Task 3:

Goal: Write a query to output all such symmetric pairs in ascending order by the value of X.

Table: Functions with two columns: X (Integer) and Y (Integer).

Definition of Symmetric Pair: Two pairs (X, Y) and (Y, X) are said to be symmetric pairs IF X \neq Y, AND X = Y. **Sample Input (Functions):** | X | Y | |----| 20 | 20 | 20 | 21 | 20 | 22 | 21 | 20 | 22 | 23 | 23 | 23 | 21 | 20 |

Sample Output: 20 20 20 21 22 23

Let's analyze the sample output against the input:

- **20 20:** This is a pair where X = Y.
- 20 21: The symmetric counterpart (21, 20) also exists in the input. Here X != Y.
- 22 23: The symmetric counterpart (23, 22) also exists in the input. Here X != Y.

This implies the true conditions for a symmetric pair are:

- 1. Pairs where X = Y.
- 2. Pairs where X != Y, AND the pair (Y, X) also exists in the table. And for the second case (X != Y), we should only output one of the pairs, specifically the one where X < Y to avoid duplicates (e.g., if we output 20 21, we don't output 21 20). The sample output confirms this: 20 21 is output, not 21 20. 22 23 is output, not 23 22.

So, the refined definition of a symmetric pair for outputting, based on the sample output, is: A pair (X, Y) is considered for output if:

- X = Y (e.g., (20, 20)).
- X < Y AND (Y, X) also exists in the Functions table (e.g., (20, 21) and (21, 20) are both in Functions, so we output (20, 21)).

SQL Query Strategy:

- 1. Handle the X = Y case directly.
- 2. Handle the X != Y case by self-joining the Functions table to find pairs (F1.X, F1.Y) where F1.X < F1.Y and (F1.Y, F1.X) exists as (F2.X, F2.Y).
- 3. Combine these two sets of results using UNION.
- 4. Order the final result by X in ascending order.

```
SELECT
    f1.X,
    f1.Y
FROM
    Functions f1
JOIN
    Functions f2 ON f1.X = f2.Y AND f1.Y = f2.X
WHERE
    f1.X < f1.Y -- To get only one of the (X,Y) or (Y,X) symmetric
pairs where X != Y
UNION
SELECT
    Χ,
    Υ
FROM
    Functions
WHERE
    X = Y -- To include pairs where X = Y
ORDER BY
```

Let's test this logic with the sample input:

- 1. WHERE X = Y part:
 - (20, 20) -> This will be selected.
- 2. Self-join part (f1.X < f1.Y):
 - Consider (20, 21) (f1). Does (21, 20) (f2) exist? Yes. And 20 < 21. So (20, 21) is selected.
 - Consider (20, 22) (f1). Does (22, 20) (f2) exist? No. So (20, 22) is NOT selected from this
 - Consider (22, 23) (f1). Does (23, 22) (f2) exist? Yes. And 22 < 23. So (22, 23) is selected.
 - Consider (21, 20) (f1). 21 < 20 is false. So this is not selected. (We already got 20,21).
 - Consider (23, 22) (f1). 23 < 22 is false. So this is not selected. (We already got 22,23).

Combined and Ordered:

```
1. (20, 20)
```

- 2. (20, 21)
- 3. (22, 23)

Х;

This exactly matches the Sample Output.

Therefore, the SQL query is:

```
SELECT
    f1.X,
    f1.Y
FROM
    Functions f1
JOIN
    Functions f2 ON f1.X = f2.Y AND f1.Y = f2.X
WHERE
    f1.X < f1.Y
UNION
SELECT
    Χ,
    Υ
FROM
    Functions
WHERE
    X = Y
ORDER BY
```