

## a) Some of the test cases for the question are:

1. For  $x=10$ :

Explanation- The value of the answer would be 5 as:

$$1! = 1 \text{ (not divisible by 10)}$$

$$2! = 2 \text{ (not divisible by 10)}$$

$$3! = 6 \text{ (not divisible by 10)}$$

$$4! = 24 \text{ (not divisible by 10)}$$

$$5! = 120$$

since,  $120/10 = 12$ . Hence, it is divisible by 10. So,  $i=5$  for  $x=10$ .

2. For  $x=9$ :

Explanation- The value of the answer would be 6 as:

$$1! = 1 \text{ (not divisible by 9)}$$

$$2! = 2 \text{ (not divisible by 9)}$$

$$3! = 6 \text{ (not divisible by 9)}$$

$$4! = 24 \text{ (not divisible by 9)}$$

$$5! = 120 \text{ (not divisible by 9)}$$

$$6! = 720$$

since,  $720/9 = 80$ . Hence, it is divisible by 9. So,  $i=6$  for  $x=9$ .

3. For  $x=8$ :

Explanation- The value of the answer would be 4 as:

$$1! = 1 \text{ (not divisible by 8)}$$

$$2! = 2 \text{ (not divisible by 8)}$$

$$3! = 6 \text{ (not divisible by 8)}$$

$$4! = 24$$

since,  $24/8 = 3$ . Hence, it is divisible by 8. So,  $i=4$  for  $x=8$ .

4. For  $x=12$ :

Explanation- The value of the answer would be 4 as:

$$1! = 1 \text{ (not divisible by 12)}$$

$$2! = 2 \text{ (not divisible by 12)}$$

$$3! = 6 \text{ (not divisible by 12)}$$

$$4! = 24$$

since,  $24/12 = 2$ . Hence, it is divisible by 12. So,  $i=4$  for  $x=12$ .

5. For  $x=15$ :

Explanation- The value of the answer would be 5 as:

$$1! = 1 \text{ (not divisible by 12)}$$

$$2! = 2 \text{ (not divisible by 12)}$$

$$3! = 6 \text{ (not divisible by 12)}$$

$$4! = 24 \text{ (not divisible by 12)}$$

$$5! = 120$$

since,  $120/15 = 8$ . Hence, it is divisible by 15. So,  $i=5$  for  $x=15$ .

## b) What value of $x$ , results in the factorial calculation to overflow?

Explanation- The max capacity of a 64 bit register is  $2^{64} - 1 = 1.84 * 10^{19}$

And the largest factorial just under this is  $20! = 2.43 * 10^{18}$

Considering, 21, we won't have to use this as 21 can be satisfied by 7!

Similarly, 22 can be satisfied by 11! But as 23 is a prime number, it can only be satisfied by 23! which will cause overflow. Hence, for 64-bit: the x that will cause overflow will be equal to 23.

Similarly, for:

32-bit,  $x=13$

16-bit,  $x=11$

8-bit,  $x=7$ .