$$Y_t = 10 + \epsilon_t - 0.65\,\epsilon_{t-1} - 0.24\,\epsilon_{t-2}$$

(a) $\hat{Y}_t(1) = E(Y_{t+1} \mid H_t) = \mu + \theta_1 \epsilon_t + \theta_2 \epsilon_{t-1}$

$$\epsilon_t = Y_t - \hat{Y}_{t-1}(1)$$

$$\epsilon_{t-1} = Y_{t-1} - \hat{Y}_{t-2}(1)$$

$$\hat{Y}_t(1) = 10 - 0.65\left[Y_t - \hat{Y}_{t-1}(1)\right] - 0.24\left[Y_{t-1} - \hat{Y}_{t-2}(1)\right]$$

$$\hat{Y}_t(2) = E(Y_{t+2} \mid H_t) = \mu + \theta_2 \epsilon_t$$

$$= 10 - 0.24\left[Y_t - \hat{Y}_{t-1}(1)\right]$$

$$\hat{Y}_t(3) = \mu = 10$$

(b) $Y_{99} = 13$, $Y_{100} = 11$, $Y_{101} = 12$, $\hat{Y}_{98}(1) = 11$, $\hat{Y}_{99}(1) = 10$

$$\hat{Y}_{100}(1) = 10 - 0.65\,\epsilon_{100} - 0.24\,\epsilon_{99}$$

$$= 10 - 0.65[11-10] - 0.24[13-11]$$

$$= 8.87$$

$$\hat{Y}_{100}(2) = 10 - 0.24\,\epsilon_{100} = 10 - 0.24[11-10] = 9.76$$

$$\hat{Y}_{100}(3) = \mu = 10$$

$$\hat{Y}_{101}(1) = \mu + \theta_1 \epsilon_{101} + \theta_2 \epsilon_{100}$$

$$= 10 - 0.65[12 - 8.87] - 0.24[11-10] = 7.7255$$

5.R

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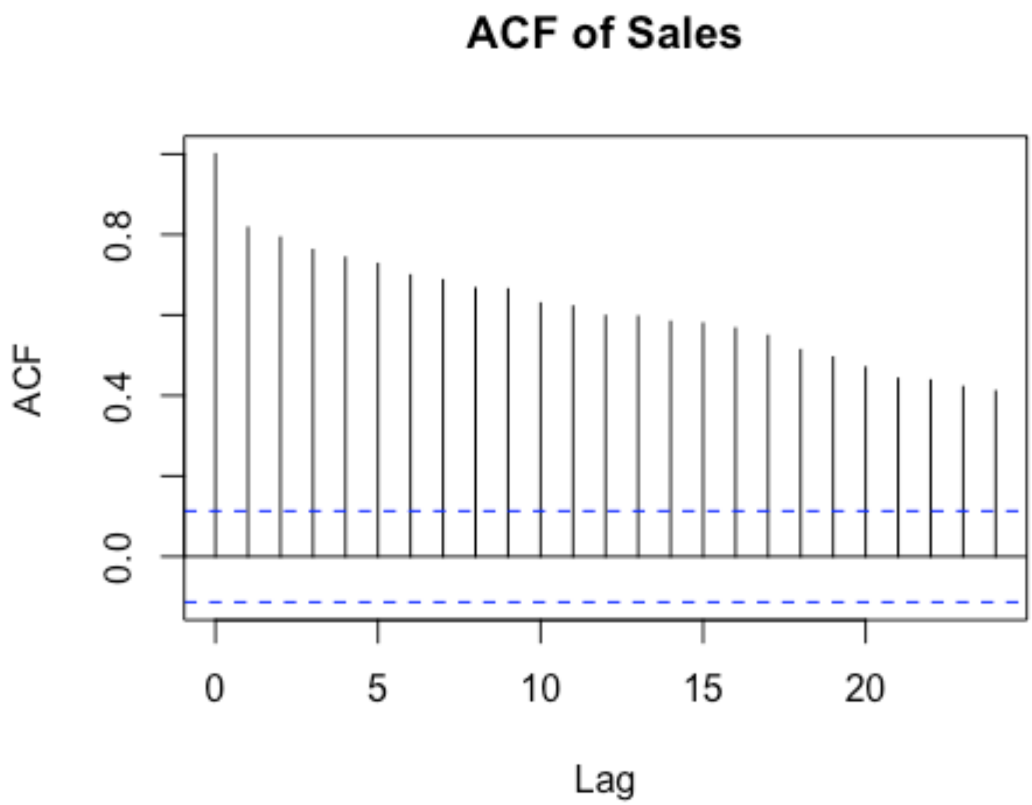
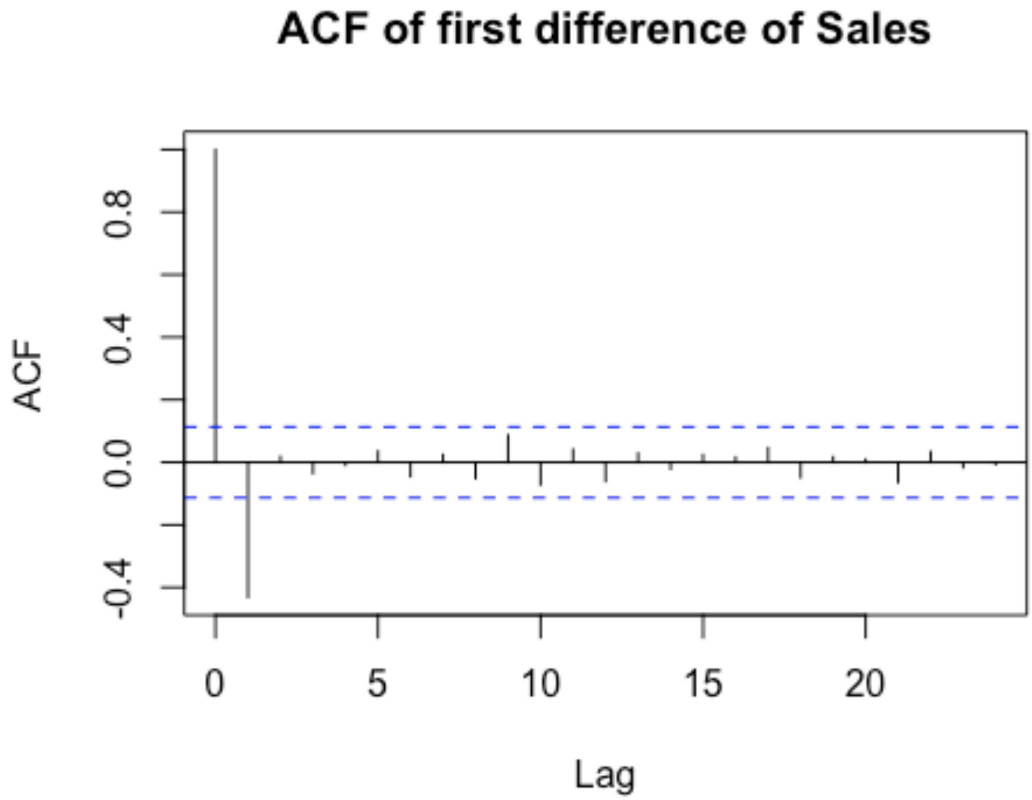
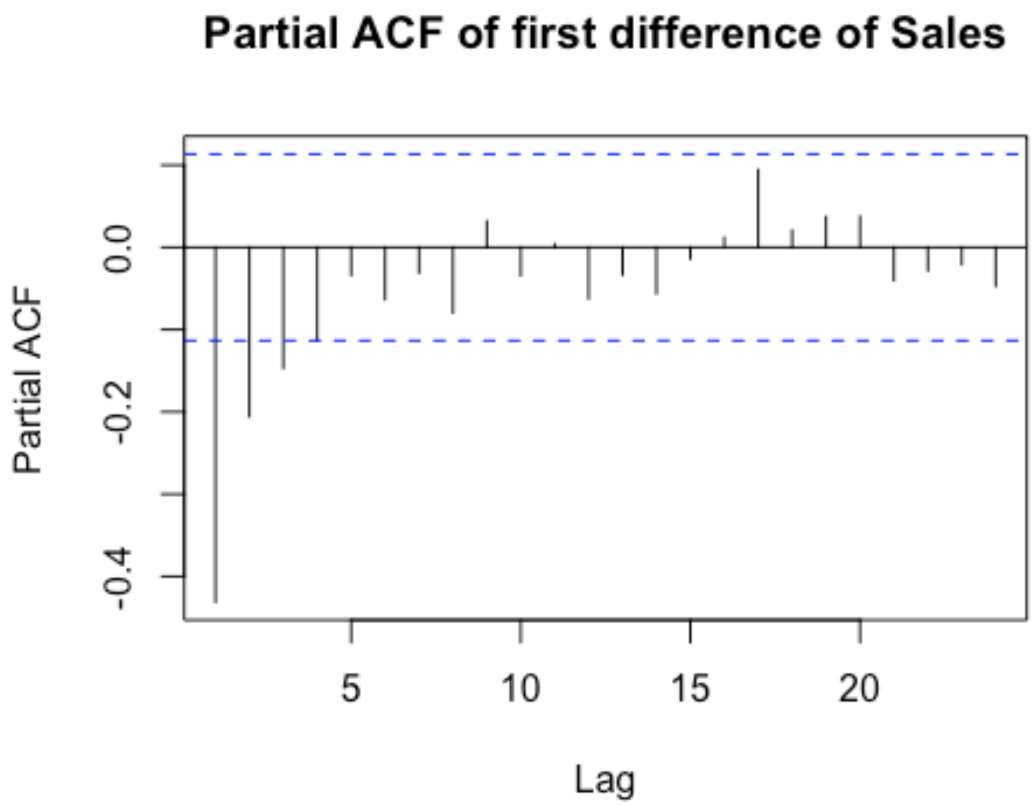
```
setwd("~/Documents/GWU/Forecasting/Assignment 5")
library(greybox)

## Warning: package 'greybox' was built under R version 4.1.2

## Package "greybox", v1.0.5 loaded.

library(forecast)

## Warning: package 'forecast' was built under R version 4.1.2

## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo

##
## Attaching package: 'forecast'

## The following object is masked from 'package:greybox':
##
##     forecast

data <- read.table(file="SALES.txt", header=TRUE)

sales <- ts(data$SALES, start=c(1,1980))

acf(sales,plot=TRUE, main="ACF of Sales")
```




```r
# As we can see the ACF is decaying very slowly with values at many lags
above the 2 standard error line hence we can conclude that this series is
non-stationary

D_sales <- diff(data$SALES)

acf(D_sales, plot=TRUE, main="ACF of first difference of Sales")
```

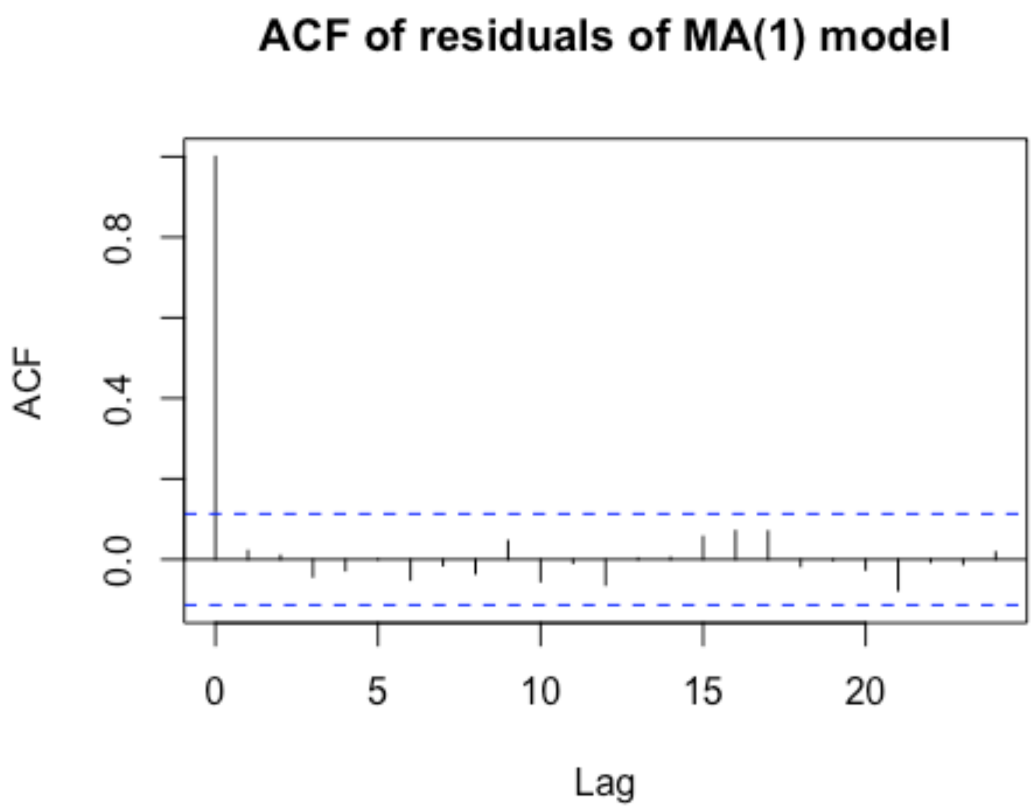



```
# We can see that the ACF at lag 1 is outside the 2 standard error bounds
however the series chops off to 0 right after hence we can say it is
stationary

pacf(D_sales, main="Partial ACF of first difference of Sales")
```




```
# As the Partial ACF is decaying slowly we will use a Moving Average model
for this series
# From the ACF plot we can see the values cuts off to 0 after lag 1, hence we
will use moving average series of order 1

fit=Arima(sales, order=c(0,1,1))
fit

## Series: sales
## ARIMA(0,1,1)
##
## Coefficients:
##           ma1
##       -0.5998
## s.e.   0.0501
##
## sigma^2 = 11204:  log likelihood = -1817.92
## AIC=3639.84   AICc=3639.88   BIC=3647.24

# M(t) = M(t-1) - 0.5998*ε(t-1)

epsilon = fit$residuals
```

```r
acf(epsilon, plot=TRUE, main="ACF of residuals of MA(1) model")
```




```r
Box.test(epsilon)

##
##  Box-Pierce test
##
## data:  epsilon
## X-squared = 0.13563, df = 1, p-value = 0.7127

# As we can see no residuals except at lag 0 are outside the 2 standard error
bounds
# The p-value value from the box test is 0.7127 so we cannot reject that this
series is white noise
# Based on the above two factors we can conclude that the residuals are white
noise

# One step ahead forecast

# M(t+1) = 4976 - 0.5998*-31.6270266 = 4994.969

# Two step ahead forecast
```

```
# M(t+2) = 4994.969 - 0.5998*ε(t+1) = 4994.969
# As we don't have the error from one step in the future the forecast will be the previous step value
```