# RMT, Symmetry and Chaos

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## Work Done under DSKC Internship

Literature review of Random Matrix Theory: Wigner's Surmise and Semi-Circular Law for a 2x2 Hermitian Matrix and Computational for a large GOE, GUE.

Numerically verified Wigner's surmise and Semi-Circular Law for a very large Hermitian Ensemble using Python and learnt optimisation techniques in Python like Numba and multiprocessing. Also have little experience with FORTRAN, c++, C.

Treatment (classical) of Coulomb gas (1D) using RMT and comparison with usual statistical treatment (where we use a differential equation to find the expected energy values). Computational Project.

Numerically verified that the eigenvalue spectrum given by RMT is equivalent to the energy levels output by the usual statistical treatment. (solving the differential equation)
Literature Review of behaviour of GOE to GUE transitions using Python.

## Work Done from November 2023 to Febuary 2024

**mainly Literature Review, Some Computational Review**
Studied the symmetry conditions of various Ensembles, mainly GOE and GUE. Reviewed the Symmetry Conditions for the Hamiltonian of a QKR under transitioning to chaotic conditions. (Focusing on the breaking of symmetry parameters $\alpha$ and $\kappa$.)

Studied the QKR Hamiltonian using RMT to try to explain the seemingly spontaneous symmetry breaking needed to transition to the chaotic regime, both quasi-analytically and numerically (using Python), by building transition models for Time evolution operator by application of floquet's theorem.

Looked at two transitions;

1. GOE $\rightarrow$ GUE, where the model will be (For studying P breaking),

$$(H_{GOE} + H_{GUE})/\sqrt{1+\eta^2}$$

2. 2GOE $\rightarrow$ 1GOE, where the model will be (for studying breakage of T),

$$(H_{2GOE} + H_{1GOE}/1+\eta^2)$$

$\eta$ just a number that varies from 0 to $\infty$ (in a very interesting by ML Mehta, it was proved that through varying a particular symmetry constant represented by a number between 0 and 1 ($\infty$ being the fringe case also representing something similar to 1), we can go break symmetry, basically go from GOE TO GUE.

Through Poissoinian statistics, normalization constraints and symmetry conditions, along with the Hamiltonian being used, partial symmetry breaking through each of these transitions was determined.

Transition model which seems to agree with the literature using Poisson Statistics (using partition matrices): $H_0 = H_{0:1} \bigoplus ... \bigoplus H_{l-1:l}$. ($GOE \rightarrow 2GUE$ for full symmetry breaking; Apprently no exact solution for eigenvalue distributions for these transitions exists)

Explained the QKR symmetry breaking however why the 'breaking' is needed is still not explained since the ensembles really just come with symmetry constraints, so really just a model.

Conjecture: Symmetry breaking is a condition needed for the chaotic regime.

Found literature in both dynamical measures of chaos and RMT supporting this claim.

# Possible Future Work

Expand on the numerical analysis of other statistical systems like real gases and use RMT to compare results.

Since P and T are being broken down, my hypothesis is that PT symmetry should be conserved, explore that avenue using RMT exnsemble transitions and dynamical measures of chaos.

Study Floquet Systems and OTOC Dynamical systems of chaos and symmetry conditions for chaos (separately and together with RMT)