Week 2 Exercises

Help Center

2.17

Solve the recurrence. $A_N = A_{N-1} - \frac{2A_{N-1}}{N} + 2\left(1 - \frac{2A_{N-1}}{N}\right) \quad \text{for } N > 0 \text{ with } A_0 = 0.$

This recurrence describes the following random process: A set of N elements collect into "2-nodes" and "3-nodes." At each step each 2-node is likely to turn into a 3-node with probability $2/N$ and each 3-node is likely to turn into two 2-nodes with probability $3/N$ What is the average number of 2-nodes after N steps?

2.69

Plot the periodic part of the solution to the recurrence
$a_N = 3a_{\lfloor N/3 \rfloor} + N \quad \text{for } N > 3 \text{ with } a_1 = a_2 = a_3 = 1$ for $1 \le N \le 972$.

3.20

Solve the recurrence $a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3} \quad \text{for } n > 2$ with $a_0 = a_1 = 0$ and $a_2 = 1$.
Solve the same recurrence with the initial condition on a_1 changed to $a_1 = 1$.

3.28

Find an expression for $[z^n]\frac{1}{\sqrt{1-z}} \ln \frac{1}{1-z}$. *Hint.* Expand $(1-z)^{-\alpha}$ and differentiate with respect to α.

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