Assignment - 1

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1. Using Cayley-Hamilton, compute A^{-1}

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$ Also compute A^8

$$\Rightarrow |A - \lambda I| = \begin{vmatrix} 1-\lambda & 2 \\ 2 & -1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(-1-\lambda) - 4 = 0$$

$$-1 - \lambda + \lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 - 5 = 0$$

By Cayley-Hamilton theorem

$$A^2 - 5I = 0$$

$$|A| = -5 \neq 0 \Rightarrow A^{-1} \text{ exists}$$

$$A^2 - 5I = 0$$

Multiplying A^{-1} on both sides,

$$A - 5A^{-1} = 0$$

$$5A^{-1} = A$$

$$A^{-1} = \frac{1}{5} A$$

$$A^{-1} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix}$$

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To find A^8:

$$A^2 - 5I = 0$$

$$A^2 = 5I$$

$$(A^2)^4 = (5I)^4$$

$$A^8 = 5^4 I$$

$$= 625\ I \quad = 625\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^8 = \begin{bmatrix} 625 & 0 \\ 0 & 625 \end{bmatrix}$$

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2. Using cayley-hamilton theorem, find A^{-1}, A^{-2}, A^{-3}

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$

=>

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 4-\lambda & 6 & 6 \\ 1 & 3-\lambda & 2 \\ -1 & -4 & -3-\lambda \end{vmatrix} = 0$$

$$4-\lambda\left[(-9+\lambda^2)+8\right] - 1(-18-6\lambda+24)$$
$$-1(-18+6\lambda+12) = 0$$

$$4\lambda^2 - 4 - \lambda^3 + \lambda - 6 + 6\lambda + 6 - 6\lambda = 0$$

$$\lambda^3 - 4\lambda^2 - \lambda + 4 = 0$$

By cayley Hamilton theorem, we know

$$A^3 - 4A^2 - A + 4 = 0$$

$$A(A^2 - 4A - I) = -4$$

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$$A^{-1} = -\frac{1}{4}(A^2 - 4A - I) \quad \text{—(1)}$$

$$A^2 = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix} \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 16+6-6 & 24+18-24 & 24+12-18 \\ 4+3-2 & 6+9-8 & 6+6-6 \\ -4-4+3 & -6-12+12 & -6-8+9 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 18 & 18 \\ 5 & 7 & 6 \\ -5 & -6 & -5 \end{bmatrix}$$

$$A^{-1} = -\frac{1}{4}\left\{\begin{bmatrix} 16 & 18 & 18 \\ 5 & 7 & 6 \\ -5 & -6 & -5 \end{bmatrix} - 4\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right\}$$

$$= -\frac{1}{4}\begin{bmatrix} -1 & -6 & -6 \\ 1 & -6 & -2 \\ -1 & 10 & 6 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1/4 & 3/2 & 3/2 \\ -1/4 & 3/2 & 1/2 \\ 1/4 & -5/2 & -3/2 \end{bmatrix}$$

(1) $A^{-1} = -\frac{1}{4}(A^2 - 4A - I)$

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Multiply A^{-1}

$$A^{-2} = -\frac{1}{4}\left[A - 4I - A^{-1}\right]$$

$$A^{-2} = -\frac{1}{4}\left\{\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1/4 & 3/2 & 3/2 \\ -1/4 & 3/2 & 1/2 \\ 1/4 & -5/2 & -3/2 \end{bmatrix}\right\}$$

$$A^{-2} = -\frac{1}{4}\begin{bmatrix} -1/4 & 9/2 & 9/2 \\ 5/4 & -5/2 & 3/2 \\ -5/4 & -3/2 & -11/2 \end{bmatrix} = \frac{1}{16}\begin{bmatrix} 1 & -18 & -18 \\ -5 & 10 & -6 \\ 5 & 6 & 22 \end{bmatrix}$$

$$A^{-3} = A^{-2}\cdot A^{-1} = \frac{1}{16}\cdot\frac{1}{4}\begin{bmatrix} 1 & -18 & -18 \\ -5 & 10 & -6 \\ 5 & 6 & 22 \end{bmatrix}\begin{bmatrix} 1 & 6 & 6 \\ -1 & 6 & 2 \\ 1 & -10 & -6 \end{bmatrix}$$

$$= \frac{1}{64}\begin{bmatrix} 1+18-18 & 6-108+180 & 6-36+108 \\ -5-10-6 & -30+60+60 & -30+20+36 \\ 5-6+22 & 30+36-220 & 30+12-132 \end{bmatrix}$$

$$\Rightarrow A^{-3} = \begin{bmatrix} 1/64 & 78/64 & 78/64 \\ -21/64 & 90/64 & 26/64 \\ 21/64 & -154/64 & -90/64 \end{bmatrix}$$

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3. Find the eigen space of $T: R^3 \to R^3$

$$T(x,y,z) = (2x+y-2z,\ 2x+3y-4z,\ x+y-z)$$

$$\Rightarrow T = \begin{bmatrix} 2x+y-2z \\ 2x+3y-4z \\ x+y-z \end{bmatrix} = \begin{bmatrix} 2 & 1 & -2 \\ 2 & 3 & -4 \\ 1 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Matrix of linear transformation

$$A = \begin{bmatrix} 2 & 1 & -2 \\ 2 & 3 & -4 \\ 1 & 1 & -1 \end{bmatrix}$$

Characteristic equation of A is

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$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 1 & -2 \\ 2 & 3-\lambda & -4 \\ 1 & 1 & -1-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)((3-\lambda)(-1-\lambda)+4)-2(-1-\lambda+2)+1(-4+6-2\lambda)$$

$$\Rightarrow (2-\lambda)(-3-3\lambda+\cancel{\lambda}+\lambda^2+4)-2(1-\lambda)+1(2-2\lambda)=0$$

$$\Rightarrow (2-\lambda)(\lambda^2-2\lambda+1)=0$$

$$(2-\lambda)(\lambda-1)^2=0$$

$$\lambda=2, \quad 1$$

Eigen values : $\lambda=2,1$

For $\lambda=2$:

$$(A-2I)x=0$$

$$\left(\begin{bmatrix} 2 & 1 & -2 \\ 2 & 3 & -4 \\ 1 & 1 & -1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow A-2I = \begin{bmatrix} 0 & 1 & -2 \\ 2 & 1 & -4 \\ 1 & 1 & -3 \end{bmatrix}$$

$$\approx \begin{bmatrix} 1 & 1 & -3 \\ 2 & 1 & -4 \\ 0 & 1 & -2 \end{bmatrix} \approx \begin{bmatrix} 1 & 1 & -3 \\ 0 & -1 & 2 \\ 0 & 1 & -2 \end{bmatrix} \quad R_2 \to R_2 - 2R_1$$

$$\approx \begin{bmatrix} 1 & 1 & -3 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

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$\rho = 2$

$3 - 2 = 1$ free variables

$x_3 = k$

$-x_2 + 2x_3 = 0$

$x_2 = 2k$

$x_1 + x_2 - 3x_3 = 0$

$x_1 = -2k + 3k = k$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} k \\ 2k \\ k \end{bmatrix} = k \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

Eigen ~~value~~ space corresponding to $\lambda = 2$

$$E_2 = \left\{ k \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} ; k \in \mathbb{R} \right\} \quad \text{or } E_2 = \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right)$$

For $A = 1$:

$(A - I)x = 0$

$$\left(\begin{bmatrix} 2 & 1 & -2 \\ 2 & 3 & -4 \\ 1 & 1 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A - I \Rightarrow \begin{bmatrix} 1 & 1 & -2 \\ 2 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

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Reducing to echelon

$$\begin{bmatrix} 1 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad R_2 \to R_2 - 2R_1, \quad R_3 \to R_3 - R_1$$

$\rho = 1$

$3 - 1 = 2$ free variables

$x_2 = s$

$x_3 = t$

$x_1 + x_2 - 2x_3 = 0$

$x_1 = -s + 2t$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -s+2t \\ s \\ t \end{bmatrix}$$

$$= s\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

Eigen space corresponding to $\lambda = 1$ is

$$E_1 = \left\{ s\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \middle| s, t \in R \right\}$$

$$E_1 = \text{Span}\left(\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}\right)$$

4. Find the Eigen space of $T: \mathbb{R}^3 \to \mathbb{R}^3$

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a) $T = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$\Rightarrow \quad A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0$$

$\Rightarrow (2-\lambda)(6-3\lambda-2\lambda+\lambda^2-2) - 1(4-2\lambda-2) + 1(2-3+\lambda) = 0$

$\Rightarrow (2-\lambda)(\lambda^2-5\lambda+4) + 2\lambda - 2 - 1 + \lambda = 0$

$\Rightarrow (2-\lambda)(\lambda-4)(\lambda-1) + 3(\lambda-1) = 0$

$\Rightarrow (\lambda-1)((2-\lambda)(\lambda-4)+3) = 0$

$(\lambda-1)(\lambda^2-6\lambda+5) = 0$

$(\lambda-1)(\lambda(\lambda-5) - 1(\lambda-5)) = 0$

$(\lambda-1)(\lambda-5)(\lambda-1) = 0$

$\lambda = 1, 5$

Eigen values are $\lambda = 1, 5$

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for $\lambda = 1$

$(A - I)x = 0$

$$\left(\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A - I = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$\rho = 1$

$3 - 1 = 2$ free variables

$x_2 = s$

$x_3 = t$

$x_1 + 2x_2 + x_3 = 0$

$x_1 = -2s - t$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2s - t \\ s \\ t \end{bmatrix}$$

$$= s \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore$ Eigen space $\lambda = 1$

$$E_1 = \left\{ s \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \,\middle|\, s, t \in R \right\}$$

$$E_1 = \text{span}\left(\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right)$$

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For $\lambda = 5$

$(A - 5I)\, x = 0$

$$\left(\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A - 5I = \begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix} \approx \begin{bmatrix} -3 & 2 & 1 \\ 0 & -4 & 4 \\ 0 & 8 & -8 \end{bmatrix} \begin{array}{l} R_2 \to 3R_2 + R_1 \\ R_3 \to 3R_3 + R_1 \end{array}$$

$$= \begin{bmatrix} -3 & 2 & 1 \\ 0 & -4 & 4 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3 + 2R_2$$

$\rho = 2$

$3 - 2 = 1$ free variable

$x_3 = k$

$-4x_2 + 4x_3 = 0$

$x_2 = k$

$-3x_1 + 2x_2 + x_3 = 0$

$-3x_1 = -2k - k$

$x_1 = k$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} k \\ k \\ k \end{bmatrix} = k\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Eigen space for $\lambda = 5$

$$E_5 = \left\{ k\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \,\middle|\, k \in \mathbb{R} \right\}$$

$$E_5 = \text{span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)$$

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b) $T = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{vmatrix} = 0$$

$(1-\lambda)\ (5-5\lambda-\lambda+\lambda^2-1) - 1(1-\lambda-3)$
$+3(1-15+3\lambda) = 0$

$(1-\lambda)\ (4-6\lambda+\lambda^2) - 1(-2-\lambda) + 3(-14+3\lambda) = 0$

$4-6\lambda+\lambda^2-4\lambda+6\lambda^2-\lambda^3+2+\lambda-42+9\lambda=0$

$\lambda^3 - 7\lambda^2 + 36 = 0$

Let $\lambda = -2$

$-8-28+36 = 0$

-2	1	-7	0	36
	0	-2	18	-36
	1	-9	18	0

$\lambda^2 - 9\lambda + 18 = 0$

$\lambda(\lambda-6) - 3(\lambda-6) = 0$

$(\lambda-6)(\lambda-3) = 0$

$\lambda = 6, 3, -2.$

Eigen values are $\lambda = -2, 3, 6$

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For $\lambda = -2$

$(A+2I)\ x = 0$

$$\left(\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A+2I = \begin{bmatrix} 3 & 1 & 3 \\ 1 & 7 & 1 \\ 3 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 3 & 1 & 3 \\ 0 & 20 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} R_2 \to 3R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$\rho = 2$

$3 - 2 = 1$ free variable

$x_3 = k$

$20x_2 = 0$

$x_2 = 0$

$3x_1 + x_2 + 3x_3 = 0$

$x_1 = -k$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -k \\ 0 \\ k \end{bmatrix} = k \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Eigen space $\lambda = -2$

$$E_{-2} = \left\{ k \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \middle|\ k \in \mathbb{R} \right\}$$

$$E_{-2} = \text{span}\left(\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right)$$

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For $\lambda = 3$

$(A - 3I)x = 0$

$$\left(\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A - 3I = \begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & -2 \end{bmatrix} \approx \begin{bmatrix} -2 & 1 & 3 \\ 0 & 5 & 5 \\ 0 & 5 & 5 \end{bmatrix} \begin{matrix} R_2 \to 2R_2 + R_1 \\ R_3 \to 2R_3 + 3R_1 \end{matrix}$$

$$\approx \begin{bmatrix} -2 & 1 & 3 \\ 0 & 5 & 5 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$\rho = 2$

1 free variable

$x_3 = k$

$5x_2 + 5x_3 = 0$

$x_2 = -k$

$-2x_1 + x_2 + 3x_3 = 0$

$x_1 = k$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} k \\ -k \\ k \end{bmatrix} = k\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

~~Esp~~ Eigen space $\lambda = 3$

$$E_3 = \left\{ k\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \middle| \, k \in R \right\}$$

or

$$E_3 = \text{span}\left(\left(\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}\right)\right)$$

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For $\lambda = 6$

$$(A - 6I)x = 0$$

$$\left(\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix} - 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A - 6I = \begin{bmatrix} -5 & 1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -5 \end{bmatrix}$$

$$\approx \begin{bmatrix} -5 & 1 & 3 \\ 0 & -4 & 8 \\ 0 & 8 & -16 \end{bmatrix} \quad \begin{matrix} R_2 \to 5R_2 + R_1 \\ R_3 \to 5R_3 + 3R_1 \end{matrix}$$

$$\approx \begin{bmatrix} -5 & 1 & 3 \\ 0 & -4 & 8 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3 + 2R_1$$

1 free variable

$x_3 = k$

$-4x_2 + 8x_3 = 0$

$x_2 = 2k$

$-5x_1 + x_2 + 3x_3 = 0$

$x_1 = k$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} k \\ 2k \\ k \end{bmatrix} = k\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

Eigen space for $\lambda = 6$

$$E_6 = \left\{ k\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \;\middle|\; k \in R \right\}$$

$$E_6 = \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}\right)$$

5. Find minimal polynomial of

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$$A = \begin{bmatrix} 4 & -2 & 2 \\ 6 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$$

2) $|A - \lambda I| = 0$

$$\begin{vmatrix} 4-\lambda & -2 & 2 \\ 6 & -3-\lambda & 4 \\ 3 & -2 & 3-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)(\lambda^2-1) - 6(2\lambda-2) + 3(2\lambda-2) = 0$$

$$(4-\lambda)(\lambda+1)(\lambda-1) - 6(\lambda-1) = 0$$

$$(\lambda-1)\left[(4-\lambda)(\lambda+1) - 6\right] = 0$$

$$(\lambda-1)\left[4\lambda + 4 - \lambda^2 - \lambda - 6\right] = 0$$

$$(\lambda-1)\left[\lambda^2 - 3\lambda + 2\right] = 0$$

$$(\lambda-1)\left[\lambda^2 - 2\lambda - \lambda + 2\right] = 0$$

$$(\lambda-1)\left(\lambda(\lambda-2) - 1(\lambda-2)\right) = 0$$

$$(\lambda-1)^2(\lambda-2) = 0$$

$$\lambda = 1, 1, 2$$

$f(x) = (x-1)^2(x-2)$ is characteristic polynomial

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The minimal ~~opening~~ polynomial divides the characteristic polynomial

$m(x) = (x-1)(x-2)$

To verify cayley-Hamilton theorem

$M(A) = 0$

$(A-I)(A-2I) = 0$

$$= \begin{bmatrix} 3 & -2 & 2 \\ 6 & -4 & 4 \\ 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} 2 & -2 & 2 \\ 6 & -5 & 4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6-12+6 & -6+10-4 & 6-8+2 \\ 12-24+12 & -12+20-8 & 12-16+4 \\ 6-12+6 & -6+10-4 & 6-8+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ LHS = RHS

$A - I \neq 0$ & $A - 2I \neq 0$

There exists $m(x)$ which is monic polynomial of lowest degree & satisfies cayley Hamilton theorem

$m(x) = (x-1)(x-2)$ is minimal polynomial