Week 5 Exercises

Help Center

6.6

What proportion of the forests with N nodes have no trees consisting of a single node? For $N = 1, 2, 3,$ and 4, the answers are $0, 1/2, 2/5,$ and $3/7$, respectively.

6.27

For $N = 2^n - 1$, what is the probability that a perfectly balanced tree structure (all 2^n external nodes on level n) will be built, if all $N!$ key insertion sequences are equally likely?

6.43

Internal nodes in binary trees fall into one of three classes: they have either two, one, or zero external children. What fraction of the nodes are of each type, in a random binary search tree of N nodes?

7.29

An *arrangement* of N elements is a sequence formed from a subset of the elements. Prove that the EGF for arrangements is $e^z/(1-z)$. Express the coefficients as a simple sum and interpret that sum combinatorially.

7.45

Find the CGF for the total number of inversions in all involutions of length N. Use this to find the average number of inversions in an involution.

7.61

Use asymptotics from generating functions (see Section 5.5) or a direct argument to show that the probability for a random permutation to have j cycles of length k is asymptotic to the Poisson distribution $e^{-\lambda}\lambda^j/j!$ with $\lambda = 1/k$.

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