2b) The equations given for optical flow are (u, v):

$u(x, y) = a_1 x + b_1 y + c_1$

$v(x, y) = a_2 x + b_2 y + c_2$

These equations describe how each pixel in the image moves from one frame to the next.

a_1, b_1, c_1 and a_2, b_2, c_2 are the affine parameters we need to estimate.

Lucas Kanade algorithm works effectively for motion tracking when the motion is affine.

For each pixel, we assume that optical flow (u, v) and motion field is constant within a small neighbourhood 'n'.

For every point $(k, l) \in n$, we get

$$I_x(k,l)\,u + I_y(k,l)\,v + I_t(k,l) = 0$$

Let size of neighbour n be $n \times n$.

$$\underbrace{\begin{bmatrix} I_x(1,1), & I_y(1,1) \\ I_x(k,l), & I_y(k,l) \\ \vdots & \vdots \\ I_x(n,n), & I_y(n,n) \end{bmatrix}}_{\substack{\text{known} \\ X}} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{\text{unknown}} = \underbrace{\begin{bmatrix} I_t(1,1) \\ I_t(k,l) \\ \vdots \\ I_t(n,n) \end{bmatrix}}_{\substack{\text{known} \\ Y}}$$

From above we can solve for X & Y, we get $X^T X B = X^T Y$

This can be represented in matrix format as

$$\begin{bmatrix} \Sigma_w I_x I_x & \Sigma_w I_x I_y \\ \Sigma_w I_x I_y & \Sigma_w I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -\Sigma_w I_x I_t \\ -\Sigma_w I_y I_t \end{bmatrix}$$

known unknown known

Let pixel (x, y) is displaced by $(x+v, y+u)$

$$E(u,v) = \Sigma\left[I(x+u, y+v) - T(x,y)\right]^2$$

$$= \Sigma\left[I(x,y) + u I_x(x,y) + v I_y(x,y) - T(x,y)\right]^2$$

$$= \Sigma\left[u I_x(x,y) + v I_y(x,y) + D(x,y)\right]^2$$

Finding partial derivative & equating to zero we get

$$\frac{\partial E}{\partial u} = \Sigma\left[u I_x(x,y) + v I_y(x,y) + D(x,y)\right] \times I_x(x,y) = 0$$

$$\frac{\partial E}{\partial v} = \Sigma\left[u I_x(x,y) + v I_y(x,y) + D(x,y)\right] \times I_y(x,y) = 0$$

This can be written as

$$\Sigma \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\Sigma \begin{bmatrix} I_x D \\ I_y D \end{bmatrix}$$

Can be summarized as

computing I_x, I_y, I_t for images & if determinant of x is zero or not and if det $|A| = 0$ then pixel was least squares

The general form can be written as

$$\begin{bmatrix} I_x(1,1) & I_y(1,1) & 1 \\ \vdots & \vdots & \vdots \\ I_x(n,n) & I_y(n,n) & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} = \begin{bmatrix} I(1,1) \\ \vdots \\ I_t(n,n) \end{bmatrix}$$

same for $\begin{bmatrix} I_x(1,1) & I_y(1,1) & 1 \\ \vdots & \vdots & \\ I_x(n,n) & I_y(n,n) & 1 \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} = \begin{bmatrix} I_t(1,1) \\ \vdots \\ I_t(n,n) \end{bmatrix}$

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