3 and 4

(C) 1 and 3 (D) 2 and 4

MCQ 1.1.43 The open loop transfer function of a *ufb* system is

$$G(s)H(s) = \frac{K}{s(s+4)(s^2+4s+20)}$$

The root locus of the system has

(A) 3 real break point.

(B) 1 real and 2 complex break point.

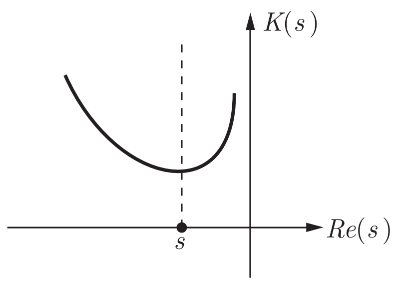
(C) only one real break point.

(D) No one break point.

MCQ 1.1.44 The open loop transfer function of a control system is given below.

$$G(s)H(s) = \frac{Ks}{(s^2 - s + 4.25)}$$

The system gain $K$ as a function of $s$ along real axis is shown below.




In the root locus plot, point $s$, corresponding to gain plot, is

(A) $s = -2.06$; Break in point
(B) $s = -1.25$; Break in point
(C) $s = -2.06$; Break away point
(D) $s = -4.25$; Break away point

**Common Data For Q. 45 and 46**

The open loop transfer function of a system is

$$G(s)H(s) = \frac{K(s-1)}{(s+1)(s+2)}$$

MCQ 1.1.45 What is the gain $K$ for which the closed loop system has a pole at $s = 0$ ?

(A) $K = 0$
(B) $K = 2$
(C) $K = 10$
(D) $K = \infty$

MCQ 1.1.46 Based on the above result, other pole of the system is

(A) $s = -3 + j0$
(B) $s = -5 + j0$
(C) $s = -13 + j0$
(D) None

**Common Data For Q. 47 to 49**

The open loop transfer function of a system is given below.

$$G(s)H(s) = \frac{K(s+2)}{s(s+1)(s+30)}$$

MCQ 1.1.47 What is the value of gain $K$ for which the closed loop system has two poles with real part $-2$ ?

(A) $K = 0$
(B) $K = 30$
(C) $K = 84.24$
(D) $K = \infty$

MCQ 1.1.48 Based on the above result, third pole of the closed loop system is

(A) $s = -30$ (B) $s = -31$

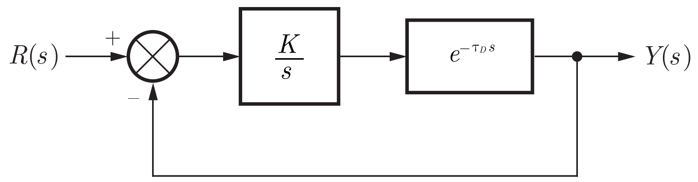
(C) $s = -27$ (D) None

MCQ 1.1.49 Complex poles of the system are

(A) $s = -2 \pm j2.245$ (B) $s = -2 \pm j3.467$

(C) $s = -2 \pm j1.497$ (D) None

MCQ 1.1.50 Consider the system with delay time $(\tau_D)$ shown below.




Suppose delay time $\tau_D = 1\,\text{sec}$. In root locus plot of the system, the break-away and break-in points are respectively

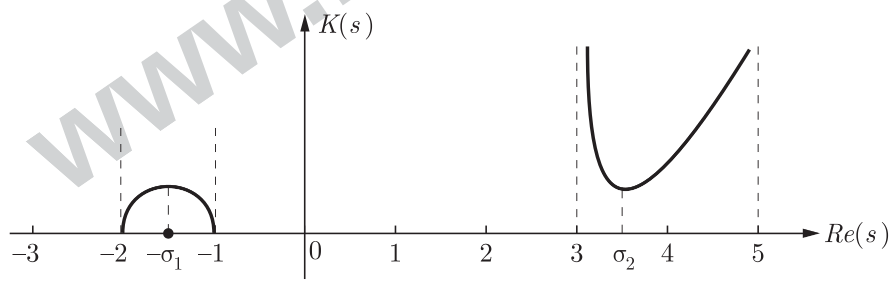
(A) 0, 4.83

(B) 4.83, 0

(C) $-0.83$, 4.83

(D) 4.83, $-0.83$

MCQ 1.1.51 Variation of system gain $K$ along the real axis of $s$-plane for the root locus of a system is shown below.




Which of the following options is correct for the root locus plot of the system?

(A) $(-\sigma_1)$ is break in and $\sigma_2$ is break away point.

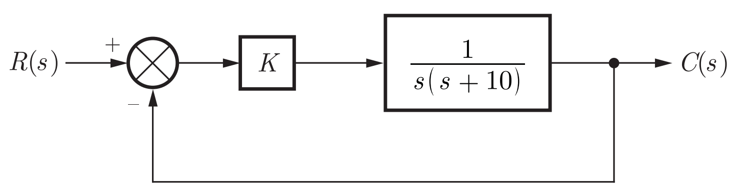
(B) $(-\sigma_1)$ is break away and $\sigma_2$ is break in point.

(C) Both $(-\sigma_1)$ and $\sigma_2$ are break away points.

(D) Both $(-\sigma_1)$ and $\sigma_2$ are break in points.

**Common Data For Q. 52 to 54**

Consider the system given below.



MCQ 1.1.52 Poles of the system for $K = 5$ are

(A) $s = -9.47$ (B) $s = -0.53$

(C) Both A and B (D) None

MCQ 1.1.53 What will be the change in pole $(s = -9.47)$ location for a 10% change in $K$ ?

(A) Pole moves to left by 0.056

(B) Pole moves to right by 0.056

(C) Pole moves to left by 0.029

(D) Pole moves to right by 0.029

MCQ 1.1.54 A *ufb* system has an open loop transfer function,

$$G(s)H(s) = \frac{K(s+1)}{s(s-1)}$$

Root locus for the system is a circle. Centre and radius of the circle are respectively

(A) $(0,0)$, 2 (B) $(0,0)$, $\sqrt{2}$

(C) $(-1,0)$, 2 (D) $(-1,0)$, $\sqrt{2}$

MCQ 1.1.55 The open loop transfer function of a system is

$$G(s)H(s) = \frac{K(s+3)}{s(s+2)}$$

The root locus of the system is a circle. The equation of circle is

(A) $(\sigma+4)^2 + \omega^2 = 4$ (B) $(\sigma-3)^2 + \omega^2 = 3$

(C) $(\sigma+3)^2 + \omega^2 = (\sqrt{3})^2$ (D) $(\sigma-4)^2 + \omega^2 = (2)^2$

MCQ 1.1.56 Consider the open loop transfer function of a system given below.

$$G(s)H(s) = \frac{K}{(s^2+2s+2)(s^2+6s+10)}$$

The break-away point in root locus plot for the system is/are

(A) 3 real

(B) only real

(C) 1 real, 2 complex

(D) None

MCQ 1.1.57 The open loop transfer function of a *ufb* system is given below.

$$G(s)H(s) = \frac{K}{s(s+4)(s+5)}$$

Consider the following statements for the system.

1. Root locus plot cross $j\omega$-axis at $s = \pm j2\sqrt{5}$
2. Gain margin for $K = 18$ is 20 dB.
3. Gain margin for $K = 1800$ is $-20$ dB
4. Gain $K$ at breakaway point is 13.128

Which of the following is correct ?

(A) 1 and 2 (B) 1, 2 and 3

(C) 2, 3 and 4 (D) All

MCQ 1.1.58 The open loop transfer function of a control system is

$$G(s)H(s) = \frac{K(s+1)}{s(s-1)(s^2+4s+16)}$$

Consider the following statements for the system

1. Root locus of the system cross $j\omega$-axis for $K = 35.7$
2. Root locus of the system cross $j\omega$-axis for $K = 23.3$
3. Break away point is $s = 0.45$
4. Break in point is $s = -2.26$

Which of the following statement is correct ?

(A) 1, 3 and 4 (B) 2, 3 and 4

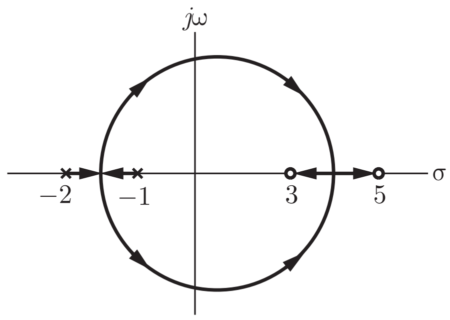
(C) 3 and 4 (D) all

*************

# EXERCISE 1.2

**Common Data For Q. 1 and 2**

A root locus of *ufb* system is shown below.




QUES 1.2.1 The breakaway point for the root locus of system is ______

QUES 1.2.2 At break-away point, the value of gain $K$ is ______

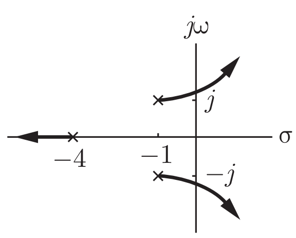
QUES 1.2.3 The forward-path transfer function of a *ufb* system is

$$G(s) = \frac{K(s+2)}{(s+3)(s^2+2s+2)}$$

The angle of departure from the complex poles is $\pm\phi_D$; where $\phi_D =$ ______ degree.

**Common Data For Q. 4 and 5**

The root locus for a ufb system is shown below.




QUES 1.2.4 The root locus crosses the imaginary axis at $\pm ja$; where $a =$ _______

QUES 1.2.5 The value of gain for which the closed-loop transfer function will have a pole on the real axis at $-5$, will be _______

QUES 1.2.6 The open-loop transfer function a system is

$$G(s)H(s) = \frac{K(s+8)}{s(s+4)(s+12)(s+20)}$$

A closed loop pole will be located at $s = -10$, if the value of $K$ is ______

QUES 1.2.7 Characteristic equation of a closed-loop system is $s(s+1)(s+2)+K=0$. What will be the centroid of the asymptotes in root-locus ?

QUES 1.2.8 A unity feedback control system has an open-loop transfer function

$$G(s) = \frac{K}{s(s^2+7s+12)}$$

The gain $K$ for which $s = -1+j1$ will lie on the root locus of this system is ______

**Common Data For Q. 9 and 10**

The open loop transfer function of a control system is

$$G(s)H(s) = \frac{K(s^2+2s+10)}{(s^2+6s+10)}$$



QUES 1.2.9 The angle of departure at the complex poles will be $\pm\phi_D$; where $\phi_D =$ ______ degree.

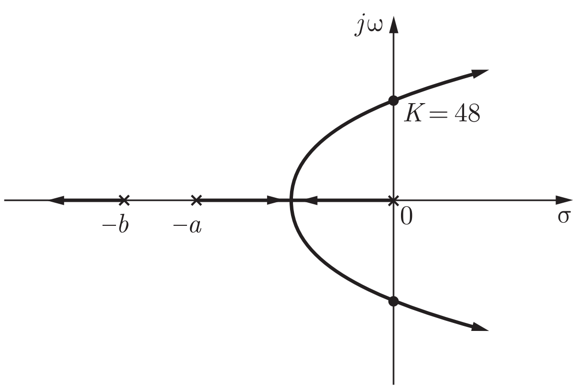
QUES 1.2.10 The angle of arrival at complex zeros is $\pm\phi_A$; where $\phi_A =$ _______ degrees.

QUES 1.2.11 The open loop transfer function of a unity feedback system is shown below.

$$G(s)H(s) = \frac{10K}{(s+2)(s+10)}$$

What is the value of $K$ for which the root locus cross the line of constant damping $\xi = \frac{1}{\sqrt{2}}$ ?

QUES 1.2.12 The root locus plot of a system is shown below.




The gain margin for $K = 12$ is ______ dB.

QUES 1.2.13 A unity feedback control system has open loop transfer function,

$$G(s)H(s) = \frac{K}{s(s+10)(s+20)}$$

The value of $K$ at the breakaway point is _______

QUES 1.2.14 The root sensitivity of the system at $s = -9.47$ is ______

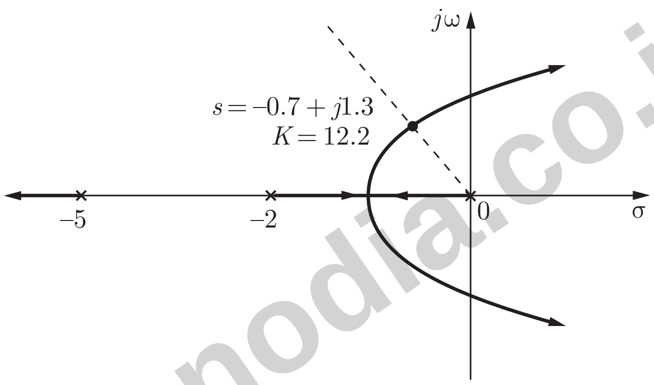
QUES 1.2.15 The open loop transfer function of a system is

$$G(s)H(s) = \frac{K(s^2+4)}{s(s+2)}$$

The value of $K$ at breakaway point is _______

**Common Data For Q. 16 and 17**

The root locus plot for a system is given below.



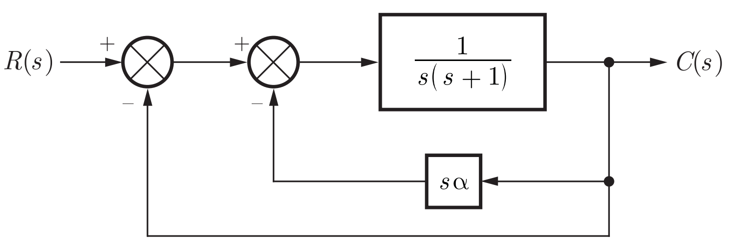



QUES 1.2.16 Damping ratio for $K = 12.2$ is $\xi =$ ______

QUES 1.2.17 Peak over shoot of the system response for $K = 12.2$ is $M_P =$ ______

**Common Data For Q. 18 and 19**

The block diagram of a control system is given below.




QUES 1.2.18 The root locus of the system is plotted as the value of parameter $\alpha$ is varied. The break away point is $s =$ _______

QUES 1.2.19 The value of $\alpha$ for which transient response have critical damping, is ______

QUES 1.2.20 The open loop transfer function of a *ufb* system is

$$G(s)H(s)=\frac{K(s+1)}{s^2(s+9)}$$

In the root locus of the system, as parameter $K$ is varied from 0 to $\infty$, the gain $K$ when all three roots are real and equal is _______

QUES 1.2.21 The open loop transfer function of a system is

$$G(s)H(s)=\frac{K}{(s+1)(s+5)}$$

What is the value of $K$, so that the point $s=-3+j5$ lies on the root locus ?

***********

# EXERCISE 1.3

MCQ 1.3.1 If the gain $(K)$ of a system becomes zero, the roots will

(A) move away from zeros (B) move away from the poles

(C) coincide with the zeros (D) coincide with the poles

MCQ 1.3.2 The root locus plot is symmetrical about the real axis because

(A) roots occur simultaneously in LH and RH planes

(B) complex roots occur in conjugate pairs

(C) all roots occur in pairs

(D) none of these

MCQ 1.3.3 The break away points of the root locus occur at

(A) imaginary axis

(B) real axis

(C) multiple roots of characteristic equation

(D) either (A) or (B)

MCQ 1.3.4 The algebraic sum of the angles of the vectors from all poles and zeros to the point on any root locus segment is

(A) $180^\circ$

(B) $150^\circ$

(C) $180^\circ$ or its odd multiple

(D) $80^\circ$ or its odd multiple

MCQ 1.3.5 The root locus is

(A) an algebraic method (B) a graphical method

(C) combination of both (D) none of these

MCQ 1.3.6 The root locus is a

(A) time-domain approach

(B) frequency domain approach

(C) combination of both

(D) none of these

MCQ 1.3.7 The root locus can be applied to

(A) only linear systems

(B) only nonlinear systems

(C) both linear and nonlinear systems

(D) none of these

MCQ 1.3.8 The root locus can be used to determine
(A) the absolute stability of a system of a system
(B) the relative stability of a system
(C) both absolute and relative stabilities of a system
(D) none of these

MCQ 1.3.9 The root locus always starts at the
(A) open-loop poles (B) open-loop zeros
(C) closed-loop poles (D) closed-loop zeros

MCQ 1.3.10 The root locus always terminates on the
(A) open-loop zeros (B) closed-loop zeros
(C) roots of the characteristic equation (D) none of these

MCQ 1.3.11 The root locus gives the locus of
(A) open-loop poles
(B) closed-loop poles
(C) both open-loop and closed-loop poles
(D) none of these

MCQ 1.3.12 An open-loop transfer function has 4 poles and 1 zero. The number of branches of root locus is
(A) 4 (B) 1
(C) 5 (D) 3

MCQ 1.3.13 The open-loop transfer function of a control system has 5 poles and 3 zeros. The number of asymptotes is equal to
(A) 5 (B) 3
(C) 2 (D) 8

MCQ 1.3.14 Angles of asymptotes are measured at the centroid with respect to
(A) negative real axis (B) positive real axis
(C) imaginary axis (D) none of these

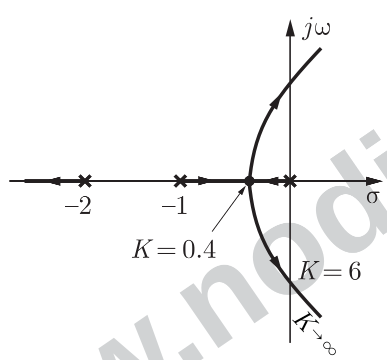
MCQ 1.3.15 Break points can be
(A) only real (B) only complex
(C) real or complex (D) none of these

MCQ 1.3.16 The angle of departure from a real pole is always
(A) $0^\circ$
(B) $180^\circ$
(C) either $0^\circ$ or $180^\circ$
(D) to be calculated for each problem

MCQ 1.3.17 The angle of arrival at a real zero is always
(A) $0^\circ$
(B) $180^\circ$
(C) either $0^\circ$ or $180^\circ$
(D) to be calculated for each problem

MCQ 1.3.18 A unit feedback system has open-loop poles at $s = -2 \pm j2$, $s = -1$, and $s = 0$; and a zero at $s = -3$. The angles made by the root-locus asymptotes with the real axis, and the point of intersection of the asymptotes are, respectively,
(A) $(60^\circ, -60^\circ, 180^\circ)$ and $-3/2$
(B) $(60^\circ, -60^\circ, 180^\circ)$ and $-2/3$
(C) $(45^\circ, -45^\circ, 180^\circ)$ and $-2/3$
(D) $(45^\circ, -45^\circ, 180^\circ)$ and $-4/3$

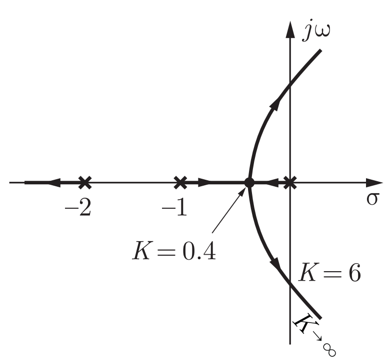
MCQ 1.3.19 Root locus plot of a feedback system as gain $K$ is varied, is shown in below. The system response to step input is non-oscillatory for






(A) $0 < K < 0.4$
(B) $0.4 < K < 6$
(C) $6 < K < \infty$
(D) None of the answers in (A), (B) and (C) is correct

MCQ 1.3.20 Consider the root locus plot shown in below.




(P) Adding a zero between $s = -1$ and $s = -2$ would move the root locus to the left.
(Q) Adding a pole at $s = 0$ would move the root locus to the right.

Which of the following is the correct answer ?
(A) None of the above statements is true
(B) Statement (i) is true but statement (ii) is false
(C) Statement (i) is false but statement (ii) is true
(D) Both the statements are true

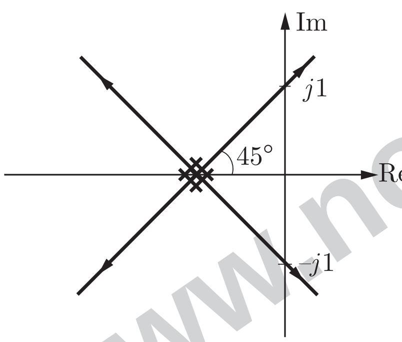


MCQ 1.3.21 Consider the root locus plot of unity-feedback system with open-loop transfer function,

$$G(s) = \frac{K(s+5)}{s(s+2)(s+4)(s^2+2s+2)}$$

The meeting point of the asymptotes on the real axis occurs at
(A) $-1.2$
(B) $-0.85$
(C) $-1.05$
(D) $-0.75$

MCQ 1.3.22 The root locus plot of the characteristic equation $1+KF(s)=0$ is given in below. The value of $K$ at $s=\pm j1$ is




(A) 4
(B) 1
(C) 10
(D) None of the answers in (A), (B), and (C) is correct.

MCQ 1.3.23 In a root locus plot,
(P) there is only one intersect to the asymptotes and it is always on the real axis;
(Q) the breakaway points always lie on the real axis.
Which of the following is the correct answer ?
(A) None of the statements is true
(B) Statement (P) is true but statement (Q) is false
(C) Statement (P) is false but statement (Q) is true
(D) Both the statements are true

MCQ 1.3.24 Consider the following statements :
(P) The effect of compensating pole is to pull the root locus towards left.
(Q) The effect of compensating zero is to press the locus towards right.

(A) None of the above statements is true

(B) Statement (P) is true but statement (Q) is false

(C) Statement (P) is false but statement (Q) is true

(D) Both the statements are true

MCQ 1.3.25 The loop transfer function $GH$ of a control system is given by

$$GH = \frac{K}{s(s+1)(s+2)(s+3)}$$

Which of the following statements regarding the conditions of the system root loci diagram is/are correct.

1. There will be four asymptotes.
2. There will be three separate root loci.
3. Asymptotes will intersect at real axis at $\sigma_A = -2/3$

Select the correct answer using the codes given below :

Codes :

(A) 1 alone

(B) 2 alone

(C) 3 alone

(D) 1, 2 and 3

MCQ 1.3.26 If the characteristic equation of a closed-loop system is

$$1 + \frac{K}{s(s+1)(s+2)} = 0$$

the centroid of the asymptotes in root-locus will be

(A) zero (B) 2

(C) $-1$ (D) $-2$

MCQ 1.3.27 **Assertion (A) :** The number of separate loci or poles of the closed loop system corresponding to $G(s)H(s) = \dfrac{K(s+4)}{s(s+1)(s+3)}$ is three.

**Reason (R) :** Number of separate loci is equal to number of finite poles of $G(s)H(s)$ if the latter is more than the number of finite zeros of $G(s)H(s)$.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false

(D) A is false but R is true

MCQ 1.3.28 Which one of the following is correct ?

The value of the system gain at any point on a root locus can be obtained as a

(A) product of lengths of vectors from the poles to that point

(B) product of lengths of vectors from the zeros to that point

(C) ratio of product of lengths of vectors from poles to that point to the product of length of vectors from zeros to that point

(D) product of lengths of vectors from all poles to zeros

MCQ 1.3.29 Which one of the following is not a property of root loci ?

(A) The root locus is symmetrical about $j\omega$ axis.

(B) They start from the open loop poles and terminate at the open loop zeros.

(C) The breakaway points are determined from $dk/ds = 0$.

(D) Segments of the real axis are part of the root locus, if and only if, the total number of real poles and zeros to their right is odd.

MCQ 1.3.30 The addition of open loop zero pulls the root-loci towards :

(A) The left and therefore system becomes more stable

(B) The right and therefore system becomes unstable

(C) Imaginary axis and therefore system becomes marginally stable

(D) The left and therefore system becomes unstable

MCQ 1.3.31 Which one of the following is correct ?
The root locus is the path of the roots of the characteristic equation traced out in the $s$-plane

(A) as the input of the system is changed

(B) as the output of the system is changed

(C) as a system parameter is changed

(D) as the sensitivity is changed

***********

# SOLUTIONS 1.1

SOL 1.1.1 Correct option is (D).

We check the validity of root locus for each of the given options.

Option (A) :

Root locus is always symmetric about real axis. This condition is not satisfied for option (A). A point on the real axis lies on the root locus if the total number of poles and zeros to the right of this point is odd. This is also not satisfied by (A). Thus, option (A) is not a root locus diagram.

Option (B) & Option (C) :

These does not satisfy the condition that, a point on the real axis lies on the root locus if the total number of poles and zeros to the right of this point is odd. Thus, option (B) and (C) are also not root locus diagram.

Option (D) :

This is symmetric about real axis and every point of locus satisfy the condition that number of poles and zeros in right of any point on locus be odd. Thus, the sketch given in option (D) can be considered as root locus for a system.

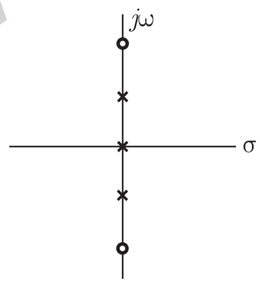


SOL 1.1.2 Correct option is (D).

Here, option (2) and option (3) both are not symmetric about real axis. So, both can not be root locus.

SOL 1.1.3 Correct option is (A).

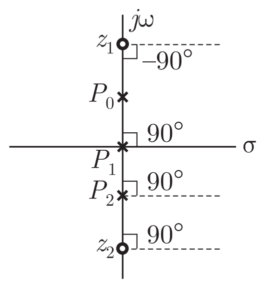
Here, pole-zero location is given as




The angle of departure of the root locus branch from a complex pole is given by

$$\phi_D = \pm[180° + \phi]$$

where $\phi$ is net angle contribution at this pole due to all other poles and zeros, as shown in figure below.



From the pole-zero plot, we have

$$\phi = \phi_Z - \phi_P = [-90^\circ + 90^\circ] - [90^\circ + 90^\circ] = -180^\circ$$

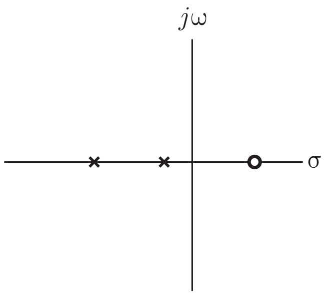
So, the departure angle is

$$\phi_D = \pm[180^\circ - 180^\circ] = \pm 0^\circ$$

Therefore, the departure angle for pole $P_0$ is $0^\circ$. Thus, root locus branch will depart at $0^\circ$. Only option (A) satisfies this condition.

SOL 1.1.4 Correct option is (A).

Given open loop pole-zero plot is




From the given plot, we have

Number of poles, $P = 2$

Number of zeros, $Z = 1$

Since, the number of branches of root locus is equal to number of poles, so we have

Number of branches $= 2$

Thus (B) and (D) are not correct.

Again, the branch of root locus always starts from open loop pole and ends either at an open loop zero (or) infinite. Thus, (C) is incorrect and remaining Correct option is (A).

SOL 1.1.5 Correct option is (C).

Root locus plot starts from poles and ends at zeros (or) infinite. Only option (C) satisfies this condition. No need to check further.

SOL 1.1.6 Correct option is (A).

Root locus always starts from open loop pole, and ends at open loop zero (or) infinite. Only option (A) satisfies this condition.

We can find the root locus of given plot as follows

Number of poles, $P = 2$

Number of zeros, $Z = 0$

So, we have number of asymptotes

$$P - Z = 2$$

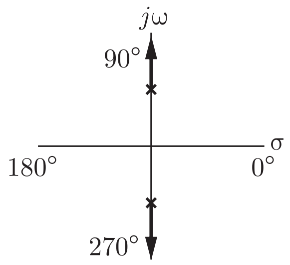
Also, the angle of asymptotes is obtained as

$$\phi_a = \frac{(2q+1)180^\circ}{P-Z};\ q = 0, 1$$

$$\phi_a = \frac{(0+1)180^\circ}{2} = 90^\circ;\ q = 0$$

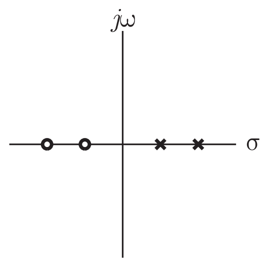
and $$\phi_a = \frac{(2+1)180^\circ}{P-Z} = \frac{3\times 180^\circ}{2} = 270^\circ;\ q = 1$$

Hence, we sketch the root locus plot as




SOL 1.1.7

Correct option is (A).
An open loop pole-zero plot is given as




Root locus always starts from open loop poles and ends at open loop zeros or infinite along with asymptotes. So, the options (C) and (D) are wrong. Again, a point on the real axis lies on the root locus if the total number of poles and zeros to the right of this point is odd. This is not satisfied by (B). Thus, the remaining Correct option is (A).

SOL 1.1.8

Correct option is (C).
Forward path open loop transfer function of given *ufb* system is

$$G(s) = \frac{K(s+2)(s+6)}{s^2+8s+25}$$

So, we have the characteristic equation as

$$s^2 + 8s + 25 = 0$$

or
$$s = \frac{-8 \pm \sqrt{64-100}}{2} = -4 \pm j3$$

or
$$s = -4 + j3;\ s = -4 - j3$$

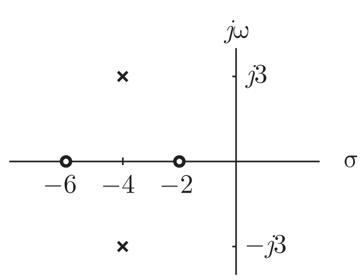
i.e. poles of the system are

$$s = -4 + j3;\ s = -4 - j3$$

Also, from the given transfer function, we have zeros of the system as

$$s = -2;\ s = -6$$

Thus, we get the pole-zero plot as shown below.




Also, we have the condition that root locus starts from poles and ends with zeros. Only option (C) satisfies this condition.

SOL 1.1.9 Correct option is (B).

Forward path open loop transfer function of given *ufb* system is

$$G(s) = \frac{K(s^2+4)}{s^2+1}$$

So, we obtain the poles of the system as

$$s^2+1 = 0$$

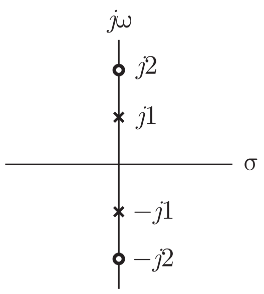
or $$s = \pm j1$$

Also, zeros of the system are obtained as

$$s^2+4 = 0$$

or $$s = \pm j2$$

Therefore, the pole-zero plot of the system is




Since, we have the condition that root locus starts from poles and ends with zeros. Thus, Correct option is (B).

SOL 1.1.10 Correct option is (A).

Forward path open loop transfer function of given *ufb* system is

$$G(s) = \frac{K(s^2+1)}{s^2}$$

So, we obtain the zeros of the system as

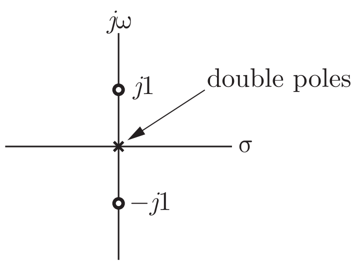
$$s^2+1 = 0$$

or $$s = \pm j1$$

Also, the poles of the system are

$$s = 0;\ s = 0$$

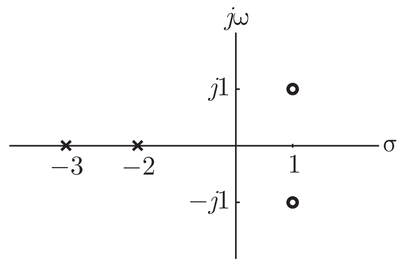
So, we have the pole-zero plot for the system as




Hence, option (B) and (D) may not be correct option. A point on the real axis lies on the root locus if the total number of poles and zeros to the right of this point is odd. This is not satisfied by (C) because at origin there are double pole. Thus, remaining Correct option is (A).

SOL 1.1.11 Correct option is (C).

Given open loop pole zero plot of the system




From above plot, we have the zeros and poles as

Zeros: $s = 1 + j1$ and $s = 1 - j1$

Poles: $s = -2$ and $s = -3$

So, the transfer function of the system is obtained as

$$G(s) = \frac{K\{s-(1+j1)\}\{s-(1-j1)\}}{\{s-(-2)\}\{s-(-3)\}}$$

$$= \frac{K\{(s-1)-j1\}\{(s-1)+j1\}}{(s+2)(s+3)}$$

$$= \frac{K\{(s-1)^2-(j1)^2\}}{(s+2)(s+3)}$$

or
$$G(s) = \frac{K(s^2-2s+2)}{(s+2)(s+3)}$$

SOL 1.1.12 Correct option is (C).

Root locus lies on real axis where number of poles and zeros are odd in number from that right side.

Hence, for the given pole-zero plot, root locus lies between poles $(-2)$ and $(-3)$ on real axis. From the given option, we have two points

$$\sigma = -1.29,\ \sigma = -2.43$$

Since, $\sigma = -1.29$ does not lie on root locus, so it can not be a break point. Therefore, the possible break point is $\sigma = -2.43$ which lies between $-2$ and $-3$. Now, we check wether the point is break away or break in.

On root locus, it may be seen easily that $\sigma = -2.43$ lies on root locus and locus start from poles $(-2)$ and $(-3)$. Therefore, at $\sigma = -2.43$ it must break apart. Thus, this point is break away point, i.e. the break point is

$$\text{Break away at } \sigma = -2.43$$

ALTERNATIVE METHOD :

Gain $K$ will be maximum at break away point and minimum at break in point. We can also check maxima and minima for gain $K$. The point, at which multiple roots are present, are known as break point. These are obtained from

$$\frac{dK}{ds} = 0 \qquad ...(1)$$

Now, we have the characteristic equation as

$$1 + G(s)H(s) = 0$$

or
$$1 + \frac{K(s^2-2s+2)}{(s+2)(s+3)} = 0$$

So,
$$K = \frac{-(s+2)(s+3)}{(s^2-2s+2)}$$

$$= \frac{-(s^2+5s+6)}{s^2-2s+2} \qquad ...(2)$$

Differentiating equation (2) w.r.t. $s$ and applying it to equation (1), we have

$$\frac{dK}{ds} = \frac{-(s^2-2s+2)(2s+5)+(s^2+5s+6)(2s-2)}{(s^2-2s+2)^2} = 0$$

or $\quad 7s^2+8s-22=0$

Solving the above expressions, we get

$$s = +1.29 \text{ and } s = -2.43$$

The point $s=-2.43$ is maxima for gain $K$, so $s=-2.43$ is break away point.

SOL 1.1.13 Correct option is (A).

Forward path transfer function of given *ufb* system is

$$G(s) = \frac{K(s+1)(s+2)}{(s+5)(s+6)}$$

So, we have the characteristics equation

$$1+G(s)H(s) = 0$$

or
$$1+\frac{K(s+1)(s+2)}{(s+5)(s+6)} = 0$$

or $\quad (s^2+11s+30)+K(s^2+3s+2) = 0$

or
$$K = -\frac{(s^2+11s+30)}{s^2+3s+2} \qquad ...(1)$$

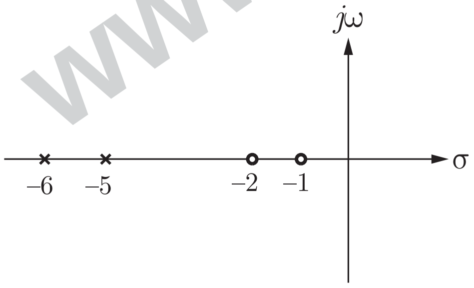
Differentiating above equation with respect to $s$ and equating to zero, we get

$$\frac{dK}{ds} = -\frac{(s^2+3s+2)(2s+11)+(s^2+11s+30)(2s+3)}{(s^2+3s+2)^2} = 0$$

or $\quad 8s^2+56s+68=0$

or $\quad s=-5.437$ and $s=-1.563$

Now, we have the pole-zero plot for the given system as shown below.




From the diagram, we note that root locus lies on real axis from $-6$ to $-5$ and from $-2$ to $-1$ because of odd number of pole and zero constrain.

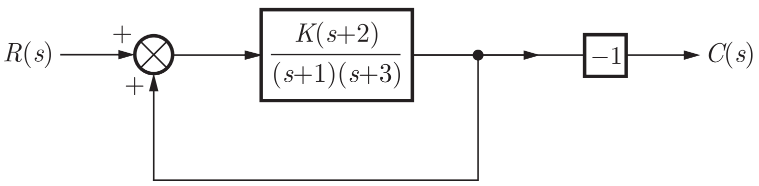
Now, we further note that locus starts from $-6$ and $-5$ (poles of the system).

Thus, locus must break apart at $s=-5.437$, i.e. it is break away point.

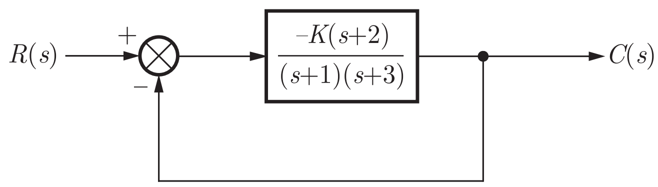
Again, the locus end at $-2$ and $-1$ (zeros of the system), thus there must be a break in at $s=-1.563$.

SOL 1.1.14 Correct option is (A).

The given system is shown below.



We redraw the block diagram after moving take off point as shown below.




So, the forward path transfer function is

$$G(s) = \frac{-K(s+2)}{(s+1)(s+3)}$$

Root locus is plotted for $K = 0$ to $K = \infty$. But, here the gain $K$ is negative. So, we will plot for $K = -\infty$ to $K = 0$. This is called complementary root locus.

For this case, the root locus on the real axis is found to the left of an even count of real poles and real zeros of $G(s)$. Also, the plot will start from pole and ends on zero. Only option (A) satisfies the condition for given system.

SOL 1.1.15 Correct option is (D).

For given system, forward path transfer function is

$$G(s) = \frac{K(s+6)}{(s+3)(s+5)}$$

So, the characteristic equation for closed loop transfer function is

$$1 + G(s)H(s) = 0$$

or $$1 + \frac{K(s+6)}{(s+3)(s+5)} = 0$$

or $$K = \frac{-(s^2+8s+15)}{(s+6)}$$

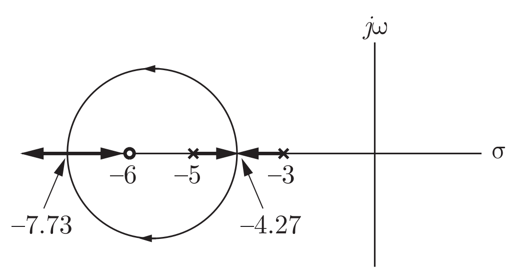
Differentiating the above expression with respect to $s$ and equating it to zero, we have

$$\frac{dK}{ds} = \frac{-(s+6)(2s+8)+(s^2+8s+15)}{(s+6)^2} = 0$$

or $$s^2 + 12s + 33 = 0$$

or $$s = -7.73 \text{ and } s = -4.27$$

Thus, we obtain the root locus for the system as shown below.




Observing the root locus, we can easily say that $s = -4.27$ is break away point and $s = -7.73$ is break in point.

SOL 1.1.16 Correct option is (B).

For the given system, we have the open loop transfer function

$$G(s)H(s) = \frac{K}{s(s+1)(s+2)}$$

In option (A) and option (C), the root locus on the real axis is found to the left of an even count of real poles and real zeros of $GH$. So, these can not be the root locus diagram. Now, we have the characteristic equation

$$1 + G(s)H(s) = 0$$

or
$$1 + \frac{K}{s(s+1)(s+2)} = 0$$

or
$$s^3 + 3s^2 + 2s + K = 0$$

So, we have the Routh's array for the system

| | | |
|---|---|---|
| $s^3$ | 1 | 2 |
| $s^2$ | 3 | $K$ |
| $s^1$ | $\frac{6-K}{3}$ | |
| $s^0$ | $K$ | |

At $K = 6$, $s^1$ row is zero, thus using auxiliary equation, we get

$$3s^2 + 6 = 0$$

or
$$s = \pm j\sqrt{2}$$

Root locus cut on $j\omega$ axis at $s = \pm j\sqrt{2}$ for $K = 6$. Since, the root locus given in option (D) does not cut $j\omega$ axis. So, it is not the root locus for given system. Therefore, the remaining Correct option is (B).

SOL 1.1.17 Correct option is (D).

For given *ufb* system, forward transfer function is

$$G(s) = \frac{K}{s^2}$$

Angle of departure or angle of asymptote for multiple poles is

$$\phi_a = \frac{(2q+1)180^\circ}{r};$$

where $r$ = number of multiple poles

$q = 0, 1, 2, ....(r-1)$

For the given system, we have

$r = 2$; (2 multiple poles at origin)

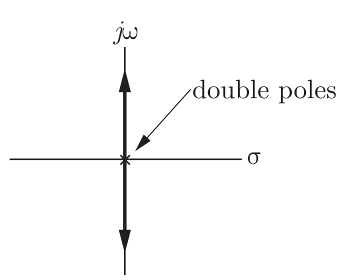
$q = 0, 1$

So, we obtain the angle of departure as

$$\phi_a = \frac{(0+1)180^\circ}{2} = 90^\circ \text{ for } q = 0$$

and
$$\phi_a = \frac{(2+1)180^\circ}{2} = 270^\circ \text{ for } q = 1$$

Hence, root locus plot will be as shown below.



SOL 1.1.18

Correct option is (C).

For given *ufb* system, open loop transfer function is

$$G(s) = \frac{K(s+3)(s+4)}{(s+1)(s+2)}$$

Given the two points,

$$s_1 = -2 + j3;\ s_2 = -2 + j\frac{1}{\sqrt{2}}$$

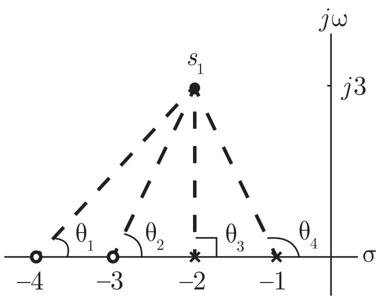
If any point lies on root locus, it satisfies the characteristic equation of the system, i.e.

$$q(s) = 1 + G(s)H(s) = 0$$

or $\quad |G(s)H(s)| = 1 \qquad$ (Magnitude)

and $\quad \angle G(s)H(s) = \pm 180^\circ \qquad$ (Phase)

Now, we consider the point $s_1 = -2 + j3$ as shown in the diagram below.

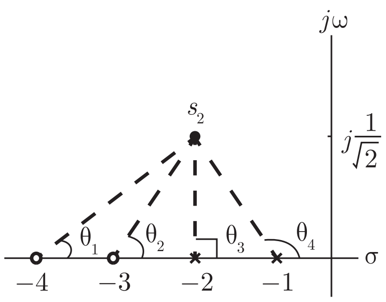



We check the point for the phase condition. At $s = s_1 = -2 + j3$, we have

$$\angle G(s)H(s)\big|_{s=s_1} = \phi_1$$
$$= \theta_1 + \theta_2 + \theta_3 + \theta_4$$
$$= \tan^{-1}\frac{3}{2} + \tan^{-1}\frac{3}{1} - 90^\circ - \tan^{-1}\left(\frac{3}{-1}\right)$$
$$= \tan^{-1}\frac{3}{2} + \tan^{-1}3 - 90^\circ - (180^\circ - \tan^{-1}3)$$
$$= 56.30 + 71.56 - 270^\circ + 71.56$$

or $\quad \phi_1 = -70.56 \neq \pm 180^\circ$

Hence, point $s_1$ does not lie on root locus. Again, we consider the point $s_2 = -2 + j\frac{1}{\sqrt{2}}$ as shown in the diagram below.




At the given point $s_2$, we have

$$\phi_2 = \theta_1 + \theta_2 - \theta_3 - \theta_4$$
$$= \tan^{-1}\frac{1}{2\sqrt{2}} + \tan^{-1}\frac{1}{\sqrt{2}} - 90^\circ - \left(180^\circ - \tan^{-1}\frac{1}{\sqrt{2}}\right)$$
$$= \tan^{-1}\frac{1}{2\sqrt{2}} + \tan^{-1}\frac{1}{\sqrt{2}} - 90^\circ - 180^\circ + \tan^{-1}\frac{1}{\sqrt{2}}$$
$$\phi_2 = -180^\circ$$

i.e. phase condition is satisfied. Hence, point $s_2$ lies on root locus.

SOL 1.1.19 Correct option is (A).

For given *ufb* system, open loop transfer function is

$$G(s) = \frac{K(s+\alpha)}{s^2(s+\beta)};\ \beta > \alpha > 0$$

So, we have the poles and zeros for the system as

Zero: $s = -\alpha$

Poles: $s = 0,\ 0,\ -\beta$

So, the departure angles at double poles on origin are obtained as

$$\phi = \frac{(2q+1)180^\circ}{r};\ r = 2,\ q = 0,\ 1$$

or $\phi = 90^\circ$ and $270^\circ$

To get intersection with imaginary axis, we use Routh's criteria. The characteristic equation for the system is given as

$$s^3 + \beta s^2 + Ks + K\alpha = 0$$

So, we have the Routh's array as

| | | |
|---|---|---|
| $s^3$ | 1 | $K$ |
| $s^2$ | $\beta$ | $K\alpha$ |
| $s^1$ | $\frac{\beta-\alpha}{\beta}$ | |
| $s^0$ | $K\alpha$ | |

Here, for any value of $K$, $s^1$ row of Routh array will not be zero. Thus, system is stable for all positive value of $K$, and hence root locus does not cross $j\omega$ axis. Therefore, root locus completely lies in left half of $s$-plane. Based on these results we say that Correct option is (A).

SOL 1.1.20 Correct option is (C).

Characteristic equation of given feedback control system is

$$(s^2+4s+4)(s^2+11s+30) + Ks^2 + 4K = 0$$

or
$$1 + \frac{K(s^2+4)}{(s^2+4s+4)(s^2+11s+30)} = 0 \qquad ...(1)$$

Since, the characteristic equation of a system is defined as

$$1 + G(s)H(s) = 0 \qquad ...(2)$$

Comparing equations (1) and (2), we get open loop transfer function as

$$G(s)H(s) = \frac{K(s^2+4)}{(s^2+4s+4)(s^2+11s+30)}$$
$$= \frac{K(s^2+4)}{(s+2)(s+2)(s+5)(s+6)}$$

So, the open loop poles and zeros of the system are

Poles: $s = -2, -2$ and $s = -6, -5$

Zeros: $s = \pm j2$

The point at which asymptotes meet (centroid) is given by

$$\sigma_A = \frac{\text{Sum of Re}[P] - \text{Sum of Re}[Z]}{(P-Z)}$$
$$= \frac{(-2-2-5-6)-0}{4-2} = \frac{-15}{2} = -7.5$$

This is the point on real axis. So, the asymptotes meet at $(-7.5, 0)$

SOL 1.1.21 Correct option is (D).

Open loop transfer function for given system is

$$G(s)H(s)=\frac{K}{s(s+4)(s^2+4s+5)}$$

So, we have the characteristic equation

$$1+\frac{K}{s(s+4)(s^2+4s+5)}=0$$

or $$K=-s(s+4)(s^2+4s+5) \qquad ...(1)$$

Differentiating the above equation with respect to $s$ and equating it to zero, we have

$$\frac{dK}{ds}=-\left[(2s+4)(s^2+4s+5)+(s^2+4s)(2s+4)\right]=0$$

or $$-(2s+4)(s^2+4s+5+s^2+4s)=0$$

or $$-(s+2)(2s^2+8s+5)=0 \qquad ...(2)$$

Solving the above equation, we get

$$s=-2 \text{ and } s=-0.775,\ -3.225$$

Now, we check for maxima and minima value of gain $K$ at above point. If gain is maximum, then that point will be break away point. If gain is minimum, then that point will be break in point. Again, differentiating equation (2) with respect to $s$, we get

$$\frac{d^2K}{ds^2}=-[(2s^2+8s+5)+(s+2)(4s+8)]$$
$$=-(6s^2+24s+21)$$

For $s=-0.775$ and $s=-3.225$, we have

$$\frac{d^2K}{ds^2}=-6.0<0$$

So, the points $s=-0.775$ and $-3.225$ are maxima points. Hence, $s=-0.775$ and $s=-3.225$ are break away points. Again, for $s=-2$, we have

$$\frac{d^2K}{ds^2}=+3>0$$

So, the point $s=-2$ is minima points. Hence, $s=-2$ is break in point. Thus, there is two break away points $(s=-.0775,-3.225)$ and one break in point $(s=-2)$.



SOL 1.1.22 Correct option is (B).

For a *ufb* system, forward transfer function is

$$G(s)=\frac{1}{s(s+\alpha)}$$

So, the characteristic equation of system is obtained as

$$1+G(s)H(s)=0$$

or $$1+\frac{1}{s(s+\alpha)}=0$$

or $$s^2+\alpha s+1=0$$

or $$1+\frac{\alpha s}{(s^2+1)}=0$$

Open loop transfer function, as $\alpha$ is varied, is

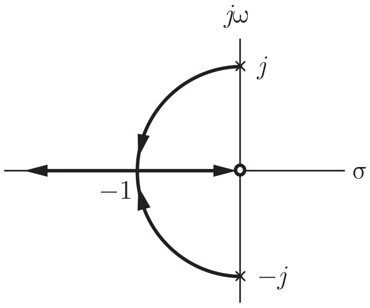
$$G(s)H(s)=\frac{\alpha s}{s^2+1}$$

So, we have the open loop poles and zeros as

$$\text{zero}: s=0$$

and poles : $s^2+1=0 \Rightarrow s=\pm j$

Therefore, we sketch the root locus for the system as




ALTERNATIVE METHOD :

The closed loop transfer function for the system is

$$T(s)=\frac{G(s)}{1+G(s)H(s)}=\frac{1}{s^2+\alpha s+1}$$

So,
$$\frac{G(s)}{1+G(s)H(s)}=\frac{\frac{1}{s^2+1}}{1+\frac{\alpha s}{s^2+1}}$$

or
$$G(s)H(s)=\frac{\alpha s}{s^2+1}$$

Now, we sketch the root locus by following the steps described in previous method.

SOL 1.1.23 Correct option is (A).

Forward path transfer function of given *ufb* system is

$$G(s)=\frac{K(s+\alpha)(s+3)}{s(s^2-1)};\ \alpha=5 \text{ and } K>0$$

or
$$G(s)=\frac{K(s+5)(s+3)}{s(s^2-1)}$$

So, we have the open loop poles and zeros for the system as

Zeros : $s=-5,\ s=-3$

Poles : $s=0,\ s=1,\ s=-1$

Locus branches start from poles and ends on zeros or infinite along asymptote. Here, number of asymptotes is

$$P-Z=3-2=1$$

Observing all the given options, we conclude that only option (A) has one asymptote. Now, the angle of asymptotes is given as

$$\phi_a=\frac{(2q+1)180^\circ}{(P-Z)};\ q=0,\ 1,\ 2,\ ....,\ (P-Z-1)$$
$$=\frac{(0+1)180^\circ}{1}=180^\circ$$

Only option (A) satisfies these conditions.

SOL 1.1.24 Correct option is (C).

Forward path transfer function of the given system is

$$G(s)=\frac{K(s+\alpha)(s+3)}{s(s^2-1)};\ K=10 \text{ and } \alpha>0$$

So, the characteristic equation of the system is obtained as

$$1 + G(s)H(s) = 0$$

or $$1 + \frac{10(s+\alpha)(s+3)}{s(s^2-1)} = 0$$

or $$s^3 - s + 10[s^2 + (\alpha+3)s + 3\alpha] = 0$$

or $$s(s^2 + 10s + 29) + \alpha 10(s+3) = 0 \qquad ...(1)$$

or $$1 + \frac{\alpha 10(s+3)}{s(s^2+10s+29)} = 0 \qquad ...(2)$$

So, we have the open loop gain as $\alpha$ is varied,

$$G(s)H(s) = \frac{\alpha 10(s+3)}{s(s^2+10s+29)} \qquad ...(3)$$

Therefore, the number of asymptotes are

$$P - Z = 3 - 1 = 2$$

So, two root branches will go to infinite along asymptotes as $\alpha \to \infty$. Now, from equation (1) we have

$$s^3 + 10s^2 + (29 + 10\alpha)s + 30\alpha = 0$$

So, we form the Routh's array as

| | | |
|---|---|---|
| $s^3$ | 1 | $29 + 10\alpha$ |
| $s^2$ | 10 | $30\alpha$ |
| $s^1$ | $29 + 7\alpha$ | |
| $s^0$ | $30\alpha$ | |

For $\alpha > 0$, $s^1$ row can not be zero. Hence, root locus does not intersect $j\omega$ axis for $\alpha > 0$. Only option (C) satisfies these conditions.



SOL 1.1.25 Correct option is (B).

Given the open loop transfer function,

$$G(s)H(s) = \frac{K(s+6)}{(s+2)(s+4)}$$

So, we have the open loop poles and zeros as

$$\text{Poles} : s = -2 \text{ and } s = -4$$

$$\text{Zeros} : s = -6$$

Therefore, the number of asymptotes is

$$P - Z = 2 - 1 = 1$$

So, the characteristic (1) is correct.

Now, we have the characteristic equation for the system

$$(s+2)(s+4) + K(s+6) = 0$$

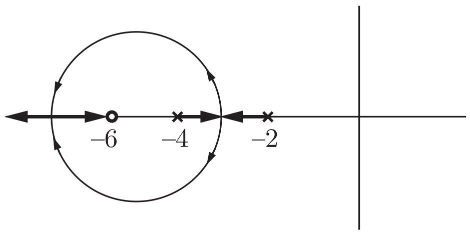
or $$s^2 + (6+K)s + 8 + 6K = 0$$

For the characteristic equation, we form the Routh's array as

| | | |
|---|---|---|
| $s^2$ | 1 | $8 + 6K$ |
| $s^1$ | $6 + K$ | |
| $s^0$ | $8 + 6K$ | |

Root locus is plotted for $K = 0$ to $\infty$. i.e. $K > 0$.

Here, for $K > 0$ root locus does not intersect $j\omega$ axis because $s^1$ row will not be zero. Thus, characteristic (2) is incorrect.
For the given system, we have two poles and one zero. So, one imaginary zero lies on infinite. Therefore, the characteristic (4) is incorrect.
Hence, (B) must be correct option. But, we check further for characteristic (3) as follows. We sketch the root locus for given system as




It has two real axis intersections. So, characteristic (3) is correct.

SOL 1.1.26 Correct option is (C).
Forward path transfer function of given *ufb* system is

$$G(s) = \frac{K(s+3)}{s(s+1)(s+2)(s+4)}$$

So, the open loop poles and zeros are

$$\text{zero: } s = -3 \qquad \text{(i.e. } Z = 1)$$

and $$\text{poles: } s = 0,\ s = -1,\ s = -2,\ s = -4 \qquad \text{(i.e. } P = 4)$$

So, we obtain the angle of asymptotes as

$$\phi_a = \frac{(2q+1)180^\circ}{(P-Z)};\ q = 0,\ 1,\ 2, \ldots..(P-Z-1)$$

$$= \frac{(0+1)180^\circ}{(4-1)} = \frac{180}{3} = 60^\circ;\ q = 0$$

$$= \frac{(2+1)180^\circ}{(4-1)} = 180^\circ;\ q = 1$$

$$= \frac{(4+1)180^\circ}{(4-1)} = 300^\circ;\ q = 2$$

Thus, $$\phi_a = \begin{cases} 60^\circ = \frac{\pi}{3}; & q = 0 \\ 180^\circ = \pi; & q = 1 \\ 300^\circ = \frac{5\pi}{3}; & q = 2 \end{cases}$$

SOL 1.1.27 Correct option is (B).
For the given system, we have the open loop transfer function

$$G(s)H(s) = \frac{K(s+10)(s^2+20s+500)}{s(s+20)(s+50)(s^2+4s+5)}$$

For the open loop transfer function, we obtain

$$\text{Separate loci} = \text{Number of open loop poles} = 5$$

$$\text{Asymptotes} = \text{Number of OLP} - \text{Number of OLZ} = 5 - 3 = 2$$

Matching the two parameters of root locus, we say that Correct option is (B). But, we check further for other characteristic as follows.

$$\text{Loci on real axis} = \text{number of poles that lie on real axis}$$

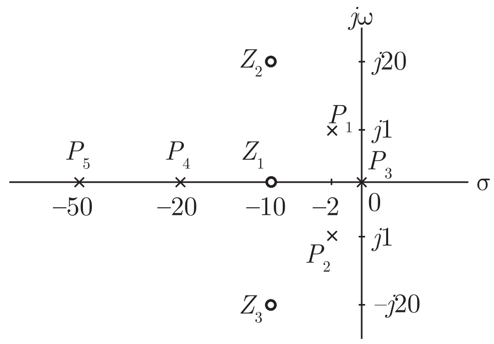
$$= 3;\ (s = 0,\ s = -20,\ s = -50)$$

Also, we have the open loop poles and zeros for the system as

$$\text{zeros: } s = -10,\ s = -10 \pm j20$$

poles: $s = 0,\ s = -20,\ s = -50,\ s = -2 \pm j1$

So, we have the pole zero plot as shown below.




So, we obtain the centroid as

$$\sigma_A = \frac{(0-20-50-2-2)-(-10-10-10)}{5-3}$$

$$= -22$$

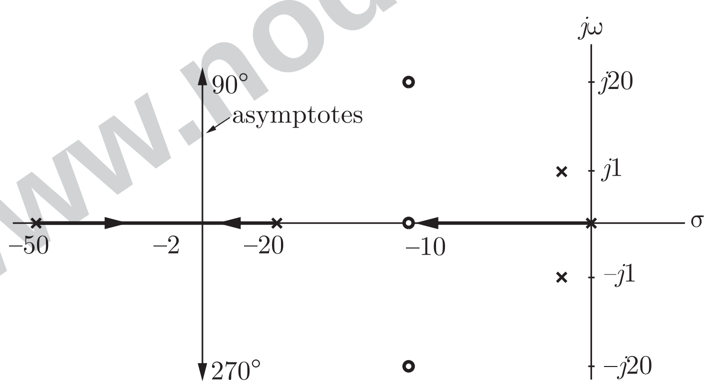
Also, the angle of asymptotes is given as

$$\phi_a = \frac{(2q+1)180^\circ}{P-Z};\ P-Z=2,\ q=0,\ 1$$

$$= \frac{(0+1)180^\circ}{2} = 90^\circ;\ q=0$$

$$= \frac{(2+1)180^\circ}{2} = 270^\circ;\ q=1$$

Therefore, we get the root locus for the system as




Here, root locus lies only in the region on real axis that is in left of an odd count of real poles and real zeros.

Hence, root locus lies between $-20$ and $-50$ and break away point will also be in this region. Thus, there will be only one break away point.

SOL 1.1.28 Correct option is (B).

The loci starts from $s = -1$ and 0, and ends at $s = -3$ and $\infty$. Hence, poles are $-1, 0$, and zeros are $-3,\ \infty$. Thus, the transfer function of the system is

$$\frac{K(s+3)}{s(s+1)}$$

SOL 1.1.29 Correct option is (D).

The characteristic equation of the given system is

$$s^2 + 5Ks + 9 = 0$$

or $$1+\frac{K5s}{s^2+9}=0$$

Since, we defined the characteristic equation as

$$1+G(s)H(s)=0$$

So, open loop transfer function of the system is

$$G(s)H(s)=\frac{5Ks}{s^2+9}$$

Therefore, we have the open loop poles and zeros of the system,

poles: $s=\pm j3$

zeros: $s=0$

Option (A) and option (B) are incorrect because root locus are starting from zeros. On real axis, loci exist to the left of odd number of real poles and real zeros. Hence, only Correct option is (D).

SOL 1.1.30 Correct option is (C).

Open loop transfer function of given *ufb* system is

$$G(s)=\frac{K(1-s)}{s(s+3)}=\frac{-K(s-1)}{s(s+3)}$$

So, we have the open loop poles and zeros as

poles: $s=0,\ s=-3$

zeros: $s=1$

Here, gain $K$ is negative, so root locus will be complementary root locus and is found to the left of an even count of real poles and real zeros of $GH$. Hence, option (A) and option (D) are incorrect. Option (B) is also incorrect because it does not satisfy this condition. Thus, option (C) gives the correct root locus diagram.

SOL 1.1.31 Correct option is (A).

Open loop transfer function of the given system is

$$G(s)=\frac{K(s+\frac{2}{3})}{s^2(s+2)}$$

So, we have the open loop poles and zeros as

poles: $s=0,\ s=0,\ s=-2$

zero: $s=-\frac{2}{3}$

Therefore, the number of asymptotes is given as

$$P-Z = \text{Number of OLP} - \text{Number of OLZ}$$
$$=3-1=2$$

So, we obtain the angle of asymptotes

$$\phi_a=\frac{(2q+1)180^\circ}{P-Z};\ P-Z=2,\ q=0,1$$

or $$\phi_a=\frac{(0+1)180^\circ}{2}=90^\circ \text{ for } q=0$$

and $$\phi_a=\frac{(2+1)180^\circ}{2}=270^\circ \text{ for } q=1$$

The centroid is obtained as

$$\sigma_A=\frac{\text{Sum of Re}[P]-\text{Sum of Re}[Z]}{P-Z}$$

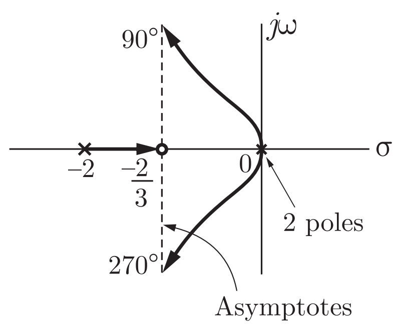
$$= \frac{(0-2)-\left(-\frac{2}{3}\right)}{3-1} = -\frac{2}{3}$$

Also, we obtain the angle of departure at double pole (at origin) as

$$\phi_D = \frac{(2q+1)180^\circ}{r}; \; r = 2, \; q = 0, \, 1$$

$$= 90^\circ, \, 270^\circ$$

Thus, from above analysis, we sketch the root locus as




From root locus, it can be observed easily that for all values of gain $K$ ( $K = 0$ to $\infty$) root locus lie only in left half of $s$-plane.

SOL 1.1.32 Correct option is (C).

Characteristic equation of the given closed loop system is

$$s(s+1)(s+3) + K(s+2) = 0; \quad K > 0$$

or
$$1 + \frac{K(s+2)}{s(s+1)(s+3)} = 0$$

So, the open loop transfer function is given as

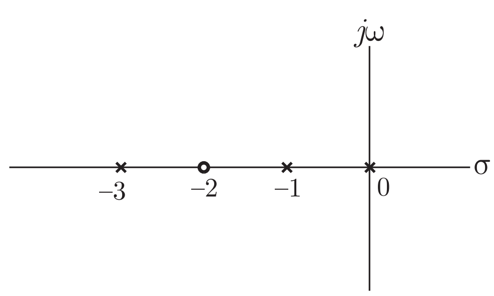
$$G(s)H(s) = \frac{K(s+2)}{s(s+1)(s+3)}$$

Therefore, we have the open loop poles and zeros as

poles: $s = 0, \; s = -1, \; s = -3$

zero: $s = -2$

So, we obtain the pole zero plot for the system as




For the pole-zero location, we obtain the following characteristic of root locus

Number of asymptotes: $P - Z = 3 - 1 = 2$

Angles of asymptotes: $\phi_a = \frac{(2q+1)180^\circ}{P-Z}; \; P - Z = 2, \; q = 0, \, 1$

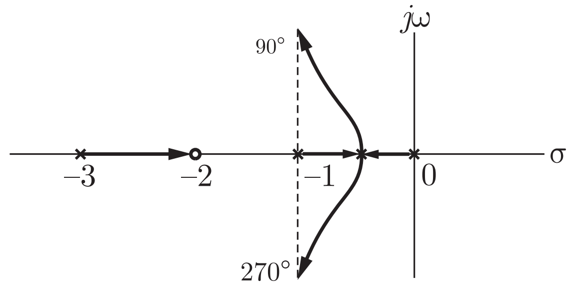
$\phi_a = 90^\circ$ and $270^\circ$

Centroid: $\sigma_A = \frac{\text{Sum of Re}[P] - \text{Sum of Re}[Z]}{P - Z}$

$$= \frac{(0-1-3)-(-2)}{3-1} = -\frac{2}{2}$$

$$= -1$$

Thus, from above analysis, we sketch the root locus as




For the root locus, we conclude the following points

1. The break away point lies in the range,

$$-1 < \mathrm{Re}[s] < 0$$

2. Two of its roots tends to infinite along the asymptotes $\mathrm{Re}[s] = -1$.
3. Root locus lies only in left half of $s$-plane.

SOL 1.1.33 Correct option is (B).

For closed loop system, given characteristic equation is

$$(s^2-4)(s+1) + K(s-1) = 0 \qquad ...(1)$$

or
$$1 + \frac{K(s-1)}{(s^2-4)(s+1)} = 0 \qquad ...(2)$$

Since, we define the characteristic equation as

$$1 + G(s)H(s) = 0$$

So, the open loop transfer function of the system is obtained as

$$G(s)H(s) = \frac{K(s-1)}{(s^2-4)(s+1)}$$

For the given system, we have the open loop poles and zeros as

poles: $s = 2,\ s = -2,\ s = -1$

zeros: $s = 1$

So, we obtain the following characteristic for root locus

Number of branches of loci: $P$ = number of OLP $= 3$

Number of asymptotes: $P - Z$ = number of OLP − number of OLZ $= 3 - 1 = 2$

Angle of asymptotes:
$$\phi_a = \frac{(2q+1)180^\circ}{P-Z};\ P - Z = 2,\ q = 0, 1$$

$$\phi_a = \frac{(0+1)180^\circ}{2} = 90^\circ \text{ for } q = 0$$

$$\phi_a = \frac{(2+1)180^\circ}{2} = 270^\circ \text{ for } q = 1$$

Centroid:
$$\sigma_A = \frac{\text{Sum of Re}[P] - \text{Sum of Re}[Z]}{P-Z}$$

$$= \frac{(-1-2+2)-(1)}{3-1} = -\frac{2}{2}$$

$$= -1$$

Root locus on the real axis is found to the left of an odd count of real poles and zeros of $GH$. From equation (2), we have

$$K = -\frac{(s^2-4)(s+1)}{(s-1)}$$

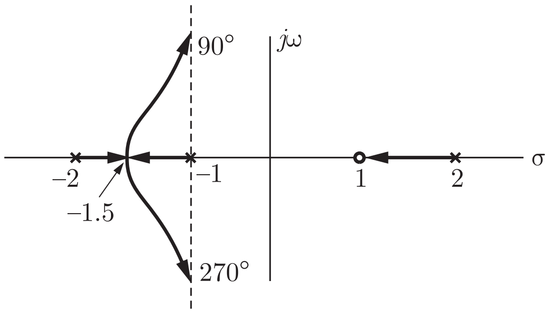
Now, we obtain the break-away point (point of maxima) as

$$\frac{dK}{ds} = 0$$

or $$\frac{-(s-1)[(s^2-4)+(s+1)(2s)]+(s^2-4)(s+1)}{(s-1)^2} = 0$$

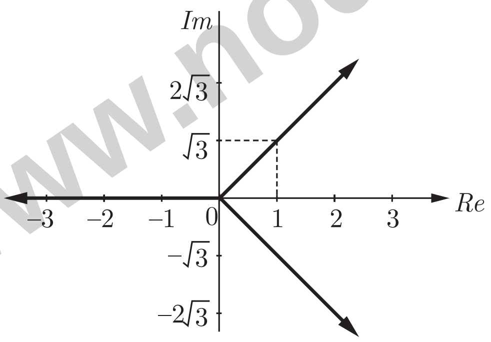
or $$s = -1.5$$

This is a break away point. From above analysis, we sketch the root locus as




SOL 1.1.34 Correct option is (A).

Given root locus plot,




From given plot, we can observe that centroid (point where asymptotes intersect on real axis) is origin and all three root locus branches also start from origin and goes to infinite along with asymptotes. Therefore, there is no any zero and three poles are at origin.

So, option (A) must be correct.

$$G(s) = \frac{K}{s^3}$$

Now, we verify the above result as follows. Using phase condition, we have

$$\angle G(s)H(s)\big|_{s=s_0} = \pm 180^\circ$$

From given plot, for a given point on root locus, we have

$$\angle G(s)H(s)\big|_{s=(1,\sqrt{3})} = -3\tan^{-1}\left(\frac{y}{x}\right)$$
$$= -3\tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$$
$$= -3\times 60^\circ = -180^\circ$$

SOL 1.1.35 Correct option is (B).

Given open loop transfer function,

$$G(s) = \frac{K}{s^2}$$

So, we have the open loop poles $s = 0, 0$; i.e.

Number of poles, $P = 2$

Number of zeros, $Z = 0$

Therefore, root loci starts $(K = 0)$ from $s = 0$ and $s = 0$. Since, there is no open loop zero, root loci terminate $(K = \infty)$ at infinity. Now, we obtain the characteristics of root locus as

Angle of asymptotes:

$$\phi_a = \frac{(2q+1)180^\circ}{P-Z}; \ P - Z = 2 - 0 = 2, \ q = 0, 1$$

$$= \frac{(0+1)180^\circ}{2} = 90^\circ \text{ for } q = 0$$

$$= \frac{(2+1)180^\circ}{2} = 270^\circ \text{ for } q = 1$$

Centroid: $$\sigma_A = \frac{\text{Sum of Re}[P] - \text{Sum of Re}[Z]}{P-Z} = \frac{0-0}{2} = 0$$

Break-away point:

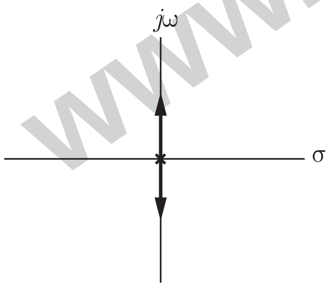
$$1 + \frac{K}{s^2} = 0$$

or $$K = -s^2$$

So, $$\frac{dK}{ds} = -2s = 0$$

$$s = 0$$

Thus, from the above analysis, we have the root locus plot as




SOL 1.1.36 Correct option is (B).

Open loop transfer function of given *ufb* system is

$$G(s) = \frac{2(s+\alpha)}{s(s+2)(s+10)}$$

So, we have the characteristic equation as

$$1 + \frac{2(s+\alpha)}{s(s+2)(s+10)} = 0$$

or $$s(s+2)(s+10) + 2s + 2\alpha = 0$$

or $$s^3 + 12s^2 + 22s + 2\alpha = 0$$

or $$1 + \frac{2\alpha}{s^3 + 2s^2 + 22s} = 0$$

Therefore, we get the open loop transfer function as $\alpha$ varies,

$$G(s)H(s) = \frac{2\alpha}{s^3 + 12s^2 + 22s}$$

So, the number of open loop poles and zeros are

Number of poles, $P = 3$

Number of zeros, $Z = 0$

Also, we obtain the angle of asymptotes as

$$\phi_a = \frac{(2q+1)180^\circ}{P-Z}; \; P-Z = 3; \; q = 0, 1, 2$$

$$= \frac{(0+1)180^\circ}{3} = 60^\circ \text{ for } q = 0$$

$$= \frac{(2+1)180^\circ}{3} = 180^\circ \text{ for } q = 1$$

$$= \frac{(4+1)180^\circ}{3} = 300^\circ \text{ for } q = 2$$

$$\phi_a = 60^\circ, 180^\circ, 300^\circ$$

SOL 1.1.37 Correct option is (C).

The intercept point (centroid) of asymptotes is defined as

$$\sigma_A = \frac{\text{Sum of Re}[P] - \text{Sum of Re}[Z]}{P-Z}$$

Since, we have the open loop poles and zeros as

Poles: $s = 0$, $s = -2$, $s = -10$

Zeros: No any zero

Therefore, we get

$$\sigma_A = \frac{(0-2-10)-0}{3-0} = -4$$

SOL 1.1.38 Correct option is (C).

For the given system, we have the open loop transfer function

$$G(s)H(s) = \frac{2\alpha}{s^3 + 12s^2 + 22s}$$

So, we obtain the gain $(K)$ for the system as

$$K = \frac{-(s^3 + 12s^2 + 22s)}{2}$$

For break-away point (maxima point), we have

$$\frac{dK}{ds} = 0$$

$$\frac{-(3s^2 + 24s + 22)}{2} = 0$$

or $$-3s^2 - 24s - 22 = 0$$

So, $$s = -1.056, -6.943$$

Thus, the break-away points are

$$s = -1.056 \text{ and } -6.943$$

SOL 1.1.39 Correct option is (A).

The characteristics equation of given system is

$$s^3 + 2s^2 + Ks + K = 0$$

or $$s^3 + 2s^2 + K(s+1) = 0$$

or $$1 + \frac{K(s+1)}{s^2(s+2)} = 0$$

So, we have the open loop transfer function

$$G(s)H(s) = \frac{K(s+1)}{s^2(s+2)}$$

For the system, we have open loop poles and zeros

zeros: $s = -1$; Number of zeros: $Z = 1$

poles: $s = 0,\ s = 0,\ s = -2$; Number of poles: $P = 3$

Root loci starts $(K = 0)$ at $s = 0$, $s = 0$ and $s = -2$. One of root loci terminates at $s = -1$ and other two terminates at infinity. So, we have the characteristic of root loci as given below.

Number of asymptotes:

$$P - Z = 2$$

Angle of asymptotes:

$$\phi_a = \frac{(2q+1)180^\circ}{P-Z};\ P - Z = 2,\ q = 0, 1$$

$$= \frac{(0+1)180^\circ}{2} = 90^\circ;\ q = 0$$

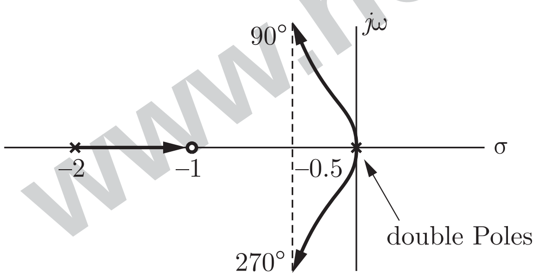
$$= \frac{(2+1)180^\circ}{2} = 270^\circ;\ q = 1$$

Intercept point (centroid) of asymptotes on real axis:

$$\sigma_A = \frac{\text{Sum of Re}[P] - \text{Sum of Re}[Z]}{P-Z}$$

$$= \frac{(0+0-2)-(-1)}{3-1} = -\frac{1}{2} = -0.5$$

So, we get the root locus for the system as




SOL 1.1.40 Correct option is (C).

From the root locus plot, we can observe that the difference between the values of $K$ at the break point and at the point of intersection of the root locus with the imaginary axis gives the range of $K$. Since, the poles are complex conjugate in this region, so the system has damped oscillatory response. Hence, we first find value of $K$ for the point of intersection with imaginary axis and then determine value of $K$ at the break away point. For the given system, we have the Routh's array as

| | | |
|---|---|---|
| $s^3$ | 1 | 8 |
| $s^2$ | 6 | $K$ |
| $s^1$ | $\frac{48-K}{6}$ | |
| $s^0$ | $K$ | |

The characteristic equation is given by

$$1+G(s)H(s)=0$$

or $$1+\frac{K}{s(s+2)(s+4)}=0$$

or $$s^3+6s^2+8s+K=0 \qquad ...(1)$$

For intersection of root locus with imaginary axis, $s^1$ row should be zero, i.e.

$$\frac{48-K}{6}=0$$

or $$K=48$$

The breakaway point is given by

$$\frac{dK}{ds}=0 \qquad ...(2)$$

From equation (1), we have

$$K=-(s^3+6s^2+8s)$$

Substituting it in equation (2), we get

$$\frac{dK}{ds}=-(3s^2+12s+8)=0$$

or $$3s^2+12s+8=0$$

or $$s=-2\pm 1.15$$

So, $$s=-3.15 \text{ and } s=-0.85$$

From the plot, we can observe that $s=-0.85$ is the actual break away point out of these two points. For $s_0=-0.85$, the value of $K$ is obtained as

$$K=\frac{\Pi(\text{Phasor lengths from } s_0 \text{ to the } OLP)}{\Pi(\text{Phasor lengths form } s_0 \text{ to the } OLZ)}$$

Here, no single zero in the system, hence

$$K=0.85\times 1.15\times 3.15=3.08$$

Therefore, the range of $K$ for damped oscillatory system is

$$3.08<K<48$$

SOL 1.1.41 Correct option is (B).

1. From given root locus plot, we can see that for all positive values of $K$ ($K=0$ to $\infty$) system poles lie in left half of $s$-plane, hence system is stable for all positive values of $K$.
2. Only for $K=7.464$ and $K=0.573$, poles of the system are real and repeated. For range of $K$ : $0.573<K<7.464$ poles of the system are complex conjugate in LH of $s$-plane.
3. System has damped oscillatory response for complex conjugate poles in left half which is possible only for $0.573<K<7.464$.
4. When the system has real and distinct poles, then response is overdamped. Hence, from root locus plot, real and distinct poles are possible for $0<K<0.573$ and also for $K>7.464$.

SOL 1.1.42 Correct option is (D).

For low frequencies, we have

$$e^{-s}\approx 1-s$$

So, the open loop transfer function is

$$G(s)H(s)=\frac{K(1-s)}{s(s+2)}=\frac{-K(s-1)}{s(s+2)}$$

or $$|G(s)H(s)| = \left|\frac{K(1-s)}{s(s+2)}\right| = 1$$

or $$K = \frac{s(s+2)}{1-s}$$

The break points are given by solution of

$$\frac{dK}{ds} = 0$$

or $$\frac{d}{ds}\left[\frac{s(s+2)}{1-s}\right] = 0$$

or $$(1-s)(2s+2) - s(s+2)(-1) = 0$$

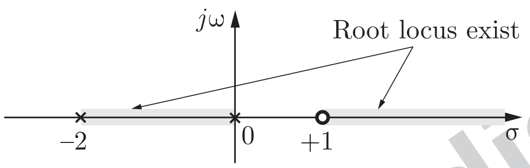
or $$s^2 - 2s - 2 = 0$$

Therefore, the break points are

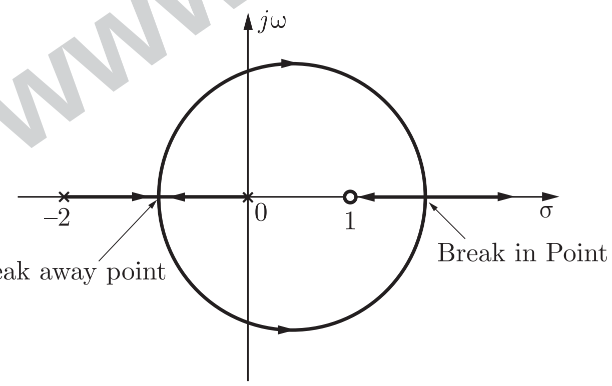
$$s = \frac{+2 \pm \sqrt{4+8}}{2} = 1 \pm \sqrt{3}$$

$$= 2.73, -0.73 \text{ or } -0.73$$

Since, $G(s)H(s)$ is negative, so the root locus will be complementary root locus and will exist at any point on the real axis, if the total number of poles and zeros to the right of that point is even.




Root locus will exist on real axis between $s = -2$ and 0 and also for $s > +1$. Hence, break away point will be $s = -0.73$ and break in point will be $s = +2.73$




SOL 1.1.43 Correct option is (B).

The open loop transfer function is

$$G(s)H(s) = \frac{K}{s(s+4)(s^2+4s+20)}$$

So, $$|G(s)H(s)| = \left|\frac{K}{s(s+4)(s^2+4s+20)}\right| = 1$$

or $$K = s(s+4)(s^2+4s+20)$$

The break points are given by the solution of

$$\frac{dK}{ds} = 0$$

or $$-(4s^3 + 24s^2 + 72s + 80) = 0$$

or $$s^3+6s^2+18s+20=0$$

or $$(s+2)(s^2+4s+10)=0$$

Hence, solving the above equation, we get the break points as

$$s=-2 \text{ and } s=-2\pm j2.45$$

i.e. the root locus has one real break point and two complex break point.

ALTERNATIVE METHOD :

For the open loop transfer function of the form,

$$G(s)H(s)=\frac{K}{s(s+a)(s^2+bs+c)}$$

if $$\left(\frac{-a}{2}\right)=\left(\frac{-b}{2}\right)$$

then, number of break points $=3$

These may be all three real (1 real + 2 real).

If $$\left(\frac{-a}{2}\right)\neq\left(\frac{-b}{2}\right)$$

then, number of break points $=1$ real

if $$\left(\frac{-a}{2}\right)=\left(-\frac{b}{2}\right)$$

then, we check $$\left|\frac{-a}{2}\right|\times x=c$$

Here, if $x\le 5\Rightarrow 3$ real break point

and if $x>5\Rightarrow 1$ real and 2 complex

For given problem, we have

$$G(s)H(s)=\frac{K}{s(s+a)(s^2+bs+c)}$$

$$=\frac{K}{s(s+4)(s^2+4s+20)}$$

Here, we have $$a=4,\ b=4,\ c=20$$

So, $$-\frac{a}{2}=-2=\frac{-b}{2}$$

Hence, there are three break points.

and $$\left|-\frac{a}{2}\right|\times x=c$$

or $$|(-2)|\times x=20\Rightarrow x=10>5$$

Thus, there are 1 real and 2 complex break points.

SOL 1.1.44 Correct option is (A).

The open loop transfer function is

$$G(s)H(s)=\frac{Ks}{(s^2-s+4.25)}$$

Here, $$|G(s)H(s)|=\left|\frac{Ks}{s^2-s+4.25}\right|=1$$

or $$K=\frac{s^2-s+4.25}{s}$$

The break point is given by the solution of

$$\frac{dK}{ds}=0$$

or $$\frac{d}{ds}\left[\frac{s^2-s+4.25}{s}\right]=0$$

or $$s^2 - 4.25 = 0$$

or $$s^2 = 4.25$$

So, $$s = \pm 2.06$$

The critical point, $s = -2.06$ is break point that belongs to the root locus. The other critical point $s = 2.06$ belongs to the complementary root locus. From the given gain $(K)$ plot, we can see that at point $s = -2.06$, gain has a minima. Hence, this point will be break in point. Because, minimum value of gain $K$ is achieved at break in point and maximum value at break away point.

SOL 1.1.45 Correct option is (B).

The characteristic equation is given by

$$1 + G(s)H(s) = 0$$

or $$1 + \frac{K(s-1)}{(s+1)(s+2)} = 0$$

or $$s^2 + (3+K)s + (2-K) = 0 \qquad ...(1)$$

It is required that one pole should lie at $s = 0$. Let another pole lies at $s = -P$, then required equation is

$$(s+0)(s+P) = 0$$

or $$s^2 + Ps = 0 \qquad ...(2)$$

On comparing equations (1) and (2), we get

$$2 - K = 0$$

or $$K = 2$$

SOL 1.1.46 Correct option is (B).

Substituting $K = 2$ in equation (1) in previous solution, we have

$$s^2 + 5s = 0 \qquad ...(1)$$

It is required that one pole at $s = 0$ and other pole at $s = -P$. So, the required equation is

$$s(s+P) = 0$$

$$s^2 + Ps = 0 \qquad ...(2)$$

Comparing equations (1) and (2), we get

$$P = 5$$

Hence, other pole is located at $(-5, 0)$.

SOL 1.1.47 Correct option is (C).

The characteristic equation of the system is given by

$$1 + G(s)H(s) = 0$$

or $$1 + \frac{K(s+2)}{s(s+1)(s+30)} = 0$$

or $$s(s+1)(s+30) + K(s+2) = 0$$

or $$s^3 + 31s^2 + (30+K)s + 2K = 0 \qquad ...(1)$$

We require the two poles with real part of $-2$, i.e.

$$s = -2 \pm j\omega$$

Assume other pole is $s = -P$, then required equation is

$$(s+P)(s+2+j\omega)(s+2-j\omega) = 0$$

or $$(s+P)\left[(s+2)^2-(j\omega)^2\right]=0$$

or $$(s+P)(s^2+4s+4+\omega^2)=0$$

or $$s^3+(4+P)s^2+(4+4P+\omega^2)s+P(4+\omega^2)=0 \qquad ...(2)$$

Now, comparing equations (1) and (2), we have

$$P(4+\omega^2)=2K \qquad ...(3)$$

and $$(4+4P+\omega^2)=30+K \qquad ...(4)$$

and $$(4+P)=31 \Rightarrow P=27 \qquad ...(5)$$

Solving equations (3), (4), and (5), we get

$$27(4+\omega^2)=2K$$

or $$(112+\omega^2)=30+K$$

or $$\omega^2=\frac{2K}{27}-4=(30+K-112)$$

or $$2K-108=27K-2214$$

or $$25K=2106$$

So, $$K=\frac{2106}{25}=84.24$$

SOL 1.1.48 Correct option is (C).

From previous solution, we have third pole at

$$s=-P$$

Since, $$P=27$$

Hence, third pole of closed loop system is at

$$s=-27$$

SOL 1.1.49 Correct option is (C).

From previous solution, we have

$$P=27$$

and $$K=\frac{2106}{25}$$

Then, from equation (3) in above solution, we have

$$\omega^2=\frac{2K}{27}-4=2.24$$

or $$\omega=\pm 1.497$$

Therefore, complex poles are

$$s=-2\pm j\omega$$

$$=-2\pm j1.497$$

SOL 1.1.50 Correct option is (C).

For delay time $\tau_D=1\,\text{sec}$, the characteristic equation of the system is

$$1+\frac{Ke^{-s}}{s}=0;\ K\geq 0$$

Now, we have the approximation

$$e^{-s}\cong\frac{1-s/2}{1+s/2}=\frac{2-s}{2+s}$$

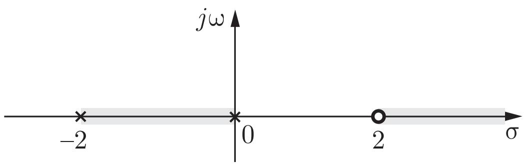
So, the characteristic equation becomes

$$1-\frac{K(s-2)}{s(s+2)}=0 \qquad ...(1)$$

Therefore, the open loop transfer function is

$$G(s)H(s) = \frac{-K(s-2)}{s(s+2)}$$

Since, $G(s)H(s)$ is negative, so the root locus will be complementary root locus and will exist at any point on the real axis, if the total number of poles and zeros to the right of that point is even.




So, the root locus (complementary) for the given system will exist on real axis in the region

$$-2 < s < 0 \quad \text{and} \quad s > 2$$

The break points of the system are given by solution of

$$\frac{dK}{ds} = 0 \qquad ...(2)$$

From equation (1), we have

$$K = \frac{s(s+2)}{(s-2)}$$

Substituting it in equation (2), we get

$$\frac{d}{ds}\left[\frac{s(s+2)}{(s-2)}\right] = 0$$

or $$(s-2)(2s+2) - s(s+2) = 0$$

or $$2s^2 - 2s - 4 - s^2 - 2s = 0$$

or $$s^2 - 4s - 4 = 0$$

So, $$s = \frac{4 \pm 5.657}{2} = 4.83, -0.83$$

Hence, break away point is $s = -0.83$ and beak in point is $s = 4.83$.



SOL 1.1.51 Correct option is (B).

The sketch shows the variation of gain with respect to real axis, the maxima is found at $-\sigma_1$ and minima is found at $\sigma_2$.

Maxima indicates the breakaway point and minima indicates the break in point. Hence, $(-\sigma_1)$ is breakaway point and $\sigma_2$ is break in point.

SOL 1.1.52 Correct option is (C).

For the given system, the characteristic equation is

$$1 + G(s)H(s) = 0$$

or $$1 + \frac{K}{s(s+10)} = 0$$

or $$s^2 + 10s + 5 = 0 \qquad (K = 5)$$

So, the poles (roots) of system are

$$s = \frac{-10 \pm 8.944}{2}$$

or $$s = -9.47 \text{ and } s = -0.53$$

SOL 1.1.53 Correct option is (B).

The actual change in the closed loop poles can be given by root sensitivity as converting the partial change to finite change, i.e.

$$S_K^s = \frac{K}{s}\frac{\Delta s}{\Delta K}$$

Hence, change in poles location is given as

$$\Delta s = sS_K^s \frac{\Delta K}{K} \qquad ...(1)$$

Given that % change in $K$ is 10. So, we have

$$\frac{\Delta K}{K} \times 100 = 10$$

or $$\frac{\Delta K}{K} = 0.1 \qquad ...(2)$$

Also, we have

$$S_K^s = -0.059 \text{ at } s = -9.47 \qquad ...(3)$$

Substituting value of equation (2) and (3) in equation (1), we get

$$\Delta s = (-9.47)(-0.059)(0.1)$$
$$= 0.056$$

Since, the change $\Delta s$ is positive, so it moves in right side. Hence, the pole will move to the right by 0.056 units for a 10% change in $K$.

SOL 1.1.54 Correct option is (D).

For the given system, the open loop poles and zeros are

poles: $s = 0$ and $s = 1$

zero: $s = -1$

So, we have the characteristic equation

$$1 + \frac{K(s+1)}{s(s-1)} = 0$$

or $$K = -\frac{s(s-1)}{s+1}$$

The break points are given by solution of

$$\frac{dK}{ds} = 0$$

So, $$\frac{d}{ds}\left[\frac{-s(s-1)}{s+1}\right] = 0$$

or $$(s+1)(2s-1) - s(s-1) = 0$$

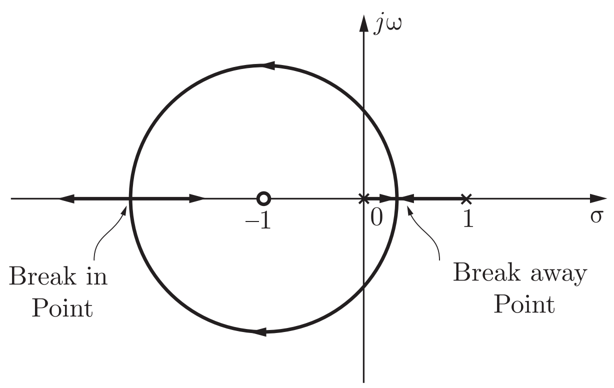
or $$2s^2 - s + 2s - 1 - s^2 + s = 0$$

or $$s^2 + 2s - 1 = 0$$

or $$s = \frac{-2 \pm 2.828}{2}$$

$$= -1 \pm 1.414 \qquad ...(1)$$

The root locus for the system is given below.



From the root locus, we get

$$\text{centre of circle} = (-1, 0)$$

and $\quad \text{radius of circle} = 1.414 = \sqrt{2}$

SOL 1.1.55 Correct option is (C).

For a system with open loop transfer function $G(s)H(s)$, the criterion of root locus is

$$\angle G(s)H(s) = 180^\circ$$

or, $\quad \angle(s+3) - \angle s - \angle(s+2) = 180^\circ$

Substituting $s = \sigma + j\omega$ in above equation, we get

$$\angle(\sigma + j\omega + 3) - \angle(\sigma + j\omega) - \angle(\sigma + j\omega + 2) = 180^\circ$$

or $\quad \tan^{-1}\left(\frac{\omega}{\sigma+3}\right) - \tan^{-1}\left(\frac{\omega}{\sigma}\right) = 180^\circ + \tan^{-1}\left(\frac{\omega}{\sigma+2}\right) \qquad ...(1)$

or $\quad \tan\left[\tan^{-1}\left(\frac{\omega}{\sigma+3}\right) - \tan^{-1}\left(\frac{\omega}{\sigma}\right)\right] = \tan\left[180^\circ + \tan^{-1}\left(\frac{\omega}{\sigma+2}\right)\right]$

or $\quad \dfrac{\frac{\omega}{\sigma+3} - \frac{\omega}{\sigma}}{1 + \left(\frac{\omega}{\sigma+3}\right)\left(\frac{\omega}{\sigma}\right)} = \dfrac{0 + \frac{\omega}{\sigma+2}}{1 - (0)\left(\frac{\omega}{\sigma+2}\right)}$

or $\quad \dfrac{-3\omega}{\sigma(\sigma+3) + \omega^2} = \dfrac{\omega}{\sigma+2}$

or $\quad -3(\sigma+2) = \sigma(\sigma+3) + \omega^2$

or $\quad (\sigma^2 + 6\sigma + 9) + \omega^2 = -6 + 9$

or $\quad (\sigma+3)^2 + \omega^2 = (\sqrt{3})^2$

This is the equation of circle.

SOL 1.1.56 Correct option is (D).

Given open loop transfer function of the system,

$$G(s)H(s) = \frac{K}{(s+1+j)(s+1-j)(s+3+j)(s+3-j)}$$

The root locus starts from

$$s_1 = -1 + j$$
$$s_2 = -1 - j$$
$$s_3 = -3 - j$$
$$s_4 = -3 + j$$

Since, there is no zero, all root loci end at infinity. So, we have

Number of open loop poles, $\quad P = 4$

Number of open loop zeros, $\quad Z = 0$

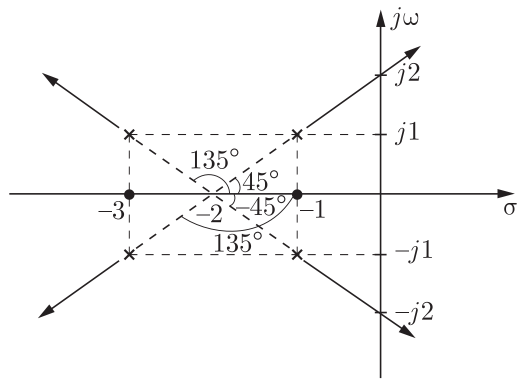
Therefore, number of asymptotes is 4 with angles of

$$\phi_A = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$
$$= \pm 45^\circ, \pm 135^\circ$$

Also, the point of intersection (centroid) of asymptotes with real axis is

$$\sigma_A = \frac{\Sigma \mathrm{Re}[P] - \Sigma \mathrm{Re}[Z]}{P - Z}$$
$$= \frac{(-1)+(-1)+(-3)+(-3)-0}{4-0} = -2$$

So, we get the root locus of the system as shown below.




Thus, there is no breakaway point.

SOL 1.1.57 Correct option is (D).

The characteristic equation of the system is

$$1+G(s)H(s)=0$$

or $$1+\frac{K}{s(s+4)(s+5)}=0$$

or $$s^3+9s^2+20s+K=0 \qquad ...(1)$$

Intersection of root loci with $j\omega$ axis is determined using Routh's array. For the given system, we form the Routh's array as

| | | |
|---|---|---|
| $s^3$ | 1 | 20 |
| $s^2$ | 9 | $K$ |
| $s^1$ | $\frac{180-K}{9}$ | |
| $s^0$ | $K$ | |

The critical gain before the closed loop system goes to instability is $K_c=180$ and the auxiliary equation is

$$9s^2+180=0$$

or $$s^2=-20$$

or $$s=\pm j2\sqrt{5}$$

Hence, root loci intersect with $j\omega$-axis at $s=\pm j2\sqrt{5}$. The gain margin for $K=18$ is given by,

$$GM \text{ (in dB)} = 20\log_{10}\left(\frac{K_c}{K}\right)$$

$$= 20\log_{10}\frac{180}{18} = 20 \text{ dB}$$

The gain margin for $K=1800$ is given by

$$GM \text{ (in dB)} = 20\log_{10}\left(\frac{180}{1800}\right) = -20 \text{ dB}$$

The break away point is given by solution of

$$\frac{dK}{ds}=0$$

From equation (1), we have

$$K=-(s^3+9s^2+20s)$$

So, $$\frac{dK}{ds}=-(3s^2+18s+20)=0$$

or $$s=\frac{-18\pm 9.165}{6}$$

$$= -4.5275 \text{ and } -1.4725$$

Point $s = -1.4725$ lies on root locus. So, break away point is $s = -1.4725$. The value of $K$ at break away point is

$$\begin{aligned} K &= \left| s(s+4)(s+5) \right| \\ &= 13.128 \qquad (s = -1.4725) \end{aligned}$$

SOL 1.1.58 Correct option is (D).

The characteristic equation of the system is

$$1 + \frac{K(s+1)}{s(s-1)(s^2+4s+16)} = 0 \qquad ...(1)$$

or $s^4 + 3s^3 + 12s^2 + (K-16)s + K = 0$

Intersection of root loci with $j\omega$-axis is determined using Routh's array which is shown below.

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 12 | $K$ |
| $s^3$ | 3 | $K-16$ | |
| $s^2$ | $\frac{52-K}{3}$ | $K$ | |
| $s^1$ | $\frac{-K^2+59K-832}{52-K}$ | | |
| $s^0$ | $K$ | | |

The root locus cross the $j\omega$-axis, if $s^1$ row is completely zero, i.e.

$$-K^2 + 59K - 832 = 0$$

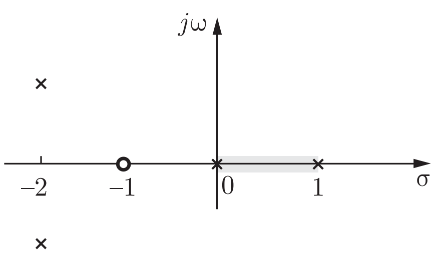
or $$K^2 - 59K + 832 = 0$$

or $$K = \frac{59 \pm 12.37}{2} = 35.7,\ 23.3$$

Hence, root locus cross $j\omega$-axis two times and the break points are given by solution of

$$\frac{dK}{ds} = 0$$

Also, we can directly check option using pole-zero plot.




So, break away point will lie on real axis from $s = 0$ to 1 and break in point will lie on real axis for $s < -1$. Hence,

$$s = 0.45 \text{ is break away point}$$

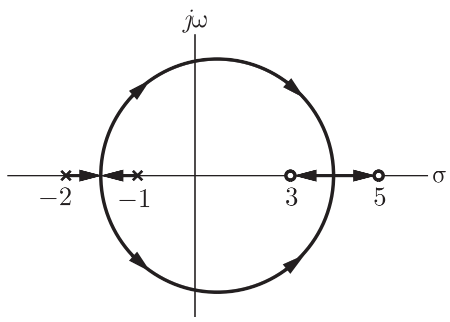
and $$s = -2.26 \text{ is break in point}$$

***********

# SOLUTIONS 1.2

SOL 1.2.1 Correct answer is $-1.45$.

Given root locus of *ufb* system is




Here, root locus branches meet between $-1$ and $-2$ and go apart. Hence, break-away point will lie between $-1$ and $-2$. For this system, the open loop poles and zeros are

zeros: $s = 3$ and $s = 5$;

poles: $s = -1$ and $s = -2$

So, the transfer function of given system will be

$$G(s) = \frac{K(s-3)(s-5)}{(s+1)(s+2)}$$

Therefore, the characteristic equation is obtained as

$$1 + G(s)H(s) = 0$$

or $$1 + \frac{K(s-3)(s-5)}{(s+1)(s+2)} = 0$$

or $$K = \frac{-(s^2+3s+2)}{(s^2-8s+15)} \qquad \text{...(1)}$$

Differentiating equation (1) with respect to $s$ and equating to zero, we have

$$\frac{dK}{ds} = \frac{-(s^2-8s+15)(2s+3)+(s^2+3s+2)(2s-8)}{(s^2-8s+15)^2} = 0$$

or $\quad 11s^2 - 26s - 61 = 0$

or $$s = +3.9 \text{ and } s = -1.45$$

Thus, $s = -1.45$ is break-away point and $s = +3.9$ is break-in point.

SOL 1.2.2 Correct answer is 8.62.

Break-away and break-in points always satisfy characteristic equation. So, we substitute $s = -1.45$ in equation (1) to obtain

$$K = \frac{-[(-1.45)^2 + 3(-1.45) + 2]}{[(-1.45)^2 - 8(-1.45) + 15]}$$

$$= -\frac{(-0.2475)}{28.7025}$$

$$= 8.62 \times 10^{-3}$$

SOL 1.2.3 Correct answer is 108.4.

Forward path transfer function of given *ufb* system is

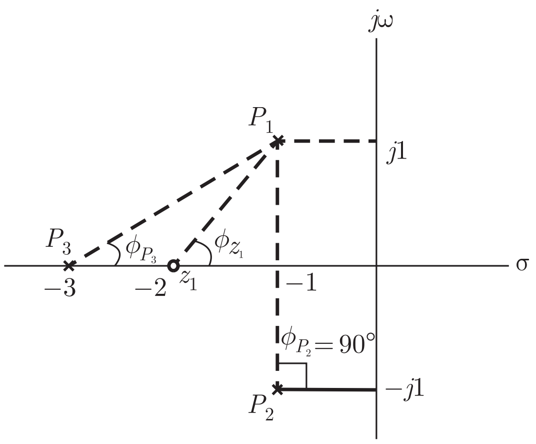
$$G(s) = \frac{K(s+2)}{(s+3)(s^2+2s+2)}$$

So, we have the open loop poles and zeros as

$$\text{zero: } s = -2$$

$$\text{poles: } s = -3 \text{ and } s = -1 \pm j1$$

Therefore, we get the pole-zero plot as




Angle of departure at pole $P_1$ is given by

$$\phi_D = \pm[180° + \phi]$$

where $\phi$ is net angle contribution at pole $P_1$ due to all other poles and zeros.

$$\phi = \phi_Z - \phi_P$$
$$= \phi_{Z1} - [\phi_{P2} + \phi_{P3}]$$

where $\phi_{Z1} = \tan^{-1}1;\ \phi_{P2} = 90°;\ \phi_{P3} = \tan^{-1}\frac{1}{2}$

So, $\phi = \tan^{-1}1 - \left[90° + \tan^{-1}\frac{1}{2}\right]$

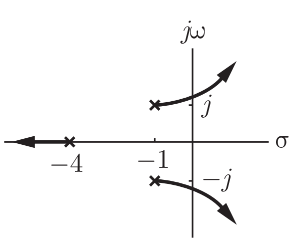
Therefore, we obtain the departure angle as

$$\phi_D = \pm[180° + \phi]$$
$$= \pm\left[180° + \tan^{-1}1 - 90° - \tan^{-1}\frac{1}{2}\right]$$
$$= \pm[180 + 45 - 90 - 26.56]$$
$$\phi_D = \pm 108.4°$$

Hence, departure angle for pole $P_1$ is $+108.4°$ and departure angle for pole $P_2$ is $-108.4°$ because $P_1$ and $P_2$ are complex conjugate.

SOL 1.2.4 Correct answer is 3.162.

Given root locus is shown below.




It does not have any zero and have poles at $s = -4$ and $s = -1 \pm j1$. So, the open loop transfer function is

$$G(s) = \frac{K}{(s+4)(s^2+2s+2)}$$
$$= \frac{K}{s^3+6s^2+10s+8}$$

Therefore, we get the closed loop transfer function as

$$T(s) = \frac{\frac{K}{s^3+6s^2+10s+8}}{1+\frac{K}{s^3+6s^2+10s+8}}$$
$$= \frac{K}{s^3+6s^2+10s+8+K}$$

The characteristics equation of the system is given as

$$s^3+6s^2+10s+8+K = 0$$

So, we have the Routh's array as

| | | |
|---|---|---|
| $s^3$ | 1 | 10 |
| $s^2$ | 6 | $8+K$ |
| $s^1$ | $\frac{52-K}{6}$ | |
| $s^0$ | $8+K$ | |

Root locus will cut imaginary axis, if element in $s^1$ is zero, i.e.

$$\frac{52-K}{6} = 0 \Rightarrow K = 52$$

So, we have the auxiliary equation

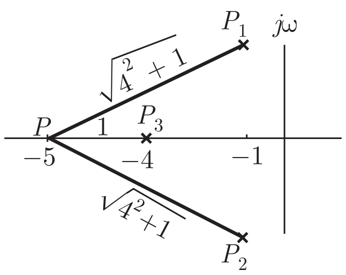
$$6s^2+(8+52) = 0$$

or $$s^2 = -\frac{60}{6} = -10$$

or $$s = \pm j3.162$$

SOL 1.2.5 Correct answer is 17.

For the given system, we have the pole-zero plot as shown below.




Gain $K$ at any $(s = s_0)$ point on root locus is given by

$$K\big|_{s=s_0} = \frac{\text{Product of phasors drawn from } OLP \text{ at that point}}{\text{Product of phasors drawn from } OLZ \text{ at that point}}$$

Since, no any zero is present in the given system. So, we obtain

$$K = \text{Product of phasors drawn from } OLP \text{ at that point}$$

So, $$K\big|_{s=-5} = (PP_3)\times(PP_1)\times(PP_2)$$
$$= 1\times\sqrt{4^2+1}\times\sqrt{4^2+1}$$
$$= 4^2+1 = 17 = 17$$

SOL 1.2.6 Correct answer is 600.

Open loop transfer function of given system is

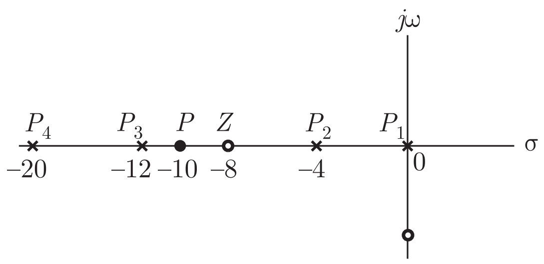
$$G(s)H(s) = \frac{K(s+8)}{s(s+4)(s+12)(s+20)}$$

For the given system, we have the open loop poles and zeros as

Zeros : $Z = -8$

Poles : $P_1 = 0;\ P_2 = -4;\ P_3 = -12;\ P_4 = -20$

So, we get the pole-zero plot for the given system as




Therefore, we obtain the value of $K$ at $s = -10$ as

$$K\Big|_{s=-10} = \frac{\pi(\text{Phasors drawn from } OLP \text{ at } s = -10)}{\pi(\text{Phasors drawn from } OLZ \text{ at } s = -10)}$$

$$= \frac{(PP_1)\times(PP_2)\times(PP_3)\times(PP_4)}{(PZ)}$$

$$= \frac{10\times 6\times 2\times 10}{2} = 600$$



SOL 1.2.7 Correct answer is $-1$.

Characteristic equation of given closed loop system is

$$s(s+1)(s+2) + K = 0$$

or

$$1 + \frac{K}{s(s+1)(s+2)} = 0$$

Since, the characteristic equation for a system is defined as

$$1 + G(s)H(s) = 0$$

So, we get open loop transfer function of the system as

$$G(s)H(s) = \frac{K}{s(s+1)(s+2)}$$

Therefore, we have

Poles: $s = 0,\ s = -1,\ s = -2$

Zeros: No zero

Thus, the centroid is obtained as

$$\sigma_A = \frac{\text{Sum of Re}[P] - \text{Sum of Re}[Z]}{P - Z}$$

$$= \frac{(0-1-2)-(0)}{3-0} = -\frac{3}{3} = -1$$

SOL 1.2.8 Correct answer is 10.

Given the open loop transfer function,

$$G(s) = \frac{K}{s(s^2+7s+12)};\ H(s) = 1$$

So, we have the characteristic equation

$$1 + G(s)H(s) = 0$$

or $$1+\frac{K}{s(s^2+7s+12)}=0$$

or $$s^3+7s^2+12s+K=0 \qquad ...(1)$$

If point $s=-1+j1$ lies on root locus, then it satisfies characteristic equation. Substituting $s=-1+j1$ in equation (1), we get

$$(-1+j)^3+7(-1+j)^2+12(-1+j)+K=0$$

or $$-10+K=0$$

So, $$K=+10$$

SOL 1.2.9 Correct answer is 71.56.

Given the open loop transfer function,

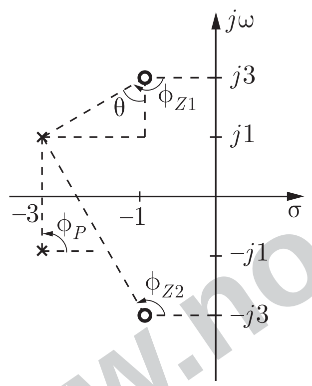
$$G(s)H(s)=\frac{K(s^2+2s+10)}{(s^2+6s+10)}$$

So, we have the open loop poles and zeros as

$$\text{Poles: } s^2+6s+10=0 \Rightarrow s=-3\pm j1$$

$$\text{zeros } s^2+2s+10=0 \Rightarrow s=-1\pm j3$$

Therefore, we get the pole-zero plot in $s$-plane as




Angle of departure at complex pole is given by

$$\phi_D=\pm[180^\circ+\phi]$$

where $\phi$ is the net angle contribution at this pole due to all other poles and zeros. So, we have

$$\phi=\phi_{Z_1}+\phi_{Z_2}-\phi_P$$

where

$$\phi_{Z_1}=-(90^\circ+\theta)$$

$$=-\left(90^\circ+\tan^{-1}\frac{2}{2}\right)=-135^\circ;$$

$$\phi_{Z_2}=180^\circ-\tan^{-1}\frac{4}{2}=116.56^\circ;$$

and $$\phi_P=90^\circ$$

Therefore, we get

$$\phi=-135^\circ+116.56^\circ-90^\circ$$

$$=-108.44^\circ$$

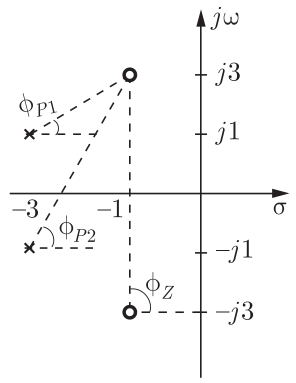
Hence, angle of departure will be

$$\phi_D=\pm[180^\circ-108.44^\circ]$$

$$=\pm[71.56^\circ]$$

SOL 1.2.10 Correct answer is 198.43.

The pole zero plot of the system is




The angle of arrival at complex zero is given by

$$\phi_A = \pm[180^\circ - \phi]$$

where, $\phi$ is the net angle contribution at this zero due to all other poles and zeros. So, we have

$$\phi = \phi_Z - \phi_{P_1} - \phi_{P_2}$$

where

$$\phi_Z = 90^\circ$$

$$\phi_{P_1} = \tan^{-1}\frac{2}{2} = 45^\circ$$

$$\phi_{P_2} = \tan^{-1}\frac{4}{2} = 63.43^\circ$$

Hence, $$\phi = 90^\circ - 45^\circ - 63.43^\circ$$

$$= -18.43^\circ$$

Thus, we obtain the angle of arrival as

$$\phi_A = \pm[180^\circ - \phi]$$

$$= \pm[180^\circ + 18.43^\circ]$$

$$= \pm[198.43^\circ]$$

SOL 1.2.11 Correct answer is 5.2.

The poles of the open loop transfer function are

$$s = -2 \text{ and } s = -10$$

So, the root loci starts at $s = -2$ and $s = -10$. Also, we have

Number of poles, $P = 2$

Number of zeros, $Z = 0$

Hence, number of asymptotes is obtained as

$$P - Z = 2$$

Therefore, the angle of asymptotes is given by

$$\phi_a = \frac{(2q+1)180^\circ}{P-Z}; \; q = 0, 1, 2, \text{-----}(P - Z - 1)$$

So, $\phi_a = 90^\circ; \; q = 0$

and $\phi_a = 270^\circ; \; q = 1$

Again, the open loop transfer function is

$$G(s)H(s) = \frac{10K}{(s+2)(s+10)}$$

So, $$|G(s)H(s)| = \left|\frac{10K}{(s+2)(s+10)}\right| = 1$$

or $$K = \frac{(s+2)(s+10)}{10} \qquad ...(1)$$

The break away point is given by solution of

$$\frac{dK}{ds} = 0$$

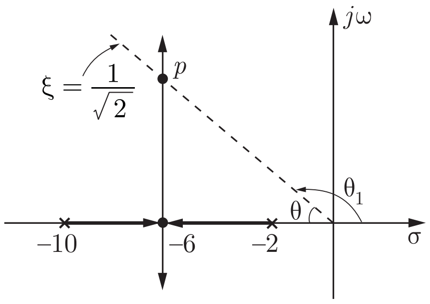
Substituting equation (1) in the above expression, we have

$$(s+2)+(s+10) = 0$$

or $$2s+12 = 0$$

or $$s = -6$$

The point $s = (-6, j0)$ is the break-away point. Therefore, root locus will be as shown below.




From the root locus plot, we have

$$\xi = \frac{1}{\sqrt{2}} = \cos\theta$$

and $$\theta = 45^\circ$$

Root locus will cross constant $\xi = \frac{1}{\sqrt{2}}$ line at point $p$. The intersection point $p$ is $(-6, j\omega)$. At the point $p$, angle of function is

$$\theta_1 = 135^\circ$$

which is given by

$$135^\circ = \tan^{-1}\left(\frac{\omega}{-6}\right)$$

or $$135^\circ = 180^\circ - \tan^{-1}\frac{\omega}{6}$$

or $$\tan^{-1}45^\circ = \frac{\omega}{6}$$

$$\omega = 6$$

Hence, point $p$ will be $(-6+j6)$. So, the gain $K$ at point $p$ is

$$K = \frac{|s+2||s+10|}{10}\bigg|_{s=-6+j6}$$

$$= \frac{|-4+j6||4+j6|}{10} = 5.2$$

SOL 1.2.12 Correct answer is 12.04.

From root locus, intersection with imaginary axis indicates the marginal stability. So, for marginal stability the value of $K = 48$. So, the gain margin is given by

$$\text{Gain margin } (GM) = \frac{\text{Value of } K \text{ for marginal stability}}{\text{desired value of } K} = \frac{48}{12} = 4$$

In decibel, we get

$$GM \text{ (in dB)} = 20\log_{10}4 = 12.04 \text{ dB}$$

SOL 1.2.13 Correct answer is 385.

For the given system, we have

$$1+G(s)H(s)=0$$

or $$|G(s)H(s)|=\left|\frac{K}{s(s+10)(s+20)}\right|=1$$

or $$K=s(s+10)(s+20)$$

The break points of the system are given by solution of

$$\frac{dK}{ds}=0$$

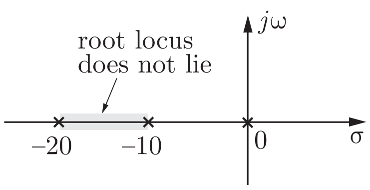
or $$\frac{d}{ds}[s^3+30s^2+200s]=0$$

or $$3s^2+60s+200=0$$

So, $$s=\frac{-60\pm 34.64}{2\times 3}$$

$$=-15.773,\ -4.226$$

Therefore, we get the pole zero plot as




Hence, break away point is

$$s=-4.226$$

The value of $K$ at point $s_0$ is given by

$$|G(s)H(s)|=1 \text{ at } s=s_0$$

or $$\left.\frac{K}{|s||s+10||s+20|}\right|_{s=s_0}=1$$

So, $$K=\left.\frac{|s||s+10||s+20|}{1}\right|_{s=s_0}$$

where $s_0=-4.226$ is break away point, given as

$$K=4.226\times 5.774\times 15.774$$

$$=384.9\approx 385$$

SOL 1.2.14 Correct answer is $-0.059$.

The root sensitivity is defined as the ratio of the fractional change in a closed loop pole to the fraction change in a system parameters, such as gain $K$. We calculate the sensitivity of a closed loop pole, $s$, to gain $K$,

$$S_K^s=\frac{\frac{\delta s}{s}}{\frac{\delta K}{K}}=\frac{K}{s}\frac{\delta s}{\delta K} \qquad ...(1)$$

where $s$ is the current pole location and $K$ is the current gain. The characteristic equation for the system is

$$s^2+10s+K=0 \qquad ...(2)$$

Differentiating equation (2) with respect to $K$, we have

$$2s\frac{\delta s}{\delta K}+10\frac{\delta s}{\delta K}+1=0$$

or $$\frac{\delta s}{\delta K}=\frac{-1}{2s+10}$$

From equation (1), the root sensitivity is obtained as

$$S_K^s = \frac{K}{s}\frac{-1}{2s+10}$$

$$= \frac{-K}{s(2s+10)}$$

The root sensitivity at $s = -9.47$ is given by substituting $s = -9.47$ and corresponding $K = 5$ in above expression, we get

$$S_K^s = -\frac{5}{(-9.47)(-18.94+10)}$$

$$s_K^s = -0.059$$

SOL 1.2.15 Correct answer is 0.2.

For the given system, we have the characteristic equation

$$1 + G(s)H(s) = 0$$

or $$1 + \frac{K(s^2+4)}{s(s+2)} = 0$$

or $$K = -\frac{(s^2+2s)}{s^2+4}$$

The break points of the root locus are given by solution of

$$\frac{dK}{ds} = 0$$

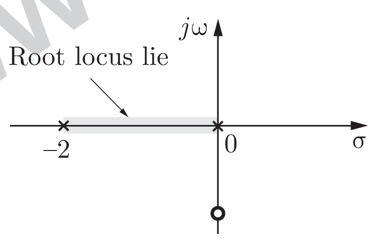
or $$\frac{-\left[(s^2+4)(2s+2)-(s^2+2s)(2s)\right]}{(s^2+4)^2} = 0$$

or $$2s^2 - 8s - 8 = 0$$

or $$s^2 - 4s - 4 = 0$$

So, $$s = 4.82,\ -0.82$$

The root locus will be on real axis at any point, if total number of poles and zeros are odd to the right of that point.




Hence, breakaway point should lie between $s = 0$ and $-2$. So, breakaway point is $s = -0.82$. Now, the open loop transfer function is

$$G(s)H(s) = \frac{K(s^2+4)}{s(s+2)}$$

So, we obtain the value of $K$ at break-away point as

$$|G(s)H(s)| = 1 \text{ at } s = -0.82$$

or $$\frac{K\left|(-0.82)^2+4\right|}{|(-0.82)||(-0.82+2)|} = 1$$

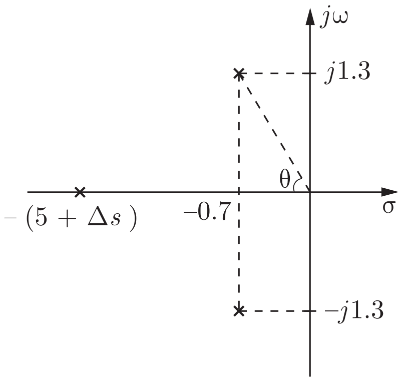
or $$K = \frac{0.82 \times 1.18}{4.67} = 0.2$$

SOL 1.2.16 Correct answer is 0.47.

From root locus, for $K = 12.2$ poles are complex conjugate and are given by

$$s = -0.7 \pm j1.3$$

For $K = 12.2$, the pole zero plot is




From the pole-zero plot, we have

$$\theta = \tan^{-1}\left(\frac{1.3}{0.7}\right) = 61.69$$

So, the damping ratio is given by

$$\begin{aligned}\xi &= \cos\theta \\ &= \cos(61.69) \\ &= 0.47\end{aligned}$$

SOL 1.2.17 Correct answer is 0.187.

The peak overshoot $(M_P)$ is defined as

$$M_P = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$$

For $K = 12.2$, damping ratio is $\xi = 0.47$. So, we have

$$\begin{aligned}M_P &= e^{\frac{-0.47\pi}{\sqrt{1-(0.47)^2}}} \\ &= 0.187\end{aligned}$$

SOL 1.2.18 Correct answer is $-1$.

The overall transfer function of the system is

$$\begin{aligned}\frac{C(s)}{R(s)} &= \frac{\frac{1}{s(s+1)}}{1+\frac{1}{s(s+1)}(\alpha s+1)} \\ &= \frac{1}{s^2+(\alpha+1)s+1}\end{aligned}$$

So, we have the characteristic equation as

$$s^2+(\alpha+1)s+1 = 0$$

or $$s^2+s+1+\alpha s = 0$$

or $$1+\frac{\alpha s}{s^2+s+1} = 0$$

Comparing it to $1+G(s)H(s) = 0$, we have the open loop transfer function

$$G(s)H(s) = \frac{\alpha s}{s^2+s+1}$$

Also, we have

$$\alpha = \frac{-(s^2+s+1)}{s}$$

The break points are given by solution of

$$\frac{d\alpha}{ds} = 0$$

So, $$\frac{-[s(2s+1)-(s^2+s+1)]}{s^2} = 0$$

or $$s^2 - 1 = 0$$

or $$s = \pm 1$$

The point $s = +1$ does not lie on root locus and $s = -1$ lies on root locus. Hence, break away point is $s = -1$.

SOL 1.2.19 Correct answer is 1.

We know that, on the break away point system has multiple poles. If break away point is on real axis, then multiple poles should be real and equal, and in this case, system have critically damped response. Hence, value of parameter $\alpha$ for critical damping will be equal to value of $\alpha$ at break away point. For the given system, breakaway point is

$$s = -1$$

So, the value of $\alpha$ at $s = -1$ is obtained as

$$|G(s)H(s)| = 1$$

or $$\left.\frac{\alpha|s|}{|s^2+s+1|}\right|_{s=-1} = 1$$

$$\alpha = \frac{|(-1)^2+(-1)+1|}{|(-1)|} = 1$$

SOL 1.2.20 Correct answer is 27.

The root locus plot gives the location of the closed loop poles for different values of parameter gain $K$. So, we have the characteristic equation as

$$1 + \frac{K(s+1)}{s^2(s+9)} = 0$$

or $$s^3 + 9s^2 + Ks + K = 0 \qquad ...(1)$$

For all the roots to be equal and real, we require

$$(s+P)^3 = s^3 + 3Ps^2 + 3P^2 s + P^3 = 0 \qquad ...(2)$$

On comparing equations (1) and (2), we get

$$3P = 9 \Rightarrow P = 3$$

and $$K = P^3 = (3)^3 = 27$$

SOL 1.2.21 Correct answer is 29.

First we check if point lies on root locus. For this, we use angle criterion

$$\angle G(s)H(s)\big|_{s=s_0} = \pm 180$$

Since, we have

$$G(s)H(s)\big|_{s=-3+j5} = \frac{K}{(-3+j5+1)(-3+j5+5)} = \frac{K}{(-2+j5)(2+j5)}$$

So, $$\angle G(s)H(s)\big|_{s=-3+j5} = -\tan^{-1}\left(\frac{5}{-2}\right) - \tan^{-1}\left(\frac{5}{2}\right)$$
$$= -180^\circ + \tan^{-1}\frac{5}{2} - \tan^{-1}\frac{5}{2}$$
$$= -180^\circ$$

i.e. the given point satisfies angle criterion. Now, using magnitude condition, we have

$$|G(s)H(s)|_{s=-3+j5} = 1$$

or
$$\left|\frac{K}{(-2+j5)(2+j5)}\right| = 1$$

or
$$\frac{K}{\sqrt{4+25}\sqrt{4+25}} = 1$$

Thus,
$$K = 29$$

***********

# SOLUTIONS 1.3

SOL 1.3.1 Correct option is (D).

SOL 1.3.2 Correct option is (B).

SOL 1.3.3 Correct option is (C).

SOL 1.3.4 Correct option is (C).

SOL 1.3.5 Correct option is (B).

SOL 1.3.6 Correct option is (A).

SOL 1.3.7 Correct option is (A).

SOL 1.3.8 Correct option is (C).

SOL 1.3.9 Correct option is (A).

SOL 1.3.10 Correct option is (A).

SOL 1.3.11 Correct option is (B).

SOL 1.3.12 Correct option is (A).
The number of branches is equal to the order of the polynomial.
Here, order of the system is 4. Hence the number of branches is 4.

SOL 1.3.13 Correct option is (C).
The number of asymptotes $=\#$ open loop poles $-\#$ open loop zeros

$$= 5 - 3 = 2$$

SOL 1.3.14 Correct option is (B).

SOL 1.3.15 Correct option is (C).

SOL 1.3.16 Correct option is (C).

SOL 1.3.17 Correct option is (C).

SOL 1.3.18 Correct option is (B).

The open-loop poles are,

$$s = -2 \pm j2,\ s = -1 \text{ and } s = 0 \text{ and the open loop zero is } s = -3$$

$\#$ open loop poles $P = 4$

and $\#$ open loop zeros $Z = 1$

The angle of asymptotes is given by

$$\phi_A = \frac{(2q+1)180^\circ}{P-Z};\ q = 0,\ 1,......(P-Z-1)$$

So, $\phi_A = 60^\circ;\ q = 0$

and $\phi_A = 180^\circ;\ q = 1$

and $\phi_A = 300^\circ - 60^\circ;\ q = 2$

The centroid is $\sigma_A \dfrac{\Sigma\text{real of poles} - \Sigma\text{real of zeros}}{P-Z}$

$$= \frac{\{(-2)+(-2)+(-1)+0\}-(-3)}{4-1}$$

$$= -\frac{2}{3}$$

SOL 1.3.19 Correct option is (A).

When the system has real and different poles then response becomes non oscillatory. From root locus plot, it can be observed that for $0 < K < 0.4$ system has real and different poles.

SOL 1.3.20 Correct option is (D).

Due to addition of zero to the open loop transfer function, root locus move to left half. And due to addition of pole to the open loop transfer function, root locus move to right half.

SOL 1.3.21 Correct option is (D).

The meeting point of asymptotes on the real axis is a centroid which is given by

$$\sigma_A = \frac{\Sigma\text{real of poles} - \Sigma\text{real of zeros}}{\#\text{ poles} - \#\text{ zeros}}$$

$$= \frac{\{0+(-2)+(-4)+(-1)+(-1)\}-(-5)}{5-1}$$

$$= \frac{-3}{4} = -0.75$$

SOL 1.3.22 Correct option is (A).

From the root locus we can observe that the four open loop poles lie as $s = -1$. So, open loop transfer function is,

$$G(s) = \frac{K}{(s+1)^4}$$

The characteristic equation is

$$1 + \frac{K}{(s+1)^4} = 0$$

or $s^4 + 4s^3 + 6s^2 + 4s + 1 + K = 0$

The Routh's array is

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 6 | $(K+1)$ |
| $s^3$ | 4 | 4 | |
| $s^2$ | 5 | $(K+1)$ | |
| $s^1$ | $\frac{16-4K}{5}$ | | |
| $s^0$ | $K+1$ | | |

For intersection of $j\omega$-axis, $s^1$ row should be completely zero.

i.e. $$\frac{16-4K}{5} = 0$$

or $$K = 4$$

SOL 1.3.23 Correct option is (B).

The intersection of asymptotes is always on the real axis because it is given by

$$\sigma_A = \frac{\Sigma\text{real part of } OLP - \Sigma\text{real part of } OLZ}{\# OLP - \# OLZ}$$

The breakaway point is determined by solution of $\frac{dK}{ds} = 0$.

So, it can be real or complex.

SOL 1.3.24 Correct option is (A).

The effect of compensating pole is to pull the root locus towards right half of $s$-plane. The effect of compensating zero is to pull the root locus towards left half of $s$-plane.

SOL 1.3.25 Correct option is (A).

(1) There will be four asymptotes because,

$$\# OLP - \# OLZ = 4 - 0 = 4$$

(2) There will be four separate root because the order of polynomial is four.

(3) Asymptotes will intersect at $\sigma_A$,

$$\sigma_A = \frac{(0-1-2-3)-(0)}{4-0} = -\frac{6}{4} = -\frac{3}{2}$$

SOL 1.3.26 Correct option is (C).

The given characteristic equation is

$$1 + \frac{K}{s(s+1)(s+2)} = 0$$

or $$1 + G(s)H(s) = 0$$

So, the open loop transfer function is

$$G(s)H(s) = \frac{K}{s(s+1)(s+2)}$$

The centroid $\sigma_A$ is,

$$\sigma_A = \frac{(0-1-2)-(0)}{3-0} = -1$$

SOL 1.3.27 Correct option is (A).

SOL 1.3.28 Correct option is (C).

The value of gain $K$ at any point $s_0$ on the root locus is given by

$$K\Big|_{s=s_0} = \frac{\Pi(\text{Lengths of vectors from } OLP \text{ to the point } s_0)}{\Pi(\text{Lengths of vectors from } OLZ \text{ to the point } s_0)}$$

SOL 1.3.29 Correct option is (A).
The root locus is symmetrical about real axis, not $j\omega$- axis.

SOL 1.3.30 Correct option is (A).

SOL 1.3.31 Correct option is (C).

***********

Eighth Edition

# GATE

## ELECTRONICS & COMMUNICATION

## Electromagnetics

Vol 10 of 10


**RK Kanodia**

**Ashish Murolia**



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GATE Electronics & Communication Vol 10, 8e
Electromagnetics
RK Kanodia & Ashish Murolia



MRP 590.00

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Printed by Nodia and Company, Jaipur

To Our Parents

## *Preface to the Series*

For almost a decade, we have been receiving tremendous responses from GATE aspirants for our earlier books: GATE Multiple Choice Questions, GATE Guide, and the GATE Cloud series. Our first book, GATE Multiple Choice Questions (MCQ), was a compilation of objective questions and solutions for all subjects of GATE Electronics & Communication Engineering in one book. The idea behind the book was that Gate aspirants who had just completed or about to finish their last semester to achieve his or her B.E/B.Tech need only to practice answering questions to crack GATE. The solutions in the book were presented in such a manner that a student needs to know fundamental concepts to understand them. We assumed that students have learned enough of the fundamentals by his or her graduation. The book was a great success, but still there were a large ratio of aspirants who needed more preparatory materials beyond just problems and solutions. This large ratio mainly included average students.

Later, we perceived that many aspirants couldn't develop a good problem solving approach in their B.E/B.Tech. Some of them lacked the fundamentals of a subject and had difficulty understanding simple solutions. Now, we have an idea to enhance our content and present two separate books for each subject: one for theory, which contains brief theory, problem solving methods, fundamental concepts, and points-to-remember. The second book is about problems, including a vast collection of problems with descriptive and step-by-step solutions that can be understood by an average student. This was the origin of *GATE Guide* (the theory book) and *GATE Cloud* (the problem bank) series: two books for each subject. *GATE Guide* and *GATE Cloud* were published in three subjects only.

Thereafter we received an immense number of emails from our readers looking for a complete study package for all subjects and a book that combines both *GATE Guide* and *GATE Cloud*. This encouraged us to present GATE Study Package (a set of 10 books: one for each subject) for GATE Electronic and Communication Engineering. Each book in this package is adequate for the purpose of qualifying GATE for an average student. Each book contains brief theory, fundamental concepts, problem solving methodology, summary of formulae, and a solved question bank. The question bank has three exercises for each chapter: 1) Theoretical MCQs, 2) Numerical MCQs, and 3) Numerical Type Questions (based on the new GATE pattern). Solutions are presented in a descriptive and step-by-step manner, which are easy to understand for all aspirants.

We believe that each book of GATE Study Package helps a student learn fundamental concepts and develop problem solving skills for a subject, which are key essentials to crack GATE. Although we have put a vigorous effort in preparing this book, some errors may have crept in. We shall appreciate and greatly acknowledge all constructive comments, criticisms, and suggestions from the users of this book. You may write to us at rajkumar.kanodia@gmail.com and ashish.murolia@gmail.com.

Acknowledgements

We would like to express our sincere thanks to all the co-authors, editors, and reviewers for their efforts in making this project successful. We would also like to thank Team NODIA for providing professional support for this project through all phases of its development. At last, we express our gratitude to God and our Family for providing moral support and motivation.

We wish you good luck !

R. K. Kanodia

Ashish Murolia

# SYLLABUS

## GATE Electronics & Communications:

Electromagnetics :

Elements of vector calculus: divergence and curl; Gauss' and Stokes' theorems, Maxwell's equations: differential and integral forms. Wave equation, Poynting vector. Plane waves: propagation through various media; reflection and refraction; phase and group velocity; skin depth. Transmission lines: characteristic impedance; impedance transformation; Smith chart; impedance matching; S parameters, pulse excitation. Waveguides: modes in rectangular waveguides; boundary conditions; cut-off frequencies; dispersion relations. Basics of propagation in dielectric waveguide and optical fibers. Basics of Antennas: Dipole antennas; radiation pattern; antenna gain.

## IES Electronics & Telecommunication

Electromagnetic Theory

Analysis of electrostatic and magnetostatic fields; Laplace's and Poisson's equations; Boundary value problems and their solutions; Maxwell's equations; application to wave propagation in bounded and unbounded media; Transmission lines : basic theory, standing waves, matching applications, microstrip lines; Basics of wave guides and resonators; Elements of antenna theory.

## IES Electrical

EM Theory

Electric and magnetic fields. Gauss's Law and Amperes Law. Fields in dielectrics, conductors and magnetic materials. Maxwell's equations. Time varying fields. Plane-Wave propagating in dielectric and conducting media. Transmission lines.

**********

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## CHAPTER 1 VECTOR ANALYSIS

## CHAPTER 2 ELECTROSTATIC FIELDS

## CHAPTER 3 ELECTRIC FIELD IN MATTER

## CHAPTER 4 MAGNETOSTATIC FIELDS



## CHAPTER 5 MAGNETIC FIELDS IN MATTER

## CHAPTER 7 ELECTROMAGNETIC WAVES

***********

# CHAPTER 6

## TIME VARYING FIELDS AND MAXWELL EQUATIONS

### 6.1 INTRODUCTION

Maxwell's equations are very popular and they are known as Electromagnetic Field Equations. The main aim of this chapter is to provide sufficient background and concepts on Maxwell's equations. They include:

- Faraday's law of electromagnetic induction for three different cases: time-varying magnetic field, moving conductor with static magnetic field, and the general case of moving conductor with time-varying magnetic field.
- Lenz's law which gives direction of the induced current in the loop associated with magnetic flux change.
- Concept of self and mutual inductance
- Maxwell's equations for static and time varying fields in free space and conductive media in differential and integral form

### 6.2 FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

According to Faraday's law of electromagnetic induction, emf induced in a conductor is equal to the rate of change of flux linkage in it. Here, we will denote the induced emf by $V_{emf}$. Mathematically, the induced emf in a closed loop is given as

$$V_{emf} = \frac{-d\Phi}{dt} = -\frac{d}{dt}\int_S B \cdot dS \qquad \text{...(6.1)}$$

where $\Phi$ is the total magnetic flux through the closed loop, $B$ is the magnetic flux density through the loop and $S$ is the surface area of the loop. If the closed path is taken by an $N$-turn filamentary conductor, the induced emf becomes

$$V_{emf} = -N\frac{d\Phi}{dt}$$

#### 6.2.1 Integral Form of Faraday's Law

We know that the induced emf in the closed loop can be written in terms of electric field as

$$V_{emf} = \oint_L E \cdot dL \qquad \text{...(6.2)}$$

From equations (6.1) and (6.2), we get

$$\oint_L E \cdot dL = -\frac{d}{dt}\int_S B \cdot dS \qquad \text{...(6.3)}$$

This equation is termed as the integral form of Faraday's law.

### 6.2.2 Differential Form of Faraday's Law

Applying Stoke's theorem to equation (6.3), we obtain

$$\int_S (\nabla \times E) \cdot dS = -\frac{d}{dt}\int_S B \cdot dS$$

Thus, equating the integrands in above equation, we get

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

This is the differential form of Faraday's law.

## 6.3 LENZ'S LAW

The negative sign in Faraday equation is due to Lenz's law which states that the direction of emf induced opposes the cause producing it. To understand the Lenz's law, consider the two conducting loops placed in magnetic fields with increasing and decreasing flux densities respectively as shown in Figure 6.1.

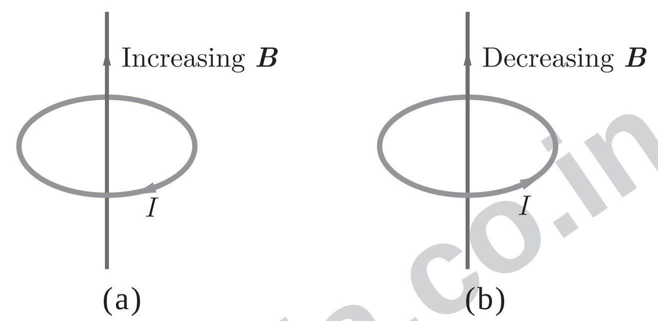



Figure 6.1: Determination of Direction of Induced Current in a Loop according to Lenz's Law (a) $B$ in Upward Direction Increasing with Time (b) $B$ in Upward Direction Decreasing with Time

**METHODOLOGY: TO DETERMINE THE POLARITY OF INDUCED EMF**

To determine the polarity of induced emf (direction of induced current), we may follow the steps given below.

Step 1: Obtain the direction of magnetic flux density through the loop. In both the Figures 6.1(a),(b) the magnetic field is directed upward.

Step 2: Deduce whether the field is increasing or decreasing with time along its direction. In Figure 6.1(a), the magnetic field directed upward is increasing, whereas in Figure 6.1(b), the magnetic field directed upward is decreasing with time.

Step 3: For increasing field assign the direction of induced current in the loop such that it produces the field opposite to the given magnetic field direction. Whereas for decreasing field assign the direction of induced current in the loop such that it produces the field in the same direction that of the given magnetic field. In Figure 6.1(a), using right hand rule we conclude that any current flowing in clockwise direction in the loop will cause a magnetic field directed downward and hence, opposes the increase in flux (i.e. opposes the field that causes it). Similarly in Figure 6.1(b), using right hand rule, we conclude that any current flowing in anti-clockwise direction in the loop will cause a magnetic field directed upward and hence, opposes the decrease in flux (i.e. opposes the field that causes it).

Step 4: Assign the polarity of induced emf in the loop corresponding to the obtained direction of induced current.



## 6.4 MOTIONAL AND TRANSFORMER EMFS

According to Faraday's law, for a flux variation through a loop, there will be induced emf in the loop. The variation of flux with time may be caused in following three ways:

### 6.4.1 Stationary Loop in a Time Varying Magnetic Field

For a stationary loop located in a time varying magnetic field, the induced emf in the loop is given by

$$V_{\text{emf}} = \oint_L E \cdot dL = -\int_S \frac{\partial B}{\partial t} \cdot dS$$

This emf is induced by the time-varying current (producing time-varying magnetic field) in a stationary loop is called *transformer emf*.

### 6.4.2 Moving Loop in Static Magnetic Field

When a conducting loop is moving in a static field, an emf is induced in the loop. This induced emf is called *motional emf* and given by

$$V_{\text{emf}} = \oint_L E_m \cdot dL = \oint_L (u \times B) \cdot dL$$

where $u$ is the velocity of loop in magnetic field. Using Stoke's theorem in above equation, we get

$$\nabla \times E_m = \nabla \times (u \times B)$$

### 6.4.3 Moving Loop in Time Varying Magnetic Field

This is the general case of induced emf when a conducting loop is moving in time varying magnetic field. Combining the above two results, total emf induced is

$$V_{\text{emf}} = \oint_L E \cdot dL = -\int_S \frac{\partial B}{\partial t} \cdot dS + \oint_L (u \times B) \cdot dL$$

or,

$$V_{\text{emf}} = \underbrace{-\int_S \frac{\partial B}{\partial t} \cdot dS}_{\text{transformer emf}} + \underbrace{\oint_L (u \times B) \cdot dL}_{\text{motional emf}}$$

Using Stoke's theorem, we can write the above equation in differential form as

$$\nabla \times E = -\frac{\partial B}{\partial t} + \nabla \times (u \times B)$$

## 6.5 INDUCTANCE

An inductance is the inertial property of a circuit caused by an induced reverse voltage that opposes the flow of current when a voltage is applied. A circuit or a part of circuit that has inductance is called an inductor. A device can have either self inductance or mutual inductance.

### 6.5.1 Self Inductance

Consider a circuit carrying a varying current which produces varying magnetic field which in turn produces induced emf in the circuit to oppose the change in flux. The emf induced is called emf of self-induction because the change in flux is produced by the circuit itself. This phenomena is called self-induction and the property of the circuit to produce self-induction is known as *self inductance*.

#### Self Inductance of a Coil

Suppose a coil with $N$ number of turns carrying current $I$. Let the current induces the total magnetic flux $\Phi$ passing through the loop of the coil. Thus, we have

$$N\Phi \propto I$$

or $$N\Phi = LI$$

or $$L = \frac{N\Phi}{I}$$

where $L$ is a constant of proportionality known as self inductance.

#### Expression for Induced EMF in terms of Self Inductance

If a variable current $i$ is introduced in the circuit, then magnetic flux linked with the circuit also varies depending on the current. So, the self-inductance of the circuit can be written as

$$L = \frac{d\Phi}{di} \quad \text{...(6.4)}$$

Since, the change in flux through the coil induces an emf in the coil given by

$$V_{emf} = -\frac{d\Phi}{dt} \quad \text{...(6.5)}$$

So, from equations (6.4) and (6.5), we get

$$V_{emf} = -L\frac{di}{dt}$$

### 6.5.2 Mutual Inductance

Mutual inductance is the ability of one inductor to induce an emf across another inductor placed very close to it. Consider two coils carrying current $I_1$ and $I_2$ as shown in Figure 6.2. Let $B_2$ be the magnetic flux density produced due to the current $I_2$ and $S_1$ be the cross sectional area of coil 1. So, the magnetic flux due to $B_2$ will link with the coil 1, that is, it will pass through the surface $S_1$. Total magnetic flux produced by coil 2 that passes through coil 1 is called mutual flux and given as

$$\Phi_{12} = \int_{S_1} B_2 \cdot dS$$

We define the mutual inductance $M_{12}$ as the ratio of the flux linkage on coil 1 to current $I_2$, i.e.

$$M_{12} = \frac{N_1\Phi_{12}}{I_2}$$

where $N_1$ is the number turns in coil 1. Similarly, the mutual inductance $M_{21}$ is defined as the ratio of flux linkage on coil 2 (produced by current in coil 1) to current $I_1$, i.e.

$$M_{21} = \frac{N_2\Phi_{21}}{I_1}$$

The unit of mutual inductance is Henry (H). If the medium surrounding the circuits is linear, then

$$M_{12} = M_{21}$$

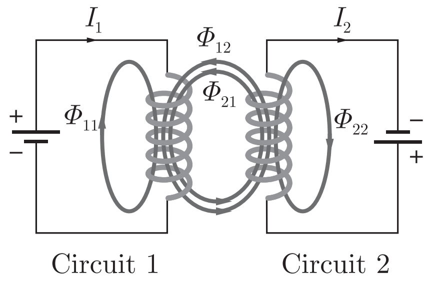



Figure 6.2 : Mutual Inductance between Two Current Carrying Coils

### Expression for Induced EMF in terms of Mutual Inductance

If a variable current $i_2$ is introduced in coil 2 then, the magnetic flux linked with coil 1 also varies depending on current $i_2$. So, the mutual inductance can be given as

$$M_{12} = \frac{d\Phi_{12}}{di_2} \qquad ...(6.6)$$

The change in the magnetic flux linked with coil 1 induces an emf in coil 1 given as

$$(V_{emf})_1 = -\frac{d\Phi_{12}}{dt} \qquad ...(6.7)$$

So, from equations (6.6) and (6.7) we get

$$(V_{emf})_1 = -M_{12}\frac{di_2}{dt}$$

This is the induced emf in coil 1 produced by the current $i_2$ in coil 2. Similarly, the induced emf in the coil 2 due to a varying current in the coil 1 is given as

$$(V_{emf})_2 = -M_{21}\frac{di_1}{dt}$$

## 6.6 MAXWELL'S EQUATIONS

The set of four equations which have become known as Maxwell's equations are those which are developed in the earlier chapters and associated with them the name of other investigators. These equations describe the sources and the field vectors in the broad fields to electrostatics, magnetostatics and electro-magnetic induction.

### 6.6.1 Maxwell's Equations for Time Varying Fields

The four Maxwell's equation include Faraday's law, Ampere's circuital law, Gauss's law, and conservation of magnetic flux. There is no guideline for giving numbers to the various Maxwell's equations. However, it is customary to call the Maxwell's equation derived from Faraday's law as the first Maxwell's equation.

#### Maxwell's First Equation : Faraday's Law

The electromotive force around a closed path is equal to the time derivative of the magnetic displacement through any surface bounded by the path.

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \text{(Differential form)}$$

or

$$\oint_L \boldsymbol{E} \cdot d\boldsymbol{L} = -\frac{\partial}{\partial t}\int_S \boldsymbol{B} \cdot d\boldsymbol{S} \qquad \text{(Integral form)}$$

### Maxwell's Second Equation: Modified Ampere's Circuital law

The magnetomotive force around a closed path is equal to the conduction plus the time derivative of the electric displacement through any surface bounded by the path. i.e.

$$\nabla \times H = J + \frac{\partial D}{\partial t} \qquad \text{(Differential form)}$$

$$\oint_L H \cdot dL = \int_S \left(J + \frac{\partial D}{\partial t}\right) \cdot dS \qquad \text{(Integral form)}$$

### Maxwell's Third Equation : Gauss's Law for Electric Field

The total electric displacement through any closed surface enclosing a volume is equal to the total charge within the volume. i.e.,

$$\nabla \cdot D = \rho_v \qquad \text{(Differential form)}$$

or,
$$\oint_S D \cdot dS = \int_v \rho_v dv \qquad \text{(Integral form)}$$

This is the Gauss' law for static electric fields.

### Maxwell's Fourth Equation : Gauss's Law for Magnetic Field

The net magnetic flux emerging through any closed surface is zero. In other words, the magnetic flux lines do not originate and end anywhere, but are continuous. i.e.,

$$\nabla \cdot B = 0 \qquad \text{(Differential form)}$$

or,
$$\oint_S B \cdot dS = 0 \qquad \text{(Integral form)}$$

This is the Gauss' law for static magnetic fields, which confirms the non-existence of magnetic monopole. Table 6.1 summarizes the Maxwell's equation for time varying fields.

Table 6.1: Maxwell's Equation for Time Varying Field

| S.N. | Differential form | Integral form | Name |
|---|---|---|---|
| 1. | $\nabla \times E = -\frac{\partial B}{\partial t}$ | $\oint_L E \cdot dL = -\frac{\partial}{\partial t}\int_S B \cdot dS$ | Faraday's law of electromagnetic induction |
| 2. | $\nabla \times H = J + \frac{\partial D}{\partial t}$ | $\oint_L H \cdot dL = \int_S \left(J + \frac{\partial D}{\partial t}\right) \cdot dS$ | Modified Ampere's circuital law |
| 3. | $\nabla \cdot D = \rho_v$ | $\oint_S D \cdot dS = \int_v \rho_v dv$ | Gauss' law of Electrostatics |
| 4. | $\nabla \cdot B = 0$ | $\oint_S B \cdot dS = 0$ | Gauss' law of Magnetostatic (non-existence of magnetic mono-pole) |

## 6.6.2 Maxwell's Equations for Static Fields

For static fields, all the field terms which have time derivatives are zero, i.e.

$$\frac{\partial B}{\partial t} = 0$$

and
$$\frac{\partial D}{\partial t} = 0$$

Therefore, for a static field the four Maxwell's equations described above reduces to the following form.

Table 6.2: Maxwell's equation for static field

| S.N. | Differential Form | Integral form | Name |
|---|---|---|---|
| 1. | $\nabla \times E = 0$ | $\oint_L E \cdot dL = 0$ | Faraday's law of electromagnetic induction |
| 2. | $\nabla \times H = J$ | $\oint_L H \cdot dL = \int_S J \cdot dS$ | Modified Ampere's circuital law |
| 3. | $\nabla \cdot D = \rho_v$ | $\oint_S D \cdot dS = \int_v \rho_v dv$ | Gauss' law of Electrostatics |
| 4. | $\nabla \cdot B = 0$ | $\oint_S B \cdot dS = 0$ | Gauss' law of Magnetostatic (non-existence of magnetic mono-pole) |

### 6.6.3 Maxwell's Equations in Phasor Form

In a time-varying field, the field quantities $E(x,y,z,t)$, $D(x,y,z,t)$, $B(x,y,z,t)$, $H(x,y,z,t)$, $J(x,y,z,t)$ and $\rho_v(x,y,z,t)$ can be represented in their respective phasor forms as below:

$$E = \mathrm{Re}\{E_s e^{j\omega t}\} \quad ...(6.8a)$$

$$D = \mathrm{Re}\{D_s e^{j\omega t}\} \quad ...(6.8b)$$

$$B = \mathrm{Re}\{B_s e^{j\omega t}\} \quad ...(6.8c)$$

$$H = \mathrm{Re}\{H_s e^{j\omega t}\} \quad ...(6.8d)$$

$$J = \mathrm{Re}\{J_s e^{j\omega t}\} \quad ...(6.8e)$$

and $$\rho_v = \mathrm{Re}\{\rho_{vs} e^{j\omega t}\} \quad ...(6.8f)$$

where $E_s, D_s, B_s, H_s, J_s$ and $\rho_{vs}$ are the phasor forms of respective field quantities. Using these relations, we can directly obtain the phasor form of Maxwell's equations as described below.

#### Maxwell's First Equation: Faraday's Law

In time varying field, first Maxwell's equation is written as

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad ...(6.9)$$

Now, from equation (6.8a) we can obtain

$$\nabla \times E = \nabla \times \mathrm{Re}\{E_s e^{j\omega t}\} = \mathrm{Re}\{\nabla \times E_s e^{j\omega t}\}$$

and using equation (6.8c) we get

$$\frac{\partial B}{\partial t} = \frac{\partial}{\partial t}\mathrm{Re}\{B_s e^{j\omega t}\} = \mathrm{Re}\{j\omega B_s e^{j\omega t}\}$$

Substituting the two results in equation (6.9) we get

$$\mathrm{Re}\{\nabla \times E_s e^{j\omega t}\} = -\mathrm{Re}\{j\omega B_s e^{j\omega t}\}$$

Hence, $$\nabla \times E_s = -j\omega B_s \quad \text{(Differential form)}$$

or, $$\oint_L E_s \cdot dL = -j\omega \int_S B_s \cdot dS \quad \text{(Integral form)}$$

#### Maxwell's Second Equation: Modified Ampere's Circuital Law

In time varying field, second Maxwell's equation is written as

$$\nabla \times H = J + \frac{\partial D}{\partial t} \quad ...(6.10)$$

From equation (6.8d) we can obtain

$$\nabla \times H = \nabla \times \mathrm{Re}\{H_s e^{j\omega t}\} = \mathrm{Re}\{\nabla \times H_s e^{j\omega t}\}$$

From equation (6.8e), we have

$$J = \mathrm{Re}\{J_s e^{j\omega t}\}$$

and using equation (6.8b) we get

$$\frac{\partial D}{\partial t} = \frac{\partial}{\partial t}\mathrm{Re}\{D_s e^{j\omega t}\} = \mathrm{Re}\{j\omega D_s e^{j\omega t}\}$$

Substituting these results in equation (6.10) we get

$$\mathrm{Re}\{\nabla\times H_s e^{j\omega t}\} = \mathrm{Re}\{J_s e^{j\omega t} + j\omega D_s e^{j\omega t}\}$$

Hence, $$\nabla\times H_s = J_s + j\omega D_s \qquad \text{(Differential form)}$$

or, $$\oint_L H_s\cdot dL = \int_S (J_s + j\omega D_s)\cdot dS \qquad \text{(Integral form)}$$

### Maxwell's Third Equation : Gauss's Law for Electric Field

In time varying field, third Maxwell's equation is written as

$$\nabla\cdot D = \rho_v \qquad ...(6.11)$$

From equation (6.8b) we can obtain

$$\nabla\cdot D = \nabla\cdot\mathrm{Re}\{D_s e^{j\omega t}\} = \mathrm{Re}\{\nabla\cdot D_s e^{j\omega t}\}$$

and from equation (6.8f) we have

$$\rho_v = \mathrm{Re}\{\rho_{vs} e^{j\omega t}\}$$

Substituting these two results in equation (6.11) we get

$$\mathrm{Re}\{\nabla\cdot D_s e^{j\omega t}\} = \mathrm{Re}\{\rho_{vs} e^{j\omega t}\}$$

Hence, $$\nabla\cdot D_s = \rho_{vs} \qquad \text{(Differential form)}$$

or, $$\oint_S D_s\cdot dS = \int_v \rho_{vs}\,dv \qquad \text{(Integral form)}$$

### Maxwell's Fourth Equation : Gauss's Law for Magnetic Field

$$\nabla\cdot B = 0 \qquad ...(6.12)$$

From equation (6.8c) we can obtain

$$\nabla\cdot B = \nabla\cdot\mathrm{Re}\{B_s e^{j\omega t}\} = \mathrm{Re}\{\nabla\cdot B_s e^{j\omega t}\}$$

Substituting it in equation (6.12) we get

$$\mathrm{Re}\{\nabla\cdot B_s e^{j\omega t}\} = 0$$

Hence, $$\nabla\cdot B_s = 0 \qquad \text{(Differential form)}$$

or, $$\oint_S B_s\cdot dS = 0 \qquad \text{(Integral form)}$$

Table 6.3 summarizes the Maxwell's equations in phasor form.

Table 6.3: Maxwell's Equations in Phasor Form

| S.N. | Differential form | Integral form | Name |
|---|---|---|---|
| 1. | $\nabla\times E_s = -j\omega B_s$ | $\oint_L E_s\cdot dL = -j\omega\int_S B_s\cdot dS$ | Faraday's law of electromagnetic induction |
| 2. | $\nabla\times H_s = J_s + j\omega D_s$ | $\oint_L H_s\cdot dL = \int_S (J_s + j\omega D_s)\cdot dS$ | Modified Ampere's circuital law |
| 3. | $\nabla\cdot D_s = \rho_{vs}$ | $\oint_S D_s\cdot dS = \int_v \rho_{vs}\,dv$ | Gauss' law of Electrostatics |
| 4. | $\nabla\cdot B_s = 0$ | $\oint_S B_s\cdot dS = 0$ | Gauss' law of Magnetostatic (non-existence of magnetic mono-pole) |

## 6.7 MAXWELL'S EQUATIONS IN FREE SPACE

For electromagnetic fields, free space is characterised by the following parameters:

1. Relative permittivity, $\varepsilon_r = 1$
2. Relative permeability, $\mu_r = 1$
3. Conductivity, $\sigma = 0$
4. Conduction current density, $J = 0$
5. Volume charge density, $\rho_v = 0$

As we have already obtained the four Maxwell's equations for time-varying fields, static fields, and harmonic fields; these equations can be easily written for the free space by just replacing the variables to their respective values in free space.

### 6.7.1 Maxwell's Equations for Time Varying Fields in Free Space

By substituting the parameters, $J = 0$ and $\rho_v = 0$ in the Maxwell's equations given in Table 6.1, we get the Maxwell's equation for time-varying fields in free space as summarized below:

Table 6.4: Maxwell's Equations for Time Varying Fields in Free Space

| S.N. | Differential form | Integral form | Name |
|---|---|---|---|
| 1. | $\nabla \times E = -\frac{\partial B}{\partial t}$ | $\oint_L E \cdot dL = -\frac{\partial}{\partial t}\int_S B \cdot dS$ | Faraday's law of electromagnetic induction |
| 2. | $\nabla \times H = \frac{\partial D}{\partial t}$ | $\oint_L H \cdot dL = \int_S \frac{\partial D}{\partial t} \cdot dS$ | Modified Ampere's circuital law |
| 3. | $\nabla \cdot D = 0$ | $\oint_S D \cdot dS = 0$ | Gauss' law of Electrostatics |
| 4. | $\nabla \cdot B = 0$ | $\oint_S B \cdot dS = 0$ | Gauss' law of Magnetostatic (non-existence of magnetic mono-pole) |

### 6.7.2 Maxwell's Equations for Static Fields in Free Space

Substituting the parameters, $J = 0$ and $\rho_v = 0$ in the Maxwell's equation given in Table 6.2, we get the Maxwell's equation for static fields in free space as summarized below.

Table 6.5: Maxwell's Equations for Static Fields in Free Space

| S.N. | Differential Form | Integral Form | Name |
|---|---|---|---|
| 1. | $\nabla \times E = 0$ | $\oint_L E \cdot dL = 0$ | Faraday's law of electromagnetic induction |
| 2. | $\nabla \times H = 0$ | $\oint_L H \cdot dL = 0$ | Modified Ampere's circuital law |
| 3. | $\nabla \cdot D = 0$ | $\oint_S D \cdot dS = 0$ | Gauss' law of Electrostatics |
| 4. | $\nabla \cdot B = 0$ | $\oint_S B \cdot dS = 0$ | Gauss' law of Magnetostatic (non-existence of magnetic mono-pole) |

Thus, all the four Maxwell's equation vanishes for static fields in free space.

### 6.7.3 Maxwell's Equations for Time Harmonic Fields in Free Space

Again, substituting the parameters, $J = 0$ and $\rho_v = 0$ in the Maxwell's equations given in Table 6.3, we get the Maxwell's equation for time harmonic fields in free space as summarized below.

Table 6.6 : Maxwell's Equations for Time-Harmonic Fields in Free Space

| S.N. | Differential form | Integral form | Name |
|---|---|---|---|
| 1. | $\nabla \times E_s = -j\omega B_s$ | $\oint_L E_s \cdot dL = -j\omega \int_S B_s \cdot dS$ | Faraday's law of electromagnetic induction |
| 2. | $\nabla \times H_s = j\omega D_s$ | $\oint_L H_s \cdot dL = \int_S j\omega D_s \cdot dS$ | Modified Ampere's circuital law |
| 3. | $\nabla \cdot D_s = 0$ | $\oint_S D_s \cdot dS = 0$ | Gauss' law of Electrostatics |
| 4. | $\nabla \cdot B_s = 0$ | $\oint_S B_s \cdot dS = 0$ | Gauss' law of Magnetostatic (non-existence of magnetic mono-pole) |

***********

# EXERCISE 6.1



MCQ 6.1.1 A perfect conducting sphere of radius $r$ is such that it's net charge resides on the surface. At any time $t$, magnetic field $B(r,t)$ inside the sphere will be

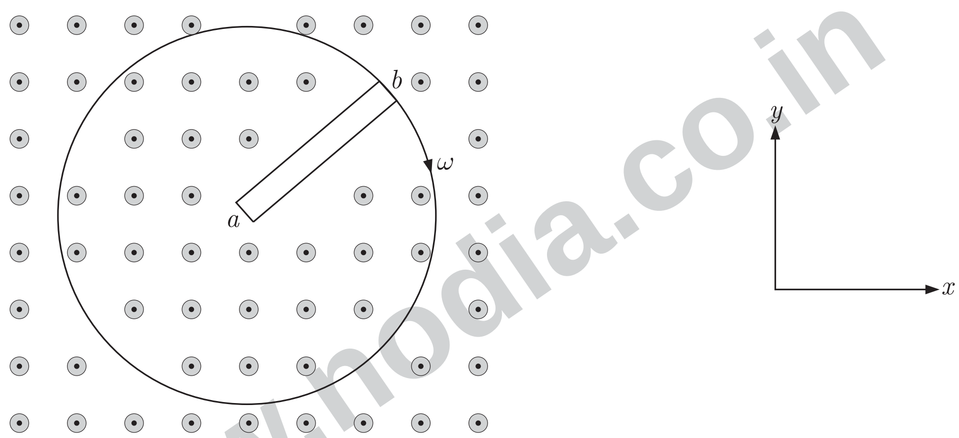
(A) 0

(B) uniform, independent of $r$

(C) uniform, independent of $t$

(D) uniform, independent of both $r$ and $t$

MCQ 6.1.2 A straight conductor $ab$ of length $l$ lying in the $xy$ plane is rotating about the centre $a$ at an angular velocity $\omega$ as shown in the figure.




If a magnetic field $B$ is present in the space directed along $a_z$ then which of the following statement is correct ?

(A) $V_{ab}$ is positive

(B) $V_{ab}$ is negative

(C) $V_{ba}$ is positive

(D) $V_{ba}$ is zero

MCQ 6.1.3 **Assertion (A) :** A small piece of bar magnet takes several seconds to emerge at bottom when it is dropped down a vertical aluminum pipe where as an identical unmagnetized piece takes a fraction of second to reach the bottom.

**Reason (R) :** When the bar magnet is dropped inside a conducting pipe, force exerted on the magnet by induced eddy current is in upward direction.

(A) Both A and R are true and R is correct explanation of A.

(B) Both A and R are true but R is not the correct explanation of A.

(C) A is true but R is false.

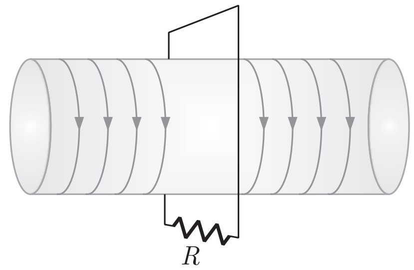
(D) A is false but R is true.

MCQ 6.1.4 Self inductance of a long solenoid having $n$ turns per unit length will be proportional to

(A) $n$ (B) $1/n$

(C) $n^2$ (D) $1/n^2$

MCQ 6.1.5 A wire with resistance $R$ is looped on a solenoid as shown in figure.

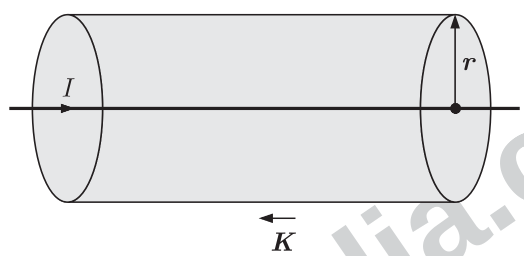



If a constant current is flowing in the solenoid then the induced current flowing in the loop with resistance $R$ will be

(A) non uniform (B) constant

(C) zero (D) none of these

MCQ 6.1.6 A long straight wire carries a current $I = I_0 \cos(\omega t)$. If the current returns along a coaxial conducting tube of radius $r$ as shown in figure then magnetic field and electric field inside the tube will be respectively.



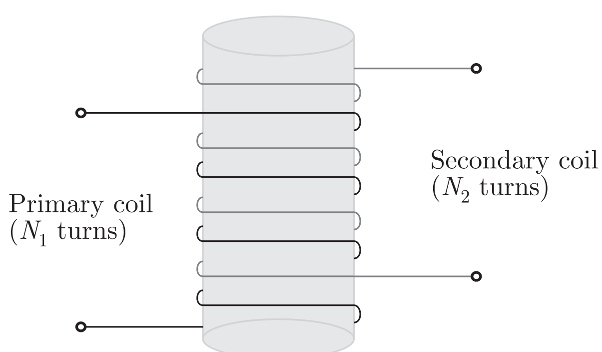

(A) radial, longitudinal (B) circumferential, longitudinal

(C) circumferential, radial (D) longitudinal, circumferential



MCQ 6.1.7 Assertion (A) : Two coils are wound around a cylindrical core such that the primary coil has $N_1$ turns and the secondary coils has $N_2$ turns as shown in figure. If the same flux passes through every turn of both coils then the ratio of emf induced in the two coils is

$$\frac{V_{\text{emf}\,2}}{V_{\text{emf}\,1}} = \frac{N_2}{N_1}$$




Reason (R) : In a primitive transformer, by choosing the appropriate no. of turns, any desired secondary emf can be obtained.

(A) Both A and R are true and R is correct explanation of A.

(B) Both A and R are true but R is not the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

MCQ 6.1.8 In a non magnetic medium electric field $E = E_0 \cos \omega t$ is applied. If the permittivity of medium is $\varepsilon$ and the conductivity is $\sigma$ then the ratio of the amplitudes of the conduction current density and displacement current density will be

(A) $\mu_0 / \omega\varepsilon$ (B) $\sigma / \omega\varepsilon$

(C) $\sigma\mu_0 / \omega\varepsilon$ (D) $\omega\varepsilon / \sigma$

MCQ 6.1.9 In a medium, the permittivity is a function of position such that $\frac{\nabla \varepsilon}{\varepsilon} \approx 0$. If the volume charge density inside the medium is zero then $\nabla \cdot \mathbf{E}$ is roughly equal to

(A) $\varepsilon \mathbf{E}$ (B) $-\varepsilon \mathbf{E}$

(C) 0 (D) $-\nabla \varepsilon \cdot \mathbf{E}$

MCQ 6.1.10 In free space, the electric field intensity at any point $(r, \theta, \phi)$ in spherical coordinate system is given by

$$\mathbf{E} = \frac{\sin\theta \cos(\omega t - kr)}{r} \mathbf{a}_\theta$$

The phasor form of magnetic field intensity in the free space will be

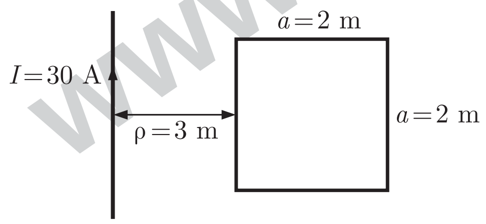
(A) $\frac{k \sin\theta}{\omega \mu_0 r} e^{-jkr} \mathbf{a}_\phi$ (B) $-\frac{k \sin\theta}{\omega \mu_0 r} e^{-jkr} \mathbf{a}_\phi$

(C) $\frac{k\omega\mu_0}{r} e^{-jkr} \mathbf{a}_\phi$ (D) $\frac{k \sin\theta}{r} e^{-jkr} \mathbf{a}_\phi$



Common Data For Q. 11 and 12 :

A conducting wire is formed into a square loop of side 2 m. A very long straight wire carrying a current $I = 30$ A is located at a distance 3 m from the square loop as shown in figure.




MCQ 6.1.11 If the loop is pulled away from the straight wire at a velocity of 5 m/s then the induced e.m.f. in the loop after 0.6 sec will be

(A) 5 μvolt (B) 2.5 μvolt

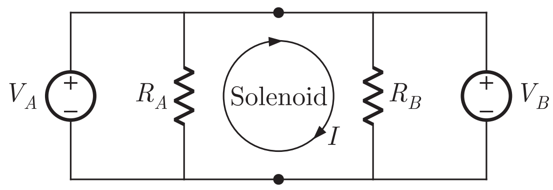
(C) 25 μvolt (D) 5 mvolt

MCQ 6.1.12 If the loop is pulled downward in the parallel direction to the straight wire, such that distance between the loop and wire is always 3 m then the induced e.m.f. in the loop at any time $t$ will be

(A) linearly increasing with $t$ (B) always 0

(C) linearly decreasing with $t$ (D) always constant but not zero.

MCQ 6.1.13 Two voltmeters $A$ and $B$ with internal resistances $R_A$ and $R_B$ respectively is connected to the diametrically opposite points of a long solenoid as shown in figure. Current in the solenoid is increasing linearly with time. The correct relation between the voltmeter's reading $V_A$ and $V_B$ will be

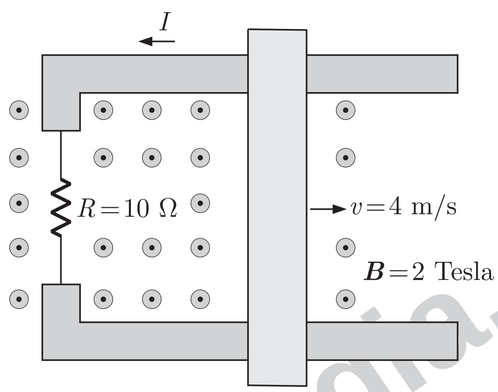



(A) $V_A = V_B$ (B) $V_A = -V_B$

(C) $\frac{V_A}{V_B} = \frac{R_A}{R_B}$ (D) $\frac{V_A}{V_B} = -\frac{R_A}{R_B}$

### Common Data For Q. 14 and 15 :

Two parallel conducting rails are being placed at a separation of 5 m with a resistance $R = 10\,\Omega$ connected across it's one end. A conducting bar slides frictionlessly on the rails with a velocity of 4 m/s away from the resistance as shown in the figure.




MCQ 6.1.14 If a uniform magnetic field $B = 2$ Tesla pointing out of the page fills entire region then the current $I$ flowing in the bar will be

(A) 0 A (B) $-40$ A

(C) 4 A (D) $-4$ A

MCQ 6.1.15 The force exerted by magnetic field on the sliding bar will be

(A) 4 N, opposes it's motion

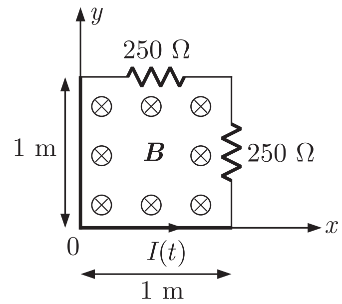
(B) 40 N, opposes it's motion

(C) 40 N, in the direction of it's motion

(D) 0

MCQ 6.1.16 Two small resistor of $250\,\Omega$ each is connected through a perfectly conducting filament such that it forms a square loop lying in $x$-$y$ plane as shown in the figure. Magnetic flux density passing through the loop is given as

$$B = -7.5\cos(120\pi t - 30^\circ)\,a_z$$

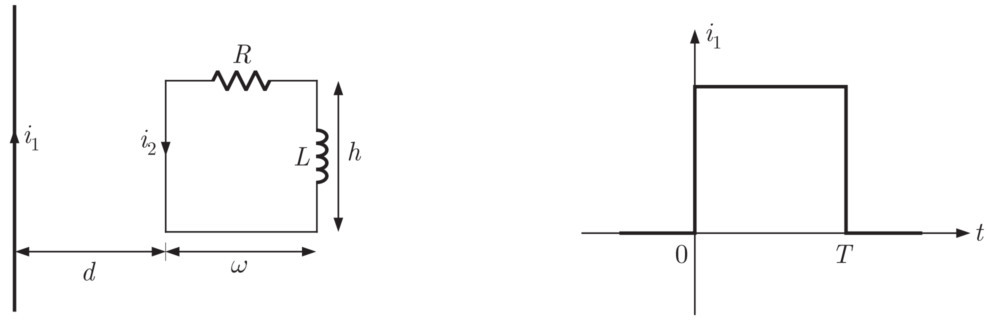


The induced current $I(t)$ in the loop will be

(A) $0.02\sin(120\pi t - 30^\circ)$ (B) $2.8\times10^3\sin(120\pi t - 30^\circ)$

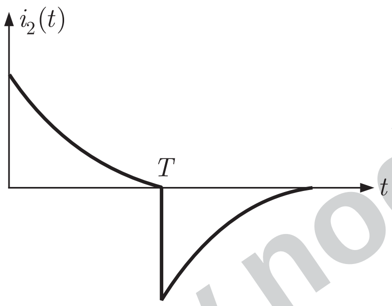
(C) $-5.7\sin(120\pi t - 30^\circ)$ (D) $5.7\sin(120\pi t - 30^\circ)$

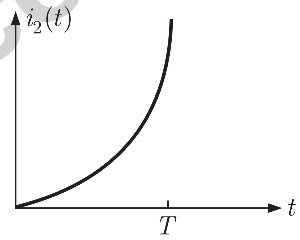
MCQ 6.1.17 A rectangular loop of self inductance $L$ is placed near a very long wire carrying current $i_1$ as shown in figure (a). If $i_1$ be the rectangular pulse of current as shown in figure (b) then the plot of the induced current $i_2$ in the loop versus time $t$ will be (assume the time constant of the loop, $\tau \gg L/R$)

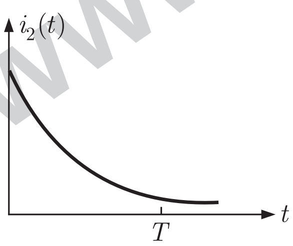


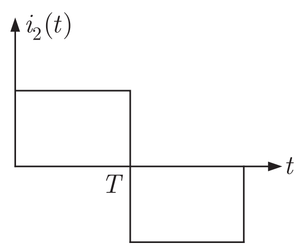

(a) (b)



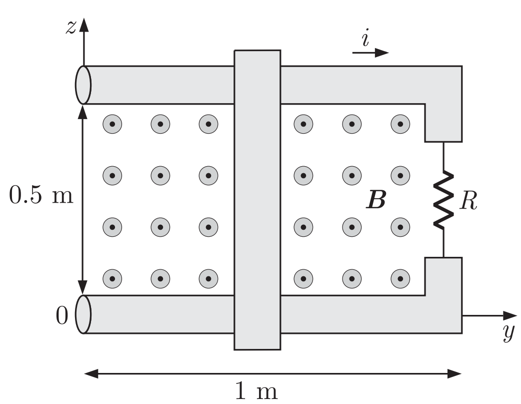
(A)



(B)



(C)



(D)



MCQ 6.1.18 Two parallel conducting rails is placed in a varying magnetic field $B = 0.2\cos\omega t a_x$. A conducting bar oscillates on the rails such that it's position is given by $y = 0.5(1-\cos\omega t)$ m. If one end of the rails are terminated in a resistance $R = 5\,\Omega$, then the current $i$ flowing in the rails will be



(A) $0.01\omega\sin\omega t(1+2\cos\omega t)$ (B) $-0.01\omega\sin\omega t(1+2\cos\omega t)$
(C) $0.01\omega\cos\omega t(1+2\sin\omega t)$ (D) $0.05\omega\sin\omega t(1+2\sin\omega t)$

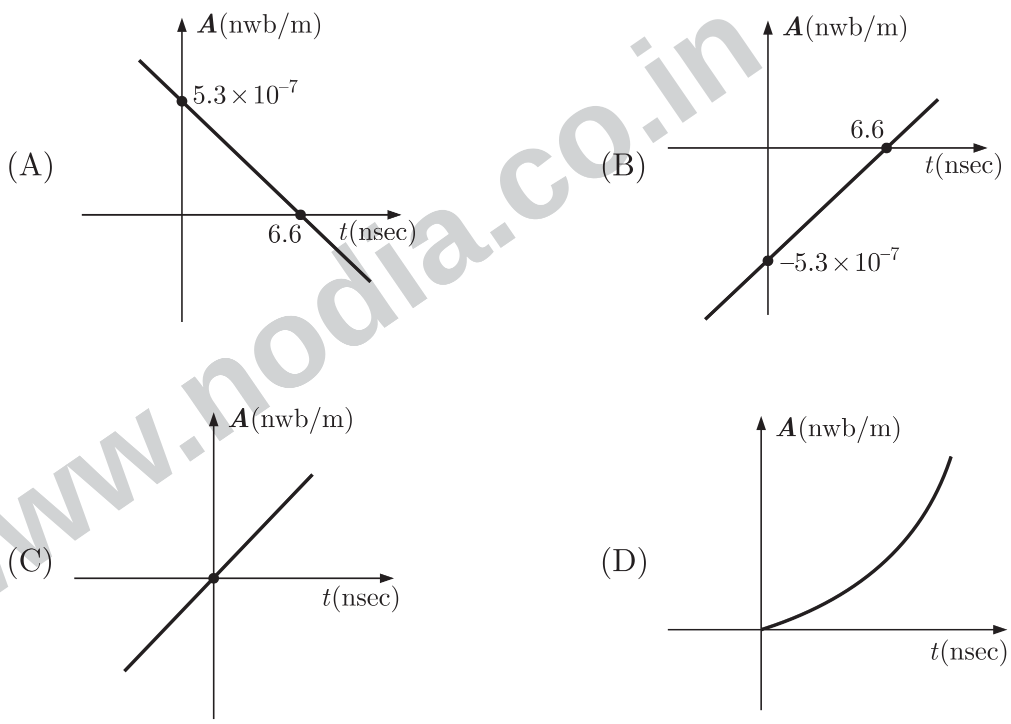
MCQ 6.1.19 Electric flux density in a medium ($\varepsilon_r = 10$, $\mu_r = 2$) is given as

$$\boldsymbol{D} = 1.33\sin(3\times 10^8 t - 0.2x)\boldsymbol{a}_y \ \mu\text{C/m}^2$$

Magnetic field intensity in the medium will be

(A) $10^{-5}\sin(3\times 10^8 t - 0.2x)\boldsymbol{a}_y$ A/m
(B) $2\sin(3\times 10^8 t - 0.2x)\boldsymbol{a}_y$ A/m
(C) $-4\sin(3\times 10^8 t - 0.2x)\boldsymbol{a}_y$ A/m
(D) $4\sin(3\times 10^8 t - 0.2x)\boldsymbol{a}_y$ A/m

MCQ 6.1.20 A current filament located on the $x$-axis in free space with in the interval $-0.1 < x < 0.1$ m carries current $I(t) = 8t$ A in $\boldsymbol{a}_x$ direction. If the retarded vector potential at point $P(0,0,2)$ be $\boldsymbol{A}(t)$ then the plot of $\boldsymbol{A}(t)$ versus time will be






Common Data For Q. 21 and 22 :

In a region of electric and magnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$, respectively, the force experienced by a test charge $qC$ are given as follows for three different velocities.

| Velocity m/sec | Force, N |
|---|---|
| $\boldsymbol{a}_x$ | $q(\boldsymbol{a}_y + \boldsymbol{a}_z)$ |
| $\boldsymbol{a}_y$ | $q\boldsymbol{a}_y$ |
| $\boldsymbol{a}_z$ | $q(2\boldsymbol{a}_y + \boldsymbol{a}_z)$ |

MCQ 6.1.21 What will be the magnetic field $\boldsymbol{B}$ in the region ?

(A) $\boldsymbol{a}_x$ (B) $\boldsymbol{a}_x - \boldsymbol{a}_y$
(C) $\boldsymbol{a}_z$ (D) $\boldsymbol{a}_y - \boldsymbol{a}_z$

MCQ 6.1.22 What will be electric field $E$ in the region ?

(A) $a_x - a_z$ (B) $a_y - a_z$

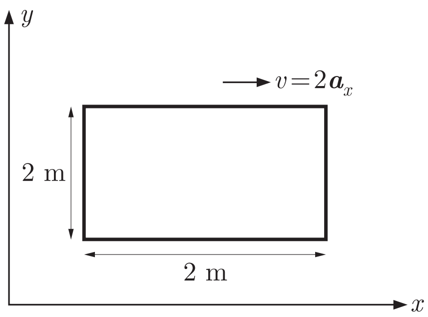
(C) $a_y + a_z$ (D) $a_y + a_z - a_x$

MCQ 6.1.23 In a non-conducting medium ($\sigma = 0$, $\mu_r = \varepsilon_r = 1$), the retarded potentials are given as $V = y(x - ct)$ volt and $A = y(\frac{x}{c} - t)a_x$ Wb/m where $c$ is velocity of waves in free space. The field (electric and magnetic) inside the medium satisfies Maxwell's equation if

(A) $J = 0$ only (B) $\rho_v = 0$ only

(C) $J = \rho_v = 0$ (D) Can't be possible

MCQ 6.1.24 In Cartesian coordinates magnetic field is given by $B = -2/x\, a_z$. A square loop of side 2 m is lying in $xy$ plane and parallel to the $y$-axis. Now, the loop is moving in that plane with a velocity $v = 2a_x$ as shown in the figure.




What will be the circulation of the induced electric field around the loop ?

(A) $\frac{16}{x(x+2)}$ (B) $\frac{8}{x}$

(C) $\frac{8}{x(x+2)}$ (D) $\frac{x(x+2)}{16}$

Common Data For Q. 25 to 27 :

In a cylindrical coordinate system, magnetic field is given by

$$B = \begin{cases} 0 & \text{for } \rho < 4\text{ m} \\ 2\sin\omega t\, a_z & \text{for } 4 < \rho < 5\text{ m} \\ 0 & \text{for } \rho > 5\text{ m} \end{cases}$$

MCQ 6.1.25 The induced electric field in the region $\rho < 4$ m will be

(A) 0 (B) $\frac{2\omega\cos\omega t}{\rho}a_\phi$

(C) $-2\cos\omega t a_\phi$ (D) $\frac{1}{2\sin\omega t}a_\phi$

MCQ 6.1.26 The induced electric field at $\rho = 4.5$ m is

(A) 0 (B) $-\frac{17\omega\cos\omega t}{18}$

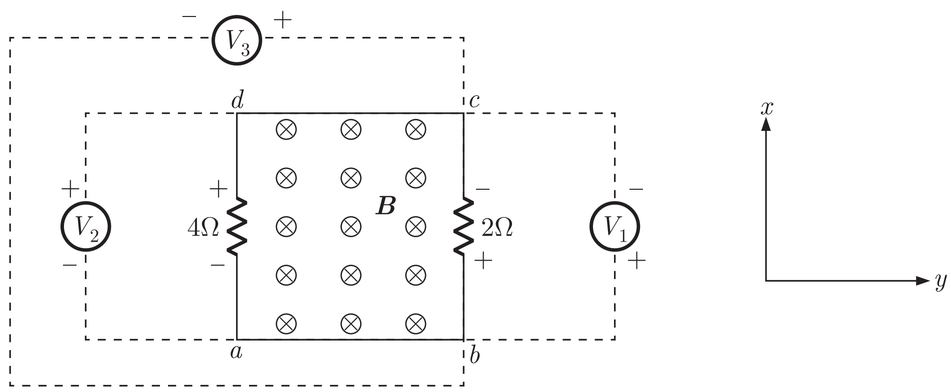
(C) $\frac{4\omega\cos\omega t}{9}$ (D) $-\frac{17\omega\cos\omega t}{4}$

MCQ 6.1.27 The induced electric field in the region $\rho > 5$ m is

(A) $-\frac{18}{\rho}\omega\cos\omega t a_\phi$ (B) $\frac{-9\omega\cos\omega t}{\rho}a_\phi$

(C) $-9\rho\cos\omega t a_\phi$ (D) $\frac{9\omega\cos\omega t}{\rho}a_\phi$

MCQ 6.1.28 Magnetic flux density, $B = 0.1t\,a_z$ Tesla threads only the loop *abcd* lying in the plane *xy* as shown in the figure.



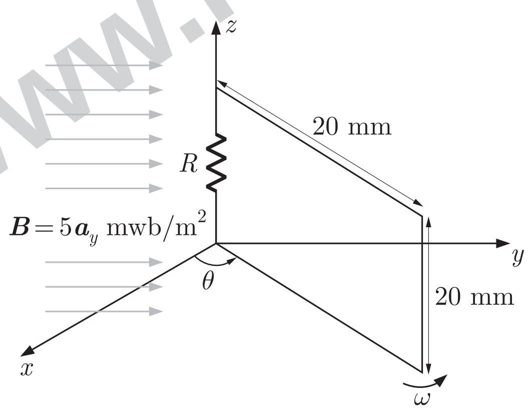

Consider the three voltmeters $V_1$, $V_2$ and $V_3$, connected across the resistance in the same *xy* plane. If the area of the loop *abcd* is $1\text{ m}^2$ then the voltmeter readings are

| | $V_1$ | $V_2$ | $V_3$ |
|---|---|---|---|
| (A) | 66.7 mV | 33.3 mV | 66.7 mV |
| (B) | 33.3 mV | 66.7 mV | 33.3 mV |
| (C) | 66.7 mV | 66.7 mV | 33.3 mV |
| (D) | 33.3 mV | 66.7 mV | 66.7 mV |



Common Data For Q. 29 and 30 :

A square wire loop of resistance $R$ rotated at an angular velocity $\omega$ in the uniform magnetic field $B = 5a_y\text{ mWb/m}^2$ as shown in the figure.



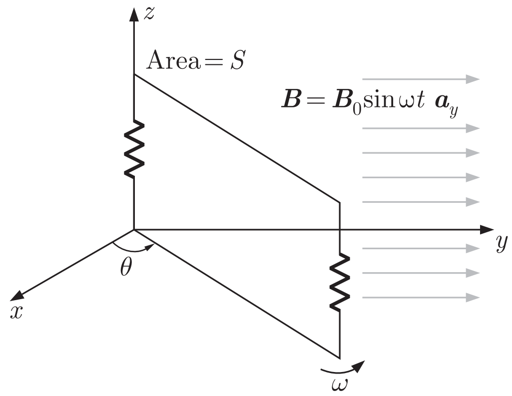

MCQ 6.1.29 If the angular velocity, $\omega = 2\text{ rad/sec}$ then the induced e.m.f. in the loop will be

(A) $2\sin\theta\,\mu\text{V/m}$
(B) $2\cos\theta\,\mu\text{V/m}$
(C) $4\cos\theta\,\mu\text{V/m}$
(D) $4\sin\theta\,\mu\text{V/m}$

MCQ 6.1.30 If resistance, $R = 40\text{ m}\Omega$ then the current flowing in the square loop will be

(A) $0.2\sin\theta$ mA
(B) $0.1\sin\theta$ mA
(C) $0.1\cos\theta$ mA
(D) $0.5\sin\theta$ mA

MCQ 6.1.31 In a certain region magnetic flux density is given as $B = B_0\sin\omega t\,a_y$. A rectangular loop of wire is defined in the region with it's one corner at origin and one side along $z$-axis as shown in the figure.



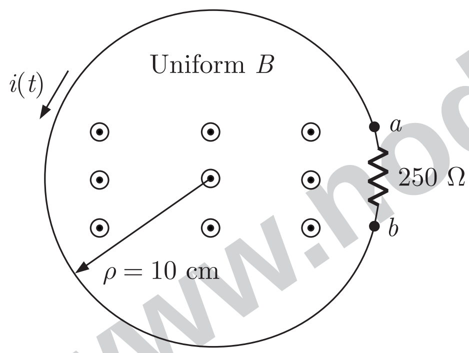

If the loop rotates at an angular velocity $\omega$ (same as the angular frequency of magnetic field) then the maximum value of induced e.m.f in the loop will be

(A) $\frac{1}{2}B_0S\omega$ (B) $2B_0S\omega$

(C) $B_0S\omega$ (D) $4B_0S\omega$

## Common Data For Q. 32 and 33 :

Consider the figure shown below. Let $B = 10\cos 120\pi t$ Wb/m$^2$ and assume that the magnetic field produced by $i(t)$ is negligible






MCQ 6.1.32 The value of $v_{ab}$ is

(A) $-118.43\cos 120\pi t$ V (B) $118.43\cos 120\pi t$ V

(C) $-118.43\sin 120\pi t$ V (D) $118.43\sin 120\pi t$ V

MCQ 6.1.33 The value of $i(t)$ is

(A) $-0.47\cos 120\pi t$ A (B) $0.47\cos 120\pi t$ A

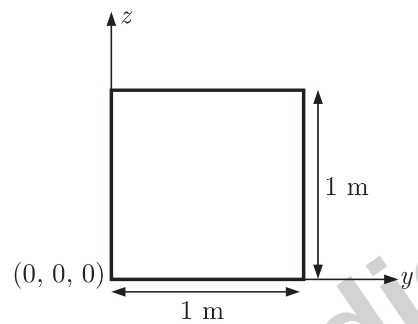
(C) $-0.47\sin 120\pi t$ A (D) $0.47\sin 120\pi t$ A

***********

# EXERCISE 6.2

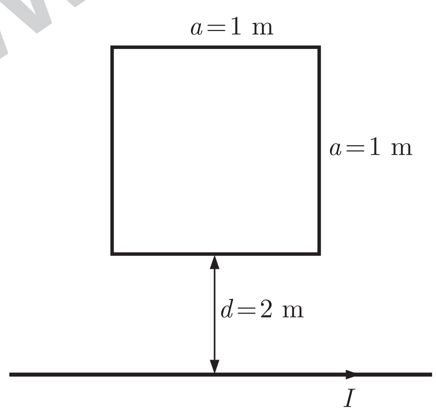
QUES 6.2.1 A small conducting loop is released from rest with in a vertical evacuated cylinder. What is the voltage induced (in mV) in the falling loop ?
(Assume earth magnetic field $= 10^{-6}$ T at a constant angle of $10°$ below the horizontal)

QUES 6.2.2 A square loop of side 1 m is located in the plane $x = 0$ as shown in figure. A non-uniform magnetic flux density through it is given as $B = 4z^3t^2 a_x$, The emf induced in the loop at time $t = 2$ sec will be ____ Volt.





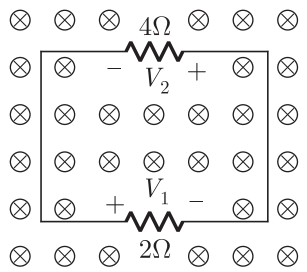

QUES 6.2.3 A very long straight wire carrying a current $I = 5$ A is placed at a distance of 2 m from a square loop as shown. If the side of the square loop is 1 m then the total flux passing through the square loop will be ____ $\times 10^{-7}$ wb



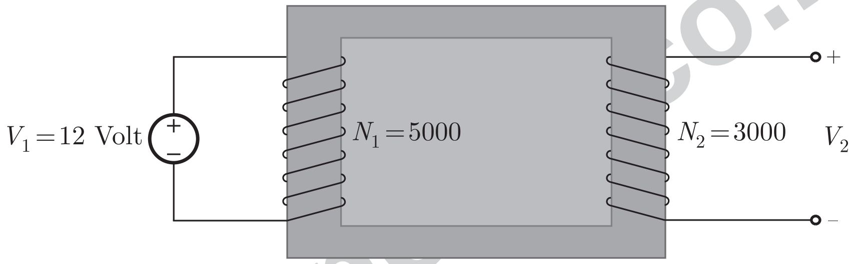

QUES 6.2.4 In a medium where no D.C. field is present, the conduction current density at any point is given as $J_d = 20\cos(1.5\times10^8 t)a_y$ A/m$^2$. Electric flux density in the medium will be $D_0\sin(1.5\times10^8 t)a_y$ nC/m$^2$ such that $D_0 =$ ____

QUES 6.2.5 A conducting medium has permittivity, $\varepsilon = 4\varepsilon_0$ and conductivity, $\sigma = 1.14\times10^8$ s/m. The ratio of magnitude of displacement current and conduction current in the medium at 50 GHz will be _____ $\times 10^{-8}$.

QUES 6.2.6 In a certain region magnetic flux density is given as $B = 0.1ta_z$ Wb/m$^2$. An electric loop with resistance $2\,\Omega$ and $4\,\Omega$ is lying in $x$-$y$ plane as shown in the figure. If the area of the loop is $1\,\text{m}^2$ then, the voltage drop $V_1$ across the $2\,\Omega$ resistance is ____ mV.




QUES 6.2.7 A magnetic core of uniform cross section having two coils (Primary and secondary) wound on it as shown in figure. The no. of turns of primary coil is 5000 and no. of turns of secondary coil is 3000. If a voltage source of 12 volt is connected across the primary coil then what will be the voltage (in Volt) across the secondary coil ?

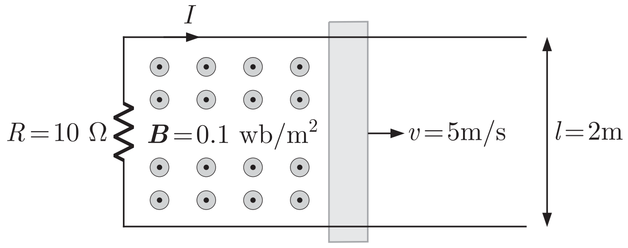



QUES 6.2.8 Magnetic field intensity in free space is given as

$$H = 0.1\cos(15\pi y)\sin(6\pi \times 10^9 t - bx)a_z \text{ A/m}$$

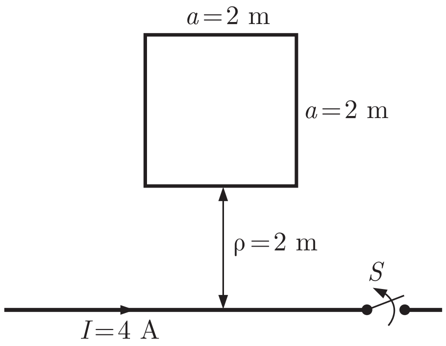
It satisfies Maxwell's equation when $|b| =$ ____

QUES 6.2.9 Two parallel conducting rails are being placed at a separation of $2\,\text{m}$ as shown in figure. One end of the rail is being connected through a resistor $R = 10\,\Omega$ and the other end is kept open. A metal bar slides frictionlessly on the rails at a speed of $5\,\text{m/s}$ away from the resistor. If the magnetic flux density $B = 0.1\,\text{Wb/m}^2$ pointing out of the page fills entire region then the current $I$ flowing in the resistor will be ____ Ampere.

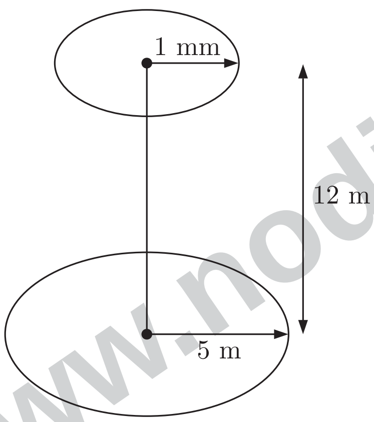



QUES 6.2.10 An infinitely long straight wire with a closed switch $S$ carries a uniform current $I = 4\,\text{A}$ as shown in figure. A square loop of side $a = 2\,\text{m}$ and resistance

$R = 4\,\Omega$ is located at a distance $2\,\text{m}$ from the wire. Now at any time $t = t_0$ the switch is open so the current $I$ drops to zero. What will be the total charge (in nC) that passes through a corner of the square loop after $t = t_0$ ?




QUES 6.2.11 A circular loop of radius $5\,\text{m}$ carries a current $I = 2\,\text{A}$. If another small circular loop of radius $1\,\text{mm}$ lies a distance $12\,\text{m}$ above the large circular loop such that the planes of the two loops are parallel and perpendicular to the common axis as shown in figure then total flux through the small loop will be _ _ _ _ fermi-weber.






QUES 6.2.12 A non magnetic medium at frequency $f = 1.6 \times 10^8\,\text{Hz}$ has permittivity $\varepsilon = 54\varepsilon_0$ and resistivity $\rho = 0.77\,\Omega\text{-m}$. What will be the ratio of amplitudes of conduction current to the displacement current ?

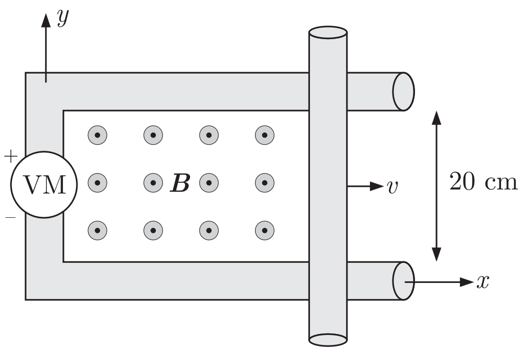
QUES 6.2.13 In a certain region a test charge is moving with an angular velocity $2\,\text{rad/sec}$ along a circular path of radius $2\,\text{m}$ centred at origin in the $x$-$y$ plane. If the magnetic flux density in the region is $\boldsymbol{B} = 2\boldsymbol{a}_z\,\text{Wb/m}^2$ then the electric field viewed by an observer moving with the test charge is _ _ _ _ V/m in $\boldsymbol{a}_\rho$ direction.

## Common Data For Q. 13 and 14 :

In a non uniform magnetic field $\boldsymbol{B} = 8x^2\boldsymbol{a}_z$ Tesla, two parallel rails with a separation of $20\,\text{cm}$ and connected with a voltmeter at it's one end is located in $x$-$y$ plane as shown in figure. The Position of the bar which is sliding on the rails is given as

$$x = t(1 + 0.4t^2)$$

QUES 6.2.14 What will be the voltmeter reading (in volt) at $t = 0.4\,\text{sec}$ ?




QUES 6.2.15 What will be the voltmeter reading (in volt) at $x = 12\,\text{cm}$ ?

QUES 6.2.16 In a non conducting medium $(\sigma = 0)$ magnetic field intensity at any point is given by $H = \cos(10^{10}t - bx)a_z$ A/m . The permittivity of the medium is $\varepsilon = 0.12\,\text{nF/m}$ and permeability of the medium is $\mu = 3 \times 10^{-5}\,\text{H/m}$. D.C. field is not present in medium. Field satisfies Maxwell's equation, if $|b| =$ ____

QUES 6.2.17 Electric field in free space in given as

$$E = 5\sin(10\pi y)\cos(6\pi \times 10^9 - bx)a_z$$

It satisfies Maxwell's equation for $|b| =$ ?

QUES 6.2.18 8 A current is flowing along a straight wire from a point charge situated at the origin to infinity and passing through the point $(2,2,2)$. The circulation of the magnetic field intensity around the closed path formed by the triangle having the vertices $(2,0,0)$, $(0,2,0)$ and $(0,0,2)$ is equal to ____ Ampere.

QUES 6.2.19 A 50 turn rectangular loop of area $64\,\text{cm}^2$ rotates at 60 revolution per seconds in a magnetic field $B = 0.25\sin 377t\ \text{Wb/m}^2$ directed normal to the axis of rotation. What is the rms value of the induced voltage (in volt) ?

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# EXERCISE 6.3

MCQ 6.3.1 Match List I with List II and select the correct answer using the codes given below (Notations have their usual meaning)

| | List-I | | List-II |
|---|---|---|---|
| a | Ampere's circuital law | 1. | $\nabla \cdot D = \rho_v$ |
| b | Faraday's law | 2. | $\nabla \cdot B = 0$ |
| c | Gauss's law | 3. | $\nabla \times E = -\frac{\partial B}{\partial t}$ |
| d | Non existence of isolated magneticharge | 4. | $\nabla \times H = J + \frac{\partial D}{\partial t}$ |

Codes :

| | a | b | c | d |
|---|---|---|---|---|
| (A) | 4 | 3 | 2 | 1 |
| (B) | 4 | 1 | 3 | 2 |
| (C) | 2 | 3 | 1 | 4 |
| (D) | 4 | 3 | 1 | 2 |

MCQ 6.3.2 Magneto static fields is caused by

(A) stationary charges

(B) steady currents

(C) time varying currents

(D) none of these

MCQ 6.3.3 Let $A$ be magnetic vector potential and $E$ be electric field intensity at certain time in a time varying $EM$ field. The correct relation between $E$ and $A$ is

(A) $E = -\frac{\partial A}{\partial t}$ (B) $A = -\frac{\partial E}{\partial t}$

(C) $E = \frac{\partial A}{\partial t}$ (D) $A = \frac{\partial E}{\partial t}$

MCQ 6.3.4 A closed surface $S$ defines the boundary line of magnetic medium such that the field intensity inside it is $B$. Total outward magnetic flux through the closed surface will be

(A) $B \cdot S$ (B) 0

(C) $B \times S$ (D) none of these

MCQ 6.3.5 The total magnetic flux through a conducting loop having electric field $E = 0$ inside it will be

(A) 0

(B) constant

(C) varying with time only

(D) varying with time and area of the surface both

MCQ 6.3.6 A cylindrical wire of a large cross section made of super conductor carries a current $I$. The current in the superconductor will be confined.

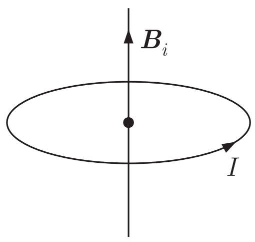
(A) inside the wire

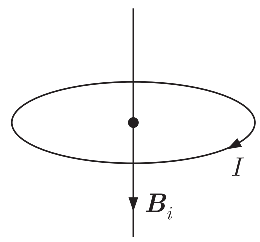
(B) to the axis of cylindrical wire

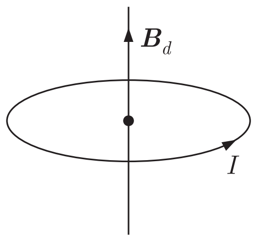
(C) to the surface of the wire

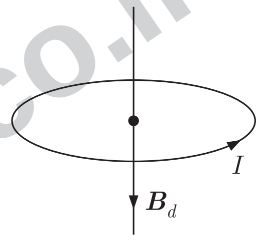
(D) none of these

MCQ 6.3.7 If $B_i$ denotes the magnetic flux density increasing with time and $B_d$ denotes the magnetic flux density decreasing with time then which of the configuration is correct for the induced current $I$ in the stationary loop ?

(A)



(B)



(C)



(D)





MCQ 6.3.8 A circular loop is rotating about $z$-axis in a magnetic field $B = B_0 \cos\omega t a_y$. The total induced voltage in the loop is caused by

(A) Transformer emf

(B) motion emf.

(C) Combination of (A) and (B)

(D) none of these

MCQ 6.3.9 For static magnetic field,

(A) $\nabla \times B = \rho$

(B) $\nabla \times B = \mu J$

(C) $\nabla \cdot B = \mu_0 J$

(D) $\nabla \times B = 0$

MCQ 6.3.10 Displacement current density is

(A) $D$

(B) $J$

(C) $\partial D / \partial t$

(D) $\partial J / \partial t$

MCQ 6.3.11 The time varying electric field is

(A) $E = -\nabla V$

(B) $E = -\nabla V - \dot{A}$

(C) $E = -\nabla V - B$

(D) $E = -\nabla V - D$

MCQ 6.3.12 A field can exist if it satisfies

(A) Gauss's law

(B) Faraday's law

(C) Coulomb's law

(D) All Maxwell's equations

MCQ 6.3.13 Maxwell's equations give the relations between

(A) different fields

(B) different sources

(C) different boundary conditions

(D) none of these

MCQ 6.3.14 If $E$ is a vector, then $\nabla \cdot \nabla \times E$ is

(A) 0 (B) 1

(C) does not exist (D) none of these

MCQ 6.3.15 The Maxwell's equation, $\nabla \cdot B = 0$ is due to

(A) $B = \mu H$ (B) $B = \frac{H}{\mu}$

(C) non-existence of a mono pole (D) none of these

MCQ 6.3.16 For free space,

(A) $\sigma = \infty$ (B) $\sigma = 0$

(C) $J \neq 0$ (D) none of these

MCQ 6.3.17 For time varying EM fields

(A) $\nabla \times H = J$ (B) $\nabla \times H = \dot{D} + J$

(C) $\nabla \times E = 0$ (D) none of these

***********

# EXERCISE 6.4



MCQ 6.4.1 A magnetic field in air is measured to be $B = B_0\left(\frac{x}{x^2+y^2}a_y - \frac{y}{x^2+y^2}a_x\right)$ What current distribution leads to this field ?
[Hint : The algebra is trivial in cylindrical coordinates.]

(A) $J = \frac{B_0 z}{\mu_0}\left(\frac{1}{x^2+y^2}\right), r \neq 0$
(B) $J = -\frac{B_0 z}{\mu_0}\left(\frac{2}{x^2+y^2}\right), r \neq 0$
(C) $J = 0, r \neq 0$
(D) $J = \frac{B_0 z}{\mu_0}\left(\frac{1}{x^2+y^2}\right), r \neq 0$

MCQ 6.4.2 For static electric and magnetic fields in an inhomogeneous source-free medium, which of the following represents the correct form of Maxwell's equations ?

(A) $\nabla \cdot E = 0$, $\nabla \times B = 0$
(B) $\nabla \cdot E = 0$, $\nabla \cdot B = 0$
(C) $\nabla \times E = 0$, $\nabla \times B = 0$
(D) $\nabla \times E = 0$, $\nabla \cdot B = 0$

MCQ 6.4.3 If $C$ is closed curve enclosing a surface $S$, then magnetic field intensity $H$, the current density $J$ and the electric flux density $D$ are related by

(A) $\iint_S H \cdot dS = \oint_C \left(J + \frac{\partial D}{\partial t}\right) \cdot dl$
(B) $\int_S H \cdot dl = \oiint_S \left(J + \frac{\partial D}{\partial t}\right) \cdot dS$
(C) $\oiint_S H \cdot dS = \int_C \left(J + \frac{\partial D}{\partial t}\right) \cdot dl$
(D) $\oint_C H \cdot dl = \iint_S \left(J + \frac{\partial D}{\partial t}\right) \cdot dS$

MCQ 6.4.4 The unit of $\nabla \times H$ is

(A) Ampere
(B) Ampere/meter
(C) Ampere/meter$^2$
(D) Ampere-meter

MCQ 6.4.5 The Maxwell equation $\nabla \times H = J + \frac{\partial D}{\partial t}$ is based on

(A) Ampere's law
(B) Gauss' law
(C) Faraday's law
(D) Coulomb's law

MCQ 6.4.6 A loop is rotating about they $y$-axis in a magnetic field $B = B_0 \cos(\omega t + \phi)\, a_x$ T. The voltage in the loop is

(A) zero
(B) due to rotation only
(C) due to transformer action only
(D) due to both rotation and transformer action

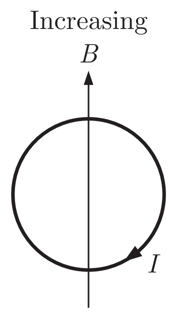
MCQ 6.4.7 The credit of defining the following current is due to Maxwell

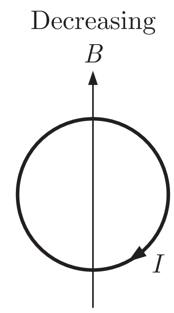
(A) Conduction current
(B) Drift current
(C) Displacement current
(D) Diffusion current

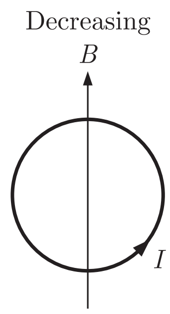
MCQ 6.4.8 A varying magnetic flux linking a coil is given by $\Phi = 1/3\lambda t^3$. If at time $t = 3$ s, the emf induced is 9 V, then the value of $\lambda$ is.

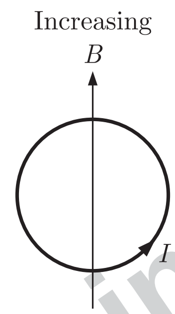
(A) zero
(B) 1 Wb/s$^2$
(C) $-1$ Wb/s$^2$
(D) 9 Wb/s$^2$

MCQ 6.4.9 Assuming that each loop is stationary and time varying magnetic field $B$, induces current $I$, which of the configurations in the figures are correct ?

1.



2.



3.



4.



(A) 1, 2, 3 and 4 (B) 1 and 3 only
(C) 2 and 4 only (D) 3 and 4 only

MCQ 6.4.10 Assertion (A) : For time varying field the relation $E = -\nabla V$ is inadequate.
Reason (R) : Faraday's law states that for time varying field $\nabla \times E = 0$
(A) Both Assertion (A) and Reason (R) are individually true and Reason (R) is the correct explanation of Assertion (A)
(B) Both Assertion (A) and Reason (R) are individually true but Reason (R) is not the correct explanation of Assertion (A)
(C) Assertion (A) is true but Reason (R) is false
(D) Assertion (A) is false but Reason (R) is true

MCQ 6.4.11 Who developed the concept of time varying electric field producing a magnetic field ?
(A) Gauss (B) Faraday
(C) Hertz (D) Maxwell

MCQ 6.4.12 A single turn loop is situated in air, with a uniform magnetic field normal to its plane. The area of the loop is 5 $m^2$ and the rate of charge of flux density is 2 $Wb/m^2/s$. What is the emf appearing at the terminals of the loop ?
(A) $-5$ V (B) $-2$ V
(C) $-0.4$ V (D) $-10$ V

MCQ 6.4.13 Which of the following equations results from the circuital form of Ampere's law ?
(A) $\nabla \times E = -\frac{\partial B}{\partial t}$ (B) $\nabla \cdot B = 0$
(C) $\nabla \cdot D = \rho$ (D) $\nabla \times H = J + \frac{\partial D}{\partial t}$

MCQ 6.4.14 Assertion (A) : Capacitance of a solid conducting spherical body of radius $a$ is given by $4\pi\varepsilon_0 a$ in free space.

Reason (R) : $\nabla \times H = j\omega\varepsilon E + J$

(A) Both A and R are individually true and R is the correct explanation of A.

(B) Both A and R are individually true but R is not the correct explanation of A.

(C) A is true but R is false

(D) A is false but R is true

MCQ 6.4.15 Two conducting thin coils $X$ and $Y$ (identical except for a thin cut in coil $Y$) are placed in a uniform magnetic field which is decreasing at a constant rate. If the plane of the coils is perpendicular to the field lines, which of the following statement is correct ? As a result, emf is induced in

(A) both the coils (B) coil $Y$ only

(C) coil $X$ only (D) none of the two coils

MCQ 6.4.16 Assertion (A) : Time varying electric field produces magnetic fields.
Reason (R) : Time varying magnetic field produces electric fields.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

MCQ 6.4.17 Match List I (Electromagnetic Law) with List II (Different Form) and select the correct answer using the code given below the lists :

| | List-I | | List-II |
|---|---|---|---|
| a. | Ampere's law | 1. | $\nabla \cdot D = \rho_v$ |
| b. | Faraday's law | 2. | $\nabla \cdot J = -\frac{\partial h}{\partial t}$ |
| c. | Gauss law | 3. | $\nabla \times H = J + \frac{\partial D}{\partial t}$ |
| d. | Current | 4. | $\nabla \times E = -\frac{\partial B}{\partial t}$ |

Codes :

| | a | b | c | d |
|---|---|---|---|---|
| (A) | 1 | 2 | 3 | 4 |
| (B) | 3 | 4 | 1 | 2 |
| (C) | 1 | 4 | 3 | 2 |
| (D) | 3 | 2 | 1 | 4 |

MCQ 6.4.18 Two metal rings 1 and 2 are placed in a uniform magnetic field which is decreasing with time with their planes perpendicular to the field. If the rings are identical except that ring 2 has a thin air gap in it, which one of the following statements is correct ?

(A) No e.m.f is induced in ring 1

(B) An e.m.f is induced in both the rings

(C) Equal Joule heating occurs in both the rings

(D) Joule heating does not occur in either ring.

MCQ 6.4.19 Which one of the following Maxwell's equations gives the basic idea of radiation ?

(A) $\left.\begin{array}{l}\nabla\times H=\partial D/\partial t\\ \nabla\times E=\partial B/\partial t\end{array}\right\}$
(B) $\left.\begin{array}{l}\nabla\times E=-\partial B/\partial t\\ \nabla\cdot D=-\partial B/\partial t\end{array}\right\}$

(C) $\left.\begin{array}{l}\nabla\cdot D=\rho\\ \nabla\cdot D=0\end{array}\right\}$
(D) $\left.\begin{array}{l}\nabla\cdot B=\rho\\ \nabla\times H=(\partial D/\partial t)\end{array}\right\}$

MCQ 6.4.20 Which one of the following is NOT a correct Maxwell equation ?

(A) $\nabla\times H=\frac{\partial D}{\partial t}+J$
(B) $\nabla\times E=\frac{\partial H}{\partial t}$

(C) $\nabla\cdot D=\rho$
(D) $\nabla\cdot B=0$

MCQ 6.4.21 Match List I (Maxwell equation) with List II (Description) and select the correct answer :

List I

a. $\oint B\cdot dS=0$

b. $\oint D\cdot dS=\int_v\rho_v dv$

c. $\oint E\cdot dl=-\int\frac{\partial B}{\partial t}\cdot dS$

d. $\oint H\cdot dl=\int\frac{\partial(D+J)}{\partial t}\cdot dS$

List II

1. The mmf around a closed path is equal to the conduction current plus the time derivative of the electric displacement current through any surface bounded by the path.
2. The emf around a closed path is equal to the time derivative is equal to the time derivative of the magnetic displacement through any surface bounded by the path.
3. The total electric displacement through the surface enclosing a volume is equal to total charge within the volume
4. The net magnetic flux emerging through any closed surface is zero.

Codes :

| | a | b | c | d |
|---|---|---|---|---|
| (A) | 1 | 3 | 2 | 4 |
| (B) | 4 | 3 | 2 | 1 |
| (C) | 4 | 2 | 3 | 1 |
| (D) | 1 | 2 | 3 | 4 |

MCQ 6.4.22 The equation of continuity defines the relation between
(A) electric field and magnetic field
(B) electric field and charge density
(C) flux density and charge density
(D) current density and charge density

MCQ 6.4.23 What is the generalized Maxwell's equation $\nabla \times H = J_c + \frac{\partial D}{\partial t}$ for the free space ?

(A) $\nabla \times H = 0$ (B) $\nabla \times H = J_c$

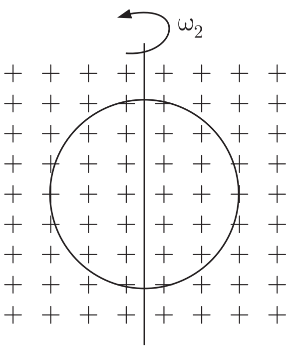
(C) $\nabla \times H = \frac{\partial D}{\partial t}$ (D) $\nabla \times H = D$

MCQ 6.4.24 Magnetic field intensity is $H = 3a_x + 7ya_y + 2xa_z$ A/m. What is the current density $J$ A/m$^2$ ?

(A) $-2a_y$ (B) $-7a_z$

(C) $3a_x$ (D) $12a_y$

MCQ 6.4.25 A circular loop placed perpendicular to a uniform sinusoidal magnetic field of frequency $\omega_1$ is revolved about an axis through its diameter at an angular velocity $\omega_2$ rad/sec ($\omega_2 < \omega_1$) as shown in the figure below. What are the frequencies for the e.m.f induced in the loop ?




(A) $\omega_1$ and $\omega_2$ (B) $\omega_1, \omega_2 + \omega_2$ and $\omega_2$

(C) $\omega_2, \omega_1 - \omega_2$ and $\omega_2$ (D) $\omega_1 - \omega_2$ and $\omega_1 + \omega_2$

MCQ 6.4.26 Which one of the following is not a Maxwell's equation ?

(A) $\nabla \times H = (\sigma + j\omega\varepsilon)E$ (B) $F = Q(E + v \times B)$

(C) $\oint_c H \cdot dl = \oint_s J \cdot dS + \oint_s \frac{\partial D}{\partial t} \cdot dS$ (D) $\oint_S B \cdot dS = 0$

MCQ 6.4.27 Consider the following three equations :

1. $\nabla \times E = -\frac{\partial B}{\partial t}$

2. $\nabla \times H = J + \frac{\partial D}{\partial t}$

3. $\nabla \cdot B = 0$

Which of the above appear in Maxwell's equations ?

(A) 1, 2 and 3 (B) 1 and 2

(C) 2 and 3 (D) 1 and 3

MCQ 6.4.28 In free space, if $\rho_v = 0$, the Poisson's equation becomes

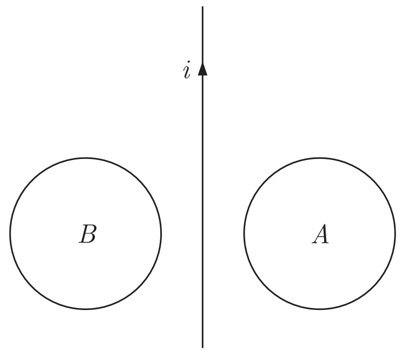
(A) Maxwell's divergence equation $\nabla \cdot B = 0$

(B) Laplacian equation $\nabla^2 V = 0$

(C) Kirchhoff's voltage equation $\Sigma V = 0$

(D) None of the above

MCQ 6.4.29 A straight current carrying conductor and two conducting loops A and B are shown in the figure given below. What are the induced current in the two loops ?




(A) Anticlockwise in A and clockwise in B
(B) Clockwise in A and anticlockwise in B
(C) Clockwise both in A and B
(D) Anticlockwise both in A and B

MCQ 6.4.30 Which one of the following equations is not Maxwell's equation for a static electromagnetic field in a linear homogeneous medium ?

(A) $\nabla \cdot B = 0$ (B) $\nabla \times D = \vec{0}$
(C) $\oint_c B \cdot dl = \mu_0 I$ (D) $\nabla^2 A = \mu_0 J$



MCQ 6.4.31 Match List I with List II and select the correct answer using the codes given below :

| | List I | | List II |
|---|---|---|---|
| a | Continuity equation | 1. | $\nabla \times H = J + \dfrac{\partial D}{\partial t}$ |
| b | Ampere's law | 2. | $J = \dfrac{\partial D}{\partial t}$ |
| c | Displacement current | 3. | $\nabla \times E = -\dfrac{\partial B}{\partial t}$ |
| d | Faraday's law | 4. | $\nabla \times J = -\dfrac{\partial \rho_v}{\partial t}$ |

Codes :

| | a | b | c | d |
|---|---|---|---|---|
| (A) | 4 | 3 | 2 | 1 |
| (B) | 4 | 1 | 2 | 3 |
| (C) | 2 | 3 | 4 | 1 |
| (D) | 2 | 1 | 4 | 3 |

MCQ 6.4.32 The magnetic flux through each turn of a 100 turn coil is $(t^3 - 2t)$ milli-Webers where $t$ is in seconds. The induced e.m.f at $t = 2$ s is

(A) 1 V (B) $-1$ V
(C) 0.4 V (D) $-0.4$ V

MCQ 6.4.33 Match List I (Type of field denoted by $A$) with List II (Behaviour) and select the correct answer using the codes given below :

| | List I | | List II |
|---|---|---|---|
| a | A static electric field in a charge free region | 1. | $\nabla \cdot A = 0$<br>$\nabla \times A \neq 0$ |
| b | A static electric field in a charged region | 2. | $\nabla \cdot A \neq 0$<br>$\nabla \times A = 0$ |
| c | A steady magnetic field in a current carrying conductor | 3. | $\nabla \cdot A \neq 0$<br>$\nabla \times A \neq 0$ |
| d | A time-varying electric field in a charged medium with time-varying magnetic field | 4. | $\nabla \cdot A = 0$<br>$\nabla \times A = 0$ |

Codes :

| | a | b | c | d |
|---|---|---|---|---|
| (A) | 4 | 2 | 3 | 1 |
| (B) | 4 | 2 | 1 | 3 |
| (C) | 2 | 4 | 3 | 1 |
| (D) | 2 | 4 | 1 | 3 |

MCQ 6.4.34 Which one of the following pairs is not correctly matched ?

(A) Gauss Theorem : $\oint_s D \cdot ds = \oint_v \nabla \cdot D dv$

(B) Gauss's Law : $\oint D \cdot ds = \oint_v \rho dv$

(C) Coulomb's Law : $V = -\frac{d\phi_m}{dt}$

(D) Stoke's Theorem : $\oint \xi \cdot dl = \oint_s (\nabla \times \xi) \cdot ds$

MCQ 6.4.35 Maxwell equation $\nabla \times E = -(\partial B / \partial t)$ is represented in integral form as

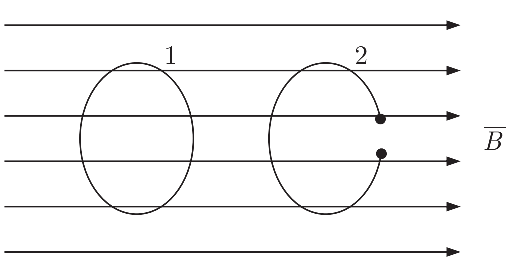
(A) $\oint E \cdot dl = -\frac{\partial}{\partial t} \oint B \cdot dl$

(B) $\oint E \cdot dl = -\frac{\partial}{\partial t} \int_s B \cdot ds$

(C) $\oint E \times dl = -\frac{\partial}{\partial t} \oint B \cdot dl$

(D) $\oint E \times dl = -\frac{\partial}{\partial t} \int_s B \cdot dl$

MCQ 6.4.36 Two conducting coils 1 and 2 (identical except that 2 is split) are placed in a uniform magnetic field which decreases at a constant rate as in the figure. If the planes of the coils are perpendicular to the field lines, the following statements are made :




1. an e.m.f is induced in the split coil 2
2. e.m.fs are induced in both coils

3. equal Joule heating occurs in both coils
4. Joule heating does not occur in any coil

Which of the above statements is/are true ?

(A) 1 and 4 (B) 2 and 4

(C) 3 only (D) 2 only

MCQ 6.4.37 For linear isotropic materials, both $E$ and $H$ have the time dependence $e^{j\omega t}$ and regions of interest are free of charge. The value of $\nabla \times H$ is given by

(A) $\sigma E$ (B) $j\omega\varepsilon E$

(C) $\sigma E + j\omega\varepsilon E$ (D) $\sigma E - j\omega\varepsilon E$

MCQ 6.4.38 Which of the following equations is/are not Maxwell's equations(s) ?

(A) $\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$ (B) $\nabla \cdot D = \rho_v$

(C) $\nabla \cdot E = -\frac{\partial B}{\partial t}$ (D) $\oint H \cdot dl = \int_s \left(\sigma E + \varepsilon \frac{\partial E}{\partial t}\right) \cdot ds$

Select the correct answer using the codes given below :

(A) 2 and 4 (B) 1 alone

(C) 1 and 3 (D) 1 and 4

MCQ 6.4.39 Assertion (A) : The relationship between Magnetic Vector potential $A$ and the current density $J$ in free space is

$$\nabla \times (\nabla \times A) = \mu_0 J$$

For a magnetic field in free space due to a $dc$ or slowly varying current is $\nabla^2 A = -\mu_0 J$

Reason (R) : For magnetic field due to $dc$ or slowly varying current $\nabla \cdot A = 0$.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

MCQ 6.4.40 Given that $\nabla \times H = J + \frac{\partial D}{\partial t}$

Assertion (A) : In the equation, the additional term $\frac{\partial D}{\partial t}$ is necessary.

Reason (R) : The equation will be consistent with the principle of conservation of charge.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

MCQ 6.4.41 A circular loop is rotating about the $y$-axis as a diameter in a magnetic field $B = B_0 \sin \omega t a_x$ Wb/m$^2$. The induced emf in the loop is

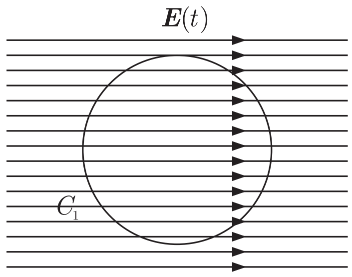
(A) due to transformer emf only

(B) due to motional emf only

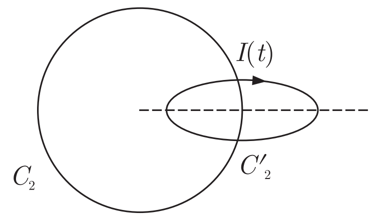
(C) due to a combination of transformer and motional emf

(D) zero

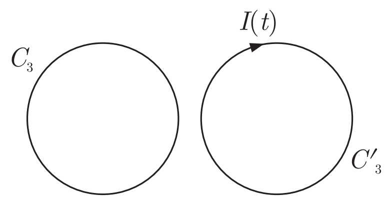
MCQ 6.4.42 Consider coils $C_1, C_2, C_3$ and $C_4$ (shown in the given figures) which are placed in the time-varying electric field $E(t)$ and electric field produced by the coils $C'_2, C'_3$ and $C'_4$ carrying time varying current $I(t)$ respectively :

1.


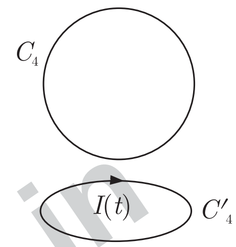
Time varying electric field $\boldsymbol{E}(t)$ parallel to the plane of coil $C_1$

2.


Coil planes are orthogonal

3.


Co-planer coils

4.


Coil planes are orthogonal

The electric field will induce an emf in the coils

(A) $C_1$ and $C_2$ (B) $C_2$ and $C_3$

(C) $C_1$ and $C_3$ (D) $C_2$ and $C_4$

MCQ 6.4.43 Match List I (Law/quantity) with List II (Mathematical expression) and select the correct answer :

| | List I | | List II |
|---|---|---|---|
| a. | Gauss's law | 1. | $\nabla \cdot D = \rho$ |
| b. | Ampere's law | 2. | $\nabla \times E = -\frac{\partial B}{\partial t}$ |
| c. | Faraday's law | 3. | $\mathcal{P} = E \times H$ |
| d. | Poynting vector | 4. | $F = q(E + v \times B)$ |
| | | 5. | $\nabla \times H = J_c + \frac{\partial D}{\partial t}$ |

Codes :

| | a | b | c | d |
|---|---|---|---|---|
| (A) | 1 | 2 | 4 | 3 |
| (B) | 3 | 5 | 2 | 1 |
| (C) | 1 | 5 | 2 | 3 |
| (D) | 3 | 2 | 4 | 1 |

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# SOLUTIONS 6.1

SOL 6.1.1 Option (C) is correct.

From Faraday's law, the relation between electric field and magnetic field is

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

Since the electric field inside a conducting sphere is zero.

i.e. $E = 0$

So the rate of change in magnetic flux density will be

$$\frac{\partial B}{\partial t} = -(\nabla \times E) = 0$$

Therefore $B(r, t)$ will be uniform inside the sphere and independent of time.

SOL 6.1.2 Option (A) is correct.

Electric field intensity experienced by the moving conductor $ab$ in the presence of magnetic field $B$ is given as

$$E = v \times B \qquad \text{where } v \text{ is the velocity of the conductor.}$$

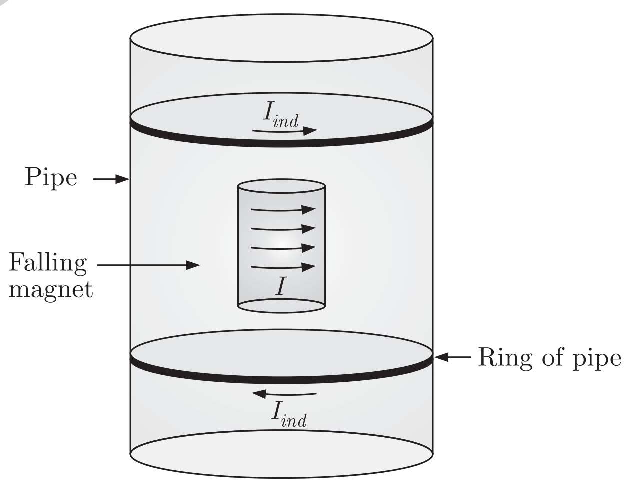
So, electric field will be directed from $b$ to $a$ as determined by right hand rule for the cross vector. Therefore, the voltage difference between the two ends of the conductor is given as

$$V_{ab} = -\int_a^b E \cdot dl$$

Thus, the positive terminal of voltage will be $a$ and $V_{ab}$ will be positive.

SOL 6.1.3 Option (A) is correct.

Consider a magnet bar being dropped inside a pipe as shown in figure.




Suppose the current $I$ in the magnet flows counter clockwise (viewed from above) as shown in figure. So near the ends of pipe, it's field points upward. A ring of pipe below the magnet experiences an increasing upward flux as the magnet approaches and hence by Lenz's law a current will be induced in it such as to produce downward flux.

Thus, $I_{ind}$ must flow clockwise which is opposite to the current in the magnet.

Since opposite currents repel each other so, the force exerted on the magnet due to the induced current is directed upward. Meanwhile a ring above the magnet experiences a decreasing upward flux; so it's induced current parallel to $I$ and it attracts magnet upward. And flux through the rings next to the magnet bar is constant. So no current is induced in them.
Thus, for all we can say that the force exerted by the eddy current (induced current according to Lenz's law) on the magnet is in upward direction which causes the delay to reach the bottom. Whereas in the cases of unmagnetized bar no induced current is formed. So it reaches in fraction of time.
Thus, A and R both true and R is correct explanation of A.

SOL 6.1.4 Option (C) is correct.
The magnetic flux density inside a solenoid of $n$ turns per unit length carrying current $I$ is defined as

$$B = \mu_0 nI$$

Let the length of solenoid be $l$ and its cross sectional radius be $r$. So, the total magnetic flux through the solenoid is

$$\Phi = (\mu_0 nI)(\pi r^2)(nl) \qquad (1)$$

Since the total magnetic flux through a coil having inductance $L$ and carrying current $I$ is given as

$$\Phi = LI$$

So comparing it with equation (1) we get,

$$L = \mu_0 n^2 I\pi^2 l$$

and as for a given solenoid, radius $r$ and length $l$ is constant therefore

$$L \propto n^2$$

SOL 6.1.5 Option (C) is correct.
The magnetic flux density inside the solenoid is defined as

$$B = \mu_0 nI$$

where $n \rightarrow$ no. of turns per unit length
$I \rightarrow$ current flowing in it.

So the total magnetic flux through the solenoid is

$$\Phi = \int B \cdot dS = (\mu_0 nI)(\pi a^2)$$

where $a \rightarrow$ radius of solenoid

Induced emf in a loop placed in a magnetic field is defined as

$$V_{emf} = -\frac{d\Phi}{dt}$$

where $\Phi$ is the total magnetic flux passing through the loop. Since the resistance $R$ is looped over the solenoid so total flux through the loop will be equal to the total flux through the solenoid and therefore the induced emf in the loop of resistance will be

$$V_{emf} = -\pi a^2 \mu_0 n \frac{dI}{dt}$$

Since current $I$ flowing in the solenoid is constant so, the induced emf is

$$V_{emf} = 0$$

and therefore the induced current in the loop will be zero.

SOL 6.1.6 Option (B) is correct.

It will be similar to the current in a solenoid.
So, the magnetic field will be in circumferential while the electric field is longitudinal.

SOL 6.1.7 Option (B) is correct.
In Assertion (A) the magnetic flux through each turn of both coils are equal So, the net magnetic flux through the two coils are respectively

$$\Phi_1 = N_1\Phi$$

and $$\Phi_2 = N_2\Phi$$

where $\Phi$ is the magnetic flux through a single loop of either coil and $N_1$, $N_2$ are the total no. of turns of the two coils respectively.
Therefore the induced emf in the two coils are

$$V_{\text{emf}\,1} = -\frac{d\Phi_1}{dt} = -N_1\frac{d\Phi}{dt}$$

$$V_{\text{emf}\,2} = -\frac{d\Phi_2}{dt} = -N_2\frac{d\Phi}{dt}$$

Thus, the ratio of the induced emf in the two loops are

$$\frac{V_{\text{emf}\,2}}{V_{\text{emf}\,1}} = \frac{N_2}{N_1}$$

Now, in Reason (R) : a primitive transformer is similar to the cylinder core carrying wound coils. It is the device in which by choosing the appropriate no. of turns, any desired secondary emf can be obtained.
So, both the statements are correct but R is not the explanation of A.

SOL 6.1.8 Option (B) is correct.
Electric flux density in the medium is given as

$$D = \varepsilon E = \varepsilon E_0 \cos\omega t \qquad (E = E_0\cos\omega t)$$

Therefore the displacement current density in the medium is

$$J_d = \frac{\partial D}{\partial t} = -\omega\varepsilon E_0 \sin\omega t$$

and the conduction current density in the medium is

$$J_c = \sigma E = \sigma E_0 \cos\omega t$$

So, the ratio of amplitudes of conduction current density and displacement current density is

$$\frac{|J_c|}{|J_d|} = \frac{\sigma}{\omega\varepsilon}$$

SOL 6.1.9 Option (C) is correct.
Given the volume charge density, $\rho_v = 0$
So, from Maxwell's equation we have

$$\nabla\cdot D = \rho_v$$

$$\nabla\cdot D = 0 \qquad (1)$$

Now, the electric flux density in a medium is defined as

$$D = \varepsilon E \qquad \text{(where } \varepsilon \text{ is the permittivity of the medium)}$$

So, putting it in equation (1) we get,

$$\nabla\cdot(\varepsilon E) = 0$$

or, $$E\cdot(\nabla\varepsilon) + \varepsilon(\nabla\cdot E) = 0$$

and since $$\frac{\nabla\varepsilon}{\varepsilon} \approx 0 \Rightarrow \nabla\varepsilon \approx 0 \qquad \text{(given)}$$

Therefore, $$\nabla\cdot E \approx 0$$

SOL 6.1.10 Option (A) is correct.
Given the electric field intensity in time domain as

$$E = \frac{\sin\theta\cos(\omega t - kr)}{r}a_\theta$$

So, the electric field intensity in phasor form is given as

$$E_s = \frac{\sin\theta}{r}e^{-jkr}a_\theta$$

and
$$\nabla \times E_s = \frac{1}{r}\frac{\partial}{kdr}(rE_{\theta s})a_\phi = (-jk)\frac{\sin\theta}{r}e^{-jkr}a_\phi$$

Therefore, from Maxwell's equation we get the magnetic field intensity as

$$H_s = -\frac{\nabla \times E_s}{j\omega r_0} = \frac{k}{\omega r_0}\frac{\sin\theta}{r}e^{-jkr}a_\phi$$

SOL 6.1.11 Option (B) is correct.
Magnetic flux density produced at a distance $\rho$ from a long straight wire carrying current $I$ is defined as

$$B = \frac{\mu_0 I}{2\pi\rho}a_\phi$$

where $a_\phi$ is the direction of flux density as determined by right hand rule. So, the magnetic flux density produced by the straight conducting wire linking through the loop is normal to the surface of the loop.
Now consider a strip of width $d\rho$ of the square loop at distance $\rho$ from the wire for which the total magnetic flux linking through the square loop is given as

$$\Phi = \int_S B \cdot dS$$

$$= \frac{\mu_0 I}{2\pi}\int_\rho^{\rho+a}\frac{1}{\rho}(ad\rho) \qquad \text{(area of the square loop is } dS = ad\rho)$$

$$= \frac{\mu_0 Ia}{2\pi}\ln\left(\frac{\rho+a}{\rho}\right)$$

The induced emf due to the change in flux (when pulled away) is given as

$$V_{\text{emf}} = -\frac{d\Phi}{dt} = -\frac{\mu_0 Ia}{2\pi}\frac{d}{dt}\left[\ln\left(\frac{\rho+a}{\rho}\right)\right]$$

Therefore,
$$V_{\text{emf}} = -\frac{\mu_0 Ia}{2\pi}\left(\frac{1}{\rho+a}\frac{d\rho}{dt} - \frac{1}{\rho}\frac{d\rho}{dt}\right)$$

Given
$$\frac{d\rho}{dt} = \text{velocity of loop} = 5\,\text{m/s}$$

and since the loop is currently located at $3\,\text{m}$ distance from the straight wire, so after $0.6\,\text{sec}$ it will be at

$$\rho = 3 + (0.6) \times v \qquad (v \to \text{velocity of the loop})$$

$$= 3 + 0.6 \times 5 = 6\,\text{m}$$

So,
$$V_{\text{emf}} = -\frac{\mu_0 \times (30) \times 2}{2\pi}\left[\frac{1}{8}(5) - \frac{1}{6}(5)\right] \qquad (a = 2\,\text{m}, I = 30\,\text{A})$$

$$= 25 \times 10^{-7}\,\text{volt} = 2.5\,\mu\text{volt}$$

SOL 6.1.12 Option (B) is correct.
Since total magnetic flux through the loop depends on the distance from the straight wire and the distance is constant. So the flux linking through the loop will be constant, if it is pulled parallel to the straight wire. Therefore the induced emf in the loop is

$$V_{\text{emf}} = -\frac{d\Phi}{dt} = 0 \qquad (\Phi \text{ is constant})$$

SOL 6.1.13 Option (D) is correct.

Total magnetic flux through the solenoid is given as

$$\Phi = \mu_0 nI$$

where $n$ is the no. of turns per unit length of solenoid and $I$ is the current flowing in the solenoid.

Since the solenoid carries current that is increasing linearly with time

i.e. $$I \propto t$$

So the net magnetic flux through the solenoid will be

$$\Phi \propto t$$

or, $$\Phi = kt$$ where $k$ is a constant.

Therefore the emf induced in the loop consisting resistances $R_A$, $R_B$ is

$$V_{emf} = -\frac{d\Phi}{dt}$$

$$V_{emf} = -k$$

and the current through $R_1$ and $R_2$ will be

$$I_{ind} = -\frac{k}{R_1 + R_2}$$

Now according to Lenz's law the induced current $I$ in a loop flows such as to produce a magnetic field that opposes the change in $B(t)$.

i.e. the induced current in the loop will be opposite to the direction of current in solenoid (in anticlockwise direction).

So, $$V_A = I_{ind} R_A = -\frac{kR_A}{R_A + R_B}$$

and $$V_B = -I_{ind} R_B = \left(\frac{kR_B}{R_A + R_B}\right)$$

Thus, the ratio of voltmeter readings is

$$\frac{V_A}{V_B} = -\frac{R_A}{R_B}$$

SOL 6.1.14 Option (D) is correct.

Induced emf in the conducting loop formed by rail, bar and the resistor is given by

$$V_{emf} = -\frac{d\Phi}{dt}$$

where $\Phi$ is total magnetic flux passing through the loop.

The bar is located at a distance $x$ from the resistor at time $t$. So the total magnetic flux passing through the loop at time $t$ is

$$\Phi = \int B \cdot dS = \mathrm{B}lx$$ where $l$ is separation between the rails

Now the induced emf in a loop placed in magnetic field is defined as

$$V_{emf} = -\frac{d\Phi}{dt}$$

where $\Phi$ is the total magnetic flux passing through the loop. Therefore the induced emf in the square loop is

$$V_{emf} = -\frac{d}{dt}(\mathrm{B}lx) = -\mathrm{B}l\frac{dx}{dt} \qquad (\Phi = \mathrm{B}lx)$$

Since from the given figure, we have

$$l = 5\text{ m}$$

$$B = 2\text{ T}$$

and $dx/dt \rightarrow$ velocity of bar $= 4\text{ m/s}$

So, induced emf is

$$V_{emf} = -(2)(5)(4) = -40\text{ volt}$$

Therefore the current in the bar loop will be

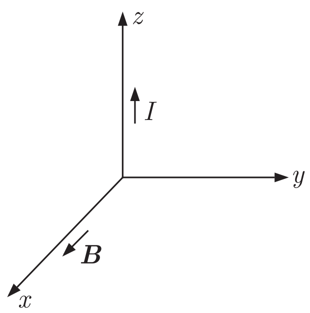
$$I = \frac{V_{emf}}{R} = -\frac{40}{10} = -4\text{ A}$$

SOL 6.1.15 Option (B) is correct.

As obtained in the previous question the current flowing in the sliding bar is

$$I = -4\text{ A}$$

Now we consider magnetic field acts in $\boldsymbol{a}_x$ direction and current in the sliding bar is flowing in $+\boldsymbol{a}_z$ direction as shown in the figure.




Therefore, the force exerted on the bar is

$$\boldsymbol{F} = \int I d\boldsymbol{l} \times \boldsymbol{B} = \int_0^5 (-4dz\boldsymbol{a}_z) \times (2\boldsymbol{a}_x)$$

$$= -8\boldsymbol{a}_y [z]_0^5 = -40\boldsymbol{a}_y\text{ N}$$

i.e. The force exerted on the sliding bar is in opposite direction to the motion of the sliding bar.

SOL 6.1.16 Option (C) is correct.

Given the magnetic flux density through the square loop is

$$\boldsymbol{B} = 7.5\cos(120\pi t - 30^\circ)\,\boldsymbol{a}_z$$

So the total magnetic flux passing through the loop will be

$$\Phi = \oint_S \boldsymbol{B} \cdot d\boldsymbol{S}$$

$$= [-7.5\cos(120\pi t - 30^\circ)\,\boldsymbol{a}_z](1 \times 1)(-\boldsymbol{a}_z)$$

$$= 7.5\cos(120\pi t - 30^\circ)$$

Now, the induced emf in the square loop is given by

$$V_{emf} = -\frac{d\Phi}{dt} = 7.5 \times 120\pi\sin(120\pi t - 30^\circ)$$

The polarity of induced emf (according to Lenz's law) will be such that induced current in the loop will be in opposite direction to the current $I(t)$ shown in the figure. So we have

$$I(t) = -\frac{V_{emf}}{R}$$

$$= -\frac{7.5 \times 120\pi}{500}\sin(120\pi t - 30^\circ) \qquad (R = 250 + 250 = 500\,\Omega)$$

$$= -5.7\sin(120\pi t - 30^\circ)$$

SOL 6.1.17 Option (A) is correct.

Consider the mutual inductance between the rectangular loop and straight wire be $M$. So applying KVL in the rectangular loop we get,

$$M\frac{di_1}{dt} = L\frac{di_2}{dt} + Ri_2 \qquad ...(1)$$

Now from the shown figure (b), the current flowing in the straight wire is given as

$$i_1 = I_1 u(t) - I_1 u(t-T) \quad (I_1 \text{ is amplitude of the current})$$

or, $$\frac{di_1}{dt} = I_1\delta(t) - I_1\delta(t-T) \qquad (2)$$

So, at $t = 0$ $$\frac{di_1}{dt} = I_1$$

and $$MI_1 = L\frac{di_2}{dt} + Ri_2 \qquad \text{(from equation (1))}$$

Solving it we get

$$i_2 = \frac{M}{L}I_1 e^{-(R/L)t} \qquad \text{for } 0 < t < T$$

Again in equation (2) at $t = T$ we have

$$\frac{di_1}{dt} = -I_1$$

and $$-MI_1 = L\frac{di_2}{dt} + Ri_2 \qquad \text{(from equation (1))}$$

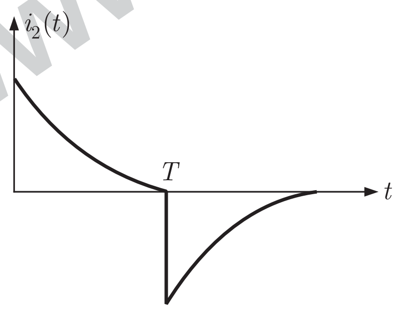
Solving it we get

$$i_2 = -\frac{M}{L}I_1 e^{-(R/L)(t-T)} \qquad \text{for } t > T$$

Thus, the current in the rectangular loop is

$$i_2 = \begin{cases} \frac{M}{L}I_1 e^{-(R/L)t} & 0 < t < T \\ -\frac{M}{L}I_1 e^{-(R/L)(t-T)} & t > T \end{cases}$$

Plotting $i_2$ versus $t$ we get




SOL 6.1.18 Option (A) is correct.

Total magnetic flux passing through the loop formed by the resistance, bar and the rails is given as:

$$\Phi = \int_S B \cdot dS$$
$$= B \cdot S = [0.2\cos\omega t a_x] \cdot [0.5(1-y)a_x]$$
$$= 0.1[1 - 0.5(1-\cos\omega t)]\cos\omega t \quad (y = 0.5(1-\cos\omega t)\,\text{m})$$
$$= 0.05\cos\omega t(1+\cos\omega t) = 0.05(\cos\omega t + \cos^2\omega t)$$

So, the induced emf in the loop is

$$V_{\text{emf}} = -\frac{d\Phi}{dt}$$

and as determined by Lenz's law, the induced current will be flowing in opposite direction to the current $i$. So the current $i$ in the loop will be

$$i = -\frac{V_{emf}}{R} = -\frac{1}{R}\left(-\frac{d\Phi}{dt}\right)$$
$$= \frac{0.05}{5}[-\omega\sin\omega t - 2\omega\cos\omega t\sin\omega t]$$
$$= -0.01\omega\sin\omega t(1 + 2\cos\omega t)$$

SOL 6.1.19 Option (D) is correct.

Given the electric flux density in the medium is

$$D = 1.33\sin(3\times10^8 t - 0.2x)a_y\ \mu\text{C/m}^2$$

So, the electric field intensity in the medium is given as

$$E = \frac{D}{\varepsilon} \qquad \text{where } \varepsilon \text{ is the permittivity of the medium}$$

or,
$$E = \frac{D}{\varepsilon_r\varepsilon_0} = \frac{1.33\times10^{-6}\sin(3\times10^8 t - 0.2x)}{10\times8.85\times10^{-12}}a_y \quad (\varepsilon_r = 10)$$
$$= 1.5\times10^4\sin(3\times10^8 t - 0.2x)a_y$$

Now, from maxwell's equation we have

$$\nabla\times E = -\frac{\partial B}{\partial t}$$

or,
$$\frac{\partial B}{\partial t} = -\nabla\times E$$
$$= -\frac{\partial E_y}{\partial x}a_z$$
$$= -(-0.2)\times(1.5\times10^4)\cos(3\times10^8 t - 0.2x)a_y$$
$$= 3\times10^3\cos(3\times10^8 t - 0.2x)a_y$$

Integrating both sides, we get the magnetic flux density in the medium as

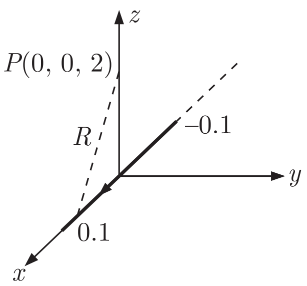
$$B = \int 3\times10^3\cos(3\times10^8 t - 0.2x)a_y$$
$$= \frac{3\times10^3}{3\times10^8}\sin(3\times10^8 t - 0.2x)a_y$$
$$= 10^{-5}\sin(3\times10^8 t - 0.2x)a_y \text{ Tesla}$$

Therefore the magnetic field intensity in the medium is

$$H = \frac{B}{\mu} = \frac{B}{\mu_r\mu_0} = \frac{10^{-5}\sin(3\times10^8 t - 0.2x)}{2\times4\pi\times10^{-7}} \qquad \mu_r = 2$$

Thus
$$H = 4\sin(3\times10^8 t - 0.2x)a_y \text{ A/m}$$

SOL 6.1.20 Option (B) is correct.




The magnetic vector potential for a direct current flowing in a filament is given as

$$A = \int\frac{\mu_0 I}{4\pi R}a_x dx$$

Here current $I(t)$ flowing in the filament shown in figure is varying with time as

$$I(t) = 8t \text{ A}$$

So, the retarded vector potential at the point $P$ will be given as

$$A = \int \frac{\mu_0 I(t - R/c)}{4\pi R} a_x dx$$

where $R$ is the distance of any point on the filamentary current from $P$ as shown in the figure and $c$ is the velocity of waves in free space. So, we have

$$R = \sqrt{x^2 + 4} \text{ and } c = 3 \times 10^8 \text{ m/s}$$

Therefore,
$$A = \int_{x=-0.1}^{0.1} \frac{\mu_0 8(t - R/c)}{4\pi R} a_x dx$$

$$= \frac{8\mu_0}{4\pi}\left[\int_{-0.1}^{0.1} \frac{t}{\sqrt{x^2+4}} dx - \int_{-0.1}^{0.1} \frac{1}{c} dx\right]$$

$$= 8 \times 10^{-7} t\left[\ln(x + \sqrt{x^2+4})\right]_{-0.1}^{0.1} - \frac{8 \times 10^{-7}}{3 \times 10^8}[x]_{-0.1}^{0.1}$$

$$= 8 \times 10^{-7} t \ln\left(\frac{0.1 + \sqrt{4.01}}{-0.1 + \sqrt{4.01}}\right) - 0.53 \times 10^{-15}$$

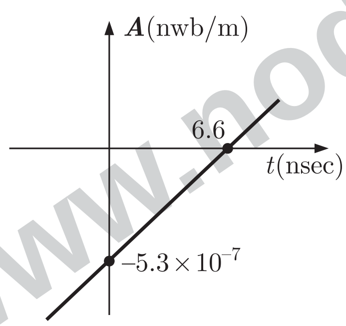
$$= 8 \times 10^{-8} t - 0.53 \times 10^{-15}$$

or,
$$A = (80t - 5.3 \times 10^{-7}) a_x \text{ nWb/m} \qquad (1)$$

So, when $A = 0$ $\qquad t = 6.6 \times 10^{-9} = 6.6 \text{ n sec}$

and when $t = 0$ $\qquad A = -5.3 \times 10^{-7} \text{ nWb/m}$

From equation (1) it is clear that $A$ will be linearly increasing with respect to time. Therefore the plot of $A$ versus $t$ is




NOTE :
Time varying potential is usually called the retarded potential.

SOL 6.1.21 Option (A) is correct.

The force experienced by a test charge $q$ in presence of both electric field $E$ and magnetic field $B$ in the region will be evaluated by using Lorentz force equation as

$$F = q(E + v \times B)$$

So, putting the given three forces and their corresponding velocities in above equation we get the following relations

$$q(a_y + a_z) = q(E + a_x \times B) \qquad (1)$$

$$qa_y = q(E + a_y \times B) \qquad (2)$$

$$q(2a_y + a_z) = q(E + a_z \times B) \qquad (3)$$

Subtracting equation (2) from (1) we get

$$a_z = (a_x - a_y) \times B \qquad (4)$$

and subtracting equation (1) from (3) we get

$$a_y = (a_z - a_x) \times B \qquad (5)$$

Now we substitute $B = B_x a_x + B_y a_y + B_z a_z$ in eq (4) to get

$$a_z = B_y a_z - B_z a_y + B_x a_z - B_z a_x$$

So, comparing the $x, y$ and $z$ components of the two sides we get

$$B_x + B_y = 1$$

and $$B_z = 0$$

Again by substituting $B = B_x a_x + B_y a_y + B_z a_z$ in eq (5), we get

$$a_y = B_x a_y - B_y a_x - B_y a_z + B_z a_y$$

So, comparing the $x, y$ and $z$ components of the two sides we get

$$B_x + B_z = 1$$

and $$B_y = 0$$

as calculated above $B_z = 0$, therefore $B_x = 1$

Thus, the magnetic flux density in the region is

$$B = a_x \text{ Wb/m}^2 \qquad (B_x = 1,\ B_y = B_z = 0)$$

SOL 6.1.22 Option (C) is correct.

As calculated in previous question the magnetic flux density in the region is

$$B = a_x \text{ Wb/m}^2$$

So, putting it in Lorentz force equation we get

$$F = q(E + V \times B)$$

or, $$q(a_y + a_z) = q(E + a_x \times a_x)$$

Therefore, the electric field intensity in the medium is

$$E = a_y + a_z \text{ V/m}$$

SOL 6.1.23 Option (C) is correct.

Given

Retarded scalar potential, $V = y(x - ct)$ volt

and retarded vector potential, $A = y\left(\frac{x}{c} - t\right)a_x$ Wb/m

Now the magnetic flux density in the medium is given as

$$B = \nabla \times A$$

$$= -\frac{\partial A_y}{\partial y} a_z = \left(t - \frac{x}{c}\right) a_z \text{ Tesla} \qquad (1)$$

So, the magnetic field intensity in the medium is

$$H = \frac{B}{\mu_0} \qquad (\mu_0 \text{ is the permittivity of the medium})$$

$$= \frac{1}{\mu_0}\left(t - \frac{x}{c}\right) a_z \text{ A/m} \qquad (2)$$

and the electric field intensity in the medium is given as

$$E = -\nabla V - \frac{\partial A}{\partial t}$$

$$= -(x - ct) a_y - y a_x + y a_x = (ct - x) a_y \qquad (3)$$

So, the electric flux density in the medium is

$$D = \varepsilon_0 E \qquad (\varepsilon_0 \text{ is the permittivity of the medium})$$

$$= \varepsilon_0 (ct - x) a_y \text{ C/m}^2 \qquad (4)$$

Now we determine the condition for the field to satisfy all the four Maxwell's equation.

(a) $$\nabla \cdot D = \rho_v$$

or, $$\rho_v = \nabla \cdot [\varepsilon_0 (ct - x) a_y] \qquad (\text{from equation (4)})$$

$$= 0$$

It means the field satisfies Maxwell's equation if $\rho_v = 0$.

(b) $\nabla \cdot B = 0$

Now, $\nabla \cdot B = \nabla \cdot \left[\left(t - \frac{x}{c}\right)a_z\right] = 0$ (from equation (1))

So, it already, satisfies Maxwell's equation

(c) $\nabla \times H = J + \frac{\partial D}{\partial t}$

Now, $\nabla \times H = -\frac{\partial H_z}{\partial x}a_y = \frac{1}{\mu_0 c}a_y = \sqrt{\frac{\varepsilon_0}{\mu_0}}a_y$ (from equation (2))

and from equation (4) we have

$$\frac{\partial D}{\partial t} = \varepsilon_0 c a_y = \sqrt{\frac{\varepsilon_0}{\mu_0}}a_y \qquad \left(\text{Since in free space } c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\right)$$

Putting the two results in Maxwell's equation, we get the condition

$$J = 0$$

(d) $\nabla \times E = -\frac{\partial B}{\partial t}$

Now $\nabla \times E = \frac{\partial E_y}{\partial x}a_z = -a_z$

$$\frac{\partial B}{\partial t} = a_z$$

So, it already satisfies Maxwell's equation. Thus, by combining all the results we get the two required conditions as $J = 0$ and $\rho_v = 0$ for the field to satisfy Maxwell's equation.

SOL 6.1.24 Option (A) is correct.

Given the magnetic flux density through the loop is

$$B = -2/x\, a_z$$

So the total magnetic flux passing through the loop is given as

$$\Phi = \int B \cdot dS = \int_x^{x+2}\int_y^{y+2}\left(-\frac{2}{x}a_z\right)\cdot(-dxdya_z)$$

$$= \left(2\ln\frac{x+2}{x}\right)(2) = 4\ln\left(\frac{x+2}{x}\right)$$

Therefore, the circulation of induced electric field in the loop is

$$\oint_C E \cdot dl = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left[4\ln\left(\frac{x+2}{x}\right)\right]$$

$$= -\frac{4}{\left(\frac{x+2}{x}\right)}\frac{d}{dt}\left(\frac{x+2}{x}\right)$$

$$= -\frac{4x}{x+2}\left(-\frac{2}{x^2}\frac{dx}{dt}\right)$$

$$= \frac{8}{x(x+2)}(2) = \frac{16}{x(x+2)} \qquad \left(\frac{dx}{dt} = v = 2a_x\right)$$

SOL 6.1.25 Option (A) is correct.

As the magnetic flux density for $\rho < 4$ is $B = 0$ so, the total flux passing through the closed loop defined by $\rho = 4$ m is

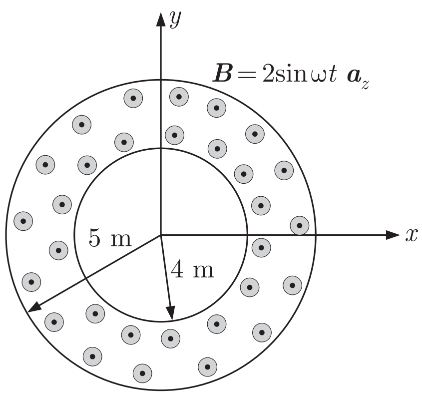
$$\Phi = \int B \cdot dS = 0$$

So, the induced electric field circulation for the region $\rho < 4$ m is given as

$$\oint_C E \cdot dl = -\frac{d\Phi}{dt} = 0$$

or, $\quad E = 0 \quad$ for $\rho < 4$ m

SOL 6.1.26 Option (B) is correct.
As the magnetic field for the region $\rho < 4$ m and $\rho > 5$ m is zero so we get the distribution of magnetic flux density as shown in figure below.




At any distance $\rho$ from origin in the region $4 < \rho < 5$ m, the circulation of induced electric field is given as

$$\oint_C E \cdot dl = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(\int B \cdot dS\right)$$

$$= -\frac{d}{dt}\left[2\sin\omega t(\pi\rho^2 - \pi 4^2)\right]$$

$$= -2\omega\cos\omega t(\pi\rho^2 - 16\pi)$$

or,
$$E(2\pi\rho) = -2\omega\cos\omega t(\pi\rho^2 - 16\pi)$$

$$E = -\frac{2(\rho^2 - 16)\omega\cos\omega t}{2\rho}$$

So, the induced electric field intensity at $\rho = 4.5$ m is

$$E = -\frac{2}{4.5}((4.5)^2 - 16)\omega\cos\omega t$$

$$= -\frac{17}{18}\omega\cos\omega t$$

SOL 6.1.27 Option (B) is correct.
For the region $\rho > 5$ m the magnetic flux density is 0 and so the total magnetic flux passing through the closed loop defined by $\rho = 5$ m is

$$\Phi = \int_0^5 B \cdot dS = \int_0^4 B \cdot dS + \int_4^5 B \cdot dS$$

$$= 0 + \int_4^5 (2\sin\omega t)a_z \cdot dS$$

$$= (2\sin\omega t)\left[\pi(5)^2 - \pi(4)^2\right] = 18\pi\sin\omega t$$

So, the circulation of magnetic flux density for any loop in the region $\rho > 5$ m is

$$\oint E \cdot dl = -\frac{d\psi}{dt}$$

$$E(2\pi\rho) = -\frac{d}{dt}(18\pi\sin\omega t)$$

$$= -18\pi\omega\cos\omega t$$

So, the induced electric field intensity in the region $\rho > 5$ m is

$$E = \frac{-18\pi\omega\cos\omega t}{2\pi\rho}a_\phi$$

$$= -\frac{9}{\rho}\omega\cos\omega t\boldsymbol{a}_{\phi}$$

SOL 6.1.28 Option (D) is correct.

The distribution of magnetic flux density and the resistance in the circuit are same as given in section A (Q. 31) so, as calculated in the question, the two voltage drops in the loop due to magnetic flux density $\boldsymbol{B} = 0.1t\,\boldsymbol{a}_z$ are

$$V_1 = 33.3\text{ mV}$$

and $$V_2 = 66.67\text{ mV} = 66.7\text{ mV}$$

Now $V_3$ (voltmeter) which is directly connected to terminal $cd$ is in parallel to both $V_2$ and $V_1$. It must be kept in mind that the loop formed by voltmeter $V_3$ and resistance $2\,\Omega$ also carries the magnetic flux density crossing through it. So, in this loop the induced emf will be produced which will be same as the field produced in loop $abcd$ at the enclosed fluxes will be same.

Therefore as calculated above induced emf in the loop of $V_3$ is

$$V_{\text{emf}} = 100\text{ mV}$$

According to lenz's law it's polarity will be opposite to $V_3$ and so

$$-V_{\text{emf}} = V_1 + V_3$$

or, $$V_3 = 100 - 33.3 = 66.7\text{ mV}$$

SOL 6.1.29 Option (D) is correct.

The induced emf in a closed loop is defined as

$$V_{\text{emf}} = -\frac{d\Phi}{dt}$$

where $\Phi$ is the total magnetic flux passing through the square loop

At any time $t$, angle between $\boldsymbol{B}$ and $d\boldsymbol{S}$ is $\theta$ since $\boldsymbol{B}$ is in $\boldsymbol{a}_y$ direction so the total magnetic flux passing through the square loop is

$$\begin{aligned}\Phi &= \int \boldsymbol{B}\cdot d\boldsymbol{S}\\ &= (B)(S)\cos\theta\\ &= (5\times10^{-3})(20\times10^{-3}\times20\times10^{-3})\cos\theta\\ &= 2\times10^{-6}\cos\theta\end{aligned}$$

Therefore the induced emf in the loop is

$$\begin{aligned}V_{\text{emf}} &= -\frac{d\Phi}{dt}\\ &= -2\times10^{-6}\frac{d}{dt}(\cos\theta)\\ &= 2\times10^{-6}\sin\theta\frac{d\theta}{dt}\end{aligned}$$

and as $$\frac{d\theta}{dt} = \text{angular velocity} = 2\text{ rad/sec}$$

So, $$\begin{aligned}V_{\text{emf}} &= (2\times10^{-6})\sin\theta(2)\\ &= 4\times10^{-6}\sin\theta\text{ V/m} = 4\sin\theta\,\mu\text{V/m}\end{aligned}$$

SOL 6.1.30 Option (B) is correct.

As calculated in previous question the induced emf in the closed square loop is

$$V_{\text{emf}} = 4\sin\theta\,\mu\text{V/m}$$

So the induced current in the loop is

$$I = \frac{V_{\text{emf}}}{R}$$ where $R$ is the resistance in the loop.

$$= \frac{4\sin\theta \times 10^{-6}}{40 \times 10^{-3}} \qquad (R = 40\ \text{m}\Omega)$$
$$= 0.1\sin\theta\ \text{mA}$$



SOL 6.1.31 Option (C) is correct.

The total magnetic flux through the square loop is given as

$$\Phi = \oint B \cdot dS = (B_0 \sin\omega t)(S)\cos\theta$$

So, the induced emf in the loop is

$$V_{\text{emf}} = -\frac{d\Phi}{dt} = -\frac{d}{dt}[(B_0\sin\omega t)(S)\cos\theta]$$
$$= -B_0 S \frac{d}{dt}[\sin\omega t\cos\omega t] \qquad (\theta = \omega t)$$
$$= -B_0 S\cos 2\omega t$$

Thus, the maximum value of induced emf is

$$|V_{emf}| = B_0 S\omega$$

SOL 6.1.32 Option (C) is correct.

e.m.f. induced in the loop due to the magnetic flux density is given as

$$V_{emf} = -\frac{\partial\Phi}{\partial t} = -\frac{\partial}{\partial t}(10\cos 120\pi t)(\pi\rho^2)$$
$$= -\pi(10 \times 10^{-2})^2 \times (120\pi)(-10\sin 120\pi t)$$
$$= 12\pi^2 \sin 120\pi t$$

As determined by Lenz's law the polarity of induced e.m.f will be such that $b$ is at positive terminal with respect to $a$.

i.e. $$V_{ba} = V_{emf} = 12\pi^2\sin 120\pi t$$

or $$V_{ab} = -12\pi^2\sin 120\pi t$$
$$= -118.43\sin 120\pi t\ \text{Volt}$$

SOL 6.1.33 Option (D) is correct.

As calculated in previous question, the voltage induced in the loop is

$$V_{ab} = -12\pi^2\sin 120\pi t$$

Therefore, the current flowing in the loop is given as

$$I(t) = -\frac{V_{ab}}{250} = \frac{12\pi^2\sin 120\pi t}{250}$$
$$= 0.47\sin 120\pi t$$

***********

# SOLUTIONS 6.2

SOL 6.2.1 Correct answer is 0.

As the conducting loop is falling freely So, the flux through loop will remain constant. Therefore, the voltage induced in the loop will be zero.

SOL 6.2.2 Correct answer is $-4$.

The magnetic flux density passing through the loop is given as

$$B = 4z^3t^2\boldsymbol{a}_x$$

Since the flux density is directed normal to the plane $x = 0$ so the total magnetic flux passing through the square loop located in the plane $x = 0$ is

$$\Phi = \int B \cdot dS = \int_{y=0}^{1}\int_{z=0}^{1}(4z^3t^2)\,dydz = t^2 \qquad (dS = (dydz)\,\boldsymbol{a}_x)$$

Induced emf in a loop placed in magnetic field is defined as

$$V_{\text{emf}} = -\frac{d\Phi}{dt}$$

where $\Phi$ is the total magnetic flux passing through the loop. So the induced emf in the square loop is

$$V_{\text{emf}} = -\frac{d(t^2)}{dt} = -2t \qquad (\Phi = t^2)$$

Therefore at time $t = 2\,\text{sec}$ the induced emf is

$$V_{\text{emf}} = -4\text{ volt}$$

SOL 6.2.3 Correct answer is 4.05 .

Magnetic flux density produced at a distance $\rho$ from a long straight wire carrying current $I$ is defined as

$$B = \frac{\mu_0 I}{2\pi\rho}\boldsymbol{a}_\phi$$

where $\boldsymbol{a}_\phi$ is the direction of flux density as determined by right hand rule. So the flux density produced by straight wire at a distance $\rho$ from it is

$$B = \frac{\mu_0 I}{2\pi\rho}\boldsymbol{a}_n \qquad (\boldsymbol{a}_n \text{ is unit vector normal to the loop})$$

Therefore the total magnet flux passing through the loop is

$$\Phi = \int B \cdot dS = \int_d^{d+a}\frac{\mu_0 I}{2\pi\rho}a\,d\rho \qquad (dS = a\,d\rho\,\boldsymbol{a}_n)$$

where $d\rho$ is width of the strip of loop at a distance $\rho$ from the straight wire. Thus,

$$\Phi = \int_2^3\left(\frac{\mu_0 I}{2\pi}\right)\frac{d\rho}{\rho} = \frac{\mu_0 I}{2\pi}\ln\left(\frac{3}{2}\right) = \frac{\mu_0(5)}{2\pi}\ln(1.5)$$

$$= (2\times10^{-7})(5)\ln(1.5) = 4.05\times10^{-7}\text{ Wb}$$

SOL 6.2.4 Correct answer is 133.3 .

The displacement current density in a medium is equal to the rate of change in electric flux density in the medium.

$$J_d = \frac{\partial D}{\partial t}$$

Since the displacement current density in the medium is given as

$$J_d = 20\cos(1.5\times10^8 t)a_y \text{ A/m}^2$$

So, the electric flux density in the medium is

$$D = \int J_d\,dt + C \qquad (C \to \text{constant})$$
$$= \int 20\cos(1.5\times10^8 t)a_y\,dt + C$$

As there is no D.C. field present in the medium so, we get $C = 0$ and thus,

$$D = \frac{20\sin(1.5\times10^8 t)}{1.5\times10^8}a_y = 1.33\times10^{-7}\sin(1.5\times10^8 t)a_y$$
$$= 133.3\sin(1.5\times10^8 t)a_y \text{ nC/m}^2$$

Since, from the given problem we have the flux density

$$D = D_0\sin(1.5\times10^8 t)a_y \text{ nC/m}^2$$

So, we get

$$D_0 = 133.3$$

SOL 6.2.5 Correct answer is 9.75 .

The ratio of magnitudes of displacement current to conduction current in any medium having permittivity $\varepsilon$ and conductivity $\sigma$ is given as

$$\left|\frac{\text{Displacement current}}{\text{Conduction current}}\right| = \frac{\omega\varepsilon}{\sigma}$$

where $\omega$ is the angular frequency of the current in the medium.

Given frequency, $f = 50\text{ GHz}$

Permittivity, $\varepsilon = 4\varepsilon_0 = 4\times8.85\times10^{-12}$

Conductivity, $\sigma = 1.14\times10^8\text{ s/m}$

So, $\omega = 2\pi f = 2\pi\times50\times10^9 = 100\pi\times10^9$

Therefore, the ratio of magnitudes of displacement current to the conduction current is

$$\left|\frac{I_d}{I_c}\right| = \frac{100\pi\times10^9\times4\times8.85\times10^{-12}}{1.14\times10^8} = 9.75\times10^{-8}$$

SOL 6.2.6 Correct answer is 33.3 .

Given magnetic flux density through the square loop is

$$B = 0.1ta_z \text{ Wb/m}^2$$

So, total magnetic flux passing through the loop is

$$\Phi = B\cdot dS = (0.1t)(1) = 0.1t$$

The induced emf (voltage) in the loop is given as

$$V_{emf} = -\frac{d\phi}{dt} = -0.1\text{ Volt}$$

As determined by Lenz's law the polarity of induced emf will be such that

$$V_1 + V_2 = -V_{emf}$$

Therefore, the voltage drop in the $2\,\Omega$ resistance is

$$V_1 = \left(\frac{2}{2+4}\right)(-V_{emf}) = \frac{0.1}{3} = 33.3\text{ mV}$$

SOL 6.2.7 Correct answer is 7.2 .

Voltage, $V_1 = -N_1\dfrac{d\Phi}{dt}$

where $\Phi$ is total magnetic flux passing through it.

Again $$V_2 = -N_2\frac{d\Phi}{dt}$$

Since both the coil are in same magnetic field so, change in flux will be same for both the coil.

Comparing the equations (1) and (2) we get

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$V_2 = V_1\frac{N_2}{N_1} = (12)\frac{3000}{5000} = 7.2 \text{ volt}$$

SOL 6.2.8 Correct answer is 41.6 .

In phasor form the magnetic field intensity can be written as

$$H_s = 0.1\cos(15\pi y)e^{-jbx}a_z \text{ A/m}$$

Similar as determined in MCQ 42 using Maxwell's equation we get the relation

$$(15\pi)^2 + b^2 = \omega^2\pi_0\varepsilon_0$$

Here $$\omega = 6\pi \times 10^9$$

So, $$(15\pi)^2 + b^2 = \left(\frac{6\pi \times 10^9}{3 \times 10^8}\right)^2$$

$$(15\pi)^2 + b^2 = 400\pi^2$$

$$b^2 = 175\pi^2 \quad \Rightarrow \quad b = \pm 41.6 \text{ rad/m}$$

So, $$|b| = 41.6 \text{ rad/m}$$

SOL 6.2.9 Correct answer is 0.01 .

Induced emf. in the conducting loop formed by rail, bar and the resistor is given by

$$V_{emf} = -\frac{d\Phi}{dt}$$

where $\Phi$ is total magnetic flux passing through the loop.

Consider the bar be located at a distance $x$ from the resistor at time $t$. So the total magnetic flux passing through the loop at time $t$ is

$$\Phi = \int B \cdot dS = Blx \qquad \text{(area of the loop is } S = lx\text{)}$$

Now the induced emf in a loop placed in magnetic field is defined as

$$V_{emf} = -\frac{d\Phi}{dt}$$

where $\Phi$ is the total magnetic flux passing through the loop. Therefore the induced emf in the square loop is

$$V_{emf} = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt} \qquad (\Phi = Blx)$$

Since from the given figure, we have

$$l = 2 \text{ m} \quad \text{and} \quad B = 0.1 \text{ Wb/m}^2$$

and $$dx/dt = \text{velocity of bar} = 5 \text{ m/s}$$

So, induced emf is

$$V_{emf} = -(0.1)(2)(5) = -1 \text{ volt}$$

According to Lenz's law the induced current $I$ in a loop flows such as to produce magnetic field that opposes the change in $B(t)$. As the bar moves away from the resistor the change in magnetic field will be out of the page so the induced current will be in the same direction of $I$ shown in figure.

Thus, the current in the loop is

$$I = -\frac{V_{emf}}{R} = -\frac{(-1)}{10} = 0.01 \text{ A} \qquad (R = 10\Omega)$$



SOL 6.2.10 Correct answer is 277.

Magnetic flux density produced at a distance $\rho$ from a long straight wire carrying current $I$ is defined as

$$B = \frac{\mu_0 I}{2\pi\rho}\boldsymbol{a}_\phi$$

where $\boldsymbol{a}_\phi$ is the direction of flux density as determined by right hand rule. Since the direction of magnetic flux density produced at the loop is normal to the surface of the loop So, total flux passing through the loop is given by

$$\Phi = \int_S B \cdot dS = \int_{\rho=2}^{4}\left(\frac{\mu_0 I}{2\pi\rho}\right)(a d\rho) \qquad (dS = a d\rho)$$

$$= \frac{\mu_0 I a}{2\pi}\int_2^4 \frac{d\rho}{\rho}$$

$$= \frac{\mu_0 I 2}{2\pi}\ln 2 = \frac{\mu_0 I}{\pi}\ln(2)$$

The current flowing in the loop is $I_{loop}$ and induced e.m.f. is $V_{emf}$.

So, $$V_{emf} = I_{loop} R = -\frac{d\Phi}{dt}$$

$$\frac{dQ}{dt}(R) = -\frac{\mu_0}{\pi}\ln(2)\frac{dI}{dt}$$

where $Q$ is the total charge passing through a corner of square loop.

$$\frac{dQ}{dt} = -\frac{\mu_0}{4\pi}\ln(2)\frac{dI}{dt} \qquad (R = 4\,\Omega)$$

$$dQ = -\frac{\mu_0}{4\pi}\ln(2)\,dI$$

Therefore the total charge passing through a corner of square loop is

$$Q = -\frac{\mu_0}{4\pi}\ln(2)\int_4^0 dI$$

$$= -\frac{\mu_0}{4\pi}\ln(2)(0-4)$$

$$= \frac{4\times 4\pi\times 10^{-7}}{4\pi}\ln(2)$$

$$= 2.77\times 10^{-7}\text{ C} = 277\text{ nC}$$

SOL 6.2.11 Correct answer is 44.9 .

Since the radius of small circular loop is negligible in comparison to the radius of the large loop. So, the flux density through the small loop will be constant and equal to the flux on the axis of the loops.

So, $$B = \frac{\mu_0 I}{2}\frac{R^2}{(z^2+R^2)^{3/2}}\boldsymbol{a}_z$$

where $R \rightarrow$ radius of large loop $= 5$ m

$z \rightarrow$ distance between the loops $= 12$ m

$$B = \frac{\mu_0\times 2}{2}\times\frac{(5)^2}{[(12)^2+(5)^2]^{3/2}}\boldsymbol{a}_z = \frac{25\mu_0}{(13)^3}\boldsymbol{a}_z$$

Therefore, the total flux passing through the small loop is

$$\Phi = \int B\cdot dS = \frac{25\mu_0}{(13)^3}\times\pi r^2$$ where $r$ is radius of small circular loop.

$$= \frac{25\times 4\pi\times 10^{-7}}{(13)^3}\times\pi(10^{-3})^2 = 44.9\text{ fWb}$$

SOL 6.2.12 Correct answer is 2.7 .

Electric field in any medium is equal to the voltage drop per unit length.

i.e. $$E = \frac{V}{d}$$

where $V \rightarrow$ potential difference between two points.

$d \rightarrow$ distance between the two points.

The voltage difference between any two points in the medium is

$$V = V_0 \cos 2\pi ft$$

So the conduction current density in the medium is given as

$$J_c = \sigma E \qquad (\sigma \rightarrow \text{conductivity of the medium})$$

$$= \frac{E}{\rho} \qquad (\rho \rightarrow \text{resistivity of the medium})$$

$$= \frac{V}{\rho d} = \frac{V_0 \cos 2\pi ft}{\rho d} \qquad (V = V_0 \cos 2\pi ft)$$

or, $$|J_c| = \frac{V_0}{\rho d}$$

and displacement current density in the medium is given as

$$J_d = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} = \varepsilon \frac{\partial}{\partial t}\left[\frac{V_0 \cos(2\pi ft)}{d}\right] \qquad (V = V_0 \cos 2\pi ft)$$

$$= \frac{\varepsilon V_0}{d}[-2\pi ft \sin(2\pi ft)]$$

or, $$|J_d| = \frac{2\pi f \varepsilon V_0}{d}$$

Therefore, the ratio of amplitudes of conduction current and displacement current in the medium is

$$\frac{|I_c|}{|I_d|} = \frac{|J_C|}{|J_d|} = \frac{(V_0)/(\rho d)}{(d)/(2\pi f \varepsilon V_0)} = \frac{1}{2\pi f \varepsilon \rho}$$

$$= \frac{1}{2\pi \times (1.6 \times 10^8) \times (54 \times 8.85 \times 10^{-12}) \times 0.77}$$

$$= 2.7$$

SOL 6.2.13 Correct answer is 8.

Let the test charge be $q$ coulomb So the force presence of experienced by the test charge in the presence of magnetic field is

$$F = q(v \times B)$$

and the force experienced can be written in terms of the electric field intensity as

$$F = qE$$

Where $E$ is field viewed by observer moving with test charge.

Putting it in Eq. (i)

$$qE = q(v \times B)$$

$$E = (\omega \rho \boldsymbol{a}_\phi) \times (2\boldsymbol{a}_z)$$

where $\omega$ is angular velocity and $\rho$ is radius of circular loop.

$$= (2)(2)(2)\boldsymbol{a}_\rho = 8\boldsymbol{a}_\rho \text{ V/m}$$

SOL 6.2.14 Correct answer is $-0.35$ .

As shown in figure the bar is sliding away from origin.

Now when the bar is located at a distance $dx$ from the voltmeter, then, the vector area of the loop formed by rail and the bar is

$$dS = (20 \times 10^{-2})(dx)\,\boldsymbol{a}_z$$

So, the total magnetic flux passing through the loop is

$$\Phi = \int_S B \cdot dS$$
$$= \int_0^x (8x^2 a_z)(20 \times 10^{-2} dx a_z)$$
$$= \frac{1.6\left[t(1+0.4t^2)\right]^3}{3}$$

Therefore, the induced e.m.f. in the loop is given as

$$V_{emf} = -\frac{d\Phi}{dt} = -\frac{1.6}{3} \times 3(t+0.4t^3)^2 \times (1+1.2t^2)$$
$$V_{emf} = -1.6\left[(0.4)+(0.4)^4\right]^2 \times \left[1+(1.2)(0.4)^2\right] \qquad (t = 0.4 \text{ sec})$$
$$= -0.35 \text{ volt}$$

Since the voltmeter is connected in same manner as the direction of induced emf (determined by Lenz's law).

So the voltmeter reading will be

$$V = V_{emf} = -0.35 \text{ volt}$$

SOL 6.2.15 Correct answer is $-23.4$ .

Since the position of bar is give as

$$x = t(1+0.4t^2)$$

So for the position $x = 12$ cm we have

$$0.12 = t(1+0.4t^2)$$

or, $$t = 0.1193 \text{ sec}$$

As calculated in previous question, the induced emf in the loop at a particular time $t$ is

$$V_{emf} = -(1.6)\left[t+0.4t^3\right]^2(1+1.2t^2)$$

So, at $t = 0.1193$ sec,

$$V_{emf} = -1.6\left[(0.1193)+0.4(0.1193)^3\right]^2\left[1+(1.2)(0.1193)^2\right]$$
$$= -0.02344 = -23.4 \text{ mV}$$

Since the voltmeter is connected in same manner as the direction of induced emf as determined by Lenz's law. Therefore, the voltmeter reading at $x = 12$ cm will be

$$V = V_{emf} = -23.4 \text{ mvolt}$$

SOL 6.2.16 Correct answer is $\pm 600$.

Given the magnetic field intensity in the medium is

$$H = \cos(10^{10}t - bx)a_z \text{ A/m}$$

Now from the Maxwell's equation, we have

$$\nabla \times H = \frac{\partial D}{\partial t}$$

or, $$\frac{\partial D}{\partial t} = -\frac{\partial H_z}{\partial x}a_y = -b\sin(10^{10}t - bx)a_y$$

$$D = \int -b\sin(10^{10}t - bx)dt + C$$ where $C$ is a constant.

Since no D.C. field is present in the medium so, we get $C = 0$ and therefore,

$$D = \frac{b}{10^{10}}\cos(10^{10}t - bx)a_y \text{ C/m}^2$$

and the electric field intensity in the medium is given as

$$E = \frac{D}{\varepsilon} = \frac{b}{0.12 \times 10^{-9} \times 10^{10}}\cos(10^{10t} - bx)a_y \quad (\varepsilon = 0.12 \text{ nF/m})$$

Again From the Maxwell's equation

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

or,
$$\frac{\partial B}{\partial t} = -\nabla \times \left[\frac{b}{1.2}\cos(10^{10t} - bx)a_y\right]$$
$$= -\frac{b^2}{1.2}\sin(10^{10t} - bx)a_z$$

So, the magnetic flux density in the medium is

$$B = -\int \frac{b^2}{1.2}\sin(10^{10t} - bx)a_z dt$$
$$= \frac{b^2}{(1.2)\times 10^{10}}\cos(10^{10}t - bx)a_z \qquad (1)$$

We can also determine the value of magnetic flux density as :

$$B = \mu H$$
$$= (3\times 10^{-5})\cos(10^{10}t - bx)a_z \qquad (2)$$

Comparing the results of equation (1) and (2) we get,

$$\frac{b^2}{(1.2)\times 10^{10}} = 3\times 10^{-5}$$
$$b^2 = 3.6\times 10^5$$
$$b = \pm 600 \text{ rad/m}$$

SOL 6.2.17 Correct answer is 54.414 .

Given the electric field in time domain as

$$E = 5\sin(10\pi y)\cos(6\pi\times 10^9 - bx)a_z$$

Comparing it with the general equation for electric field intensity given as

$$E = E_0\cos(\omega t - \beta x)a_z$$

We get, $\omega = 6\pi\times 10^9$

Now in phasor form, the electric field intensity is

$$E_s = 5\sin(10\pi y)e^{-jbx}a_z \qquad (1)$$

From Maxwell's equation we get the magnetic field intensity as

$$H_s = -\frac{1}{j\omega\mu_0}(\nabla\times E_s) = \frac{j}{\omega\mu_0}\left[\frac{\partial E_{sz}}{\partial y}a_x - \frac{\partial E_{sz}}{\partial x}a_y\right]$$
$$= \frac{j}{\omega\mu_0}[50\pi\cos(10\pi y)e^{-jbx}a_x + j5b\sin(10\pi y)a_y]e^{-jbx}$$

Again from Maxwell's equation we have the electric field intensity as

$$E_s = \frac{1}{j\omega\varepsilon_0}(\nabla\times H_s) = \frac{1}{j\omega\varepsilon_0}\left[\frac{\partial H_{sy}}{\partial x} - \frac{\partial H_{sx}}{\partial y}\right]a_z$$
$$= \frac{1}{\omega^2\mu_0\varepsilon_0}[(j5b)(-jb)\sin(10\pi y)e^{-jbx} + (50\pi)(10\pi)\sin(10\pi y)e^{-jbx}]a_z$$
$$= \frac{1}{\omega^2\mu_0\varepsilon_0}[5b^2 + 500\pi^2]\sin 10\pi y e^{-jbx}a_z$$

Comparing this result with equation (1) we get

$$\frac{1}{\omega^2\mu_0\varepsilon_0}(5b^2 + 500\pi^2) = 5$$

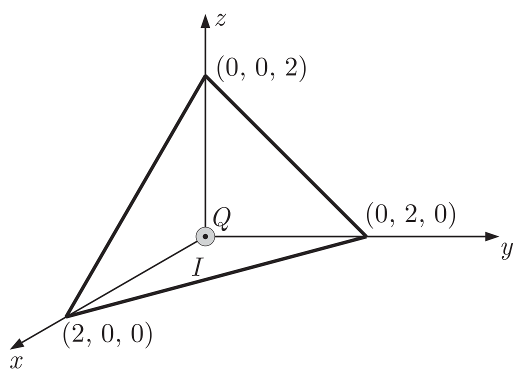
or,
$$b^2 + 100\pi^2 = \omega^2\mu_0\varepsilon_0$$
$$b^2 + 100\pi^2 = (6\pi\times 10^9)^2\times\frac{1}{(3\times 10^8)^2} \quad \left(\omega = 6\pi\times 10^9, \sqrt{\mu_0\varepsilon_0} = \frac{1}{c}\right)$$
$$b^2 + 100\pi^2 = 400\pi^2$$
$$b^2 = 300\pi^2$$

$$b = \pm\sqrt{300}\,\pi \text{ rad/m}$$

So, $$|b| = \sqrt{300}\,\pi = 54.414 \text{ rad/m}$$

SOL 6.2.18 Correct answer is 7.

Let the point change located at origin be $Q$ and the current $I$ is flowing out of the page through the closed triangular path as shown in the figure.




As the current $I$ flows away from the point charge along the wire, the net charge at origin will change with increasing time and given as

$$\frac{dQ}{dt} = -I$$

So the electric field intensity will also vary through the surface and for the varying field circulation of magnetic field intensity around the triangular loop is defined as

$$\oint H \cdot dl = [I_d]_{enc} + [I_c]_{enc}$$

where $[I_c]_{enc}$ is the actual flow of charge called enclosed conduction current and $[I_d]_{enc}$ is the current due to the varying field called enclosed displacement current which is given as

$$[I_d]_{enc} = \frac{d}{dt}\int_S (\varepsilon_0 E) \cdot dS = \frac{d}{dt}\int_S D \cdot dS \qquad (1)$$

From symmetry the total electric flux passing through the triangular surface is

$$\int_S D \cdot dS = \frac{Q}{8}$$

So, $$[I_d]_{enc} = \frac{d}{dt}\left(\frac{Q}{8}\right) = \frac{1}{8}\frac{dQ}{dt} = -\frac{I}{8} \qquad \text{(from equation (1))}$$

whereas $$[I_c]_{enc} = I$$

So, the net circulation of the magnetic field intensity around the closed triangular loop is

$$\oint_C H \cdot dl = [I_d]_{enc} + [I_c]_{enc}$$

$$= -\frac{I}{8} + I = \frac{7}{8}(8) = 7 \text{ A} \qquad (I = 8 \text{ A})$$

SOL 6.2.19 Correct answer is 21.33 .

As calculated in previous question the maximum induced voltage in the rotating loop is given as

$$|V_{emf}| = B_0 S\omega$$

From the given data, we have

$$B_0 = 0.25 \text{ Wb/m}^2$$

$$S = 64 \text{ cm}^2 = 64 \times 10^{-4} \text{ m}^2$$

and $\omega = 60 \times 2\pi = 377$ rad/sec (In one revolution $2\pi$ radian is covered)

So, the r.m.s. value of the induced voltage is

$$\begin{aligned}[V_{\rm emf}]_{r.m.s} &= \frac{1}{\sqrt{2}}\lvert V_{\rm emf}\rvert = \frac{1}{\sqrt{2}}B_0 S\omega \\ &= \frac{1}{\sqrt{2}}(0.25 \times 64 \times 10^{-4} \times 377) \\ &= 0.4265\end{aligned}$$

Since the loop has 50 turns so net induced voltage will be 50 times the calculated value.

i.e. $$\begin{aligned}[V_{\rm emf}]_{r.m.s} &= 50 \times (0.4265) \\ &= 21.33 \text{ volt}\end{aligned}$$

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# SOLUTIONS 6.3



SOL 6.3.1 Option (D) is correct.

SOL 6.3.2 Option (B) is correct.
The line integral of magnetic field intensity along a closed loop is equal to the current enclosed by it.

i.e. $$\int H \cdot dl = I_{enc}$$

So, for the constant current, magnetic field intensity will be constant i.e. magnetostatic field is caused by steady currents.

SOL 6.3.3 Option (A) is correct.
From Faraday's law the electric field intensity in a time varying field is defined as

$$\nabla \times E = -\frac{\partial B}{\partial t}$$ where $B$ is magnetic flux density in the EM field.

and since the magnetic flux density is equal to the curl of magnetic vector potential

i.e. $$B = \nabla \times A$$

So, putting it in equation (1), we get

$$\nabla \times E = -\frac{\partial}{\partial t}(\nabla \times A)$$

or $$\nabla \times E = \nabla \times \left(-\frac{\partial}{\partial t}A\right)$$

Therefore, $$E = -\frac{\partial A}{\partial t}$$

SOL 6.3.4 Option (B) is correct.
Since total magnetic flux through a surface $S$ is defined as

$$\Phi = \int_S B \cdot dS$$

From Maxwell's equation it is known that curl of magnetic flux density is zero

$$\nabla \cdot B = 0$$

$$\int_S B \cdot dS = \int_v (\nabla \cdot B) dv = 0 \qquad \text{(Stokes Theorem)}$$

Thus, net outwards flux will be zero for a closed surface.

SOL 6.3.5 Option (B) is correct.
From the integral form of Faraday's law we have the relation between the electric field intensity and net magnetic flux through a closed loop as

$$\oint E \cdot dl = -\frac{d\Phi}{dt}$$

Since electric field intensity is zero ($E = 0$) inside the conducting loop. So, the rate of change in net magnetic flux through the closed loop is

$$\frac{d\Phi}{dt} = 0$$

i.e. $\Phi$ is constant and doesn't vary with time.

SOL 6.3.6 Option (C) is correct.
A superconductor material carries zero magnetic field and zero electric field inside it.

i.e. $B = 0$ and $E = 0$

Now from Ampere-Maxwell equation we have the relation between the magnetic flux density and electric field intensity as

$$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

So, $J = 0$ $(B = 0,\ E = 0)$

Since the net current density inside the superconductor is zero so all the current must be confined at the surface of the wire.

SOL 6.3.7 Option (C) is correct.
According to Lenz's law the induced current $I$ in a loop flows such as to produce a magnetic field that opposes the change in $B(t)$.
Now the configuration shown in option (A) and (B) for increasing magnetic flux $B_i$, the change in flux is in same direction to $B_i$ as well as the current $I$ flowing in the loop produces magnetic field in the same direction so it does not follow the Lenz's law.
For the configuration shown in option (D), as the flux $B_d$ is decreasing with time so the change in flux is in opposite direction to $B_d$ as well as the current $I$ flowing in the loop produces the magnetic field in opposite direction so it also does not follow the Lenz's law.
For the configuration shown in option (C), the flux density $B_d$ is decreasing with time so the change in flux is in opposite direction to $B_d$ but the current $I$ flowing in the loop produces magnetic field in the same direction to $B_d$ (opposite to the direction of change in flux density). Therefore this is the correct configuration.

SOL 6.3.8 Option (C) is correct.
Induced emf in a conducting loop is given by

$$V_{emf} = -\frac{d\Phi}{dt}$$ where $\Phi$ is total magnetic flux passing through the loop.

Since, the magnetic field is non-uniform so the change in flux will be caused by it and the induced emf due to it is called transformer emf.
Again the field is in $a_y$ direction and the loop is rotating about $z$-axis so flux through the loop will also vary due to the motion of the loop. This causes the emf which is called motion emf. Thus, total induced voltage in the rotating loop is caused by the combination of both the transformer and motion emf.

SOL 6.3.9 Option (B) is correct.

SOL 6.3.10 Option (C) is correct.

SOL 6.3.11 Option (B) is correct.

SOL 6.3.12 Option (D) is correct.

SOL 6.3.13 Option (A) is correct.

SOL 6.3.14 Option (A) is correct.

SOL 6.3.15 Option (C) is correct.

SOL 6.3.16 Option (B) is correct.

SOL 6.3.17 Option (B) is correct.

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# SOLUTIONS 6.4

SOL 6.4.1 Option (C) is correct.

Given, the magnetic flux density in air as

$$B = B_0\left(\frac{x}{x^2+y^2}a_y - \frac{y}{x^2+y^2}a_x\right) \quad ...(1)$$

Now, we transform the expression in cylindrical system, substituting

$$x = r\cos\phi \quad \text{and} \quad y = r\sin\phi$$

$$a_x = \cos\phi a_r - \sin\phi a_\phi$$

and $$a_y = \sin\phi a_r + \cos\phi a_\phi$$

So, we get $$B = B_0 a_\phi$$

Therefore, the magnetic field intensity in air is given as

$$H = \frac{B}{\mu_0} = \frac{B_0 a_\phi}{\mu_0}, \text{ which is constant}$$

So, the current density of the field is

$$J = \nabla \times H = 0 \quad \text{(since } H \text{ is constant)}$$

SOL 6.4.2 Option (D) is correct.

Maxwell equations for an EM wave is given as

$$\nabla \cdot B = 0$$

$$\nabla \cdot E = \frac{\rho_v}{\varepsilon}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = \frac{\partial D}{\partial t} + J$$

So, for static electric magnetic fields

$$\nabla \cdot B = 0$$

$$\nabla \cdot E = \rho_v/\varepsilon$$

$$\nabla \times E = 0 \quad \left(\frac{\partial B}{\partial t} = 0\right)$$

$$\nabla \times H = J \quad \left(\frac{\partial D}{\partial t} = 0\right)$$

SOL 6.4.3 Option (D) is correct.

$$\nabla \times H = J + \frac{\partial D}{\partial t} \quad \text{Maxwell Equations}$$

$$\iint_S (\nabla \times H)\cdot dS = \iint_S \left(J + \frac{\partial D}{\partial t}\right)\cdot dS \quad \text{Integral form}$$

$$\oint H \cdot dl = \iint_S \left(J + \frac{\partial D}{\partial t}\right)\cdot dS \quad \text{Stokes Theorem}$$

SOL 6.4.4 Option (C) is correct.

From Maxwells equations we have

$$\nabla \times H = \frac{\partial D}{\partial t} + J$$

Thus, $\nabla \times H$ has unit of current density $J$ (i.e., $A/m^2$)

SOL 6.4.5 Option (A) is correct.

This equation is based on Ampere's law as from Ampere's circuital law we have

$$\oint_l H \cdot dl = I_{\text{enclosed}}$$

or,
$$\oint_l H \cdot dl = \int_S J \cdot dS$$

Applying Stoke's theorem we get

$$\int_S (\nabla \times H) \cdot dS = \int_S J \cdot dS$$

$$\nabla \times H = J$$

Then, it is modified using continuity equation as

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

SOL 6.4.6 Option (D) is correct.

When a moving circuit is put in a time varying magnetic field induced emf have two components. One due to time variation of magnetic flux density $B$ and other due to the motion of circuit in the field.

SOL 6.4.7 Option (C) is correct.

From maxwell equation we have

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

The term $\frac{\partial D}{\partial t}$ defines displacement current.

SOL 6.4.8 Option (C) is correct.

Emf induced in a loop carrying a time varying magnetic flux $\Phi$ is defined as

$$V_{emf} = -\frac{d\Phi}{dt}$$

$$9 = -\frac{d}{dt}\left(\frac{1}{3}\lambda t^3\right)$$

$$9 = -\lambda t^2$$

at time, $t = 3\,\text{s}$, we have

$$9 = -\lambda(3)^2$$

$$\lambda = -1\,\text{Wb/s}^2$$

SOL 6.4.9 Option (B) is correct.

According to Lenz's law the induced emf (or induced current) in a loop flows such as to produce a magnetic field that opposed the change in $B$. The direction of the magnetic field produced by the current is determined by right hand rule.

Now, in figure (1), $B$ directed upwarded increases with time where as the field produced by current $I$ is downward so, it obey's the Lenz's law.

In figure (2), $B$ directed upward is decreasing with time whereas the field produced by current $I$ is downwards (i.e. additive to the change in $B$) so, it doesn't obey Lenz's law.

In figure (3), $B$ directed upward is decreasing with time where as current $I$ produces the field directed upwards (i.e. opposite to the change in $B$) So, it also obeys Lenz's law.

In figure (4), $B$ directed upward is increasing with time whereas current $I$ produces field directed upward (i.e. additive to the change in $B$) So, it

doesn't obey Lenz's law.
Thus, the configuration 1 and 3 are correct.

SOL 6.4.10 Option (C) is correct.
Faraday's law states that for time varying field,

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

Since, the curl of gradient of a scalar function is always zero

i.e. $\nabla \times (\nabla V) = 0$

So, the expression for the field, $E = -\nabla V$ must include some other terms is

$$E = -\nabla V - \frac{\partial A}{\partial t}$$

i.e. A is true but R is false.

SOL 6.4.11 Option (B) is correct.
Faraday develops the concept of time varying electric field producing a magnetic field. The law he gave related to the theory is known as Faraday's law.

SOL 6.4.12 Option (D) is correct.
Given, the area of loop

$$S = 5\,\text{m}^2$$

Rate of change of flux density,

$$\frac{\partial B}{\partial t} = 2\,\text{Wb/m}^2/\text{S}$$

So, the emf in the loop is

$$V_{\text{emf}} = -\frac{\partial}{\partial t}\int B \cdot dS$$

$$= (5)(-2) = -10\text{ V}$$

SOL 6.4.13 Option (D) is correct.
The modified Maxwell's differential equation.

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

This equation is derived from Ampere's circuital law which is given as

$$\oint H \cdot dl = I_{enc}$$

$$\oint (\nabla \times H) \cdot dS = \int J dS$$

$$\nabla \times H = J$$

SOL 6.4.14 Option (B) is correct.
Electric potential of an isolated sphere is defined as

$$C = 4\pi\varepsilon_0 a \qquad \text{(free space)}$$

The Maxwell's equation in phasor form is written as

$$\nabla \times H = j\omega\varepsilon E + \sigma E = j\omega\varepsilon E + J \qquad (J = \sigma E)$$

So A and R both are true individually but R is not the correct explanation of A.

SOL 6.4.15 Option (A) is correct.
If a coil is placed in a time varying magnetic field then the e.m.f. will induce in coil. So here in both the coil e.m.f. will be induced.

SOL 6.4.16 Option (B) is correct.

Both the statements are individually correct but R is not explanation of A.

SOL 6.4.17 Option (B) is correct.

| | | |
|---|---|---|
| Ampere's law | $\nabla \times H = J + \frac{\partial D}{\partial t}$ | (a → 3) |
| Faraday' law | $\nabla \times E = \frac{\partial B}{\partial t}$ | (b → 4) |
| Gauss law | $\nabla \cdot D = \rho_v$ | (c → 1) |
| Current continuity | $\nabla \cdot J = -\frac{\partial \rho}{\partial t}$ | (d → 2) |

SOL 6.4.18 Option (B) is correct.

Since, the magnetic field perpendicular to the plane of the ring is decreasing with time so, according to Faraday's law emf induced in both the ring is

$$V_{\text{emf}} = -\frac{\partial}{\partial t}\int B \cdot dS$$

Therefore, emf will be induced in both the rings.

SOL 6.4.19 Option (A) is correct.

The Basic idea of radiation is given by the two Maxwell's equation

$$\nabla \times H = \frac{\partial D}{\partial t}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

SOL 6.4.20 Option (B) is correct.

The correct Maxwell's equation are

$$\nabla \times H = J + \frac{\partial D}{\partial t} \qquad \nabla \cdot D = \rho$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad \nabla \cdot B = 0$$

SOL 6.4.21 Option (B) is correct.

In List I

a. $\oint B \cdot dS = 0$

The surface integral of magnetic flux density over the closed surface is zero or in other words, net outward magnetic flux through any closed surface is zero. (a → 4)

b. $\oint D \cdot dS = \int_v \rho_v dv$

Total outward electric flux through any closed surface is equal to the charge enclosed in the region. (b → 3)

c. $\int E \cdot dl = -\int \frac{\partial B}{\partial t} dS$

i.e. The line integral of the electric field intensity around a closed path is equal to the surface integral of the time derivative of magnetic flux density (c → 2)

d. $\oint H \cdot dS = \int \left(\frac{\partial D}{\partial t} + J\right) da$

i.e. The line integral of magnetic field intensity around a closed path is equal to the surface integral of sum of the current density and time derivative of electric flux density. (d → 1)

SOL 6.4.22 Option (D) is correct.

The continuity equation is given as

$$\nabla \cdot J = -\rho_v$$

i.e. it relates current density $(J)$ and charge density $\rho_v$.

SOL 6.4.23 Option (C) is correct.

Given Maxwell's equation is

$$\nabla \times H = J_c + \frac{\partial D}{\partial t}$$

For free space, conductivity, $\sigma = 0$ and so,

$$J_c = \sigma E = 0$$

Therefore, we have the generalized equation

$$\nabla \times H = \frac{\partial D}{\partial t}$$

SOL 6.4.24 Option (A) is correct.

Given the magnetic field intensity,

$$H = 3a_x + 7y a_y + 2x a_z$$

So from Ampere's circuital law we have

$$J = \nabla \times H$$

$$= \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3 & 7y & 2x \end{vmatrix}$$

$$= a_x(0) - a_y(2-0) + a_z(0) = -2a_y$$

SOL 6.4.25 Option (A) is correct.

The emf in the loop will be induced due to motion of the loop as well as the variation in magnetic field given as

$$V_{\text{emf}} = -\int \frac{\partial B}{\partial t} dS + \oint (v \times B) dl$$

So, the frequencies for the induced e.m.f. in the loop is $\omega_1$ and $\omega_2$.

SOL 6.4.26 Option (B) is correct.

$F = Q(E + v \times B)$ is Lorentz force equation.

SOL 6.4.27 Option (A) is correct.

All of the given expressions are Maxwell's equation.

SOL 6.4.28 Option (B) is correct.

Poission's equation for an electric field is given as

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

where, $V$ is the electric potential at the point and $\rho_v$ is the volume charge density in the region. So, for $\rho_v = 0$ we get,

$$\nabla^2 V = 0$$

Which is Laplacian equation.

SOL 6.4.29 Option (A) is correct.

The direction of magnetic flux due to the current '$i$' in the conductor is determined by right hand rule. So, we get the flux through $A$ is pointing into the paper while the flux through $B$ is pointing out of the paper.

According to Lenz's law the induced e.m.f. opposes the flux that causes it.

So again by using right hand rule we get the direction of induced e.m.f. is anticlockwise in $A$ and clockwise in $B$.

SOL 6.4.30 Option (D) is correct.

$$\nabla^2 A = -\mu_0 J$$

This is the wave equation for static electromagnetic field.
i.e. It is not Maxwell's equation.

SOL 6.4.31 Option (B) is correct.

Continuity equation $\quad \nabla \times J = -\dfrac{\partial \rho_v}{\partial t}$ (a → 4)

Ampere's law $\quad \nabla \times H = J + \dfrac{\partial D}{\partial t}$ (b → 1)

Displacement current $\quad J = \dfrac{\partial D}{\partial t}$ (c → 2)

Faraday' law $\quad \nabla \times E = -\dfrac{\partial B}{\partial t}$ (d → 3)

SOL 6.4.32 Option (B) is correct.
Induced emf in a coil of $N$ turns is defined as

$$V_{emf} = -N\frac{d\Phi}{dt}$$

where $\Phi$ is flux linking the coil. So, we get

$$\begin{aligned} V_{emf} &= -100\frac{d}{dt}(t^3 - 2t) \\ &= -100(3t^2 - 2) \\ &= -100(3(2)^2 - 2) = -1000 \text{ mV} \qquad (\text{at } t = 2 \text{ s}) \\ &= -1 \text{ V} \end{aligned}$$



SOL 6.4.33 Option (B) is correct.
A static electric field in a charge free region is defined as

$$\nabla \cdot E = 0 \qquad (a \to 4)$$

and $\quad \nabla \times E = 0$

A static electric field in a charged region have

$$\nabla \cdot E = \frac{\rho_v}{\varepsilon} \neq 0 \qquad (b \to 2)$$

and $\quad \nabla \times E = 0$

A steady magnetic field in a current carrying conductor have

$$\nabla \cdot B = 0 \qquad (c \to 1)$$

$$\nabla \times B = \mu_0 J \neq 0$$

A time varying electric field in a charged medium with time varying magnetic field have

$$\nabla \times E = -\frac{\partial B}{\partial t} \neq 0 \qquad (d \to 3)$$

$$\nabla \cdot E = \frac{\rho_v}{\varepsilon} \neq 0$$

SOL 6.4.34 Option (C) is correct.

$$V = -\frac{d\Phi_m}{dt}$$

It is Faraday's law that states that the change in flux through any loop induces e.m.f. in the loop.

SOL 6.4.35 Option (B) is correct.

From stokes theorem, we have

$$\int (\nabla \times E) \cdot dS = \oint E \cdot dl \qquad (1)$$

Given, the Maxwell's equation

$$\nabla \times E = -(\partial B / \partial t)$$

Putting this expression in equation (1) we get,

$$\oint E \cdot dl = -\frac{\partial}{\partial t}\int_s B \cdot dS$$

SOL 6.4.36 Option (D) is correct.

Since, the flux linking through both the coil is varying with time so, emf are induced in both the coils.

Since, the loop 2 is split so, no current flows in it and so joule heating does not occur in coil 2 while the joule heating occurs in closed loop 1 as current flows in it.

Therefore, only statement 2 is correct.

SOL 6.4.37 Option (C) is correct.

The electric field intensity is

$$E = E_0 e^{j\omega t} \qquad \text{where } E_0 \text{ is independent of time}$$

So, from Maxwell's equation we have

$$\nabla \times H = J + \frac{\varepsilon\,\partial E}{\partial t}$$

$$= \sigma E + \varepsilon (j\omega) E_0 e^{j\omega t} = \sigma E + j\omega\varepsilon E$$

SOL 6.4.38 Option (C) is correct.

Equation (1) and (3) are not the Maxwell's equation.

SOL 6.4.39 Option (A) is correct.

From the Maxwell's equation for a static field (DC) we have

$$\nabla \times B = \mu_0 J$$

$$\nabla \times (\nabla \times A) = \mu_0 J$$

$$\nabla(\nabla \cdot A) - \nabla^2 A = \mu_0 J$$

For static field (DC), $\nabla \cdot A = 0$

therefore we have, $\nabla^2 A = -\mu_0 J$

So, both A and R are true and R is correct explanation of A.

SOL 6.4.40 Option (A) is correct.

For a static field, Maxwells equation is defined as

$$\nabla \times H = J$$

and since divergence of the curl is zero

i.e. $$\nabla \cdot (\nabla \times H) = 0$$

$$\nabla \cdot J = 0$$

but in the time varying field, from continuity equation (conservation of charges)

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t} \neq 0$$

So, an additional term is included in the Maxwell's equation.

i.e. $$\nabla \times H = J + \frac{\partial D}{\partial t}$$

where $\frac{\partial D}{\partial t}$ is displacement current density which is a necessary term.

Therefore A and R both are true and R is correct explanation of A.

SOL 6.4.41 Option (C) is correct.

Since, the circular loop is rotating about the $y$-axis as a diameter and the flux lines is directed in $\boldsymbol{a}_x$ direction. So, due to rotation magnetic flux changes and as the flux density is function of time so, the magnetic flux also varies w.r.t time and therefore the induced e.m.f. in the loop is due to a combination of transformer and motional e.m.f. both.

SOL 6.4.42 Option (A) is correct.

For any loop to have an induced e.m.f., magnetic flux lines must link with the coil.

Observing all the given figures we conclude that loop $C_1$ and $C_2$ carries the flux lines through it and so both the loop will have an induced e.m.f.

SOL 6.4.43 Option (C) is correct.

| | | |
|---|---|---|
| Gauss's law | $\nabla \cdot D = \rho$ | (a → 1) |
| Ampere's law | $\nabla \times H = J_c + \frac{\partial D}{\partial t}$ | (b → 5) |
| Faraday's law | $\nabla \times E = -\frac{\partial B}{\partial t}$ | (c → 2) |
| Poynting vector | $\mathcal{P} = E \times H$ | (d → 3) |

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Eighth Edition

# GATE

## ELECTRONICS & COMMUNICATION

## Network Analysis

Vol 3 of 10


**RK Kanodia**

**Ashish Murolia**



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GATE Electronics & Communication Vol 3, 8e
Network Analysis
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MRP 690.00

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Printed by Nodia and Company, Jaipur

To Our Parents

# *Preface to the Series*

For almost a decade, we have been receiving tremendous responses from GATE aspirants for our earlier books: GATE Multiple Choice Questions, GATE Guide, and the GATE Cloud series. Our first book, GATE Multiple Choice Questions (MCQ), was a compilation of objective questions and solutions for all subjects of GATE Electronics & Communication Engineering in one book. The idea behind the book was that Gate aspirants who had just completed or about to finish their last semester to achieve his or her B.E/B.Tech need only to practice answering questions to crack GATE. The solutions in the book were presented in such a manner that a student needs to know fundamental concepts to understand them. We assumed that students have learned enough of the fundamentals by his or her graduation. The book was a great success, but still there were a large ratio of aspirants who needed more preparatory materials beyond just problems and solutions. This large ratio mainly included average students.

Later, we perceived that many aspirants couldn't develop a good problem solving approach in their B.E/B.Tech. Some of them lacked the fundamentals of a subject and had difficulty understanding simple solutions. Now, we have an idea to enhance our content and present two separate books for each subject: one for theory, which contains brief theory, problem solving methods, fundamental concepts, and points-to-remember. The second book is about problems, including a vast collection of problems with descriptive and step-by-step solutions that can be understood by an average student. This was the origin of *GATE Guide* (the theory book) and *GATE Cloud* (the problem bank) series: two books for each subject. *GATE Guide* and *GATE Cloud* were published in three subjects only.

Thereafter we received an immense number of emails from our readers looking for a complete study package for all subjects and a book that combines both *GATE Guide* and *GATE Cloud*. This encouraged us to present GATE Study Package (a set of 10 books: one for each subject) for GATE Electronic and Communication Engineering. Each book in this package is adequate for the purpose of qualifying GATE for an average student. Each book contains brief theory, fundamental concepts, problem solving methodology, summary of formulae, and a solved question bank. The question bank has three exercises for each chapter: 1) Theoretical MCQs, 2) Numerical MCQs, and 3) Numerical Type Questions (based on the new GATE pattern). Solutions are presented in a descriptive and step-by-step manner, which are easy to understand for all aspirants.

We believe that each book of GATE Study Package helps a student learn fundamental concepts and develop problem solving skills for a subject, which are key essentials to crack GATE. Although we have put a vigorous effort in preparing this book, some errors may have crept in. We shall appreciate and greatly acknowledge all constructive comments, criticisms, and suggestions from the users of this book. You may write to us at rajkumar.kanodia@gmail.com and ashish.murolia@gmail.com.

Acknowledgements

We would like to express our sincere thanks to all the co-authors, editors, and reviewers for their efforts in making this project successful. We would also like to thank Team NODIA for providing professional support for this project through all phases of its development. At last, we express our gratitude to God and our Family for providing moral support and motivation.

We wish you good luck !

R. K. Kanodia
Ashish Murolia

# SYLLABUS

### GATE Electronics & Communications

## Networks:

Network graphs: matrices associated with graphs; incidence, fundamental cut set and fundamental circuit matrices. Solution methods: nodal and mesh analysis. Network theorems: superposition, Thevenin and Norton's maximum power transfer, Wye-Delta transformation. Steady state sinusoidal analysis using phasors. Linear constant coefficient differential equations; time domain analysis of simple RLC circuits, Solution of network equations using Laplace transform: frequency domain analysis of RLC circuits. 2-port network parameters: driving point and transfer functions. State equations for networks.

### IES Electronics & Telecommunication

## Network Theory

Network analysis techniques; Network theorems, transient response, steady state sinusoidal response; Network graphs and their applications in network analysis; Tellegen's theorem. Two port networks; Z, Y, h and transmission parameters. Combination of two ports, analysis of common two ports. Network functions : parts of network functions, obtaining a network function from a given part. Transmission criteria : delay and rise time, Elmore's and other definitions effect of cascading. Elements of network synthesis.

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## CHAPTER 12 MAGNETICALLY COUPLED CIRCUITS

## CHAPTER 13 FREQUENCY RESPONSE



## CHAPTER 14 CIRCUIT ANALYSIS USING LAPLACE TRANSFORM

## CHAPTER 15 TWO PORT NETWORK



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# CHAPTER 5

## CIRCUIT THEOREMS

### 5.1 INTRODUCTION

In this chapter we study the methods of simplifying the analysis of more complicated circuits. We shall learn some of the circuit theorems which are used to reduce a complex circuit into a simple equivalent circuit. This includes Thevenin theorem and Norton theorem. These theorems are applicable to linear circuits, so we first discuss the concept of circuit linearity.

### 5.2 LINEARITY

A system is linear if it satisfies the following two properties

#### Homogeneity Property

The homogeneity property requires that if the input (excitation) is multiplied by a constant, then the output (response) is multiplied by the same constant. For a resistor, for example, Ohm's law relates the input $I$ to the output $V$,

$$V = IR$$

If the current is increased by a constant $k$, then the voltage increases correspondingly by $k$, that is,

$$kIR = kV$$

#### Additivity Property

The additivity property requires that the response to a sum of inputs is the sum of the responses to each input applied separately. Using the voltage-current relationship of a resistor, if

$$V_1 = I_1 R \qquad \text{(Voltage due to current } I_1\text{)}$$

and $$V_2 = I_2 R \qquad \text{(Voltage due to current } I_2\text{)}$$

then, applying current $(I_1 + I_2)$ gives

$$V = (I_1 + I_2) R = I_1 R + I_2 R$$
$$= V_1 + V_2$$

These two properties defining a linear system can be combined into a single statement as

> For any linear resistive circuit, any output voltage or current, denoted by the variable $y$, is related linearly to the independent sources(inputs), i.e.,
>
> $$y = a_1 x_1 + a_2 x_2 + ... + a_n x_n$$
>
> where $x_1, x_2 .... x_n$ are the voltage and current values of the independent sources in the circuit and $a_1$ through $a_m$ are properly dimensioned constants.

Thus, a linear circuit is one whose output is linearly related (or directly proportional) to its input. For example, consider the linear circuit shown in figure 5.2.1. It is excited by an input voltage source $V_s$, and the current through load $R$ is taken as output(response).

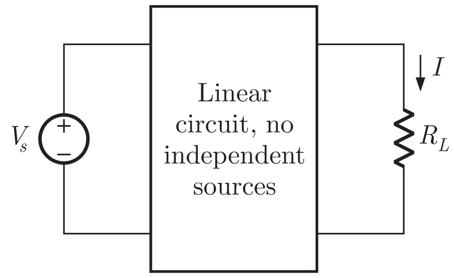



Fig. 5.2.1 A Linear Circuit

Suppose $V_s = 5\text{ V}$ gives $I = 1\text{ A}$. According to the linearity principle, $V_s = 10\text{ V}$ will give $I = 2\text{ A}$. Similarly, $I = 4\text{ mA}$ must be due to $V_s = 20\text{ mV}$. Note that ratio $V_s / I$ remains constant, since the system is linear.

NOTE :
We know that the relationship between power and voltage (or current) is not linear. Therefore, linearity does not applicable to power calculations..

## 5.3 SUPERPOSITION

The number of circuits required to solve a network. using superposition theorem is equal to the number of independent sources present in the network. It states that

In any linear circuit containing multiple independent sources the total current through or voltage across an element can be determined by algebraically adding the voltage or current due to each independent source acting alone with all other independent sources set to zero.

An independent voltage source is set to zero by replacing it with a 0 V source(short circuit) and an independent current source is set to zero by replacing it with 0 A source(an open circuit). The following methodology illustrates the procedure of applying superposition to a given circuit

METHODOLOGY

1. Consider one independent source (either voltage or current) at a time, short circuit all other voltage sources and open circuit all other current sources.
2. Dependent sources can not be set to zero as they are controlled by other circuit parameters.
3. Calculate the current or voltage due to the single source using any method (KCL, KVL, nodal or mesh analysis).
4. Repeat the above steps for each source.
5. Algebraically add the results obtained by each source to get the total response.

NOTE :
Superposition theorem can not be applied to power calculations since power is not a linear quantity.

## 5.4 SOURCE TRANSFORMATION

It states that an independent voltage source $V_s$ in series with a resistance $R$ is equivalent to an independent current source $I_s = V_s / R$, in parallel with a resistance $R$.

or

An independent current source $I_s$ in parallel with a resistance $R$ is equivalent to an independent voltage source $V_s = I_s R$, in series with a resistance $R$.

Figure 5.4.1 shows the source transformation of an independent source. The following points are to be noted while applying source transformation.

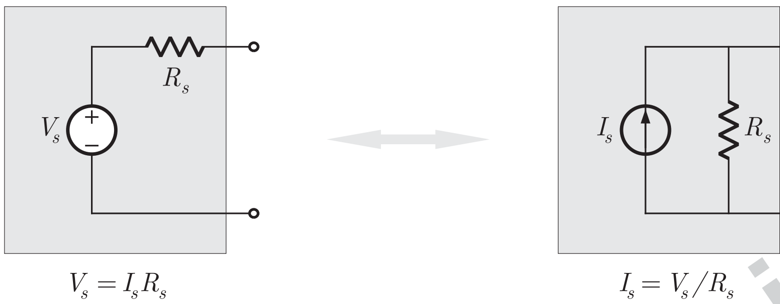



Fig. 5.4.1 Source Transformation of Independent Source

1. Note that head of the current source arrow corresponds to the +ve terminal of the voltage source. The following figure illustrates this


Fig. 5.4.2 Source Transformation of Independent Source

2. Source conversion are equivalent at their external terminals only i.e. the voltage-current relationship at their external terminals remains same. The two circuits in figure 5.4.3a and 5.4.3b are equivalent, provided they have the same voltage-current relation at terminals $a$-$b$

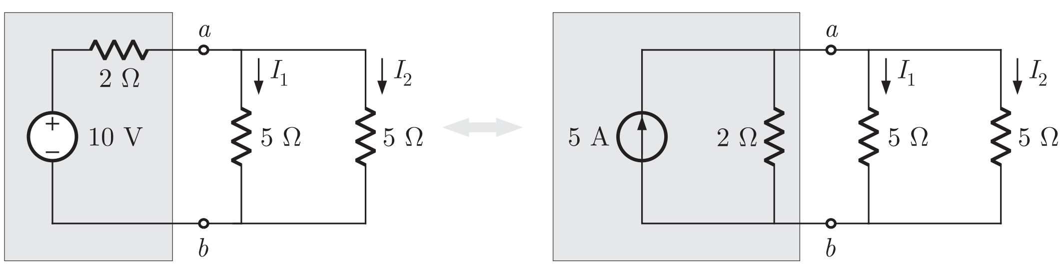



Fig. 5.4.3 An example of source transformation (a) Circuit with a voltage source (b) Equivalent circuit when the voltage source is transformed into current sources

3. Source transformation is not applicable to ideal voltage sources as $R_s = 0$ for an ideal voltage source. So, equivalent current source value $I_s = V_s / R \to \infty$. Similarly it is not applicable to ideal current source

because for an ideal current source $R_s = \infty$, so equivalent voltage source value will not be finite.

### 5.4.1 Source Transformation For Dependent Source

Source transformation is also applicable to dependent source in the same manner as for independent sources. It states that

An dependent voltage source $V_x$ in series with a resistance $R$ is equivalent to a dependent current source $I_x = V_x / R$, in parallel with a resistance $R$, keeping the controlling voltage or current unaffected.

or,

A dependent current source $I_x$ in parallel with a resistance $R$ is equivalent to an dependent voltage source $V_x = I_x R$, in series with a resistance $R$, keeping the controlling voltage or current unaffected.

Figure 5.4.4 shows the source transformation of an dependent source.


Fig. 5.4.4 Source Transformation of Dependent Sources



## 5.5 THEVENIN'S THEOREM

It states that any network composed of ideal voltage and current sources, and of linear resistors, may be represented by an equivalent circuit consisting of an ideal voltage source, $V_{Th}$, in series with an equivalent resistance, $R_{Th}$ as illustrated in the figure 5.5.1.

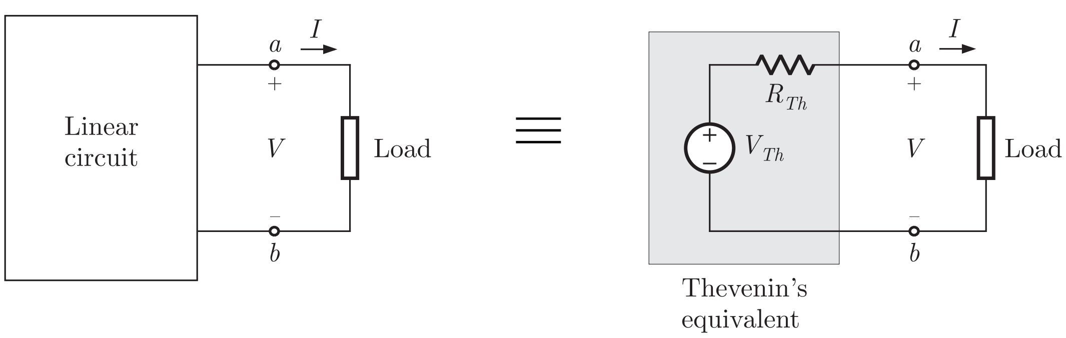



Fig. 5.5.1 Illustration of Thevenin Theorem

where $V_{Th}$ is called Thevenin's equivalent voltage or simply Thevenin voltage and $R_{Th}$ is called Thevenin's equivalent resistance or simply Thevenin resistance.

The methods of obtaining Thevenin equivalent voltage and resistance are given in the following sections.

### 5.5.1 Thevenin's Voltage



The equivalent Thevenin voltage $(V_{Th})$ is equal to the open-circuit voltage present at the load terminals (with the load removed). Therefore, it is also denoted by $V_{oc}$

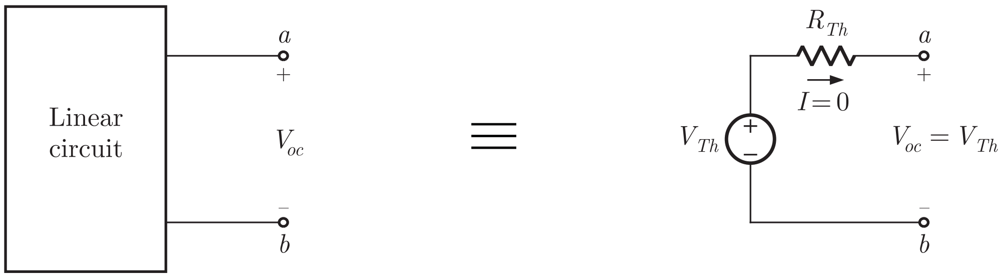



Fig. 5.5.2 Equivalence of Open circuit and Thevenin Voltage

Figure 5.5.2 illustrates that the open-circuit voltage, $V_{oc}$, and the Thevenin voltage, $V_{Th}$, must be the same because in the circuit consisting of $V_{Th}$ and $R_{Th}$, the voltage $V_{oc}$ must equal $V_{Th}$, since no current flows through $R_{Th}$ and therefore the voltage across $R_{Th}$ is zero. Kirchhoff's voltage law confirms that

$$V_{Th} = R_{Th}(0) + V_{oc} = V_{oc}$$

The procedure of obtaining Thevenin voltage is given in the following methodology.

METHODOLOGY 1

1. Remove the load i.e open circuit the load terminals.
2. Define the open-circuit voltage $V_{oc}$ across the open load terminals.
3. Apply any preferred method (KCL, KVL, nodal analysis, mesh analysis etc.) to solve for $V_{oc}$.
4. The Thevenin voltage is $V_{Th} = V_{oc}$.

NOTE :
Note that this methodology is applicable with the circuits containing both the dependent and independent source.

If a circuit contains dependent sources only, i.e. there is no independent source present in the network then its open circuit voltage or Thevenin voltage will simply be zero.

NOTE :
For the Thevenin voltage we may use the terms Thevenin voltage or open circuit voltage interchangeably.

### 5.5.2 Thevenin's Resistance

Thevenin resistance is the input or equivalent resistance at the open circuit terminals $a, b$ when all independent sources are set to zero(voltage sources replaced by short circuits and current sources replaced by open circuits).

We consider the following cases where Thevenin resistance $R_{Th}$ is to be determined.

Case 1: Circuit With Independent Sources only

If the network has no dependent sources, we turn off all independent sources. $R_{Th}$ is the input resistance or equivalent resistance of the network looking between terminals $a$ and $b$, as shown in figure 5.5.3.

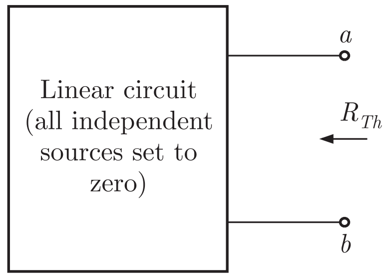



Fig 5.5.3 Circuit for Obtaining $R_{Th}$

Case 2: Circuit With Both Dependent and Independent Sources

Different methods can be used to determine Thevenin equivalent resistance of a circuit containing dependent sources. We may follow the given two methodologies. Both the methods are also applicable to circuit with independent sources only(case 1).

Using Test Source

METHODOLOGY 2

1. Set all independent sources to zero(Short circuit independent voltage source and open circuit independent current source).
2. Remove the load, and put a test source $V_{test}$ across its terminals. Let the current through test source is $I_{test}$. Alternatively, we can put a test source $I_{test}$ across load terminals and assume the voltage across it is $V_{test}$. Either method would give same result.
3. Thevenin resistance is given by $R_{Th} = V_{test}/I_{test}$.

NOTE :
We may use $V_{test} = 1$ V or $I_{test} = 1$ A.

Using Short Circuit Current

$$R_{Th} = \frac{\text{open circuit voltage}}{\text{short circuit current}} = \frac{V_{oc}}{I_{sc}}$$

METHODOLOGY 3

1. Connect a short circuit between terminal $a$ and $b$.
2. Be careful, do not set independent sources zero in this method because we have to find short circuit current.
3. Now, obtain the short circuit current $I_{sc}$ through terminals $a$, $b$.
4. Thevenin resistance is given as $R_{Th} = V_{oc}/I_{sc}$ where $V_{oc}$ is open circuit voltage or Thevenin voltage across terminal $a,b$ which can be obtained by same method given previously.

## 5.5.3 Circuit Analysis Using Thevenin Equivalent

Thevenin's theorem is very important in circuit analysis. It simplifies a circuit. A large circuit may be replaced by a single independent voltage source and a single resistor. The equivalent network behaves the same way externally as the original circuit. Consider a linear circuit terminated by a load $R_L$, as shown in figure 5.5.5. The current $I_L$ through the load and the voltage $V_L$ across the load are easily determined once the Thevenin equivalent of the circuit at the load's terminals is obtained.

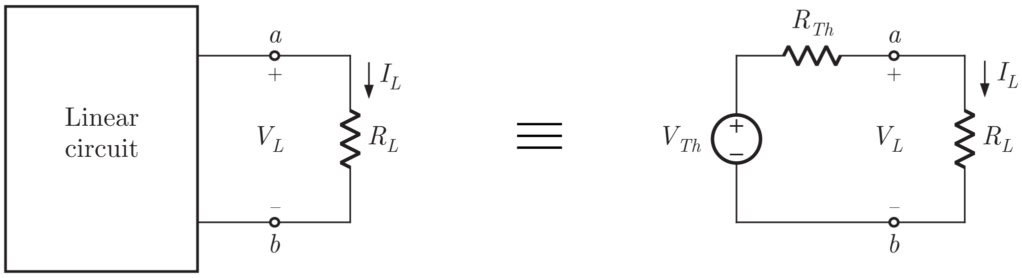



Fig. 5.5.5 A Circuit with a Load and its Equivalent Thevenin Circuit

Current through the load $R_L$

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Voltage across the load $R_L$

$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

## 5.6 NORTON'S THEOREM

Any network composed of ideal voltage and current sources, and of linear resistors, may be represented by an equivalent circuit consisting of an ideal current source, $I_N$, in parallel with an equivalent resistance, $R_N$ as illustrated in figure 5.6.1.

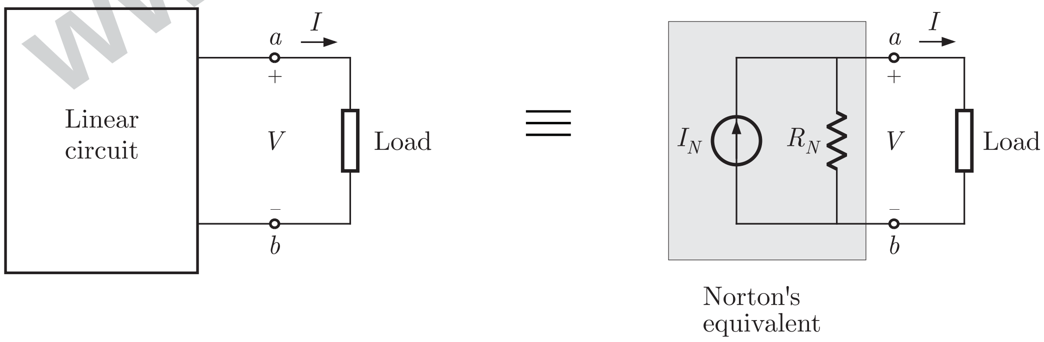



Fig. 5.6.1 Illustration of Norton Theorem

where $I_N$ is called Norton's equivalent current or simply Norton current and $R_N$ is called Norton's equivalent resistance. The methods of obtaining Norton equivalent current and resistance are given in the following sections.

### 5.6.1 Norton's Current

The Norton equivalent current is equal to the short-circuit current that would flow when the load replaced by a short circuit. Therefore, it is also called short circuit current $I_{sc}$.

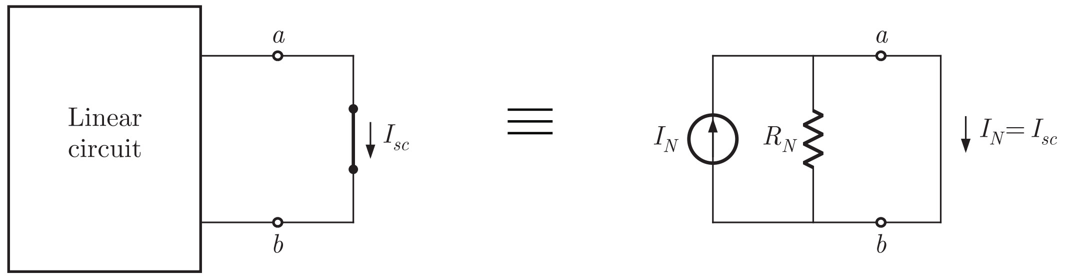



Fig 5.6.2 Equivalence of Short Circuit Current and Norton Current

Figure 5.6.2 illustrates that if we replace the load by a short circuit, then current flowing through this short circuit will be same as Norton current $I_N$

$$I_N = I_{sc}$$

The procedure of obtaining Norton current is given in the following methodology. Note that this methodology is applicable with the circuits containing both the dependent and independent source.

METHODOLOGY

1. Replace the load with a short circuit.
2. Define the short circuit current, $I_{sc}$, through load terminal.
3. Obtain $I_{sc}$ using any method (KCL, KVL, nodal analysis, loop analysis).
4. The Norton current is $I_N = I_{sc}$.

If a circuit contains dependent sources only, i.e. there is no independent source present in the network then the short circuit current or Norton current will simply be zero.



### 5.6.2 Norton's Resistance

Norton resistance is the input or equivalent resistance seen at the load terminals when all independent sources are set to zero(voltage sources replaced by short circuits and current sources replaced by open circuits) i.e. Norton resistance is same as Thevenin's resistance

$$R_N = R_{Th}$$

So, we can obtain Norton resistance using same methodologies as for Thevenin resistance. Dependent and independent sources are treated the same way as in Thevenin's theorem.

NOTE :
For the Norton current we may use the term Norton current or short circuit current interchangeably.

### 5.6.3 Circuit Analysis Using Norton's Equivalent

As discussed for Thevenin's theorem, Norton equivalent is also useful in circuit analysis. It simplifies a circuit. Consider a linear circuit terminated by a load $R_L$, as shown in figure 5.6.4. The current $I_L$ through the load and the voltage $V_L$ across the load are easily determined once the Norton equivalent of the circuit at the load's terminals is obtained,

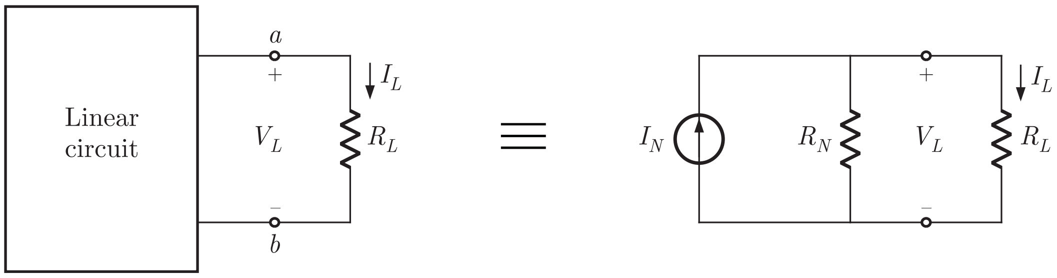



Fig. 5.6.4 A circuit with a Load and its Equivalent Norton Circuit

Current through load $R_L$ is,

$$I_L = \frac{R_N}{R_L + R_L} I_N$$

Voltage across load $R_L$ is,

$$V_L = R_L I_L = \frac{R_L R_N}{R_{Th} + R_L} I_N$$

## 5.7 TRANSFORMATION BETWEEN THEVENIN & NORTON'S EQUIVALENT CIRCUITS

From source transformation it is easy to find Norton's and Thevenin's equivalent circuit from one form to another as following


Fig. 5.7.1 Source Transformation of Thevenin and Norton Equivalents



## 5.8 MAXIMUM POWER TRANSFER THEOREM

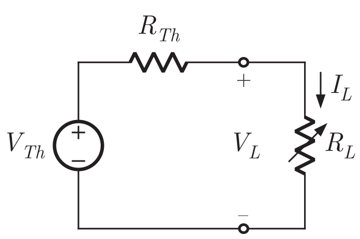
Maximum power transfer theorem states that a load resistance $R_L$ will receive maximum power from a circuit when the load resistance is equal to Thevenin's/Norton's resistance seen at load terminals.

i.e. $\quad R_L = R_{Th}$, $\quad$ (For maximum power transfer)

In other words a network delivers maximum power to a load resistance $R_L$ when $R_L$ is equal to Thevenin equivalent resistance of the network.

PROOF :

Consider the Thevenin equivalent circuit of figure 5.8.1 with Thevenin voltage $V_{Th}$ and Thevenin resistance $R_{Th}$.



Fiig. 5.8.1 A Circuit Used for Maximum Power Transfer

We assume that we can adjust the load resistance $R_L$. The power absorbed by the load, $P_L$, is given by the expression

$$P_L = I_L^2 R_L \tag{5.8.1}$$

and that the load current is given as,

$$I_L = \frac{V_{Th}}{R_L + R_{Th}} \tag{5.8.2}$$

Substituting $I_L$ from equation (5.8.2) into equation (5.8.1)

$$P_L = \frac{V_{Th}^2}{(R_L + R_{Th})^2} R_L \tag{5.8.3}$$

To find the value of $R_L$ that maximizes the expression for $P_L$ (assuming that $V_{Th}$ and $R_{Th}$ are fixed), we write

$$\frac{dP_L}{dR_L} = 0$$

Computing the derivative, we obtain the following expression :

$$\frac{dP_L}{dR_L} = \frac{V_{Th}^2 (R_L + R_{Th})^2 - 2V_{Th}^2 R_L (R_L + R_{Th})}{(R_L + R_{Th})^4}$$

which leads to the expression

$$(R_L + R_{Th})^2 - 2R_L (R_L + R_{Th}) = 0$$

or $$R_L = R_{Th}$$

Thus, in order to transfer maximum power to a load, the equivalent source and load resistances must be matched, that is, equal to each other.

$$R_L = R_{Th}$$

The maximum power transferred is obtained by substituting $R_L = R_{Th}$ into equation (5.8.3)

$$P_{\max} = \frac{V_{Th}^2 R_{Th}}{(R_{Th} + R_{Th})^2} = \frac{V_{Th}^2}{4R_{Th}} \tag{5.8.4}$$

or, $$P_{\max} = \frac{V_{Th}^2}{4R_L}$$



### If the Load resistance $R_L$ is fixed :

Now consider a problem where the load resistance $R_L$ is fixed and Thevenin resistance or source resistance $R_s$ is being varied, then

$$P_L = \frac{V_{Th}^2}{(R_L + R_s)^2} R_L$$

To obtain maximum $P_L$ denominator should be minimum or $R_s = 0$. This can be solved by differentiating the expression for the load power, $P_L$, with respect to $R_s$ instead of $R_L$.

The step-by-step methodology to solve problems based on maximum power transfer is given as following :

METHODOLOGY

1. Remove the load $R_L$ and find the Thevenin equivalent voltage $V_{Th}$ and resistance $R_{Th}$ for the remainder of the circuit.
2. Select $R_L = R_{Th}$, for maximum power transfer.
3. The maximum average power transfer can be calculated using $P_{\max} = V_{Th}^2 / 4R_{Th}$.

## 5.9 RECIPROCITY THEOREM

The reciprocity theorem is a theorem which can only be used with single source circuits (either voltage or current source). The theorem states the following

### 5.9.1 Circuit With a Voltage Source

In any linear bilateral network, if a single voltage source $V_a$ in branch $a$ produces a current $I_b$ in another branch $b$, then if the voltage source $V_a$ is removed(i.e. short circuited) and inserted in branch $b$, it will produce a current $I_b$ in branch $a$.

In other words, it states that the ratio of response(output) to excitation(input) remains constant if the positions of output and input are interchanged in a reciprocal network. Consider the network shown in figure 5.9.1a and b. Using reciprocity theorem we my write

$$\frac{V_1}{I_1} = \frac{V_2}{I_2} \tag{5.9.1}$$

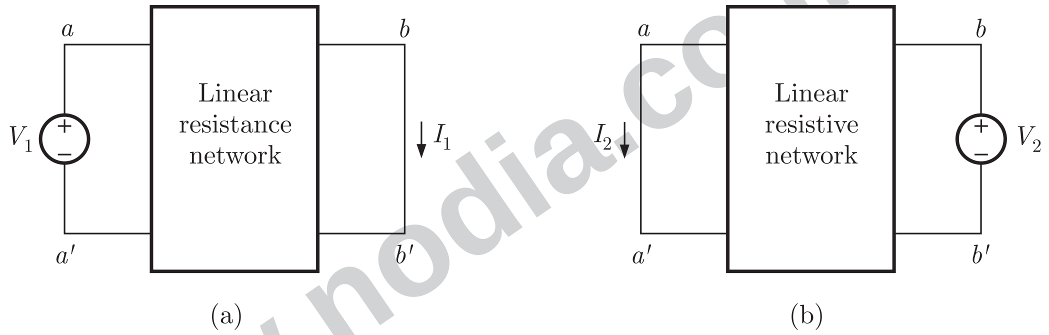



Fig. 5.9.1 Illustration of Reciprocity Theorem for a Voltage Source



When applying the reciprocity theorem for a voltage source, the following steps must be followed:

1. The voltage source is replaced by a short circuit in the original location.
2. The polarity of the voltage source in the new location have the same correspondence with branch current, in each position, otherwise a –ve sign appears in the expression (5.9.1).

This can be explained in a better way through following example.

### 5.9.2 Circuit With a Current Source

In any linear bilateral network, if a single current source $I_a$ in branch $a$ produces a voltage $V_b$ in another branch $b$, then if the current source $I_a$ is removed(i.e. open circuited) and inserted in branch $b$, it will produce a voltage $V_b$ in open-circuited branch $a$.

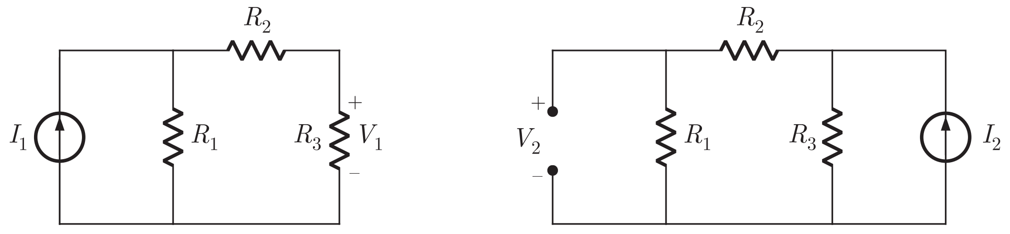



Fig. 5.9.2 Illustration of Reciprocity Theorem for a Current Source

Again, the ratio of voltage and current remains constant. Consider the network shown in figure 5.9.2a and 5.9.2b. Using reciprocity theorem we my write

$$\frac{V_1}{I_1} = \frac{V_2}{I_2} \quad (5.9.2)$$

When applying the reciprocity theorem for a current source, the following conditions must be met:

1. The current source is replaced by an open circuit in the original location.
2. The direction of the current source in the new location have the same correspondence with voltage polarity, in each position, otherwise a –ve sign appears in the expression (5.9.2).

## 5.10 SUBSTITUTION THEOREM

If the voltage across and the current through any branch of a dc bilateral network are known, this branch can be replaced by any combination of elements that will maintain the same voltage across and current through the chosen branch.

For example consider the circuit of figure 5.10.1 .

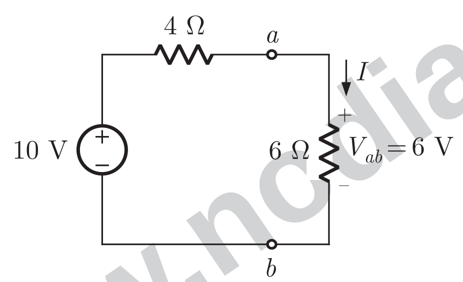



Fig 5.10.1 A Circuit having Voltage $V_{ab} = 6$ V and Current $I = 1$ A in Branch $ab$

The voltage $V_{ab}$ and the current $I$ in the circuit are given as

$$V_{ab} = \left(\frac{6}{6+4}\right)10 = 6 \text{ V}$$

$$I = \frac{10}{6+4} = 1 \text{ A}$$

The $6\,\Omega$ resistor in branch $a$-$b$ may be replaced with any combination of components, provided that the terminal voltage and current must be the same.

We see that the branches of figure 5.10.2a-e are each equivalent to the original branch between terminals a and b of the circuit in figure 5.10.1.

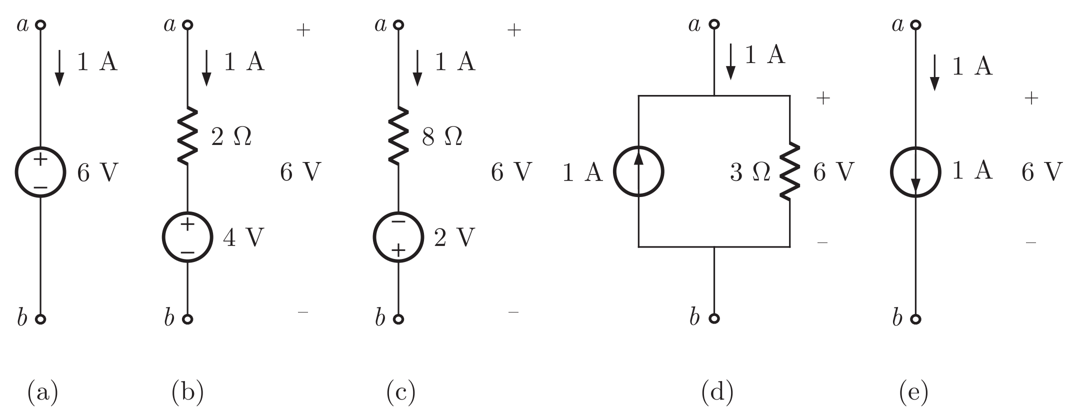


Fig. 5.10.2 Equivalent Circuits for Branch *ab*



Also consider that the response of the remainder of the circuit of figure 5.10.1 is unchanged by substituting any one of the equivalent branches.

## 5.11 MILLMAN'S THEOREM

Millman's theorem is used to reduce a circuit that contains several branches in parallel where each branch has a voltage source in series with a resistor as shown in figure 5.11.1.

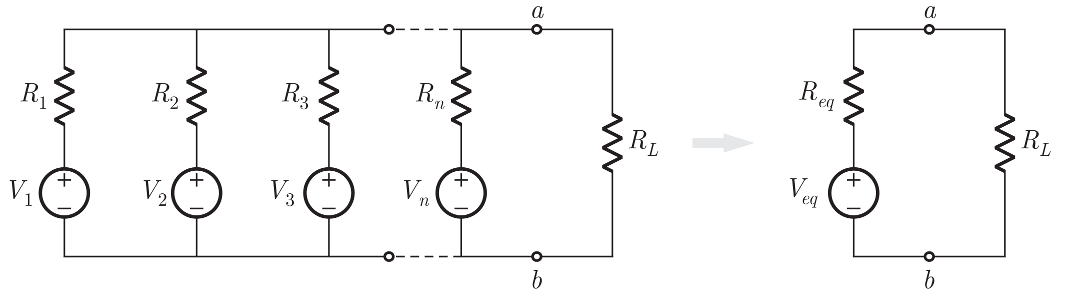



Fig. 5.11.1 Illustration of Millman's Theorem

Mathematically

$$V_{eq} = \frac{V_1 G_1 + V_2 G_2 + V_3 G_3 + V_4 G_4 + \ldots + V_n G_n}{G_1 + G_2 + G_3 + G_4 + \ldots + G_n}$$

$$R_{eq} = \frac{1}{G_{eq}} = \frac{1}{G_1 + G_2 + G_3 + \ldots + G_n}$$

where conductances

$$G_1 = \frac{1}{R_1}, G_2 = \frac{1}{R_2}, G_3 = \frac{1}{R_3}, G_4 = \frac{1}{R_4}, \ldots G_n = \frac{1}{R_n}$$

In terms of resistances

$$V_{eq} = \frac{V_1/R_1 + V_2/R_2 + V_3/R_3 + V_4/R_4 + \ldots + V_n R_n}{1/R_1 + 1/R_2 + 1/R_3 + 1/R_4 + \ldots + 1/R_n}$$

$$R_{eq} = \frac{1}{G_{eq}} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3 + \ldots + 1/R_n}$$

## 5.12 TELLEGEN'S THEOREM

Tellegen's theorem states that the sum of the power dissipations in a lumped network at any instant is always zero. This is supported by Kirchhoff's voltage and current laws. Tellegen's theorem is valid for any lumped network which may be linear or non-linear, passive or active, time-varying or time-invariant.

For a network with $n$ branches, the power summation equation is,

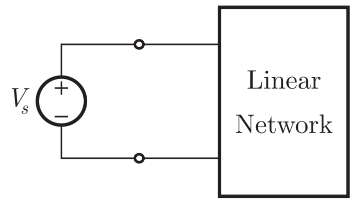
$$\sum_{k=1}^{k=n} V_k I_k = 0$$

One application of Tellegen's theorem is checking the quantities obtained when a circuit is analyzed. If the individual branch power dissipations do not add up to zero, then some of the calculated quantities are incorrect.

***********

# EXERCISE 5.1

MCQ 5.1.1 The linear network in the figure contains resistors and dependent sources only. When $V_s = 10$ V, the power supplied by the voltage source is 40 W. What will be the power supplied by the source if $V_s = 5$ V ?



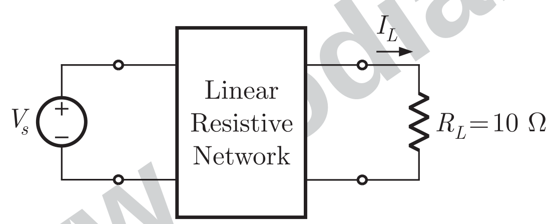

(A) 20 W

(B) 10 W

(C) 40 W

(D) can not be determined

MCQ 5.1.2 In the circuit below, it is given that when $V_s = 20$ V, $I_L = 200$ mA. What values of $I_L$ and $V_s$ will be required such that power absorbed by $R_L$ is 2.5 W ?



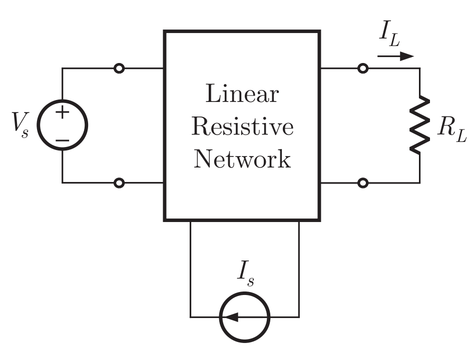

(A) 1 A, 2.5 V

(B) 0.5 A, 2 V

(C) 0.5 A, 50 V

(D) 2 A, 1.25 V

MCQ 5.1.3 For the circuit shown in figure below, some measurements are made and listed in the table.




| | $V_s$ | $I_s$ | $I_L$ |
|---|---|---|---|
| 1. | 14 V | 6 A | 2 A |
| 2. | 18 V | 2 A | 6 A |

Which of the following equation is true for $I_L$ ?

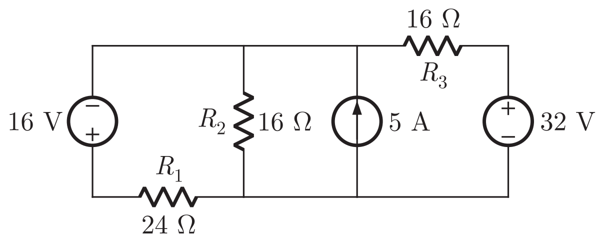
(A) $I_L = 0.6V_s + 0.4I_s$

(B) $I_L = 0.2V_s - 0.3I_s$

(C) $I_L = 0.2V_s + 0.3I_s$

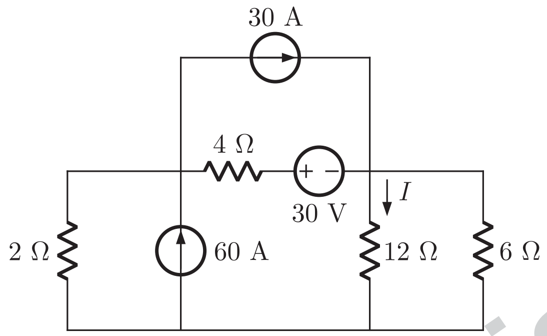
(D) $I_L = 0.4V_s - 0.6I_s$

MCQ 5.1.4 In the circuit below, the voltage drop across the resistance $R_2$ will be equal to




(A) 46 volt (B) 38 volt
(C) 22 volt (D) 14 volt

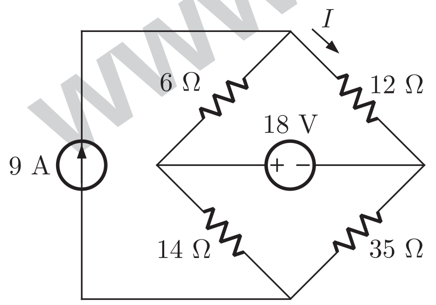
MCQ 5.1.5 In the circuit below, current $I = I_1 + I_2 + I_3$, where $I_1$, $I_2$ and $I_3$ are currents due to 60 A, 30 A and 30 V sources acting alone. The values of $I_1$, $I_2$ and $I_3$ are respectively




(A) 8 A, 8 A, $-4$ A (B) 12 A, 12 A, $-5$ A
(C) 4 A, 4 A, $-1$ A (D) 2 A, 2 A, $-4$ A



MCQ 5.1.6 In the circuit below, current $I$ is equal to sum of two currents $I_1$ and $I_2$. What are the values of $I_1$ and $I_2$ ?

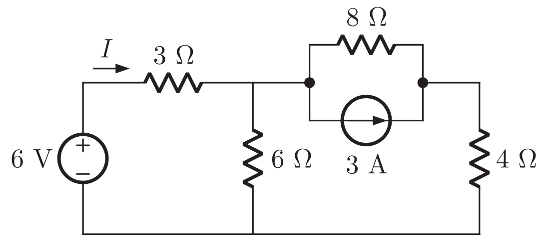



(A) 6 A, 1 A (B) 9 A, 6 A
(C) 3 A, 1 A (D) 3 A, 4 A

MCQ 5.1.7 A network consists only of independent current sources and resistors. If the values of all the current sources are doubled, then values of node voltages
(A) remains same
(B) will be doubled
(C) will be halved
(D) changes in some other way.

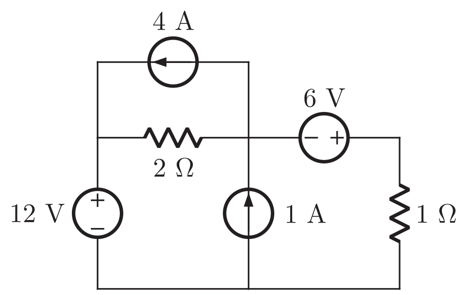
MCQ 5.1.8 Consider a network which consists of resistors and voltage sources only. If the values of all the voltage sources are doubled, then the values of mesh current will be
(A) doubled (B) same
(C) halved (D) none of these

MCQ 5.1.9 The value of current $I$ in the circuit below is equal to



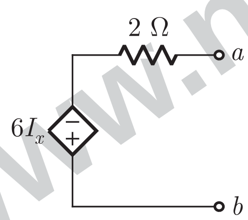

(A) $\frac{2}{7}$ A
(B) 1 A
(C) 2 A
(D) 4 A

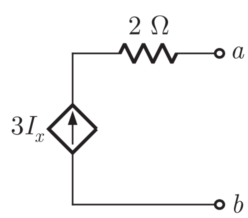
MCQ 5.1.10 In the circuit below, the 12 V source

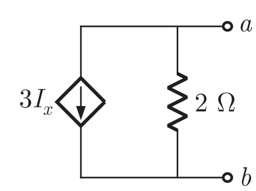


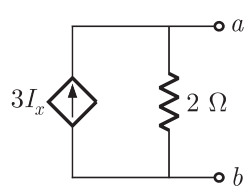

(A) absorbs 36 W
(B) delivers 4 W
(C) absorbs 100 W
(D) delivers 36 W

MCQ 5.1.11 Which of the following circuits is equivalent to the circuit shown below ?



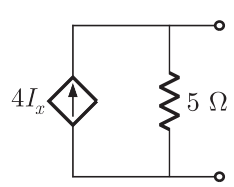

(A)



(B)


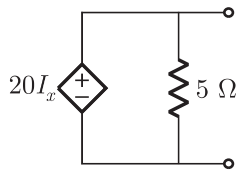

(C)


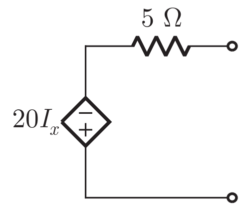

(D) None of these

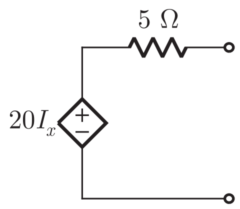
MCQ 5.1.12 Consider a dependent current source shown in figure below.



The source transformation of above is given by

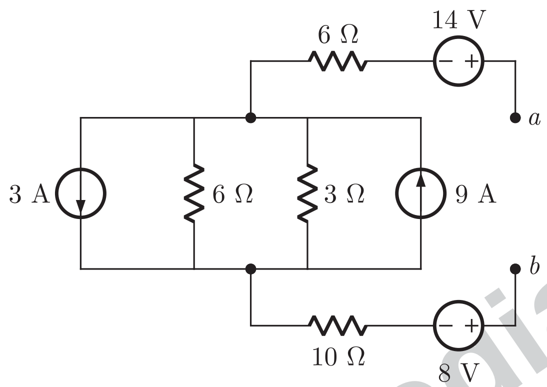
(A) 


(B) 

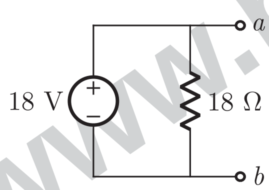

(C) 

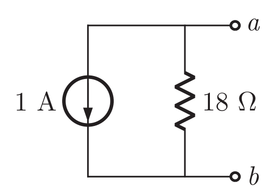

(D) Source transformation does not applicable to dependent sources

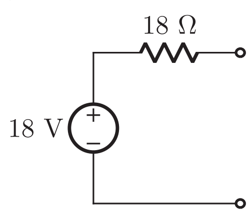
MCQ 5.1.13 Consider a circuit shown in the figure

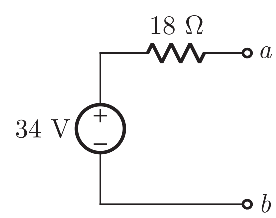



Which of the following circuit is equivalent to the above circuit ?

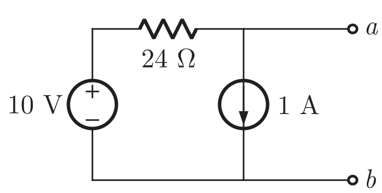
(A) 


(B) 


(C) 

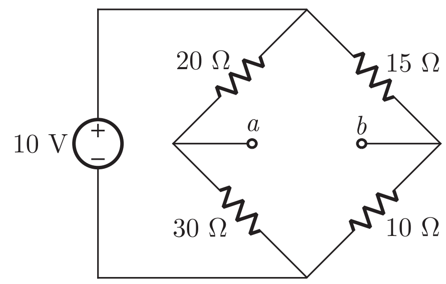

(D) 


MCQ 5.1.14 For the circuit shown in the figure the Thevenin voltage and resistance seen from the terminal $a$-$b$ are respectively




(A) 34 V, 0 Ω
(B) 20 V, 24 Ω
(C) 14 V, 0 Ω
(D) $-14$ V, 24 Ω

MCQ 5.1.15 In the following circuit, Thevenin voltage and resistance across terminal $a$ and $b$ respectively are




(A) 10 V, 18 Ω (B) 2 V, 18 Ω
(C) 10 V, 18.67 Ω (D) 2 V, 18.67 Ω

MCQ 5.1.16 The value of $R_{Th}$ and $V_{Th}$ such that the circuit of figure (B) is the Thevenin equivalent circuit of the circuit shown in figure (A), will be equal to

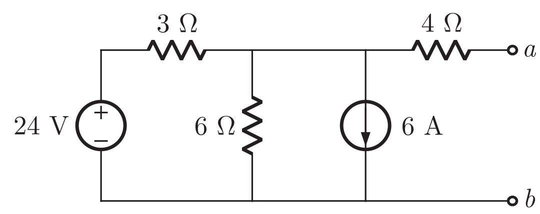



Fig.(A)

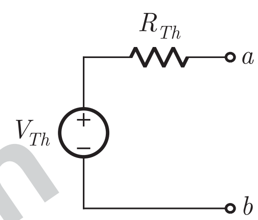



Fig.(B)

(A) $R_{Th} = 6\,\Omega$, $V_{Th} = 4$ V
(B) $R_{Th} = 6\,\Omega$, $V_{Th} = 28$ V
(C) $R_{Th} = 2\,\Omega$, $V_{Th} = 24$ V
(D) $R_{Th} = 10\,\Omega$, $V_{Th} = 14$ V

MCQ 5.1.17 What values of $R_{Th}$ and $V_{Th}$ will cause the circuit of figure (B) to be the equivalent circuit of figure (A) ?

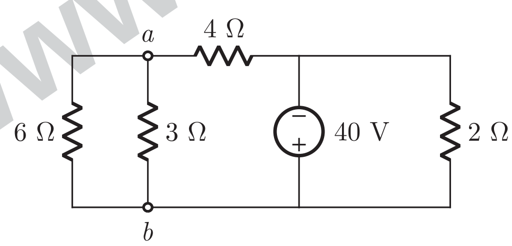



Fig.(A)

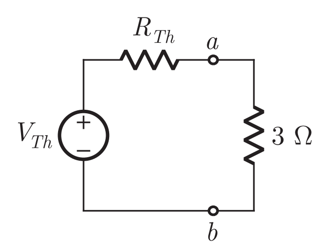



Fig.(B)

(A) 2.4 Ω, $-24$ V (B) 3 Ω, 16 V
(C) 10 Ω, 24 V (D) 10 Ω, $-24$ V

Common Data For Q. 18 and 19 :

Consider the two circuits shown in figure (A) and figure (B) below

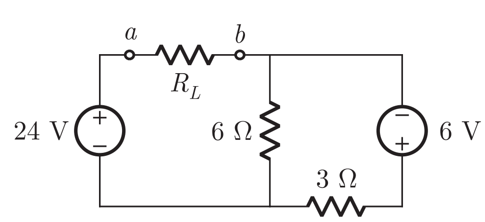



Fig.(A)

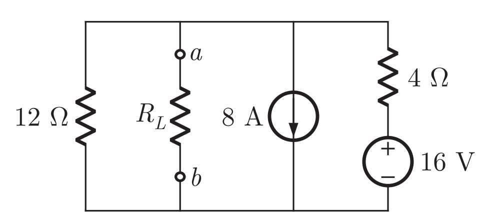



Fig.(B)

MCQ 5.1.18 The value of Thevenin voltage across terminals $a$-$b$ of figure (A) and figure (B) respectively are

(A) 30 V, 36 V (B) 28 V, $-12$ V

(C) 18 V, 12 V (D) 30 V, $-12$ V

MCQ 5.1.19 The value of Thevenin resistance across terminals $a$-$b$ of figure (A) and figure (B) respectively are

(A) zero, 3 Ω (B) 9 Ω, 16 Ω

(C) 2 Ω, 3 Ω (D) zero, 16 Ω

MCQ 5.1.20 For a network having resistors and independent sources, it is desired to obtain Thevenin equivalent across the load which is in parallel with an ideal current source. Then which of the following statement is true ?

(A) The Thevenin equivalent circuit is simply that of a voltage source.

(B) The Thevenin equivalent circuit consists of a voltage source and a series resistor.

(C) The Thevenin equivalent circuit does not exist but the Norton equivalent does exist.

(D) None of these

MCQ 5.1.21 The Thevenin equivalent circuit of a network consists only of a resistor (Thevenin voltage is zero). Then which of the following elements might be contained in the network ?

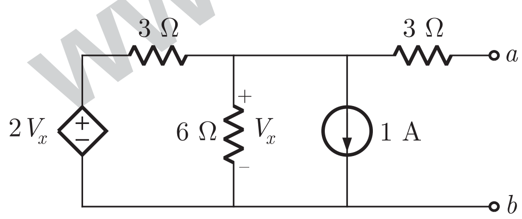
(A) resistor and independent sources

(B) resistor only

(C) resistor and dependent sources

(D) resistor, independent sources and dependent sources.

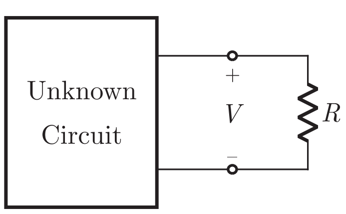
MCQ 5.1.22 For the circuit shown in the figure, the Thevenin's voltage and resistance looking into $a$-$b$ are




(A) 2 V, 3 Ω (B) 2 V, 2 Ω

(C) 6 V, $-9$ Ω (D) 6 V, $-3$ Ω

MCQ 5.1.23 For the following circuit, values of voltage $V$ for different values of $R$ are given in the table.




| $R$ | $V$ |
|---|---|
| 3 Ω | 6 V |
| 8 Ω | 8 V |

The Thevenin voltage and resistance of the unknown circuit are respectively.

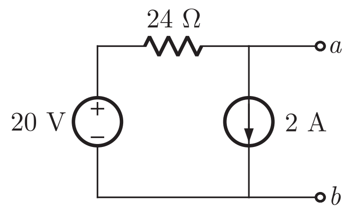
(A) 14 V, 4 Ω

(B) 4 V, 1 Ω

(C) 14 V, 6 Ω

(D) 10 V, 2 Ω

MCQ 5.1.24 In the circuit shown below, the Norton equivalent current and resistance with respect to terminal $a$-$b$ is



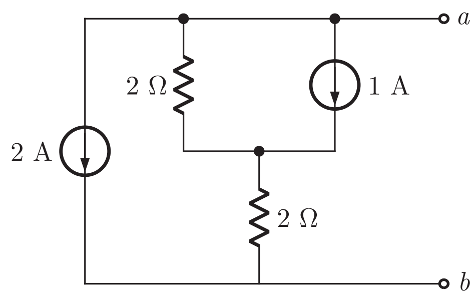

(A) $\frac{17}{6}$ A, 0 Ω

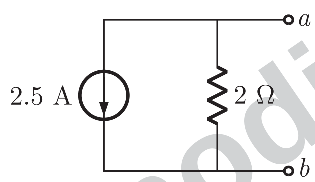
(B) 2 A, 24 Ω

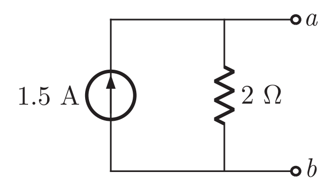
(C) $-\frac{7}{6}$ A, 24 Ω

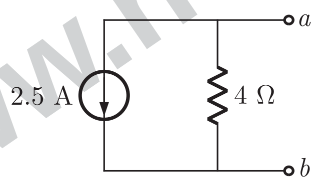
(D) $-2$ A, 24 Ω

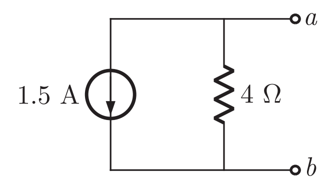
MCQ 5.1.25 The Norton equivalent circuit for the circuit shown in figure is given by




(A) 

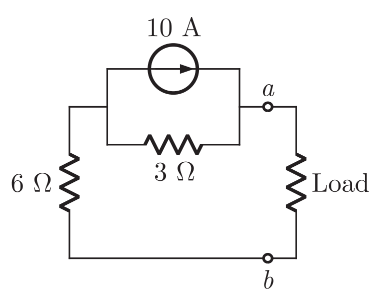

(B) 


(C) 


(D) 




MCQ 5.1.26 What are the values of equivalent Norton current source ($I_N$) and equivalent resistance ($R_N$) across the load terminal of the circuit shown in figure ?




| | $I_N$ | $R_N$ |
|---|---|---|
| (A) | 10 A | 2 Ω |
| (B) | 10 A | 9 Ω |
| (C) | 3.33 A | 9 Ω |
| (D) | 6.66 A | 2 Ω |

MCQ 5.1.27 For a network consisting of resistors and independent sources only, it is desired to obtain Thevenin's or Norton's equivalent across a load which is in parallel with an ideal voltage sources.

Consider the following statements :

1. Thevenin equivalent circuit across this terminal does not exist.
2. The Thevenin equivalent circuit exists and it is simply that of a voltage source.
3. The Norton equivalent circuit for this terminal does not exist.

Which of the above statements is/are true ?

(A) 1 and 3 (B) 1 only

(C) 2 and 3 (D) 3 only

MCQ 5.1.28 For a network consisting of resistors and independent sources only, it is desired to obtain Thevenin's or Norton's equivalent across a load which is in series with an ideal current sources.

Consider the following statements

1. Norton equivalent across this terminal is not feasible.
2. Norton equivalent circuit exists and it is simply that of a current source only.
3. Thevenin's equivalent circuit across this terminal is not feasible.

Which of the above statements is/are correct ?

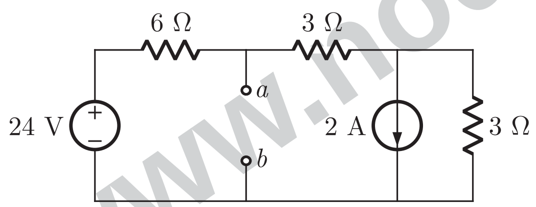
(A) 1 and 3

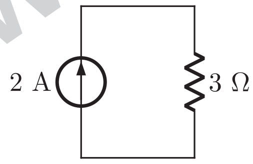
(B) 2 and 3

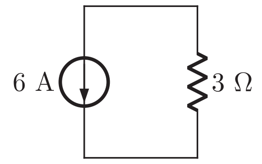
(C) 1 only

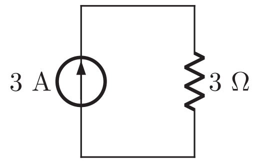
(D) 3 only

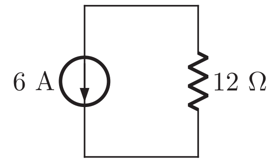
MCQ 5.1.29 The Norton equivalent circuit of the given network with respect to the terminal $a$-$b$, is



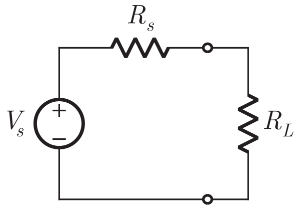

(A) 


(B) 


(C) 


(D) 

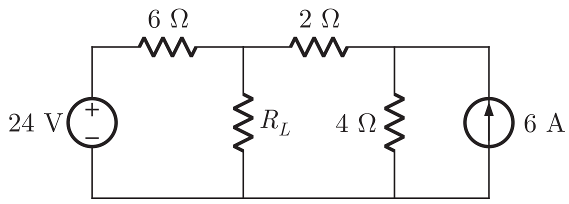

MCQ 5.1.30 In the circuit below, if $R_L$ is fixed and $R_s$ is variable then for what value of $R_s$ power dissipated in $R_L$ will be maximum ?




(A) $R_S = R_L$ (B) $R_S = 0$

(C) $R_S = R_L/2$ (D) $R_S = 2R_L$

MCQ 5.1.31 In the circuit shown below the maximum power transferred to $R_L$ is $P_{max}$, then




(A) $R_L = 12\,\Omega,\ P_{max} = 12$ W
(B) $R_L = 3\,\Omega,\ P_{max} = 96$ W
(C) $R_L = 3\,\Omega,\ P_{max} = 48$ W
(D) $R_L = 12\,\Omega,\ P_{max} = 24$ W

MCQ 5.1.32 In the circuit shown in figure (A) if current $I_1 = 2$ A, then current $I_2$ and $I_3$ in figure (B) and figure (C) respectively are

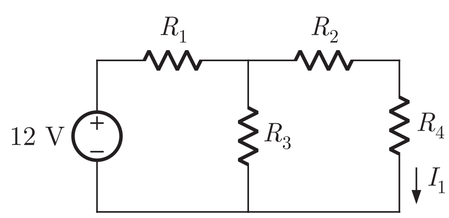



Fig.(A)

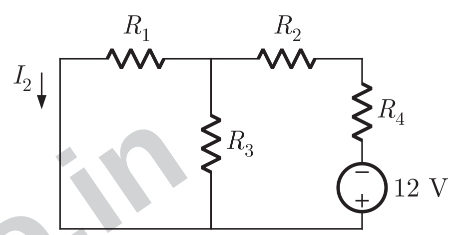



Fig.(B)

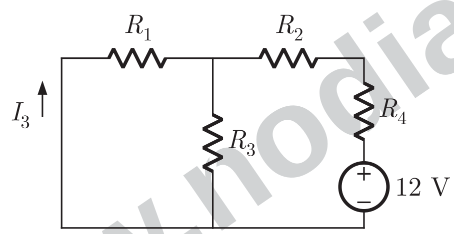



Fig.(C)



(A) 2 A, 2 A
(B) $-2$ A, 2 A
(C) 2 A, $-2$ A
(D) $-2$ A, $-2$ A

MCQ 5.1.33 In the circuit of figure (A), if $I_1 = 20$ mA, then what is the value of current $I_2$ in the circuit of figure (B) ?

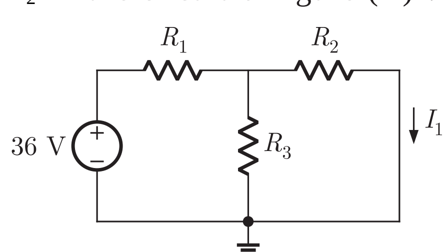



Fig.(A)

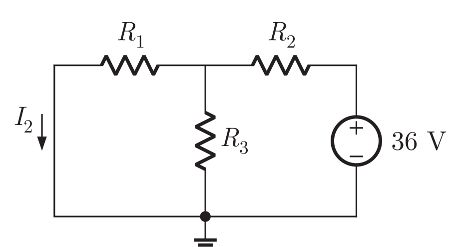



Fig.(B)

(A) 40 mA
(B) $-20$ mA
(C) 20 mA
(D) $R_1$, $R_2$ and $R_3$ must be known

MCQ 5.1.34 If $V_1 = 2$ V in the circuit of figure (A), then what is the value of $V_2$ in the circuit of figure (B) ?

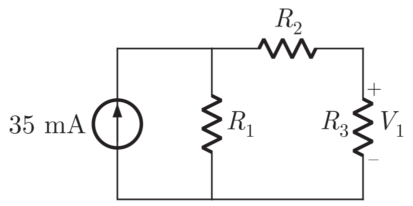



Fig.(A)

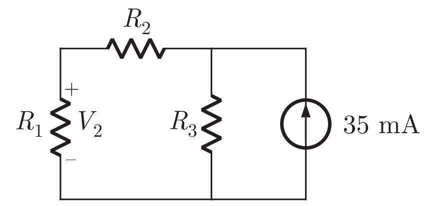
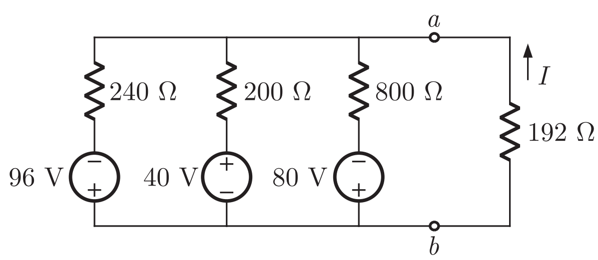



Fig.(B)

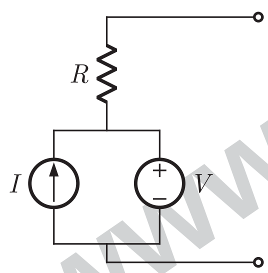
(A) 2 V
(B) $-2$ V
(C) 4 V
(D) $R_1$, $R_2$ and $R_3$ must be known

MCQ 5.1.35 The value of current $I$ in the circuit below is equal to

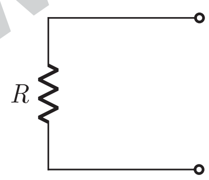



(A)100 mA
(B) 10 mA
(C) 233.34 mA
(D) none of these

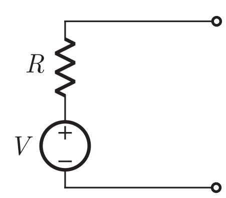
MCQ 5.1.36 A simple equivalent circuit of the two-terminal network shown in figure is



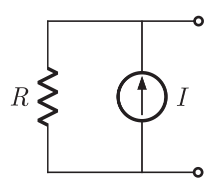

(A)



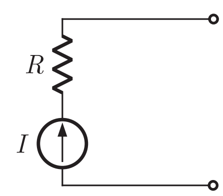

(B)



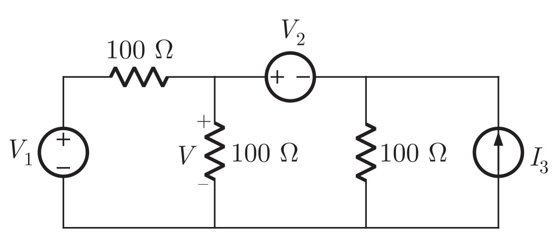

(C)




(D)

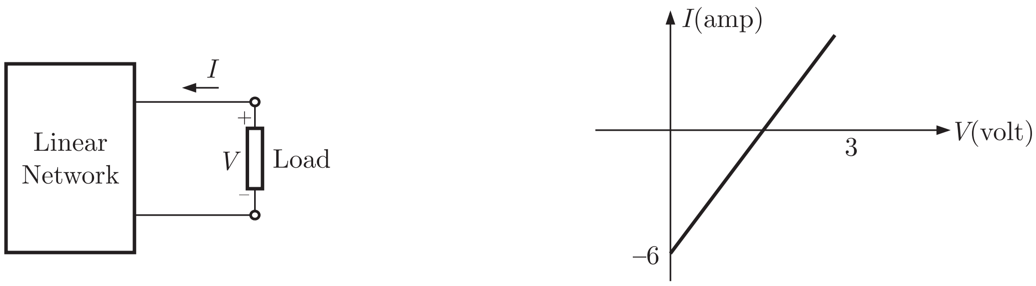



MCQ 5.1.37 If $V = AV_1 + BV_2 + CI_3$ in the following circuit, then values of $A$, $B$ and $C$ respectively are



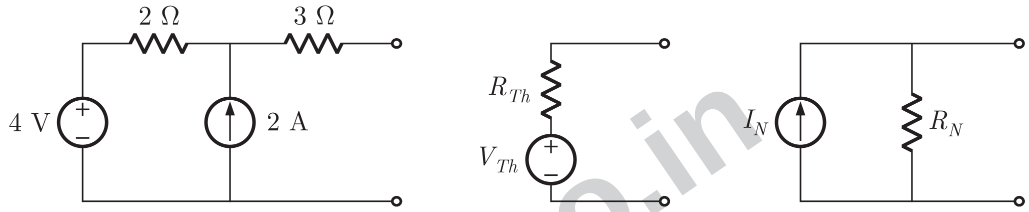

(A) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$
(B) $\frac{1}{3}, \frac{1}{3}, \frac{100}{3}$
(C) $\frac{1}{2}, \frac{1}{2}, \frac{1}{3}$
(D) $\frac{1}{3}, \frac{2}{3}, \frac{100}{3}$

MCQ 5.1.38 For the linear network shown below, $V$-$I$ characteristic is also given in the figure. The value of Norton equivalent current and resistance respectively are




(A) 3 A, 2 Ω (B) 6 Ω, 2 Ω
(C) 6 A, 0.5 Ω (D) 3 A, 0.5 Ω

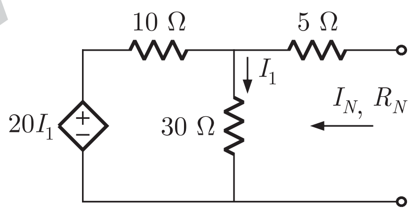
MCQ 5.1.39 In the following circuit a network and its Thevenin and Norton equivalent are given.




The value of the parameter are

| | $V_{Th}$ | $R_{Th}$ | $I_N$ | $R_N$ |
|---|---|---|---|---|
| (A) | 4 V | 2 Ω | 2 A | 2 Ω |
| (B) | 4 V | 2 Ω | 2 A | 3 Ω |
| (C) | 8 V | 1.2 Ω | $\frac{30}{3}$ A | 1.2 Ω |
| (D) | 8 V | 5 Ω | $\frac{8}{5}$ A | 5 Ω |

MCQ 5.1.40 For the following circuit the value of equivalent Norton current $I_N$ and resistance $R_N$ are




(A) 2 A, 20 Ω (B) 2 A, –20 Ω
(C) 0 A, 20 Ω (D) 0 A, –20 Ω

MCQ 5.1.41 Consider the following circuits shown below

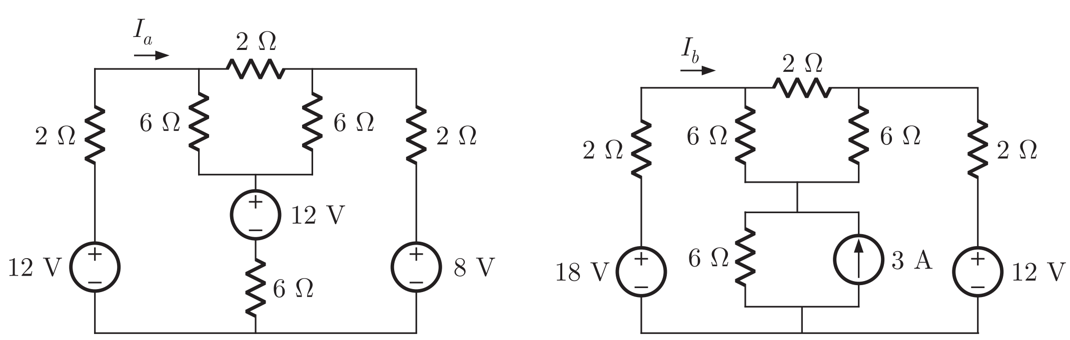



Fig (A) Fig (B)

The relation between $I_a$ and $I_b$ is

(A) $I_b = I_a + 6$

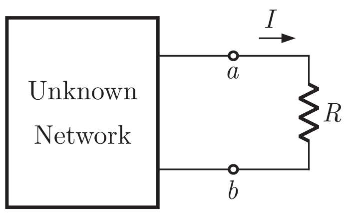
(B) $I_b = I_a + 2$

(C) $I_b = 1.5 I_a$

(D) $I_b = I_a$

## Common Data For Q. 42 and 43 :

In the following circuit, some measurements were made at the terminals $a$, $b$ and given in the table below.




| $R$ | $I$ |
|---|---|
| 3 Ω | 2 A |
| 5 Ω | 1.6 A |

MCQ 5.1.42 The Thevenin equivalent of the unknown network across terminal $a$-$b$ is

(A) 3 Ω, 14 V (B) 5 Ω, 16 V

(C) 16 Ω, 38 V (D) 10 Ω, 26 V

MCQ 5.1.43 The value of $R$ that will cause $I$ to be 1 A, is

(A) 22 Ω (B) 16 Ω

(C) 8 Ω (D) 11 Ω

MCQ 5.1.44 In the circuit shown in fig (A) if current $I_1 = 2.5$ A then current $I_2$ and $I_3$ in fig (B) and (C) respectively are

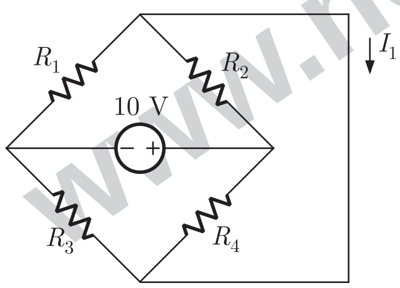



Fig.(A)

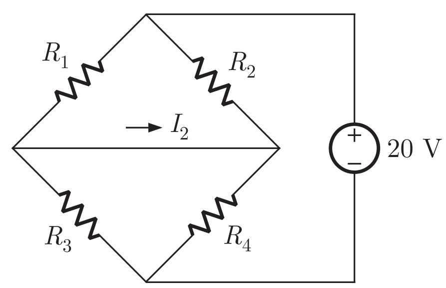



Fig.(B)

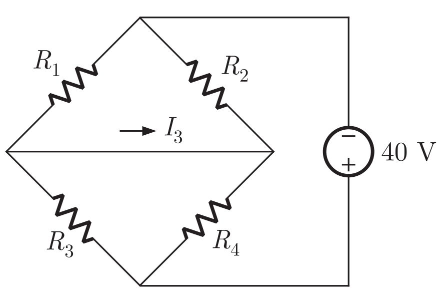
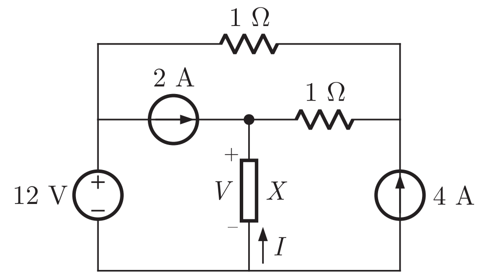



Fig.(C)

(A) 5 A, 10 A (B) −5 A, 10 A

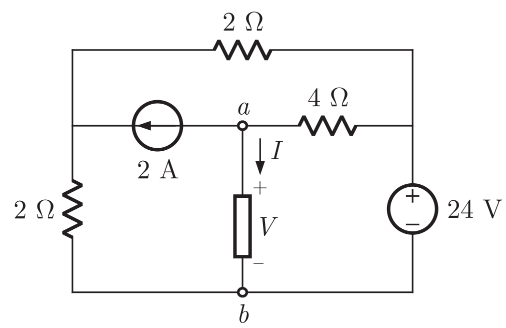
(C) 5 A, −10 A (D) −5 A, −10 A

MCQ 5.1.45 The $V$-$I$ relation of the unknown element $X$ in the given network is $V = AI + B$. The value of $A$ (in ohm) and $B$ (in volt) respectively are

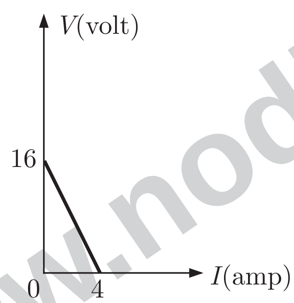



(A) 2, 20
(B) 2, 8
(C) 0.5, 4
(D) 0.5, 16

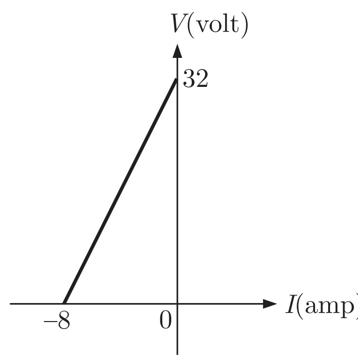
MCQ 5.1.46 For the following network the $V$-$I$ curve with respect to terminals $a$-$b$, is given by



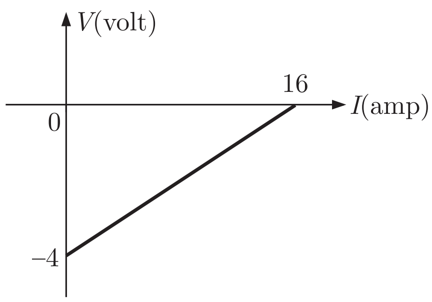

(A)



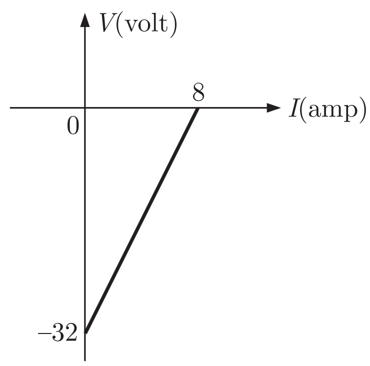

(B)



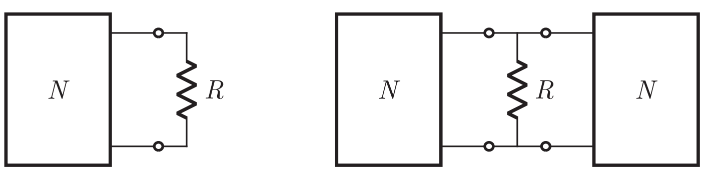

(C)




(D)




MCQ 5.1.47 A network $N$ feeds a resistance $R$ as shown in circuit below. Let the power consumed by $R$ be $P$. If an identical network is added as shown in figure, the power consumed by $R$ will be




(A) equal to $P$
(B) less than $P$
(C) between $P$ and $4P$
(D) more than $4P$

MCQ 5.1.48 A certain network consists of a large number of ideal linear resistors, one of which is $R$ and two constant ideal source. The power consumed by $R$ is $P_1$ when only the first source is active, and $P_2$ when only the second source is active. If both sources are active simultaneously, then the power consumed by $R$ is

(A) $P_1 \pm P_2$
(B) $\sqrt{P_1} \pm \sqrt{P_2}$
(C) $(\sqrt{P_1} \pm \sqrt{P_2})^2$
(D) $(P_1 \pm P_2)^2$

MCQ 5.1.49 If the $60\,\Omega$ resistance in the circuit of figure (A) is to be replaced with a current source $I_s$ and $240\,\Omega$ shunt resistor as shown in figure (B), then magnitude and direction of required current source would be

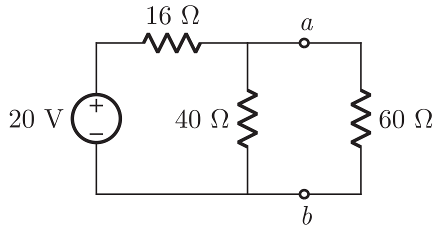


Fig.(A)

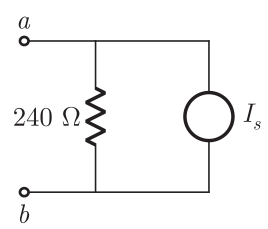
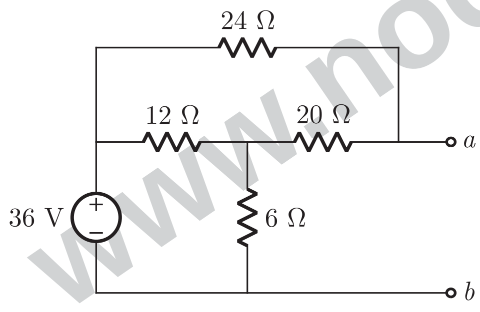


Fig.(B)

(A) 200 mA, upward
(B) 150 mA, downward
(C) 50 mA, downward
(D) 150 mA, upward

MCQ 5.1.50 The Thevenin's equivalent of the circuit shown in the figure is

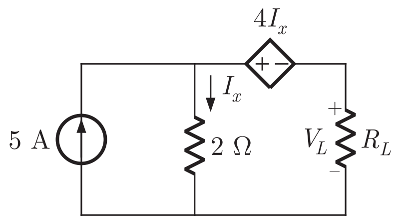



(A) 4 V, $48\,\Omega$
(B) 24 V, $12\,\Omega$
(C) 24 V, $24\,\Omega$
(D) 12 V, $12\,\Omega$

MCQ 5.1.51 The voltage $V_L$ across the load resistance in the figure is given by

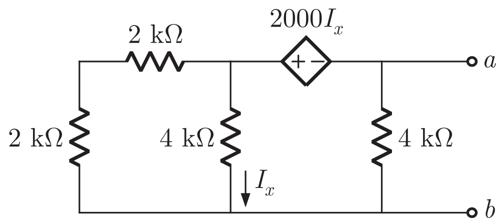
$$V_L = V\left(\frac{R_L}{R + R_L}\right)$$

$V$ and $R$ will be equal to




(A) $-10$ V, $2\,\Omega$
(B) 10 V, $2\,\Omega$
(C) $-10$ V, $-2\,\Omega$
(D) none of these

MCQ 5.1.52 In the circuit given below, viewed from $a$-$b$, the circuit can be reduced to an equivalent circuit as



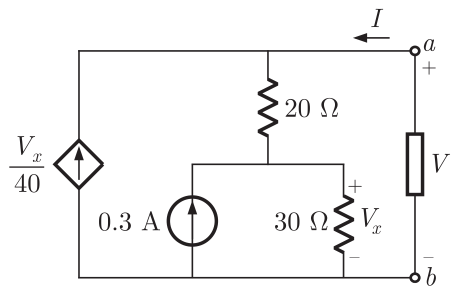

(A) 10 volt source in series with 2 kΩ resistor

(B) 1250 Ω resistor only

(C) 20 V source in series with 1333.34 Ω resistor

(D) 800 Ω resistor only

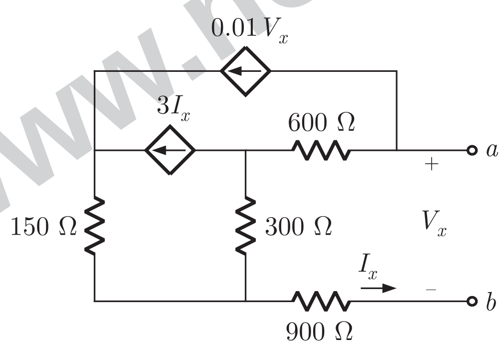
MCQ 5.1.53 The $V$-$I$ equation for the network shown in figure, is given by




(A) $7V = 200I + 54$ (B) $V = 100I + 36$

(C) $V = 200I + 54$ (D) $V = 50I + 54$

MCQ 5.1.54 In the following circuit the value of open circuit voltage and Thevenin resistance at terminals $a, b$ are




(A) $V_{oc} = 100$ V, $R_{Th} = 1800\,\Omega$

(B) $V_{oc} = 0$ V, $R_{Th} = 270\,\Omega$

(C) $V_{oc} = 100$ V, $R_{Th} = 90\,\Omega$

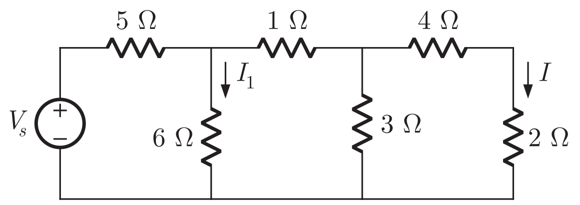
(D) $V_{oc} = 0$ V, $R_{Th} = 90\,\Omega$

***********

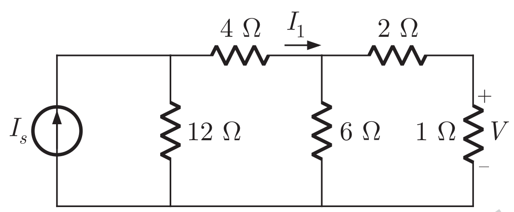
# EXERCISE 5.2



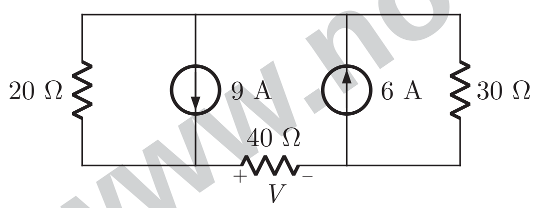
QUES 5.2.1 In the given network, if $V_s = V_0$, $I = 1\text{ A}$ . If $V_s = 2V_0$ then what is the value of $I_1$ (in Amp) ?



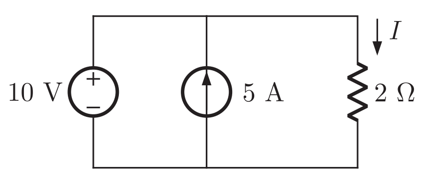

QUES 5.2.2 In the given network, if $I_s = I_0$ then $V = 1$ volt. What is the value of $I_1$ (in Amp) if $I_s = 2I_0$ ?



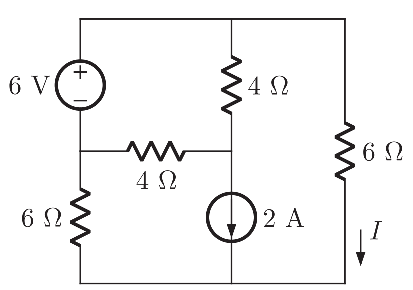

QUES 5.2.3 In the circuit below, the voltage $V$ across the $40\,\Omega$ resistor would be equal to _ _ _ _ Volts.



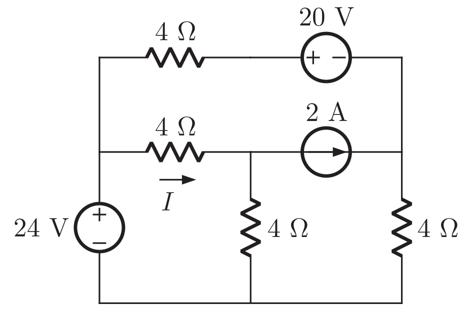

QUES 5.2.4 The value of current $I$ flowing through $2\,\Omega$ resistance in the given circuit, equals to _ _ _ _ Amp.



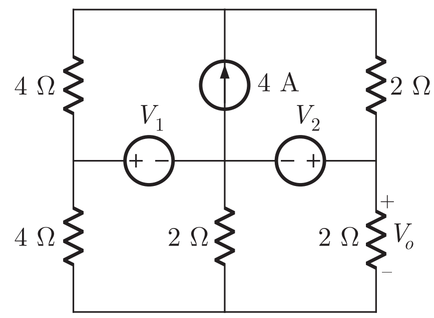

QUES 5.2.5 In the given circuit, the value of current $I$ will be _ _ _ _ _Amps.



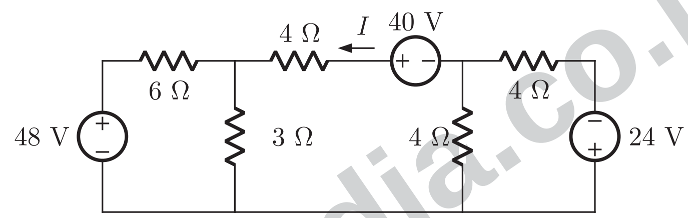

QUES 5.2.6 What is the value of current $I$ in the given network (in Amp) ?



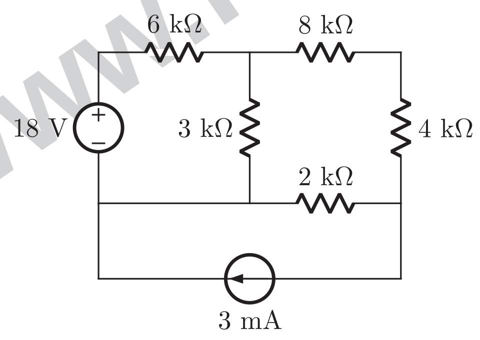

QUES 5.2.7 In the given network if $V_1 = V_2 = 0$, then what is the value of $V_o$ (in volts) ?



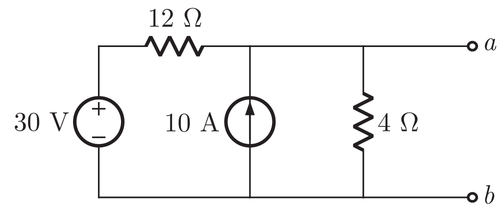

QUES 5.2.8 What is the value of current $I$ in the circuit shown below (in Amp) ?




QUES 5.2.9 How much power is being dissipated by the $4\,\text{k}\Omega$ resistor in the network (in mW) ?

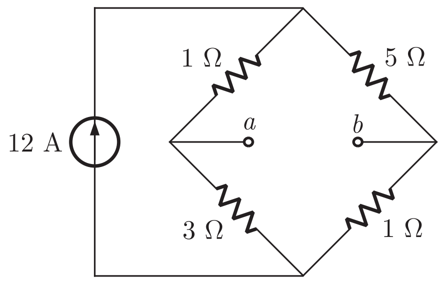



QUES 5.2.10 Thevenin equivalent resistance $R_{Th}$ between the nodes $a$ and $b$ in the following circuit is _ _ _ _ Ω.




Common Data For Q. 11 and 12 :

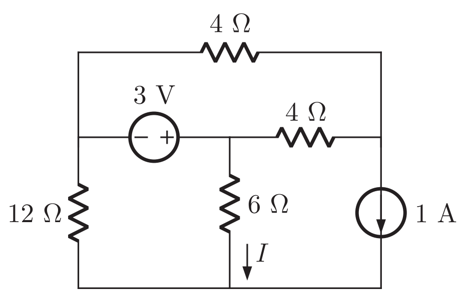
Consider the circuit shown in the figure.



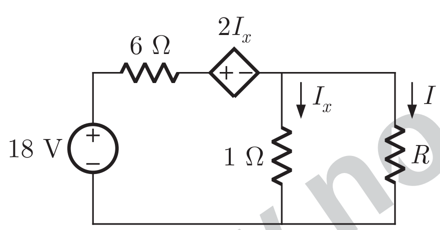

QUES 5.2.11 The equivalent Thevenin voltage across terminal $a$-$b$ is____ Volts.

QUES 5.2.12 The Norton equivalent current with respect to terminal $a$-$b$ is ____ Amps

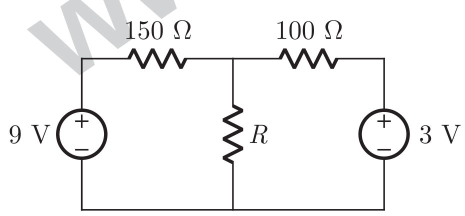
QUES 5.2.13 In the circuit given below, what is the value of current $I$ (in Amp) through $6\,\Omega$ resistor



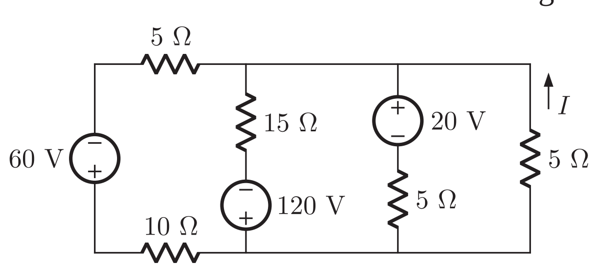

QUES 5.2.14 For the circuit below, what value of $R$ will cause $I = 3$ A (in Ω) ?



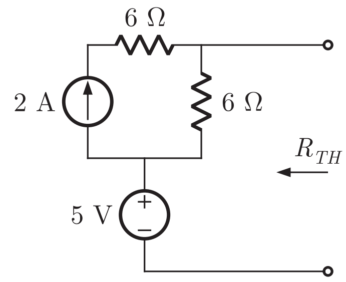

QUES 5.2.15 The maximum power that can be transferred to the resistance $R$ in the circuit is ____ mili watts.



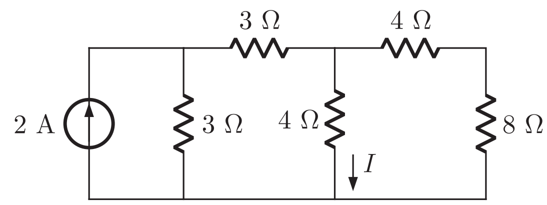

QUES 5.2.16 The value of current $I$ in the following circuit is equal to _____Amp.



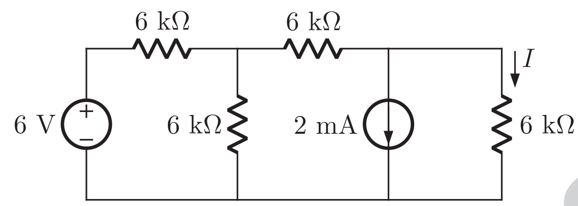

QUES 5.2.17 For the following circuit the value of $R_{Th}$ is____Ω.



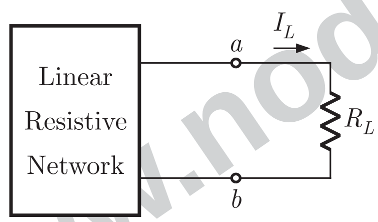

QUES 5.2.18 What is the value of current $I$ in the given network (in Amp) ?




QUES 5.2.19 The value of current $I$ in the figure is _____ mA.



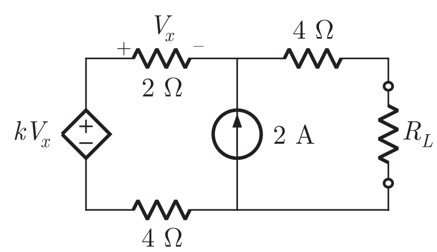

QUES 5.2.20 For the circuit of figure, some measurements were made at the terminals $a$-$b$ and given in the table below.



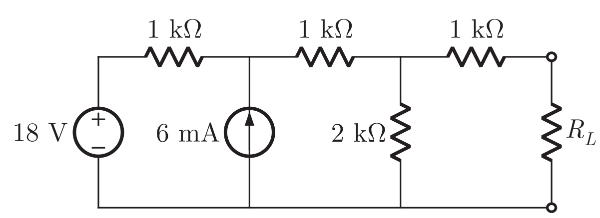

| $R_L$ | $I_L$ |
|---|---|
| 2 Ω | 10 A |
| 10 Ω | 6 A |

What is the value of $I_L$ (in Amps) for $R_L = 20\,\Omega$ ?

QUES 5.2.21 In the circuit below, for what value of $k$, load $R_L = 2\,\Omega$ absorbs maximum power ?

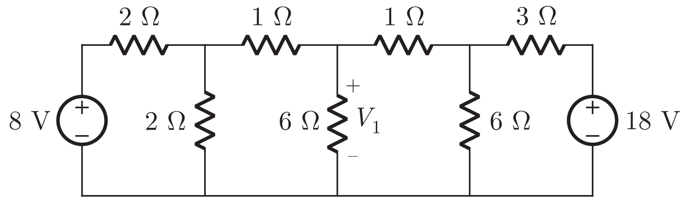



QUES 5.2.22 In the circuit shown below, the maximum power that can be delivered to the load $R_L$ is equal to _____ mW.

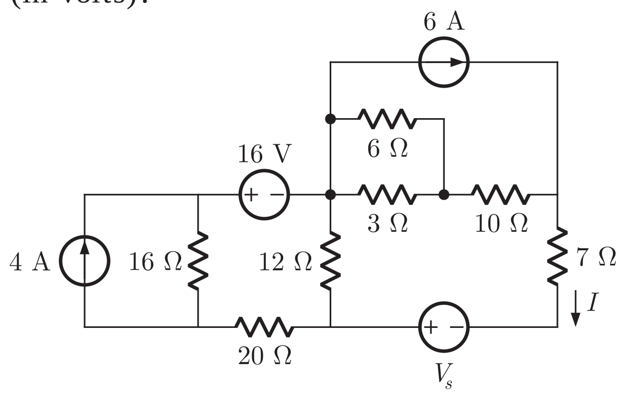



QUES 5.2.23 A practical DC current source provide 20 kW to a $50\,\Omega$ load and 20 kW to a $200\,\Omega$ load. The maximum power, that can drawn from it, is ____ kW.

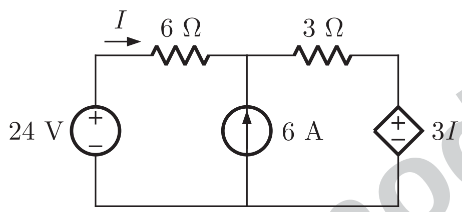
QUES 5.2.24 In the following circuit the value of voltage $V_1$ is _ _ _ _ Volts.



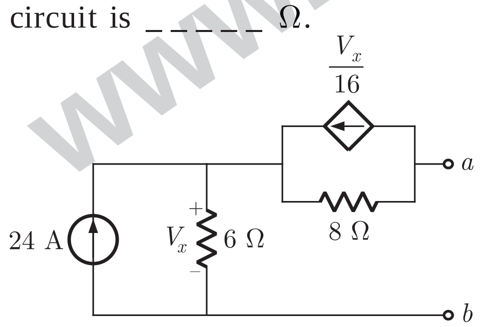

QUES 5.2.25 If $I = 5$ A in the circuit below, then what is the value of voltage source $V_s$ (in volts)?



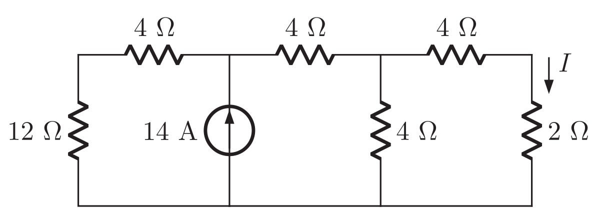

QUES 5.2.26 For the following circuit, what is the value of current $I$ (in Amp) ?



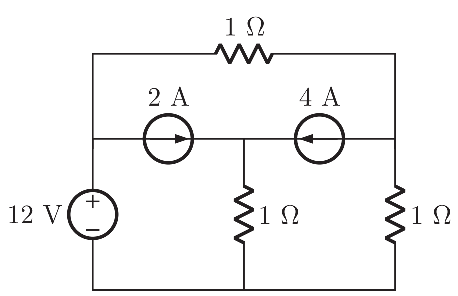

QUES 5.2.27 The Thevenin equivalent resistance between terminal $a$ and $b$ in the following circuit is _ _ _ _ _ Ω.



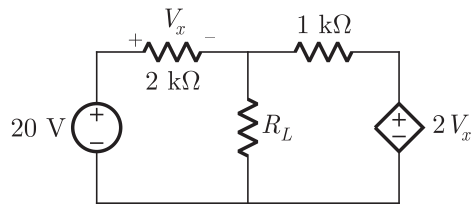

QUES 5.2.28 In the circuit shown below, what is the value of current $I$ (in Amps) ?



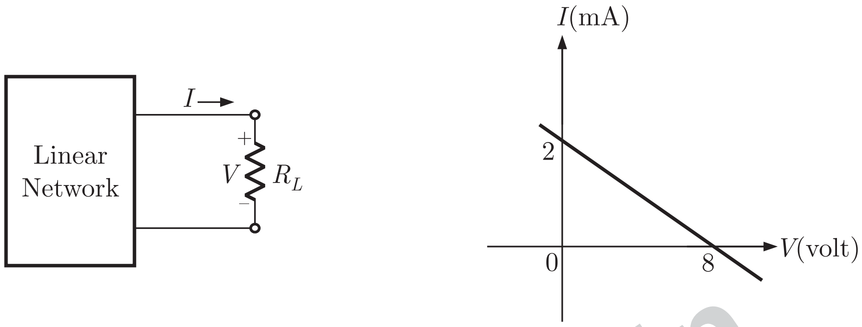

QUES 5.2.29 The power delivered by 12 V source in the given network is _ _ _ _ watts.



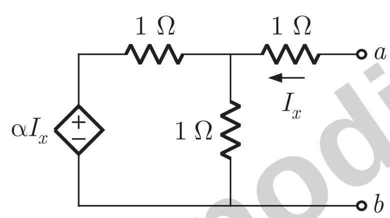
QUES 5.2.30 In the circuit shown, what value of $R_L$ (in Ω) maximizes the power delivered to $R_L$ ?



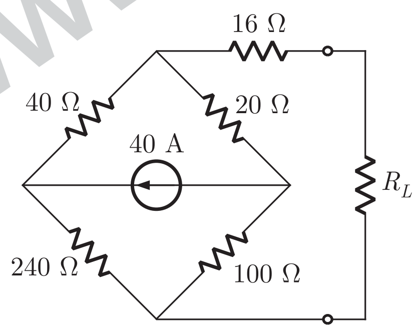

QUES 5.2.31 The $V$-$I$ relation for the circuit below is plotted in the figure. The maximum power that can be transferred to the load $R_L$ will be ______ mW




QUES 5.2.32 In the following circuit equivalent Thevenin resistance between nodes $a$ and $b$ is $R_{Th} = 3\,\Omega$. The value of $\alpha$ is_____

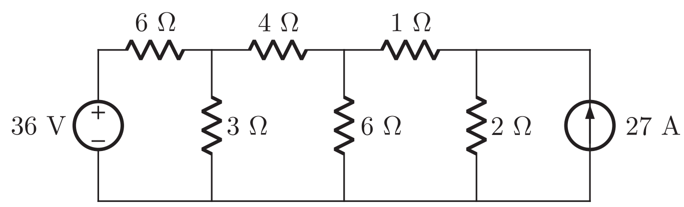



QUES 5.2.33 The maximum power that can be transferred to the load resistor $R_L$ from the current source in the figure is ____ watts.




## Common Data For Q. 34 and 35

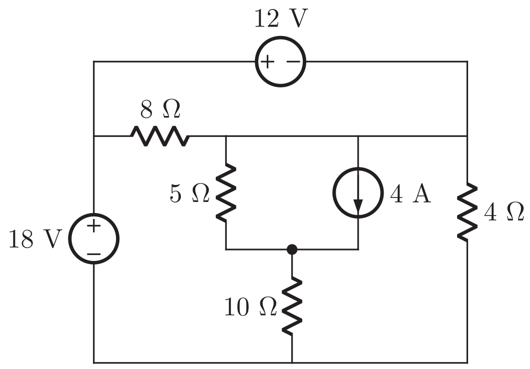
An electric circuit is fed by two independent sources as shown in figure.



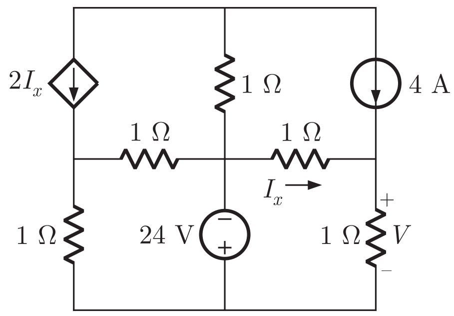

QUES 5.2.34 The power supplied by 36 V source will be _____ watts.

QUES 5.2.35 The power supplied by 27 A source will be _____ watts.

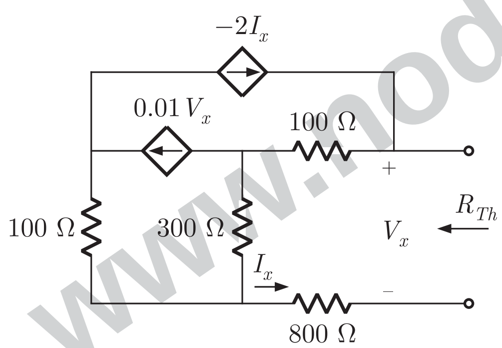
QUES 5.2.36 In the circuit shown in the figure, what is the power dissipated in $4\,\Omega$ resistor (in watts)



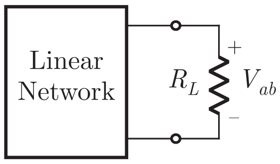

QUES 5.2.37 What is the value of voltage $V$ in the following network (in volts) ?




QUES 5.2.38 For the circuit shown in figure below the value of $R_{Th}$ is ____$\Omega$.




QUES 5.2.39 Consider the network shown below :




The power absorbed by load resistance $R_L$ is shown in table :

| | | |
|---|---|---|
| $R_L$ | 10 kΩ | 30 kΩ |
| $P$ | 3.6 mW | 4.8 mW |

The value of $R_L$ (in kΩ), that would absorb maximum power, is____

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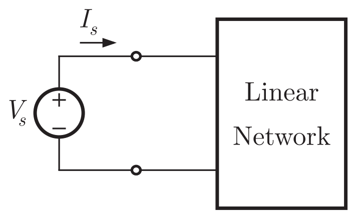
# SOLUTIONS 5.1

SOL 5.1.1 Option (B) is correct.




For, $V_s = 10\text{ V},\ P = 40\text{ W}$

So, $I_s = \dfrac{P}{V_s} = \dfrac{40}{10} = 4\text{ A}$

Now, $V_s' = 5\text{ V}$, so $I_s' = 2\text{ A}$ (From linearity)

New value of the power supplied by source is

$$P_s' = V_s' I_s' = 5 \times 2 = 10\text{ W}$$

**Note:** Linearity does not apply to power calculations.

SOL 5.1.2 Option (C) is correct.

From linearity, we know that in the circuit $\dfrac{V_s}{I_L}$ ratio remains constant

$$\frac{V_s}{I_L} = \frac{20}{200 \times 10^{-3}} = 100$$

Let current through load is $I_L'$ when the power absorbed is 2.5 W, so

$$P_L = (I_L')^2 R_L$$

$$2.5 = (I_L')^2 \times 10$$

$$I_L' = 0.5\text{ A}$$

$$\frac{V_s}{I_L} = \frac{V_s'}{I_L'} = 100$$

So, $V_s' = 100 I_L' = 100 \times 0.5 = 50\text{ V}$

Thus required values are

$$I_L' = 0.5\text{ A},\ V_s' = 50\text{ V}$$

SOL 5.1.3 Option (D ) is correct.

From linearity,

$$I_L = AV_s + BI_s, \quad A \text{ and } B \text{ are constants}$$

From the table $2 = 14A + 6B$ ...(1)

$6 = 18A + 2B$ ...(2)

Solving equation (1) & (2)

$$A = 0.4,\ B = -0.6$$

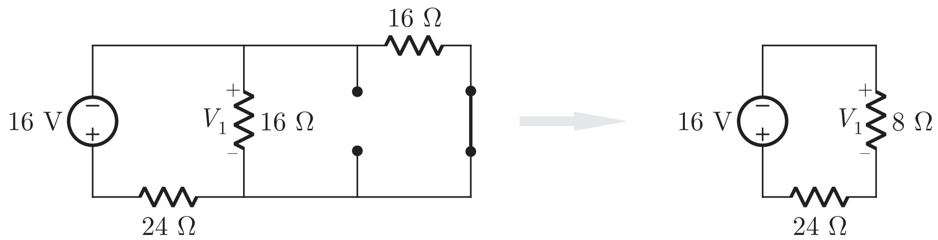
So, $I_L = 0.4V_s - 0.6I_s$

SOL 5.1.4 Option (B) is correct.

The circuit has 3 independent sources, so we apply superposition theorem to obtain the voltage drop.

**Due to 16 V source only :** (Open circuit 5 A source and Short circuit 32 V source)

Let voltage across $R_2$ due to 16 V source only is $V_1$.

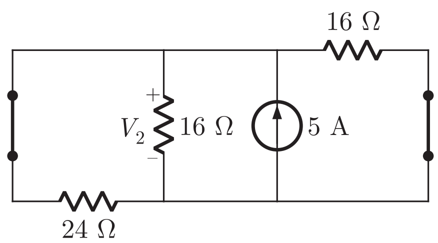



Using voltage division

$$V_1 = -\frac{8}{24+8}(16)$$
$$= -4 \text{ V}$$

**Due to 5 A source only :** (Short circuit both the 16 V and 32 V sources)

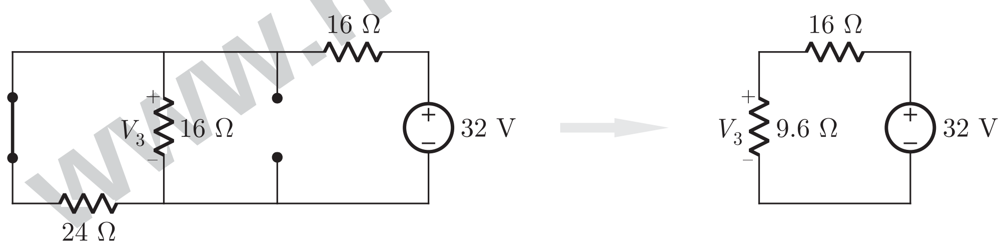
Let voltage across $R_2$ due to 5 A source only is $V_2$.




$$V_2 = (24\,\Omega\,||\,16\,\Omega\,||\,16\,\Omega) \times 5$$
$$= 6 \times 5 = 30 \text{ volt}$$

**Due to 32 V source only :** (Short circuit 16 V source and open circuit 5 A source)

Let voltage across $R_2$ due to 32 V source only is $V_3$




Using voltage division

$$V_3 = \frac{9.6}{16+9.6}(32) = 12 \text{ V}$$

By superposition, the net voltage across $R_2$ is

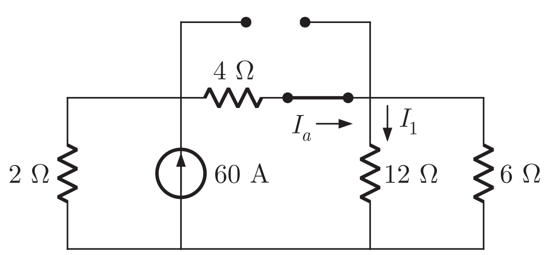
$$V = V_1 + V_2 + V_3 = -4 + 30 + 12 = 38 \text{ volt}$$

ALTERNATIVE METHOD :

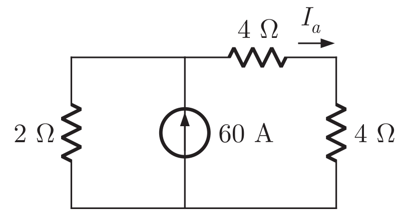
The problem may be solved by applying a node equation at the top node.

SOL 5.1.5 Option (C) is correct

**Due to 60 A Source Only :** (Open circuit 30 A and short circuit 30 V sources)




$$12\,\Omega\,||\,6\,\Omega = 4\,\Omega$$



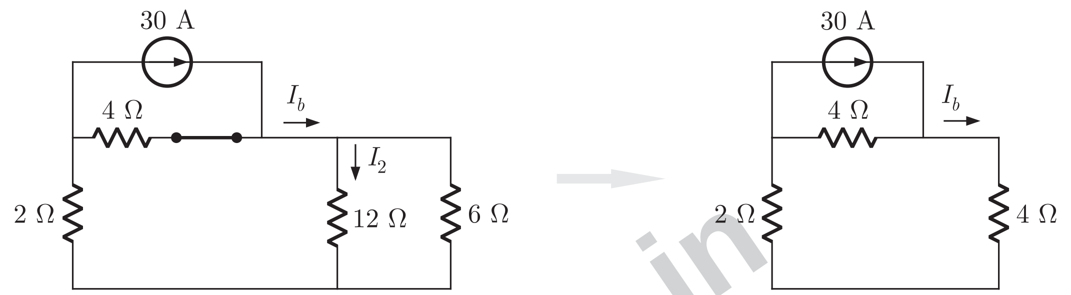

Using current division

$$I_a = \frac{2}{2+8}(60) = 12\text{ A}$$

Again, $I_a$ will be distributed between parallel combination of $12\,\Omega$ and $6\,\Omega$

$$I_1 = \frac{6}{12+6}(12) = 4\text{ A}$$

Due to 30 A source only : (Open circuit 60 A and short circuit 30 V sources)



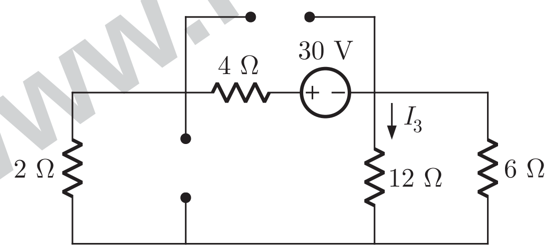

Using current division

$$I_b = \frac{4}{4+6}(30) = 12\text{ A}$$

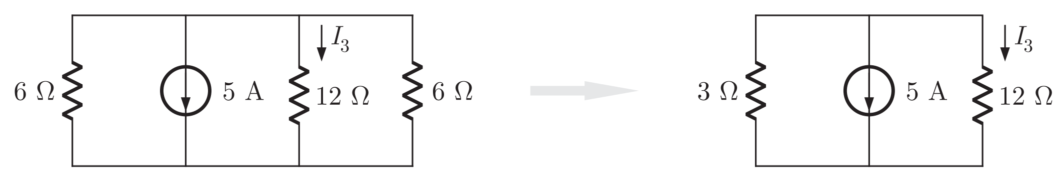
$I_b$ will be distributed between parallel combination of $12\,\Omega$ and $6\,\Omega$

$$I_2 = \frac{6}{12+6}(12) = 4\text{ A}$$

Due to 30 V Source Only : (Open circuit 60 A and 30 A sources)




Using source transformation




Using current division

$$I_3 = -\frac{3}{12+3}(5) = -1\text{ A}$$

SOL 5.1.6 Option (C) is correct.

Using superposition, $\qquad I = I_1 + I_2$

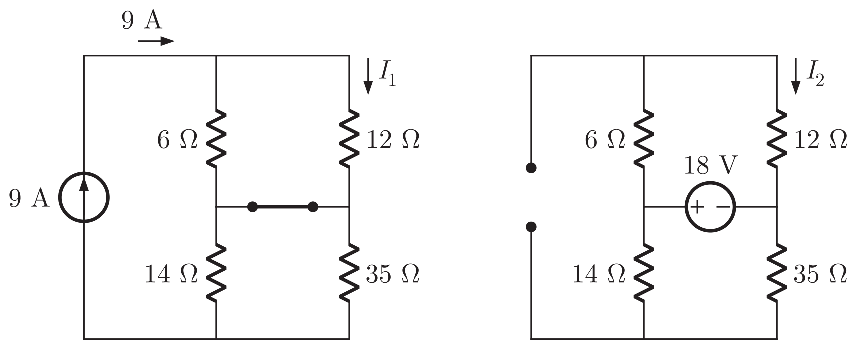
Let $I_1$ is the current due to 9 A source only. (i.e. short 18 V source)

$$I_1 = \frac{6}{6+12}(9) = 3\text{ A} \qquad \text{(current division)}$$

Let $I_2$ is the current due to 18 V source only (i.e. open 9 A source)

$$I_2 = \frac{18}{6+12} = 1\text{ A}$$

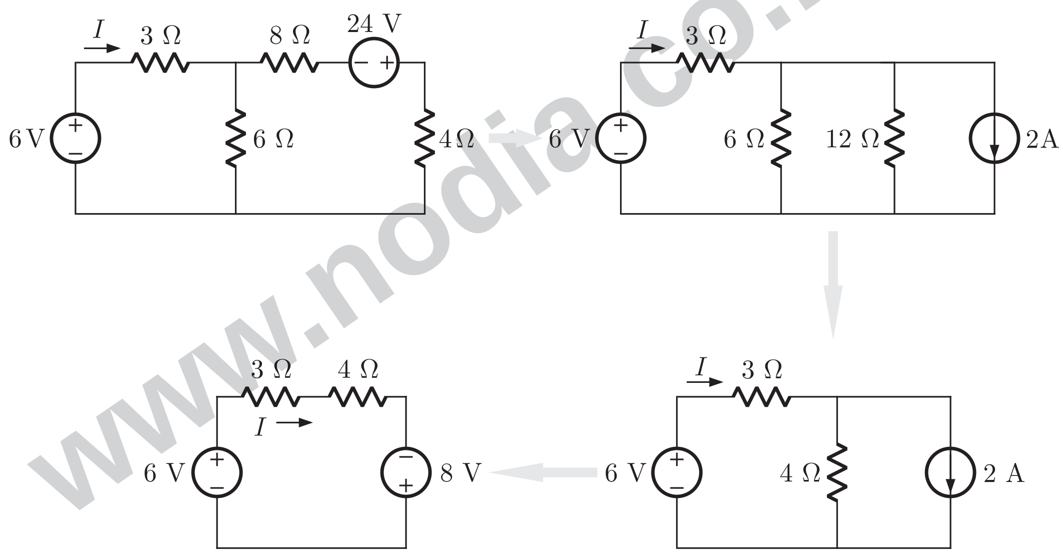
So, $I_1 = 3\text{ A},\ I_2 = 1\text{ A}$




SOL 5.1.7 Option (B) is correct.
From superposition theorem, it is known that if all source values are doubled, then node voltages also be doubled.

SOL 5.1.8 Option (A) is correct.
From the principal of superposition, doubling the values of voltage source doubles the mesh currents.

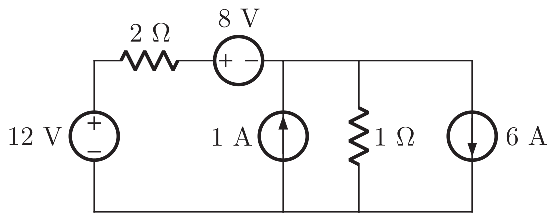
SOL 5.1.9 Option (C) is correct.
Using source transformation, we can obtain $I$ in following steps.



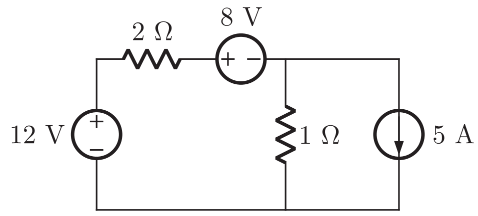

$$I = \frac{6+8}{3+4} = \frac{14}{7} = 2\text{ A}$$

ALTERNATIVE METHOD :
Try to solve the problem by obtaining Thevenin equivalent for right half of the circuit.

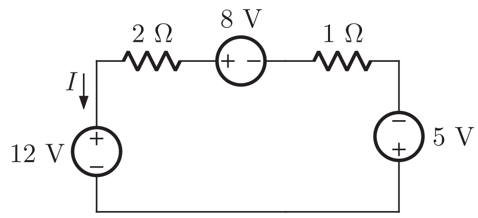
SOL 5.1.10 Option (D) is correct.
Using source transformation of 4 A and 6 V source.




Adding parallel current sources




Source transformation of 5 A source




Applying KVL around the anticlockwise direction

$$-5 - I + 8 - 2I - 12 = 0$$

$$-9 - 3I = 0$$

$$I = -3\text{ A}$$

Power absorbed by 12 V source

$$P_{12\text{ V}} = 12 \times I \qquad \text{(Passive sign convention)}$$

$$= 12 \times -3 = -36\text{ W}$$

or, 12 V source supplies 36 W power.



SOL 5.1.11 Option (B) is correct.

We know that source transformation also exists for dependent source, so


Current source values

$$I_s = \frac{6I_x}{2} = 3I_x \text{ (downward)}$$

$$R_s = 2\,\Omega$$

SOL 5.1.12 Option (C) is correct.

We know that source transformation is applicable to dependent source also.

Values of equivalent voltage source

$$V_s = (4I_x)(5) = 20I_x$$

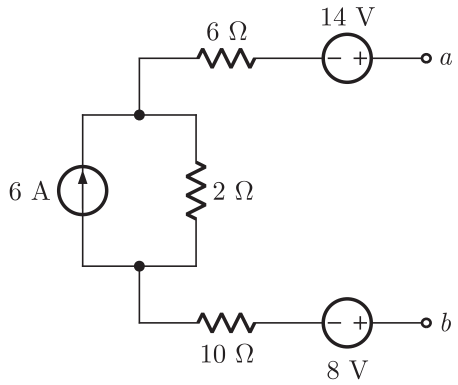
$$R_s = 5\,\Omega$$


SOL 5.1.13 Option (C) is correct.

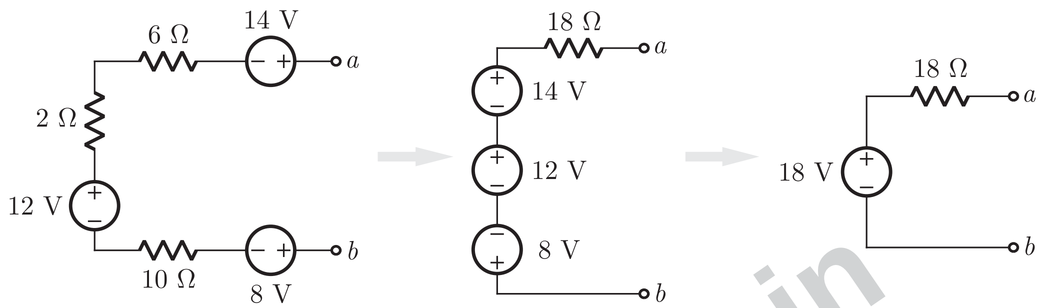
Combining the parallel resistance and adding the parallel connected current sources.

$$9\text{ A} - 3\text{ A} = 6\text{ A (upward)}$$

$$3\,\Omega \,||\, 6\,\Omega = 2\,\Omega$$



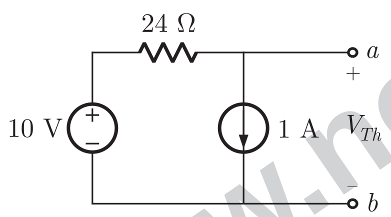

Source transformation of 6 A source




SOL 5.1.14 Option (D) is correct.

Thevenin Voltage : (Open Circuit Voltage)

The open circuit voltage between $a$-$b$ can be obtained as



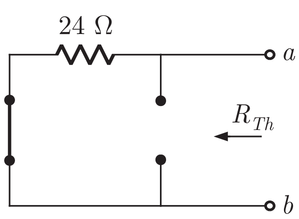

Writing KCL at node $a$

$$\frac{V_{Th} - 10}{24} + 1 = 0$$

$$V_{Th} - 10 + 24 = 0 \text{ or } V_{Th} = -14 \text{ volt}$$

Thevenin Resistance :

To obtain Thevenin's resistance, we set all independent sources to zero i.e., short circuit all the voltage sources and open circuit all the current sources.




$$R_{Th} = 24\,\Omega$$

SOL 5.1.15 Option (B) is correct.

Thevenin Voltage :

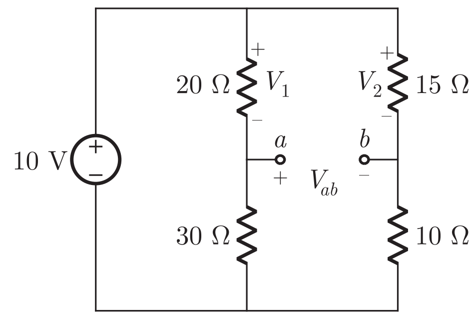
Using voltage division $\quad V_1 = \frac{20}{20+30}(10) = 4 \text{ volt}$

and, $\quad V_2 = \frac{15}{15+10}(10) = 6 \text{ volt}$

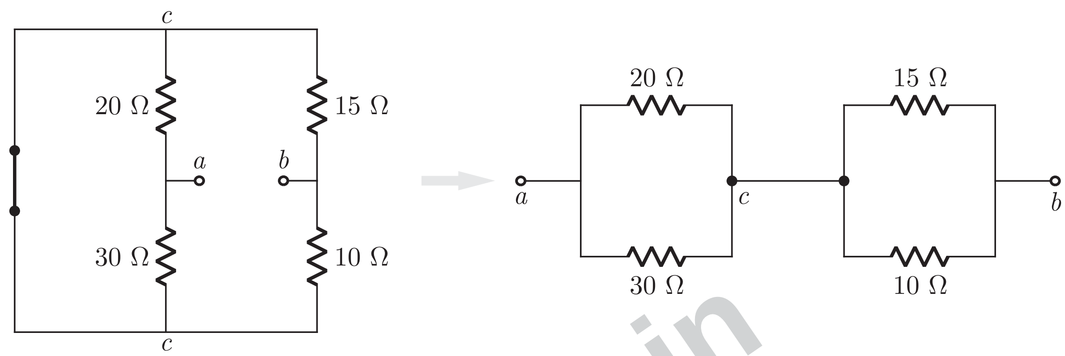
Applying KVL, $V_1 - V_2 + V_{ab} = 0$

$$4 - 6 + V_{ab} = 0$$

$$V_{Th} = V_{ab} = -2 \text{ volt}$$




Thevenin Resistance :

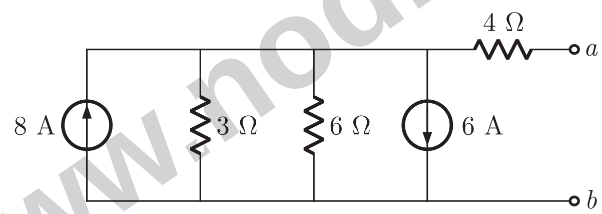



$$R_{ab} = [20\,\Omega\,||\,30\,\Omega] + [15\,\Omega\,||\,10\,\Omega] = 12\,\Omega + 6\,\Omega = 18\,\Omega$$

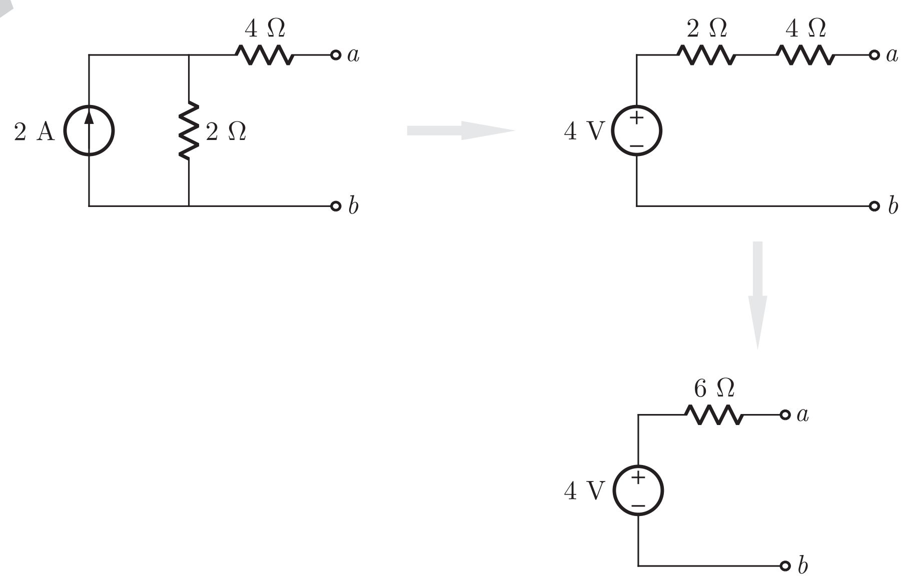
$$R_{Th} = R_{ab} = 18\,\Omega$$

SOL 5.1.16 Option (A) is a correct.

Using source transformation of 24 V source



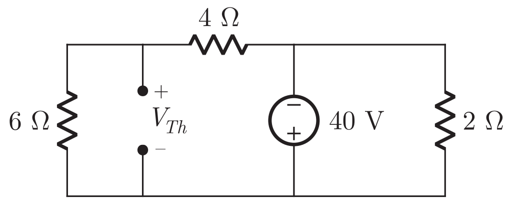

Adding parallel connected sources




So, $V_{Th} = 4\text{ V},\ R_{Th} = 6\,\Omega$

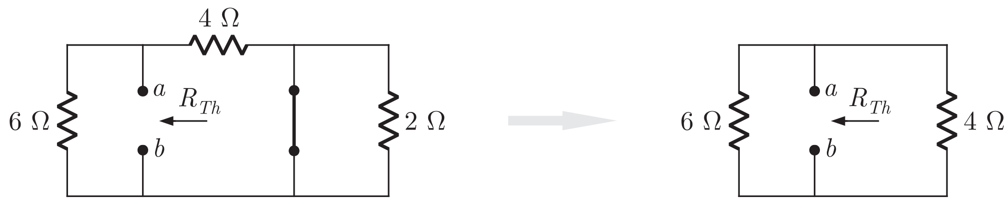
SOL 5.1.17 Option (A) is correct.

Thevenin Voltage: (Open Circuit Voltage)




$$V_{Th} = \frac{6}{6+4}(-40) = -24 \text{ volt} \qquad \text{(using voltage division)}$$

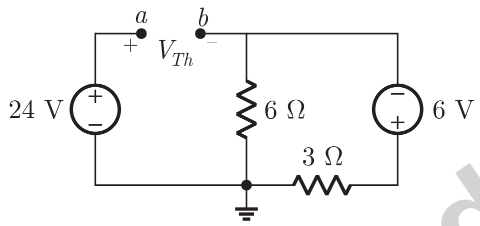
Thevenin Resistance :




$$R_{Th} = 6\,\Omega\,||\,4\,\Omega = \frac{6\times 4}{6+4} = 2.4\,\Omega$$

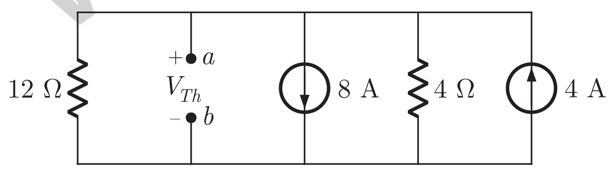
SOL 5.1.18 Option (B) is correct.

For the circuit of figure (A)




$$V_{Th} = V_a - V_b$$
$$V_a = 24 \text{ V}$$
$$V_b = \frac{6}{6+3}(-6) = -4 \text{ V} \qquad \text{(Voltage division)}$$
$$V_{Th} = 24 - (-4) = 28 \text{ V}$$

For the circuit of figure (B), using source transformation

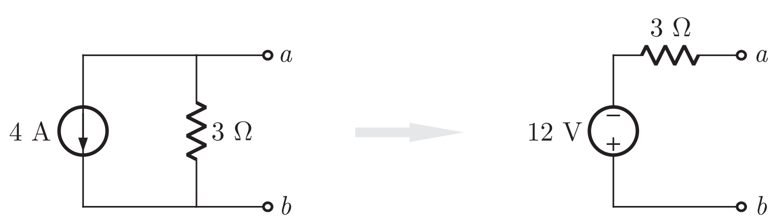



Combining parallel resistances,

$$12\,\Omega\,||\,4\,\Omega = 3\,\Omega$$

Adding parallel current sources,

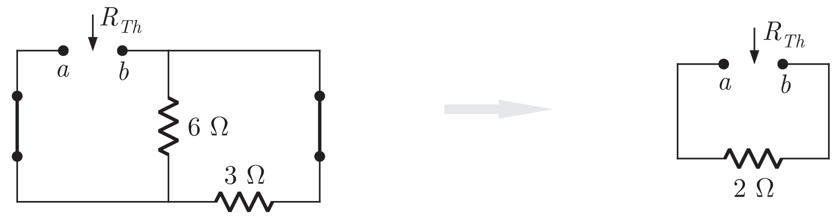
$$8 - 4 = 4 \text{ A (downward)}$$




$$V_{Th} = -12 \text{ V}$$

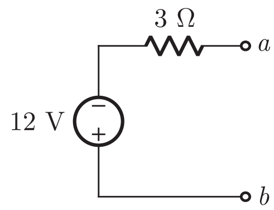
SOL 5.1.19 Option (C) is correct.

For the circuit for fig (A)



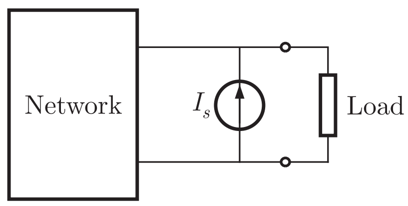

$$R_{Th} = R_{ab} = 6\,\Omega \,||\, 3\,\Omega = 2\,\Omega$$

For the circuit of fig (B), as obtained in previous solution.




$$R_{Th} = 3\,\Omega$$

SOL 5.1.20 Option (B) is correct.



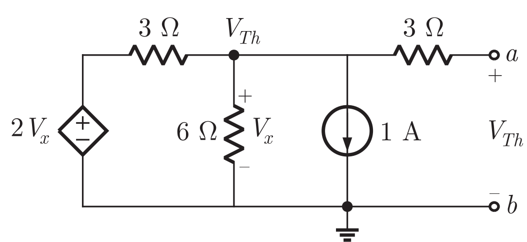

The current source connected in parallel with load does not affect Thevenin equivalent circuit. Thus, Thevenin equivalent circuit will contain its usual form of a voltage source in series with a resistor.

SOL 5.1.21 Option (C) is correct.

The network consists of resistor and dependent sources because if it has independent source then there will be an open circuit Thevenin voltage present.

SOL 5.1.22 Option (D) is correct.

Thevenin Voltage (Open Circuit Voltage) :




Applying KCL at top middle node

$$\frac{V_{Th} - 2V_x}{3} + \frac{V_{Th}}{6} + 1 = 0$$

$$\frac{V_{Th} - 2V_{Th}}{3} + \frac{V_{Th}}{6} + 1 = 0 \qquad (V_{Th} = V_x)$$

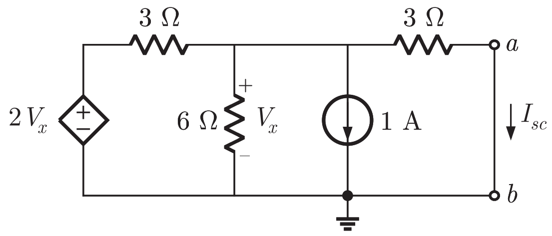
$$-2V_{Th} + V_{Th} + 6 = 0$$

$$V_{Th} = 6 \text{ volt}$$

Thevenin Resistance :

$$R_{Th} = \frac{\text{Open circuit voltage}}{\text{Short circuit current}} = \frac{V_{Th}}{I_{sc}}$$

To obtain Thevenin resistance, first we find short circuit current through $a$-$b$



Writing KCL at top middle node

$$\frac{V_x - 2V_x}{3} + \frac{V_x}{6} + 1 + \frac{V_x - 0}{3} = 0$$

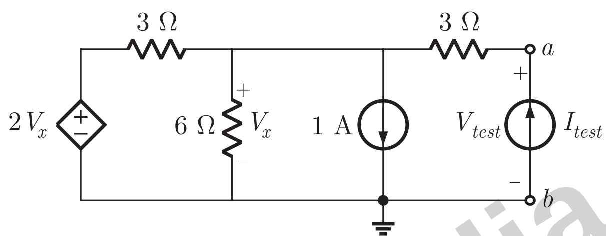
$$-2V_x + V_x + 6 + 2V_x = 0 \text{ or } V_x = -6 \text{ volt}$$

$$I_{sc} = \frac{V_x - 0}{3} = -\frac{6}{3} = -2 \text{ A}$$

Thevenin's resistance, $R_{Th} = \frac{V_{Th}}{I_{sc}} = -\frac{6}{2} = -3\,\Omega$

ALTERNATIVE METHOD :

Since dependent source is present in the circuit, we put a test source across $a$-$b$ to obtain Thevenin's equivalent.





By applying KCL at top middle node

$$\frac{V_x - 2V_x}{3} + \frac{V_x}{6} + 1 + \frac{V_x - V_{test}}{3} = 0$$

$$-2V_x + V_x + 6 + 2V_x - 2V_{test} = 0$$

$$2V_{test} - V_x = 6 \qquad ...(1)$$

We have $I_{test} = \frac{V_{test} - V_x}{3}$

$$3I_{test} = V_{test} - V_x$$

$$V_x = V_{test} - 3I_{test}$$

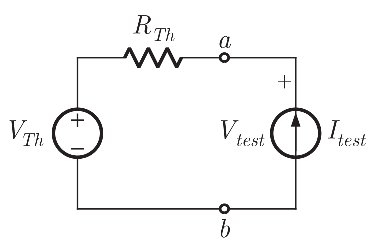
Put $V_x$ into equation (1)

$$2V_{test} - (V_{test} - 3I_{test}) = 6$$

$$2V_{test} - V_{test} + 3I_{test} = 6$$

$$V_{test} = 6 - 3I_{test} \qquad ...(2)$$

For Thevenin's equivalent circuit



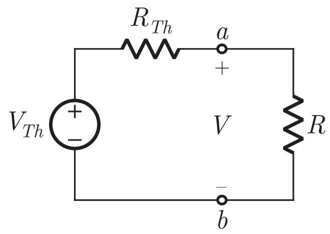
$$\frac{V_{test} - V_{Th}}{R_{Th}} = I_{test}$$

$$V_{test} = V_{Th} + R_{Th} I_{test} \qquad ...(3)$$

Comparing equation (2) and (3)

$$V_{Th} = 6 \text{ V},\ R_{Th} = -3\,\Omega$$

SOL 5.1.23 Option (D) is correct.




Using voltage division

$$V = V_{Th}\left(\frac{R}{R+R_{Th}}\right)$$

From the table,

$$6 = V_{Th}\left(\frac{3}{3+R_{Th}}\right) \quad ...(1)$$

$$8 = V_{Th}\left(\frac{8}{8+R_{Th}}\right) \quad ...(2)$$

Dividing equation (1) and (2), we get

$$\frac{6}{8} = \frac{3(8+R_{Th})}{8(3+R_{Th})}$$

$$6+2R_{Th} = 8+R_{Th}$$

$$R_{Th} = 2\,\Omega$$

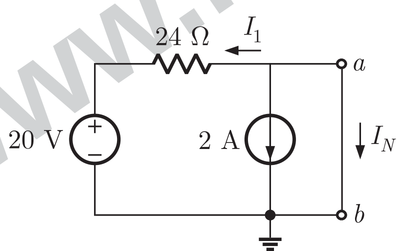
Substituting $R_{Th}$ into equation (1)

$$6 = V_{Th}\left(\frac{3}{3+2}\right) \text{ or } V_{Th} = 10\text{ V}$$

SOL 5.1.24 Option (C) is correct.

Norton Current : (Short Circuit Current)

The Norton equivalent current is equal to the short-circuit current that would flow when the load replaced by a short circuit as shown below




Applying KCL at node $a$

$$I_N + I_1 + 2 = 0$$

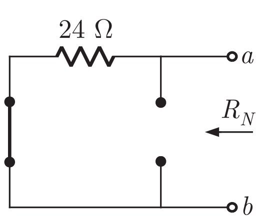
Since $$I_1 = \frac{0-20}{24} = -\frac{5}{6}\text{ A}$$

So, $$I_N - \frac{5}{6} + 2 = 0$$

$$I_N = -\frac{7}{6}\text{ A}$$

Norton Resistance :

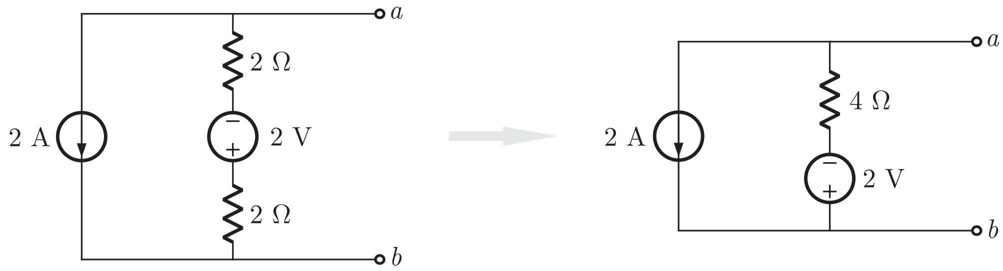
Set all independent sources to zero (i.e. open circuit current sources and short circuit voltage sources) to obtain Norton's equivalent resistance $R_N$.



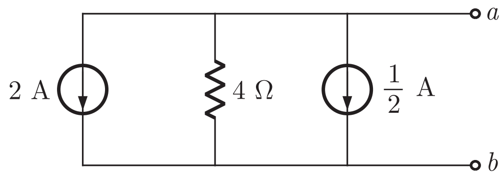

$$R_N = 24\,\Omega$$

SOL 5.1.25 Option (C) is correct.

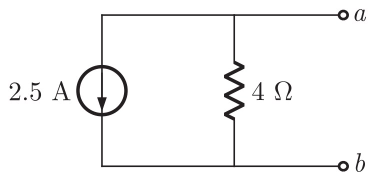
Using source transformation of 1 A source




Again, source transformation of 2 V source




Adding parallel current sources

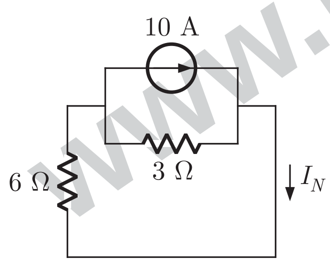



ALTERNATIVE METHOD :

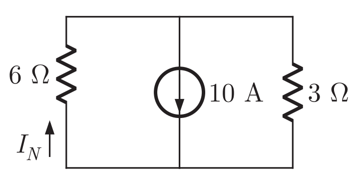
Try to solve the problem using superposition method.

SOL 5.1.26 Option (C) is correct.

Short circuit current across terminal $a$-$b$ is



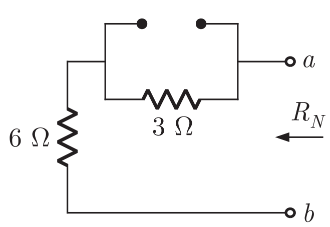

For simplicity circuit can be redrawn as




$$I_N = \frac{3}{3+6}(10) \qquad \text{(Current division)}$$

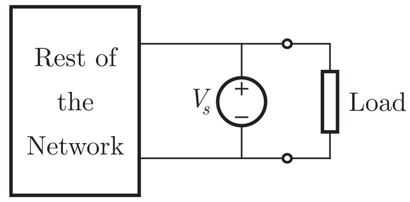
$$= 3.33 \text{ A}$$

Norton's equivalent resistance




$$R_N = 6 + 3 = 9\,\Omega$$

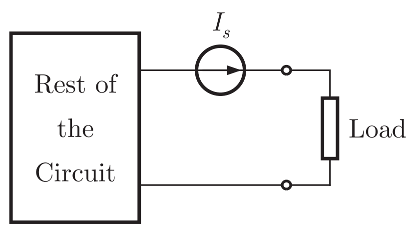
SOL 5.1.27 Option (C) is correct.




The voltage across load terminal is simply $V_s$ and it is independent of any other current or voltage. So, Thevenin equivalent is $V_{Th} = V_s$ and $R_{Th} = 0$ (Voltage source is ideal).

Norton equivalent does not exist because of parallel connected voltage source.

SOL 5.1.28 Option (B) is correct.

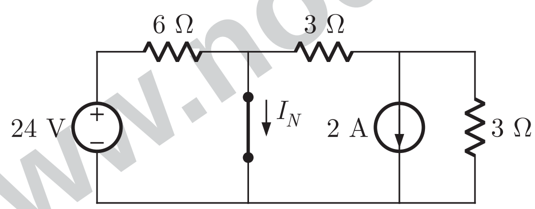



The output current from the network is equal to the series connected current source only, so $I_N = I_s$. Thus, effect of all other component in the network does not change $I_N$.

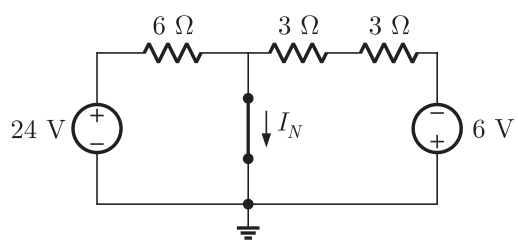
In this case Thevenin's equivalent is not feasible because of the series connected current source.

SOL 5.1.29 Option (C) is correct.

Norton Current : (Short Circuit Current)




Using source transformation



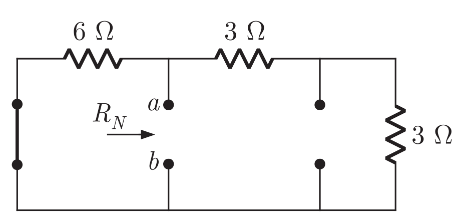

Nodal equation at top center node

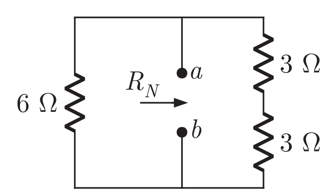
$$\frac{0-24}{6} + \frac{0-(-6)}{3+3} + I_N = 0$$

$$-4 + 1 + I_N = 0$$

$$I_N = 3\text{ A}$$

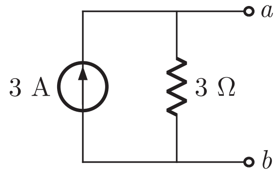
Norton Resistance :



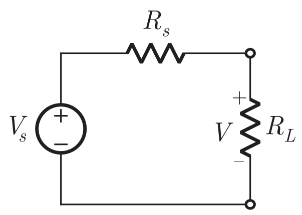



$$R_N = R_{ab} = 6\,||\,(3+3) = 6\,||\,6 = 3\,\Omega$$

So, Norton equivalent will be




SOL 5.1.30 Option (B) is correct.




$$V = V_s\left(\frac{R_L}{R_s + R_L}\right)$$

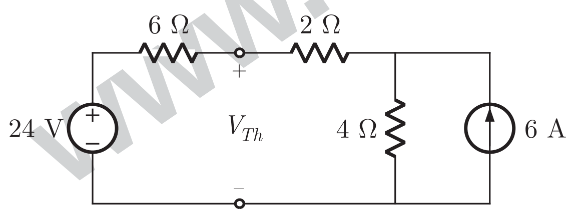
Power absorbed by $R_L$

$$P_L = \frac{(V)^2}{R_L} = \frac{V_s^2 R_L}{(R_s + R_L)^2}$$

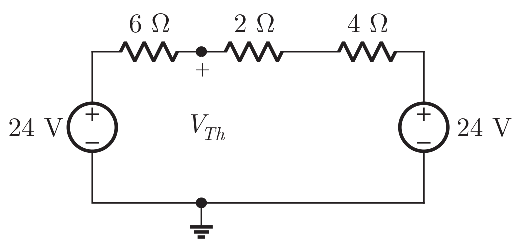
From above expression, it is known that power is maximum when $R_s = 0$

NOTE :
Do not get confused with maximum power transfer theorem. According to maximum power transfer theorem if $R_L$ is variable and $R_s$ is fixed then power dissipated by $R_L$ is maximum when $R_L = R_s$.

SOL 5.1.31 Option (C) is correct.
We solve this problem using maximum power transfer theorem. First, obtain Thevenin equivalent across $R_L$.
Thevenin Voltage : (Open circuit voltage)



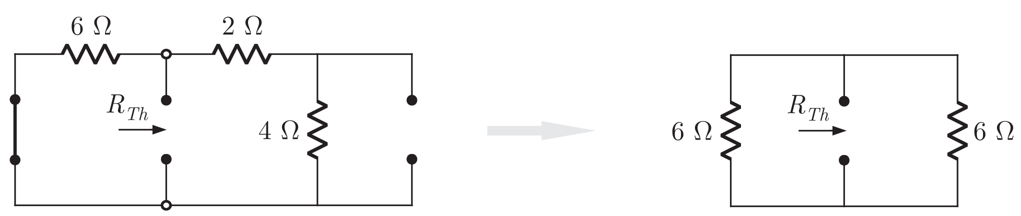

Using source transformation




Using nodal analysis $\dfrac{V_{Th} - 24}{6} + \dfrac{V_{Th} - 24}{2+4} = 0$

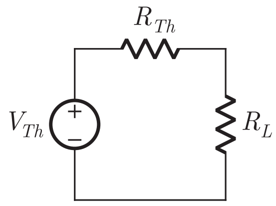
$$2V_{Th} - 48 = 0 \Rightarrow V_{Th} = 24 \text{ V}$$

Thevenin Resistance :



$$R_{Th} = 6\,\Omega\,||\,6\,\Omega = 3\,\Omega$$

Circuit becomes as




For maximum power transfer

$$R_L = R_{Th} = 3\,\Omega$$

Value of maximum power

$$P_{\max} = \frac{(V_{Th})^2}{4R_L} = \frac{(24)^2}{4\times 3} = 48\text{ W}$$

SOL 5.1.32 Option (D) is correct.

This can be solved by reciprocity theorem. But we have to take care that the polarity of voltage source have the same correspondence with branch current in each of the circuit.

In figure (B) and figure (C), polarity of voltage source is reversed with respect to direction of branch current so

$$\frac{V_1}{I_1} = -\frac{V_2}{I_2} = -\frac{V_3}{I_3}$$

$$I_2 = I_3 = -2\text{ A}$$

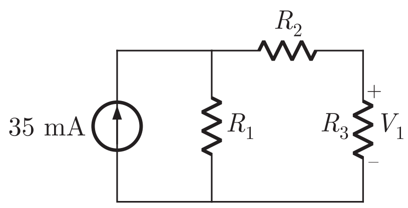
SOL 5.1.33 Option (C) is correct.

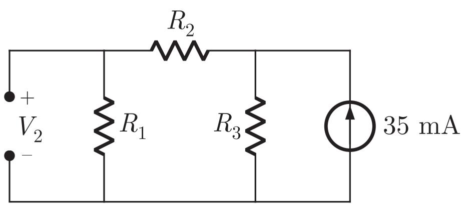
According to reciprocity theorem in any linear bilateral network when a single voltage source $V_a$ in branch $a$ produces a current $I_b$ in branches $b$, then if the voltage source $V_a$ is removed(i.e. branch $a$ is short circuited) and inserted in branch $b$, then it will produce a current $I_b$ in branch $a$.

So, $$I_2 = I_1 = 20\text{ mA}$$

SOL 5.1.34 Option (A) is correct.

According to reciprocity theorem in any linear bilateral network when a single current source $I_a$ in branch $a$ produces a voltage $V_b$ in branches $b$, then if the current source $I_a$ is removed(i.e. branch $a$ is open circuited) and inserted in branch $b$, then it will produce a voltage $V_b$ in branch $a$.




So, $$V_2 = 2\text{ volt}$$

SOL 5.1.35 Option (A) is correct.

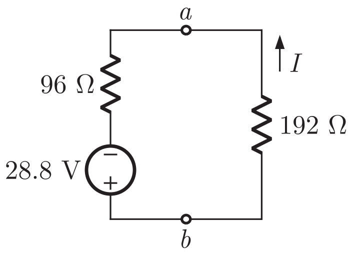
We use Millman's theorem to obtain equivalent resistance and voltage across $a$-$b$.

$$V_{ab} = \frac{-\frac{96}{240} + \frac{40}{200} + \frac{-80}{800}}{\frac{1}{240} + \frac{1}{200} + \frac{1}{800}} = -\frac{144}{5} = -28.8\text{ V}$$

The equivalent resistance

$$R_{ab} = \frac{1}{\frac{1}{240} + \frac{1}{200} + \frac{1}{800}} = 96\,\Omega$$

Now, the circuit is reduced as




$$I = \frac{28.8}{96+192} = 100 \text{ mA}$$

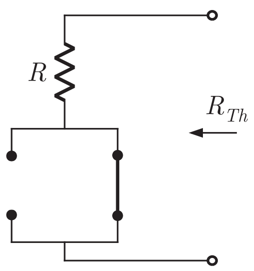
SOL 5.1.36 Option (B) is correct.

Thevenin Voltage: (Open circuit voltage):

The open circuit voltage will be equal to $V$, i.e. $V_{Th} = V$

Thevenin Resistance:

Set all independent sources to zero i.e. open circuit the current source and short circuit the voltage source as shown in figure



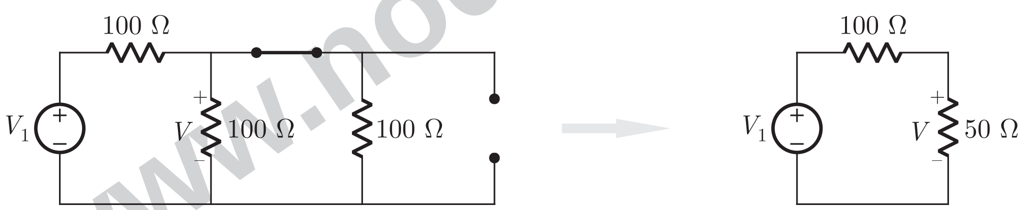

Open circuit voltage $= V_1$



SOL 5.1.37 Option (B) is correct.

$V$ is obtained using super position.

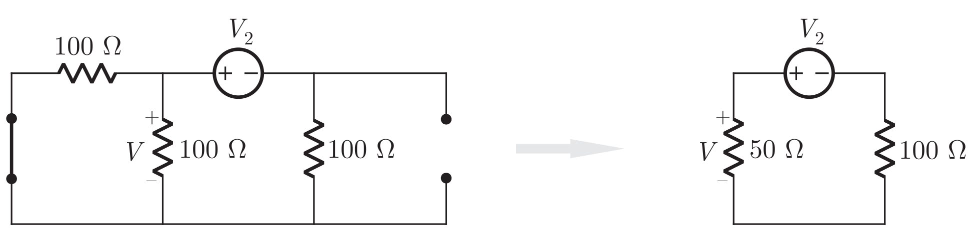
Due to source $V_1$ only : (Open circuit source $I_3$ and short circuit source $V_2$)




$$V = \frac{50}{100+50}(V_1) = \frac{1}{3}V_1 \qquad \text{(using voltage division)}$$

so, $$A = \frac{1}{3}$$

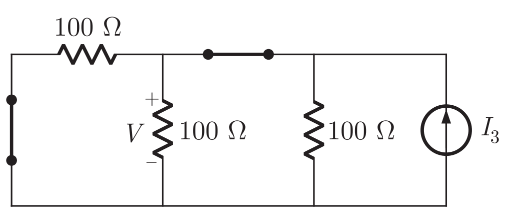
Due to source $V_2$ only : (Open circuit source $I_3$ and short circuit source $V_1$)




$$V = \frac{50}{100+50}(V_2) = \frac{1}{3}V_2 \qquad \text{(Using voltage division)}$$

So, $$B = \frac{1}{3}$$

Due to source $I_3$ only : (short circuit sources $V_1$ and $V_2$)



$$V = I_3[100 \,||\, 100 \,||\, 100] = I_3\left(\frac{100}{3}\right)$$

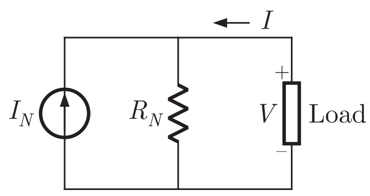
So, $$C = \frac{100}{3}$$

ALTERNATIVE METHOD :

Try to solve by nodal method, taking a supernode corresponding to voltage source $V_2$.

SOL 5.1.38 Option (C) is correct.

The circuit with Norton equivalent




So, $$I_N + I = \frac{V}{R_N}$$

$$I = \frac{V}{R_N} - I_N \qquad \text{(General form)}$$

From the given graph, the equation of line

$$I = 2V - 6$$

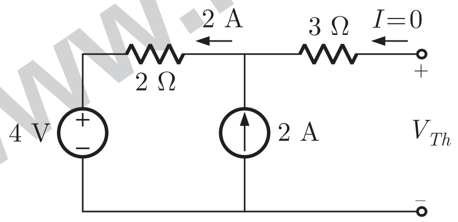
Comparing with general form

$$\frac{1}{R_N} = 2 \text{ or } R_N = 0.5\,\Omega$$

$$I_N = 6\text{ A}$$

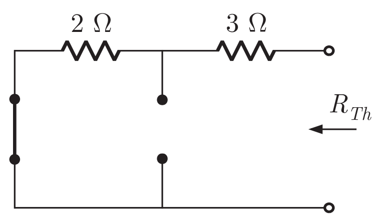
SOL 5.1.39 Option (D) is correct.

Thevenin voltage: (Open circuit voltage)




$$V_{Th} = 4 + (2 \times 2) = 4 + 4 = 8\text{ V}$$

Thevenin Resistance:




$$R_{Th} = 2 + 3 = 5\,\Omega = R_N$$

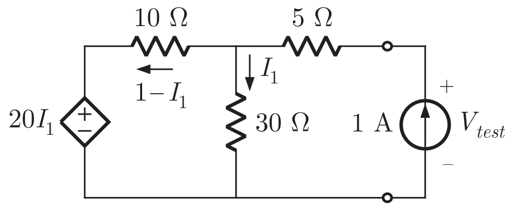
Norton Current:

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{8}{5}\text{ A}$$

SOL 5.1.40 Option (C) is correct.

Norton current, $I_N = 0$ because there is no independent source present in the circuit.

To obtain Norton resistance we put a 1 A test source across the load terminal as shown in figure.




Norton or Thevenin resistance

$$R_N = \frac{V_{test}}{1}$$

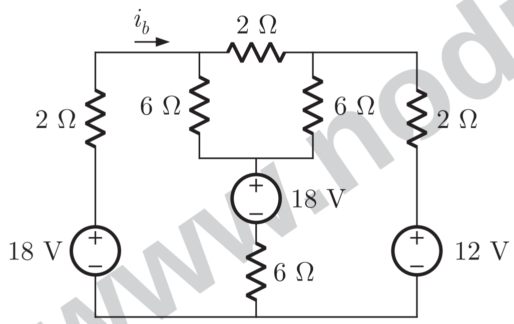
Writing KVL in the left mesh

$$20I_1 + 10(1 - I_1) - 30I_1 = 0$$
$$20I_1 - 10I_1 - 30I_1 + 10 = 0$$
$$I_1 = 0.5 \text{ A}$$

Writing KVL in the right mesh

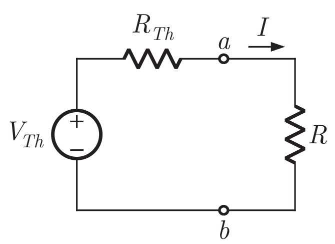
$$V_{test} - 5(1) - 30I_1 = 0$$
$$V_{test} - 5 - 30(0.5) = 0$$
$$V_{test} - 5 - 15 = 0$$
$$R_N = \frac{V_{test}}{1} = 20\,\Omega$$

SOL 5.1.41 Option (C) is correct.
In circuit (b) transforming the 3 A source in to 18 V source all source are 1.5 times of that in circuit (a) as shown in figure.




Using principal of linearity, $I_b = 1.5I_a$

SOL 5.1.42 Option (B) is correct.




$$I = \frac{V_{Th}}{R + R_{Th}}$$

From the table, $$2 = \frac{V_{Th}}{3 + R_{Th}} \qquad ...(1)$$

$$1.6 = \frac{V_{Th}}{5 + R_{Th}} \qquad ...(2)$$

Dividing equation (1) and (2), we get

$$\frac{2}{1.6} = \frac{5 + R_{Th}}{3 + R_{Th}}$$
$$6 + 2R_{Th} = 8 + 1.6R_{Th}$$
$$0.4R_{Th} = 2$$

$$R_{Th} = 5\,\Omega$$

Substituting $R_{Th}$ into equation (1)

$$2 = \frac{V_{Th}}{3+5}$$

$$V_{Th} = 2(8) = 16\text{ V}$$

SOL 5.1.43 Option (D) is correct.

We have, $$I = \frac{V_{Th}}{R_{Th}+R}$$

$$V_{Th} = 16\text{ V},\ R_{Th} = 5\,\Omega$$

$$I = \frac{16}{5+R} = 1$$

$$16 = 5+R$$

$$R = 11\,\Omega$$

SOL 5.1.44 Option (B) is correct.

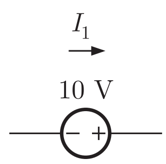



Fig.(A)

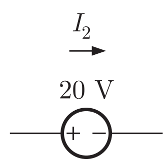



Fig.(B)

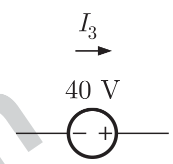



Fig.(C)

It can be solved by reciprocity theorem. Polarity of voltage source should have same correspondence with branch current in each of the circuit. Polarity of voltage source and current direction are shown below

So, $$\frac{V_1}{I_1} = -\frac{V_2}{I_2} = \frac{V_3}{I_3}$$

$$\frac{10}{2.5} = -\frac{20}{I_2} = \frac{40}{I_3}$$

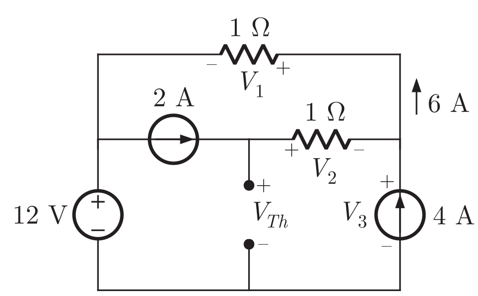
$$I_2 = -5\text{ A}$$

$$I_3 = 10\text{ A}$$

SOL 5.1.45 Option (A) is correct.

To obtain $V$-$I$ equation we find the Thevenin equivalent across the terminal at which $X$ is connected.

Thevenin Voltage : (Open Circuit Voltage)




$$V_1 = 6\times 1 = 6\text{ V}$$

$$12 + V_1 - V_3 = 0 \qquad \text{(KVL in outer mesh)}$$

$$V_3 = 12+6 = 18\text{ V}$$

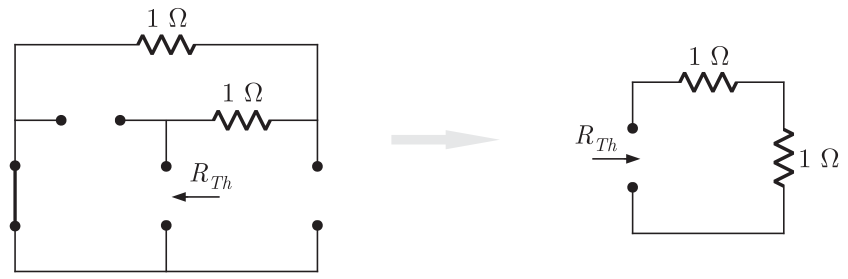
$$V_{Th} - V_2 - V_3 = 0 \qquad \text{(KVL in Bottom right mesh)}$$

$$V_{Th} = V_2 + V_3$$

$(V_2 = 2\times 1 = 2\text{ V})$

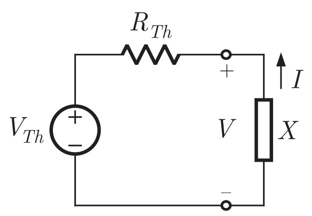
$$V_{Th} = 2+18 = 20\text{ V}$$

Thevenin Resistance :



$$R_{Th} = 1 + 1 = 2\,\Omega$$

Now, the circuit becomes as



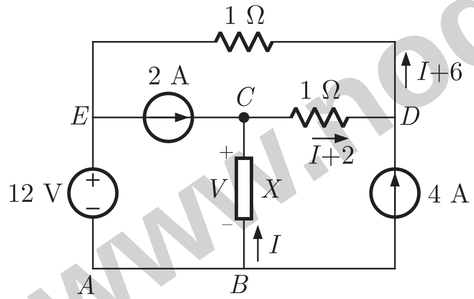
$$I = \frac{V - V_{Th}}{R_{Th}}$$

$$V = R_{Th} I + V_{Th}$$

so $$A = R_{Th} = 2\,\Omega$$

$$B = V_{Th} = 20\text{ V}$$

ALTERNATIVE METHOD :





In the mesh $ABCDEA$, we have KVL equation as

$$V - 1(I+2) - 1(I+6) - 12 = 0$$

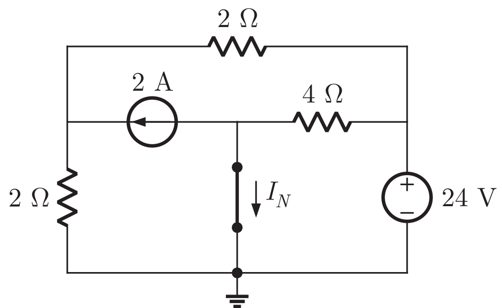
$$V = 2I + 20$$

So, $$A = 2, \quad B = 2$$

SOL 5.1.46 Option (A) is correct.

To obtain $V$-$I$ relation, we obtain either Norton equivalent or Thevenin equivalent across terminal $a$-$b$.

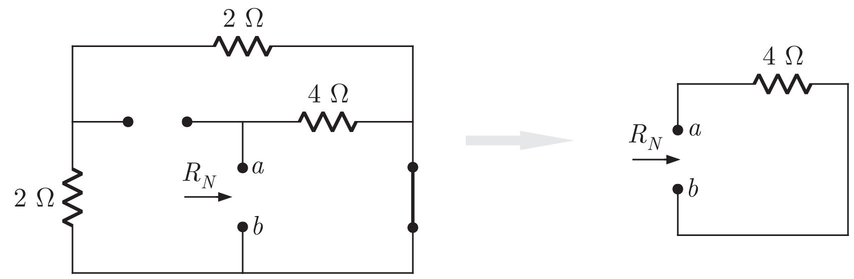
Norton Current (short circuit current) :



Applying nodal analysis at center node

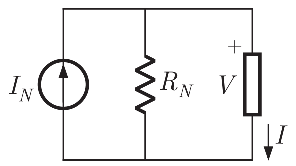
$$I_N + 2 = \frac{24}{4} \text{ or } I_N = 6 - 2 = 4\text{ A}$$

Norton Resistance :




$$R_N = 4\,\Omega \qquad \text{(Both } 2\,\Omega \text{ resistor are short circuited)}$$

Now, the circuit becomes as

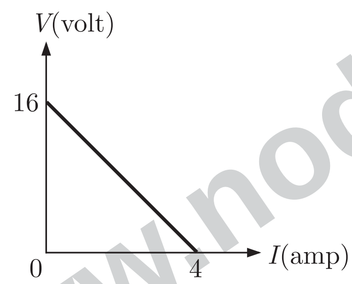



$$I_N = \frac{V}{R_N} + I$$

$$4 = \frac{V}{4} + I$$

$$16 = V + 4I$$

or $$V = -4I + 16$$

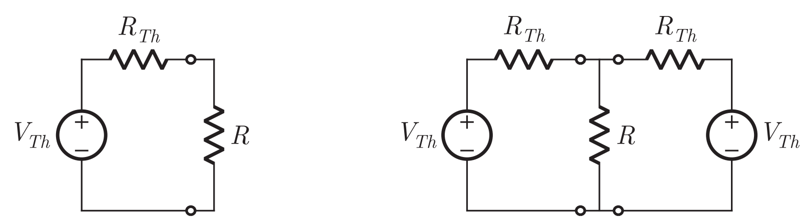



ALTERNATIVE METHOD :

Solve by writing nodal equation at the center node.

SOL 5.1.47 Option (C) is correct.

Let Thevenin equivalent of both networks are as shown below.




$$P = \left(\frac{V_{Th}}{R_{Th} + R}\right)^2 R \qquad \text{(Single network } N\text{)}$$

$$P' = \left(\frac{V_{Th}}{R + \frac{R_{Th}}{2}}\right)^2 R = 4\left(\frac{V_{Th}}{2R + R_{Th}}\right)^2 R \qquad \text{(Two } N \text{ are added)}$$

Thus $P < P' < 4P$

SOL 5.1.48 Option (C) is correct.

$$I_1 = \sqrt{\frac{P_1}{R}} \text{ and } I_2 = \sqrt{\frac{P_2}{R}}$$

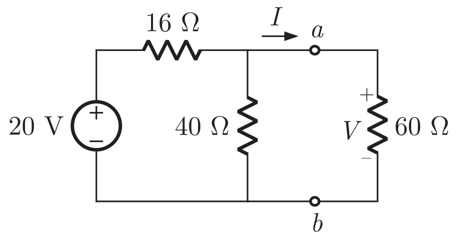
Using superposition $$I = I_1 \pm I_2 = \sqrt{\frac{P_1}{R}} \pm \sqrt{\frac{P_2}{R}}$$

$$I^2 R = (\sqrt{P_1} \pm \sqrt{P_2})^2$$

SOL 5.1.49 Option (B) is correct.

From the substitution theorem we know that any branch within a circuit can be replaced by an equivalent branch provided that replacement branch has the same current through it and voltage across it as the original branch.

The voltage across the branch in the original circuit




$$V = \frac{40\,||\,60}{(40\,||\,60)+16}(20) = \frac{24}{40}\times 20 = 12\text{ V}$$

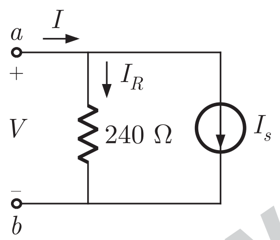
Current entering terminal $a$-$b$ is

$$I = \frac{V}{R} = \frac{12}{60} = 200\text{ mA}$$

In fig(B), to maintain same voltage $V = 12$ V current through $240\,\Omega$ resistor must be

$$I_R = \frac{12}{240} = 50\text{ mA}$$

Using KCL at terminal $a$, as shown

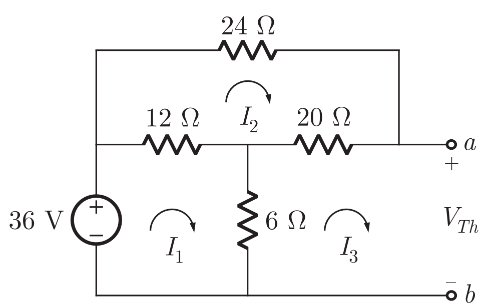



$$I = I_R + I_S$$
$$200 = 50 + I_s$$
$$I_s = 150\text{ mA},\quad \text{down wards}$$

SOL 5.1.50 Option (B) is correct.

Thevenin voltage : (Open Circuit Voltage)

In the given problem, we use mesh analysis method to obtain Thevenin voltage




$$I_3 = 0 \qquad (a\text{-}b \text{ is open circuit})$$

Writing mesh equations

Mesh 1: $36 - 12(I_1 - I_2) - 6(I_1 - I_3) = 0$

$$36 - 12I_1 + 12I_2 - 6I_1 = 0 \qquad (I_3 = 0)$$

$$3I_1 - 2I_2 = 6 \qquad ...(1)$$

Mesh 2: $-24I_2 - 20(I_2 - I_3) - 12(I_2 - I_1) = 0$

$$-24I_2 - 20I_2 - 12I_2 + 12I_1 = 0 \qquad (I_3 = 0)$$

$$14I_2 = 3I_1 \qquad ...(2)$$

From equation (1) and (2)

$$I_1 = \frac{7}{3}\,\text{A}, \quad I_2 = \frac{1}{2}\,\text{A}$$

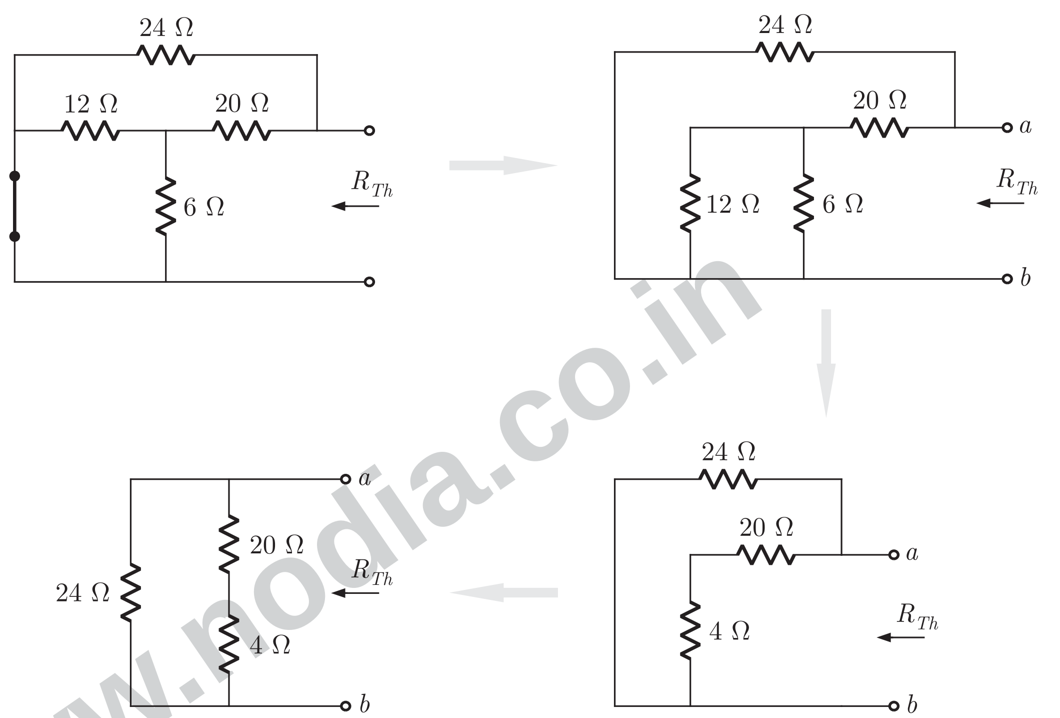
Mesh 3: $\quad -6(I_3 - I_1) - 20(I_3 - I_2) - V_{Th} = 0$

$$-6\left[0 - \frac{7}{3}\right] - 20\left[0 - \frac{1}{2}\right] - V_{Th} = 0$$

$$14 + 10 = V_{Th}$$

$$V_{Th} = 24 \text{ volt}$$

Thevenin Resistance :




$$R_{Th} = (20 + 4)\,||\,24\,\Omega = 24\,\Omega\,||\,24\,\Omega = 12\,\Omega$$

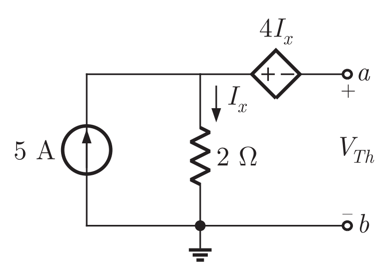
ALTERNATIVE METHOD :

$V_{Th}$ can be obtained by writing nodal equation at node $a$ and at center node.

SOL 5.1.51 Option (C) is correct.

We obtain Thevenin's equivalent across load terminal.

Thevenin Voltage : (Open Circuit Voltage)




Using KCL at top left node

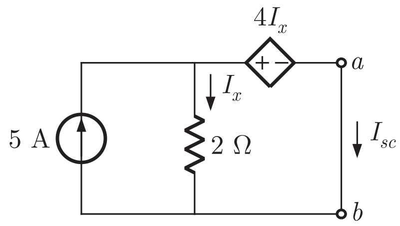
$$5 = I_x + 0 \text{ or } I_x = 5\,\text{A}$$

Using KVL $\quad 2I_x - 4I_x - V_{Th} = 0$

$$2(5) - 4(5) = V_{Th} \text{ or } V_{Th} = -10 \text{ volt}$$

Thevenin Resistance :

First we find short circuit current through $a$-$b$




Using KCL at top left node

$$5 = I_x + I_{sc}$$
$$I_x = 5 - I_{sc}$$

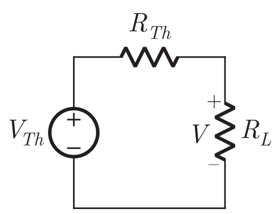
Applying KVL in the right mesh

$$2I_x - 4I_x + 0 = 0 \text{ or } I_x = 0$$

So, $$5 - I_{sc} = 0 \text{ or } I_{sc} = 5\text{ A}$$

Thevenin resistance, $$R_{Th} = \frac{V_{Th}}{I_{sc}} = -\frac{10}{5} = -2\,\Omega$$

Now, the circuit becomes as




$$V = V_{Th}\left(\frac{R}{R+R_L}\right) \qquad \text{(Using voltage division)}$$

So, $$V = V_{Th} = -10\text{ volt}$$
$$R = R_{Th} = -2\,\Omega$$

SOL 5.1.52 Option (D) is correct.

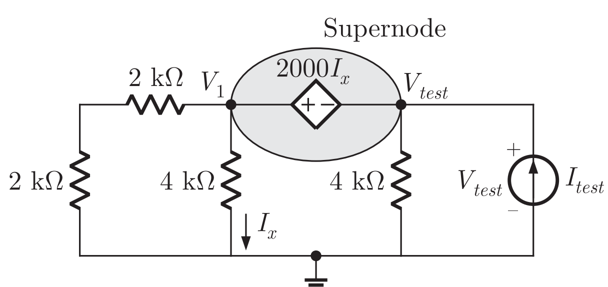
We obtain Thevenin equivalent across terminal $a$-$b$.

Thevenin Voltage :

Since there is no independent source present in the network, Thevenin voltage is simply zero i.e. $V_{Th} = 0$

Thevenin Resistance :

Put a test source across terminal $a$-$b$




$$R_{Th} = \frac{V_{test}}{I_{test}}$$

For the super node

$$V_1 - V_{test} = 2000I_x$$
$$V_1 - V_{test} = 2000\left(\frac{V_1}{4000}\right) \qquad (I_x = V_1/4000)$$
$$\frac{V_1}{2} = V_{test} \text{ or } V_1 = 2V_{test}$$

Applying KCL to the super node

$$\frac{V_1 - 0}{4k} + \frac{V_1}{4k} + \frac{V_{test}}{4k} = I_{test}$$

$$2V_1 + V_{test} = 4 \times 10^3 I_{test}$$

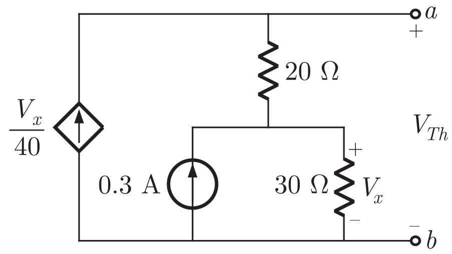
$$2(2V_{test}) + V_{test} = 4 \times 10^3 I_{test} \qquad (V_1 = 2V_{test})$$

$$\frac{V_{test}}{I_{test}} = \frac{4 \times 10^3}{5} = 800\,\Omega$$

SOL 5.1.53 Option (C) is correct.

Equation for $V$-$I$ can be obtained with Thevenin equivalent across $a$-$b$ terminals.

Thevenin Voltage: (Open circuit voltage)




Writing KCL at the top node

$$\frac{V_x}{40} = \frac{V_{Th} - V_x}{20}$$

$$V_x = 2V_{Th} - 2V_x$$

$$3V_x = 2V_{Th} \Rightarrow V_x = \frac{2}{3}V_{Th}$$

KCL at the center node

$$\frac{V_x - V_{Th}}{20} + \frac{V_x}{30} = 0.3$$

$$3V_x - 3V_{Th} + 2V_x = 18$$

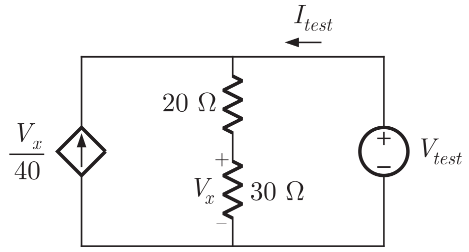
$$5V_x - 3V_{Th} = 18$$

$$5\left(\frac{2}{3}\right)V_{Th} - 3V_{Th} = 18 \qquad \left(V_x = \frac{2}{3}V_{Th}\right)$$

$$10V_{Th} - 9V_{Th} = 54 \text{ or } V_{Th} = 54 \text{ volt}$$

Thevenin Resistance :

When a dependent source is present in the circuit the best way to obtain Thevenin resistance is to remove all independent sources and put a test source across $a$-$b$ terminals as shown in figure.




$$R_{Th} = \frac{V_{test}}{I_{test}}$$

KCL at the top node

$$\frac{V_x}{40} + I_{test} = \frac{V_{test}}{20 + 30}$$

$$\frac{V_x}{40} + I_{test} = \frac{V_{test}}{50} \qquad ...(1)$$

$$V_x = \frac{30}{30+20}(V_{test}) = \frac{3}{5}V_{test} \qquad \text{(using voltage division)}$$

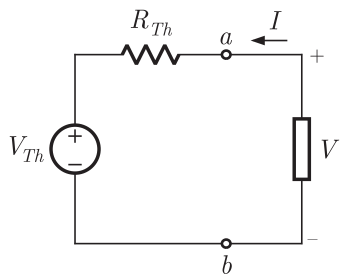
Substituting $V_x$ into equation (1), we get

$$\frac{3V_{test}}{5(40)} + I_{test} = \frac{V_{test}}{50}$$

$$I_{test} = V_{test}\left(\frac{1}{50} - \frac{3}{200}\right) = \frac{V_{test}}{200}$$

$$R_{Th} = \frac{V_{test}}{I_{test}} = 200\,\Omega$$

The circuit now reduced as

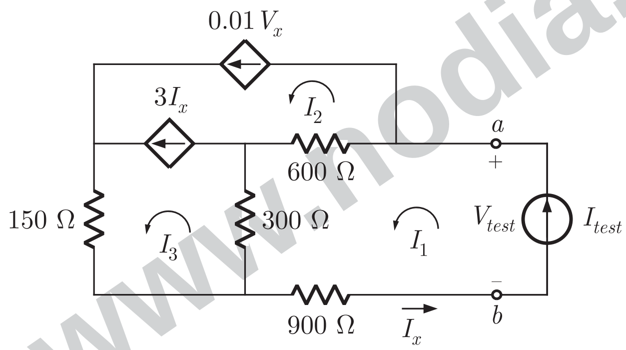



$$I = \frac{V - V_{Th}}{R_{Th}} = \frac{V - 54}{200}$$

$$V = 200I + 54$$

SOL 5.1.54 Option (D) is correct.

To obtain Thevenin resistance put a test source across the terminal $a$, $b$ as shown.




$$V_{test} = V_x,\ I_{test} = I_x$$

Writing loop equation for the circuit

$$V_{test} = 600(I_1 - I_2) + 300(I_1 - I_3) + 900(I_1)$$

$$V_{test} = (600 + 300 + 900)I_1 - 600I_2 - 300I_3$$

$$V_{test} = 1800I_1 - 600I_2 - 300I_3 \quad ...(1)$$

The loop current are given as,

$$I_1 = I_{test},\quad I_2 = 0.3V_s,\quad \text{and}\quad I_3 = 3I_{test} + 0.2V_s$$

Substituting theses values into equation (1),

$$V_{test} = 1800I_{test} - 600(0.01V_s) - 300(3I_{test} + 0.01V_s)$$

$$V_{test} = 1800I_{test} - 6V_s - 900I_{test} - 3V_s$$

$$10V_{test} = 900I_{test} \text{ or } V_{test} = 90I_{test}$$

Thevenin resistance

$$R_{Th} = \frac{V_{test}}{I_{test}} = 90\,\Omega$$

Thevenin voltage or open circuit voltage will be zero because there is no independent source present in the network, i.e. $V_{oc} = 0$ V

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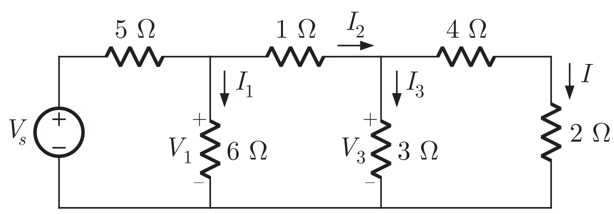
# SOLUTIONS 5.2

SOL 5.2.1 Correct answer is 3.

We solve this problem using principal of linearity.




In the left, $4\,\Omega$ and $2\,\Omega$ are in series and has same current $I = 1$ A.

$$V_3 = 4I + 2I \qquad \text{(using KVL)}$$
$$= 6I = 6 \text{ V}$$
$$I_3 = \frac{V_3}{3} = \frac{6}{3} = 2 \text{ A} \qquad \text{(using ohm's law)}$$
$$I_2 = I_3 + I \qquad \text{(using KCL)}$$
$$= 2 + 1 = 3 \text{ A}$$
$$V_1 = (1) I_2 + V_3 \qquad \text{(using KVL)}$$
$$= 3 + 6 = 9 \text{ V}$$
$$I_1 = \frac{V_1}{6} = \frac{9}{6} = \frac{3}{2} \text{ A} \qquad \text{(using ohm's law)}$$

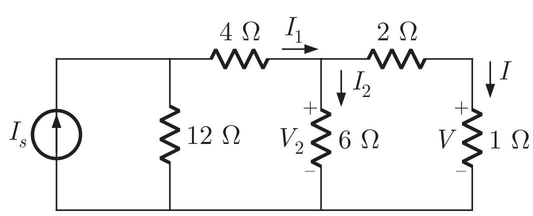
Applying principal of linearity

For $V_s = V_0$, $\quad I_1 = \frac{3}{2}$ A

So for $V_s = 2V_0$, $\quad I_1 = \frac{3}{2} \times 2 = 3$ A

SOL 5.2.2 Correct answer is 3.

We solve this problem using principal of linearity.




$$I = \frac{V}{1} = \frac{1}{1} = 1 \text{ A} \qquad \text{(using ohm's law)}$$
$$V_2 = 2I + (1) I = 3 \text{ V} \qquad \text{(using KVL)}$$
$$I_2 = \frac{V_2}{6} = \frac{3}{6} = \frac{1}{2} \text{ A} \qquad \text{(using ohm's law)}$$
$$I_1 = I_2 + I \qquad \text{(using KCL)}$$
$$= \frac{1}{2} + 1 = \frac{3}{2} \text{ A}$$

Applying principal of superposition

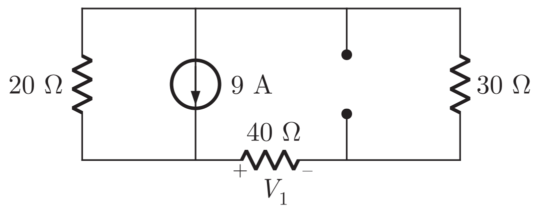
When $I_s = I_0$, and $V = 1$ V, $\quad I_1 = \frac{3}{2}$ A

So, if $I_s = 2I_0$, $$I_1 = \frac{3}{2} \times 2 = 3 \text{ A}$$

SOL 5.2.3 Correct answer is 160.

We solve this problem using superposition.

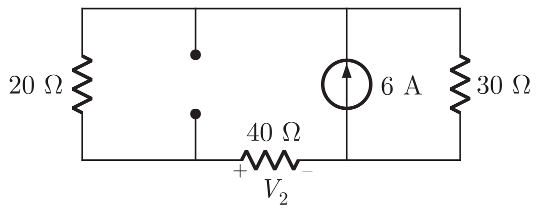
Due to 9 A source only : (Open circuit 6 A source)




Using current division

$$\frac{V_1}{40} = \frac{20}{20 + (40 + 30)}(9) \Rightarrow V_1 = 80 \text{ volt}$$

Due to 6 A source only : (Open circuit 9 A source)




Using current division,

$$\frac{V_2}{40} = \frac{30}{30 + (40 + 20)}(6) \Rightarrow V_2 = 80 \text{ volt}$$

From superposition,

$$V = V_1 + V_2 = 80 + 80 = 160 \text{ volt}$$

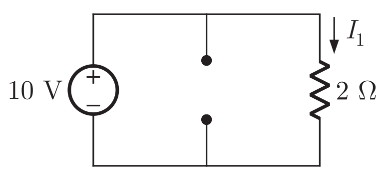
ALTERNATIVE METHOD :

The problem may be solved by transforming both the current sources into equivalent voltage sources and then applying voltage division.

SOL 5.2.4 Correct answer is 5.

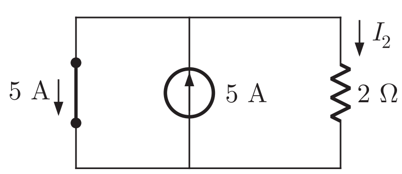
Using super position, we obtain $I$.

Due to 10 V source only : (Open circuit 5 A source)




$$I_1 = \frac{10}{2} = 5 \text{ A}$$

Due to 5 A source only : (Short circuit 10 V source)




$$I_2 = 0$$

$$I = I_1 + I_2 = 5 + 0 = 5 \text{ A}$$

ALTERNATIVE METHOD :

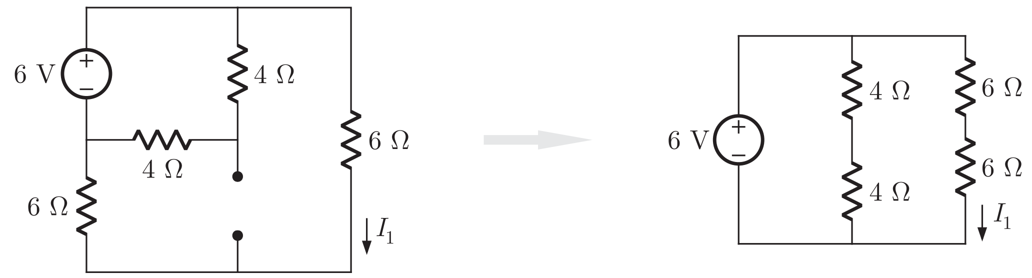
We can see that voltage source is in parallel with resistor and current source so voltage across parallel branches will be 10 V and $I = 10/2 = 5$ A

SOL 5.2.5

Correct answer is $-0.5$ .

Applying superposition,

Due to 6 V source only : (Open circuit 2 A current source)




$$I_1 = \frac{6}{6+6} = 0.5\text{ A}$$

Due to 2 A source only : (Short circuit 6 V source)


$$I_2 = \frac{6}{6+6}(-2) \qquad \text{(using current division)}$$

$$= -1\text{ A}$$

$$I = I_1 + I_2 = 0.5 - 1 = -0.5\text{ A}$$

ALTERNATIVE METHOD :

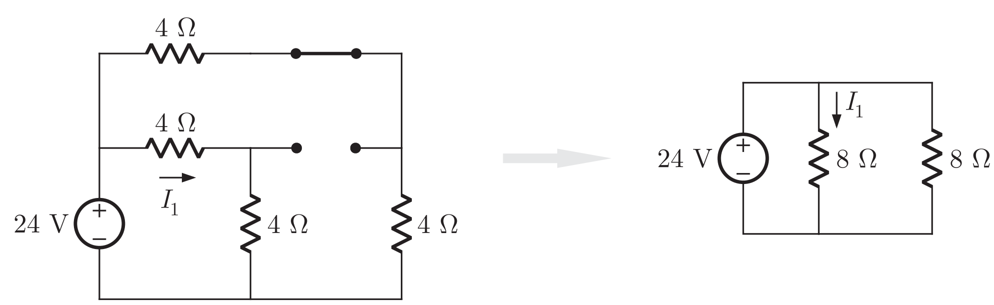
This problem may be solved by using a single KVL equation around the outer loop.

SOL 5.2.6

Correct answer is 4.

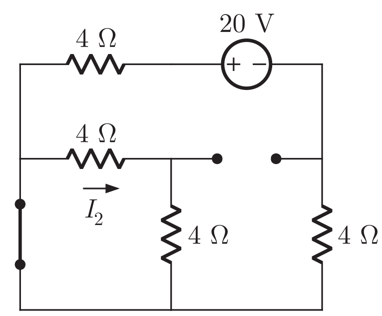
Applying superposition,

Due to 24 V Source Only : (Open circuit 2 A and short circuit 20 V source)



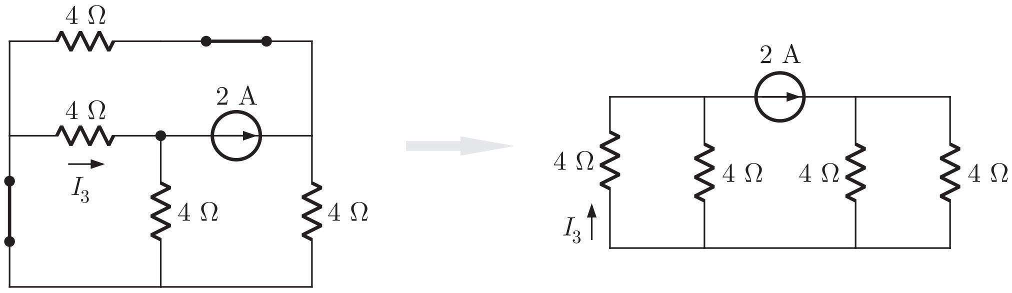

$$I_1 = \frac{24}{8} = 3\text{ A}$$

Due to 20 V source only : (Short circuit 24 V and open circuit 2 A source)



So $I_2 = 0$ (Due to short circuit)

Due to 2 A source only : (Short circuit 24 V and 20 V sources)




$$I_3 = \frac{4}{4+4}(2) \quad \text{(using current division)}$$

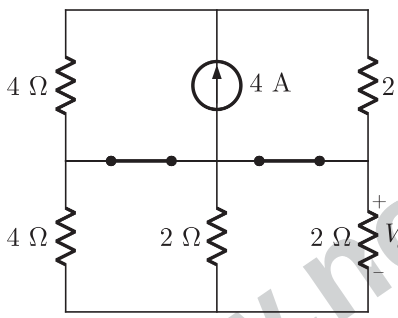
$$= 1\,\text{A}$$

So $I = I_1 + I_2 + I_3 = 3 + 0 + 1 = 4\,\text{A}$

Alternate Method: We can see that current in the middle $4\,\Omega$ resistor is $I - 2$, therefore $I$ can be obtained by applying KVL in the bottom left mesh.

SOL 5.2.7 Correct answer is 0.

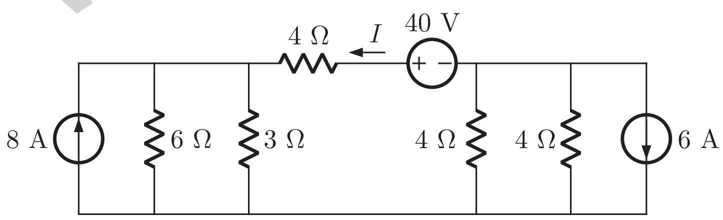
$V_1 = V_2 = 0$ (short circuit both sources)



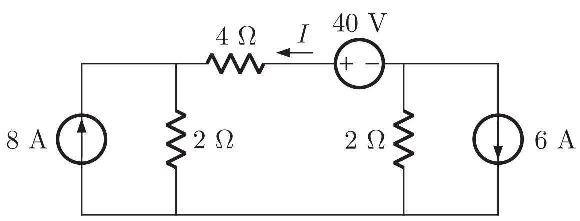

$V_o = 0$

SOL 5.2.8 Correct answer is 1.5 .

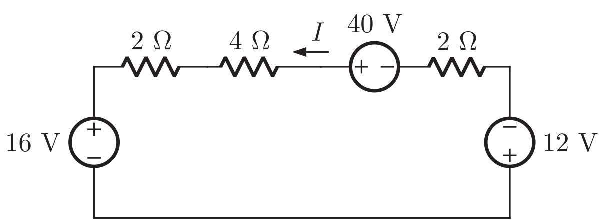
Using source transformation of 48 V source and the 24 V source




using parallel resistances combination




Source transformation of 8 A and 6 A sources



Writing KVL around anticlock wise direction

$$-12 - 2I + 40 - 4I - 2I - 16 = 0$$

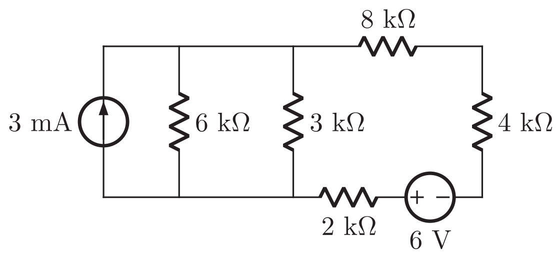
$$12 - 8I = 0$$

$$I = \frac{12}{8} = 1.5 \text{ A}$$

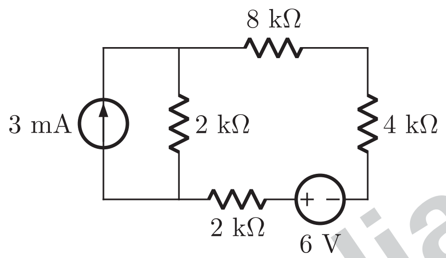
SOL 5.2.9 Correct answer is 2.25 .

We apply source transformation as follows.

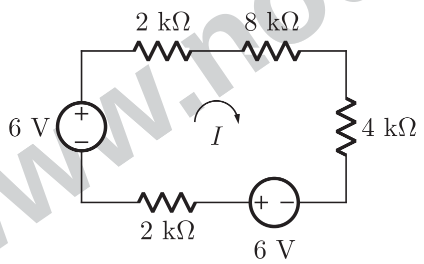
Transforming 3 mA source into equivalent voltage source and 18 V source into equivalent current source.




6 kΩ and 3 kΩ resistors are in parallel and equivalent to 2 Ω.




Again transforming 3 mA source

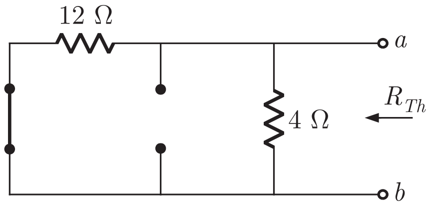



$$I = \frac{6+6}{2+8+4+2} = \frac{3}{4} \text{ mA}$$

$$P_{4\,\text{k}\Omega} = I^2(4 \times 10^3) = \left(\frac{3}{4}\right)^2 \times 4 = 2.25 \text{ mW}$$

SOL 5.2.10 Correct answer is 3.

Set all independent sources to zero (i.e. open circuit current sources and short circuit voltage sources) to obtain $R_{Th}$




$$R_{Th} = 12\,\Omega\,||\,4\,\Omega = 3\,\Omega$$

SOL 5.2.11 Correct answer is 16.8 .

Using current division

$$I_1 = \frac{(5+1)}{(5+1)+(3+1)}(12) = \frac{6}{6+4}(12) = 7.2 \text{ A}$$

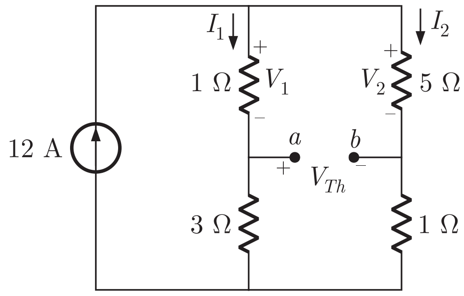
$$V_1 = I_1 \times 1 = 7.2 \text{ V}$$

$$I_2 = \frac{(3+1)}{(3+1)+(5+1)}(12) = 4.8 \text{ A}$$

$$V_2 = 5I_2 = 5 \times 4.8 = 24 \text{ V}$$

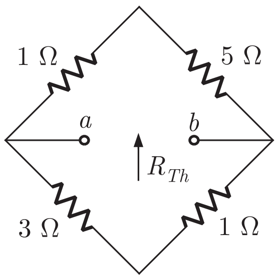
$$V_{Th} + V_1 - V_2 = 0 \qquad \text{(KVL)}$$

$$V_{Th} = V_2 - V_1 = 24 - 7.2 = 16.8 \text{ V}$$



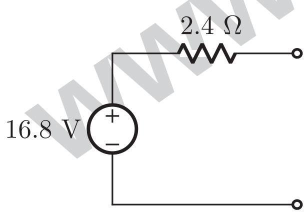

SOL 5.2.12 Correct answer is 7.

We obtain Thevenin's resistance across $a$-$b$ and then use source transformation of Thevenin's circuit to obtain equivalent Norton circuit.

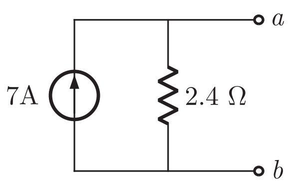



$$R_{Th} = (5+1)\,||\,(3+1) = 6\,||\,4 = 2.4\,\Omega$$

Thevenin's equivalent is



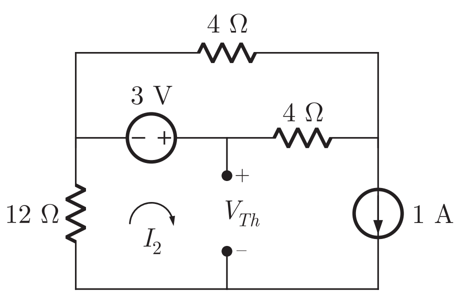

Norton equivalent




SOL 5.2.13 Correct answer is $-0.5$ .

Current $I$ can be easily calculated by Thevenin's equivalent across $6\,\Omega$.

Thevenin Voltage : (Open Circuit Voltage)

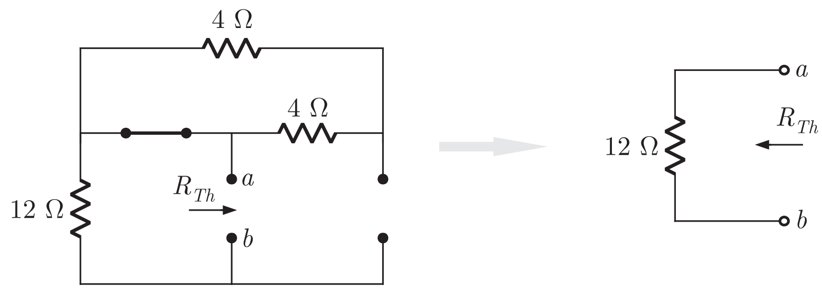


In the bottom mesh $\qquad I_2 = 1\text{ A}$

In the bottom left mesh $\quad -V_{Th} - 12I_2 + 3 = 0$

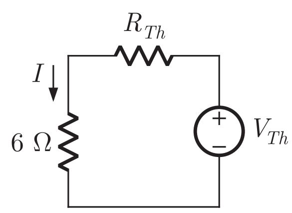
$$V_{Th} = 3 - (12)(1) = -9\text{ V}$$

Thevenin Resistance :




$R_{Th} = 12\,\Omega \qquad$ (both $4\,\Omega$ resistors are short circuit)

so, circuit becomes as



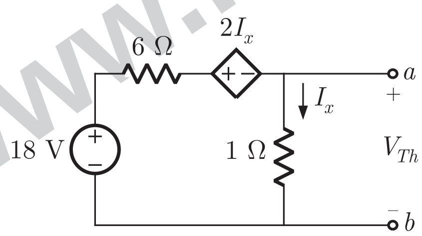

$$I = \frac{V_{Th}}{R_{Th}+6} = \frac{-9}{12+6} = -\frac{9}{18} = -0.5\text{ A}$$

Note: The problem can be solved easily by a single node equation. Take the nodes connecting the top $4\,\Omega$, $3\text{ V}$ and $4\,\Omega$ as supernode and apply KCL.

SOL 5.2.14 Correct answer is 0.

We obtain Thevenin's equivalent across $R$.

Thevenin Voltage : (Open circuit voltage)



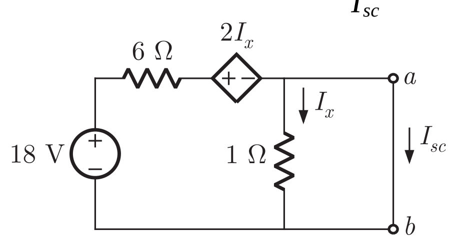

Applying KVL $\qquad 18 - 6I_x - 2I_x - (1)I_x = 0$

$$I_x = \frac{18}{9} = 2\text{ A}$$

$$V_{Th} = (1)I_x = (1)(2) = 2\text{ V}$$

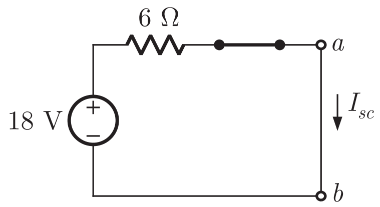
Thevenin Resistance :

$R_{Th} = \dfrac{V_{Th}}{I_{sc}} \qquad I_{sc} \rightarrow$ Short circuit current




$I_x = 0 \qquad$ (Due to short circuit)

So dependent source also becomes zero.

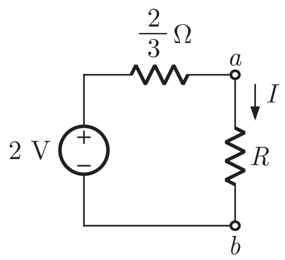



$$I_{sc} = \frac{18}{6} = 3\text{ A}$$

Thevenin resistance,

$$R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{2}{3}\,\Omega$$

Now, the circuit becomes as



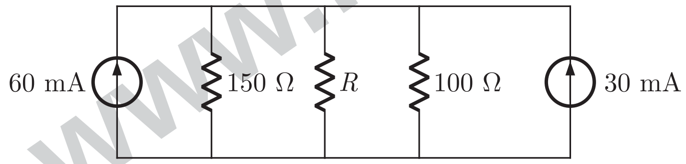

$$I = \frac{2}{\frac{2}{3}+R} = 3$$

$$2 = 2 + 3R$$

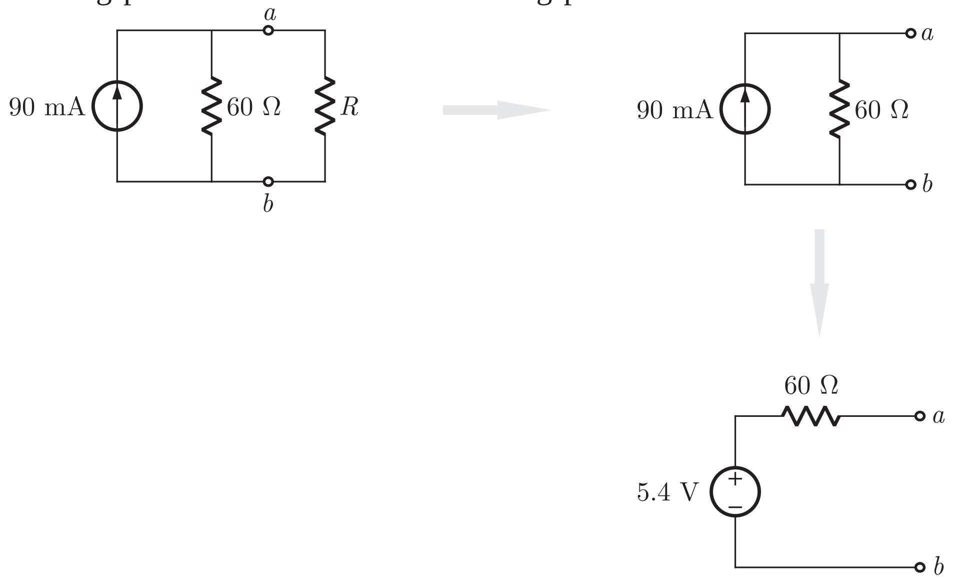
$$R = 0$$

SOL 5.2.15 Correct answer is 121.5 .

We obtain Thevenin's equivalent across $R$. By source transformation of both voltage sources



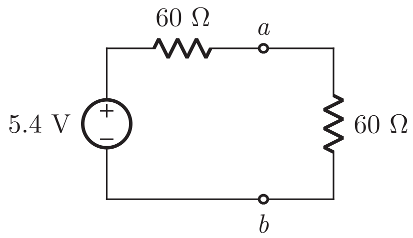

Adding parallel sources and combining parallel resistances




Here, $V_{Th} = 5.4\text{ V}, \quad R_{Th} = 60\,\Omega$

For maximum power transfer

$$R = R_{Th} = 60\,\Omega$$




Maximum Power absorbed by $R$

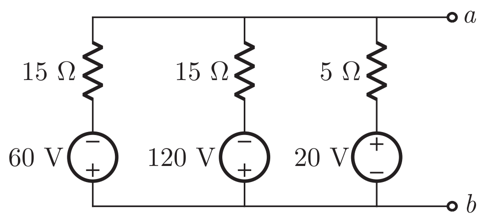
$$P = \frac{(V_{Th})^2}{4R} = \frac{(5.4)^2}{4 \times 60} = 121.5 \text{ mW}$$

ALTERNATIVE METHOD :

Thevenin voltage (open circuit voltage) may be obtained using node voltage method also.

SOL 5.2.16 Correct answer is 3.

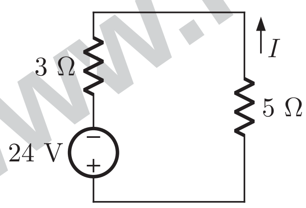
First we obtain equivalent voltage and resistance across terminal $a$-$b$ using Millman's theorem.




$$V_{ab} = \frac{-\frac{60}{15} + (-\frac{120}{15}) + \frac{20}{5}}{\frac{1}{15} + \frac{1}{15} + \frac{1}{5}} = -24 \text{ V}$$

$$R_{ab} = \frac{1}{\frac{1}{15} + \frac{1}{15} + \frac{1}{5}} = 3\,\Omega$$

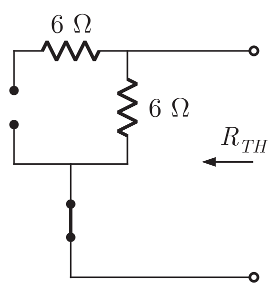
So, the circuit is reduced as




$$I = \frac{24}{3+5} = 3 \text{ A}$$

SOL 5.2.17 Correct answer is 6.

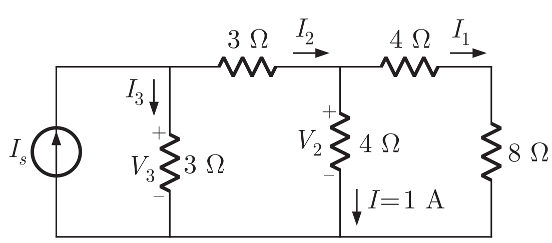
Set all independent sources to zero as shown,




$$R_{Th} = 6\,\Omega$$

SOL 5.2.18 Correct answer is 0.5 .

We solve this problem using linearity and taking assumption that $I = 1$ A.



In the circuit, $V_2 = 4I = 4\text{ V}$ (Using Ohm's law)

$$I_2 = I + I_1 \quad \text{(Using KCL)}$$

$$= 1 + \frac{V_2}{4+8} = 1 + \frac{4}{12} = \frac{4}{3}\text{ A}$$

$$V_3 = 3I_2 + V_2 \quad \text{(Using KVL)}$$

$$= 3 \times \frac{4}{3} + 4 = 8\text{ V}$$

$$I_s = I_3 + I_2 \quad \text{(Using KCL)}$$

$$= \frac{V_3}{3} + I_2 = \frac{8}{3} + \frac{4}{3} = 4\text{ A}$$

Applying superposition

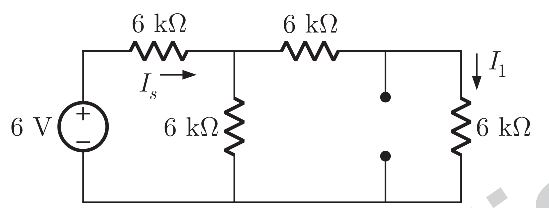
When $I_s = 4\text{ A}$, $I = 1\text{ A}$

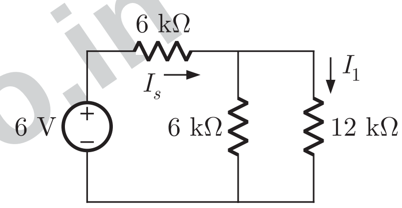
But actually $I_s = 2\text{ A}$, So $I = \frac{1}{4} \times 2 = 0.5\text{ A}$

SOL 5.2.19 Correct answer is $-1$.

Solving with superposition,

Due to 6 V Source Only : (Open Circuit 2 mA source)

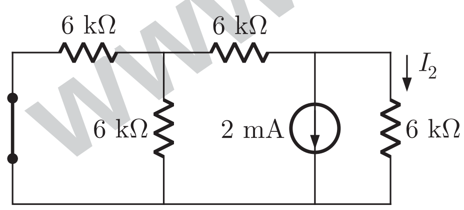



$$I_s = \frac{6}{6 + 6\,||\,12} = \frac{6}{6+4} = 0.6\text{ mA}$$

$$I_1 = \frac{6}{6+12}(I_s) = \frac{6}{18} \times 0.6 = 0.2\text{ mA} \quad \text{(Using current division)}$$

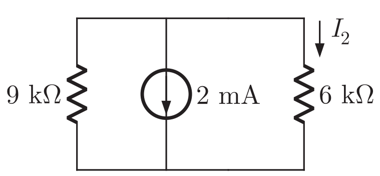
Due to 2 mA source only : (Short circuit 6 V source) :




Combining resistances,

$$6\text{ k}\Omega\,||\,6\text{ k}\Omega = 3\text{ k}\Omega$$

$$3\text{ k}\Omega + 6\text{ k}\Omega = 9\text{ k}\Omega$$




$$I_2 = \frac{9}{9+6}(-2) = -1.2\text{ mA} \quad \text{(Current division)}$$

$$I = I_1 + I_2 \quad \text{(Using superposition)}$$

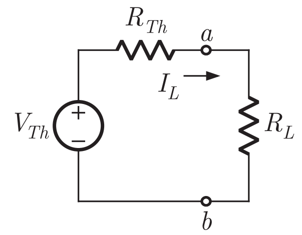
$$= 0.2 - 1.2 = -1\text{ mA}$$

ALTERNATIVE METHOD :

Try to solve the problem using source conversion.

SOL 5.2.20 Correct answer is 4.

We find Thevenin equivalent across $a$-$b$.




$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

From the data given in table

$$10 = \frac{V_{Th}}{R_{Th} + 2} \quad ...(1)$$

$$6 = \frac{V_{Th}}{R_{Th} + 10} \quad ...(2)$$

Dividing equation (1) and (2), we get

$$\frac{10}{6} = \frac{R_{Th} + 10}{R_{Th} + 2}$$

$$10R_{Th} + 20 = 6R_{Th} + 60$$

$$4R_{Th} = 40 \Rightarrow R_{Th} = 10\,\Omega$$

Substituting $R_{Th}$ into equation (1)

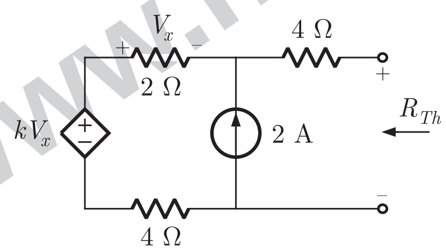
$$10 = \frac{V_{Th}}{10 + 2}$$

$$V_{Th} = 10(12) = 120 \text{ V}$$

For $R_L = 20\,\Omega$, $\quad I_L = \frac{V_{Th}}{R_{Th} + R_L}$

$$= \frac{120}{10 + 20} = 4 \text{ A}$$

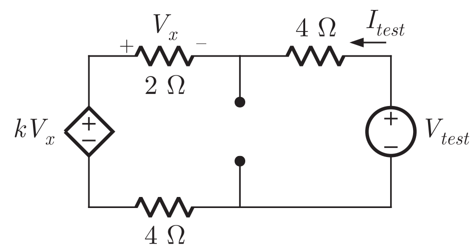
SOL 5.2.21 Correct answer is 4.




For maximum power transfer

$$R_{Th} = R_L = 2\,\Omega$$

To obtain $R_{Th}$ set all independent sources to zero and put a test source across the load terminals.




$$R_{Th} = \frac{V_{test}}{I_{test}}$$

Using KVL,

$$V_{test} - 4I_{test} - 2I_{test} - kV_x - 4I_{test} = 0$$

$$V_{test} - 10I_{test} - k(-2I_{test}) = 0 \quad (V_x = -2I_{test})$$

$$V_{test} = (10 - 2k) I_{test}$$

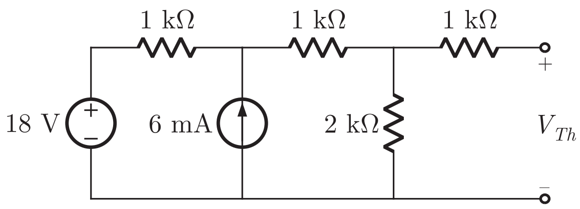
$$R_{Th} = \frac{V_{test}}{I_{test}} = 10 - 2k = 2$$

$$8 = 2k \text{ or } k = 4$$

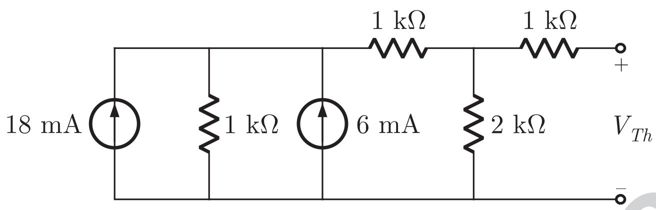
SOL 5.2.22 Correct answer is 18.

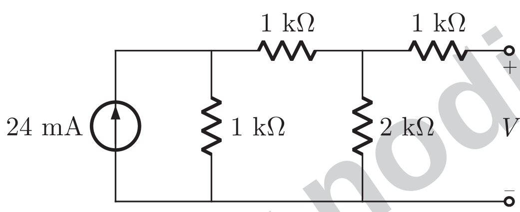
To calculate maximum power transfer, first we will find Thevenin equivalent across load terminals.

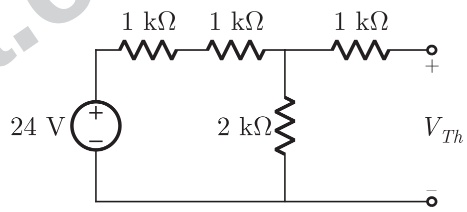
Thevenin Voltage: (Open Circuit Voltage)




Using source transformation



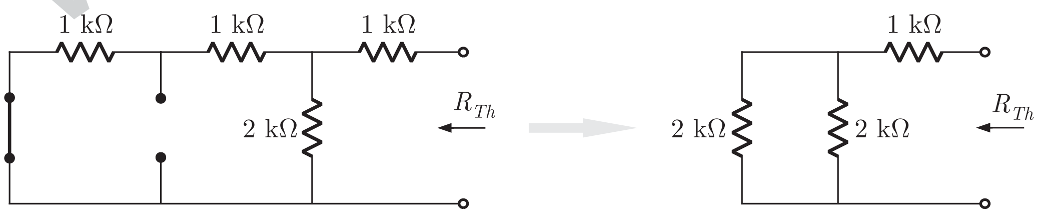




$$V_{Th} = \frac{2}{2+2}(24) \qquad \text{(Using voltage division)}$$

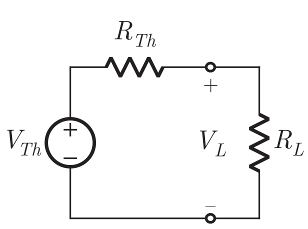
$$= 12 \text{ V}$$

Thevenin Resistance :




$$R_{Th} = 1 + 2 \,||\, 2 = 1 + 1 = 2 \text{ k}\Omega$$

Circuit becomes as




$$V_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

For maximum power transfer $R_L = R_{Th}$

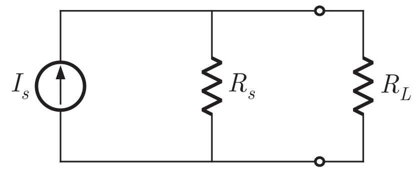
$$V_L = \frac{V_{Th}}{2R_{Th}} \times R_{Th} = \frac{V_{Th}}{2}$$

So maximum power absorbed by $R_L$

$$P_{\max} = \frac{V_L^2}{R_L} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(12)^2}{4 \times 2} = 18 \text{ mW}$$

SOL 5.2.23 Correct answer is 22.5 .

The circuit is as shown below




When $R_L = 50\,\Omega$, power absorbed in load will be

$$\left(\frac{R_s}{R_s + 50} I_s\right)^2 50 = 20 \text{ kW} \qquad \text{...(1)}$$

When $R_L = 200\,\Omega$, power absorbed in load will be

$$\left(\frac{R_s}{R_s + 200} I_s\right)^2 200 = 20 \text{ kW} \qquad \text{...(2)}$$

Dividing equation (1) and (2), we have

$$(R_s + 200)^2 = 4(R_s + 50)^2$$

$$R_s = 100\,\Omega \text{ and } I_s = 30 \text{ A}$$

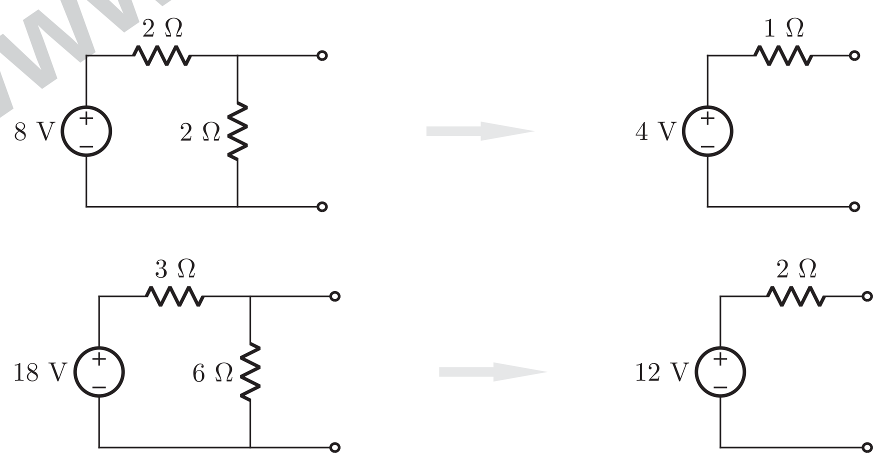
From maximum power transfer, the power supplied by source current $I_s$ will be maximum when load resistance is equal to source resistance i.e. $R_L = R_s$ . Maximum power is given as

$$P_{\max} = \frac{I_s^2 R_s}{4} = \frac{(30)^2 \times 100}{4} = 22.5 \text{ kW}$$

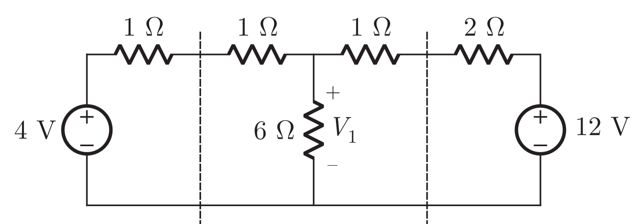


SOL 5.2.24 Correct answer is 6.

If we solve this circuit directly by nodal analysis, then we have to deal with three variables. We can replace the left most and write most circuit by their Thevenin equivalent as shown below.




Now the circuit becomes as shown



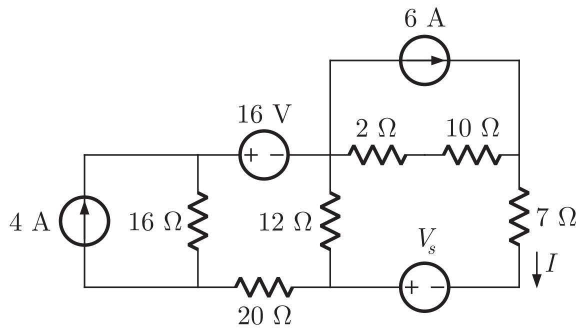

Writing node equation at the top center node

$$\frac{V_1 - 4}{1+1} + \frac{V_1}{6} + \frac{V_1 - 12}{1+2} = 0$$

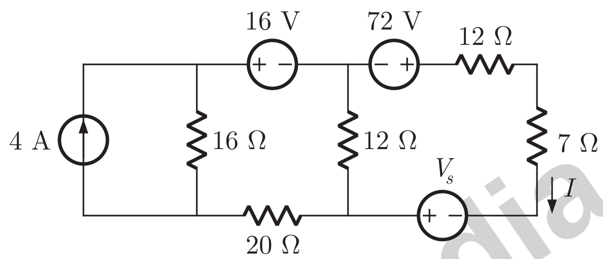
$$\frac{V_1+4}{2}+\frac{V_1}{6}+\frac{V_1-12}{3} = 0$$
$$3V_1 - 12 + V_1 + 2V_1 - 24 = 0$$
$$6V_1 = 36$$
$$V_1 = 6\text{ V}$$

SOL 5.2.25 Correct answer is 56.

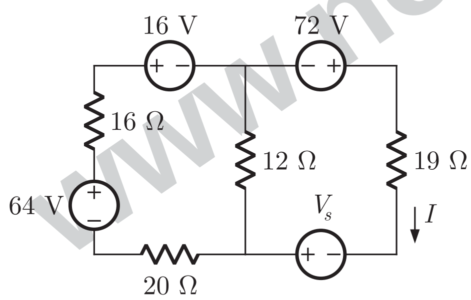
$6\,\Omega$ and $3\,\Omega$ resistors are in parallel, which is equivalent to $2\,\Omega$.



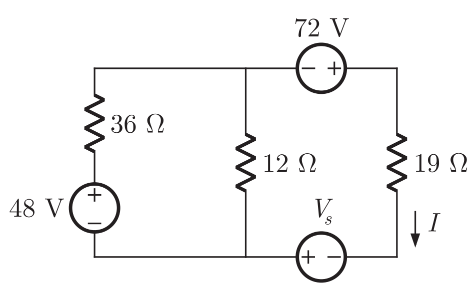

Using source transformation of 6 A source



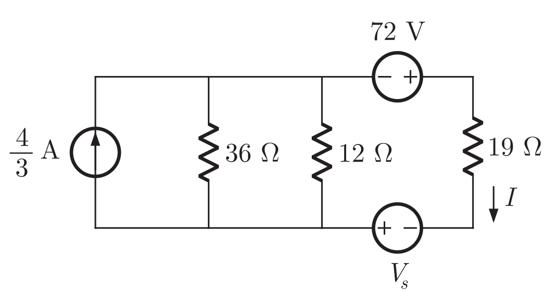

Source transform of 4 A source

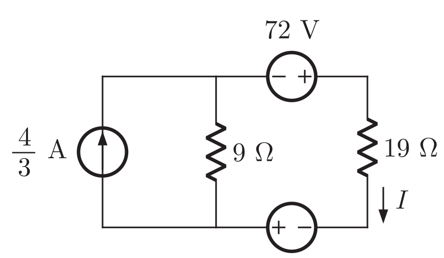



Adding series resistors and sources on the left

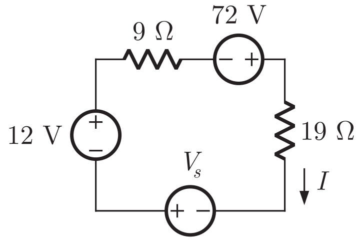



Source transformation of 48 V source



Source transformation of $\frac{4}{3}$ A source.



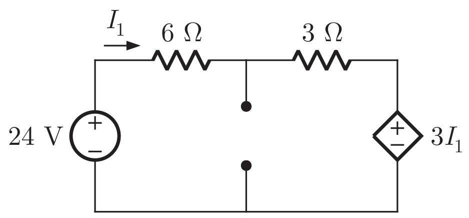

$$I = \frac{12 + 72 + V_s}{19 + 9}$$

$$V_s = (28 \times I) - 12 - 72 = (28 \times 5) - 12 - 72 = 56 \text{ V}$$

SOL 5.2.26 Correct answer is 0.5 .

We obtain $I$ using superposition.

Due to 24 V source only : (Open circuit 6 A)

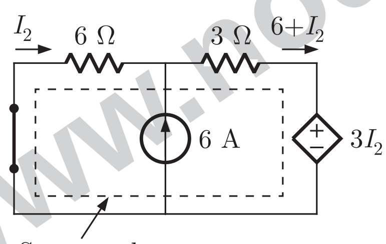



Applying KVL

$$24 - 6I_1 - 3I_1 - 3I_1 = 0$$

$$I_1 = \frac{24}{12} = 2 \text{ A}$$

Due to 6 A source only : (Short circuit 24 V source)




Applying KVL to supermesh

$$-6I_2 - 3(6 + I_2) - 3I_2 = 0$$

$$6I_2 + 18 + 3I_2 + 3I_2 = 0$$

$$I_2 = -\frac{18}{12} = -\frac{3}{2} \text{ A}$$

From superposition, $I = I_1 + I_2$

$$= 2 - \frac{3}{2} = \frac{1}{2} = 0.5 \text{ A}$$

ALTERNATIVE METHOD :

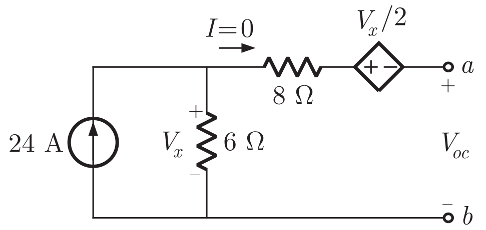
Note that current in 3 Ω resistor is $(I + 6)$ A, so by applying KVL around the outer loop, we can find current $I$.

SOL 5.2.27 Correct answer is 11.

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{\text{Open circuit voltage}}{\text{short circuit}}$$

Thevenin Voltage: (Open Circuit Voltage $V_{oc}$)

Using source transformation of the dependent source




Applying KCL at top left node

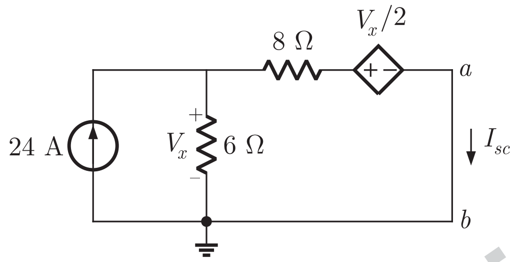
$$24 = \frac{V_x}{6} \Rightarrow V_x = 144\text{ V}$$

Using KVL, $$V_x - 8I - \frac{V_x}{2} - V_{oc} = 0$$

$$144 - 0 - \frac{144}{2} = V_{oc}$$

$$V_{oc} = 72\text{ V}$$

Short circuit current ($I_{sc}$):




Applying KVL in the right mesh

$$V_x - 8I_{sc} - \frac{V_x}{2} = 0$$

$$\frac{V_x}{2} = 8I_{sc}$$

$$V_x = 16I_{sc}$$

KCL at the top left node

$$24 = \frac{V_x}{6} + \frac{V_x - V_x/2}{8}$$

$$24 = \frac{V_x}{6} + \frac{V_x}{16}$$

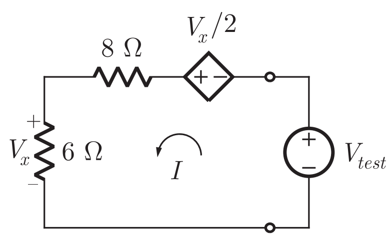
$$V_x = \frac{1152}{11}\text{ V}$$

$$I_{sc} = \frac{V_x}{16} = \frac{1152}{11 \times 16} = \frac{72}{11}\text{ A}$$

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{72}{\frac{72}{11}} = 11\,\Omega$$

ALTERNATIVE METHOD :

We can obtain Thevenin equivalent resistance without calculating the Thevenin voltage (open circuit voltage). Set all independent sources to zero (i.e. open circuit current sources and short circuit voltage sources) and put a test source $V_{test}$ between terminal $a$-$b$ as shown



$$R_{Th} = \frac{V_{test}}{I_{test}}$$

$$6I + 8I - \frac{V_x}{2} - V_{test} = 0 \quad \text{(KVL)}$$

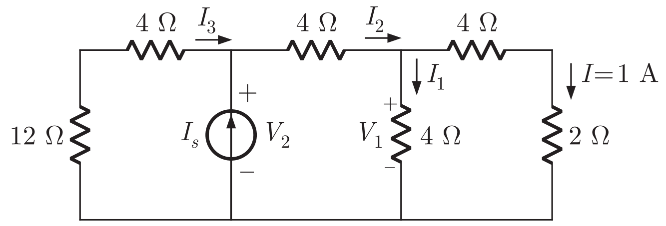
$$14I - \frac{6I}{2} - V_{test} = 0 \quad V_x = 6I_{test} \text{ (Using Ohm's law)}$$

$$11I = V_{test}$$

So $$R_{Th} = \frac{V_{test}}{I_{test}} = 11\,\Omega$$

SOL 5.2.28 Correct answer is 4.

We solve this problem using linearity and assumption that $I = 1\,\text{A}$.




$$V_1 = 4I + 2I \quad \text{(Using KVL)}$$
$$= 6\text{ V}$$
$$I_2 = I_1 + I \quad \text{(Using KCL)}$$
$$= \frac{V_1}{4} + I = \frac{6}{4} + 1 = 2.5\text{ A}$$
$$V_2 = 4I_2 + V_1 \quad \text{(Using KVL)}$$
$$= 4(2.5) + 6 = 16\text{ V}$$
$$I_s + I_3 = I_2 \quad \text{(Using KCL)}$$
$$I_s - \frac{V_2}{4+12} = I_2$$
$$I_s = \frac{16}{16} + 2.5 = 3.5\text{ A}$$

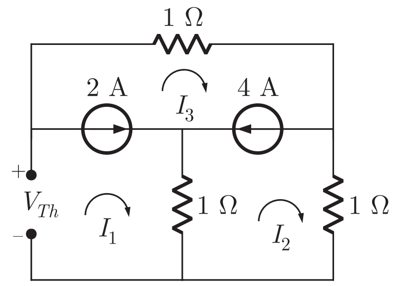
When $I_s = 3.5\,\text{A}$, $I = 1\,\text{A}$

But $I_s = 14\,\text{A}$, so $I = \frac{.1}{3.5} \times 14 = 4\,\text{A}$

SOL 5.2.29 Correct answer is 120.

This problem will easy to solve if we obtain Thevenin equivalent across the 12 V source.

Thevenin Voltage : (Open Circuit Voltage)




Mesh currents are

Mesh 1: $I_1 = 0$ (due to open circuit)

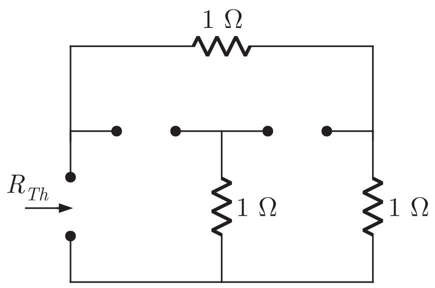
Mesh 2: $I_1 - I_3 = 2$ or $I_3 = -2\,\text{A}$

Mesh 3: $I_3 - I_2 = 4$ or $I_2 = -6\,\text{A}$

Mesh equation for outer loop

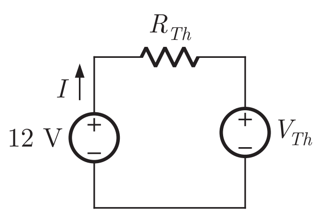
$$V_{Th} - 1 \times I_3 - 1 \times I_2 = 0$$
$$V_{Th} - (-2) - (-6) = 0$$
$$V_{Th} + 2 + 6 = 0$$
$$V_{Th} = -8 \text{ V}$$

Thevenin Resistance :




$$R_{Th} = 1 + 1 = 2\,\Omega$$

circuit becomes as

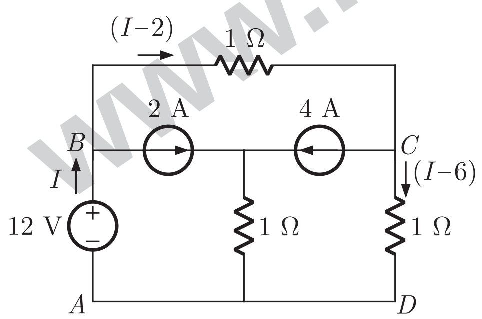



$$I = \frac{12 - V_{Th}}{R_{Th}} = \frac{12 - (-8)}{2} = 10 \text{ A}$$

Power supplied by 12 V source

$$P_{12\text{ V}} = 10 \times 12 = 120 \text{ W}$$

ALTERNATIVE METHOD :




KVL in the loop $ABCDA$

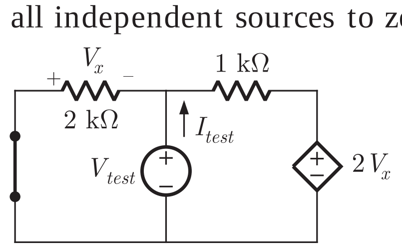
$$12 - 1(I-2) - 1(I-6) = 0$$
$$2I = 20$$
$$I = 10 \text{ A}$$

Power supplied by 12 V source

$$P_{12\text{ V}} = 10 \times 12 = 120 \text{ W}$$

SOL 5.2.30 Correct answer is 286.

For maximum power transfer $R_L = R_{Th}$. To obtain Thevenin resistance set all independent sources to zero and put a test source across load terminals.



$$R_{Th} = \frac{V_{test}}{I_{test}}$$

Writing KCL at the top center node

$$\frac{V_{test}}{2k} + \frac{V_{test} - 2V_x}{1k} = I_{test} \quad ...(1)$$

Also, $V_{test} + V_x = 0$ (KVL in left mesh)

so $V_x = - V_{test}$

Substituting $V_x = - V_{test}$ into equation (1)

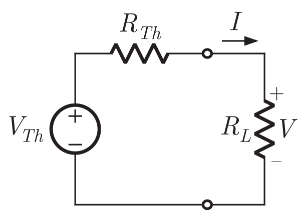
$$\frac{V_{test}}{2k} + \frac{V_{test} - 2(-V_{test})}{1k} = I_{test}$$

$$V_{test} + 6V_{test} = 2I_{test}$$

$$R_{Th} = \frac{V_{test}}{I_{test}} = \frac{2}{7}\,\text{k}\Omega \simeq 286\,\Omega$$

SOL 5.2.31 Correct answer is 4.

Redrawing the circuit in Thevenin equivalent form




$$I = \frac{V_{Th} - V}{R_{Th}}$$

or, $V = - R_{Th} I + V_{Th}$ (General form)

From the given graph

$$V = - 4I + 8$$

So, by comparing $R_{Th} = 4\,\text{k}\Omega, \quad V_{Th} = 8\text{ V}$

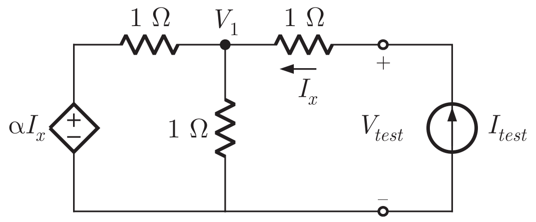
For maximum power transfer $R_L = R_{Th}$

Maximum power absorbed by $R_L$

$$P_{\max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(8)^2}{4 \times 4} = 4\text{ mW}$$

SOL 5.2.32 Correct answer is 3.

To fine out Thevenin equivalent of the circuit put a test source between node $a$ and $b$,




$$R_{Th} = \frac{V_{test}}{I_{test}}$$

Writing node equation at $V_1$

$$\frac{V_1 - \alpha I_x}{1} + \frac{V_1}{1} = I_x$$

$$2V_1 = (1 + \alpha) I_x \quad ...(1)$$

$I_x$ is the branch current in $1\,\Omega$ resistor given as

$$I_x = \frac{V_{test} - V_1}{1}$$

$$V_1 = V_{test} - I_x$$

Substituting $V_1$ into equation (1)

$$2(V_{test} - I_x) = (1+\alpha)I_x$$

$$2V_{test} = (3+\alpha)I_x$$

$$2V_{test} = (3+\alpha)I_{test} \qquad (I_x = I_{test})$$

$$R_{Th} = \frac{V_{test}}{I_{test}} = \frac{3+\alpha}{2} = 3$$

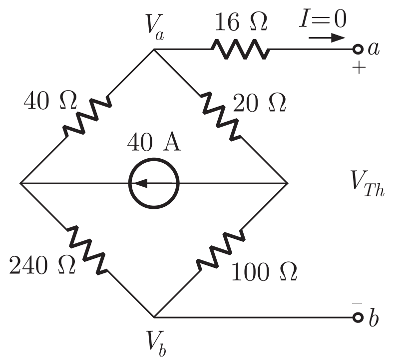
$$3+\alpha = 6$$

$$\alpha = 3\,\Omega$$

SOL 5.2.33 Correct answer is 16.

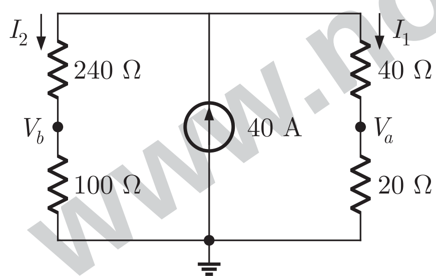
We obtain Thevenin equivalent across the load terminals

Thevenin Voltage : (Open circuit voltage)




$$V_{Th} = V_a - V_b$$

Rotating the circuit, makes it simple




$$I_1 = \frac{340}{340+60}(40) = 34\text{ A} \qquad \text{(Current division)}$$

$$V_a = 20I_1 = 20\times 34 = 680\text{ V} \qquad \text{(Ohm's Law)}$$

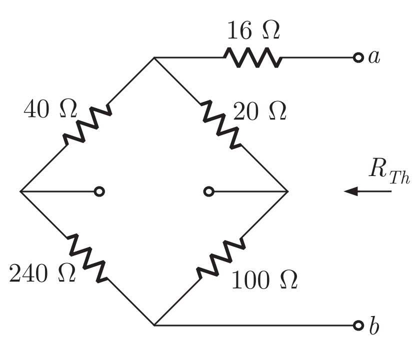
Similarly,

$$I_2 = \frac{60}{60+340}(40) = 6\text{ A} \qquad \text{(Current division)}$$

$$V_b = 100I_2 = 100\times 6 = 600\text{ V} \qquad \text{(Ohm's Law)}$$

Thevenin voltage $V_{Th} = 680 - 600 = 80$ V

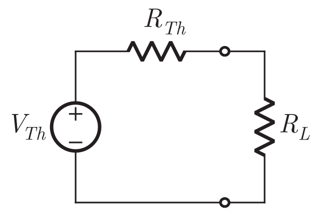
Thevenin Resistance :




$$R_{Th} = 16 + (240+40)\,||\,(20+100)$$

$$= 16 + (280\,||\,120) = 16 + 84 = 100\,\Omega$$

Now, circuit reduced as




For maximum power transfer

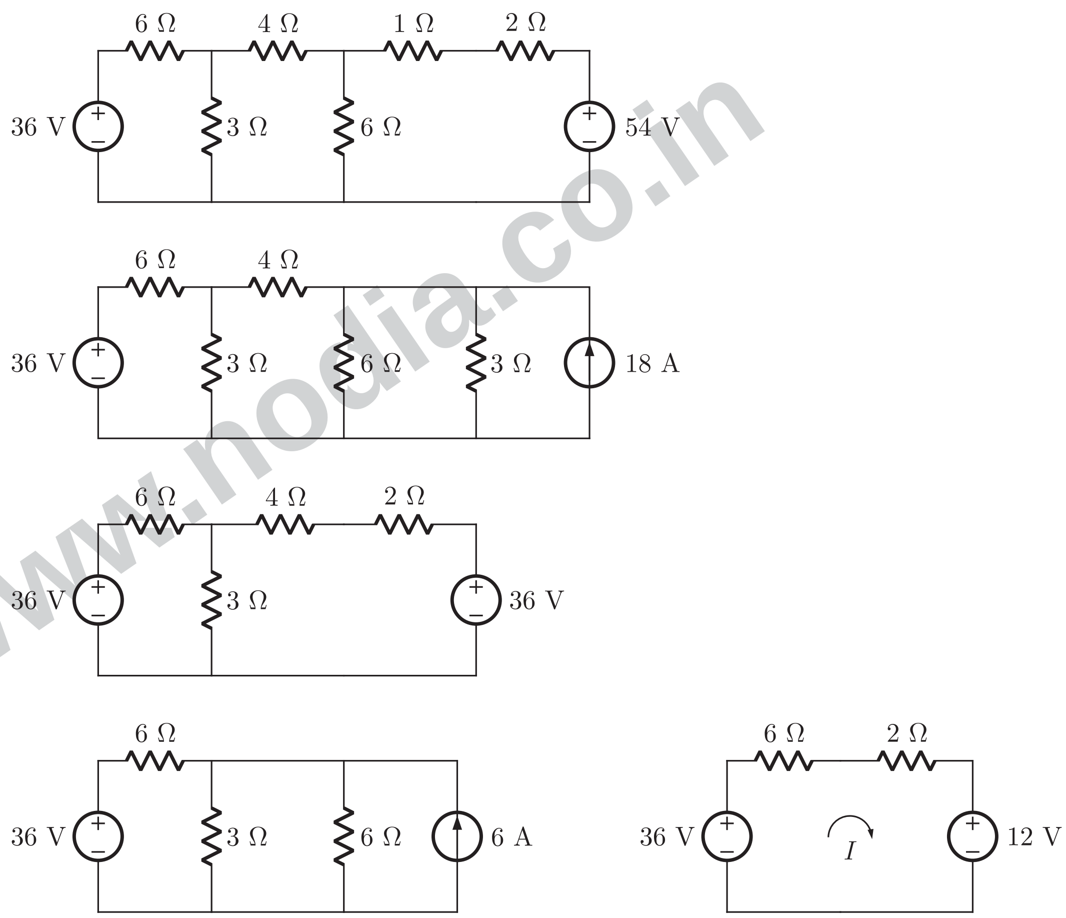
$$R_L = R_{Th} = 100\,\Omega$$

Maximum power transferred to $R_L$

$$P_{\max} = \frac{(V_{Th})^2}{4R_L} = \frac{(80)^2}{4 \times 100} = 16\text{ W}$$

SOL 5.2.34 Correct answer is 108.

We use source transformation as follows



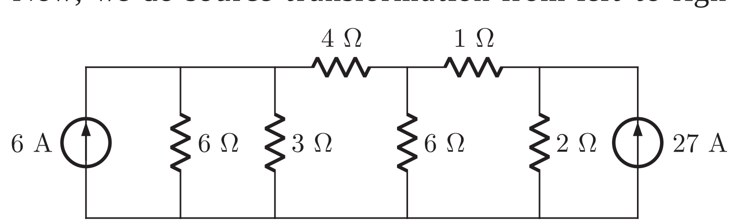

$$I = \frac{36 - 12}{6 + 2} = 3\text{ A}$$

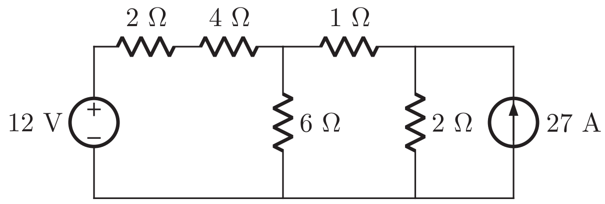
Power supplied by 36 V source

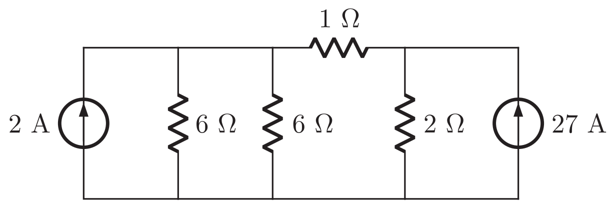
$$P_{36\text{ V}} = 3 \times 36 = 108\text{ W}$$

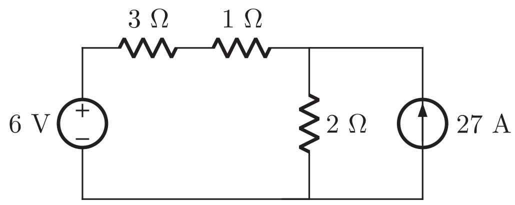
SOL 5.2.35 Correct answer is 1026.

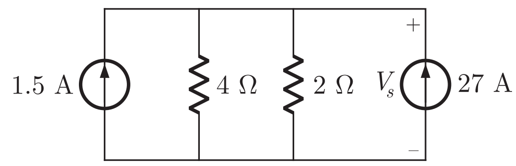
Now, we do source transformation from left to right as shown






$$\begin{aligned} V_s &= (27+1.5)(4\,\Omega \,||\, 2\,\Omega) \\ &= 28.5 \times \frac{4}{3} \\ &= 38 \text{ V} \end{aligned}$$

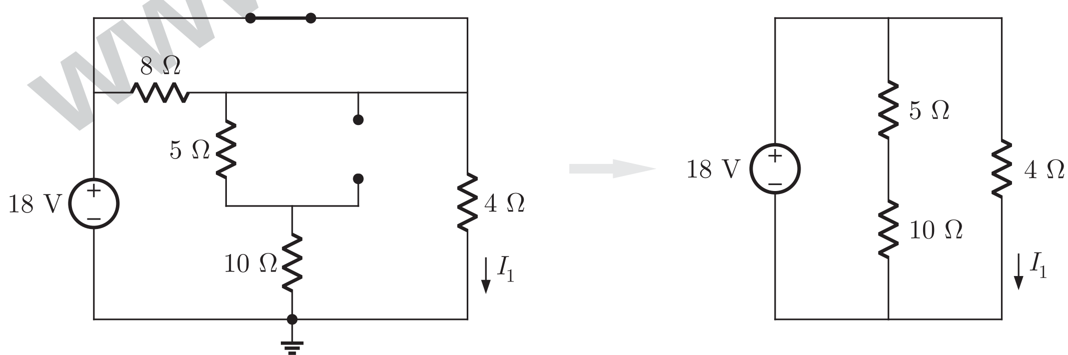
Power supplied by 27 A source

$$\begin{aligned} P_{27\text{ A}} &= V_s \times 27 = 38 \times 27 \\ &= 1026 \text{ W} \end{aligned}$$

SOL 5.2.36 Correct answer is 9.

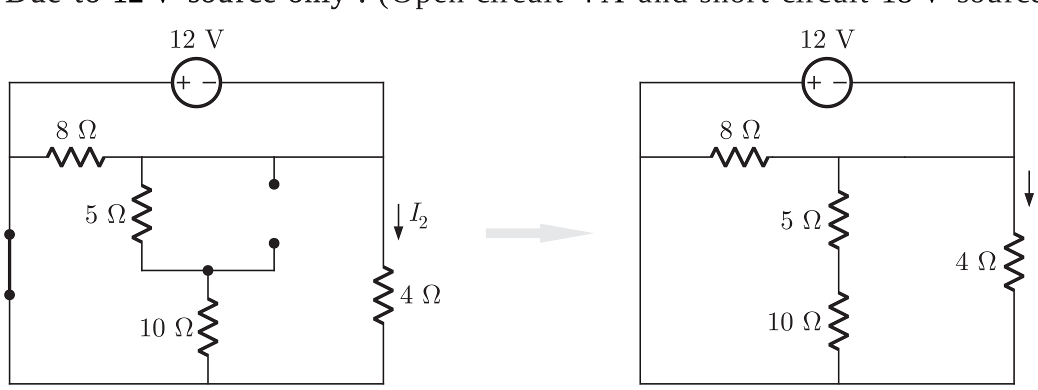
First, we find current $I$ in the $4\,\Omega$ resistors using superposition.

Due to 18 V source only : (Open circuit 4 A and short circuit 12 V source)



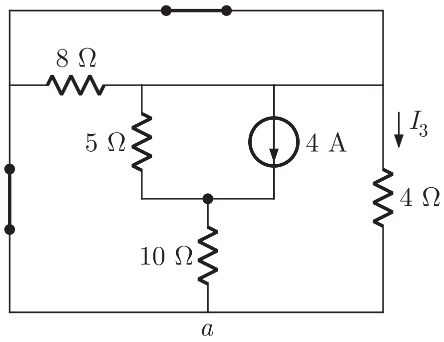

$$I_1 = \frac{18}{4} = 4.5 \text{ A}$$

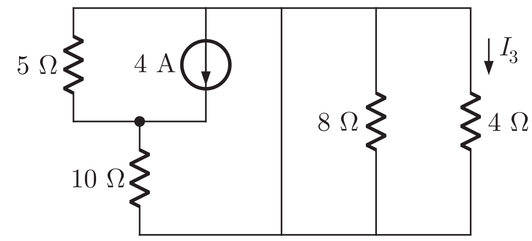
Due to 12 V source only : (Open circuit 4 A and short circuit 18 V source)



$$I_2 = -\frac{12}{4} = -3\text{ A}$$

Due to 4 A source only : (Short circuit 12 V and 18 V sources)




$$I_3 = 0 \qquad \text{(Due to short circuit)}$$

So,
$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ &= 4.5 - 3 + 0 \\ &= 1.5\text{ A} \end{aligned}$$

Power dissipated in $4\,\Omega$ resistor

$$P_{4\,\Omega} = I^2(4) = (1.5)^2 \times 4 = 9\text{ W}$$

Alternate Method: Let current in $4\,\Omega$ resistor is $I$, then by applying KVL around the outer loop

$$18 - 12 - 4I = 0$$

$$I = \frac{6}{4} = 1.5\text{ A}$$

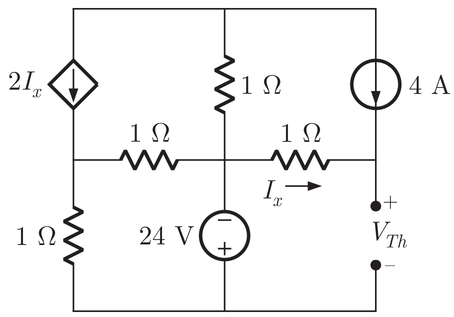
So, power dissipated in $4\,\Omega$ resistor

$$\begin{aligned} P_{4\,\Omega} &= I^2(4) = (1.5)^2 \times 4 \\ &= 9\text{ W} \end{aligned}$$

SOL 5.2.37 Correct answer is $-10$.

Using, Thevenin equivalent circuit

Thevenin Voltage : (Open Circuit Voltage)




$$I_x = -4\text{ A} \qquad \text{(due to open circuit)}$$

Writing KVL in bottom right mesh

$$-24 - (1)I_x - V_{Th} = 0$$

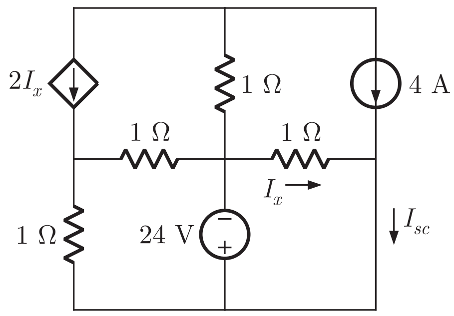
$$V_{Th} = -24 + 4 = -20\text{ V}$$

Thevenin Resistance :

$$R_{Th} = \frac{\text{open circuit voltage}}{\text{short circuit current}} = \frac{V_{oc}}{I_{sc}}$$

$$V_{oc} = V_{Th} = -20\text{ V}$$

$I_{sc}$ is obtained as follows




$$I_x = -\frac{24}{1} = -24\text{ A}$$

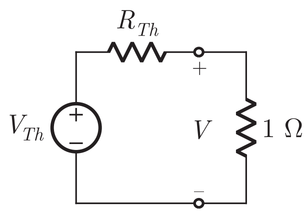
$$I_x + 4 = I_{sc} \qquad \text{(using KCL)}$$

$$-24 + 4 = I_{sc}$$

$$I_{sc} = -20\text{ A}$$

$$R_{Th} = \frac{-20}{-20} = 1\,\Omega$$

The circuit is as shown below




$$V = \frac{1}{1+R_{Th}}(V_{Th}) = \frac{1}{1+1}(-20) = -10\text{ volt} \quad \text{(Using voltage division)}$$

ALTERNATIVE METHOD :

Note that current in bottom right most $1\,\Omega$ resistor is $(I_x + 4)$, so applying KVL around the bottom right mesh,

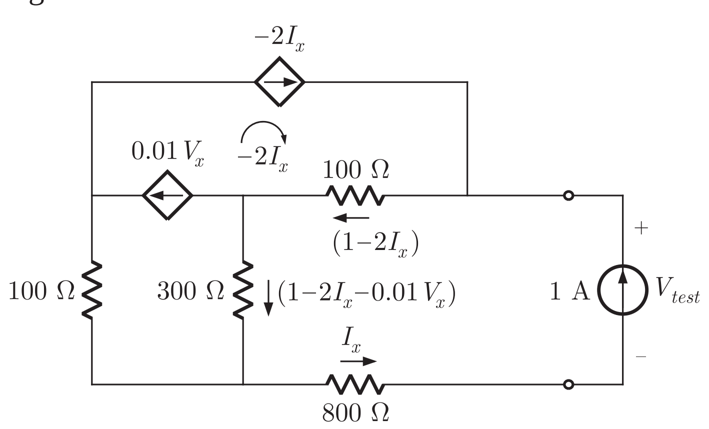
$$-24 - I_x - (I_x + 4) = 0$$

$$I_x = -14\text{ A}$$

So, $$V = 1 \times (I_x + 4) = -14 + 4 = -10\text{ V}$$

SOL 5.2.38 Correct answer is 100.

Writing currents into $100\,\Omega$ and $300\,\Omega$ resistors by using KCL as shown in figure.




$$I_x = 1\text{ A}, V_x = V_{test}$$

Writing mesh equation for bottom right mesh.

$$V_{test} = 100(1-2I_x) + 300(1-2I_x - 0.01V_x) + 800$$

$$= 100\text{ V}$$

$$R_{Th} = \frac{V_{test}}{1} = 100\,\Omega$$

SOL 5.2.39 Correct answer is 30.

For $R_L = 10\,\text{k}\Omega$, $V_{ab1} = \sqrt{10\text{k} \times 3.6\text{m}} = 6$ V

For $R_L = 30\,\text{k}\Omega$, $V_{ab2} = \sqrt{30\text{k} \times 4.8\text{m}} = 12$ V

$$V_{ab1} = \frac{10}{10 + R_{Th}} V_{Th} = 6 \qquad \text{...(1)}$$

$$V_{ab2} = \frac{30}{30 + R_{Th}} V_{Th} = 12 \qquad \text{...(2)}$$

Dividing equation (1) and (2), we get $R_{Th} = 30\,\text{k}\Omega$. Maximum power will be transferred when $R_L = R_{Th} = 30\,\text{k}\Omega$.

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**Eighth Edition**

# GATE

## ELECTRONICS & COMMUNICATION

## Signals and Systems

Vol 7 of 10

**RK Kanodia**
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GATE Electronics & Communication Vol 7, 8e
Signals and Systems
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MRP 690.00

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Printed by Nodia and Company, Jaipur

# *Preface to the Series*

For almost a decade, we have been receiving tremendous responses from GATE aspirants for our earlier books: GATE Multiple Choice Questions, GATE Guide, and the GATE Cloud series. Our first book, GATE Multiple Choice Questions (MCQ), was a compilation of objective questions and solutions for all subjects of GATE Electronics & Communication Engineering in one book. The idea behind the book was that Gate aspirants who had just completed or about to finish their last semester to achieve his or her B.E/B.Tech need only to practice answering questions to crack GATE. The solutions in the book were presented in such a manner that a student needs to know fundamental concepts to understand them. We assumed that students have learned enough of the fundamentals by his or her graduation. The book was a great success, but still there were a large ratio of aspirants who needed more preparatory materials beyond just problems and solutions. This large ratio mainly included average students.

Later, we perceived that many aspirants couldn't develop a good problem solving approach in their B.E/B.Tech. Some of them lacked the fundamentals of a subject and had difficulty understanding simple solutions. Now, we have an idea to enhance our content and present two separate books for each subject: one for theory, which contains brief theory, problem solving methods, fundamental concepts, and points-to-remember. The second book is about problems, including a vast collection of problems with descriptive and step-by-step solutions that can be understood by an average student. This was the origin of *GATE Guide* (the theory book) and *GATE Cloud* (the problem bank) series: two books for each subject. *GATE Guide* and *GATE Cloud* were published in three subjects only.

Thereafter we received an immense number of emails from our readers looking for a complete study package for all subjects and a book that combines both *GATE Guide* and *GATE Cloud*. This encouraged us to present GATE Study Package (a set of 10 books: one for each subject) for GATE Electronic and Communication Engineering. Each book in this package is adequate for the purpose of qualifying GATE for an average student. Each book contains brief theory, fundamental concepts, problem solving methodology, summary of formulae, and a solved question bank. The question bank has three exercises for each chapter: 1) Theoretical MCQs, 2) Numerical MCQs, and 3) Numerical Type Questions (based on the new GATE pattern). Solutions are presented in a descriptive and step-by-step manner, which are easy to understand for all aspirants.

We believe that each book of GATE Study Package helps a student learn fundamental concepts and develop problem solving skills for a subject, which are key essentials to crack GATE. Although we have put a vigorous effort in preparing this book, some errors may have crept in. We shall appreciate and greatly acknowledge all constructive comments, criticisms, and suggestions from the users of this book. You may write to us at rajkumar.kanodia@gmail.com and ashish.murolia@gmail.com.

Acknowledgements

We would like to express our sincere thanks to all the co-authors, editors, and reviewers for their efforts in making this project successful. We would also like to thank Team NODIA for providing professional support for this project through all phases of its development. At last, we express our gratitude to God and our Family for providing moral support and motivation.

We wish you good luck !

R. K. Kanodia

Ashish Murolia

# SYLLABUS

### GATE Electronics & Communications:

Definitions and properties of Laplace transform, continuous-time and discrete-time Fourier series, continuous-time and discrete-time Fourier Transform, DFT and FFT, z-transform. Sampling theorem. Linear Time-Invariant (LTI) Systems: definitions and properties; causality, stability, impulse response, convolution, poles and zeros, parallel and cascade structure, frequency response, group delay, phase delay. Signal transmission through LTI systems.

### IES Electronics & Telecommunication

Classification of signals and systems: System modelling in terms of differential and difference equations; State variable representation; Fourier series; Fourier transforms and their application to system analysis; Laplace transforms and their application to system analysis; Convolution and superposition integrals and their applications; Z-transforms and their applications to the analysis and characterisation of discrete time systems; Random signals and probability, Correlation functions; Spectral density; Response of linear system to random inputs.

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## CHAPTER 1 CONTINUOUS TIME SIGNALS

## CHAPTER 2 CONTINUOUS TIME SYSTEMS



## CHAPTER 3 DISCRETE TIME SIGNALS

## CHAPTER 7 THE CONTINUOUS TIME FOURIER TRANSFORM



## CHAPTER 8 THE DISCRETE TIME FOURIER TRANSFORM

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# CHAPTER 6

## THE Z-TRANSFORM

### 6.1 INTRODUCTION

As we studied in previous chapter, the Laplace transform is an important tool for analysis of continuous time signals and systems. Similarly, $z$-transforms enables us to analyze discrete time signals and systems in the $z$-domain.

Like, the Laplace transform, it is also classified as bilateral $z$-transform and unilateral $z$-transform.

The bilateral or two-sided $z$-transform is used to analyze both causal and non-causal LTI discrete systems, while the unilateral $z$-transform is defined only for causal signals.

NOTE :
The properties of $z$-transform are similar to those of the Laplace transform.

### 6.1.1 The Bilateral or Two-Sided $z$-transform

The $z$-transform of a discrete-time sequence $x[n]$, is defined as

$$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \tag{6.1.1}$$

Where, $X(z)$ is the transformed signal and $\mathcal{Z}$ represents the $z$-transformation. $z$ is a complex variable. In polar form, $z$ can be expressed as

$$z = re^{j\Omega}$$

where $r$ is the magnitude of $z$ and $\Omega$ is the angle of $z$. This corresponds to a circle in $z$ plane with radius $r$ as shown in figure 6.1.1 below

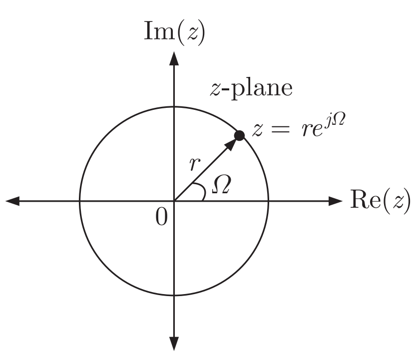



Figure 6.1.1 $z$-plane

NOTE :
The signal $x[n]$ and its $z$-transform $X(z)$ are said to form a $z$-transform pair denoted as

$$x[n] \xleftrightarrow{z} X(z)$$

### 6.1.2 The Unilateral or One-sided $z$-transform

The $z$-transform for causal signals and systems is referred to as the unilateral $z$-transform. For a causal sequence

$$x[n] = 0, \text{ for } n < 0$$

Therefore, the unilateral $z$-transform is defined as

$$X(z) = \sum_{n=0}^{\infty} x[n] z^{-n} \tag{6.1.2}$$

NOTE :
For causal signals and systems, the unilateral and bilateral $z$-transform are the same.

## 6.2 EXISTENCE OF -TRANSFORM

Consider the bilateral $z$-transform given by equation (6.1.1)

$$X[z] = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

The $z$-transform exists when the infinite sum in above equation converges. For this summation to be converged $|x[n]z^{-n}|$ must be absolutely summable.

Substituting $z = re^{j\Omega}$

$$X[z] = \sum_{n=-\infty}^{\infty} x[n] (re^{j\Omega})^{-n}$$

or,

$$X[z] = \sum_{n=-\infty}^{\infty} \{x[n] r^{-n}\} e^{-j\Omega n}$$

Thus for existence of $z$-transform

$$|X(z)| < \infty$$

$$\sum_{n=-\infty}^{\infty} x[n] r^{-n} < \infty \tag{6.2.1}$$

## 6.3 REGION OF CONVERGENCE

The existence of $z$-transform is given from equation (6.2.1). The values of $r$ for which $x[n]r^{-n}$ is absolutely summable is referred to as region of convergence. Since, $z = re^{j\Omega}$ so $r = |z|$. Therefore we conclude that the range of values of the variable $|z|$ for which the sum in equation (6.1.1) converges is called the region of convergence. This can be explained through the following examples.

### 6.3.1 Poles and Zeros of Rational $z$-transforms

The most common form of $z$-transform is a rational function. Let $X(z)$ be the $z$-transform of sequence $x[n]$, expressed as a ratio of two polynomials $N(z)$ and $D(z)$.

$$X(z) = \frac{N(z)}{D(z)}$$

The roots of numerator polynomial i.e. values of $z$ for which $X(z) = 0$ is referred to as zeros of $X(z)$. The roots of denominator polynomial for which $X(z) = \infty$ is referred to as poles of $X(z)$. The representation of $X(z)$ through its poles and zeros in the $z$-plane is called pole-zero plot of $X(z)$.
For example consider a rational transfer function $X(z)$ given as

$$H(z) = \frac{z}{z^2 - 5z + 6}$$

$$= \frac{z}{(z-2)(z-3)}$$

Now, the zeros of $X(z)$ are roots of numerator that is $z = 0$ and poles are roots of equation $(z-2)(z-3) = 0$ which are given as $z = 2$ and $z = 3$. The poles and zeros of $X(z)$ are shown in pole-zero plot of figure 6.3.1.

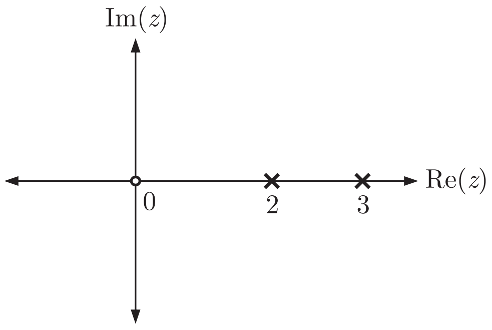



Figure 6.3.1 Pole-zero plot of $X(z)$

NOTE :
In pole-zero plot poles are marked by a small cross '×' and zeros are marked by a small dot 'o' as shown in figure 6.3.1.

### 6.3.2 Properties of ROC

The various properties of ROC are summarized as follows. These properties can be proved by taking appropriate examples of different DT signals.

PROPERTY 1

The ROC is a concentric ring in the $z$-plane centered about the origin.

PROOF :
The $z$-transform is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

Put $z = re^{j\Omega}$

$$X(z) = X(re^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n] r^{-n} e^{-j\Omega n}$$

$X(z)$ converges for those values of $z$ for which $x[n] r^{-n}$ is absolutely summbable that is

$$\sum_{n=-\infty}^{\infty} x[n] r^{-n} < \infty$$

Thus, convergence is dependent only on $r$, where, $r = |z|$
The equation $z = re^{j\Omega}$, describes a circle in $z$-plane. Hence the ROC will consists of concentric rings centered at zero.

PROPERTY 2

The ROC cannot contain any poles.

PROOF :
ROC is defined as the values of $z$ for which $z$-transform $X(z)$ converges. We know that $X(z)$ will be infinite at pole, and, therefore $X(z)$ does not converge at poles. Hence the region of convergence does not include any pole.

PROPERTY 3

If $x[n]$ is a finite duration two-sided sequence then the ROC is entire $z$-plane except at $z = 0$ and $z = \infty$.

PROOF :

A sequence which is zero outside a finite interval of time is called 'finite duration sequence'. Consider a finite duration sequence $x[n]$ shown in figure 6.3.2a; $x[n]$ is non-zero only for some interval $N_1 \le n \le N_2$.

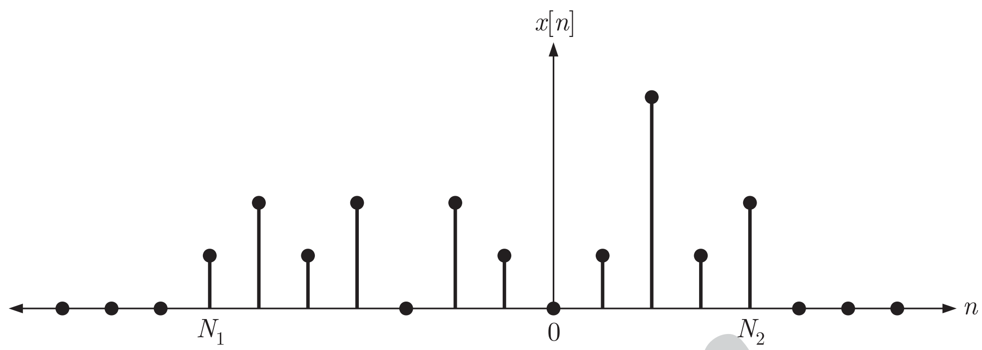



Figure 6.3.2a A Finite Duration Sequence

The $z$-transform of $x[n]$ is defined as

$$X(z) = \sum_{n=N_1}^{N_2} x[n] z^{-n}$$

This summation converges for all finite values of $z$. If $N_1$ is negative and $N_2$ is positive, then $X(z)$ will have both positive and negative powers of $z$. The negative powers of $z$ becomes unbounded (infinity) if $|z| \to 0$. Similarly positive powers of $z$ becomes unbounded (infinity) if $|z| \to \infty$. So ROC of $X(z)$ is entire $z$-plane except possible $z = 0$ and/or $z = \infty$.



NOTE :
Both $N_1$ and $N_2$ can be either positive or negative.

PROPERTY 4

If $x[n]$ is a right-sided sequence, and if the circle $|z| = r_0$ is in the ROC, then all values of $z$ for which $|z| > r_0$ will also be in the ROC.

PROOF :

A sequence which is zero prior to some finite time is called the *right-sided sequence*. Consider a right-sided sequence $x[n]$ shown in figure 6.3.2b; that is;

$x[n] = 0$ for $n < N_1$.

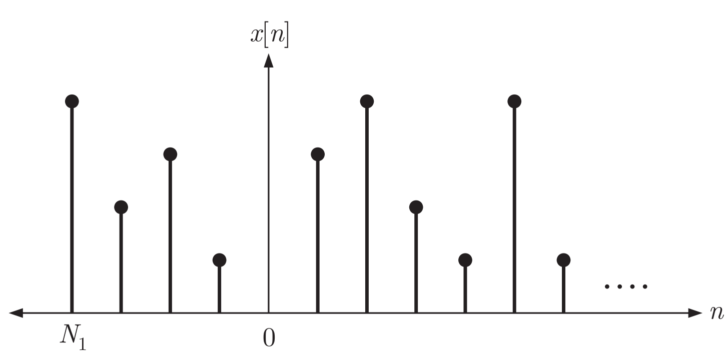



Figure 6.3.2b A Right - Sided Sequence

Let the $z$-transform of $x[n]$ converges for some value of $|z|$ (i.e. $|z| = r_0$). From the condition of convergence we can write

$$\left|\sum_{n=-\infty}^{\infty} x[n]z^{-n}\right| < \infty$$

$$\sum_{n=-\infty}^{\infty} |x[n]| r_0^{-n} < \infty$$

The sequence is right sided, so limits of above summation changes as

$$\sum_{n=N_1}^{\infty} |x[n]| r_0^{-n} < \infty \tag{6.3.1}$$

Now if we take another value of $z$ as $|z| = r_1$ with $r_1 < r_0$, then $x[n]r_1^{-n}$ decays faster than $x[n]r_0^{-n}$ for increasing $n$. Thus we can write

$$\sum_{n=N_1}^{\infty} |x[n]| z^{-n} = \sum_{n=N_1}^{\infty} |x[n]| z^{-n} r_0^{-n} r_0^{n}$$

$$= \sum_{n=N_1}^{\infty} |x[n]| r_0^{-n} \left(\frac{z}{r_0}\right)^{-n} \tag{6.3.2}$$

From equation (6.3.1) we know that $x[n]r_0^{-n}$ is absolutely summable. Let, it is bounded by some value $M_x$, then equation (6.3.2) becomes as

$$\sum_{n=N_1}^{\infty} |x[n]| z^{-n} \le M_x \sum_{n=N_1}^{\infty} \left(\frac{z}{r_0}\right)^{-n} \tag{6.3.3}$$

The right hand side of above equation converges only if

$$\left|\frac{z}{r_0}\right| > 1 \text{ or } |z| > r_0$$

Thus, we conclude that if the circle $|z| = r_0$ is in the ROC, then all values of $z$ for which $|z| > r_0$ will also be in the ROC. The ROC of a right-sided sequence is illustrated in figure 6.3.2c.

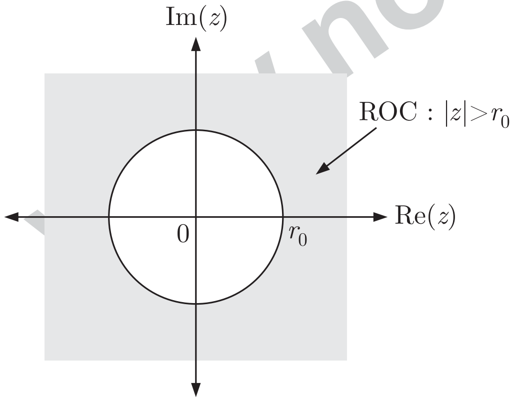



Figure 6.3.2c ROC of a right-sided sequence

PROPERTY 5

If $x[n]$ is a left-sided sequence, and if the circle $|z| = r_0$ is in the ROC, then all values of $z$ for which $|z| < r_0$ will also be in the ROC.

PROOF :

A sequence which is zero after some finite time interval is called a 'left-sided signal'. Consider a left-sided signal $x[n]$ shown in figure 6.3.2d; that is $x[n] = 0$ for $n > N_2$.

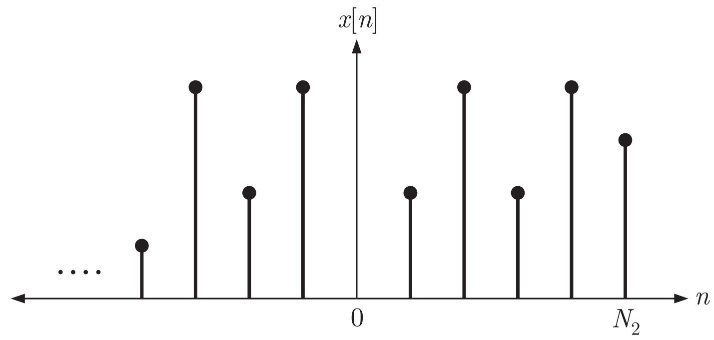



Figure 6.3.2d A left-sided sequence

Let $z$-transform of $x[n]$ converges for some values of $|z|$ (i.e. $|z| = r_0$). From the condition of convergence we write

$$\left|\sum_{n=-\infty}^{\infty} x[n] z^{-n}\right| < \infty$$

or
$$\sum_{n=-\infty}^{\infty} |x[n]| r_0^{-n} < \infty \qquad (6.3.4)$$

The sequence is left sided, so the limits of summation changes as

$$\sum_{n=-\infty}^{N_2} |x[n]| r_0^{-n} < \infty \qquad (6.3.5)$$

Now if take another value of $z$ as $|z| = r_1$, then we can write

$$\sum_{n=-\infty}^{N_2} |x[n]| z^{-n} = \sum_{n=-\infty}^{N_2} |x[n]| z^{-n} r_0^{-n} r_0^{n}$$
$$= \sum_{n=-\infty}^{N_2} |x[n]| r_0^{-n} \left(\frac{r_0}{z}\right)^n \qquad (6.3.6)$$

From equation (6.3.4), we know that $x[n] r_0^{-n}$ is absolutely summable. Let it is bounded by some value $M_x$, then equation (6.3.6) becomes as

$$\sum_{n=-\infty}^{N_2} |x[n]| z^{-n} \le M_x \sum_{n=-\infty}^{N_2} \left(\frac{r_0}{z}\right)^n$$

The above summation converges if $\left|\frac{r_0}{z}\right| > 1$ (because $n$ is increasing negatively), so $|z| < r_0$ will be in ROC.

The ROC of a left-sided sequence is illustrated in figure 6.3.2e.

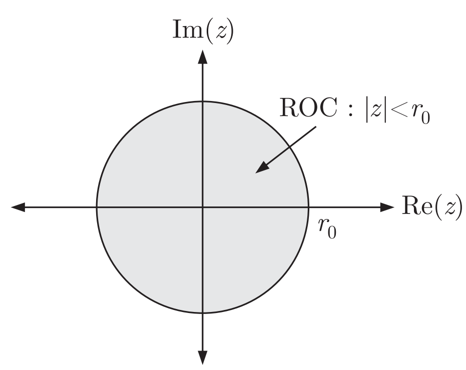



Figure 6.3.2e ROC of a Left - Sided Sequence

PROPERTY 6

If $x[n]$ is a two-sided signal, and if the circle $|z| = r_0$ is in the ROC, then the ROC consists of a ring in the $z$-plane that includes the circle $|z| = r_0$

PROOF :

A sequence which is defined for infinite extent for both $n > 0$ and $n < 0$ is called 'two-sided sequence'. A two-sided signal $x[n]$ is shown in figure 6.3.2f.

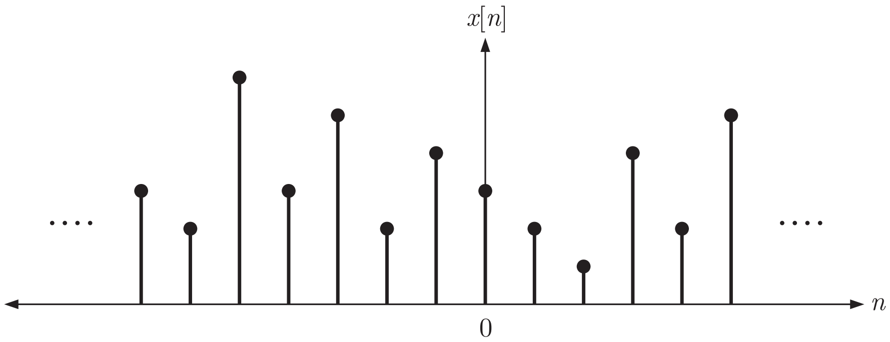



Figure 6.3.2f A Two - Sided Sequence

For any time $N_0$, a two-sided sequence can be divided into sum of left-sided and right-sided sequences as shown in figure 6.3.2g.

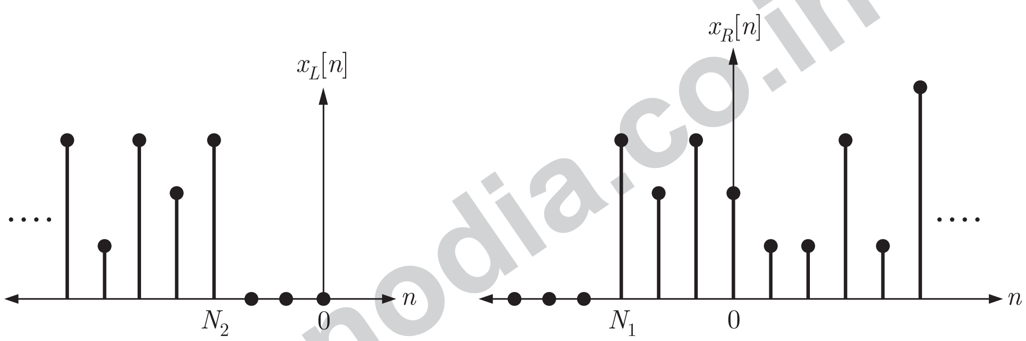
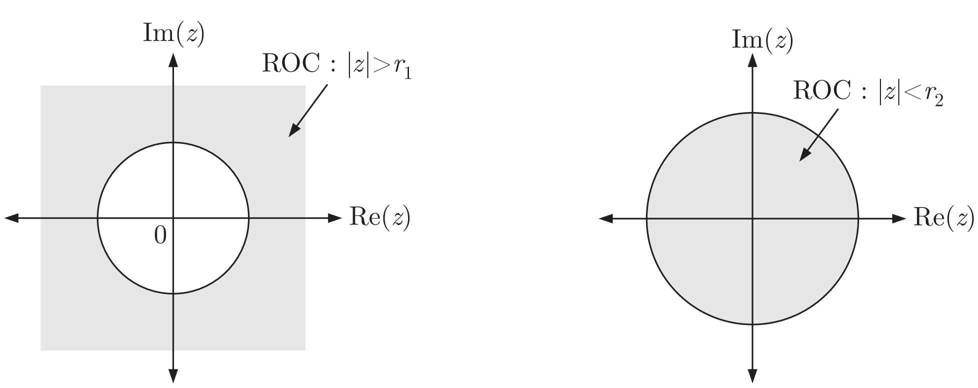



Figure 6.3.2g A Two Sided Sequence Divided into Sum of a Left - Sided and Right - Sided Sequence



The $z$-transform of $x[n]$ converges for the values of $z$ for which the transform of both $x_R[n]$ and $x_L[n]$ converges. From property 4, the ROC of a right-sided sequence is a region which is bounded on the inside by a circle and extending outward to infinity i.e. $|z| > r_1$. From property 5, the ROC of a left sided sequence is bounded on the outside by a circle and extending inward to zero i.e. $|z| < r_2$. So the ROC of combined signal includes intersection of both ROCs which is ring in the $z$-plane.

The ROC for the right-sided sequence $x_R[n]$, the left-sequence $x_L[n]$ and their combination which is a two sided sequence $x[n]$ are shown in figure 6.3.2h.



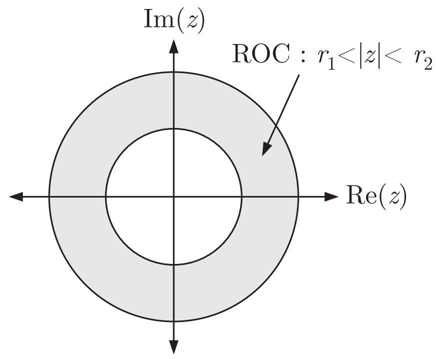



Figure 6.3.2h ROC of a left-sided sequence, a right-sided sequence and two sided sequence

PROPERTY 7

If the $z$-transform $X(z)$ of $x[n]$ is rational, then its ROC is bounded by poles or extends to infinity.

PROOF :
The exponential DT signals also have rational $z$-transform and the poles of $X(z)$ determines the boundaries of ROC.

PROPERTY 8

If the $z$-transform $X(z)$ of $x[n]$ is rational and $x[n]$ is a right-sided sequence then the ROC is the region in the $z$-plane outside the outermost pole i.e. ROC is the region outside a circle with a radius greater than the magnitude of largest pole of $X(z)$.

PROOF :
This property can be be proved by taking property 4 and 7 together.

PROPERTY 9

If the $z$-transform $X(z)$ of $x[n]$ is rational and $x[n]$ is a left-sided sequence then the ROC is the region in the $z$-plane inside the innermost pole i.e. ROC is the region inside a circle with a radius equal to the smallest magnitude of poles of $X(z)$.

PROOF :
This property can be be proved by taking property 5 and 7 together.

### -Transform of Some Basic Functions

Z-transform of basic functions are summarized in the Table 6.1 with their respective ROCs.

## 6.4 THE INVERSE -TRANSFORM

Let $X(z)$ be the $z$-transform of a sequence $x[n]$. To obtain the sequence $x[n]$ from its $z$-transform is called the inverse $z$-transform. The inverse $z$-transform is given as

TABLE 6.1 : $z$-Transform of Basic Discrete Time Signals

| | DT sequence $x[n]$ | $z$-transform | ROC |
|---|---|---|---|
| 1. | $\delta[n]$ | 1 | entire $z$-plane |
| 2. | $\delta[n-n_0]$ | $z^{-n_0}$ | entire $z$-plane, except $z=0$ |
| 3. | $u[n]$ | $\frac{1}{1-z^{-1}}=\frac{z}{z-1}$ | $\lvert z\rvert>1$ |
| 4. | $\alpha^n u[n]$ | $\frac{1}{1-\alpha z^{-1}}=\frac{z}{z-\alpha}$ | $\lvert z\rvert>\lvert\alpha\rvert$ |
| 5. | $\alpha^{n-1}u[n-1]$ | $\frac{z^{-1}}{1-\alpha z^{-1}}=\frac{1}{z-\alpha}$ | $\lvert z\rvert>\lvert\alpha\rvert$ |
| 6. | $nu[n]$ | $\frac{z^{-1}}{(1-z^{-1})^2}=\frac{z}{(z-1)^2}$ | $\lvert z\rvert>1$ |
| 7. | $n\alpha^n u[n]$ | $\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}=\frac{\alpha z}{(z-\alpha)^2}$ | $\lvert z\rvert>\alpha$ |
| 8. | $\cos(\Omega_0 n)u[n]$ | $\frac{1-z^{-1}\cos\Omega_0}{1-2z^{-1}\cos\Omega_0+z^{-2}}$ or $\frac{z[z-\cos\Omega_0]}{z^2-2z\cos\Omega_0+1}$ | $\lvert z\rvert>1$ |
| 9. | $\sin(\Omega_0 n)u[n]$ | $\frac{z^{-1}\sin\Omega_0}{1-2z^{-1}\cos\Omega_0+z^{-2}}$ or $\frac{z\sin\Omega_0}{z^2-2z\cos\Omega_0+1}$ | $\lvert z\rvert>1$ |
| 10. | $\alpha^n\cos(\Omega_0 n)u[n]$ | $\frac{1-\alpha z^{-1}\cos\Omega_0}{1-2\alpha z^{-1}\cos\Omega_0+\alpha^2z^{-2}}$ or $\frac{z[z-\alpha\cos\Omega_0]}{z^2-2\alpha z\cos\Omega_0+\alpha^2}$ | $\lvert z\rvert>\lvert\alpha\rvert$ |
| 11. | $\alpha^n\sin(\Omega_0 n)u[n]$ | $\frac{\alpha z^{-1}\sin\Omega_0}{1-2\alpha z^{-1}\cos\Omega_0+\alpha^2z^{-2}}$ or $\frac{\alpha z\sin\Omega_0}{z^2-2\alpha z\cos\Omega_0+\alpha^2}$ | $\lvert z\rvert>\alpha$ |
| 12. | $r\alpha^n\sin(\Omega_0 n+\theta)u[n]$ with $\alpha\in R$ | $\frac{A+Bz^{-1}}{1+2\gamma z^{-1}+\alpha^2z^{-2}}$ or $\frac{z(Az+B)}{z^2+2\gamma z+\gamma^2}$ | $\lvert z\rvert\le\lvert\alpha\rvert^{(a)}$ |

$$x[n] = \frac{1}{2\pi j}\oint X(z)z^{n-1}dz$$

This method involves the contour integration, so difficult to solve. There are other commonly used methods to evaluate the inverse $z$-transform given as follows

1. Partial fraction method
2. Power series expansion

### 6.4.1 Partial Fraction Method

If $X(z)$ is a rational function of $z$ then it can be expressed as follows.

$$X(z) = \frac{N(z)}{D(z)}$$

It is convenient if we consider $X(z)/z$ rather than $X(z)$ to obtain the inverse $z$-transform by partial fraction method.

Let $p_1$, $p_2$, $p_3$....$p_n$ are the roots of denominator polynomial, also the poles of $X(z)$. Then, using partial fraction method $X(z)/z$ can be expressed as

$$\frac{X(z)}{z} = \frac{A_1}{z-p_1} + \frac{A_2}{z-p_2} + \frac{A_3}{z-p_3} + \ldots + \frac{A_n}{z-p_n}$$

$$X(z) = A_1\frac{z}{z-p_1} + A_2\frac{z}{z-p_2} + \ldots + \frac{z}{z-p_n}$$

Now, the inverse $z$-transform of above equation can be obtained by comparing each term with the standard $z$-tranform pair given in table 6.1. The values of coefficients $A_1$, $A_2$, $A_3$....$A_n$ depends on whether the poles are real & distinct or repeated or complex. Three cases are given as follows

#### Case I : Poles are Simple and Real

$X(z)/z$ can be expanded in partial fraction as

$$\frac{X(z)}{z} = \frac{A_1}{z-p_1} + \frac{A_2}{z-p_2} + \frac{A_3}{z-p_3} + \ldots + \frac{A_n}{z-p_n} \qquad (6.4.1)$$

where $A_1$, $A_2$,... $A_n$ are calculated as follows

$$A_1 = (z-p_1)\frac{X(z)}{z}\Big|_{z=p_1}$$

$$A_2 = (z-p_2)\frac{X(z)}{z}\Big|_{z=p_2}$$

In general,

$$A_i = (z-p_i)X(z)\Big|_{z=p_i} \qquad (6.4.2)$$

#### Case II : If Poles are Repeated

In this case $X(z)/z$ has a different form. Let $p_k$ be the root which repeats $l$ times, then the expansion of equation must include terms

$$\frac{X(z)}{z} = \frac{A_{1k}}{z-p_k} + \frac{A_{2k}}{(z-p_k)^2} + \ldots$$
$$+ \frac{A_{ik}}{(z-p_k)^i} + \ldots + \frac{A_{lk}}{(z-p_k)^l} \qquad (6.4.3)$$

The coefficient $A_{ik}$ are evaluated by multiplying both sides of equation (6.4.3) by $(z-p_k)^l$, differentiating $(l-i)$ times and then evaluating the resultant equation at $z = p_k$.

Thus,

$$C_{ik} = \frac{1}{\underline{|(l-i)}}\frac{d^{l-i}}{dz^{l-i}}\left[(z-p_k)^l\frac{X(z)}{z}\right]\Big|_{z=p_k} \qquad (6.4.4)$$

Case III : Complex Poles

If $X(z)$ has complex poles then partial fraction of the $X(z)/z$ can be expressed as

$$\frac{X(z)}{z} = \frac{A_1}{z-p_1} + \frac{A_1^*}{z-p_1^*} \qquad (6.4.5)$$

where $A_1^*$ is complex conjugate of $A_1$ and $p_1^*$ is complex conjugate of $z_1$. The coefficients are obtained by equation (6.4.2)

### 6.4.2 Power Series Expansion Method

Power series method is also convenient in calculating the inverse $z$-transform. The $z$-transform of sequence $x[n]$ is given as

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

Now, $X(z)$ is expanded in the following form

$$X(z) = .. + x[-2]z^2 + x[-1]z^1 + x[0] + x[1]z^{-1} + x[2]z^{-2} + ...$$

To obtain inverse $z$-transform(i.e.$x[n]$), represent the given $X(z)$ in the form of above power series. Then by comparing we can get

$$x[n] = \{...x[-2], x[-1], x[0], x[1], x[2], ...\}$$

## 6.5 PROPERTIES OF Z-TRANSFORM

The unilateral and bilateral $z$-transforms possess a set of properties, which are useful in the analysis of DT signals and systems. The proofs of properties are given for bilateral transform only and can be obtained in a similar way for the unilateral transform.

### 6.5.1 Linearity

Like Laplace transform, the linearity property of $z$ transform states that, the linear combination of DT sequences in the time domain is equivalent to linear combination of their $z$ transform.

| | | |
|---|---|---|
| Let | $x_1[n] \xleftrightarrow{\mathcal{Z}} X_1(z)$, | with ROC: $R_1$ |
| and | $x_2[n] \xleftrightarrow{\mathcal{Z}} X_2(z)$, | with ROC: $R_2$ |
| then, | $ax_1[n] + bx_2[n] \xleftrightarrow{\mathcal{Z}} aX_1(z) + bX_2(z)$, | |
| | | with ROC: at least $R_1 \cap R_2$ |

for both unilateral and bilateral $z$-transform.

PROOF :

The $z$-transform of signal $\{ax_1[n] + bx_2[n]\}$ is given by equation (6.1.1) as follows

$$\begin{aligned}\mathcal{Z}\{ax_1[n] + bx_2[n]\} &= \sum_{n=-\infty}^{\infty}\{ax_1[n] + bx_2[n]\}z^{-n} \\ &= a\sum_{n=-\infty}^{\infty}x_1[n]z^{-n} + b\sum_{n=-\infty}^{\infty}x_2[n]z^{-n} \\ &= aX_1(z) + bX_2(z)\end{aligned}$$

Hence, $ax_1[n] + bx_2[n] \xleftrightarrow{\mathcal{Z}} aX_1(z) + bX_2(z)$

ROC : Since, the $z$-transform $X_1(z)$ is finite within the specified ROC, $R_1$.

Similarly, $X_2(z)$ is finite within its ROC, $R_2$. Therefore, the linear combination $aX_1(z)+bX_2(z)$ should be finite at least within region $R_1 \cap R_2$.

NOTE :
In certain cases, due to the interaction between $x_1[n]$ and $x_2[n]$, which may lead to cancellation of certain terms, the overall ROC may be larger than the intersection of the two regions. On the other hand, if there is no common region between $R_1$ and $R_2$, the z-transform of $ax_1[n]+bx_2[n]$ does not exist.

### 6.5.2 Time Shifting

For the bilateral $z$-transform

If $\quad x[n] \xleftrightarrow{\mathcal{Z}} X(z), \qquad$ with ROC $R_x$

then $\quad x[n-n_0] \xleftrightarrow{\mathcal{Z}} z^{-n_0}X(z),$

and $\quad x[n+n_0] \xleftrightarrow{\mathcal{Z}} z^{n_0}X(z),$

with ROC : $R_x$ except for the possible deletion or addition of $z=0$ or $z=\infty$.

PROOF :
The bilateral $z$-transform of signal $x[n-n_0]$ is given by equation (6.1.1) as follows

$$\mathcal{Z}\{x[n-n_0]\} = \sum_{n=-\infty}^{\infty} x[n-n_0]z^{-n}$$

Substituting $n-n_0=\alpha$ on RHS, we get

$$\mathcal{Z}\{x[n-n_0]\} = \sum_{\alpha=-\infty}^{\infty} x[\alpha]z^{-(n_0+\alpha)}$$

$$= \sum_{\alpha=-\infty}^{\infty} x[\alpha]z^{-n_0}z^{-\alpha} = z^{-n_0}\sum_{\alpha=-\infty}^{\infty} x[\alpha]z^{-\alpha}$$

$$\mathcal{Z}\{x[n-n_0]\} = z^{-n_0}X[z]$$

Similarly we can prove

$$\mathcal{Z}\{x[n+n_0]\} = z^{n_0}X[z]$$

ROC : The ROC of shifted signals is altered because of the terms $z^{n_0}$ or $z^{-n_0}$, which affects the roots of the denominator in $X(z)$.

TIME SHIFTING FOR UNILATERAL $z$-TRANSFORM

For the unilateral $z$-transform

If $\quad x[n] \xleftrightarrow{\mathcal{Z}} X(z), \qquad$ with ROC $R_x$

then $\quad x[n-n_0] \xleftrightarrow{\mathcal{Z}} z^{-n_0}\left(X(z)+\sum_{m=1}^{n_0} x[-m]z^{m}\right),$

and $\quad x[n+n_0] \xleftrightarrow{\mathcal{Z}} z^{n_0}\left(X(z)-\sum_{m=0}^{n_0-1} x[m]z^{-m}\right),$

with ROC : $R_x$ except for the possible deletion or addition of $z=0$ or $z=\infty$.

PROOF :
The unilateral $z$-transform of signal $x[n-n_0]$ is given by equation (6.1.2) as follows

$$\mathcal{Z}\{x[n-n_0]\} = \sum_{n=0}^{\infty} x[n-n_0] z^{-n}$$

Multiplying RHS by $z^{n_0}$ and $z^{-n_0}$

$$\mathcal{Z}\{x[n-n_0]\} = \sum_{n=0}^{\infty} x[n-n_0] z^{-n} z^{n_0} z^{-n_0}$$

$$= z^{-n_0} \sum_{n=0}^{\infty} x[n-n_0] z^{-(n-n_0)}$$

Substituting $n - n_0 = \alpha$

Limits; when $n \to 0$, $\alpha \to -n_0$

when $n \to +\infty$, $\alpha \to +\infty$

Now, $$\mathcal{Z}\{x[n-n_0]\} = z^{-n_0} \sum_{\alpha=-n_0}^{\infty} x[\alpha] z^{-\alpha}$$

$$= z^{-n_0} \sum_{\alpha=-n_0}^{-1} x[\alpha] z^{-\alpha} + z^{-n_0} \sum_{\alpha=0}^{\infty} x[\alpha] z^{-\alpha}$$

or, $$\mathcal{Z}\{x[n-n_0]\} = z^{-n_0} \sum_{\alpha=0}^{\infty} x[\alpha] z^{-\alpha} + z^{-n_0} \sum_{\alpha=-n_0}^{-1} x[\alpha] z^{-\alpha}$$

or, $$\mathcal{Z}\{x[n-n_0]\} = z^{-n_0} \sum_{\alpha=0}^{\infty} x[\alpha] z^{-\alpha} + z^{-n_0} \sum_{\alpha=1}^{n_0} x[-\alpha] z^{\alpha}$$

Changing the variables as $\alpha \to n$ and $\alpha \to m$ in first and second summation respectively

$$\mathcal{Z}\{x[n-n_0]\} = z^{-n_0} \sum_{n=0}^{\infty} x[n] z^{-n} + z^{-n_0} \sum_{m=1}^{n_0} x[-m] z^{m}$$

$$= z^{-n_0} X[z] + z^{-n_0} \sum_{m=1}^{n_0} x[-m] z^{m}$$

In similar way, we can also prove that

$$x[n+n_0] \xleftrightarrow{\mathcal{Z}} z^{n_0}\left(X(z) - \sum_{m=0}^{n_0-1} x[m] z^{-m}\right)$$

### 6.5.3 Time Reversal

Time reversal property states that time reflection of a DT sequence in time domain is equivalent to replacing $z$ by $1/z$ in its $z$-transform.

| | | |
|---|---|---|
| If | $x[n] \xleftrightarrow{\mathcal{Z}} X(z)$, | with ROC : $R_x$ |
| then | $x[-n] \xleftrightarrow{\mathcal{Z}} X\left(\frac{1}{z}\right)$, | with ROC : $1/R_x$ |
| for bilateral $z$-transform. | | |

PROOF :

The bilateral $z$-transform of signal $x[-n]$ is given by equation (6.1.1) as follows

$$\mathcal{Z}\{x[-n]\} = \sum_{n=-\infty}^{\infty} x[-n] z^{-n}$$

Substituting $-n = k$ on the RHS, we get

$$\mathcal{Z}\{x[-n]\} = \sum_{k=\infty}^{-\infty} x[k] z^{k} = \sum_{k=-\infty}^{\infty} x[k] (z^{-1})^{-k} = X\left(\frac{1}{z}\right)$$

Hence, $$x[-n] \xleftrightarrow{\mathcal{Z}} X\left(\frac{1}{z}\right)$$

ROC : $z^{-1} \in R_x$ or $z \in 1/R_x$

### 6.5.4 Differentiation in the $z$-domain

This property states that multiplication of time sequence $x[n]$ with $n$ corresponds to differentiation with respect to $z$ and multiplication of result by $-z$ in the $z$-domain.

| | | |
|---|---|---|
| If | $x[n] \xleftrightarrow{\mathcal{Z}} X(z)$, | with ROC : $R_x$ |
| then | $nx[n] \xleftrightarrow{\mathcal{Z}} -z\dfrac{dX(z)}{dz}$, | with ROC : $R_x$ |

For both unilateral and bilateral $z$-transforms.

PROOF :

The bilateral $z$-transform of signal $x[n]$ is given by equation (6.1.1) as follows

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

Differentiating both sides with respect to $z$ gives

$$\frac{dX(z)}{dz} = \sum_{n=-\infty}^{\infty} x[n]\frac{dz^{-n}}{dz} = \sum_{n=-\infty}^{\infty} x[n](-nz^{-n-1})$$

Multiplying both sides by $-z$, we obtain

$$-z\frac{dX(z)}{dz} = \sum_{n=-\infty}^{\infty} nx[n]z^{-n}$$

Hence, $\qquad nx[n] \xleftrightarrow{\mathcal{Z}} -z\dfrac{dX(z)}{dz}$

ROC : This operation does not affect the ROC.

### 6.5.5 Scaling in $z$-Domain

Multiplication of a time sequence with an exponential sequence $a^n$ corresponds to scaling in $z$-domain by a factor of $a$.

| | | |
|---|---|---|
| If | $x[n] \xleftrightarrow{\mathcal{Z}} X(z)$, | with ROC : $R_x$ |
| then | $a^n x[n] \xleftrightarrow{\mathcal{Z}} X\left(\dfrac{z}{a}\right)$, | with ROC : $\lvert a \rvert R_x$ |

for both unilateral and bilateral transform.

PROOF :

The bilateral $z$-transform of signal $x[n]$ is given by equation (6.1.1) as

$$\mathcal{Z}\{a^n x[n]\} = \sum_{n=-\infty}^{\infty} a^n x[n]z^{-n} = \sum_{n=-\infty}^{\infty} x[n][a^{-1}z]^{-n}$$

$$a^n x[n] \xleftrightarrow{\mathcal{Z}} X\left(\frac{z}{a}\right)$$

ROC : If $z$ is a point in the ROC of $X(z)$ then the point $\lvert a \rvert z$ is in the ROC of $X(z/a)$.

### 6.5.6 Time Scaling

As we discussed in Chapter 2, there are two types of scaling in the DT domain decimation(compression) and interpolation(expansion).

#### Time Compression

Since the decimation (compression) of DT signals is an irreversible process (because some data may lost), therefore the $z$-transform of $x[n]$ and its decimated sequence $y[n] = x[an]$ not be related to each other.

Time Expansion

In the discrete time domain, time expansion of sequence $x[n]$ is defined as

$$x_k[n] = \begin{cases} x[n/k] & \text{if } n \text{ is a multiple of integer } k \\ 0 & \text{otherwise} \end{cases} \qquad (6.5.1)$$

Time-scaling property of $z$-transform is derived only for time expansion which is given as

If $\quad x[n] \xleftrightarrow{\mathcal{Z}} X(z), \quad$ with ROC : $R_x$

then $\quad x_k[n] \xleftrightarrow{\mathcal{Z}} X_k(z) = X(z^k), \quad$ with ROC : $(R_x)^{1/k}$

for both the unilateral and bilateral $z$-transform.

PROOF :

The unilateral $z$-transform of expanded sequence $x_k[n]$ is given by

$$\begin{aligned} \mathcal{Z}\{x_k[n]\} &= \sum_{n=0}^{\infty} x_k[n] z^{-n} \\ &= x_k[0] + x_k[1]z^{-1} + ... + x_k[k]z^{-k} \\ &\quad + x_k[k+1]z^{-(k+1)} + ... x_k[2k]z^{-2k} + ... \end{aligned}$$

Since the expanded sequence $x_k[n]$ is zero everywhere except when $n$ is a multiple of $k$. This reduces the above transform as follows

$$\mathcal{Z}\{x_k[n]\} = x_k[0] + x_k[k]z^{-k} + x_k[2k]z^{-2k} + x_k[3k]z^{-3k} + ...$$

As defined in equation 6.5.1, interpolated sequence is

$$x_k[n] = x[n/k]$$

$n = 0 \qquad x_k[0] = x[0],$

$n = k \qquad x_k[k] = x[1]$

$n = 2k \qquad x_k[2k] = x[2]$

Thus, we can write

$$\begin{aligned} \mathcal{Z}\{x_k[n]\} &= x[0] + x[1]z^{-k} + x[2]z^{-2k} + x[3]z^{-3k} + ... \\ &= \sum_{n=0}^{\infty} x[n](z^k)^{-n} = X(z^k) \end{aligned}$$

NOTE :

Time expansion of a DT sequence by a factor of $k$ corresponds to replacing $z$ as $z^k$ in its $z$-transform.

## 6.5.7 Time Differencing

If $\quad x[n] \xleftrightarrow{\mathcal{Z}} X(z), \quad$ with ROC : $R_x$

then $\quad x[n] - x[n-1] \xleftrightarrow{\mathcal{Z}} (1 - z^{-1}) X(z), \quad$ with the ROC : $R_x$ except for the possible deletion of $z = 0$, for both unilateral and bilateral transform.

PROOF :

The $z$-transform of $x[n] - x[n-1]$ is given by equation (6.1.1) as follows

$$\mathcal{Z}\{x[n] - x[n-1]\} = \sum_{n=-\infty}^{\infty} \{x[n] - x[n-1]\} z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x[n]z^{-n} - \sum_{n=-\infty}^{\infty} x[n-1]z^{-n}$$

In the second summation, substituting $n-1 = r$

$$\mathcal{Z}\{x[n]-x[n-1]\} = \sum_{n=-\infty}^{\infty} x[n]z^{-n} - \sum_{r=-\infty}^{\infty} x[r]z^{-(r+1)}$$

$$= \sum_{n=-\infty}^{\infty} x[n]z^{-n} - z^{-1}\sum_{r=-\infty}^{\infty} x[r]z^{-r}$$

$$= X(z) - z^{-1}X(z)$$

Hence,$x[n]-x[n-1] \xleftrightarrow{\mathcal{Z}} (1-z^{-1})X(z)$

### 6.5.8 Time Convolution

Time convolution property states that convolution of two sequence in time domain corresponds to multiplication in $z$-domain.

Let $x_1[n] \xleftrightarrow{\mathcal{Z}} X_1(z)$, ROC : $R_1$

and $x_2[n] \xleftrightarrow{\mathcal{Z}} X_2(z)$, ROC : $R_2$

then the convolution property states that

$x_1[n] * x_2[n] \xleftrightarrow{\mathcal{Z}} X_1(z)X_2(z)$, ROC : at least $R_1 \cap R_2$

for both unilateral and bilateral $z$-transforms.



PROOF :

As discussed in chapter 4, the convolution of two sequences is given by

$$x_1[n] * x_2[n] = \sum_{k=-\infty}^{\infty} x_1[k]x_2[n-k]$$

Taking the $z$-transform of both sides gives

$$x_1[n] * x_2[n] \xleftrightarrow{\mathcal{Z}} \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} x_1[k]x_2[n-k]z^{-n}$$

Interchanging the order of the two summations, we get

$$x_1[n] * x_2[n] \xleftrightarrow{\mathcal{Z}} \sum_{k=-\infty}^{\infty} x_1[k]\sum_{n=-\infty}^{\infty} x_2[n-k]z^{-n}$$

Substituting $n-k = \alpha$ in the second summation

$$x[n] * x_2[n] \xleftrightarrow{\mathcal{Z}} \sum_{k=-\infty}^{\infty} x_1[k]\sum_{\alpha=-\infty}^{\infty} x_2[\alpha]z^{-(\alpha+k)}$$

or $$x[n] * x_2[n] \xleftrightarrow{\mathcal{Z}} \left(\sum_{k=-\infty}^{\infty} x_1[k]z^{-k}\right)\left(\sum_{\alpha=-\infty}^{\infty} x_2[\alpha]z^{-\alpha}\right)$$

$$x_1[n] * x_2[n] \xleftrightarrow{\mathcal{Z}} X_1(z)X_2(z)$$

### 6.5.9 Conjugation Property

If $x[n] \xleftrightarrow{\mathcal{Z}} X(z)$, with ROC : $R_x$

then $x^*[n] \xleftrightarrow{\mathcal{Z}} X^*(z^*)$, with ROC : $R_x$

If $x[n]$ is real, then $X(z) = X^*(z^*)$

PROOF :

The $z$-transform of signal $x^*[n]$ is given by equation (6.1.1) as follows

$$\mathcal{Z}\{x^*[n]\} = \sum_{n=-\infty}^{\infty} x^*[n] z^{-n} = \sum_{n=-\infty}^{\infty} [x[n](z^*)^{-n}]^* \qquad (6.5.2)$$

Let $z$-transform of $x[n]$ is $X(z)$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

by taking complex conjugate on both sides of above equation

$$X^*(z) = \sum_{n=-\infty}^{\infty} [x[n] z^{-n}]^*$$

Replacing $z \to z^*$, we will get

$$X^*(z^*) = \sum_{n=-\infty}^{\infty} [x[n](z^*)^{-n}]^* \qquad (6.5.3)$$

Comparing equation (6.5.2) and (6.5.3)

$$\mathcal{Z}\{x^*[n]\} = X^*(z^*) \qquad (6.5.4)$$

For real $x[n]$, $x^*[n] = x[n]$, so

$$\mathcal{Z}\{x^*[n]\} = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = X(z) \qquad (6.5.5)$$

Comparing equation (6.5.4) and (6.5.5)

$$X(z) = X^*(z^*)$$

### 6.5.10 Initial Value Theorem

If $x[n] \xleftrightarrow{\mathcal{Z}} X(z)$, with ROC : $R_x$

then initial-value theorem states that,

$$x[0] = \lim_{z \to \infty} X(z)$$

The initial-value theorem is valid only for the unilateral Lapalce transform

PROOF :

For a causal signal $x[n]$

$$X(z) = \sum_{n=0}^{\infty} x[n] z^{-n}$$
$$= x[0] + x[1]z^{-1} + x[2]z^{-2} + \ldots$$

Taking limit as $z \to \infty$ on both sides we get

$$\lim_{z \to \infty} X(z) = \lim_{z \to \infty} (x[0] + x[1]z^{-1} + x[2]z^{-2} + \ldots) = x[0]$$
$$x[0] = \lim_{z \to \infty} X(z)$$

### 6.5.11 Final Value Theorem

If $x[n] \xleftrightarrow{\mathcal{Z}} X(z)$, with ROC : $R_x$

then final-value theorem states that

$$x[\infty] = \lim_{z \to 1} (z-1) X(z)$$

Final value theorem is applicable if $X(z)$ has no poles outside the unit circle. This theorem can be applied to either the unilateral or bilateral $z$-transform.

PROOF :

$$\mathcal{Z}\{x[n+1]\} - \mathcal{Z}\{x[n]\} = \lim_{k\to\infty}\sum_{n=0}^{k}\{x[n+1]-x[n]\}z^{-n} \quad (6.5.6)$$

From the time shifting property of unilateral $z$-transform discussed in section 6.5.2

$$x[n+n_0] \xleftrightarrow{\mathcal{Z}} z^{n_0}\left(X(z) - \sum_{m=0}^{n_0-1}x[m]z^{-m}\right)$$

For $n_0 = 1$
$$x[n+1] \xleftrightarrow{\mathcal{Z}} z\left(X(z) - \sum_{m=0}^{0}x[m]z^{-m}\right)$$

$$x[n+1] \xleftrightarrow{\mathcal{Z}} z(X(z) - x[0])$$

Put above transformation in the equation (6.5.6)

$$zX[z] - zx[0] - X[z] = \lim_{k\to\infty}\sum_{n=0}^{k}(x[n+1]-x[n])z^{-n}$$

$$(z-1)X[z] - zx[0] = \lim_{k\to\infty}\sum_{n=0}^{k}(x[n+1]-x[n])z^{-n}$$

Taking limit as $z \to 1$ on both sides we get

$$\lim_{z\to 1}(z-1)X[z] - x[0] = \lim_{k\to\infty}\sum_{n=0}^{k}x[n+1]-x[n]$$

$$\lim_{z\to 1}(z-1)X[z] - x[0] = \lim_{k\to\infty}\{(x[1]-x[0]) + (x[2]-x[1]) + (x[3]-x[2]) + \ldots$$
$$\ldots + (x[k+1]-x[k])$$

$$\lim_{z\to 1}(z-1)X[z] - x[0] = x[\infty] - x[0]$$

Hence, $$x[\infty] = \lim_{z\to 1}(z-1)X(z)$$

Summary of Properties

Let,
$$x[n] \xleftrightarrow{\mathcal{Z}} X(z), \quad \text{with ROC } R_x$$
$$x_1[n] \xleftrightarrow{\mathcal{Z}} X_1(z), \quad \text{with ROC } R_1$$
$$x_2[n] \xleftrightarrow{\mathcal{Z}} X_2(z), \quad \text{with ROC } R_2$$

The properties of $z$-transforms are summarized in the following table.

TABLE 6.2 Properties of $z$-transform

| Properties | Time domain | $z$-transform | ROC |
|---|---|---|---|
| Linearity | $ax_1[n]+bx_2[n]$ | $aX_1(z)+bX_2(z)$ | at least $R_1 \cap R_2$ |
| Time shifting (bilateral or non-causal) | $x[n-n_0]$ | $z^{-n_0}X(z)$ | $R_x$ except for the possible deletion or addition of $z=0$ or $z=\infty$ |
| | $x[n+n_0]$ | $z^{n_0}X(z)$ | |
| Time shifting (unilateral or causal) | $x[n-n_0]$ | $z^{-n_0}\left(X(z) + \sum_{m=1}^{n_0}x[-m]z^{m}\right)$ | $R_x$ except for the possible deletion or addition of $z=0$ or $z=\infty$ |
| | $x[n+n_0]$ | $z^{n_0}\left(X(z) - \sum_{m=0}^{n_0-1}x[m]z^{-m}\right)$ | |

| Properties | Time domain | $z$-transform | ROC |
|---|---|---|---|
| Time reversal | $x[-n]$ | $X\left(\frac{1}{z}\right)$ | $1/R_x$ |
| Differentiation in $z$ domain | $nx[n]$ | $-z\frac{dX(z)}{dz}$ | $R_x$ |
| Scaling in $z$ domain | $a^n x[n]$ | $X\left(\frac{z}{a}\right)$ | $\|a\|R_x$ |
| Time scaling (expansion) | $x_k[n] = x[n/k]$ | $X(z^k)$ | $(R_x)^{1/k}$ |
| Time differencing | $x[n] - x[n-1]$ | $(1 - z^{-1})X(z)$ | $R_x$, except for the possible deletion of the origin |
| Time convolution | $x_1[n] * x_2[n]$ | $X_1(z)X_2(z)$ | at least $R_1 \cap R_2$ |
| Conjugations | $x^*[n]$ | $X^*(z^*)$ | $R_x$ |
| Initial-value theorem | | $x[0] = \lim_{z \to \infty} X(z)$ | provided $x[n] = 0$ for $n < 0$ |
| Final-value theorem | | $x[\infty] = \lim_{n \to \infty} x[n] = \lim_{z \to 1}(z-1)X(z)$ | provided $x[\infty]$ exists |

## 6.6 ANALYSIS OF DISCRETE LTI SYSTEMS USING Z-TRANSFORM

The $z$-transform is very useful tool in the analysis of discrete LTI system. As the Laplace transform is used in solving differential equations which describe continuous LTI systems, the $z$-transform is used to solve difference equation which describe the discrete LTI systems.

Similar to Laplace transform, for CT domain, the $z$-transform gives transfer function of the LTI discrete systems which is the ratio of the $z$-transform of the output variable to the $z$-transform of the input variable. These applications are discussed as follows

### 6.6.1 Response of LTI Continuous Time System

As discussed in chapter 4 (section 4.8), a discrete-time LTI system is always described by a linear constant coefficient difference equation given as follows

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

$$a_N y[n-N] + a_{N-1}y[n-(N-1)] + ....... + a_1 y[n-1] + a_0 y[n]$$

$$= b_M x[n-M] + b_{M-1}x[n-(M-1)] + ..... + b_1 x[n-1] + b_0 x[n] \qquad (6.6.1)$$

where, $N$ is order of the system.

The time-shift property of $z$-transform $x[n-n_0] \overset{z}{\longleftrightarrow} z^{-n_0}X(z)$, is used to solve the above difference equation which converts it into an algebraic equation. By taking $z$-transform of above equation

$$a_N z^{-N} Y(z) + a_{N-1} z^{-(N-1)} Y(z)] + ....... + a_1 z^{-1} + a_0 Y(z)$$

$$= b_M z^{-M} X(z) + b_{M-1} z^{-(M-1)} X(z) + ..... + b_1 z^{-1} X(x) + b_0 X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{b_M z^{-N} + b_{M-1} z^{M-1} + ..... + b_1 + b_0}{a_N z^N + a_{N-1} z^{N-1} + ..... + a_1 + a_0}$$

this equation can be solved for $Y(z)$ to find the response $y[n]$.The solution or total response $y[n]$ consists of two parts as discussed below.

1. Zero-input Response or Free Response or Natural Response

The zero input response $y_{zi}[n]$ is mainly due to initial output in the system. The zero-input response is obtained from system equation (6.6.1) when input $x[n] = 0$.

By substituting $x[n] = 0$ and $y[n] = y_{zi}[n]$ in equation (6.6.1), we get

$$a_N y[n-N] + a_{N-1} y[n-(N-1)] + ....... + a_1 y[n-1] + a_0 y[n] = 0$$

On taking $z$-transform of the above equation with given initial conditions, we can form an equation for $Y_{zi}(z)$. The zero-input response $y_{zi}[n]$ is given by inverse $z$-transform of $Y_{zi}(z)$.

NOTE :
The zero input response is also called the natural response of the system and it is denoted as $y_N[n]$.

2. Zero-State Response or Forced Response

The zero-state response $y_{zs}[n]$ is the response of the system due to input signal and with zero initial conditions. The zero-state response is obtained from the difference equation (6.6.1) governing the system for specific input signal $x[n]$ for $n \geq 0$ and with zero initial conditions.

Substituting $y[n] = y_{zs}[n]$ in equation (6.6.1) we get,

$$a_N y_{zs}[n-N] + a_{N-1} y_{zs}[n-(N-1)] + ....... + a_1 y_{zs}[n-1] + a_0 y_{zs}[n]$$
$$= b_M x[n-M] + b_{M-1} x[n-(M-1)] + ..... + b_1 x[n-1] + b_0 x[n]$$

Taking $z$-transform of the above equation with zero initial conditions for output (i.e., $y[-1] = y[-2]... = 0$ we can form an equation for $Y_{zs}(z)$.

The zero-state response $y_{zs}[n]$ is given by inverse $z$-transform of $Y_{zs}(z)$.

NOTE :
The zero state response is also called the forced response of the system and it is denoted as $y_F[n]$.

Total Response

The total response $y[n]$ is the response of the system due to input signal and initial output. The total response can be obtained in following two ways :

Taking $z$-transform of equation (6.6.1) with non-zero initial conditions for both input and output, and then substituting for $X(z)$ we can form an equation for $Y(z)$. The total response $y[n]$ is given by inverse Laplace transform of $Y(s)$.

Alternatively, that total response $y[n]$ is given by sum of zero-input response $y_{zi}[n]$ and zero-state response $y_{zs}[n]$.

Therefore total response,

$$y[n] = y_{zi}[n] + y_{zs}[n]$$

### 6.6.2 Impulse Response and Transfer Function

System function or transfer function is defined as the ratio of the $z$-transform of the output $y[n]$ and the input $x[n]$ with zero initial conditions.

Let $x[n] \xleftrightarrow{\mathcal{Z}} X(z)$ is the input and $y[n] \xleftrightarrow{\mathcal{L}} Y(z)$ is the output of an LTI discrete time system having impulse response $h(n) \xleftrightarrow{\mathcal{L}} H(z)$. The response

$y[n]$ of the discrete time system is given by convolution sum of input and impulse response as

$$y[n] = x[n] * h[n]$$

By applying convolution property of $z$-transform we obtain

$$Y(z) = X(z)H(z)$$
$$H(z) = \frac{Y(z)}{X(z)}$$

where, $H(z)$ is defined as the transfer function of the system. It is the $z$-transform of the impulse response.

Alternatively we can say that the inverse $z$-transform of transfer function is the impulse response of the system.

Impulse response

$$h[n] = \mathcal{Z}^{-1}\{H(z)\} = \mathcal{Z}^{-1}\left\{\frac{Y(z)}{X(z)}\right\}$$

## 6.7 STABILITY AND CAUSALITY OF LTI DISCRETE SYSTEMS USING Z -TRANSFORM

$z$-transform is also used in characterization of LTI discrete systems. In this section, we derive a $z$-domain condition to check the stability and causality of a system directly from its $z$-transfer function.

### 6.7.1 Causality

A linear time-invariant discrete time system is said to be causal if the impulse response $h[n] = 0$, for $n < 0$ and it is therefore right-sided. The ROC of such a system $H(z)$ is the exterior of a circle. If $H(z)$ is rational then the system is said to be causal if

1. The ROC is the exterior of a circle outside the outermost pole ; and
2. The degree of the numerator polynomial of $H(z)$ should be less than or equal to the degree of the denominator polynomial.

### 6.7.2 Stability

An LTI discrete-time system is said to be BIBO stable if the impulse response $h[n]$ is summable. That is

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

$z$-transform of $h[n]$ is given as

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n}$$

Let $z = e^{j\Omega}$ (which describes a unit circle in the $z$-plane), then

$$|H[e^{j\Omega}]| = \left|\sum_{n=-\infty}^{\infty} h[n]e^{-j\Omega n}\right|$$
$$\le \sum_{n=-\infty}^{\infty} |h[n]e^{-j\Omega n}|$$
$$= \sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

which is the condition for the stability. Thus we can conclude that

STABILITY OF LTI DISCRETE SYSTEM

An LTI system is stable if the ROC of its system function $H(z)$ contains the unit circle $|z| = 1$

### 6.7.3 Stability and Causality

As we discussed previously, for a causal system with rational transfer function $H(z)$, the ROC is outside the outermost pole. For the BIBO stability the ROC should include the unit circle $|z| = 1$. Thus, for the system to be causal and stable theses two conditions are satisfied if all the poles are within the unit circle in the $z$-plane.

STABILITY AND CAUSALITY OF LTI DISCRETE SYSTEM

An LTI discrete time system with the rational system function $H(z)$ is said to be both causal and stable if all the poles of $H(z)$ lies inside the unit circle.

## 6.8 BLOCK DIAGRAM REPRESENTATION



In $z$-domain, the input-output relation of an LTI discrete time system is represented by the transfer function $H(z)$ ,which is a rational function of $z$, as shown in equation

$$H(z) = \frac{Y(z)}{X(z)}$$
$$= \frac{b_0 z^M + b_1 z^{M-1} + b_2 z^{M-2} + ... + b_{M-1} z + b_M}{a_0 z^N + a_1 z^{N-1} + a_2 z^{N-2} + ... + a_{N-1} z + a_N}$$

where, $N$ = Order of the system, $M \le N$ and $a_0 = 1$

The above transfer function is realized using unit delay elements, unit advance elements, adders and multipliers. Basic elements of block diagram with their $z$-domain representation is shown in table 6.3.

TABLE 6.3 : Basic Elements of Block Diagram

| Elements of Block diagram | Time Domain Representation | $s$-domain Representation |
|---|---|---|
| Adder | $x_1[n] \rightarrow \underset{\uparrow \, x_2[n]}{\oplus} \rightarrow x_1[n]+x_2[n]$ | $X_1(z) \rightarrow \underset{\uparrow \, X_2(z)}{\oplus} \rightarrow X_1(z)+X_2(z)$ |
| Constant multiplier | $x[n] \rightarrow \boxed{a \triangleright} \rightarrow ax[n]$ | $X(z) \rightarrow \boxed{a \triangleright} \rightarrow aX(z)$ |
| Unit delay element | $x[n] \rightarrow \boxed{z^{-1}} \rightarrow x[n-1]$ | $X(z) \rightarrow \boxed{z^{-1}} \rightarrow z^{-1}X(z)$ |
| Unit advance element | $x[n] \rightarrow \boxed{z} \rightarrow x[n+1]$ | $X(z) \rightarrow \boxed{z} \rightarrow zX(z)$ |

The different types of structures for realizing discrete time systems are same as we discussed for the continuous-time system in the previous chapter.

### 6.8.1 Direct Form I Realization

Consider the difference equation governing the discrete time system with $a_0 = 1$,

$$y[n] + a_1 y[n-1] + a_2 y[n-2] + .... + a_N y[n-N]$$
$$= b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + ... + b_M x[n-M]$$

Taking $\mathcal{Z}$ transform of the above equation we get,

$$Y(z) = -a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z) - ... - a_N z^{-N} Y(z) +$$
$$b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z) + ... + b_M z^{-M} X(z) \qquad (6.8.1)$$

The above equation of $Y(z)$ can be directly represented by a block diagram as shown in figure 6.8.1a. This structure is called direct form-I structure. This structure uses separate delay elements for both input and output of the system. So, this realization uses more memory.

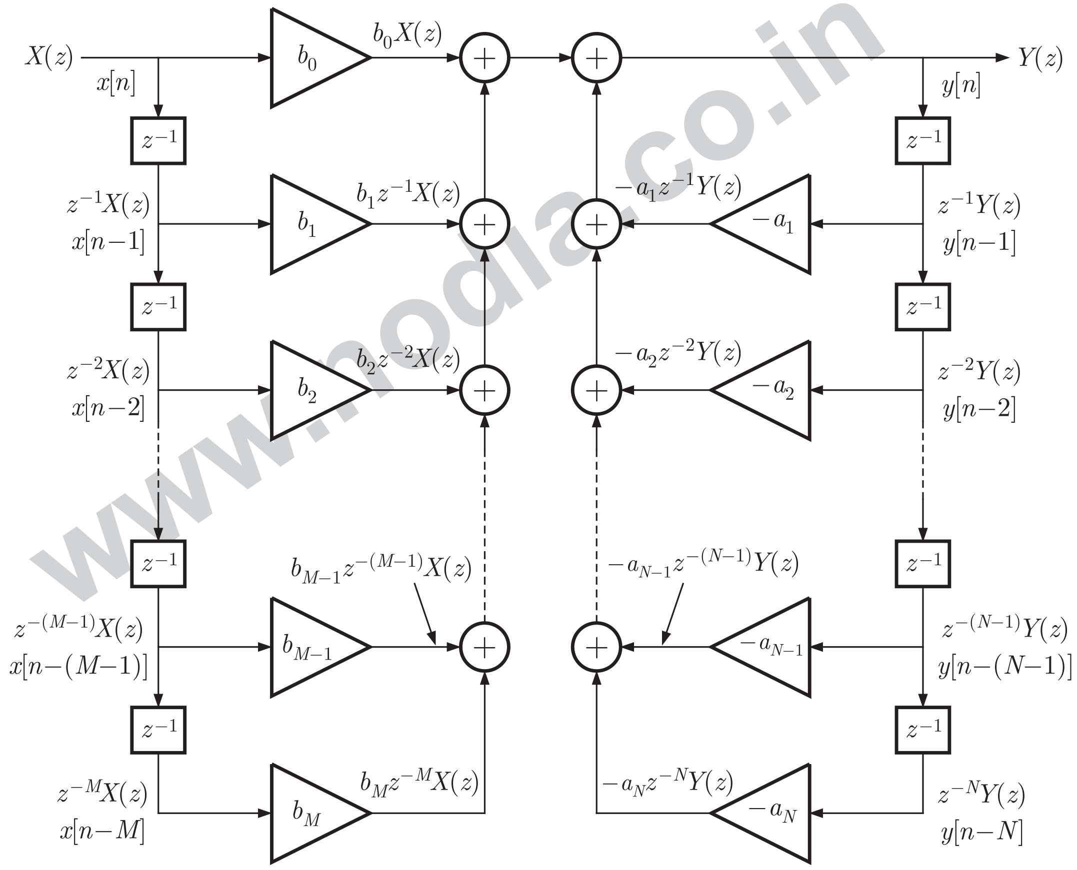



Figure 6.8.1a General structure of direct form-realization

For example consider a discrete LTI system which has the following impulse response

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 2z^{-1} + 2z^{-2}}{1 + 4z^{-1} + 3z^{-2}}$$

$$Y(z) + 4z^{-1} Y(z) + 3z^{-2} Y(z) = 1X(z) + 2z^{-1} X(z) + 2z^{-2} X(z)$$

Comparing with standard form of equation (6.8.1), we get $a_1 = 4$, $a_2 = 3$ and $b_0 = 1$, $b_1 = 2$, $b_2 = 2$. Now put these values in general structure of Direct form-I realization we get

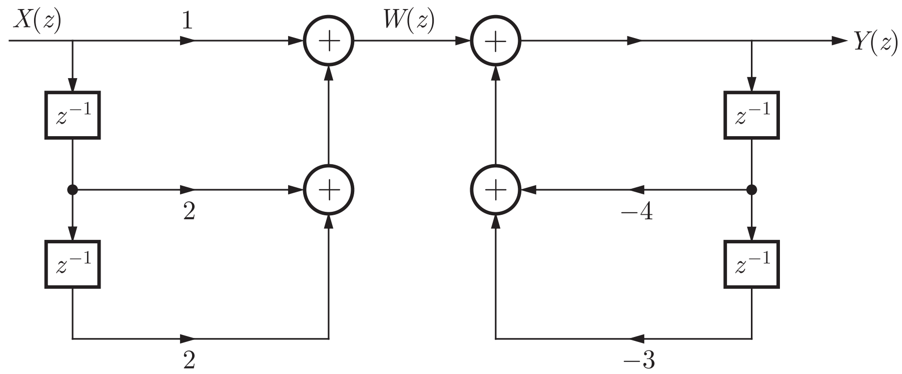



Figure 6.8.1b

### 6.8.2 Direct Form II Realization

Consider the general difference equation governing a discrete LTI system

$$y[n]+a_1y[n-1]+a_2y[n-2]+....+a_Ny[n-N]$$
$$=b_0x[n]+b_1x[n-1]+b_2x[n-2]+...+b_Mx[n-M]$$

Taking $\mathcal{Z}$ transform of the above equation we get,

$$Y(z)=-a_1z^{-1}Y(z)-a_2z^{-2}Y(z)-...-a_Nz^{-N}Y(z)+$$
$$b_0X(z)+b_1z^{-1}X(z)+b_2z^{-2}X(z)+...+b_Mz^{-M}X(z)$$

It can be simplified as,

$$Y(z)[1+a_1z^{-1}+a_2z^{-2}+...+a_Nz^{-N}]=X(z)[b_0+b_1z^{-1}+b_2z^{-2}+...+b_Mz^{-M}]$$

Let, $\dfrac{Y(z)}{X(z)}=\dfrac{W(z)}{X(z)}\times\dfrac{Y(z)}{W(z)}$

where,

$$\frac{W(z)}{X(z)}=\frac{1}{1+a_1z^{-1}+a_2z^{-2}+...+a_Nz^{-N}} \qquad (6.8.2)$$

$$\frac{Y(z)}{W(z)}=b_0+b_1z^{-1}+b_2z^{-2}+...+b_Mz^{-M} \qquad (6.8.3)$$

Equation (6.8.2) can be simplified as,

$$W(z)+a_1z^{-1}W(z)+a_2z^{-2}W(z)+...+a_Nz^{-N}W(z)=X(z)$$

$$W(z)=X(z)-a_1z^{-1}W(z)-a_2z^{-2}W(z)-...-a_Nz^{-N}W(z) \qquad (6.8.4)$$

Similarly by simplifying equation (6.8.3), we get

$$Y(z)=b_0W(z)+b_1z^{-1}W(z)+b_2z^{-2}W(z)+...+b_Mz^{-M}W(z) \qquad (6.8.5)$$

Equation (6.8.4) and (6.8.5) can be realized together by a direct structure called direct form-II structure as shown in figure 6.8.2a. It uses less number of delay elements then the Direct Form I structure.

For example, consider the same transfer function $H(z)$ which is discussed above

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1+2z^{-1}+2z^{-2}}{1+4z^{-1}+3z^{-2}}$$

Let $\dfrac{Y(z)}{X(z)}=\dfrac{Y(z)}{W(z)}\times\dfrac{W(z)}{X(z)}$

where, $\dfrac{W(z)}{X(z)}=\dfrac{1}{1+4z^{-1}+3z^{-2}}$,

$$\frac{Y(z)}{W(z)}=1+2z^{-1}+2z^{-2}$$

so, $W(z)=X(z)-4z^{-1}W(z)-3z^{-2}W(z)$

and $Y(z)=1W(z)+2z^{-1}W(z)+2z^{-2}W(z)$

Comparing these equations with standard form of equation (6.8.4) and (6.8.5), we have $a_1=4$, $a_2=3$ and $b_0=1, b_1=2, b_2=2$. Substitute these values in general structure of Direct form II , we get as shown in figure 6.8.2b

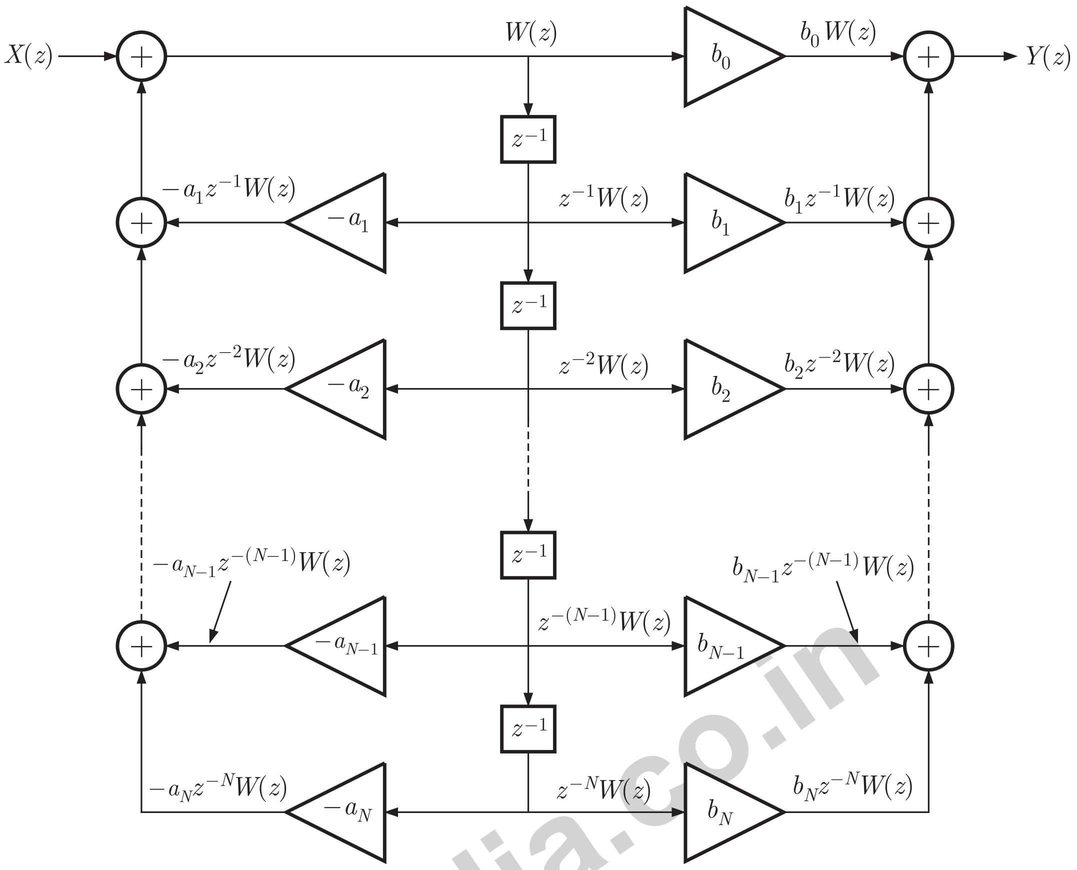



Figure 6.8.2a General structure of direct form-II realization

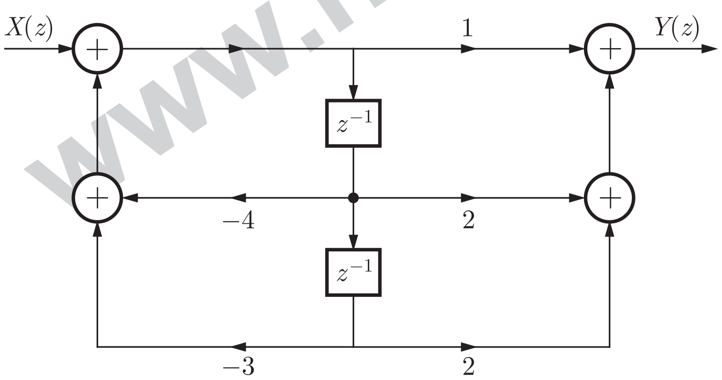



Figure 6.8.2b

## 6.8.3 Cascade Form

The transfer function $H(z)$ of a discrete time system can be expressed as a product of several transfer functions. Each of these transfer functions is realized in direct form-I or direct form II realization and then they are cascaded.

Consider a system with transfer function

$$\begin{aligned} H(z) &= \frac{(b_{k0}+b_{k1}z^{-1}+b_{k2}z^{-2})(b_{m0}+b_{m1}z^{-1}+b_{m2}z^{-2})}{(1+a_{k1}z^{-1}+a_{k2}z^{-2})(1+a_{m1}z^{-1}+a_{m2}z^{-2})} \\ &= H_1(z)H_2(z) \end{aligned}$$

where $$H_1(z) = \frac{b_{k0}+b_{k1}z^{-1}+b_{k2}z^{-2}}{1+a_{k1}z^{-1}+a_{k2}z^{-2}}$$

$$H_2(z) = \frac{b_{m0} + b_{m1}z^{-1} + b_{m2}z^{-2}}{1 + a_{m1}z^{-1} + a_{m2}z^{-2}}$$

Realizing $H_1(z)$ and $H_2(z)$ in direct form II and cascading we obtain cascade form of the system function $H(z)$ as shown in figure 6.8.3.

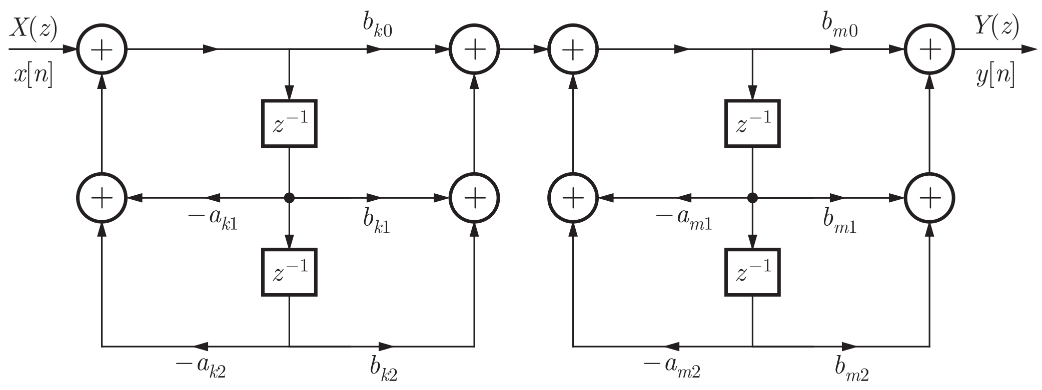



Figure 6.8.3 Cascaded form realization of discrete LTI system

### 6.8.4 Parallel Form

The transfer function $H(z)$ of a discrete time system can be expressed as the sum of several transfer functions using partial fractions. Then the individual transfer functions are realized in direct form I or direct form II realization and connected in parallel for the realization of $H(z)$. Let us consider the transfer function

$$H(z) = c + \frac{c_1}{1 - p_1 z^{-1}} + \frac{c_2}{1 - p_z z^{-1}} + \dots\dots \frac{c_N}{1 - p_n z^{-1}}$$

Now each factor in the system is realized in direct form II and connected in parallel as shown in figure 6.8.4.

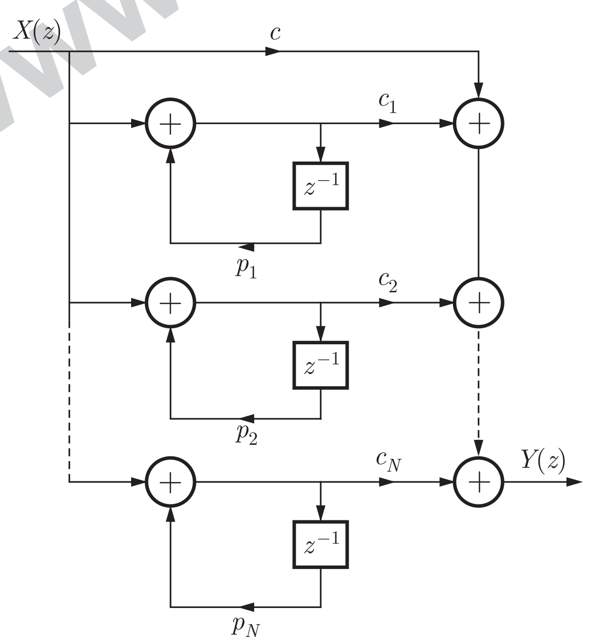



Figure 6.8.4 Parallel form realization of discrete LTI system

## 6.9 RELATIONSHIP BETWEEN s-PLANE & z-PLANE

There exists a close relationship between the Laplace and $z$-transforms. We know that a DT sequence $x[n]$ is obtained by sampling a CT signal $x(t)$ with a sampling interval $T$ , the CT sampled signal $x_s(t)$ is written as follows

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$$

where $x(nT)$ are sampled value of $x(t)$ which equals the DT sequence $x[n]$. Taking the Laplace transform of $x_s(t)$ , we have

$$X(s) = L\{x_s(t)\} = \sum_{n=-\infty}^{\infty} x(nT) L\{\delta(t - nT)\}$$

$$= \sum_{n=-\infty}^{\infty} X(nT) e^{-nTs} \qquad (6.9.1)$$

The $z$-transform of $x[n]$ is given by

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \qquad (6.9.2)$$

Comparing equation (6.9.1) and (6.9.2)

$$X(s) = X(z)\big|_{z=e^{sT}} \qquad x[n] = x(nT)$$

***********

# EXERCISE 6.1

MCQ 6.1.1 The $z$-transform and its ROC of a discrete time sequence

$$x[n] = \begin{cases} -\left(\frac{1}{2}\right)^n, & n < 0 \\ 0, & n \geq 0 \end{cases}$$

will be

(A) $\frac{2z}{2z-1}$, $|z| > \frac{1}{2}$

(B) $\frac{z}{z-2}$, $|z| < \frac{1}{2}$

(C) $\frac{2z}{2z-1}$, $|z| < \frac{1}{2}$

(D) $\frac{2z^{-1}}{z-1}$, $|z| > \frac{1}{2}$

MCQ 6.1.2 The ROC of $z$-transform of the discrete time sequence $x[n] = \left(\frac{1}{2}\right)^{|n|}$ is

(A) $\frac{1}{2} < |z| < 2$

(B) $|z| > 2$

(C) $-2 < |z| < 2$

(D) $|z| < \frac{1}{2}$

MCQ 6.1.3 Consider a discrete-time signal $x[n] = \left(\frac{1}{3}\right)^n u[n] + \left(\frac{1}{2}\right)^n u[-n-1]$. The ROC of its $z$-transform is

(A) $3 < |z| < 2$

(B) $|z| < \frac{1}{2}$

(C) $|z| > \frac{1}{3}$

(D) $\frac{1}{3} < |z| < \frac{1}{2}$

MCQ 6.1.4 For a signal $x[n] = [\alpha^n + \alpha^{-n}]u[n]$, the ROC of its $z$-transform would be

(A) $|z| > \min\left(|\alpha|, \frac{1}{|\alpha|}\right)$

(B) $|z| > |\alpha|$

(C) $|z| > \max\left(|\alpha|, \frac{1}{|\alpha|}\right)$

(D) $|z| < |\alpha|$

MCQ 6.1.5 Match List I (discrete time sequence) with List II ($z$-transform) and choose the correct answer using the codes given below the lists:

| | List-I (Discrete Time Sequence) | | List-II ($z$-Transform) |
|---|---|---|---|
| P. | $u[n-2]$ | 1. | $\frac{1}{z^{-2}(1-z^{-1})}$, $|z| < 1$ |
| Q. | $-u[-n-3]$ | 2. | $\frac{-z^{-1}}{1-z^{-1}}$, $|z| < 1$ |
| R. | $u[n+4]$ | 3. | $\frac{1}{z^{-4}(1-z^{-1})}$, $|z| > 1$ |
| S. | $u[-n]$ | 4. | $\frac{z^{-2}}{1-z^{-1}}$, $|z| > 1$ |

Codes :

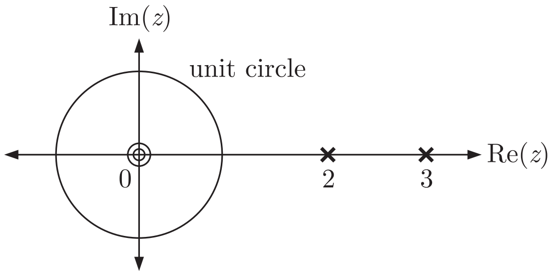
| | P | Q | R | S |
|---|---|---|---|---|
| (A) | 1 | 4 | 2 | 3 |
| (B) | 2 | 4 | 1 | 3 |
| (C) | 4 | 1 | 3 | 2 |
| (D) | 4 | 2 | 3 | 1 |

MCQ 6.1.6 The $z$-transform of signal $x[n] = e^{jn\pi} u[n]$ is

(A) $\frac{z}{z+1}$, ROC: $|z| > 1$ (B) $\frac{z}{z-j}$, ROC: $|z| > 1$

(C) $\frac{z}{z^2+1}$, ROC: $|z| < 1$ (D) $\frac{1}{z+1}$, ROC: $|z| < 1$

MCQ 6.1.7 Consider the pole zero diagram of an LTI system shown in the figure which corresponds to transfer function $H(z)$.




Match List I (The impulse response) with List II (ROC which corresponds to above diagram) and choose the correct answer using the codes given below: {Given that $H(1) = 1$}

| | List-I (Impulse Response) | | List-II (ROC) |
|---|---|---|---|
| P. | $[(-4)2^n + 6(3)^n]u[n]$ | 1. | does not exist |
| Q. | $(-4)2^n u[n] + (-6)3^n u[-n-1]$ | 2. | $\lvert z \rvert > 3$ |
| R. | $(4)2^n u[-n-1] + (-6)3^n u[-n-1]$ | 3. | $\lvert z \rvert < 2$ |
| S. | $4(2)^n u[-n-1] + (-6)3^n u[n]$ | 4. | $2 < \lvert z \rvert < 3$ |

Codes :

| | P | Q | R | S |
|---|---|---|---|---|
| (A) | 4 | 1 | 3 | 2 |
| (B) | 2 | 1 | 3 | 4 |
| (C) | 1 | 4 | 2 | 3 |
| (D) | 2 | 4 | 3 | 1 |

MCQ 6.1.8 The $z$-transform of a signal $x[n]$ is $X(z) = e^z + e^{1/z}$, $|z| \neq 0$. $x[n]$ would be

(A) $\delta[n] + \frac{1}{n!}$ (B) $u[n] + \frac{1}{n!}$

(C) $u[n-1] + n!$ (D) $\delta[n] + (n-1)!$

### Common Data For Q. 9 to 11:

Consider a discrete time signal $x[n]$ and its $z$-transform $X(z)$ given as

$$X(z) = \frac{z^2 + 5z}{z^2 - 2z - 3}$$

MCQ 6.1.9 If ROC of $X(z)$ is $|z| < 1$, then signal $x[n]$ would be

(A) $[-2(3)^n + (-1)^n]u[-n-1]$ (B) $[2(3)^n - (-1)^n]u[n]$

(C) $-2(3)^n u[-n-1] - (-1)^n u[n]$ (D) $[2(3)^n + 1]u[n]$

MCQ 6.1.10 If ROC of $X(z)$ is $|z| > 3$, then signal $x[n]$ would be

(A) $[2(3)^n - (-1)^n u[n]$ (B) $[-2(3)^n + (-1)^n]u[-n-1]$

(C) $-2(3)^n u[-n-1] - (-1)^n u[n]$ (D) $[2(3)^n + 1]u[n]$

MCQ 6.1.11 If ROC of $X(z)$ is $1 < |z| < 3$, the signal $x[n]$ would be

(A) $[2(3)^n - (-1)^n]u[n]$ (B) $[-2(3)^n + (-1)^n]u[-n-1]$

(C) $-2(3)^n u[-n-1] - (-1)^n u[n]$ (D) $[2(3)^n + (-1)^n]u[-n-1]$

MCQ 6.1.12 Consider a DT sequence $x[n] = x_1[n] + x_2[n]$ where, $x_1[n] = (0.7)^n u[n-1]$ and $x_2[n] = (-0.4)^n u[n-2]$. The region of convergence of $z$-transform of $x[n]$ is

(A) $0.4 < |z| < 0.7$ (B) $|z| > 0.7$

(C) $|z| < 0.4$ (D) none of these

MCQ 6.1.13 The $z$-transform of a DT signal $x[n]$ is $X(z) = \frac{z}{8z^2 - 2z - 1}$. What will be the $z$-transform of $x[n-4]$ ?

(A) $\frac{(z+4)}{8(z+4)^2 - 2(z+4) - 1}$ (B) $\frac{z^5}{8z^2 - 2z - 1}$

(C) $\frac{4z}{128z^2 - 8z - 1}$ (D) $\frac{1}{8z^5 - 2z^4 - z^3}$

MCQ 6.1.14 Let $x_1[n]$, $x_2[n]$ and $x_3[n]$ be three discrete time signals and $X_1(z)$, $X_2(z)$ and $X_3(z)$ are their $z$-transform respectively given as

$$X_1(z) = \frac{z^2}{(z-1)(z-0.5)},$$

$$X_2(z) = \frac{z}{(z-1)(z-0.5)}$$

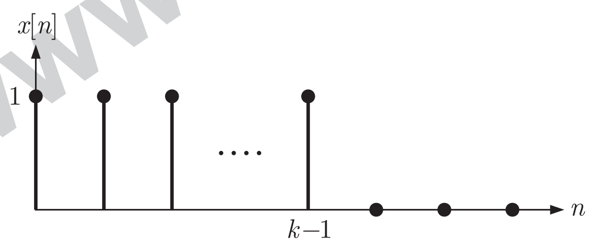
and $$X_3(z) = \frac{1}{(z-1)(z-0.5)}$$

Then $x_1[n]$, $x_2[n]$ and $x_3[n]$ are related as

(A) $x_1[n-2] = x_2[n-1] = x_3[n]$ (B) $x_1[n+2] = x_2[n+1] = x_3[n]$

(C) $x_1[n] = x_2[n-1] = x_3[n-2]$ (D) $x_1[n+1] = x_2[n-1] = x_3[n]$

MCQ 6.1.15 The $z$-transform of the discrete time signal $x[n]$ shown in the figure is




(A) $\frac{z^{-k}}{1-z^{-1}}$ (B) $\frac{z^{-k}}{1+z^{-1}}$

(C) $\frac{1-z^{-k}}{1-z^{-1}}$ (D) $\frac{1+z^{-k}}{1-z^{-1}}$

MCQ 6.1.16 Consider the unilateral $z$-transform pair $x[n] \xleftrightarrow{z} X(z) = \frac{z}{z-1}$. The $z$-transform of $x[n-1]$ and $x[n+1]$ are respectively

(A) $\frac{z^2}{z-1}, \frac{1}{z-1}$ (B) $\frac{1}{z-1}, \frac{z^2}{z-1}$

(C) $\frac{1}{z-1}, \frac{z}{z-1}$ (D) $\frac{z}{z-1}, \frac{z^2}{z-1}$

MCQ 6.1.17 A discrete time causal signal $x[n]$ has the $z$-transform

$$X(z) = \frac{z}{z-0.4}, \quad \text{ROC}: |z| > 0.4$$

The ROC for $z$-transform of the even part of $x[n]$ will be

(A) same as ROC of $X(z)$ (B) $0.4 < |z| < 2.5$
(C) $|z| > 0.2$ (D) $|z| > 0.8$

MCQ 6.1.18 Match List I (Discrete time sequence) with List II ($z$-transform) and select the correct answer using the codes given below the lists.

| List-I (Discrete time sequence) | List-II ($z$-transform) |
|---|---|
| P. $n(-1)^n u[n]$ | 1. $\frac{z^{-1}}{(1-z^{-1})^2}$, ROC: $|z| > 1$ |
| Q. $-nu[-n-1]$ | 2. $\frac{1}{(1+z^{-1})}$, ROC: $|z| > 1$ |
| R. $(-1)^n u[n]$ | 3. $\frac{z^{-1}}{(1-z^{-1})^2}$, ROC: $|z| < 1$ |
| S. $nu[n]$ | 4. $-\frac{z^{-1}}{(1+z^{-1})^2}$, ROC: $|z| > 1$ |

Codes :

| | P | Q | R | S |
|---|---|---|---|---|
| (A) | 4 | 1 | 2 | 3 |
| (B) | 4 | 3 | 2 | 1 |
| (C) | 3 | 1 | 4 | 2 |
| (D) | 2 | 4 | 1 | 3 |

MCQ 6.1.19 A discrete time sequence is defined as $x[n] = \frac{1}{n}(-2)^{-n}u[-n-1]$. The $z$-transform of $x[n]$ is

(A) $\log\left(z+\frac{1}{2}\right)$, ROC: $|z| < \frac{1}{2}$ (B) $\log\left(z-\frac{1}{2}\right)$, ROC: $|z| < \frac{1}{2}$

(C) $\log(z-2)$, ROC: $|z| > 2$ (D) $\log(z+2)$, ROC: $|z| < 2$

MCQ 6.1.20 Consider a $z$-transform pair $x[n] \overset{z}{\longleftrightarrow} X(z)$ with ROC $R_x$. The $z$ transform and its ROC for $y[n] = a^n x[n]$ will be

(A) $X\left(\frac{z}{a}\right)$, ROC: $|a|R_x$ (B) $X(z+a)$, ROC: $R_x$

(C) $z^{-a}X(z)$, ROC: $R_x$ (D) $X(az)$, ROC: $|a|R_x$

MCQ 6.1.21 Let $X(z)$ be the $z$-transform of a causal signal $x[n] = a^n u[n]$ with ROC: $|z| > a$. Match the discrete sequences $S_1$, $S_2$, $S_3$ and $S_4$ with ROC of their $z$-transforms $R_1$, $R_2$ and $R_3$.

| Sequences | ROC |
|---|---|
| $S_1$: $x[n-2]$ | $R_1$: $|z| > a$ |
| $S_2$: $x[n+2]$ | $R_2$: $|z| < a$ |
| $S_3$: $x[-n]$ | $R_3$: $|z| < \frac{1}{a}$ |
| $S_4$: $(-1)^n x[n]$ | |

(A) $(S_1, R_1), (S_2, R_2), (S_3, R_3), (S_4, R_3)$
(B) $(S_1, R_1), (S_2, R_1), (S_3, R_3), (S_4, R_1)$
(C) $(S_1, R_2), (S_2, R_1), (S_3, R_2), (S_4, R_3)$
(D) $(S_1, R_1), (S_2, R_2), (S_3, R_2), (S_4, R_3)$

MCQ 6.1.22 Consider a discrete time signal $x[n] = \alpha^n u[n]$ and its $z$-transform $X(z)$. Match List I (discrete signals) with List II ($z$-transform) and select the correct answer using the codes given below:

| | List-I (Discrete time signal) | | List-II ($z$-transform) |
|---|---|---|---|
| P. | $x[n/2]$ | 1. | $z^{-2}X(z)$ |
| Q. | $x[n-2]u[n-2]$ | 2. | $X(z^2)$ |
| R. | $x[n+2]u[n]$ | 3. | $X(z/\beta^2)$ |
| S. | $\beta^{2n}x[n]$ | 4. | $\alpha^2 X(z)$ |

Codes :

| | P | Q | R | S |
|---|---|---|---|---|
| (A) | 1 | 2 | 4 | 3 |
| (B) | 2 | 4 | 1 | 3 |
| (C) | 1 | 4 | 2 | 3 |
| (D) | 2 | 1 | 4 | 3 |

MCQ 6.1.23 The $z$-transform of a discrete sequence $x[n]$ is $X(z)$, then the $z$-transform of $x[2n]$ will be

(A) $X(2z)$ (B) $X\left(\frac{z}{2}\right)$

(C) $\frac{1}{2}\left[X(\sqrt{z}) + X(-\sqrt{z})\right]$ (D) $X(\sqrt{z})$

MCQ 6.1.24 Consider a signal $x[n]$ and its $z$ transform $X(z)$ given as

$$X(z) = \frac{4z}{8z^2 - 2z - 1}$$

The $z$-transform of the sequence $y[n] = x[0] + x[1] + x[2] + ..... + x[n]$ will be

(A) $\frac{4z^2}{(z-1)(8z^2-2z-1)}$ (B) $\frac{4z(z-1)}{8z^2-2z-1}$

(C) $\frac{4z^2}{(z+1)(8z^2-2z-1)}$ (D) $\frac{4z(z+1)}{8z^2-2z-1}$

MCQ 6.1.25 What is the convolution of two DT sequence $x[n] = \{-\underset{\uparrow}{1}, 2, 0, 3\}$ and $h[n] = \{2, 0, \underset{\uparrow}{3}\}$

(A) $\{-2, -4, \underset{\uparrow}{3}, 6, 9\}$ (B) $\{-2, 4, -\underset{\uparrow}{3}, 12, 0, 9\}$

(C) $\{9, 6, \underset{\uparrow}{3}, -4, -2\}$ (D) $\{-\underset{\uparrow}{3}, 6, 7, 4, 6\}$

MCQ 6.1.26 If $x[n] \overset{\mathcal{Z}}{\longleftrightarrow} X(z)$ be a $z$-transform pair, then which of the following is true?

(A) $x^*[n] \overset{\mathcal{Z}}{\longleftrightarrow} X^*(-z)$ (B) $x^*[n] \overset{\mathcal{Z}}{\longleftrightarrow} -X^*(z)$

(C) $x^*[n] \overset{\mathcal{Z}}{\longleftrightarrow} X^*(z^*)$ (D) $x^*[n] \overset{\mathcal{Z}}{\longleftrightarrow} X^*(-z^*)$

MCQ 6.1.27 A discrete-time system with input $x[n]$ and output $y[n]$ is governed by following difference equation

$$y[n] - \frac{1}{2}y[n-1] = x[n], \text{ with initial condition } y[-1] = 3$$

The impulse response of the system

(A) $\frac{5}{2}\left(\frac{n}{2} - 1\right)$, $n \geq 0$ (B) $\frac{5}{2}\left(\frac{1}{2}\right)^n$, $n \geq 0$

(C) $\frac{5}{2}\left(\frac{1}{2}\right)^{n-1}$, $n \geq 0$ (D) $\frac{5}{2}\left(\frac{1}{2}\right)^{n+1}$, $n \geq 0$

MCQ 6.1.28 Consider a causal system with impulse response $h[n] = (2)^n u[n]$. If $x[n]$ is the input and $y[n]$ is the output to this system, then which of the following difference equation describes the system ?

(A) $y[n] + 2y[n+1] = x[n]$ (B) $y[n] - 2y[n-1] = x[n]$

(C) $y[n] + 2y[n-1] = x[n]$ (D) $y[n] - \frac{1}{2}y[n-1] = x[n]$

MCQ 6.1.29 The impulse response of a system is given as $h[n] = \delta[n] - (-\frac{1}{2})^n u[n]$. For an input $x[n]$ and output $y[n]$, the difference equation that describes the system is

(A) $y[n] + 2y[n-1] = 2x[n]$ (B) $y[n] + 0.5y[n-1] = 0.5x[n-1]$

(C) $y[n] + 2ny[n-1] = x[n]$ (D) $y[n] - 0.5y[n-1] = 0.5x[n-1]$

MCQ 6.1.30 The input-output relationship of a system is given as $y[n] - 0.4y[n-1] = x[n]$ where, $x[n]$ and $y[n]$ are the input and output respectively. The zero state response of the system for an input $x[n] = (0.4)^n u[n]$ is

(A) $n(0.4)^n u[n]$ (B) $n^2(0.4)^n u[n]$

(C) $(n+1)(0.4)^n u[n]$ (D) $\frac{1}{n}(0.4)^n u[n]$

MCQ 6.1.31 A discrete time system has the following input-output relationship $y[n] - \frac{1}{2}y[n-1] = x[n]$. If an input $x[n] = u[n]$ is applied to the system, then its zero state response will be

(A) $\left[\frac{1}{2} - (2)^n\right]u[n]$ (B) $\left[2 - \left(\frac{1}{2}\right)^n\right]u[n]$

(C) $\left[\frac{1}{2} - \left(\frac{1}{2}\right)^n\right]u[n]$ (D) $[2 - (2)^n]u[n]$

MCQ 6.1.32 Consider the transfer function of a system

$$H(z) = \frac{2z(z-1)}{z^2 + 4z + 4}$$

For an input $x[n] = 2\delta[n] + \delta[n+1]$, the system output is

(A) $2\delta[n+1] + 6(2)^n u[n]$ (B) $2\delta[n] - 6(-2)^n u[n]$

(C) $2\delta[n+1] - 6(-2)^n u[n]$ (D) $2\delta[n+1] + 6\left(\frac{1}{2}\right)^n u[n]$

MCQ 6.1.33 The transfer function of a discrete time LTI system is given as

$$H(z) = \frac{z}{z^2 + 1}, \quad \text{ROC}: |z| > 1$$

Consider the following statements

1. The system is causal and BIBO stable.
2. The system is causal but BIBO unstable.
3. The system is non-causal and BIBO unstable.
4. Impulse response $h[n] = \sin\left(\frac{\pi}{2}n\right)u[n]$

Which of the above statements are true ?

(A) 1 and 4 (B) 2 and 4

(C) 1 only (D) 3 and 4

MCQ 6.1.34 Which of the following statement is not true?
An LTI system with rational transfer function $H(z)$ is

(A) causal if the ROC is the exterior of a circle outside the outermost pole.

(B) stable if the ROC of $H(z)$ includes the unit circle $|z| = 1$.

(C) causal and stable if all the poles of $H(z)$ lie inside unit circle.

(D) none of above

MCQ 6.1.35 If $h[n]$ denotes the impulse response of a causal system, then which of the following system is not stable?

(A) $h[n] = n\left(\frac{1}{3}\right)^n u[n]$ (B) $h[n] = \frac{1}{3}\delta[n]$

(C) $h[n] = \delta[n] - \left(-\frac{1}{3}\right)^n u[n]$ (D) $h[n] = [(2)^n - (3)^n]u[n]$

MCQ 6.1.36 A causal system with input $x[n]$ and output $y[n]$ has the following relationship

$$y[n] + 3y[n-1] + 2y[n-2] = 2x[n] + 3x[n-1]$$

The system is

(A) stable (B) unstable

(C) marginally stable (D) none of these

MCQ 6.1.37 A causal LTI system is described by the difference equation $y[n] = x[n] + y[n-1]$

Consider the following statement

1. Impulse response of the system is $h[n] = u[n]$
2. The system is BIBO stable
3. For an input $x[n] = (0.5)^n u[n]$, system output is $y[n] = 2u[n] - (0.5)^n u[n]$

Which of the above statements is/are true?

(A) 1 and 2 (B) 1 and 3

(C) 2 and 3 (D) 1, 2 and 3

MCQ 6.1.38 Match List I (system transfer function) with List II (property of system) and choose the correct answer using the codes given below

| | List-I (System transfer function) | | List-II (Property of system) |
|---|---|---|---|
| P. | $H(z) = \frac{z^3}{(z-1.2)^3}$, ROC: $\lvert z \rvert > 1.2$ | 1. | Non causal but stable |
| Q. | $H(z) = \frac{z^2}{(z-1.2)^3}$, ROC: $\lvert z \rvert < 1.2$ | 2. | Neither causal nor stable |
| R. | $H(z) = \frac{z^4}{(z-0.8)^3}$, ROC: $\lvert z \rvert < 0.8$ | 3. | Causal but not stable |
| S. | $H(z) = \frac{z^3}{(z-0.8)^3}$, ROC: $\lvert z \rvert > 0.8$ | 4. | Both causal and stable |

Codes :

| | P | Q | R | S |
|---|---|---|---|---|
| (A) | 4 | 2 | 1 | 3 |
| (B) | 1 | 4 | 2 | 3 |
| (C) | 3 | 1 | 2 | 4 |
| (D) | 3 | 2 | 1 | 4 |

MCQ 6.1.39 The transfer function of a DT feedback system is

$$H(z) = \frac{P}{1 + P\left(\frac{z}{z-0.9}\right)}$$

The range of $P$, for which the system is stable will be

(A) $-1.9 < P < -0.1$

(B) $P < 0$

(C) $P > -1$

(D) $P > -0.1$ or $P < -1.9$

MCQ 6.1.40 Consider three stable LTI systems $S_1, S_2$ and $S_3$ whose transfer functions are

$$S_1 : H(z) = \frac{z - \frac{1}{2}}{2z^2 + \frac{1}{2}z - \frac{3}{16}}$$

$$S_2 : H(z) = \frac{z + 1}{-\frac{2}{3}z^{-3} - \frac{1}{2}z^{-2} + \frac{4}{3} + z}$$

$$S_3 : H(z) = \frac{1 + \frac{1}{2}z^{-2} - \frac{4}{3}z^{-1}}{z^{-1}(1 - \frac{1}{3}z^{-1})(1 - \frac{1}{2}z^{-1})}$$

Which of the above systems is/are causal?

(A) $S_1$ only
(B) $S_1$ and $S_2$
(C) $S_1$ and $S_3$
(D) $S_1, S_2$ and $S_3$

MCQ 6.1.41 The $z$-transform of $\delta[n-k]$, $k > 0$ is

(A) $z^k$, $z > 0$
(B) $z^{-k}$, $z > 0$
(C) $z^k$, $z \neq 0$
(D) $z^{-k}$, $z \neq 0$

MCQ 6.1.42 The $z$-transform of $\delta[n+k], k > 0$ is

(A) $z^{-k}$, $z \neq 0$
(B) $z^k$, $z \neq 0$
(C) $z^{-k}$, all $z$
(D) $z^k$, all $z$

MCQ 6.1.43 The $z$-transform of $u[n]$ is

(A) $\frac{1}{1 - z^{-1}}$, $|z| > 1$
(B) $\frac{1}{1 - z^{-1}}$, $|z| < 1$
(C) $\frac{z}{1 - z^{-1}}$, $|z| < 1$
(D) $\frac{z}{1 - z^{-1}}$, $|z| > 1$

MCQ 6.1.44 The $z$-transform of $\left(\frac{1}{4}\right)^n (u[n] - u[n-5])$ is

(A) $\frac{z^5 - (0.25)^5}{z^4(z - 0.25)}$, $|z| > 0.25$
(B) $\frac{z^5 - (0.25)^5}{z^4(z - 0.25)}$, $|z| > 0.5$
(C) $\frac{z^5 - (0.25)^5}{z^3(z - 0.25)}$, $|z| < 0.25$
(D) $\frac{z^5 - (0.25)^5}{z^4(z - 0.25)}$, all $z$

MCQ 6.1.45 The $z$-transform of $\left(\frac{1}{4}\right)^n u[-n]$ is

(A) $\frac{4z}{4z - 1}$, $|z| > \frac{1}{4}$
(B) $\frac{4z}{4z - 1}$, $|z| < \frac{1}{4}$
(C) $\frac{1}{1 - 4z}$, $|z| > \frac{1}{4}$
(D) $\frac{1}{1 - 4z}$, $|z| < \frac{1}{4}$

MCQ 6.1.46 The $z$-transform of $3^n u[-n-1]$ is

(A) $\frac{z}{3 - z}$, $|z| > 3$
(B) $\frac{z}{3 - z}$, $|z| < 3$
(C) $\frac{3}{3 - z}$, $|z| > 3$
(D) $\frac{3}{3 - z}$, $|z| < 3$

MCQ 6.1.47 The $z$-transform of $\left(\frac{2}{3}\right)^{|n|}$ is

(A) $\frac{-5z}{(2z-3)(3z-2)}$, $-\frac{3}{2} < |z| < -\frac{2}{3}$
(B) $\frac{-5z}{(2z-3)(3z-2)}$, $\frac{2}{3} < |z| < \frac{3}{2}$
(C) $\frac{5z}{(2z-3)(3z-2)}$, $\frac{2}{3} < |z| < \frac{2}{3}$
(D) $\frac{5z}{(2z-3)(3z-2)}$, $-\frac{3}{2} < |z| < -\frac{2}{3}$

MCQ 6.1.48 The $z$-transform of $\cos\left(\frac{\pi}{3}n\right)u[n]$ is

(A) $\frac{z(2z-1)}{2(z^2-z+1)}$, $0<|z|<1$
(B) $\frac{z(2z-1)}{2(z^2-z+1)}$, $|z|>1$
(C) $\frac{z(1-2z)}{2(z^2-z+1)}$, $0<|z|<1$
(D) $\frac{z(1-2z)}{2(z^2-z+1)}$, $|z|>1$

MCQ 6.1.49 The $z$-transform of $\{3, 0, 0, 0, 0, \underset{\uparrow}{6}, 1, -4\}$

(A) $3z^5+6+z^{-1}-4z^{-2}$, $0\le|z|<\infty$
(B) $3z^5+6+z^{-1}-4z^{-2}$, $0<|z|<\infty$
(C) $3z^{-5}+6+z-4z^2$, $0<|z|<\infty$
(D) $3z^{-5}+6+z-4z^2$, $0\le|z|<\infty$

MCQ 6.1.50 The $z$-transform of $x[n]=\{2, 4, \underset{\uparrow}{5}, 7, 0, 1\}$

(A) $2z^2+4z+5+7z+z^3$, $z\ne\infty$
(B) $2z^{-2}+4z^{-1}+5+7z+z^3$, $z\ne\infty$
(C) $2z^{-2}+4z^{-1}+5+7z+z^3$, $0<|z|<\infty$
(D) $2z^2+4z+5+7z^{-1}+z^{-3}$, $0<|z|<\infty$

MCQ 6.1.51 The $z$-transform of $x[n]=\{\underset{\uparrow}{1}, 0, -1, 0, 1, -1\}$ is

(A) $1+2z^{-2}-4z^{-4}+5z^{-5}$, $z\ne0$
(B) $1-z^{-2}+z^{-4}-z^{-5}$, $z\ne0$
(C) $1-2z^2+4z^4-5z^5$, $z\ne0$
(D) $1-z^2+z^4-z^5$, $z\ne0$

MCQ 6.1.52 The time signal corresponding to $\frac{z^2-3z}{z^2+\frac{3}{2}z-1}$, $\frac{1}{2}<|z|<2$ is

(A) $-\frac{1}{2^n}u[n]-2^{n+1}u[-n-1]$
(B) $-\frac{1}{2^n}u[n]-2^{n+1}u[n+1]$
(C) $\frac{1}{2^n}u[n]+2^{n+1}u[n+1]$
(D) $\frac{1}{2^n}u[n]-2^{-n-1}u[-n-1]$

MCQ 6.1.53 The time signal corresponding to $\frac{3z^2-\frac{1}{4}z}{z^2-16}$, $|z|>4$ is

(A) $\left[\frac{49}{32}(-4)^n+\frac{47}{32}4^n\right]u[n]$
(B) $\left[\frac{49}{32}4^n+\frac{47}{32}4^n\right]u[n]$
(C) $\frac{49}{32}(-4)^n u[-n]+\frac{47}{32}4^n u[n]$
(D) $\frac{49}{32}4^n u[n]+\frac{47}{32}(-4)^n u[-n]$

MCQ 6.1.54 The time signal corresponding to $\frac{2z^4-2z^3-2z^2}{z^2-1}$, $|z|>1$ is

(A) $2\delta[n-2]+[1-(-1)^n]u[n-2]$
(B) $2\delta[n+2]+[1-(-1)^n]u[n+2]$
(C) $2\delta[n+2]+[(-1)^n-1]u[n+2]$
(D) $2\delta[n-2]+[(-1)^n-1]u[n-2]$

MCQ 6.1.55 The time signal corresponding to $1+2z^{-6}+4z^{-8}$, $|z|>0$ is

(A) $\delta[n]+2\delta[n-6]+4\delta[n-8]$
(B) $\delta[n]+2\delta[n+6]+4\delta[n+8]$
(C) $\delta[-n]+2\delta[-n+6]+4\delta[-n+8]$
(D) $\delta[-n]+2\delta[-n-6]+4\delta[-n-8]$

MCQ 6.1.56 The time signal corresponding to $\sum_{k=5}^{10}\frac{1}{k}z^{-k}$, $|z|>0$ is

(A) $\sum_{k=5}^{10}\frac{1}{k}\delta[n+k]$
(B) $\sum_{k=5}^{10}\frac{1}{k}\delta[n-k]$
(C) $\sum_{k=5}^{10}\frac{1}{k}\delta[-n+k]$
(D) $\sum_{k=5}^{10}\frac{1}{k}\delta[-n-k]$

MCQ 6.1.57 The time signal corresponding to $(1+z^{-1})^3$, $|z|>0$ is

(A) $\delta[-n]+3\delta[-n-1]+3\delta[-n-2]+\delta[-n-3]$
(B) $\delta[-n]+3\delta[-n+1]+3\delta[-n+2]+\delta[-n+3]$
(C) $\delta[n]+3\delta[n+1]+3\delta[n+2]+\delta[n+3]$
(D) $\delta[n]+3\delta[n-1]+3\delta[n-2]+\delta[n-3]$

MCQ 6.1.58 The time signal corresponding to $z^6+z^2+3+2z^{-3}+z^{-4}$, $|z|>0$ is

(A) $\delta[n+6]+\delta[n+2]+3\delta[n]+2\delta[n-3]+\delta[n-4]$
(B) $\delta[n-6]+\delta[n-2]+3\delta[n]+2\delta[n+3]+\delta[n+4]$
(C) $\delta[-n+6]+\delta[-n+2]+3\delta[-n]+2\delta[-n+3]+\delta[-n+4]$
(D) $\delta[-n-6]+\delta[-n-2]+3\delta[-n]+2\delta[-n-3]+\delta[-n-4]$

MCQ 6.1.59 The time signal corresponding to $\frac{1}{1-\frac{1}{4}z^{-2}}$, $|z|>\frac{1}{2}$ is

(A) $\begin{cases}2^{-n}, & n \text{ even and } n \geq 0\\ 0, & \text{otherwise}\end{cases}$
(B) $\left(\frac{1}{4}\right)^{2n}u[n]$
(C) $\begin{cases}2^{-n}, & n \text{ odd}, n>0\\ 0, & n \text{ even}\end{cases}$
(D) $2^{-n}u[n]$

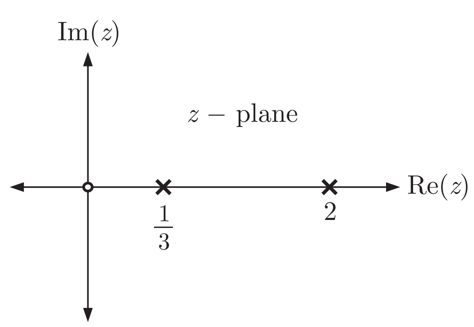
MCQ 6.1.60 The time signal corresponding to $\frac{1}{1-\frac{1}{4}z^{-2}}$, $|z|<\frac{1}{2}$ is

(A) $-\sum_{k=0}^{\infty}2^{2(k+1)}\delta[-n-2(k+1)]$
(B) $-\sum_{k=0}^{\infty}2^{2(k+1)}\delta[-n+2(k+1)]$
(C) $-\sum_{k=0}^{\infty}2^{2(k+1)}\delta[n+2(k+1)]$
(D) $-\sum_{k=0}^{\infty}2^{2(k+1)}\delta[n-2(k+1)]$

MCQ 6.1.61 The time signal corresponding to $\ln(1+z^{-1})$, $|z|>0$ is

(A) $\frac{(-1)^{k-1}}{k}\delta[n-k]$
(B) $\frac{(-1)^{k-1}}{k}\delta[n+k]$
(C) $\frac{(-1)^{k}}{k}\delta[n-k]$
(D) $\frac{(-1)^{k}}{k}\delta[n+k]$

MCQ 6.1.62 $X[z]$ of a system is specified by a pole zero pattern as following :



Consider three different solution of $x[n]$

$$x_1[n] = \left[2^n - \left(\frac{1}{3}\right)^n\right]u[n]$$

$$x_2[n] = -2^n u[n-1] - \frac{1}{3^n}u[n]$$

$$x_3[n] = -2^n u[n-1] + \frac{1}{3^n}u[-n-1]$$

Correct solution is

(A) $x_1[n]$ (B) $x_2[n]$

(C) $x_3[n]$ (D) All three

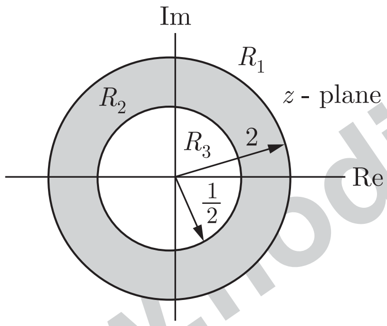
MCQ 6.1.63 Consider three different signal

$$x_1[n] = \left[2^n - \left(\frac{1}{2}\right)^n\right]u[n]$$

$$x_2[n] = -2^n u[-n-1] + \frac{1}{2^n}u[-n-1]$$

$$x_3[n] = -2^n u[-n-1] - \frac{1}{2^n}u[n]$$

Following figure shows the three different region. Choose the correct for the ROC of signal




| | $R_1$ | $R_2$ | $R_3$ |
|---|---|---|---|
| (A) | $x_1[n]$ | $x_2[n]$ | $x_3[n]$ |
| (B) | $x_2[n]$ | $x_3[n]$ | $x_1[n]$ |
| (C) | $x_1[n]$ | $x_3[n]$ | $x_2[n]$ |
| (D) | $x_3[n]$ | $x_2[n]$ | $x_1[n]$ |

MCQ 6.1.64 Given the $z$-transform

$$X(z) = \frac{1 + \frac{7}{6}z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{3}z^{-1})}$$

For three different ROC consider there different solution of signal $x[n]$ :

(a) $|z| > \frac{1}{2}$, $x[n] = \left[\frac{1}{2^{n-1}} - \left(\frac{-1}{3}\right)^n\right]u[n]$

(b) $|z| < \frac{1}{3}$, $x[n] = \left[\frac{-1}{2^{n-1}} + \left(\frac{-1}{3}\right)^n\right]u[-n+1]$

(c) $\frac{1}{3} < |z| < \frac{1}{2}$, $x[n] = -\frac{1}{2^{n-1}}u[-n-1] - \left(\frac{-1}{3}\right)^n u[n]$

Correct solution are

(A) (a) and (b)

(B) (a) and (c)

(C) (b) and (c)

(D) (a), (b), (c)

MCQ 6.1.65 The $X(z)$ has poles at $z = \frac{1}{2}$ and $z = -1$. If $x[1] = 1, x[-1] = 1$, and the ROC includes the point $z = \frac{3}{4}$. The time signal $x[n]$ is

(A) $\frac{1}{2^{n-1}}u[n] - (-1)^n u[-n-1]$ (B) $\frac{1}{2^n}u[n] - (-1)^n u[-n-1]$

(C) $\frac{1}{2^{n-1}}u[n] + u[-n+1]$ (D) $\frac{1}{2^n}u[n] + u[-n+1]$

MCQ 6.1.66 If $x[n]$ is right-sided, $X(z)$ has a signal pole and $x[0] = 2$, $x[2] = \frac{1}{2}$, then $x[n]$ is

(A) $\frac{u[-n]}{2^{n-1}}$ (B) $\frac{u[n]}{2^{n-1}}$

(C) $\frac{u[-n]}{2^{n+1}}$ (D) $a\frac{u[-n]}{2^{n+1}}$

MCQ 6.1.67 The $z$-transform of $\left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{4}\right)^n u[-n-1]$ is

(A) $\frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}}, \quad \frac{1}{4} < |z| < \frac{1}{2}$

(B) $\frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-\frac{1}{4}z^{-1}}, \quad \frac{1}{4} < |z| < \frac{1}{2}$

(C) $\frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}}, \quad |z| > \frac{1}{2}$

(D) None of the above



Common Data For Q. 68 - 73:

Given the $z$-transform pair $x[n] \xleftrightarrow{z} \frac{z^2}{z^2-16}, \quad |z| < 4$

MCQ 6.1.68 The $z$-transform of the signal $x[n-2]$ is

(A) $\frac{z^4}{z^2-16}$ (B) $\frac{(z+2)^2}{(z+2)^2-16}$

(C) $\frac{1}{z^2-16}$ (D) $\frac{(z-2)^2}{(z-2)^2-16}$

MCQ 6.1.69 The $z$-transform of the signal $y[n] = \frac{1}{2^n}x[n]$ is

(A) $\frac{(z+2)^2}{(x+2)^2-16}$ (B) $\frac{z^2}{z^2-4}$

(C) $\frac{(z-2)^2}{(z-2)^2-16}$ (D) $\frac{z^2}{z^2-64}$

MCQ 6.1.70 The $z$-transform of the signal $x[-n] * x[n]$ is

(A) $\frac{z^2}{16z^2-257z^4-16}$ (B) $\frac{-16z^2}{(z^2-16)^2}$

(C) $\frac{z^2}{257z^2-16z^4-16}$ (D) $\frac{16z^2}{(z^2-16)^2}$

MCQ 6.1.71 The $z$-transform of the signal $nx[n]$ is

(A) $\frac{32z^2}{(z^2-16)^2}$ (B) $\frac{-32z^2}{(z^2-16)^2}$

(C) $\frac{32z}{(z^2-16)^2}$ (D) $\frac{-32z}{(z^2-16)^2}$

MCQ 6.1.72 The $z$-transform of the signal $x[n+1]+x[n-1]$ is

(A) $\frac{(z+1)^2}{(z+1)^2-16}+\frac{(z-1)^2}{(z-1)^2-16}$ (B) $\frac{z(z^2+1)}{z^2-16}$

(C) $\frac{z^2(-1+z)}{z^2-16}$ (D) None of the above

MCQ 6.1.73 The $z$-transform of the signal $x[n]*x[n-3]$ is

(A) $\frac{z^{-3}}{(z^2-16)^2}$ (B) $\frac{z^7}{(z^2-16)^2}$

(C) $\frac{z^5}{(z^2-16)^2}$ (D) $\frac{z}{(z^2-16)^2}$

## Common Data For Q. 74 - 78:

Given the $z$-transform pair $3^n n^2 u[n] \overset{z}{\longleftrightarrow} X(z)$

MCQ 6.1.74 The time signal corresponding to $X(2z)$ is

(A) $n^2 3^n u[2n]$ (B) $\left(-\frac{3}{2}\right)^n n^2 u[n]$

(C) $\left(\frac{3}{2}\right)^n n^2 u[n]$ (D) $6^n n^2 u[n]$

MCQ 6.1.75 The time signal corresponding to $X(z^{-1})$ is

(A) $n^2 3^{-n} u[-n]$ (B) $n^2 3^{-n} u[-n]$

(C) $\frac{1}{n^2}3^{\frac{1}{n}}u[n]$ (D) $\frac{1}{n^2}3^{\frac{1}{n}}u[-n]$

MCQ 6.1.76 The time signal corresponding to $\frac{d}{dz}X(z)$ is

(A) $(n-1)^3 3^{n-1} u[n-1]$

(B) $n^3 3^n u[n-1]$

(C) $(1-n)^3 3^{n-1} u[n-1]$

(D) $(n-1)^3 3^{n-1} u[n]$

MCQ 6.1.77 The time signal corresponding to $\left(\frac{z^2-z^{-2}}{2}\right)X(z)$ is

(A) $\frac{1}{2}(x[n+2]-x[n-2])$ (B) $x[n+2]-x[n-2]$

(C) $\frac{1}{2}x[n-2]-x[n+2])$ (D) $x[n-2]-x[n+2]$

MCQ 6.1.78 The time signal corresponding to $\{X(z)\}^2$ is

(A) $[x[n]]^2$ (B) $x[n]*x[n]$

(C) $x(n)*x[-n]$ (D) $x[-n]*x[-n]$

MCQ 6.1.79 A causal system has

Input, $x[n]=\delta[n]+\frac{1}{4}\delta[n-1]-\frac{1}{8}\delta[n-2]$ and

Output, $y[n]=\delta[n]-\frac{3}{4}\delta[n-1]$

The impulse response of this system is

(A) $\frac{1}{3}\left[5\left(\frac{-1}{2}\right)^n-2\left(\frac{1}{4}\right)^n\right]u[n]$ (B) $\frac{1}{3}\left[5\left(\frac{1}{2}\right)^n+2\left(\frac{-1}{4}\right)^n\right]u[n]$

(C) $\frac{1}{3}\left[5\left(\frac{1}{2}\right)^n-2\left(\frac{-1}{4}\right)^n\right]u[n]$ (D) $\frac{1}{3}\left[5\left(\frac{1}{2}\right)^n+2\left(\frac{1}{4}\right)^n\right]u[n]$

MCQ 6.1.80 A causal system has

Input, $x[n] = (-3)^n u[n]$

Output, $y[n] = [4(2)^n - (\frac{1}{2})^n] u[n]$

The impulse response of this system is

(A) $\left[7\left(\frac{1}{2}\right)^n - 10\left(\frac{1}{2}\right)^n\right] u[n]$

(B) $\left[7(2^n) - 10\left(\frac{1}{2}\right)^n\right] u[n]$

(C) $\left[10\left(\frac{1}{2}\right)^2 - 7(2)^n\right] u[n]$

(D) $\left[10(2^n) - 7\left(\frac{1}{2}\right)^n\right] u[n]$

MCQ 6.1.81 A system has impulse response $h[n] = (\frac{1}{2})^n u[n]$. The output $y[n]$ to the input $x[n]$ is given by $y[n] = 2\delta[n-4]$. The input $x[n]$ is

(A) $2\delta[-n-4] - \delta[-n-5]$

(B) $2\delta[n+4] - \delta[n+5]$

(C) $2\delta[-n+4] - \delta[-n+5]$

(D) $2\delta[n-4] - \delta[n-5]$

MCQ 6.1.82 A system is described by the difference equation

$$y[n] = x[n] - x[n-2] + x[n-4] - x[n-6]$$

The impulse response of system is

(A) $\delta[n] - 2\delta[n+2] + 4\delta[n+4] - 6\delta[n+6]$

(B) $\delta[n] + 2\delta[n-2] - 4\delta[n-4] + 6\delta[n-6]$

(C) $\delta[n] - \delta[n-2] + \delta[n-4] - \delta[n-6]$

(D) $\delta[n] - \delta[n+2] + \delta[n+4] - \delta[n+6]$

MCQ 6.1.83 The impulse response of a system is given by $h[n] = \frac{3}{4^n} u[n-1]$. The difference equation representation for this system is

(A) $4y[n] - y[n-1] = 3x[n-1]$

(B) $4y[n] - y[n+1] = 3x[n+1]$

(C) $4y[n] + y[n-1] = -3x[n-1]$

(D) $4y[n] + y[n+1] = 3x[n+1]$

MCQ 6.1.84 The impulse response of a system is given by $h[n] = \delta[n] - \delta[n-5]$. The difference equation representation for this system is

(A) $y[n] = x[n] - x[n-5]$

(B) $y[n] = x[n] - x[n+5]$

(C) $y[n] = x[n] + 5x[n-5]$

(D) $y[n] = x[n] - 5x[n+5]$

MCQ 6.1.85 Consider the following three systems

$$y_1[n] = 0.2y[n-1] + x[n] - 0.3x[n-1] + 0.02x[n-2]$$

$$y_2[n] = x[n] - 0.1x[n-1]$$

$$y_3[n] = 0.5y[n-1] + 0.4x[n] - 0.3x[n-1]$$

The equivalent system are

(A) $y_1[n]$ and $y_2[n]$

(B) $y_2[n]$ and $y_3[n]$

(C) $y_3[n]$ and $y_1[n]$

(D) all

MCQ 6.1.86 The $z$-transform function of a stable system is $H(z) = \frac{2 - \frac{3}{2}z^{-1}}{(1 - 2z^{-1})(1 + \frac{1}{2}z^{-1})}$. The impulse response $h[n]$ is

(A) $2^n u[-n+1] - \left(\frac{1}{2}\right)^n u[n]$ (B) $-2^n u[-n-1] + \left(\frac{-1}{2}\right)^n u[n]$

(C) $-2^n u[-n-1] - \left(\frac{-1}{2}\right)^n u[n]$ (D) $2^n u[n] - \left(\frac{1}{2}\right)^n u[n]$

MCQ 6.1.87 The transfer function of a causal system is $H(z) = \frac{5z^2}{z^2 - z - 6}$. The impulse response is

(A) $(3^n + (-1)^n 2^{n+1}) u[n]$

(B) $(3^{n+1} + 2(-2)^n) u[n]$

(C) $(3^{n-1} + (-1)^n 2^{n+1}) u[n]$

(D) $(3^{n-1} - (-2)^{n+1}) u[n]$

MCQ 6.1.88 The transfer function of a system is given by $H(z) = \frac{z(3z-2)}{z^2 - z - \frac{1}{4}}$. The system is

(A) causal and stable

(B) causal, stable and minimum phase

(C) minimum phase

(D) none of the above

MCQ 6.1.89 The $z$-transform of a signal $x[n]$ is $X(z) = \frac{3}{1 - \frac{10}{3}z^{-1} + z^{-2}}$. If $X(z)$ converges on the unit circle, $x[n]$ is

(A) $-\frac{1}{3^{n-1}(8)}u[n] - \frac{3^{n+3}}{8}u[-n-1]$ (B) $\frac{1}{3^{n-1}(8)}u[n] - \frac{3^{n+3}}{(8)}u[-n]$

(C) $\frac{1}{3^{n-1}(8)}u[n] - \frac{3^{n+3}}{(8)}u[-n]$ (D) $-\frac{1}{3^{n-1}(8)}u[n] - \frac{3^{n+3}}{(8)}u[-n]$

MCQ 6.1.90 The transfer function of a system is $H(z) = \frac{4z^{-1}}{(1 - \frac{1}{4}z^{-1})^2}$, $|z| > \frac{1}{4}$. The $h[n]$ is

(A) stable

(B) causal

(C) stable and causal

(D) none of the above

MCQ 6.1.91 The transfer function of a system is given as

$$H(z) = \frac{2(z + \frac{1}{2})}{(z - \frac{1}{2})(z - \frac{1}{3})}$$

Consider the two statements

Statement (1) : System is causal and stable.

Statement (2) : Inverse system is causal and stable.

The correct option is

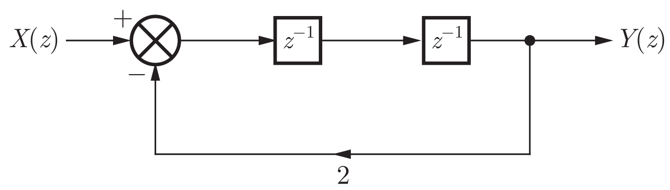
(A) (1) is true

(B) (2) is true

(C) Both (1) and (2) are true

(D) Both are false

MCQ 6.1.92 The impulse response of the system shown below is



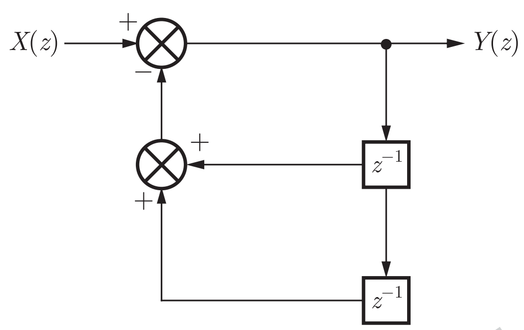

(A) $2^{(\frac{n}{2}-2)}(1+(-1)^n)u[n]+\frac{1}{2}\delta[n]$

(B) $\frac{2^n}{2}(1+(-1)^n)u[n]+\frac{1}{2}\delta[n]$

(C) $2^{(\frac{n}{2}-2)}(1+(-1)^n)u[n]-\frac{1}{2}\delta[n]$

(D) $\frac{2^n}{2}[1+(-1)^n]u[n]-\frac{1}{2}\delta[n]$

MCQ 6.1.93 The system diagram for the transfer function $H(z)=\frac{z}{z^2+z+1}$. is shown below.




The system diagram is a

(A) Correct solution

(B) Not correct solution

(C) Correct and unique solution

(D) Correct but not unique solution



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# EXERCISE 6.2

QUES 6.2.1 Consider a DT signal which is defined as follows

$$x[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

The $z$-transform of $x[n]$ will be $\frac{az}{az-1}$ such that the value of $a$ is _______

QUES 6.2.2 If the $z$-transform of a sequence $x[n] = \{1, 1, -1, \underset{\uparrow}{-1}\}$ is $X(z)$, then what is the value of $X(1/2)$ ?

QUES 6.2.3 The $z$-transform of a discrete time signal $x[n]$ is $X(z) = \frac{z+1}{z(z-1)}$. Then, $x[0] + x[1] + x[2] =$ _______

QUES 6.2.4 If $x[n] = \alpha^n u[n]$, then the $z$-transform of $x[n+3]u[n]$ will be $\alpha^k\left(\frac{z}{z-k}\right)$, where $k =$ _______

QUES 6.2.5 The inverse $z$-transform of a function $X(z) = \frac{z^{-9}}{z-\alpha}$ is $\alpha^{n-k}u[n-k]$ where the value of $k$ is _______

QUES 6.2.6 Let $x[n] \overset{z}{\longleftrightarrow} X(z)$ be a $z$-transform pair, where $X(z) = \frac{z^{-2}}{z-3}$. What will be the value of $x[5]$ ?

QUES 6.2.7 The $z$-transform of a discrete time sequence $y[n] = n[n+1]u[n]$ is $\frac{kz^k}{(z-1)^{k+1}}$ such that the value of $k$ is _______

QUES 6.2.8 A signal $x[n]$ has the following $z$-transform $X(z) = \log(1-2z)$, ROC: $|z| < \frac{1}{2}$. Let the signal be

$$x[n]\ \frac{1}{n}\left(\frac{1}{2}\right)^n u[a-n]$$

what is the value of $a$ in the expression ?

QUES 6.2.9 Let $x[n] \overset{z}{\longleftrightarrow} X(z)$ be a $z$-transform pair. Consider another signal $y[n]$ defined as

$$y[n] = \begin{cases} x[n/2], & \text{if } n \text{ is even} \\ 0, & \text{if } n \text{ is odd} \end{cases}$$

The $z$-transform of $y[n]$ is $X(z^k)$ such that the value of $k$ is _______

QUES 6.2.10 Let $X(z)$ be $z$-transform of a discrete time sequence $x[n] = (-\frac{1}{2})^2 u[n]$. Consider another signal $y[n]$ and its $z$-transform $Y(z)$ given as $Y(z) = X(z^3)$. What is the value of $y[n]$ at $n = 4$ ?

QUES 6.2.11 Let $h[n] = \{1, 2, 0, -1, 1\}$ and $x[n] = \{1, 3, -1, -2\}$ be two discrete time sequences. What is the value of convolution $y[n] = h[n] * x[n]$ at $n = 4$ ?

QUES 6.2.12 A discrete time sequence is defined as follows

$$x[n] = \begin{cases} 1, & n \text{ is even} \\ 0, & \text{otherwise} \end{cases}$$

What is the final value of $x[n]$ ?

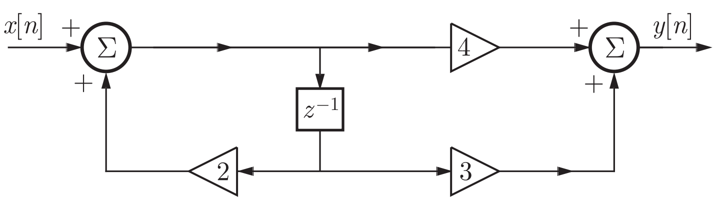
QUES 6.2.13 Let $X(z)$ be the $z$-transform of a DT signal $x[n]$ given as

$$X(z) = \frac{0.5z^2}{(z-1)(z-0.5)}$$

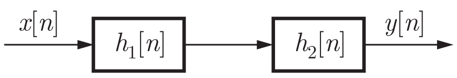
The initial value of $x[n]$ is _______

QUES 6.2.14 The signal $x[n] = (0.5)^n u[n]$ is when applied to a digital filter, it yields the following output $y[n] = \delta[n] - 2\delta[n-1]$. If impulse response of the filter is $h[n]$, then what will be the value of sample $h[1]$ ?

QUES 6.2.15 The transfer function for the system realization shown in the figure will be $\frac{k(z+1)-1}{z-2}$ such that the value of $k$ is _______



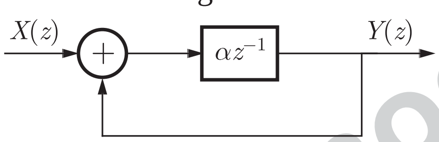

QUES 6.2.16 Consider a cascaded system shown in the figure




where, $h_1[n] = \delta[n] + \frac{1}{2}\delta[n-1]$ and $h_2[n] = \left(\frac{1}{2}\right)^n u[n]$

If an input $x[n] = \cos(n\pi)$ is applied, then output $y[n] = k\cos n\pi$ where the constant $k$ is _______

QUES 6.2.17 The block diagram of a discrete time system is shown in the figure below




The system is BIBO stable for $|\alpha| <$ _______

QUES 6.2.18 Let $x[n] = \delta[n-1] + \delta[n+2]$. If unilateral $z$-transform of the signal $x[n]$ be

$$X(z) = z^k$$

then, the value of constant $k$ is _______

QUES 6.2.19 The unilateral $z$-transform of signal $x[n] = u[n+4]$ is $\frac{1}{1+a/z}$ such that the value of $a$ is _______

QUES 6.2.20 If $z$-transform is given by $X(z) = \cos(z^{-3})$, $|z| > 0$, then what will be the value of $x[12]$ ?

QUES 6.2.21 The $z$-transform of an anticausal system is $X(z) = \frac{12-21z}{3-7z+12z^2}$. What will be the value of $x[0]$ ?

QUES 6.2.22 The system $y[n] = cy[n-1] - 0.12y[n-2] + x[n-1] + x[n-2]$ is stable if $|c| <$ _______

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# EXERCISE 6.3

MCQ 6.3.1 The $z$-transform is used to analyze

(A) discrete time signals and system

(B) continuous time signals and system

(C) both (A) and (B)

(D) none

MCQ 6.3.2 Which of the following expression is correct for the bilateral $z$-transform of $x[n]$ ?

(A) $\sum_{n=0}^{\infty} x[n]z^{n}$

(B) $\sum_{n=0}^{\infty} x[n]z^{-n}$

(C) $\sum_{n=-\infty}^{\infty} x[n]z^{n}$

(D) $\sum_{n=-\infty}^{\infty} x[n]z^{-n}$

MCQ 6.3.3 The unilateral $z$-transform of sequence $x[n]$ is defined as

(A) $\sum_{n=0}^{\infty} x[n]z^{n}$

(B) $\sum_{n=-\infty}^{\infty} x[n]z^{n}$

(C) $\sum_{n=0}^{\infty} x[n]z^{-n}$

(D) $\sum_{n=-\infty}^{\infty} x[n]z^{-n}$

MCQ 6.3.4 The $z$-transform of a causal signal $x[n]$ is given by

(A) $\sum_{n=-\infty}^{\infty} x[n]z^{n}$

(B) $\sum_{n=0}^{\infty} x[n]z^{n}$

(C) $\sum_{n=-\infty}^{\infty} x[n]z^{-n}$

(D) $\sum_{n=0}^{\infty} x[n]z^{-n}$

MCQ 6.3.5 For a signal $x[n]$, its unilateral $z$-transform is equivalent to the bilateral $z$-transform of

(A) $x[n]r[n]$

(B) $x[n]\delta[n]$

(C) $x[n]u[n]$

(D) none of these

MCQ 6.3.6 The ROC of $z$-transform $X(z)$ is defined as the range of values of $z$ for which $X(z)$

(A) zero

(B) diverges

(C) converges

(D) none

MCQ 6.3.7 In the $z$-plane the ROC of $z$-transform $X(z)$ consists of a

(A) strip

(B) parabola

(C) rectangle

(D) ring

MCQ 6.3.8 If $x[n]$ is a right-sided sequence, and if the circle $z = r_0$ is in the ROC, then

(A) the values of $z$ for which $z > r_0$ will also be in the ROC

(B) the values of $z$ for which $z < r_0$ will also be in the ROC

(C) both (A) & (B)

(D) none of these

MCQ 6.3.9 The ROC does not contain any

(A) poles

(B) 1's

(C) zeros

(D) none

MCQ 6.3.10 Let $x[n] \xleftrightarrow{z} X(z)$ be a $z$-transform pair. If $x[n] = \delta[n]$, then the ROC of $X(z)$ is

(A) $|z| < 1$ (B) $|z| > 1$

(C) entire $z$-plane (D) none of the above

MCQ 6.3.11 The ROC of $z$-transform of unit-step sequence $u[n]$, is

(A) entire $z$-plane (B) $|z| < 1$

(C) $|z| > 1$ (D) none of the above

MCQ 6.3.12 The ROC of the unilateral $z$-transform of $\alpha^n$ is

(A) $|z| > |\alpha|$

(B) $|z| < |\alpha|$

(C) $|z| < 1$

(D) $|z| > 1$

MCQ 6.3.13 Which of the following statement about ROC is not true ?

(A) ROC never lies exactly at the boundary of a circle

(B) ROC consists of a circle in the $z$-plane centred at the origin

(C) ROC of a right handed finite sequence is the entire $z$-plane except $z = 0$

(D) ROC contains both poles and zeroes

MCQ 6.3.14 The $z$-transform of unit step sequence is

(A) 1 (B) $\frac{1}{z-1}$

(C) $\frac{z}{z-1}$ (D) 0

MCQ 6.3.15 The ROC for the $z$-transform of the sequence $x[n] = u[-n]$ is

(A) $|z| > 0$ (B) $|z| < 1$

(C) $|z| > 1$ (D) does not exist

MCQ 6.3.16 Let $x[n] \xleftrightarrow{z} X(z)$, then unilateral $z$-transform of sequence $x_1[n] = x[n-1]$ will be

(A) $X_1(z) = z^{-1}X(z) + x[0]$

(B) $X_1(z) = z^{-1}X(z) - x[1]$

(C) $X_1(z) = z^{-1}X(z) - x[-1]$

(D) $X_1(z) = z^{-1}X[z] + x[-1]$

MCQ 6.3.17 Let $x[n] \xleftrightarrow{z} X(z)$, the bilateral $z$-transform of $x[n-n_0]$ is given by

(A) $zX(z)$ (B) $z^{n_0}X(z)$

(C) $z^{-n_0}X(z)$ (D) $\frac{1}{z}X(z)$

MCQ 6.3.18 If the ROC of $z$-transform of $x[n]$ is $R_x$ then the ROC of $z$-transform of $x[-n]$ is

(A) $R_x$

(B) $-R_x$

(C) $1/R_x$

(D) none of these

MCQ 6.3.19 If $X(z) = \mathcal{Z}\{x[n]\}$, then $X(z) = \mathcal{Z}\{a^{-n}x[n]\}$ will be

(A) $X(az)$ (B) $X\left(\frac{z}{a}\right)$

(C) $X\left(\frac{a}{z}\right)$ (D) $X\left(\frac{1}{az}\right)$

MCQ 6.3.20 If $x[n]$ and $y[n]$ are two discrete time sequences, then the $z$-transform of correlation of the sequences $x[n]$ and $y[n]$ is

(A) $X(z^{-1})\,Y(z^{-1})$ (B) $X(z)\,Y(z^{-1})$

(C) $X(z)*Y(z)$ (D) $X^*(z)\,Y^*(z^{-1})$

MCQ 6.3.21 If $X(z)=\mathcal{Z}\{x[n]\}$, then, value of $x[0]$ is equal to

(A) $\lim_{z\to 0} zX(z)$ (B) $\lim_{z\to 1}(z-1)X(z)$

(C) $\lim_{z\to\infty} X(z)$ (D) $\lim_{z\to 0} X(z)$

MCQ 6.3.22 The choice of realization of structure depends on

(A) computational complexity

(B) memory requirements

(C) parallel processing and pipelining

(D) all the above

MCQ 6.3.23 Which of the following schemes of system realization uses separate delays for input and output samples ?

(A) parallel form (B) cascade form

(C) direct form-I (D) direct form-II

MCQ 6.3.24 The direct form-I and II structures of IIR system will be identical in

(A) all pole system

(B) all zero system

(C) both (A) and (B)

(D) first order and second order systems

MCQ 6.3.25 The number of memory locations required to realize the system,

$$H(z)=\frac{1+3z^{-2}+2z^{-3}}{1+2z^{-2}+z^{-4}}$$ is

(A) 5 (B) 7

(C) 2 (D) 10

MCQ 6.3.26 The mapping $z=e^{sT}$ from $s$-plane to $z$-plane, is

(A) one to one (B) many to one

(C) one to many (D) many to many

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# EXERCISE 6.4



MCQ 6.4.1
IES EC 2007

What is the $z$-transform of the signal $x[n] = \alpha^n u[n]$ ?

(A) $X(z) = \dfrac{1}{z-1}$ (B) $X(z) = \dfrac{1}{1-z}$

(C) $X(z) = \dfrac{z}{z-\alpha}$ (D) $X(z) = \dfrac{1}{z-\alpha}$

MCQ 6.4.2
GATE EC 1998

The $z$-transform of the time function $\sum_{k=0}^{\infty}\delta[n-k]$ is

(A) $\dfrac{z-1}{z}$ (B) $\dfrac{z}{z-1}$

(C) $\dfrac{z}{(z-1)^2}$ (D) $\dfrac{(z-1)^2}{z}$

MCQ 6.4.3
GATE EC 1999

The $z$-transform $F(z)$ of the function $f(nT) = a^{nT}$ is

(A) $\dfrac{z}{z-a^T}$ (B) $\dfrac{z}{z+a^T}$

(C) $\dfrac{z}{z-a^{-T}}$ (D) $\dfrac{z}{z+a^{-T}}$

MCQ 6.4.4
GATE EE 2006

The discrete-time signal $x[n] \xleftrightarrow{\mathcal{Z}} X(z) = \sum_{n=0}^{\infty} \frac{3^n}{2+n} z^{2n}$, where $\longleftrightarrow$ denotes a transform-pair relationship, is orthogonal to the signal

(A) $y_1[n] \leftrightarrow Y_1(z) = \sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n z^{-n}$ ( B )
$y_2[n] \leftrightarrow Y_2(z) = \sum_{n=0}^{\infty}(5^n - n) z^{-(2n+1)}$

(C) $y_3[n] \leftrightarrow Y_3(z) = \sum_{n=-\infty}^{\infty} 2^{-|n|} z^{-n}$ (D) $y_4[n] \leftrightarrow Y_4(z) = 2z^{-4} + 3z^{-2} + 1$

MCQ 6.4.5
IES E & T 1994

Which one of the following is the region of convergence (ROC) for the sequence $x[n] = b^n u[n] + b^{-n} u[-n-1]$; $|b| < 1$ ?

(A) Region $|z| < 1$

(B) Annular strip in the region $b > |z| > \dfrac{1}{b}$

(C) Region $|z| > 1$

(D) Annular strip in the region $b < |z| < \dfrac{1}{b}$

MCQ 6.4.6
IES EC 2002

Assertion (A) : The signals $a^n u[n]$ and $-a^n u[-n-1]$ have the same $z$-transform, $z/(z-a)$.

Reason (R) : The Reason of Convergence (ROC) for $a^n u[n]$ is $|z| > |a|$, whereas the ROC for $a^n u[-n-1]$ is $|z| < |a|$.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

MCQ 6.4.7
IES EC 2006

Which one of the following is the correct statement ?

The region of convergence of $z$-transform of $x[n]$ consists of the values of $z$ for which $x[n]r^{-n}$ is

(A) absolutely integrable (B) absolutely summable

(C) unity (D) $< 1$

MCQ 6.4.8
GATE EC 2009

The ROC of $z$-transform of the sequence $x[n] = \left(\frac{1}{3}\right)^n u[n] - \left(\frac{1}{2}\right)^n u[-n-1]$ is

(A) $\frac{1}{3} < |z| < \frac{1}{2}$ (B) $|z| > \frac{1}{2}$

(C) $|z| < \frac{1}{3}$ (D) $2 < |z| < 3$

MCQ 6.4.9
GATE EC 2005

The region of convergence of $z-$ transform of the sequence $\left(\frac{5}{6}\right)^n u[n] - \left(\frac{6}{5}\right)^n u[-n-1]$ must be

(A) $|z| < \frac{5}{6}$ (B) $|z| > \frac{5}{6}$

(C) $\frac{5}{6} < |z| < \frac{6}{5}$ (D) $\frac{6}{5} < |z| < \infty$

MCQ 6.4.10
GATE IN 2008

The region of convergence of the $z$-transform of the discrete-time signal $x[n] = 2^n u[n]$ will be

(A) $|z| > 2$ (B) $|z| < 2$

(C) $|z| > \frac{1}{2}$ (D) $|z| < \frac{1}{2}$

MCQ 6.4.11
GATE EC 2001

The region of convergence of the $z-$ transform of a unit step function is

(A) $|z| > 1$ (B) $|z| < 1$

(C) (Real part of $z$) $> 0$ (D) (Real part of $z$) $< 0$

MCQ 6.4.12
IES EC 2005

Match List I (Discrete Time signal) with List II (Transform) and select the correct answer using the codes given below the lists :

| | List I | | List II |
|---|---|---|---|
| A. | Unit step function | 1. | 1 |
| B. | Unit impulse function | 2. | $\frac{z-\cos\omega T}{z^2 - 2z\cos\omega T + 1}$ |
| C. | $\sin\omega t,\ t = 0, T, 2T$ | 3. | $\frac{z}{z-1}$ |
| D. | $\cos\omega t,\ t = 0, T, 2T, .....$ | 4. | $\frac{z\sin\omega T}{z^2 - 2z\cos\omega T + 1}$ |

Codes :

| | A | B | C | D |
|---|---|---|---|---|
| (A) | 2 | 4 | 1 | 3 |
| (B) | 3 | 1 | 4 | 2 |
| (C) | 2 | 1 | 4 | 3 |
| (D) | 3 | 4 | 1 | 2 |

MCQ 6.4.13
IES EC 2006

What is the inverse $z$-transform of $X(z)$

(A) $\frac{1}{2\pi j}\oint X(z) z^{n-1} dz$ (B) $2\pi j \oint X(z) z^{n+1} dz$

(C) $\frac{1}{2\pi j}\oint X(z) z^{1-n} dz$ (D) $2\pi j \oint X(z) z^{-(n+1)} dz$

MCQ 6.4.14
IES E & T 1997

Which one of the following represents the impulse response of a system defined by $H(z) = z^{-m}$ ?

(A) $u[n-m]$ (B) $\delta[n-m]$

(C) $\delta[m]$ (D) $\delta[m-n]$

MCQ 6.4.15
IES EC 2008

If $X(z)$ is $\frac{1}{|1-z^{-1}|}$ with $|z|>1$, then what is the corresponding $x[n]$ ?

(A) $e^{-n}$ (B) $e^{n}$

(C) $u[n]$ (D) $\delta(n)$

MCQ 6.4.16
GATE EC 2007

The $z$ –transform $X(z)$ of a sequence $x[n]$ is given by $X[z]=\frac{0.5}{1-2z^{-1}}$. It is given that the region of convergence of $X(z)$ includes the unit circle. The value of $x[0]$ is

(A) $-0.5$ (B) 0

(C) 0.25 (D) 05

MCQ 6.4.17
GATE EE 2005

If $u(t)$ is the unit step and $\delta(t)$ is the unit impulse function, the inverse $z$ -transform of $F(z)=\frac{1}{z+1}$ for $k>0$ is

(A) $(-1)^k\delta(k)$ (B) $\delta(k)-(-1)^k$

(C) $(-1)^k u(k)$ (D) $u(k)-(-1)^k$

MCQ 6.4.18
IES EC 2002

For a $z$-transform $X(z)=\frac{(2z-\frac{5}{6})}{(z-\frac{1}{2})(z-\frac{1}{3})}$

Match List I (The sequences) with List II (The region of convergence ) and select the correct answer using the codes given below the lists :

| | List I | | List II |
|---|---|---|---|
| A. | $[(1/2)^n+(1/3)^n]u[n]$ | 1. | $(1/3)<\lvert z\rvert<(1/2)$ |
| B. | $(1/2)^n u[n]-(1/3)^n u[-n-1]$ | 2. | $\lvert z\rvert<(1/3)$ |
| C. | $-(1/2)^n u[-n-1]+(1/3)^n u[n]$ | 3. | $\lvert z\rvert<1/3\ \&\ \lvert z\rvert>1/2$ |
| D. | $-[(1/2)^n+(1/3)^n]u[-n-1]$ | 4. | $\lvert z\rvert>1/2$ |

Codes :

| | A | B | C | D |
|---|---|---|---|---|
| (A) | 4 | 2 | 1 | 3 |
| (B) | 1 | 3 | 4 | 2 |
| (C) | 4 | 3 | 1 | 2 |
| (D) | 1 | 2 | 4 | 3 |

MCQ 6.4.19
IES EC 2005

Which one of the following is the inverse $z$-transform of

$$X(z)=\frac{z}{(z-2)(z-3)},\quad |z|<2\ ?$$

(A) $[2^n-3^n]u[-n-1]$ (B) $[3^n-2^n]u[-n-1]$

(C) $[2^n-3^n]u[n+1]$ (D) $[2^n-3^n]u[n]$

MCQ 6.4.20
GATE EE 2008

Given $X(z)=\frac{z}{(z-a)^2}$ with $|z|>a$, the residue of $X(z)z^{n-1}$ at $z=a$ for $n\geq 0$ will be

(A) $a^{n-1}$ (B) $a^n$

(C) $na^n$ (D) $na^{n-1}$

MCQ 6.4.21
GATE IN 2004

Given $X(z)=\frac{\frac{1}{2}}{1-az^{-1}}+\frac{\frac{1}{3}}{1-bz^{-1}}$, $|a|$ and $|b|<1$ with the ROC specified as $|a|<|z|<|b|$, then $x[0]$ of the corresponding sequence is given by

(A) $\frac{1}{3}$ (B) $\frac{5}{6}$

(C) $\frac{1}{2}$ (D) $\frac{1}{6}$

MCQ 6.4.22
IES EC 2002

If $X(z) = \frac{z + z^{-3}}{z + z^{-1}}$ then $x[n]$ series has

(A) alternate 0's (B) alternate 1's

(C) alternate 2's (D) alternate $-1$'s

MCQ 6.4.23
GATE EC 2010

Consider the $z$-transform $x(z) = 5z^2 + 4z^{-1} + 3;\ 0 < |z| < \infty$. The inverse $z$ - transform $x[n]$ is

(A) $5\delta[n+2] + 3\delta[n] + 4\delta[n-1]$ (B) $5\delta[n-2] + 3\delta[n] + 4\delta[n+1]$

(C) $5u[n+2] + 3u[n] + 4u[n-1]$ (D) $5u[n-2] + 3u[n] + 4u[n+1]$

MCQ 6.4.24
GATE IN 2003

The sequence $x[n]$ whose $z$-transform is $X(z) = e^{1/z}$ is

(A) $\frac{1}{n!}u[n]$ (B) $\frac{1}{-n!}u[-n]$

(C) $(-1)^n \frac{1}{n!}u[n]$ (D) $\frac{1}{-(n+1)!}u[-n-1]$

MCQ 6.4.25
GATE EC 2006

If the region of convergence of $x_1[n] + x_2[n]$ is $\frac{1}{3} < |z| < \frac{2}{3}$ then the region of convergence of $x_1[n] - x_2[n]$ includes

(A) $\frac{1}{3} < |z| < 3$ (B) $\frac{2}{3} < |z| < 3$

(C) $\frac{3}{2} < |z| < 3$ (D) $\frac{1}{3} < |z| < \frac{2}{3}$

MCQ 6.4.26
IES EC 2000

Match List I with List II and select the correct answer using the codes given below the lists :

| List I | | List II | |
|---|---|---|---|
| A. | $\alpha^n u[n]$ | 1. | $\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$, ROC : $\lvert z \rvert > \lvert \alpha \rvert$ |
| B. | $-\alpha^n u[-n-1]$ | 2. | $\frac{1}{(1-\alpha z^{-1})}$, ROC : $\lvert z \rvert > \lvert \alpha \rvert$ |
| C. | $-n\alpha^n u[-n-1]$ | 3. | $\frac{1}{(1-\alpha z^{-1})}$, ROC : $\lvert z \rvert < \lvert \alpha$ |
| D. | $n\alpha^n u[n]$ | 4. | $\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$, ROC : $\lvert z \rvert < \lvert \alpha$ |

Codes :

| | A | B | C | D |
|---|---|---|---|---|
| (A) | 2 | 4 | 3 | 1 |
| (B) | 1 | 3 | 4 | 2 |
| (C) | 1 | 4 | 3 | 2 |
| (D) | 2 | 3 | 4 | 1 |

MCQ 6.4.27
IES EC 2007

Algebraic expression for $z$-transform of $x[n]$ is $X[z]$. What is the algebraic expression for $z$-transform of $\{e^{j\omega_0 n}x[n]\}$ ?

(A) $X(z - z_0)$ (B) $X(e^{-j\omega_0}z)$

(C) $X(e^{j\omega_0}z)$ (D) $X(z)e^{j\omega_0 z}$

MCQ 6.4.28
IES E & T 1997

Given that $F(z)$ and $G(z)$ are the one-sided $z$-transforms of discrete time functions $f(nT)$ and $g(nT)$, the $z$-transform of $\sum f(kT)g(nT - kT)$ is given by

(A) $\sum f(nT)g(nT)z^{-n}$ (B) $\sum f(nT)z^{-n} \sum g(nT)z^{-n}$

(C) $\sum f(kT)g(nT-kT)z^{-n}$ (D) $\sum f(nT-kT)g(nT)z^{-n}$

MCQ 6.4.29
IES E & T 1997

Match List-I ($x[n]$) with List-II ($X(z)$) and select the correct answer using the codes given below the Lists:

| | List-I | | List-II |
|---|---|---|---|
| A. | $a^n u[n]$ | 1. | $\frac{az}{(z-a)^2}$ |
| B. | $a^{n-2}u[n-2]$ | 2. | $\frac{ze^{-j}}{ze^{-j}-a}$ |
| C. | $e^{jn}a^n$ | 3. | $\frac{z}{z-a}$ |
| D. | $na^n u[n]$ | 4. | $\frac{z^{-1}}{z-a}$ |

Codes :

| | A | B | C | D |
|---|---|---|---|---|
| (A) | 3 | 2 | 4 | 1 |
| (B) | 2 | 3 | 4 | 1 |
| (C) | 3 | 4 | 2 | 1 |
| (D) | 1 | 4 | 2 | 3 |

MCQ 6.4.30
IES EC 2005

The output $y[n]$ of a discrete time LTI system is related to the input $x[n]$ as given below :

$$y[n] = \sum_{k=0}^{\infty} x[k]$$

Which one of the following correctly relates the $z$-transform of the input and output, denoted by $X(z)$ and $Y(z)$, respectively ?

(A) $Y(z) = (1-z^{-1})X(z)$
(B) $Y(z) = z^{-1}X(z)$
(C) $Y(z) = \frac{X(z)}{1-z^{-1}}$
(D) $Y(z) = \frac{dX(z)}{dz}$

MCQ 6.4.31
IES EC 2010

Convolution of two sequence $x_1[n]$ and $x_2[n]$ is represented as

(A) $X_1(z) * X_2(z)$
(B) $X_1(z)X_2(z)$
(C) $X_1(z) + X_2(z)$
(D) $X_1(z)/X_2(z)$

MCQ 6.4.32
GATE EC 1999

The $z$-transform of a signal is given by $C(z) = \frac{1z^{-1}(1-z^{-4})}{4(1-z^{-1})^2}$. Its final value is

(A) 1/4
(B) zero
(C) 1.0
(D) infinity

MCQ 6.4.33
IES E & T 1996

Consider a system described by the following difference equation:

$y(n+3) + 6y(n+2) + 11y(n+1) + 6y(n) = r(n+2) + 9r(n+1) + 20r(n)$

where $y$ is the output and $r$ is the input. The transfer function of the system is

(A) $\frac{2z^2+z+20}{3z^3+2z^2+z+6}$
(B) $\frac{z^2+9z+20}{z^3+6z^2+6z+11}$
(C) $\frac{z^3+6z^2+6z+11}{z^2+9z+20}$
(D) none of the above

MCQ 6.4.34
IES E & T 1998

If the function $H_1(z) = (1+1.5z^{-1}-z^{-2})$ and $H_2(z) = z^2+1.5z-1$, then

(A) the poles and zeros of the functions will be the same
(B) the poles of the functions will be identical but not zeros
(C) the zeros of the functions will be identical but not the poles
(D) neither the poles nor the zeros of the two functions will be identical

MCQ 6.4.35
IES EC 1999

The state model

$$x[k+1] = \begin{bmatrix} 0 & 1 \\ -\beta & -\alpha \end{bmatrix} x[k] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u[k]$$

$$y[k] = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix}$$

is represented in the difference equation as

(A) $c[k+2] + \alpha c[k+1] + \beta c[k] = u[k]$

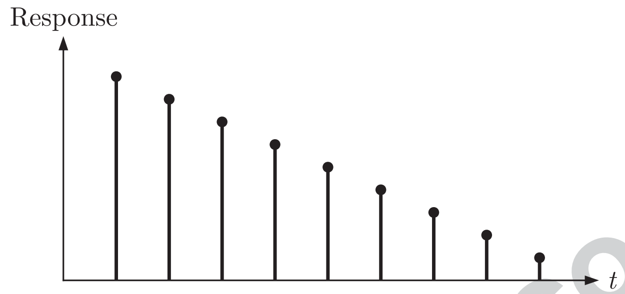
(B) $c[k+1] + \alpha c[k] + \beta c[k-1] = u[k-1]$

(C) $c[k-2] + \alpha c[k-1] + \beta c[k] = u[k]$

(D) $c[k-1] + \alpha c[k] + \beta c[k+1] = u[k+1]$

MCQ 6.4.36
IES EC 2000

The impulse response of a discrete system with a simple pole shown in the figure below. The pole of the system must be located on the




(A) real axis at $z = -1$

(B) real axis between $z = 0$ and $z = 1$

(C) imaginary axis at $z = j$

(D) imaginary axis between $z = 0$ and $z = j$

MCQ 6.4.37
IES EC 2001

Which one of the following digital filters does have a linear phase response ?

(A) $y[n] + y[n-1] = x[n] - x[n-1]$

(B) $y[n] = 1/6(3x[n] + 2x[n-1] + x[n-2])$

(C) $y[n] = 1/6(x[n] + 2x[n-1] + 3x[n-2])$

(D) $y[n] = 1/4(x[n] + 2x[n-1] + x[n-2])$

MCQ 6.4.38
IES EC 2001

The poles of a digital filter with linear phase response can lie

(A) only at $z = 0$

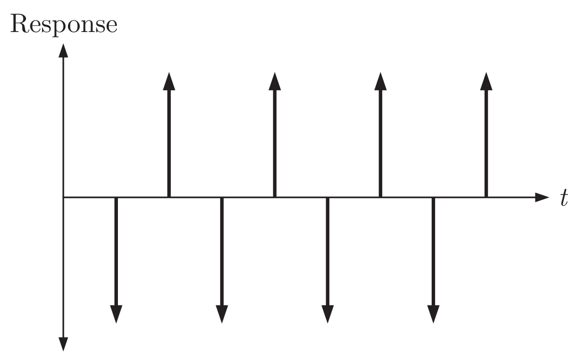
(B) only on the unit circle

(C) only inside the unit circle but not at $z = 0$

(D) on the left side of $\text{Real}(z) = 0$ line

MCQ 6.4.39
IES EC 2001

The impulse response of a discrete system with a simple pole is shown in the given figure



The pole must be located

(A) on the real axis at $z = 1$ (B) on the real axis at $z = -1$

(C) at the origin of the z-plane (D) at $z = \infty$

MCQ 6.4.40
IES EC 2002

The response of a linear, time-invariant discrete-time system to a unit step input $u[n]$ is the unit impulse $\delta[n]$. The system response to a ramp input $nu[n]$ would be

(A) $u[n]$ (B) $u[n-1]$

(C) $n\delta[n]$ (D) $\sum_{k=0}^{\infty} k\delta[n-k]$

MCQ 6.4.41
IES EC 2002

A system can be represented in the form of state equations as

$$s[n+1] = As[n] + Bx[n]$$
$$y[n] = Cs[n] + Dx[n]$$

where $A, B, C$ and $D$ are matrices, $s[n]$ is the state vector. $x[n]$ is the input and $y[n]$ is the output. The transfer function of the system $H(z) = Y(z)/X(z)$ is given by

(A) $A(zI - B)^{-1}C + D$ (B) $B(zI - C)^{-1}D + A$

(C) $C(zI - A)^{-1}B + D$ (D) $D(zI - A)^{-1}C + B$

MCQ 6.4.42
IES EC 2004

What is the number of roots of the polynomial $F(z) = 4z^3 - 8z^2 - z + 2$, lying outside the unit circle ?

(A) 0 (B) 1

(C) 2 (D) 3

MCQ 6.4.43
IES EC 2004

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

Which one of the following systems is inverse of the system given above ?

(A) $x[n] = y[n] - y[n-1]$ (B) $x[n] = y[n]$

(C) $x[n] = y[n+4]$ (D) $x[n] = ny[n]$

MCQ 6.4.44
IES EC 2006

For the system shown, $x[n] = k\delta[n]$, and $y[n]$ is related to $x[n]$ as $y[n] - \frac{1}{2}y[n-1] = x[n]$


What is $y[n]$ equal to ?

(A) k (B) $(1/2)^n k$

(C) $nk$ (D) $2^n$

MCQ 6.4.45
IES EC 2010

Unit step response of the system described by the equation $y[n] + y[n-1] = x[n]$ is

(A) $\frac{z^2}{(z+1)(z-1)}$ (B) $\frac{z}{(z+1)(z-1)}$

(C) $\frac{z+1}{z-1}$ (D) $\frac{z(z-1)}{(z+1)}$

MCQ 6.4.46
IES EC 2011

Unit step response of the system described by the equation $y[n] + y[n-1] = x[n]$ is

(A) $\frac{z^2}{(z+1)(z-1)}$ (B) $\frac{z}{(z+1)(z-1)}$

(C) $\frac{(z+1)}{(z-1)}$ (D) $\frac{z(z-1)}{(z+1)}$

MCQ 6.4.47
IES EC 2011

System transformation function $H(z)$ for a discrete time LTI system expressed in state variable form with zero initial conditions is

(A) $c(zI-A)^{-1}b+d$ (B) $c(zI-A)^{-1}$

(C) $(zI-A)^{-1}z$ (D) $(zI-A)^{-1}$

MCQ 6.4.48
GATE EC 2009

A system with transfer function $H(z)$ has impulse response $h(.)$ defined as $h(2)=1, h(3)=-1$ and $h(k)=0$ otherwise. Consider the following statements.

S1 : $H(z)$ is a low-pass filter.

S2 : $H(z)$ is an FIR filter.

Which of the following is correct?

(A) Only S2 is true

(B) Both S1 and S2 are false

(C) Both S1 and S2 are true, and S2 is a reason for S1

(D) Both S1 and S2 are true, but S2 is not a reason for S1

MCQ 6.4.49
GATE EC 2004

The $z$-transform of a system is $H(z)=\frac{z}{z-0.2}$. If the ROC is $|z|<0.2$, then the impulse response of the system is

(A) $(0.2)^n u[n]$

(B) $(0.2)^n u[-n-1]$

(C) $-(0.2)^n u[n]$

(D) $-(0.2)^n u[-n-1]$

MCQ 6.4.50
GATE EC 2003

A sequence $x(n)$ with the $z$-transform $X(z)=z^4+z^2-2z+2-3z^{-4}$ is applied as an input to a linear, time-invariant system with the impulse response $h[n]=2\delta[n-3]$ where

$$\delta[n]=\begin{cases}1, & n=0\\ 0, & \text{otherwise}\end{cases}$$

The output at $n=4$ is

(A) $-6$ (B) zero

(C) 2 (D) $-4$

MCQ 6.4.51
GATE EE 2009

The z-transform of a signal $x[n]$ is given by $4z^{-3}+3z^{-1}+2-6z^2+2z^3$

It is applied to a system, with a transfer function $H(z)=3z^{-1}-2$

Let the output be $y[n]$. Which of the following is true ?

(A) $y[n]$ is non causal with finite support

(B) $y[n]$ is causal with infinite support

(C) $y[n]=0; |n|>3$

(D) $\mathrm{Re}[Y(z)]_{z=e^{j\theta}}=-\mathrm{Re}[Y(z)]_{z=e^{-j\theta}}$

$\mathrm{Im}[Y(z)]_{z=e^{j\theta}}=\mathrm{Im}[Y(z)]_{z=e^{-j\theta}}; \quad -\pi\le\theta<\pi$

MCQ 6.4.52
GATE EE 2008

$H(z)$ is a transfer function of a real system. When a signal $x[n]=(1+j)^n$ is the input to such a system, the output is zero. Further, the Region of convergence (ROC) of $(1-\frac{1}{2}z^{-1})$ H(z) is the entire Z-plane (except $z=0$). It can then be

inferred that $H(z)$ can have a minimum of

(A) one pole and one zero

(B) one pole and two zeros

(C) two poles and one zero

(D) two poles and two zeros

MCQ 6.4.53
GATE IN 2004

A discrete-time signal, $x[n]$, suffered a distortion modeled by an LTI system with $H(z) = (1 - az^{-1})$, a is real and $|a| > 1$. The impulse response of a stable system that exactly compensates the magnitude of the distortion is

(A) $\left(\frac{1}{a}\right)^n u[n]$

(B) $-\left(\frac{1}{a}\right)^n u[-n-1]$

(C) $a^n u[n]$

(D) $a^n u[-n-1]$

MCQ 6.4.54
IES E & T 1998

Assertion (A) : A linear time-invariant discrete-time system having the system function

$$H(z) = \frac{z}{z + \frac{1}{2}} \text{ is a stable system.}$$

Reason (R) : The pole of $H(z)$ is in the left-half plane for a stable system.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT a correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

MCQ 6.4.55
IES EC 1999

Assertion (A) : An LTI discrete system represented by the difference equation $y[n+2] - 5y[n+1] + 6y[n] = x[n]$ is unstable.
Reason (R) : A system is unstable if the roots of the characteristic equation lie outside the unit circle.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

MCQ 6.4.56
IES EC 1999

Consider the following statements regarding a linear discrete-time system

$$H(z) = \frac{z^2 + 1}{(z + 0.5)(z - 0.5)}$$

1. The system is stable
2. The initial value $h(0)$ of the impulse response is $-4$
3. The steady-state output is zero for a sinusoidal discrete time input of frequency equal to one-fourth the sampling frequency.

Which of these statements are correct ?

(A) 1, 2 and 3

(B) 1 and 2

(C) 1 and 3

(D) 2 and 3

MCQ 6.4.57
IES EC 2005

Assertion (A) : The discrete time system described by $y[n] = 2x[n] + 4x[n-1]$ is unstable, (here $y[n]$ is the output and $x[n]$ the input)
Reason (R) : It has an impulse response with a finite number of non-zero samples.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

MCQ 6.4.58

GATE EC 2002

If the impulse response of discrete - time system is $h[n] = -5^n u[-n-1]$, then the system function $H(z)$ is equal to

(A) $\frac{-z}{z-5}$ and the system is stable

(B) $\frac{z}{z-5}$ and the system is stable

(C) $\frac{-z}{z-5}$ and the system is unstable

(D) $\frac{z}{z-5}$ and the system is unstable

MCQ 6.4.59

GATE IN 2010

$H(z)$ is a discrete rational transfer function. To ensure that both $H(z)$ and its inverse are stable its

(A) poles must be inside the unit circle and zeros must be outside the unit circle.

(B) poles and zeros must be inside the unit circle.

(C) poles and zeros must be outside the unit circle

(D) poles must be outside the unit circle and zeros should be inside the unit circle

MCQ 6.4.60

IES EC 2002

**Assertion (A)** : The stability of the system is assured if the Region of Convergence (ROC) includes the unit circle in the $z$-plane.

**Reason (R)** : For a causal stable system all the poles should be outside the unit circle in the $z$-plane.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false

(D) A is false but R is true

MCQ 6.4.61

IES EC 2002

**Assertion (A)** : For a rational transfer function $H(z)$ to be causal, stable and causally invertible, both the zeros and the poles should lie within the unit circle in the $z$-plane.

**Reason (R)** : For a rational system, ROC bounded by poles.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true



MCQ 6.4.62

GATE EC 2010

The transfer function of a discrete time LTI system is $H(z) = \dfrac{2 - \frac{3}{4}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$

Consider the following statements:

S1: The system is stable and causal for ROC: $|z| > 1/2$

S2: The system is stable but not causal for ROC: $|z| < 1/4$

S3: The system is neither stable nor causal for ROC: $1/4 < |z| < 1/2$

Which one of the following statements is valid ?

(A) Both S1 and S2 are true (B) Both S2 and S3 are true

(C) Both S1 and S3 are true (D) S1, S2 and S3 are all true

MCQ 6.4.63

GATE EC 2004

A causal LTI system is described by the difference equation

$$2y[n] = \alpha y[n-2] - 2x[n] + \beta x[n-1]$$

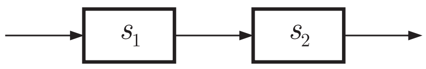
The system is stable only if

(A) $|\alpha| = 2$, $|\beta| < 2$ (B) $|\alpha| > 2$, $|\beta| > 2$

(C) $|\alpha| < 2$, any value of $\beta$ (D) $|\beta| < 2$, any value of $\alpha$

MCQ 6.4.64
IES EC 2000

Two linear time-invariant discrete time systems $s_1$ and $s_2$ are cascaded as shown in the figure below. Each system is modelled by a second order difference equation. The difference equation of the overall cascaded system can be of the order of

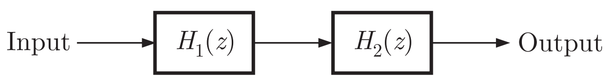



(A) 0, 1, 2, 3 or 4
(B) either 2 or 4
(C) 2
(D) 4

MCQ 6.4.65
IES EC 2000

Consider the compound system shown in the figure below. Its output is equal to the input with a delay of two units. If the transfer function of the first system is given by

$$H_1(z) = \frac{z - 0.5}{z - 0.8},$$



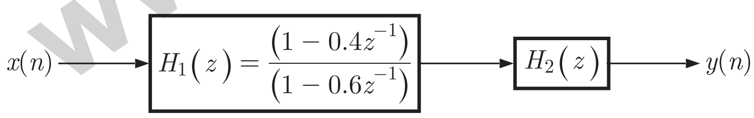

then the transfer function of the second system would be

(A) $H_2(z) = \dfrac{z^{-2} - 0.2z^{-3}}{1 - 0.4z^{-1}}$ (B) $H_2(z) = \dfrac{z^{-2} - 0.8z^{-3}}{1 - 0.5z^{-1}}$

(C) $H_2(z) = \dfrac{z^{-1} - 0.2z^{-3}}{1 - 0.4z^{-1}}$ (D) $H_2(z) = \dfrac{z^{-2} + 0.8z^{-3}}{1 + 0.5z^{-1}}$

MCQ 6.4.66
GATE EC 2011

Two systems $H_1(z)$ and $H_2(z)$ are connected in cascade as shown below. The overall output $y[n]$ is the same as the input $x[n]$ with a one unit delay. The transfer function of the second system $H_2(z)$ is




(A) $\dfrac{1 - 0.6z^{-1}}{z^{-1}(1 - 0.4z^{-1})}$ (B) $\dfrac{z^{-1}(1 - 0.6z^{-1})}{(1 - 0.4z^{-1})}$

(C) $\dfrac{z^{-1}(1 - 0.4z^{-1})}{(1 - 0.6z^{-1})}$ (D) $\dfrac{1 - 0.4\,z^{-1}}{z^{-1}(1 - 0.6z^{-1})}$

MCQ 6.4.67
GATE EC 2010

Two discrete time system with impulse response $h_1[n] = \delta[n-1]$ and $h_2[n] = \delta[n-2]$ are connected in cascade. The overall impulse response of the cascaded system is

(A) $\delta[n-1] + \delta[n-2]$ (B) $\delta[n-4]$
(C) $\delta[n-3]$ (D) $\delta[n-1]\delta[n-2]$

MCQ 6.4.68
GATE EE 2009

A cascade of three Linear Time Invariant systems is causal and unstable. From this, we conclude that

(A) each system in the cascade is individually causal and unstable
(B) at least on system is unstable and at least one system is causal
(C) at least one system is causal and all systems are unstable
(D) the majority are unstable and the majority are causal

MCQ 6.4.69
IES EC 2001

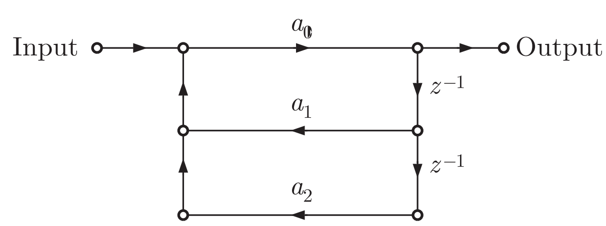
The minimum number of delay elements required in realizing a digital filter with the transfer function

$$H(z) = \frac{1 + az^{-1} + bz^{-2}}{1 + cz^{-1} + dz^{-2} + ez^{-3}}$$

(A) 2 (B) 3
(C) 4 (D) 5

MCQ 6.4.70
GATE IN 2004

A direct form implementation of an LTI system with $H(z) = \frac{1}{1 - 0.7z^{-1} + 0.13z^{-2}}$ is shown in figure. The value of $a_0$, $a_1$ and $a_2$ are respectively




(A) 1.0, 0.7 and $-0.13$ (B) $-0.13, 0.7$ and 1.0
(C) 1.0, $-$ 0.7 and 0.13 (D) 0.13, $-$ 0.7 and 1.0

MCQ 6.4.71
GATE IN 2010

A digital filter having a transfer function $H(z) = \frac{p_0 + p_1 z^{-1} + p_3 z^{-3}}{1 + d_3 z^{-3}}$ is implemented

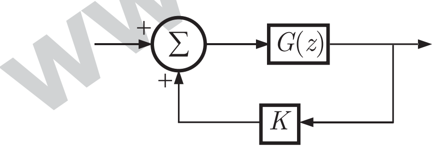
using Direct Form-I and Direct Form-II realizations of IIR structure. The number of delay units required in Direct Form-I and Direct Form-II realizations are, respectively

(A) 6 and 6 (B) 6 and 3
(C) 3 and 3 (D) 3 and 2



MCQ 6.4.72
GATE EE 2007

Consider the discrete-time system shown in the figure where the impulse response of $G(z)$ is $g(0) = 0, g(1) = g(2) = 1, g(3) = g(4) = \cdots = 0$

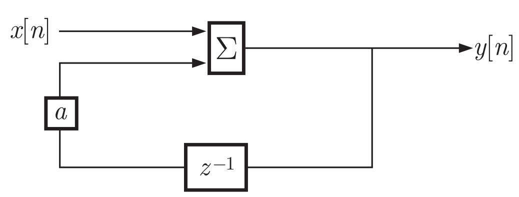



This system is stable for range of values of $K$

(A) $[-1, \frac{1}{2}]$ (B) $[-1, 1]$
(C) $[-\frac{1}{2}, 1]$ (D) $[-\frac{1}{2}, 2]$

MCQ 6.4.73
GATE IN 2006

In the IIR filter shown below, $a$ is a variable gain. For which of the following cases, the system will transit from stable to unstable condition ?




(A) $0.1 < a < 0.5$
(B) $0.5 < a < 1.5$
(C) $1.5 < a < 2.5$
(D) $2 < a < \infty$

MCQ 6.4.74
IES EC 2002

The poles of an analog system are related to the corresponding poles of the digital system by the relation $z = e^{sT}$. Consider the following statements.

1. Analog system poles in the left half of $s$-plane map onto digital system poles inside the circle $|z| = 1$.
2. Analog system zeros in the left half of $s$-plane map onto digital system zeros inside the circle $|z| = 1$.
3. Analog system poles on the imaginary axis of $s$-plane map onto digital system zeros on the unit circle $|z| = 1$.
4. Analog system zeros on the imaginary axis of $s$-plane map onto digital system zeros on the unit circle $|z| = 1$.

Which of these statements are correct ?

(A) 1 and 2 (B) 1 and 3
(C) 3 and 4 (D) 2 and 4

MCQ 6.4.75
IES EC 2007

Which one of the following rules determines the mapping of $s$-plane to $z$ -plane ?

(A) Right half of the $s$-plane maps into outside of the unit circle in $z$-plane
(B) Left half of the $s$-plane maps into inside of the unit circle
(C) Imaginary axis in $s$-plane maps into the circumference of the unit circle
(D) All of the above

MCQ 6.4.76
IES EC 2008

Assertion (A) : The $z$-transform of the output of an ideal sampler is given by

$$\mathcal{Z}[f(t)] = K_0 + \frac{K_1}{z} + \frac{K_2}{z^2} + .... + \frac{K_n}{z^n}$$

Reason (R) : The relationship is the result of application of $z = e^{-sT}$, where $T$ stands for the time gap between the samples.

(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

MCQ 6.4.77
IES EC 2010

$z$ and Laplace transform are related by

(A) $s = \ln z$ (B) $s = \frac{\ln z}{T}$
(C) $s = z$ (D) $\frac{T}{\ln z}$

MCQ 6.4.78
IES EC 2010

Frequency scaling [relationship between discrete time frequency ($\Omega$) and continuous time frequency ($\omega$)] is defined as

(A) $\omega = 2\Omega$ (B) $\omega = 2T_S/\Omega$
(C) $\Omega = 2\omega/T_S$ (D) $\Omega = \omega T_S$

MCQ 6.4.79
GATE IN 2004

A casual, analog system has a transfer function $H(s) = \frac{a}{s^2+a^2}$. Assuming a sampling
time of $T$ seconds, the poles of the transfer function $H(z)$ for an equivalent digital system obtained using impulse in variance method are at

(A) $(e^{aT}, e^{-aT})$ (B) $\left(j\frac{a}{T}, -j\frac{a}{T}\right)$
(C) $(e^{jaT}, e^{-jaT})$ (D) $(e^{aT/2}, e^{-aT/2})$

***********

# SOLUTIONS 6.1

SOL 6.1.1 Option (C) is correct.

The $z$-transform is

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = \sum_{n=-\infty}^{-1} -\left(\frac{1}{2}\right)^n z^{-n}$$

$$= -\sum_{n=-\infty}^{-1}\left(\frac{1}{2}z^{-1}\right)^n = -\sum_{n=-\infty}^{-1}(2z)^{-n}$$

$$= -\sum_{m=1}^{\infty}(2z)^m \qquad \text{Let } -n = m \text{ so,}$$

The above series converges if $|2z| < 1$ or $|z| < \frac{1}{2}$

$$X(z) = -\frac{2z}{1-2z} = \frac{2z}{2z-1}, \qquad |z| < \frac{1}{2}$$

SOL 6.1.2 Option (A) is correct.

We have $\quad x[n] = \left(\frac{1}{2}\right)^{|n|} = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{2}\right)^{-n} u[-n-1]$

$z$-transform is $\quad X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$

$$= \sum_{n=-\infty}^{\infty}\left(\frac{1}{2}\right)^{-n} z^{-n}u[-n-1] + \sum_{n=-\infty}^{\infty}\left(\frac{1}{2}\right)^n z^{-n}u[n]$$

$$= \sum_{n=-\infty}^{-1}\left(\frac{1}{2}\right)^{-n} z^{-n} + \sum_{n=0}^{\infty}\left(\frac{1}{2z}\right)^n$$

$$= \underbrace{\sum_{n=1}^{\infty}\left(\frac{1}{2}z\right)^n}_{\text{II}} + \underbrace{\sum_{n=0}^{\infty}\left(\frac{1}{2z}\right)^n}_{\text{I}}$$

$$= \frac{1}{1-\frac{1}{2z}} + \frac{\frac{1}{2}z}{1-\frac{1}{2}z} = \frac{z}{z-\frac{1}{2}} - \frac{z}{z-2}$$

Series I converges, if $\left|\frac{1}{2}z\right| < 1$ or $|z| < 2$

Series II converges, if $\left|\frac{1}{2z}\right| < 1$ or $|z| > \frac{1}{2}$

ROC is intersection of both, therefore ROC : $\frac{1}{2} < |z| < 2$

SOL 6.1.3 Option (D) is correct.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$= \sum_{n=-\infty}^{\infty}\left(\frac{1}{2}\right)^n z^{-n}u[-n-1] + \sum_{n=-\infty}^{\infty}\left(\frac{1}{3}\right)^n z^{-n}u[n]$$

$$= \sum_{n=-\infty}^{-1}\left(\frac{1}{2}\right)^n z^{-n} + \sum_{n=0}^{\infty}\left(\frac{1}{3z}\right)^n$$

$$= \sum_{n=1}^{\infty}(2z)^n + \sum_{n=0}^{\infty}\left(\frac{1}{3z}\right)^n = \underbrace{\frac{2z}{1-2z}}_{\text{I}} + \underbrace{\frac{1}{1-\frac{1}{3}z^{-1}}}_{\text{II}}$$

Series I converges, when $|2z| < 1$ or $|z| < \frac{1}{2}$

Series II converges, when $\left|\frac{1}{3z}\right| < 1$ or $|z| > \frac{1}{3}$

So ROC of $X(z)$ is intersection of both ROC: $\frac{1}{3} < |z| < \frac{1}{2}$

SOL 6.1.4 Option (C) is correct.

$z$-transform of $x[n]$

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} x[n] z^{-n} \\ &= \sum_{n=-\infty}^{\infty} \alpha^n z^{-n} u[n] + \sum_{n=-\infty}^{\infty} \alpha^{-n} z^{-n} u[n] \\ &= \sum_{n=0}^{\infty} (\alpha z^{-1})^n + \sum_{n=0}^{\infty} (\alpha z)^{-n} = \underbrace{\frac{1}{1-\alpha z^{-1}}}_{\text{I}} + \underbrace{\frac{1}{1-(\alpha z)^{-1}}}_{\text{II}} \end{aligned}$$

Series I converges, if $\alpha z^{-1} < 1$ or $|z| > |\alpha|$

Series II converges, if $(\alpha z)^{-1} < 1$ or $\alpha z > 1$ or $|z| > \frac{1}{|\alpha|}$

So ROC is interaction of both

$$\text{ROC} : |z| > \max\left(|\alpha|, \frac{1}{|\alpha|}\right)$$

SOL 6.1.5 Option (C) is correct.

(P $\rightarrow$ 4) $\quad x_1[n] = u[n-2]$

$$X_1(z) = \sum_{n=-\infty}^{\infty} u[n-2] z^{-n} = \sum_{n=2}^{\infty} z^{-n} = \frac{z^{-2}}{1-z^{-1}}, \quad |z| > 1$$

(Q $\rightarrow$ 1) $\quad x_2[n] = -u[-n-3]$

$$\begin{aligned} X_2[z] &= -\sum_{n=-\infty}^{\infty} u[-n-3] z^{-n} \\ &= -\sum_{n=-\infty}^{-3} z^{-n} = -\sum_{m=3}^{\infty} z^m \qquad \text{Let, } n = -m \\ &= \frac{-z^3}{1-z} = \frac{1}{z^{-2}(1-z^{-1})}, \quad |z| < 1 \end{aligned}$$

(R $\rightarrow$ 3) $\quad x_3[n] = (1)^n u[n+4]$

$$\begin{aligned} X_3(z) &= \sum_{n=-\infty}^{\infty} u[n+4] z^{-n} = \sum_{n=-4}^{\infty} z^{-n} \\ &= \frac{z^4}{1-z^{-1}} = \frac{1}{z^{-4}(1-z^{-1})}, \quad |z| > 1 \end{aligned}$$

(S $\rightarrow$ 2) $\quad x_4[n] = (1)^n u[-n]$

$$\begin{aligned} &= \sum_{n=-\infty}^{\infty} u[-n] z^{-n} \\ &= \sum_{n=-\infty}^{0} z^{-n} = \sum_{m=0}^{\infty} z^m = \frac{1}{1-z} = \frac{-z^{-1}}{1-z^{-1}}, \quad |z| < 1 \end{aligned}$$

SOL 6.1.6 Option (A) is correct.

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} e^{jn\pi} z^{-n} u[n] = \sum_{n=0}^{\infty} (e^{j\pi} z^{-1})^n \\ &= \frac{1}{1-e^{j\pi} z^{-1}}, \quad |z| > 1 \\ &= \frac{z}{z-e^{j\pi}} = \frac{z}{z+1} \qquad \because e^{j\pi} = -1 \end{aligned}$$

SOL 6.1.7 Option (D) is correct.

We can write, transfer function

$$H(z) = \frac{Az^2}{(z-2)(z-3)}$$

$$H(1) = \frac{A}{(-1)(-2)} = 1 \text{ or } A = 2$$

so, $$H(z) = \frac{2z^2}{(z-2)(z-3)}$$

$$\frac{H(z)}{z} = \frac{2z}{(z-2)(z-3)}$$

$$H(z) = \frac{-4z}{(z-2)} + \frac{6z}{(z-3)} \qquad \text{From partial fraction}$$

We can see that for ROC: $|z| > 3$, the system is causal and unstable because ROC is exterior of the circle passing through outermost pole and does not include unit circle.

so, $$h[n] = [(-4)2^n + (6)(3)^n]u[n], \ |z| > 3 \qquad (P \to 2)$$

For ROC $2 < |z| < 3$, The sequence corresponding to pole at $z = 2$ corresponds to right-sided sequence while the sequence corresponds to pole at $z = 3$ corresponds to left sided sequence

$$h[n] = (-4)2^n u[n] + (-6)3^n u[-n-1] \qquad (Q \to 4)$$

For ROC: $|z| < 2$, ROC is interior to circle passing through inner most pole, hence the system is non causal.

$$h[n] = (4)2^n u[-n-1] + (-6)3^n u[-n-1] \qquad (R \to 3)$$

For the response

$$h[n] = 4(2)^n u[-n-1] + (-6)3^n u[n]$$

ROC: $|z| < 2$ and $|z| > 3$ which does not exist $\qquad (S \to 1)$

SOL 6.1.8 Option (A) is correct.

$$X(z) = e^z + e^{1/z}$$

$$X(z) = \left(1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + .....\right) + \left(1 + \frac{1}{z} + \frac{1}{2!}\frac{1}{z^2} + .....\right)$$

$$= \left(1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + ....\right) + \left(1 + z^{-1} + \frac{z^{-2}}{2!} + ....\right)$$

Taking inverse $z$-transform

$$x[n] = \delta[n] + \frac{1}{n!}$$

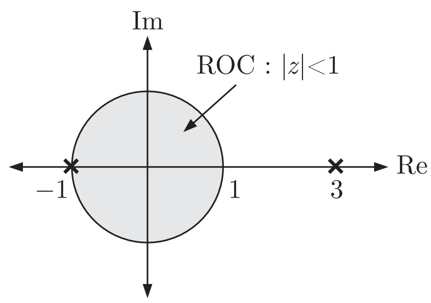
SOL 6.1.9 Option (A) is correct.

$$X(z) = \frac{z^2 + 5z}{z^2 - 2z - 3} = \frac{z(z+5)}{(z-3)(z+1)}$$

$$\frac{X(z)}{z} = \frac{z+5}{(z-3)(z+1)} = \frac{2}{z-3} - \frac{1}{z+1} \qquad \text{By partial fraction}$$

Thus $$X(z) = \frac{2z}{z-3} - \frac{z}{z+1}$$

Poles are at $z = 3$ and $z = -1$



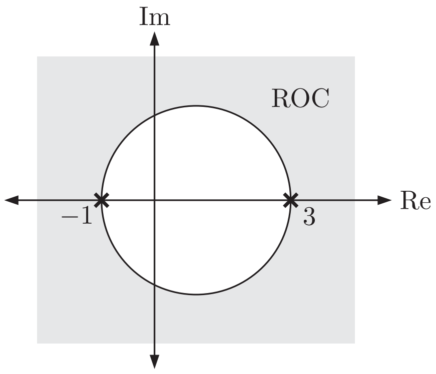

ROC : $|z| < 1$, which is not exterior of circle outside the outermost pole $z = 3$. So, $x[n]$ is anticausal given as

$$x[n] = [-2(3)^n + (-1)^n]u[-n-1]$$

SOL 6.1.10 Option (A) is correct.

$$X(z) = \frac{2z}{z-3} - \frac{z}{z+1}$$

If $|z| > 3$, ROC is exterior of a circle outside the outer most pole, $x[n]$ is causal.

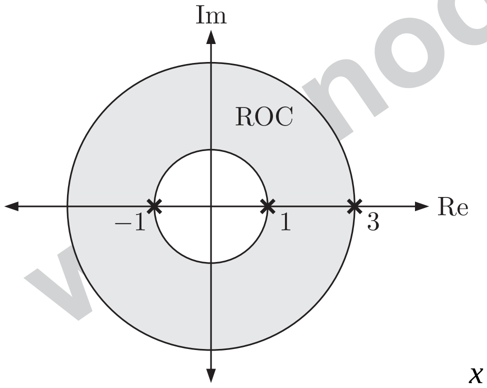



$$x[n] = [2(3)^n - (-1)^n]u[n]$$

SOL 6.1.11 Option (C) is correct.

$$X(z) = \frac{2z}{z-3} - \frac{z}{z+1}$$

If ROC is $1 < |z| < 3$, $x[n]$ is two sided with anticausal part $\frac{2z}{z-3}$, $|z| < 3$ and causal part $\frac{-z}{z+1}$, $|z| > 1$




$$x[n] = -2(3)^n u[-n-1] - (-1)^n u[n]$$

SOL 6.1.12 Option (D) is correct.

$$X_1(z) = \sum_{n=-\infty}^{\infty}(0.7)^n z^{-n} u[n-1] = \sum_{n=1}^{\infty}(0.7z^{-1})^n$$

$$= \frac{0.7z^{-1}}{1-0.7z^{-1}}$$

ROC : $|0.7z^{-1}| < 1$ or $|z| > 0.7$

$$X_2(z) = \sum_{n=-\infty}^{\infty}(-0.4)^n z^{-n} u[-n-2] = \sum_{n=-\infty}^{-2}(-0.4)^n z^{-n}$$

$$= \sum_{m=2}^{\infty}(-0.4)^{-m} z^m \qquad \text{Let } n = -m$$

$$= \sum_{m=2}^{\infty}[(-0.4)^{-1}z]^m = \frac{-(0.4)^{-1}z}{1+(0.4)^{-1}z}$$

ROC : $|(0.4)^{-1}z| < 1$ or $|z| < 0.4$

The ROC of $z$-transform of $x[n]$ is intersection of both which does not exist.

SOL 6.1.13 Option (D) is correct.

If $\qquad x[n] \xleftrightarrow{z} X(z)$

From time shifting property

$$x[n-n_0] \xleftrightarrow{z} z^{-n_0} X(z)$$

So $\qquad z(x[n-4]) = z^{-4} X(z) = \frac{1}{8z^5 - 2z^4 - z^3}$

SOL 6.1.14 Option (A) is correct.

We can see that

$$X_1(z) = z^1 X_2(z) = z^2 X_3(z)$$

or $\qquad z^{-2} X_1(z) = z^{-1} X_2(z) = X_3(z)$

So $\qquad x_1[n-2] = x_2[n-1] = x_3[n]$

SOL 6.1.15 Option (C) is correct.

$x[n]$ can be written in terms of unit sequence as

$$x[n] = u[n] - u[n-k]$$

so $\qquad X(z) = \frac{z}{z-1} - z^{-k}\frac{z}{z-1} = \frac{1-z^{-k}}{1-z^{-1}}$

SOL 6.1.16 Option (C) is correct.

For positive shift

If, $\qquad x[n] \xleftrightarrow{z} X(z)$

then, $\qquad x[n-n_0] \xleftrightarrow{z} z^{-n_0} X(z),\ n_0 \geq 0$

So $\qquad x[n-1] \xleftrightarrow{z} z^{-1}\left(\frac{z}{z-1}\right) = \frac{1}{z-1}$

For negative shift

$$x[n+n_0] \xleftrightarrow{z} z^{n_0}\left(X(z) - \sum_{m=0}^{n_0-1} x[n] z^{-m}\right),\ n_0 > 0$$

$$x[n+1] \xleftrightarrow{z} z(X(z) - x[0])$$

We know that $x[n] = u[n]$ so $x[0] = 1$

and $\qquad x[n+1] \xleftrightarrow{z} z(X(z) - 1) = z\left(\frac{z}{z-1} - 1\right) = \frac{z}{z-1}$

SOL 6.1.17 Option (B) is correct.

Even part of $x[n]$, $\qquad x_e[n] = \frac{1}{2}(x[n] + x[-n])$

$z$- transform of $x_e[n]$, $X_e(z) = \frac{1}{2}\left[X(z) + X\left(\frac{1}{z}\right)\right] \qquad \because\ x[-n] \xleftrightarrow{z} X\left(\frac{1}{z}\right)$

$$= \underbrace{\frac{1}{2}\left(\frac{z}{z-0.4}\right)}_{\text{I}} + \underbrace{\frac{1}{2}\left(\frac{1/z}{1/z-0.4}\right)}_{\text{II}}$$

Region of convergence for I series is $|z| > 0.4$ and for II series it is $|z| < 2.5$. Therefore, $X_e(z)$ has ROC $0.4 < |z| < 2.5$

SOL 6.1.18 Option (B) is correct.

(P $\rightarrow$ 4) $\qquad y[n] = n(-1)^n u[n]$

We know that

$$(-1)^n u[n] \xleftrightarrow{z} \frac{1}{1+z^{-1}}, \qquad |z| > 1$$

If $\qquad x[n] \xleftrightarrow{z} X(z)$

then, $\qquad nx[n] \xleftrightarrow{z} -z\frac{dX(z)}{dz} \qquad$ ($z$-domain differentiation) so, $$n(-1)^n u[n] \xleftrightarrow{z} -z\frac{d}{dz}\left[\frac{1}{1+z^{-1}}\right], \quad \text{ROC}: |z| > 1$$

$$Y(z) = \frac{-z^{-1}}{(1+z^{-1})^2}, \quad \text{ROC}: |z| > 1$$

(Q → 3) $$y[n] = -nu[-n-1]$$

We know that,

$$u[-n-1] \xleftrightarrow{z} \frac{-1}{(1-z^{-1})}, \quad \text{ROC}: |z| < 1$$

Again applying $z$-domain differentiation property

$$-nu[-n-1] \xleftrightarrow{z} z\frac{d}{dz}\left[\frac{-1}{1-z^{-1}}\right], \quad \text{ROC}: |z| < 1$$

$$Y(z) = \frac{z^{-1}}{(1-z^{-1})^2}, \quad \text{ROC}: |z| < 1$$

(R → 2) $$y[n] = (-1)^n u[n]$$

$$Y(z) = \sum_{n=-\infty}^{\infty}(-1)^n z^{-n} u[n] = \sum_{n=0}^{\infty}(-z^{-1})^n$$

$$= \frac{1}{1+z^{-1}}, \quad \text{ROC}: |z| > 1$$

(S → 1) $$y[n] = nu[n]$$

We know that $$u[n] \xleftrightarrow{z} \frac{1}{1-z^{-1}}, \quad \text{ROC}: |z| > 1$$

so, $$nu[n] \xleftrightarrow{z} z\frac{d}{dz}\left(\frac{1}{1-z^{-1}}\right), \quad \text{ROC}: |z| > 1$$

$$Y(z) = \frac{z^{-1}}{(1-z^{-1})^2}, \quad \text{ROC}: |z| > 1$$

SOL 6.1.19 Option (A) is correct.

Its difficult to obtain $z$-transform of $x[n]$ directly due to the term $1/n$.

Let $$y[n] = nx[n] = (-2)^{-n} u[-n-1]$$

So $z$-transform of $y[n]$

$$Y(z) = \frac{-z}{z+\frac{1}{2}}, \quad \text{ROC}: |z| < \frac{1}{2}$$

Since $$y[n] = nx[n]$$

so, $$Y(z) = -z\frac{dX(z)}{dz} \qquad \text{(Differentiation in } z\text{-domain)}$$

$$-z\frac{dX(z)}{dz} = \frac{-z}{z+\frac{1}{2}}$$

$$\frac{dX(z)}{dz} = \frac{1}{z+\frac{1}{2}}$$

or $$X(z) = \log\left(z+\frac{1}{2}\right), \quad \text{ROC}: |z| < \frac{1}{2}$$

SOL 6.1.20 Option (A) is correct.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}, \quad \text{ROC}: R_x$$

$$Y(z) = \sum_{n=-\infty}^{\infty} y[n] z^{-n} = \sum_{n=-\infty}^{\infty} a^n x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x[n]\left(\frac{z}{a}\right)^{-n} = X\left(\frac{z}{a}\right), \quad \text{ROC}: aR_x$$

SOL 6.1.21 Option (B) is correct.

Using time shifting property of $z$-transform

If, $$x[n] \xleftrightarrow{z} X(z), \quad \text{ROC}: R_x$$

then, $$x[n-n_0] \xleftrightarrow{z} z^{-n_0}X(z)$$
with same ROC except the possible deletion or addition of $z=0$ or $z=\infty$.

So, ROC for $x[n-2]$ is $R_x$ $(S_1, R_1)$

Similarly for $x[n+2]$, ROC : $R_x$ $(S_2, R_1)$

Using time-reversal property of $z$-transform

If, $$x[n] \xleftrightarrow{z} X(z), \quad \text{ROC}: R_x$$

then, $$x[-n] \xleftrightarrow{z} X\left(\frac{1}{z}\right), \quad \text{ROC}: \frac{1}{R_x}$$

For $S_3$, $$x[-n] \xleftrightarrow{z} X\left(\frac{1}{z}\right),$$

Because $z$ is replaced by $1/z$, so ROC would be $|z| < \frac{1}{a}$ $(S_3, R_3)$

$S_4 : (-1)^n x[n]$

Using the property of scaling in $z$-domain, we have

If, $$x[n] \xleftrightarrow{z} X(z), \quad \text{ROC}: R_x$$

then, $$\alpha^n x[n] \xleftrightarrow{z} X\left(\frac{z}{\alpha}\right)$$

$z$ is replaced by $z/\alpha$ so ROC will be $\frac{R_x}{|\alpha|}$

Here $$(-1)^n x[n] \xleftrightarrow{z} X\left(\frac{z}{1}\right), \quad \alpha = 1$$

so, $$\text{ROC}: |z| > a \qquad (S_4, R_1)$$

SOL 6.1.22 Option (D) is correct.

Time scaling property :

If, $$x[n] \xleftrightarrow{z} X(z)$$

then, $$x[n/2] \xleftrightarrow{z} X(z^2) \qquad (P \to 2)$$

Time shifting property :

If, $$x[n] \xleftrightarrow{z} X(z)$$

then, $$x[n-2]u[n-2] \xleftrightarrow{z} z^{-2}X(z) \qquad (Q \to 1)$$

For $x[n+2]u[n]$ we can not apply time shifting property directly.

Let, $$y[n] = x[n+2]u[n]$$
$$= \alpha^{n+2}u[n+2]u[n] = \alpha^{n+2}u[n]$$

so, $$Y(z) = \sum_{n=-\infty}^{\infty} y[n]z^{-n} = \sum_{n=0}^{\infty} \alpha^{n+2}z^{-n}$$
$$= \alpha^2 \sum_{n=0}^{\infty} (\alpha z^{-1})^n = \alpha^2 X(z) \qquad (R \to 4)$$

Let, $$g[n] = \beta^{2n}x[n]$$
$$G(z) = \sum_{n=-\infty}^{\infty} g[n]z^{-n} = \sum_{n=-\infty}^{\infty} \beta^{2n}\alpha^n z^{-n}u[n]$$
$$= \sum_{n=0}^{\infty} \alpha^n \left(\frac{z}{\beta^2}\right)^n = X\left(\frac{z}{\beta^2}\right) \qquad (S \to 3)$$

SOL 6.1.23 Option (C) is correct.

Let, $$y[n] = x[2n]$$
$$Y(z) = \sum_{n=-\infty}^{\infty} x[2n]z^{-n}$$
$$= \sum_{k=-\infty}^{\infty} x[k]z^{-k/2} \qquad \text{Put } 2n = k \text{ or } n = \frac{k}{2}, \ k \text{ is even}$$

Since $k$ is even, so we can write

$$\begin{aligned}Y(z) &= \sum_{k=-\infty}^{\infty}\left[\frac{x[k]+(-1)^k x[k]}{2}\right]z^{-k/2}\\ &= \frac{1}{2}\sum_{k=-\infty}^{\infty}x[k]z^{-k/2}+\frac{1}{2}\sum_{k=-\infty}^{\infty}x[k](-z^{1/2})^k\\ &= \frac{1}{2}\left[X(\sqrt{z})+X(-\sqrt{z})\right]\end{aligned}$$

SOL 6.1.24 Option (A) is correct.

From the accumulation property we know that

If, $x[n] \xleftrightarrow{\mathcal{Z}} X(z)$

then, $\sum_{k=-\infty}^{n}x[k] \xleftrightarrow{\mathcal{Z}} \frac{z}{(z-1)}X(z)$

Here, $y[n] = \sum_{k=0}^{n}x[k]$

$$Y(z) = \frac{z}{(z-1)}X(z) = \frac{4z^2}{(z-1)(8z^2-2z-1)}$$

SOL 6.1.25 Option (B) is correct.

By taking $z$-transform of both the sequences

$$X(z) = (-1+2z^{-1}+0+3z^{-3})$$
$$H(z) = 2z^2+3$$

Convolution of sequences $x[n]$ and $h[n]$ is given as

$$y[n]\ x[n]*h[n]$$

Applying convolution property of $z$-transform, we have

$$\begin{aligned}Y(z) &= X(z)H(z)\\ &= (-1+2z^{-1}+3z^{-3})(2z^2+3) = -2z^2+4z-3+12z^{-1}+9z^{-3}\end{aligned}$$

or, $y[n] = \{-2, 4, \underset{\uparrow}{-3}, 12, 0, 9\}$

SOL 6.1.26 Option (C) is correct.

The $z$-transform of signal $x^*[n]$ is given as follows

$$\mathcal{Z}\{x^*[n]\} = \sum_{n=-\infty}^{\infty}x^*[n]z^{-n} = \sum_{n=-\infty}^{\infty}[x[n](z^*)^{-n}]^* \quad ...(1)$$

Let $z$-transform of $x[n]$ is $X(z)$

$$X(z) = \sum_{n=-\infty}^{\infty}x[n]z^{-n}$$

Taking complex conjugate on both sides of above equation

$$X^*(z) = \sum_{n=-\infty}^{\infty}[x[n]z^{-n}]^*$$

Replacing $z \to z^*$, we will get

$$X^*(z^*) = \sum_{n=-\infty}^{\infty}[x[n](z^*)^{-n}]^* \quad ...(2)$$

Comparing equation (1) and (2)

$$\mathcal{Z}\{x^*[n]\} = X^*(z^*)$$

SOL 6.1.27 Option (B) is correct.

By taking $z$-transform on both sides of given difference equation

$$Y(z)-\frac{1}{2}z^{-1}[Y(z)+y[-1]z] = X(z)$$

Let impulse response is $H(z)$, so the impulse input is $X(z)=1$

$$H(z)-\frac{1}{2}z^{-1}[H(z)+3z] = 1$$

$$H(z)[1 - \tfrac{1}{2}z^{-1}] = \frac{5}{2}$$

$$H(z) = \frac{5/2}{1 - \frac{1}{2}z^{-1}} = \frac{5}{2}\left(\frac{z}{z - \frac{1}{2}}\right)$$

$$h[n] = \frac{5}{2}\left(\frac{1}{2}\right)^n, \quad n \geq 0$$

SOL 6.1.28 Option (B) is correct.

$$h[n] = (2)^n u[n]$$

Taking $z$-transform

$$H(z) = \frac{z}{z-2} = \frac{Y(z)}{X(z)}$$

so, $(z-2)\,Y(z) = zX(z)$

or, $(1 - 2z^{-1})\,Y(z) = X(z)$

Taking inverse $z$-transform

$$y[n] - 2y[n-1] = x[n]$$

SOL 6.1.29 Option (B) is correct.

$$h[n] = \delta[n] - \left(\frac{-1}{2}\right)^n u[n]$$

$z$-transform of $h[n]$

$$H(z) = 1 - \frac{z}{z + \frac{1}{2}} = \frac{\frac{1}{2}}{z + \frac{1}{2}} = \frac{Y(z)}{X(z)}$$

$$\left(z + \frac{1}{2}\right)Y(z) = \frac{1}{2}X(z)$$

$$\left(1 + \frac{1}{2}z^{-1}\right)Y(z) = \frac{1}{2}z^{-1}X(z)$$

Taking inverse $z$-transform

$$y[n] + \frac{1}{2}y[n-1] = \frac{1}{2}x[n-1]$$

$$y[n] + 0.5y[n-1] = 0.5x[n-1]$$

SOL 6.1.30 Option (C) is correct.

We have $y[n] - 0.4y[n-1] = (0.4)^n u[n]$

Zero state response refers to the response of system with zero initial condition.

So, by taking $z$-transform

$$Y(z) - 0.4z^{-1}Y(z) = \frac{z}{z - 0.4}$$

$$Y(z) = \frac{z^2}{(z-0.4)^2}$$

Taking inverse $z$-transform

$$y[n] = (n+1)(0.4)^n u[n]$$

SOL 6.1.31 Option (B) is correct.

Zero state response refers to response of the system with zero initial conditions.

Taking $z$-transform

$$Y(z) - \frac{1}{2}z^{-1}Y(z) = X(z)$$

$$Y(z) = \left(\frac{z}{z - 0.5}\right)X(z)$$

For an input $x[n] = u[n]$, $X(z) = \frac{z}{z-1}$

so, $$Y(z) = \frac{z}{(z-0.5)}\frac{z}{(z-1)} = \frac{z^2}{(z-1)(z-0.5)}$$

$$\frac{Y(z)}{z} = \frac{z}{(z-1)(z-0.5)}$$

$$= \frac{2}{z-1} - \frac{1}{z-0.5} \quad \text{By partial fraction}$$

Thus $$Y(z) = \frac{2z}{z-1} - \frac{z}{z-0.5}$$

Taking inverse $z$-transform

$$y[n] = 2u[n] - (0.5)^n u[n]$$

SOL 6.1.32 Option (C) is correct.

Input, $x[n] = 2\delta[n] + \delta[n+1]$

By taking $z$-transform

$$X(z) = 2 + z$$

$$\frac{Y(z)}{X(z)} = H(z), \quad Y(z) \text{ is } z\text{-transform of output } y[n]$$

$$Y(z) = H(z)X(z)$$

$$= \frac{2z(z-1)}{(z+2)^2}(z+2)$$

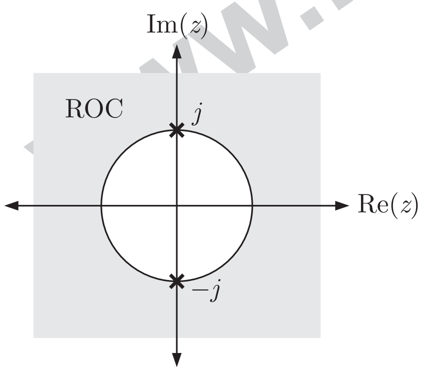
$$= \frac{2z(z-1)}{(z+2)} = 2z - \frac{6z}{z+2}$$

Taking inverse $z$-transform

$$y[n] = 2\delta[n+1] - 6(-2)^n u[n]$$

SOL 6.1.33 Option (B) is correct.

Poles of the system function are at $z = \pm j$ ROC is shown in the figure.




Causality :

We know that a discrete time LTI system with transfer function $H(z)$ is causal if and only if ROC is the exterior of a circle outside the outer most pole.

For the given system ROC is exterior to the circle outside the outer most pole ($z = \pm j$). The system is causal.

Stability :

A discrete time LTI system is stable if and only if ROC of its transfer function $H(z)$ includes the unit circle $|z| = 1$.

The given system is unstable because ROC does not include the unit circle.

Impulse Response :

$$H(z) = \frac{z}{z^2+1}$$

We know that

$$\sin(\Omega_0 n)u[n] \overset{z}{\longleftrightarrow} \frac{z\sin\Omega_0}{z^2 - 2z\cos\Omega_0 + 1},\ |z| > 1$$

Here $\quad z^2 + 1 = z^2 - 2z\cos\Omega_0 + 1$

So $\quad 2z\cos\Omega_0 = 0$ or $\Omega_0 = \frac{\pi}{2}$

Taking the inverse Laplace transform of $H(z)$

$$h[n] = \sin\left(\frac{\pi}{2}n\right)u[n]$$

SOL 6.1.34 Option (D) is correct.
Statement (A), (B) and (C) are true.

SOL 6.1.35 Option (D) is correct.
First we obtain transfer function ($z$-transform of $h[n]$) for all the systems and then check for stability

(A) $\quad H(z) = \frac{\frac{1}{3}z}{(z-\frac{1}{3})^2}$

Stable because all poles lies inside unit circle.

(B) $\quad h[n] = \frac{1}{3}\delta[n]$

$$\sum|h[n]| = \frac{1}{3} \quad \text{(absolutely summable)}$$

Thus this is also stable.

(C) $\quad h[n] = \delta[n] - \left(\frac{-1}{3}\right)^n u[n]$

$$H(z) = 1 - \frac{z}{z+\frac{1}{3}}$$

Pole is inside the unit circle, so the system is stable.

(D) $\quad h[n] = [(2)^n - (3)^n]u[n]$

$$H(z) = \frac{z}{z-2} - \frac{z}{z-3}$$

Poles are outside the unit circle, so it is unstable.



SOL 6.1.36 Option (B) is correct.
By taking $z$-transform

$$(1 + 3z^{-1} + 2z^{-2})Y(z) = (2 + 3z^{-1})X(z)$$

So, transfer function

$$H(z) = \frac{Y(z)}{X(z)} = \frac{(2+3z^{-1})}{(1+3z^{-1}+2z^{-2})} = \frac{2z^2+3z}{z^2+3z+2}$$

or $\quad \frac{H(z)}{z} = \frac{2z+3}{z^2+3z+2} = \frac{1}{z+2} + \frac{1}{z+1} \quad$ By partial fraction

Thus $\quad H(z) = \frac{z}{z+2} + \frac{z}{z+1}$

Both the poles lie outside the unit circle, so the system is unstable.

SOL 6.1.37 Option (B) is correct.

$$y[n] = x[n] + y[n-1]$$

Put $x[n] = \delta[n]$ to obtain impulse response $h[n]$

$$h[n] = \delta[n] + h[n-1]$$

For $n = 0$, $\quad h[0] = \delta[0] + h[-1]$

$h[0] = 1 \quad$ ( $h[-1] = 0$, for causal system)

$n = 1$, $\quad h[1] = \delta[1] + h[0]$

$$h[1] = 1$$

$n = 2$, $$h[2] = \delta[2] + h[1]$$

$$h[2] = 1$$

In general form

$$h[n] = u[n]$$ Thus, statement 1 is true.

Let $$x[n] \overset{z}{\longleftrightarrow} X(z)$$

$$X(z) = \frac{z}{z - 0.5}$$

$$h[n] \overset{z}{\longleftrightarrow} H(z)$$

$$H(z) = \frac{z}{z - 1}$$

Output $$Y(z) = H(z)X(z)$$

$$= \left(\frac{z}{z-1}\right)\left(\frac{z}{z-0.5}\right) = \frac{2z}{z-1} - \frac{z}{z-0.5}$$ By partial fraction

Inverse $z$-transform

$$y[n] = 2u[n] - (0.5)^n u[n]$$ Statement 3 is also true.

$$H(z) = \frac{z}{z-1}$$

System pole lies at unit circle $|z| = 1$, so the system is not BIBO stable.

SOL 6.1.38 Option (C) is correct.

(P $\rightarrow$ 3) ROC is exterior to the circle passing through outer most pole at $z = 1.2$, so it is causal. ROC does not include unit circle, therefore it is unstable.

(Q $\rightarrow$ 1) ROC is not exterior to the circle passing through outer most pole at $z = 1.2$, so it is non causal. But ROC includes unit circle, so it is stable.

(R $\rightarrow$ 2), ROC is not exterior to circle passing through outermost pole $z = 0.8$, so it is not causal. ROC does not include the unit circle, so it is unstable also.

(S $\rightarrow$ 4), ROC contains unit circle and is exterior to circle passing through outermost pole, so it is both causal and stable.

SOL 6.1.39 Option (D) is correct.

$$H(z) = \frac{P(z-0.9)}{z - 0.9 + Pz}$$

$$= \frac{P(z-0.9)}{(1+P)z - 0.9} = \frac{P}{1+P}\left(\frac{z-0.9}{z - \frac{0.9}{1+P}}\right)$$

Pole at $$z = \frac{0.9}{1+P}$$

For stability pole lies inside the unit circle, so

$$|z| < 1$$

or $$\left|\frac{0.9}{1+P}\right| < 1$$

$$0.9 < |1+P|$$

$$P > -0.1 \text{ or } P < -1.9$$

SOL 6.1.40 Option (A) is correct.

For a system to be causal and stable, $H(z)$ must not have any pole outside the unit circle $|z| = 1$.

$S_1$ : $$H(z) = \frac{z - \frac{1}{2}}{z^2 + \frac{1}{2}z - \frac{3}{16}} = \frac{z - \frac{1}{2}}{(z - \frac{1}{4})(z + \frac{3}{4})}$$

Poles are at $z = 1/4$ and $z = -3/4$, so it is causal.

$S_2$ : $$H(z) = \frac{z+1}{(z+\frac{4}{3})(1-\frac{1}{2}z^{-3})}$$

one pole is at $z = -4/3$, which is outside the unit circle, so it is not causal.

$S_3$ : one pole is at $z = \infty$, so it is also non-causal.

SOL 6.1.41 Option (D) is correct.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = \sum_{n=-\infty}^{\infty} \delta[n-k]z^{-n} = z^{-k},\ z \neq 0$$

ROC : We can find that $X(z)$ converges for all values of $z$ except $z = 0$, because at $z = 0$ $X(z) \to \infty$.

SOL 6.1.42 Option (D) is correct.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = \sum_{n=-\infty}^{\infty} \delta[n+k]z^{-n} = z^{k},\ \text{all } z$$

ROC : We can see that above summation converges for all values of $z$.

SOL 6.1.43 Option (A) is correct.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = \sum_{n=-\infty}^{\infty} u[n]z^{-n}$$
$$= \underbrace{\sum_{n=0}^{\infty} z^{-n}}_{\mathrm{I}} = \frac{1}{1-z^{-1}}$$

ROC : Summation I converges if $|z| > 1$, because when $|z| < 1$, then $\sum_{n=0}^{\infty} z^{-n} \to \infty$.

SOL 6.1.44 Option (D) is correct.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$
$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n (u[n]-u[n-5])z^{-n}$$
$$= \underbrace{\sum_{n=0}^{4} \left(\frac{1}{4}z^{-1}\right)^n}_{\mathrm{I}} \qquad u[n]-u[n-5] = 1,\ \text{for } 0 \le n \le 4$$
$$= \frac{1-(\frac{1}{4}z^{-1})^5}{1-(\frac{1}{4}z^{-1})} = \frac{z^5-(0.25)^5}{z^4(z-0.5)},\quad \text{all } z$$

ROC : Summation I converges for all values of $z$ because $n$ has only four value.

SOL 6.1.45 Option (D) is correct.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n u[-n]z^{-n}$$
$$= \sum_{n=-\infty}^{0} \left(\frac{1}{4}z^{-1}\right)^n = \sum_{n=-\infty}^{0} (4z)^{-n}$$
$$= \underbrace{\sum_{m=0}^{\infty} (4z)^m}_{\mathrm{I}} = \frac{1}{1-4z},\quad |z| < \frac{1}{4} \qquad \text{Taking } n \to -m,$$

ROC : Summation I converges if $|4z| < 1$ or $|z| < \frac{1}{4}$.

SOL 6.1.46 Option (B) is correct.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = \sum_{n=-\infty}^{\infty} 3^n u[-n-1]z^{-n}$$
$$= \sum_{n=-\infty}^{-1} (3z^{-1})^n = \underbrace{\sum_{n=1}^{\infty} \left(\frac{1}{3}z\right)^n}_{\mathrm{I}} \qquad u[-n-1] = 1,\ n \le -1$$

$$= \frac{\frac{1}{3}z}{1-\frac{1}{3}z} = \frac{z}{3-z}, \quad |z| < 3$$

ROC : Summation I converges when $\left|\frac{1}{3}z\right| < 1$ or $|z| < 3$

SOL 6.1.47 Option (B) is correct.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=-\infty}^{\infty} \left(\frac{2}{3}\right)^{|n|} z^{-n}$$
$$= \sum_{n=-\infty}^{-1} \left(\frac{2}{3}\right)^{-n} z^{-n} + \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^{n} z^{-n}$$

In first summation taking $n = -m$,

$$X(z) = \sum_{m=1}^{\infty} \left(\frac{2}{3}\right)^{m} z^{m} + \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^{n} z^{-n}$$
$$= \underbrace{\sum_{m=1}^{\infty} \left(\frac{2}{3}z\right)^{m}}_{\text{I}} + \underbrace{\sum_{n=0}^{\infty} \left(\frac{2}{3}z^{-1}\right)^{n}}_{\text{II}}$$
$$= \frac{\frac{2}{3}z}{(1-\frac{2}{3}z)} + \frac{1}{(1-\frac{2}{3}z^{-1})}$$
$$= \frac{-1}{(1-\frac{3}{2}z^{-1})} + \frac{1}{(1-\frac{2}{3}z^{-1})}$$

ROC : Summation I converges if $\left|\frac{2}{3}z\right| < 1$ or $|z| < \frac{3}{2}$ and summation II converges if $\left|\frac{2}{3}z^{-1}\right| < 1$ or $|z| > \frac{2}{3}$. ROC of $X(z)$ would be intersection of both, that is $\frac{2}{3} < |z| < \frac{3}{2}$

SOL 6.1.48 Option (B) is correct.

$$x[n] = \cos\left(\frac{\pi}{3}n\right) u[n] = \frac{e^{j(\frac{\pi}{3})n} + e^{-j(\frac{\pi}{3})n}}{2} u[n]$$
$$= \frac{1}{2} e^{j(\frac{\pi}{3})n} u[n] + \frac{1}{2} e^{-j(\frac{\pi}{3})n} u[n]$$
$$X[z] = \frac{1}{2}\left[\underbrace{\frac{1}{1-e^{\frac{j\pi}{3}}z^{-1}}}_{\text{I}} + \underbrace{\frac{1}{1-e^{-\frac{j\pi}{3}}z^{-1}}}_{\text{II}}\right]$$
$$a^n u[n] \xleftrightarrow{\mathcal{Z}} \frac{1}{1-az^{-1}}, \quad |z| > |a|$$
$$= \frac{1}{2}\left[\frac{2 - z^{-1}[e^{-\frac{j\pi}{3}} + e^{\frac{j\pi}{3}}]}{1 - z^{-1}(e^{\frac{j\pi}{3}} + e^{-\frac{j\pi}{3}}) + z^{-2}}\right], \quad |z| > 1$$
$$= \frac{z}{2} \frac{(2z-1)}{(z^2 - z + 1)}, \quad |z| > 1$$

ROC : First term in $X(z)$ converges for $|z| > \left|e^{\frac{j\pi}{3}}\right| \Rightarrow |z| > 1$. Similarly II term also converges for $|z| > \left|e^{-\frac{j\pi}{3}}\right| \Rightarrow |z| > 1$, so ROC would be simply $|z| > 1$.

SOL 6.1.49 Option (B) is correct.

$$x[n] = 3\delta[n+5] + 6\delta[n] + \delta[n-1] - 4\delta[n-2]$$
$$X[z] = 3z^5 + 6 + z^{-1} - 4z^{-2}, \qquad 0 < |z| < \infty$$

$\delta[n \pm n_0] \xleftrightarrow{\mathcal{Z}} z^{\pm n_0}$

ROC : $X(z)$ is finite over entire $z$ plane except $z = 0$ and $z = \infty$ because when $z = 0$ negative power of $z$ becomes infinite and when $z \to \infty$ the positive powers of $z$ tends to becomes infinite.

SOL 6.1.50 Option (D) is correct.

$$x[n] = 2\delta[n+2] + 4\delta[n+1] + 5\delta[n] + 7\delta[n-1] + \delta[n-3]$$
$$X(z) = 2z^2 + 4z + 5 + 7z^{-1} + z^{-3}, \quad 0 < |z| < \infty \quad \delta[n \pm n_0] \xleftrightarrow{\mathcal{Z}} z^{\pm n_0}$$

ROC is same as explained in previous question.

SOL 6.1.51 Option (B) is correct.

$$x[n] = \delta[n] - \delta[n-2] + \delta[n-4] - \delta[n-5]$$

$$X(z) = 1 - z^{-2} + z^{-4} - z^{-5}, \quad z \neq 0 \qquad \delta[n \pm n_0] \xleftrightarrow{\mathcal{Z}} z^{\pm n_0}$$

ROC : $X(z)$ has only negative powers of $z$, therefore transform $X(z)$ does not converges for $z = 0$.

SOL 6.1.52 Option (A) is correct.

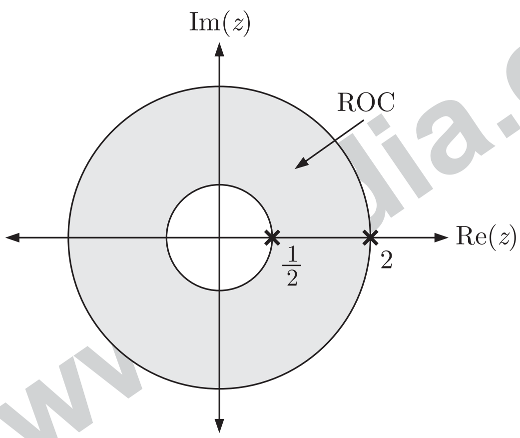
Using partial fraction expansion, $X(z)$ can be simplified as

$$X(z) = \frac{z^2 - 3z}{z^2 + \frac{3}{2}z - 1} = \frac{1 - 3z^{-1}}{1 + \frac{3}{2}z^{-1} - z^{-2}}$$

$$\frac{(1 - 3z^{-1})}{(1 + 2z^{-1})(1 - \frac{1}{2}z^{-1})}$$

$$= \underbrace{\frac{2}{1 + 2z^{-1}}}_{\mathrm{I}} - \underbrace{\frac{1}{1 - \frac{1}{2}z^{-1}}}_{\mathrm{II}}$$

Poles are at $z = -2$ and $z = \frac{1}{2}$. We obtain the inverse $z$-transform using relationship between the location of poles and region of convergence as shown in the figure.




ROC : $\frac{1}{2} < |z| < 2$ has a radius less than the pole at $z = -2$ therefore the I term of $X(z)$ corresponds to a left sided signal

$$\frac{2}{1 + 2z^{-1}} \xleftrightarrow{\mathcal{Z}^{-1}} -2(2)^n u[-n-1] \qquad \text{(left-sided signal)}$$

While, the ROC has a greater radius than the pole at $z = \frac{1}{2}$, so the second term of $X(z)$ corresponds to a right sided sequence.

$$\frac{1}{1 - \frac{1}{2}z^{-1}} \xleftrightarrow{\mathcal{Z}^{-1}} \frac{1}{2^n}u[n] \qquad \text{(right-sided signal)}$$

So, the inverse $z$-transform of $X(z)$ is

$$x[n] = -2(2)^n u[-n-1] - \frac{1}{2^n}u[n]$$

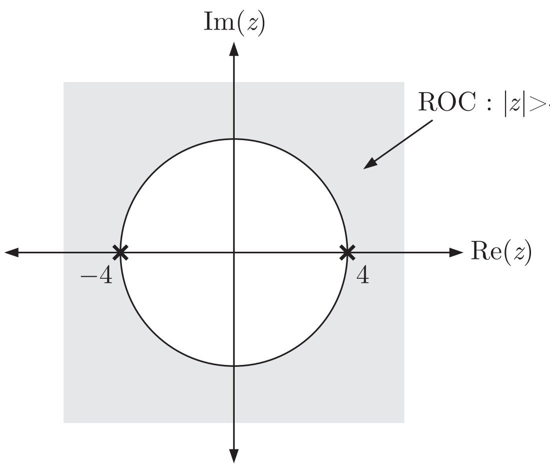
SOL 6.1.53 Option (A) is correct.

Using partial fraction expansion $X(z)$ can be simplified as follows

$$X(z) = \frac{3z^2 - \frac{1}{4}z}{z^2 - 16} = \frac{3 - \frac{1}{4}z^{-1}}{1 - 16z^{-1}}$$

$$= \frac{\frac{49}{32}}{1 + 4z^{-1}} + \frac{\frac{47}{32}}{1 - 4z^{-1}}$$

Poles are at $z = -4$ and $z = 4$. Location of poles and ROC is shown in the figure below




ROC : $|z| > 4$ has a radius greater that the pole at $z = -4$ and $z = 4$, therefore both the terms of $X(z)$ corresponds to right sided sequences. Taking inverse $z$- transform we have

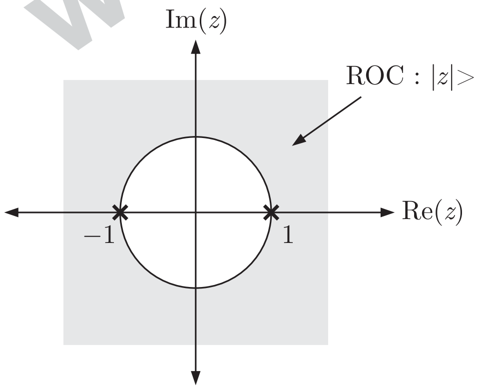
$$x[n] = \left[\frac{49}{32}(-4)^n + \frac{47}{32}4^n\right]u[n]$$

SOL 6.1.54 Option (C) is correct.

Using partial fraction expansion $X(z)$ can be simplified as

$$X(z) = \frac{2z^4 - 2z^3 - 2z^2}{z^2 - 1}$$
$$= \left[\frac{2 - 2z^{-1} - 2z^{-2}}{1 - z^{-2}}\right]z^2$$
$$= \left[2 + \frac{1}{1 + z^{-1}} + \frac{-1}{1 - z^{-1}}\right]z^2$$

Poles are at $z = -1$ and $z = 1$. Location of poles and ROC is shown in the following figure




ROC : $|z| > 1$ has radius grater than both the poles at $z = -1$ and $z = 1$, therefore both the terms in $X(z)$ corresponds to right sided sequences.

$$\frac{1}{1 + z^{-1}} \overset{z^{-1}}{\longleftrightarrow} (-1)^n u[n] \qquad \text{(right-sided)}$$
$$\frac{1}{1 - z^{-1}} \overset{z^{-1}}{\longleftrightarrow} u[n] \qquad \text{(right-sided)}$$

Now, using time shifting property the complete inverse $z$-transform of $X(z)$ is

$$x[n] = 2\delta[n+2] + ((-1)^n - 1)u[n+2]$$

SOL 6.1.55 Option (A) is correct.

We have, $X(z) = 1 + 2z^{-6} + 4z^{-8}, \quad |z| > 0$

Taking inverse $z$-transform we get

$$x[n] = \delta[n] + 2\delta[n-6] + 4\delta[n-8] \qquad z^{-n_0} \xleftrightarrow{\mathcal{Z}^{-1}} \delta[n-n_0]$$

SOL 6.1.56 Option (B) is correct.

Since $x[n]$ is right sided,

$$x[n] = \sum_{k=5}^{10} \frac{1}{k}\delta[n-k] \qquad z^{-n_0} \xleftrightarrow{\mathcal{Z}^{-1}} \delta[n-n_0]$$

SOL 6.1.57 Option (D) is correct.

We have , $X(z) = (1+z^{-1})^3 = 1 + 3z^{-1} + 3z^{-2} + z^{-3}, \quad |z| > 0$

Since $x[n]$ is right sided signal, taking inverse $z$-transform we have

$$x[n] = \delta[n] + 3\delta[n-1] + 3\delta[n-2] + \delta[n-3] \qquad z^{-n_0} \xleftrightarrow{\mathcal{Z}^{-1}} \delta[n-n_0]$$

SOL 6.1.58 Option (A) is correct.

We have, $X(z) = z^6 + z^2 + 3 + 2z^{-3} + z^{-4}, \quad |z| > 0$

Taking inverse $z$ transform we get

$$x[n] = \delta[n+6] + \delta[n+2] + 3\delta[n] + 2\delta[n-3] + \delta[n-4]$$

SOL 6.1.59 Option (A) is correct.

$$X(z) = \frac{1}{1-\frac{1}{4}z^{-2}}, \quad |z| > \frac{1}{2}$$

The power series expansion of $X(z)$ with $|z| > \frac{1}{2}$ or $\left|\frac{1}{4}z^{-2}\right| < 1$ is written as

$$X(z) = 1 + \frac{z^{-2}}{4} + \left(\frac{z^{-2}}{4}\right)^2 + \left(\frac{z^{-2}}{4}\right)^3 + \dots\dots$$

$$= \sum_{k=0}^{\infty}\left(\frac{1}{4}z^{-2}\right)^k = \sum_{k=0}^{\infty}\left(\frac{1}{4}\right)^k z^{-2k}$$

Series converges for $\left|\frac{1}{4}z^{-2}\right| < 1$ or $|z| > \frac{1}{2}$. Taking inverse $z$-transform we get

$$x[n] = \sum_{k=0}^{\infty}\left(\frac{1}{4}\right)^k \delta[n-2k] \qquad z^{-2k} \xleftrightarrow{\mathcal{Z}^{-1}} \delta[n-2k]$$

$$= \begin{cases}\left(\frac{1}{4}\right)^{\frac{n}{2}}, & n \text{ even and } n \geq 0\\ 0, & n \text{ odd}\end{cases}$$

$$= \begin{cases}2^{-n}, & n \text{ even and } n \geq 0\\ 0, & n \text{ odd}\end{cases}$$

SOL 6.1.60 Option (C) is correct.

$$X(z) = \frac{1}{1-\frac{1}{4}z^{-2}}, \quad |z| < \frac{1}{2}$$

Since ROC is left sided so power series expansion of $X(z)$ will have positive powers of $z$, we can simplify above expression for positive powers of $z$ as

$$X(z) = \frac{-4z^2}{1-(2z)^2}, \quad |z| < \frac{1}{2}$$

The power series expansion of $X(z)$ with $|z| < \frac{1}{2}$ or $|4z^2| < 1$ is written as

$$X(z) = -4z^2[1 + (2z)^2 + (2z)^4 + (2z)^6 + \dots\dots]$$

$$X(z) = -4z^2\sum_{k=0}^{\infty}(2z)^{2k} = -\sum_{k=0}^{\infty}2^{2(k+1)}z^{2(k+1)}$$

Taking inverse $z$-transform, we get

$$x[n] = -\sum_{k=0}^{\infty}2^{2(k+1)}\delta[n+2(k+1)] \qquad z^{2(k+1)} \xleftrightarrow{\mathcal{Z}^{-1}} \delta[n+2(k+1)]$$

SOL 6.1.61 Option (A) is correct.

Using Taylor's series expansion for a right-sided signal, we have

$$\ln(1+\alpha) = \alpha - \frac{\alpha^2}{2} + \frac{\alpha^3}{3} - \frac{\alpha^4}{4} + ....... = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} (\alpha)^k$$

$$X(z) = \ln(1+z^{-1}) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} (z^{-1})^k$$

Taking inverse $z$-transform we get

$$x[n] = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} \delta[n-k] \qquad z^{-k} \xleftrightarrow{\mathcal{Z}^{-1}} \delta[n-k]$$

SOL 6.1.62 Option (D) is correct.

From the given pole-zero pattern

$$X(z) = \frac{Az}{(z-\frac{1}{3})(z-2)}, \qquad A \rightarrow \text{Some constant}$$

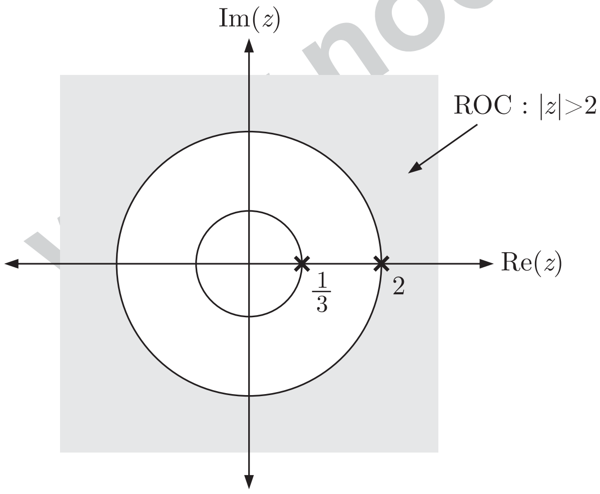
Using partial fraction expansion, we write

$$\frac{X(z)}{z} = \frac{\alpha}{z-\frac{1}{3}} + \frac{\beta}{z-2}, \quad \alpha \text{ and } \beta \text{ are constants.}$$

$$X(z) = \underbrace{\frac{\alpha}{(1-\frac{1}{3}z^{-1})}}_{\mathrm{I}} + \underbrace{\frac{\beta}{(1-2z^{-1})}}_{\mathrm{II}} \qquad ...(1)$$

Poles are at $z = \frac{1}{3}$ and $z = 2$. We obtain the inverse $z$-transform using relationship between the location of poles and region of convergence as shown in following figures.

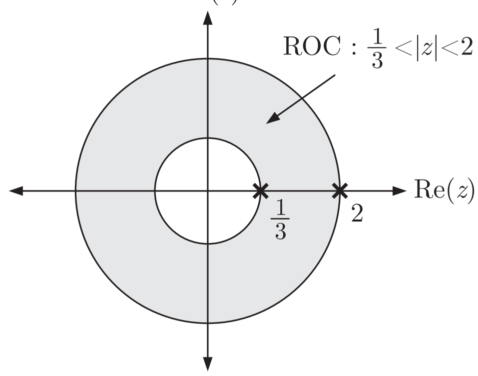
ROC : $|z| > 2$




ROC is exterior to the circle passing through right most pole so both the term in equation (1) corresponds to right sided sequences

$$x_1[n] = \alpha\left(\frac{1}{3}\right)^n u[n] + \beta(2)^n u[n]$$

ROC : $\frac{1}{3} < |z| < 2$




Since ROC has greater radius than the pole at $z = \frac{1}{3}$, so first term in equation (1) corresponds to right-sided sequence

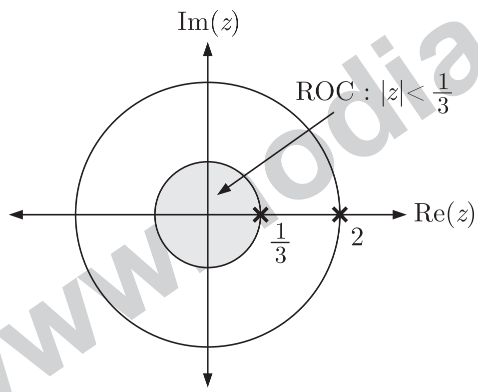
$$\frac{\alpha}{(1 - \frac{1}{3}z^{-1})} \xleftrightarrow{z^{-1}} \alpha\left(\frac{1}{3}\right)^n u[n] \qquad \text{(right-sided)}$$

ROC $|z| < 2$ has radius less than the pole at $z = 2$, so the second term in equation (1) corresponds to left sided sequence.

$$\frac{\beta}{(1 - 2z^{-1})} \xleftrightarrow{z^{-1}} \beta(2)^n u[n-1] \qquad \text{(left-sided)}$$

So, $$x_2[n] = \alpha\left(\frac{1}{3}\right)^n u[n] + \beta(2)^n u[n-1]$$

ROC : $|z| < \frac{1}{3}$






ROC is left side to both the poles of $X(z)$, so they corresponds to left sided signals.

$$x_3[n] = \alpha\left(\frac{1}{3}\right)^n u[-n-1] + \beta(2)^n u[-n-1]$$

All gives the same $z$-transform with different ROC so, all are the solution.

SOL 6.1.63 Option (C) is correct.

The $z$-transform of all the signal is same given as

$$X(z) = \frac{1}{1 - 2z^{-1}} - \frac{1}{1 - \frac{1}{2}z^{-1}}$$

Poles are at $z = 2$ and $z = \frac{1}{2}$. Now consider the following relationship between ROC and location of poles.

1. Since $x_1[n]$ is right-sided signal, so ROC is region in $z$-plane having radius greater than the magnitude of largest pole. So, $|z| > 2$ and $|z| > \frac{1}{2}$ gives $R_1 : |z| > 2$
2. Since $x_2[n]$ is left-sided signal, so ROC is the region inside a circle having radius equal to magnitude of smallest pole. So, $|z| < 2$ and $|z| < \frac{1}{2}$ gives $R_2 : |z| < \frac{1}{2}$
3. Since $x_3[n]$ is double sided signal, So ROC is the region in $z$-plane such

as $|z| > \frac{1}{2}$ and $|z| < 2$ which gives $R_3 : \frac{1}{2} < |z| < 2$

SOL 6.1.64 Option (B) is correct.

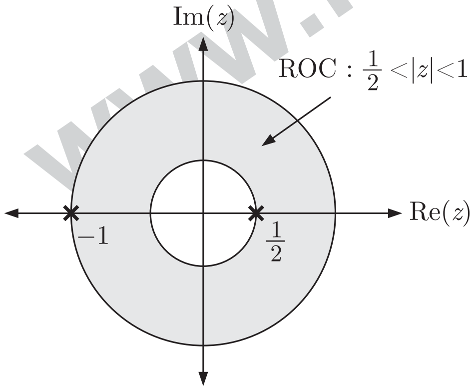
We have $$X(z) = \frac{1 + \frac{7}{6}z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{3}z^{-1})} = \frac{2}{1 - \frac{1}{2}z^{-1}} + \frac{-1}{1 + \frac{1}{3}z^{-1}}$$

$X(z)$ has poles at $z = \frac{1}{2}$ and $z = -\frac{1}{3}$, we consider the different ROC's and location of poles to obtain the inverse $z$-transform.

1. ROC $|z| > \frac{1}{2}$ is exterior to the cicle which passes through outtermost pole, so both the terms in equation (1) contributes to right sided sequences.
$$x[n] = \frac{2}{2^n}u[n] - \left(\frac{-1}{3}\right)^n u[n]$$
2. ROC $|z| < \frac{1}{3}$ is interior to the circle passing through left most poles, so both the terms in equation (1) corresponds to left sided sequences.
$$x[n] = \left[\frac{-2}{2^n} + \left(\frac{-1}{3}\right)^n\right]u[-n-1]$$
3. ROC $\frac{1}{3} < |z| < \frac{1}{2}$ is interior to the circle passing through pole at $z = \frac{1}{2}$ so the first term in equation (1) corresponds to a right sided sequence, while the ROC is exterior to the circle passing through pole at $z = -\frac{1}{3}$, so the second term corresponds to a left sided sequence. Therefore, inverse $z$-transform is
$$x[n] = -\frac{2}{2^n}u[-n-1] - \left(\frac{-1}{3}\right)^n u[n]$$

SOL 6.1.65 Option (A) is correct.

The location of poles and the ROC is shown in the figure. Since the ROC includes the point $z = \frac{3}{4}$, ROC is $\frac{1}{2} < |z| < 1$




$$X(z) = \frac{A}{1 - \frac{1}{2}z^{-1}} + \frac{B}{1 + z^{-1}}$$

ROC is exterior to the pole at $z = \frac{1}{2}$, so this term corresponds to a right-sided sequence, while ROC is interior to the pole at $z = -1$ so the second term corresponds to a left sided sequence. Taking inverse $z$-transform we get
$$x[n] = \frac{A}{2^n}u[n] + B(-1)^n u[-n-1]$$

For $n = 1$, $\quad x[1] = \frac{A}{2}(1) + B \times 0 = 1 \Rightarrow \frac{A}{2} = 1 \Rightarrow A = 2$

For $n = -1$, $\quad x[-1] = A \times 0 + B(-1) = 1 \Rightarrow B = -1$

So, $$x[n] = \frac{1}{2^{n-1}}u[n] - (-1)^n u[-n-1]$$

SOL 6.1.66 Option (B) is correct.

Let, $x[n] = Cp^n u[n]$ (right-sided sequence having a single pole)

$$x[0] = 2 = C$$

$$x[2] = \frac{1}{2} = 2p^2 \Rightarrow p = \frac{1}{2},$$

So, $x[n] = 2\left(\frac{1}{2}\right)^n u(n)$

SOL 6.1.67 Option (D) is correct.

$$X(z) = \sum_{n=0}^{\infty}\left(\frac{1}{2}z^{-1}\right)^n + \sum_{n=-\infty}^{-1}\left(\frac{1}{4}z^{-1}\right)^n$$

$$= \underbrace{\sum_{n=0}^{n}\left(\frac{1}{2}z^{-1}\right)^n}_{\text{I}} + \underbrace{\sum_{m=1}^{\infty}(4z)^m}_{\text{II}} = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}}$$

ROC : Summation I converges if $\left|\frac{1}{2}z^{-1}\right| < 1$ or $|z| > \frac{1}{2}$ and summation II converges if $|4z| < 1$ or $|z| < \frac{1}{4}$. ROC would be intersection of both which does not exist.

SOL 6.1.68 Option (C) is correct.

$$x[n] \xleftrightarrow{z} \frac{z^2}{z^2-16}, \quad \text{ROC } |z| < 4$$

$$x[n-2] \xleftrightarrow{z} z^{-2}\left(\frac{z^2}{z^2-16}\right) = \frac{1}{z^2-16} \quad \text{(Time shifting property)}$$

SOL 6.1.69 Option (B) is correct.

$$x[n] \xleftrightarrow{z} \frac{z^2}{z^2-16}, \quad \text{ROC } |z| < 4$$

$$\frac{1}{2^n}x[n] \xleftrightarrow{z} \frac{(2z)^2}{(2z)^2-16} = \frac{z^2}{z^2-4} \quad \text{(Scaling in } z\text{-domain)}$$

SOL 6.1.70 Option (C) is correct.

$$x[n] \xleftrightarrow{z} \frac{z^2}{z^2-16}, \quad \text{ROC } |z| < 4$$

$$x[-n] \xleftrightarrow{z} \frac{\left(\frac{1}{z}\right)^2}{\left(\frac{1}{z}\right)^2-16} \quad \text{(Time reversal property)}$$

$$x[-n] * x[n] \xleftrightarrow{z} \left[\frac{\left(\frac{1}{z}\right)^2}{\left(\frac{1}{z}\right)^2-16}\right]\left[\frac{z^2}{z^2-16}\right] \quad \text{(Time convolution property)}$$

$$\xleftrightarrow{z} \frac{z^2}{257z^2-16z^4-16}$$

SOL 6.1.71 Option (A) is correct.

$$x[n] \xleftrightarrow{z} \frac{z^2}{z^2-16}, \quad \text{ROC } |z| < 4$$

$$nx[n] \xleftrightarrow{z} -z\frac{d}{dz}\frac{z^2}{z^2-16} \quad \text{(Differentiation in } z\text{-domain)}$$

$$\xleftrightarrow{z} \frac{32z^2}{(z^2-16)^2}$$

SOL 6.1.72 Option (B) is correct.

$$x[n] \xleftrightarrow{z} \frac{z^2}{z^2-16}, \quad \text{ROC } |z| < 4$$

$$x[n+1] \xleftrightarrow{z} zX(z) \quad \text{(Time shifting)}$$

$$x[n-1] \xleftrightarrow{z} z^{-1}X(z) \quad \text{(Time shifting)}$$

$$x[n+1] + x[n-1] \xleftrightarrow{z} (z+z^{-1})X(z) \quad \text{(Linearity)}$$

$$\xleftrightarrow{\mathcal{Z}} \frac{z(z^2)+z^{-1}(z^2)}{z^2-16}=\frac{z(z^2+1)}{z^2-16}$$



SOL 6.1.73 Option (D) is correct.

$$x[n] \xleftrightarrow{\mathcal{Z}} \frac{z^2}{z^2-16}, \quad \text{ROC } |z|<4$$

$$x[n-3] \xleftrightarrow{\mathcal{Z}} z^{-3}\left(\frac{z^2}{z^2-16}\right)=\frac{z^{-1}}{z^2-16} \quad \text{(Time shifting property)}$$

$$x[n]*x[n-3] \xleftrightarrow{\mathcal{Z}} \left(\frac{z^2}{z^2-16}\right)\left(\frac{z^{-1}}{z^2-16}\right) \quad \text{(Time convolution property)}$$

$$\xleftrightarrow{\mathcal{Z}} \frac{z}{(z^2-16)^2}$$

SOL 6.1.74 Option (C) is correct.

We have, $$X(z) \xleftrightarrow{\mathcal{Z}^{-1}} 3^n n^2 u[n]$$

$$X(2z) \xleftrightarrow{\mathcal{Z}^{-1}} \frac{1}{2^n}\{3^n n^2 u[n]\} \quad \text{(Scaling in } z\text{-domain)}$$

SOL 6.1.75 Option (B) is correct.

$$X(z) \xleftrightarrow{\mathcal{Z}^{-1}} 3^n n^2 u[n]$$

$$X\left(\frac{1}{z}\right) \xleftrightarrow{\mathcal{Z}^{-1}} 3^{(-n)}(-n)^2 u[-n] \quad \text{(Time reversal)}$$

$$\xleftrightarrow{\mathcal{Z}^{-1}} 3^{-n} n^2 u[-n]$$

SOL 6.1.76 Option (C) is correct.

$$X(z) \xleftrightarrow{\mathcal{Z}^{-1}} 3^n n^2 u[n]$$

$$-z\frac{d}{dz}X(z) \xleftrightarrow{\mathcal{Z}^{-1}} nx[n] \quad \text{(Differentiation in } z\text{-domain)}$$

$$z^{-1}\left[-z\frac{d}{dz}X(z)\right] \xleftrightarrow{\mathcal{Z}^{-1}} (n-1)x[n-1] \quad \text{(Time shifting)}$$

So, $$\frac{dX(z)}{dz} \xleftrightarrow{\mathcal{Z}^{-1}} -(n-1)x[n-1] \qquad -z^{-1}\left[-z\frac{d}{dz}X(z)\right]=\frac{dX(z)}{dz}$$

$$\xleftrightarrow{\mathcal{Z}^{-1}} -(n-1)3^{n-1}(n-1)^2u[n-1]$$

$$\xleftrightarrow{\mathcal{Z}^{-1}} -(n-1)^3 3^{n-1}u[n-1]$$

SOL 6.1.77 Option (A) is correct.

$$\frac{1}{2}z^2X(z) \xleftrightarrow{\mathcal{Z}^{-1}} \frac{1}{2}x[n+2] \quad \text{(Time shifting)}$$

$$\frac{1}{2}z^{-2}X(z) \xleftrightarrow{\mathcal{Z}^{-1}} \frac{1}{2}x[n-2] \quad \text{(Time shifting)}$$

$$\frac{z^2-z^{-2}}{2}X(z) \xleftrightarrow{\mathcal{Z}^{-1}} \frac{1}{2}(x[n+2]-x[n-2]) \quad \text{(Linearity)}$$

SOL 6.1.78 Option (B) is correct.

$$X(z) \xleftrightarrow{\mathcal{Z}^{-1}} 3^n n^2 u[n]$$

$$X(z)X(z) \xleftrightarrow{\mathcal{Z}^{-1}} x[n]*x[n] \quad \text{(Time convolution)}$$

SOL 6.1.79 Option (A) is correct.

$$X(z)=1+\frac{z^{-1}}{4}-\frac{z^{-2}}{8}, \quad Y(z)=1-\frac{3z^{-1}}{4}$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{\frac{5}{3}}{1+\frac{1}{2}z^{-1}}+\frac{-\frac{2}{3}}{1-\frac{1}{4}z^{-1}}$$

For a causal system impulse response is obtained by taking right-sided inverse

$z$-transform of transfer function $H(z)$. Therefore,

$$h[n] = \frac{1}{3}\left[5\left(\frac{-1}{2}\right)^n - 2\left(\frac{1}{4}\right)^n\right]u[n]$$

SOL 6.1.80 Option (D) is correct.

We have $\quad x[n] = (-3)^n u[n]$

and $\quad y[n] = \left[4(2)^n - \left(\frac{1}{2}\right)^n\right]u[n]$

Taking $z$ transform of above we get

$$X(z) = \frac{1}{1+3z^{-1}}$$

and $\quad Y(z) = \frac{4}{1-2z^{-1}} - \frac{1}{1-\frac{1}{2}z^{-1}} = \frac{3}{(1-2z^{-1})(1-\frac{1}{2}z^{-1})}$

Thus transfer function is

$$H(z) = \frac{Y(z)}{X(z)} = \frac{10}{1-2z^{-1}} + \frac{-7}{1-\frac{1}{2}z^{-1}}$$

For a causal system impulse response is obtained by taking right-sided inverse $z$-transform of transfer function $H(z)$. Therefore,

$$h[n] = \left[10(2)^n - 7\left(\frac{1}{2}\right)^n\right]u[n]$$

SOL 6.1.81 Option (D) is correct.

We have $\quad h[n] = (\frac{1}{2})^n u[n]$

and $\quad y[n] = 2\delta[n-4]$

Taking $z$-transform of above we get

$$H(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$$

and $\quad Y(z) = 2z^{-4}$

Now $\quad X(z) = \frac{Y(z)}{H(z)} = 2z^{-4} - z^{-5}$

Taking inverse $z$-transform we have

$$x[n] = 2\delta[n-4] - \delta[n-5]$$

SOL 6.1.82 Option (C) is correct.

We have, $\quad y[n] = x[n] - x[n-2] + x[n-4] - x[n-6]$

Taking $z$-transform we get

$$Y(z) = X(z) - z^{-2}X(z) + z^{-4}X(z) - z^{-6}X(z)$$

or $\quad H(z) = \frac{Y(z)}{X(z)} = (1 - z^{-2} + z^{-4} - z^{-6})$

Taking inverse $z$-transform we have

$$h[n] = \delta[n] - \delta[n-2] + \delta[n-4] - \delta[n-6]$$

SOL 6.1.83 Option (A) is correct.

We have $\quad h[n] = \frac{3}{4}\left(\frac{1}{4}\right)^{n-1} u[n-1]$

Taking $z$-transform we get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{3}{4}z^{-1}}{1-\frac{1}{4}z^{-1}} \qquad (\tfrac{1}{4})^{n-1}u[n-1] \overset{z}{\longleftrightarrow} z^{-1}\left(\frac{1}{1-\frac{1}{4}z^{-1}}\right)$$

or, $\quad Y(z) - \frac{1}{4}z^{-1}Y(z) = \frac{3}{4}z^{-1}X(z)$

Taking inverse $z$-transform we have

$$y[n]-\frac{1}{4}y[n-1]=\frac{3}{4}x[n-1]$$

SOL 6.1.84 Option (A) is correct.

We have, $h[n]=\delta[n]-\delta[n-5]$

Taking $z$-transform we get

$$H(z)=\frac{Y(z)}{X(z)}=1-z^{-5}$$

or $$Y(z)=X(z)-z^{-5}X(z)$$

Taking inverse $z$-transform we get

$$y[n]=x[n]-x[n-5]$$

SOL 6.1.85 Option (A) is correct.

Taking $z$ transform of all system we get

$$Y_1(z)=0.2z^{-1}Y(z)+X(z)-0.3z^{-1}X(z)+0.02z^{-2}X(z)$$

$$H_1(z)=\frac{Y_1(z)}{X(z)}=\frac{1-0.3z^{-1}+0.02z^{-2}}{1-0.2z^{-1}}$$

$$=\frac{(1-0.2z^{-1})(1-0.1z^{-1})}{(1-0.2z^{-1})}=(1-0.1z^{-1})$$

$$Y_2(z)=X(z)-0.1z^{-1}X(z)$$

$$H_2(z)=\frac{Y_2(z)}{X(z)}=(1-0.1z^{-1})$$

$$Y_3(z)=0.5z^{-1}Y(z)+0.4X(z)-0.3z^{-1}X(z)$$

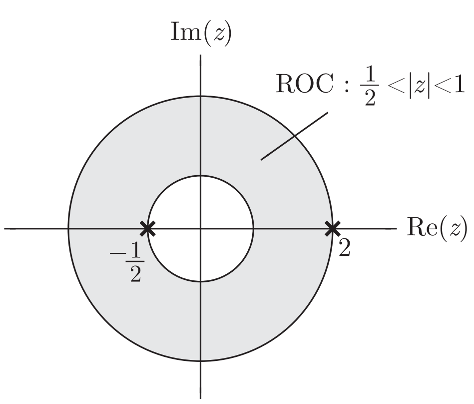
$$H_3(z)=\frac{Y_3(z)}{X(z)}=\frac{0.4-0.3z^{-1}}{1-0.5z^{-1}}$$

$H_1(z)=H_2(z)$, so $y_1$ and $y_2$ are equivalent.

SOL 6.1.86 Option (B) is correct.

We have $$H(z)=\frac{1}{1-2z^{-1}}+\frac{1}{1+\frac{1}{2}z^{-1}}$$

Poles of $H(z)$ are at $z=2$ and $z=-\frac{1}{2}$. Since $h[n]$ is stable, so ROC includes unit circle $|z|=1$ and for the given function it must be $\frac{1}{2}<|z|<2$. The location of poles and ROC is shown in the figure below




Consider the following two cases :

1. ROC is interior to the circle passing through pole at $z=2$, so this term corresponds to a left-sided signal.

$$\frac{1}{1-2z^{-1}}\xleftrightarrow{\mathcal{Z}^{-1}}-(2)^n u[-n-1] \qquad \text{(left-sided)}$$

2. ROC is exterior to the circle passing through pole at $z = -\frac{1}{2}$, so this term corresponds to a right-sided signal.

$$\frac{1}{1+\frac{1}{2}z^{-1}} \xleftrightarrow{\mathcal{Z}^{-1}} \left(\frac{-1}{2}\right)^n u[n] \qquad \text{(right-sided)}$$

Impulse response,

$$h[n] = -(2)^n u[-n-1] + \left(\frac{-1}{2}\right)^n u[n]$$

SOL 6.1.87 Option (B) is correct.

We have
$$H(z) = \frac{5z^2}{z^2 - z - 6} = \frac{5z^2}{(z-3)(z+2)}$$
$$= \frac{5}{(1-3z^{-1})(1+2z^{-1})} = \frac{3}{1-3z^{-1}} + \frac{2}{1+2z^{-1}}$$

Since $h[n]$ is causal, therefore impulse response is obtained by taking right-sided inverse $z$-transform of the transfer function $X(z)$

$$h[n] = [3^{n+1} + 2(-2)^n]u[n]$$

SOL 6.1.88 Option (D) is correct.

Zero at : $z = 0, \frac{2}{3}$, poles at $z = \frac{1 \pm \sqrt{2}}{2}$

(1) For a causal system all the poles of transfer function lies inside the unit circle $|z| = 1$. But, for the given system one of the pole does not lie inside the unit circle, so the system is not causal and stable.

(2) Not all poles and zero are inside unit circle $|z| = 1$, the system is not minimum phase.

SOL 6.1.89 Option (A) is correct.

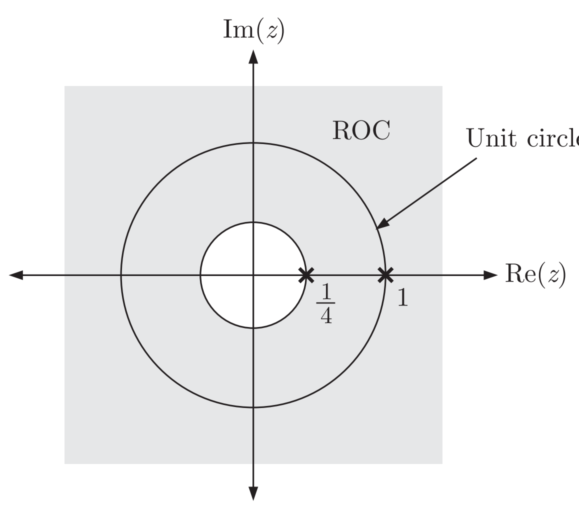
$$X(z) = \frac{-\frac{3}{8}}{1-\frac{1}{3}z^{-1}} + \frac{\frac{27}{8}}{1-3z^{-1}}$$

Poles are at $z = \frac{1}{3}$ and $z = 3$. Since $X(z)$ converges on $|z| = 1$, so ROC must include this circle. Thus for the given signal ROC : $\frac{1}{3} < |z| < 3$

ROC is exterior to the circle passing through the pole at $z = \frac{1}{3}$ so this term will have a right sided inverse $z$-transform. On the other hand ROC is interior to the circle passing through the pole at $z = 3$ so this term will have a left sided inverse $z$-transform.

$$x[n] = -\frac{1}{3^{n-1}8}u[n] - \frac{3^{n+3}}{8}u[-n-1]$$

SOL 6.1.90 Option (C) is correct.




Since ROC includes the unit circle $z = 1$, therefore the system is both stable and causal.

SOL 6.1.91 Option (C) is correct.

(1) Pole of system $z = -\frac{1}{2}, \frac{1}{3}$ lies inside the unit circle $|z| = 1$, so the system is causal and stable.

(2) Zero of system $H(z)$ is $z = -\frac{1}{2}$, therefore pole of the inverse system is at $z = -\frac{1}{2}$ which lies inside the unit circle, therefore the inverse system is also causal and stable.

SOL 6.1.92 Option (C) is correct.

Writing the equation from given block diagram we have

$$[2Y(z) + X(z)]z^{-2} = Y(z)$$

or

$$H(z) = \frac{z^{-2}}{1-2z^{-2}} = -\frac{1}{2} + \frac{\frac{1}{4}}{1-\sqrt{2}\,z^{-1}} + \frac{\frac{1}{4}}{1+\sqrt{2}\,z^{-1}}$$

Taking inverse laplace transform we have

$$h[n] = -\frac{1}{2}\delta[n] + \frac{1}{4}\{(\sqrt{2})^n + (-\sqrt{2})^n\}u[n]$$

SOL 6.1.93 Option (D) is correct.

$$Y(z) = X(z)z^{-1} - \{Y(z)z^{-1} + Y(z)z^{-2}\}$$

$$\frac{Y(z)}{X(z)} = \frac{z^{-1}}{1+z^{-1}+z^{-2}} = \frac{z}{z^2+z+1}$$

So this is a solution but not unique. Many other correct diagrams can be drawn.

***********

# SOLUTIONS 6.2

SOL 6.2.1 Correct answer is 2.

$z$-transform is given as

$$X(z) = \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n z^{-n} = \sum_{n=0}^{\infty}\left(\frac{1}{2z}\right)^n$$

$$= \frac{1}{1-\frac{1}{2z}} = \frac{2z}{2z-1} \quad \text{...(1)}$$

From the given question, we have

$$X(z) = \frac{az}{az-1} \quad \text{...(2)}$$

So, by comparing equations (1) and (2), we get $a = 2$

SOL 6.2.2 Correct answer is $-1.125$ .

The $z$-transform of given sequence is

$$X(z) = z^3 + z^2 - z^1 - z^0$$

$$= z^3 + z^2 - z - 1$$

Now $$X\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 1 = -1.125$$

SOL 6.2.3 Correct answer is 3.

$$X(z) = \frac{z+1}{z(z-1)} = -\frac{1}{z} + \frac{2}{z-1}$$

$$= -\frac{1}{z} + 2z^{-1}\left(\frac{z}{z-1}\right) \quad \text{By partial fraction}$$

Taking inverse $z$-transform

$$x[n] = -\delta[n-1] + 2u[n-1]$$

$$x[0] = -0 + 0 = 0$$

$$x[1] = -1 + 2 = 1$$

$$x[2] = -0 + 2 = 2$$

Thus, we obtain

$$x[0] + x[1] + x[2] = 3$$

SOL 6.2.4 Correct answer is 3.

$$x[n] = \alpha^n u[n]$$

Let, $$y[n] = x[n+3]u[n] = \alpha^{n+3}u[n+3]u[n]$$

$$= \alpha^{n+3}u[n] \quad u[n+3]u[n] = u[n]$$

$$Y(z) = \sum_{n=-\infty}^{\infty} y[n]z^{-n} = \sum_{n=-\infty}^{\infty}\alpha^{n+3}z^{-n}u[n] = \sum_{n=0}^{\infty}\alpha^{n+3}z^{-n}$$

$$= \alpha^3\sum_{n=0}^{\infty}(\alpha z^{-1})^n = \alpha^3\frac{1}{1-\alpha z^{-1}} = \alpha^3\left(\frac{z}{z-\alpha}\right) \quad \text{...(1)}$$

From the given question, we have

$$Y(z) = \alpha^k\left(\frac{z}{z-\alpha}\right) \quad \text{...(2)}$$

So, by comparing equations (1) and (2), we get $k = 3$

NOTE :
Do not apply time shifting property directly because $x[n]$ is a causal signal.

SOL 6.2.5 Correct answer is 10.

We know that

$$\alpha^n u[n] \xleftrightarrow{z} \frac{z}{z-\alpha}$$

$$\alpha^{n-10} u[n-10] \xleftrightarrow{z} \frac{z^{-10} z}{z-\alpha} \qquad \text{(time shifting property)}$$

So, $$x[n] = \alpha^{n-10} u[n-10] \qquad ...(1)$$

From the given question, we have

$$x[n] = \alpha^{n-k} u[n-k] \qquad ...(2)$$

So, by comparing equations (1) and (2), we get

$$k = 10$$

SOL 6.2.6 Correct answer is 9.

We know that $$a^n u[n] \xleftrightarrow{z} \frac{z}{z-a}$$

or $$3^n u[n] \xleftrightarrow{z} \frac{z}{z-3}$$

$$3^{n-3} u[n-3] \xleftrightarrow{z} z^{-3}\left(\frac{z}{z-3}\right)$$

So $$x[n] = 3^{n-3} u[n-3]$$

$$x[5] = 3^2 u[2] = 9$$

SOL 6.2.7 Correct answer is 2.

$$y[n] = n[n+1] u[n]$$

$$y[n] = n^2 u[n] + n u[n]$$

We know that $$u[n] \xleftrightarrow{z} \frac{z}{z-1}$$

Applying the property of differentiation in $z$-domain

If, $$x[n] \xleftrightarrow{z} X(z)$$

then, $$n x[n] \xleftrightarrow{z} -z\frac{d}{dz} X(z)$$

so, $$n u[n] \xleftrightarrow{z} -z\frac{d}{dz}\left(\frac{z}{z-1}\right)$$

or, $$n u[n] \xleftrightarrow{z} \frac{z}{(z-1)^2}$$

Again by applying the above property

$$n(n u[n]) \xleftrightarrow{z} -z\frac{d}{dz}\left[\frac{z}{(z-1)^2}\right]$$

$$n^2 u[n] \xleftrightarrow{z} \frac{z(z+1)}{(z-1)^3}$$

So $$Y(z) = \frac{z}{(z-1)^2} + \frac{z(z+1)}{(z-1)^3} = \frac{2z^2}{(z-1)^3} \qquad ...(1)$$

From the given question, we have

$$x[n] = \frac{kz^k}{(z-1)^{k+1}} \qquad ...(2)$$

So, by comparing equations (1) and (2), we get

$$k = 2$$

SOL 6.2.8 Correct answer is $-1$.

Given that $$X(z) = \log(1-2z), \ |z| < \frac{1}{2}$$

Differentiating

$$\frac{dX(z)}{dz} = \frac{-2}{1-2z} = \frac{z^{-1}}{1-\frac{1}{2}z^{-1}}$$

or,
$$\frac{zdX(z)}{dz} = \frac{1}{1-\frac{1}{2}z^{-1}}$$

From $z$-domain differentiation property

$$nx[n] \overset{z}{\longleftrightarrow} -z\frac{dX(z)}{dz}$$

so,
$$nx[n] \overset{z}{\longleftrightarrow} \frac{-1}{1-\frac{1}{2}z^{-1}}$$

From standard $z$-transform pair, we have

$$\left(\frac{1}{2}\right)^n u[-n-1] \overset{z}{\longleftrightarrow} \frac{-1}{1-\frac{1}{2}z^{-1}}$$

Thus
$$nx[n] = \left(\frac{1}{2}\right)^n u[-n-1]$$

or,
$$x[n] = \frac{1}{n}\left(\frac{1}{2}\right)^n u[-n-1] \qquad \text{...(1)}$$

From the given question, we have

$$x[n] = \frac{1}{n}\left(\frac{1}{2}\right)^n u[a-n] \qquad \text{...(2)}$$

So, by comparing equations (1) and (2), we get $a = -1$

SOL 6.2.9 Correct answer is 2.

$$Y(z) = \sum_{n=-\infty}^{\infty} x\left[\frac{n}{2}\right] z^{-n}$$

$$= \sum_{k=-\infty}^{\infty} x[k] z^{-2k} \qquad \text{Put } \frac{n}{2} = k \text{ or } n = 2k$$

$$= X(z^2) \qquad \text{...(1)}$$

From the given question, we have

$$Y(z) = X(z^k) \qquad \text{...(2)}$$

So, by comparing equations (1) and (2), we get $k = 2$

SOL 6.2.10 Correct answer is 0.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$Y(z) = X(z^3) = \sum_{n=-\infty}^{\infty} x[n](z^3)^{-n} = \sum_{n=-\infty}^{\infty} x[n] z^{-3n}$$

$$= \sum_{k=-\infty}^{\infty} x[k/3] z^{-k} \qquad \text{Put } 3n = k \text{ or } n = k/3$$

Thus
$$y[n] = x[n/3]$$

$$y[n] = \begin{cases} (-0.5)^{n/3}, & n = 0, 3, 6, .... \\ 0, & \text{otherwise} \end{cases}$$

Thus
$$y[4] = 0$$

SOL 6.2.11 Correct answer is $-6$.

By taking $z$-transform of $x[n]$ and $h[n]$

$$H(z) = 1 + 2z^{-1} - z^{-3} + z^{-4}$$

$$X(z) = 1 + 3z^{-1} - z^{-2} - 2z^{-3}$$

From the convolution property of $z$-transform

$$Y(z) = H(z)X(z)$$

$$Y(z) = 1 + 5z^{-1} + 5z^{-2} - 5z^{-3} - 6z^{-4} + 4z^{-5} + z^{-6} - 2z^{-7}$$

Sequence is
$$y[n] = \{1, 5, 5, -5, -6, 4, 1, -2\}$$

$$y[4] = -6$$



SOL 6.2.12 Correct answer is 0.5 .

$x[n]$ can be written as

$$x[n] = \frac{1}{2}[u[n] + (-1)^n u[n]]$$

$z$-transform of $x[n]$

$$X(z) = \frac{1}{2}\left[\frac{1}{1-z^{-1}} + \frac{1}{1+z^{-1}}\right]$$

From final value theorem

$$x(\infty) = \lim_{z \to 1}(z-1)X(z)$$

$$= \frac{1}{2}\lim_{z \to 1}(z-1)\left[\frac{z}{z-1} + \frac{z}{z+1}\right]$$

$$= \frac{1}{2}\lim_{z \to 1}\left[z + \frac{z(z-1)}{(z+1)}\right]$$

$$= \frac{1}{2}(1) = 0.5$$

SOL 6.2.13 Correct answer is 0.5 .

From initial value theorem

$$x[0] = \lim_{z \to \infty} X(z)$$

$$= \lim_{z \to \infty}\frac{0.5z^2}{(z-1)(z-0.5)}$$

$$= \lim_{z \to \infty}\frac{0.5}{\left(1-\frac{1}{z}\right)\left(1-\frac{0.5}{z}\right)} = 0.5$$

SOL 6.2.14 Correct answer is $-2.5$ .

Taking $z$ transform of input and output

$$X(z) = \frac{z}{z-0.5}$$

$$Y(z) = 1 - 2z^{-1} = \frac{z-2}{z}$$

Transfer function of the filter

$$H(z) = Y(z)/X(z)$$

$$= \left(\frac{z-2}{z}\right)\left(\frac{z-0.5}{z}\right) = \frac{z^2 - 2.5z + 1}{z^2}$$

$$= 1 - 2.5z^{-1} + z^{-2}$$

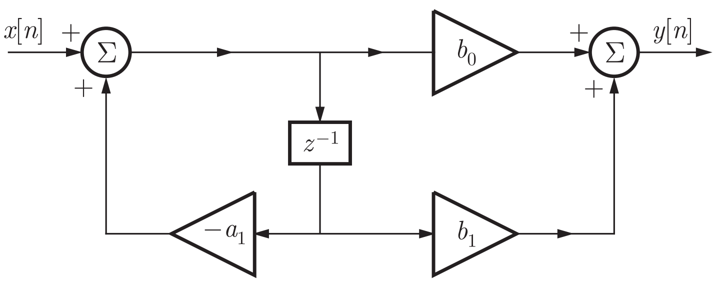
Taking inverse $z$-transform

$$h[n] = \{\underset{\uparrow}{1}, -2.5, 1\}$$

Therefore $h[1] = -2.5$

SOL 6.2.15 Correct answer is 4.

Comparing the given system realization with the generic first order direct form II realization



Difference equation for above realization is

$$y[n]+a_1y[n-1]=b_0x[n]+b_1x[n-1]$$

Here $a_1=-2, b_1=3, b_0=4$

So $\quad y[n]-2y[n-1]=4x[n]+3x[n-1]$

Taking $z$-transform on both sides

$$Y(z)-2z^{-1}Y(z)=4X(z)+3z^{-1}X(z)$$

Transfer function

$$H(z)=\frac{Y(z)}{X(z)}=\frac{4+3z^{-1}}{1-2z^{-1}}=\frac{4z+3}{z-2} \quad ...(1)$$

From the given question, we have

$$H(z)=\frac{Y(z)}{X(z)}=\frac{k(z+1)-1}{z-2} \quad ...(2)$$

So, by comparing equations (1) and (2), we get

$$k=4$$

SOL 6.2.16 Correct answer is 0.3333 .

The $z$-transform of each system response

$$H_1(z)=1+\frac{1}{2}z^{-1},\quad H_2(z)=\frac{z}{z-\frac{1}{2}}$$

The overall system function

$$H(z)=H_1(z)H_2(z)$$

$$=\left(1+\frac{1}{2}z^{-1}\right)\left(\frac{z}{z-\frac{1}{2}}\right)=\frac{(z+\frac{1}{2})}{(z-\frac{1}{2})}$$

Input, $\quad x[n]=\cos(n\pi)$

So, $z=-1$ and $H(z=-1)=\dfrac{-1+\frac{1}{2}}{-1-\frac{1}{2}}=\dfrac{1}{3}$

Output of system

$$y[n]=H(z=-1)x[n]=\frac{1}{3}\cos n\pi \quad ...(1)$$

From the given question, we have

$$y[n]=k\cos n\pi \quad ...(2)$$

So, by comparing equations (1) and (2), we get

$$k=\frac{1}{3}=0.3333$$

SOL 6.2.17 Correct answer is 1.

From the given block diagram

$$Y(z)=\alpha z^{-1}X(z)+\alpha z^{-1}Y(z)$$

$$Y(z)(1-\alpha z^{-1})=\alpha z^{-1}X(z)$$

Transfer function

$$\frac{Y(z)}{X(z)}=\frac{\alpha z^{-1}}{1-\alpha z^{-1}}$$

For stability poles at $z=1$ must be inside the unit circle.

So $\quad |\alpha|<1$

SOL 6.2.18 Correct answer is $-2$.

$$X^+(z)=\sum_{n=0}^{\infty}x[n]z^{-n}=\sum_{n=0}^{\infty}\delta[n-2]z^{-n}=z^{-2} \quad ...(1)$$

$$\sum_{n=-\infty}^{\infty}f[n]\delta[n-n_0]=f[n_0]$$

From the given question, we have

$$X^+(z)=z^{-2} \quad ...(2)$$

So, by comparing equations (1) and (2), we get

$$k = -2$$

SOL 6.2.19 Correct answer is $-1$.

$$X^+(z) = \sum_{n=0}^{\infty} x[n]z^{-n} = \sum_{n=0}^{\infty} z^{-n} = \frac{1}{1-z^{-1}} \qquad ...(1)$$

From the given question, we have

$$X^+(z) = \frac{1}{1+a/z} \qquad ...(2)$$

So, by comparing equations (1) and (2), we get

$$k = -1$$

SOL 6.2.20 Correct answer is 0.0417 .

We know that,

$$\cos\alpha = 1 - \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} - \frac{\alpha^6}{6!} + \frac{\alpha^8}{8!} - ..........$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}\alpha^{2k}$$

Thus, $$X(z) = \cos(z^{-3}) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}(z^{-3})^{2k}, \quad |z| > 0$$

Taking inverse $z$-transform we get

$$x[n] = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}\delta[n-6k]$$

Now for $n = 12$ we get, $12 - 6k = 0 \;\Rightarrow\; k = 2$

Thus, $$x[12] = \frac{(-1)^2}{4!} = \frac{1}{24} = 0.0417$$

SOL 6.2.21 Correct answer is 4.

For anticausal signal initial value theorem is given as,

$$x[0] = \lim_{z\to\infty} X(z) = \lim_{z\to\infty} \frac{12-21z}{3-7z+12z^2} = \frac{12}{3} = 4$$

SOL 6.2.22 Correct answer is 1.12 .

Taking $z$-transform on both sides

$$Y(z) = cz^{-1}Y(z) - 0.12z^{-2}Y(z) + z^{-1}X(z) + z^{-2}X(z)$$

Transfer function,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}+z^{-2}}{1-cz^{-1}+0.12z^{-2}} = \frac{z+1}{z^2-cz+0.12}$$

Poles of the system are $z = \dfrac{c \pm \sqrt{c^2-0.48}}{2}$.

For stability poles should lie inside the unit circle, so $|z| < 1$

$$\left|\frac{c \pm \sqrt{c^2-0.48}}{2}\right| < 1$$

Solving this inequality, we get $|c| < 1.12$.

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# SOLUTIONS 6.3

| Answers | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | (B) | 5. | (C) | 9. | (A) | 13. | (D) | 17. | (C) | 21. | (C) | 25. | (B) |
| 2. | (D) | 6. | (C) | 10. | (C) | 14. | (C) | 18. | (A) | 22. | (D) | 26. | (B) |
| 3. | (C) | 7. | (D) | 11. | (C) | 15. | (B) | 19. | (A) | 23. | (C) | | |
| 4. | (D) | 8. | (A) | 12. | (A) | 16. | (D) | 20. | (B) | 24. | (C) | | |

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# SOLUTIONS 6.4



SOL 6.4.1 Option (C) is correct.

$z$-transform of $x[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = \sum_{n=-\infty}^{\infty} \alpha^n u[n]z^{-n}$$

$$= \sum_{n=0}^{\infty} (\alpha z^{-1})^n = \frac{1}{1-\alpha z^{-1}} = \frac{z}{z-\alpha}$$

SOL 6.4.2 Option (B) is correct.

We have $$x[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

$$X(z) = \sum_{k=0}^{\infty} x[n]z^{-n} = \sum_{n=-\infty}^{\infty}\left[\sum_{k=0}^{\infty}\delta[n-k]z^{-n}\right]$$

Since $\delta[n-k]$ defined only for $n = k$ so

$$X(z) = \sum_{k=0}^{\infty} z^{-k} = \frac{1}{(1-1/z)} = \frac{z}{(z-1)}$$

SOL 6.4.3 Option (A) is correct.

We have $$f(nT) = a^{nT}$$

Taking $z$-transform we get

$$F(z) = \sum_{n=-\infty}^{\infty} a^{nT} z^{-n} = \sum_{n=-\infty}^{\infty} (a^T)^n z^{-n}$$

$$= \sum_{n=0}^{\infty}\left(\frac{a^T}{z}\right)^n = \frac{z}{z-a^T}$$

SOL 6.4.4 Option ( ) is correct.

SOL 6.4.5 Option (A) is correct.

$$x[n] = b^n u[n] + b^{-n} u[-n-1]$$

$z$-transform of $x[n]$ is given as

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} b^n u[n]z^{-n} + \sum_{n=-\infty}^{\infty} b^{-n} u[-n-1]z^{-n}$$

$$= \sum_{n=0}^{\infty} b^n z^{-n} + \sum_{n=-\infty}^{-1} b^{-n} z^{-n}$$

In second summation, Let $n = -m$

$$X(z) = \sum_{n=0}^{\infty} b^n z^{-n} + \sum_{m=1}^{\infty} b^m z^m$$

$$= \underbrace{\sum_{n=0}^{\infty} (bz^{-1})^n}_{\mathrm{I}} + \underbrace{\sum_{m=1}^{\infty} (bz)^m}_{\mathrm{II}}$$

Summation I converges, if $|bz^{-1}| < 1$ or $|z| > |b|$

Summation II converges, if $|bz| < 1$ or $|z| < \frac{1}{|b|}$

since $|b| < 1$ so from the above two conditions ROC : $|z| < 1$.

SOL 6.4.6 Option (B) is correct.

$z$-transform of signal $a^n u[n]$ is

$$X(z) = \sum_{n=-\infty}^{\infty} a^n u[n] z^{-n} = \underbrace{\sum_{n=0}^{\infty} (az^{-1})^n}_{\mathrm{I}} \qquad u[n] = 1,\ n \geq 0$$

$$= \frac{1}{1-az^{-1}} = \frac{z}{z-a}$$

Similarly, $z$-transform of signal $a^n u[-n-1]$ is

$$X(z) = \sum_{n=-\infty}^{\infty} -a^n u[-n-1] z^{-n}$$

$$= -\sum_{n=-\infty}^{-1} a^n z^{-n} \qquad \because u[-n-1] = 1,\ n \leq -1$$

Let $n = -m$, then

$$X(z) = -\sum_{m=1}^{\infty} a^{-m} z^m = -\underbrace{\sum_{m=1}^{\infty} (a^{-1}z)^m}_{\mathrm{II}}$$

$$= -\frac{a^{-1}z}{1-a^{-1}z} = \frac{z}{z-a}$$

$z$-transform of both the signal is same.

(A) is true

ROC : To obtain ROC we find the condition for convergences of $X(z)$ for both the transform.

Summation I converges if $|a^{-1}z| < 1$ or $|z| > a$, so ROC for $a^n u[n]$ is $|z| > |a|$

Summation II converges if $|a^{-1}z| < 1$ or $|z| < |a|$, so ROC for $-a^n u[-n-1]$ is $|z| < |a|$.

(R) is true, but (R) is NOT the correct explanation of (A).

SOL 6.4.7 Option (B) is correct.

$z$- transform of $x[n]$ is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x[n] r^{-n} e^{-j\Omega n} \qquad \text{Putting } z = re^{j\Omega}$$

$z$-transform exists if $|X(z)| < \infty$

$$\sum_{n=-\infty}^{\infty} |x[n] r^{-n} e^{-j\Omega n}| < \infty$$

or

$$\sum_{n=-\infty}^{\infty} |x[n] r^{-n}| < \infty$$

Thus, $z$-transform exists if $x[n] r^{-n}$ is absolutely summable.

SOL 6.4.8 Option (A) is correct.

$$x[n] = \left(\frac{1}{3}\right)^n u[n] - \left(\frac{1}{2}\right)^n u[-n-1]$$

Taking $z$ transform we have

$$X(z) = \sum_{n=0}^{n=\infty} \left(\frac{1}{3}\right)^n z^{-n} - \sum_{n=-\infty}^{n=-1} \left(\frac{1}{2}\right)^n z^{-n}$$

$$= \sum_{n=0}^{n=\infty} \left(\frac{1}{3} z^{-1}\right)^n - \sum_{n=-\infty}^{n=-1} \left(\frac{1}{2} z^{-1}\right)^n$$

First term gives

$$\frac{1}{3} z^{-1} < 1 \to \frac{1}{3} < |z|$$

Second term gives

$$\frac{1}{2} z^{-1} > 1 \to \frac{1}{2} > |z|$$

Thus its ROC is the common ROC of both terms. that is

$$\frac{1}{3} < |z| < \frac{1}{2}$$



SOL 6.4.9 Option (C) is correct.

Here $x_1[n] = \left(\frac{5}{6}\right)^n u[n]$

$$X_1(z) = \frac{1}{1-\left(\frac{5}{6}z^{-1}\right)} \qquad \text{ROC}: R_1 \rightarrow |z| > \frac{5}{6}$$

$$x_2[n] = -\left(\frac{6}{5}\right)^n u[-n-1]$$

$$X_1(z) = 1 - \frac{1}{1-\left(\frac{6}{5}z^{-1}\right)} \qquad \text{ROC}: R_2 \rightarrow |z| < \frac{6}{5}$$

Thus ROC of $x_1[n] + x_2[n]$ is $R_1 \cap R_2$ which is $\frac{5}{6} < |z| < \frac{6}{5}$

SOL 6.4.10 Option (A) is correct.

$$x[n] = 2^n u[n]$$

$z$-transform of $x[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = \sum_{n=-\infty}^{\infty} 2^n u[n] z^{-n}$$

$$= \sum_{n=0}^{\infty} (2z^{-1})^n = 1 + 2z^{-1} + (2z^{-1})^2 + \dots$$

$$= \frac{1}{1-2z^{-1}}$$

the above series converges if $|2z^{-1}| < 1$ or $|z| > 2$

SOL 6.4.11 Option (A) is correct.

We have $h[n] = u[n]$

$$H(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = \sum_{n=0}^{\infty} 1z^{-n} = \sum_{n=0}^{\infty} (z^{-1})^n$$

$H(z)$ is convergent if

$$\sum_{n=0}^{\infty} (z^{-1})^n < \infty$$

and this is possible when $|z^{-1}| < 1$. Thus ROC is $|z^{-1}| < 1$ or $|z| > 1$

SOL 6.4.12 Option (B) is correct.

(Please refer to table 6.1 of the book Gate Guide signals & Systems by same authors)

(A) $u[n] \xleftrightarrow{z} \frac{z}{z-1}$ (A → 3)

(B) $\delta[n] \xleftrightarrow{z} 1$ (B → 1)

(C) $\sin \omega t \big|_{t=nT} \xleftrightarrow{z} \frac{z \sin \omega T}{z^2 - 2z\cos\omega T + 1}$ (C → 4)

(D) $\cos \omega t \big|_{t=nT} \xleftrightarrow{z} \frac{z - \cos\omega T}{z^2 - 2z\cos\omega t + 1}$ (D → 2)

SOL 6.4.13 Option (A) is correct.

Inverse $z$-transform of $X(z)$ is given as

$$x[n] = \frac{1}{2\pi j}\oint X(z) z^{n-1} dz$$

SOL 6.4.14 Option (B) is correct.

$$H(z) = z^{-m}$$

so $h[n] = \delta[n-m]$

SOL 6.4.15 Option (C) is correct.

We know that

$$\alpha^n u[n] \xleftrightarrow{z} \frac{1}{1-\alpha z^{-1}}$$

For $\alpha = 1$, $$u[n] \xleftrightarrow{z} \frac{1}{1-z^{-1}}$$

SOL 6.4.16 Option (B) is correct.

$$X(z) = \frac{0.5}{1-2z^{-1}}$$

Since ROC includes unit circle, it is left handed system

$$x[n] = -(0.5)(2)^{-n} u[-n-1]$$

$$x(0) = 0$$

If we apply initial value theorem

$$x(0) = \lim_{z \to \infty} X(z) = \lim_{z \to \infty} \frac{0.5}{1-2z^{-1}} = 0.5$$

That is wrong because here initial value theorem is not applicable because signal $x[n]$ is defined for $n < 0$.

SOL 6.4.17 Option (B) is correct.

$z$-transform $$F(z) = \frac{1}{z+1} = 1 - \frac{z}{z+1} = 1 - \frac{1}{1+z^{-1}}$$

so, $$f(k) = \delta(k) - (-1)^k$$

Thus $$(-1)^k \xleftrightarrow{z} \frac{1}{1+z^{-1}}$$

SOL 6.4.18 Option (C) is correct.

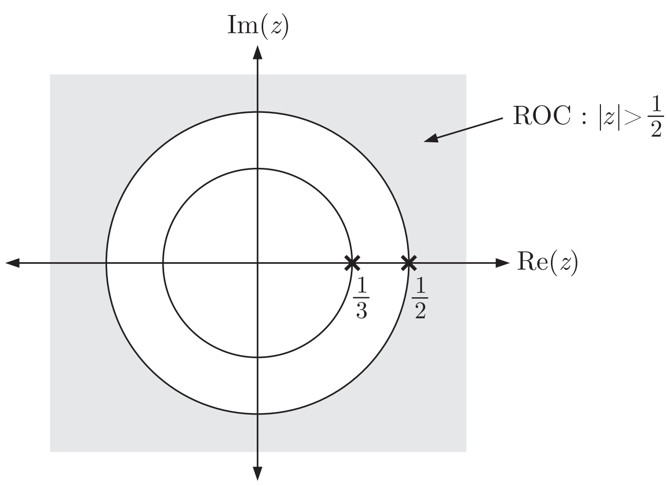
$$X(z) = \frac{z\left(2z-\frac{5}{6}\right)}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}$$

$$\frac{X(z)}{z} = \frac{\left(2z-\frac{5}{6}\right)}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)} = \frac{1}{\left(z-\frac{1}{2}\right)} + \frac{1}{\left(z-\frac{1}{3}\right)}$$

or $$X(z) = \underbrace{\frac{z}{\left(z-\frac{1}{2}\right)}}_{\text{term I}} + \underbrace{\frac{z}{\left(z-\frac{1}{3}\right)}}_{\text{term II}} \qquad ...(1)$$

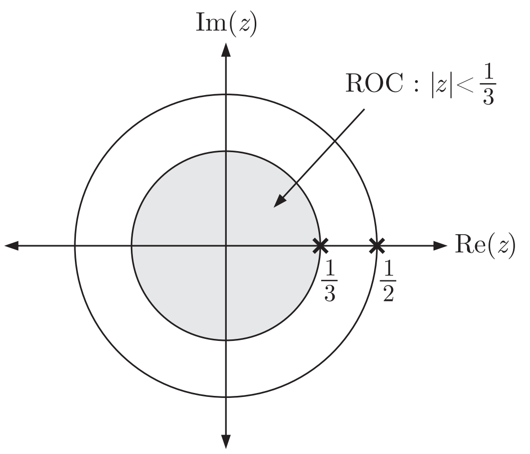
Poles of $X(z)$ are at $z = \frac{1}{2}$ and $z = \frac{1}{3}$

ROC : $|z| > \frac{1}{2}$ Since ROC is outside to the outer most pole so both the terms in equation (1) corresponds to right sided sequence.



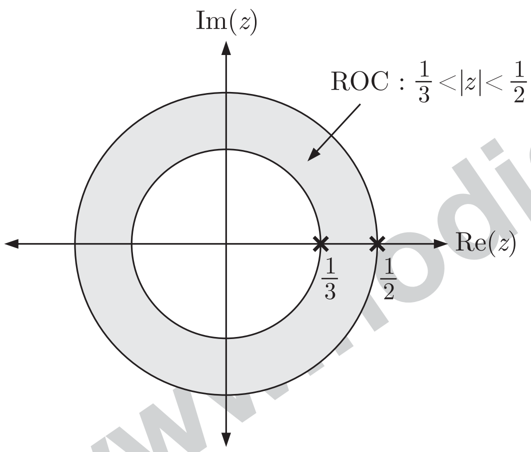

So, $$x[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{3}\right)^n u[n] \qquad (A \to 4)$$

ROC : $|z| < \frac{1}{3}$ :Since ROC is inside to the innermost pole so both the terms in equation (1) corresponds to left sided signals.




So, $$x[n] = -\left(\frac{1}{2}\right)^n u[-n-1] - \left(\frac{1}{3}\right)^n u[-n-1] \qquad (D \to 2)$$

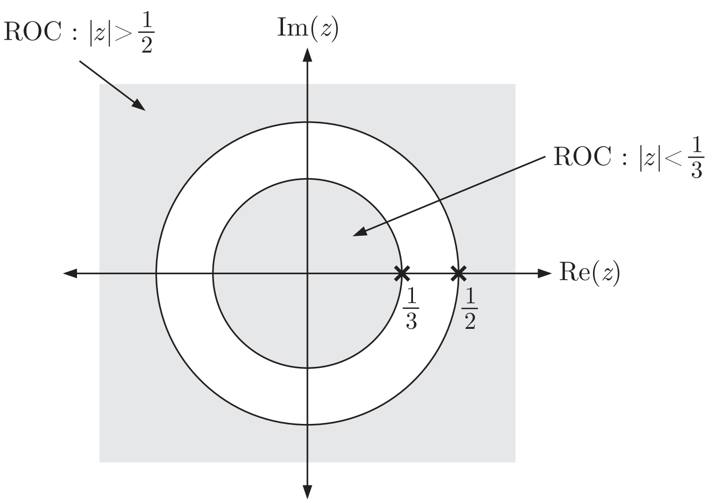
ROC : $\frac{1}{3} < |z| < \frac{1}{2}$ : ROC is outside to the pole $z = \frac{1}{3}$, so the second term of equation (1) corresponds to a causal signal. ROC is inside to the pole at $z = \frac{1}{2}$, so First term of equation (1) corresponds to anticausal signal.






So, $$x[n] = -\left(\frac{1}{2}\right)^n u[-n-1] + \left(\frac{1}{3}\right)^n u[n] \qquad (C \to 1)$$

ROC : $|z| < \frac{1}{3}$ & $|z| > \frac{1}{2}$ : ROC : $|z| < \frac{1}{3}$ is inside the pole at $z = \frac{1}{3}$ so second term of equation (1) corresponds to anticausal signal. On the other hand, ROC : $|z| > \frac{1}{2}$ is outside to the pole at $z = \frac{1}{2}$, so the first term in equation (1) corresponds to a causal signal.




So, $$x[n] = \left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{3}\right)^n u[-n-1] \qquad (B \to 3)$$

SOL 6.4.19 Option (A) is correct.

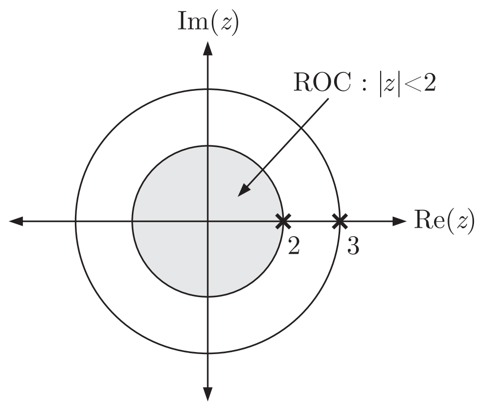
Given, $$X(z) = \frac{z}{(z-2)(z-3)}, \ |z| < 2$$

$$\frac{X(z)}{z} = \frac{1}{(z-2)(z-3)} = \frac{1}{z-3} - \frac{1}{z-2} \qquad \text{By partial fraction}$$

or, $$X(z) = \frac{z}{z-3} - \frac{z}{z-2} \qquad ...(1)$$

Poles of $X(z)$ are $z = 2$ and $z = 3$

ROC : $|z| < 2$




Since ROC is inside the innermost pole of $X(z)$, both the terms in equation (1) corresponds to anticausal signals.

$$x[n] = -3^n u[-n-1] + 2^n u[-n-n] = (2^n - 3^n) u[-n-1]$$

SOL 6.4.20 Option (D) is correct.

Given that $$X(z) = \frac{z}{(z-a)^2}, \ |z| > a$$

Residue of $X(z) z^{n-1}$ at $z = a$ is

$$= \frac{d}{dz}(z-a)^2 X(z) z^{n-1}\Big|_{z=a}$$

$$= \frac{d}{dz}(z-a)^2 \frac{z}{(z-a)^2} z^{n-1}\Bigg|_{z=a}$$

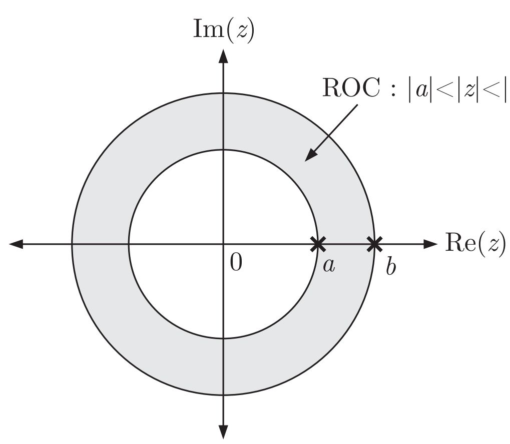
$$= \frac{d}{dz} z^n\Big|_{z=a} = n z^{n-1}\Big|_{z=a} = n a^{n-1}$$

SOL 6.4.21 Option (C) is correct.

$$X(z) = \frac{\frac{1}{2}}{1-az^{-1}} + \frac{\frac{1}{3}}{1-bz^{-1}}, \qquad \text{ROC} : |a| < |z| < |b|$$

Poles of the system are $z = a$, $z = b$

ROC : $|a| < |z| < |b|$



Since ROC is outside to the pole at $z = a$, therefore the first term in $X(z)$ corresponds to a causal signal.

$$\frac{\frac{1}{2}}{1 - az^{-1}} \xleftrightarrow{\mathcal{Z}^{-1}} \frac{1}{2}(a)^n u[n]$$

ROC is inside to the pole at $z = b$, so the second term in $X(z)$ corresponds to a anticausal signal.

$$\frac{\frac{1}{3}}{1 - bz^{-1}} \xleftrightarrow{\mathcal{Z}^{-1}} -\frac{1}{3}(b)^n u[-n-1]$$

$$x[n] = \frac{1}{2}(a)^n u[n] - \frac{1}{3}(b)^n u[-n-1]$$

$$x[0] = \frac{1}{2}u[0] - \frac{1}{3}u[-1] = \frac{1}{2}$$

SOL 6.4.22 Option (A) is correct.

$$X(z) = \frac{(z + z^{-3})}{(z + z^{-1})} = \frac{z(1 + z^{-4})}{z(1 + z^{-2})}$$
$$= (1 + z^{-4})(1 + z^{-2})^{-1}$$

Writing binomial expansion of $(1 + z^{-2})^{-1}$, we have

$$X(z) = (1 + z^{-4})(1 - z^{-2} + z^{-4} - z^{-6} + \dots)$$
$$= 1 - z^{-2} + 2z^{-4} - 2z^{-6} + \dots$$

For a sequence $x[n]$, its $z$-transform is

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

Comparing above two

$$x[n] = \delta[n] - \delta[n-2] + 2\delta[n-4] - 2\delta[n-6] + \dots$$
$$= \{1, 0, -1, 0, 2, 0, -2, \dots\}$$

$x[n]$ has alternate zeros.

SOL 6.4.23 Option (A) is correct.

We know that $\alpha Z^{\pm a} \xleftrightarrow{\mathcal{Z}^{-1}} \alpha\delta[n \pm a]$

Given that $X(z) = 5z^2 + 4z^{-1} + 3$

Inverse $z$-transform $x[n] = 5\delta[n+2] + 4\delta[n-1] + 3\delta[n]$

SOL 6.4.24 Option (A) is correct.

$$X(z) = e^{1/z}$$

$$X(z) = e^{1/z} = 1 + \frac{1}{z} + \frac{1}{\lfloor 2\, z^2} + \frac{1}{\lfloor 3\, z^3} + \dots$$

$z$-transform of $x[n]$ is given by

$$X(z) = \sum_{n=0}^{\infty} x[n] z^{-n} = x[0] + \frac{x[1]}{z} + \frac{x[2]}{z^2} + \frac{x[3]}{z^3} + \dots$$

Comparing above two

$$\{x[0], x[1], x[2], x[3] \dots\} = \left\{1, \frac{1}{\lfloor 1}, \frac{1}{\lfloor 2}, \frac{1}{\lfloor 3}, \dots\right\}$$

$$x[n] = \frac{1}{\lfloor n} u[n]$$

SOL 6.4.25 Option (D) is correct.

The ROC of addition or subtraction of two functions $x_1[n]$ and $x_2[n]$ is $R_1 \cap R_2$. We have been given ROC of addition of two function and has been asked ROC of subtraction of two function. It will be same.

SOL 6.4.26 Option (D) is correct.

(A) $$x[n] = \alpha^n u[n]$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} = \sum_{n=0}^{\infty} \alpha^n z^{-n} \qquad u[n] = 1,\ n \geq 0$$

$$= \sum_{n=0}^{\infty} (\alpha z^{-1})^n$$

$$= \frac{1}{1-\alpha z^{-1}}, \quad |\alpha z^{-1}| < 1 \text{ or } |z| > |\alpha| \qquad (A \to 2)$$

(B) $$x[n] = -\alpha^n u[-n-1]$$

$$X(z) = -\sum_{n=-\infty}^{\infty} \alpha^n u[-n-1]z^{-n}$$

$$= -\sum_{n=-\infty}^{-1} \alpha^n z^{-n} \qquad u[-n-1] = 1,\ n \leq -1$$

Let $n = -m$, $$X(z) = -\sum_{m=1}^{\infty} \alpha^{-m} z^m = -\sum_{m=1}^{\infty} (\alpha^{-1}z)^m$$

$$= \frac{-\alpha^{-1}z}{1-\alpha^{-1}z}, \quad |\alpha^{-1}z| < 1 \text{ or } |z| < |\alpha|$$

$$= \frac{1}{(1-\alpha z^{-1})}, \quad |z| < |\alpha| \qquad (B \to 3)$$

(C) $$x[n] = -n\alpha^n u[-n-1]$$

We have,

$$-\alpha^n u[-n-1] \overset{Z}{\longleftrightarrow} \frac{1}{(1-\alpha z^{-1})}, \quad |z| < |\alpha|$$

From the property of differentiation in $z$-domain

$$-n\alpha^n u[-n-1] \overset{Z}{\longleftrightarrow} -z\frac{d}{dz}\left[\frac{1}{1-\alpha z^{-1}}\right], \quad |z| < |\alpha|$$

$$\overset{Z}{\longleftrightarrow} \frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}, \quad |z| < |\alpha| \qquad (C \to 4)$$

(D) $$x[n] = n\alpha^n u[n]$$

We have, $$\alpha^n u[n] \overset{Z}{\longleftrightarrow} \frac{1}{(1-\alpha z^{-1})}, \quad |z| > |\alpha|$$

From the property of differentiation in $z$-domain

$$n\alpha^n u[n] \overset{Z}{\longleftrightarrow} -z\frac{d}{dz}\left[\frac{1}{(1-\alpha z^{-1})}\right], \quad |z| > |\alpha|$$

$$\overset{Z}{\longleftrightarrow} \frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}, \quad |z| > |\alpha| \qquad (D \to 1)$$

SOL 6.4.27 Given that, $z$ transform of $x[n]$ is

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$z$-transform of $\{x[n]e^{j\omega_0 n}\}$

$$Y(z) = \sum_{n=-\infty}^{\infty} x[n]e^{j\omega_0 n}z^{-n} = \sum_{n=-\infty}^{\infty} x[n](ze^{-j\omega_0})^{-n} = X(z')\big|_{z' = ze^{-j\omega_0}}$$

so, $$Y(z) = X(ze^{-j\omega_0})$$

SOL 6.4.28 Option (A) is correct.

SOL 6.4.29 Option (C) is correct.

We know that,

$$a^n u[n] \overset{Z}{\longleftrightarrow} \frac{z}{z-a} \qquad (A \to 3)$$

From time shifting property

$$a^{n-2}u[n-2] \overset{z}{\longleftrightarrow} z^{-2}\frac{z}{z-a} \qquad (B \to 4)$$

From the property of scaling in $z$-domain

If, $\qquad x[n] \overset{z}{\longleftrightarrow} X(z)$

then, $\qquad \alpha^n x[n] \overset{z}{\longleftrightarrow} X\left(\frac{z}{\alpha}\right)$

so $$(e^j)^n a^n \overset{z}{\longleftrightarrow} \frac{\left(\frac{z}{e^j}\right)}{\left(\frac{z}{e^j}-a\right)} = \frac{ze^{-j}}{ze^{-j}-a} \qquad (C \to 2)$$

From the property of differentiation in $z$-domain

If, $\qquad a^n u[n] \overset{z}{\longleftrightarrow} \frac{z}{z-a}$

then, $$na^n u[n] \overset{z}{\longleftrightarrow} \frac{d}{dz}\left(\frac{z}{z-a}\right) = \frac{az}{(z-a)^2} \qquad (D \to 1)$$

SOL 6.4.30

SOL 6.4.31 Option (C) is correct.

The convolution of a signal $x[n]$ with unit step function $u[n]$ is given by

$$y[n] = x[n] * u[n] = \sum_{k=0}^{\infty} x[k]$$

Taking $z$-transform

$$Y(z) = X(z)\frac{1}{1-z^{-1}}$$

SOL 6.4.32 Option (B) is correct.

From the property of $z$-transform.

$$x_1[n] * x_2[n] \overset{z}{\longleftrightarrow} X_1(z)X_2(z)$$

SOL 6.4.33 Option (C) is correct.

Given $z$ transform

$$C(z) = \frac{z^{-1}(1-z^{-4})}{4(1-z^{-1})^2}$$

Applying final value theorem

$$\lim_{n\to\infty} f(n) = \lim_{z\to 1}(z-1)f(z)$$

$$\begin{aligned}\lim_{z\to 1}(z-1)F(z) &= \lim_{z\to 1}(z-1)\frac{z^{-1}(1-z^{-4})}{4(1-z^{-1})^2}\\ &= \lim_{z\to 1}\frac{z^{-1}(1-z^{-4})(z-1)}{4(1-z^{-1})^2}\\ &= \lim_{z\to 1}\frac{z^{-1}z^{-4}(z^4-1)(z-1)}{4z^{-2}(z-1)^2}\\ &= \lim_{z\to 1}\frac{z^{-3}}{4}\frac{(z-1)(z+1)(z^2+1)(z-1)}{(z-1)^2}\\ &= \lim_{z\to 1}\frac{z^{-3}}{4}(z+1)(z^2+1) = 1\end{aligned}$$

SOL 6.4.34 Option (C) is correct.

$$\begin{aligned}H_1(z) &= 1 + 1.5z^{-1} - z^{-2}\\ &= 1 + \frac{3}{2z} - \frac{1}{z^2} = \frac{2z^2+3z-2}{2z^2}\end{aligned}$$

Poles $z^2 = 0 \Rightarrow z = 0$

zeros $\quad (2z^2+3z-2) = 0 \Rightarrow \left(z-\frac{1}{2}\right)(z+2) = 0 \Rightarrow z = \frac{1}{2}, z = -2$

zeros of the two systems are identical.

SOL 6.4.35 Option (D) is correct.

Taking $z$-transform on both sides of given equation.

$$z^3 Y(z) + 6z^2 Y(z) + 11z Y(z) + 6Y(z) = z^2 R(z) + 9zR(z) + 20R(z)$$

Transfer function

$$\frac{Y(z)}{R(z)} = \frac{z^2 + 9z + 20}{z^3 + 6z^2 + 11z + 6}$$

SOL 6.4.36 Option (A) is correct.

Characteristic equation of the system

$$|zI - A| = 0$$

$$zI - A = \begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ \beta & -\alpha \end{bmatrix} = \begin{bmatrix} z & -1 \\ \beta & z+\alpha \end{bmatrix}$$

$$|zI - A| = z(z+\alpha) + \beta = 0$$

$$z^2 + z\alpha + \beta = 0$$

In the given options, only option (A) satisfies this characteristic equation.

$$c[k+2] + \alpha c[k+1] + \beta c[k] = u[k]$$

$$z^2 + z\alpha + \beta = 0$$

SOL 6.4.37 Option (B) is correct.

We can see that the given impulse response is decaying exponential, i.e.

$$h[n] = a^n u[n], \quad 0 < a < 1$$

$z$-transform of $h[n]$ $\quad H(z) = \frac{z}{z-a}$

Pole of the transfer function is at $z = a$, which is on real axis between $0 < a < 1$.

SOL 6.4.38 Option (A) is correct.

$$y[n] + y[n-1] = x[n] - x[n-1]$$

Taking $z$-transform

$$Y(z) + z^{-1} Y(z) = X(z) - z^{-1} X(z)$$

$$Y(z) = \frac{(1 - z^{-1})}{(1 + z^{-1})} \qquad \text{which has a linear phase response.}$$

SOL 6.4.39 Option (A) is correct.

For the linear phase response output is the delayed version of input multiplied by a constant.

$$y[n] = kx[n - n_0]$$

$$Y(z) = kz^{-n_0} X(z) = \frac{kX(z)}{z^{n_0}}$$

Pole lies at $z = 0$

SOL 6.4.40 Option (B) is correct.

Given impulse response can be expressed in mathematical form as

$$h[n] = \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3] + \ldots.$$

By taking $z$-transform

$$H(z) = 1 - z^{-1} + z^{-2} - z^{-3} + z^{-4} - z^{-5} + \ldots.$$

$$= (1 + z^{-2} + z^{-4} + \ldots.) - (z^{-1} + z^{-3} + z^{-5} + \ldots.)$$

$$= \frac{1}{1 - z^{-2}} - \frac{z^{-1}}{1 - z^{-2}} = \frac{z^2}{z^2 - 1} - \frac{z}{z^2 - 1}$$

$$= \frac{(z^2 - z)}{(z^2 - 1)} = \frac{z(z-1)}{(z-1)(z+1)} = \frac{z}{z+1} \qquad \text{Pole at } z = -1$$

SOL 6.4.41 Option (B) is correct.

Let Impulse response of system $h[n] \overset{\mathcal{Z}}{\longleftrightarrow} H(z)$

First consider the case when input is unit step.

Input, $x_1[n] = u[n]$ or $X_1(z) = \frac{z}{(z-1)}$

Output, $y_1[n] = \delta[n]$ or $Y_1(z) = 1$

so,
$$Y_1(z) = X_1(z)H(z)$$
$$1 = \frac{z}{(z-1)}H(z)$$

Transfer function, $H(z) = \frac{(z-1)}{z}$

Now input is ramp function
$$x_2[n] = nu[n]$$
$$X_2(z) = \frac{z}{(z-1)^2}$$

Output,
$$Y_2(z) = X_2(z)H(z)$$
$$= \left[\frac{z}{(z-1)^2}\right]\left[\frac{(z-1)}{(z)}\right] = \frac{1}{(z-1)}$$
$$Y_2(z) \overset{\mathcal{Z}^{-1}}{\longleftrightarrow} y_2[n]$$
$$\frac{1}{(z-1)} \overset{\mathcal{Z}^{-1}}{\longleftrightarrow} u[n-1]$$

SOL 6.4.42 Option (C) is correct.

Given state equations
$$s[n+1] = As[n] + Bx[n] \qquad ...(1)$$
$$y[n] = Cs[n] + Dx[n] \qquad ...(2)$$

Taking $z$-transform of equation (1)
$$zS(z) = AS(z) + BX(z)$$
$$S(z)[zI - A] = BX(z) \qquad I \rightarrow \text{unit matrix}$$
$$S(z) = (zI - A)^{-1}BX(z) \qquad ...(3)$$

Now, taking $z$-transform of equation (2)
$$Y(z) = CS(z) + DX(z)$$

Substituting $S(z)$ from equation (3), we get
$$Y(z) = C(zI - A)^{-1}BX(z) + DX(z)$$

Transfer function
$$H(z) = \frac{Y(z)}{X(z)} = C(zI - A)^{-1}B + D$$

SOL 6.4.43 Option (B) is correct.
$$F(z) = 4z^3 - 8z^2 - z + 2$$
$$F(z) = 4z^2(z-2) - z(z-2)$$
$$= (4z^2 - 2)(z-2)$$
$$4z^2 - 2 = 0 \text{ and } (z-2) = 0$$
$$z = \pm\frac{1}{2} \text{ and } z = 2$$

Only one root lies outside the unit circle.

SOL 6.4.44 Option (A) is correct.

We know that convolution of $x[n]$ with unit step function $u[n]$ is given by
$$x[n] * u[n] = \sum_{k=-\infty}^{n} x[k]$$

so $$y[n] = x[n] * u[n]$$

Taking $z$-transform on both sides

$$Y(z) = X(z)\frac{z}{(z-1)} = X(z)\frac{1}{(1-z^{-1})}$$

Transfer function,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{(1-z^{-1})}$$

Now consider the inverse system of $H(z)$, let impulse response of the inverse system is given by $H_1(z)$, then we can write

$$H(z)H_1(z) = 1$$

$$H_1(z) = \frac{X(z)}{Y(z)} = 1 - z^{-1}$$

$$(1-z^{-1})Y(z) = X(z)$$

$$Y(z) - z^{-1}Y(z) = X(z)$$

Taking inverse $z$-transform

$$y[n] - y[n-1] = x[n]$$

SOL 6.4.45 Option (B) is correct.

$$y[n] - \frac{1}{2}y[n-1] = x[n]$$

Taking $z$-transform on both sides

$$Y(z) - \frac{1}{2}z^{-1}Y(z) = X(z)$$

Transfer function $$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-\frac{1}{2}z^{-1}}$$

Now, for input $x[n] = k\delta[n]$Output is

$$Y(z) = H(z)X(z)$$

$$= \frac{k}{(1-\frac{1}{2}z^{-1})} \qquad X(z) = k$$

Taking inverse $z$-transform

$$y[n] = k\left(\frac{1}{2}\right)^n u[n] = k\left(\frac{1}{2}\right)^n, \quad n \geq 0$$

SOL 6.4.46 Option (A) is correct.

$$y[n] + y[n-1] = x[n]$$

For unit step response, $x[n] = u[n]$

$$y[n] + y[n-1] = u[n]$$

Taking $z$-transform $$Y(z) + z^{-1}Y(z) = \frac{z}{z-1}$$

$$(1+z^{-1})Y(z) = \frac{z}{(z-1)}$$

$$\frac{(1+z)}{z}Y(z) = \frac{z}{(z-1)}$$

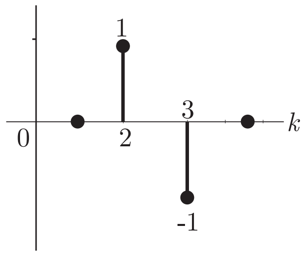
$$Y(z) = \frac{z^2}{(z+1)(z-1)}$$

SOL 6.4.47 Option (A) is correct.

SOL 6.4.48 Option (A) is correct.

SOL 6.4.49 Option (A) is correct.

We have $h(2) = 1$, $h(3) = -1$ otherwise $h[k] = 0$. The diagram of response is as follows :




It has the finite magnitude values. So it is a finite impulse response filter. Thus $S_2$ is true but it is not a low pass filter. So $S_1$ is false.

SOL 6.4.50 Option (D) is correct.

$$H(z) = \frac{z}{z-0.2} \qquad |z| < 0.2$$

We know that

$$-a^n u[-n-1] \longleftrightarrow \frac{1}{1-az^{-1}} \qquad |z| < a$$

Thus $\quad h[n] = -(0.2)^n u[-n-1]$

SOL 6.4.51 Option (B) is correct.

We have $\quad h[n] = 3\delta[n-3]$

or $\quad H(z) = 2z^{-3} \qquad$ Taking $z$ transform

$$X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$$

Now $\quad Y(z) = H(z)X(z)$

$$= 2z^{-3}(z^4 + z^2 - 2z + 2 - 3z^{-4})$$

$$= 2(z + z^{-1} - 2z^{-2} + 2z^{-3} - 3z^{-7})$$

Taking inverse $z$ transform we have

$$y[n] = 2[\delta[n+1] + \delta[n-1] - 2\delta[n-2] + 2\delta[n-3] - 3\delta[n-7]]$$

At $n = 4$,

$$y[4] = 0$$



SOL 6.4.52 Option (A) is correct.

$z$-transform of $x[n]$ is

$$X(z) = 4z^{-3} + 3z^{-1} + 2 - 6z^2 + 2z^3$$

Transfer function of the system

$$H(z) = 3z^{-1} - 2$$

Output, $\quad Y(z) = H(z)X(z)$

$$= (3z^{-1} - 2)(4z^{-3} + 3z^{-1} + 2 - 6z^2 + 2z^3)$$

$$= 12z^{-4} + 9z^{-2} + 6z^{-1} - 18z + 6z^2 - 8z^{-3} - 6z^{-1} - 4 + 12z^2 - 4z^3$$

$$= 12z^{-4} - 8z^{-3} + 9z^{-2} - 4 - 18z + 18z^2 - 4z^3$$

Or sequence $y[n]$ is

$$y[n] = 12\delta[n-4] - 8\delta[n-3] + 9\delta[n-2] - 4\delta[n]$$
$$- 18\delta[n+1] + 18\delta[n+2] - 4\delta[n+3]$$

$$y[n] \neq 0,\ n < 0$$

So $y[n]$ is non-causal with finite support.

SOL 6.4.53 Option (C) is correct.

Impulse response of given LTI system.

$$h[n] = x[n-1] * y[n]$$

Taking $z$-transform on both sides.

$$H(z) = z^{-1}X(z)\,Y(z) \qquad x[n-1] \xleftrightarrow{z} z^{-1}x(z)$$

We have $X(z) = 1 - 3z^{-1}$ and $Y(z) = 1 + 2z^{-2}$

So $$H(z) = z^{-1}(1-3z^{-1})(1+2z^{-2})$$

Output of the system for input $u[n] = \delta[n-1]$ is,

$$y(z) = H(z)U(z) \qquad U[n] \xleftrightarrow{z} U(z) = z^{-1}$$

So $$\begin{aligned} Y(z) &= z^{-1}(1-3z^{-1})(1+2z^{-2})z^{-1} \\ &= z^{-2}(1-3z^{-1}+2z^{-2}-6z^{-3}) \\ &= z^{-2} - 3z^{-3} + 2z^{-4} - 6z^{-5} \end{aligned}$$

Taking inverse z-transform on both sides we have output.

$$y[n] = \delta[n-2] - 3\delta[n-3] + 2\delta[n-4] - 6\delta[n-5]$$

SOL 6.4.54 Option (D) is correct.

$$H(z) = (1 - az^{-1})$$

We have to obtain inverse system of $H(z)$. Let inverse system has response $H_1(z)$.

$$H(z)H_1(z) = 1$$

$$H_1(z) = \frac{1}{H(z)} = \frac{1}{1-az^{-1}}$$

For stability $H(z) = (1-az^{-1})$, $|z| > a$ but in the inverse system $|z| < a$, for stability of $H_1(z)$.

so $$h_1[n] = -a^n u[-n-1]$$

SOL 6.4.55 Option (C) is correct.

$$H(z) = \frac{z}{z+\frac{1}{2}}$$

Pole, $$z = -\frac{1}{2}$$

The system is stable if pole lies inside the unit circle. Thus (A) is true, (R) is false.

SOL 6.4.56 Option (A) is correct.

Difference equation of the system.

$$y[n+2] - 5y[n+1] + 6y[n] = x[n]$$

Taking $z$-transform on both sides of above equation.

$$\begin{aligned} z^2Y(z) - 5zY(z) + 6Y(z) &= X(z) \\ (z^2 - 5z + 6)y(z) &= X(z) \end{aligned}$$

Transfer function,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{(z^2-5z+6)} = \frac{1}{(z-3)(z-2)}$$

Roots of the characteristic equation are $z = 2$ and $z = 3$

We know that an LTI system is unstable if poles of its transfer function (roots of characteristic equation) lies outside the unit circle. Since, for the given system the roots of characteristic equation lies outside the unit circle $(z = 2, z = 3)$ so the system is unstable.

SOL 6.4.57 Option (C) is correct.

System function, $$H(z) = \frac{z^2+1}{(z+0.5)(z-0.5)}$$

Poles of the system lies at $z = 0.5$, $z = -0.5$. Since, poles are within the unit

circle, therefore the system is stable.
From the initial value theorem

$$h[0] = \lim_{z\to\infty} H(z) = \lim_{z\to\infty}\frac{(z^2+1)}{(z+0.5)(z-0.5)}$$

$$= \lim_{z\to\infty}\frac{\left(1+\frac{1}{z^2}\right)}{\left(1+\frac{0.5}{z}\right)\left(1-\frac{0.5}{z}\right)} = 1$$

SOL 6.4.58 Option (D) is correct.

$$y[n] = 2x[n] + 4x[n-1]$$

Taking $z$-transform on both sides

$$Y(z) = 2X(z) + 4z^{-1}X(z)$$

Transfer Function,

$$H(z) = \frac{Y(z)}{X(z)} = 2 + 4z^{-1} = \frac{2z+4}{z}$$

Pole of $H(z)$, $z = 0$
Since Pole of $H(z)$ lies inside the unit circle so the system is stable.
(A) is not True.

$$H(z) = 2 + 4z^{-1}$$

Taking inverse $z$-transform

$$h[n] = 2\delta[n] + 4\delta[n-1] = \{2, 4\}$$

Impulse response has finite number of non-zero samples.
(R) is true.

SOL 6.4.59 Option (B) is correct.
For left sided sequence we have

$$-a^n u[-n-1] \xleftrightarrow{z} \frac{1}{1-az^{-1}} \qquad \text{where } |z| < a$$

Thus $$-5^n u[-n-1] \xleftrightarrow{z} \frac{1}{1-5z^{-1}} \qquad \text{where } |z| < 5$$

or $$-5^n u[-n-1] \xleftrightarrow{z} \frac{z}{z-5} \qquad \text{where } |z| < 5$$

Since ROC is $|z| < 5$ and it include unit circle, system is stable.

ALTERNATIVE METHOD :

$$h[n] = -5^n u[-n-1]$$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n} = \sum_{n=-\infty}^{-1} -5^n z^{-n} = -\sum_{n=-\infty}^{-1}(5z^{-1})^n$$

Let $n = -m$, then

$$H(z) = -\sum_{n=-1}^{-\infty}(5z^{-1})^{-m} = 1 - \sum_{m=0}^{\infty}(5^{-1}z)^{-m}$$

$$= 1 - \frac{1}{1-5^{-1}z}, \qquad |5^{-1}z| < 1 \text{ or } |z| < 5$$

$$= 1 - \frac{5}{5-z} = \frac{z}{z-5}$$

SOL 6.4.60 Option (B) is correct.
For a system to be stable poles of its transfer function $H(z)$ must lie inside the unit circle. In inverse system poles will appear as zeros, so zeros must be inside the unit circle.

SOL 6.4.61 Option (C) is correct.
An LTI discrete system is said to be BIBO stable if its impulse response $h[n]$

is summable, that is

$$\sum_{n=-\infty}^{\infty} h[n] < \infty$$

$z$-transform of $h[n]$ is given as

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n}$$

Let $z = e^{j\Omega}$ (which describes a unit circle in the $z$-plane), then

$$\begin{aligned} |H(z)| &= \left|\sum_{n=-\infty}^{\infty} h[n] e^{-j\Omega n}\right| \\ &= \sum_{n=-\infty}^{\infty} \left|h[n] e^{-j\Omega n}\right| \\ &= \sum_{n=-\infty}^{\infty} |h[n]| < \infty \end{aligned}$$

which is the condition of stability. So LTI system is stable if ROC of its system function includes the unit circle $|z| = 1$.

(A) is true.

We know that for a causal system, the ROC is outside the outermost pole. For the system to be stable ROC should include the unit circle $|z| = 1$. Thus, for a system to be causal & stable these two conditions are satisfied if all the poles are within the unit circle in $z$-plane.

(R) is false.

SOL 6.4.62 Option (B) is correct.

We know that for a causal system, the ROC is outside the outermost pole. For the system to be stable ROC should include the unit circle $|z| = 1$. Thus, for a system to be causal & stable these two conditions are satisfied if all the poles are within the unit circle in $z$-plane.

(A) is true.

If the $z$-transform $X(z)$ of $x[n]$ is rational then its ROC is bounded by poles because at poles $X(z)$ tends to infinity.

(R) is true but (R) is not correct explanation of (A).

SOL 6.4.63 Option (C) is correct.

We have,

$$\begin{aligned} H(z) &= \frac{2 - \frac{3}{4}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} \\ &= \frac{1}{(1 - \frac{1}{2}z^{-1})} + \frac{1}{(1 - \frac{1}{4}z^{-1})} \qquad \text{By partial fraction} \end{aligned}$$

For ROC : $|z| > 1/2$

$$h[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{4}\right)^n u[n],\ n > 0 \qquad \frac{1}{1 - z^{-1}} = a^n u[n], |z| > a$$

Thus, system is causal. Since ROC of $H(z)$ includes unit circle, so it is stable also. Hence $S_1$ is True

For ROC : $|z| < \frac{1}{4}$

$$h[n] = -\left(\frac{1}{2}\right)^n u[-n-1] + \left(\frac{1}{4}\right)^n u[n], \qquad |z| > \frac{1}{4}, |z| < \frac{1}{2}$$

System is not causal. ROC of $H(z)$ does not include unity circle, so it is not stable and $S_3$ is True.

SOL 6.4.64 Option (C) is correct.

We have $\quad 2y[n] = \alpha y[n-2] - 2x[n] + \beta x[n-1]$

Taking $z$ transform we get

$$2Y(z) = \alpha Y(z) z^{-2} - 2X(z) + \beta X(z) z^{-1}$$

or $$\frac{Y(z)}{X(z)} = \left(\frac{\beta z^{-1} - 2}{2 - \alpha z^{-2}}\right) \qquad ...(1)$$

or $$H(z) = \frac{z(\frac{\beta}{2} - z)}{(z^2 - \frac{\alpha}{2})}$$

It has poles at $\pm\sqrt{\alpha/2}$ and zero at 0 and $\beta/2$. For a stable system poles must lie inside the unit circle of $z$ plane. Thus

$$\left|\sqrt{\frac{\alpha}{2}}\right| < 1$$

or $$|\alpha| < 2$$

But zero can lie anywhere in plane. Thus, $\beta$ can be of any value.

SOL 6.4.65 Option (D) is correct.

Let $H_1(z)$ and $H_2(z)$ are the transfer functions of systems $s_1$ and $s_2$ respectively. For the second order system, transfer function has the following form

$$H_1(z) = az^{-2} + bz^{-1} + c$$

$$H_2(z) = pz^{-2} + qz^{-1} + r$$

Transfer function of the cascaded system

$$\begin{aligned} H(z) &= H_1(z) H_2(z) \\ &= (az^{-2} + bz^{-1} + c)(pz^{-2} + qz^{-1} + r) \\ &= apz^{-4} + (aq + bp) z^{-3} + (ar + cp) z^{-2} + (br + qc) z^{-1} + cr \end{aligned}$$

So, impulse response $h[n]$ will be of order 4.

SOL 6.4.66 Option (B) is correct.

Output is equal to input with a delay of two units, that is

$$y(t) = x(t-2)$$

$$Y(z) = z^{-2} X(z)$$

Transfer function,

$$H(z) = \frac{Y(z)}{X(z)} = z^{-2}$$

For the cascaded system, transfer function

$$H(z) = H_1(z) H_2(z)$$

$$z^{-2} = \frac{(z-0.5)}{(z-0.8)} H_2(z)$$

$$H_2(z) = \frac{z^{-1} - 0.8z^{-2}}{z - 0.5} = \frac{z^{-2} - 0.8z^{-3}}{1 - 0.5z^{-1}}$$

SOL 6.4.67 Option (B) is correct.

$$y[n] = x[n-1]$$

or $$Y(z) = z^{-1} X(z)$$

or $$\frac{Y(z)}{X(z)} = H(z) = z^{-1}$$

Now $$H_1(z) H_2(z) = z^{-1}$$

$$\left(\frac{1 - 0.4z^{-1}}{1 - 0.6z^{-1}}\right) H_2(z) = z^{-1}$$

$$H_2(z) = \frac{z^{-1}(1 - 0.6z^{-1})}{(1 - 0.4z^{-1})}$$

SOL 6.4.68 Option (C) is correct.

We have $\qquad h_1[n] = \delta[n-1] or\ H_1[Z] = Z^{-1}$

and $\qquad h_2[n] = \delta[n-2] or\ H_2(Z) = Z^{-2}$

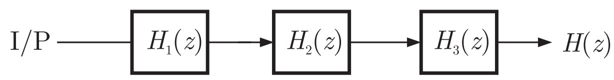
Response of cascaded system

$$H(z) = H_1(z) \cdot H_2(z) = z^{-1} \cdot z^{-2} = z^{-3}$$

or, $\qquad h[n] = \delta[n-3]$

SOL 6.4.69 Option (B) is correct.

Let three LTI systems having response $H_1(z), H_2(z)$ and $H_3(z)$ are Cascaded as showing below




Assume $\qquad H_1(z) = z^2 + z^1 + 1$ (non-causal)

$\qquad H_2(z) = z^3 + z^2 + 1$ (non-causal)

Overall response of the system

$$\begin{aligned} H(z) &= H_1(z) H_2(z) H_3(z) \\ &= (z^2 + z^1 + 1)(z^3 + z^2 + 1) H_3(z) \end{aligned}$$

To make $H(z)$ causal we have to take $H_3(z)$ also causal.

Let $\qquad H_3(z) = z^{-6} + z^{-4} + 1$

$$H(z) = (z^2 + z^1 + 1)(z^3 + z^2 + 1)(z^{-6} + z^{-4} + 1)$$

$$H(z) \to \text{causal}$$

Similarly to make $H(z)$ unstable atleast one of the system should be unstable.

SOL 6.4.70 Option (B) is correct.

$$H(z) = \frac{1 + az^{-1} + bz^{-2}}{1 + cz^{-1} + dz^{-2} + ez^{-3}}$$

We know that number of minimum delay elements is equal to the highest power of $z^{-1}$ present in the denominator of $H(z)$.

No. of delay elements $= 3$

SOL 6.4.71 Option (A) is correct.

From the given system realization, we can write

$$\left(X(z) + Y(z) z^{-2} a_2 + Y(z) a_1 z^{-1}\right) \times a_0 = Y(z)$$

System Function

$$\begin{aligned} H(z) = \frac{Y(z)}{X(z)} &= \frac{a_0}{1 - a_1 z^{-1} - a_2 z^{-2}} \\ &= \frac{1}{\frac{1}{a_0} - \frac{a_1}{a_0} z^{-1} - \frac{a_2}{a_0} z^{-2}} \end{aligned}$$

Comparing with given $H(z)$

$$\frac{1}{a_0} = 1 \Rightarrow a_0 = 1$$

$$-\frac{a_1}{a_0} = -0.7 \Rightarrow a_1 = 0.7$$

$$-\frac{a_2}{a_0} = 0.13 \Rightarrow a_2 = -0.13$$

SOL 6.4.72 Option (B) is correct.

Let, $\quad M \to$ highest power of $z^{-1}$ in numerator.

$\qquad N \to$ highest power of $z^{-1}$ in denominator

Number of delay elements in direct form-I realization equals to $M + N$

Number of delay elements in direct form-II realization equal to $N$.
Here, $M = 3$, $N = 3$
So delay element in direct form-I realization will be 6 and in direct form realization will be 3.

SOL 6.4.73 Option (A) is correct.
System response is given as

$$H(z) = \frac{G(z)}{1 - KG(z)}$$

$$g[n] = \delta[n-1] + \delta[n-2]$$

$$G(z) = z^{-1} + z^{-2}$$

So $$H(z) = \frac{(z^{-1}+z^{-2})}{1 - K(z^{-1}+z^{-2})} = \frac{z+1}{z^2 - Kz - K}$$

For system to be stable poles should lie inside unit circle.

$$|z| \le 1$$

$$z = \frac{K \pm \sqrt{K^2+4K}}{2} \le 1$$

$$K \pm \sqrt{K^2+4K} \le 2$$

$$\sqrt{K^2+4K} \le 2 - K$$

$$K^2 + 4K \le 4 - 4K + K^2$$

$$8K \le 4$$

$$K \le 1/2$$

SOL 6.4.74 Option (B) is correct.
Input-output relationship of the system

$$y[n] = x[n] + ay[n-1]$$

Taking $z$-transform

$$Y(z) = X(z) + z^{-1}aY(z)$$

Transform Function,

$$\frac{Y(z)}{X(z)} = \frac{1}{1 - z^{-1}a}$$

Pole of the system $(1 - z^{-1}a) = 0 \Rightarrow z = a$
For stability poles should lie inside the unit circle $|z| < 1$ so $|a| < 1$.

SOL 6.4.75 Option (D) is correct.
The relation ship between Laplace transform and $z$-transform is given as

$$X(s) = X(z)\big|_{z = e^{st}}$$

$$z = e^{sT} \quad \text{...(1)}$$

We know that $$z = re^{j\Omega} \quad \text{...(2)}$$

and $$s = \sigma + j\omega \quad (3)$$

From equation (1), (2) and (3), we can write

$$z = re^{j\Omega} = e^{(\sigma + j\omega)T} = e^{\sigma T}e^{j\omega T}$$

From above relation we can find that $|z| = e^{\sigma T}$ and $\Omega = \omega T$. It is concluded that,

- If $\sigma = 0$ then $|z| = 1$, the $j\omega$-axis of $s$-plane maps into unit circle.
- If $\sigma < 0$, $|z| < 1$, it implies that left half of $s$-plane maps into inside of unit circle $(|z| < 1)$.
- Similarly, if $\sigma > 0$, $|z| > 1$ which implies that right half of $s$-plane maps into outside of unit circle.

SOL 6.4.76 Option (D) is correct.

The relation ship between Laplace transform and $z$-transform is given as

$$X(s) = X(z)\big|_{z=e^{st}}$$

$$z = e^{sT} \quad \text{...(1)}$$

We know that $z = re^{j\Omega}$ ...(2)

and $s = \sigma + j\omega$ (3)

From equation (1), (2) and (3), we can write

$$z = re^{j\Omega} = e^{(\sigma + j\omega)T}$$

$$= e^{\sigma T} e^{j\omega T}$$

$$z = re^{j\Omega} = e^{\sigma T} e^{j\omega T}$$

From above relation we can find that $|z| = e^{\sigma T}$ and $\Omega = \omega T$. It is concluded that,

- If $\sigma = 0$ then $|z| = 1$, the $j\omega$-axis of $s$-plane maps into unit circle.
- If $\sigma < 0$, $|z| < 1$, it implies that left half of $s$-plane maps into inside of unit circle $(|z| < 1)$.
- Similarly, if $\sigma > 0$, $|z| > 1$ which implies that right half of $s$-plane maps into outside of unit circle.

SOL 6.4.77 Option (C) is correct.

Ideal sampler output is given by

$$f(t) = \sum_{n=0}^{\infty} K_n \delta[t - nT_s]$$

where $T_s \rightarrow$ sampling period

$n \rightarrow$ integer

$$f(t) = K_0\delta[n] + K_1\delta[n-1] + K_2\delta[n-2] + ...$$

$$\mathcal{Z}[f(t)] = K_0 + K_1 z^{-1} + K_2 z^{-2} + ... + K_n z^{-n}$$

SOL 6.4.78 Option (B) is correct.

We know that $X(s) = X(z)\big|_{z=e^{sT}}$

so, $z = e^{sT}$

$$\ln z = sT$$

$$s = \frac{\ln z}{T}$$

SOL 6.4.79 Option (D) is correct.

SOL 6.4.80 Option (C) is correct.

$$H(s) = \frac{a}{s^2 + a^2}$$

Poles in $s$-domain are at $s = \pm ja$. In $z$-domain poles will be at $z = e^{sT}$, so

$$z_1 = e^{-jaT} \text{ and } z_2 = e^{jaT}$$

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