Assignment - 1

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1BM18CS036

1. Using Cayley-Hamilton, compute A^{-1}

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$ Also compute A^8

=> $$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 2 \\ 2 & -1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(-1-\lambda) - 4 = 0$$

$$-1 - \lambda + \lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 - 5 = 0$$

By Cayley-Hamilton theorem,

$$A^2 - 5I = 0$$

$$|A| = -5 \neq 0 \Rightarrow A^{-1} \text{ exists}$$

$$A^2 - 5I = 0$$

Multiplying A^{-1} on both sides,

$$A - 5A^{-1} = 0$$

$$5A^{-1} = A$$

$$A^{-1} = \frac{1}{5} A$$

$$A^{-1} = \frac{1}{5} \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix}$$