Week 1 Exercises

1.14

Follow through the steps above to solve the recurrence $A_N = 1 + \frac{2}{N} \sum_{1 \le j \le N} A_{j-1}$ for $N > 0$.

1.15

Show that the average number of exchanges used during the first partitioning stage (before the pointers cross) is $(N-2)/6$. (Thus, by linearity of the recurrences, the average number of exchanges used by quicksort is $\frac{1}{6} C_N - \frac{1}{2} A_N$.)

1.17

If we change the first line in the quicksort implementation above to call insertion sort when `hi-lo <= M` then the total number of comparisons to sort N elements is described by the recurrence

$$C_N = \begin{cases} N + 1 + \dfrac{1}{N} \displaystyle\sum_{1 \le j \le N} (C_{j-1} + C_{N-j}) & N > M; \\ \frac{1}{4} N(N-1) & N \le M \end{cases}$$ Solve this recurrence.

1.18

Ignoring small terms (those significantly less than N) in the answer to the previous exercise, find a function $f(M)$ so that the number of comparisons is approximately $2N \ln N + f(M)N$. Plot the function $f(M)$, and find the value of M that minimizes the function.