

Longest increasing subsequence

08 October 2023 13:01

→ 8 4 6 1 9 10 2 5 7 0

[1, 2, 5, 7] } → 4 ans = 4
[4, 6, 9, 10]

[3 1 5 2 7]

ans = 3

(1, 5, 7) or (3, 5, 7)
(1, 2, 7)

(1, 5)
(1, 2)
(1, 7)
(3, 7)

~~10~~

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Brute force

① Generate all subsequences. Time = $O(2^N)$

② Using 2 for loops. Time = $O(N^2)$ ↓
Space = $O(N)$

nums = [3, 1, 5, 2, 7]

[3, 100, 4, 5, 6]

curr count = ~~2~~ 3
max count = 3

count = ~~2~~

min = 3

max = 100

3, 100, 101, 102, 4, 5, 6, 7

arr: [3, 1, 5, 2, 7]

dp: (length) [1, 1, 2, 2, 3]

for (i=0; i<n)
for (j=0 to i)

③ Binary search.

$O(N \log N)$

ans = 5

nums: 9 2 4 10 1 6 7 15 3

dp: 9 4 10 7 15
2 3 6

~~3 ans~~

ans = dp. length
= 5

nums: 2 10 3 4 7
dp: 2 10 4 7
3

ans = [2, 3, 4]
= 3

	i	i	i	i
numbr	2	10	3	4
dp	2	10	4	
		3		

[2, 10]

	0	1	2	3	4	
dp	2	4	7	10	12	X
	s		mid	s	e	
			e	mid		

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return start

s = mid + 1
e = mid - 1

$n=4$

weights:

values:

✓	✓	✗	✗
1	2	4	5
5	4	8	6

get max profit/value.

max ratio?

(greedy)

weights

values

greedy.

1. Take ratio

2. Take max value

DP

1. Explore all possibilities

Time = $O(2^n)$
optimal

general max (include, exclude).

weights: 1, 2, 4, 5
values: 5, 4, 8, 6

Base case:

if ($w == 0$ || $n == 0$)

return 0;

$w =$ Bag
5 kg

$w =$ sum capacity of bag
 $n =$ no. of items

Bag
5 kg

profit = 6

8 + 5

5 + 4

bag capacity

10 kg

ans = 10

ans = (13)

3

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Recursive over

if (currW <= totalW)

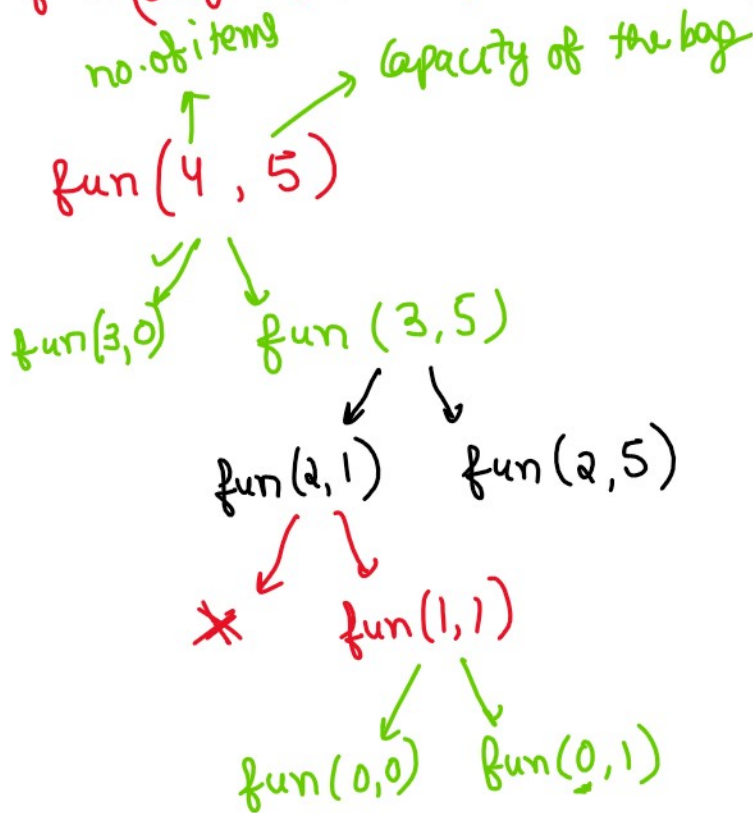
include = value[i] + fun(weights, values, W_T - w_c, n-1);

exclude = 0 + fun(weights, values, W_T, n-1);

weights
1, 2, 4, 5
5, 4, 8, 6
values

max(include, exclude)

[N] [w]
[4] {6}
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Time = $O(N \times W)$
Space = $O(N \times W)$

n=3

weights
[3, 2, 2]
values
[1, 2, 1]

Capacity = 4

values: [1.8, 1, 1]

value/weight = 0.6 0.5 0.5

$$\text{ans} = (1+1) = \underline{\underline{2}}$$

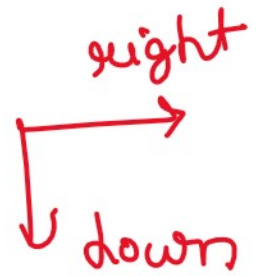
(greedy)
ans = 1.8



Min path sum

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	0	1	2	3	
0	1	2	3	8	
1	4 5	3 6	2 8	5	13
2	6 11	7	12 5	7	19



$$(1 + 2 + 3 + 1 + 5 + 7) = 19$$

μ-w Top down
Bottom up

$$\text{grid}[i][j] += \min(\text{grid}[i-1][j], \text{grid}[i][j-1])$$

(top) (left)

Time = $O(N \times M)$
Space = $O(1)$

LCS (Longest common subsequence)

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test1 := a b c d e

'ac'

ans = 2

test2 := x a z y c

test1 := a b c d e

ae / ac

ans = 2

test2 := a e c

2^{N1}

2^{N2}

$2^{N1} \times 2^{N2} = 2^{N1+N2}$

test1 := a b c d g h

(N2)

'adh'

test2

ans = 3

test2 := a e d f h x

(N1)
test1

		a	e	d	f	h	x
a	0	1	1	1	1	1	1
b	0	1	1	1	1	1	1
c	0	1	1	1	1	1	1
d	0	1	1	2	2	2	2
g	0	1	1	2	2	2	2
h	0	1	1	2	2	3	3

1. If test1[i] == test2[j]
go to diagonal (+1)

2. In other cases
max(left, top)

'abcd'

'aed'

(1,0)

h	0	1	1	2	2	3	3
---	---	---	---	---	---	---	---

'ab' 'x'

'aed'

Time $O(N1 \times N2)$
Space $O(N1 \times N2)$

↓
optimise?

$O(N^2)$

2 rows

? → single char

* → zero or more char

? = a

* = bb

a?ba

- aabbb (true)
- aaabb (false)
- aebc (true)
- abcc (false)
- acbbz (true)
- abb (true)

? = e

* = c

? = c

* = bz

? = b

* = ''

s = albmnc

p = (a?b*c

? = l

* = mn

a?ba
albmnc

albmnc
a?ba

(A)

	a	?	b	*	c
a	T	F	F	F	F
l	F	T	F	F	F
b	F	F	T	F	F
m	F	F	F	T	F
n	F	F	F	F	T
c	F	F	F	T	T

① If s[i] = p[j]
or p[j] = ?

(check diagonal)

② 'e'
check top or left

one or more char
empty sequence

Time + $O(p \times m)$
Space + $O(N \times m)$ $O(N)$