

2a) To derive the motion tracking equation from fundamental principles and to compute motion function estimates between two consecutive frames, we need to establish basic equations and assumptions used in optical flow.

Consider two images captured at times t and $t + \delta t$.

Let a point $x(x, y)$ in image 1 displaced by $x'(x + \delta x, y + \delta y)$ in image 2.

Assuming the ~~point~~ intensity of point x' doesn't change in Image 2 and Image 1 as well.

$$I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t) \rightarrow (1)$$

For the point (x, y) in the image at frame t , the intensity doesn't change when it is moved by $(\delta x, \delta y)$ later at frame $t + \delta t$.

From Taylor series expansion we can write

$$I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t) + \frac{\partial I}{\partial x}(\delta x) + \frac{\partial I}{\partial y}(\delta y) + \frac{\partial I}{\partial t}(\delta t) \rightarrow (2)$$

subtracting (1) from (2) we get

$$I_x \delta x + I_y \delta y + I_t \delta t = 0$$

Divide by δt and taking limit $\delta t \rightarrow 0$

$$I_x u + I_y v + I_t = 0$$

Optical flow can be split as $u_n + u_p$ where u_n Normal flow, u_p parallel flow

\hat{u}_n the normal optical flow can be computed as

$$\text{Direction of } \hat{u}_n = \frac{(I_x, I_y)}{\sqrt{I_x^2 + I_y^2}}$$

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$$\text{Magnitude of } |u_n| = \frac{|I_t|}{\sqrt{I_x^2 + I_y^2}}$$