Network Theory :-

1. Basics
2. Steady state A.C circuits (Resonance)
3. Theorems
4. Transients
5. Two - port
6. Magnetic coupled circuits and Graph theory
7. Filters
8. Synthesis → IES

Books :-

→ Fundamentals of electric circuits
By Alexander Sadiku

→ Engg. circuit analysis
By Hayt and kemmerly

→ Networks and systems
By D. Roy choudhury

→ Network Analysis
By Van Valkenburg

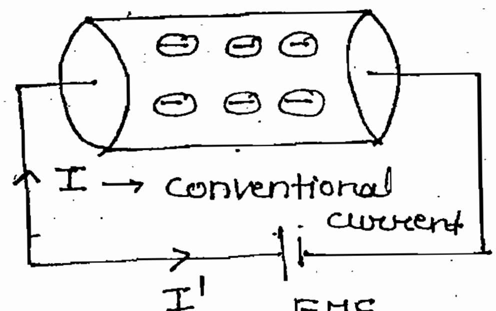
Lecture - 1

Charge :-

→ The basic quantity in the electric circuit is charge

→ The charge on the electron is given $(-1.6 \times 10^{-19} C)$

→ The flow of $\bar{e}$'s is called as current



OR

The time rate of charge is also called as current.

$$\boxed{I = \frac{dQ}{dt}} \quad C/s \text{ or } A$$

→ By using conventional current direction KVL and KCL equations are developed

→ To move the $\bar{e}$ from one point to other point in particular direction external force is required. In electric ckt external force is provided by EMF and it is given by

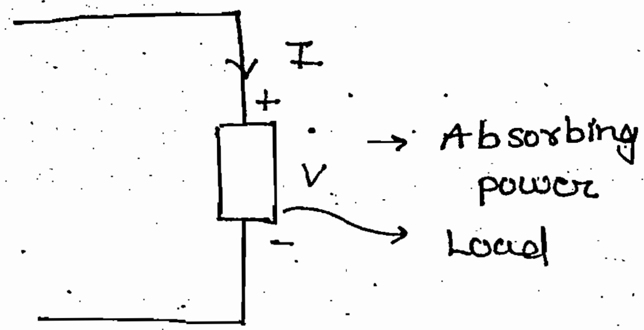
$$\boxed{V = \frac{dW}{dQ}} \quad J/C \text{ or volts}$$

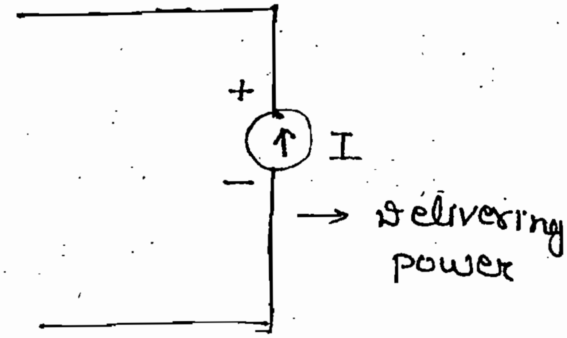
→ The time rate of energy is called as power

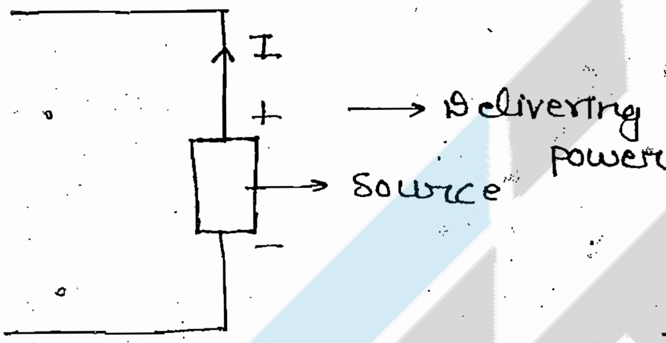
$$P = \frac{dW}{dt} \quad J/s \text{ or Watts}$$

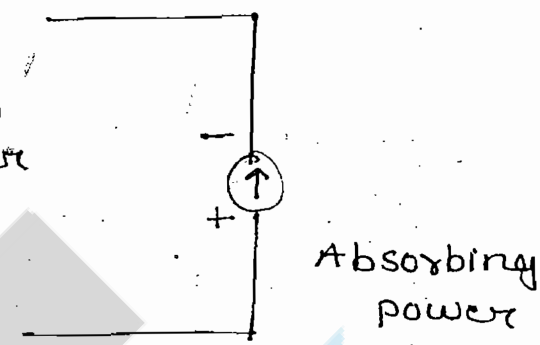
$$P = \frac{dW}{dQ} \cdot \frac{dQ}{dt}$$

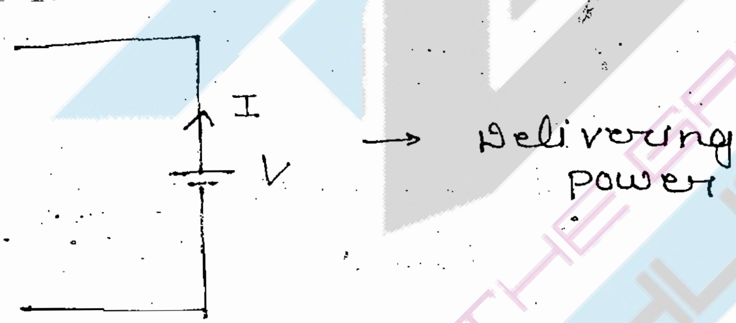
$$\boxed{P = V \cdot I}$$

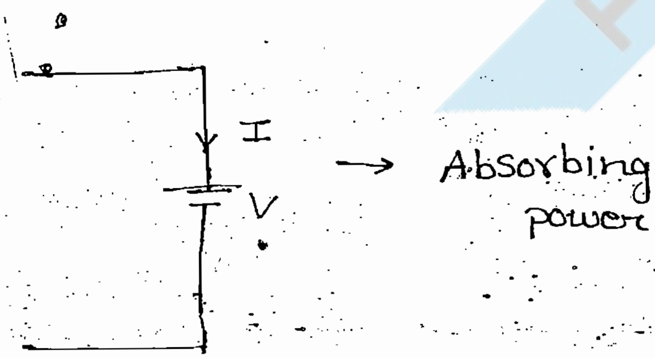










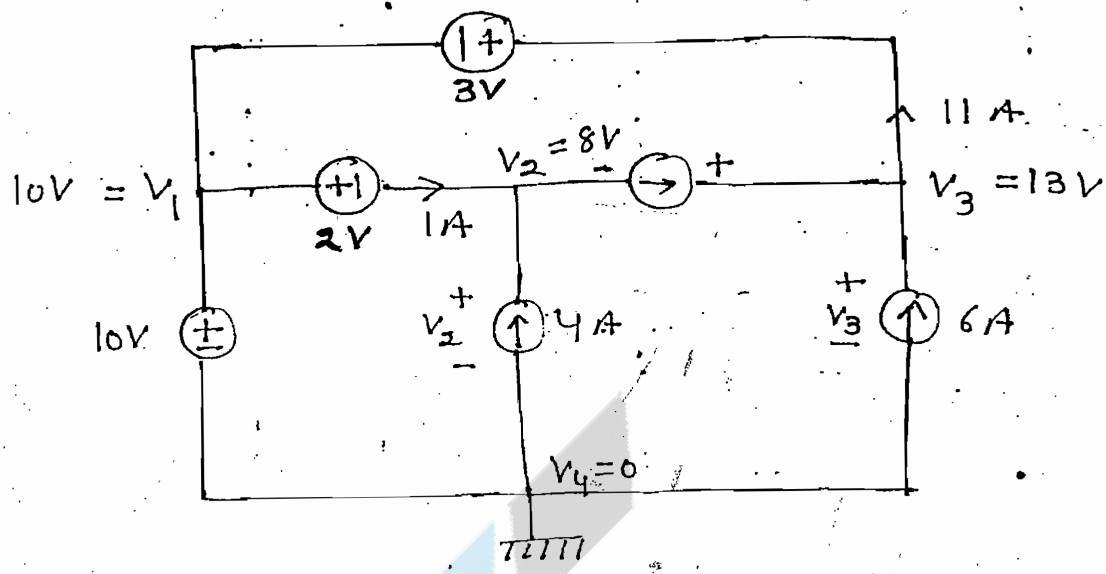


Note :-

→ When current is entering into the terminal element is absorbing power

→ When current is leaving from the terminal element is delivering power

Ques:- Find power of each element of the circuit shown



Soln:- $V_1 - V_2 = 2 \Rightarrow V_2 = 8V$

$V_3 - V_1 = 3 \Rightarrow V_3 = 13\ V$

$P_4 = 4 \times 8 = 32W$ (Delivering Power)

$P_6 = 13 \times 6 = 78W$ (" ")

$P_5 = 5 \times 5 = 25W$ (" ")

$P_{10} = 10 \times 10 = 100W$ (Absorbing Power)

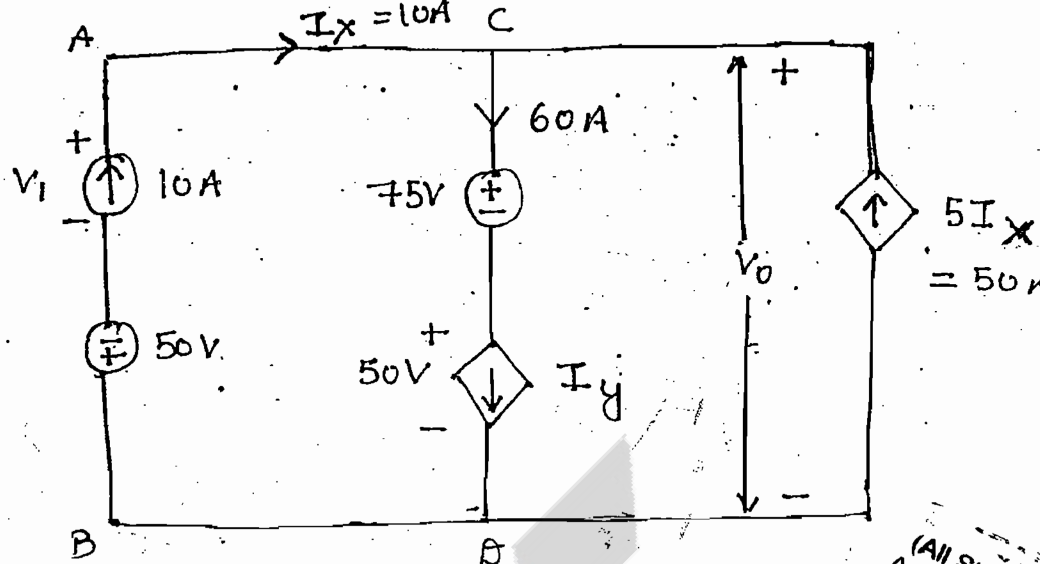
$P_3 = 3 \times 11 = 33W$ (" ")

$P_2 = 1 \times 2 = 2W$ (" ")

$(P_T)_{Absorbing} = (P_T)_{Delivering}$

$\Rightarrow 135W = 135W$

Ques:- Find power developed in a ckt when V_0 =
$V_0 = 125\ V$



Soln:-

$$V_{AB} = V_1 - 50$$

$$\Rightarrow 125 = V_1 - 50 \quad \Rightarrow \quad V_1 = 175$$

$$P_{5I_x} = 125 \times 50 = 6250\ W$$

$$P_{10} = 175 \times 10 = 1750\ W$$

$$P_T = 6250 + 1750 = 8000\ W$$

Classification of elements :-

(I) Active and Passive

(II) Linear and Non-linear

(III) Uni-direction and Bi-direction

(IV) Time varient and invarient

(V) Lumped and Distributed

Active & Passive Element :-

Active Element :-

When the element is capable of delivering energy for long time (approximately ∞ time) → A·E

OR When the element is having property of internal amplification then the element is called as active element

transistor

Elements

- Active
  - Independent
    - Ideal
      - V
      - I
    - Practical
      - V
      - I
  - Dependent
    - VCVS
    - CCVS
    - CCCS
    - VCCS
- Passive
  - R
    - LR
    - NLR
  - C
    - LC
    - NLC
  - L
    - LL
    - NLL

Voltage source , current source → Independen

Transient , op-amp → Dependent

→ During discharging capacitor (inductor) can deliver energy independtly for a short time and capacitor (inductor) is not having property of internal amplification

Passive Elements :-

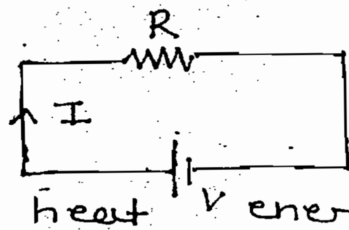
When the element is not capable of delivering energy independtly then the element is called as passive element.

eg:- Resistor, bulb, transformer

↓ It can't step-up or down power

Resistor :-

→ Resistance is a property of resistor.. It always opposes the current. By doing so it converts electric energy to heat energy



→ Resistance is nothing but a friction to flow of $\bar{e}$'s

$$P = I^2 R$$

$$W = I^2 R t$$

↓ Heat

$$R = \rho \frac{l}{a} \quad \Omega$$

ρ → Ω-m

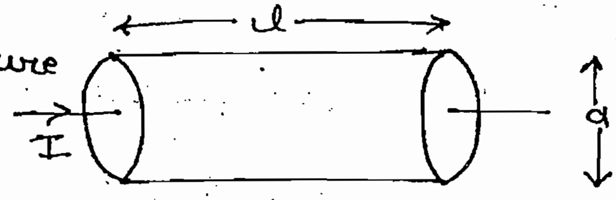
- → conductors
- → semi-conductors
- → Insulators

$$\boxed{\begin{array}{l}\text{Super conductor} \\ \rho = 0 \\ \text{eg:- Mercury at } 4.15\,K\end{array}}$$

$$R_1 = R_0 (1 + \alpha_0 t)$$

where
R_0 = Resistance at 0°C
t = Change in tempt.

Ohm's Law :-



→ At constant temperature current density is directly proportional to electric field intensity

→ At constant tempt. potential diff. across the element is directly proportional to current flowing into element.

$$J = \frac{I}{a} \quad A/m^2$$

$$E = \frac{V}{l} \quad Volts/m$$

$$\sigma = \frac{1}{\rho} \quad mho/m$$

$$R = \rho \frac{l}{a}$$

$$J \propto E$$

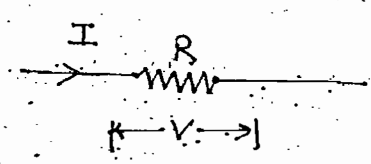
$$\Rightarrow J = \sigma E$$

$$\Rightarrow \frac{I}{a} = \frac{1}{\rho} \frac{V}{l}$$

$$\Rightarrow \boxed{\frac{V}{I} = \rho \frac{l}{a} = R}$$

$$V \propto I$$

$$V = RI$$



$$\boxed{R = \frac{V}{I} = \text{constant}}$$

Different forms of Ohm's law :—

$$J = \sigma E \quad \text{—(I)}$$

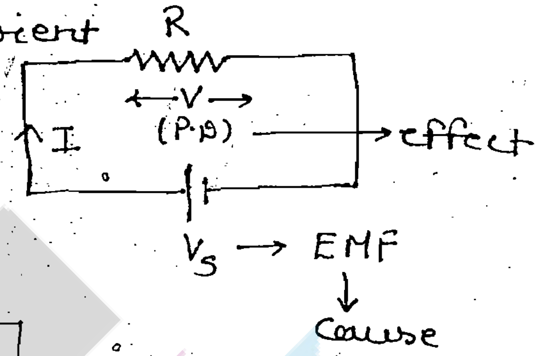
$$V = RI \quad \text{—(II)}$$

$$I = GV \quad \text{—(III)} \qquad \boxed{G = \frac{1}{R}} \text{ mho}$$

$$V = R\frac{dq}{dt} \quad \text{—(IV)}$$

→ EMF is independent on current and resistance magnitude

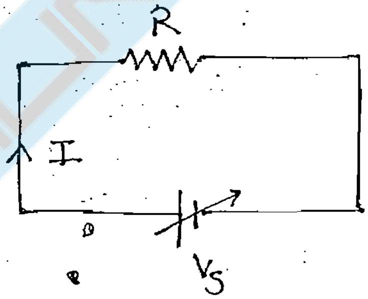
→ Potential difference depends on current and resistance magnitude



$\boxed{I \propto V_S}$ → EMF

$\boxed{V \propto I}$ → P.D / Voltage drop

→ When element properties and characteristics independent on the direction of current then the element is called as bidirectional element (bilateral)

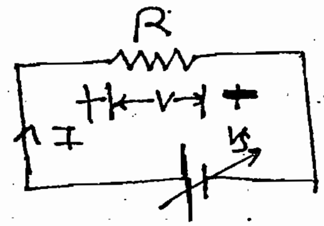


→ When element obeys ohm's law then the element is called as linear resistor

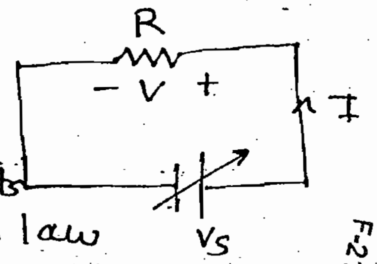
→ Every linear element should obey the bidirectional properties but not vice-versa

$I \uparrow 10\%$, $V \uparrow 10\%$

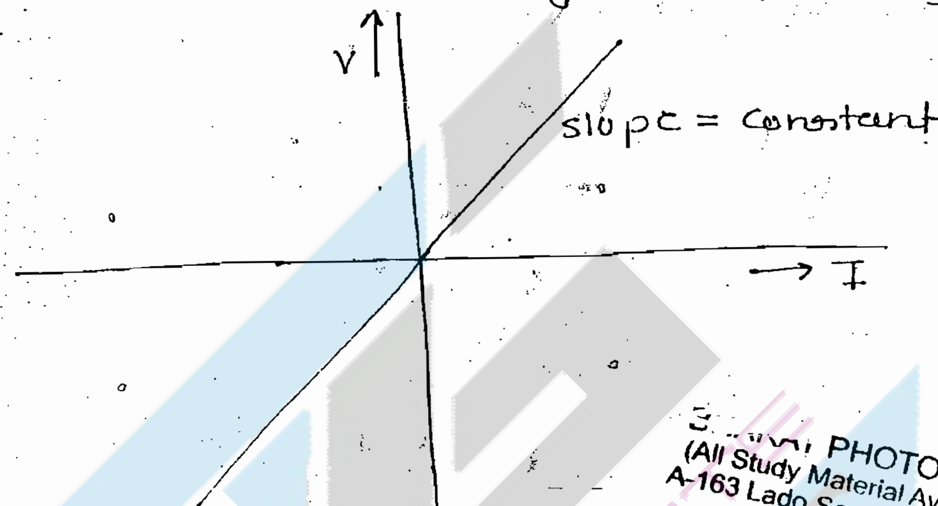
$I \uparrow 90\%$ $V \uparrow 10\%$



$R = \dfrac{V}{I} = \text{constant}$

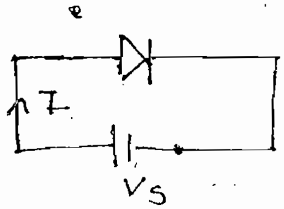


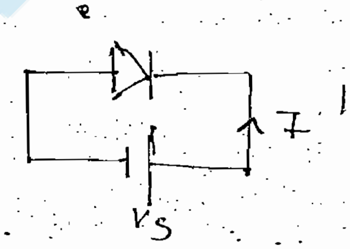
$\dfrac{V}{I}$ = constant then elements obey ohm's law



→ When element properties and characteristics depends on the direction of current then element is called as unidirectional element

→ When element does n't obeys the ohm's law then the element is called as non-linear resistor

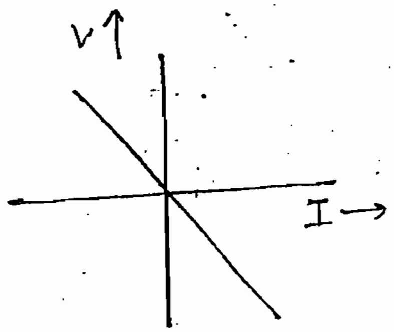




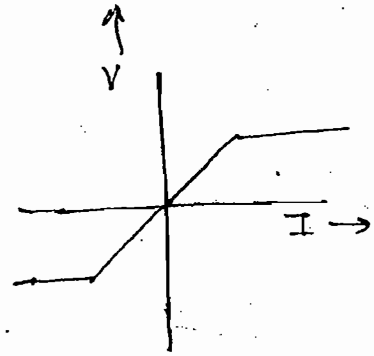
$|I| \neq |I'|$

<u>Note</u> :-

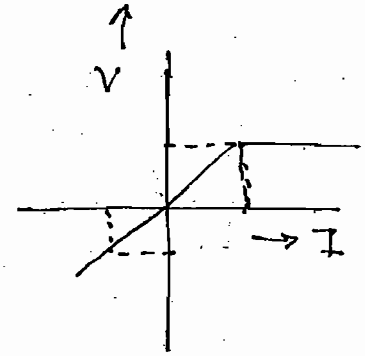
→ $\boxed{\dfrac{V}{I} = +ve}$ then element is passive



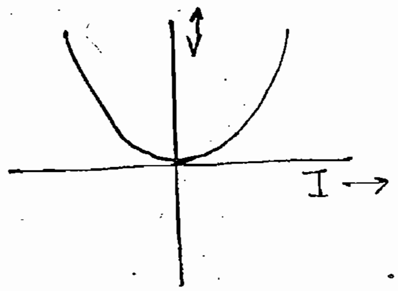
→ Linear
→ Bi-directional
→ Active



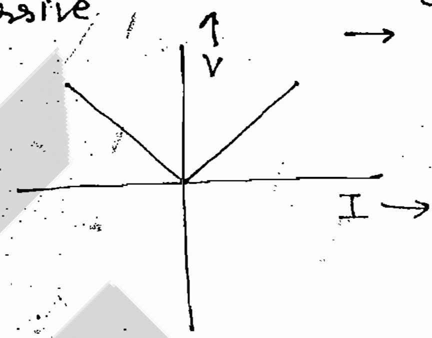
→ ~~Non~~Linear
→ Bi-directional
→ Passive



→ Non-linear
→ Unidirectional
→ Passive



→ Non-linear
→ Uni-directional
→ Active



→ Non-linear
→ Uni-directional
→ Active

Note :-

→ If in any of the quadrant $\frac{V}{I}$ = -ve then the element is active

→ Every linear element should obey the bidirectional property but not vice-versa

→ If $\frac{V}{I}$ = +ve in both coordinates then the element is passive —(I)

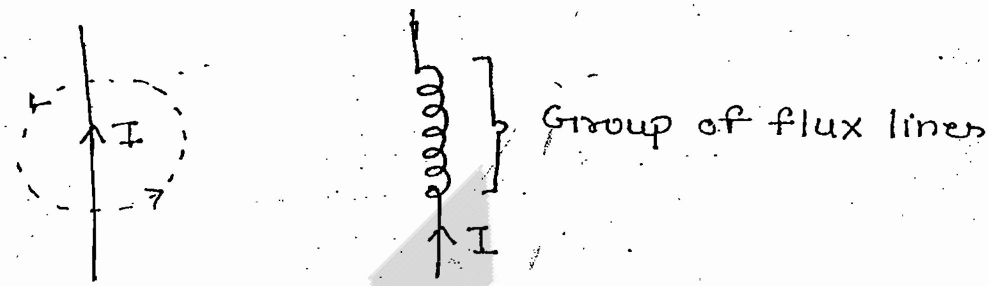
→ ~~In the~~ above

→ If $\frac{V}{I}$ = -ve either in any of the coordinates or both the coordinates then the element is active —(II)

→ In the above two cases waveform should passes through origin

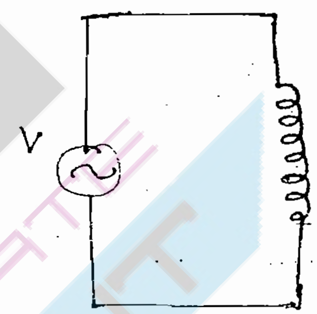
→ When the element obeys the bi-directional property characteristic should be identical in the opposite coordinates but not in the adjacent coordinates

Inductor (L) :—



Weak Magnetic Field   Strong magnetic field.

→



Faraday's Laws :—

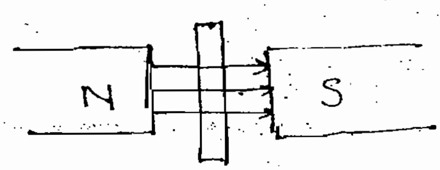
1st Law :—

When conductor cuts a magnetic lines of force an emf induced in the conductor

Second Law :—

An emf induced in the conductor is directly proportional to rate of change of flux

$e = B.l.v\sin\theta$, $e \propto \frac{d\phi}{dt}$



where

B = flux density

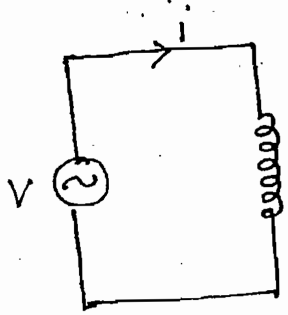
l = length of conductor

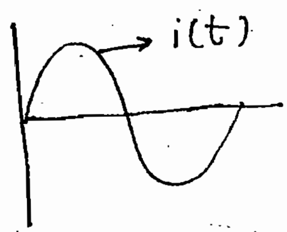
v = velocity of conductor

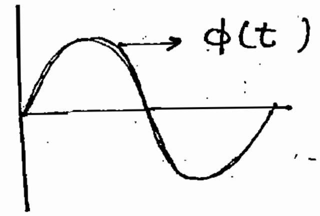
ϕ = Phase displacement b/w conductor & magnetic field

e = dynamically induced emf (eg:- generator

Inductor (L) :-







$e \propto \dfrac{d\phi}{dt}$

$\Rightarrow \boxed{e = -N\dfrac{d\phi}{dt}} \rightarrow$ Lenz's Law

$\rightarrow$ V, i, ϕ, e $\Rightarrow$ $V = N\dfrac{d\phi}{dt}$

Lenz's Law

$\rightarrow$ $\psi = N\phi$

(Flux linkage)

$V = \dfrac{d\psi}{dt}$

$\boxed{V = L\dfrac{di}{dt}}$

$\Rightarrow \boxed{L = \dfrac{V}{\frac{di}{dt}}}$

$\psi \propto \phi$
$\psi \propto i$

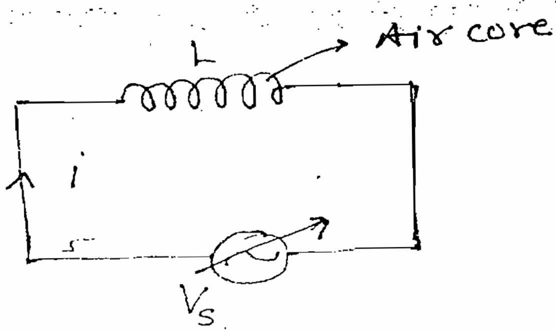
$\psi = N\phi$
$\psi = Li$

$\psi \propto i$

$\boxed{\psi = Li}$.

$\boxed{L = \dfrac{N\phi}{i}}$ Henry

$L = \dfrac{N\phi}{i} =$ Constant

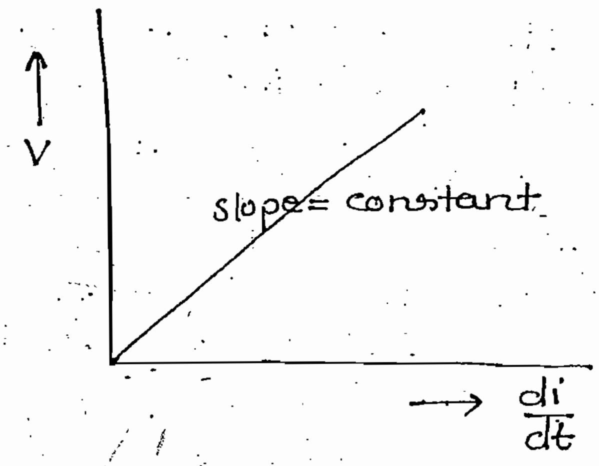


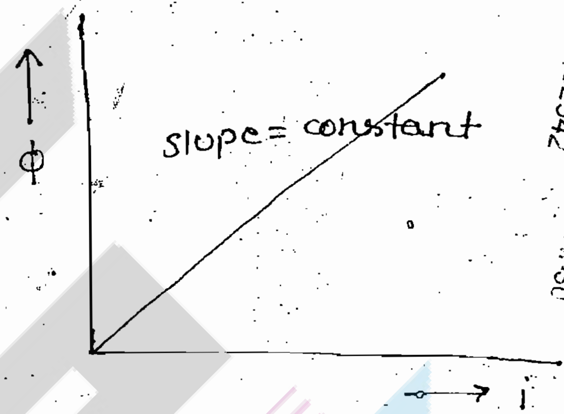
$i \uparrow 10\%, \ \phi \uparrow 10\%$

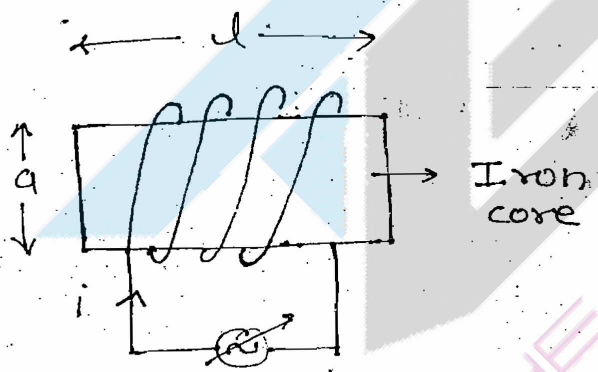
$i \uparrow 90\%, \ \phi \uparrow 90\%$

$V = L \frac{di}{dt}$

$\Rightarrow \boxed{L = \frac{V}{\frac{di}{dt}}}$







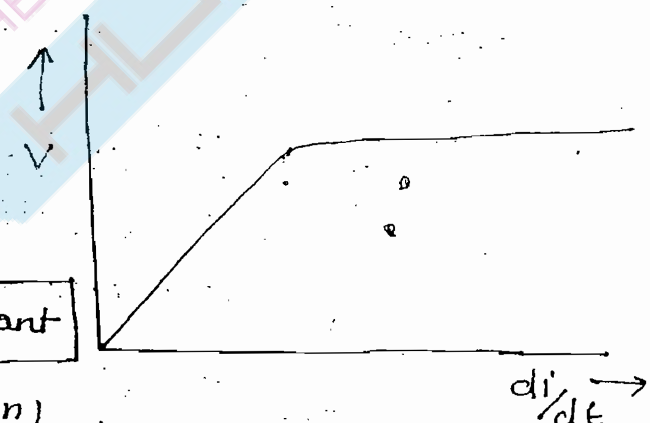
$L = \frac{N\phi}{i}$ —— (1)

If $i \uparrow 10\%$, $\phi \uparrow 10\%$

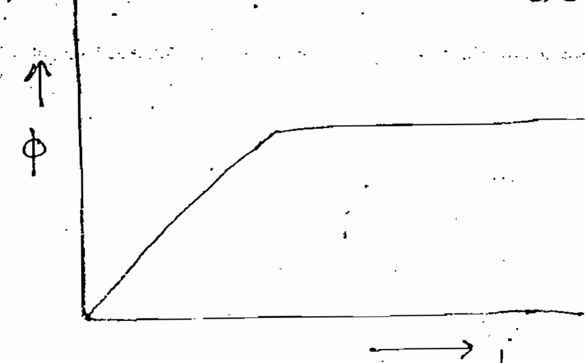
If $i \uparrow 60\%$, $\phi \uparrow 60\%$

If $i \uparrow 90\%$, $\boxed{\phi = \text{constant}}$

(saturation)



→ The flux linked with iron-core is upto a certain limit.



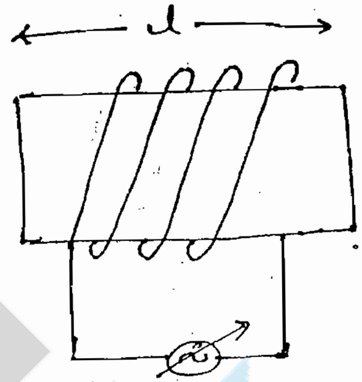
→ When inductance of a inductor is independent on the current magnitude then inductor is called as linear inductor.

eg:- Air core inductor.

→ When inductance of a inductor depends on current magnitude then inductor is called as non-linear inductor

eg:- iron core inductor.

Electric circuit	Magnetic circuit
1. $i = \frac{EMF}{R}$	$\phi = \frac{MMF}{S(\text{Reluctance})}$
2. $i = \frac{EMF}{\frac{\rho l}{a}}$	$\phi = \frac{NI}{\frac{l}{a\mu_0\mu_r}}$ — (ii)



Substitute eq-(ii) in eq-(i)

$$**\quad \boxed{L = \frac{N^2 \mu_0 \mu_r a}{l}}$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m

μ_r = Relative permeability

a = Area of cross-section of core

$$\therefore L = \frac{N^2}{\frac{l}{a\mu_0\mu_r}} \Rightarrow \boxed{L = \frac{N^2}{S}}$$

$$\rightarrow \boxed{V = L\frac{di}{dt}} \Rightarrow \boxed{i = \frac{1}{L}\int_{-\infty}^{t} V dt}$$

ohm's law 5th form & 6th form

→ The above formula is only applicable for linear inductor.

$$P = Vi$$

$$\boxed{P = L\frac{di}{dt}\, i}$$ → Instantaneous Power

$$W = \int P dt$$

$$\Rightarrow \boxed{W = \int L\frac{di}{\cancel{dt}}\, i \cdot \cancel{dt} = \frac{1}{2}Li^2}$$

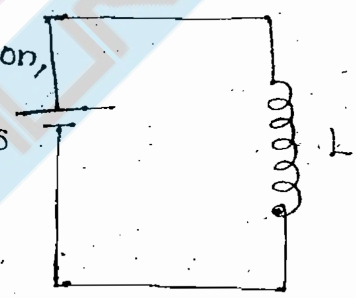
→ Power dissipation in ideal ~~capacitor~~ inductor is zero since internal resistance is zero

→ Inductor stores energy in the form of magnetic field (K.E)

→ Due to energy storage property inductor is also called as dynamic element

Conclusions :-

1. Under steady state condition, for a dc source inductor behave as a short circuit



$$V = L\frac{di}{dt}$$

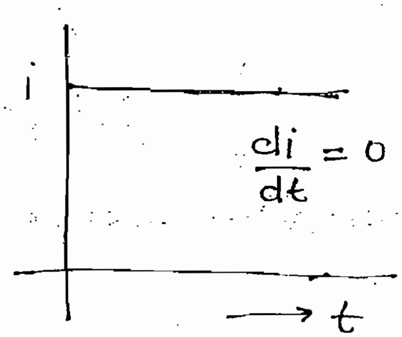
For dc, $\frac{di}{dt} = 0$

↓

$V = 0$

↓

S.C



2 Inductor does not allow sudden change of current since

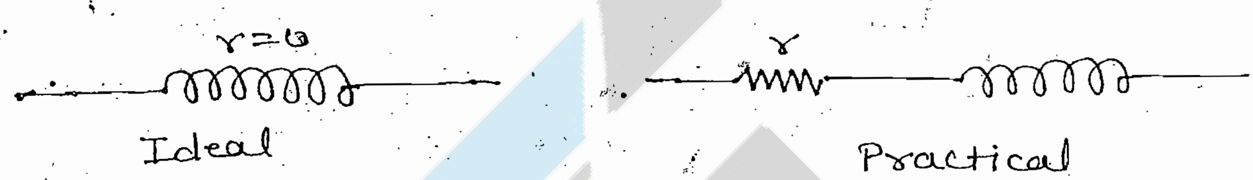
(a) For sudden change of current infinite voltage is required but practically it is not possible

(b) Practical inductive circuit having finite value of time constant i.e. $\tau = \frac{L}{R}$

$$V = L\frac{di}{dt}$$

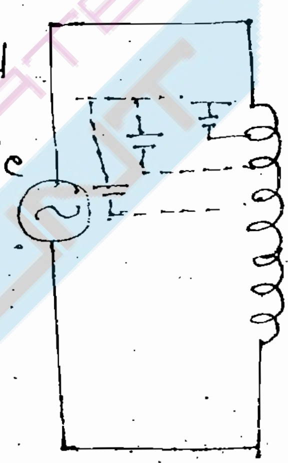
$$dt \to 0 \quad \Rightarrow \quad V = \infty$$

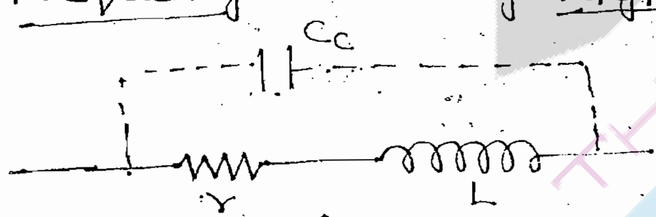
3.



$r \simeq m\Omega$

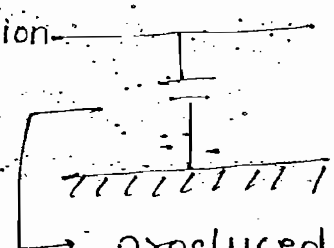
Inter-turn capacitance is present when inductor is operated at either at very high frequency or very high voltage





where C_c = inter-turn capacitance
or
• self capacitance

Transmission line



produced due to high potential diff.

$$X_L = 2\pi f L$$

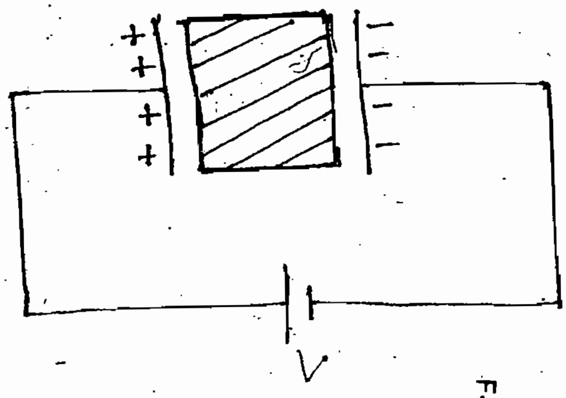
$$X_C = \frac{1}{2\pi f C}$$

Capacitor :—

$Q \propto V$

$\Rightarrow Q = CV$

$\Rightarrow \boxed{C = \frac{Q}{V}}$ C/Volt or F



Again $Q = CV$

$\Rightarrow \frac{dQ}{dt} = C\frac{dV}{dt}$

$\Rightarrow i = C\frac{dV}{dt}$

$\Rightarrow \boxed{C = \frac{i}{\frac{dV}{dt}}}$

→ Capacitor opposes rate of change of voltage

$\boxed{i = C\frac{dV}{dt}}$ → Ohm's law 7th form

$\boxed{V = \frac{1}{C}\int_{-\infty}^{t} i\,dt}$ → Ohm's law 8th form

$P = Vi$

$\Rightarrow \boxed{P = VC\frac{dV}{dt}}$ → Instantaneous power

$W = \int P dt$

$W = \int V.C\frac{dV}{dt}\, dt$

$\Rightarrow \boxed{W = \frac{1}{2}CV^2}$ → Potential Energy

→ Power dissipation in ideal capacitor = 0

→ Capacitor stores energy in the form of electric field (Potential Energy)

→ Due to energy storage property capacitor is also called as dynamic element

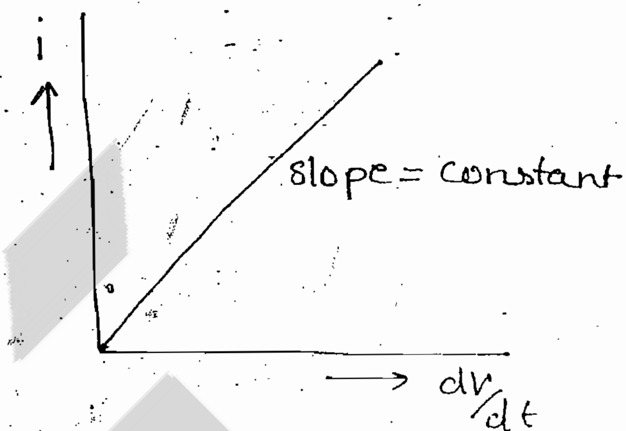
→

$$C = \frac{Q}{V} = \text{constant}$$

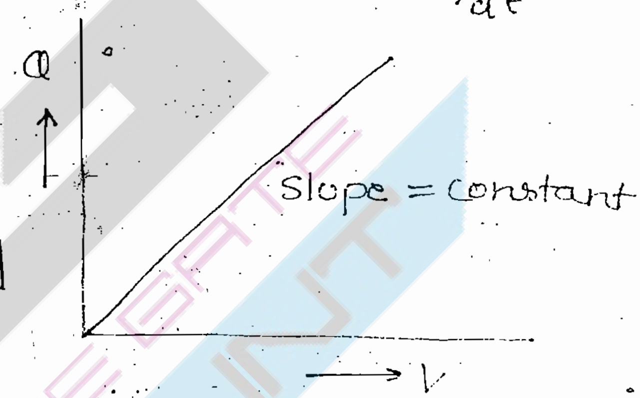
$V \uparrow 10\%$, $Q \uparrow 10\%$

$V \uparrow 90\%$, $Q \uparrow 90\%$

$$i = C\frac{dv}{dt}$$



$$\Rightarrow C = \frac{i}{\frac{dv}{dt}}$$



$$\boxed{C = \frac{Q}{V} = \text{Variable}}$$

↓ Non-linear

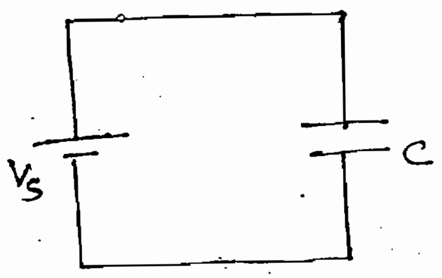
→ When capacitance of a capacitor independent on the voltage magnitude then the capacitor is called as linear capacitor.

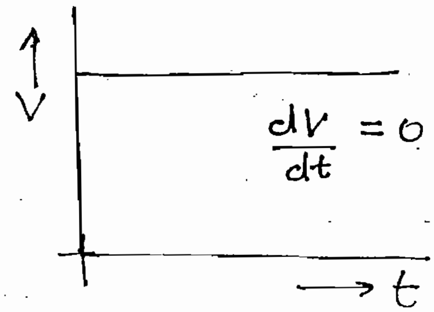
→ When capacitance of a capacitor depends on the voltage magnitude then capacitor is called as non-linear capacitor

eg:- Varactor diode

<u>Conclusion:-</u>

1. Under steady state condition for a dc source capacitor behaves as an open circuit





$$i = C\frac{dV}{dt}$$

$$\frac{dV}{dt} = 0 \longrightarrow i = 0 \implies O \cdot C$$

2. Capacitor does not allow sudden change of voltages since

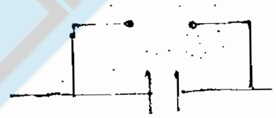
(a) For sudden change of voltages infinite current is required but practically it is not possible

(b) Practical capacitive circuit has finite value of time constant

$$(a) \quad i = C\frac{dV}{dt} = \infty \qquad dt \to 0$$

$$(b) \quad T = RC$$

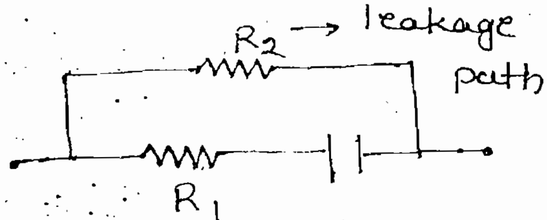
3. <u>For ideal capacitor :-</u>



(Ideal)

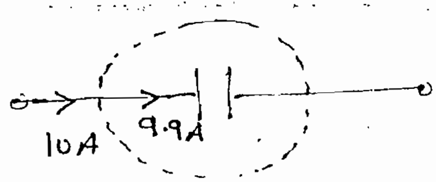
$$\boxed{\begin{aligned} R_1 &= 0 \\ R_2 &= \infty \end{aligned}}$$

<u>For Practical capacitor :-</u>



$$\boxed{\begin{aligned} R_1 &\simeq \text{very less} \\ R_2 &\simeq \text{very high} \end{aligned}}$$

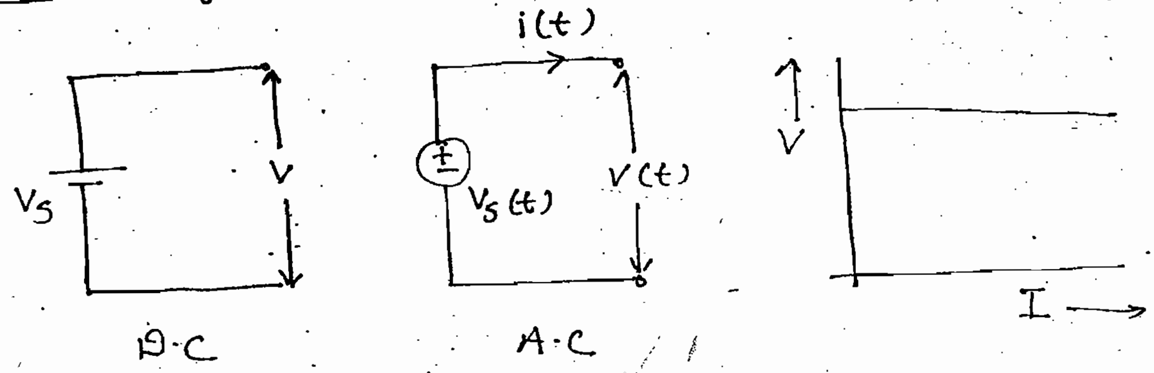
Dielectric loss — (Power loss) (Practical)



R_2 = leakage path

Lecture - 2

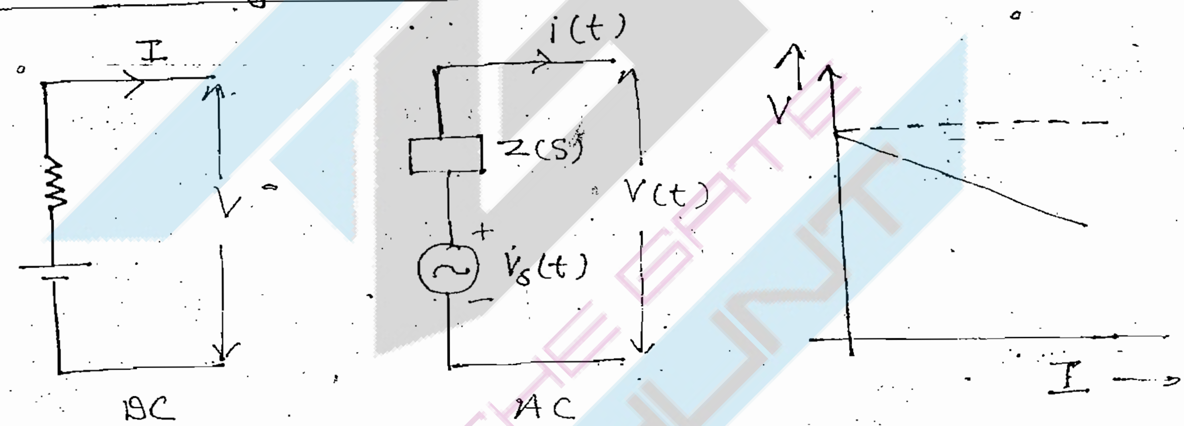
Ideal Voltage Source :-



→ $R_S = 0$

→ $V_S(t)$ → either AC or DC if t → specify then AC

Practical Voltage Source :-



$$V_S = V + IR_S$$

$$V = V_S - IR_S$$

→ Ideal voltage source delivers energy at ~~It~~ specify voltage (V) it is independent on current deliver by a source

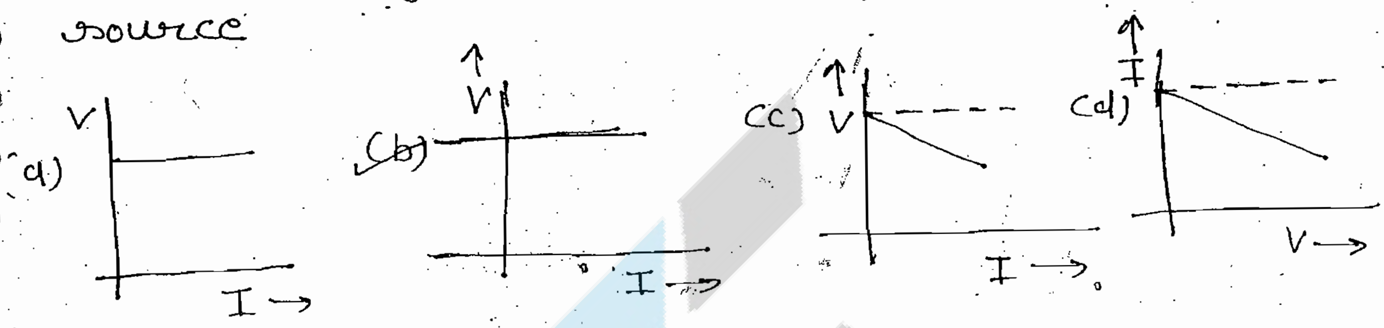
→ Internal resistance of ideal voltage source is zero

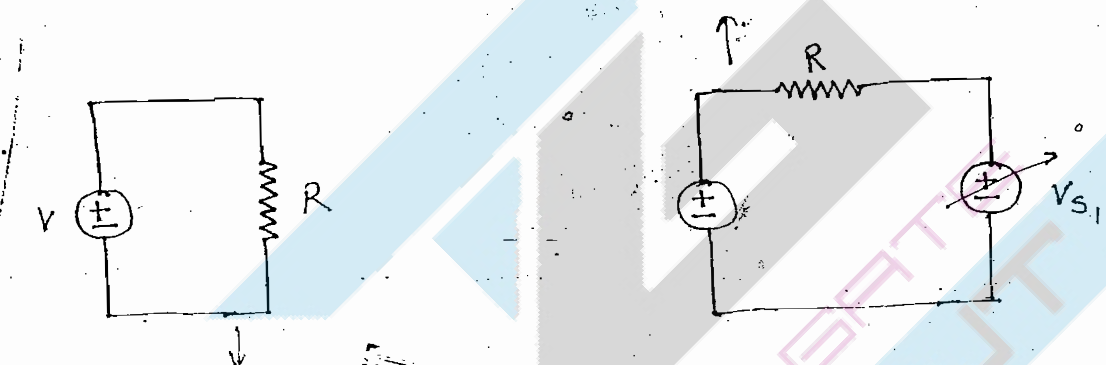
→ Practical voltage source delivers energy at ~~It~~ specified voltage (V) which depends on current deliver by the source.

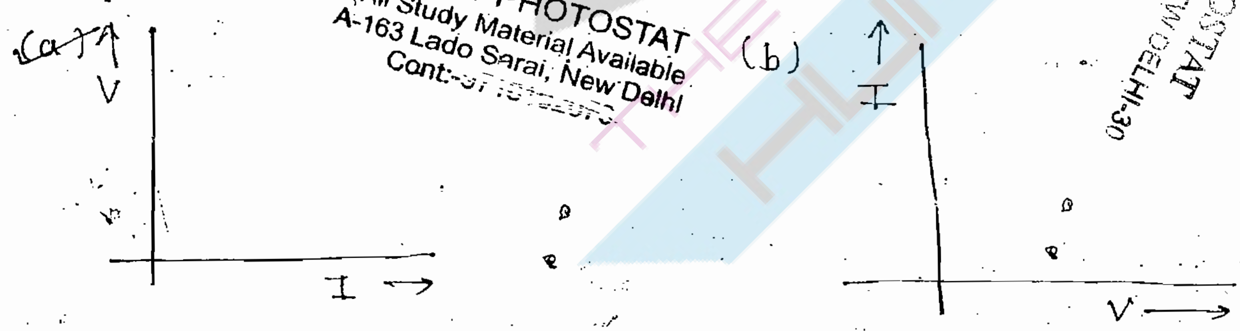
→ Linear → characteristic passes through origin and inc. linearly

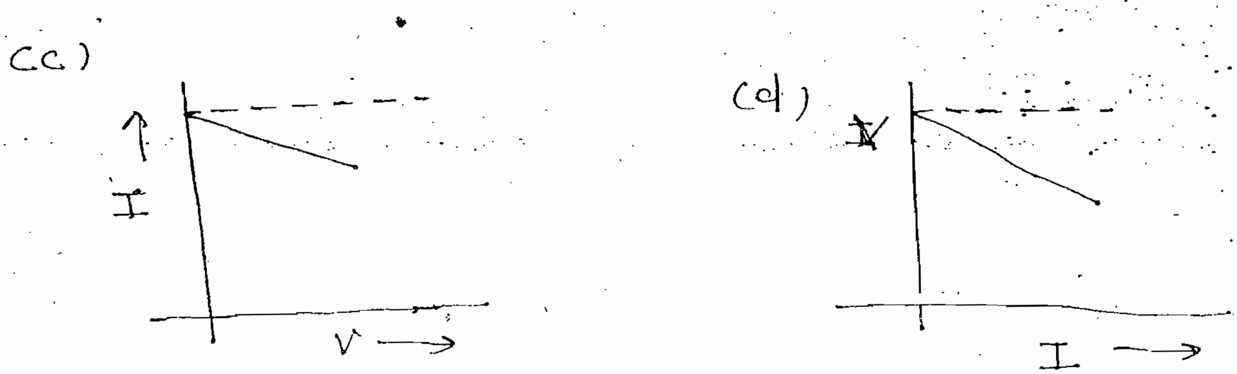
→ Independent voltage source does not obey the ohm's law. since V–I characteristic is non-linear.

Ques:- Identify V–I characteristic of ideal voltage source



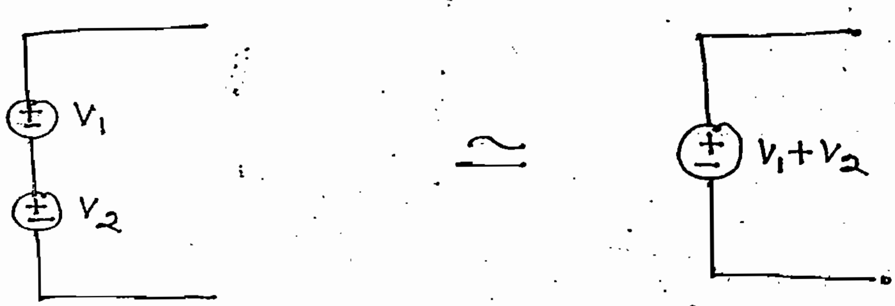




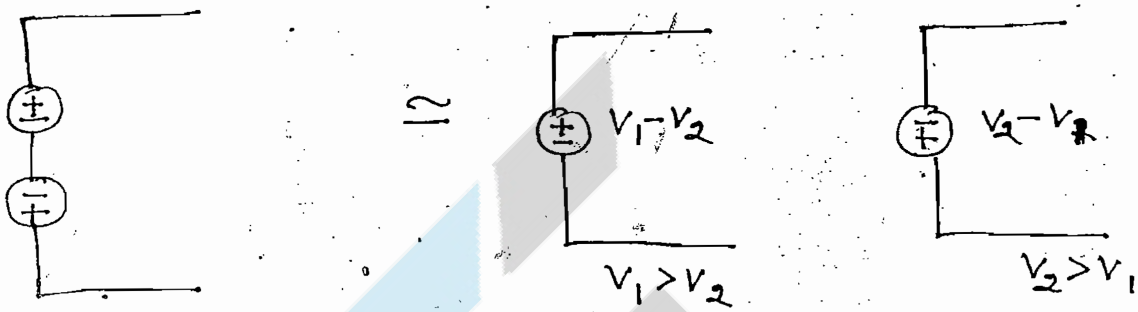


Note :-

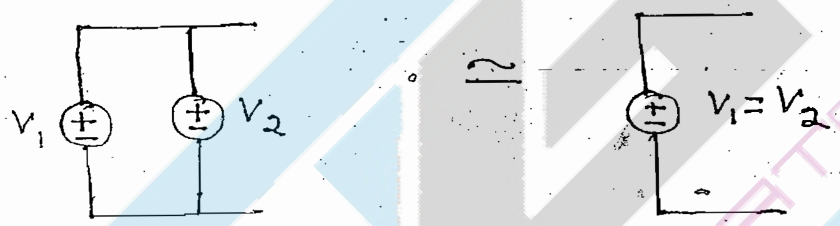
(I)



(II)

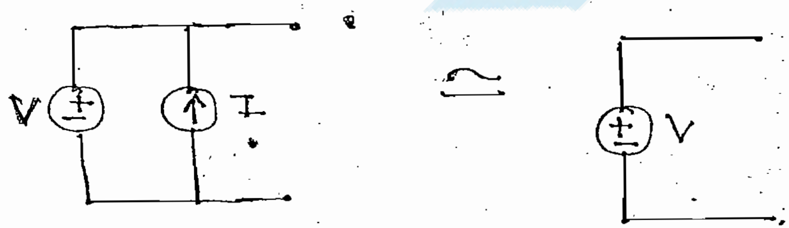


(III)



(IV)

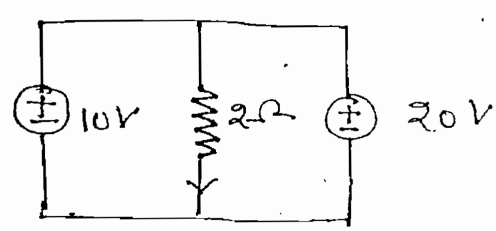
(V)



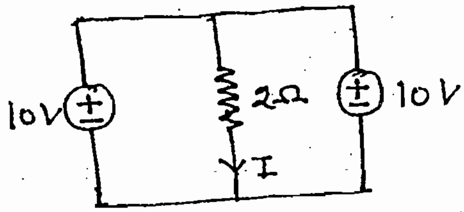
Ques :- Find current in the 2Ω resistor

(a) 5A (b) 10A

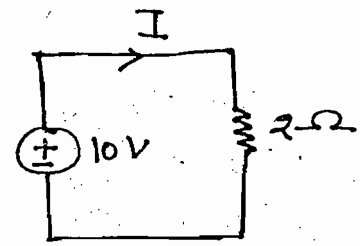
(c) 15A) (d) None or Not satisfying KVL



Note :-



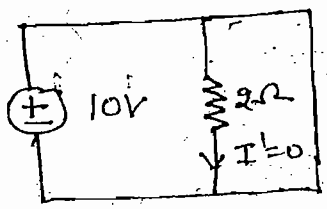
$\simeq$



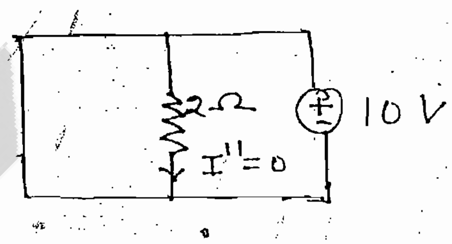
$I = 5A$

By using superposition theorem

Case - I



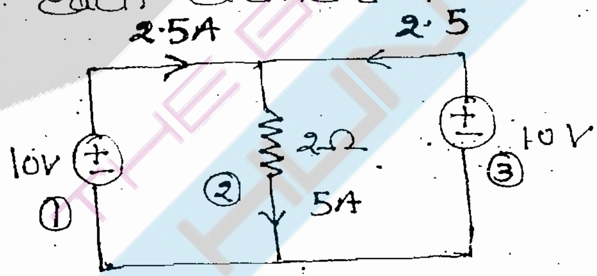
Case - II



$$I = I' + I'' = 0$$

→ For the above circuit superposition theorem can't be applied since case-(I) & case-(II) circuits are not satisfying KVL

Ques :- Find power of each element in the circuit given below



Soln :-

$P_1 = 10 \times 2.5 = 25 \text{ W}$ (Delivering)

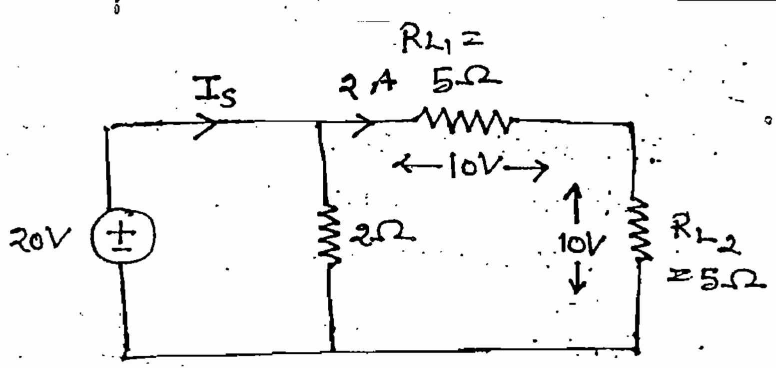
$P_3 = 10 \times 2.5 = 25 \text{ W}$ (Del.)

$P_2 = I^2 R = 5^2 \times 2 = 50 \text{ W}$ (Absorbing)

Note:—

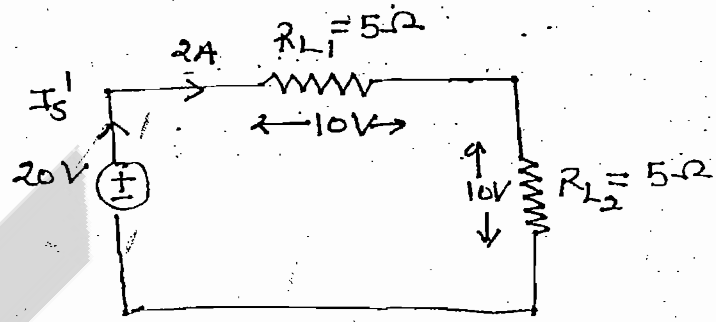
$I_S = 10 + 2 = 12A$

$P_S = 20 \times 12 = 240W$



$I_S' = 2A$

$P_S' = 20 \times 2 = 40W$



→ In the above circuit 2Ω resistance can be neglected while calculating either load current or load voltages

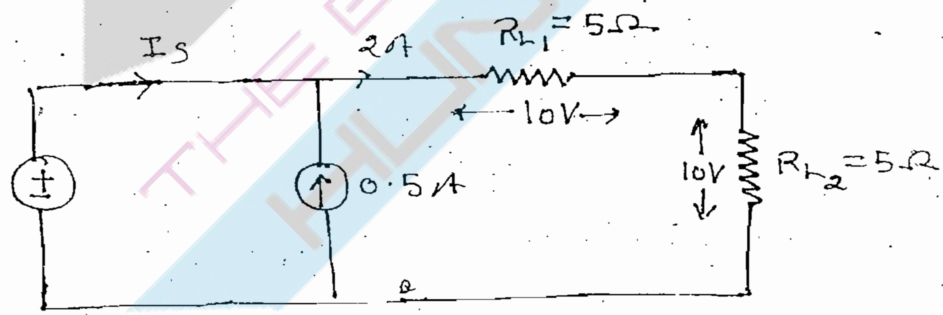
→ In the above circuit 2Ω resistance can't be neglected while calculating either source current or source power

$I_S + 0.5 = 2$

$\Rightarrow I_S = 1.5A$

$P_S = 20 \times 1.5$

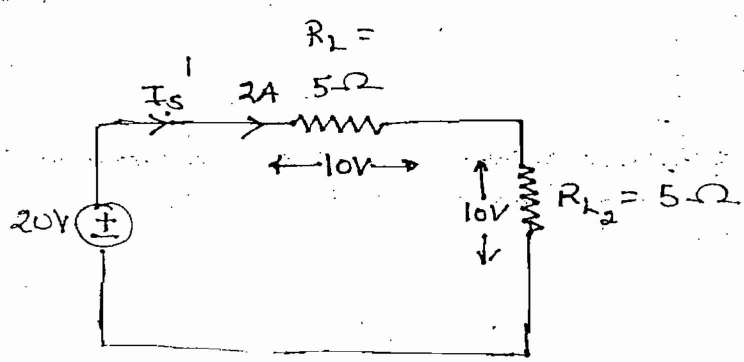
$= 30W$



$I_S' = 2A$

$P_S' = 20 \times 2$

$= 40W$

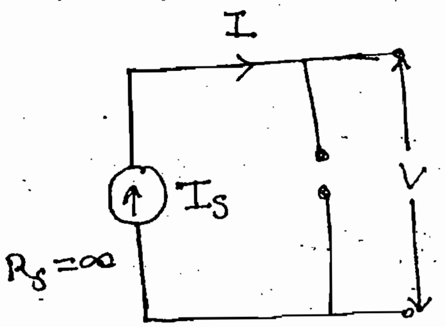


Note:-

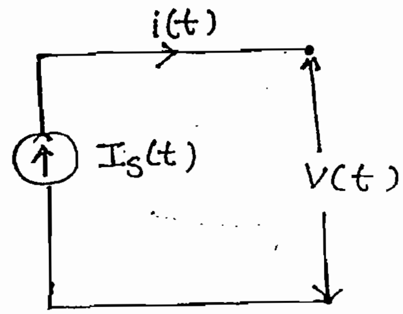
→ In the above circuit current source can be neglected while calculating either load current or load voltage

→ In the above circuit current source can't be neglected while calculating either voltage source current or voltage source power

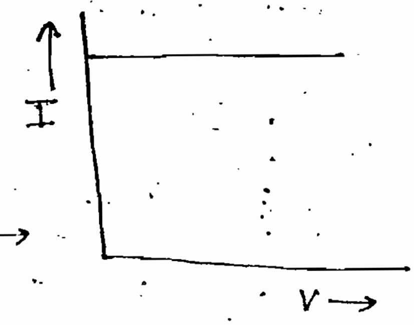
Current (I) :–



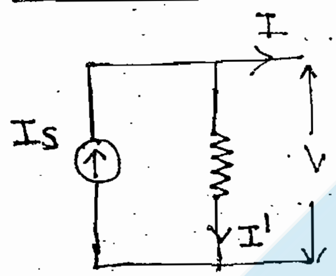
Ideal DC current source

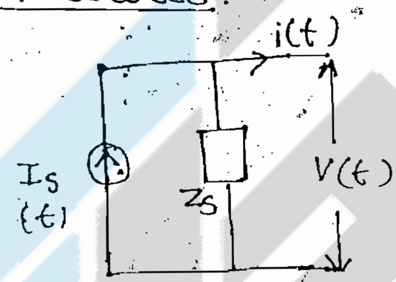


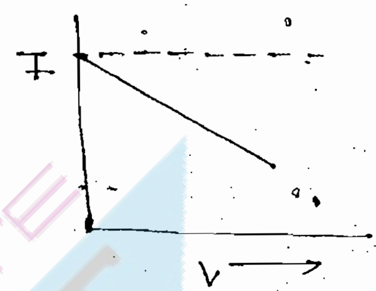
Ideal AC current source



Practical current source :–







$$I_s = I + I'$$

$$I = I_s - I'$$

$$I = I_s - \frac{V}{R_s}$$

→ Ideal current source deliver energy at specified current (I) which is independent on voltag across source

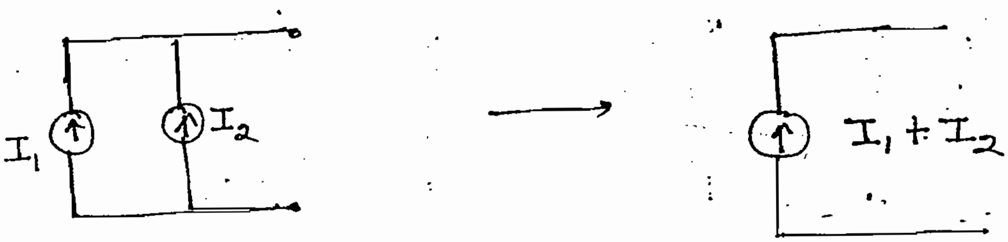
→ Internal resistance of ideal current source = ∞

→ Practical current source deliver energy at specified current (I) which depends on voltage across source

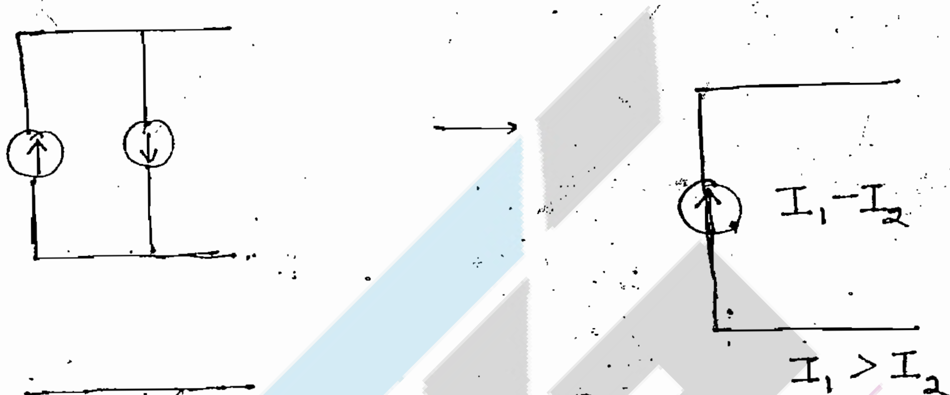
→ Independent current source doesn't obey the ohm's law since $\forall_I$ V–I characteristics are non-linear

→ In the practical system no independent current source are exist but dependent current source are exist.

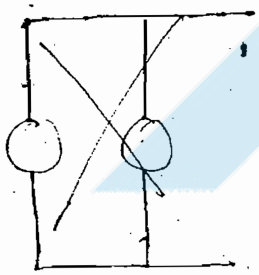
(I)



(II)

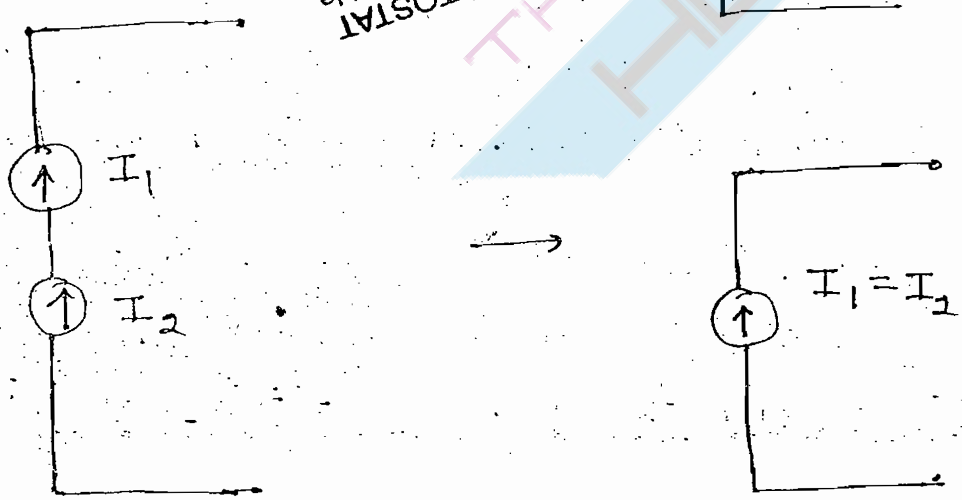


(III)

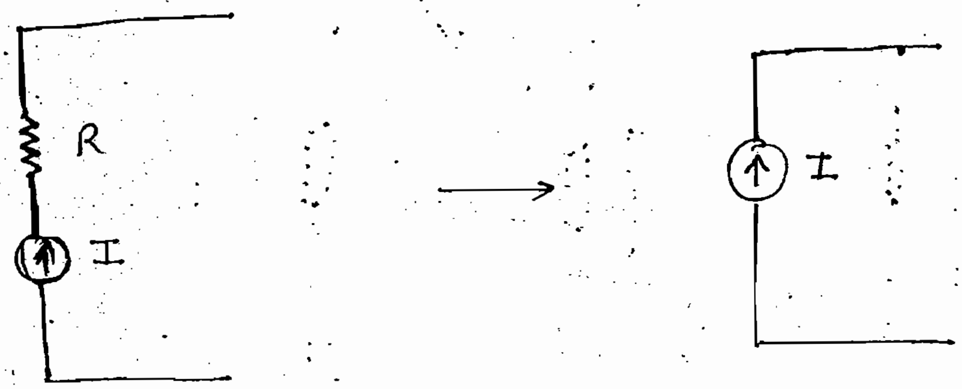


OR

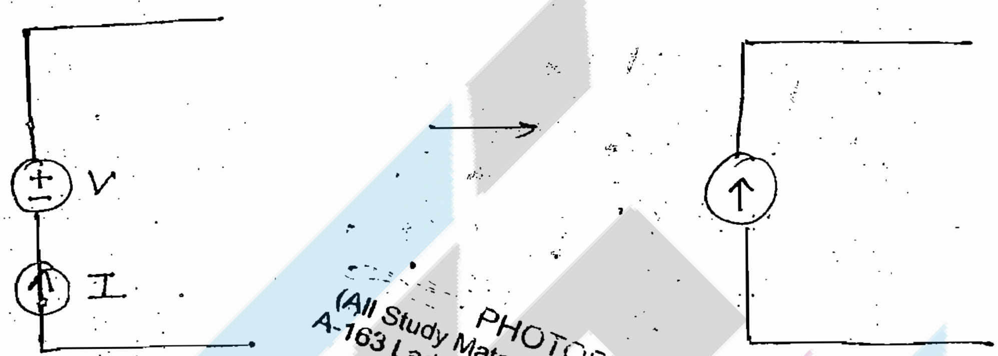
(III)



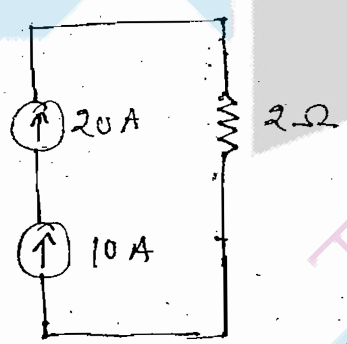
(IV)



(V)



Ques:-



(a) 10 A

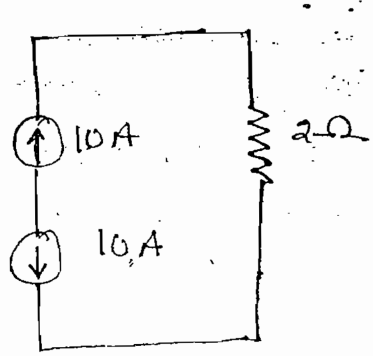
(b) 20 A

(c) 30 A

(d) None of these OR Not satisfied KCL

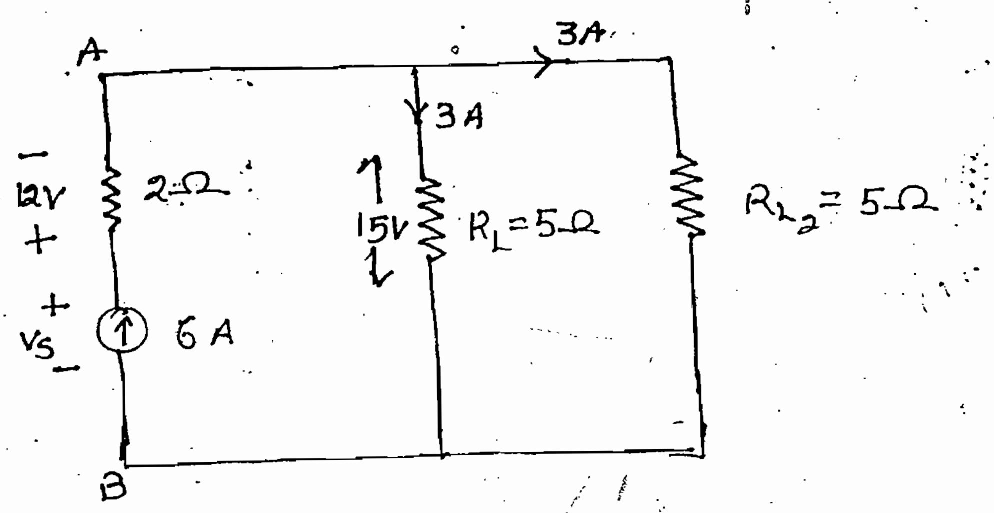
Note:- With respect to KCL current flowing through all the series element should be equal

Ques:-



What is the current through 2 Ω resistor?

Soln:- Not satisfied KCL bec. polarities are opposite.

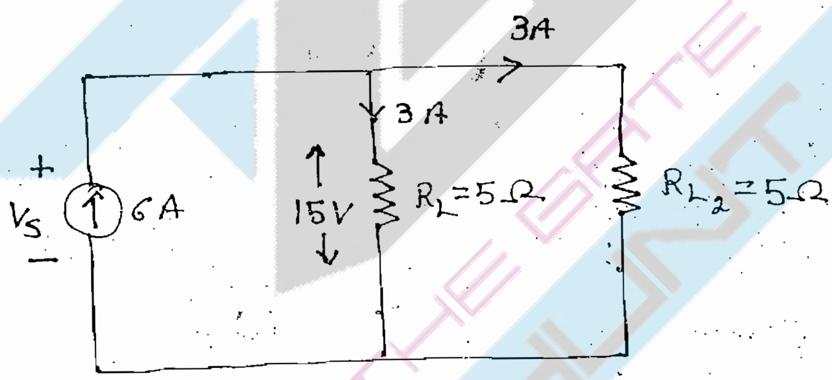


$$V_{AB} = V_S - 12$$

$$\Rightarrow 15 = V_S - 12$$

$$\Rightarrow V_S = 27$$

& $P_S = 27 \times 6 = 162\ W$



$V_S{}' = 15\ V$ $\quad P_S{}' = 15 \times 6 = 90\ W.$

Note:-

→ In the above circuit 2Ω resistance can be neglecting either by calculating load current or load voltage

→ In the above circuit 2Ω resistance cann't be neglected by calculating either voltage across current source or power of the current source.

Note:-

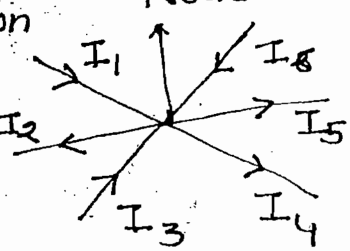
→ In above circuit voltage source can be neglected by calculating either load current or load voltage

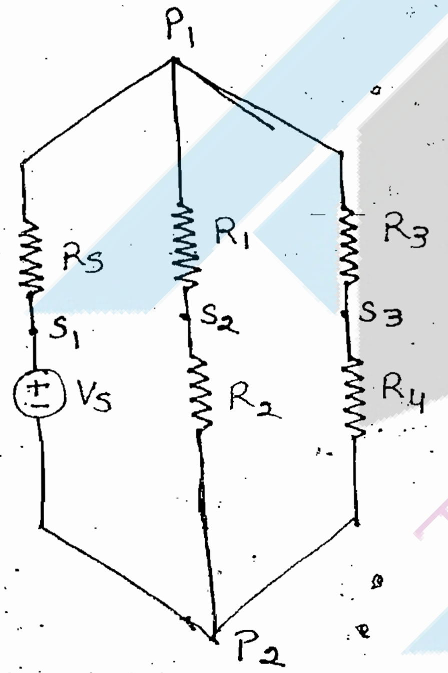
→ In the above circuit voltage source cannot be neglected ~~to~~ either calculating voltage across currer source or power of the current source

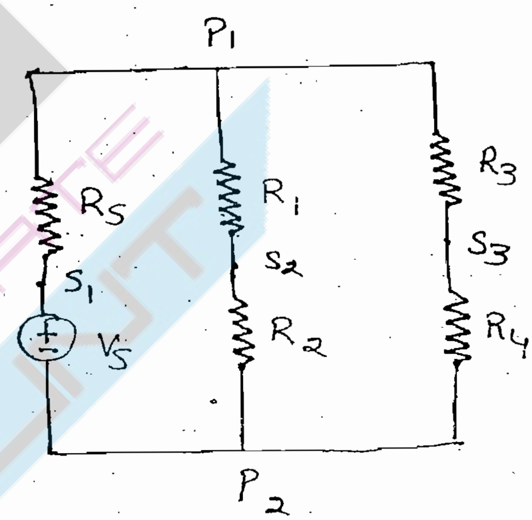
KCL:-

→ Based on law of conservation of charge

$$I_1 + I_3 + I_6 = I_2 + I_4 + I_5$$







→ KCL states that algebraic sum of currents meeting at a point is equal to zero.

→ When two elements are connected together then common point is called as simple node

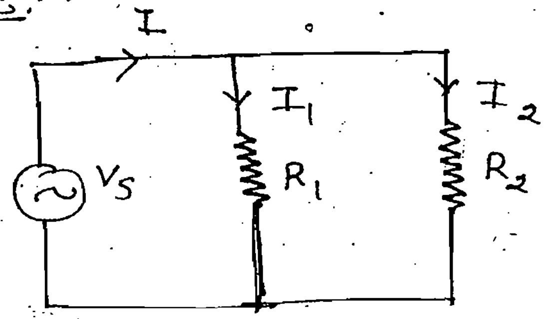
→ When more than two elements are connected together then common point is called as principal node.

Current division technique :-

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

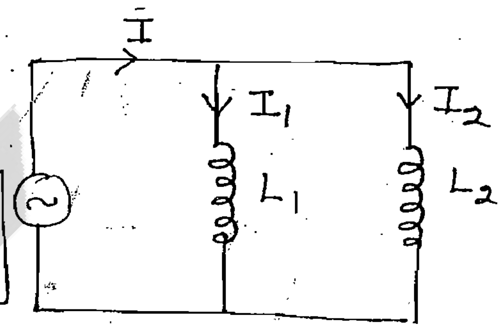
$$\boxed{I_1 = I\,\frac{R_2}{R_1+R_2}}$$

$$\boxed{I_2 = I\,\frac{R_1}{R_1+R_2}}$$



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

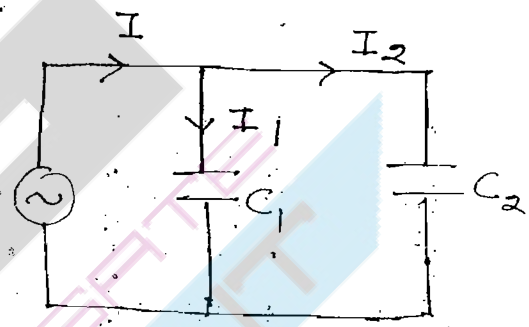
$$\boxed{I_1 = I\,\frac{L_2}{L_1+L_2}} \qquad \boxed{I_2 = I\,\frac{L_1}{L_1+L_2}}$$



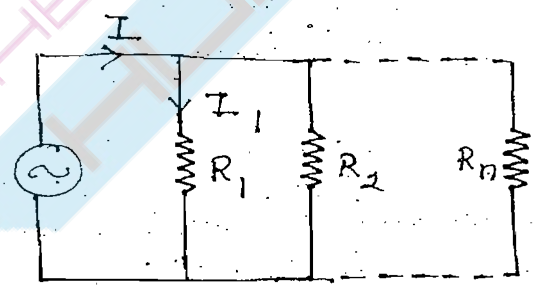
$$C_{eq} = C_1 + C_2$$

$$\boxed{I_1 = I\,\frac{C_1}{C_1+C_2}}$$

$$\boxed{I_2 = I\,\frac{C_2}{C_1+C_2}}$$



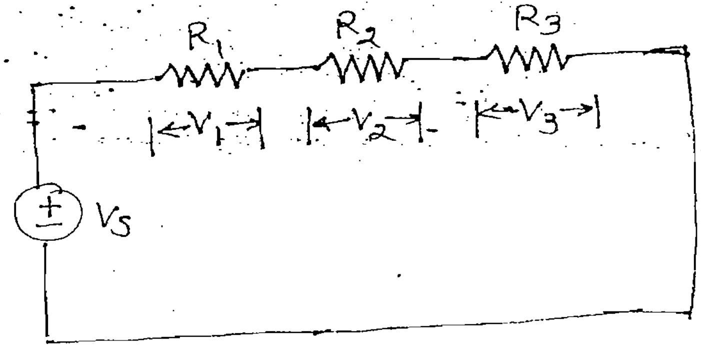
$$\boxed{I_1 = I\,\frac{1/R_1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}}$$



KVL :-

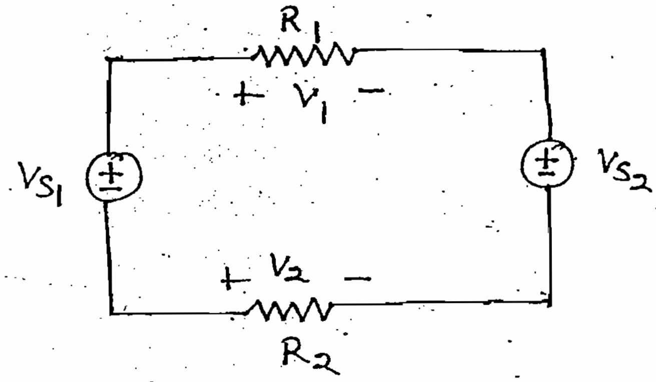
→ KVL works based on the principle of law of conservation of energy

$$V_1 + V_2 + V_3 - V_S = 0$$



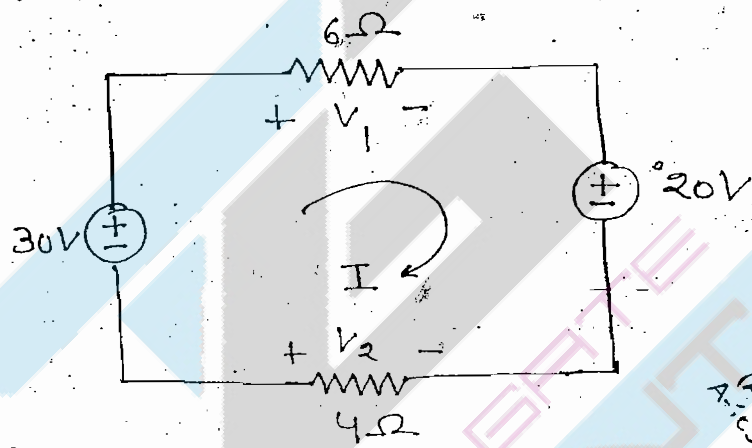
$$V_{S_1} - V_1 - V_{S_3} + V_2 = 0$$

$$V_{S_1} + V_2 = V_1 + V_{S_3}$$



→ KVL states that algebraic sum of voltages in a closed loop is equal to zero

<u>Ques:-</u> Find V_1 & V_2 of the circuit shown



<u>Soln:-</u> $V_1 = 6I \qquad V_2 = -4I$

$$30 - V_1 + 20 - V_2 = 0$$

$$\Rightarrow -30 + 6I + 20 - (-4I) = 0$$

$$\Rightarrow I = 1A$$

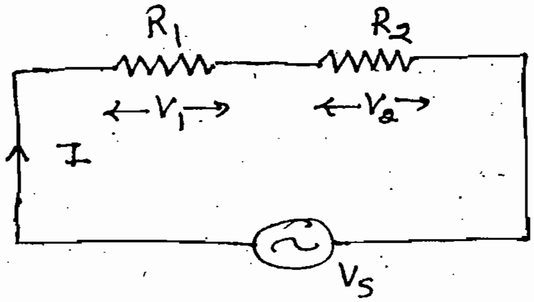
$$V_1 = 6V \quad \& \quad V_2 = -4V$$

Voltage Division Technique :-

$$R_{eq} = R_1 + R_2$$

$$\boxed{V_1 = V_S \frac{R_1}{R_1 + R_2}}$$

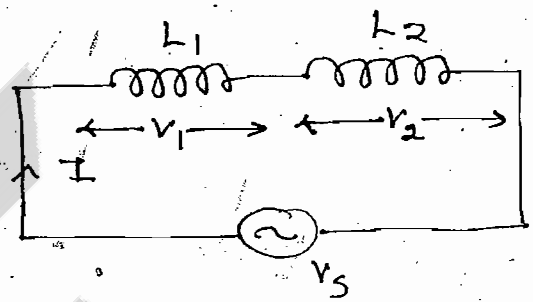
$$\boxed{V_2 = V_S \frac{R_2}{R_1 + R_2}}$$



$$L_{eq} = L_1 + L_2$$

$$\boxed{V_1 = V_S \frac{L_1}{L_1 + L_2}}$$

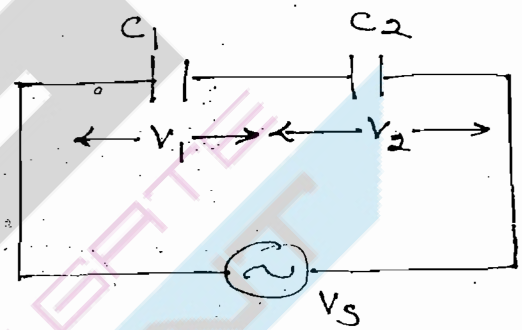
$$\boxed{V_2 = V_S \frac{L_2}{L_1 + L_2}}$$



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\boxed{V_1 = V_S \frac{C_2}{C_1 + C_2}}$$

$$\boxed{V_2 = V_S \frac{C_1}{C_1 + C_2}}$$



Conclusions :-

Field Theory → Low frequency
Field Theory → High frequency

Network Theory → Low frequency

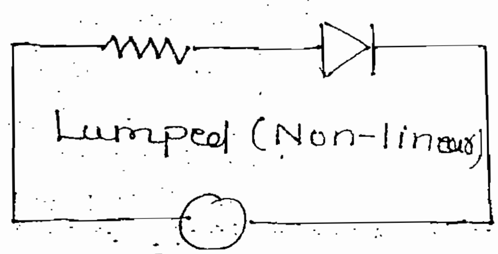
→ Field theory can be applied for low or high frequency application

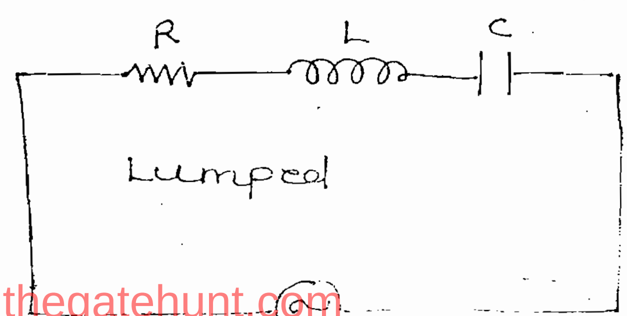
→ In field theory accurate results are obtain. but developing mathematical equation is complex

→ Network theory is applied only for low frequency applications

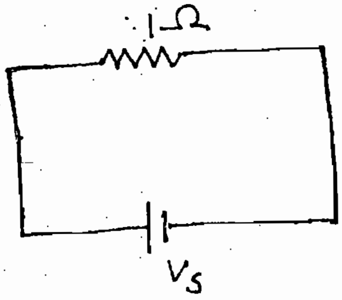
→ In the network theory approximate results are obtained and developing mathematical equation is simple

→ KVL and KCL fails for high frequencies application

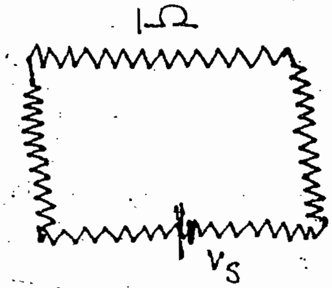




$$V_S = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

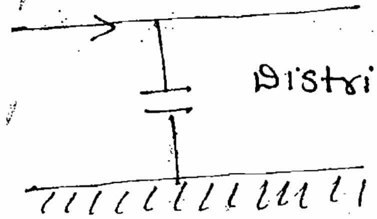


Lumped
(Linear)



Distributed

Transmission Line.



Distributed

$J = \sigma E$
(Ohm's Law

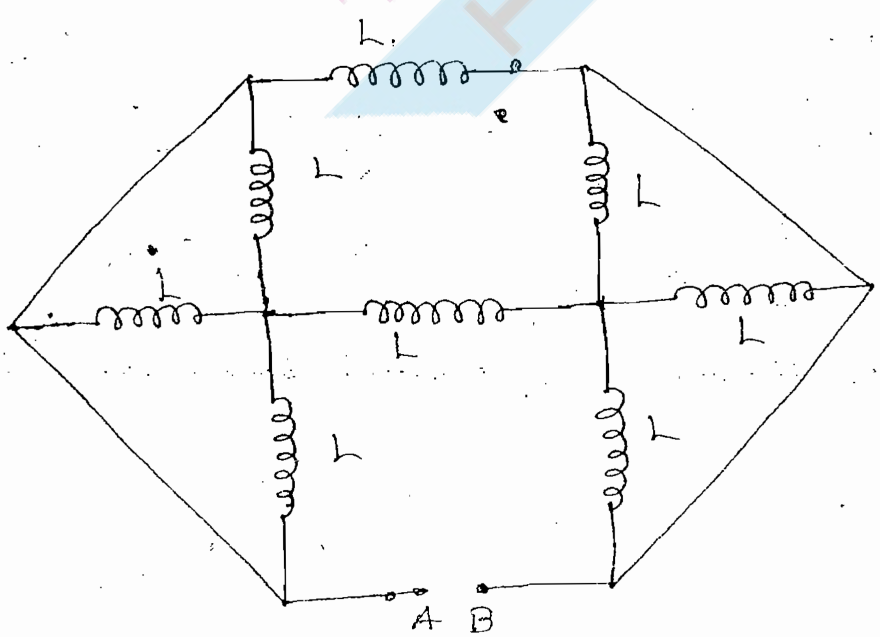
→ Ohm's law can be applied lumped (linear) and distributed parameters

→ KVL and KCL fails for distributed parameters since in the distributed parameters electrically it is not possible to separate resistance, inductance and capacitance effects

→ KVL and KCL are applied for lumped parameters (Linear, non-linear, uni-directional, bi-directional, time variant and invariant elements)

Ques:-

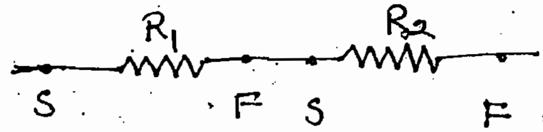
$L_{eq} = ?$



Note :–

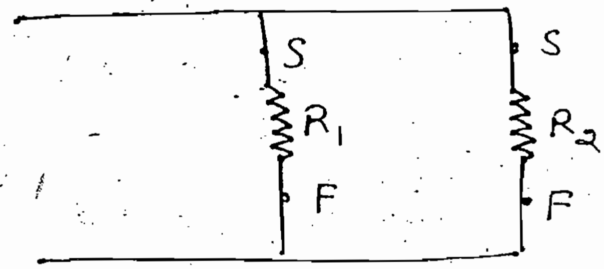
(i) one joint

(ii) $I \rightarrow$ same

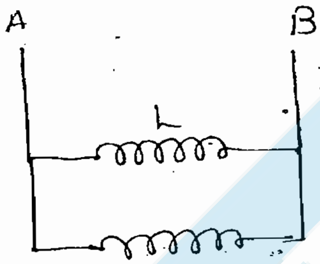


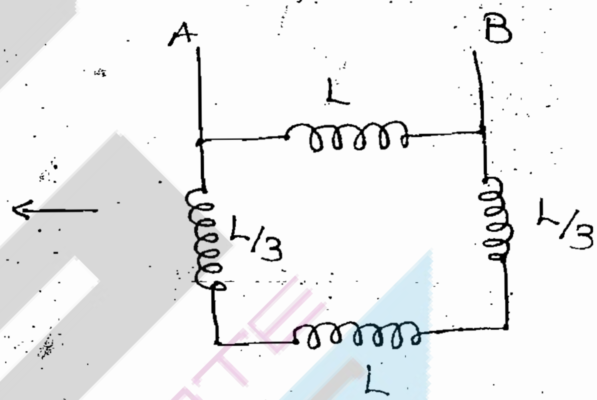
(i) Twice

(ii) $V \longrightarrow$ same



Soln :–





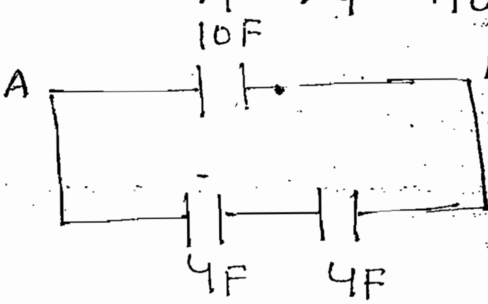
$$\frac{L}{3} + L + \frac{L}{3} = \frac{5L}{3}$$

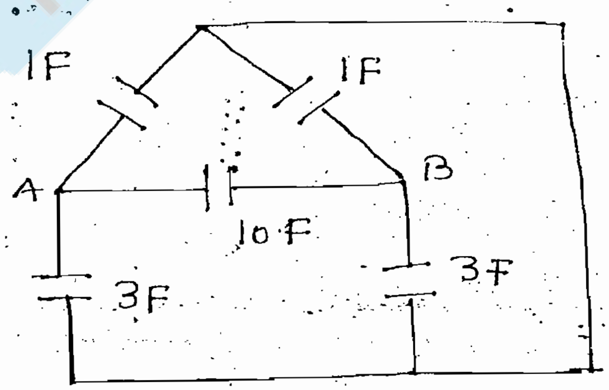
$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2} = \frac{5L}{8}, \text{Ans}$$

Ques :– Find equivalent capacitance w.r.t A & B

Soln :–

$$C_{eq} = \frac{1}{4} + \frac{1}{4} + \frac{1}{10}$$

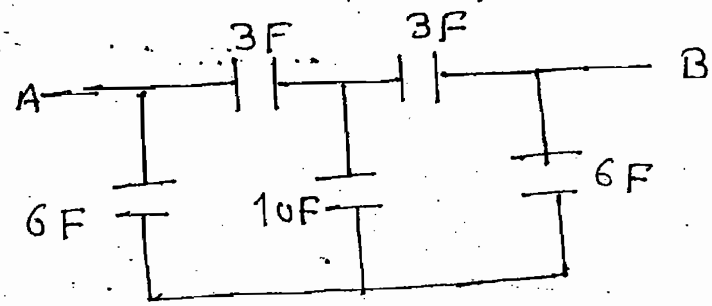




$$C_{eq} = 10 + 2 = 12\,F$$

ro

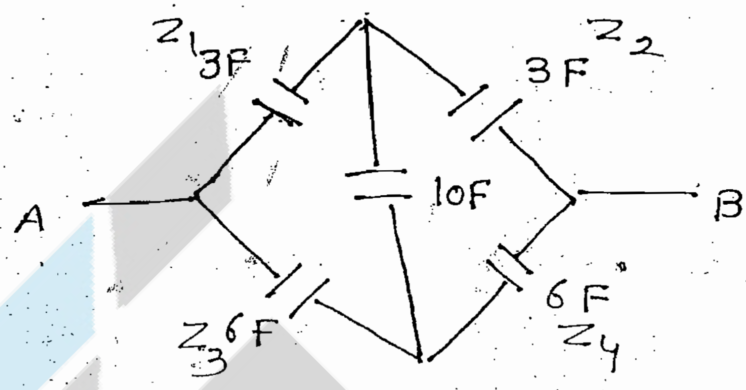
Ques:- Find equivalent capacitance w.r.t A & B



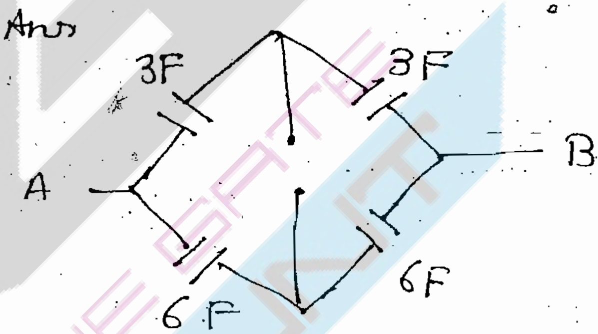
Soln:-

Balanced bridge

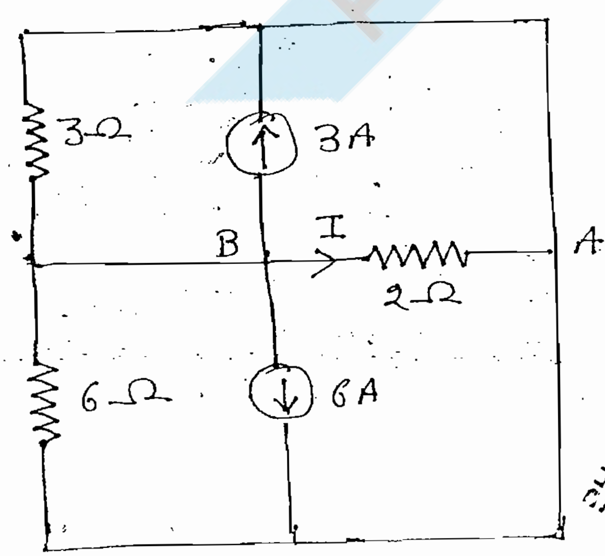
$Z_1 Z_4 = Z_2 Z_3$



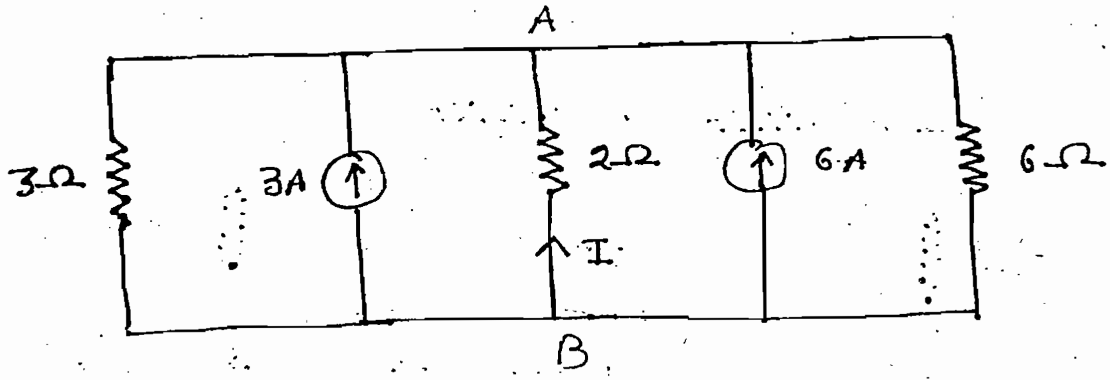
$C_{eq} = 3 + 1.5 = 4.5F$, Ans



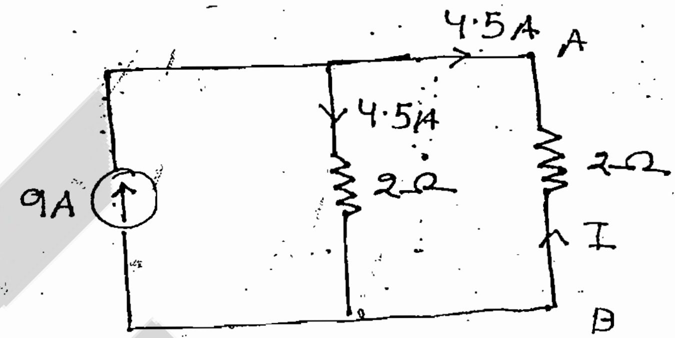
Ques:- Find I of the circuit shown



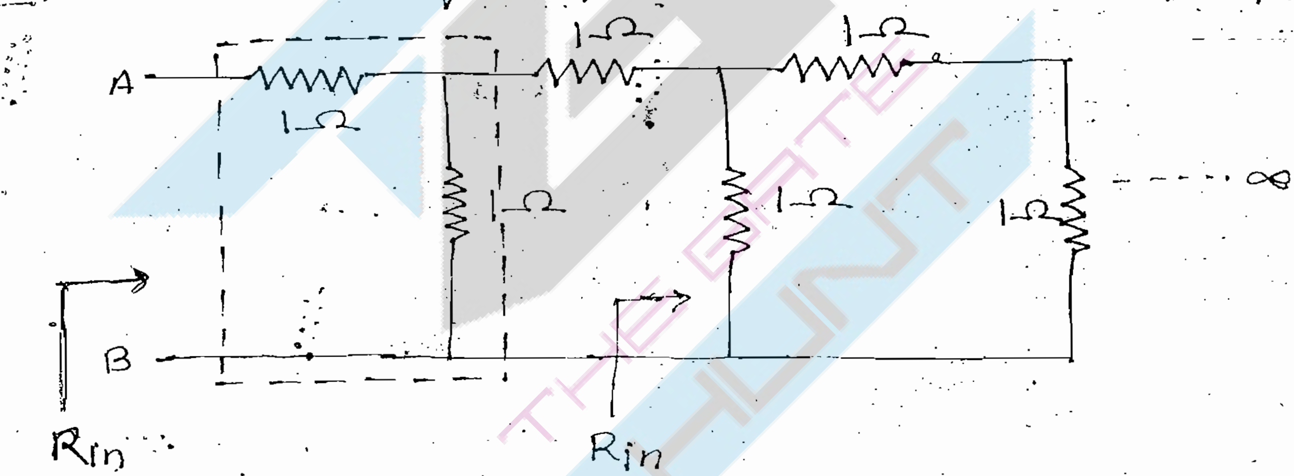
Soln :-



$$\boxed{I = -4.5\,A}$$

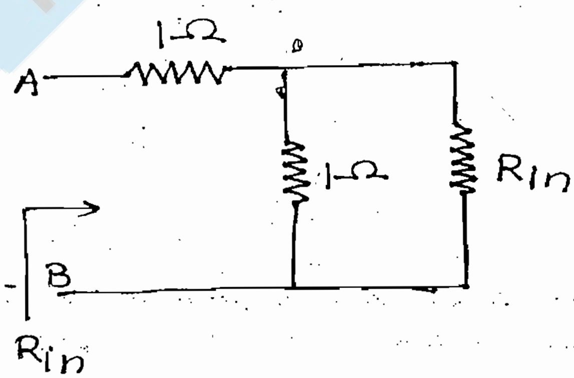


Ques:- Find the equivalent resistance w.r.t A & B



Soln:-

$$R_{in} = 1 + \frac{1 \times R_{in}}{1 + R_{in}}$$

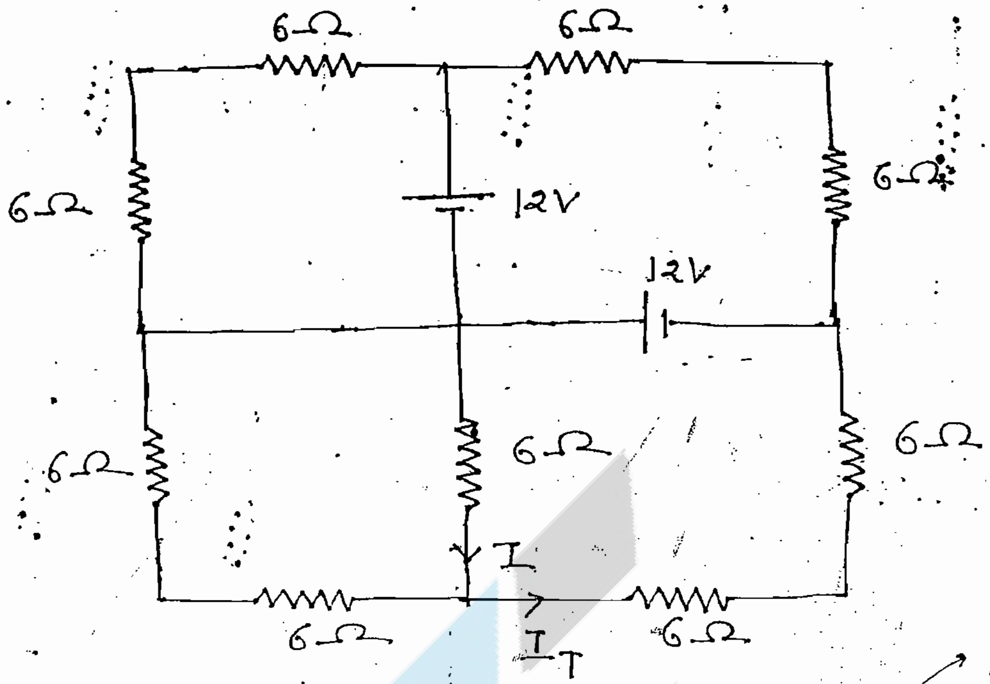


$$R_{in} = \frac{1 + R_{in} + R_{in}}{1 + R_{in}}$$

$$\Rightarrow \quad R_{in}^2 - R_{in} - 1 = 0$$

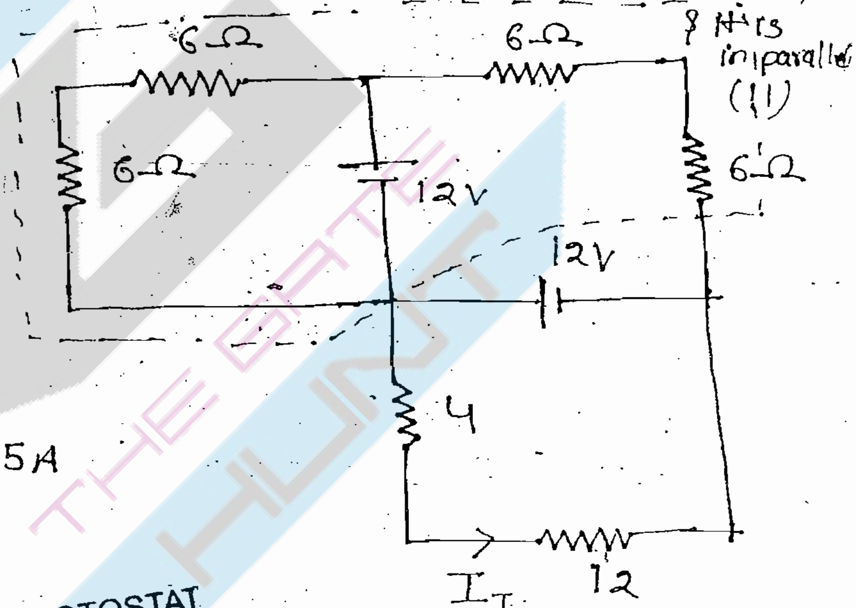
$$\boxed{R_{in} = \frac{1 + \sqrt{5}}{2}}$$, Ans

Ques:- Find I of the circuit shown



Not effect in cal: I_T (∵ V → ideal) & Hrs in parallel (||)

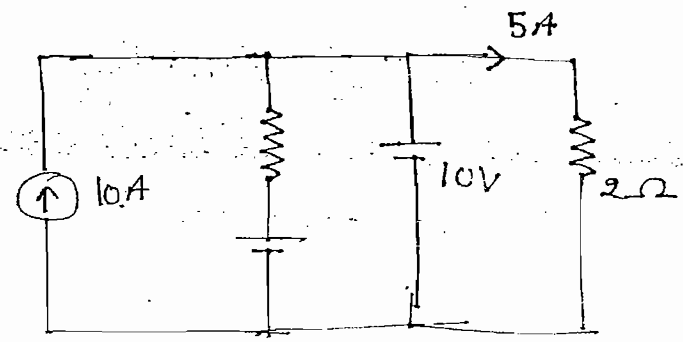
Soln:-



$$I_T = \frac{12}{4+12}$$

$$I = I_T \frac{12}{12+6}$$

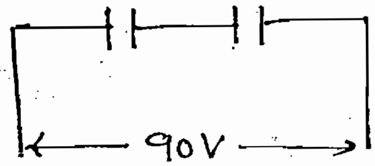
$$= \frac{12}{16} \times \frac{12}{18} = 0.5A$$



Ques:- When two capacitance 40F & 20F are connected in series with a source voltage 90V. When two capacitor charged fully then they are connected in parallel. Find voltage across capacitor in parallel branch.

(a) 40V (b) 60V (c) 45V (d) 30V

Soln:-

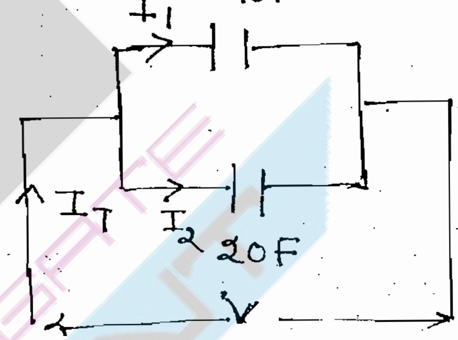


$$C_{eq} = \frac{40\times 20}{40+20} = \frac{40}{3}$$

$$Q = C_{eq}V = \frac{40}{3}\times 90$$

$$= \cancel{90}\ 1200C$$

$$Q_T = 1200 + 1200 = 2400C$$



$$Q_T = 2400$$

$$C_{eq} = 40+20 = 60$$

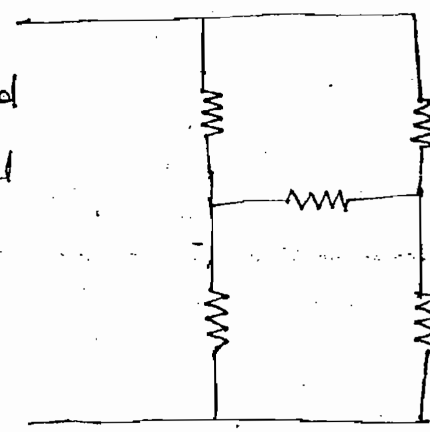
$$V = \frac{Q_T}{C_{eq}} = \frac{2400}{60}$$

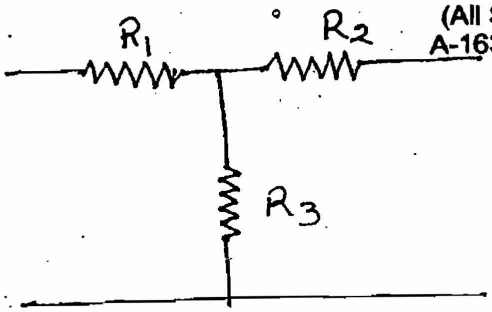
$$= 40V \text{ , Ans}$$

~~Ques~~:-

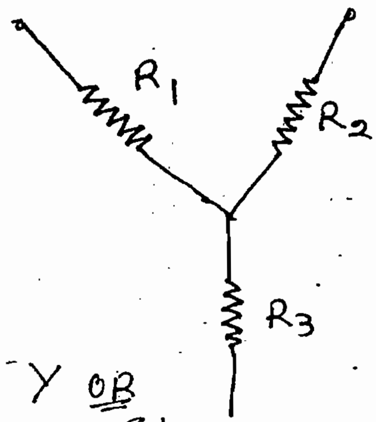
Note:-

When elements are connected not in series nor in parallel network is reduced by Star-delta transformation

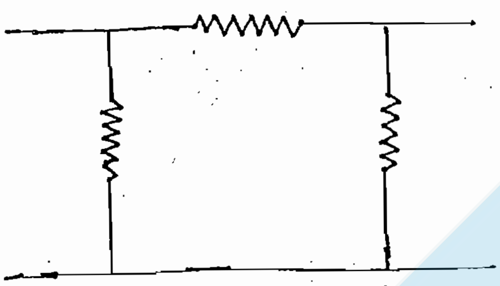




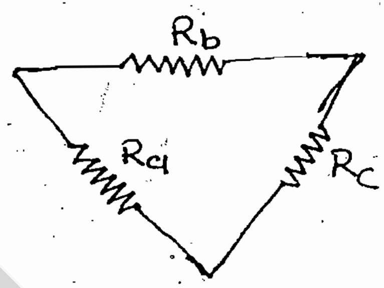
T



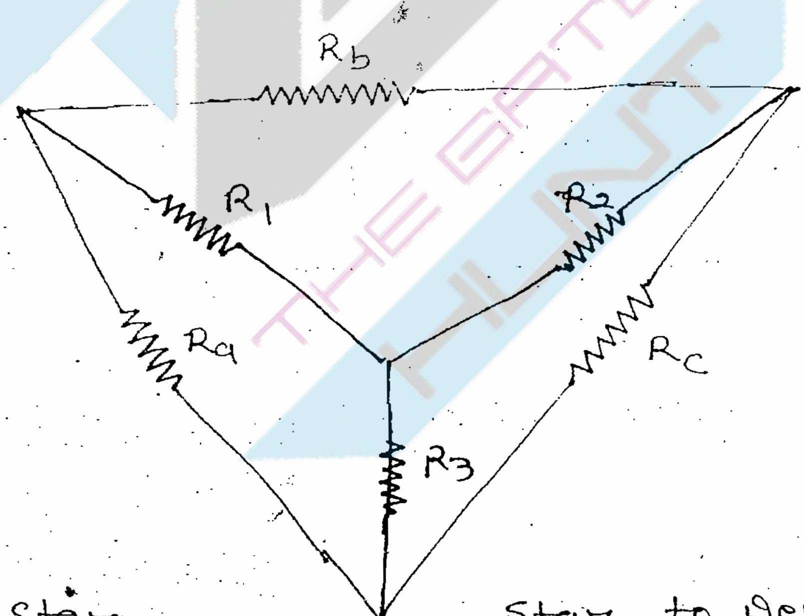
Y or Star



π



Delta or Mesh



<u>Delta to Star</u>

$$R_1 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_c}{R_a + R_b + R_c}$$

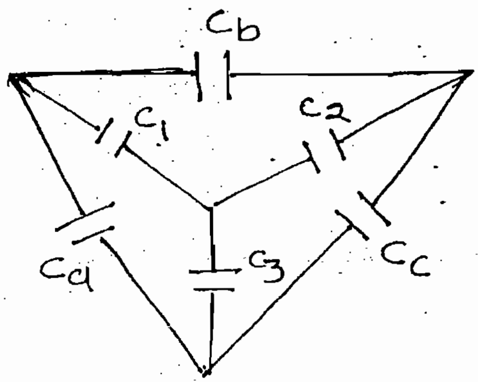
<u>Star to Delta</u>

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

→ The procedure of transformation either from delta to star or star to delta for the resistor, inductor and impedance is same



Delta to Star :—

$$\frac{1}{C_1} = \frac{\frac{1}{C_a} \cdot \frac{1}{C_b}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$

$$\frac{1}{C_2} = \frac{\frac{1}{C_b} \cdot \frac{1}{C_c}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$

$$\frac{1}{C_3} = \frac{\frac{1}{C_a} \cdot \frac{1}{C_c}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$

Star to Delta :—

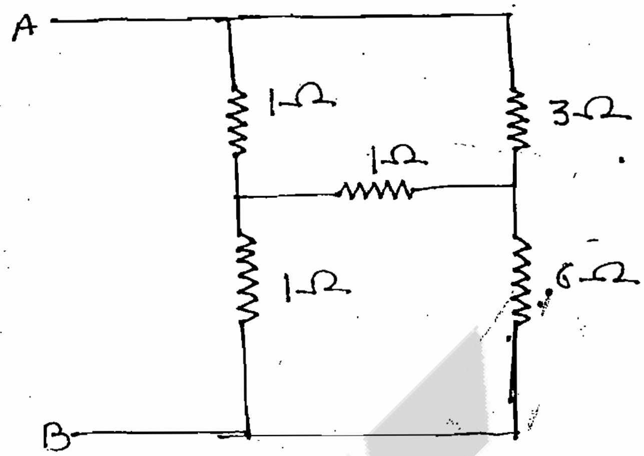
$$\frac{1}{C_a} = \frac{\frac{1}{C_1}\frac{1}{C_2} + \frac{1}{C_2}\frac{1}{C_3} + \frac{1}{C_3}\frac{1}{C_1}}{1/C_2}$$

$$\frac{1}{C_b} = \frac{\frac{1}{C_1}\frac{1}{C_2} + \frac{1}{C_2}\frac{1}{C_3} + \frac{1}{C_3}\frac{1}{C_1}}{1/C_3}$$

$$\frac{1}{C_c} = \frac{\frac{1}{C_1}\frac{1}{C_2} + \frac{1}{C_2}\frac{1}{C_3} + \frac{1}{C_3}\frac{1}{C_1}}{1/C_1}$$

Lecture - 3

Ques:- Find equivalent resistance w.r.t A & B



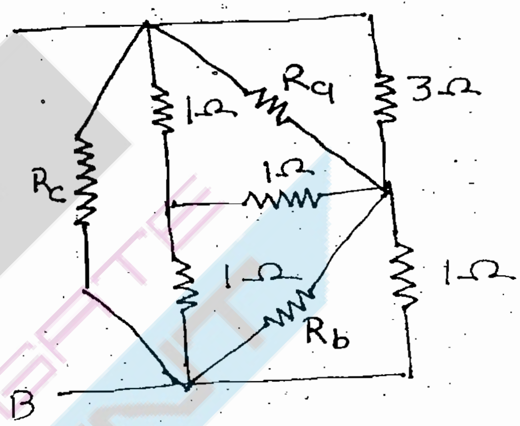
Soln:-

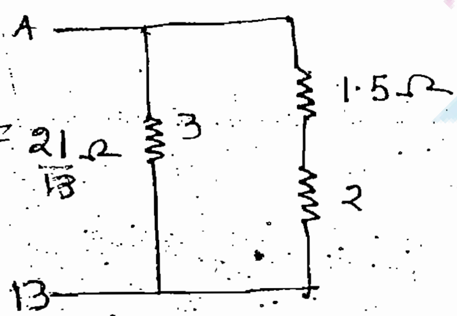
$$R_a = \frac{(1\times1)+(1\times1)+(1\times1)}{1}$$

$$= 3\Omega$$

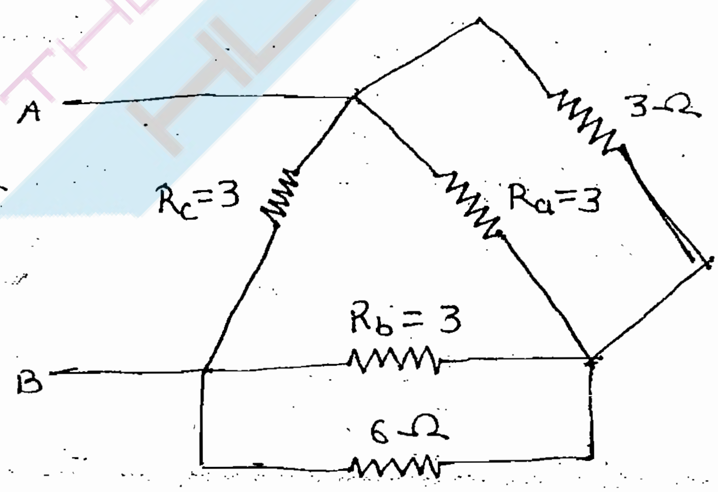
$$R_b = 3$$

$$R_c = 3$$





$R_{eq} = \frac{21}{13}\Omega$

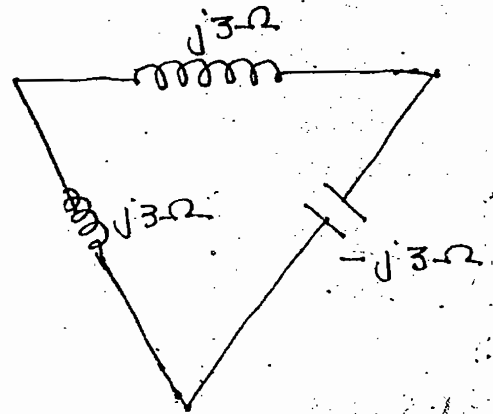


Note:-

When resistor of equal value transfer from star to delta resistance inc. by 3 times.

When capacitance of equal value transform from star to delta capacitance dec. by 3 times

Ques:- Obtain equivalent star connection of the circuit shown



Soln:- jX_L, $-jX_C$

$$Z_1 = \frac{(j3)(j3)}{j3 + j3 - j3}$$

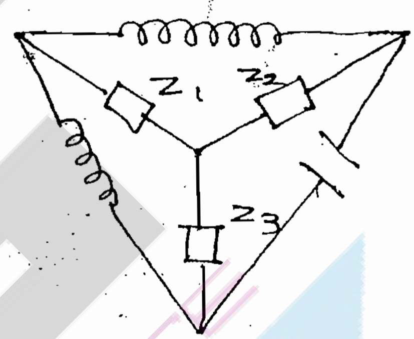
$$\boxed{Z_1 = j3}$$

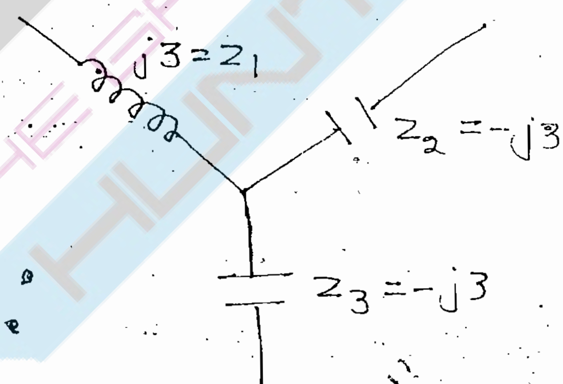
$$Z_2 = \frac{(j3)(-j3)}{j3}$$

$$\Rightarrow \boxed{Z_2 = -j3}$$

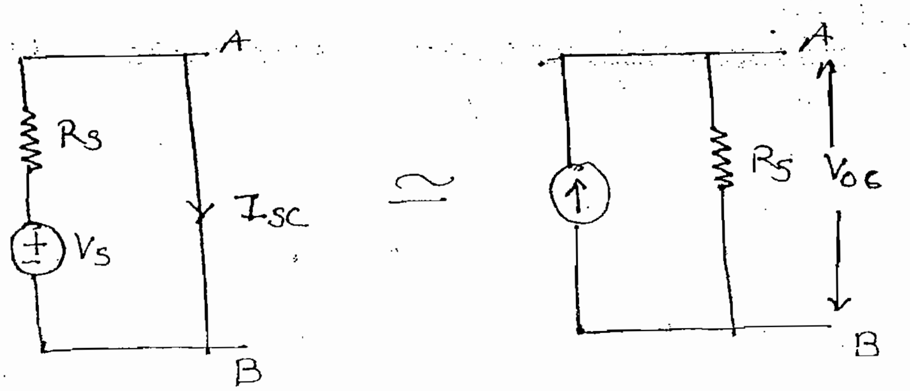
$$Z_3 = \frac{(j3)(-j3)}{j3}$$

$$\Rightarrow \boxed{Z_3 = -j3}$$





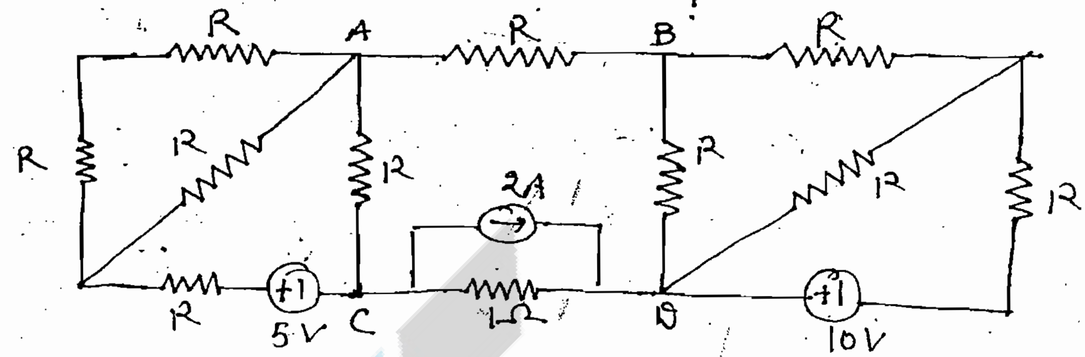
Source Transformation:-



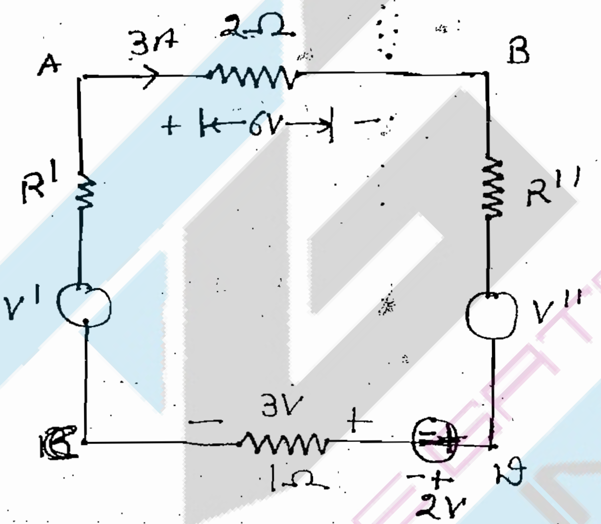
$I_S = I_{SC} = \frac{V_S}{R_S}$ | $V_S = V_{OC} = I_S R_S$

$\Rightarrow R_S = R_S$ | $R_S = R_S$

Ques:- In a circuit shown with $V_A - V_B = 6$ then find $V_C - V_D$



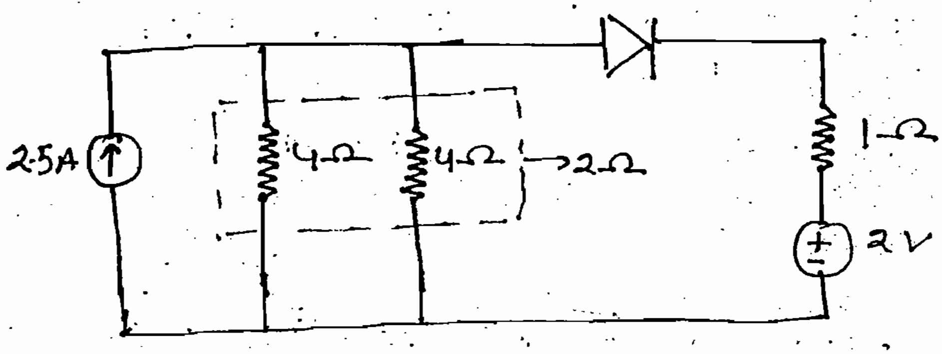
Soln:-



$V_C - V_D = -5\ V$

Ques:- Find current flown through ideal diode of the circuit

Soln:-

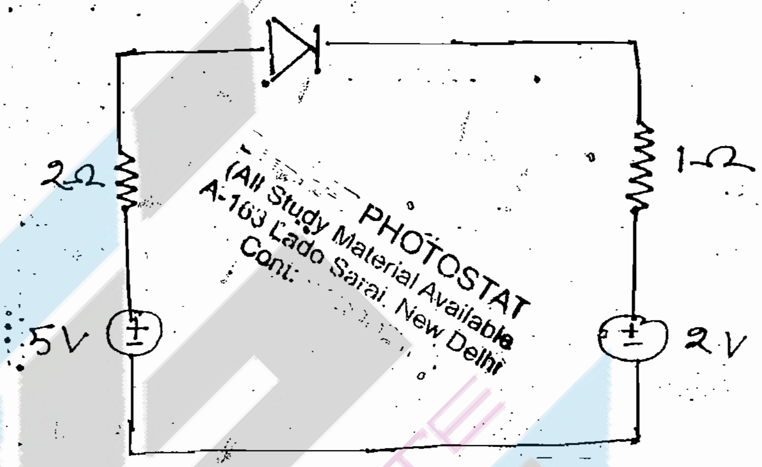


$V_{S_1} = 2.5 \times 2 = 5V$

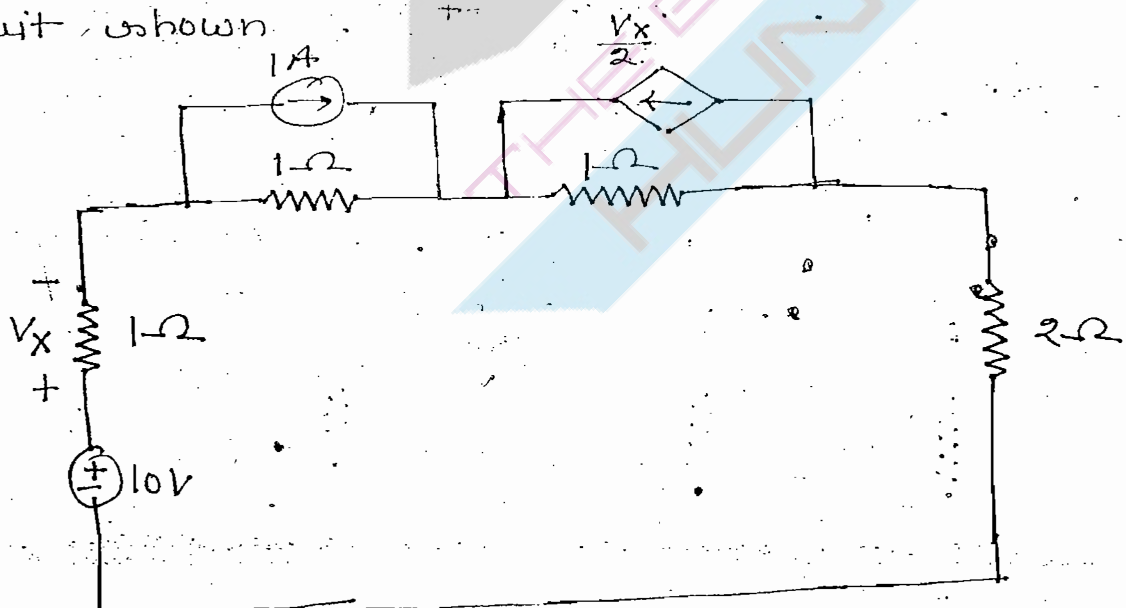
$i = \frac{V_{eq}}{R_{eq}}$

$\Rightarrow i = \frac{5-2}{2+1}$

$i = 1A$



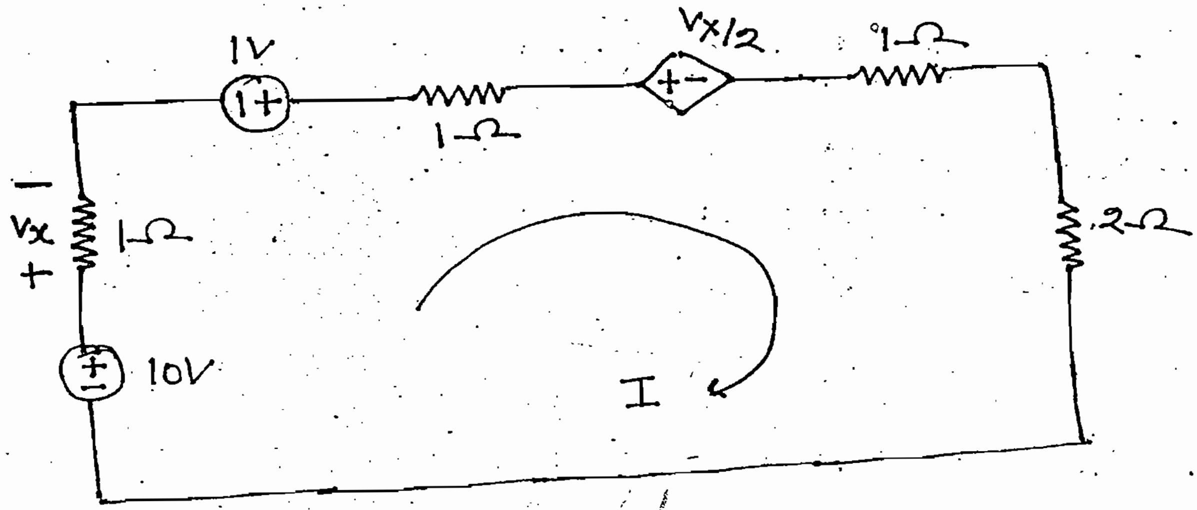
Ques:- Find current in 2Ω resistor for the circuit shown.



Note:-

While applying source transformation for dependent source wherever dependent source magnitude depends without disturbing an element transformation can be applied

Soln :-



$$-10 + 5I - 1 + \frac{V_x}{2} = 0 \quad \text{— (I)}$$

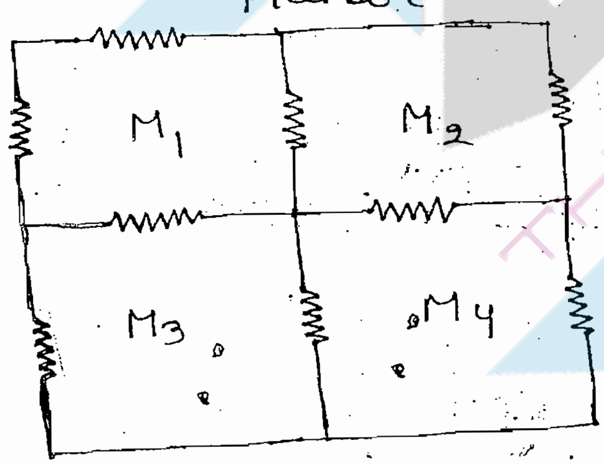
$$V_x = 1 \times I = I \quad \text{— (II)}$$

From (I) & (II)

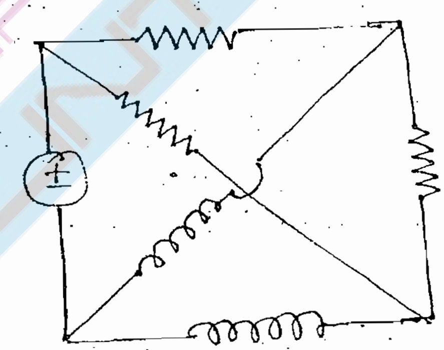
$$\boxed{I = 2A} \quad \text{Ans.}$$

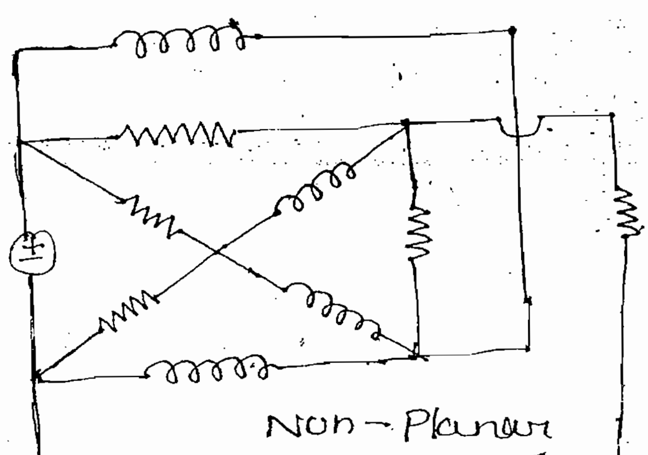
Mesh Analysis :-

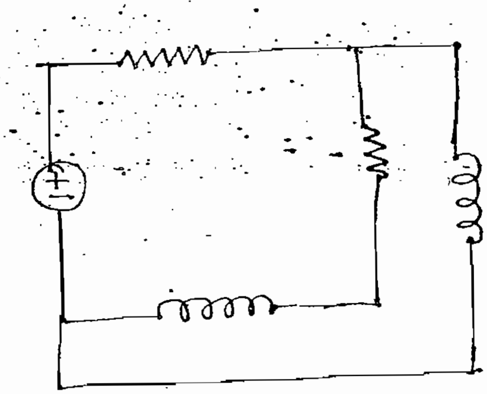
Planar



Planar







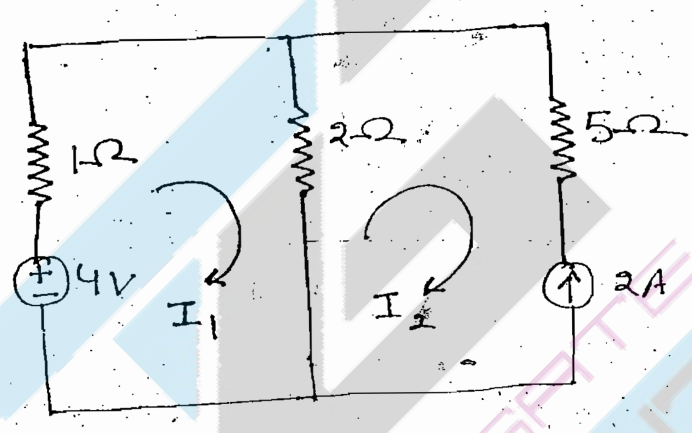
Non - Planar

→ Mesh is a loop which does not consist of any loop

→ When the network is drawn on plane without any crossover then the network is called as planar network

Procedure of Mesh Analysis :—

1. Identify total no. of meshes in the given network
2. Assign the current direction for each mesh
3. Develop KVL equation for each mesh
4. By solving KVL equations find loop currents



$$-4 + 3I_1 - 2I_2 = 0$$
$$-4 + (1 \times I_1) + 2(I_1 - I_2) = 0$$
→ (I). (same)

$I_2 = -2$ —— (II) From (I) & (II)

$$\boxed{I_1 = 0}$$

Note :—

e = Mesh No b = total no. of branches

N = total no. of nodes

$$** \boxed{e = b - (N-1)}$$

In above ques

$e = 2$

$b = 3$

$N = 2$

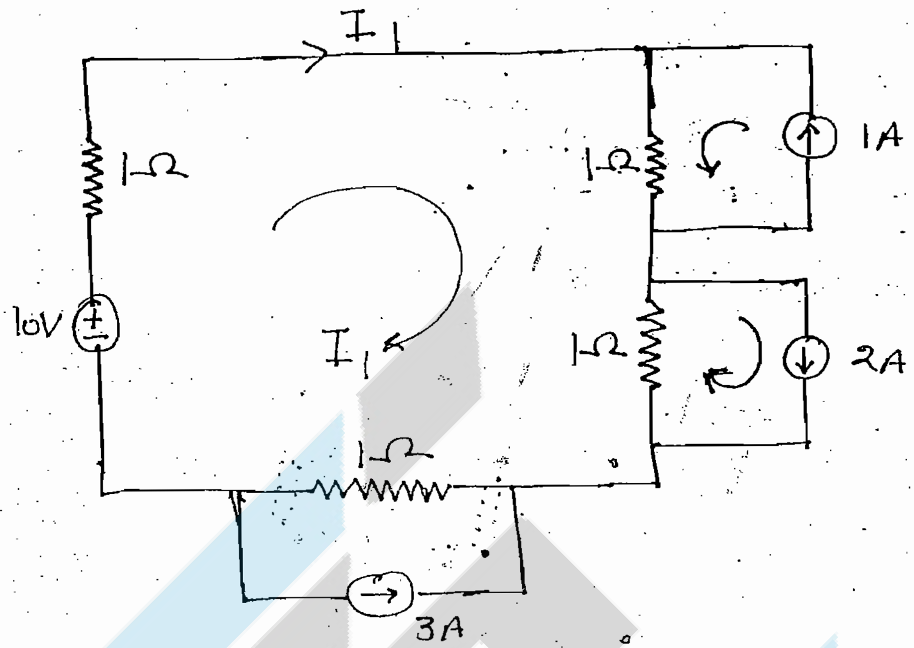
$e = 3 - (2-1)$

$$\boxed{e = 2}$$

~~e = 3 - (2~~

→ In above network to find loop current minimum one equation required

Ques:- Find I_1 of the circuit shown.

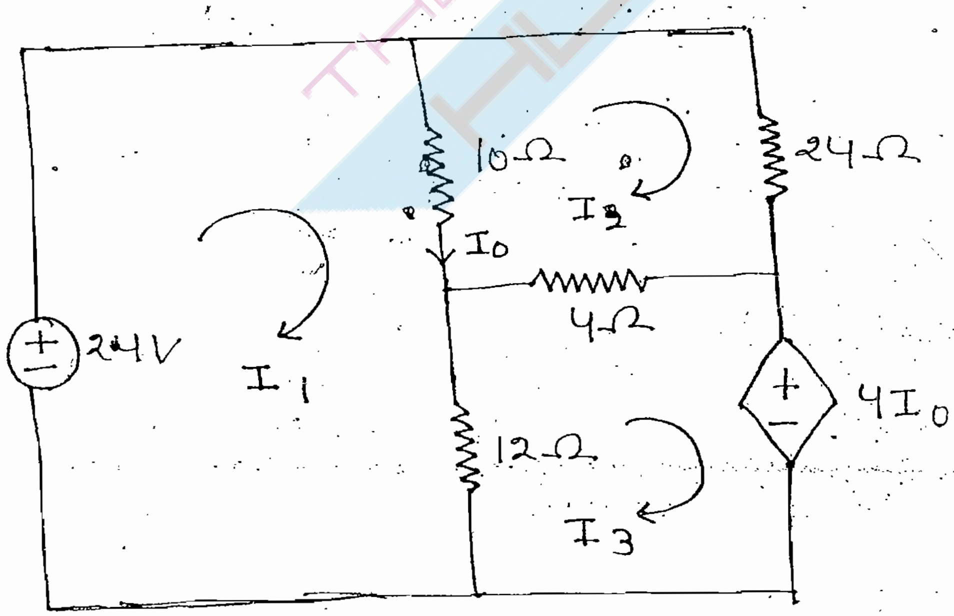


Soln:-

$$-10 + 4I + (1\times1) - (2\times1) + 3(\times1) = 0$$

$$\Rightarrow \quad I = 2A \quad \& \quad I_1 = I$$

Ques:- Find I_o



Sol^n:-

$$24 = 22I_1 - 10I_2 - 12I_3 \quad \text{——(I)}$$

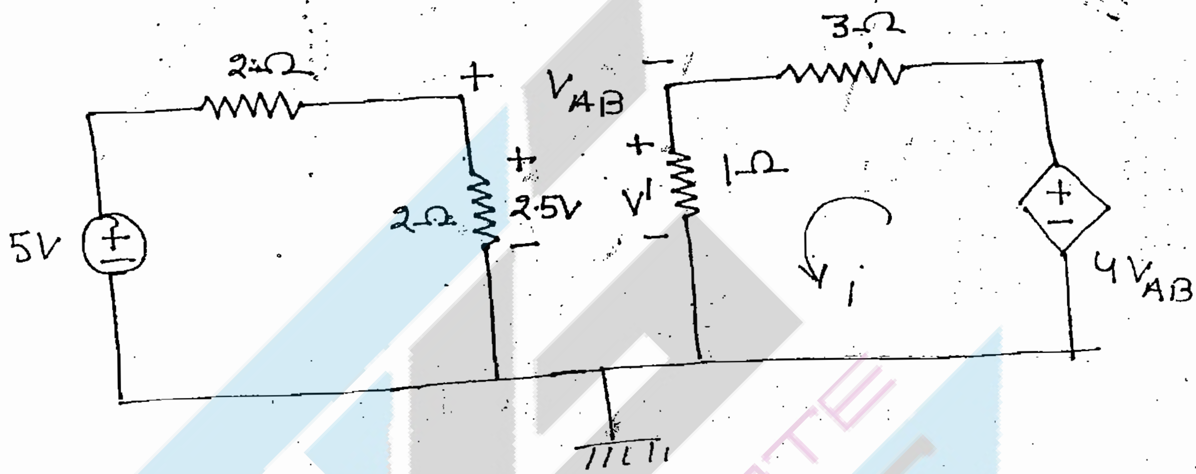
$$0 = -10I_1 + 38I_2 - 4I_3 \quad \text{——(II)}$$

$$-4I_0 = -12I_1 - 4I_2 + 16I_3 \quad \text{——(III)}$$

$$I_0 = I_1 - I_2$$

$$\boxed{I_0 = 1.5A}$$

Ques:- Find i of the ckt shown



Sol^n:-

$$i = \frac{4V_{AB}}{3+1} = V_{AB}$$

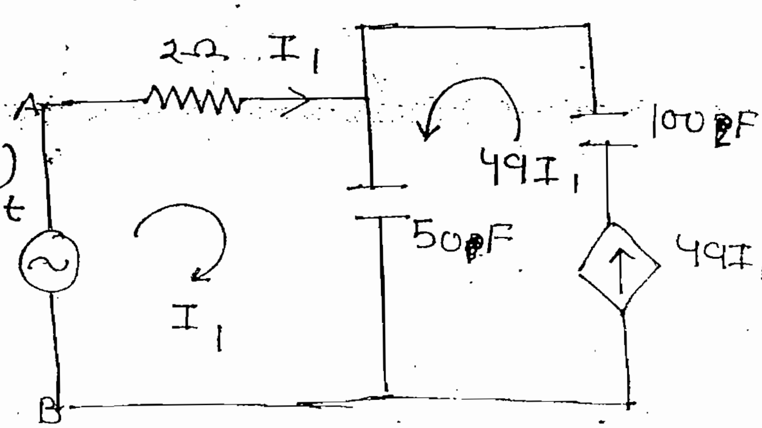
$$V' = i \times 1 = i$$

$$-2.5 + V_{AB} + V' = 0$$

$$\Rightarrow \boxed{i = 2.5A} \quad \text{Ans}$$

Ques:- Find C_{eq} w.r.t A & B

Sol^n:-



$$V_s = 2I_1 + \frac{1}{50}\int (I_1 + 49I_1)\,dt$$

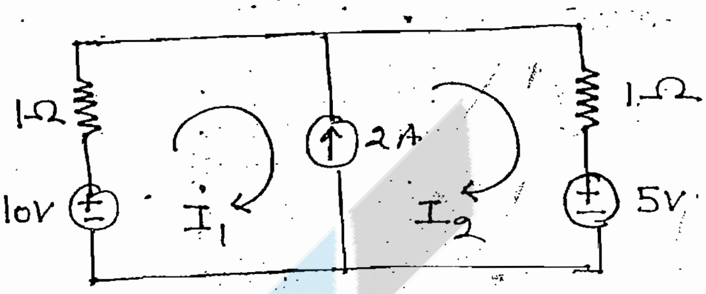
$$\Rightarrow V_s = 2I_1 + \frac{50}{50}\int I_1\,dt \quad \text{——(1)}$$

$$V_S = RI_1 + \frac{1}{C_{eq}} \int I_1 dt \quad -(II)$$

compare (I) & (II)

$$\boxed{C_{eq} = 1}$$, Ans

Ques:- Find I_1 and I_2 of the circuit shown



Note:-

When current source branch is common for two meshes it is possible to find solution using supermesh technique

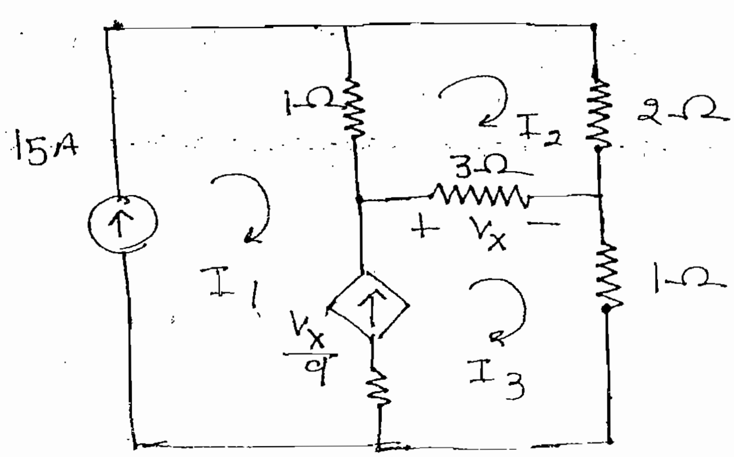
$$KVL \rightarrow -10 + (1 \times I_1) + (1 \times I_2) + 5 = 0 \quad -(I)$$

$$KCL \rightarrow I_2 - I_1 = 2 \quad -(II)$$

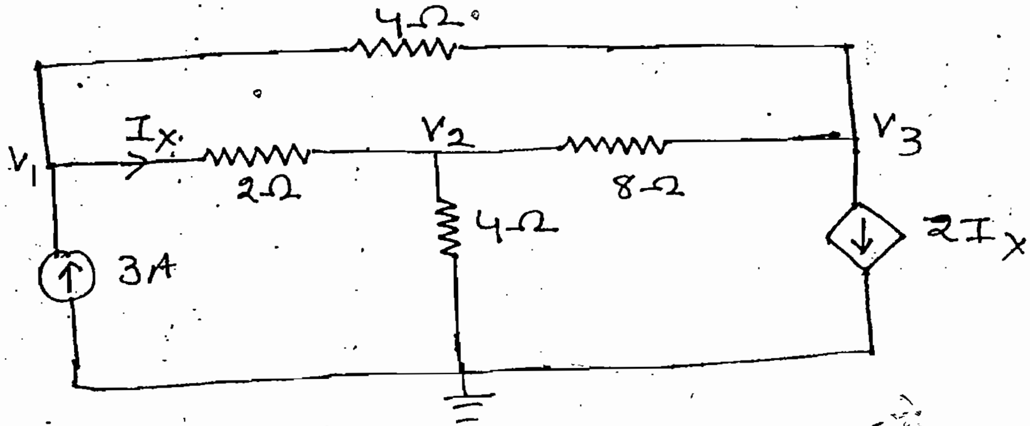
Mesh	$\longrightarrow$	KVL + ohm's law
Super Mesh	$\longrightarrow$	KVL + KCL + ohm's law

$$\Rightarrow \quad I_1 = 1.5 \quad I_2 = 3.5 \quad , \text{Ans}$$

Ques:- Find loop currents of the circuit shown!-



Ques:-



Find V_1, V_2 & V_3

Soln:-

$$\frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = 3 \quad \text{—(I)}$$

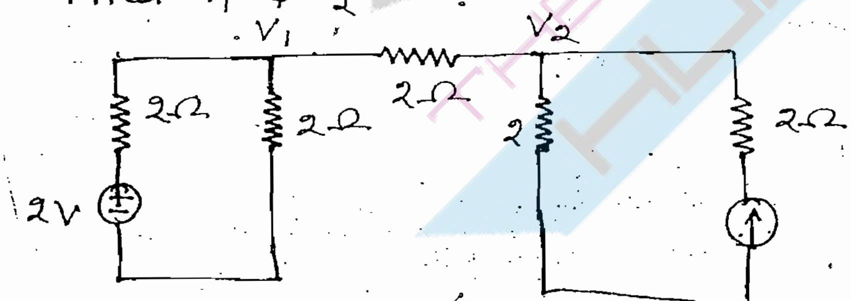
$$\frac{V_2}{4} + \frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{8} = 0 \quad \text{—(II)}$$

$$\frac{V_3 - V_1}{4} + \frac{V_3 - V_2}{8} + 2I_x = 0 \quad \text{—(III)}$$

$$I_x = \frac{V_1 - V_2}{2} \quad \text{—(IV)}$$

$\underline{V_1 = 4.8}$, $\underline{V_2 = 2.4}$, $\underline{V_3 = -2.4}$, Ans

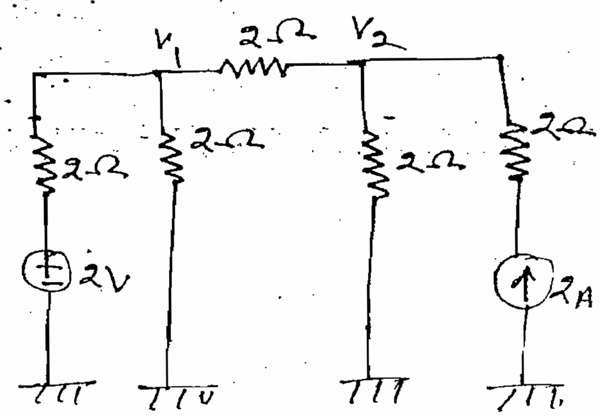
Ques:- Find V_1 & V_2



Soln:-

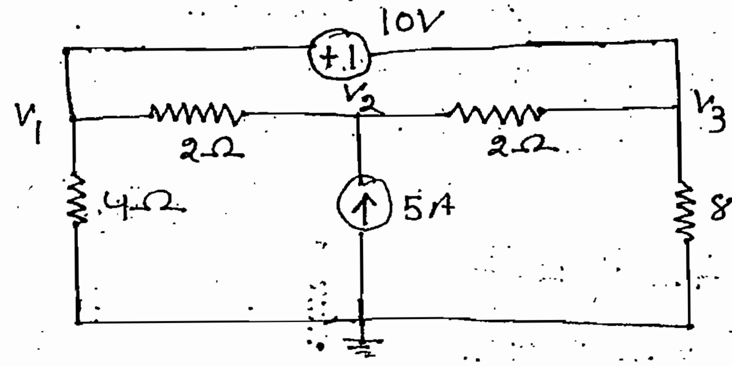
$$\frac{V_1 - 2}{2} + \frac{V_1}{2} + \frac{V_1 - V_2}{2} = 0 \quad \text{—(I)}$$

$$\frac{V_2}{\cancel{2}} + \frac{V_2 - V_1}{2} = 2 \quad \text{—(II)}$$



$$\boxed{\begin{aligned} V_1 &= 1.6\ V \\ V_2 &= 2.8\ V \end{aligned}}$$ Ans

Ques:- Find node voltages of the circuit shown.



Note:-

When ideal voltage source is connected b/w two non-reference node it is possible to find solution by using supernode technique

Soln:-

$$\frac{V_1}{4} + \frac{V_1 - V_2}{2} + \frac{V_3}{8} + \frac{V_3 - V_2}{2} = 0 \quad \text{—(I)} \rightarrow \text{KCL}$$

$$V_1 - V_3 = 10 \quad \text{—— (II)} \longrightarrow \text{KVL}$$

$$-5 = \frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{2} \quad \text{—— (III)}$$

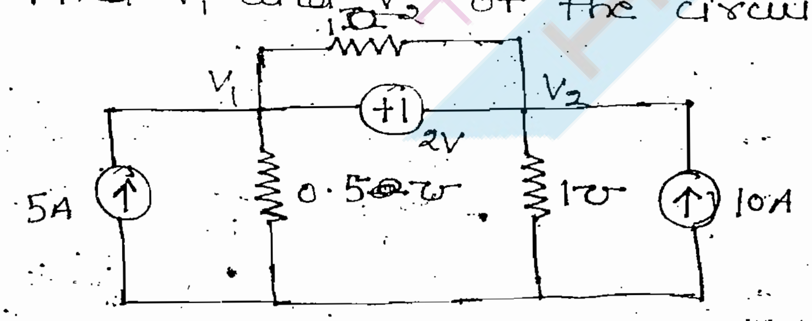
$V_1 = \cancel{16.67}\ 16.67\text{V}$ $\quad V_2 = 16.67$ $\quad V_3 = 6.67\text{V}$, Ans

Note:-

Nodal → KCL + ohm's law

Super Node →, KCL + KVL + ohm's law

Ques:- Find V_1 and V_2 of the circuit shown

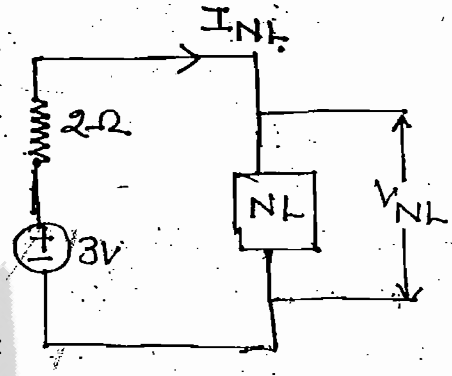


Soln:- Resistance connected in parallel with voltage source does not influence

$$10 + 5 = (V_1 \times 0.5) + (V_2 \times 1) \longrightarrow \text{KCL}$$

$$V_1 - V_2 = 2 \longrightarrow \text{KVL}$$

Ques:- The practical source of 3V and internal resistance 2Ω connected to non-linear resistor. The characteristic of non-linear resistor is given by $V_{NL} = I_{NL}^2$. Find power dissipation in the non-linear resistor.



Soln:-

$$-3 + 2I_{NL} + V_{NL} = 0$$

$$-3 + 2I_{NL} + I_{NL}^2 = 0$$

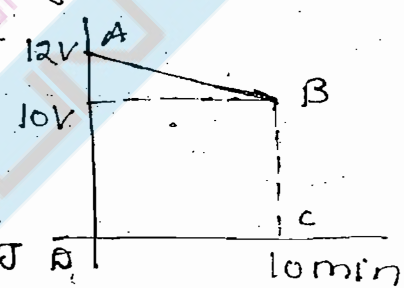
$$I_{NL}^2 + 2I_{NL} - 3 = 0$$

$$I_{NL} = 1$$

$$V_{NL} = I_{NL}^2 = (1)^2 = 1$$

$$P_{NL} = V_{NL} I_{NL} = 1 \times 1 = 1W$$

Ques:- A fully charged mobile phone is good for 10min talk time. During talktime battery delivers a constant current of 2A. The voltage characteristics of battery is as shown in figure. Find energy of battery during talk time.

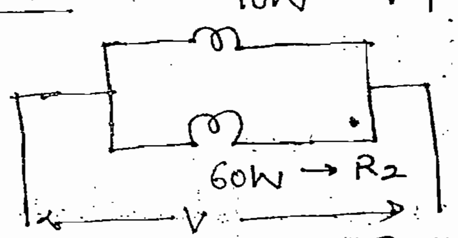


Soln:- $t = 10\text{min} = 10 \times 60 = 600s$

$$V \times t = 6600$$

$$W = Vit = 6600 \times 2 = 13.2\,kJ$$

Note:- $40W \rightarrow R_1$



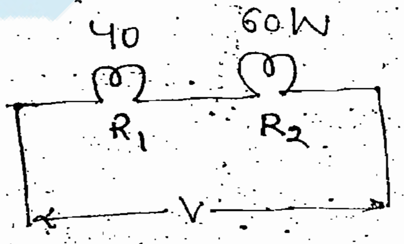
$\cancel{R_2} > \cancel{R_1}$ $\quad R_1 > R_2$

$P_1 = \dfrac{V^2}{R_1}$ $\quad P_2 = \dfrac{V^2}{R_2}$

$P_1 < P_2$

↓

More brightness



$P = \dfrac{V^2}{R}$

$P_1 = i^2 R_1$ $\quad R_1 > R_2$

$P_2 = i^2 R_2$ $\quad P_1 > P_2$

↓

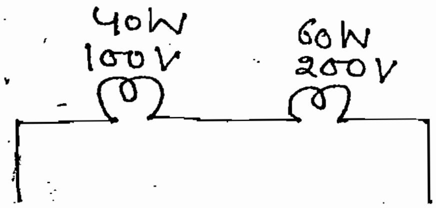
More brightness

→ When the bulbs are connected in series low rating bulb used more brightness

→ When the bulbs are connected in parallel high rating bulb used more brightness

→ In the above two cases voltages reading of bulb are equal

Ques:— In the given connection which bulb glow brightly



Soln:— $R = \frac{V^2}{P}$

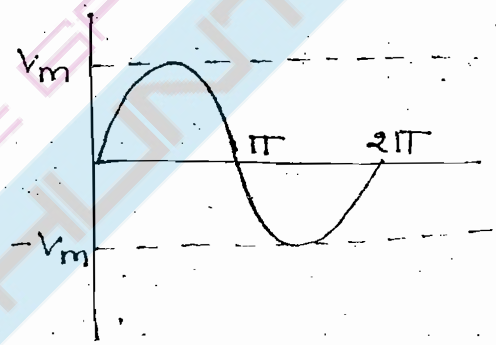
$R_1 = \frac{(100)^2}{40}$ $\quad R_2 = \frac{(200)^2}{60}$

$R_2 > R_1$ $\quad R = i^2R$

→ 60W bulb having more brightness

Steady State AC Circuits :—

$V(t) = V_m \sin\omega t$



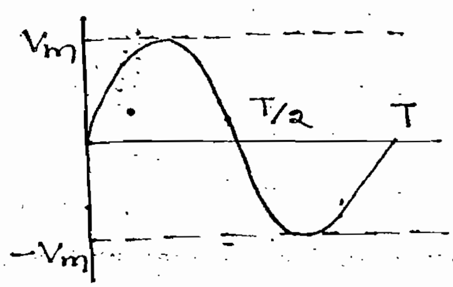
V_m = Peak or Max. value

ω = Angular frequency → rad/sec

ωt = Argument – rad

$\omega T = 2\pi$

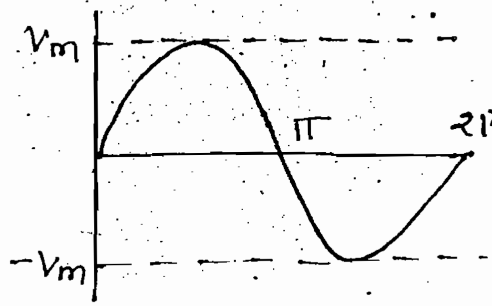
$\Rightarrow \boxed{T = \frac{2\pi}{\omega}}$ sec.



$\boxed{f = \frac{1}{T}}$

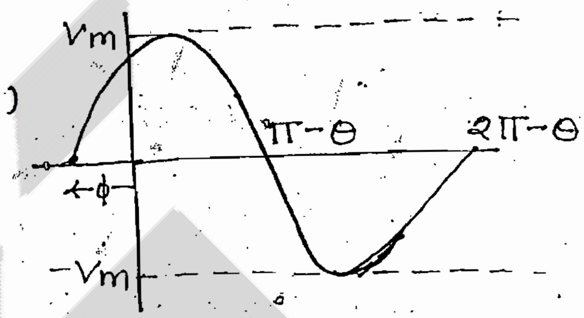
$\boxed{f = \frac{\omega}{2\pi}}$ Hz or cycles/sec.

$v(t) = V_m \sin \omega t$



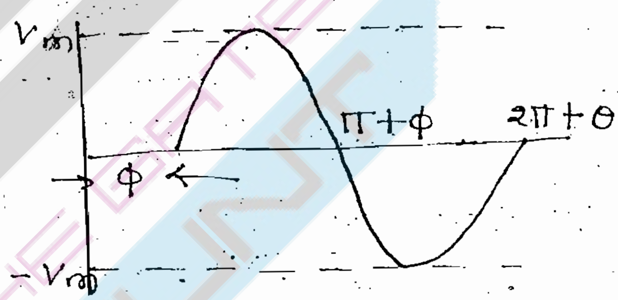
$v(t) = V_m \sin(\omega t + \theta)$

$\longrightarrow$ leading



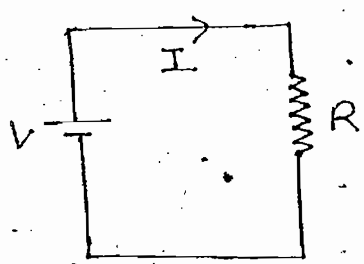
$v(t) = V_m \sin(\omega t - \theta)$

$\longrightarrow$ lagging



RMS Value :-

t



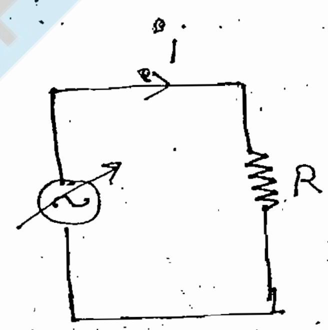
$P = I^2 R$

$W = I^2 R t$

$\downarrow$ DC

Heat

t



$P = i^2 R$

$W = i^2 R t$

$\downarrow$ AC

Heat

$\boxed{W_{AC} = W_{DC}}$

→ RMS value is defined based on heating effect of the waveform

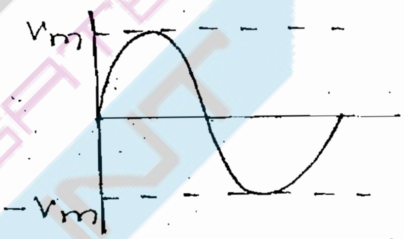
→ The voltage at which heat dissipation in A.C circuit is equal to heat dissipation in DC circuit is called as V_{RMS} provided both AC and DC circuit having equal value of resistance and operated for same time

$$\boxed{\begin{aligned} V_{RMS} &= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V^2 \omega t} \\ V_{RMS} &= \sqrt{\frac{1}{T}\int_0^{T} V^2 dt} \end{aligned}}$$

Lecture - 4

Ques:- Find RMS value of following waveforms

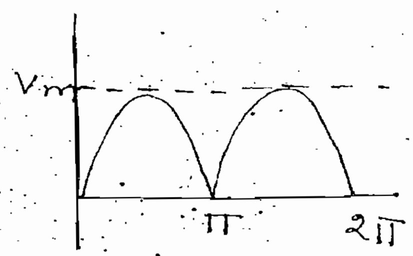
(1) $V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V^2 d\omega t}$



$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (V_m \sin\omega t)^2 dt}$

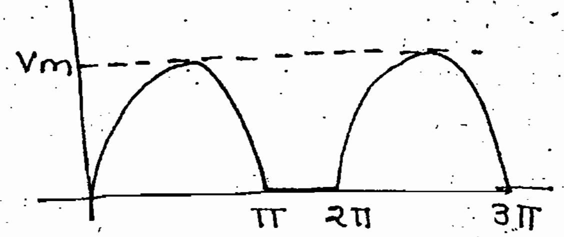
$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \left(\frac{1-\cos 2\omega t}{2}\right) dt}$

$\Rightarrow \boxed{V_{RMS} = \frac{V_m}{\sqrt{2}}}$



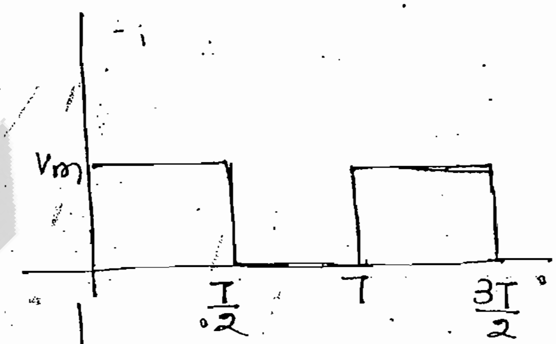
(II) $\boxed{V_{RMS} = \frac{V_m}{\sqrt{2}}}$

→ $\boxed{V_{RMS} = \dfrac{V_m}{2}}$

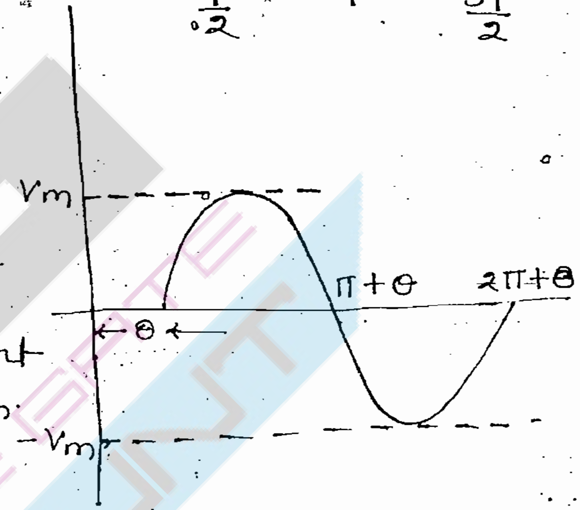


→ $\boxed{V_{RMS} = \dfrac{V_m}{\sqrt{2}}}$

$$\left[V_{RMS} = \sqrt{\frac{1}{T}\left[\int_0^{T/2} V_m^2\,dt + \int_{T/2}^{T} 0\,dt\right]} \right]$$

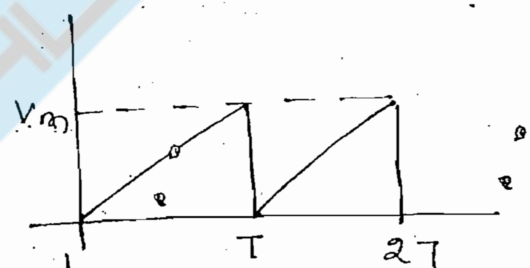


→ $\boxed{V_{RMS} = \dfrac{V_m}{\sqrt{2}}}$



Note:- RMS value is independent on the position of starting of waveform.
But it depends on the shape of waveform.

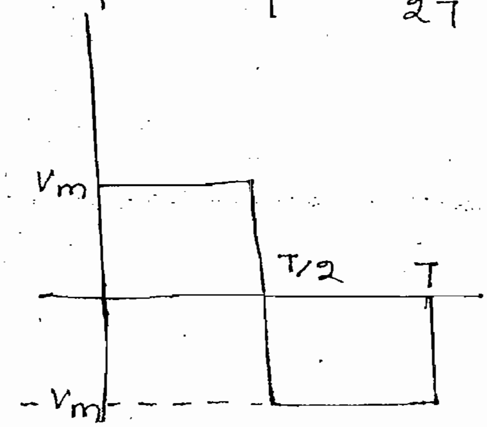
→ $\boxed{V_{RMS} = \dfrac{V_m}{\sqrt{3}}}$



$0 < t < T \quad y = mx$

$\Rightarrow \quad V = \dfrac{V_m}{T}\, t$

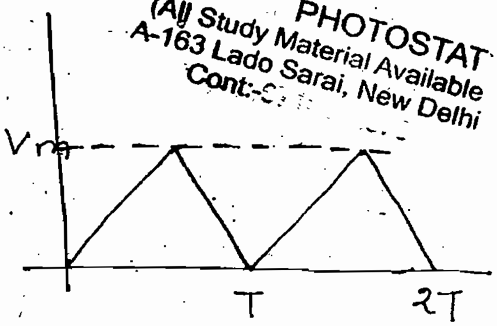
→ $\boxed{V_{RMS} = V_m}$



$\rightarrow \quad V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v^2 dt}$

$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{V_m}{T} t\right)^2 dt}$

$\Rightarrow \quad \boxed{V_{RMS} = \frac{V_m}{\sqrt{3}}}$



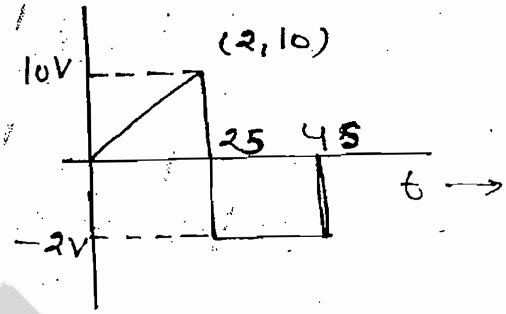
$\rightarrow \quad 0 < t < 2$

$y = mx$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-0}{2-0}$

$= 5$

$V = 5t$

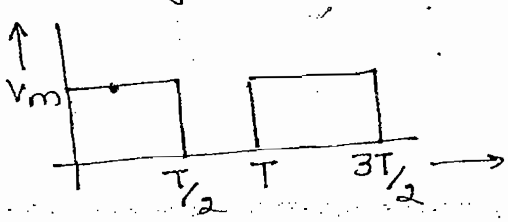


$V_{RMS} = \sqrt{\frac{1}{4}\left[\int_0^2 (5t)^2 dt + \int_2^4 (-2)^2 dt\right]}$

$\Rightarrow \quad V_{RMS} = \sqrt{\frac{1}{4}\left[25\left(t^3/3\right)^2 + (4t)_2^4\right]}$

$\Rightarrow \quad V_{RMS} = 4 - - - - - -$

Ques:- Find power dissipation in the resistor for the given voltage waveform



(a) $P_{av} = \frac{P_{peak}}{\sqrt{2}}$

(b) $P_{av} = \frac{P_{peak}}{2}$

(c) $P_{av} = P_{peak}$

(d) $P_{av} = \frac{P_{peak}}{\sqrt{3}}$

Soln:- B. $P_{peak} = \frac{V_m^2}{R}$

$P_{av} = \frac{V_{RMS}^2}{R} = \frac{(V_m/\sqrt{2})^2}{R} = \frac{V_m^2}{2R} = \frac{P_{peak}}{2}$

Note:-

→ $$P_{RMS} = \frac{V_{RMS}^2}{R}$$

→ ** $$\boxed{\frac{P_{DC}}{P_{AC}} = \frac{I_{av}^2 R}{I_{RMS}^2 R}}$$

Ques:- Find RMS value for the following function

$$V(t) = 3 + \sin 3t + \cos t$$

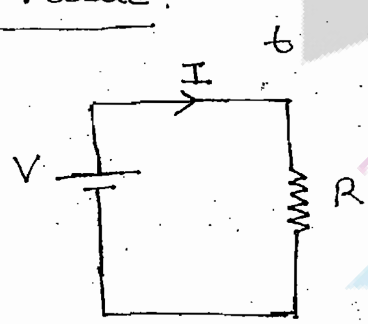
Solⁿ:- $$V_{RMS} = \sqrt{V_{RMS_1}^2 + V_{RMS_2}^2 + V_{RMS_2}^2 + \cdots + V_{RMS_n}^2}$$

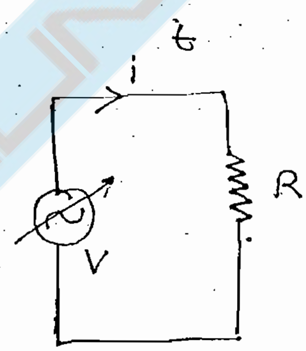
[Used when different ω are present]

$$\Rightarrow V_{RMS} = \sqrt{(3)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$\Rightarrow \boxed{V_{RMS} = \sqrt{10}}$$

Average Value:-





$$I = \frac{V}{R}$$ $$i = \frac{V}{R}$$

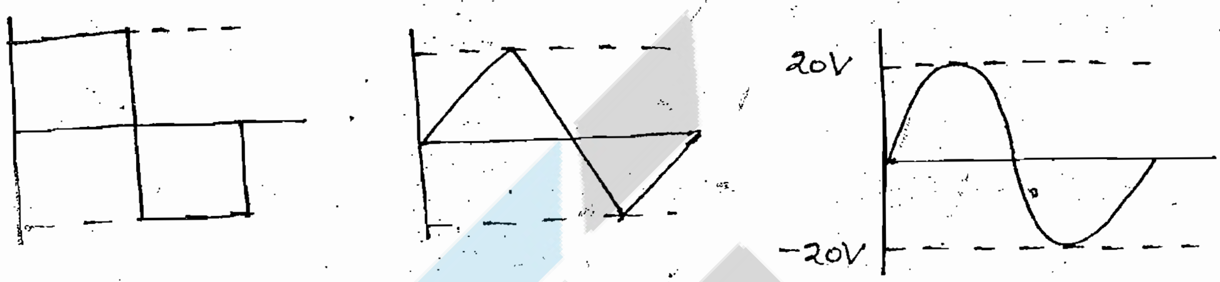
$Q = It \rightarrow DC$ | $Q = it \rightarrow AC$

$$\boxed{Q_{A\cdot C} = Q_{DC}}$$

→ Average value is defined based on charge transfer in the circuit

→ The voltage at which charge transfer in AC circuit is equal to charge transfer in DC circuit is called as V_{avg} provided both AC and DC circuit having equal value of resistance and operated for same time.

Symmetrical Wave :-



$$\text{Form factor} = \frac{V_{RMS}}{V_{av}}$$

→ Avg Value for complete symmetrical wave = 0

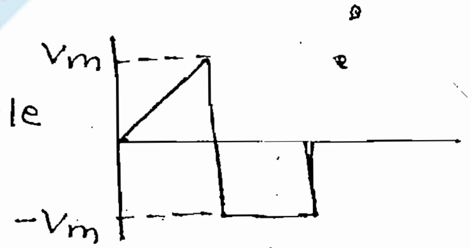
→ Hence we can find Average value only for half cycles for symmetrical waveform

$$\boxed{V_{av} = \frac{1}{\pi}\int_0^{\pi} V\,d\omega t}$$

{ For current and voltage waveform

Unsymmetrical Wave :-

→ Finding average value of unsymmetrical wave angle of complete cycle is considered



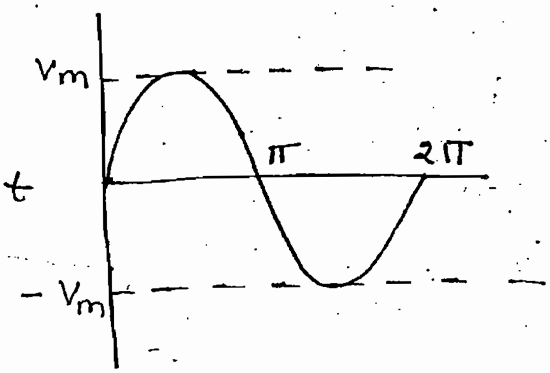
$$\boxed{V_{av} = \frac{1}{2\pi}\left[\int_0^{\pi} V\,d\omega t + \int_{\pi}^{2\pi} V\,d\omega t\right]}$$

Ques:– Find Avg. value of following waveforms.

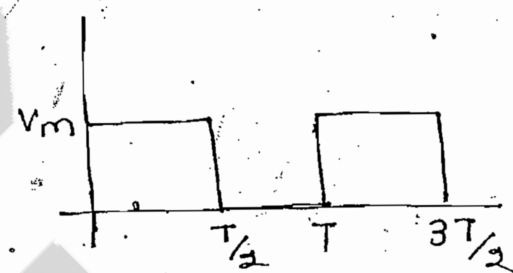
→ $V_{av} = \frac{1}{\pi}\int_0^{\pi} v\, d\omega t$

$V_{av} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \, d\omega t$

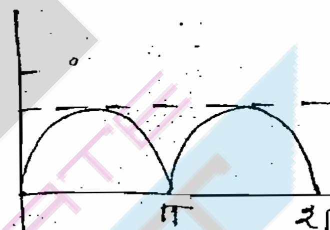
⇒ $\boxed{V_{av} = \frac{2V_m}{\pi}}$



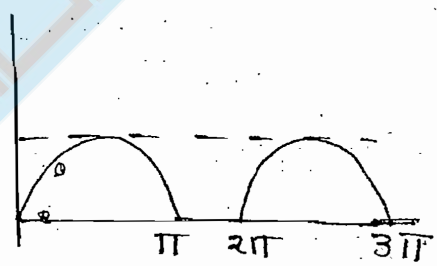
→ $V_{av} = \frac{V_m}{2}$



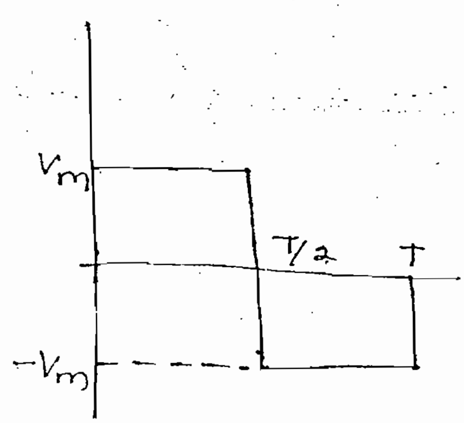
→ $V_{av} = \frac{2V_m}{\pi}$



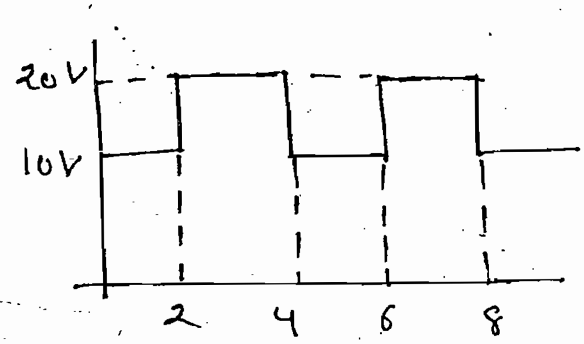
→ $V_{av} = \frac{V_m}{\pi}$



→ $V_{RMS} = V_{av} = V_m$

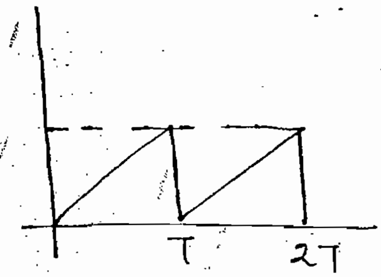


$\rightarrow V_{av} = \frac{1}{4}\left[\int_0^2 10\,dt + \int_2^4 20\,dt\right]$

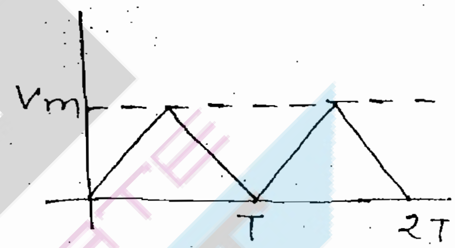


$$\boxed{V_{av} = 15}$$

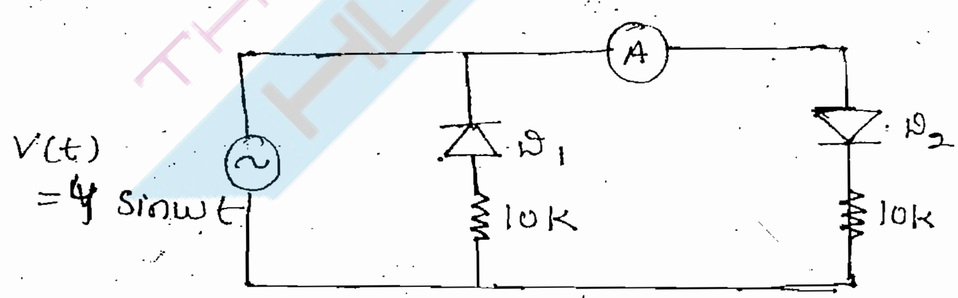
$\rightarrow$ $$\boxed{V_{av} = \frac{V_m}{2}}$$



$\rightarrow$ $$\boxed{V_{av} = \frac{V_m}{2}}$$



Ques! — When the circuit is having ideal diodes and avg value of indicating ammeter. Find reading of ammeter



Sol'n! — $V_{av} = \frac{V_m}{\pi} = \frac{4}{\pi}$

$$I_{av} = \frac{V_{av}}{10\times10^3} = \frac{4/\pi}{10\times10^3}$$

$$\Rightarrow I_{av} = \frac{0.4}{\pi}\ mA$$

Note:-

$\rightarrow$ For sine wave

$$\text{Form factor} = \frac{V_{RMS}}{V_{av}} = \frac{V_m/\sqrt{2}}{2V_m/\pi} = 1.11$$

$$\text{Peak factor} = \frac{V_m}{V_{RMS}} = \frac{V_m}{V_m/\sqrt{2}} = \sqrt{2}$$

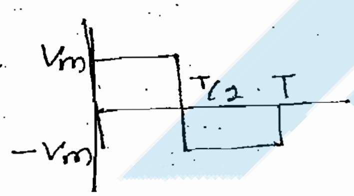
Power System:-

11 kV, 33 kV, 66 kV, 132 kV, 220 kV] $\rightarrow$ basis of form factor

$\rightarrow$ To justify above shape of waveform form factor and peak factor concepts are introduced

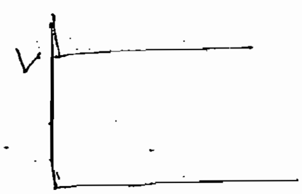
Ques:- Which of the following waveforms have form factor = peak factor ?

(a)

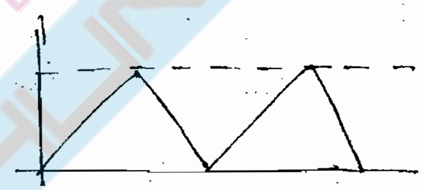


(b)

(c)



(d)



Soln:- (a) $V_{RMS} = V_{av} = V_m$

Form factor = 1

Peak factor = 1

(b) $V_{RMS} = \frac{V_m}{\sqrt{2}}$, $V_{av} = \frac{V_m}{2}$

$$\text{Form factor} = \frac{V_{RMS}}{V_{av}} = \sqrt{2}$$

$$\text{Peak factor} = \frac{V_m}{V_{RMS}} = \sqrt{2}$$

(c) $V_{RMS} = V_m = V_{av}$

$= V$

Form factor = 1

Peak factor = 1

(d) $V_{rms} = \frac{V_m}{\sqrt{3}}$

$V_{av} = V_m/2$

Form factor $= 2/\sqrt{3}$

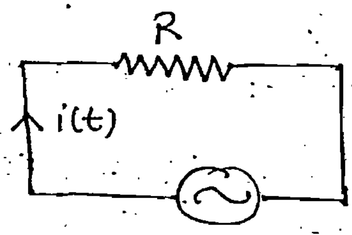
Peak factor $= \sqrt{3}$

AC Source Across Resistor :-

$$i(t) = \frac{V(t)}{R}$$

$$i(t) = \frac{V_m \sin\omega t}{R}$$

$$\Rightarrow \boxed{i(t) = I_m \sin\omega t}$$

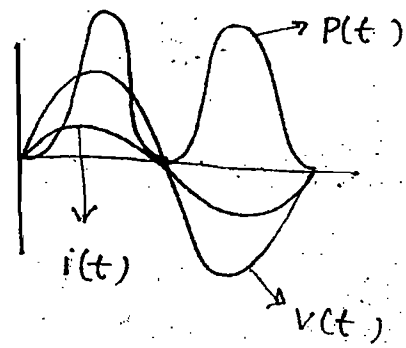


$V(t) = V_m \sin\omega t$

$$P(t) = V(t)\, i(t)$$

$$P(t) = V_m \sin\omega t \; I_m \sin\omega t$$

$$\boxed{P(t) = \frac{V_m I_m}{2}(1 - \cos 2\omega t)}$$



$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} P(t)\, d\omega t$$

$$\Rightarrow \boxed{P_{av} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = V_{RMS} I_{RMS}}$$

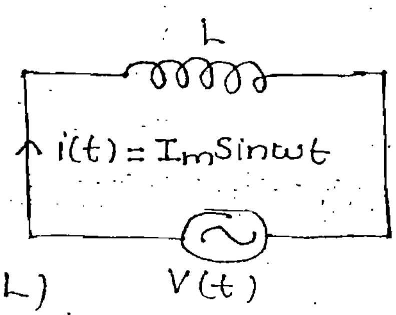
eg :- $f = 50\,Hz$ or C/sec

$\Rightarrow f_p = 100\,Hz$ (Power of frequency)

→ When voltage or current completes one cycle then power completes two cycle.

AC Source across Inductor :-

$$V = L\frac{di}{dt}$$



$$V(t) = L\frac{d}{dt}(I_m \sin\omega t)$$

$$V(t) = \omega L I_m \cos\omega t \qquad (X_L = \omega L)$$

$$V(t) = V_m \sin(\omega t + 90)$$

$$\boxed{\begin{aligned} P &= I^2R = VI\cos\theta \\ Q_L &= I^2X_L = VI\sin\theta \\ S &= I^2Z = VI^* \end{aligned}}$$

$\rightarrow$ conjugate

eg:- $V = 10\angle 40^\circ$ $\qquad$ $i = 5\angle 15^\circ$

$S = Vi$

$\Rightarrow S = 10\angle 40 \; 5\angle 15$

$\Rightarrow S = 50\angle 55$

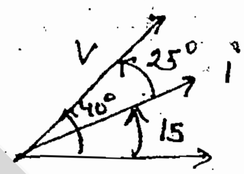
Wrong

$S = Vi^*$

$\Rightarrow S = 10\angle 40 \; 5\angle -15$

$\Rightarrow S = 50\angle 25$

Correct



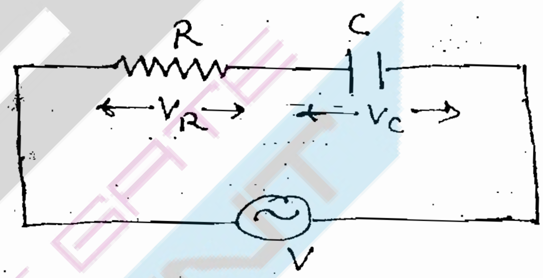
R-C Series Circuit :-

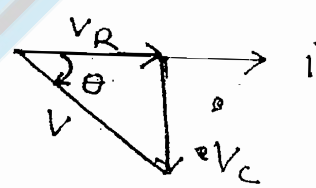
By KVL

$V = V_R\angle 0 + V_c\angle -90^\circ$

$\Rightarrow IZ = IR - jIX_C$

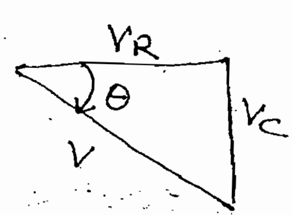
$\Rightarrow \boxed{Z = R - jX_C}$





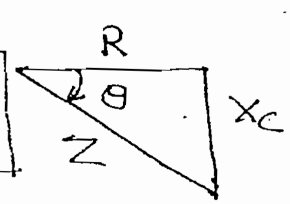
$$\boxed{V = \sqrt{V_R^2 + V_C^2}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{-V_C}{R}\right)}$$



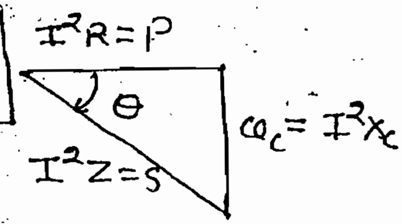
$$\boxed{Z = \sqrt{R^2 + X_C^2}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{-X_C}{R}\right)}$$



$$\boxed{S = \sqrt{P^2 + Q^2}}$$

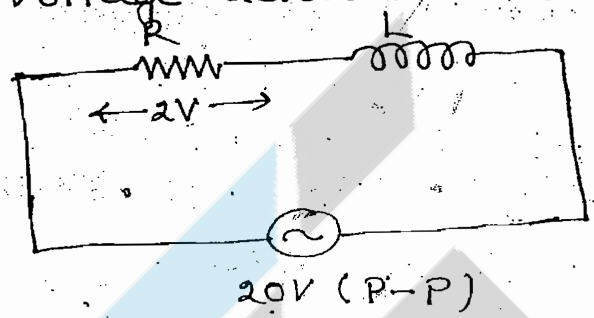
$$\boxed{\theta = \tan^{-1}\left(\frac{-Q_c}{P}\right)}$$



Power Factor :-

$$\boxed{\text{Power Factor} = \cos\theta = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S} \rightarrow \text{leading}}$$

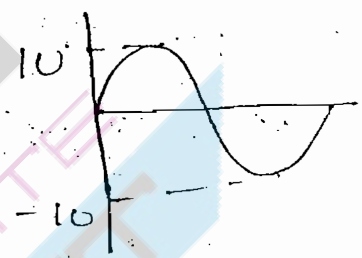
Ques:- Find voltage across inductor



Soln:-

$$V_m = 10.$$

$$V_{RMS} = \frac{10}{\sqrt{2}} = V$$



$$V = \sqrt{V_R^2 + V_L^2}$$

$$\Rightarrow \frac{10}{\sqrt{2}} = \sqrt{2^2 + V_L^2} \Rightarrow \boxed{V_L = \sqrt{46}\ V}$$

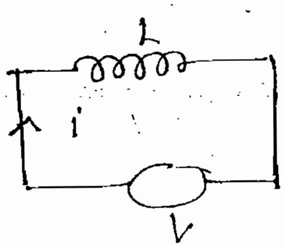
Ques:- Find circuit element for given voltage and current equations

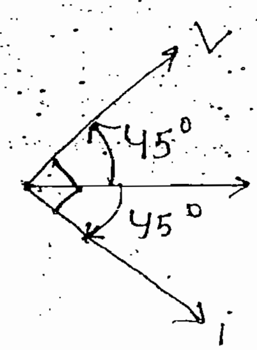
$$V(t) = 9\sin(t + 45)$$

$$i(t) = 3\sin(t - 45^\circ)$$

Soln:-

$$X_L = \frac{V}{i}$$





$$X_L = \frac{9/\sqrt{2}}{3/\sqrt{2}}$$

$$\Rightarrow X_L = 3 = \omega L \Rightarrow \boxed{L = 3} \quad (\because \omega = 1)$$

Ques:— Find circuit element for given voltage and current equations.

$$V(t) = 9\sin(t+30^\circ)$$
$$i(t) = 3\sin(2t+60^\circ)$$

Note:—

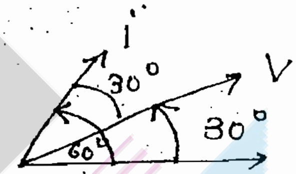
By using above equations it is not possible to design the network since frequency of voltage and current are unequal.

Ques:- Find active power, reactive power and apparent power by using following equations:—

$$V(t) = 9\sin(t+30^\circ)$$
$$i(t) = 3\sin(t+60)$$

Soln:—

$$P = VI\cos\theta$$



$$\Rightarrow P = \frac{9}{\sqrt{2}} \cdot \frac{3}{\sqrt{2}} \cos 30$$

$$\Rightarrow P = 27 \times \frac{\sqrt{3}}{2}$$

$\rightarrow$ R–C circuit (I leading)

$$Q_c = VI\sin\theta$$

$$\Rightarrow Q_c = \frac{9}{\sqrt{2}} \frac{3}{\sqrt{2}} \sin 30^\circ$$

$$\Rightarrow Q_c = \frac{27}{4}$$

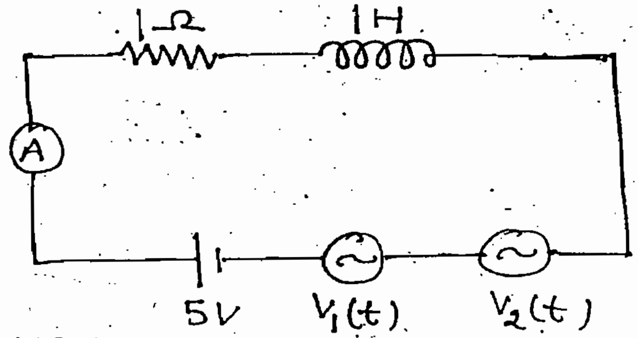
$$S = \sqrt{P^2 + Q_c^2} =$$

Alternate Way:—

$$Z = V/I$$
$$Z = \frac{9/\sqrt{2}}{3/\sqrt{2}}$$
$$Z = 3$$

$$\cos\theta = \frac{R}{Z}$$
$$\Rightarrow \cos 30^\circ = R/3 \Rightarrow R = 3\cos 30^\circ$$
$$X_c = \sqrt{Z^2 - R^2}$$
$$P = i^2 R = (3/\sqrt{2})^2 R$$
$$Q_c = I^2 X_c = (3/\sqrt{2})^2 X_c$$

Ques:- Find ammeter reading and power factor of the ckt shown



$V_1(t) = 10 \sin t$

$V_2(t) = 10\sqrt{5} \sin 2t$

Soln:- When multiple sources are present then at one time only one source is activated

For V_1 $\quad X_{L_1} = \omega_1 L = 1$

$$Z_1 = \sqrt{R^2 + X_{L_1}^2} = \sqrt{1^2+1^2} = \sqrt{2}$$

$$i_1 = \frac{V_1}{Z_1} = \frac{10/\sqrt{2}}{\sqrt{2}} = 5$$

For V_2 $\quad X_{L_2} = \omega_2 L = 2$

$$Z_2 = \sqrt{1^2+2^2} = \sqrt{5}$$

$$i_2 = \frac{V_2}{Z_2} = \frac{10\sqrt{5}/\sqrt{2}}{\sqrt{5}} = \frac{10}{\sqrt{2}}$$

For V_3. $\quad$ DC $\rightarrow$ $\omega = 0$

$$i_3 = \frac{V_3}{R} = \frac{5}{1} \Rightarrow i_3 = 5$$

When same frequency are present then directly add them But above question different frequency are present Hence use general formula $\left(i = \sqrt{i_1^2 + i_2^2 + i_3^2} = 10A\right)$ Ans

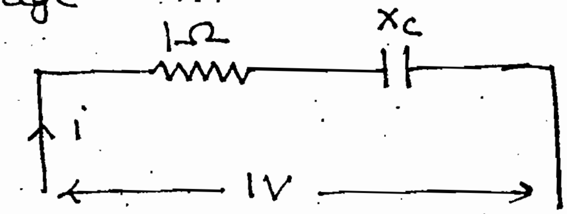
For power factor use power triangle not impedance triangle (same reason)

$$P.F = \cos\theta = P/S \Rightarrow \cos\theta = i^2R/Vi = \frac{iR}{V} = 0.55$$

$$V = \sqrt{V_1^2+V_2^2+V_3^2} = \sqrt{\left(\frac{10}{\sqrt{2}}\right)^2 + \left(\frac{10\sqrt{5}}{\sqrt{2}}\right)^2 + 5^2}$$

$=$

Ques:- In the circuit shown power dissipation in the resistor is 500 mW. Find angle of current w.r.t source voltage



Soln:-

$$P = i^2 R \Rightarrow P = \left(\frac{V}{Z}\right)^2 R$$

$$\Rightarrow P = \frac{V^2}{R^2 + X_C^2} R$$

$$\Rightarrow \frac{500}{1000} = \frac{1^2}{1^2 + X_C^2}(1) \Rightarrow X_C = 1$$

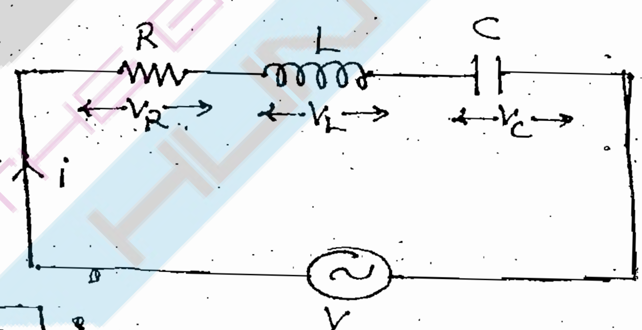
$$Z = R - jX_C$$

$$Z = 1 - j1 \Rightarrow \theta = \tan^{-1}\left(\frac{-1}{1}\right) = -45^\circ$$

$$i = \frac{V\angle 0}{Z\angle -45} = \frac{V\angle +45}{Z} \Rightarrow \theta = 45^\circ \text{ Ans}$$

R-L-C Series Circuit:-

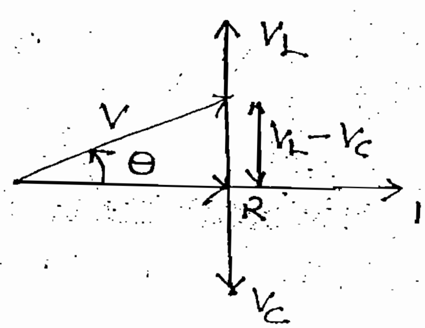
By KVL



$$V = V_R\angle 0 + V_L\angle 90 + V_C\angle -90$$

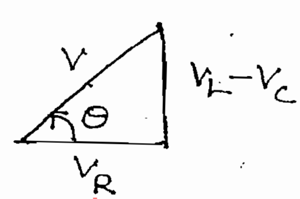
$$\Rightarrow IZ = IR + jIX_L - jX_C$$

$$\Rightarrow \boxed{Z = R + j(X_L - X_C)}$$



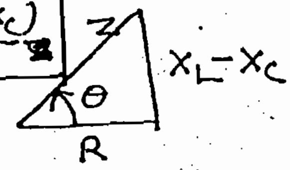
$$\boxed{V = \sqrt{V_R^2 + (V_L - V_C)^2}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)}$$



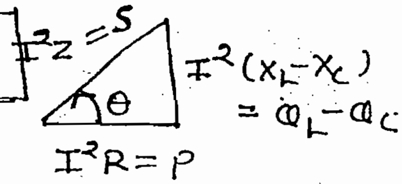
$$\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$

$$\boxed{\theta = \tan^{-1}\frac{(X_L - X_C)}{R}}$$



$$\boxed{S = \sqrt{P^2 + (Q_L - Q_C)^2}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{Q_L - Q_C}{P}\right)}$$



Power factor :-

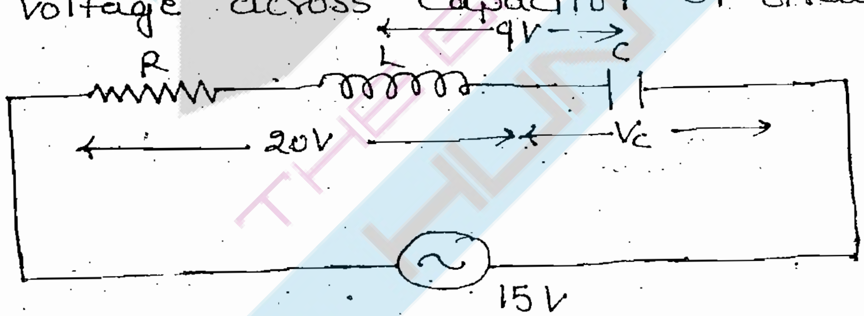
$$\cos\theta = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$$

(I) If $V_L > V_C$ $\longrightarrow$ lagging power factor

(II) If $V_L < V_C$ $\longrightarrow$ leading power factor

(III) If $V_L = V_C$ $\longrightarrow$ Unity power factor

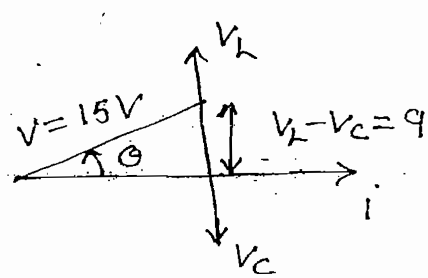
Ques:- Find voltage across capacitor of circuit shown



(a) 7 (b) 25 (c) 7 or 25 (d) 20

Soln:- $V_L - V_C = 9V$

$V = 15V$



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\Rightarrow 15^2 = V_R^2 + 9^2$$

$$V_R = \sqrt{225 - 81} = 12V$$

$$20 = \sqrt{V_R^2 + V_L^2} = \sqrt{12^2 + V_L^2} \Rightarrow V_L = 16$$

$$V_L - V_C = 9 \Rightarrow V_C = 7V \text{ Ans}$$

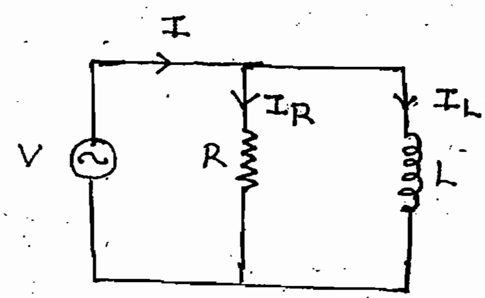
Lecture - 5

R-L Parallel circuit:-

By KCL

$I = I_R\angle 0 + I_L\angle -90^\circ$

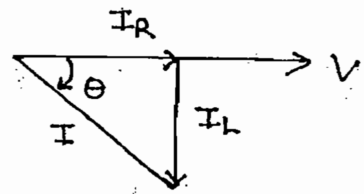
$\Rightarrow \dfrac{V}{Z} = \dfrac{V}{R} - j\dfrac{V}{X_L}$



$\Rightarrow VY = VG - jVB_L$

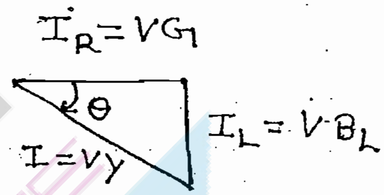
$\Rightarrow \boxed{Y = G - jB_L}$

mho mho mho



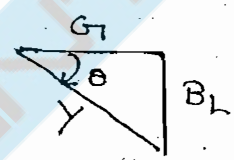
$\boxed{I = \sqrt{I_R^2 + I_L^2}}$

$\boxed{\theta = \tan^{-1}\left(\dfrac{-I_L}{I_R}\right)}$



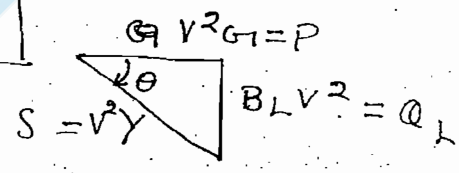
$\boxed{Y = \sqrt{G^2 + B_L^2}}$

$\boxed{\theta = \tan^{-1}\left(-\dfrac{B_L}{G}\right)}$



$\boxed{S = \sqrt{P^2 + Q_L^2}}$

$\boxed{\theta = \tan^{-1}\left(\dfrac{-Q_L}{P}\right)}$



Power factor:-

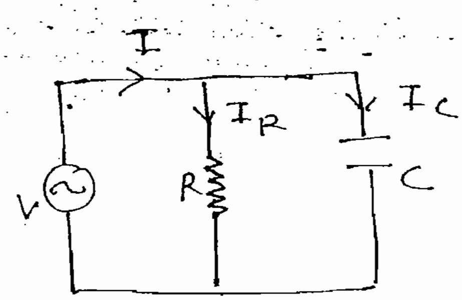
$\rightarrow$ $\boxed{\cos\theta = \dfrac{I_R}{I} = \dfrac{G}{Y} = \dfrac{P}{S} = \text{Lagging}}$

R-C Parallel circuit:-

By KCL

$I = I_R\angle 0 + I_c\angle +90$

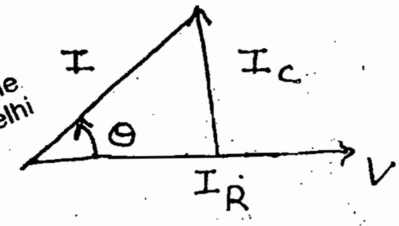
$\Rightarrow \dfrac{V}{Z} = \dfrac{V}{R} + j\dfrac{V}{X_c}$



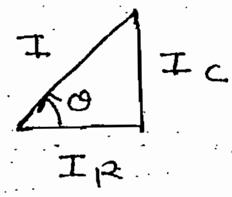
$\Rightarrow V.Y = VG + jVB_C$

$\Rightarrow \boxed{Y = G + jB_C}$

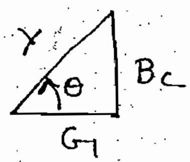
Mho Mho Mho



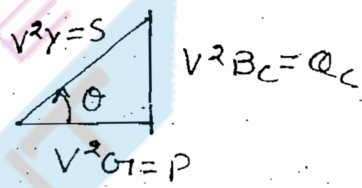
$\boxed{I = \sqrt{I_R^2 + I_C^2}}$ $\boxed{\theta = \tan^{-1}\left(\frac{I_C}{I_R}\right)}$



$\boxed{Y = \sqrt{G^2 + B_C^2}}$ $\boxed{\theta = \tan^{-1}\left(\frac{B_C}{G}\right)}$



$\boxed{S = \sqrt{P^2 + Q_C^2}}$ $\boxed{\theta = \tan^{-1}\left(\frac{Q_C}{P}\right)}$



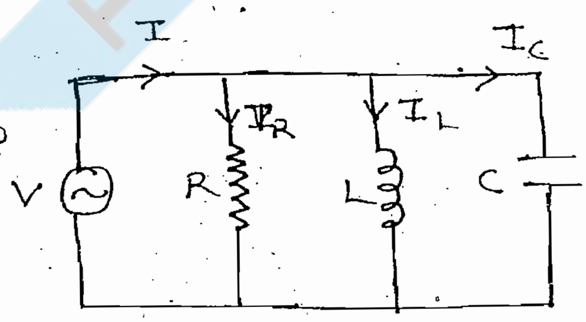
Power Factor $\Rightarrow$

$\boxed{\cos\theta = \frac{I_R}{I} = \frac{G}{Y} = \frac{P}{S} = \text{leading}}$

RLC Parallel Circuit :-

By KCL

$I = I_R\angle 0 + I_L\angle -90 + I_C\angle +90$

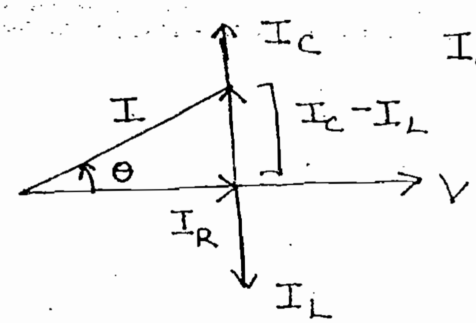


$\Rightarrow \frac{V}{Z} = \frac{V}{R} - j\frac{V}{X_L} + j\frac{V}{X_C}$

$\Rightarrow VY = V\left[G + j(B_C - B_L)\right]$

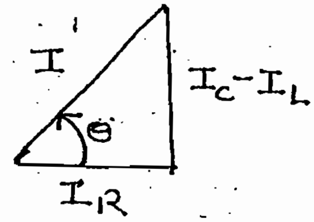
$\Rightarrow \boxed{Y = G + j(B_C - B_L)}$

$I_C > I_L$



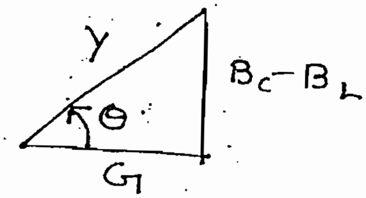
$$\boxed{I = \sqrt{I_R^2 + (I_C - I_L)^2}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{I_C - I_L}{I_R}\right)}$$



$$\boxed{Y = \sqrt{G^2 + (B_C - B_L)^2}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{B_C - B_L}{G}\right)}$$



$$\boxed{\text{Power Factor} = \cos\theta = \frac{I_R}{I} = \frac{G}{Y} = \frac{P}{S}}$$

(I) $I_C > I_L \rightarrow$ leading

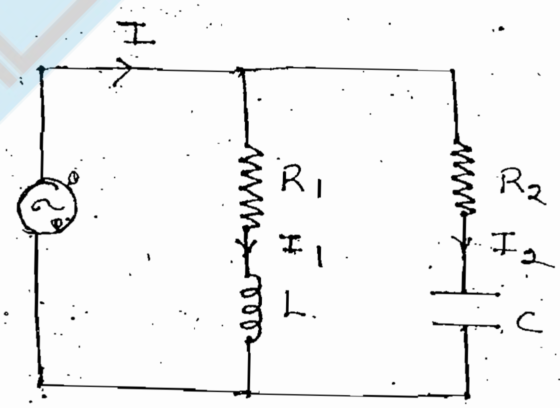
(II) $I_C < I_L \rightarrow$ lagging

(IV) $I_C = I_L \rightarrow$ Unity Power factor

Note:-

When combinational elements are present then we can't directly find conductance, admittance etc. Then following procedure is used

$$I_1 = \left(\frac{V}{R_1 + jX_L}\right)\left(\frac{R_1 - jX_L}{R_1 - jX_L}\right)$$



$$\Rightarrow \frac{V}{Z_1} = V\left[\frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2}\right]V$$

$$\Rightarrow \boxed{Y_1 = G_1 - jB_L}$$

$$I_2 = \frac{V}{R_2 - jX_C} \cdot \frac{R_2 + jX_C}{R_2 + jX_C}$$

$$\Rightarrow \frac{V}{Z_2} = V\left[\frac{R_2}{R_2^2 + X_C^2} + j\frac{X_C}{R_2^2 + X_C^2}\right]$$

$$\boxed{Y_2 = G_2 + jB_C}$$

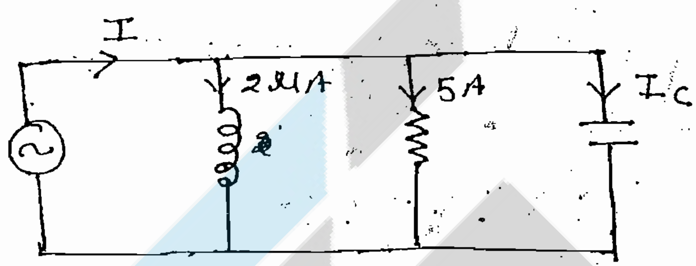
$$I = I_1 + I_2$$

$$\Rightarrow \quad VY_{eq} = VY_1 + VY_2$$

$$\Rightarrow \quad Y_{eq} = Y_1 + Y_2$$

$$\Rightarrow \quad \boxed{Y_{eq} = (G_1 + G_2) + j(B_C - B_L)}$$

Ques:- Find I_c and I of the circuit shown



Soln:- $13 = \sqrt{5^2 + I_c^2} \Rightarrow I_c = 12A$

$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$\Rightarrow \quad I = \sqrt{5^2 + (12 - 2\mu A)^2} = 13 \text{ A}$$

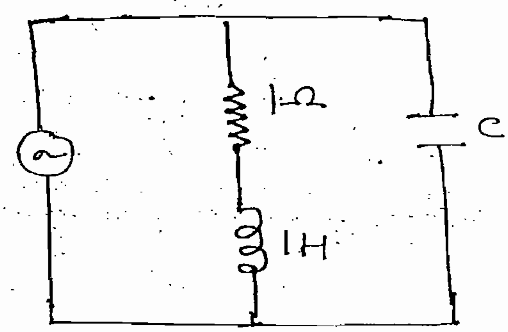
Note:-

A.C $\longrightarrow$ (KVL, KCL) $\longrightarrow$ Phasor sum

D.C $\longrightarrow$ (KVL, KCL) $\longrightarrow$ Arithmatic sum

Ques:- Find capacitance of the capacitor when power factor of the circuit is 0.8 lagging

$V(t) = \sin t$



Soln:- For Branch-1

$$X_L = \omega L = 1$$

$$G_1 = \frac{R_1}{R_1^2 + X_L^2} = \frac{1}{1^2 + 1^2} = \frac{1}{2}$$

$$B_L = \frac{X_L}{R_1^2 + X_L^2} = \frac{1}{1^2+1^2} = \frac{1}{2}$$

$$Y_1 = G_1 - jB_L$$

$$\Rightarrow Y = \frac{1}{2} - j\frac{1}{2}$$

<u>For Branch-2</u>

$$Y_2 = +jB_C$$

$$Y_2 = j\frac{1}{X_C} = j\omega C \quad \Rightarrow \quad Y_2 = \omega C$$

$$Y_{eq} = Y_1 + Y_2$$

$$\Rightarrow Y_{eq} = \frac{1}{2} + j\left(C - \frac{1}{2}\right)$$

$$\cos\theta = \frac{G_1}{\sqrt{G_1^2 + (B_C - B_L)^2}}$$

$$\Rightarrow 0.8 = \frac{1/2}{\sqrt{(1/2)^2 + (C - 1/2)^2}}$$

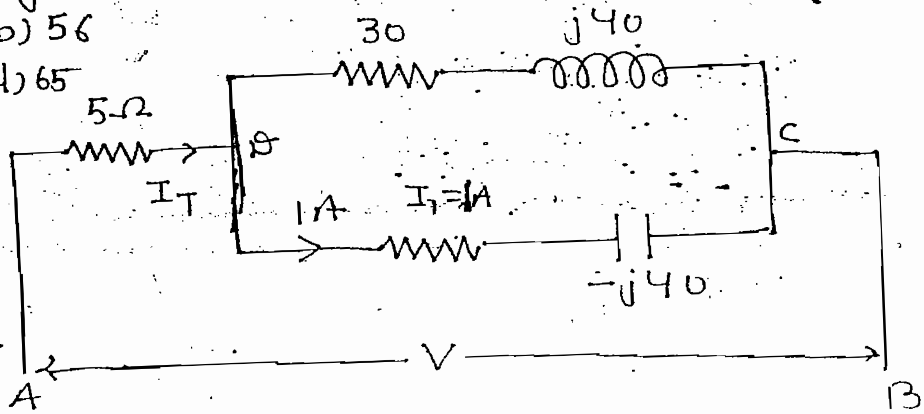
$$\Rightarrow C = \frac{7}{8} \text{ or } \frac{1}{8}$$

Since $B_L > B_C$

$\frac{1}{2} > C$ Hence, $\boxed{C = \frac{1}{8}}$ Ans

Ques:- Find voltage across A and B of the circuit shown (a) 55 (b) 56 (c) 60 (d) 65

Soln:-



Apply current division technique,

$$I_1 = I_T \frac{30 + j40}{30 + j40 + 30 - j40} =$$

$$\Rightarrow \quad 1 = I_T \frac{50\angle\tan^{-1}(4/3)}{60}$$

$$\Rightarrow \quad I_T = \frac{60}{50\angle\tan^{-1}(4/3)} = 1.2\angle\tan^{-1}(-4/3)$$

$$V_{AD} = I_T \times 5$$

$$\Rightarrow \quad V_{AD} = (1.2 \times 5)\angle\tan^{-1}(-4/3)$$

$$\Rightarrow \quad V_{AD} = 6\angle\tan^{-1}4/3$$

$$V_{DC} = (30 - j40)\,1$$

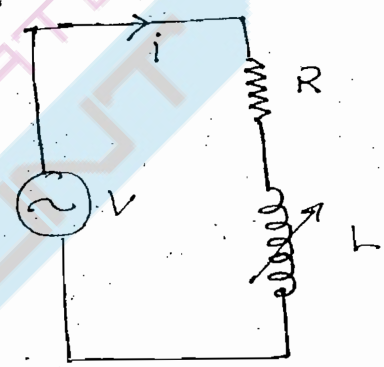
$$V_{DC} = 50\angle\tan^{-1}(-4/3)$$

Angles are same hence they can be added

$$V_{AB} = V_{AC} = \cancel{V_{AD}}\; V_{AD} + V_{DC}$$

$$\Rightarrow \quad \boxed{V_{AB} = 56\angle\tan^{-1}(-4/3)} \quad \text{Ans}$$

<u>Locus Diagram</u> :—



$$X_L = 2\pi f L$$

$$X_L = 0$$

$$Z = R \quad \Rightarrow \quad I = \frac{V}{R}$$

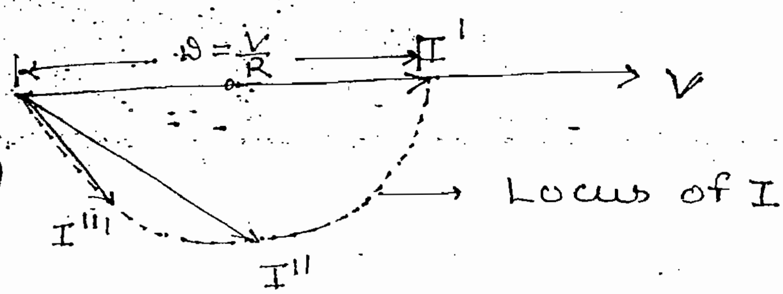
$$\Rightarrow \quad \theta = 0$$

$$X_L \uparrow \quad Z \uparrow \quad I \downarrow \qquad \boxed{\theta = \tan^{-1}\left(\frac{X_L}{R}\right)} \uparrow$$

$$X_L \simeq \infty \quad Z \simeq \infty \quad I = 0$$

→ semicircle

→ centre $\left(\frac{V}{2R}, 0\right)$



→ Locus diagram are useful for analysis and designing of the circuit eg:- filters

→ With respect to practical application it is possible to develop the following locus diagram

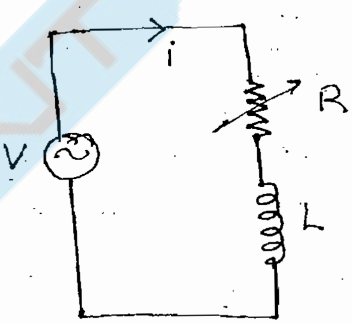
(I) current locus diagram
(II) Voltage locus diagram
(III) Impedance locus diagram
(IV) Admittance locus diagram

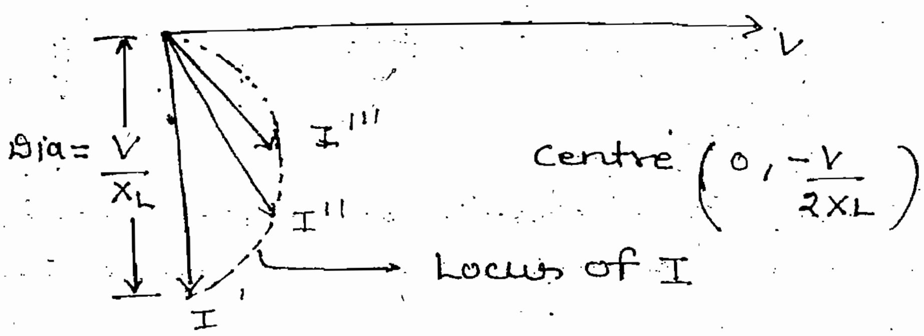
→ The path traced by terminous of current vectors by varying either any of the circuit elements or by varying source frequency is called as current locus

→ In the above circuit by keeping all the elements constant and by varying source frequency also same shape of the current locus diagram is obtained.

→ Develop current locus of the circuit shown

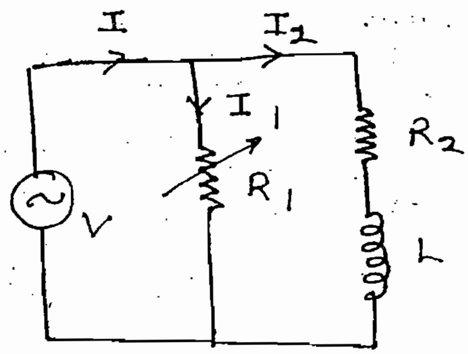
$R=0$	$R\uparrow$
$Z=X_L$	$Z\uparrow$
$I=\frac{V}{X_L}$	$I\downarrow$
$\theta=90°$	$\theta=\tan^{-1}\left(\frac{X_L}{R\uparrow}\right)\downarrow$





→ Diameter is always due to constant elemen

Ques:- Develop current locus of I_1 and I of the circuit shown:-



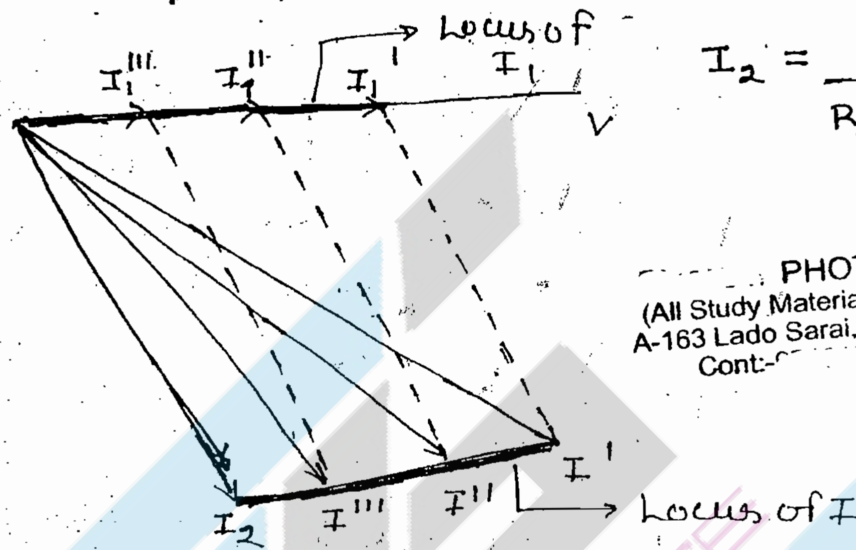
Soln:- $R_1 \simeq 0.1\Omega$

$I_1 = \frac{V}{R_1}$

$\theta_1 = 0$

$I = I_1 + I_2$

$R_1 \uparrow$	$R_1 \simeq \infty$
$I_1 \downarrow$	$I_1 = 0$
$\theta_1 = 0$	$I_2 = I$
$I_2 =$ Constant	
$I \downarrow$	



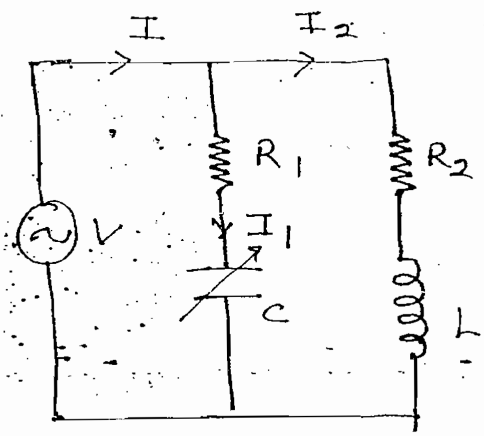
$$I_2 = \frac{V}{R_2 + jX_L}$$

Note:-

→ When power factor angle is variable the shape of current locus is semi-circle

→ When power factor angle is constant the shape of the current locus diagram is straight line.

Ques:- Develop current locus of I_1 and I of the circuit shown



Soln:- $X_C = \frac{1}{2\pi fC}$

$X_C \simeq 0$

$I_1 = \frac{V}{R_1}$

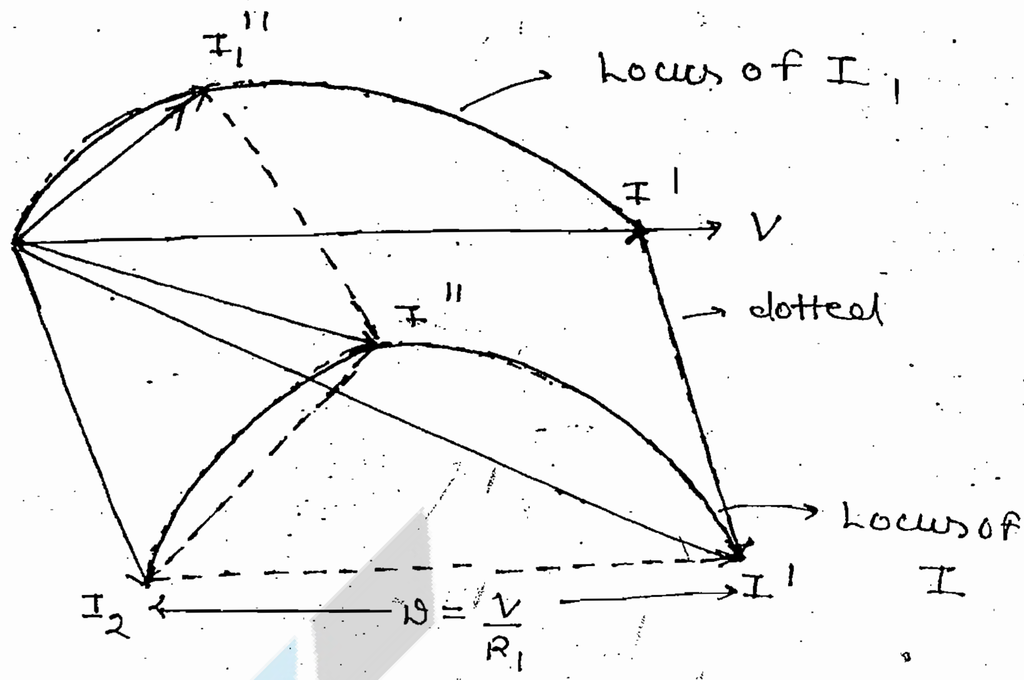
$\theta_1 = 0$

$I = I_1 + I_2$

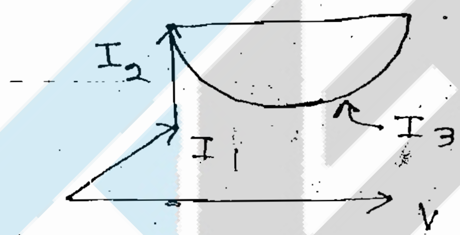
$X_C \uparrow$	$X_C \simeq \infty$
$Z_1 = \sqrt{R_1^2 + X_C^2} \uparrow$	$I_1 = 0$
$I_1 \downarrow$	$I = I_2$
$\theta_1 = \tan^{-1}\left(\frac{-X_C}{R}\right) \uparrow$	
$I_2 =$ Constant	
$I_1 \downarrow$	

$$I_2 = \frac{V}{R_2 + jX_L}$$

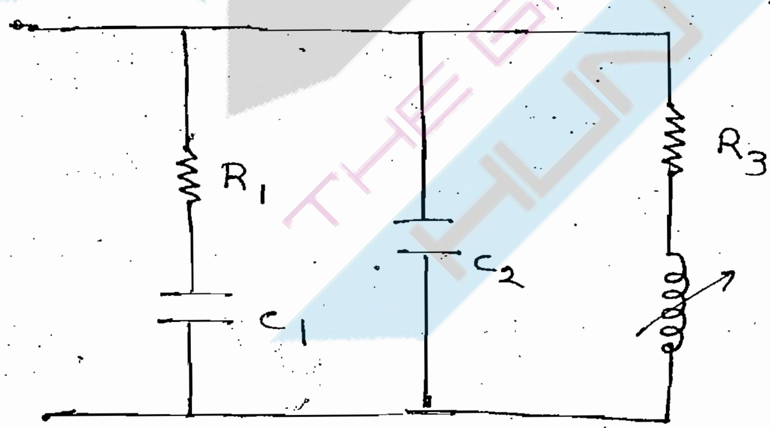
Focus only angles not sign but focus on magnitude



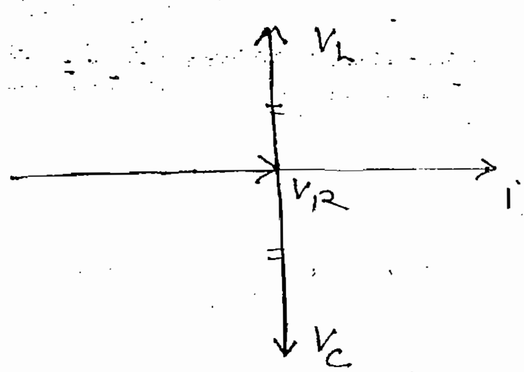
Ques:- Design a N/w for given current locus diagram

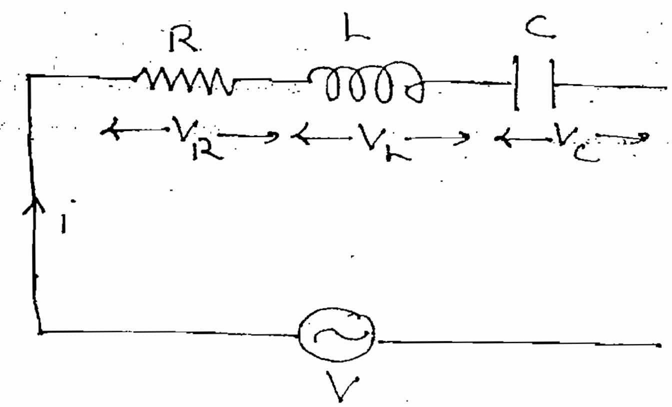


Solⁿ:-



Resonance :-





By KVL, $V = V_R\angle 0 + V_L\angle 90 + V_C\angle -90$

At resonance,

$$V_L = V_C$$

$$IX_L = IX_C$$

$$\Rightarrow \omega L = \frac{1}{\omega C}$$

$$\Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \text{ rad/sec}$$

$$\Rightarrow \boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}} \text{ Hz}$$

→ For occurrence of resonance in any circuit two energies are required. In RLC circuit inductor is having energy in the form of magnetic field capacitor is having energy in the form of electric field. When these two energies are present at particular frequency wide variations are present in the system is called as resonance

→ The circuit is said to be resonance when source current is ~~influence~~ phase with source voltage

→ The frequency at which $X_C = X_L$ is called as resonant frequency

→ The resonant frequency indicates rate at which energy transformation is done b/w inductor and capacitor

1. → $Z = R + j\underbrace{(X_L - X_C)}_{=0}$

$$Z_{min} = R$$

2. → $I_{max} = \dfrac{V}{Z_{min}} = \dfrac{V}{R}$

3. → $\cos\theta = 1$

4. → $V_R = V$

5. Net Reactive.

6. Voltage across inductor or voltage across ~~capacit~~ capacitor greater than source. This phenomenon is called as <u>voltage magnification</u>

<u>Application :-</u>

→ Oscillators

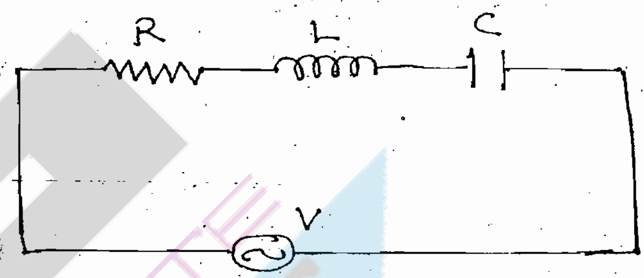
→ Filters (BP, BE)

→ Tuning circuits

→ Induction heating

<u>Variation of voltage across inductor and voltage across capacitor w.r.t frequency :-</u>

$V_C = I X_C$



$X_L = 2\pi f L \quad , \quad X_C = \frac{1}{2\pi f C}$

→ $f = 0 \quad , \quad X_L = 0 \quad , \quad X_C = \infty$

→ $X = |X_L - X_C| = \infty$

$\Rightarrow \quad Z = \infty \quad \& \quad I = 0 \quad \Rightarrow \boxed{V_C = V}$

<u>~~For inductor~~ :-</u>

→ $f\uparrow \quad X_L \uparrow \quad X_C \downarrow \quad X = |X_L - X_C| \downarrow\downarrow \quad Z \downarrow\downarrow \quad I \uparrow\uparrow \quad \boxed{V_C \uparrow}$

$(10\,\Omega) \quad (100\,\Omega) \quad (90\,\Omega)$

→ $f \uparrow\uparrow \quad X_L \uparrow\uparrow \quad X_C \downarrow\downarrow \quad Z = R + j(X_L - X_C) \uparrow\uparrow\uparrow \quad I \downarrow\downarrow\downarrow$

(Very low) $\quad V_C \downarrow$

→ $$V_C = \frac{V X_C}{\sqrt{R^2 + (X_L - X_C)^2}} \Rightarrow V_C = \frac{V \frac{1}{\omega C}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \text{——(1)}$$

Differentiate eq-(1) w.r.t ω and equal to zero we get

$$\boxed{f_c = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \left(\frac{R^2 C}{2L}\right)^2}}$$

dL

For inductor :-

→ $V_L = I X_L$

→ $X_L = 2\pi f L$ & $X_C = \frac{1}{2\pi f C}$

→ $f = 0$, $X_L = 0$, $X_C = \infty$ $X = |X_L - X_C| = \infty$

$Z = \infty$, $I = 0$, $\boxed{V_L = 0}$

→ $f\uparrow$ $X_L \uparrow$ $X_C \downarrow$ $X = |X_L - X_C| \downarrow\downarrow$ $Z \downarrow\downarrow$

(10Ω) (100Ω)

$I \uparrow\uparrow$ $\boxed{V_L \uparrow}$

→ $f\uparrow\uparrow$ $X_L \uparrow\uparrow$ $X_C \downarrow\downarrow$

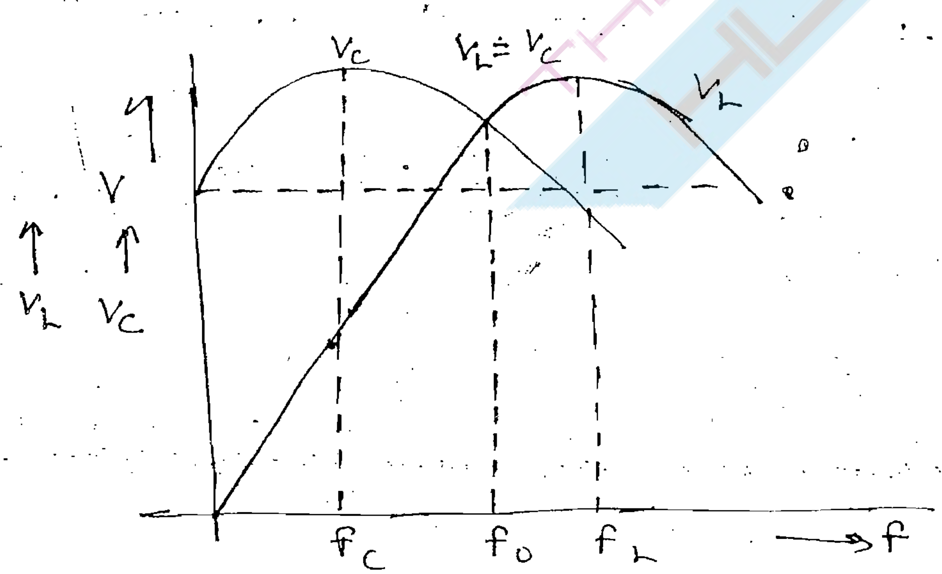
(Very low)

$Z = R + j(X_L - X_C) \uparrow\uparrow\uparrow$ $I_C \downarrow\downarrow\downarrow$ $\boxed{V_L \downarrow}$

→ $$V_L = \frac{V X_L}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V \omega L}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)}} \quad \text{— (11)}$$

Differentiate eq-(11) w.r.t ω and equal to zero

$$\boxed{f_L = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{1}{1 - \left(\frac{R^2 C}{2L}\right)}}}$$



Quality factor :—

→ Q-factor is a ratio of max. energy stored the circuit to power dissipation per cycle

$$Q =$$

$$i(t) = I_m \sin\omega t \quad —(I)$$

$$V_c(t) = \frac{1}{C}\int i(t)\,dt = \frac{1}{C}\int I_m \sin\omega t\, dt$$

$$\Rightarrow V_c(t) = \frac{-I_m}{\omega C}\cos\omega t \quad —(II)$$

$$w_g = \frac{1}{2}Li^2 + \frac{1}{2}CV_c^2$$

$$\Rightarrow w_g = \frac{1}{2}L(I_m \sin\omega t)^2 + \frac{1}{2}C\left(-\frac{I_m}{\omega C}\cos\omega t\right)^2$$

$$\Rightarrow w_g = \frac{1}{2}LI_m^2\sin^2\omega t + \frac{1}{2}C\frac{I_m^2}{\omega^2C^2}\cos^2\omega t$$

$$\Rightarrow w_g = \frac{1}{2}LI_m^2\sin^2\omega t + \frac{1}{2}LI_m^2\cos^2\omega t \qquad \left[\omega^2 = \frac{1}{LC},\ L = \frac{1}{\omega^2 C}\right]$$

$$\Rightarrow \boxed{w_g = \frac{1}{2}LI_m^2} \qquad \boxed{w_g = \frac{1}{2}CV_{cm}^2}$$

$$\left(\because V_{cm} = \frac{I_m}{\omega C}\right)$$

$$Q = \frac{\frac{1}{2}LI_m^2}{\left(\frac{I_m}{\sqrt{2}}\right)^2 R\,\frac{1}{\omega}} \qquad I^2R = \left(\frac{I_m}{\sqrt{2}}\right)^2 R$$

$$\Rightarrow \boxed{Q = \frac{\omega L}{R}} \qquad \left(\omega = \frac{1}{\sqrt{C}}\right)$$

$$\Rightarrow \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

$$\rightarrow \boxed{Q = \frac{\omega L}{R} = \frac{IX_L}{IR} = \frac{V_L}{V_R} = \frac{V_L}{V} \quad (V_R = V)}$$

$$\boxed{Q = \frac{V_L \text{ or } V_C}{V}} \quad (\because V_L = V_C)$$

$$\boxed{Q = \frac{I^2 X_L}{I^2 R} = \frac{Q_L}{P}}$$

$$\boxed{Q = \frac{X_L \text{ or } X_C}{R}} \quad (\because X_L = X_C)$$

$$\boxed{Q = \frac{X_C}{R} = \frac{1}{\omega RC}}$$

→ $Q > 1$, $X_L > R$, $X_C > R$

→ $Q \propto \frac{1}{\text{Power loss } (I^2R = P)}$

→ $Q \propto \frac{1}{BW}$

Note:-

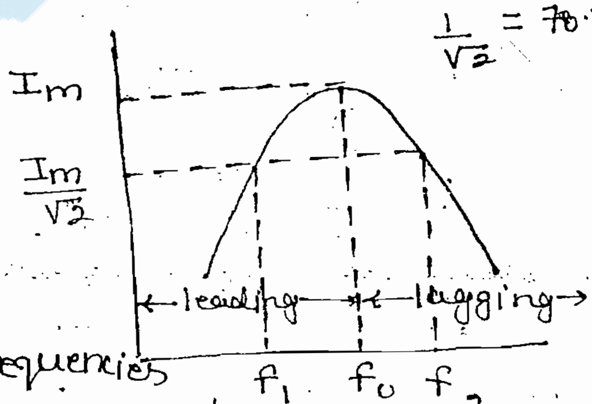
→ To obtain high efficiency circuit is designed with high Q - factor

→ To obtain wide BW circuit is designed with low Q - factor

Bandwidth:-

$\frac{1}{\sqrt{2}} = 70.7\%$

→ When the curve is developed b/w current and frequency then curve is called as resonance curve



→ BW is the range of frequencies on either side of the resonant frequencies where the current falls from max. value to 70.7% of the max. value and it is given by

$$BW = f_2 - f_1$$

where f_2 = upper cut-off frequency

f_1 = lower " " "

f_1, f_2 = 3dB points or half power frequencies

1. $f_0 \rightarrow I_m$, $Z = R$

$f_1, f_2 \rightarrow \frac{I_m}{\sqrt{2}} \Rightarrow \boxed{Z = \sqrt{2} R}$

2. $f_0 \rightarrow \cos\theta = 1$

$f_1, f_2 \rightarrow Z = R \pm jX$

$X = X_L - X_C$

$X_L = 2\pi f L$

$X_C = \frac{1}{2\pi f C}$

$Z = \sqrt{R^2 + X^2} = \sqrt{2} R$

$\Rightarrow \boxed{X = R}$

$f_1 \rightarrow X_C > X_L$

$X \rightarrow -ve$

$f_2 \rightarrow X_L > X_C$

$X \rightarrow +ve$

w.r.t

$f_1 \rightarrow Z = R - jX$ $X = R$

Impedance Angle $= \tan^{-1}\left(-\frac{X}{R}\right)$

$= -45°$

$I = \frac{V\angle 0}{Z\angle -45} = \frac{V}{Z}\angle +45$

Power factor angle $= 45$

$\Rightarrow$ Power factor $= \cos 45 = \frac{1}{\sqrt{2}} \rightarrow$ leading

w.r.t f_2 :-

$f_2 \rightarrow Z = R + jX$ $X = R$

Impedance angle $= \theta = \tan^{-1}(X/R) = +45°$

Power factor angle $= -45°$

Power factor $= \cos(-45°) = \frac{1}{\sqrt{2}} \rightarrow$ lagging

3. $f_1 \rightarrow X_C > X_L$ $X = R$

$\frac{1}{\omega_1 C} - \omega_1 L = R$ —(1)

$f_2 \longrightarrow X_L > X_C$ $X = R$

$$\omega_2 L - \frac{1}{\omega_2 C} = R \quad \text{—— (II)}$$

From (I) & (II)

$$\omega_1 \omega_2 = \frac{1}{LC} \quad \text{—— (III)}$$

$$\Rightarrow \omega_0^2 = \frac{1}{LC} \quad \text{—— (IV)}$$

From (III) & (IV)

$$\omega_0^2 = \omega_1 \omega_2$$

$$\Rightarrow \boxed{\begin{aligned} \omega_0 &= \sqrt{\omega_1 \omega_2} \\ f_0 &= \sqrt{f_1 f_2} \end{aligned}}$$

Add eq- (I) & (II)

$$\frac{1}{C}\left[\frac{1}{\omega_1} - \frac{1}{\omega_2}\right] + L\left[\omega_2 - \omega_1\right] = 2R$$

$$\Rightarrow \frac{1}{C}\left[\frac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right] + L\left[\omega_2 - \omega_1\right] = 2R$$

$$\Rightarrow L\left[\omega_2 - \omega_1\right] + L\left[\omega_2 - \omega_1\right] = 2R$$

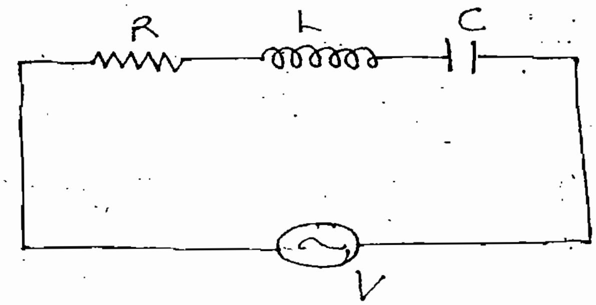
$$\boxed{\begin{aligned} BW &= \omega_2 - \omega_1 = \frac{R}{L} \text{ rad/sec} \\ BW &= f_2 - f_1 = \frac{R}{2\pi L} \text{ Hz} \end{aligned}}$$

4. $Q = \frac{\omega_0 L}{R} \Rightarrow Q = \frac{\omega_0}{R/L}$

$$\Rightarrow \boxed{Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{f_0}{f_2 - f_1}}$$

Lecture-6

By KVL



$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

Diff w.r.t t

$$V' = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C}$$

Dividing both sides of L

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = \frac{V'}{L}$$

$$\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right)i = 0 \qquad \frac{d}{dt} = D$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$2\xi\omega_n = \frac{R}{L}$$

$$2\xi\frac{1}{\sqrt{LC}} = \frac{R}{L}$$

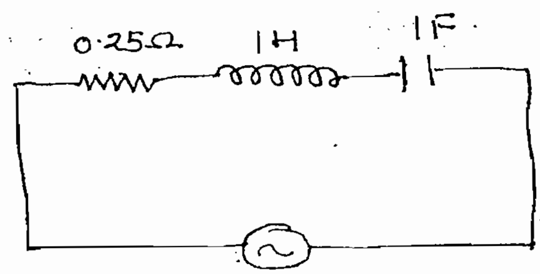
$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Damping ratio $= \xi = \frac{R}{2}\sqrt{\frac{C}{L}}$

$$\boxed{\xi = \frac{1}{2Q}}$$

 9718102070

Ques:- Find f_0, Q, ξ, B.W, f_1, f_2, I at f_0



Soln:- (i) $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi}$

(ii) $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{0.25}\sqrt{\frac{1}{1}} = 4$

(iii) $\delta = \frac{1}{2Q} = \frac{1}{8}$

(iv) $\underset{(f_2 - f_1)}{BW} = \frac{f_0}{Q} = \frac{1/2\pi}{4} = \frac{1}{8\pi}$

(v) $f_2 - f_1 = \frac{1}{8\pi}$

$f_1 f_2 = f_0^2 = \left(\frac{1}{2\pi}\right)^2$

$(f_2+f_1)^2 - (f_2-f_1)^2 = 4f_1f_2$

$f_1 =$

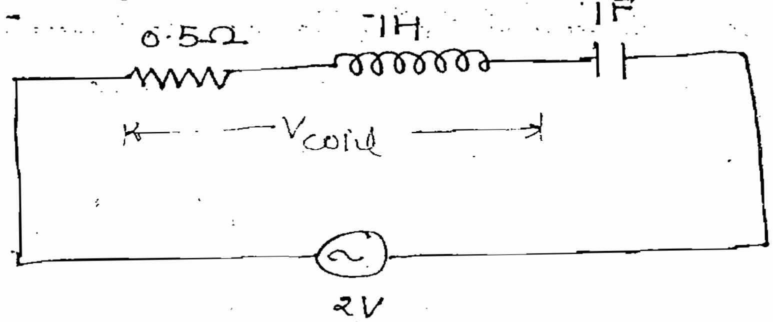
$f_2 =$

(vi) $I = \frac{V}{Z} = \frac{V}{R}$ (At Resonance)

$I = \frac{10/\sqrt{2}}{0.25}$

$I = \frac{40}{\sqrt{2}}$, Ans.

Ques:- Find voltage across the coil under resonance condition



Soln:— $V_R = V = 2$ [When nothing is given take it as RMS]

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q = \frac{1}{0.5}\sqrt{\frac{1}{1}} = 2$$

$$Q = \frac{V_L}{V}$$

$$\Rightarrow 2 = \frac{V_L}{2}$$

$$\Rightarrow V_L = 4$$

$$V_{coil} = \sqrt{V_R^2 + V_L^2}$$

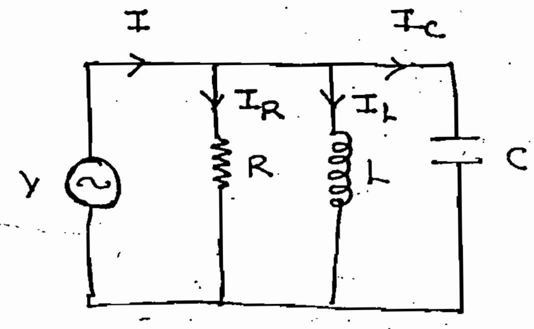
$$\Rightarrow V_{coil} = \sqrt{2^2 + 4^2}$$

$$= \sqrt{20} \text{, Ans.}$$

... HOTOSTAT
(All Study Material Available
A-163 Lado Sarai, New Delhi
Cont:-9718102070

Parallel Resonance :-

Case-(1) :-

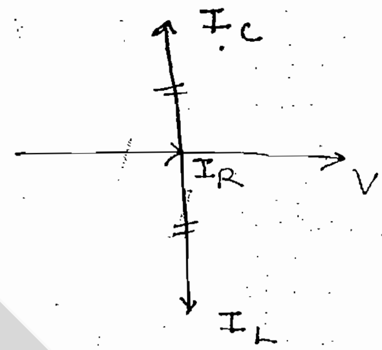


$$I_C = I_L$$

$$\frac{V}{X_C} = \frac{V}{X_L} \qquad X_L = X_L$$

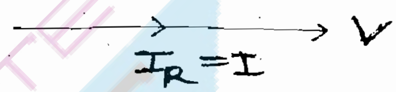
$$\Rightarrow \boxed{B_C = B_L}$$

$$\omega C = \frac{1}{\omega L}$$



$$\boxed{\begin{aligned} \omega_0 &= \frac{1}{\sqrt{LC}} \text{ rad/sec} \\ f_0 &= \frac{1}{2\pi\sqrt{LC}} \text{ Hz} \end{aligned}}$$

(I) $Y = G + j\underbrace{(B_C - B_L)}_{0}$



$$Y_{min} = G$$

(II) $Z_{max} = \frac{1}{Y_{min}}$

(III) $I_{min} = \frac{V}{Z_{max}}$

(IV) $\cos\theta = 1$

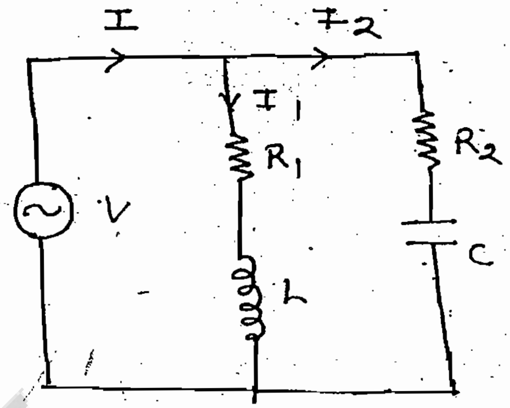
(V) $I_R = I$

(VI) Net Reactive current = 0

(VII) current in inductor or current in capacitor is greater than total current. This phenomenon is called as current magnification

viii) Parallel Resonance circuit is also called as Anti-Resonance circuit.

Case-(II):-

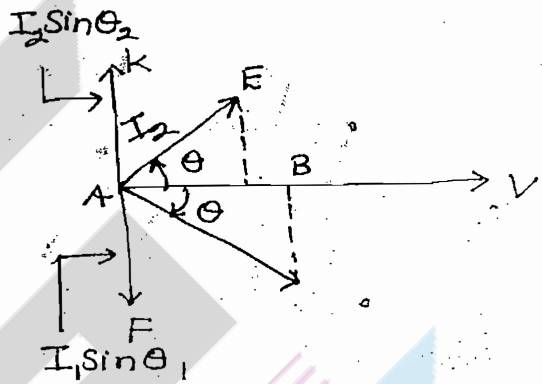


$AB = I_1 \cos\theta_1$

$AF = BC = I_1 \sin\theta_1$

$AD = I_2 \cos\theta_2$

$AK = DE = I_2 \sin\theta_2$



$B_L = B_C$

$$\frac{X_L}{R_1^2 + X_L^2} = \frac{X_C}{R_2^2 + X_C^2}$$

$$\frac{\omega L}{R_1^2 + (\omega L)^2} = \frac{1/\omega C}{R_2^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{R_1^2 - \frac{L}{C}}{R_2^2 - L/C}}}$$

$I = I_1 \cos\theta_1 + I_2 \cos\theta_2$

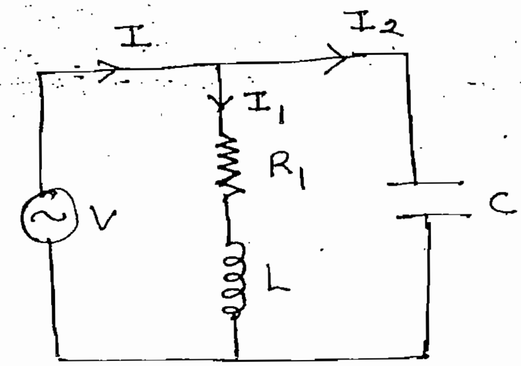
$I = VY$

$$\Rightarrow I = V\left[(G_1 + G_2) + j\underbrace{(B_C - B_L)}_{0}\right]$$

$$\Rightarrow \boxed{I = V\left[\frac{R_1}{R_1^2 + X_L^2} + \frac{R_2}{R_2^2 + X_C^2}\right]}$$

Imp

Case-(III):-

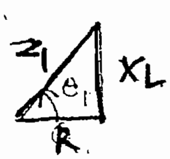


Tank ckt

$AB = I_1 \cos\theta_1$

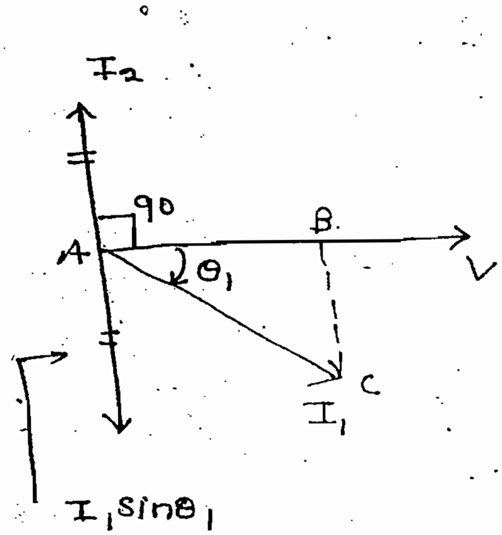
$AF = BC = I_1 \sin\theta_1$

$$\cos\theta_1 = \frac{R_1}{Z_1}$$



$$\sin\theta_1 = \frac{X_L}{Z_1}$$

$$I_2 = I_1 \sin\theta_1$$



$$\frac{V}{X_C} = \frac{V}{Z_1}\,\frac{X_L}{Z_1}$$

$$\Rightarrow Z_1^2 = X_L X_C$$

$$\Rightarrow Z_1^2 = \frac{\omega L}{\omega C}$$

$$\Rightarrow Z_1^2 = \frac{L}{C}$$

$$\Rightarrow \boxed{Z_1 = \sqrt{\frac{L}{C}}}\ \Omega$$

$$\longrightarrow V$$

$$I = I_1 \cos\theta_1$$

$$\rightarrow B_L = B_C$$

$$Z_1^2 = R_1^2 + X_L^2$$

$$\frac{L}{C} = R_1^2 + (2\pi f_0 L)^2$$

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_1^2}{L^2}}}$$

$$\rightarrow I = I_1 \cos\theta_1$$

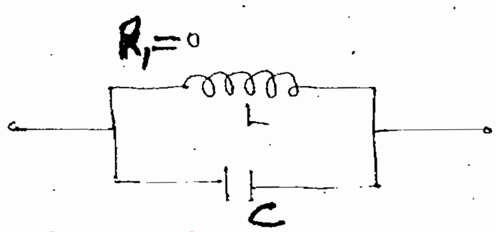
$$\Rightarrow I = \frac{V}{Z_1}\,\frac{R_1}{Z_1}$$

$$\Rightarrow I = \frac{VR_1}{Z_1^2} \quad \Rightarrow I = \frac{VR_1}{L/C}$$

$$\Rightarrow \boxed{I = \frac{V}{\frac{L}{R_1 C}}}$$

$$\Rightarrow \boxed{Z_{dyn} = \frac{L}{R_1 C}}\ \Omega$$

Ideal Tank Circuit :-



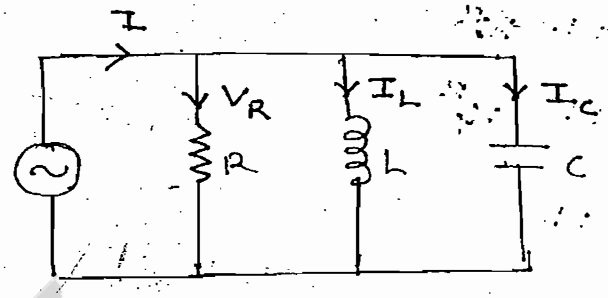
$$\boxed{\begin{aligned} Z_{dyn} &= \frac{L}{R_1 C} = \infty \\ f_0 &= \frac{1}{2\pi\sqrt{LC}} \end{aligned}}$$

or

Q - Factor :-

$$\boxed{Q = \frac{V_L \text{ or } V_C}{V}} \rightarrow \text{Series}$$

$$\boxed{Q = \frac{I_L \text{ or } I_C}{I}} \rightarrow \text{Parallel}$$



$$\boxed{Q = \frac{I_L}{I} = \frac{I_L}{I_R} = \frac{\text{Reactive component of current}}{\text{Active component of current}}}$$

This combination is valid for any combination of parallel circuit

$$\boxed{Q = \frac{I_L}{I_R} = \frac{V/X_L}{V/R} = \frac{R}{X_L} = \frac{R}{\omega L}} \left(\omega = \frac{1}{\sqrt{LC}} \right)$$

$$\boxed{Q = \frac{1/X_L}{1/R} = \frac{B_L \text{ or } B_C}{G}} \quad (B_L = B_C)$$

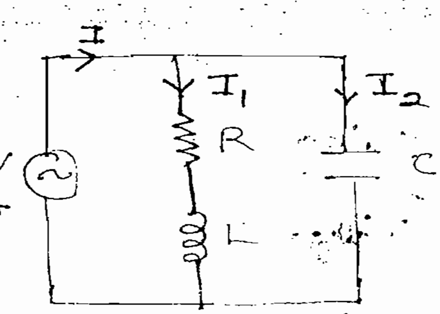
$$\boxed{Q = R\sqrt{\frac{C}{L}}}$$

$$\boxed{Q = \frac{I_C}{I} = \frac{I_C}{I_R} = \frac{V/X_C}{V/R} = \frac{R}{X_C} = R\omega C}$$

For $\boxed{Q > 1 \;,\; R > X_L \;,\; R > X_C}$

Tank Circuit :-

$$Q = \frac{\text{Reactive component of current}}{\text{Active comp. of current}}$$

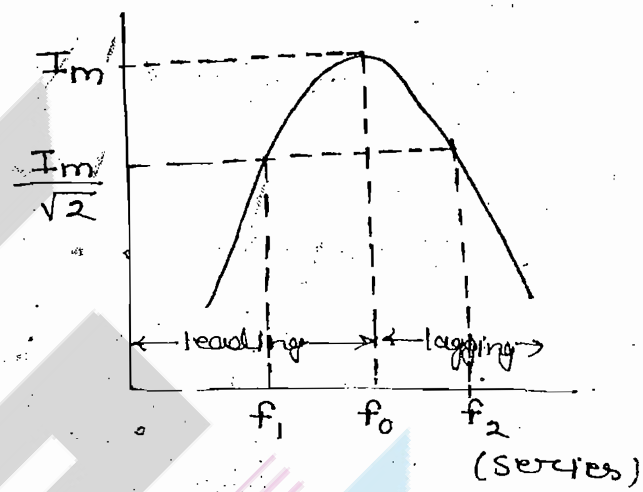


$$\boxed{Q = \frac{I_1 \sin\theta_1 \text{ or } I_2}{I}}$$

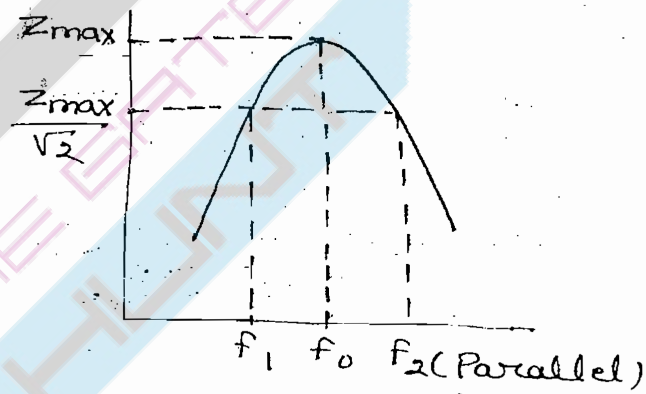
$$\boxed{Q = \frac{I_2}{I} = \frac{V/X_C}{\dfrac{V}{L/RC}} = \frac{\dfrac{1}{1/\omega C}}{\dfrac{1}{L/RC}} = \frac{\omega L}{R} = \frac{X_L}{R}}$$

For $Q > 1$, $\boxed{X_L > R}$

$$\boxed{B \cdot W = f_2 - f_1}$$

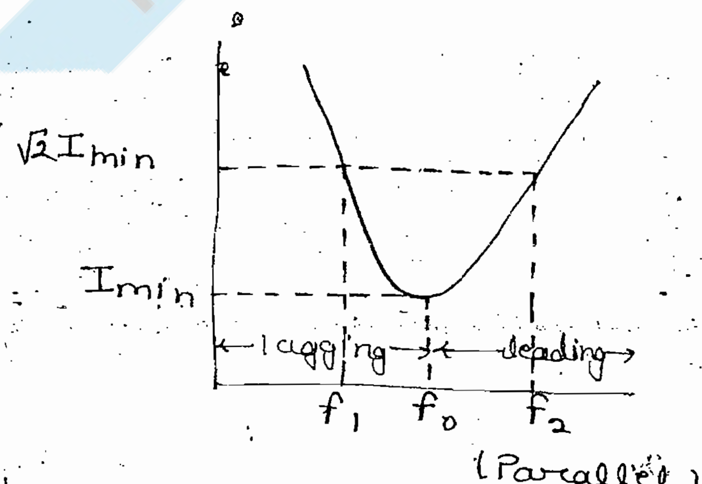


$$\boxed{B \cdot W = f_2 - f_1}$$



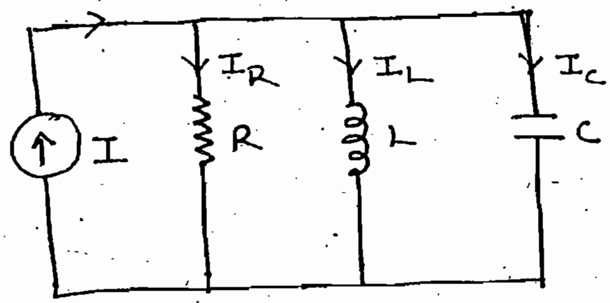
$$\boxed{B \cdot W = f_2 - f_1}$$

$$\boxed{\begin{aligned} B_L &= \frac{1}{2\pi f L} \\ B_C &= 2\pi f C \end{aligned}}$$



For parallel circuit we concentrate on the value of B_L and B_C

By KCL



$$I = \frac{V}{R} + C\frac{dV}{dt} + \frac{1}{L}\int V dt$$

Diff. w.r.t t

$$I' = \frac{1}{R}\frac{dV}{dt} + C\frac{d^2V}{dt^2} + \frac{V}{L}$$

Dividing both sides by C , we get

$$\frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{V}{LC} = \frac{I'}{C}$$

$$D = \frac{d}{dt}$$

$$\left(D^2 + \frac{1}{RC}D + \frac{1}{LC}\right)V = 0$$

Compare with $s^2 + 2\delta\omega_n s + \omega_n^2 = 0$ we get

$$2\delta\omega_n = \frac{1}{RC}, \quad \omega_n^2 = \frac{1}{LC}$$

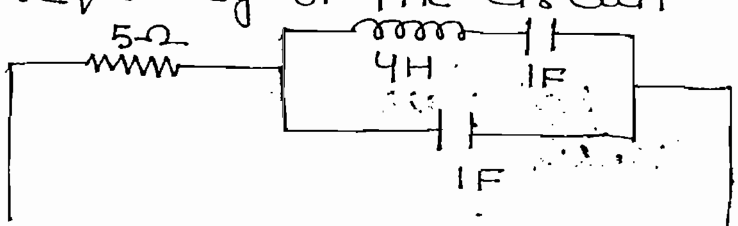
$$2\delta\frac{1}{\sqrt{LC}} = \frac{1}{RC}, \quad \omega_n = \frac{1}{\sqrt{LC}}$$

$$\boxed{Q = R\sqrt{\frac{C}{L}}}$$

$$\boxed{\text{Damping Ratio} = \delta = \frac{1}{2R}\sqrt{\frac{L}{C}}}$$

$$\boxed{\delta = \frac{1}{2Q}}$$

Ques:- Find resonant frequency of the circuit shown



Soln:- At resonance in any of the case imag. part = 0

Note:-

To find resonant frequency for any combination of the network

(1) Find Z_{eq}

(11) Equate imag. part of $Z = 0$

$$Z_1 = j(X_L - X_C) = j\left(\omega L - \frac{1}{\omega C}\right) = j\left(4\omega - \frac{1}{\omega}\right)$$

$$Z_2 = -jX_C = -\frac{j}{\omega C} = -\frac{j}{\omega}$$

$$Z'_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$\Rightarrow Z'_{eq} = \frac{j\left(4\omega - \frac{1}{\omega}\right)\left(-j/\omega\right)}{j\left(4\omega - \frac{1}{\omega}\right) - j/\omega}$$

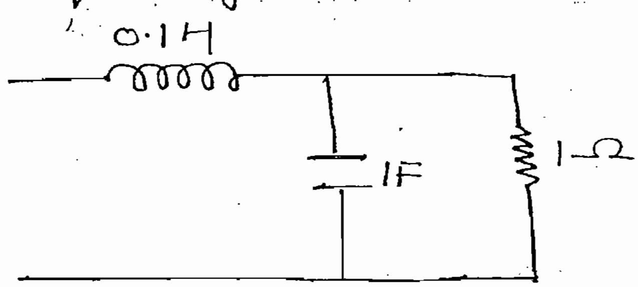
$$\Rightarrow Z'_{eq} = \frac{\left(4\omega - 1/\omega\right)\left(+1/\omega\right)}{j\left(4\omega - \frac{1}{\omega}\right) - j/\omega} \times \frac{j}{j}$$

$$\text{Im } Z'_{eq} = 0$$

$$\left(4\omega - \frac{1}{\omega}\right)\frac{1}{\omega} = 0$$

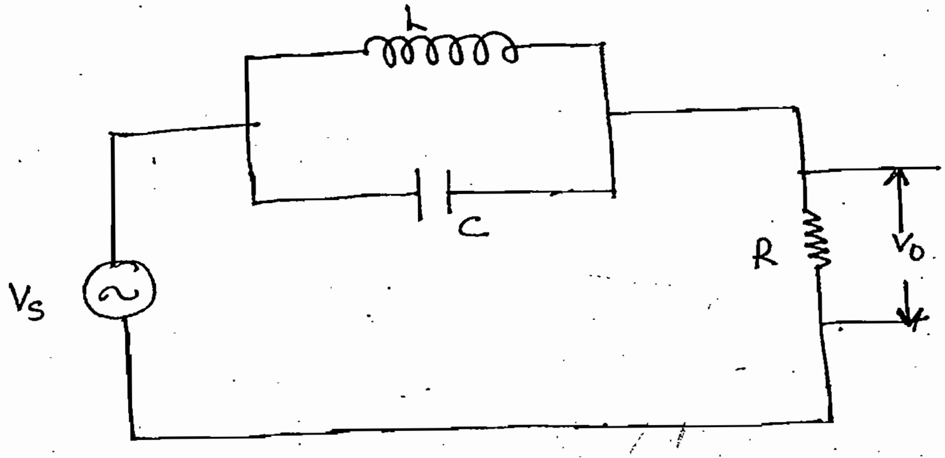
$$\Rightarrow \boxed{\omega = 0.5 \text{ rad/sec}}$$

Ques:- Find resonant frequency of the circuit shown

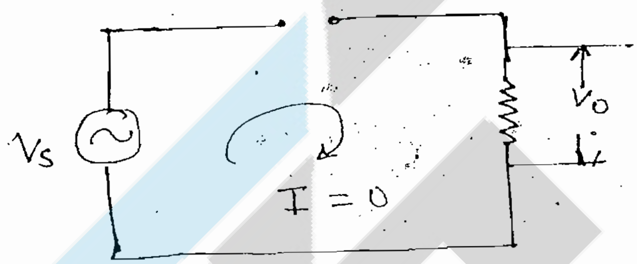


Ans:- $\omega_0 = 3$ rad/sec

Ques 1– Find V_o under resonance condition



Soln 1– Ideal tank circuit, $Z_{Dyn} = \infty$

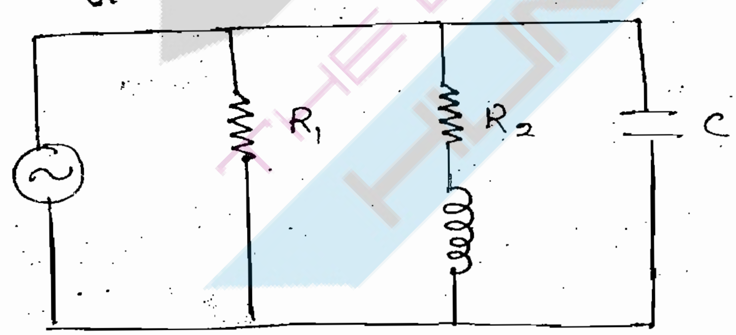


$V_o = 0$, Ans

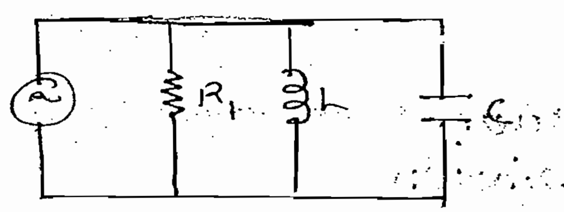
Note:–

$R_1 \uparrow \quad Q \uparrow$

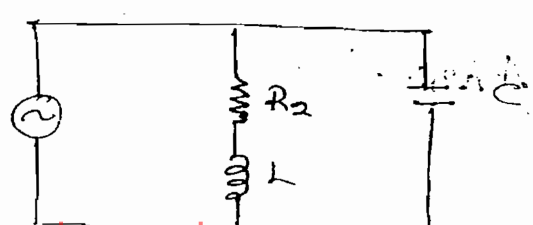
$R_2 \uparrow \quad Q \downarrow$



$Q = \dfrac{R_1}{\omega L} \leftarrow$



$Q = \dfrac{\omega L}{R_2} \leftarrow$



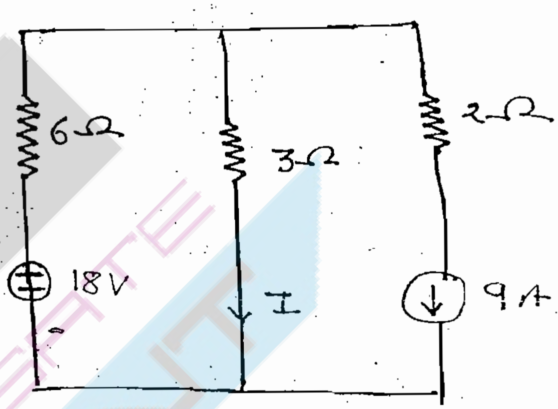
THEOREMS :-

When the N/w is having more no. of nodes and more no. of meshes, the response in any one of the branches can be easily obtained by using theorem

Superposition theorem:-

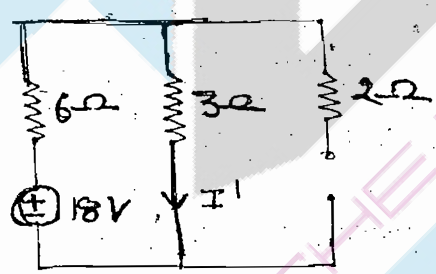
In any linear bidirectional circuit having more than one independent source the response in any of the branches is equal to algebraic sum of the responses caused by individual sources while rest of the sources are replaced by its internal resistances

Ques:- Find the value of I by using superposition theorem



Soln:- Case-(I):-

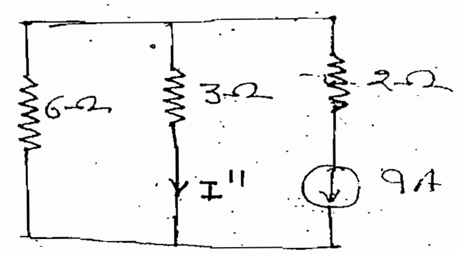
Due to 18V



$$I' = \frac{18}{6+3}$$

$$= 2A$$

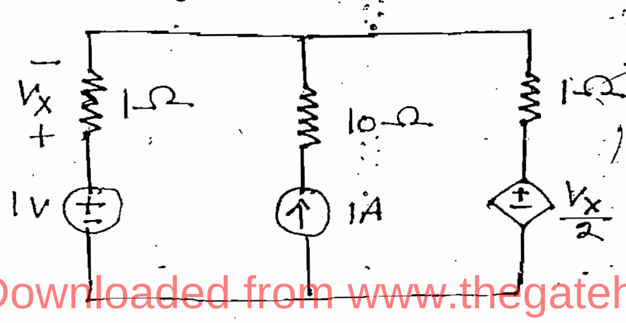
Case-(II):-



$$I'' = -9 \frac{6}{6+3} = -6$$

$$I = I' + I'' = 2-6 = -4 \text{, Ans.}$$

Ques:- Find V_x by using superposition theorem

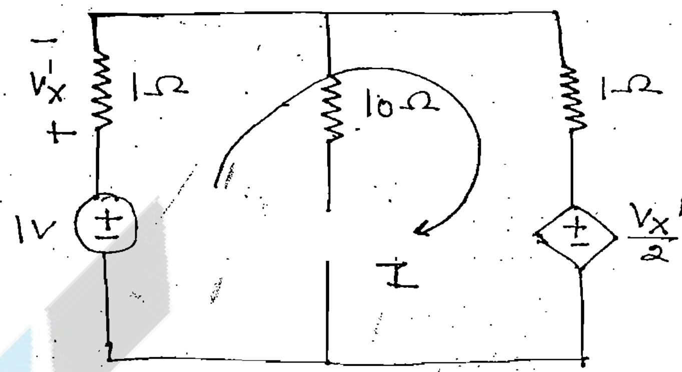


Note:-

In the above circuit while applying superposition theorem dependent sources neither replaced by open circuit nor S.C & it remains as original ckt.

Soln:- Case-(I):-

$$I = \frac{1 - \frac{V_x'}{2}}{1+1}$$



$V_x' = 1 \times I = I$

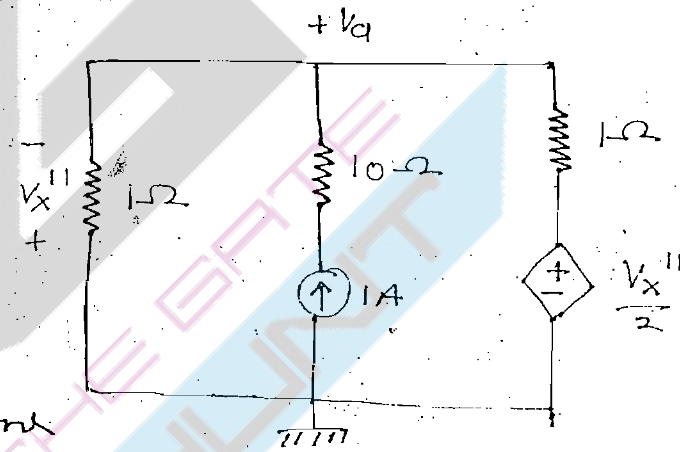
$V_x' = \frac{2}{5}$

Case-(II):-

$V_a = -V_x''$

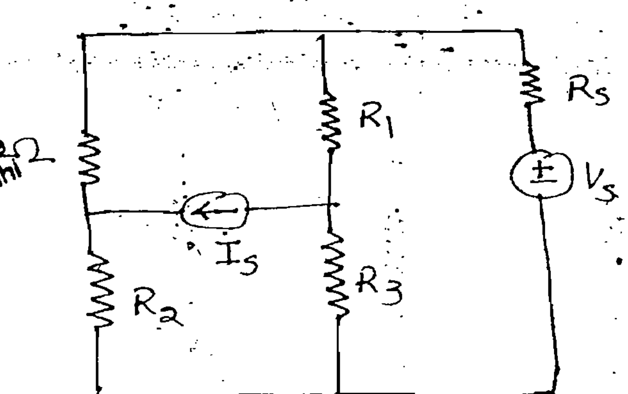
$$\frac{V_a}{1} + \frac{V_a - \frac{V_x''}{2}}{1} = 1$$

$V_x'' = -\frac{2}{5}$



$V_x = V_x' + V_x'' = 0$, Ans

ques:- In the circuit shown power dissipation in 1Ω resistor is 576 W when voltage source is acting along and power dissipation in 1Ω is resistor is 1W when current source is acting along. Find total power dissipation in 1 ohm resistor



Soln:-

$$I = \pm I' \pm I'' \checkmark$$

$$V = \pm V' \pm V'' \checkmark$$

$$P = \pm P' \pm P'' \quad \times$$

$$P = I^2 R \qquad P' = I'^2 R$$

$$I' = \sqrt{\frac{P'}{R}}$$

$$I'' = \sqrt{\frac{P''}{R}}$$

$$I = \pm I' \pm I''$$

$$\Rightarrow I = \pm \sqrt{\frac{P'}{R}} \pm \sqrt{\frac{P''}{R}}$$

$$P = I^2 R$$

$$\Rightarrow P = \left(\pm \sqrt{\frac{P'}{R}} \pm \sqrt{\frac{P''}{R}} \right)^2 R$$

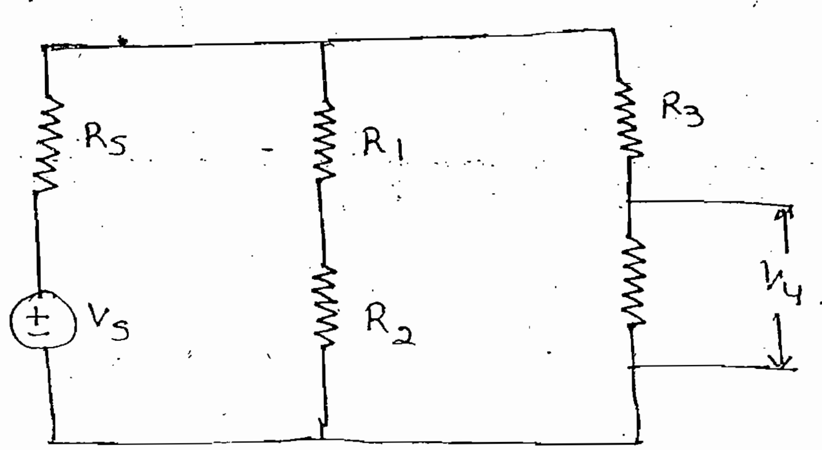
$$\Rightarrow ** \boxed{P = \left(\pm \sqrt{P'} \pm \sqrt{P''} \right)^2}$$

$$P = \left(+\sqrt{P'} - \sqrt{P''} \right)^2$$

$$= \left(+\sqrt{576} - \sqrt{1} \right)^2 = 529 \text{ W}, \text{ Ans.}$$

Ques:- In the circuit shown if the source voltage is increased by 10% then find variation of the power in the R_4 resistor.

Soln:-



Soln:—

$$P_4 = \frac{V_4^2}{R_4}$$

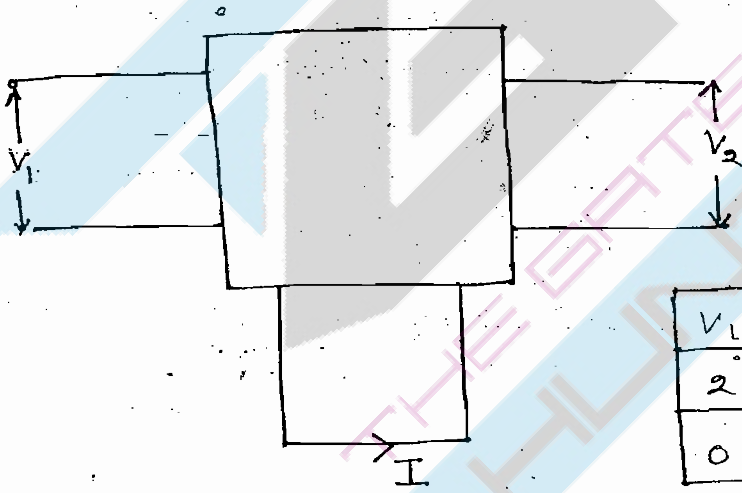
$$P_4' = \frac{(1.1V_4)^2}{R_4} = 1.21\frac{V_4^2}{R_4} = 1.21 P_4$$

Inc by 21%.

Note:—

When the N/w is having linear bidirectional element if excitation is particularly constant K the response of each element also multiplied by constant K (Homogenity Principle)

Ques:— In the circuit shown find I when $V_1 = 10V$ & $V_2 = -12V$



V_1	V_2	I
2	0	3A
0	3V	-4A

Soln:— $V_1 = 2$ $\quad I = 3A$

$V_1 = 10$ then $I' = 5\times3 = 15A$

(2x5)

5times ↑

$V_2 = 3V$ $\quad I = -4$

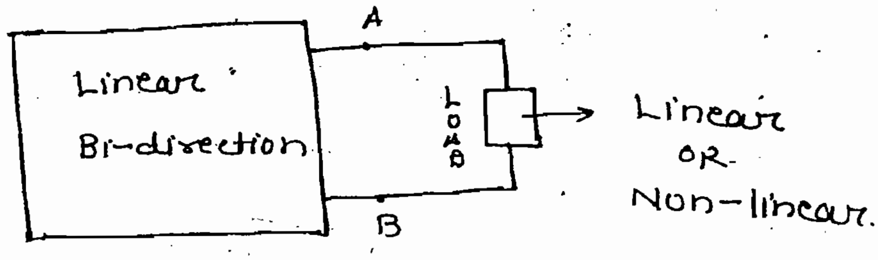
$V_2 = -12V$ then $I'' = (-4)(-4)$

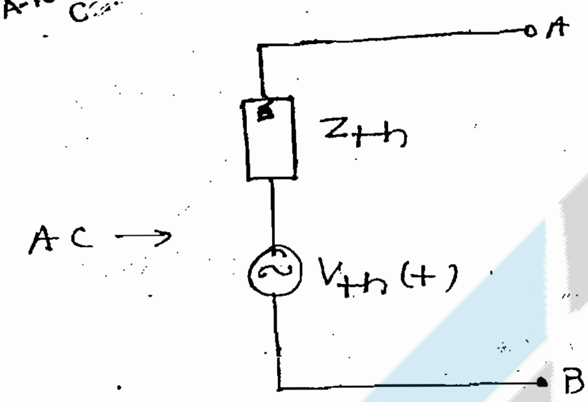
(-4x3) $\quad = 16$

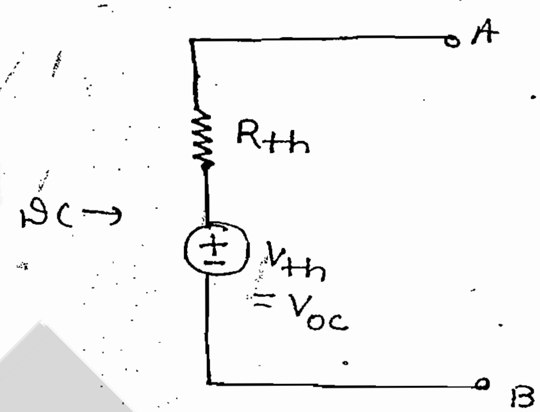
$$I = I' + I''$$

$$I = 15+16 = 31\ A \text{ , Ans}$$

Thevenins Theorem :-

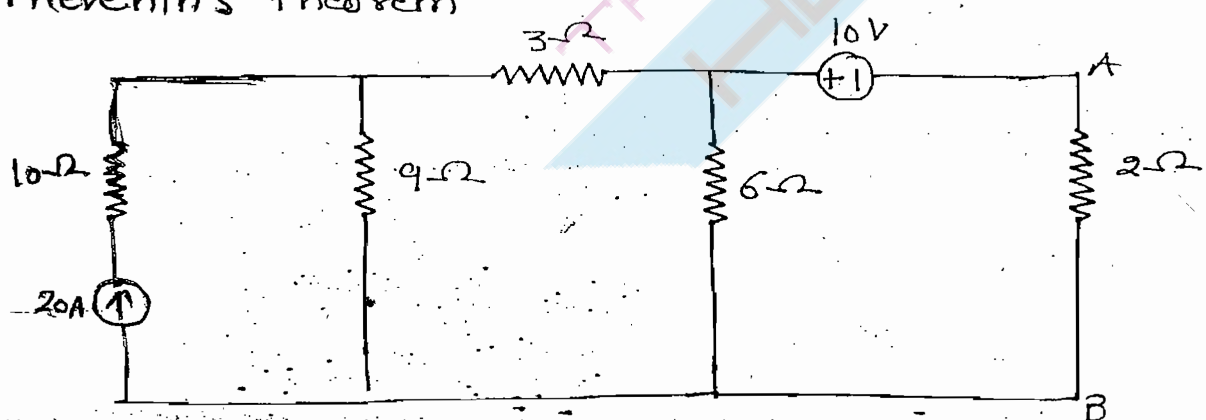






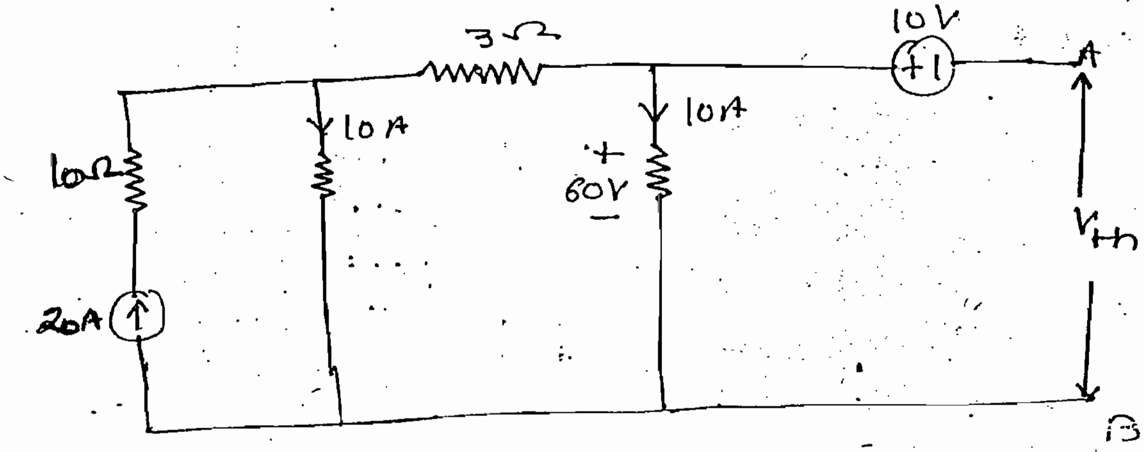
When N/w is having linear bidirectional elements and more no. of active and passive elements, it can be replaced by single equivalent circuit consisting of equivalent voltage source (V_{th}) in series with equivalent resistance (R_{th})

Ques:- Find current in the 2Ω resistor by using thevenin's theorem



Soln:- Case-(1) → (V_{th}) :-

Disconnect the load resistor and o.c voltage across the load terminals

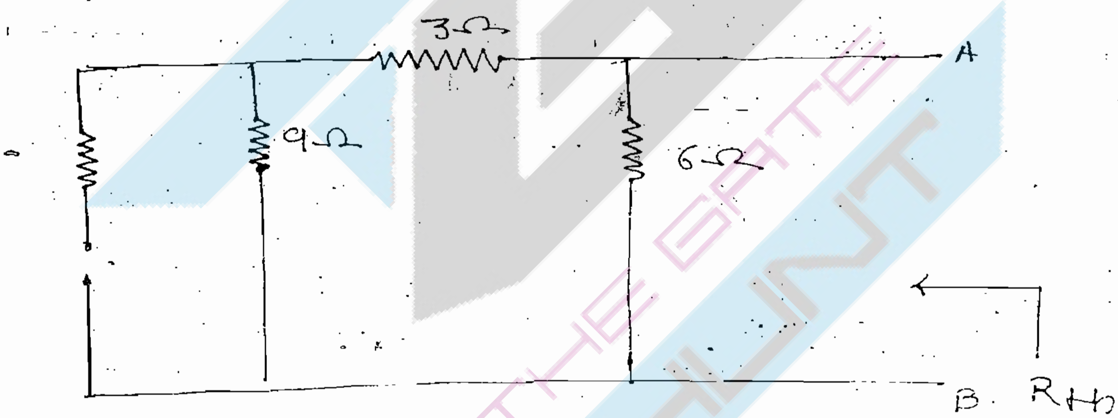


$$-60 + 10 + V_{th} = 0$$

$$\Rightarrow \quad V_{th} = 50V$$

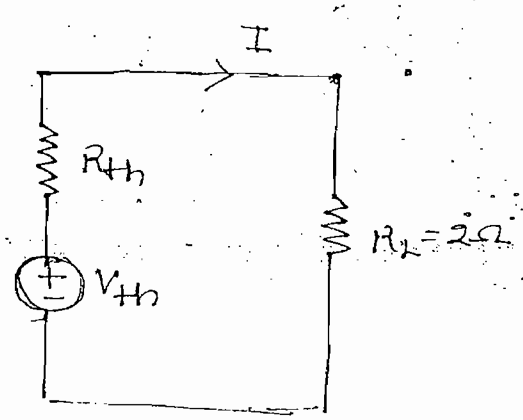
Case-(II) $\longrightarrow$ (R_{th}) :—

Deactivate all the independent sources and find eq. resistance w.r.t. load terminals



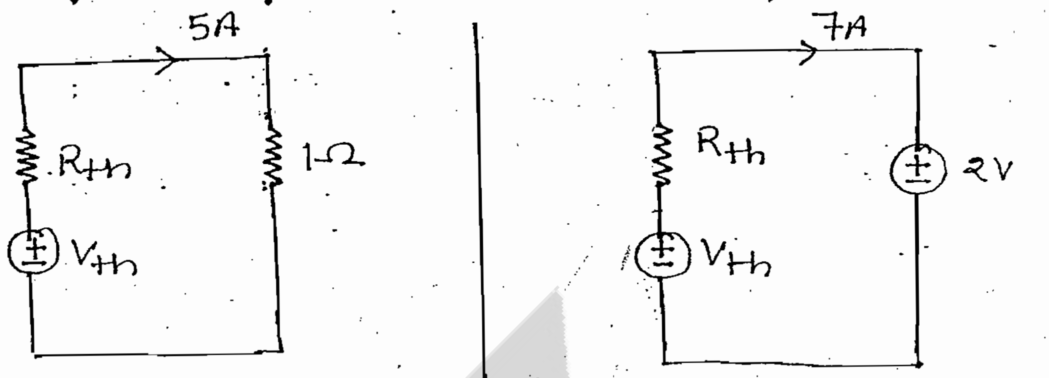
$$R_{th} = \frac{12 \times 6}{12+6} = 4\Omega$$

$$I = \frac{V_{th}}{R_{th} + R_L}$$



Ques:- When a battery charger connected to 1Ω resistor, current in the resistor is 5A. When same battery charger is connected for charging of ideal 2V battery at 7A rate. Find V_{th} & R_{th}

Soln:-

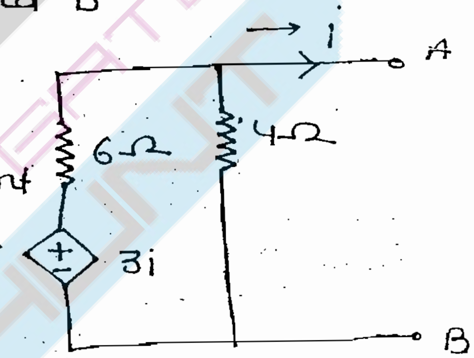


$$5 = \frac{V_{th}}{R_{th}+1} \quad \text{—(I)}$$

$$7 = \frac{V_{th}-2}{R_{th}} \quad \text{—(II)}$$

$\Rightarrow V_{th} = 12.5$ and $R_{th} = 1.5$

Ques:- Find V_{th} w.r.t A and B

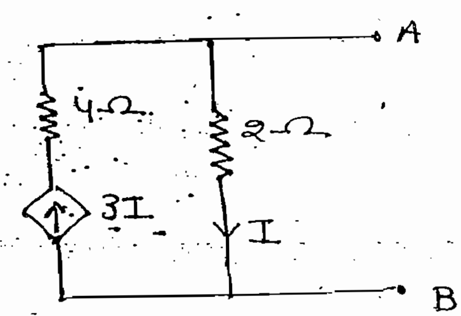


Note:- In above N/w no independent source is present Therefore $V_{th} = 0$

Ques:- Find V_{th} w.r.t A and B.

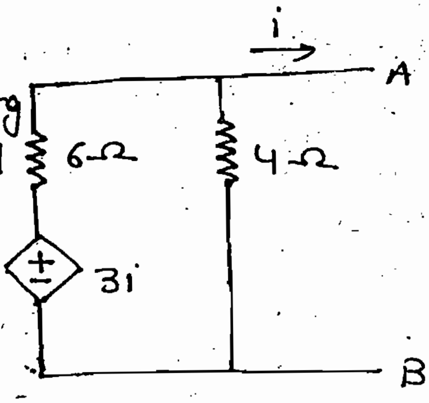
(a) 0 (b) 4 (c) 6

✓(d) None



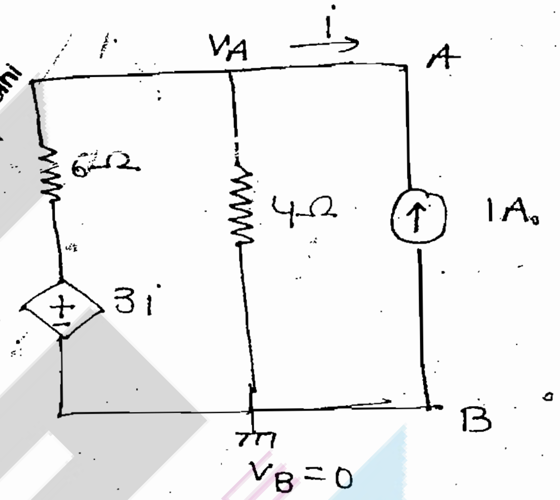
Note:- In the above ckt, it is not possible to find o.c voltage since it is not satisfying KCL

Ques:- Find R_{th} w.r.t A and B



Note:- In above N/w while finding R_{th} dependent source is replaced by neither OC nor short ckt and it remains same as original ckt

Soln:-



$$\frac{V_A - 3i}{6} + \frac{V_A}{4} = 1$$

$$\Rightarrow \quad i = -1$$

$$V_A = 1.2$$

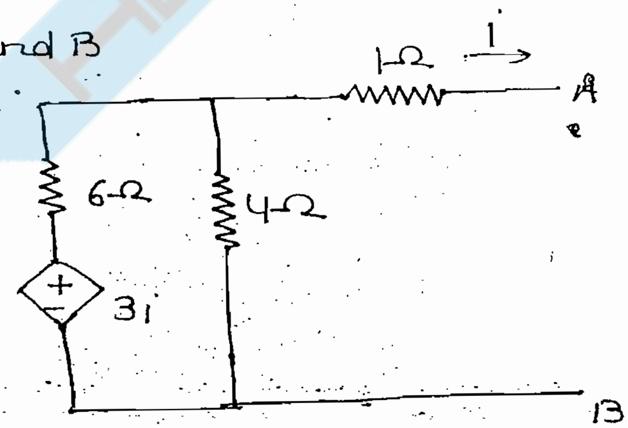
$$V_{AB} = V_A - V_B$$

$$= 1.2 - 0 = 1.2$$

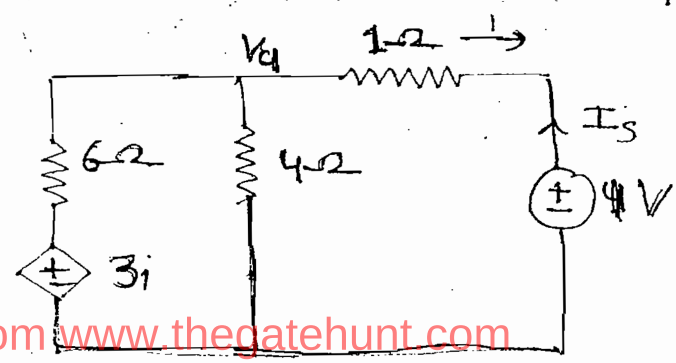
$$R_{th} = \frac{V_{AB}}{I_s} = \frac{1.2}{1} = 1.2\,\Omega \text{, Ans}$$

In parallel consider current source for simple calculation.

Ques:- Find R_{th} w.r.t A and B



Soln:- In series consider voltage source



$$\frac{V_1 - 3i}{6} + \frac{V_1}{4} + \frac{V_1 - 1}{1} = 0$$

$$i = \frac{V_1 - 1}{1} \Rightarrow \cancel{V_1 = \frac{6}{11}}$$

$$= -\frac{5}{11}$$

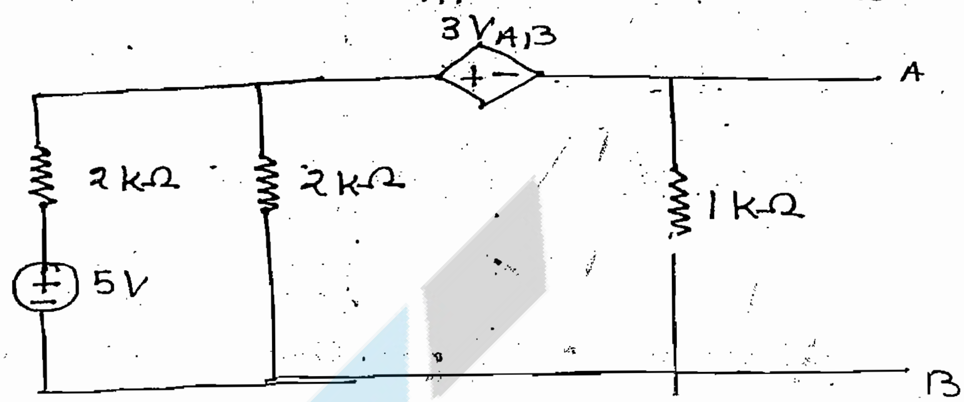
$$\& \quad V_1 = \frac{6}{11}$$

$$I_s = -i = -\left(-\frac{5}{11}\right) = \frac{5}{11}$$

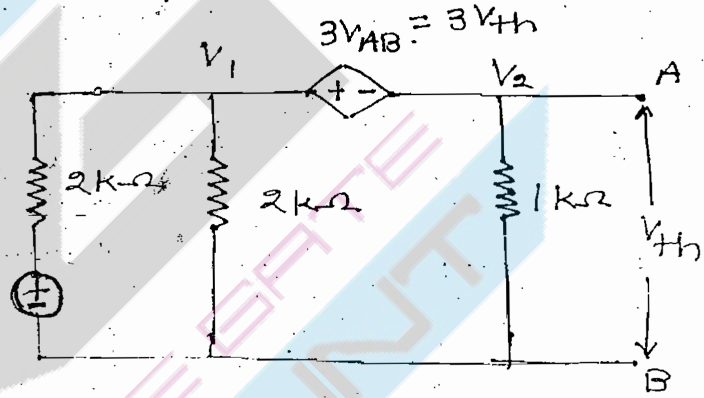
$$R_{th} = \frac{V_s}{I_s} = \frac{1}{5/11} = \frac{11}{5}\,\Omega \quad \text{, Ans}$$

$$= 2.2\,\Omega \quad \text{, Ans.}$$

Ques:- Find V_{th} and R_{th} w.r.t A and B



Soln:- Case-(I) :-



$$\frac{V_1 - 5}{2\times10^3} + \frac{V_1}{2\times10^3} + \frac{V_2}{1\times10^3} = 0 \quad \text{---(I)}$$

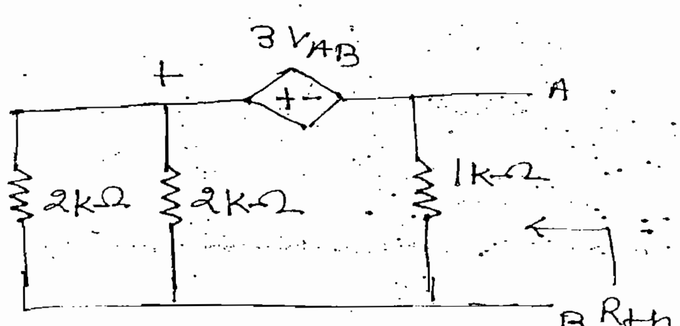
$$V_1 - V_2 = 3V_{th} \quad \text{---(II)}$$

$$V_2 = V_{th}$$

$$V_1 = 4V_{th}$$

$$V_{th} = 0.5V$$

Case-(II) :- For R_{th}



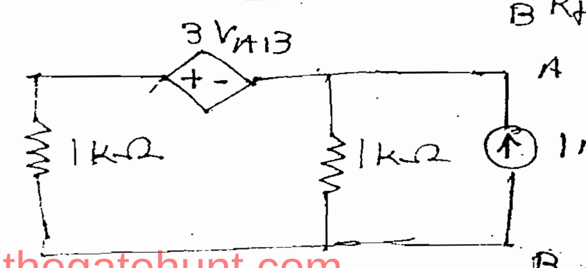
$$\frac{V_A + 3V_{AB}}{10^3} + \frac{V_A}{1\times10^3} = 1$$

$$V_{AB} = V_A - V_B = V_A$$

$$V_{AB} = 200$$

$$R_{th} = \frac{V_{AB}}{I_s} = \frac{200}{1} = 200\,\Omega$$

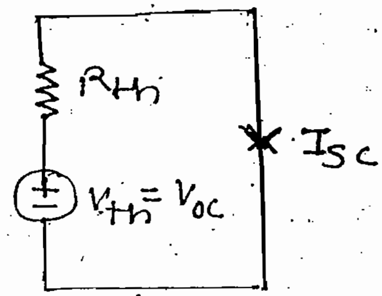
And



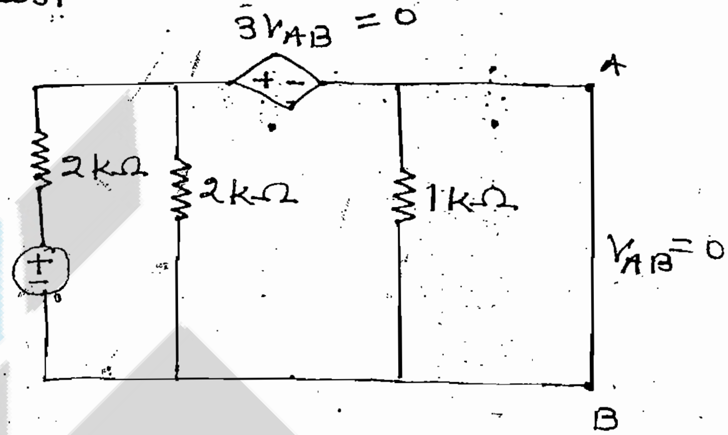
Verification:-

$$I_{sc} = \frac{V_{oc}}{R_{th}}$$

$$R_{th} = \frac{V_{oc}}{I_{sc}}$$



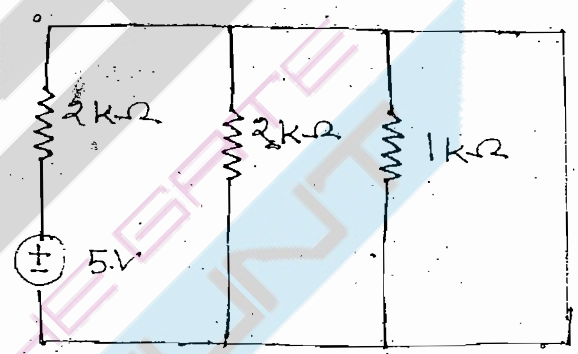
For this method atleast one independent source should be present



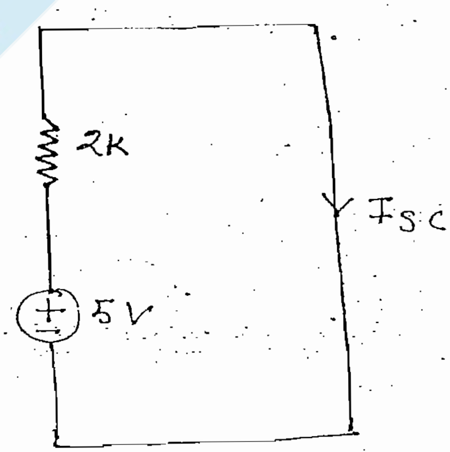
$$I_{sc} = \frac{5}{2\times10^3}$$

$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{0.5}{5/2\times10^3}$$

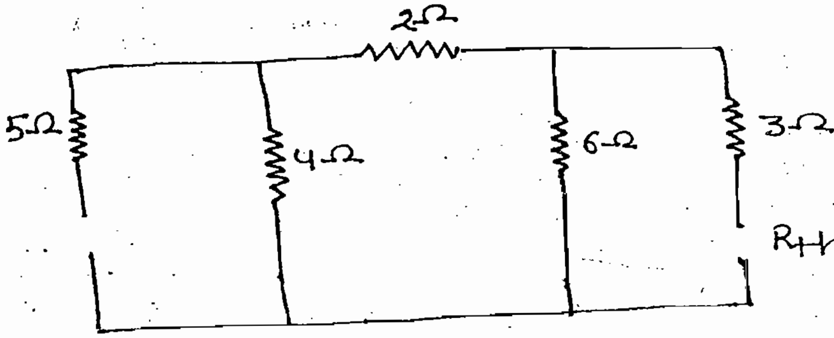
$$= 200\Omega$$



Note:- The above method of R_{th} calculation can be done provided original N/w should consist of atleast one independent source.



Case - 2 (R_{th}):-

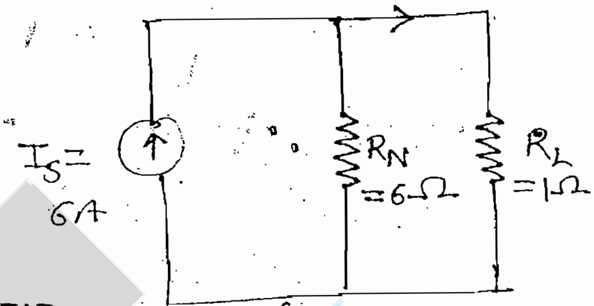


$R_{th} = R_N$

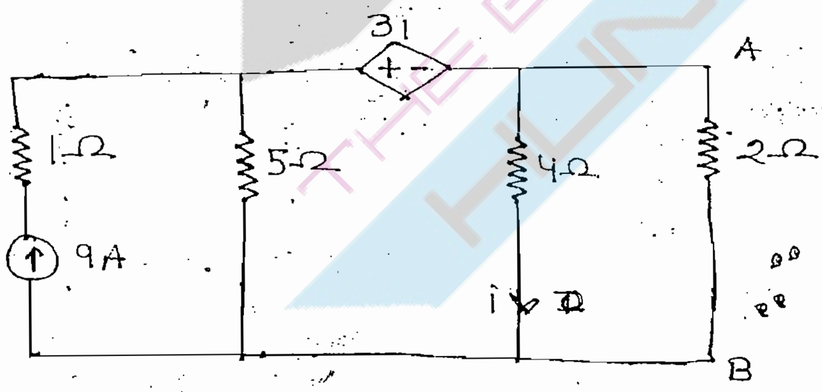
$$R_{th} = R_N = 6\Omega$$

$$I_L = 6 \times \frac{6}{6+1}$$

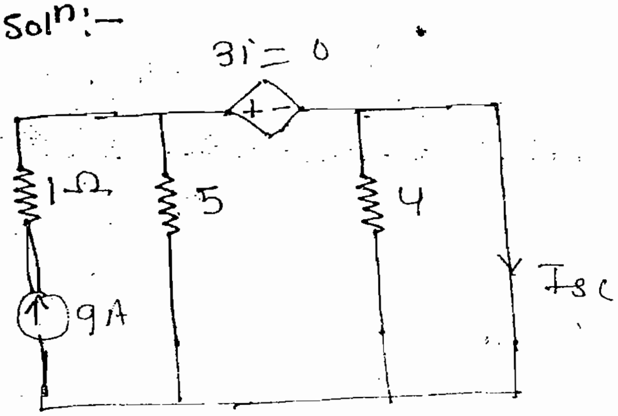
$$= \frac{36}{7} \text{ A} \quad \text{, Ans}$$



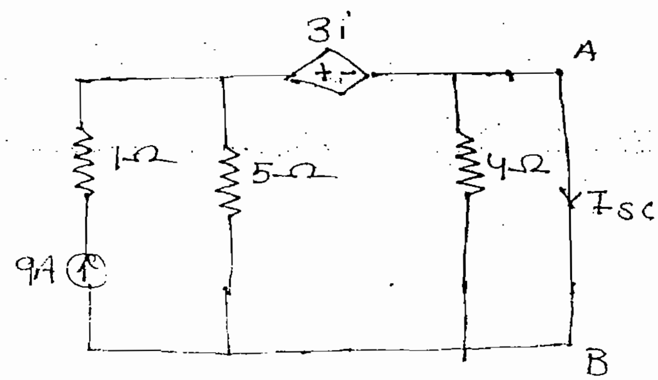
Ques:- Find S.C current w.r.t A and B



Soln:-



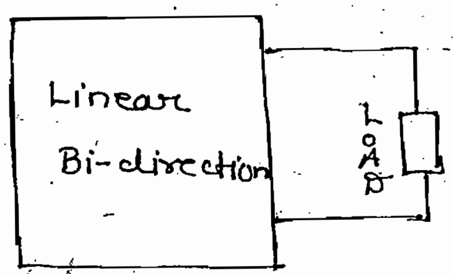
⇐

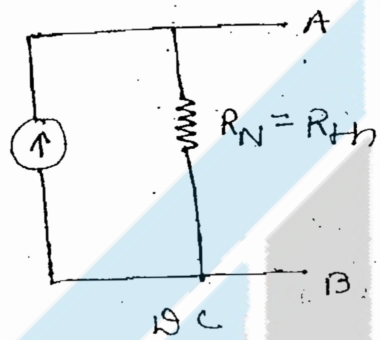


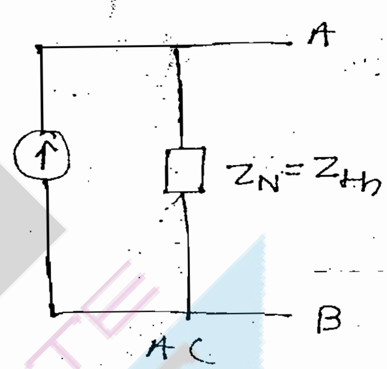
Lecture - 7

Norton's theorem :-

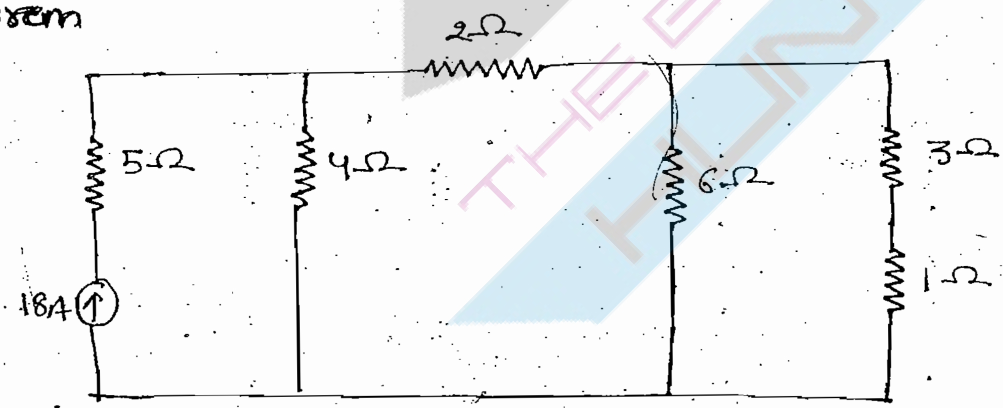
In any linear bidirection circuit having more no. of active and passive element, it can be replaced by single equivalent. ckt consisting of equivalent current source (I_N) in parallel with equivalent resistance (R_N)







Ques :- Find current in 1Ω resistor using Norton's theorem



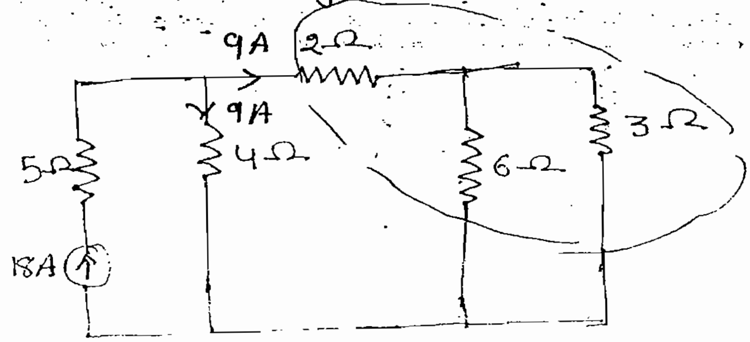
Soln :-

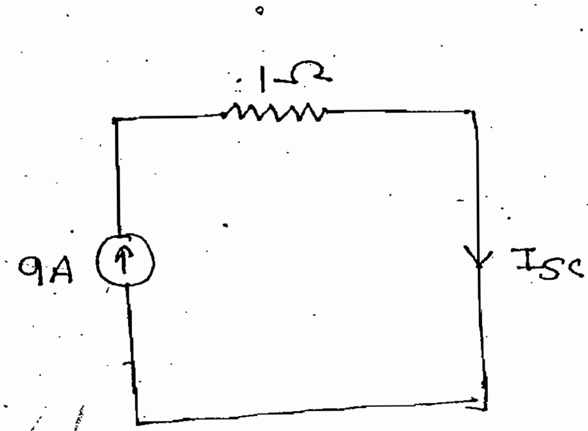
Case - (1) :-

Replace the load resistor by S.C and find SC current

$$I_{sc} = 9 \times \frac{6}{6+3}$$

$$= 6A$$



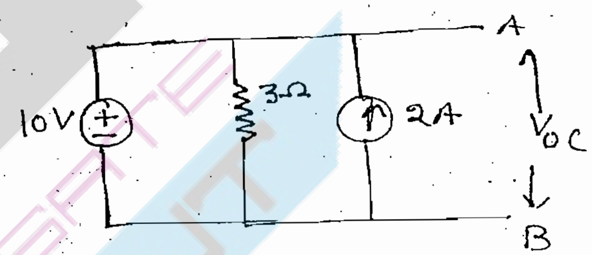


$I_{sc} = 9A$, Ans.

Ques:- Find o.c voltage and SC current w.r.t A & B

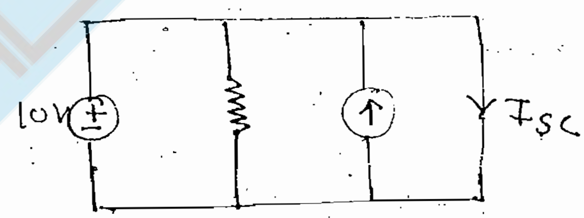
Soln:-

$V_{oc} = 10V$



For I_{sc}:-

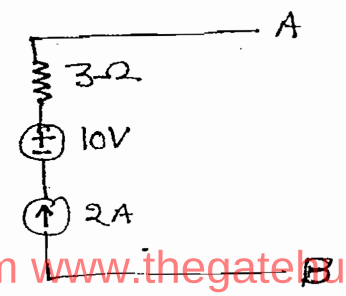
I_{sc} = not possible



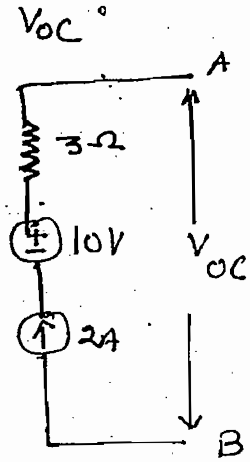
Note:-

In the above circuit it is not possible to find I_{sc} since it is not satisfying KVL

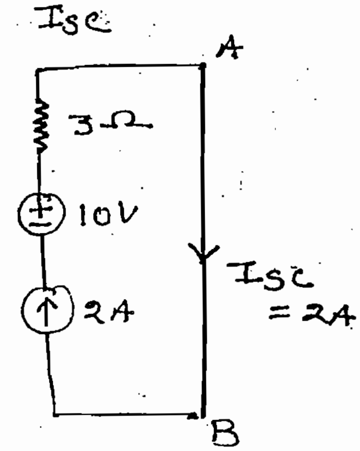
Ques:- Find V_{oc} and I_{sc} w.r.t A and B



Voc



Isc

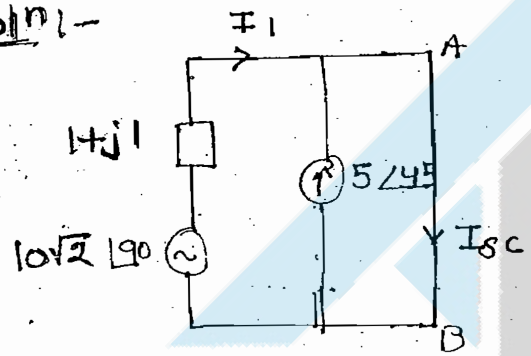


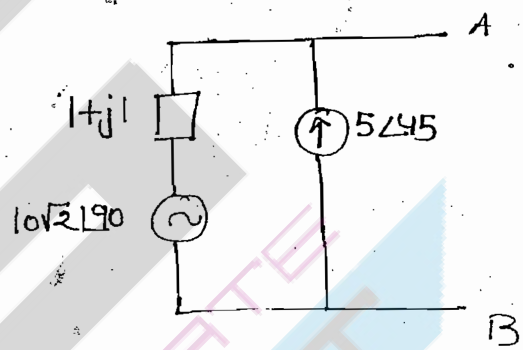
Note:-

In above circuit it is not possible to find O·C voltage since it is not satisfying KCL

Ques:- Find I_{sc} w.r.t A and B

Soln:-





$$I_1 = \frac{10\sqrt{2}\angle 90}{\sqrt{2}\angle 45} = 10\angle 45$$

$$I_1 + 5\angle 45 = I_{sc}$$

$$10\angle 45 + 5\angle 45 = I_{sc}$$

$$\Rightarrow \quad I_{sc} = 15\angle 45^\circ$$

Ques:- Find current in 4Ω resistor using the following data:

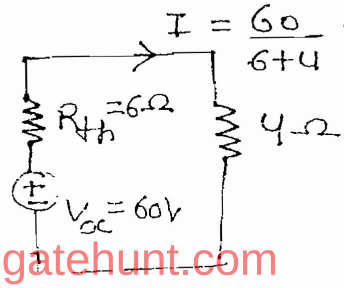
V	60	0
I	0	10A

soln:- $V_{oc} = 60$

$I_{sc} = 10A$

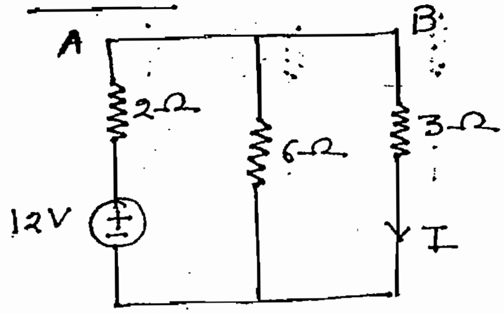
$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{60}{10} = 6\Omega$$

$$I = \frac{60}{6+4} = 6A$$



Reciprocity theorem :-

Case-(i) :-



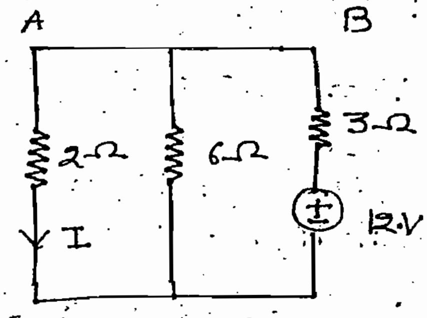
$$R_{eq} = 2 + \frac{6\times3}{6+3} = 4$$

$$I_T = \frac{12}{4} = 3A$$

$$I = 3\,\frac{6}{6+3} = 2A$$

$$\frac{\text{Response}}{\text{Excitation}} = \frac{2}{12} = \frac{1}{6}$$

Case-(ii) :-



$$R_{eq} = 3 + \frac{6\times2}{6+2}$$

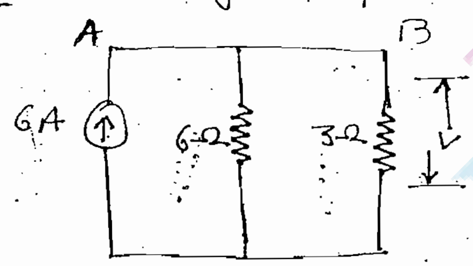
$$I_T' = \frac{12}{R_{eq}}$$

$$I = I_T'\,\frac{6}{6+2} = 2A$$

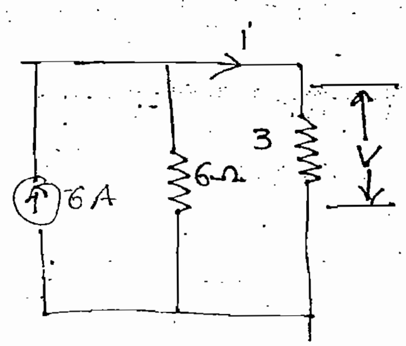
$$\frac{\text{Response}}{\text{Excitation}} = \frac{2}{12} = \frac{1}{6}$$

In above N/w after interchanging position of response and excitation the ratio of response to excitation is constant. Hence given N/w satisfy the reciprocity (Linear bi-directional element)

Ques:- Verify Reciprocity theorem of the circuit shown.



Soln:- Case-(i) :-



$$I' = 6\times\frac{6}{6+3} = 4A$$

$$V = 3\times4 = 12$$

$$\frac{\text{Response}}{\text{Excitation}} = \frac{12}{6} = 2$$

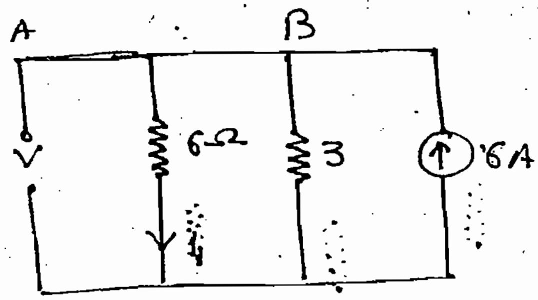
Case-(II):-

$i_1 = 6 \dfrac{3}{3+6} = 2A$

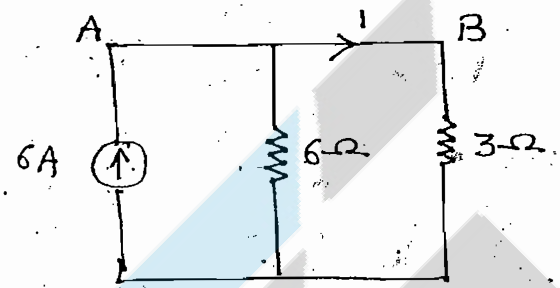
$V = 6 \times 2 = 12$

$\dfrac{\text{Response}}{\text{Excitation}} = \dfrac{12}{6} = 2$

$\rightarrow$ satisfy Reciprocity



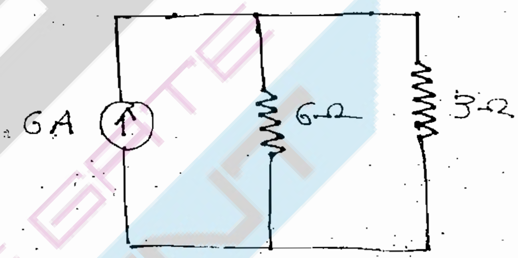
Ques:- Verify Reciprocity theorem of the ckt shown:-



Soln:-

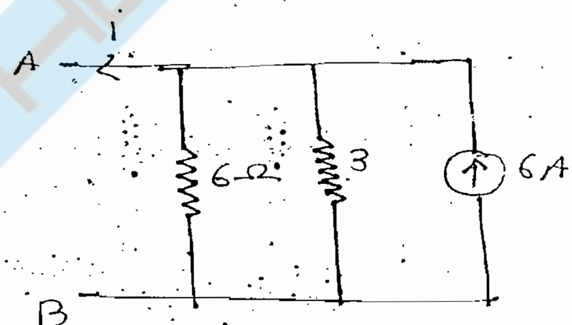
Case-(I)

$i = 6 \dfrac{6}{6+3} = 4A$



Case-(II)

$i = 0$



Note:- For above problem

(I) $\dfrac{\text{Response}}{\text{Excitation}} = \dfrac{I}{V_s} = \text{mho}$

(II) $\dfrac{\text{Response}}{\text{Excitation}} = \dfrac{V}{I_s} \rightarrow \Omega$

(III) $\dfrac{\text{Response}}{\text{Excitation}} = i/I_s$

(IV) $\dfrac{\text{Response}}{\text{Excitation}} = \dfrac{V}{V_s}$

(III), (IV) $\rightarrow$ No units

Note:-

For the above N/W Reciprocity theorem can't be applied since unit of response to excitation should be neither mho or Ω

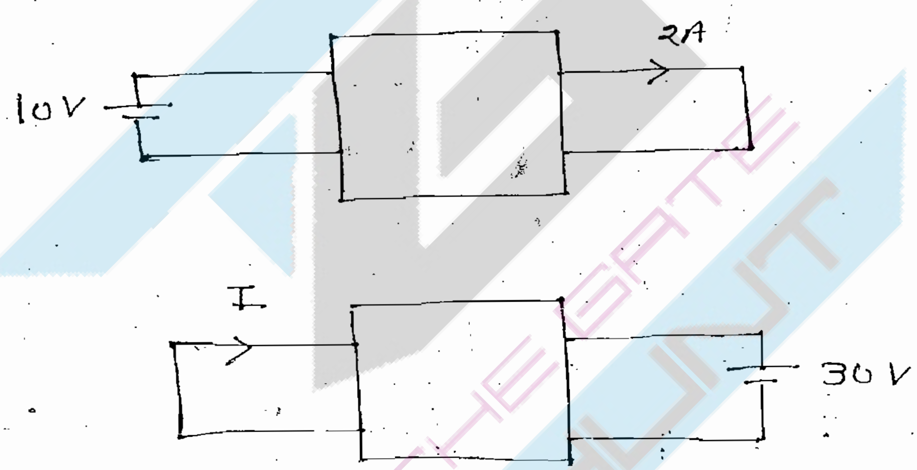
Note:-

1- To apply Reciprocity theorem unit of Res/Exc. should be either mho or ohm

2. While applying reciprocity theorem N/W should consist of only one independent source

3. While applying reciprocity theorem N/W should not consist of any dependent sources

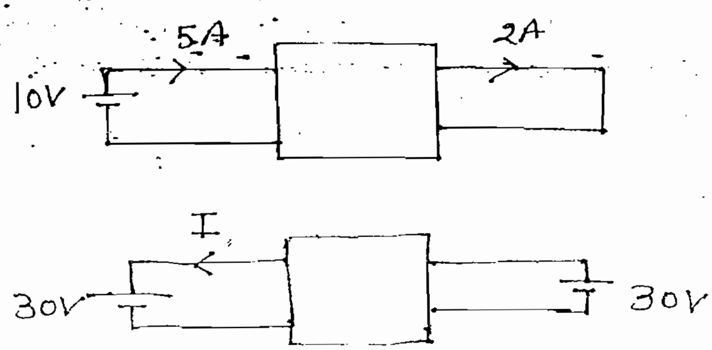
Ques:- When given N/W satisfy the Reciprocity then find the value of I



Soln:-

$$\frac{\text{Response}}{\text{Excitation}} = \frac{2}{10} = \frac{-I}{30} \Rightarrow I = -6A$$

Ques:- When given N/W satisfy the Reciprocity then find the value of I

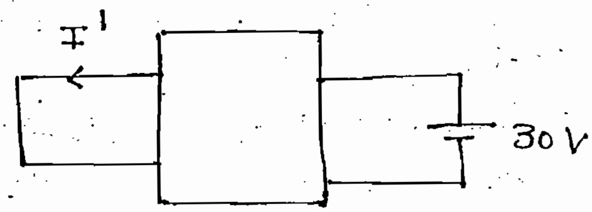


soln:- Soln by using Superposition & Reciprocity
($\because$ more than one independent sources are present)

Case-(I) (30V) :-

$$\frac{\text{Response}}{\text{Excitation}} = \frac{I'}{30} = \frac{2}{10}$$

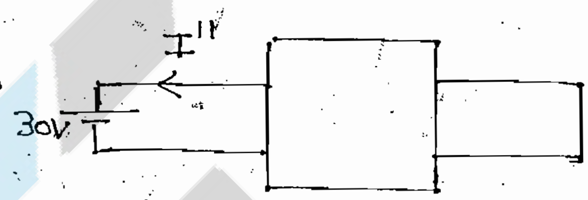
$$\Rightarrow I' = 6A$$



Case-(II) (30V) :-

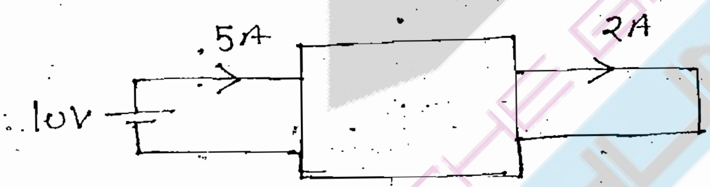
$$\frac{\text{Response}}{\text{Excitation}} = \frac{-I''}{30} = \frac{5}{10}$$

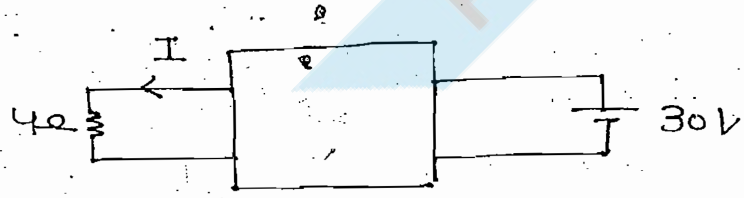
$$\Rightarrow I'' = -15$$



$$I = I' + I'' = 6 - 15 = -9A$$

Ques:- When given N/w satisfy the Reciprocity find the value of I

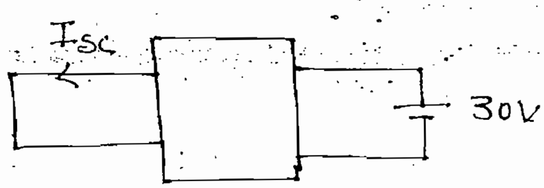




soln:- Case-(I) (I_{sc}) :- (Norton's theorem)

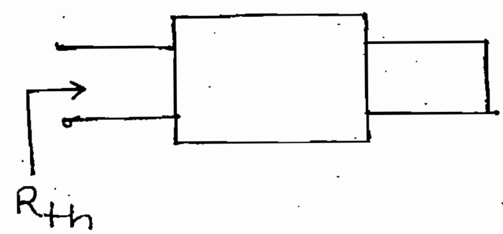
$$\frac{\text{Response}}{\text{Excitation}} = \frac{I_{sc}}{30} = \frac{2}{10}$$

$$I_{sc} = 6A$$

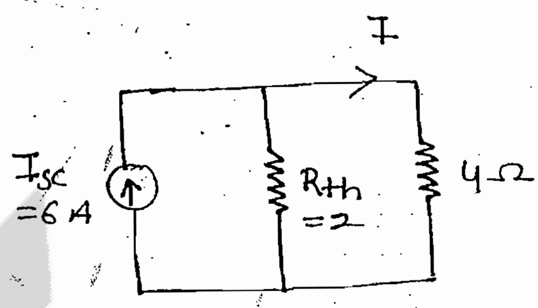


Case 2 (R_{th}) :-

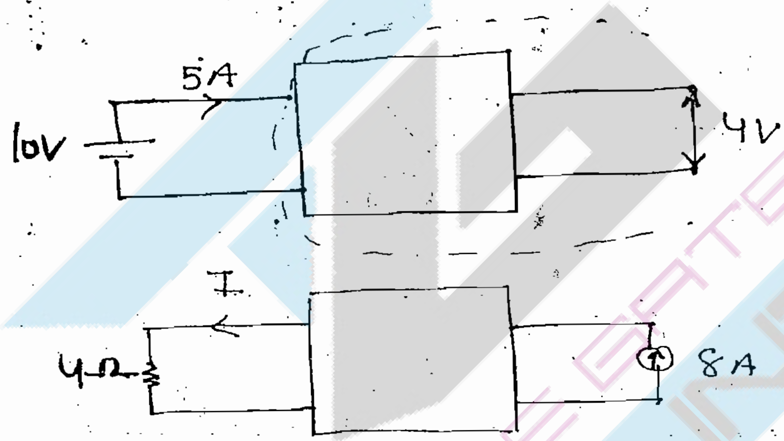
$R_{th} = \frac{10}{5} = 2\Omega$



$I = 6 \times \frac{2}{2+4} = 2A$



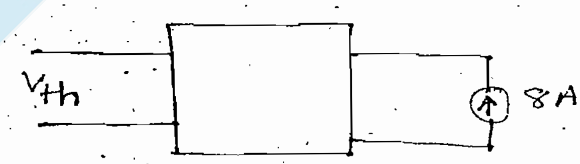
Ques:- When given N/w satisfy the Reciprocity Find the value of I :



Soln:- Case (1) (V_{th}) :-

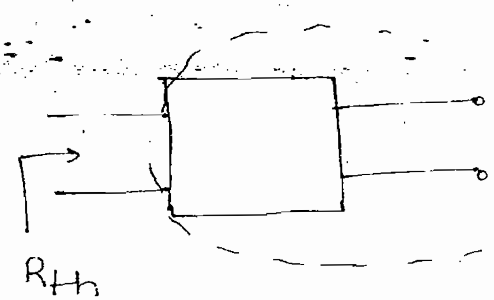
$\frac{\text{Response}}{\text{Excitation}} = \frac{V_{th}}{8} = \frac{4}{5}$

$\Rightarrow V_{th} = \frac{32}{5}$

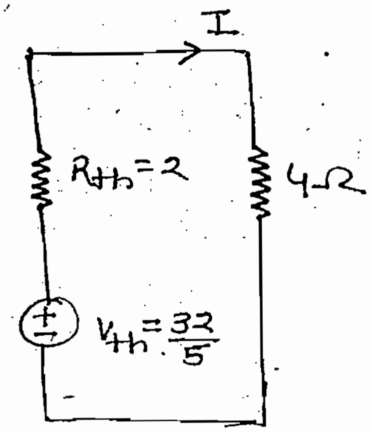


Case (II) (R_{th}) :-

$R_{th} = \frac{10}{5} = 2\Omega$

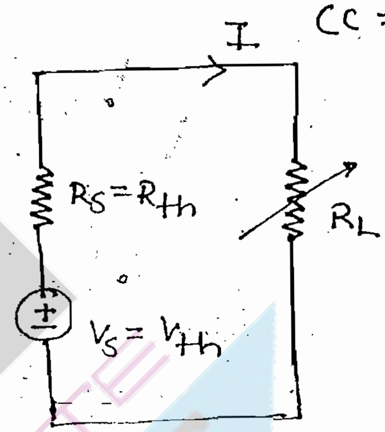


$$I = \frac{32/5}{2+4} = \frac{\cancel{32}^{16}}{5} \times \frac{1}{\cancel{6}_{3}} = \frac{16}{15}A$$



Maximum Power Transfer theorem:— ✓(variable = constant)

(C = V) ✗



$$I = \frac{V_S}{R_S + R_L}$$

$$P_L = I^2 R_L$$

$$\Rightarrow P_L = \left(\frac{V_S}{R_S + R_L}\right)^2 R_L \quad \text{——} (1)$$

Diff eq-(1) w.r.t 'R_L' and equated to zero, we get

$$R_L = R_S$$

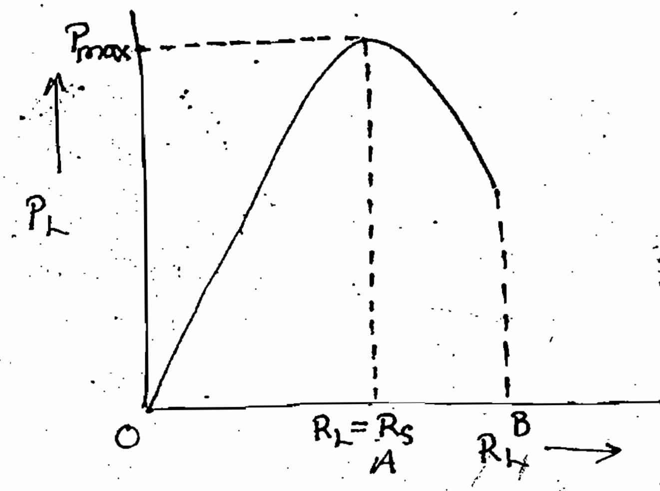
$$P_{max} = \frac{V_S^2}{(R_L + R_L)^2} R_L$$

$$\Rightarrow \boxed{P_{max} = \frac{V_S^2}{4R_L}}$$

$$\eta = \frac{o/p}{i/p} \times 100$$

$$\eta = \frac{I^2 R_L}{I^2 (R_L + R_S)} \times 100 \Rightarrow \eta = \frac{R_L}{R_L + R_L} \times 100$$

$$\Rightarrow \boxed{\eta = 50\%}$$



Case – (I) :–

OA $\Rightarrow$ $R_S > R_L$

$$\boxed{\eta < 50\%}$$

Case – (II) :–

At point A $\Rightarrow$ $\cancel{R_S =}$ $R_L = R_S$

$$\boxed{\eta = 50\%}$$

Case – (III) :–

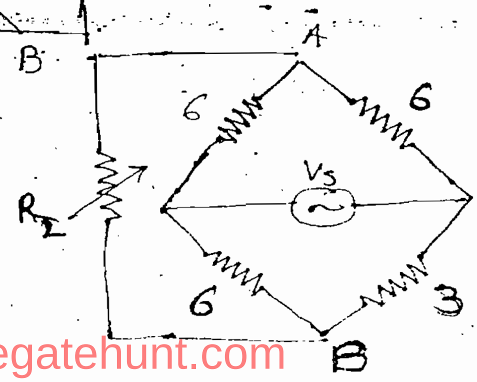
AB $\Rightarrow$ $R_L > R_S$

$$\boxed{\eta > 50\%}$$

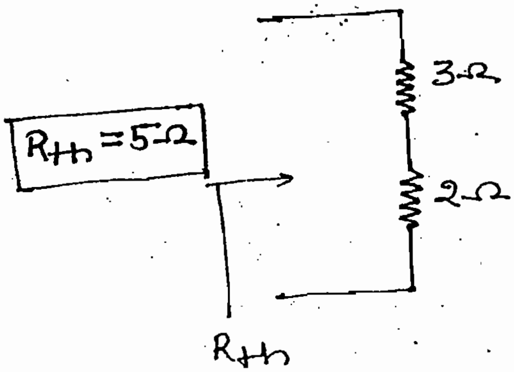
Ques :– Find R_L to obtain max. power from source to load

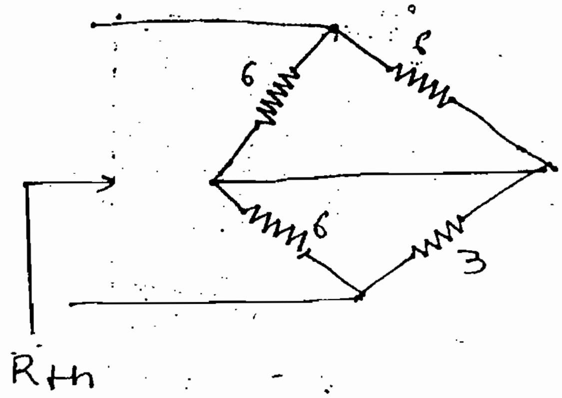
Soln :–

$\Rightarrow$



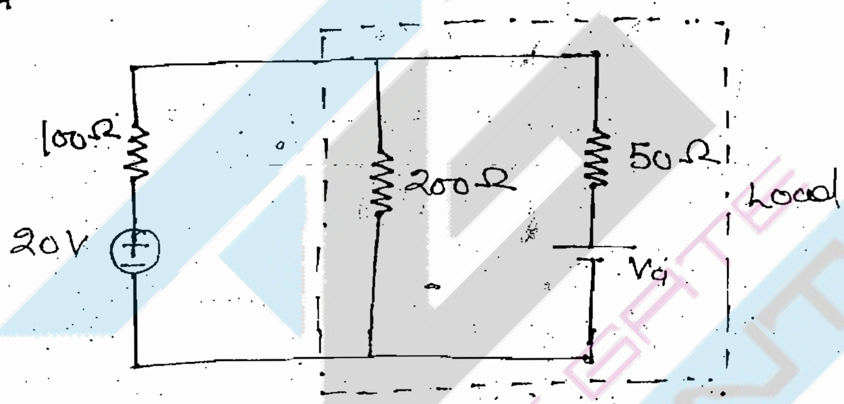
R_{th} :-



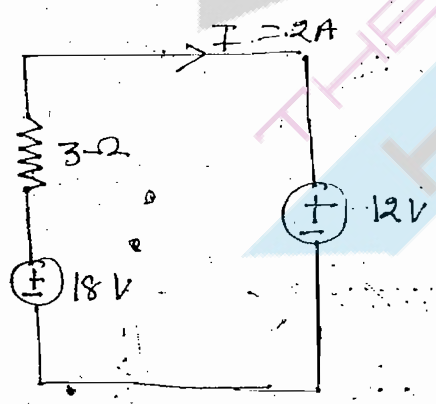


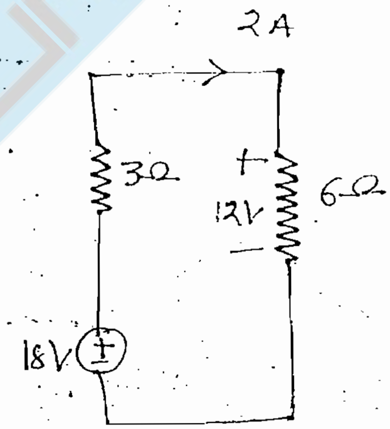
$$\boxed{R_L = R_{th} = 5\Omega}$$

Ques:- Find V_a to obtain max. power from source to load.



Note:-



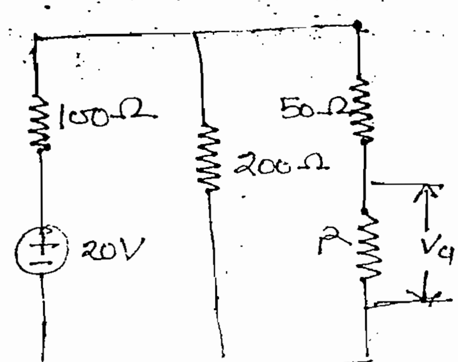


Soln:- $(R_{eq})_L = R_s = 100$

$$(R_{eq})_L = \frac{200\,(50+R)}{200+50+R}$$

$$\Rightarrow \quad 100 = \frac{200\,(50+R)}{250+R}$$

$$\Rightarrow \quad \boxed{R = 150\,\Omega}$$

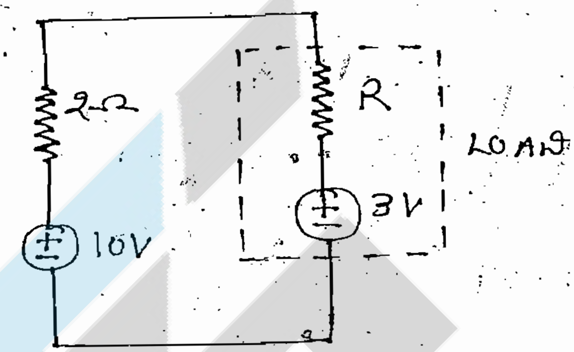


$$I_T = \frac{20}{R_S + (R_{eq.})_L}$$

$$I_T = \frac{20}{100+100} = \frac{1}{10} A$$

$$V_a = \frac{I_T}{2} R = 7.5 \text{ volts}, \text{ Ans}$$

Ques:- Find resistance R to obtain maximum power from source to load



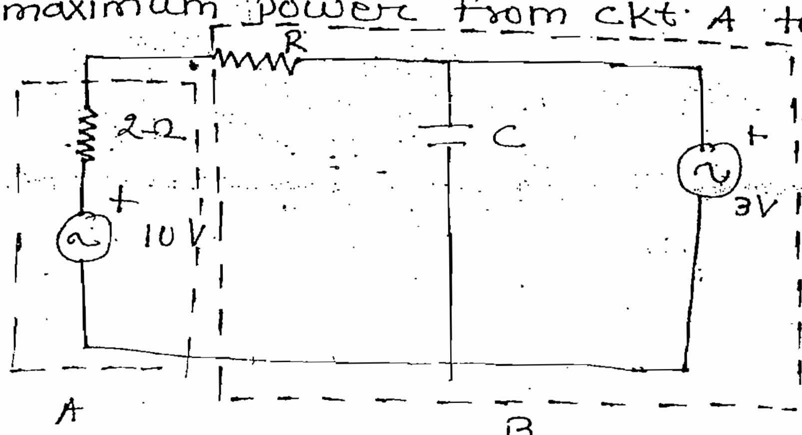
Soln:- $(R_{eq.})_L = R_S = 2\Omega$

$$I_T = \frac{10}{R_S + (R_{eq.})_L}$$

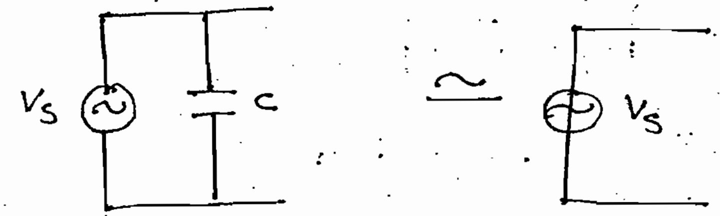
$$I_T = \frac{10}{2+2} = 2.5 A$$

$$I_T = \frac{10-3}{2+R} = 2.5 \Rightarrow \boxed{R = 0.8\Omega} \text{ Ans}$$

Ques:- In the figure shown find resistance R to obtain maximum power from ckt. A to ckt. B

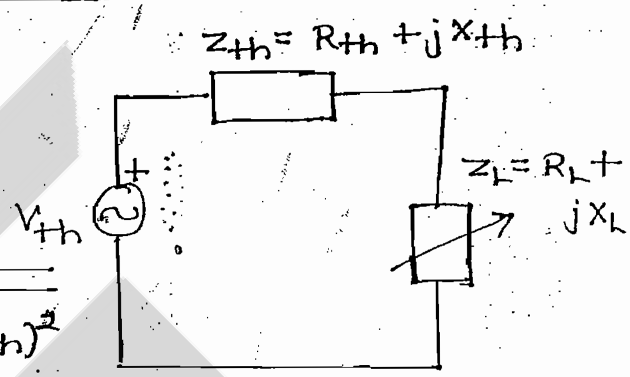


Note:-



Soln:- $R = 0.8\,\Omega$

Maximum Power Transfer theorem (For AC):-



$$i = \frac{V_{th}}{(R_L+R_{th}) + j(X_L+X_{th})}$$

$$i = \frac{V_{th}}{\sqrt{(R_L+R_{th})^2 + (X_L+X_{th})^2}}$$

$$P_L = i^2 R_L$$

$$P_L = \frac{V_{th}^2 R_L}{(R_L+R_{th})^2 + (X_L+X_{th})^2} \quad \text{---(1)}$$

→ Max Power transfer is applicable only for active power

Case (1):-

Both R_L & X_L are variable

(i) Diff eq-(1) w.r.t R_L and equated to zero

(ii) Diff eq-(ii) w.r.t X_L and equated to zero

we get

$$\boxed{\begin{aligned} R_L + jX_L &= R_{th} - jX_{th} \\ Z_L &= Z_{th}^* \end{aligned}}$$

Putting these values in eq-(1), we get

$$\boxed{\begin{aligned} P_{max} &= \frac{V_{th}^2}{4R_L} \\ \eta &= 50\% \end{aligned}}$$

Case (II) :—

Only R_L is variable $(X_L = \text{constant})$

→ Diff. eq-(I) w.r.t R_L and equated to zero

$$R_L = |Z_{th} + jX_L| = |R_{th} + j(X_{th} + X_L)|$$

$$R_L = \sqrt{R_{th}^2 + (X_L + X_{th})^2}$$

$$\eta = \frac{R_L}{R_L + R_{th}} \times 100$$

$$\boxed{\eta \geq 50\%}$$

Case (III) :—

Load impedance is only resistive $(X_L = 0)$

$$P_L = \frac{V_{th}^2 R_L}{(R_L + R_{th})^2 + X_{th}^2} \quad \text{—— (II)}$$

Diff. eq-(II) w.r.t R_L and equated to zero, we ge

$$R_L = \sqrt{R_{th}^2 + X_{th}^2}$$

$$R_L = |Z_{th}| \qquad \therefore R_L > R_{th}$$

$$\boxed{\eta > 50}$$

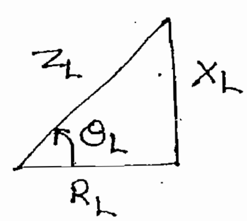
Case-(IV) :— Both R_L & X_L → variable But load impedance angle → constant

$$Z_L = R_L + jX_L$$

load impedance angle $= \theta_L = \tan^{-1}\left(\frac{X_L}{R_L}\right) = \text{constant}$

$$R_L = Z_L \cos\theta_L$$

$$X_L = Z_L \sin\theta_L$$



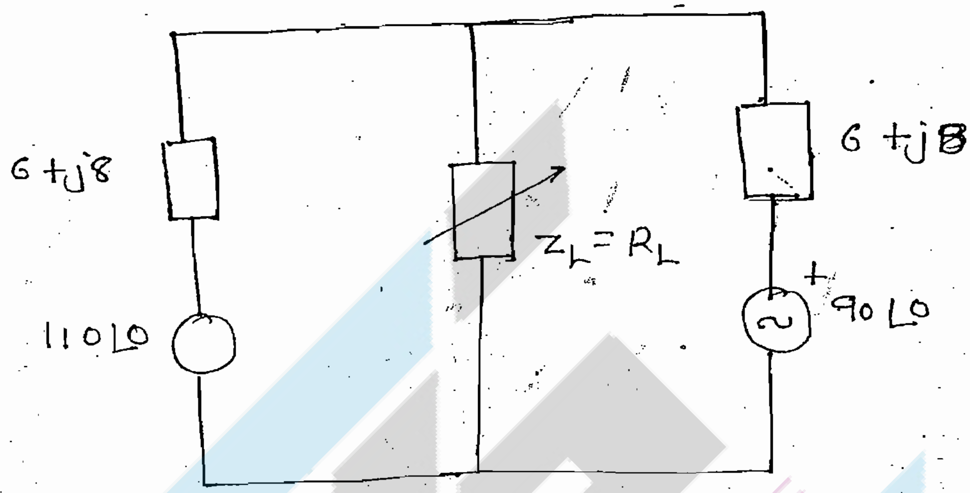
4A

$$P_L = \frac{V_{th}^2 Z_L \cos\theta_L}{(Z_L\cos\theta_L + R_{th})^2 + (Z_L\sin\theta_L + X_{th})^2} \quad \text{—— (iii)}$$

Diff. eq-(iii) w.r.t Z_L and equated to zero, we get

$$\boxed{Z_L = Z_{th}}$$

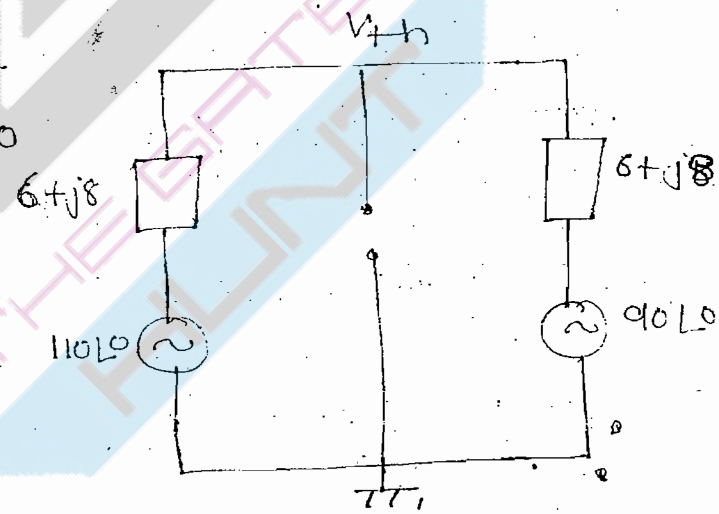
Ques:- Find max. power dissipation in load impedance



Soln:- Case-(i) :- (V_{th})

$$\frac{V_{th} - 110\angle 0^\circ}{6+j8} + \frac{V_{th} - 90\angle 0^\circ}{6+j8} = 0$$

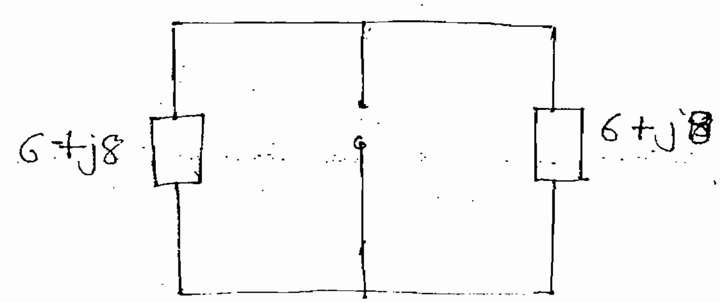
$$V_{th} = 100\angle 0^\circ$$



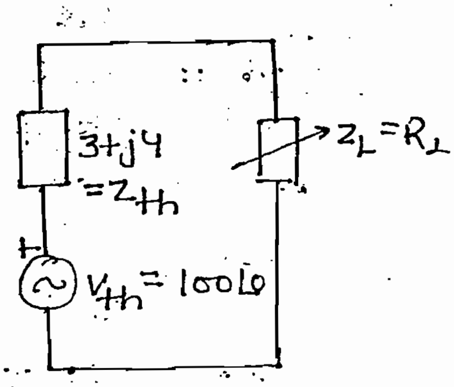
Case-(ii) (Z_{th}):-

$$Z_{th} = \frac{6+j8}{2}$$

$$Z_{th} = 3+j4$$



$$R_L = \sqrt{R_{th}^2 + X_{th}^2} = \sqrt{3^2+4^2}$$
$$= 5$$

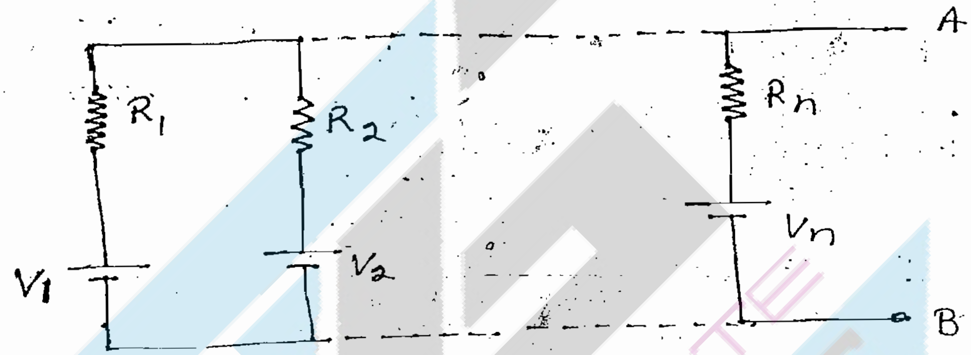


$$i = \frac{100\angle 0}{3+j4+5} = \frac{100}{8+j4}$$

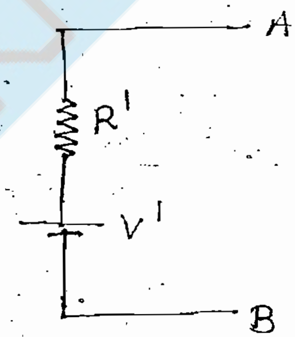
$$= \frac{100}{\sqrt{8^2+4^2}}$$

$$P_L = i^2 R_L \Rightarrow \boxed{P_L = 625\,W}$$

Millman's theorem :-



$$R' = R_{th}$$



$$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

$$\boxed{\begin{aligned} R' &= \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}} \\ R' &= \frac{1}{G_1 + G_2 + \cdots + G_n} \end{aligned}}$$

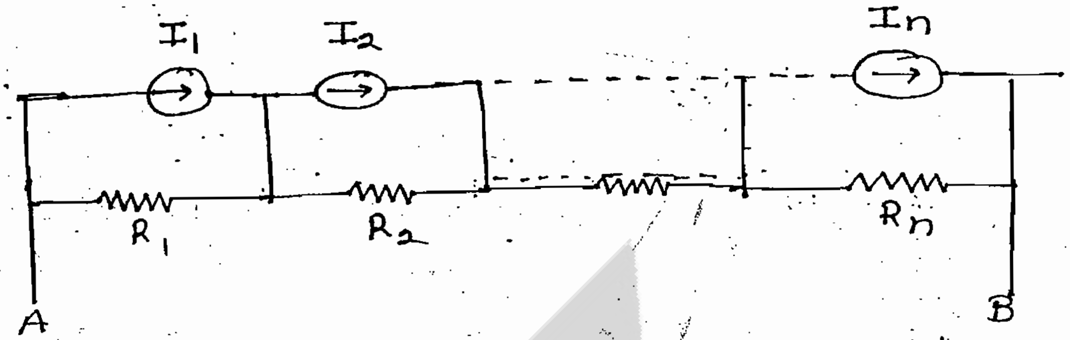
$$V_{th} = I_{sc} R_{th}$$

$$V' = I' R'$$

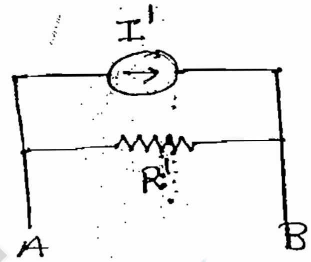
49

$$V' = \frac{V_1/R_1 + V_2/R_2 + \cdots + V_n/R_n}{1/R_1 + 1/R_2 + \cdots + 1/R_n}$$

$$\boxed{V' = \frac{V_1 G_1 + V_2 G_2 + \cdots + V_n G_n}{G_1 + G_2 + \cdots + G_n}}$$



$R' = R_{th}$



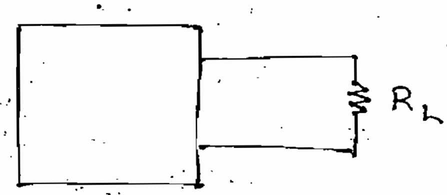
$$\boxed{R' = R_1 + R_2 + \cdots + R_{th}}$$

$$I_{sc} = \frac{V_{oc}}{R_{th}}$$

$$I' = \frac{V'}{R'}$$

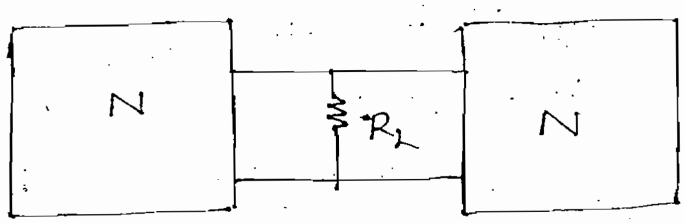
$$\boxed{I' = \frac{I_1R_1 + I_2R_2 + \cdots + I_nR_n}{R_1 + R_2 + \cdots + R_n}}$$

Ques:- When complex N/w of N is connected to load resistor power dissipation in the load resistor is P Watts. When two identical complex N/w of N are connected to same load resistor. Find power dissipation in the load resistor

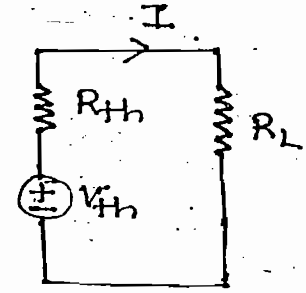


(a) P (b) 2P

(c) 4P (d) P to 4P



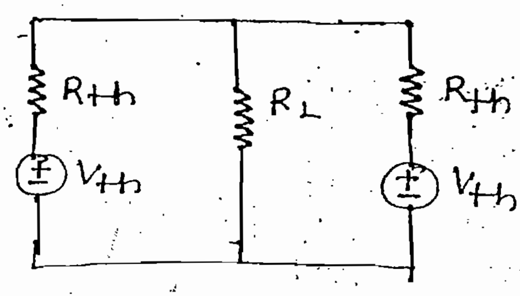
Soln:-



$$I = \frac{V_{th}}{R_L + R_{th}}$$

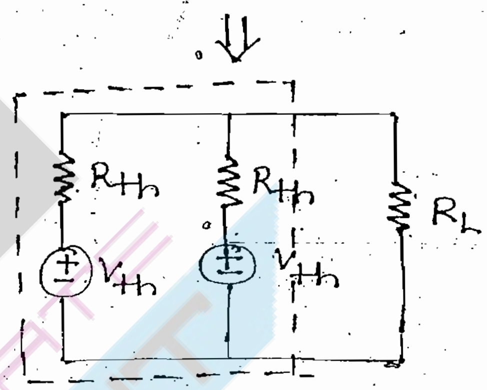
$$P = I^2 R_L$$

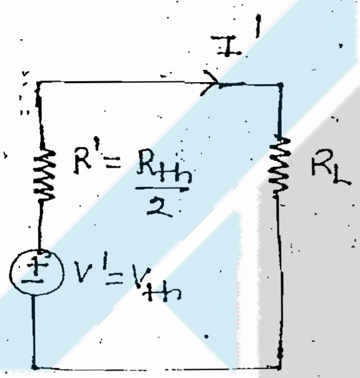
$$P = \left(\frac{V_{th}}{R_L + R_{th}}\right)^2 R_L \quad \text{—— (1)}$$



$$V' = \frac{\frac{V_{th}}{R_{th}} + \frac{V_{th}}{R_{th}}}{\frac{1}{R_{th}} + \frac{1}{R_{th}}} = V_{th}$$

$$R' = \frac{1}{1/R_{th} + 1/R_{th}} = \frac{R_{th}}{2}$$



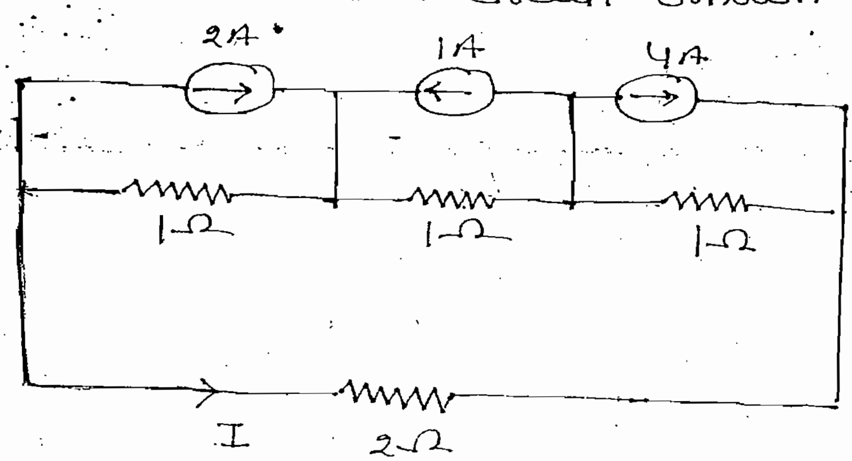


$$I' = \frac{V_{th}}{R_L + \frac{R_{th}}{2}} = \frac{2V_{th}}{2R_L + R_{th}}$$

$$P' = I'^2 R_L = \left(\frac{2V_{th}}{2R_L + R_{th}}\right)^2 R_L \quad \text{—— (11)}$$

Comparing eq-(1) & (11) we get option-d

Ques:- Find I of the circuit shown

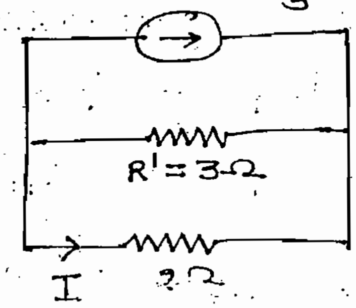


Soln:-

$$I' = \frac{(2\times1) - (1\times1) + (4\times1)}{1+1+1} = \frac{5}{3}$$

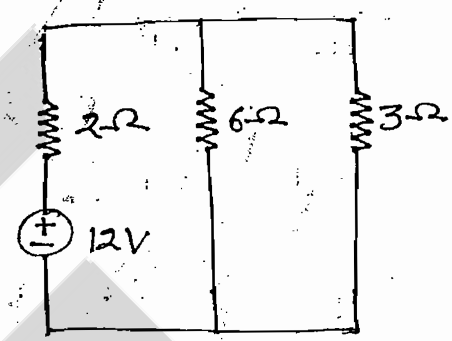
$$R' = 1+1+1 = 3\,\Omega$$

$$I = -\frac{5}{3}\,\frac{3}{3+2} = -1\,A$$



Tellegen's theorem:-

$$\sum_{k=1}^{n} V_k i_k = 0$$



Tellegen's theorem states that algebraic sum of the powers in any circuit at any instant is equal to zero (linear, non-linear, unidirectional, bidirectional, time variant and time invariant elements)

$$R_{eq} = 2 + \frac{6\times3}{6+3} = 4$$

$$I_T = \frac{12}{4} = 3A$$

$$I_6 = 3\,\frac{3}{3+6} = 1A \qquad I_3 = 3-1 = 2A$$

$$V_2 = I_T \times 2 = 6$$

$$V_6 = V_3 = 2\times3 = 6$$

$$V_2 I_T + V_6 I_6 + V_3 I_3 - V_s I_T$$

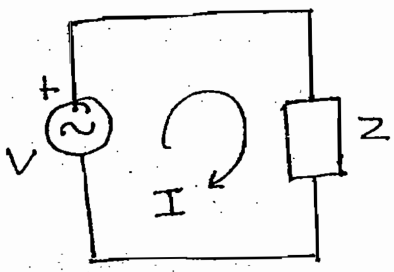
$$18 + 6 + 12 - 12\times3 = 0$$

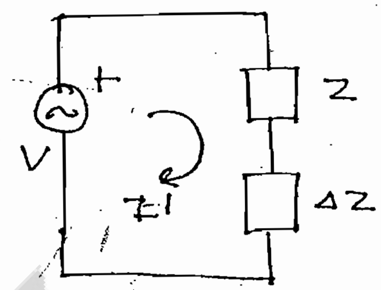
Note:-

→ Tellegen's theorem is verified by using KCL and KVL equations

→ Tellegen's theorem works based on the principle of law of conservation of energy.

Compensation theorem:-





$$I = \frac{V}{Z}$$

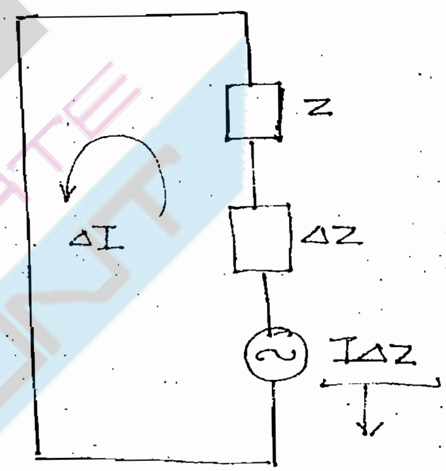
$$I' = \frac{V}{Z+\Delta Z}$$

$$\Delta I = |I - I'|$$

(Clockwise)

Modified Circuit:-

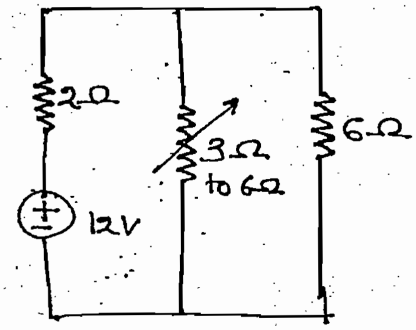
$$\boxed{\Delta I = \frac{-I\Delta Z}{Z+\Delta Z}}$$



Compensation emf
or
opposing emf

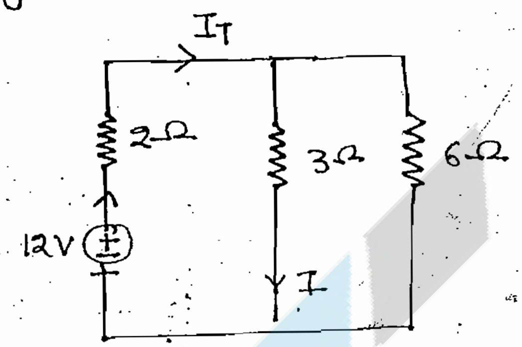
Lecture - 8

Ques:- Find change in current in the 2Ω and 6Ω resistor when resistance in the variable branch is changed from 3 to 6Ω



Soln:- Step-(I):-

Find original current circulating in the variable branch



$$R_{eq} = 2 + \frac{6 \times 3}{6+3} = 4$$

$$I_T = \frac{12}{4} = 3A$$

$$I = 3\frac{6}{6+3} = 2A$$

Step-(II):-

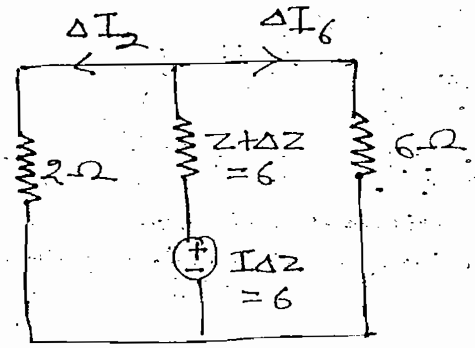
Find compensation emf

$$I\Delta Z = 2(6-3) = 6V$$

Step-(III):-

Develop modified ckt

→ While developing modified circuit deactivate all the original sources and connect the compensation emf in series to variable branch



$$R_{eq} = 6 + \frac{6 \times 2}{6+2}$$

$$I_T = \frac{6}{R_{eq}}$$

$$\Delta I_2 = I_T \frac{6}{6+2} = 0.6A$$

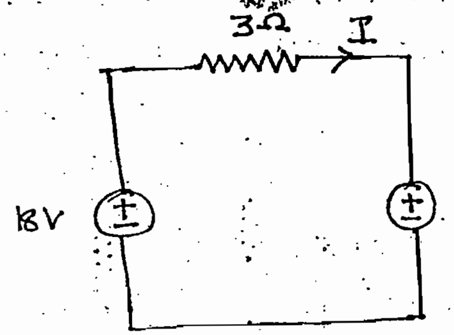
$$\Delta I_6 = I_T - \Delta I_2 = 0.2A$$

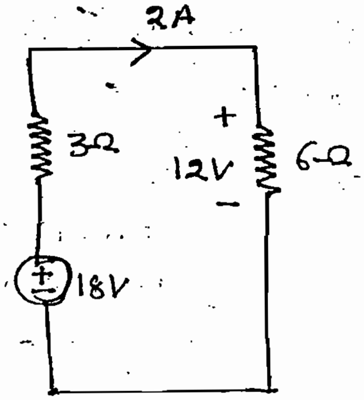
Note:-

In the bridge circuit by using compensation theorem condition for null deflection in the galvanometer is obtain

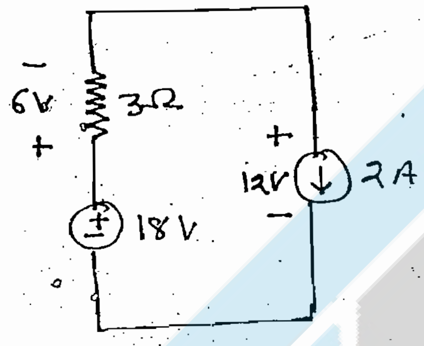
Application of comp. theorem

Substitution theorem :-

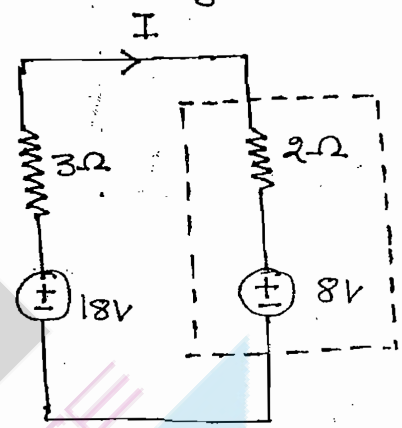




$$I = \frac{18-12}{3} = 2$$



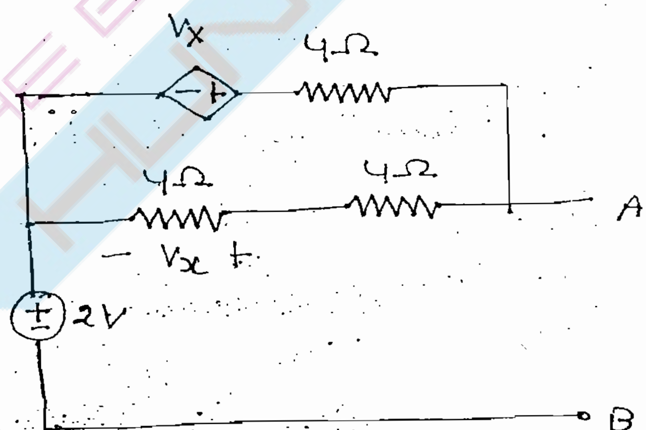
$6\Omega \rightarrow 2\Omega + 4\Omega$



$$I = \frac{18-8}{3+2} = 2A$$

→ All circuits are equivalent

Ques:- Find R_{th} w.r.t A and B



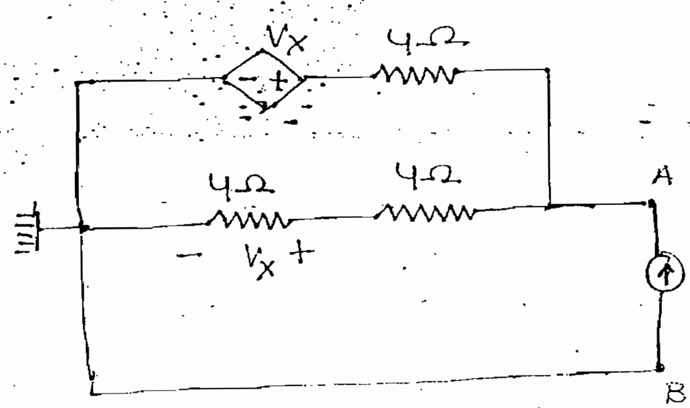
Soln:-

$$\frac{V_A}{8} + \frac{V_A - V_X}{4} = 1$$

$$V_X = \frac{V_A}{2}$$

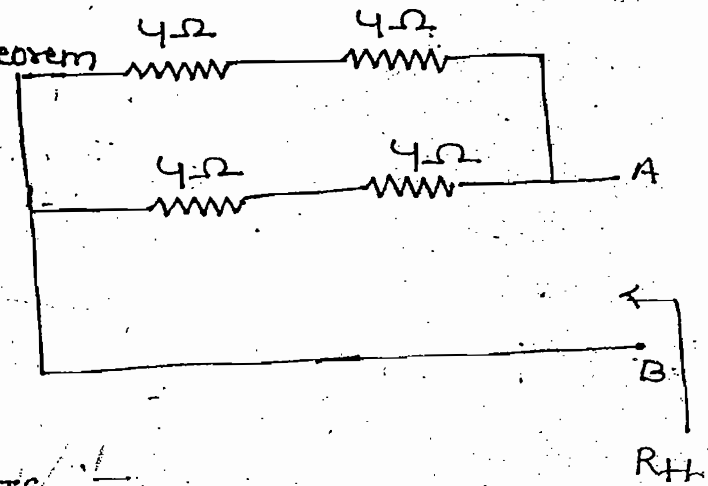
$$V_A = 4V$$

$$R_{th} = \frac{V_A}{I_S} = \frac{4}{1} = 4\Omega$$



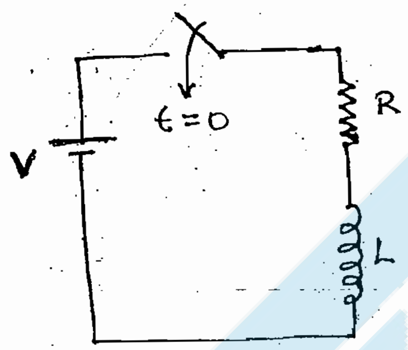
Alternate Way :-

By using substitution theorem



$$R_{th} = \frac{8\times 8}{8+8} = 4$$

TRANSIENTS :-

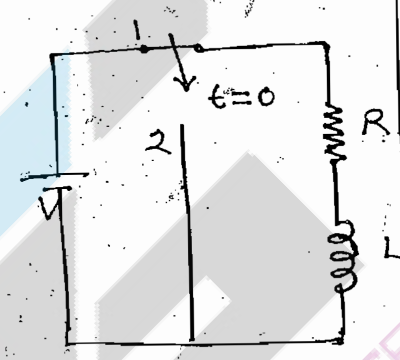


$t=0^-$, $i=0$

$t=0^+$, $i=0$
↓
(O.C)

$t=\infty$, $V_L = L\frac{di}{dt}$
$= 0$
↓
SC

$i = \frac{V}{R}$

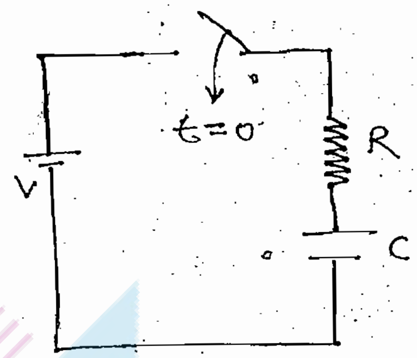


$t=0^-$, $i = I_0$

$t=0^+$, $i = I_0$

Inductor behaves as current source

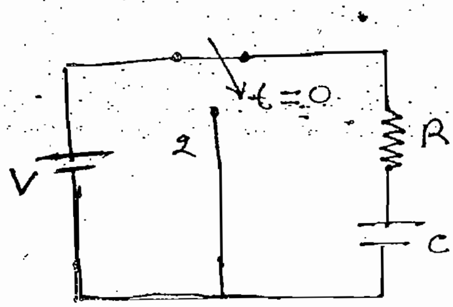
$t=\infty$, $i=0$



$t=0^-$, $V_c = 0$

$t=0^+$, $V_c = 0$
↓
(S.C)

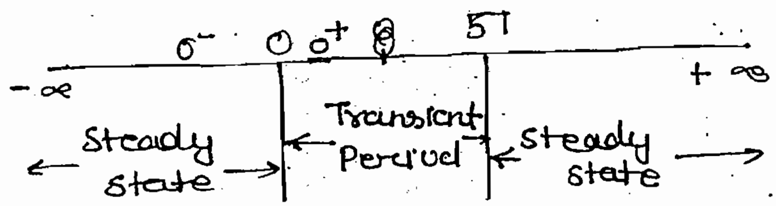
$t=\infty$, $V_c = V$
$i = 0$
(O.C)



$t=0^-$ $V_c = V_0 (V)$

$t=0^+$ $V_c = V_0$
↓
Acts as voltage source

$t=\infty$ $V_c = 0$



→ Transients are present in the circuit when circuit is subjected to any changes either by changing source magnitude or by changing circuit element provided circuit consist of any energy storage element

→ When the ckt is having only resistive element then no transients are present in the N/w since
(I) Resistor allow sudden change of current and voltage
(II) Resistor does not store any energy.

→ Transients are present in the N/w when the N/w is having any energy storage elements since
(I) Inductor does not allow sudden change of current and it stores energy in the form of magnetic field
(II) Capacitor does not allow sudden change of voltage and it stores energy in the form of electric field.

→ Due to energy storage property inductor and capacitor are known as dynamic elements (Memory elements)

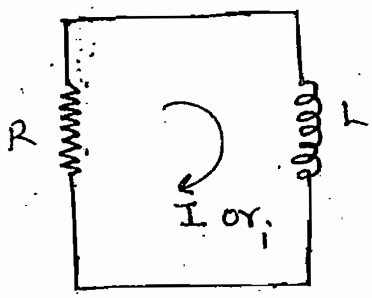
→ $t = 0^-$ indicates immediately before operate the switch

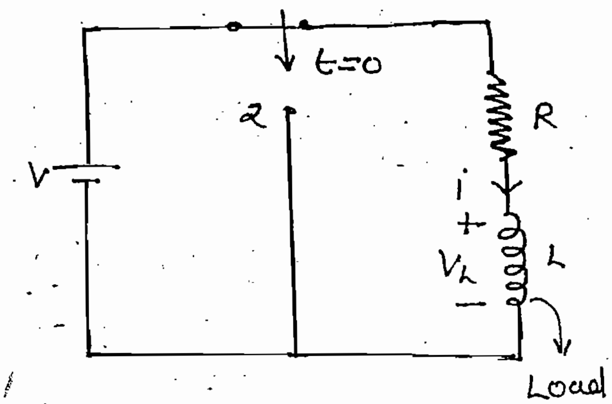
→ $t = 0^+$ indicates immediately after operating the switch

→ $t = \infty$ indicates steady state condition after operating the switch.

Source Free RL Circuit :-

$t > 0$





$$iR + L\frac{di}{dt} = 0$$

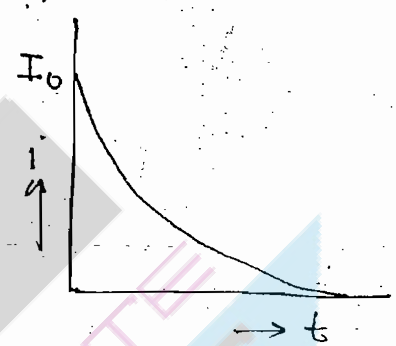
$t = 0^-$, $i = I_0$

$t = 0^+$, $i = I_0$

$$iR = -L\frac{di}{dt}$$

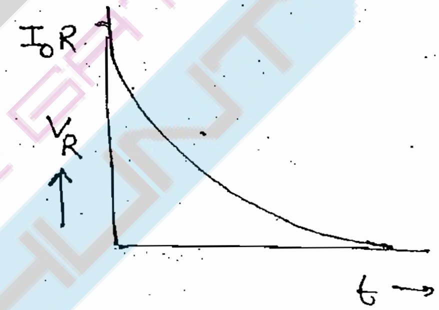
$$\int_0^t -\frac{R}{L}\,dt = \int_{I_0}^{i(t)} \frac{di}{i}$$

$$\Rightarrow \quad i(t) = I_0 e^{-\frac{Rt}{L}} \longrightarrow$$



$$V_R = iR$$

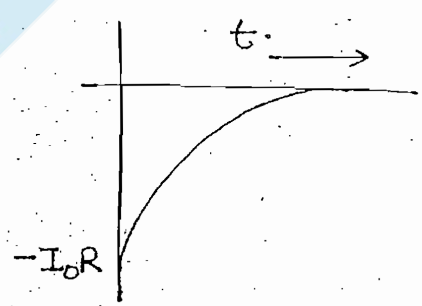
$$\Rightarrow \quad V_R = I_0 R e^{-\frac{Rt}{L}} \longrightarrow$$



$$V_L = L\frac{di}{dt}$$

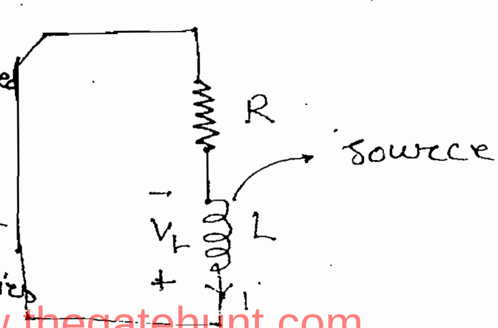
$$V_L = L\frac{d}{dt}\left(I_0 e^{-\frac{Rt}{L}}\right)$$

$$\Rightarrow \quad V_L = -I_0 R e^{-\frac{Rt}{L}} \longrightarrow$$

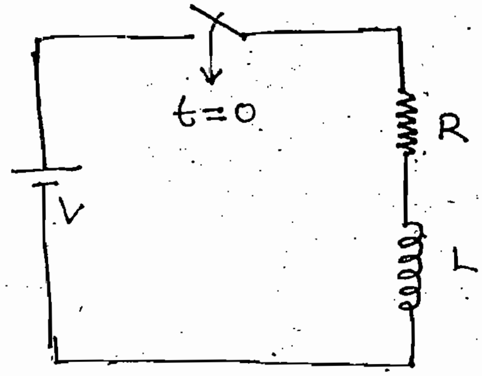


$t \to 0$ $\qquad$ $t > 0$

Note :- When switch is transferred from position 1 to position 2 current direction of the inductor remains same but voltage across inductor polarities are reversed.



RL Circuit with Source :-



By KVL

$$V = iR + L\frac{di}{dt}$$

Divide whole equation by L

$$\frac{V}{L} = \frac{iR}{L} + \frac{di}{dt}$$

$$\Rightarrow \boxed{\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}}$$

$i(t) = C.F + P.I$

C.F → Transient response or Source free response or Natural Response

$$\frac{di}{dt} + \frac{R}{L}i = 0 \Rightarrow i(t) = Ae^{-\frac{Rt}{L}}$$

P.I → Steady state response or final value or forced Response

$$i(t) = C.F + P.I$$

$$i(t) = Ae^{-\frac{Rt}{L}} + \frac{V}{R}$$

$t = 0^-$, $i = 0$

$t = 0^+$, $i = 0$

$$\Rightarrow 0 = A + \frac{V}{R} \Rightarrow A = 0 - \frac{V}{R}$$

$$\Rightarrow \boxed{A = i(0^+) - i(\infty)}$$

→ Applicable for RL or RC circuit with source.

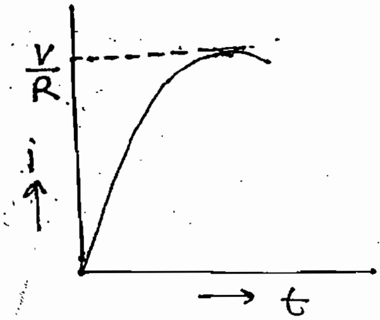
$$i(t) = C.F + P.I$$

$$i(t) = -\frac{V}{R} e^{-Rt/L} + \frac{V}{R}$$

$$\Rightarrow \boxed{i(t) = [i(0^+) - i(\infty)] e^{-Rt/L} + i(\infty)}$$

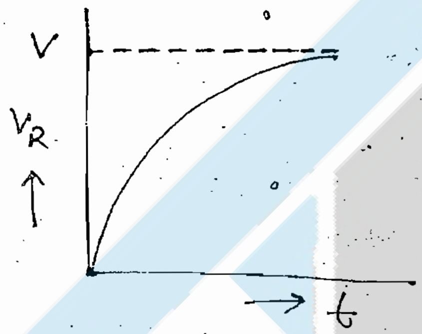
→ Valid for RL and RC circuit

$$i(t) = \frac{V}{R}(1 - e^{-Rt/L}) \rightarrow$$



$$V_R = iR$$

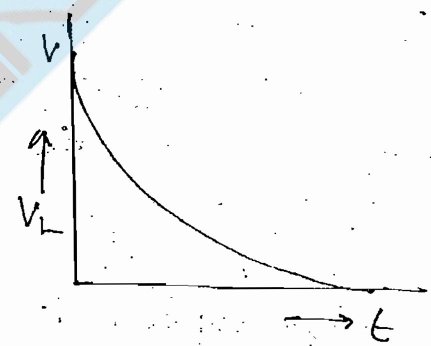
$$\Rightarrow V_R = V(1 - e^{-Rt/L})$$



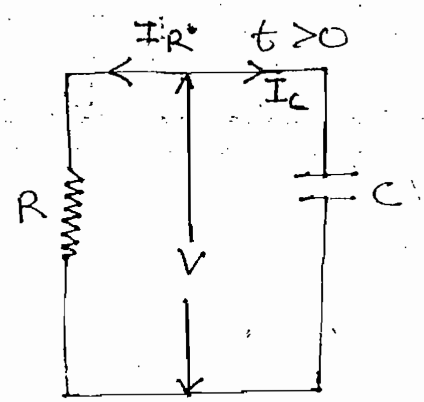
$$V_L = L\frac{di}{dt}$$

$$V_L = L\frac{d}{dt}\left[\frac{V}{R}(1 - e^{-\frac{Rt}{L}})\right]$$

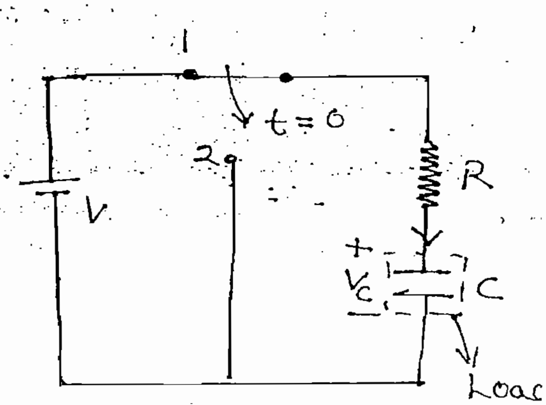
$$V_L = V_0 e^{-Rt/L}$$



<u>Source Free RC Circuit</u> :-



←



$t = 0^-$, $V_C = V_0$

$t = 0^+$, $V_C = V_0$

V = Virtual voltage source

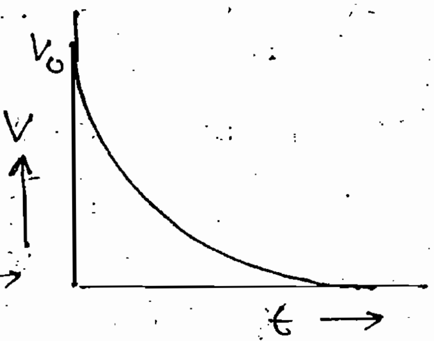
$$I_R + I_c = 0$$

$$\Rightarrow \frac{V}{R} + C\frac{dV}{dt} = 0$$

$$\Rightarrow \frac{V}{R} = -C\frac{dV}{dt}$$

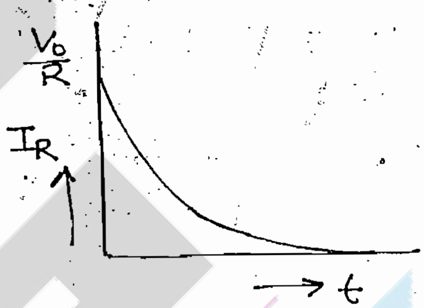
$$\Rightarrow \int_0^t -\frac{1}{RC}\,dt = \int_{V_0}^{V(t)} \frac{dV}{V}$$

$$\Rightarrow V(t) = V_0 e^{-t/RC}$$



$$I_R = \frac{V}{R}$$

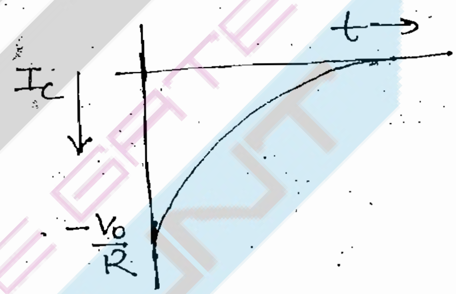
$$\Rightarrow I_R = \frac{V_0}{R} e^{-t/RC}$$



$$I_c = C\frac{dV}{dt}$$

$$I_c = C\frac{d}{dt}\left(V_0 e^{-t/RC}\right)$$

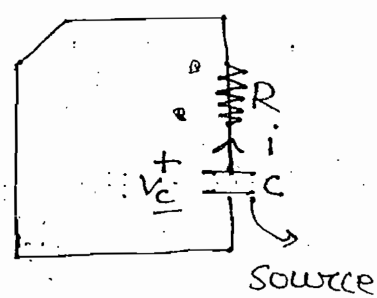
$$\Rightarrow I_c = -\frac{V_0}{R} e^{-t/RC}$$



$t > 0$

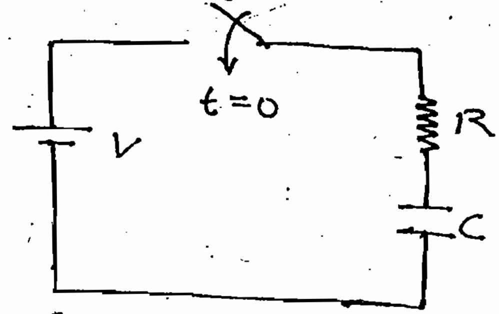
Note:-

When switch is transferred from position 1 to position 2 voltage across the capacitor will remain same but current direction of the capacitor is reversed.



$I_R = -I_c$

RC circuit with source



By KVL

$$V = iR + \frac{1}{C}\int i\,dt$$

Diff. w.r.t t

$$0 = R\frac{di}{dt} + \frac{i}{C}$$

Divide both sides by R we get

$$\frac{di}{dt} + \frac{i}{RC} = 0$$

$$i(t) = CF + PI$$

CF → Transient Response

$$\frac{di}{dt} + \frac{i}{R} = 0$$

$$\boxed{i(t) = A e^{-t/RC}}$$

PI → steady state response

$C \to$ o.c

$i = 0$

$$i(t) = CF + PI$$

$$i(t) = A e^{-t/RC} + 0$$

$$\boxed{A = i(0^+) - i(\infty)}$$

$t = 0^-$, $V_c = 0$

$t = 0^+$, $V_c = 0$

By KVL

$t = 0^+$

$$V = V_R + V_c$$
$$= iR + 0$$

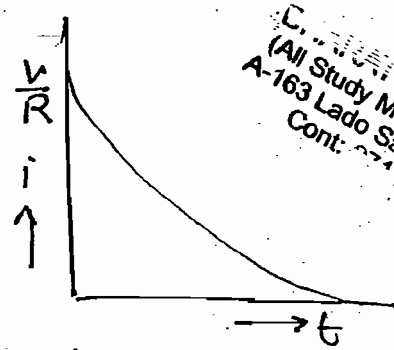
at $t \to t(0^+)$ $i = \frac{V}{R}$

$$i(t) = CF + PI$$

$$A = i(0^+) - i(\infty)$$

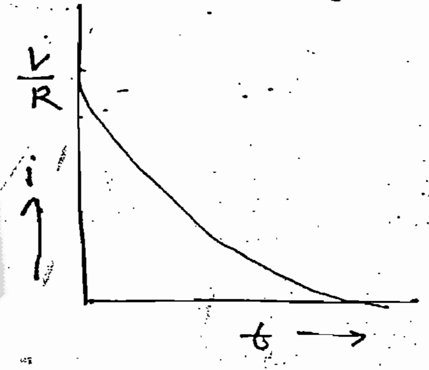
$$A = \frac{V}{R} - 0$$

$$\boxed{i(t) = \frac{V}{R}\, e^{-t/RC}}$$



$$V_R = iR$$

$$V_R = V e^{-t/RC}$$



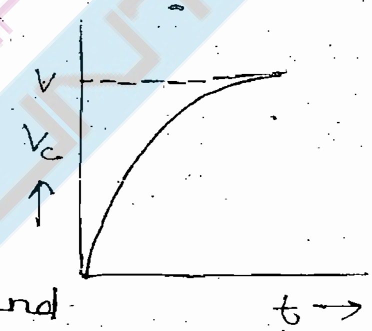
$$V_c = \frac{1}{C}\int_0^t i\,dt$$

$$V_c = \frac{1}{C}\int_0^t \frac{V}{R}\, e^{-t/RC}\, dt$$

$$V_c = -V e^{-t/RC} + V$$

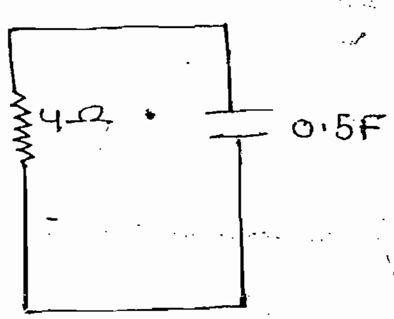
$$\boxed{V_C(t) = [V_c(0^+) - V_c(\infty)]\, e^{-t/RC} + V_c(\infty)}$$

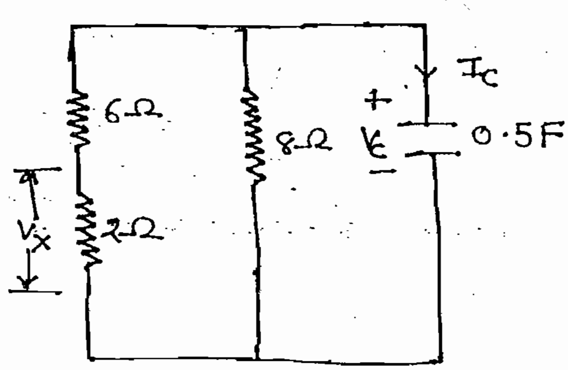
$$V_c(t) = V(1 - e^{-t/RC})$$



Ques:- Find response of V_c, I_c and V_x when initial voltage of the capacitor is 3V

Soln:-





$$V_c = V_0\, e^{-t/RC}$$

$$V_c = 3\, e^{-t/2}$$

$$V_x = V_c \frac{2}{2+6} \Rightarrow V_x = 3e^{-t/2}\, \frac{1}{4}$$

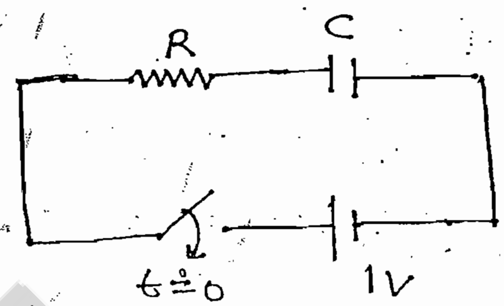
$$\Rightarrow V_x = \frac{3}{4} e^{-t/2}$$

$$I_c = C \frac{dV_c}{dt}$$

$$I_c = \frac{1}{2} \frac{d(3e^{-t/2})}{dt} = 3\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) e^{-t/2}$$

$$\Rightarrow \boxed{I_c = -\frac{3}{4} e^{-t/2}} \quad \underline{\text{Ans}}$$

Ques:- Find rate of rise of Voltage across the capacitor at $t = 0^+$



(a) RC (b) $\frac{1}{RC}$ (c) 0 (d) 2RC

Soln:-

$$V_c = -Ve^{-t/RC} + V$$

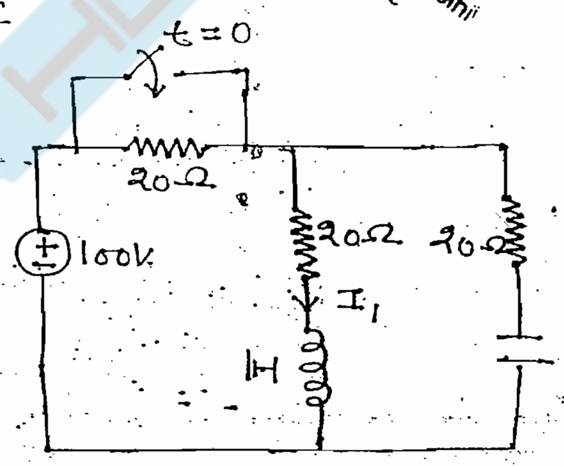
$$\frac{dV_c}{dt} = (-V)\left(-\frac{1}{RC}\right) e^{-t/RC} + 0$$

$$\frac{dV_c}{dt} = \frac{V}{RC} e^{-t/RC}$$

$$t = 0^+$$

$$\frac{dV_c}{dt}(0^+) = \frac{V}{RC} = \frac{1}{RC}$$

Ques:- Find $\frac{di_1}{dt}$ at $t = 0^+$

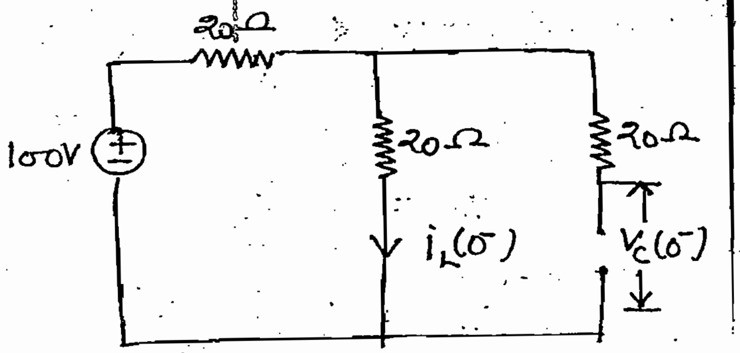


Soln:- Step-(1):-

Develop eq. circuit at $t = 0^-$ and find initial current of conductor and initial voltage of capacitor

→ In the above ckt at $t = 0^-$ for DC source inductor behaves as S.C and C behaves as as O.C

$$i_L(0^-) = \frac{100}{20+20} = 2.5A$$



$$V_C(0^-) = 2.5 \times 20$$
$$= 50$$

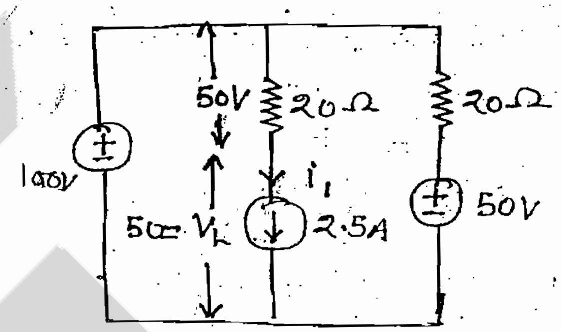
Step-2:—

Develop eq. ckt at $t=0^+$ and indicate initial current of the inductor and initial voltage of the capacitor

$t = 0^+$

$$i_L(0^+) = i_L(0^-) = 2.5A$$

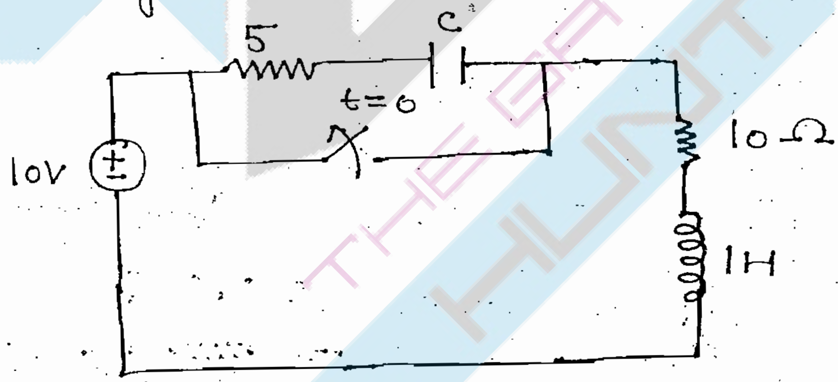
$$V_C(0^+) = V_C(0^-) = 50V$$



$$V_L = L\frac{di}{dt}$$

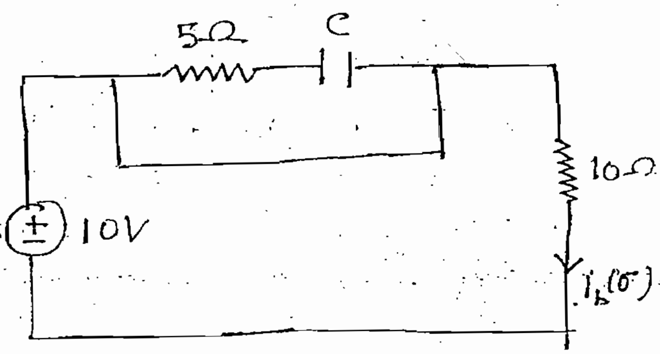
$$\Rightarrow 50 = 1\frac{di}{dt} \Rightarrow \frac{di}{dt} = 50\ A/s \quad \text{Ans}$$

Ques:- Find voltage across the switch at $t=0^+$



Soln:- At $t=0^-$

→ Develop equivalent circuit at $t=0^-$



→ In the above ckt. at $t=0^-$ capacitor is uncharged & for DC source inductor behaves as a S.C

$$i_L(0^-) = \frac{10}{10} = 1A, \quad V_C(0^-) = 0$$

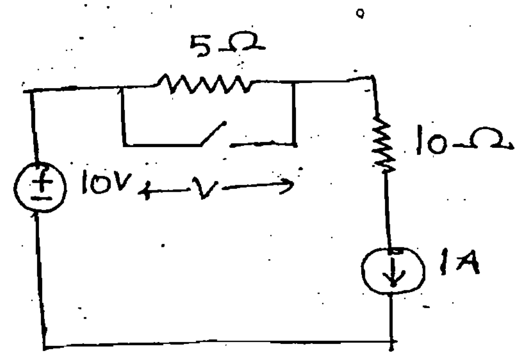
Step-2:-

At $t = 0^+$

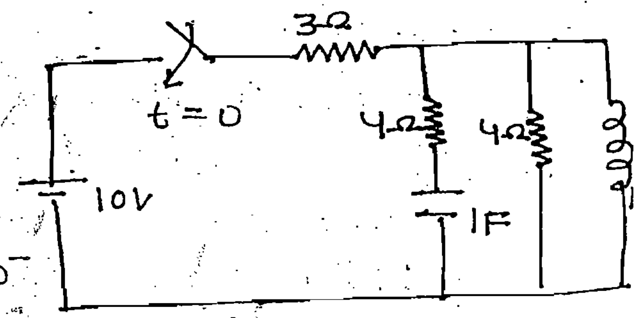
$i_L(0^+) = i_L(0^-) = 1A$

$V_c(0^+) = V_c(0^-) = 0$

$V(0^+) = 5 \times 1 = 5V$



Ques:- Find I in capacitor and voltage across inductor at $t = 0^+$



Soln:- Step (I) :-

Develop eq. ckt. at $t = 0^-$

→ In the above ckt at $t = 0^-$ capacitor and inductor are uncharged elements.

$t = 0^-$, $V_c(0^-) = 0$

$i_L(0^-) = 0$

Step-(II) :-

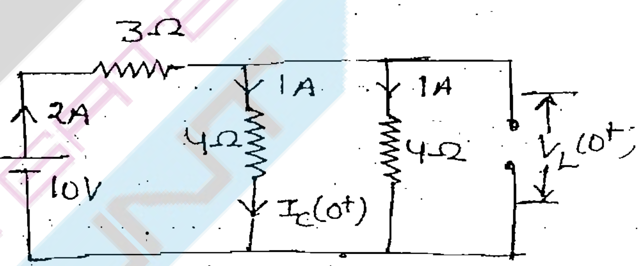
At $t = 0^+$

$i_L(0^+) = i_L(0^-) = 0$

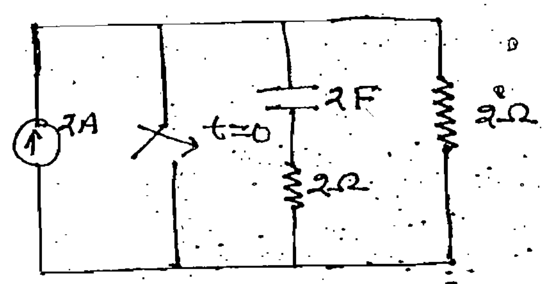
$V_c(0^+) = V_c(0^-) = 0$

$I_c(0^+) = 1A$

$V_L(0^+) = 4 \times 1 = 4V$

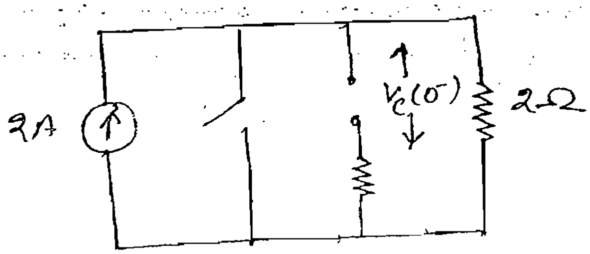


Ques:- Find initial voltage and final voltage of the capacitor



Soln:- Step (I) :-

$t = 0^-$



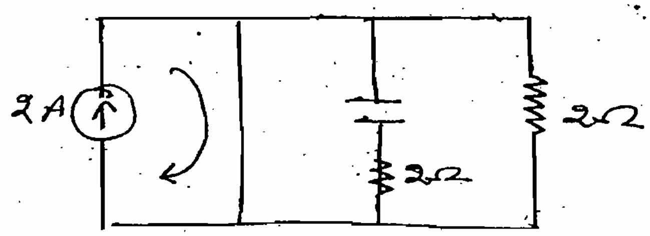
$V_c(0^-) = 2 \times 2 = 4V$

Step-(II):-

At $t = 0^+$.

$V_c(0^+) = V_c(0^-) = 4V$

$V_c(\infty) = 0$



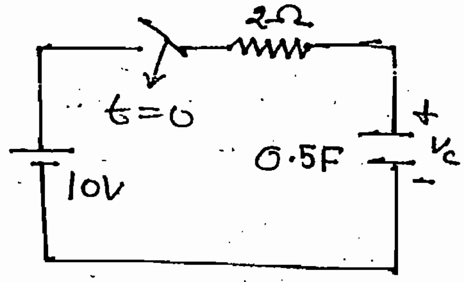
Ques:- Find

(I) Energy of Capacitor at $t=\infty$

(II) current in the capacitor at $t = 1sec$

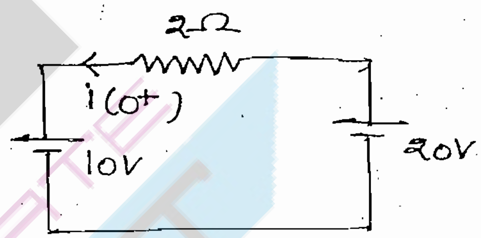
(III) Energy of the resistor from 0 to ∞ interval

When initial charge of the capacitor is 10C

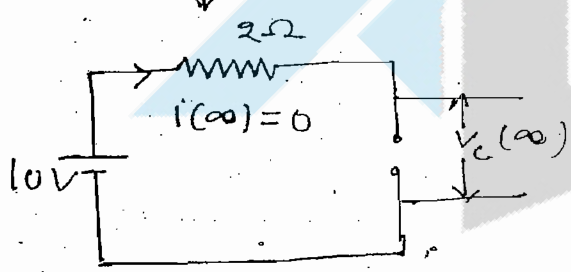


Soln:- At $t = 0^+$ $(\because Q_{ini} = 10C)$

$$V_0 = \frac{Q_0}{C} = \frac{10}{0.5} \Rightarrow V_0 = 20$$



At $t = \infty$



$V_c(\infty) = 10V$

$$W_c(\infty) = \frac{1}{2} C V_c^2(\infty) = \frac{1}{2} \cdot \frac{1}{2} \cdot (10)^2 = 25J$$

(II) $i(t) = [i(0^+) - i(\infty)] e^{-t/RC} + i(\infty)$

$\Rightarrow i(t) = [-5-0] e^{-t/1} + 0$

$i(t) = -5e^{-t}$

At $t = 1sec$

$$i(1s) = -5e^{-1} = -\frac{5}{e}$$

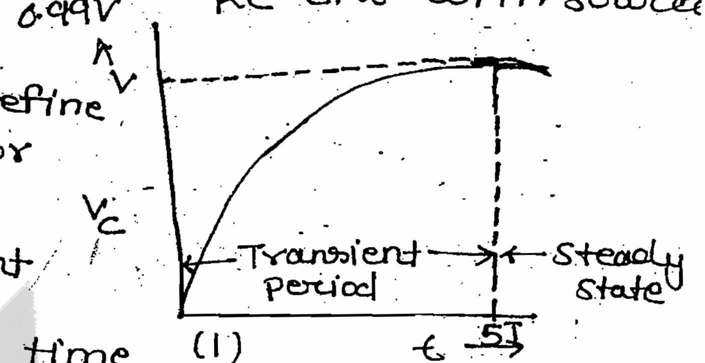
$$= -1.84$$

(iii) $W_R = \int_0^{\infty} P\,dt = \int_0^{\infty} i^2 R\,dt = \int_0^{\infty} (-5e^{-t})^2\, 2\,dt$

$\Rightarrow W_R = 25\,J$, Ans.

Time Constant :-

RC ckt with source

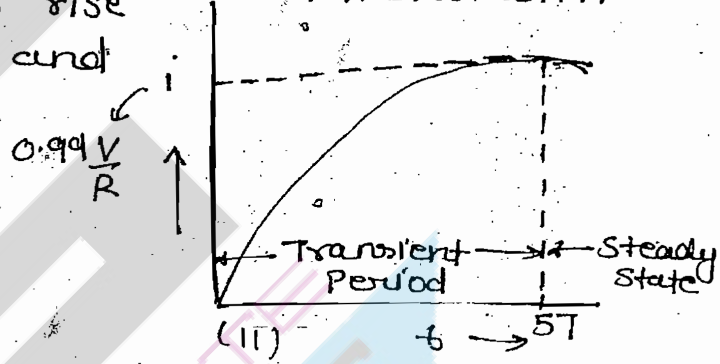


→ Time constant can be define either w.r.t charging or discharging action of energy storage element.

→ Time constant is the time taken for response to rise 63.2% of max value and it is given by

$$\boxed{\begin{aligned} T &= \frac{L}{R} \\ T &= RC \end{aligned}}\ \text{sec.}$$

RL ckt with source



$V_c(t) = V(1 - e^{-t/T})$

(I) $V_c(t) = V(1 - e^{-t/RC})$

(II) $i(t) = \frac{V}{R}(1 - e^{-Rt/L})$

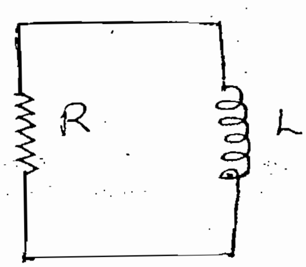
(i) $t = T$ $\quad V_c = V(1 - e^{-1}) = 0.632\,V$

$t = 5T$ $\quad V_c = V(1 - e^{-5}) = 0.99\,V$

$R_1 = 10\,\Omega$ $\quad P_1 = I^2 R_1$ $\quad t = 10\,sec$

$R_2 = 20\,\Omega$ $\quad P_2 = i^2 R_2$ $\quad t = 7\,sec$

$R_3 = 100\,\Omega$ $\quad P_3 = i^2 R_3$ $\quad t = 2\,sec.$



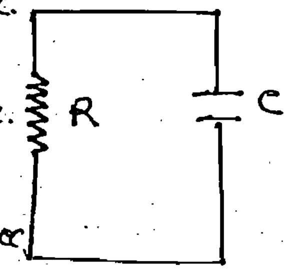
Hence $T = \frac{L}{R}$

PHOTOSTAT
(All Study Material Available
A-163 Lado Sarai, New Delhi
Cont:-

$R_1 = 10\Omega \qquad P_1 = \frac{V^2}{R_1} \qquad t = 10\,sec.$

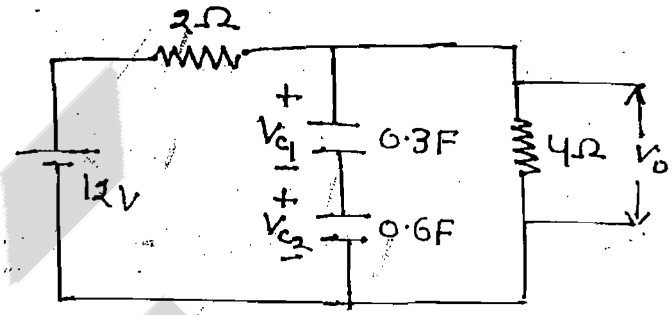
$R_2 = 20\Omega \qquad P_2 = \frac{V^2}{R_2} \qquad t = 15\,sec.$

$R_3 = 100\Omega \qquad P_3 = \frac{V^2}{R_3} \qquad t = 50\,sec$



Hence $\quad T = RC$

Ques:- Find V_0 response for $t > 0$



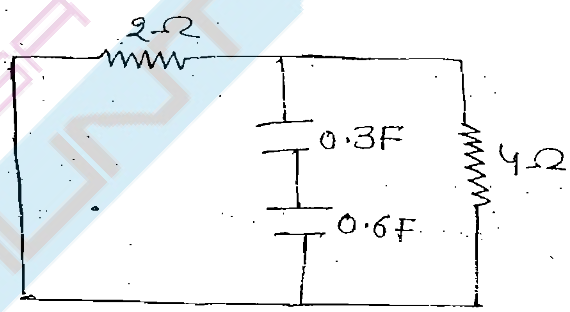
$V_{c_1}(0) = 24V$

$V_{c_2}(0) = 6V$

(a) $8 + 22e^{-3.75t}$ $\qquad$ (b) $8 + 22e^{-t/3.75}$

(c) $8 + 22e^{-t/1.2}$ $\qquad$ (d) $8 + 22e^{-1.2t}$

Soln:- For time constant, deactivate all the independent sources



$R_{eq} = \frac{2 \times 4}{2+4}$

$C_{eq} = \frac{0.3 \times 0.6}{0.3 + 0.6}$

$T = R_{eq} C_{eq} = RC$

$V(t) = [V(0^+) - V(\infty)]e^{-t/RC} + V(\infty)$

$V_0(t) = [V_0(0^+) - V_0(\infty)]e^{-t/RC} + V_0(\infty)$

$V_0 = V_{c_1} + V_{c_2}$

$t = 0^+$

$V_0(0^+) = V_{c_1}(0^+) + V_{c_2}(0^+) = 24 + 6 = 30$

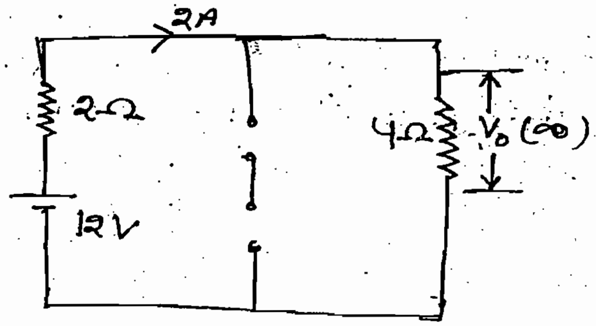
At $t = \infty$

At $t = \infty$

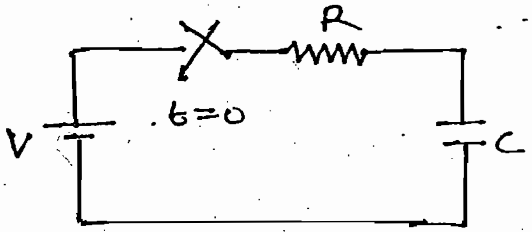
$$V_0(\infty) = 2\times 4 = 8$$

$$V_0(t) = [30-8]e^{-3.75t} + 8$$

$$= 8 + 22e^{-3.75t}$$



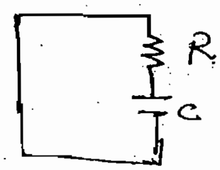
(I)



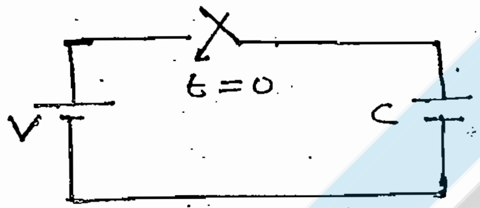
At $t=0^-$, $V_c=0$

$t=0^+$, $V_c=0$

$T=RC$



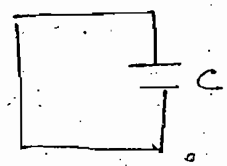
(II)



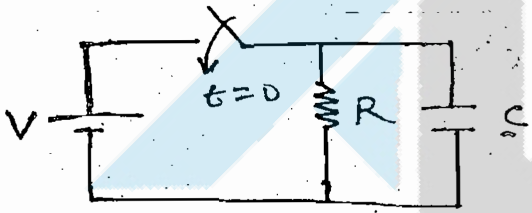
$t=0^-$, $V_c=0$

$t=0^+$, $V_c=V$

$T=RC=0$



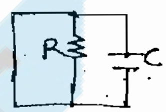
(III)

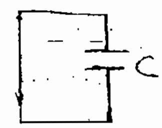


$t=0^-$, $V_c=0$

$t=0^+$, $V_c=V$

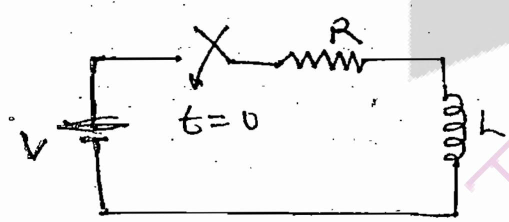
$T=RC=0$





$T=RC=0$

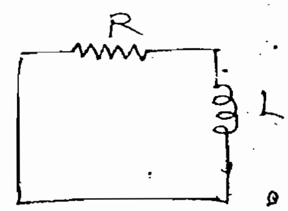
(IV)



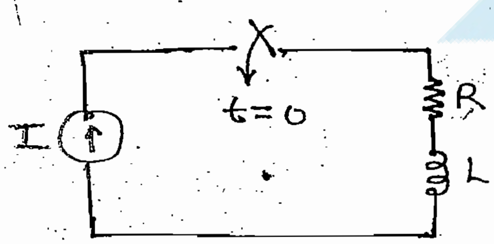
$t=0^-$, $i=0$

$t=0^+$, $i=0$

$T=L/R$



(V)

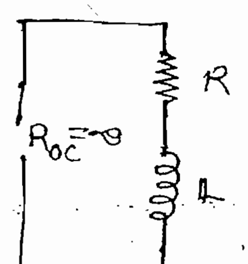


$t=0^-$, $i=0$

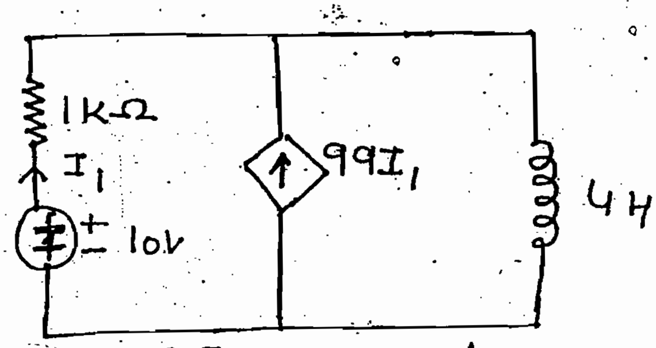
$t=0^+$, $i=I$

$R_{eq} = R_{oc}+R = \infty$

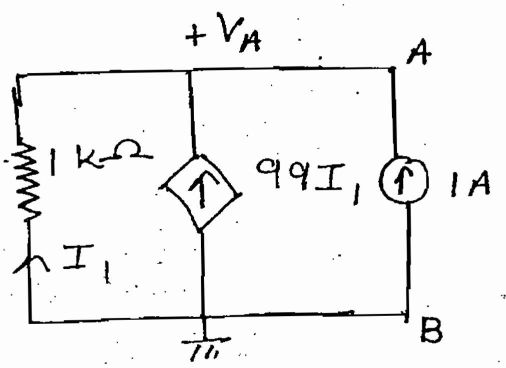
$$T = \frac{L}{R_{eq}} = 0$$

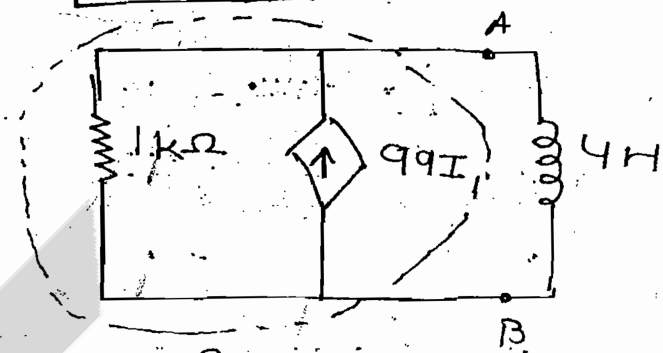


Ques:- Find time constant of the circuit shown



Soln:-





$$\frac{V_A}{1\times10^3} = 99I_1 + 1 \quad -(I)$$

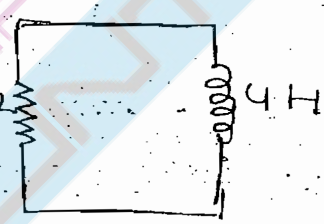
$$I_1 = -\frac{V_A}{1\times10^3} \quad -(II)$$

From (I) & (II)

$$V_A = 10V$$

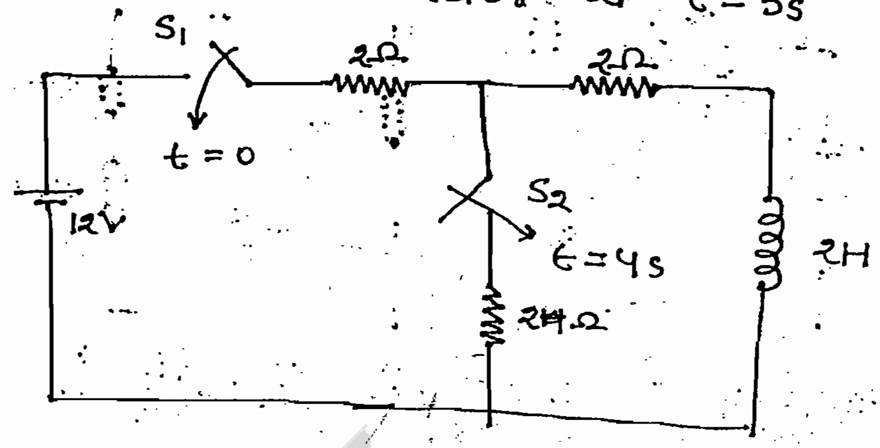
$$R_{th} = \frac{V_A}{I_S} = \frac{10}{1} = 10\Omega$$

$$\tau = \frac{L}{R_{th}} = \frac{4}{10}$$



Lecture – 10

Ques:– Find current in the inductor at $t = 5s$



Solⁿ:– S_1

$$i(t) = \frac{V}{R}\left(1 - e^{-Rt/L}\right)$$

$$i(t) = \frac{12}{4}\left(1 - e^{-2t}\right) \Rightarrow i(t) = 3\left(1 - e^{-2t}\right)$$

$t > 4s$, S_1 and S_2 $\longrightarrow$ operated

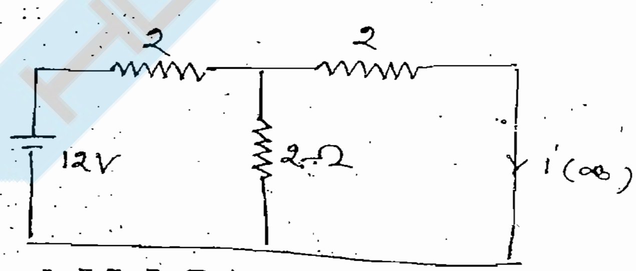
$$i(t) = [i(0^+) - i(\infty)]e^{-Rt/L} + i(\infty)$$

$$\Rightarrow i(t) = [i(4) - i(\infty)]e^{-\frac{R'}{L}(t-4)} + i(\infty) \quad \text{—(I)}$$

$t = 4$

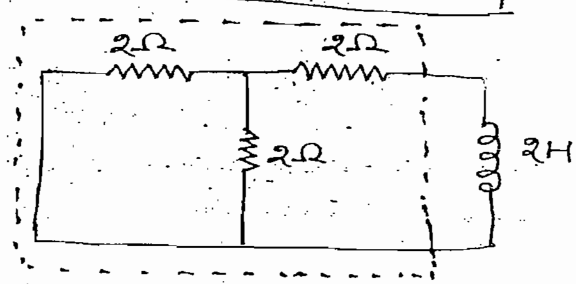
$$i(4) = 3\left[1 - e^{-2(4)}\right] \simeq 2.99A$$

$t = \infty$



$$i(\infty) = 2A$$

Time constant for $T > 4s$



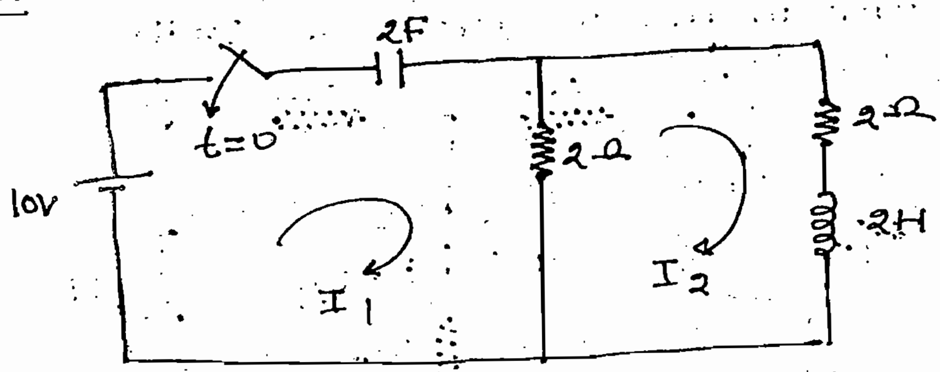
$$R' = 3 \quad \text{—(II)}$$

From –(I) & (II)

$$i(t) = [2.99 - 2]e^{-3/2(t-4)} + 2$$

$t = 5$ sec.

Ques:-



Find $i_1(0^+)$, $i_2(0^+)$, $V_c(0^+)$, $\frac{di_1(0^+)}{dt}$, $\frac{di_2(0^+)}{dt}$, $\frac{d^2i_1(0^+)}{dt^2}$, $\frac{d^2i_2(0^+)}{dt^2}$

Soln:- At $t=0^-$

$i_1(0^-) = 0 \quad i_2(0^-) = 0 \quad V_c(0^-) = 0$

At $t=0^+$

$i_2(0^+) = 0 \quad V_c(0^+) = 0 \quad i_1(0^+) = \frac{10}{2} = 5A$

(1) — $4I_2 + 2\frac{di_2}{dt} - 2I_1 = 0 \quad \rightarrow t=0^+$

$$4(0) + 2\frac{di_2}{dt} - 2(5) = 0$$

$$\Rightarrow \frac{di_2}{dt} = 5\ A/s$$

$$-10 + \frac{1}{2}\int i_1 dt + 2(I_1 - I_2) = 0$$

Diff. w.r.t t

$$0 + \frac{i_1}{2} + 2\left[\frac{di_1}{dt} - \frac{di_2}{dt}\right] = 0 \quad \text{—(II)}$$

$$\Rightarrow \frac{5}{2} + 2\left[\frac{di_1}{dt} - 5\right] = 0$$

$$\Rightarrow \frac{5}{2} - 10 + \frac{2di_1}{dt} = 0$$

$$\Rightarrow \frac{di_1}{dt} = \frac{15}{4} = 3.75\ A/s$$

Diff eq-(I) w.r.t 't'

$$4\frac{di_2}{dt} + 2\frac{d^2i_2}{dt^2} - 2\frac{di_1}{dt^2} = 0$$

$$\Rightarrow \frac{d^2i_2}{dt^2} = -6.25 \text{ A/s}^2$$

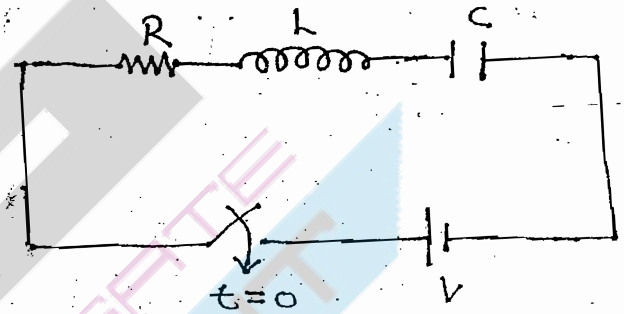
Diff eq-(II) w.r.t 't'

$$\frac{1}{2}\frac{di_1}{dt} + 2\left[\frac{d^2i_1}{dt^2} - \frac{d^2i_2}{dt^2}\right] = 0$$

$$\Rightarrow \frac{d^2i_1}{dt^2} = -7.18 \text{ A/s}^2$$

RLC Series Circuit with DC Excitation :-

By KVL



$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

Diff w.r.t 't'

$$0 = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C}$$

Divide both sides by L we get

$$\boxed{\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0}$$

$$\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right)i = 0 \qquad \left(\because \frac{d}{dt} = D\right)$$

$$D_1, D_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$\boxed{D_1, D_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}}$$

Case-(I) :-

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$

Overdamping

$$\alpha = \frac{-R}{2L} \qquad \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)}$$

$$\boxed{i(t) = c_1 e^{(\alpha-\beta)t} + c_2 e^{(\alpha+\beta)t}}$$

Case-(II) :-

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC} \longrightarrow \text{Critical damping}$$

$$\boxed{i(t) = (c_1 + c_2 t)\, e^{\alpha t}}$$

Case-(III) :-

$$\left(\frac{R}{2L}\right)^2 < \frac{1}{LC} \longrightarrow \text{Underdamping}$$

$$\alpha = \frac{R}{2L}$$

$$\beta = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$\boxed{i(t) = (c_1 \cos \beta t + c_2 \sin \beta t)\, e^{\alpha t}}$$

Damping coefficient $= \frac{R}{2L}$

Time constant $= \frac{1}{\text{Damping Coefficient}} = \frac{2L}{R}$

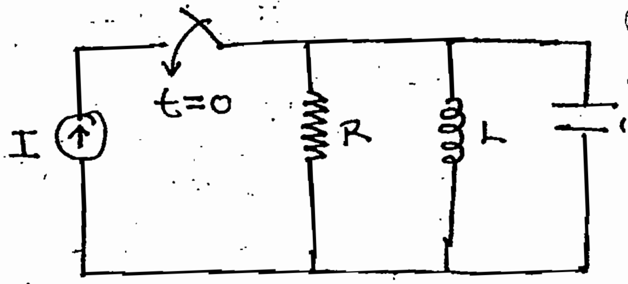
Case-(IV) :-

$$R = 0 \longrightarrow \text{Underdamping}$$

$$\alpha = 0, \quad \beta = \frac{1}{\sqrt{LC}}$$

$$\boxed{i(t) = C_1 \cos\beta t + C_2 \sin\beta t}$$

RLC Parallel Circuit with DC Excitation:—



$$I = \frac{V}{R} + C\frac{dV}{dt} + \frac{1}{L}\int V dt$$

Diff. w.r.t t

$$0 = \frac{1}{R}\cdot\frac{dV}{dt} + C\frac{d^2V}{dt^2} + \frac{V}{L}$$

Divide the whole equation by C, we get

$$\frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{V}{LC} = 0$$

$$\left(D^2 + \frac{1}{RC}D + \frac{1}{LC}\right)V = 0. \qquad \left(\because D = \frac{d}{dt}\right)$$

$$D_1, D_2 = \frac{-\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - \frac{4}{LC}}}{2}$$

$$D_1, D_2 = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

Case—(I):—

$$\left(\frac{1}{2RC}\right) > \frac{1}{LC} \longrightarrow \text{over damping}$$

$$\alpha = -\frac{1}{2RC}$$

$$\beta = \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$\boxed{V(t) = C_1 e^{(\alpha-\beta)t} + C_2 e^{(\alpha+\beta)t}}$$

Case—(II):—

$$\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC} \longrightarrow \text{Critical damping}$$

$$\boxed{v(t) = (C_1 + C_2 t)e^{\alpha t}}$$

Case-(III) :-

$$\left(\frac{1}{2RC}\right)^2 < \frac{1}{LC} \longrightarrow \text{Under damping}$$

$$\alpha = -\frac{1}{2RC} \qquad \beta = \sqrt{\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2}$$

$$\boxed{V(t) = (C_1 \cos\beta t + C_2 \sin\beta t)\, e^{\alpha t}}$$

Damping coefficient $= \frac{1}{2RC}$

Time constant $= \frac{1}{\text{Damping coefficient}} = 2RC$

Case-(IV) :-

$G = 0 \qquad \frac{1}{R} = 0 \Rightarrow R = \infty$

$\longrightarrow$ Underdamping

$\alpha = 0 \quad , \quad \beta = \frac{1}{\sqrt{LC}}$

$$\boxed{V(t) = C_1 \cos\beta t + C_2 \sin\beta t}$$

Note :-

1. Steady state current response of RLC series circuit with DC excitation = 0

2. At $t = \infty$

   $L \rightarrow$ Uncharged $(I = 0)$

   $C \rightarrow$ Charged $(V_C = V)$

3. $$\boxed{\xi = \frac{R}{2}\sqrt{\frac{C}{L}}} \longrightarrow \text{Series}$$

   $\xi > 1 \rightarrow$ overdamping $\qquad \xi < 1 \rightarrow$ Underdamping

   $\xi = 1 \rightarrow$ Critical damping $\qquad \xi = 0 \rightarrow$ Undamping

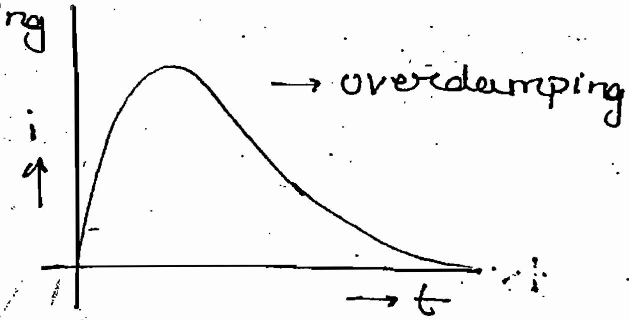
$$\boxed{\zeta = \frac{1}{2R}\sqrt{\frac{L}{C}}} \rightarrow \text{Parallel}$$

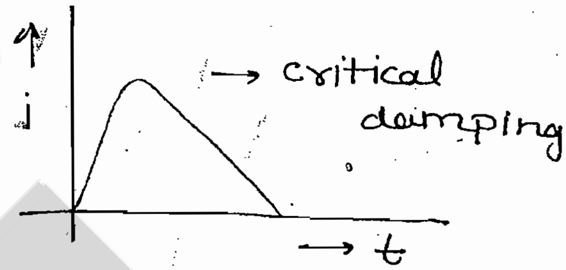
4. For over and critical damping system no oscillations are present

$\frac{1}{2R}\sqrt{\frac{L}{C}} \geq 1 \rightarrow$ Parallel

$\frac{R}{2}\sqrt{\frac{C}{L}} \geq 1 \rightarrow$ Series

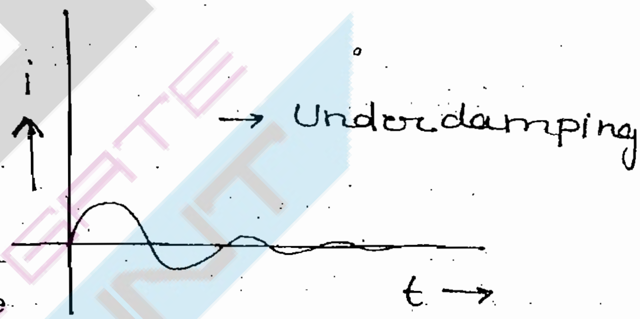
$\zeta \geq 1$





→ In the underdamping system more than 1 oscillation are present

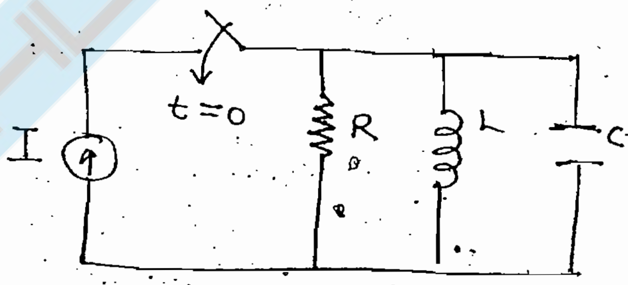
$\zeta < 1$



5. At $t = \infty$

$L \rightarrow$ Charged ($I_L = I$)

$C \rightarrow$ Uncharged ($V_c = 0$)

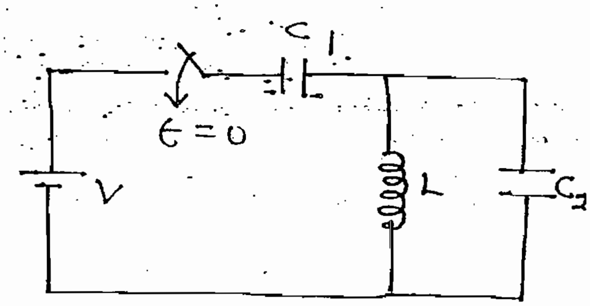


6. $t = \infty$

$C_1 \rightarrow$ charged ($V_{C_1} = V$)

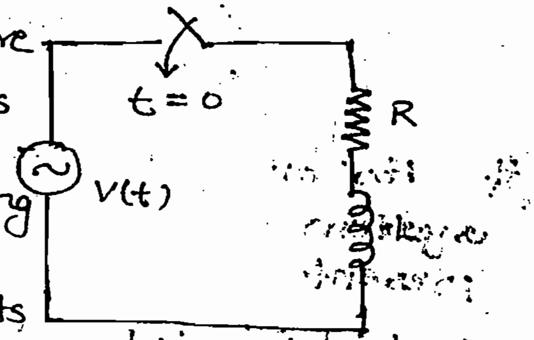
$C_2 \rightarrow$ Uncharged ($V_{C_2} = 0$)

$L \rightarrow$ Uncharged ($I_L = 0$)



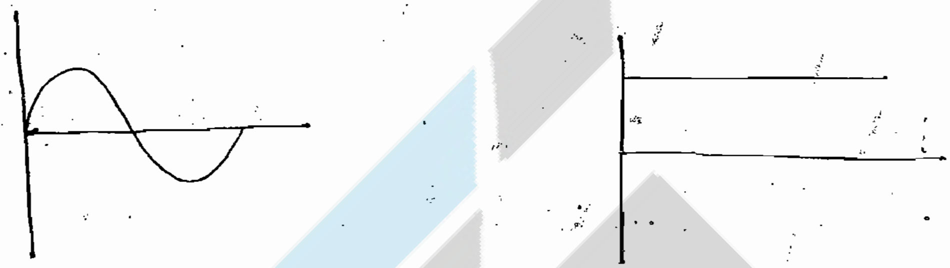
A.C Transients :-

→ DC transients intensity is more than intensity of AC transients



→ Based on selection of operating frequency and switching operation and circuit elements it is possible to ~~op~~ obtain transient free response. But in DC circuit it is not possible to obtain transient free response

$V(t) = V_m \sin(\omega t + \theta)$



By KVL

$$V = iR + L\frac{di}{dt}$$

Divide both sides by L, we get

$$\boxed{\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}}$$

$$i(t) = \underbrace{C.F}_{\text{Transient Response}} + P.I$$

C.F :-

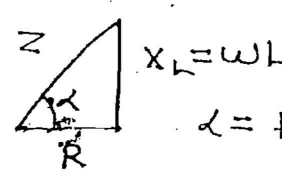
$$\frac{di}{dt} + \frac{R}{L}i = 0$$

$$\Rightarrow i(t) = A e^{-\frac{Rt}{L}}$$

P.I → Steady state response

$$i = \frac{V}{Z\angle\alpha} = \frac{V\angle-\alpha}{Z}$$

$$\boxed{i(t) = \frac{V_m}{Z}\sin(\omega t + \theta - \alpha)}$$



$\alpha = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$i(t) = C.F + P.I$$

$$i(t) = Ae^{-\frac{R}{L}t} + \frac{V_m}{Z}\sin(\omega t + \theta - \alpha)$$

$t = 0^-$, $i = 0$

$t = 0^+$, $i = 0$

$$0 = A + \frac{V_m}{Z}\sin(\theta - \alpha)$$

$$\Rightarrow \boxed{A = -\frac{V_m}{Z}\sin(\theta - \alpha)}$$

$$\boxed{i(t) = \underbrace{-\frac{V_m}{Z}\sin(\theta - \alpha)e^{-\frac{R}{L}t}}_{T.R.} + \underbrace{\frac{V_m}{Z}\sin(\omega t + \theta - \alpha)}_{S.R}}$$

Case-(I) :-

$V(t) = V_m \sin(\omega t + \theta)$, $t = 0$

$$\theta - \alpha = 0$$

$$\theta = \alpha \Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)}$$

→ condition for transient free response

Case-(II) :-

$V(t) = V_m \cos(\omega t + \theta)$, $t = 0$

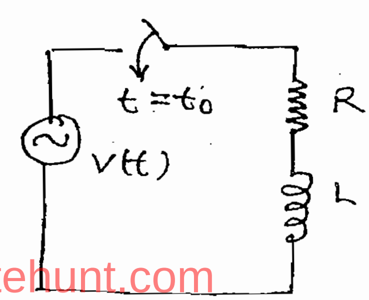
$$\boxed{\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) + \frac{\pi}{2}}$$

→ condition for transient free response

Case-(III) :-

$t = t_0^-$ $i = 0$

$t = t_0$ $i = 0$



$$0 = A + \frac{V_m}{Z}\sin(\omega t_0 + \theta - \alpha)$$

$$\Rightarrow \boxed{A = -\frac{V_m}{Z}\sin(\omega t_0 + \theta - \alpha)}$$

$$\boxed{i(t) = \underbrace{-\frac{V_m}{Z}\sin(\omega t_0 + \theta - \alpha)\, e^{-R/L(t-t_0)}}_{T.R} + \underbrace{\frac{V_m}{Z}\sin(\omega t + \theta - \alpha)}_{S.R}}$$

condition for transient free response for <u>case-(III)</u> is

$$V(t) = V_m \sin(\omega t + \theta) \quad , \; t = t_0$$

$$\omega t_0 + \theta - \alpha = 0$$

$$\Rightarrow \omega t_0 = \alpha - \theta$$

$$\Rightarrow \boxed{\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta}$$

<u>Case-(IV)</u> :-

$$V(t) = V_m \cos(\omega t + \theta) \quad t = t_0$$

$$\boxed{\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta + \frac{\pi}{2}}$$

PHOTOSTAT

	RL	RC	
(i) $V(t) = V_m \sin(\omega t + \theta)$, $t = 0$	$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$	$\theta = \tan^{-1}(\omega \tau)$	$\theta = \tan^{-1}(\omega R$
(ii) $V(t) = V_m \cos(\omega t + \theta)$, $t = 0$	$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) + \frac{\pi}{2}$	$\theta = \tan^{-1}(\omega \tau) + \frac{\pi}{2}$	$\theta = \tan^{-1}(\omega R + \frac{\pi}{2}$
(iii) $V(t) = V_m \sin(\omega t + \theta)$, $t = t_0$	$\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta$	$\omega t_0 = \tan^{-1}(\omega \tau) - \theta$	$\omega t_0 = \tan^{-1}(\omega R - \theta$
(iv) $V(t) = V_m \cos(\omega t + \theta)$, $t = t_0$	$\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta$	$\omega t_0 = \tan^{-1}(\omega \tau) - \theta + \frac{\pi}{2}$	$\omega t_0 = \tan^{-1}(\omega R - \theta + \frac{\pi}{2}$

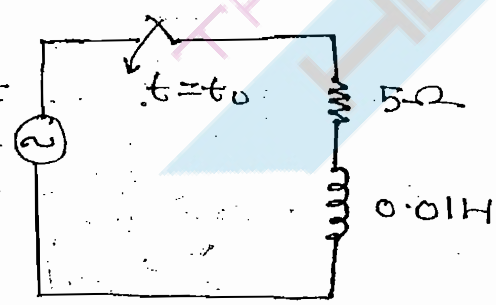
→ The above condition for series circuit for transient free response and also for parallel circuit.

→ In the RLC circuit it is not possible to obtain transient free response since circuit is having two energy storage element (simultaneously both charging and discharging action are present)

Ques:- Find t_0 to obtain transient free response

$V(t) = \sin \omega t$

$f = 50\,Hz$



Soln:-

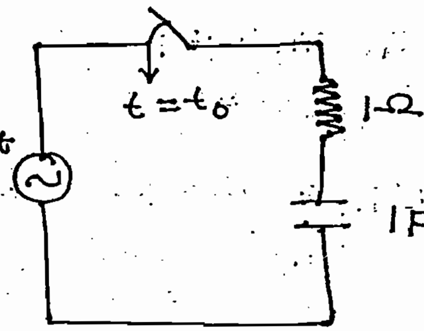
$$\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta$$

$$t_0 = \frac{\tan^{-1}\left(\frac{\omega L}{R}\right) \rightarrow rad}{\omega \rightarrow rad/s}$$

$$t_0 = \frac{32.1 \times \pi/180}{100\pi} = 1.78\,ms \text{ , Ans}$$

Ques:- Find t_0 to obtain transient free response

Soln:- $\omega t_0 = \tan^{-1}(\omega RC) - \theta + \frac{\pi}{2}$



$$\Rightarrow t_0 = \frac{\tan^{-1}(1) + \pi/2}{1}$$

$$\Rightarrow t_0 = \frac{\frac{\pi}{4} + \frac{\pi}{2}}{1} = \frac{3\pi}{4} \quad (\pi = 22/7)$$

$$\Rightarrow t_0 = 7.35\,s \quad \text{Ans}$$

Laplace :-

$$A \rightarrow \frac{1}{s}$$

$$e^{at} \rightarrow \frac{1}{s-a}$$

$$\sin\omega t \rightarrow \frac{\omega}{s^2+\omega^2}$$

$$\cos\omega t \rightarrow \frac{s}{s^2+\omega^2}$$

$$e^{at}\sin\omega t \rightarrow \frac{\omega}{(s-a)^2+\omega^2}$$

$$e^{-at}\sin\omega t \rightarrow \frac{\omega}{(s-a)^2+\omega^2}$$

$$e^{-at}\cos\omega t \rightarrow \frac{s+a}{(s+a)^2+\omega^2}$$

$$\frac{df}{dt} \Rightarrow sF(s) - f(0)$$

$$u(t) \rightarrow \frac{1}{s}$$

$$\delta(t) \rightarrow 1$$

$$\boxed{f(0^+) = \lim_{s\to\infty} sF(s) = \lim_{t\to 0} f(t)} \rightarrow \text{Initial value theorem}$$

Ques:- Find initial value of the following functions:-

$$F(S) = \frac{(S+1)(S+3)}{S(2S+2)(S+4)}$$

Soln:- $f(0^+) = \lim_{S\to\infty} SF(S)$

$$= \lim_{S\to\infty} \frac{S\cdot S^2\left(1+\frac{1}{S}\right)\left(3+\frac{3}{S}\right)}{S\cdot S^2\left(2+\frac{2}{S}\right)\left(1+\frac{4}{S}\right)} = \frac{1}{2}$$

Ques:- Find initial value of the following function

$$f(t) = 3 + \delta(t)$$

Soln:- $F(S) = \frac{3}{S} + 1$

Note:-

For the above function initial value theorem can't be applied since to apply the initial value theorem denominator power should be greater than Numerator power

Final Value Theorem:-

$$\boxed{f(\infty) = \lim_{S\to 0} SF(S) = \lim_{t\to\infty} f(t)}$$

Ques:- Find final value of the following function

(I) $F(S) = \frac{(S+1)(S+3)}{S(S+2)(S+4)}$

(II) $f(t) = 3 + e^{2t}$

Soln:- (I) $f(\infty) = \lim_{S\to 0} SF(S)$

$$= \lim_{S\to 0} \frac{S(S+1)(S+3)}{S(S+2)(S+4)} = \frac{3}{8}$$

(II) $f(t) = 3 + e^{2t}$

$F(S) = \frac{3}{S} + \frac{1}{S-2}$

$$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i\,dt$$

$$\Rightarrow \quad V_c(t) = \frac{1}{C}\int_{-\infty}^{0^-} i\,dt + \frac{1}{C}\int_{0^-}^{t} i\,dt$$

$$\Rightarrow \quad V_c(t) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{t} i\,dt$$

At $t = 0^+$

$$V_c(0^+) = V_c(0^-) + \underbrace{\frac{1}{C}\int_{0^+}^{0^+} i\,dt}_{=0}$$

$$V_c(0^+) = V_c(0^-) \quad \text{— (I)}$$

$$V_c(0^+) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{0^+} i\,dt$$

$$V_c(0^+) = V_c(0^-) + \frac{1}{C}\underbrace{\int_{0^-}^{0^+} \delta(t)\,dt}_{\text{Area}=1} \qquad i \to \delta(t)$$

$$\boxed{V_c(0^+) = \frac{1}{C}} \quad \text{— (II)}$$

$$w_c(0^+) = \frac{1}{2} C V_c^2(0^+) = \frac{1}{2} C \left(\frac{1}{C}\right)^2$$

$$\boxed{w_c(0^+) = \frac{1}{2C}} \quad \text{— (III)}$$

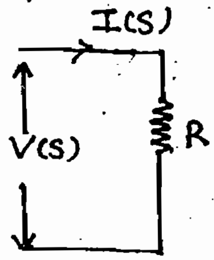
Note:—

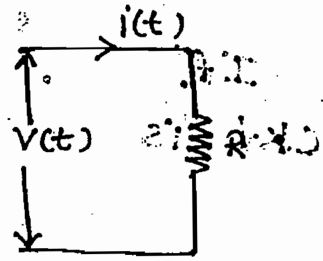
→ From eq-(I) it is concluded that capacitor does not allow instantaneous changes for given i/p.

→ From eq-(II) & (III) it is concluded that capacitor allows instantaneous changes for current impulse function.

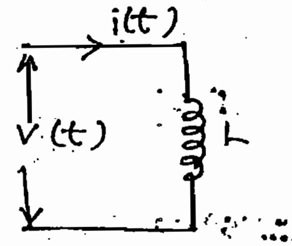
$$v(t) = i(t)R$$

$$V(s) = I(s)R$$



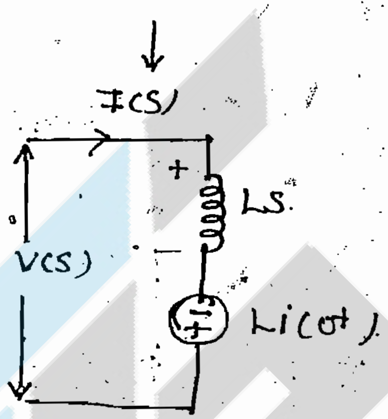


$$v = L\frac{di}{dt} \qquad s = j\omega$$



$$V(s) = L\left[sI(s) - i(0^+)\right]$$

$$V(s) = LsI(s) - Li(0^+) \longrightarrow \text{KVL}$$

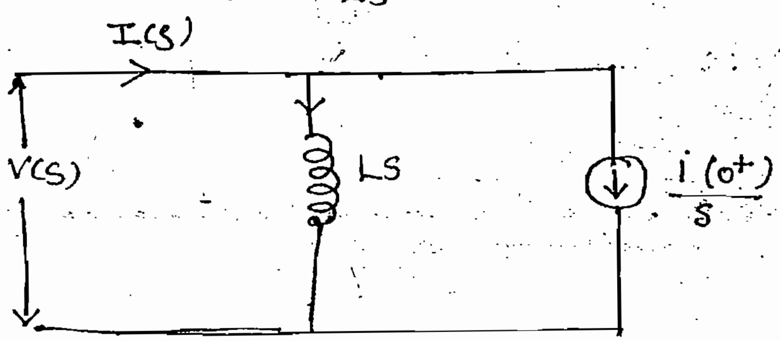


$$Z(s) = \frac{V(s)}{I(s)}$$

$$\Rightarrow \boxed{Z(s) = Ls - \frac{Li(0^+)}{I(s)}}$$

$$LsI(s) = V(s) + Li(0^+)$$

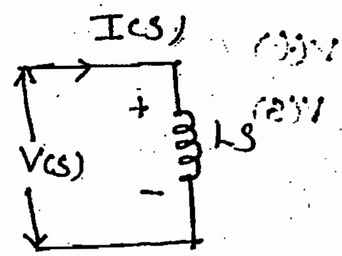
$$I(s) = \frac{V(s)}{Ls} + \frac{Li(0^+)}{Ls} \longrightarrow \text{KCL}$$



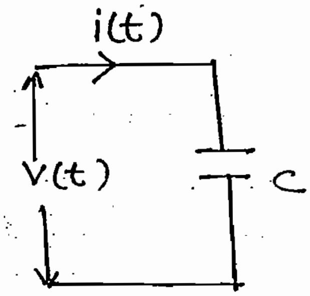
$$Y(s) = \frac{I(s)}{V(s)}$$

$$\boxed{Y(s) = \frac{1}{Ls} + \frac{i(0^+)}{sV(s)}}$$

If $i(0^+) = 0$ then eq. ckt is $\longrightarrow$



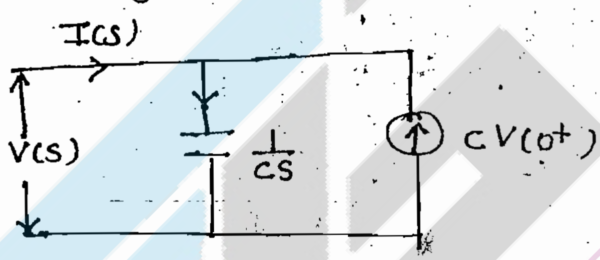
$$i = C\frac{dV}{dt}$$



$$I(s) = C\left[sV(s) - V(0^+)\right]$$

$$\Rightarrow \quad I(s) = \frac{V(s)}{1/Cs} - CV(0^+) \quad \longrightarrow \text{KCL}$$

$$s = j\omega$$

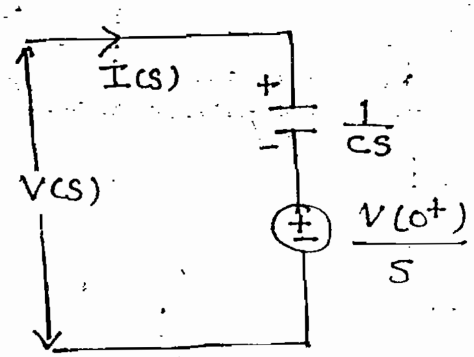


$$Y(s) = \frac{I(s)}{V(s)}$$

$$\boxed{Y(s) = Cs - \frac{CV(0^+)}{V(s)}}$$

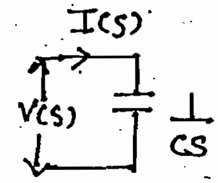
$$\frac{V(s)}{1/Cs} = I(s) + CV(0^+)$$

$$\Rightarrow \quad V(s) = \frac{1}{Cs}I(s) + \frac{CV(0^+)}{Cs} \quad \longrightarrow \text{KVL}$$

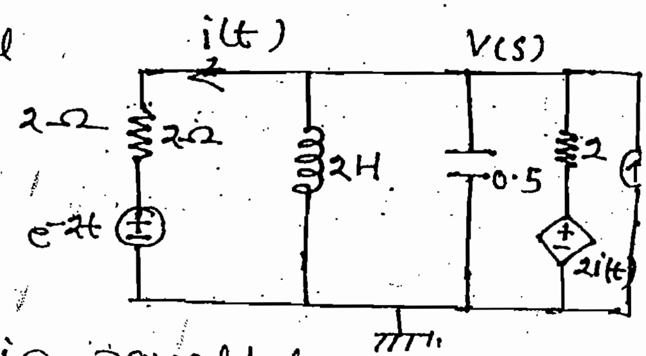


$$\boxed{Z(s) = \frac{V(s)}{I(s)} = \frac{1}{Cs} + \frac{V(0^+)}{s\,I(s)}}$$

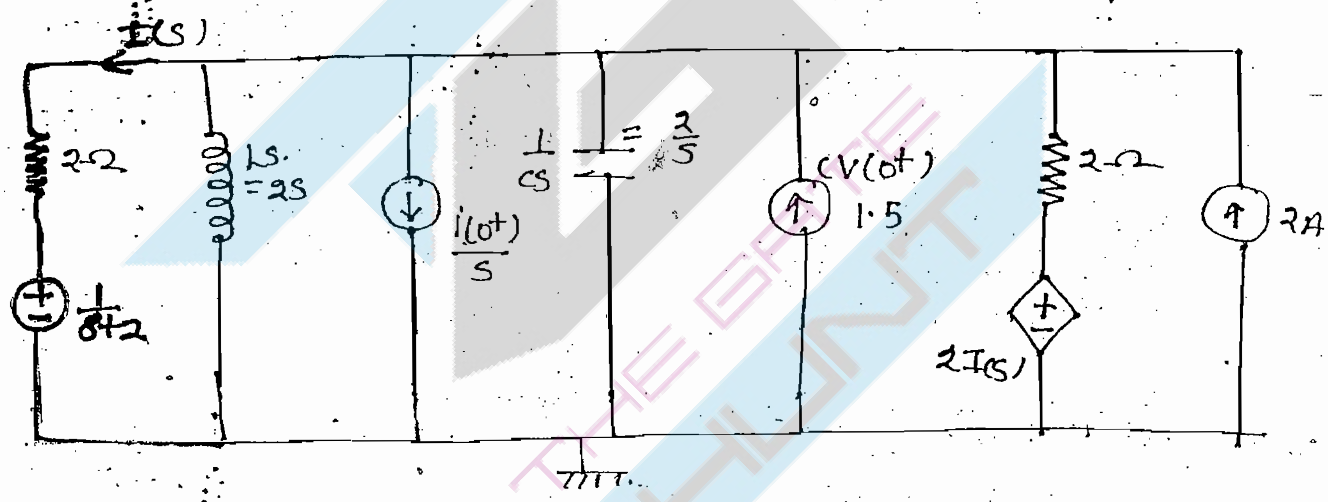
$V(0^+) = 0 \longrightarrow$



Ques:- Find V(s) when initial current of the inductor is 2A and initial voltage of the capacitor is 3V



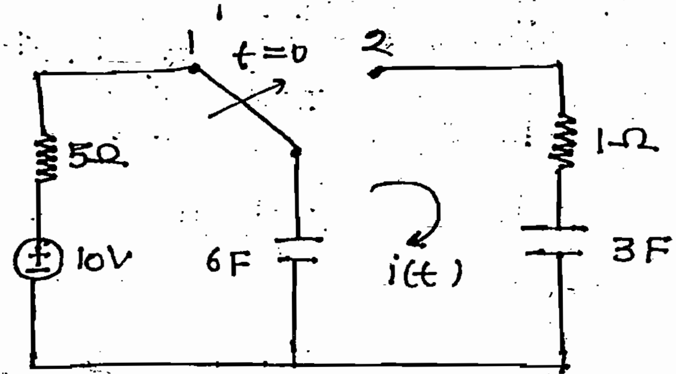
Soln:- When elements are in parallel then for simple calculation convert elements into current sources



$$\frac{V(s) - \frac{1}{s+2}}{2} + \frac{V(s)}{2s} + \frac{2}{s} + \frac{V(s)}{\frac{1}{Cs}} + \frac{V(s) - 2I(s)}{2} = 1.5 + 2 \quad \text{---(1)}$$

$$I(s) = \frac{V(s) - \frac{1}{s+2}}{2}$$

Ques:- Find $i(t)$ for $t>0$

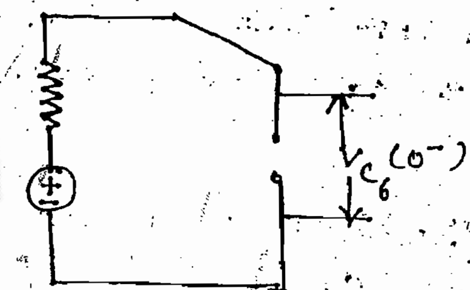


Soln:-

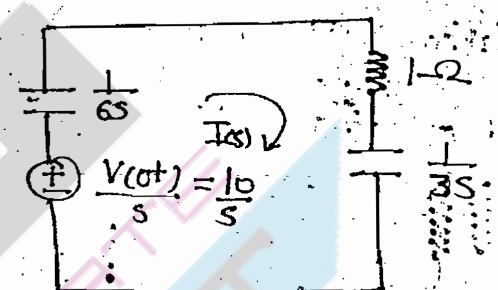
At $t = 0^-$

$V_{C_6}(0^-) = 10V$

$V_{C_3}(0^-) = 0V$



At $t>0$



$$I(s) = \frac{10/s}{\frac{1}{6s} + \frac{1}{3s} + 1}$$

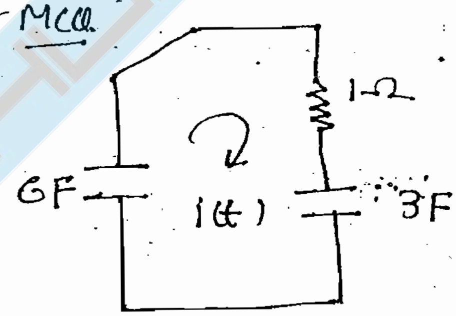
$$i(t) = L^{-1} I(s)$$

$$\Rightarrow \quad i(t) = 10\, e^{-t/2}$$

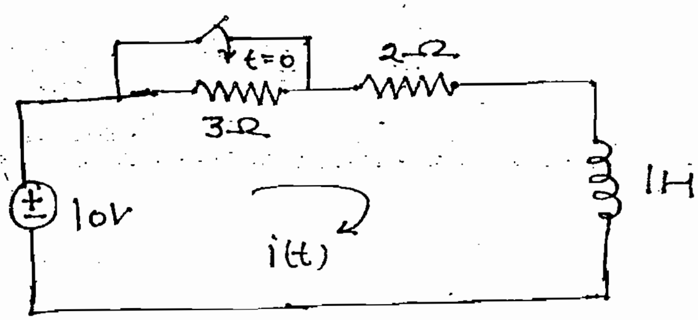
Verification :- → Method for MCQ

$$i(t) = \frac{V_0}{R} e^{-t/RC}$$

$$\boxed{i(t) = \frac{10}{1} e^{-t/2}}$$



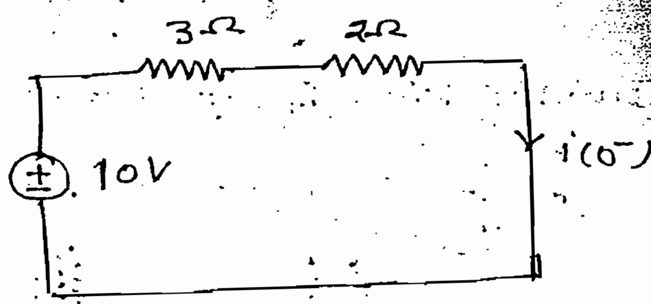
Ques:- Find $i(t)$ for $t>0$



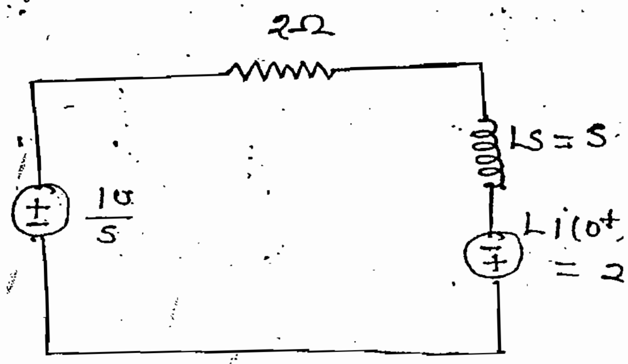
Soln:-

At $t = 0^-$

$$i(0^-) = \frac{10}{3+2} = 2A.$$



For $t > 0$



$$I(s) = \frac{\frac{10}{s}+2}{2+s}$$

$$i(t) = L^{-1} I(s)$$

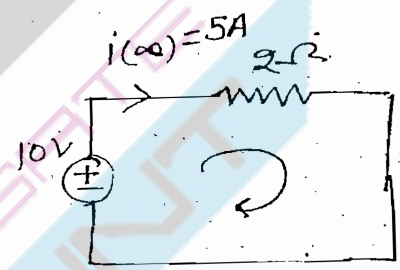
$$\boxed{i(t) = 5 - 3e^{-2t}}$$

Alternate way:-

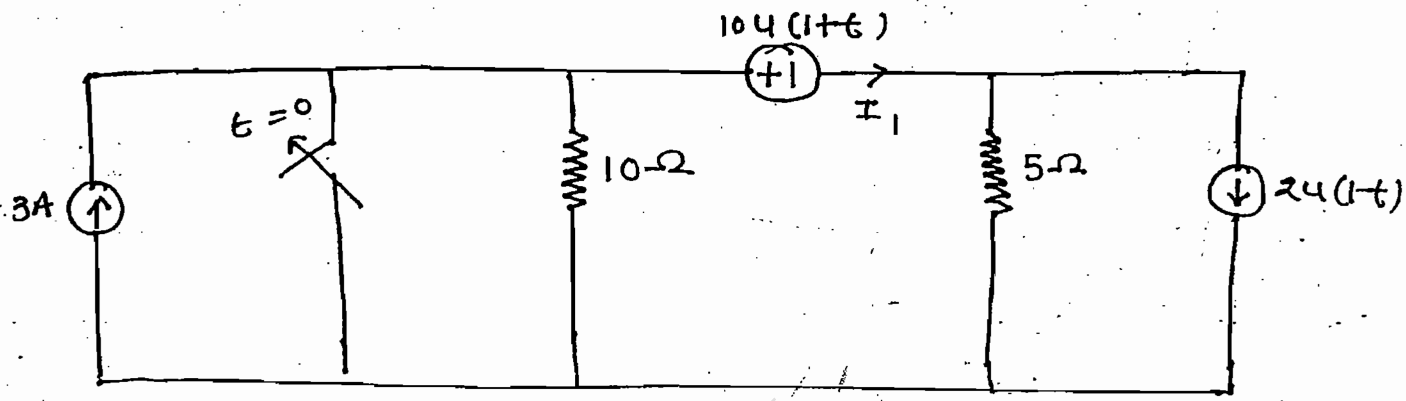
$$i(t) = [\,i(0^+) - i(\infty)\,]\,e^{-\frac{R}{L}t} + i(\infty)$$

$$i(0^+) = i(0^-) = 2$$

$$i(t) = [\,2-5\,]\,e^{-2t} + 5$$



Ques:- Find I_1 at $t = -2$ seconds.



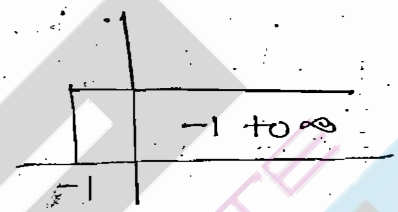
Soln:-

∞ → $-\infty$ to ∞

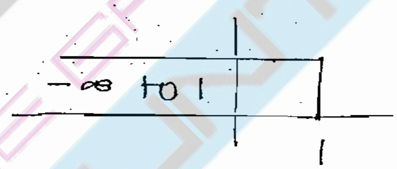
$u(t)$ → 0 to ∞

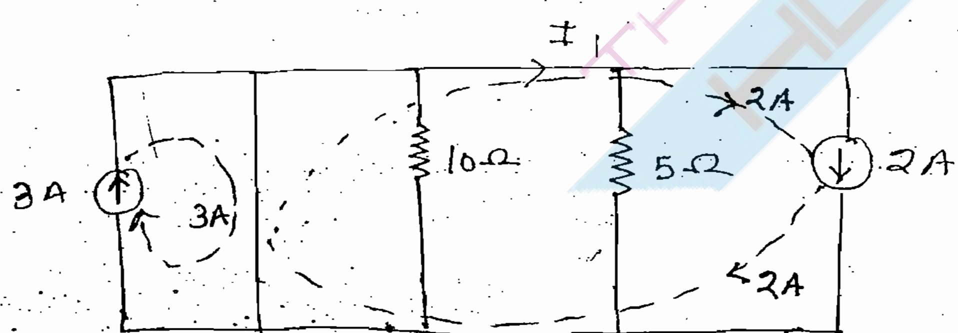
$u(-t)$ → $-\infty$ to 0

$u(1+t)$



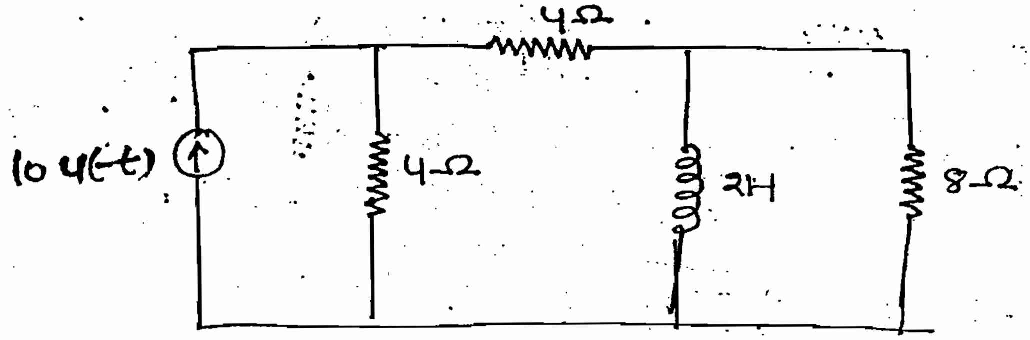
$u(1-t)$





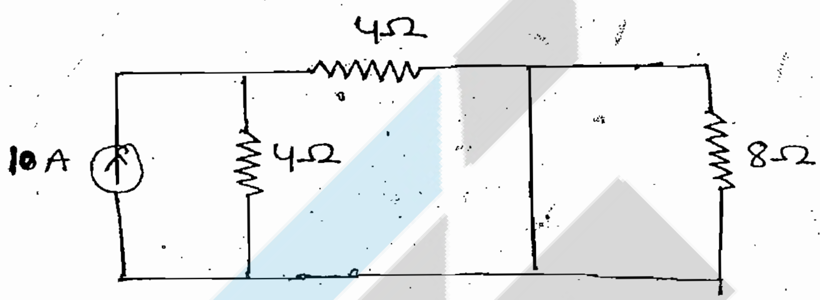
$I_1 = 2A$

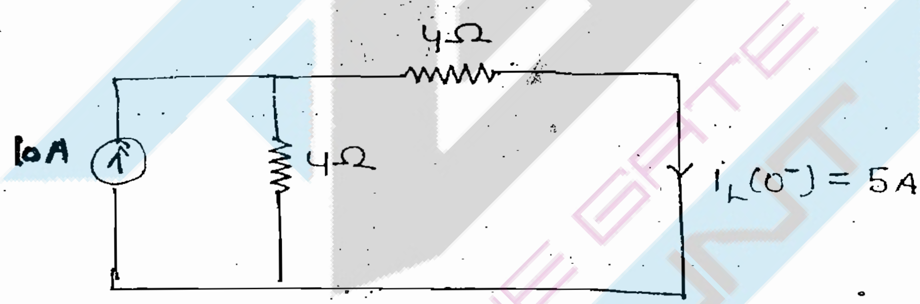
Ques:- Find current response in the inductor for $t>0$



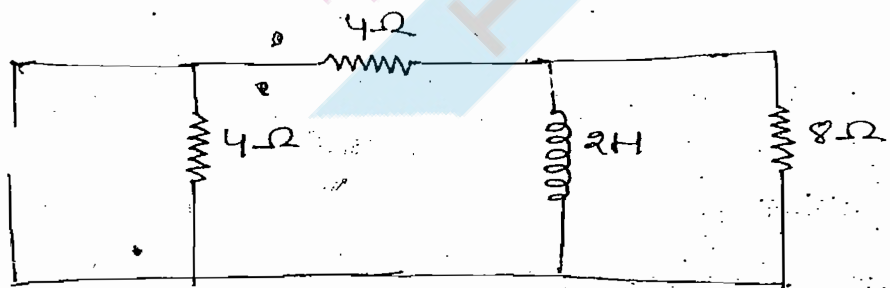
Soln:- $t = 0^-$

$u(-t) \longrightarrow -\infty$ to 0

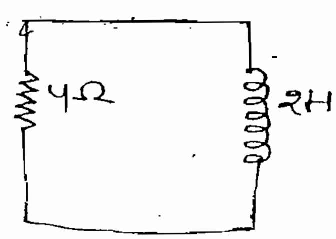




$t > 0$



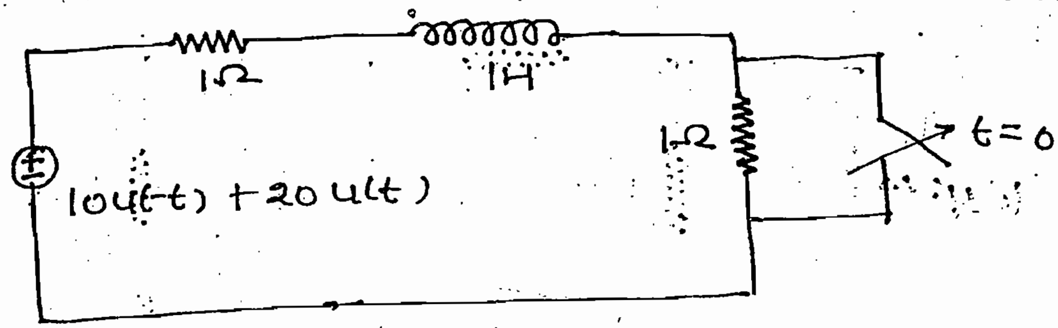
$i(t) = I_0 e^{-R/L\, t}$



$i(t) = 5e^{-2t}$, Ans

$I_0 = i_L(0) = 5A$

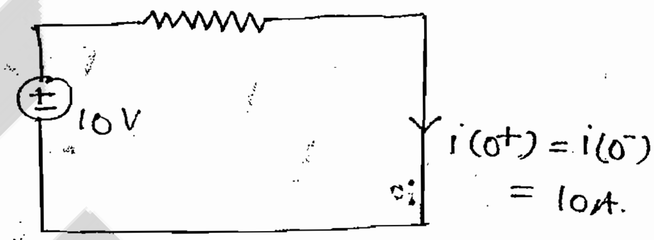
Ques:- Find current in the inductor for $t > 0$



Soln:- $i(t) = [i(0^+) - i(\infty)] e^{-\frac{R}{L}t} + i(\infty)$

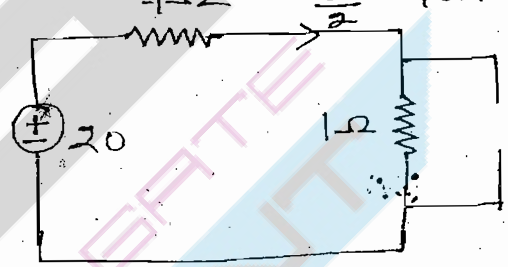
$t = 0^-$

$i(0^+) = i(0^-) = 10A$



$t = \infty$

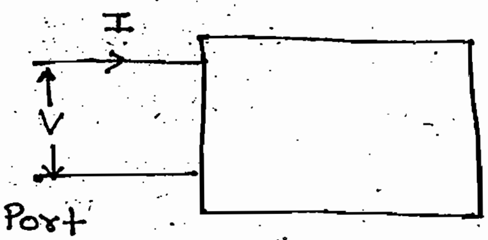
$\frac{20}{2} = 10A = i(\infty)$



$i(t) = [10-10]. e^{-R/L\, t} + 10 = 10$ Ans.

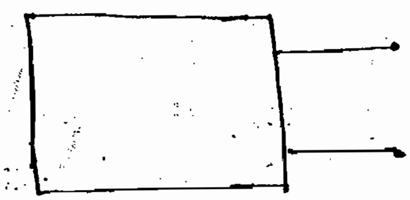
TWO PORT NETWORK

→ A pair of terminals at which signal may enter or leave from the N/w is called as port.



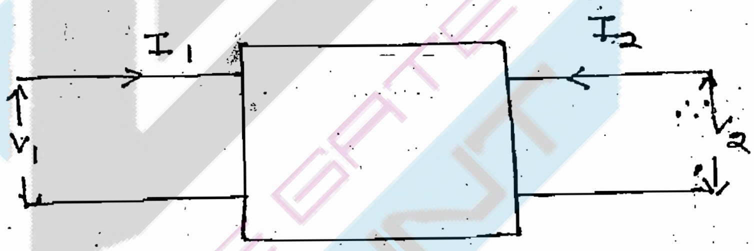
→ When the N/w is having one pair of terminals then the N/w is called as single port N/w

eg:- Motor, Generator



eg:- Generator

eg:- Transformer



Classification of Parameter:-

(I) z – Parameter (O.C Parameter)
(II) Y – Parameter (S.C Parameter)
(III) h – Parameter (Hybrid Parameter)
(IV) g – Parameter (Inverse hybrid Parameter)
(V) ABCD – Parameter (Transmission line parameter)
(VI) abcd – Parameter (Inverse transmission line parameter)

(I) **Z-Parameter :-**

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad —(I) \longrightarrow KVL$$

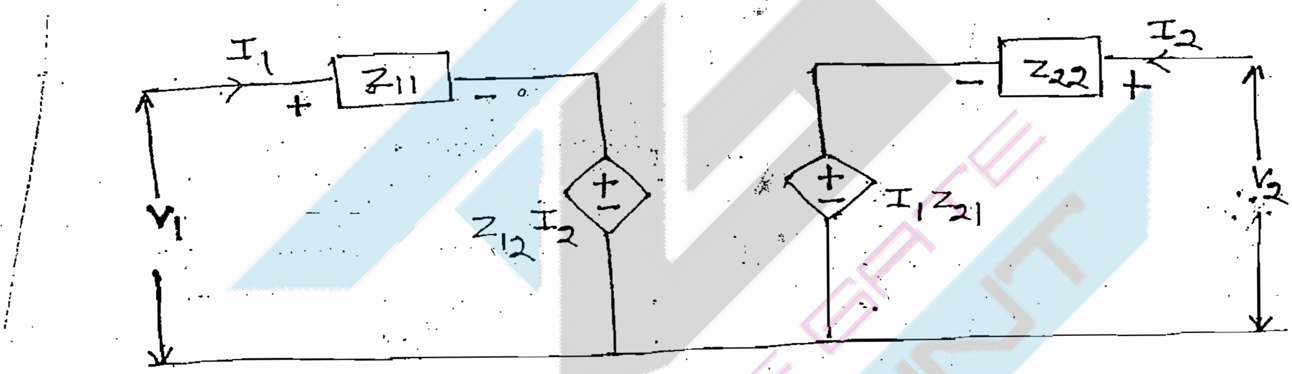
$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad —(II) \longrightarrow KVL$$

$V_1, V_2 \longrightarrow$ Dependent Variables

Source $\rightarrow I_1, I_2 \longrightarrow$ Independent Variables

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$



Z_{11} = Open ckt i/p impedance / Driving point i/p impedance

Z_{21} = Forward transfer impedance

Z_{12} = Reverse transfer impedance

Z_{22} = Open ckt o/p impedance / Driving point o/p impedance

(II) **Y-Parameter :-**

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad —(I) \longrightarrow KCL$$

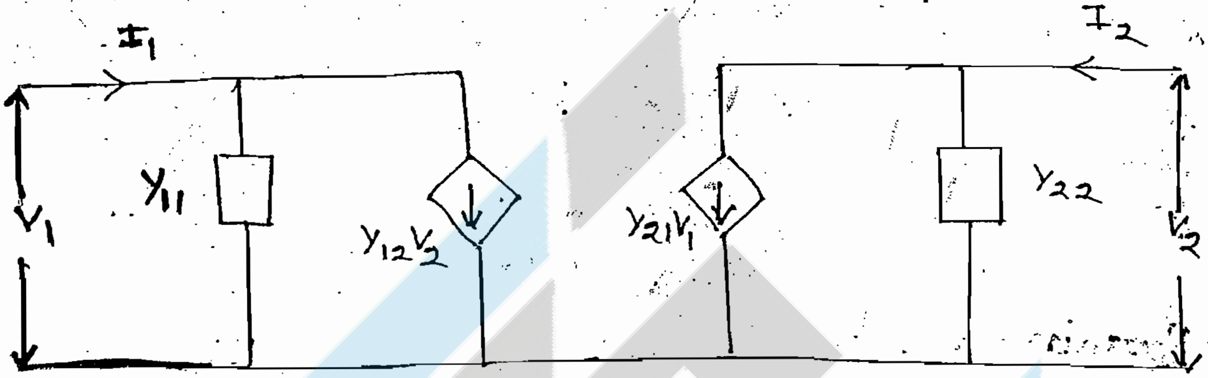
$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad —(II) \longrightarrow KCL$$

$I_1, I_2 \longrightarrow$ Dependent Variables

$V_1, V_2 \longrightarrow$ Independent Variables
(source)

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$



Y_{11} = S.C i/p admittance / Driving point i/p admittance

Y_{21} = forward transfer admittance

Y_{12} = Reverse transfer admittance

Y_{22} = S.C o/p admittance / Driving point o/p admittance

h-Parameter :-

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad \text{——(I)} \longrightarrow \text{KVL}$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad \text{——(II)} \longrightarrow \text{KCL}$$

h_{11} (Ω) $= \left.\dfrac{V_1}{I_1}\right|_{V_2=0}$ $\quad \left(h_{11} \neq Z_{11} \,,\; h_{11} = \dfrac{1}{Y_{11}} \right)$

h_{21} (unitless) $= \left.\dfrac{I_2}{I_1}\right|_{V_2=0}$ $\quad \left(h_{21} = \dfrac{Y_{21}}{Y_{11}} = \dfrac{I_2/\cancel{V_1}}{I_1/\cancel{V_1}} \right)$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad \left(h_{12} = \frac{Z_{12}}{Z_{22}} \right)$$

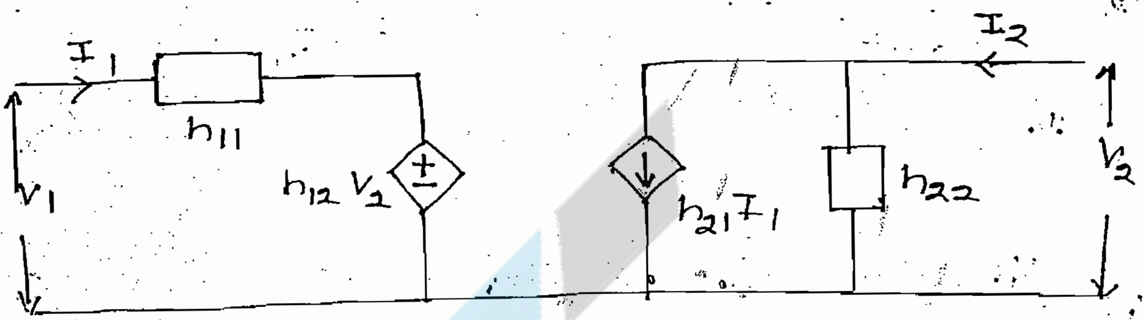
$\downarrow$

Unitless

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} \quad \left(h_{22} \neq Y_{22} \; , \; h_{22} = \frac{1}{Z_{22}} \right)$$

$\downarrow$

(mho)



g-Parameters :-

$$I_1 = g_{11} V_1 + g_{12} I_2 \quad \text{--- (I)}$$

$$V_2 = g_{21} V_1 + g_{22} I_2 \quad \text{--- (II)}$$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \quad \left(g_{11} \neq Y_{11} \; , \; g_{11} = \frac{1}{Z_{11}} \right)$$

$\downarrow$

mho

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} \quad \left(g_{21} = \frac{Z_{21}}{Z_{11}} \right)$$

$\downarrow$

Unitless

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} \quad \left(g_{12} = \frac{Y_{12}}{Y_{22}} \right)$$

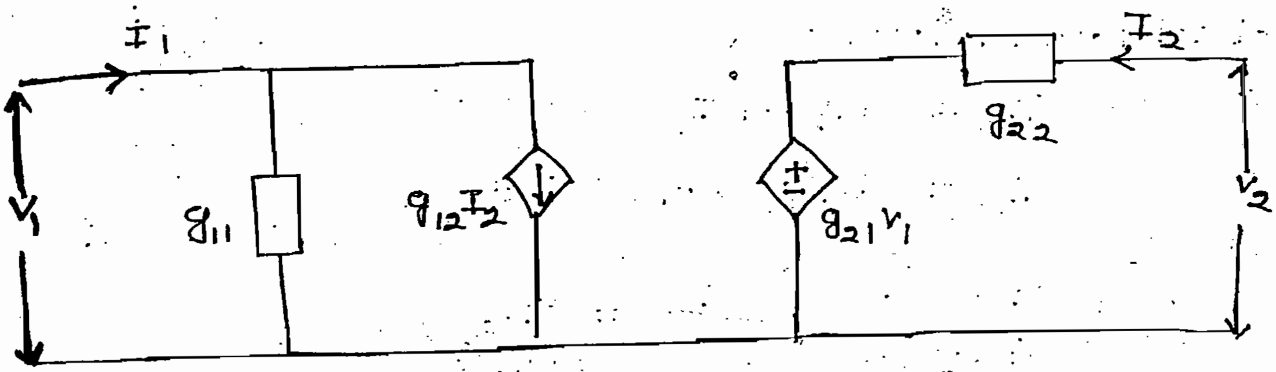
$\downarrow$

(Unitless)

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} \quad \left(g_{22} \neq Z_{22} \; , \; g_{22} = \frac{1}{Y_{22}} \right)$$

$\downarrow$

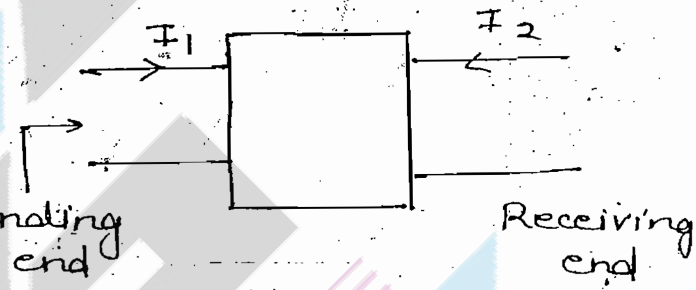
Ω



ABCD Parameters:-

$$V_1 = AV_2 - BI_2 \quad \text{—(i)}$$

$$I_1 = CV_2 - DI_2 \quad \text{—(ii)}$$



$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad \left(A = \frac{Z_{11}}{Z_{21}}\right)$$

↓
Unitless

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad \left(C = \frac{1}{Z_{21}}\right)$$

↓
mho

$$B = \left.\frac{-V_1}{I_2}\right|_{V_2=0} \qquad \left(B = -\frac{1}{Y_{21}}\right)$$

↓
Ω

$$D = \left.\frac{-I_1}{I_2}\right|_{V_2=0} \qquad \left(D = -\frac{Y_{11}}{Y_{21}}\right)$$

↓
Unitless

By using eq-(i) & (ii) it is not possible to develo eq ckt since both equations are developed w.r.t i/p

abcd Parameter:-

$$V_2 = aV_1 - bI_1 \quad \text{--- (I)}$$

$$I_2 = cV_1 - dI_1 \quad \text{--- (II)}$$

$$a = \left.\frac{V_2}{V_1}\right|_{I_1=0} \qquad \left(a = \frac{Z_{22}}{Z_{12}}\right)$$

↓

Unitless

$$c = \left.\frac{I_2}{V_1}\right|_{I_1=0} \qquad \left(c = \frac{1}{Z_{12}}\right)$$

↓

mho

$$b = \left.\frac{-V_2}{I_1}\right|_{V_1=0} \qquad \left(b = -\frac{1}{Y_{12}}\right)$$

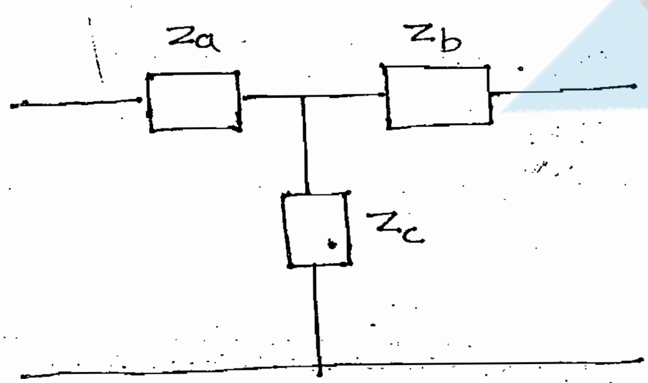
↓

Ω

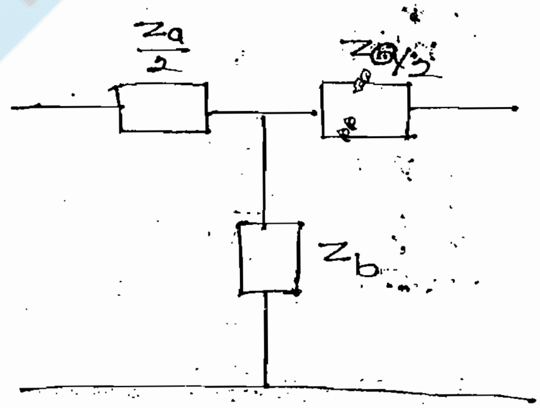
$$d = -\left.\frac{I_2}{I_1}\right|_{V_1=0} \qquad \left(d = -\frac{Y_{22}}{Y_{12}}\right)$$

↓

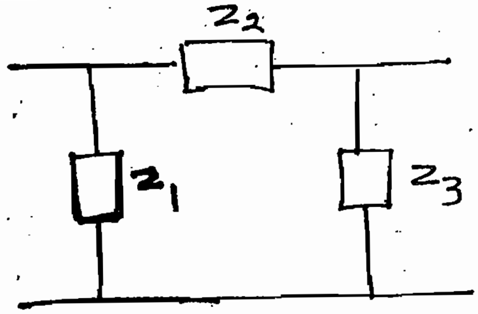
No units



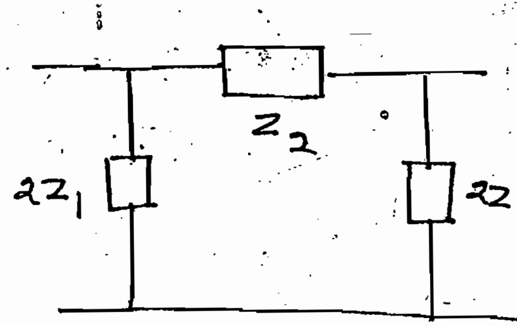
→ Unsymmetrical T-N/w



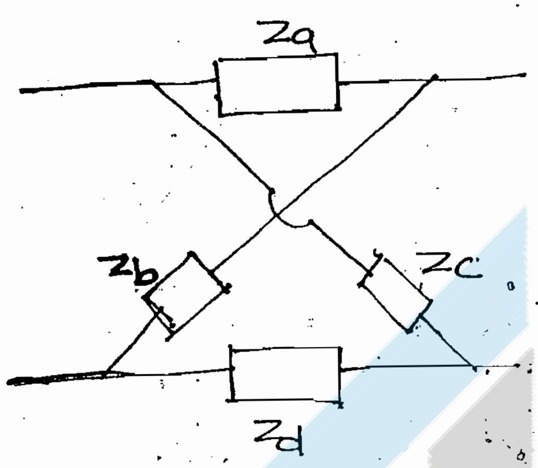
→ Symmetrical T-N/w



→ Unsymmetrical π N/ω



→ Symmetrical – π N/ω



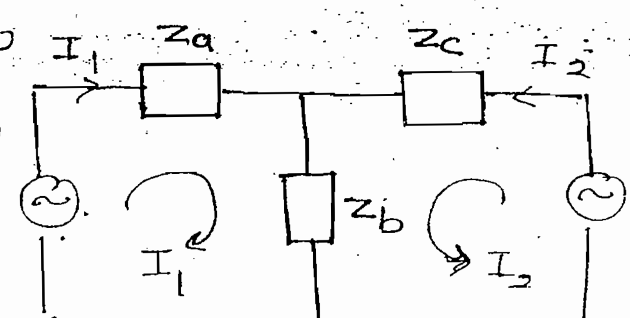
$Z_a = Z_d$, $Z_b = Z_c$ → Symmetrical Lattice N/ω

$Z_a \neq Z_d$, $Z_b \neq Z_c$ → Unsymmetrical Lattice N/ω

**

Symmetrical	Reciprocal
$Z_{11} = Z_{22}$	$Z_{12} = Z_{21}$
$Y_{11} = Y_{22}$	$Y_{12} = Y_{21}$
$h_{11}h_{22} - h_{12}h_{21} = 1$	$h_{12} = -h_{21}$
$A = D$	$AD - BC = 1$

Z-Parameter → Use T-N/ω



$V_1 = (Z_a + Z_b)I_1 + Z_b I_2$ —(I)

$V_1 = Z_{11} I_1 + Z_{12} I_2$ — (II)

$Z_{11} = Z_a + Z_b$

$Z_{12} = Z_b$

$$V_2 = Z_b I_1 + (Z_b + Z_c) I_2 \quad \text{--- (III)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (IV)}$$

$$Z_{21} = Z_b \quad \& \quad Z_{22} = Z_b + Z_c$$

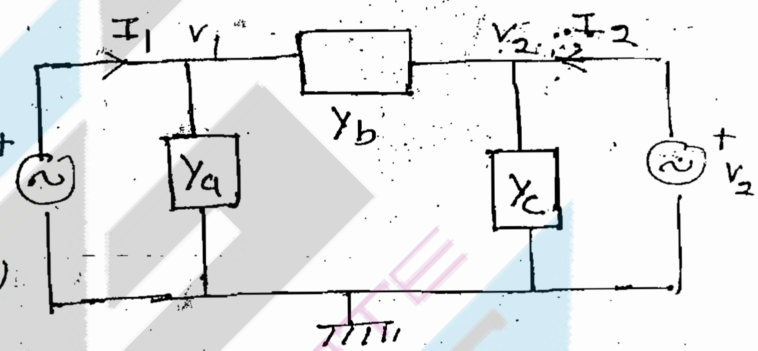
$$Z_{11} = Z_a + Z_b$$

$$Z_{11} = Z_a + Z_{12}$$

**

$$\boxed{\begin{aligned} Z_a &= Z_{11} - Z_{12} \\ Z_c &= Z_{22} - Z_{12} \\ Z_b &= Z_{12} = Z_{21} \end{aligned}}$$

For Y- Parameter → Convert complex N/w into π-N/w



$$I_1 = V_1 Y_a + (V_1 - V_2) Y_b$$

$$I_1 = (Y_a + Y_b) V_1 - Y_b V_2 \quad \text{--- (I)}$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{--- (II)}$$

$$Y_{11} = Y_a + Y_b$$

$$Y_{12} = -Y_b$$

$$I_2 = V_2 Y_c + (V_2 - V_1) Y_b$$

$$\Rightarrow \quad I_2 = -V_1 Y_b + (Y_b + Y_c) V_2 \quad \text{--- (III)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{--- (IV)}$$

$$Y_{21} = -Y_b \quad \& \quad Y_{22} = Y_b + Y_c$$

**

$$\boxed{\begin{aligned} Y_a &= Y_{11} + Y_{12} \\ Y_c &= Y_{22} + Y_{12} \\ Y_b &= -Y_{12} = -Y_{21} \end{aligned}}$$

$Z_a = Z_d \quad , Z_b = Z_c$

$\rightarrow$ symmetrical lattice N/w

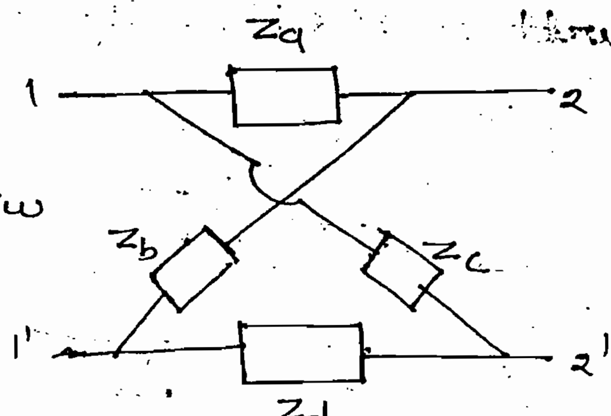


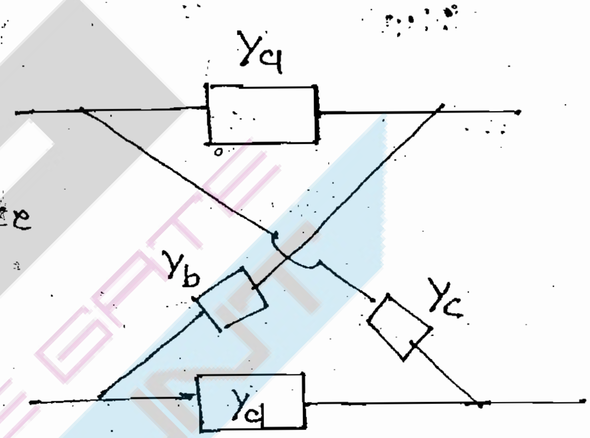
Fig - (1)

$$\boxed{\begin{aligned} Z_{11} = Z_{22} &= \frac{Z_b + Z_a}{2} \\ Z_{12} = Z_{21} &= \frac{Z_b - Z_a}{2} \end{aligned}}$$

$$\boxed{\begin{aligned} Z_b &= Z_{11} + Z_{12} \\ Z_a &= Z_{11} - Z_{12} \end{aligned}}$$

$Y_a = Y_d \quad , Y_b = Y_c$

$\rightarrow$ symmetrical lattice N/w



$$\boxed{\begin{aligned} Y_{11} = Y_{22} &= \frac{Y_b + Y_a}{2} \\ Y_{12} = Y_{21} &= \frac{Y_b - Y_a}{2} \end{aligned}}$$

$$\boxed{\begin{aligned} Y_b &= Y_{11} + Y_{12} \\ Y_a &= Y_{11} - Y_{12} \end{aligned}}$$

$\rightarrow$ Fig-(I) & (II) are same.

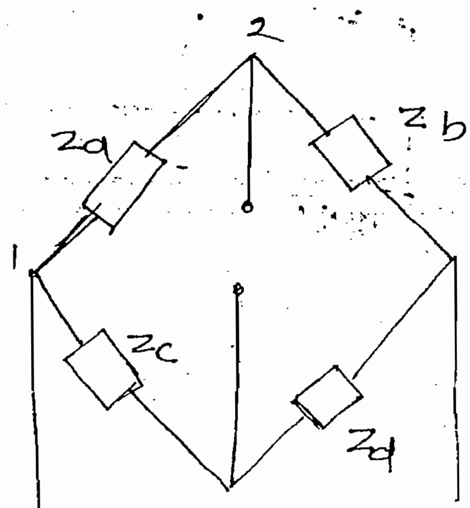
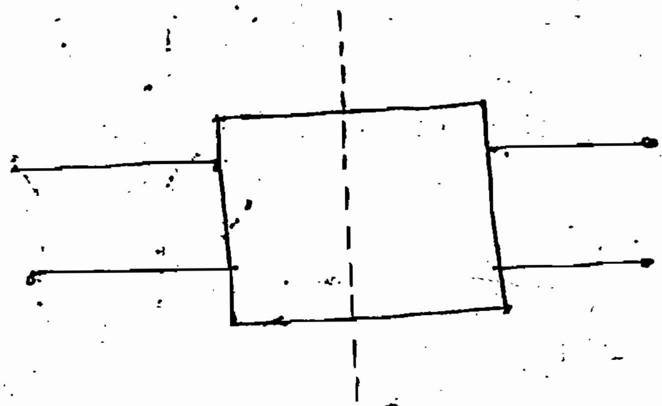


Fig-(II)

Bridge

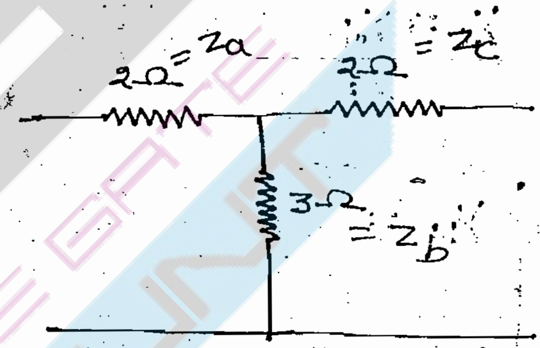
Bartlett's Bi-section theorem :-

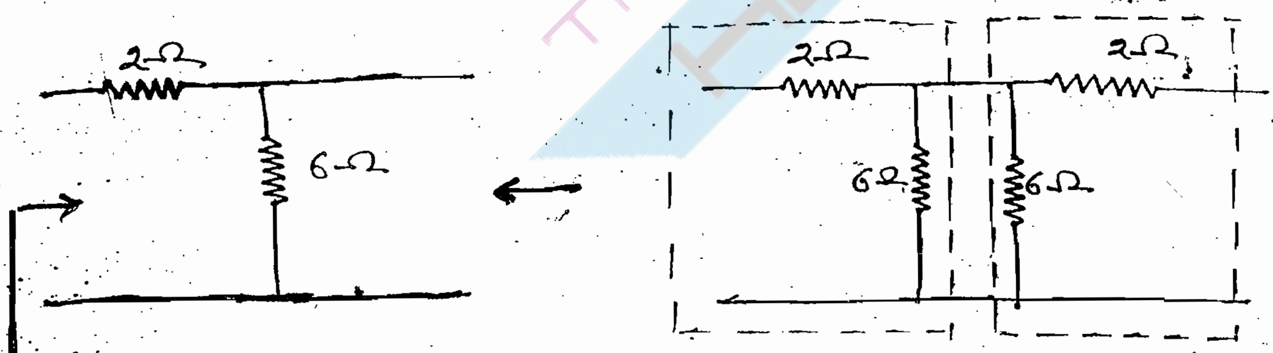
eg :- Filters



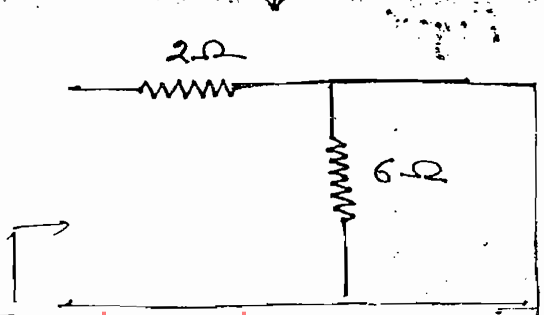
Symmetrical

$$\boxed{\begin{aligned} Z_{11} &= Z_{22} = \frac{Z_{OCH} + Z_{SCH}}{2} \\ Z_{12} &= Z_{21} = \frac{Z_{OCH} - Z_{SCH}}{2} \end{aligned}}$$





$Z_{OCH} = 2+6 = 8$



$$Z_{11} = Z_{22} = \frac{Z_{OCH} + Z_{SCH}}{2} = \frac{8+2}{2} = 5$$

$$Z_{12} = Z_{21} = \frac{Z_{OCH} - Z_{SCH}}{2} = \frac{8-2}{2} = 3$$

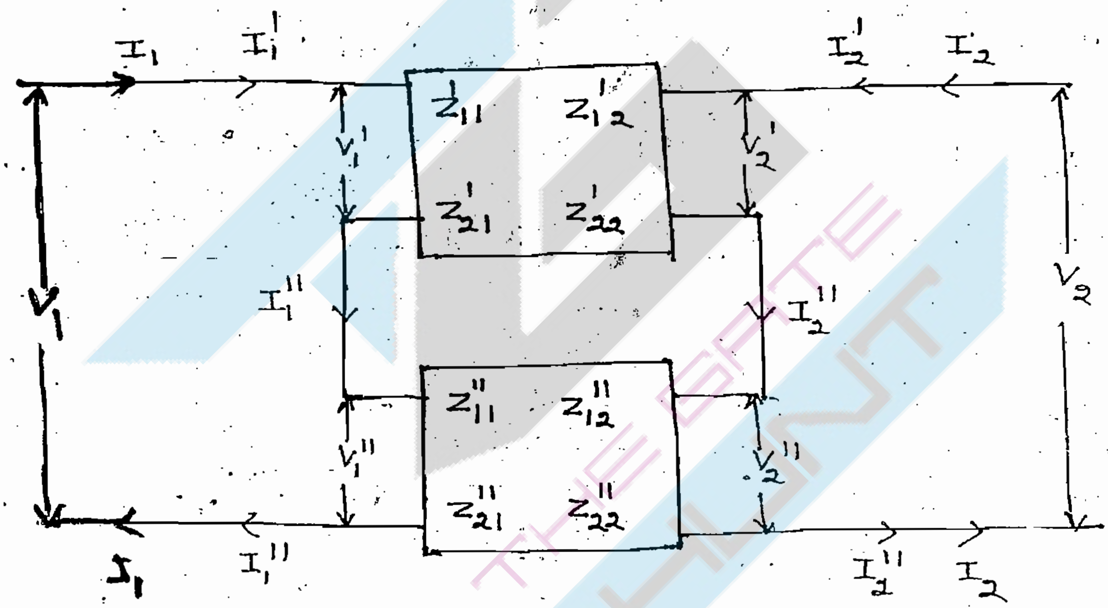
Alternate Way:-

$$Z_{11} = Z_a + Z_b = 5$$

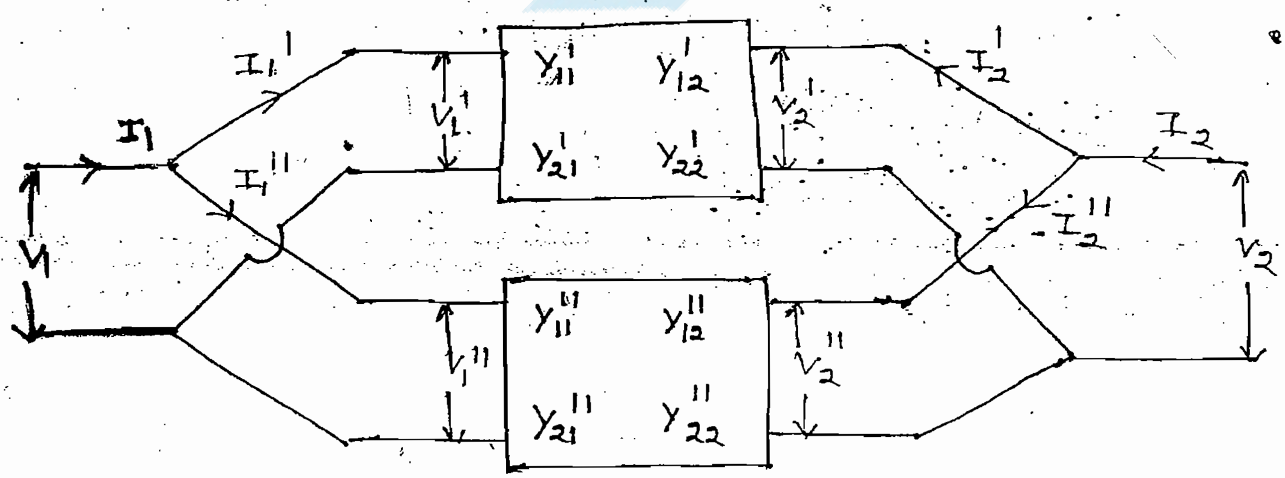
$$Z_{22} = Z_b + Z_c = 5$$

$$Z_{12} = Z_{21} = Z_b = 3$$

Series combination:-



Parallel combination:-



For series combination:—

$$V_1 = V_1' + V_1'' \qquad V_2 = V_2' + V_2''$$

$$I_1 = I_1' = I_1'' \qquad I_2 = I_2' = I_2''$$

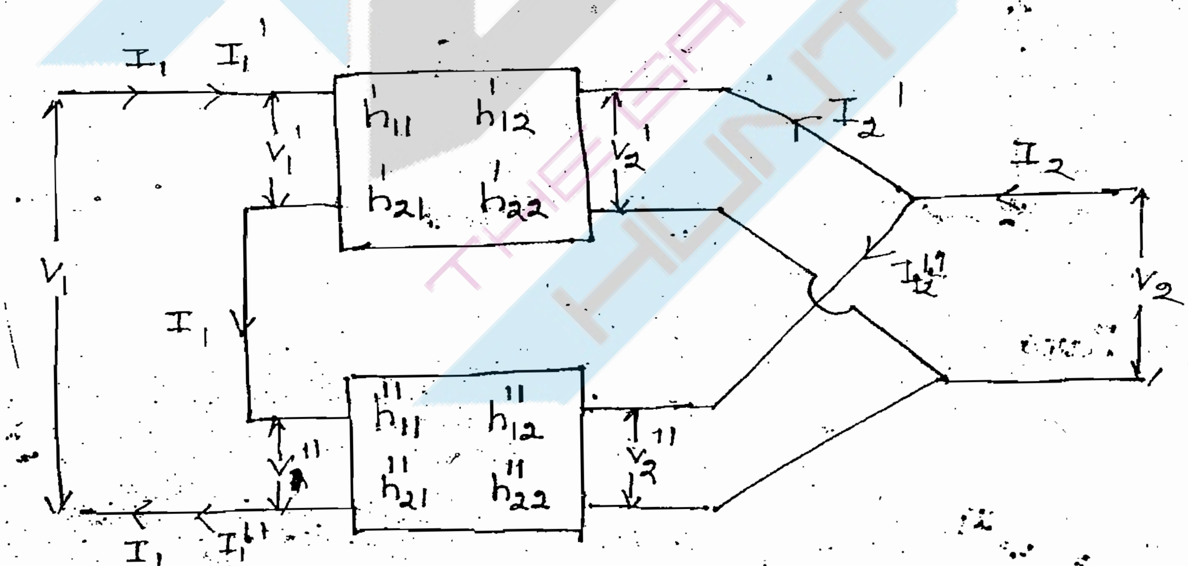
$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} z_{11}' & z_{12}' \\ z_{21}' & z_{22}' \end{bmatrix} + \begin{bmatrix} z_{11}'' & z_{12}'' \\ z_{21}'' & z_{22}'' \end{bmatrix}$$

For Parallel combination:—

$$V_1 = V_1' = V_1'' \qquad V_2 = V_2' = V_2''$$

$$I_1 = I_1' + I_1'' \qquad I_2 = I_2' + I_2''$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} y_{11}' & y_{12}' \\ y_{21}' & y_{22}' \end{bmatrix} + \begin{bmatrix} y_{11}'' & y_{12}'' \\ y_{21}'' & y_{22}'' \end{bmatrix}$$

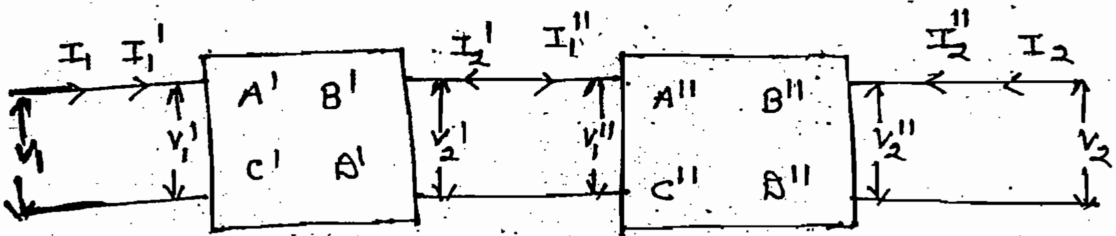


$$V_1 = V_1' + V_1'' \qquad V_2 = V_2' = V_2''$$

$$I_1 = I_1' = I_1'' \qquad I_2 = I_2' + I_2''$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} h_{11}' & h_{12}' \\ h_{21}' & h_{22}' \end{bmatrix} + \begin{bmatrix} h_{11}'' & h_{12}'' \\ h_{21}'' & h_{22}'' \end{bmatrix}$$

Cascade or Tandum Connection:-

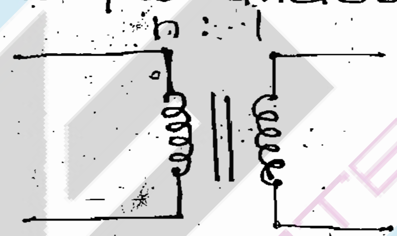


$$\boxed{\begin{array}{l} V_1 = V_1' \\ I_1 = I_1' \end{array}} \qquad \boxed{\begin{array}{l} V_2' = V_1'' \\ I_1'' = -I_2' \end{array}} \qquad \boxed{\begin{array}{l} V_2 = V_2'' \\ I_2 = I_2'' \end{array}}$$

$$\boxed{\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix}}$$

Ques:- Find Z, Y, ABCD parameters of the N/w shown:-

Ideal t/f



Soln:-

$$A = \frac{V_1}{V_2} = n$$

$$D = -\frac{I_1}{I_2} = +\frac{1}{n}$$

−ve is due to current direction

$$\boxed{\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{n}{1}}$$

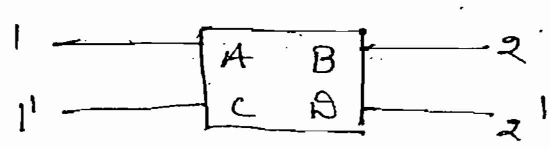
Note:-

In ideal t/f it is not possible to find impedance and admittance values since self and mutual inductance of ideal transformer are ∞

Ques:- Find O.C & S.C impedance w.r.t

(I) 1−1'

(II) 2−2'



Soln:- (I) 1-1' :-

$$Z_{i/p} = \frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2}$$

$$(Z_{i/p})_{o.c} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{A}{C}\ \Omega$$

$$(Z_{i/p})_{s.c} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{B}{D}\ \Omega$$

(II) 2-2' :-

$$(Z_{o/p})_{o.c} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$\therefore I_1 = CV_2 - DI_2 = 0$$

$$\Rightarrow \frac{V_2}{I_2} = \frac{D}{C}$$

$$(Z_{o/p})_{o.c} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{D}{C}\ \Omega$$

$$\therefore (Z_{o/p})_{s.c} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

$$V_1 = AV_2 - BI_2$$

$$\Rightarrow 0 = AV_2 - BI_2$$

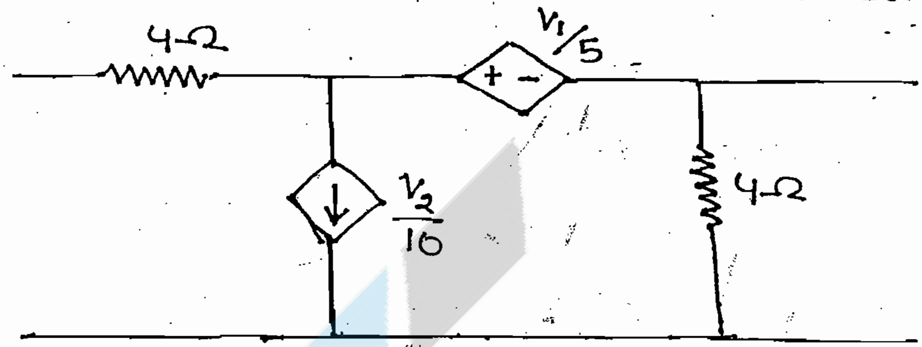
$$\Rightarrow \frac{V_2}{I_2} = \frac{B}{A}$$

$$(Z_{o/p})_{s.c} = \frac{B}{A}\ \Omega$$, Ans.

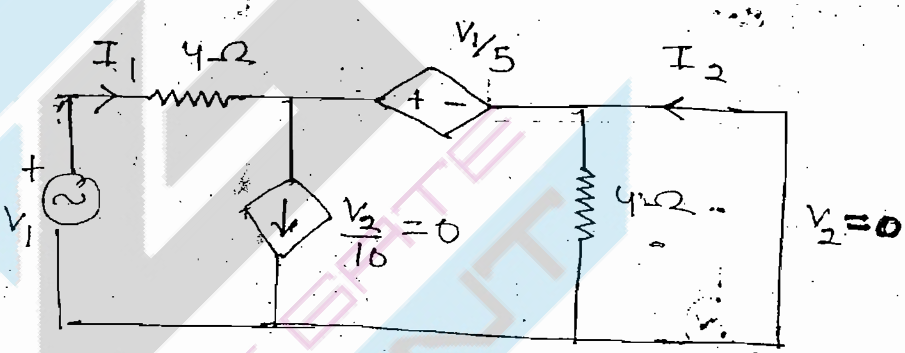
$$B = -\frac{V_1}{I_2}\bigg|_{V_2=0} \Rightarrow \frac{2s+1}{s^2} = \frac{eq-(1)}{eq-(4)}$$

$$\Rightarrow \boxed{Z(S) = \frac{1}{S}} \quad \text{Ans}$$

Ques:- Find B and D of the circuit shown



Soln:-



$$B = -\frac{V_1}{I_2}\bigg|_{V_2=0}$$

$$D = \frac{-I_1}{I_2}\bigg|_{V_2=0}$$

$$= \frac{-(-I_2)}{I_2} = 1$$

$$I_1 = -I_2$$

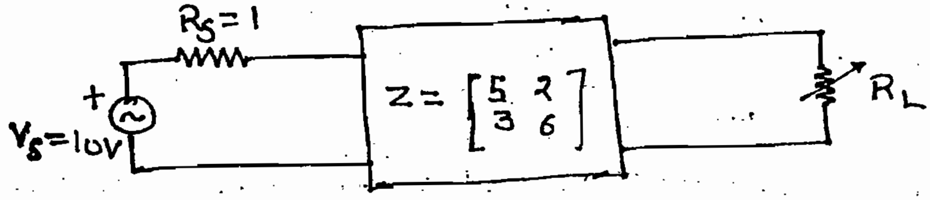
$$I_1 = \frac{V_1 - \frac{V_1}{5}}{4}$$

$$-I_2 = \frac{4V_1/5}{4}$$

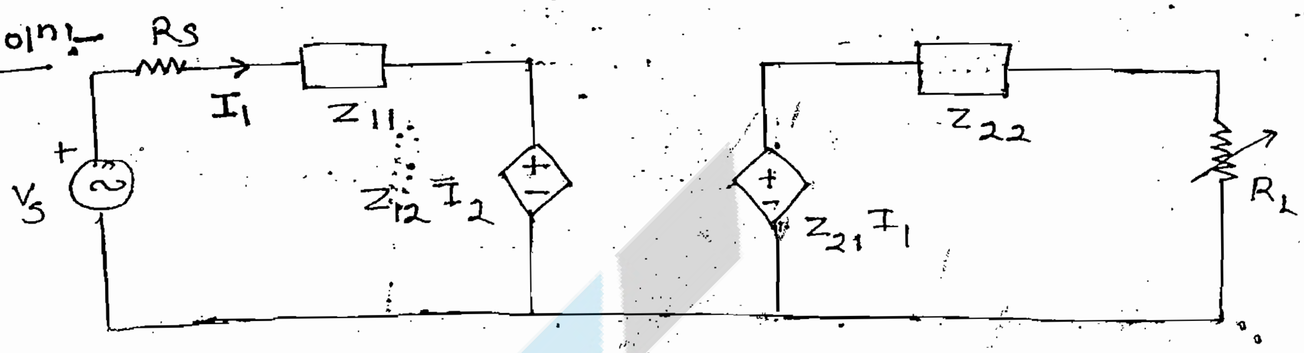
$$\Rightarrow \boxed{B = -\frac{V_1}{I_2} = 5} \quad \text{Ans}$$

Lecture-12

Ques:- Find max. power dissipation in load resistor

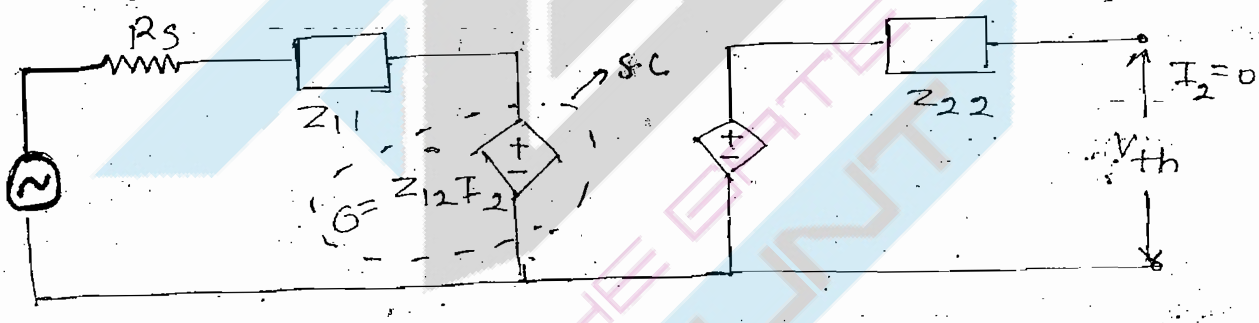


Soln:-



Case-(1):- V_{th}

Disconnect load resistor



$$I_1 = \frac{V_S}{R_S + Z_{11}} \quad \text{---(I)}$$

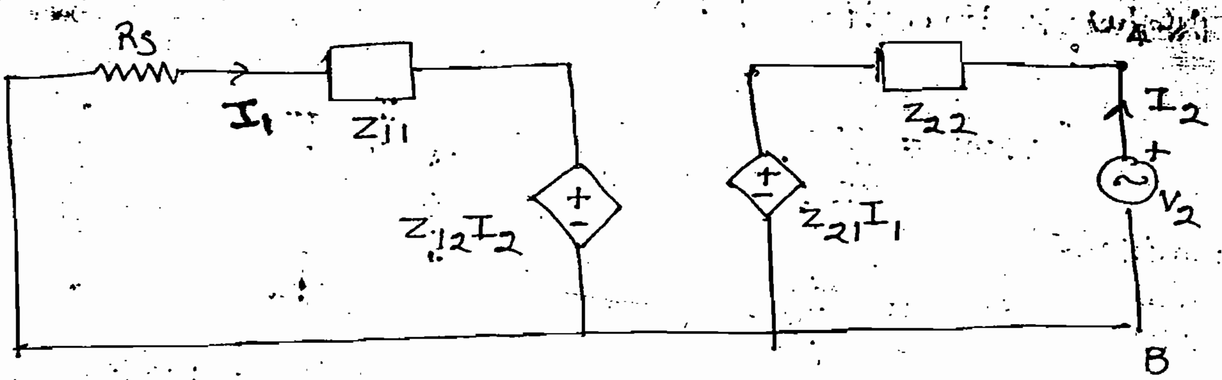
$$V_{th} = Z_{21} I_1 \quad \text{---(II)}$$

Substitute eq-(I) in eq- (II)

$$V_{th} = Z_{21} \frac{V_S}{R_S + Z_{11}}$$

$$V_{th} = \frac{3 \times 10}{1+5} = 5$$

Case-(II) (R_{th}) :—



$$Z_{th} = \frac{V_2}{I_2} \qquad I_2 = \frac{V_2 - z_{21}I_1}{z_{22}}$$

$$I_1 = -\frac{z_{12}I_2}{R_s + z_{11}} \quad \text{—(I)} \qquad I_2 z_{22} = V_2 - z_{21}I_1$$

$$V_2 = I_2 z_{22} + z_{21}I_1 \quad \text{—(II)}$$

Substitute eq-(I) in eq-(II).

$$V_2 = I_2 z_{22} - \frac{z_{12}z_{21}I_2}{R_s + z_{11}}$$

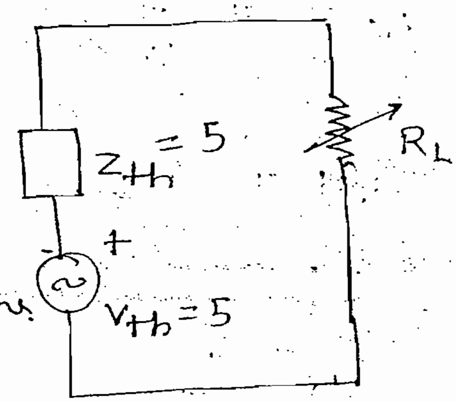
$$Z_{th} = \frac{V_2}{I_2} = z_{22} - \frac{z_{12}z_{21}}{R_s + z_{11}}$$

$$= 6 - \frac{2\times 3}{1+5} \Rightarrow Z_{th} = 5$$

$$R_L = R_{th} = 5\Omega$$

$$P_{max} = \frac{V_{th}^2}{4R_L}$$

$$= \frac{\cancel{25}\ 5}{4\times\cancel{5}} = 1.25\,W, \text{ Ans.}$$

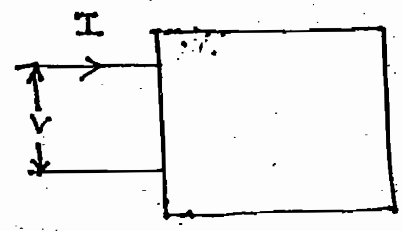


Network Functions :-

Immittance (common Name)

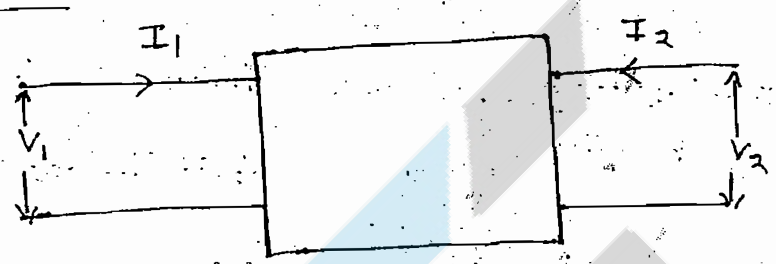
$$Z(s) = \frac{V(s)}{I(s)}$$

$$Y(s) = \frac{I(s)}{V(s)}$$



Single Port

Two Port :-



Driving Point I/p impedance

(I) $Z_{11}(s) = \frac{V_1(s)}{I_1(s)}$ $\quad Z_{22}(s) = \frac{V_2(s)}{I_2(s)}$

(II) $Y_{11}(s) = \frac{I_1(s)}{V_1(s)}$ $\quad Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$ → Driving pt admittance function

(III) $Z_{12}(s) = \frac{V_1(s)}{\cancel{V_2}(s)}$ I_2 $\quad Z_{21}(s) = \frac{V_2(s)}{I_1(s)}$ → Transfer impedance ratio

(IV) $Y_{12}(s) = \frac{I_1(s)}{V_2(s)}$ $\quad Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$ → Transfer admittance ratio

(V) $G_{12} = \frac{V_1(s)}{V_2(s)}$ $\quad G_{21} = \frac{V_2(s)}{V_1(s)}$ → Transfer voltage ratio

(VI) $\alpha_{12} = \frac{I_1(s)}{I_2(s)}$ $\quad \alpha_{21} = \frac{I_2(s)}{I_1(s)}$ → Transfer current ratio

→ N/w parameter is calculated at pre-defined condition (either O.C or S.C)

eg:- $Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \rightarrow$ O.C $\quad Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \rightarrow$ S.C.

→ To find the N/w function no pre-defined condition is required

→ To represent the N/w simultaneously four parameter are required

eg:- To develop T-N/w z_{11}, z_{12}, z_{21} & z_{22} are required.

→ By using only one N/w function it is possible to represent the N/w.

Network Synthesis:→

→ In the N/w analysis for a given N/w it is possible to find either voltage response or current response or impedance function or admittance function

→ In a N/w synthesis, for a given function N/w is designed (It is reverse procedure of N/w analysis)

→ In a N/w synthesis it is possible to design the N/w for the following functions

(I) Single port → 1. Z(S), 2. Y(S) → Immittance — Driving Point Immittance func

(II) Two port → 1. $Z_{11}(S), Z_{22}(S), Y_{11}(S), Y_{22}(S)$ ↑

2. $Z_{12}(S), Z_{21}(S), Y_{12}(S), Y_{21}(S), G_{12}, G_{21}, \alpha_{12}, \alpha_{21}$ → Transfer function

→ In the N/w synthesis for given function it is possible to design the following N/w

(I) Series (II) Parallel (III) Ladder

10 PU

(I) Series → Foster-I form
(II) Parallel → Foster-II form
Partial fraction of expansion

(III) Ladder → Cauer-I form / Cauer-II form
Continued fraction of expansion

Necessary condition to design a N/w :-

(I) F(s) should be ~~primary~~ positive real function (PRF)

$R \geq 0$
$L \geq 0$ → Positive Real
$C \geq 0$

(II) If F(s) is PRF then $\frac{1}{F(s)}$ also PRF

(III) If $F_1(s)$ and $F_2(s)$ are PRF then

$$F(s) = F_1(s) + F_2(s) \text{ are always PRF}$$

(iv) All the poles of the function should be present in the left half of the plane

(V) Imaginary poles and zeroes should be conjugate pairs

(VI) In the partial fraction of expansion residue should be positive real

(VII) Numerator and denominator polynomials should satisfy Hurwitz.

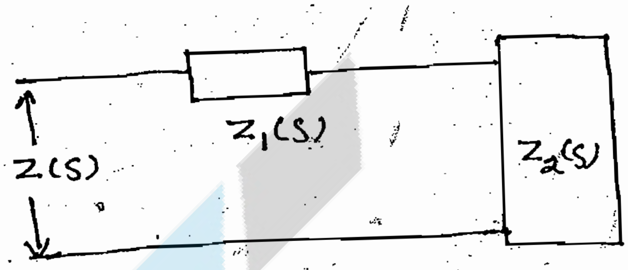
(VIII) The highest power of Numerator and denominator of polynomial should be differ at ~~by~~ atmost unity. This condition prohibits multiple pole and zero at infinity.

9 → The lowest power of Numerator and denominator should be differ by atmost unity. This conditions prohibits multiple poles and zeroes at origin

10 → Total No. of poles of a function should be equal to total no. of zeroes.

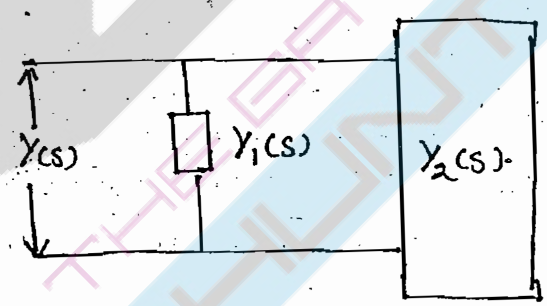
$$Z(s) = Z_1(s) + Z_2(s)$$

$$\Rightarrow \quad Z_2(s) = Z(s) - Z_1(s)$$



$$Y(s) = Y_1(s) + Y_2(s)$$

$$\Rightarrow \quad Y_2(s) = Y(s) - Y_1(s)$$



Removal of Pole at ∞ :—

$$Z(s) = \frac{b_{n+1}s^{n+1} + b_n s^n + \cdots + b_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_0}$$

Numerator Power > denominator power

$\longrightarrow$ pole exist at ∞.

By long division

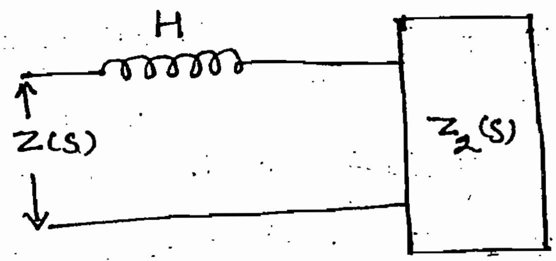
$$Z(s) = \frac{b_{n+1}s^{n+1}}{a_n s^n} + Z_2(s)$$

$$\left(H = \frac{b_{n+1}}{a_n}\right)$$

$$Z(s) = Hs + Z_2(s)$$

$$Z_2(s) = Z(s) - Hs$$

$X_L = Ls$



$$Y(s) = \underset{\downarrow}{Hs} + Y_2(s)$$

$$B_C = sC$$

$$Y_2(s) = Y(s) - Hs$$

Removal of pole at origin :-

$$Z(s) = \frac{b_0 + \cdots + b_{n-1}s^{n-1} + b_n s^n}{s(a_0 + \cdots + a_{n-1}s^{n-1} + a_n s^n)}$$

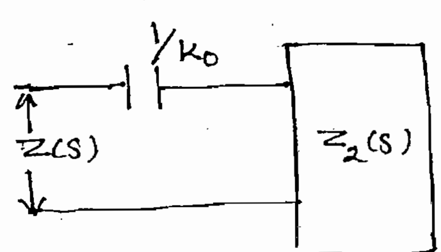
By long division

$$Z(s) = \frac{b_0}{s a_0} + Z_2(s) \quad \left(k_0 = \frac{b_0}{a_0} \right)$$

$$Z(s) = \frac{k_0}{s} + Z_2(s)$$

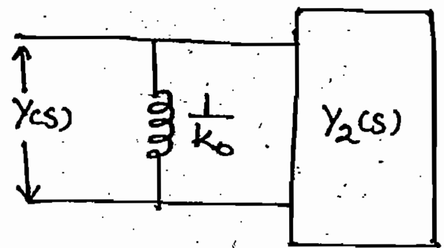
$$\Rightarrow Z_2(s) = Z(s) - \frac{k_0}{s}$$

$X_C = \frac{1}{Cs}$



$$Y(s) = \frac{k_0}{s} + Y_2(s)$$

$$Y_2(s) = Y(s) - \frac{k_0}{s}$$



Removal of conjugate Pair of Poles:—

$$Z(s) = Z_1(s) + Z_2(s)$$

$$Z_2(s) = Z(s) - Z_1(s)$$

$$Z_1(s) \longrightarrow \text{Poles} \Rightarrow \pm j\omega$$

$$Z_1(s) = \frac{k_1}{s+j\omega} + \frac{k_2}{s-j\omega} \qquad (k_1 = k_2 = k)$$

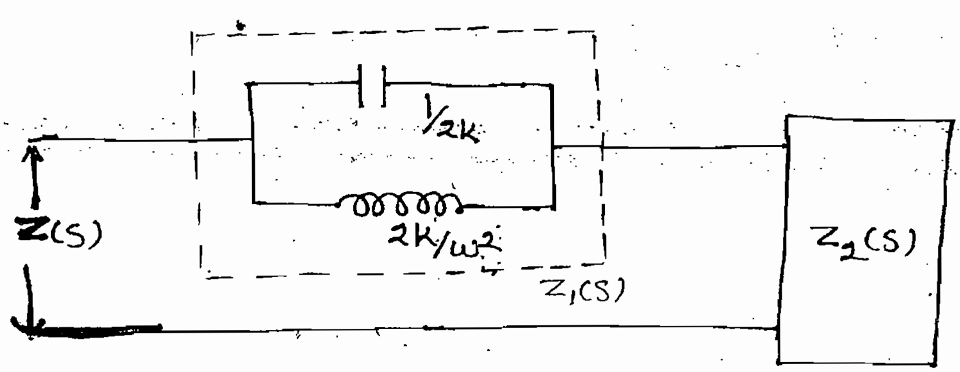
$$Z_1(s) = \frac{k}{s+j\omega} + \frac{k}{s-j\omega}$$

$$Z_1(s) = \frac{2ks}{s^2+\omega^2}$$

$$Z_1(s) = \frac{1}{\frac{s^2}{2ks} + \frac{\omega^2}{2ks}} = \frac{1}{Y_a + Y_b}$$

$B_C = sC$

$B_L = \frac{1}{Ls}$



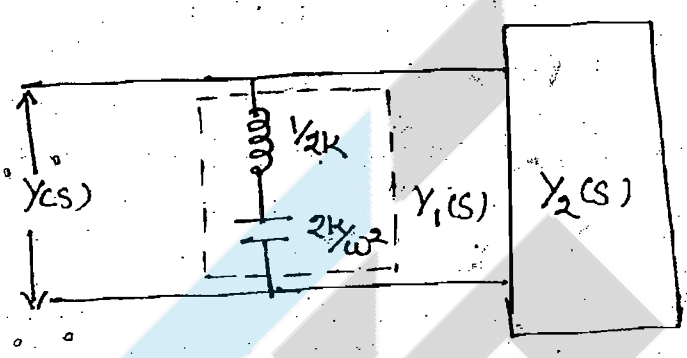
$$Y(s) = Y_1(s) + Y_2(s)$$

$$Y_2(s) = Y(s) - Y_1(s)$$

$$Y_1(s) = \frac{1}{\frac{s}{2k} + \frac{\omega^2}{2ks}} = \frac{1}{Z_a + Z_b}$$

$X_L = Ls$

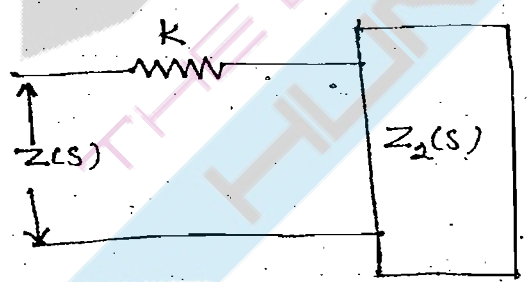
$X_C = \frac{1}{cs}$



Removal of constant:

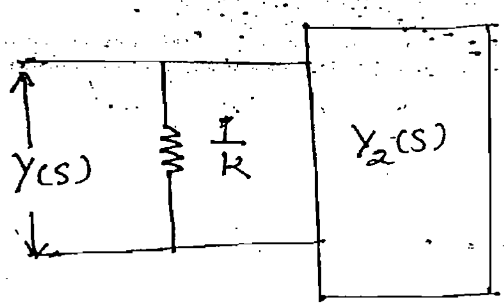
$$Z(s) = k + Z_2(s)$$

$$Z_2(s) = Z(s) - k$$



$$Y(s) = k + Y_2(s)$$

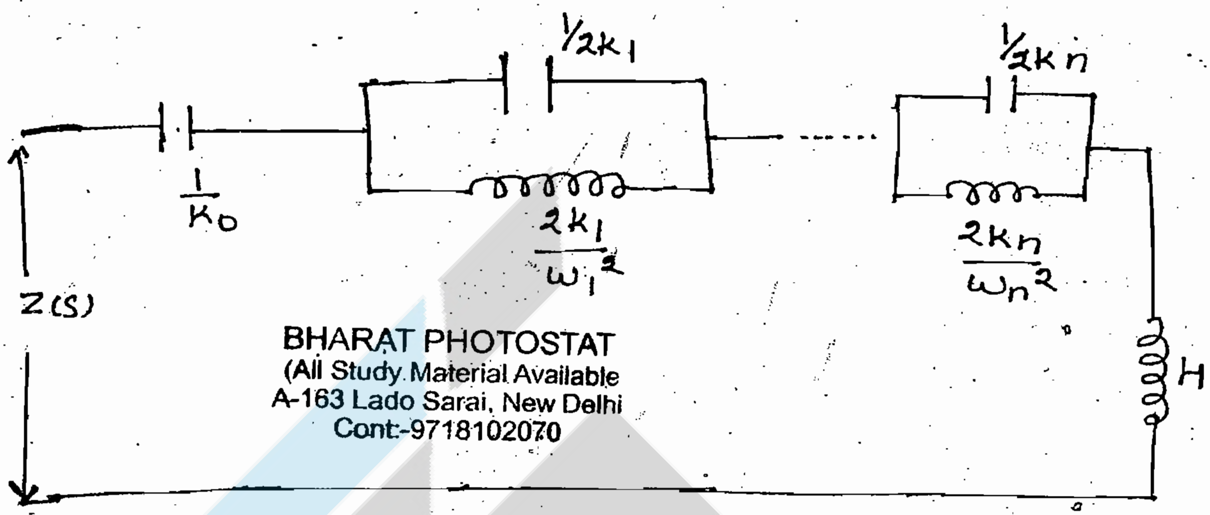
$$Y_2(s) = Y(s) - k$$



LC N/w Foster-I form :- (Series)

$$Z(s) = \frac{k_0}{s} + \sum_{i=1}^{n} \frac{2k_i s}{s^2+\omega_i^2} + Hs$$

$$X_c = \frac{1}{Cs} \qquad (X_L = Ls)$$

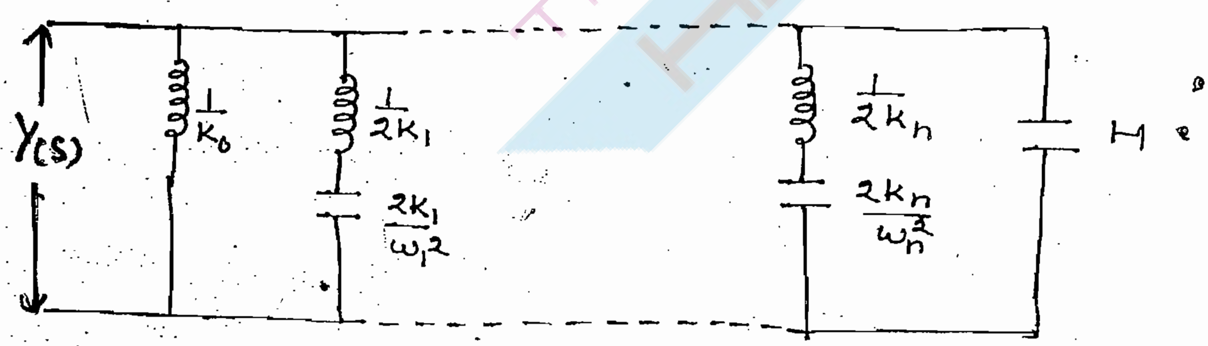


Lecture-13

LC N/w Foster-II form :- (Parallel)

$$Y(s) = \frac{k_0}{s} + \sum_{i=1}^{n} \frac{2k_i s}{s^2+\omega_i^2} + Hs$$

$$B_L = \frac{1}{Ls} \qquad B_C = sC$$



Ques:- obtain foster-I & II from Z(s)

$$Z(s) = \frac{(s^2+2)(s^2+4)}{s(s^2+3)}$$

Soln :- **Foster - I** :-

Pole $\rightarrow \infty \rightarrow Hs$

Pole $\rightarrow$ origin $\rightarrow k_0/s$

Pole $\rightarrow$ conjugate pair $\rightarrow \dfrac{2ks}{s^2+\omega^2}$

$$Z(s) = \frac{k_0}{s} + \frac{2ks}{s^2+\omega^2} + Hs$$

$$\frac{(s^2+2)(s^2+4)}{s(s^2+3)} = \frac{k_0}{s} + \frac{2ks}{s^2+3} + Hs \quad \text{(Partial Fraction)}$$

$$k_0 = \frac{8}{3} \qquad 2k \rightarrow \frac{1}{3} \qquad H \rightarrow 1$$

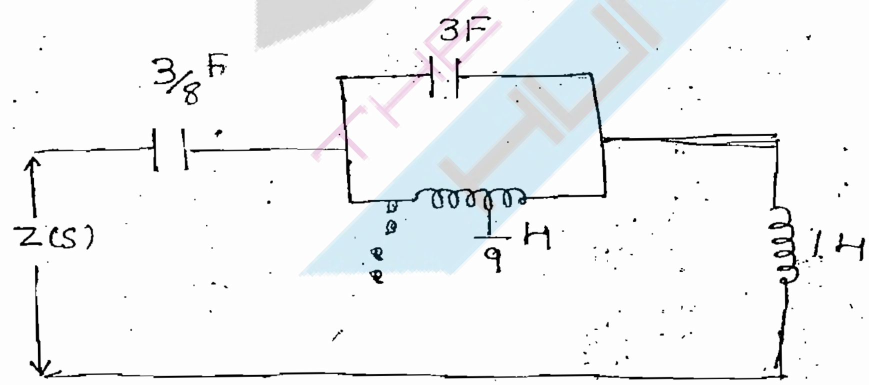
$$Z(s) = \frac{8/3}{s} + \frac{s/3}{s^2+3} + s$$

$$\Rightarrow Z(s) = \frac{1}{\frac{3}{8}s} + \frac{1}{\frac{s^2}{\frac{s}{3}} + \frac{3}{\frac{s}{3}}} + s$$

$$\Rightarrow Z(s) = \frac{1}{\frac{3}{8}s} + \frac{1}{3s + \frac{9}{s}} + s$$

$$X_C = \frac{1}{Cs} \qquad B_C = sC \qquad B_L = \frac{1}{Ls}$$

$$\frac{1}{Y_a + Y_b}$$



Foster - II :-

Note :- If F(s) is given by using same function foster - I and II form are obtained.

$\rightarrow$ If Z(s) is given by using $\frac{1}{Z(s)}$ function foster-II form is obtained.

$\rightarrow$ If Y(s) is given by using $\frac{1}{Y(s)}$ function foster-I form is obtained.

$$Y(s) = \frac{s(s^2+3)}{(s^2+2)(s^2+4)}$$

$P \to \infty$ ✗

$P \to$ origin ✗

$P \to$ conjugate pair ✓

$$Y(s) = \frac{2k_1 s}{s^2+\omega_1^2} + \frac{2k_2 s}{s^2+\omega_2^2}$$

$$\frac{s(s^2+3)}{(s^2+2)(s^2+4)} = \frac{2k_1 s}{s^2+2} + \frac{2k_2 s}{s^2+4}$$

$$2k_1 \to \frac{1}{2} \quad \$ \quad 2k_2 \to \frac{1}{2}$$

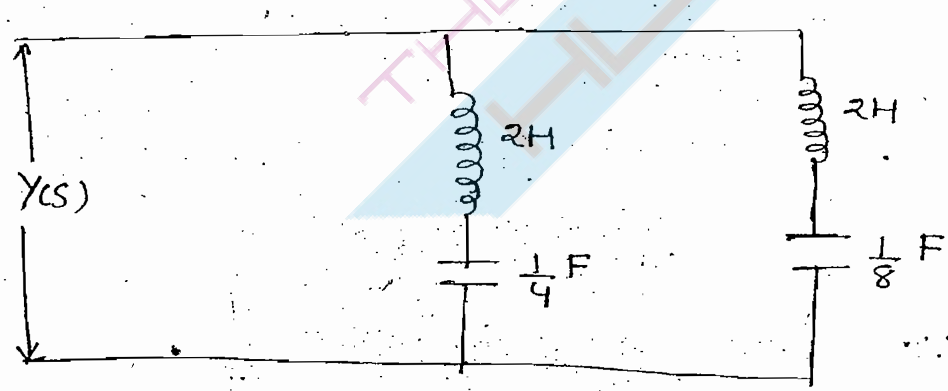
$$Y(s) = \frac{s/2}{s^2+2} + \frac{s/2}{s^2+4}$$

$$\Rightarrow Y(s) = \frac{1}{\frac{s^2}{s/2} + \frac{2}{s/2}} + \frac{1}{\frac{s^2}{\frac{s}{2}} + \frac{4}{\frac{s}{2}}}$$

$$\Rightarrow Y(s) = \frac{1}{2s + \frac{4}{s}} + \frac{1}{2s + \frac{8}{s}}$$

$X_L = Ls \quad X_C = \frac{1}{Cs} \qquad X_L \quad X_C$

$$\frac{1}{Z_a+Z_b} \qquad \frac{1}{Z_c+Z_d}$$



LC N/w Cauer-I form :- → Ladder

(1) Removal of Pole at ∞ :-

$$Z_2(s) = Z(s) - H_1 s$$
$$(X_L = Ls)$$

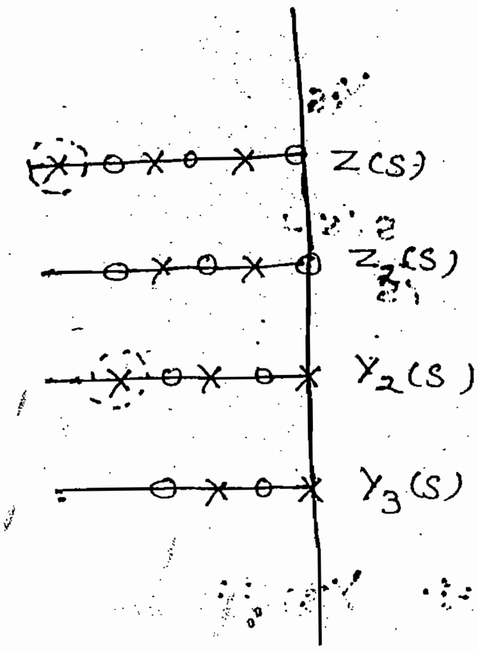
$$Y_2(s) = \frac{1}{Z_2(s)}$$

$$Y_3(s) = Y_2(s) - H_2 s$$
$$(B_C = sC)$$

$$Z_3(s) = \frac{1}{Y_3(s)}$$

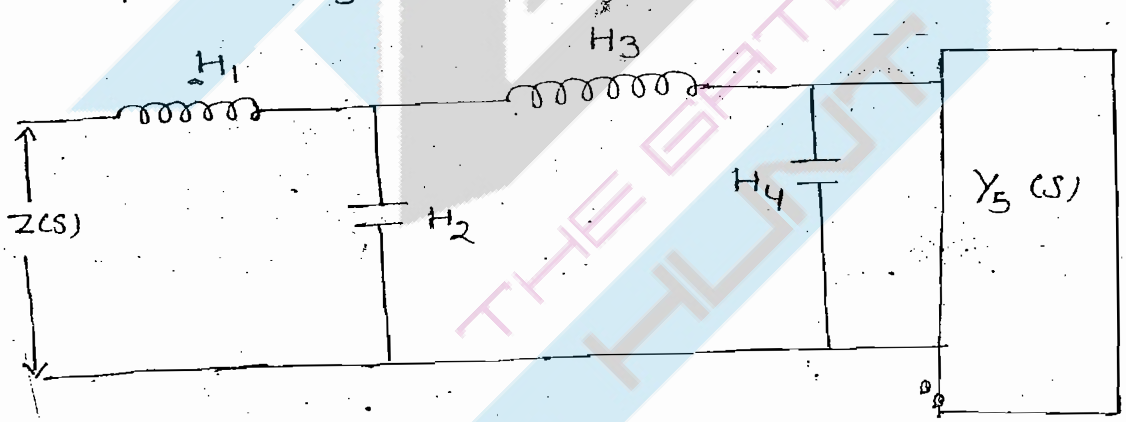
$$Z_4(s) = Z_3(s) - H_3 s$$

$$Y_4(s) = \frac{1}{Z_4(s)}$$



$$Y_5(s) = Y_5(s) - H_4 s$$

$$Y_4(s) = Y_5(s) + H_4 s$$



LC N/w Cauer-II form :-

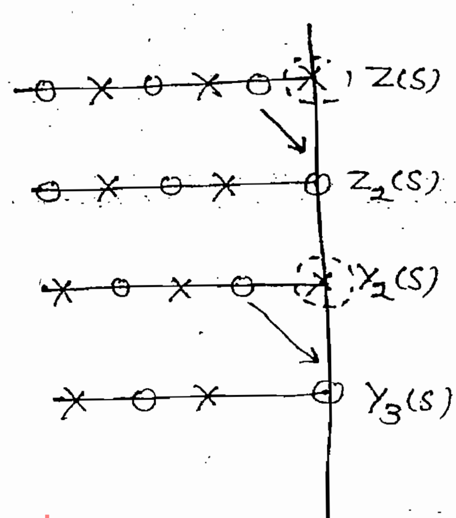
Removal of Pole at origin :-

$$Z_2(s) = Z(s) - \frac{k_{01}}{s} \rightarrow \left(X_C = \frac{1}{Cs}\right)$$

$$Y_2(s) = \frac{1}{Z_2(s)}$$

$$Y_3(s) = Y_2(s) - \frac{k_{02}}{s} \quad \left(B_L = \frac{1}{Ls}\right)$$

$$Z_3(s) = \frac{1}{Y_3(s)}$$

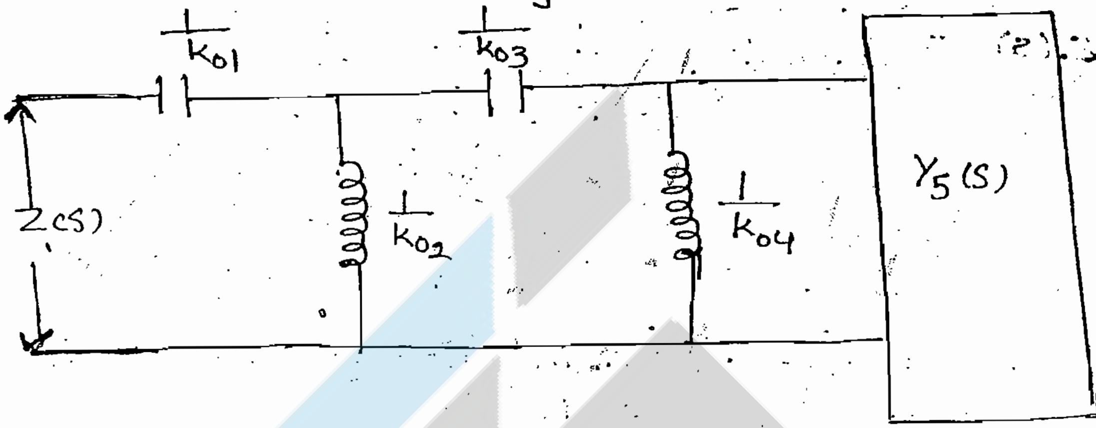


$$\therefore Z_4(s) = Z_3(s) - \frac{k_{03}}{s}$$

$$Y_4(s) = \frac{1}{Z_4(s)}$$

$$Y_5(s) = Y_4(s) - \frac{k_{04}}{s}$$

$$Y_4(s) = Y_5(s) + \frac{k_{04}}{s}$$



Ques:- Obtain Cauer - I and II form

$$Z(s) = \frac{s^3+2s}{s^4+5s^2+4}$$

Soln:- Cauer - I :-

Pole at $\infty \longrightarrow$ not present

then reciprocal

$$Y(s) = \frac{s^4+5s^2+4}{s^3+2s}$$

$$s^3+2s \,\big)\, s^4+5s^2+4 \,\big(\, s \; \begin{cases} \rightarrow B_C = sC \\ \rightarrow Y \end{cases}$$

$$s^4+2s^2$$

$$3s^2+4 \,\big)\, s^3+2s \,\big(\, \frac{s}{3} \; \begin{cases} \rightarrow X_L = Ls \\ \rightarrow Z \end{cases}$$

$$s^3+\frac{4s}{3}$$

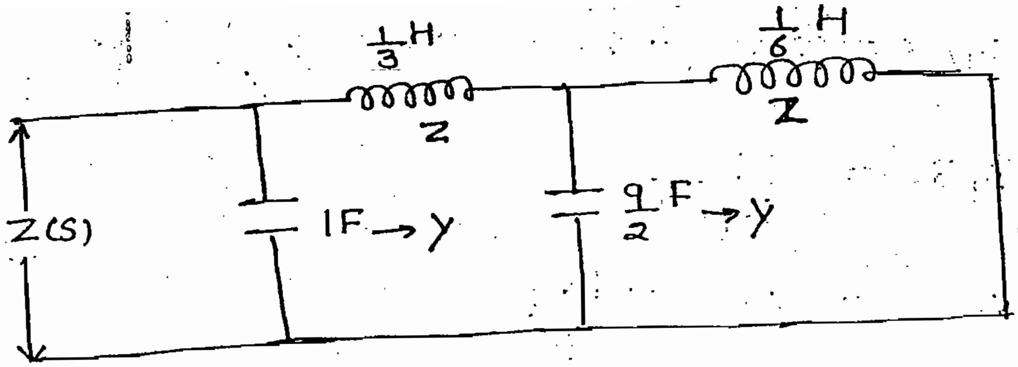
$$\frac{2s}{3} \,\big)\, 3s^2+4 \,\big(\, \frac{9}{2}s \; \begin{cases} \rightarrow B_C = sC \\ \rightarrow Y \end{cases}$$

$$3s^2$$

$$4 \,\big)\, \frac{2s}{3} \,\big(\, \frac{s}{6} \; \begin{cases} \rightarrow X_L = \\ \rightarrow Z \end{cases}$$

$$\frac{2s}{3}$$

$$\times$$



$$Z(s) = \cfrac{1}{s + \cfrac{1}{\cfrac{s}{3} + \cfrac{1}{\cfrac{9s}{2} + \cfrac{1}{\cfrac{s}{6}}}}}$$

Cauer – II Form :–

Pole → origin not present then take reciprocal

$$Z(s) = \frac{s(s^2+2)}{s^4+5s^2+4}$$

$$Y(s) = \frac{s^4+5s^2+4}{s^3+2s}$$

$2s+s^3 \overline{)\, s^4+5s^2+4\,}\left(\frac{2}{s}\right.$ → Y, $B_L = \frac{1}{Ls}$

$2s^2+4$

$3s^2+s^4 \overline{)\, 2s+s^3\,}\left(\frac{2}{3s}\right.$ → Z, $X_C = \frac{1}{Cs}$

$2s+\frac{2s^3}{3}$

$\frac{s^3}{3} \overline{)\, 3s^2+s^4\,}\left(\frac{9}{s}\right.$ → Y, B_L

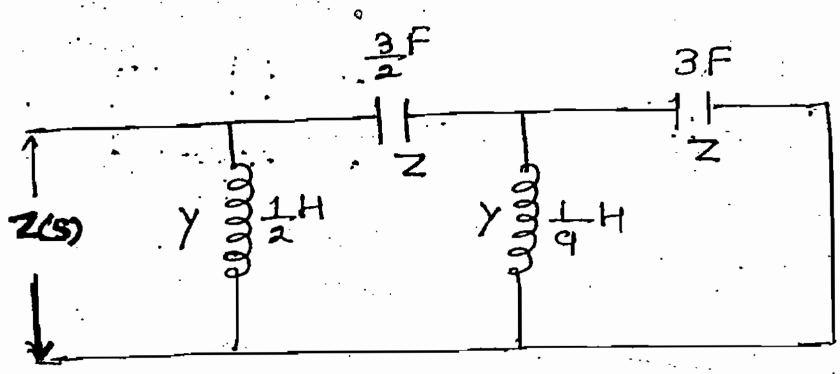
$3s^2$

$s^4 \overline{)\, \frac{s^3}{3}\,}\left(\frac{1}{3s}\right.$ → Z, X_C

$\frac{s^3}{3}$

X

PHOTOSTAT



$$Z(s) = \cfrac{1}{\cfrac{2}{s} + \cfrac{1}{\cfrac{2}{3s} + \cfrac{1}{\cfrac{9}{s} + \cfrac{1}{\cfrac{1}{3s}}}}}$$

RC N/w Foster-I form :-

$$Z_{Lc}(s) = \frac{k_0}{s} + \sum_{i=1}^{n} \frac{2k_i s}{s^2+\omega_i^2} + Hs$$

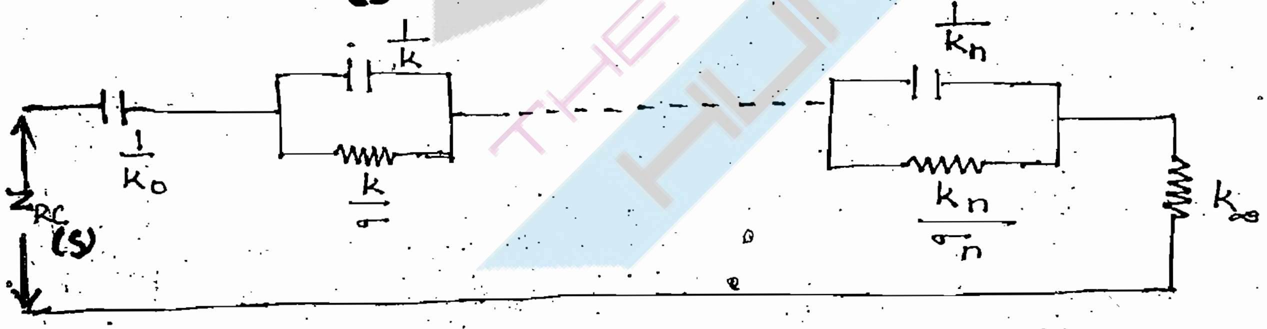
$$\frac{k}{s+\sigma}$$

$$Z_{RC}(s) = \frac{k_0}{s} + \sum_{i=1}^{n} \frac{k_i}{s+\sigma_i} + k_\infty$$

$$\frac{1}{\frac{s}{k}+\frac{\sigma}{k}} = \frac{1}{Y_a+Y_b}$$

$B_c = sC$ $\quad G_1$

$$x_c = \frac{1}{cs}$$

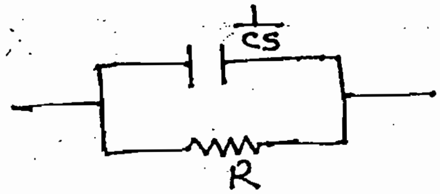


$\longrightarrow$ Series N/ω is obtained

Rc N/w Foster-II form :-

$$Z_{RC}(s) = \frac{k_0}{s} + \sum_{i=1}^{n} \frac{k_i}{s+\sigma_i}$$

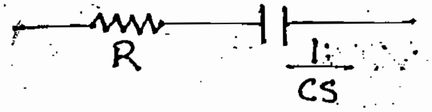
$$\rightarrow Z_{RC}(S) = \frac{R\frac{1}{CS}}{R+\frac{1}{CS}}$$



$$Z_{RC}(S) = \frac{R}{RCS+1}$$

$$Z_{RC}(S) = \frac{R}{RC\left(S+\frac{1}{RC}\right)}$$

$$\rightarrow Y_{RC}(S) = \frac{1}{R+\frac{1}{CS}}$$



$$Y_{RC}(S) = \frac{CS}{RCS+1}$$

$$Y_{RC}(S) = \frac{CS}{RC\left(S+\frac{1}{RC}\right)}$$

$$\frac{Y_{RC}(S)}{S} = \frac{1}{R\left(S+\frac{1}{RC}\right)}$$

Note:-

→ Pole-zero pattern of Z_{RC} and Y_{RC} $Y(s)$ are identical. Thereby both functions can be represented by same mathematical equation.

$$\frac{Y_{RC}(S)}{S} = \frac{k_0}{S} + \sum_{i=1}^{n} \frac{k_i}{S+\sigma_i} + k_\infty$$

$$Y_{RC}(S) = k_0 + \sum_{i=1}^{n} \frac{k_i S}{S+\sigma_i} + k_\infty S$$

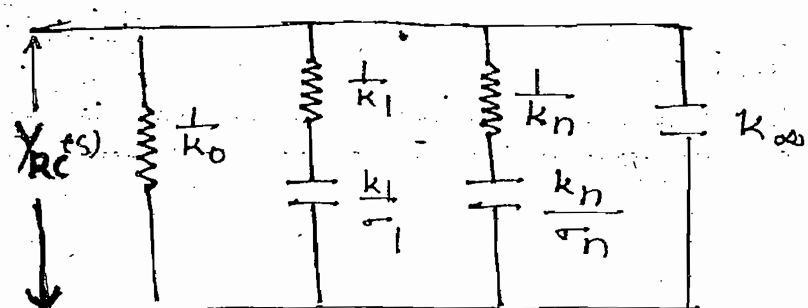
$$\frac{k_i S}{S+\sigma}$$

$$\frac{1}{\frac{S}{KS}+\frac{\sigma}{KS}}$$

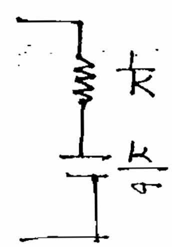
$$Y_c = \frac{1}{CS}$$

R

$$\frac{1}{Z_a+Z_b}$$



Dotted upto n



RC N/w Cauer-I form:-

LC → $Z_2(S) = Z(S) - H_1 S$ $(X_L = LS)$ $Y_2(S) = \frac{1}{Z_2(S)}$

$Y_3(S) - H_2 S$

RC → Step-I:-

Removal of constant from Z(S)

$$Z_2(S) = Z(S) - K_1$$

$$Y_2(S) = \frac{1}{Z_2(S)}$$

Step-II :-

Removal of Pole at ∞ from Y(S)

$$Y_3(S) = Y_2(S) - H_1 S$$

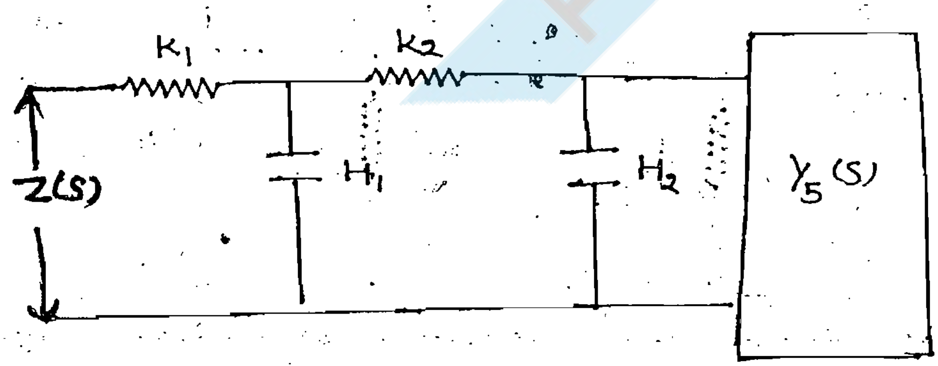
$$Z_3(S) = \frac{1}{Y_3(S)}$$

$$Z_4(S) = Z_3(S) - K_2$$

$$Y_4(S) = \frac{1}{Z_4(S)}$$

$$Y_5(S) = Y_4(S) - H_2 S$$

$$Y_4(S) = Y_5(S) + H_2 S$$



→ Ladder N/w is obtained

Note:-

Step-I and Step-II are alternatively repeated until the total function is realised

RC N/w Cauer-II form:-

LC → $Z_2(s) = Z(s) - \frac{k_{01}}{s}$ $\left(X_c = \frac{1}{Cs}\right)$ $Y_2(s) = \frac{1}{Z_2(s)}$, $Y_3(s) = \frac{k_{02}}{s}$ $\left(B_L = \frac{1}{Ls}\right)$

RC → Step-I:-

Removal of pole at origin from $Z(s)$

$$Z_2(s) = Z(s) - \frac{k_{01}}{s}$$

$$Y_2(s) = \frac{1}{Z_2(s)}$$

Step-II:-

Removal of constant from $Y(s)$

$$Y_3(s) = Y_2(s) - k_1$$

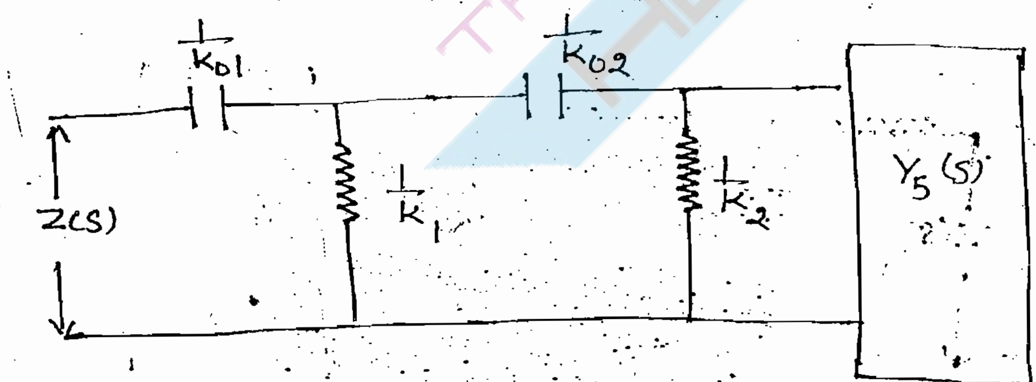
$$Z_3(s) = \frac{1}{Y_3(s)}$$

$$Z_4(s) = Z_3(s) - \frac{k_{02}}{s}$$

$$Y_4(s) = \frac{1}{Z_4(s)}$$

$$Y_5(s) = Y_4(s) - k_2$$

$$Y_4(s) = Y_5(s) + k_2$$



→ Ladder N/w is obtained.

Note:-

Step-I and II are alternatively repeated until the total function is realised.

Lecture - 14

Filters :-

W.r.t to components present in the N/w filters are classified as

(I) **Active** Filter (II) Passive Filter

→ Active filter consist of op-amp and capacitor

→ In the active filter it is possible to inc. the gain of the system

→ Generally inductor is not preferred in the active filter since its size is bulky and cost is high

→ **Passive** filter consist of LC series and parallel sections (Reactive N/w)

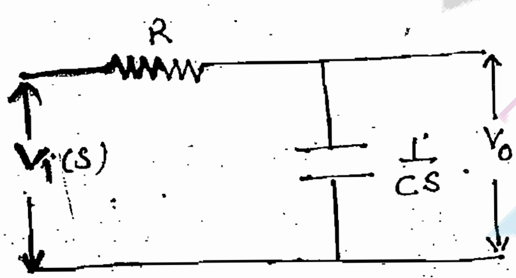
→ **Based** on the operating frequency filters are classified as.

(I) LPF (II) HPF (III) BPF (IV) BEF (Band stop)

(V) All pass

LPF:-

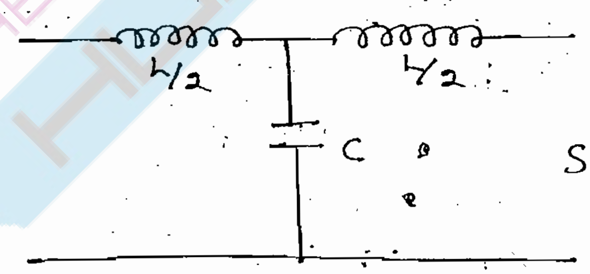
First order



$$V_o(s) = V_i(S)\,\frac{\frac{1}{CS}}{R+\frac{1}{CS}}$$

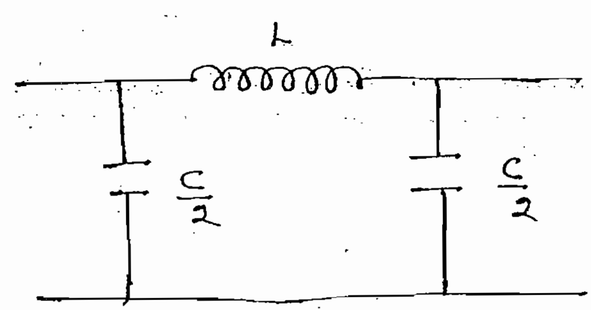
$$\Rightarrow \boxed{\frac{V_o(S)}{V_i(S)} = \frac{1}{1+RCS}}$$

Second order

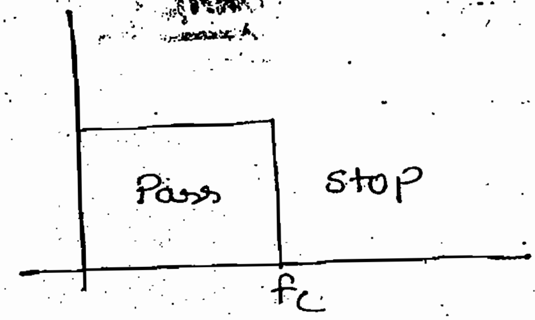


Sym-T

$X_L = 2\pi f L$ $\quad X_C = \frac{1}{2\pi f C}$



Sym-π



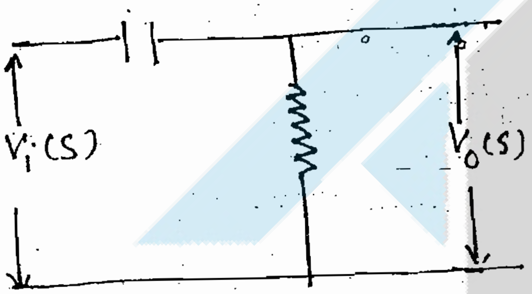
→ Inductor provide low resistance path and capacitor bypass higher frequency

L →

HPF :–

→ For lower freq. capacitor provides high impedance

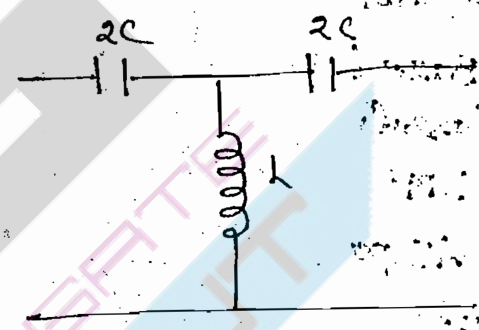
First order

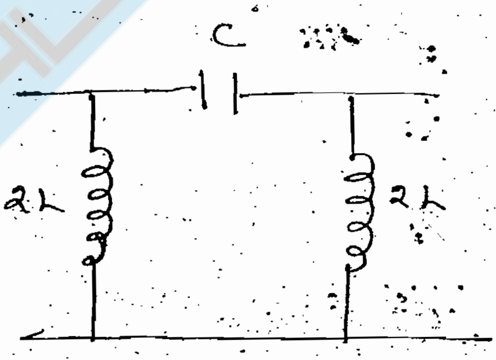


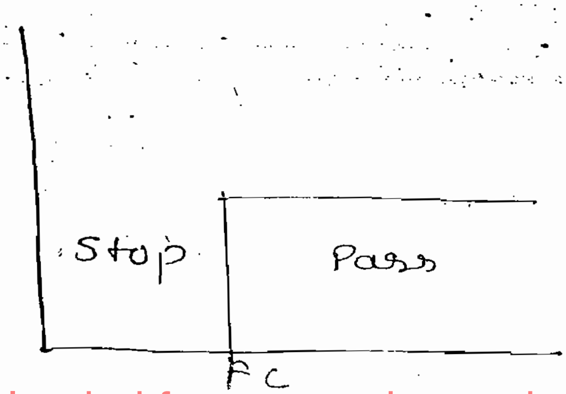
$$V_o(s) = V_i(s)\,\frac{R}{R+\frac{1}{CS}}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{RCS}{RCS+1}$$

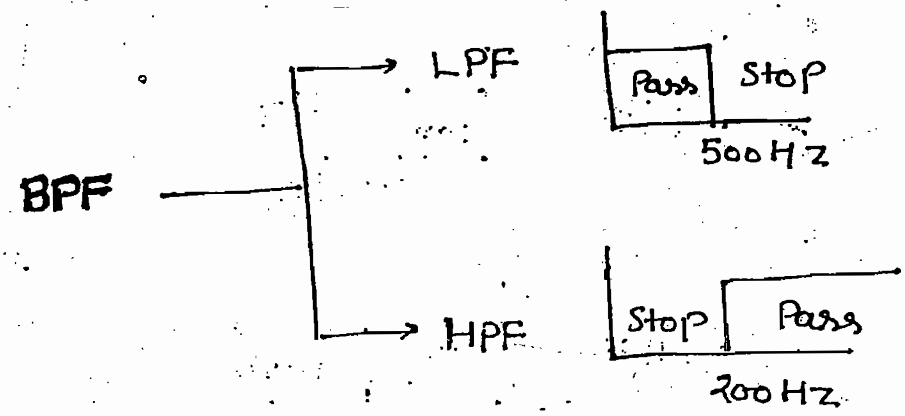
Second order

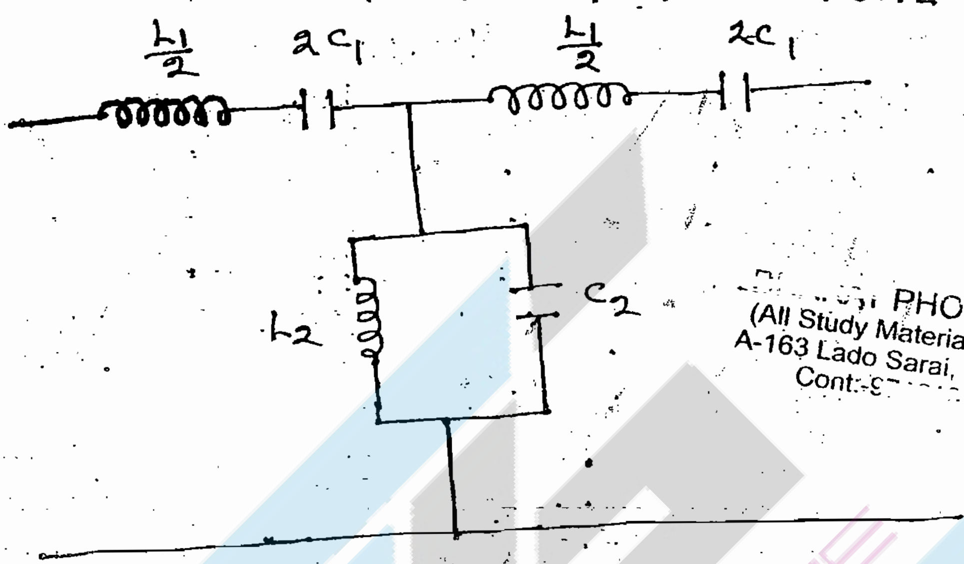


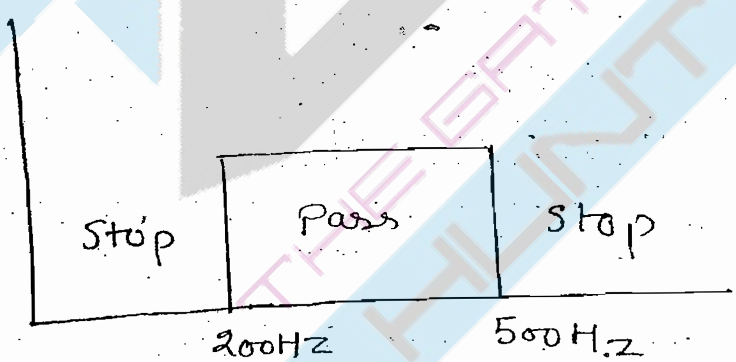




BPF :-



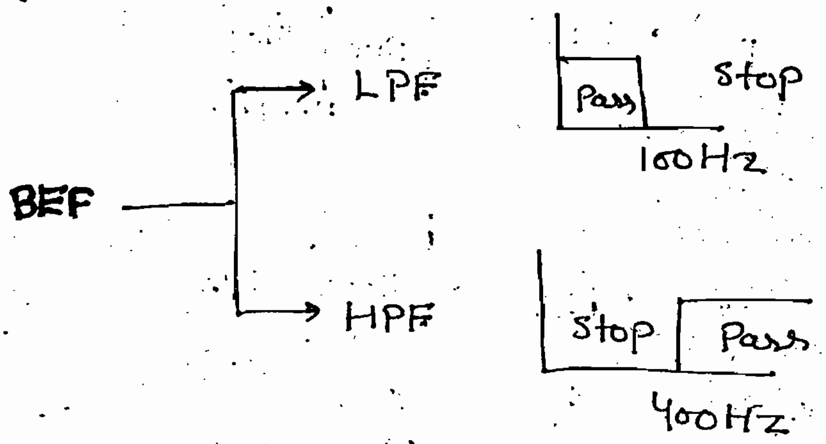


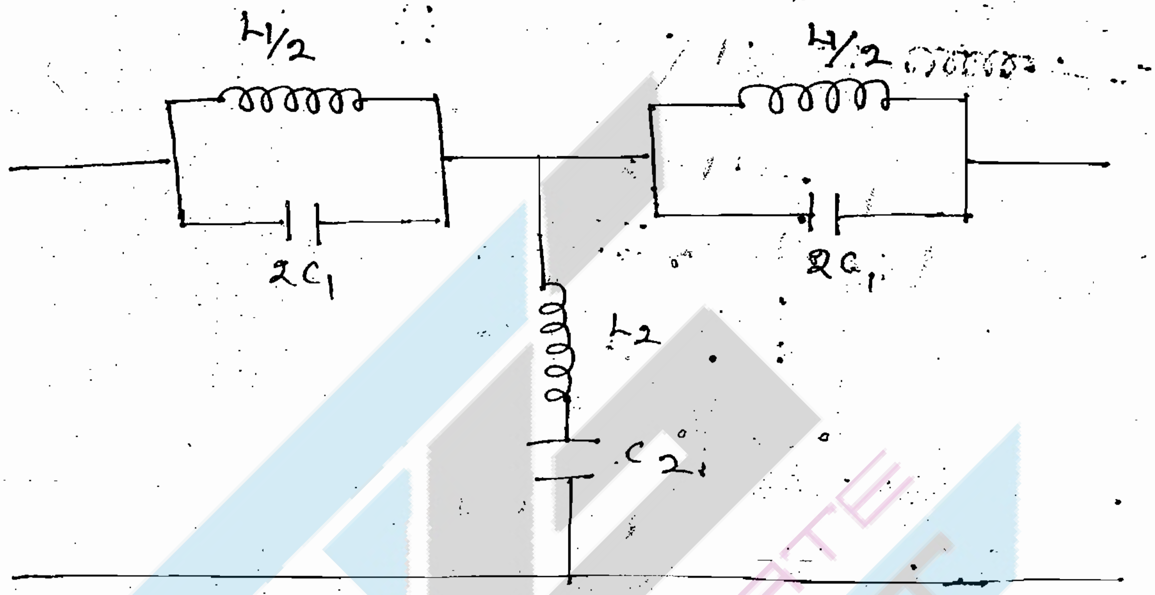


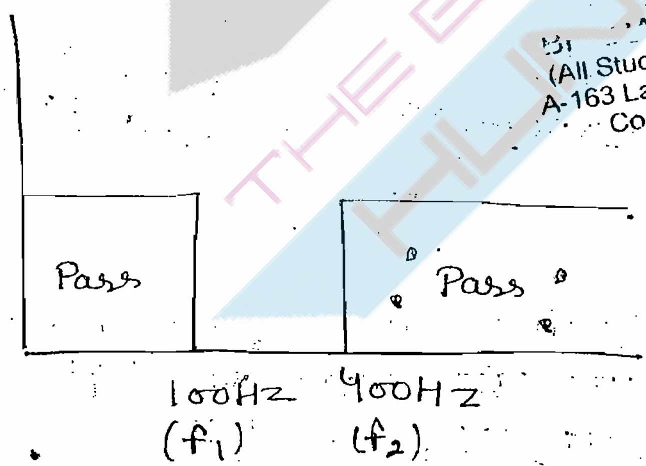
→ BPF is obtained with a combination of LPF & HPF and cut-off freq of LPF should be greate than cut-off freq of HPF

→ In the BPF first cut-off frequency is calculate w.r.t series resonance and second cut-off freq is calculated w.r.t parallel resonance

BEF :-







→ BEF is obtained with a combination of LPF and HPF and cut-off frequency of high pass filter should be greater than cut-off freq of low pass filter

Transfer Function of first order filter:-

$\frac{1}{1+Ts} \rightarrow$ LPF

$\frac{Ts}{1+Ts} \rightarrow$ HPF

$\frac{1-Ts}{1+Ts} \rightarrow$ All pass

Note:- In all pass filter zero's are present in the right half of plane and poles are present in left half of plane

Transfer Function of second order filter:-

$\frac{p}{s^2+as+b} \rightarrow$ LPF

$\frac{Ps^2}{s^2+as+b} \rightarrow$ HPF

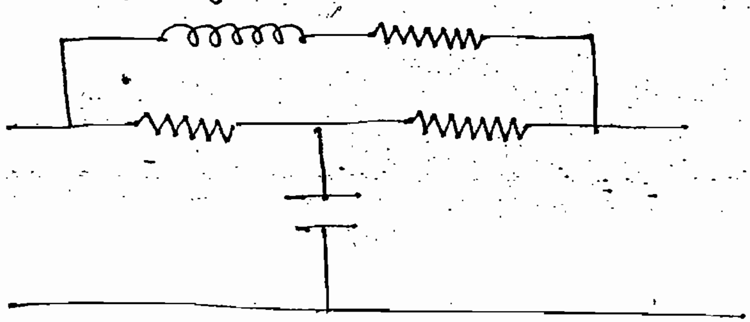
$\frac{Ps}{s^2+as+b} \rightarrow$ BPF

$\frac{Ps^2+q}{s^2+as+b} \rightarrow$ BEF

$\frac{s^2-Ps+q}{s^2+as+b} \rightarrow$ All Pass

Note:-
Poles and zeroes are symmetrical about jω-ax (All pass)

Ques:- Identify type of the filter of the ckt shown



Soln:-

Soln:-

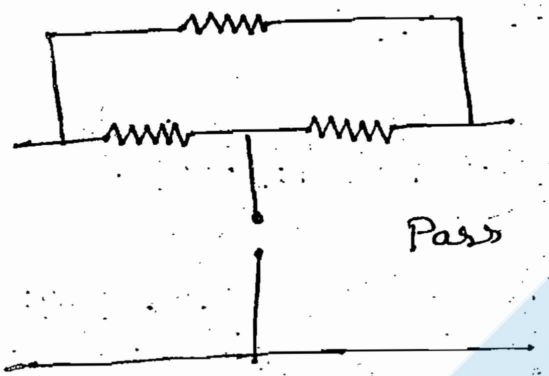
$f = 0$

$X_L = 2\pi f L = 0$

$L \to SC$

$X_C = \frac{1}{2\pi f C} = \infty$

$C \to OC$



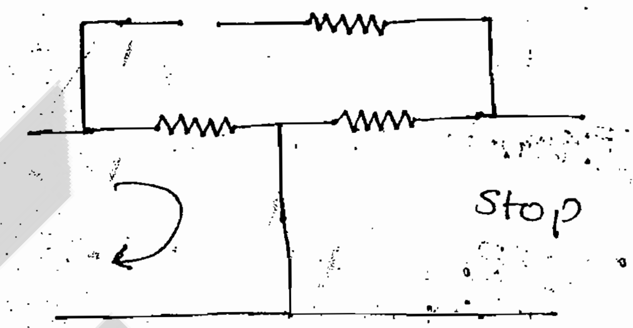
$f = \infty$

$X_L = 2\pi f L = \infty$

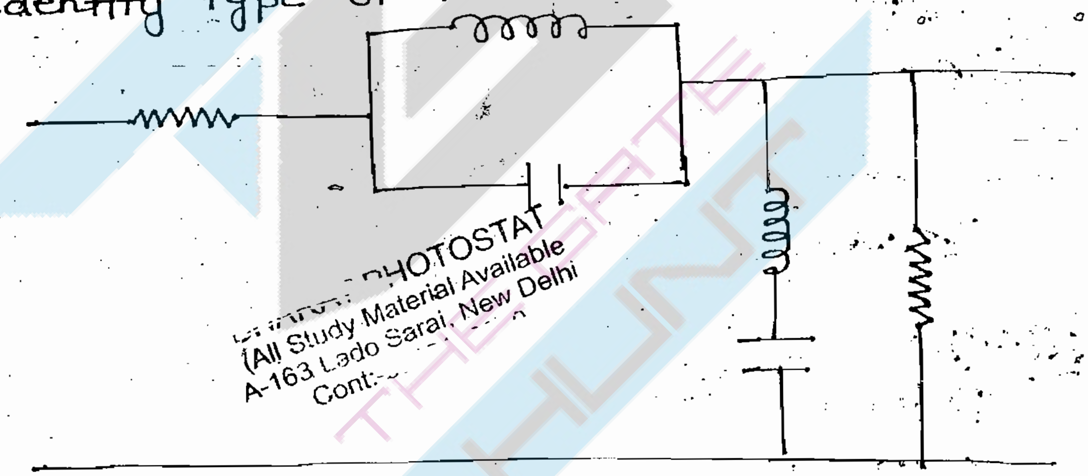
$L \to O.C$

$X_C = \frac{1}{2\pi f C} = 0$

$C \to S.C$



Ques:- Identify type of the filter of ckt. shown



Soln:-

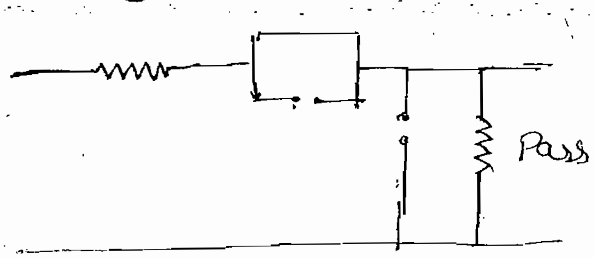
$f = 0$

$X_L = 2\pi f L = 0$

$L \to S.C$

$X_C = \frac{1}{2\pi f C} = \infty$

$C \to O.C$



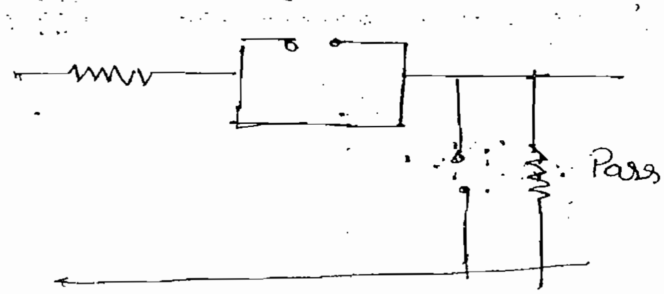
$f = \infty$

$X_L = 2\pi f L = \infty$

$L \to O.C$

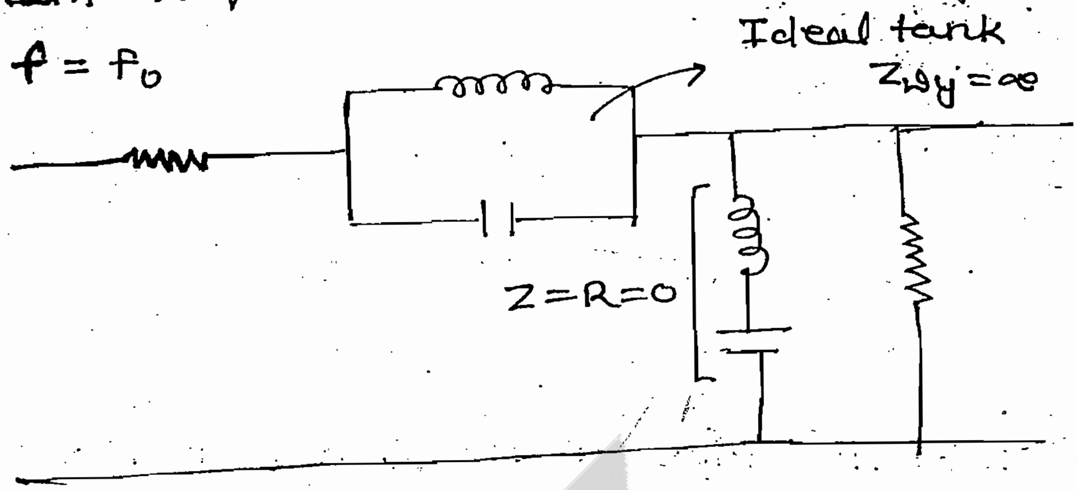
$X_C = \frac{1}{2\pi f C} = 0$

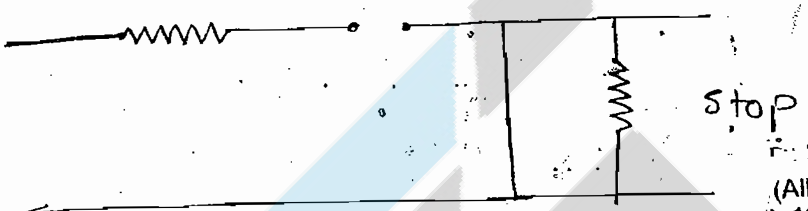
$C \to S.C$



For 'BEF & All pass filter draw eq. ckt at resonant freq.

$f = f_0$



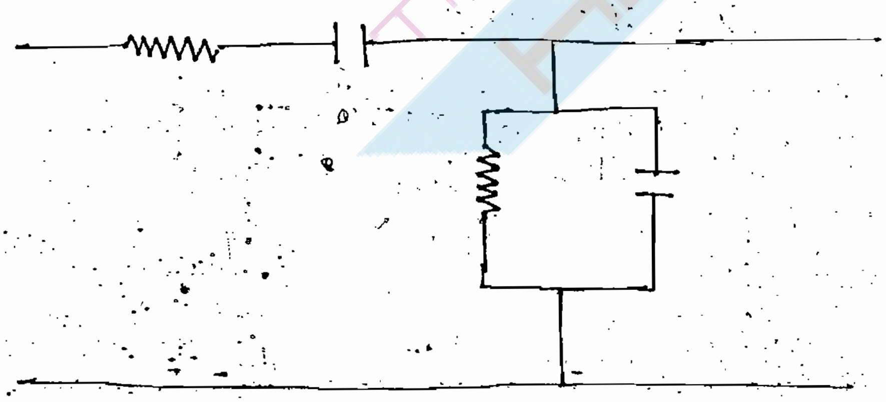


→ BEF, Ans

Note:-

When BEF eliminates only few frequency then it is also called as Notch filter

Ques:- Identify type of the filter of the ckt shown



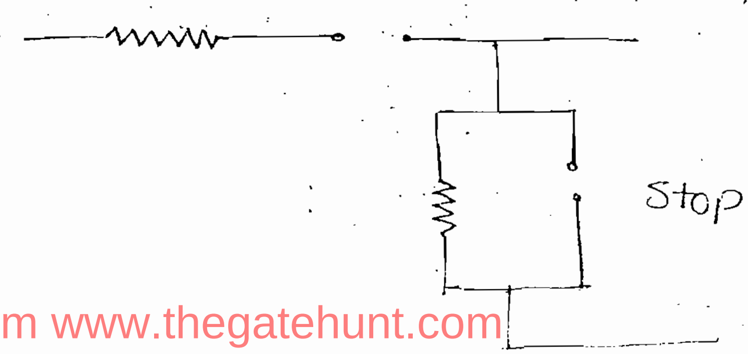
Soln:- $f = 0$

$X_L = 2\pi f L = 0$

$L \rightarrow S.C$

$X_C = \frac{1}{2\pi f C} = 0$

$C \rightarrow O.C$



1106

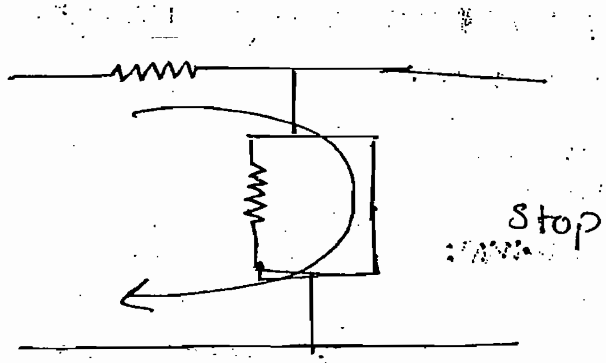
$f = \infty$

$X_L = 2\pi f L = \infty$

$L \rightarrow O.C$

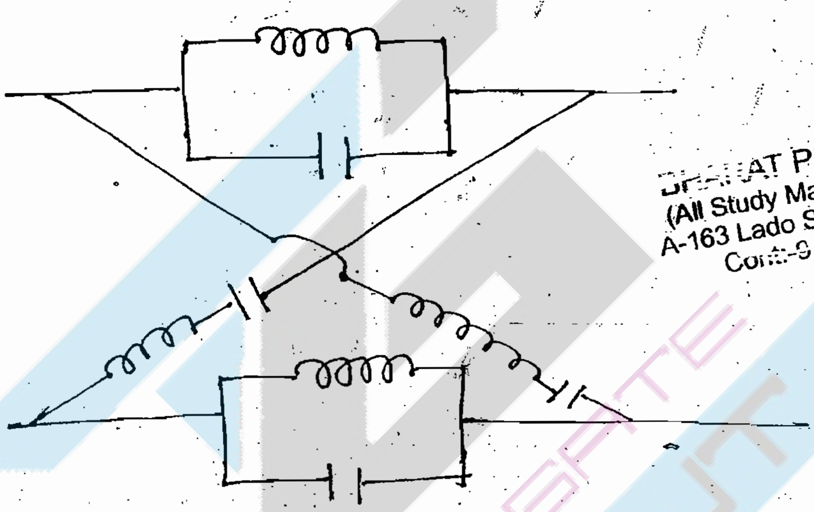
$X_c = \frac{1}{2\pi f c} = 0$

$C \rightarrow S.C$



$\longrightarrow$ BPF

Ques:- Identify type of the filter of the ckt shown



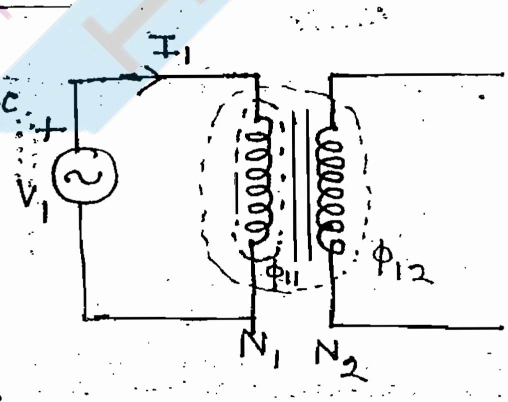
Ans:- All pass $(f = 0, f = \infty, f = f_0)$

Magnetic Coupled Circuits :-

Case-(1) :-

$I_1 \longrightarrow \phi_1$ → $\phi_{11} \rightarrow$ leakage flux

ϕ_1 → ϕ_{12} ↓ Useful flux (or) Mutual flux



$e_1 \propto \frac{d\phi_1}{dt}$

$e_1 = -N_1 \frac{d\phi_1}{dt}$

$$e_1 = -N_1 \frac{d\phi_1}{di_1} \cdot \frac{di}{dt} \quad \left(L = \frac{N\phi}{i} \right)$$

$$\boxed{e_1 = -L_1 \frac{di_1}{dt}}$$

↓

Self induced emf

$$e_2 \propto \frac{d\phi_{12}}{dt}$$

$$e_2 = -N_2 \frac{d\phi_{12}}{dt}$$

$$e_2 = -N_2 \frac{d\phi_{12}}{di_1} \frac{di_1}{dt} \quad \left(M_{21} = \frac{N_2 \phi_{12}}{i_1} \right)$$

↓

Mutual inductance of second inductor w.r.t. 1

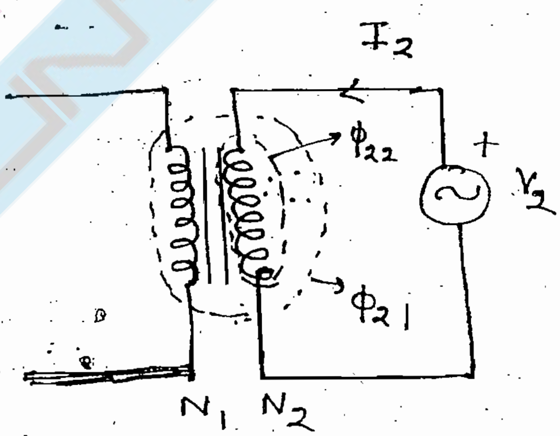
$$\boxed{e_2 = -M_{21} \frac{di_1}{dt}}$$

↓

Mutual Induced emf

Case-(II):-

$I_2 \rightarrow \phi_2$
- $\rightarrow \phi_{22} \rightarrow$ leakage flux
- $\rightarrow \phi_{21} \rightarrow$ Useful flux or Mutual flux



$$e_2 \propto \frac{d\phi_2}{dt}$$

$$e_2 = -N_2 \frac{d\phi_2}{dt}$$

$$e_2 = -N_2 \frac{d\phi_2}{di_2} \frac{di_2}{dt} \quad \left(L = \frac{N\phi}{i} \right)$$

$$\boxed{e_2 = -L_2 \frac{di_2}{dt}}$$

↓

self induced emf

$$e_1 \propto \frac{d\phi_{21}}{dt}$$

$$e_1 = -N_1 \frac{d\phi_{21}}{dt}$$

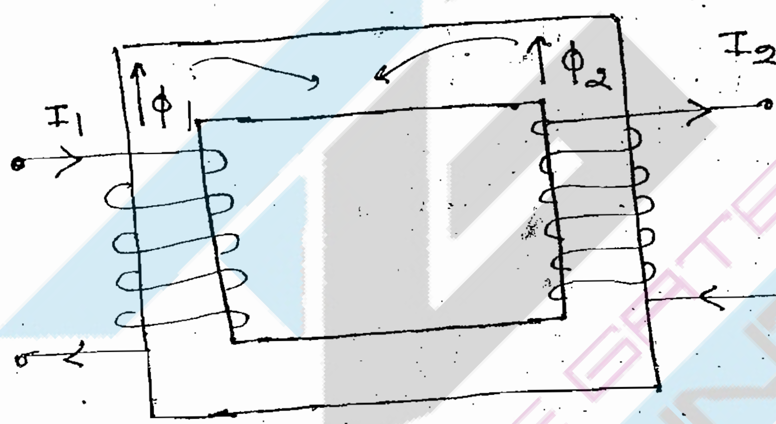
$$e_1 = -N_1 \frac{d\phi_{21}}{di_2} \cdot \frac{di_2}{dt} \qquad \left(M_{12} = \frac{N_1 \phi_{21}}{i_2} \right)$$

$$\boxed{e_1 = -M_{12} \frac{di_2}{dt}}$$

Mutual induced emf

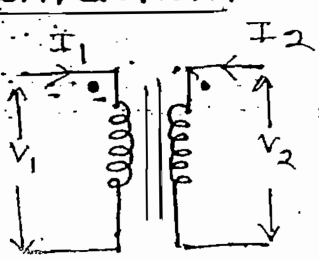
If current is flowing in any one of inductor then the sign of self & mutually induced voltage is same

Note!-

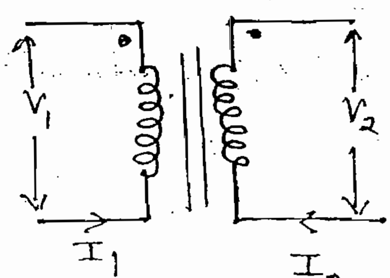


In above figure flux of the two inductors are completely closed path in opposite direction. Hence sign of mutually induced voltage is opposite to sign of self induced voltage

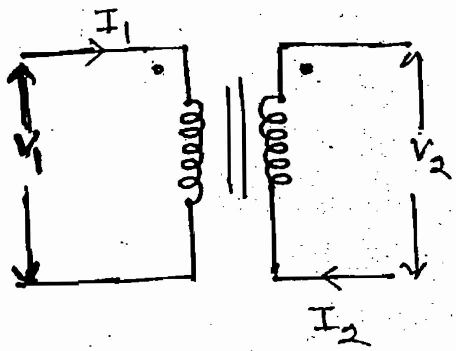
Dot convention:-



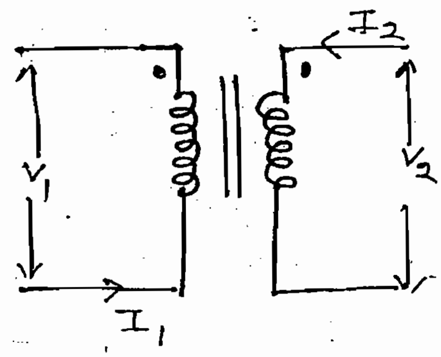
(1)



(11)



(III)



(IV)

$M_{12} = M_{21} = M$

$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Valid for (I) & (II)

$$V_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$V_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

Valid for (III) & (IV)

Note:-

→ When either both currents are entering or both are leaving at dotted terminal sign of mutually induced voltage is same as the sign of self induced voltage.

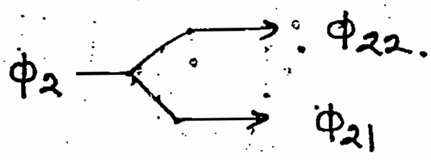
→ When one current is entering and other current is leaving at dotted terminal sign of the mutually induced voltage is opposite to sign of self induced voltage

Coefficient of Coupling / Coupling Factor :-

$$K = \frac{\text{Useful flux}}{\text{Total flux}} \longrightarrow K_1 = K_2 \rightarrow \text{conditio}$$

$\phi_1 \to \phi_{11}$, $\phi_1 \to \phi_{12}$

$$K_1 = \frac{\phi_{12}}{\phi_1}$$



$$k_2 = \frac{\phi_{21}}{\phi_2}$$

$$k = \sqrt{k_1 k_2}$$

For ideal system $k = 1$ and for practical system the range of k is 0 to 1.

$$M_{21} = \frac{N_2 \phi_{12}}{i_1}$$

$$M_{12} = \frac{N_1 \phi_{21}}{i_2}$$

$$M_{12} = M_{21} = M$$

$$M^2 = M_{12} M_{21}$$

$$\Rightarrow M^2 = \frac{N_1 \phi_{12}}{i_1} \cdot \frac{N_2 \phi_{12}}{i_1} \quad \text{—(I)}$$

$$\phi_{12} = k_1 \phi_1 \quad \text{—(II)}$$

$$\phi_{21} = k_2 \phi_2 \quad \text{—(III)}$$

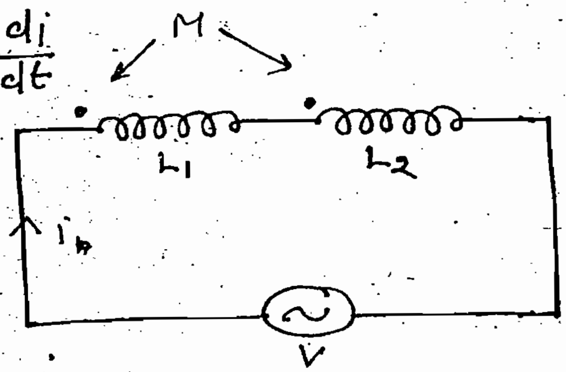
subsitute eq-(II) & (III) in eq-(I)

$$M^2 = k_1 k_2 \frac{N_1 \phi_1}{i_1} \cdot \frac{N_2 \phi_2}{i_2}$$

$$M^2 = k^2 L_1 L_2$$

$$\Rightarrow \boxed{M = k\sqrt{L_1 L_2}}$$

$$V = L_1 \frac{di}{dt} + M\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt}$$



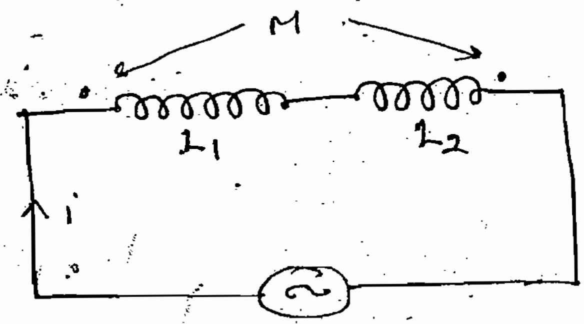
$$L_{eq}\frac{di}{dt} = (L_1 + L_2 + 2M)\frac{di}{dt}$$

$$\boxed{L_{eq} = L_1 + L_2 + 2M}$$

→ Series Aiding

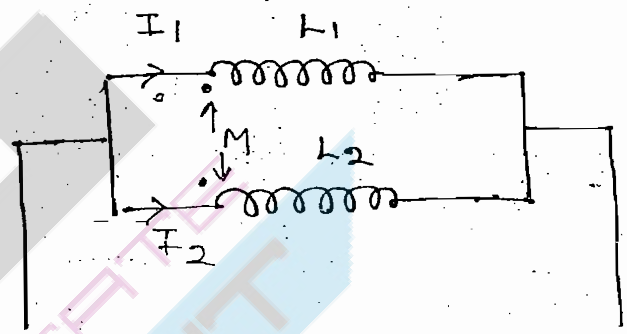
→ <u>Series opposing</u> :-

$$\boxed{L_{eq} = L_1 + L_2 - 2M}$$



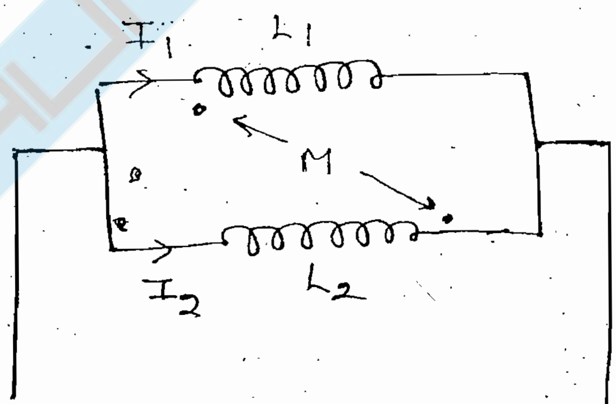
→ <u>Parallel Aiding</u> :-

$$\boxed{L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}}$$

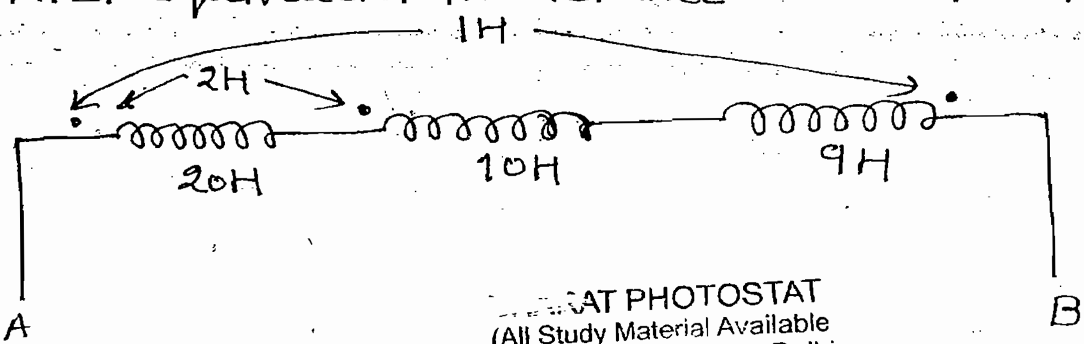


→ <u>Parallel Opposing</u> :-

$$\boxed{L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}}$$



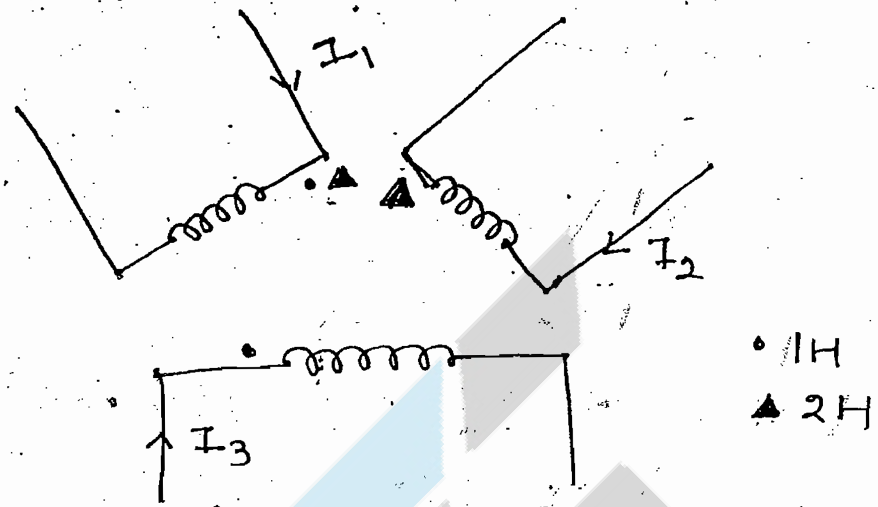
<u>Ques</u>:- Find equivalent inductance w.r.t A and B :



Soln:- $L_{eq} = L_1 + L_2 + L_3 \pm 2M_1 \pm 2M_2 \pm 2M_3$

$\Rightarrow L_{eq} = 20 + 10 + 9 + 2(2) + 0 - 2(1)$

Ques:- Develop inductance matrix of the figure shown

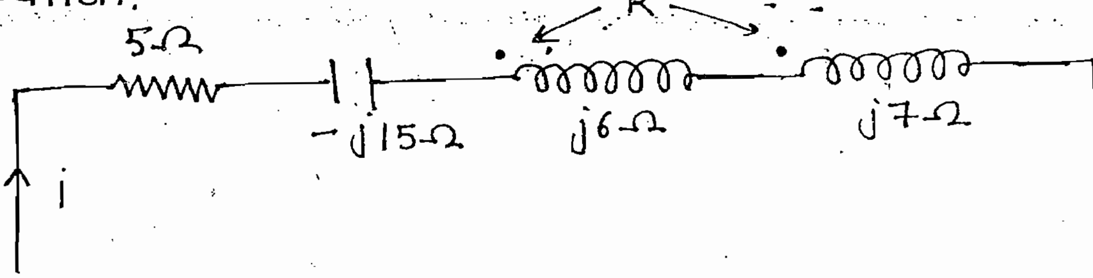


Soln:- Diagonal points denotes self inductance

$$L = \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix}$$

$$\therefore L = \begin{bmatrix} 15 & -2 & 1 \\ -2 & 20 & 0 \\ 1 & 0 & 10 \end{bmatrix}$$

Ques:- Find the value of K under resonance condition:-



Soln:- $2\pi f$ ($L_{eq} = L_1 + L_2 + 2M$)

$$M = K\sqrt{L_1 L_2}$$

$$\Rightarrow 2\pi f M = 2\pi f\, K\sqrt{L_1 L_2}$$

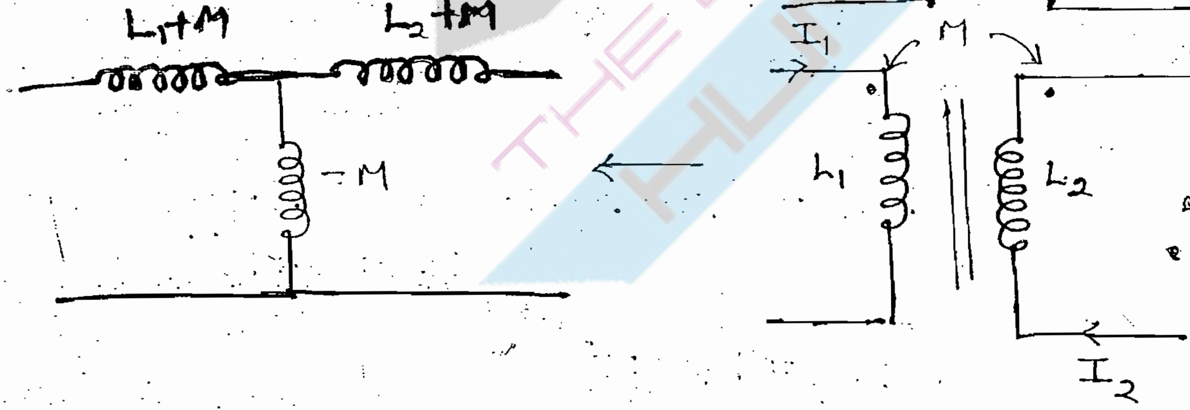
$$X_M = K\sqrt{X_1 X_2}$$

$$X_{eq} = X_1 + X_2 + 2X_M$$

$$\Rightarrow X_{eq} = X_1 + X_2 + 2K\sqrt{X_1 X_2}$$

$$\Rightarrow 15 = 6 + 7 + 2K\sqrt{6\times 7}$$

$$\Rightarrow \boxed{K = \frac{1}{\sqrt{42}}} \text{ Ans}$$

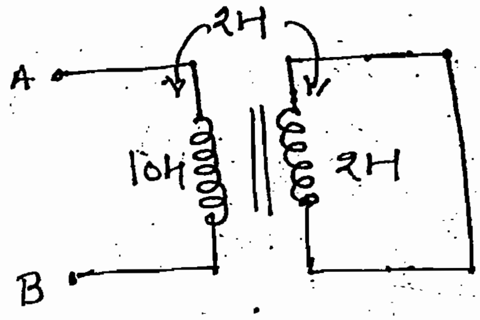


$$\boxed{W = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 \pm M I_1 I_2}$$

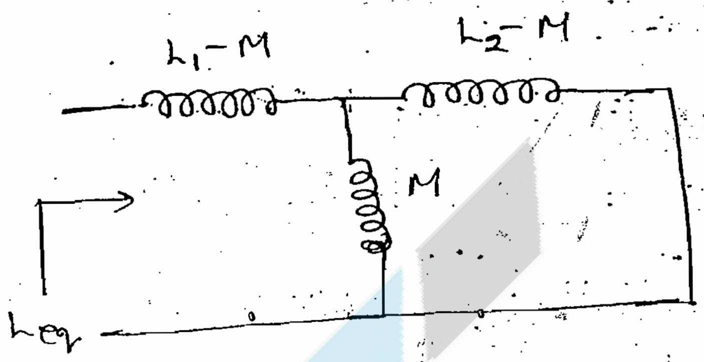
$+ \rightarrow$ fig-(I) $\quad -ve \rightarrow$ fig(II)

Energy stored in the coupled coils.

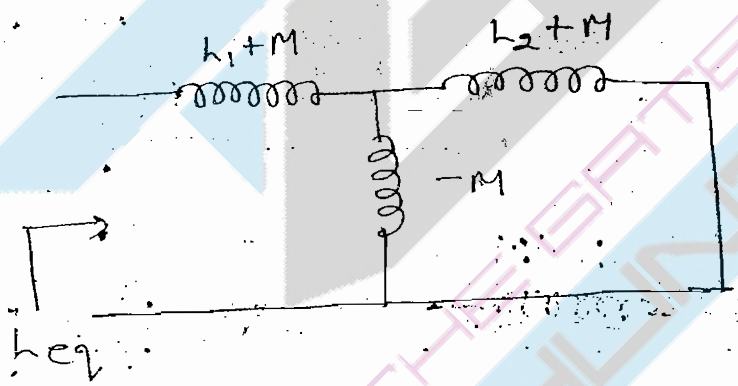
Ques:- Find L_{eq} w.r.t A and B



Soln:-



$$L_{eq} = L_1 - M + \frac{M\,(L_2 - M)}{L_2 - M + M} = L_1 - \frac{M^2}{L_2}$$

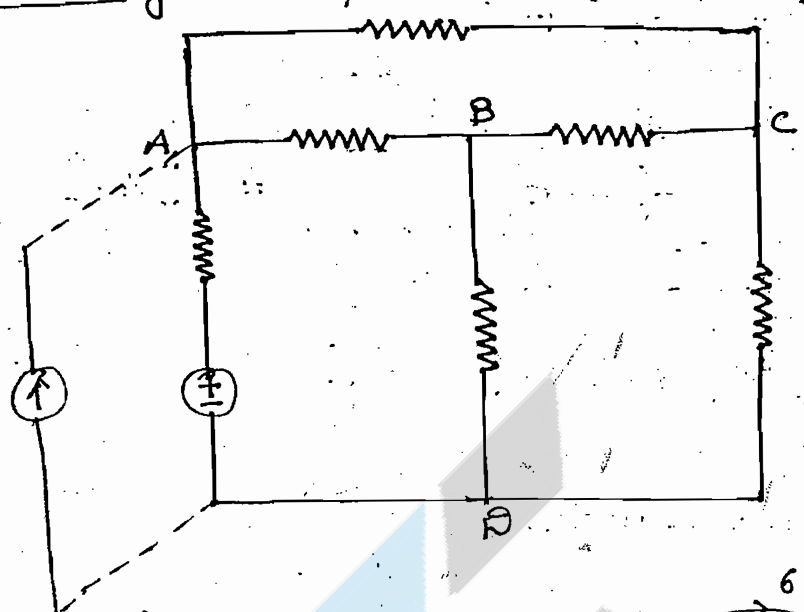


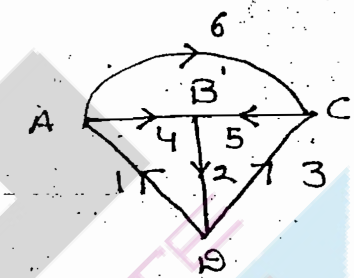
$$L_{eq} = (L_1 + M) + \frac{(-M)(L_2 + M)}{L_2 + M - M}$$

$$\boxed{L_{eq} = L_1 - \frac{M^2}{L_2}}$$

$$\Rightarrow \quad L_{eq} = 10 - \frac{2^2}{2} = 8\,H \text{ , Ans}$$

Graph Theory :-





→ N/w topology is the study of the N/w properties by investigating interconnection b/w branches and nodes, it mainly concentrate on the geometry of the N/w

→ In the N/w topology any N/w is replace by graph. To develop graph ~~of~~ each element is replace by either straight line or arc of the semi-circle, voltage source is replace by s.c and current source is replace by o.c and graph retains all the nodes of original N/w

→ $\boxed{\text{No. of branches of N/w} \geq \text{No. of branch of graph.}}$

Augmented Incidence Matrix :-

	1	2	3	4	5	6
A	-1			+1		+1
B		+1		-1	-1	
C			-1		+1	-1
D	+1	-1	+1			

$= [A_a]$

Reduced Incidence Matrix :-

	1	2	3	4	5	6
A	-1			+1		+1
B		+1		-1	-1	
C			-1		+1	-1
~~D~~						

$= [A]$

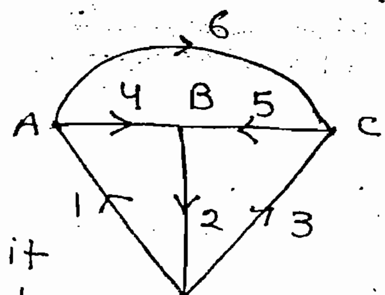
D → Ref - Node or Datum Node

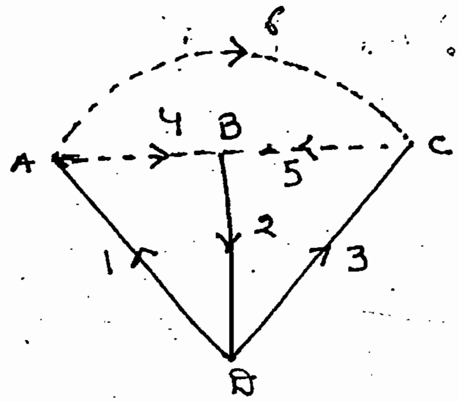
→ All the information regarding the graph can be represent mathematically inconsize form is called as Augmented incidence matrix

→ For a given graph Augmented incidence matrix is unique

Tree :-

→ Tree is a connected sub-graph. It connects the all the nodes of the N/w but it doesn't consist of any closed path





Tree {1, 2, 3}

Total Tree Branches $= N-1$
$= 4-1 = 3$

→ The set of branches which are disconnected to form a tree is called as co-tree or complementary tree

Co-Tree {4, 5, 6}

→ The branch which form a tree is called as tree branch (Twig)

→ Total no. of tree branches $= N-1$

→ The branch which is disconnected to form a tree is called as link (chord)

→ Total no. of links $(l) = b-(N-1)$

where b = total No. of branches of the N/w

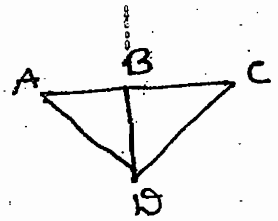
→ Tree is not unique

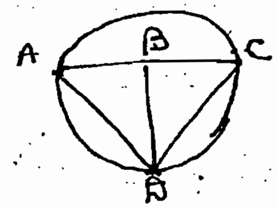
Total no. of tree $= N^{N-2} = 4^{4-2} = 16$

→ Total No of possible tree $= N^{N-2}$

Note:-

→ The above formula can be applied
(a) When connection is present b/w all the node
(b) When no repeated branch is present b/w the node





→ Total no. of possible tree for any graph $= \det(AA^T)$

where A = Reduced incidence matrix

Node Pair Voltages :-

Total no. of node pair voltages $= \dfrac{N(N-1)}{2}$

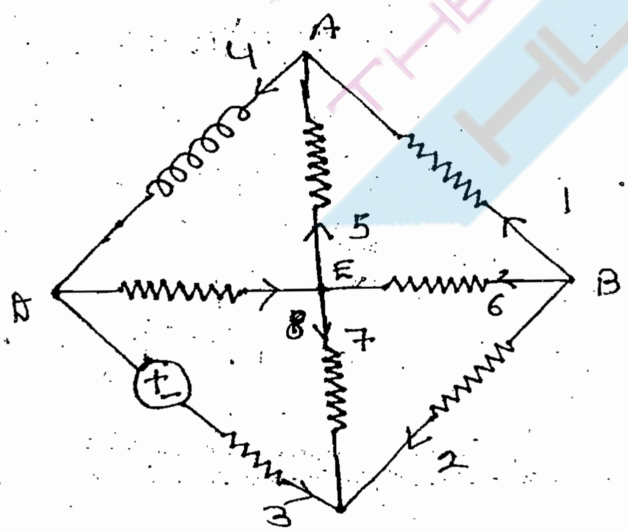
eg :- $\underbrace{V_{AB},\ V_{AC},\ V_{AD},\ V_{BC},\ V_{BD},\ V_{CD}}_{6}$

→ $\boxed{\text{Edges} = \text{Branches}}$

→ Total no. of edges (branches) $= \dfrac{N(N-1)}{2} = \dfrac{4 \times 3}{2}$

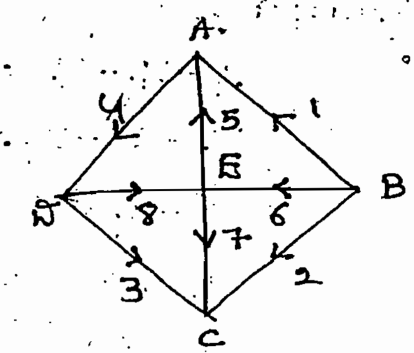
$= 6$

Ques :- Develop Tie-set matrix of the N/w shown



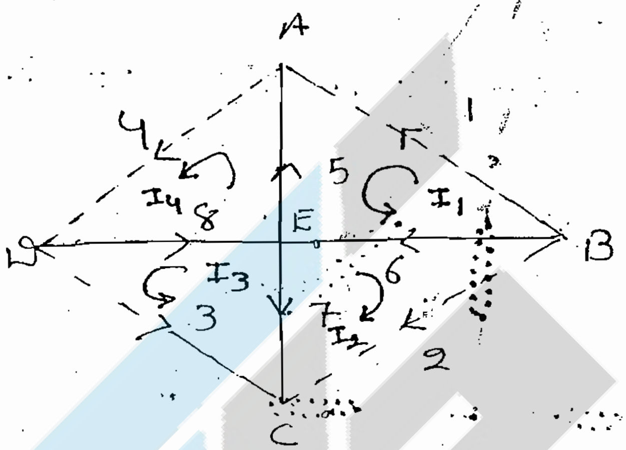
Step-1 :-

Develop the graph for the given N/w



Step-(II):-

Develop a tree for the graph



Step-(III):-

Identify total no. of basic loop / fundamental loop (f-loop) or independent loops

→ Basic loop should consist of only one link

→ Total no. of basic loop = total no. of links

i.e $l = b - (N-1)$

→ Basic loop direction is same as link current direction

$U \qquad C_b$

	V_1 1	V_2 2	V_3 3	V_4 4	V_5 5	V_6 6	V_7 7	V_8 8
I_1	+1				−1	−1		
I_2		+1				−1	−1	
I_3			+1				−1	−1
I_4				+1	+1			+1

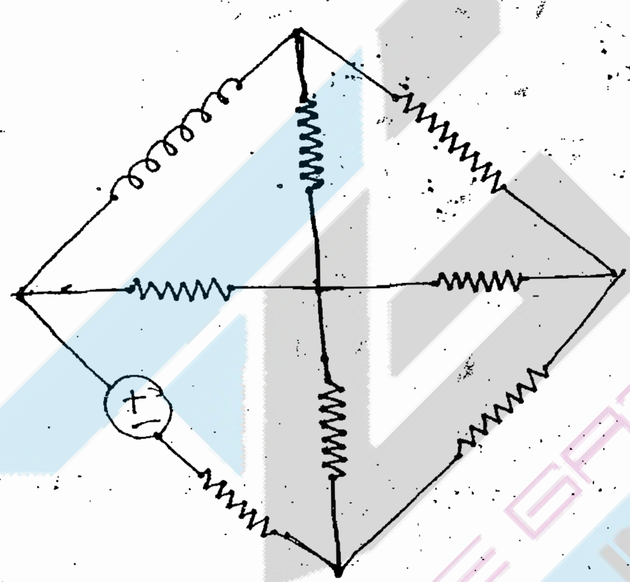
$= [C]$

KVL Equations:-

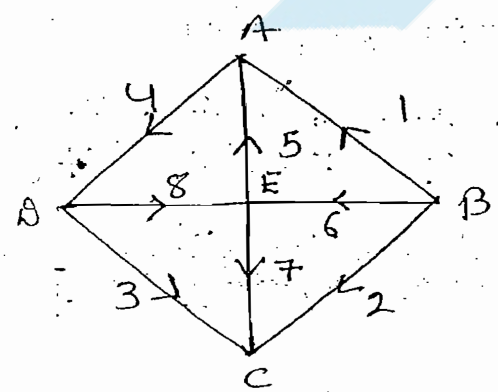
$$V_1 - V_5 - V_6 = 0$$
$$V_2 - V_6 - V_7 = 0$$
$$V_3 - V_7 - V_8 = 0$$
$$V_4 + V_5 + V_8 = 0$$

$$\boxed{[C] = [U : C_b]}$$

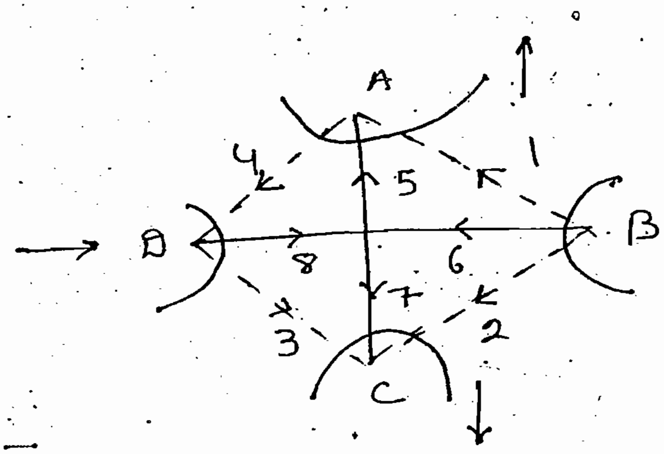
Ques:- Develop cut-set matrix of the N/w shown



Step-(I):-
Develop a graph for the given N/w



Step-(II):-
Develop a tree for a graph



Step-(III) :-

Identify total no. of basic cut-sets or fundamental cut-sets / f- cut-sets

→ Basic cut-sets should consist of only one tree branch

→ Total No. of basic cut-sets = total No. of tree branches

i.e = $N-1$

→ Basic cut-set direction as same as a tree branch current direction

	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8
	1	2	3	4	5	6	7	8
A	+1			−1	+1			
B	+1	+1				+1		
C		+1	+1				+1	
D			+1	−1				+1

$= [B]$

$$[B] = [B_{tl} \,\vdots\, U]$$

KCL Equations :-

$$I_1 - I_4 + I_5 = 0$$

$$I_1 + I_2 + I_6 = 0$$

$$I_2 + I_3 + I_7 = 0$$

$$I_3 - I_4 + I_8 = 0$$

$$[C] = [U \vdots C_b]$$

↓

Tie-set

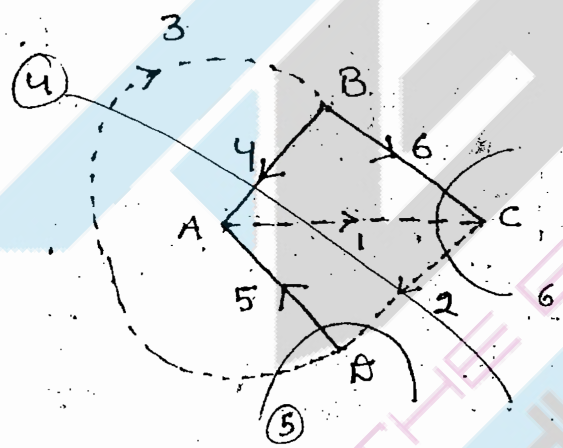
$$[B] = [B_l \vdots U]$$

↓

cut-set

$$\boxed{\begin{aligned}[C_b] &= -[B_l]^T \\ [B_l] &= -[C_b]^T\end{aligned}}$$

Ques:- Develop cut-set matrix of the graph shown

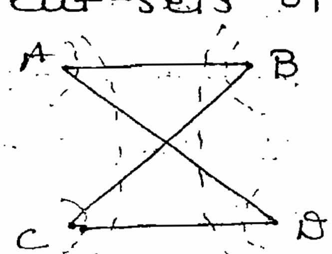


Soln:-

**

	1	2	3	4	5	6
4	−1	+1	−1	+1		
5		−1	+1		+1	
6	+1	−1				+1

Ques:- Identify total no. of cut-sets of the graph shown



(a) 3 (b) 4

(c) 5 (d) 6

Note:-

→ cut-sets may consist of one tree branch or more than one tree branch

→ Basic cut-sets consist of only one tree branc

Conclusion:-

→ Total no. of possible tree $= N^{N-2}$

→ Tie-set matrix is not unique, total no. of possible tie-set matrix $= N^{N-2}$

→ cut-set matrix is not unique. Total no. of possible cut-set matrix ~~is~~ $= N^{N-2}$

→ Rank of Tie-set matrix = total no. of links

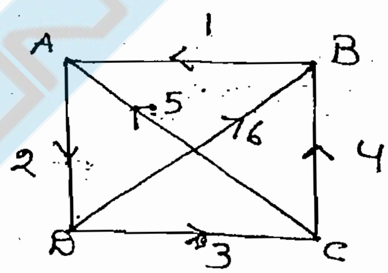
i.e. $l = b-(N-1)$

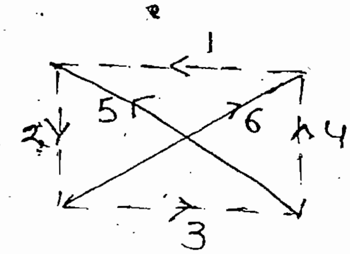
→ Rank of cut-set matrix = total no. of tree branches

$= N-1$

→ Rank of Incidence Matrix $= N-1$

→ The given figure is invalid tree since no connection is present b/w tree branches





Duality:-

$R \longleftrightarrow G$

$L \longleftrightarrow C$

$V \longleftrightarrow I$

Series ⟷ Parallel

O.C ⟷ S.C

KVL ⟷ KCL

loop $\longleftrightarrow$ Node
(Mesh)

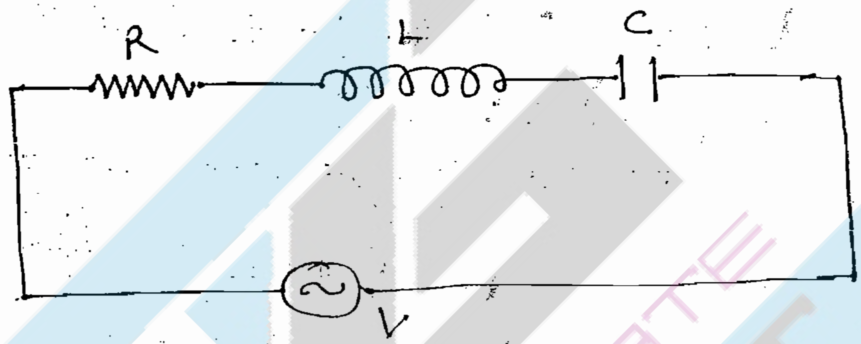
Tie-set $\longleftrightarrow$ Cut-set

Thevenin's $\longleftrightarrow$ Norton's

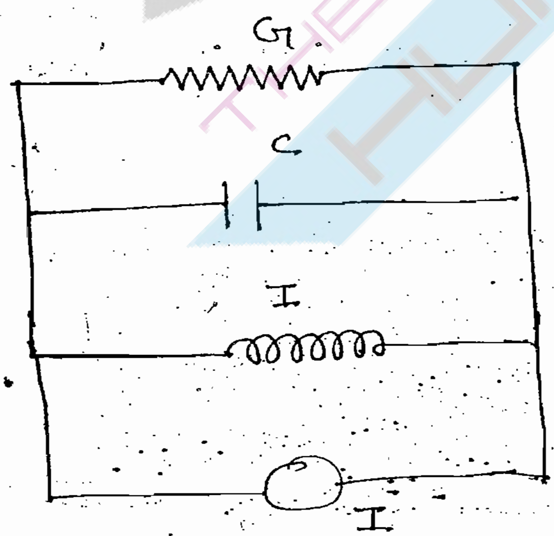
Foster-I form $\longleftrightarrow$ Foster-II form
(Series) (Parallel)

$$\frac{dV}{dt} \longleftrightarrow \frac{dI}{dt}$$

$$\int V dt \longleftrightarrow \int i dt$$



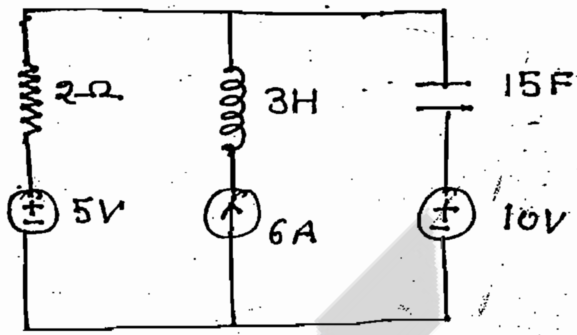
$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i dt \longrightarrow KVL$$



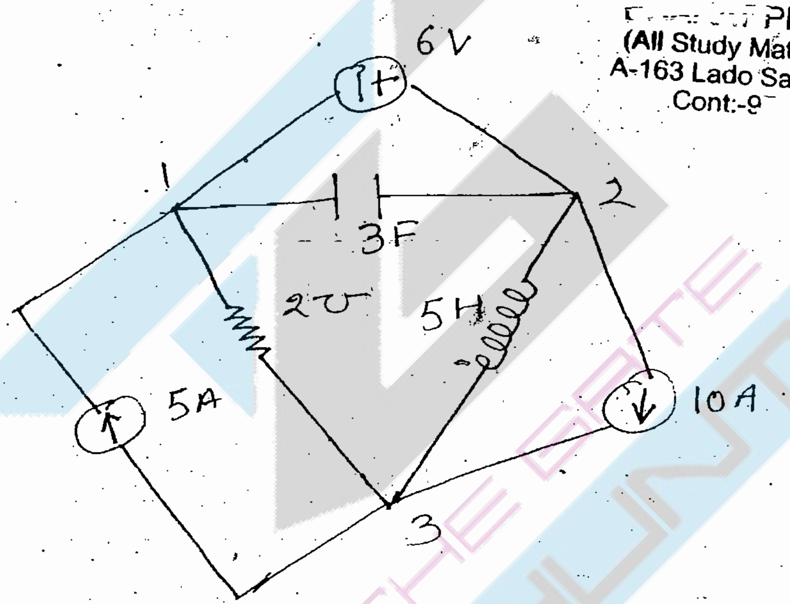
$$I = VG + C\frac{dV}{dt} + \frac{1}{L}\int V dt \longrightarrow KCL$$

Duality does not means the equivalence but it means that mathematical representation of both N/w are identicals.

Ques:- Develop dual of the N/w shown



Soln:-



Note:-

⇒ When voltage source circulate a current in clockwise direction arrowmark of the current source is indicated towards respective node

→ When current source circulate a current in clockwise direction +ve sign is assign to respecti node