LIE:-

Premises
$a \to b$
a, b
$b \to c$

Truth value Interpretation

$a \to b \in T$

$a \in T$

Modus ponens

$b \in T$

KB

$a \to b \in T$

$b \to c \in T$

$b \to c$

Simplification

$A \wedge B \in T$

A

$A \vee B \in T$

A (or) B

First Few Examples

- If I am the President then I am well-known. I am the President. So I am well-known
- If I am the President then I am well-known. I am not the President. So I am not well-known.
- If Rajat is the President then Rajat is well-known. Rajat is the President. So Rajat is well known.
- If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore Asha is not elected VP.
- If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is chosen as G-Sec. Therefore Asha is elected VP.

Deduction Using Propositional Logic: Steps

Choice of Boolean Variables a, b, c, d, … which can take values true or false.

Boolean Formulae developed using well defined connectors ~, ∧, ∨, →, etc, whose meaning (semantics) is given by their truth tables.

Codification of Sentences of the argument into Boolean Formulae.

Developing the Deduction Process as obtaining truth of a Combined Formula expressing the complete argument.

Determining the Truth or Validity of the formula and thereby proving or disproving the argument and Analyzing its truth under various Interpretations.

Deduction Using Propositional Logic: Example 1

Choice of Boolean Variables a, b, c, d, ... which can take values true or false.

Boolean Formulae developed using well defined connectors ~, ∧, V, →, etc, whose meaning (semantics) is given by their truth tables.

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If I am the President then I am well-known. I am the President. So I am well-known

Coding: Variables

a: I am the President

b: I am well-known

Coding the sentences:

F1: $a \rightarrow b$

F2: a

G: b

The final formula for deduction: $(F1 \wedge F2) \rightarrow G$, that is:

$((a \rightarrow b) \wedge a) \rightarrow b$

Deduction Using Propositional Logic: Example 1

Boolean variables a, b, c, d, … which can take values true or false. $Val(\alpha) \in \{F, T\}$

Boolean formulae developed using well defined connectors ~, ∧, V, →, etc, whose meaning (semantics) is given by their truth tables. Set of Symbols P

Codification of sentences of the argument into Boolean Formulae. $\alpha, \beta \in K_P$

Developing the Deduction Process as obtaining truth of a combined formula expressing the complete argument.

Determining the Truth or Validity of the formula and thereby proving or disproving the argument and Analyzing its truth under various interpretations.

If I am the President then I am well-known. I am the President. So I am well-known

Coding: Variables

a: I am the President

b: I am well-known

Coding the sentences:

F1: a → b $\rightarrow T$ $\quad ((a \rightarrow b) \wedge \neg a) \rightarrow \neg b.$

F2: a $\quad \neg a$

G: b $\quad \neg b.$

The final formula for deduction: (F1 ∧ F2) → G, that is: ((a → b) ∧ a) → b

a	b	a → b	(a → b) ∧ a	((a → b) ∧ a) → b
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Deduction Using Propositional Logic: Example 3

If I am the President then I am well-known. I am the President. So I am well-known

Coding: Variables

a: I am the President

b: I am well-known

Coding the sentences:

F1: $a \rightarrow b$

F2: a

G: b

The final formula for deduction: $(F1 \wedge F2) \rightarrow G$, that is: $((a \rightarrow b) \wedge a) \rightarrow b$

a	b	$a \rightarrow b$	$(a \rightarrow b) \wedge a$	$((a \rightarrow b) \wedge a) \rightarrow b$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

If Rajat is the President then Rajat is well-known. Rajat is the President. So Rajat is well known

Coding: Variables

a: Rajat is the President

b: Rajat is well-known

Coding the sentences:

F1: $a \rightarrow b$

F2: a

G: b

The final formula for deduction:

$(F1 \wedge F2) \rightarrow G$,

that is: $((a \rightarrow b) \wedge a) \rightarrow b$

Methods for Deduction in Propositional Logic

Interpretation of a Formula

Valid, non-valid, Satisfiable, Unsatisfiable

Decidable but NP-Hard 2^n

Truth Table Method

Faster Methods for validity checking:-

Tree Method

Data Structures: Binary Decision Diagrams

Symbolic Method: Natural Deduction

Soundness and Completeness of a Method

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Soundness and Completeness of a Method

NATURAL DEDUCTION:

Modus **Ponens:** $(a \rightarrow b), a$:- therefore b

Modus Tollens: **$(a \rightarrow b)$, ~b :- therefore ~a**

a.

Hypothetical Syllogism: $(a \rightarrow b), (b \rightarrow c)$:- therefore $(a \rightarrow c)$

Disjunctive Syllogism: (a V b), ~a:- therefore b

Constructive Dilemma: $(a \rightarrow b) \land (c \rightarrow d)$, (a V c) :- therefore (b V d)

Destructive Dilemma: $(a \rightarrow b) \land (c \rightarrow d)$, (~b V ~d) :- therefore (~a V ~c)

Simplification: $a \land b$:- therefore a

Conjunction: a, b:- therefore $a \land b$

Addition: a :- therefore a V b

Try to deduce with formula

Anything valid

add to KB.

Natural Deduction is Sound and Complete

Insufficiency of Propositional Logic

Wherever Mary goes, so does the lamb. Mary goes to school. So the lamb goes to school. goes(Mary, x) goes(Mary, School))

No contractors are dependable. Some engineers are contractors. Therefore some engineers are not dependable.

All dancers are graceful. Ayesha is a student. Ayesha is a dancer. Therefore some student is graceful.

Every passenger is either in first class or second class. Each passenger is in second class if and only if he or she is not wealthy. Some passengers are wealthy. Not all passengers are wealthy. Therefore some passengers are in second class.

A B

Krishna like Apple and Oranges.

Krishna likes Mountains. E.

If Krishna likes apple then Krishna likes mango D.

A

D ∧ E

If Krishna likes mango and Krishna likes Mountains then Krishna goes to forest. C.

Krishna goes to forest → C.

1. $A \wedge B$
2. E
3. $A \rightarrow D$
4. $(D \wedge E) \rightarrow C$.

1. $A \wedge B$
2. E
3. $A \rightarrow D$
4. $D \wedge E \rightarrow C$

K → C

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5. A ✓ {Simplification}
6. D 5, 3 {M.P}
6. $D \wedge E$ {Addition}
7. C {M P}

Given a sentence α, $V : \rightarrow Val(\alpha) \in$ 't'

Given a formula: $\mathbb{P} = \{p, q, r, \ldots\}$

α ?

1. VALID – TAUTOLOGY
2. SATISFIABLE
3. UNSATISFIABLE

SOUNDNESS; If $K \vdash C$; then $K \models C$.

ENTAILMENT.

COMPLETENESS :- If $K \models C$; then $K \vdash C$.

All the sentences into a form of $\boxed{\text{Clauses:-}}$

Clauses:- S, C.

$\wedge$, $\vee$, $\neg$, $\rightarrow$, $\Leftrightarrow$, $\equiv$

Given Sentence α; Convert into a clause (CNF)

$F: (F_1 \cap F_2 \cap F_3 \ldots \cap F_n)$ {CNF}

Where each $F_i : (x_i \cup x_2 \cup x_3 \ldots \cup x_n)$

Where each $x_i \in \{0, 1\}$.

$$(P \rightarrow Q) = (\neg P \cup Q).$$

$$(P \Leftrightarrow Q) = (P \rightarrow Q) \cap (Q \rightarrow P)$$

$$= (\neg P \cup Q) \cap (\neg Q \cup P)$$

$\neg, \cup, \cap$ $\quad \bigcirc \cap \bigcirc \cap \bigcirc$

$\neg$

$$\neg(\neg P) = P$$

De-morgan's law $\rightarrow$ $\neg(P \cup Q) = (\neg P \cap \neg Q)$

Resolution rule:-

$$F : (F_1 \cap F_2 \cap F_3 \cap \dots F_n \rightarrow G)$$

$\sim G$ is false

To Prove:-

$$(F_1 \cap F_2 \cap F_3 \cap F_4 \dots \cap \sim G)$$

$$\downarrow$$

$\Box \rightarrow$ NULL class

$\boxed{\text{FOL}}$

F1: ALL MAN ARE MORTAL ← α

F2: SOCRATES IS A MAN. ← β

F3: SOCRATES IS MORTAL.

$F_1 \wedge F_2 \rightarrow F_3$

D

UOD  Variables $\{x, y, z, x_1, y_1, z_1, \ldots, \text{SOCRATES}\}$

Quantifiers: $\{\forall, \exists\}$

$\{\forall x \mid \text{Men}(x) \rightarrow \text{Mortal}(x)\}$ ↳ t

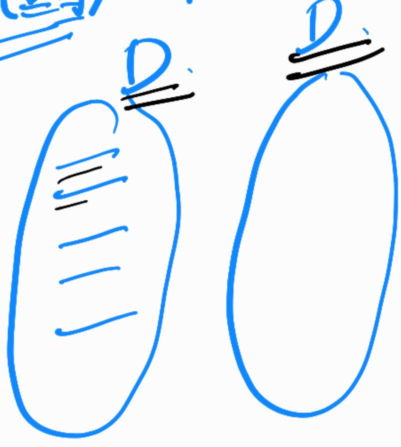
Symbols: $\{P, Q, R, S, \ldots$

$\wedge, \vee, \neg, \rightarrow, =$

Non-logical:

$\text{FOL}(P, F, C)$

$P(x,y)$: predicate function

$p(x,y)$  $D \times D \rightarrow \{t, f\}$

$\{\bot, T\}$

$P(\quad) \rightarrow D^n \rightarrow \{t, f\}$

Constants: $C: \{x_1, y_1, z_1, \ldots, \}$

$D^n \rightarrow \{D\}$

Function Symbol:-

$f(x, y) \to D$:

Sister and Rajit → Brother

Rajit and Khan → Father

$D^n \to D$

greater (x , y)

greater (y , z)

greater (x , z)