

## Week 6 Exercises

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### 8.3

How long a string of random bits should be taken to be 50% sure that there are at least 32 consecutive 0s?

### 8.14

Suppose that a monkey types randomly at a 32-key keyboard. What is the expected number of characters typed before the monkey hits upon the phrase THE QUICK BROWN FOX JUMPED OVER THE LAZY DOG?

### 8.57

Solve the recurrence for  $p_N$  given in the proof of Theorem 8.9, to within the oscillating term.

$$p_N = \frac{1}{2^N} \sum_k \binom{N}{k} p_k \quad \text{for } N > 1 \quad \text{with } p_0 = 0 \text{ and } p_1 = 1$$

### 9.5

For  $M = 365$ , how many people are needed to be 99% sure that two have the same birthday?

### 9.38

("Abel's binomial theorem.") Use the result of the previous exercise and the identity

$e^{(\alpha+\beta)C(z)} = e^{\alpha C(z)} e^{\beta C(z)}$  to prove that

$$(\alpha + \beta)(n + \alpha + \beta)^{n-1} = \alpha \beta \sum_k \binom{n}{k} (k + \alpha)^{k-1} (n - k + \beta)^{n-k-1}.$$

### 9.99

Show that the probability that a random mapping of size  $N$  has no singleton cycles is  $\sim 1/e$ , the same as for permutations (!).

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