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Minor Examination

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Subject Name : Foundation of Computer Science

Ans 1 The ~~Karatsuba~~ Fast integer Multiplication algo (Karatsuba) algorithm decreases the number of subproblems to 3 and ends up calculating the product of two n bit numbers in $\boxed{O(n^{\log_2 3})}$.

To analyze the complexity of the Karatsuba algorithm, consider the number of multiplications the algorithm performs as a function of n M(n). The algorithm multiplies together two n-bit numbers

The recurrence for this is

$$M(n) = 3M\left(\frac{n}{2}\right)$$

This will take care of the multiplication required for the algorithm. There are also $O(n)$ additions and subtractions required for Karatsuba algorithm.
∴ The overall recurrence of the Karatsuba algorithm is:

$$\boxed{T(n) = 3T\left(\frac{n}{2}\right) + \underbrace{O(n)}_{\text{addition subtractions}}}$$

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By using masters theorem on above recurrence yields that the time complexity of Fast integer Multiplication algo is $\boxed{O(n^{\log_2 3}) \approx O(n^{1.58})}$

Karatsuba recursive algorithm is most efficient when certain we put ~~values~~ certain values.!

Since the other factors like addition, subtraction and digit shifts takes time proportional to n, their cost becomes negligible as n increments. It follows that for large n, the algorithm will perform fewer shifts, single-digit additions than complex multiplications. For smaller n there will be more digit shifts and more additions. Hence will make the algorithm slower

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Ans 2

$A = \{1, 2, 3, 4\}$

$R = \{(1, 2,)\ (2, 3)\ (3, 4)\ (2, 1)\}$

$$R^0 = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$R^1 = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$R^2 = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$R^3 = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

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$$R^4 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Used Warshall's Algo to Find the the transitive closure.

$R^T = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,4)\})$

<u>Reflexive Closure</u>: Let R be a relation on set $\Delta = \{(a,a) \mid a \in A\}$
Δ is reflexive, since $a \in A$. $R_1 = R_2 \cup \Delta$ is the smallest relation on A which is reflexive and contains ~~[struck]~~ provided relation R

<u>Symmetric Closure</u>: It is the smallest possible relation which is symmetric and contains R. Let R be a relation on A. $R_1 = R \cup R^{-1}$
R_1 is a symmetric closure.

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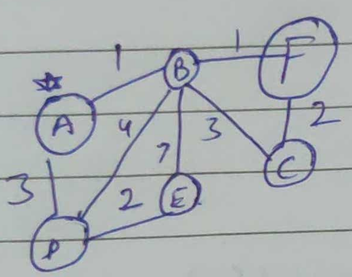
Ans3 Assuming A as root (start/First node)

We have two edges AB (1) and AD (3)

We will choose AB



Now at each step :

i) We need to find the lighest edge such that it forms tree.

ii) ~~We will~~ Add this edge to the tree and connected vertex.

iii) Then repeat until all the ~~vertexes~~ vertices are covered up.

⇒ Greedy Strategy

At each step the edge added contributes min amount possible ie weight ~~of~~ the tree.

Prim's algo

1) $\theta \leftarrow \emptyset$

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ii) For each $u \in V$

do key [u] $\leftarrow \infty$

π[u] $\leftarrow$ NiL

Insert then (Q,u)

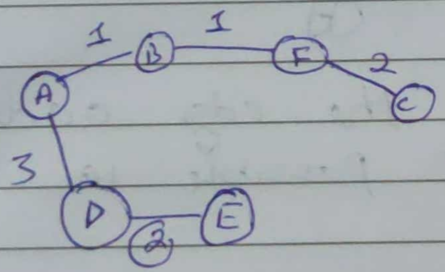
Decrease Key (Q, r, 0)
where $Q \neq \phi$

iii) do u $\leftarrow$ Extract - Minimum (Q)
for each $v \in$ Adj [u]

do if $v \in Q$ and w (u, v)

the π [v] $\leftarrow$ u

Decrease Key (Q, u, w (u, v])



AB, BF, FC, AD, DE

Total minimum weight
= 1 + 1 + 2 + 3 + 2
= 9
Ans

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