

# ASSIGNMENT-9

## Heap Sort

**AIM:** Implement the heap sort to sort given set of values using max or min heap.

### **Objectives:**

1. Understand the concept of Heap Sort.
2. Understand the types of heap sort and its applications.

### **THEORY :**

There are several types of heaps, however in this chapter, we are going to discuss binary heap. A **binary heap** is a data structure, which looks similar to a complete binary tree. Heap data structure obeys ordering properties discussed below. Generally, a Heap is represented by an array. In this chapter, we are representing a heap by **H**.

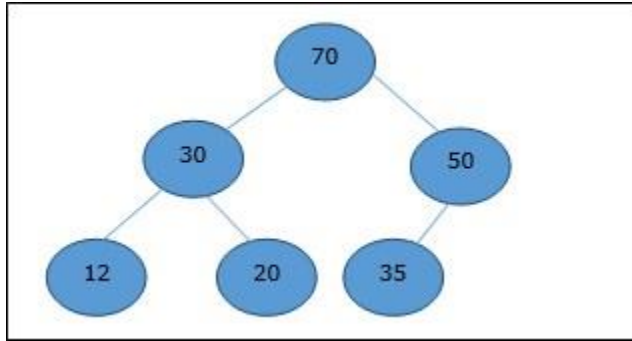
As the elements of a heap is stored in an array, considering the starting index as **1**, the position of the parent node of **i<sup>th</sup>** element can be found at  $\lfloor i/2 \rfloor$ . Left child and right child of **i<sup>th</sup>** node is at position **2i** and **2i + 1**.

A binary heap can be classified further as either a **max-heap** or a **min-heap** based on the ordering property.

### **Max-Heap**

In this heap, the key value of a node is greater than or equal to the key value of the highest child.

Hence,  $H[\text{Parent}(i)] \geq H[i]$

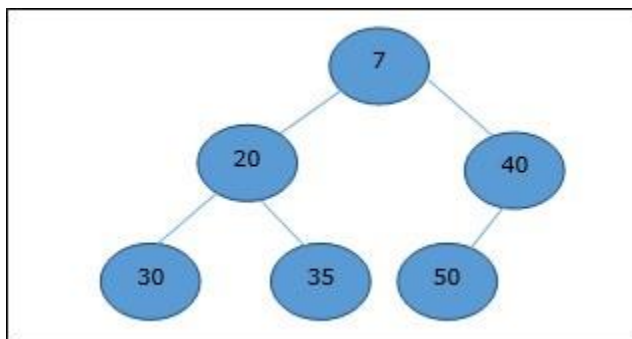


## Min-Heap

In min-heap, the key value of a node is lesser than or equal to the key value of the lowest child.

Hence,  $H[\text{Parent}(i)] \leq H[i]$

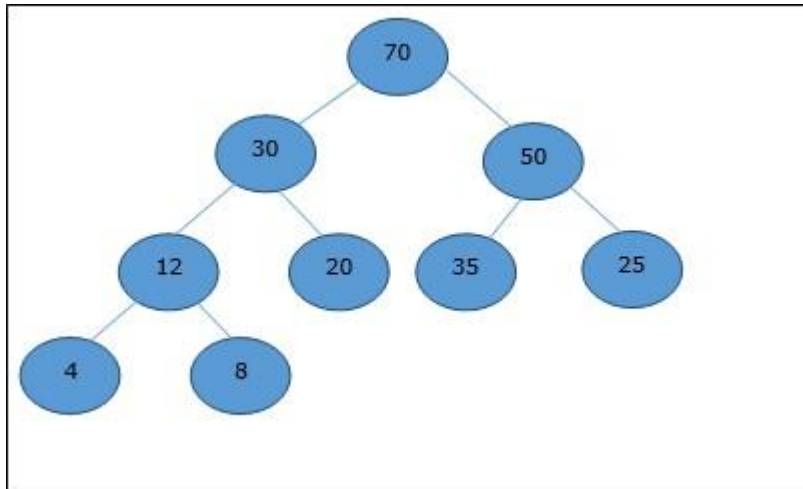
In this context, basic operations are shown below with respect to Max-Heap. Insertion and deletion of elements in and from heaps need rearrangement of elements. Hence, **Heapify** function needs to be called.



## Array Representation

A complete binary tree can be represented by an array, storing its elements using level order traversal.

Let us consider a heap (as shown below) which will be represented by an array **H**.



Considering the starting index as **0**, using level order traversal, the elements are being kept in an array as follows.

Index	0	1	2	3	4	5	6	7	8
elements	70	30	50	12	20	35	25	4	8

In this context, operations on heap are being represented with respect to Max-Heap.

To find the index of the parent of an element at index **i**, the following algorithm **Parent (numbers[], i)** is used.

**Algorithm: Parent (numbers[], i)**  
 if  $i == 1$  return NULL else  
 $[i / 2]$

The index of the left child of an element at index **i** can be found using the following algorithm, **Left-Child (numbers[], i)**.

**Algorithm: Left-Child (numbers[], i)**  
 If  $2 * i \leq \text{heapsize}$  return  
 $[2 * i]$  else return NULL

The index of the right child of an element at index **i** can be found using the following algorithm, **Right-Child (numbers[], i)**.

**Algorithm: Right-Child (numbers[], i)**  
if  $2 * i < \text{heapsize}$  return  $[2 * i + 1]$  else return NULL

## Max-Heapify

Heapify method rearranges the elements of an array where the left and right sub-tree of  $i^{\text{th}}$  element obeys the heap property.

**Algorithm:**

**Max-Heapify(numbers[], i)**  
leftchild := numbers[2i] rightchild  
:= numbers [2i + 1]  
if leftchild  $\leq$  numbers[].size and numbers[leftchild] >  
numbers[i] largest := leftchild else largest  
:= i  
if rightchild  $\leq$  numbers[].size and numbers[rightchild] >  
numbers[largest] largest := rightchild if largest  $\neq$   
i  
swap numbers[i] with numbers[largest]  
Max-Heapify(numbers, largest)

## Build-Max-Heap

When the provided array does not obey the heap property, Heap is built based on the following algorithm **Build-Max-Heap (numbers[])**.

**Algorithm:**

**Build-Max-Heap (numbers[])**  
numbers[].size := numbers[].length for i  
=  $\lfloor \text{numbers}[].\text{length}/2 \rfloor$  to 1 by -1  
Max-Heapify (numbers[], i)

## Max-Heap-Sort

**Algorithm:**

1. Build Max Heap from unordered array;
2. Find maximum element  $A[1]$ ;
3. Swap elements  $A[n]$  and  $A[1]$ :  
now max element is at the end of the array!
4. Discard node  $n$  from heap (by decrementing heap-size variable)
5. New root may violate max heap property, but its children are max heaps. Run `max_heapify` to fix this.
6. Go to Step 2 unless heap is empty.

## Min-Heapify

Heapify method rearranges the elements of an array where the left and right sub-tree of  $i^{\text{th}}$  element obeys the heap property.

### Algorithm:

```
Min-Heapify(numbers[], i)
leftchild := numbers[2i]   rightchild
:= numbers[2i + 1]
if leftchild ≤ numbers[] and numbers[leftchild] <
numbers[i]
    smallest := leftchild
else
    smallest := i
if rightchild ≤ numbers[] and numbers[rightchild] <
numbers[smallest]
    smallest := rightchild
if smallest ≠ i
    swap numbers[i] with numbers[smallest]
    Min-Heapify(numbers, smallest)
```

## Build-Min-Heap

When the provided array does not obey the heap property, Heap is built based on the following algorithm **Build-Min-Heap (numbers[])**.

### Algorithm:

```
Build-Min-Heap(numbers[])
numbers[].size := numbers[].length
for i
= [ numbers[].length/2 ] to 1 by -1
```

```
Min-Heapify (numbers[], i)
```

## Min-Heap-Sort

### Algorithm:

1. Build Min Heap from unordered array; 2. Find maximum element  $A[1]$ ; 3. Swap elements  $A[n]$  and  $A[1]$ :  
now max element is at the end of the array!
4. Discard node  $n$  from heap (by decrementing heap-size variable)
5. New root may violate min heap property, but its children are min heaps. Run `min_heapify` to fix this.
6. Go to Step 2 unless heap is empty.

**CODE-**

```
#include <iostream>
using namespace std;
void maxHeapify(int a[], int i, int n){
    int j, temp;
    temp = a[i];
    j = 2*i;
    while(j<=n){
        if(j<n && a[j+1]<a[j])
            j=j+1;
        if(temp<a[j])
            break;
        else if(temp>=a[j]){
            a[j/2] = a[j];
            j= 2*j;
        }
    }
    a[j/2] = temp;
    return;
}

void min_heapify(int a[], int i, int n){
    int j, temp;
    temp=a[i];
    j=2*i;
    while(j<=n){
        if(j<n && a[j+1]>a[j])
            j=j+1;
        if(temp>a[j])
            break;
        else if(temp<=a[j]){
            a[j/2]=a[j];
            j=2*j;
        }
    }
    a[j/2]=temp;
    return;
}

void build_maxheap(int a[], int n){
```

```
int i;
for(i=n/2 ; i>=1; i--){
    maxHeapify(a,i,n);
}
}

void max_HeapSort(int a[], int n){
    int i, temp;
    for(i=n; i>=2; i--){
        temp = a[i];
        a[i] = a[1];
        a[1] = temp;
        maxHeapify(a, 1, i-1);
    }
}

void build_minheap(int a[], int n){
    int i;
    for(i=n/2; i>=1; i--){
        min_heapify(a,i,n);
    }
}

void min_HeapSort( int a[], int n){
    int i, temp;
    for(i=n; i>=2; i--){
        temp = a[i];
        a[i] = a[1];
        a[1] = temp;
        min_heapify(a, 1, i-1);
    }
}

void print(int arr[], int n){
    cout<<"Sorted data : ";
    for(int i=1; i<=n; i++){
        cout<<"->"<<arr[i];
    }
    return;
}

int main(){
    int n, i, ch;
    cout<<"\t*** Heap Sort ***\n"<<endl;
    cout<<"Enter the number of elements to be sorted: " ;
    cin>>n;
    int arr[n];
    for(i=1; i<=n; i++) {
```



```
cout<<"Enter element "<<i<<" :";
cin>>arr[i];
}
do{
cout<<"\n\n1]Heap sort using Max Heap";
cout<<"\n2]Heap sort using Min heap";
cout<<"\n3]Exit";
cout<<"\nEnter your choice: ";
cin>>ch;
switch(ch){

case 1:
build_maxheap(arr, n);
max_HeapSort(arr, n);
print(arr, n);

break;
case 2:
build_minheap(arr, n);
min_HeapSort(arr, n);
print(arr, n);
break;
case 3:
cout<<"\nProgram Exited Successfully !!"<<endl;
}
}while(ch!=3);
return 0;
}
```

**OUTPUT-**

\*\*\* Heap Sort \*\*\*

Enter the number of elements to be sorted: 5

Enter element 1 :34

Enter element 2 :76

Enter element 3 :55

Enter element 4 :21

Enter element 5 :8

1]Heap sort using Max Heap

2]Heap sort using Min heap

3]Exit

Enter your choice: 1

Sorted data : ->76->55->34->21->8

1]Heap sort using Max Heap

2]Heap sort using Min heap

3]Exit

Enter your choice: 2

Sorted data : ->8->21->34->55->76

1]Heap sort using Max Heap

2]Heap sort using Min heap

3]Exit

Enter your choice: 3

Program Exited Successfully !!

## **Conclusion :**

We have constructed max and min heap sort to sort array and perform deletion operation on it.