Optimal Taxation of Sugary Beverages: A Mathematical Model

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Introduction

The rising prevalence of obesity, diabetes, and other diet-related health conditions has prompted policymakers to explore corrective measures aimed at improving public health. One widely discussed policy tool is the taxation of sugar-sweetened beverages (SSBs), which are major contributors to excess sugar consumption in the modern diet. These taxes are often justified on the grounds that individual consumption of sugary drinks imposes external costs on society through increased healthcare expenditures.

This paper addresses the question: *How can a government determine the optimal tax rate on sugary beverages to balance consumer utility and public health costs?* In particular, I develop a theoretical framework in which a social planner sets a tax rate on sugary drinks to internalize the negative health externalities associated with their consumption.

Model Overview and Assumptions

This paper builds a stylized theoretical model to estimate the optimal tax on sugar-sweetened beverages (SSBs), drawing inspiration from the framework developed by Allcott, Lockwood, and Taubinsky (2019). The goal is to determine the Pigouvian tax rate that internalizes health-related externalities associated with sugary drink consumption while preserving consumer welfare.

The model considers a representative consumer who allocates their income between sugary beverages and a composite good. Consumption of SSBs generates private utility but also imposes external health costs on society. The government's role, then, is to choose a tax rate that maximizes total social welfare—defined as the sum of private utility and tax revenue, minus the health costs attributable to SSB consumption.

The following simplifying assumptions underlie the model:

1. **Consumer Preferences:** Preferences are quasi-linear with Cobb-Douglas structure in logarithmic utility. The representative consumer derives utility from sugary drinks q and a numeraire good x, such that

$$U(q, x) = \alpha \ln(q) + \beta x$$

   where α and β are positive constants reflecting relative preferences.

2. **Budget Constraint:** Consumers face a linear budget constraint:

$$(p + t)q + x = I$$

   where p is the market price of sugary drinks, t is the per-unit tax imposed by the government, and I is exogenous income.

3. **Externalities:** Consumption of sugary drinks causes external health costs borne by society. These are modeled as a convex quadratic function of quantity:

$$C(q) = \delta q^2$$

   where $\delta > 0$ captures the marginal social cost per unit of consumption.

4. **Government Objective:** A benevolent social planner maximizes total welfare:

$$W = U(q, x) - C(q)$$

   The planner does not place intrinsic value on revenue redistribution and assumes lump-sum rebating of tax revenue or revenue neutrality.

5. **Market Setting:** The supply of sugary drinks is perfectly elastic (i.e., full tax pass-through), and the consumer is price-taking.

This simplified framework allows us to derive a closed-form expression for the optimal tax t^* and examine how it varies with changes in health costs and consumer preferences. The model isolates the externality-correcting role of taxation and is designed for tractable comparative statics and simulation-based policy analysis.

Mathematical Framework

We assume a representative consumer with quasi-linear preferences, choosing between sugary drinks q and a numeraire good x. The utility function is given by:

$$U(q, x) = \alpha \ln(q) + \beta x$$

The consumer faces a budget constraint:

$$(p + t)q + x = I$$

where p is the pre-tax price of sugary drinks, t is the per-unit tax, and I is income.

Substituting the constraint into the utility function:

$$\max_q \{\alpha \ln(q) + \beta(I - (p + t)q)\}$$

First-order condition:

$$\frac{\alpha}{q} - \beta(p + t) = 0 \quad \Rightarrow \quad q(t) = \frac{\alpha}{\beta(p + t)}$$

Demand for the numeraire good:

$$x(t) = I - (p + t)q(t) = I - \frac{\alpha}{\beta}$$

Let the external health cost be:

$$C(q) = \delta q^2$$

Define social welfare:

$$SW(t) = \alpha \ln(q(t)) + \beta x(t) - \delta q(t)^2$$

Substituting expressions:

$$SW(t) = \alpha \ln\left(\frac{\alpha}{\beta(p + t)}\right) + \beta\left(I - \frac{\alpha}{\beta}\right) - \delta\left(\frac{\alpha}{\beta(p + t)}\right)^2$$

The planner maximizes $SW(t)$:

$$\frac{dSW}{dt} = -\frac{\alpha}{p + t} + \frac{2\delta\alpha^2}{\beta^2(p + t)^3}$$

Set to zero:

$$\frac{\alpha}{p+t} = \frac{2\delta\alpha^2}{\beta^2(p+t)^3} \Rightarrow (p+t)^2 = \frac{2\delta\alpha}{\beta^2} \Rightarrow t^* = \sqrt{\frac{2\delta\alpha}{\beta^2}} - p$$

Comparative Statics

- $t^* \uparrow \delta$: Greater health costs raise the optimal tax.
- $t^* \uparrow \alpha$: Greater consumer preference for sugary drinks increases t^*.
- $t^* \downarrow \beta$: Greater weight on other goods reduces t^*.

Parameter Calibration and Sensitivity

To assess practical implications, we calibrate parameters using values from Allcott, Lockwood, and Taubinsky (2019):

- External cost $\delta \in \{0.4, 0.8, 1.2\}$ (cents/oz)
- $\alpha \in \{0.2, 0.4, 0.6\}$ (preference weight on sugary drinks)
- $\beta \in \{0.4, 0.6, 0.8\}$ (preference for other goods)
- Normalize $p = 1$

We use:

$$t^* = \sqrt{\frac{2\delta\alpha}{\beta^2}} - p$$

α	β	δ	Optimal Tax t^*	Implication
0.2	0.4	0.8	0.4142	Low preference and moderate health cost result in a modest corrective tax.
0.2	0.4	1.2	0.7321	Higher externality (same preference) increases the optimal tax significantly.
0.4	0.6	0.8	0.3333	Balanced preferences and moderate cost yield a moderate tax, slightly lower due to higher β.
0.6	0.4	1.2	2.0000	High preference and high health cost with low β lead to a sharply higher tax—aggressive intervention is optimal.
0.4	0.4	0.8	1.0000	Equal weighting on sugary drinks and other goods with moderate cost results in a relatively high tax.

Table 1: Updated Simulation Results and Economic Implications

Key Takeaways

- **Taxes rise with externalities (δ):** As the health-related harms of sugary drinks increase, the optimal Pigouvian tax increases proportionally to internalize the cost.
- **Higher preferences for sugary drinks (α) raise the tax:** Consumers who derive more utility from sugary beverages impose greater potential harm through consumption, which requires a higher corrective tax.
- **Higher valuation of other goods (β) lowers the tax:** If consumers strongly prefer other goods, they will naturally reduce sugary drink consumption, reducing the necessity for government intervention.
- **Tax design is context-sensitive:** Optimal tax rates depend on behavioral, economic, and health parameters. This suggests that uniform SSB taxes may be inefficient if applied without regard to local demographics or health outcomes.