Overview
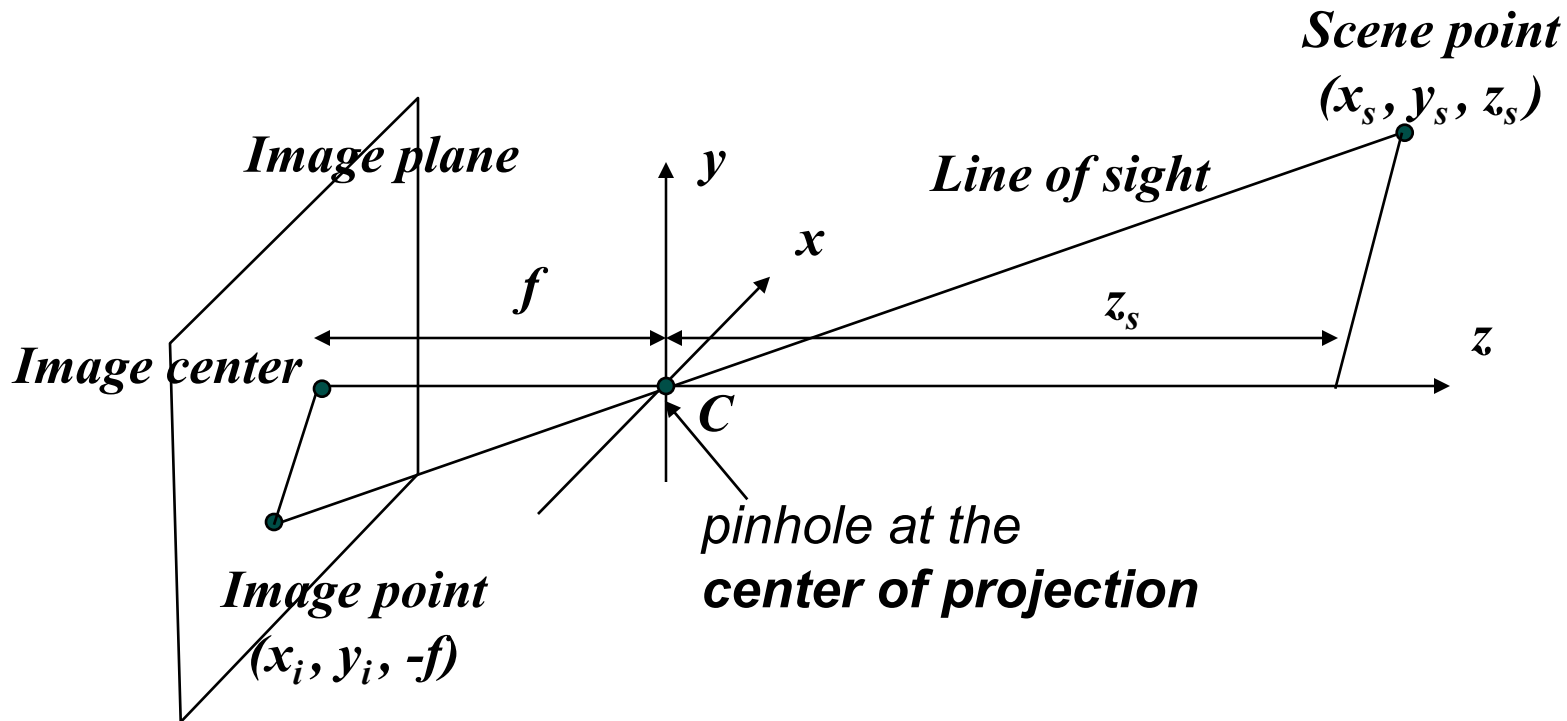
❖ perspective imaging

❖ Image formation

❖ Refraction of light

❖ Thin-lens equation

❖ Optical power and accommodation

❖ Image irradiance and scene radiance

❖ Digital images

❖ Introduction to MATLAB

Perspective Imaging - Pinhole Camera Model

❖ The point on the image plane that corresponds to a particular point in the scene is found by following the line that passes through the scene point and the center of projection

Perspective Imaging=Central Projection

❖ ***Line of sight*** to a point in the scene is the line through the center of projection to that point

❖ Image plane is parallel to the x-y plane
  - distance to image plane is f - focal length
  - this inverts the image
 - move the image plane in front of the center of projection

Perspective Imaging

Image plane

center of projection

y

x

Image point
(x_i, y_i, f)

Scene point
(x_s, y_s, z_s)

z

C

f

f
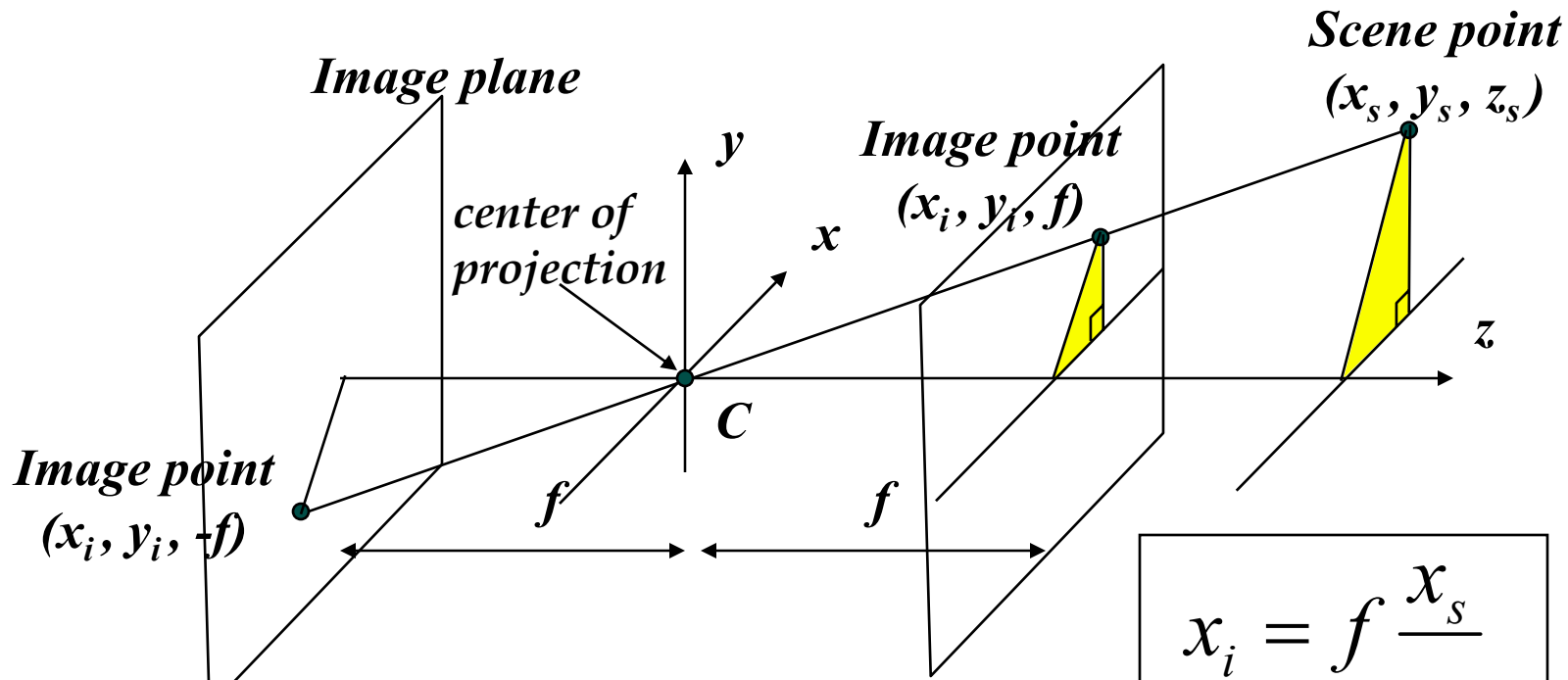
Image point
$(x_i, y_i, -f)$

❖ Fundamental equations for perspective projection onto a plane

$$x_i = f \frac{x_s}{z_s}$$

$$y_i = f \frac{y_s}{z_s}$$

Field of View

❖ As f gets smaller, image becomes more *wide angle* (more world points project onto the finite image plane)

❖ As f gets larger, image becomes more *telescopic* (smaller part of the world projects onto the finite image plane)

Field of view

f_1

f_2

f_3
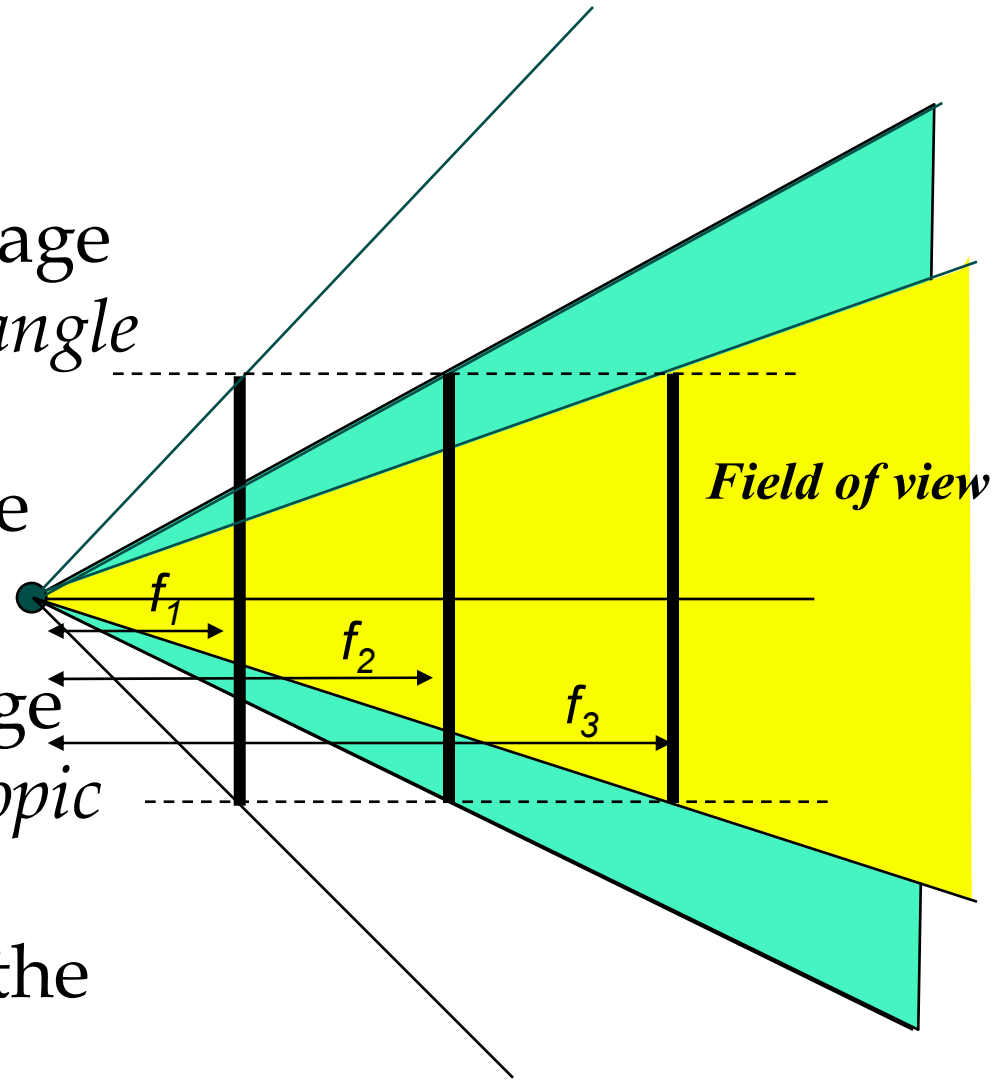
Homogenous coordinates

- ❖ Add an extra coordinate and use an equivalence relation
- ❖ for 2D
  - – equivalence relation k*(X,Y,Z) is the same as (X,Y,Z)
- ❖ for 3D
  - – equivalence relation k*(X,Y,Z,T) is the same as (X,Y,Z,T)

- ❖ Basic notion
  - – Possible to represent points "at infinity"
    - ◆ Where parallel lines intersect
    - ◆ Where parallel planes intersect
 - – Possible to write the action of a perspective camera as a matrix

The camera matrix

❖ Turn previous expression into HC's

   – HC's for 3D point are (X,Y,Z,T)

   – HC's for point in image are (U,V,W)

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

Lenses Collect More Light

❖ With a lens, diverging rays from a scene point are converged back to an image point

Scene

Image plane

Lens   *Center of projection*
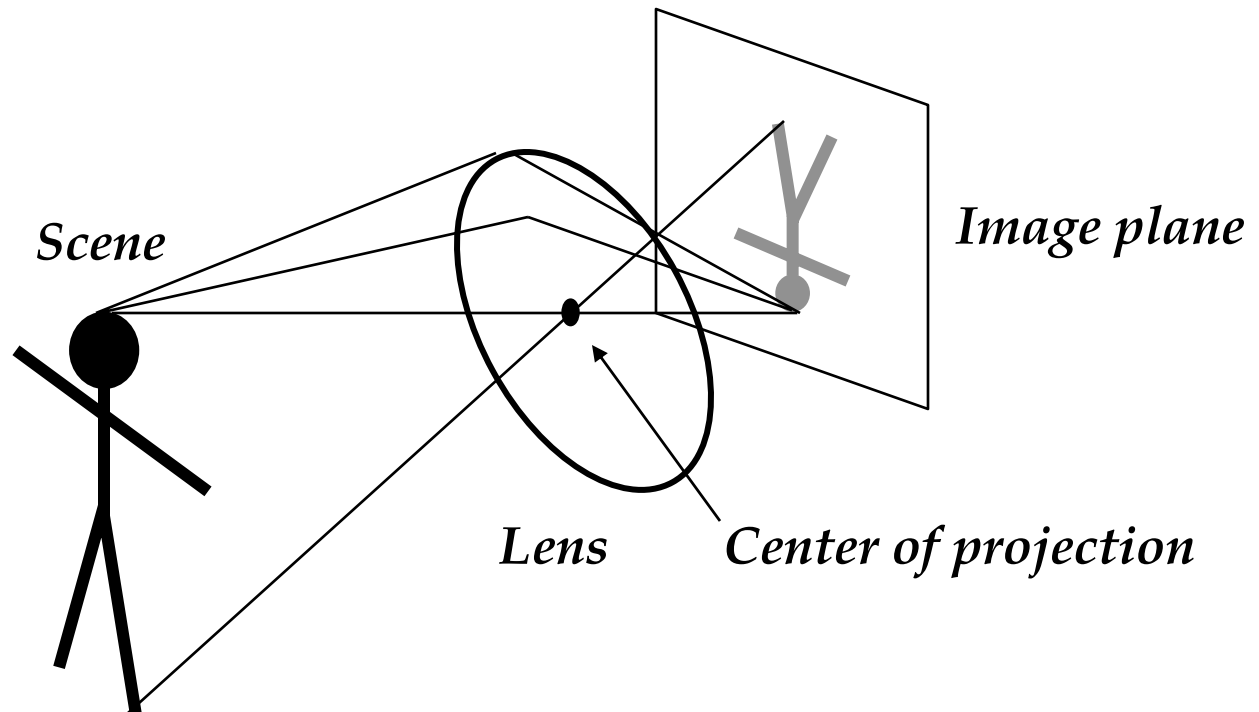
Refraction: Snell′s law

reflected ray

❖ If θ is the angle of incidence and $\theta′$ is the angle of refraction then

$$n \sin \theta = n'\sin \theta'$$

where *n* and *n′* are the refractive indices of the two media

Air
n=1

θ θ *incident ray*

Glass
n'=1.5

θ' *refracted ray*

refracted ray

❖ Refractive index is the ratio of speed of light in a vacuum to speed of light in the medium
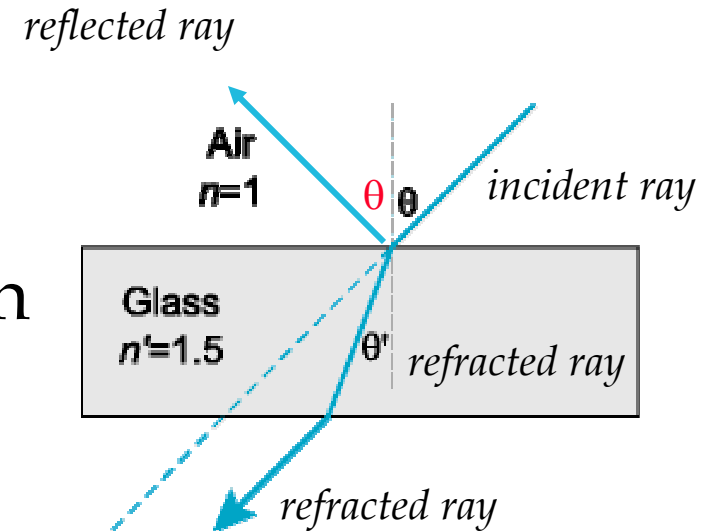
Refractive indices
glass - 1.50
water - 1.333
air - 1.000

Thin-Lens Equation

❖ Thin-lens equation

– relates the distance between the scene point being viewed and the lens to the distance between the lens and the point's image (where the rays from that point are brought into focus by the lens)

– Let M be a point being viewed
  ◆ p is the distance of M from the lens along the optical axis
  ◆ The thin lens focuses all the rays from M onto the same point, the image point m at distance q from the lens.
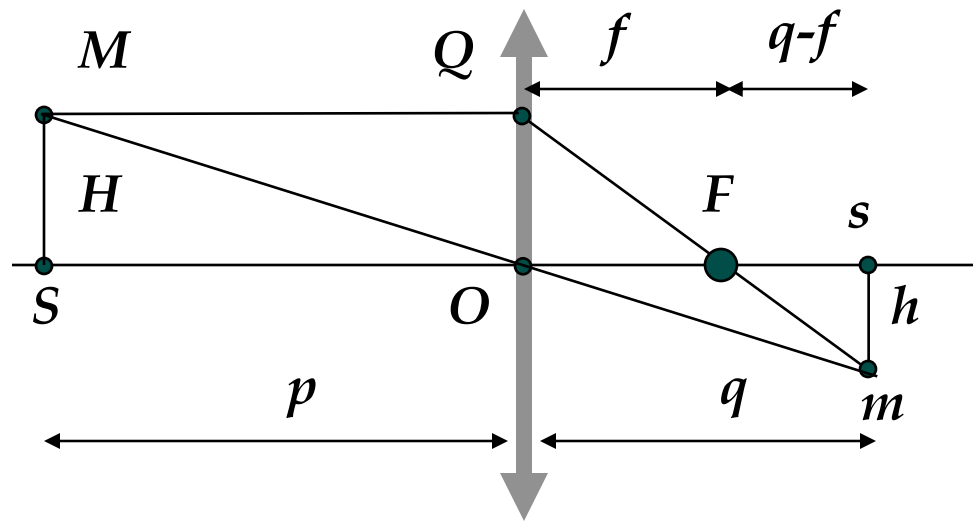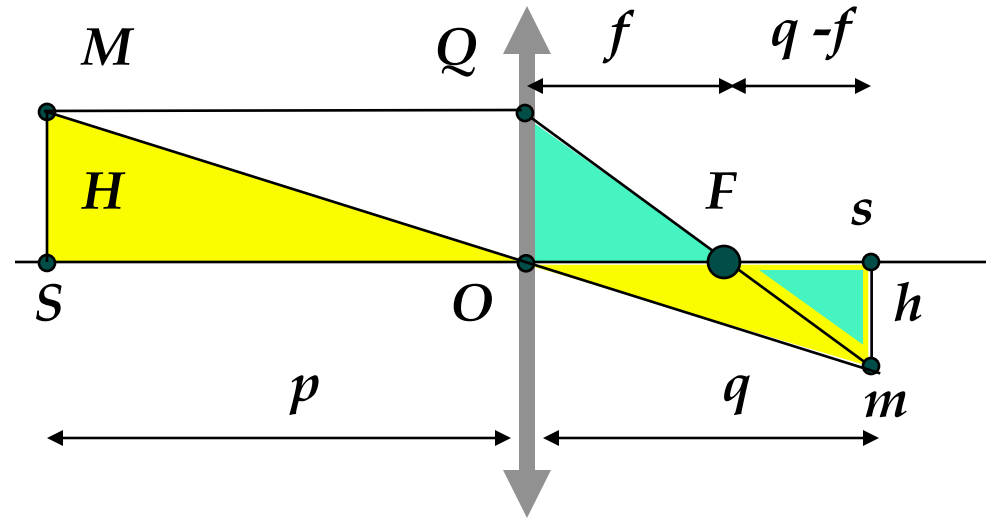
Thin-Lens Equation $\quad \dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f}$

❖ *m* can be determined by intersecting two known rays

– *MQ* is parallel to the optical axis, so it must be refracted to pass through F.

– MO passes through the lens center, so it is not bent.

❖ Note two pairs of similar triangles

– MSO and Osm (yellow)

– OQF and Fsm (green)

$$\frac{H}{p} = \frac{h}{q} = \frac{H+h}{p+q}$$

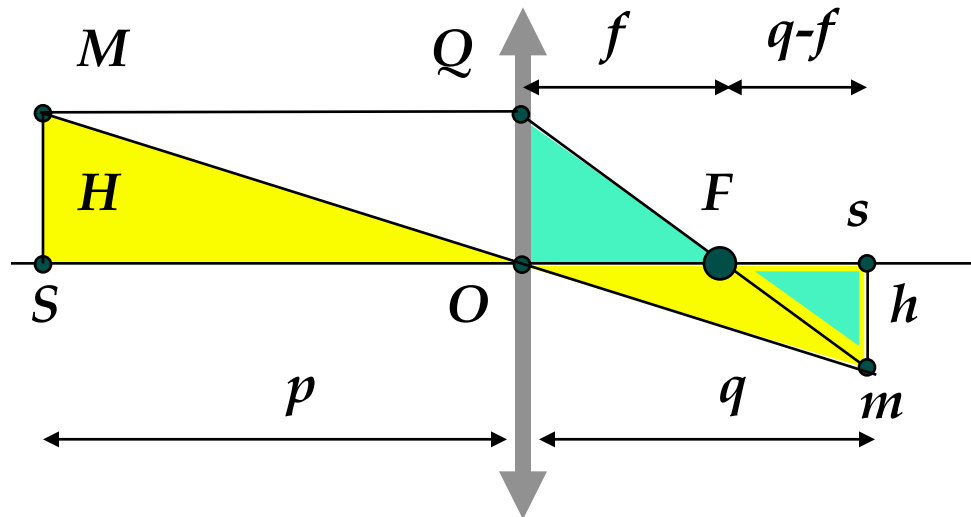$$\frac{H}{f} = \frac{h}{q-f} = \frac{H+h}{q}$$

Divide 2 equations:

$$\frac{p}{f} = \frac{p+q}{q}$$

$$\frac{1}{f} = \frac{p+q}{pq}$$

Thin-Lens Equation $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

❖ Notice that the distance behind the lens, *q*, at which a point, *M*, is brought into focus depends on *p*, the distance of that point from the lens

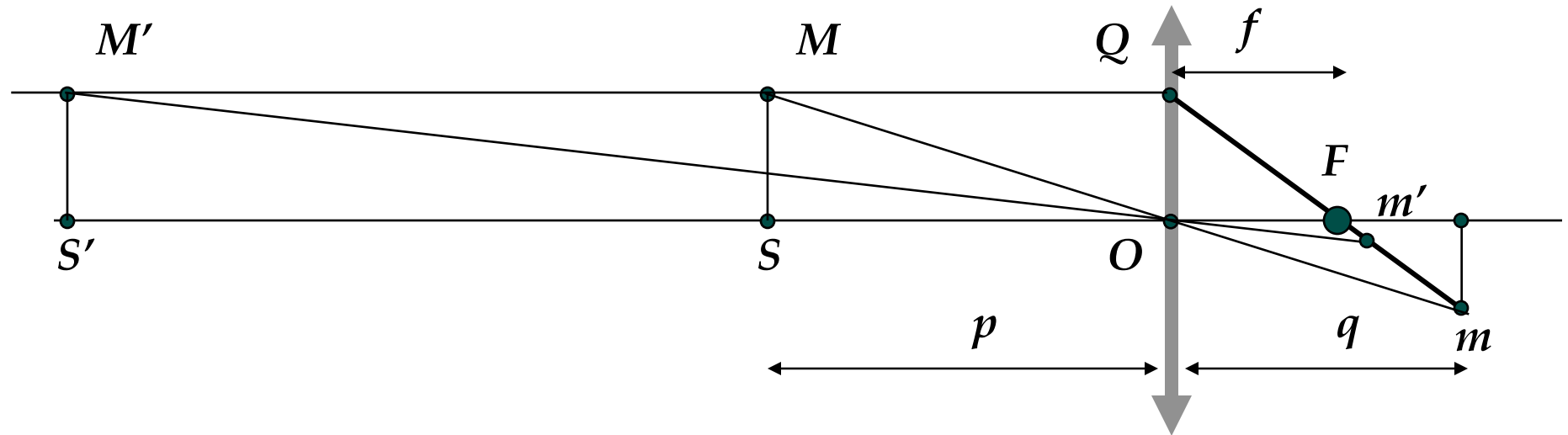 – familiar to us from rotating the focus ring of any camera

Middle school math

❖ **Invertendo.**   If a : b :: c : d then b : a :: d : c
❖ **Alternendo.**   If a : b :: c : d then a : c :: b : d
❖ **Componendo.** If a : b :: c : d then (a +b) : b :: (c +d) : d
❖ **Dividendo.** If a : b :: c : d then (a -b) : b :: (c -d) : d
❖ **Componendo and dividendo.**
   If a : b :: c : d then (a +b) : (a -b) :: (c +d) : (c -d)
   i.e., a/b = c/d => (a +b)/(a - b) = (c +d)/(c +d)
❖ If a/b = c/d = e/f = …,
   then each ratio

$$(a +c +e +...)/(b +d +f +...)$$

From: http://www.ilovemaths.com/2ratio.htm

Image formation - 13

Thin-Lens Equation $\quad \dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f}$

- ❖ As **p** gets large, **q** approaches **f**
- ❖ As **q** approaches **f**, **p** approaches infinity

Lens Equations

h_o

Object

Optical Axis

r_1

$^*r_2$

Air
$n=1.0$

Image

h_i

Glass $n=1.5$

d

f

p

q

* Negative sign convention
for radius to right of center (r_2).
For plano surface, $1/r_2 = 0$.
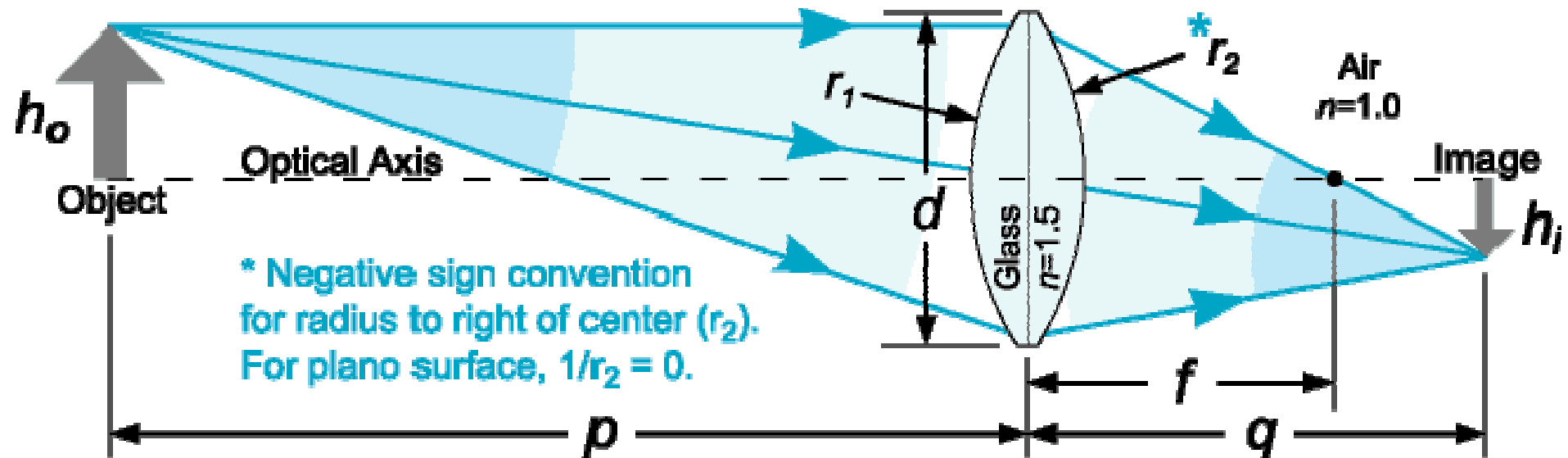
Lens Equation:
$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

Lens Maker's Equation:
$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Magnification:
$$m = \frac{h_i}{h_o} = -\frac{q}{p}$$

F-number:
$$f/\# = \frac{f}{d}$$

Some useful lens equations.

Optical Power and Accommodation

❖ Optical power of a lens - how strongly the lens bends the incoming rays

– Short focal length lens bends rays significantly

– It images a point source at infinity (large p) at distance f behind the lens. The smaller f, the more the rays must be bent to bring them into focus sooner.

– Optical power is $1/f$, with f measured in meters. The unit is called the diopter

– Human vision: when viewing faraway objects the distance from the lens to the retina is 0.017m. So the optical power of the eye is 58.8 diopters

Accommodation

❖ How does the human eye bring nearby points into focus on the retina?

– by increasing the power of the lens

– muscles attached to the lens change its shape to change the lens power

– accommodation: adjusting the focal length of the lens

– bringing points that are nearby into focus causes faraway points to go out of focus

– depth-of-field: range of distances in focus
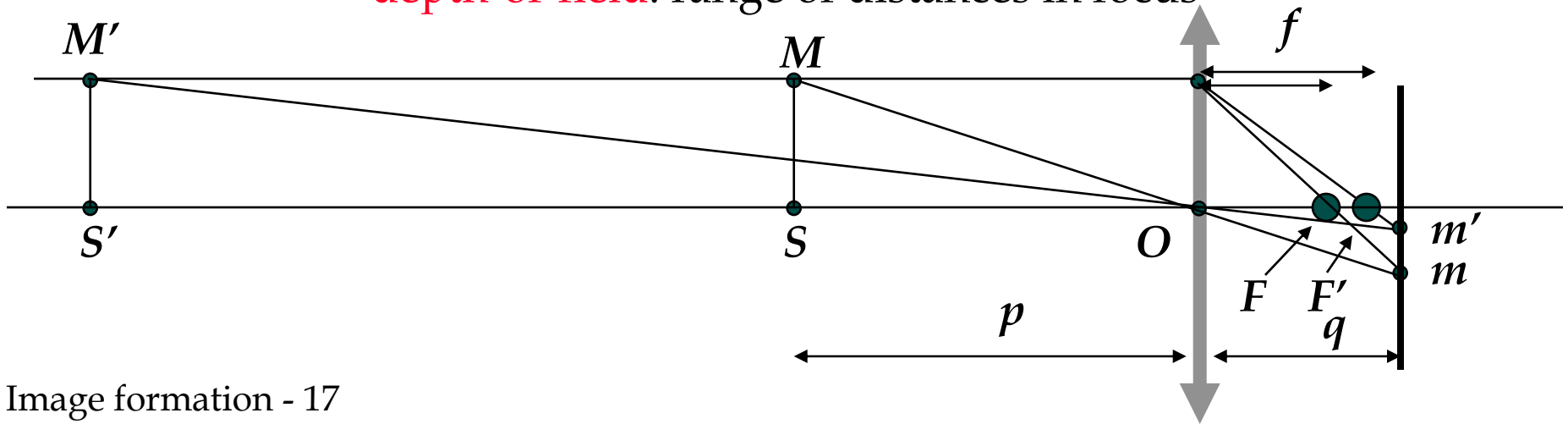
Accommodation
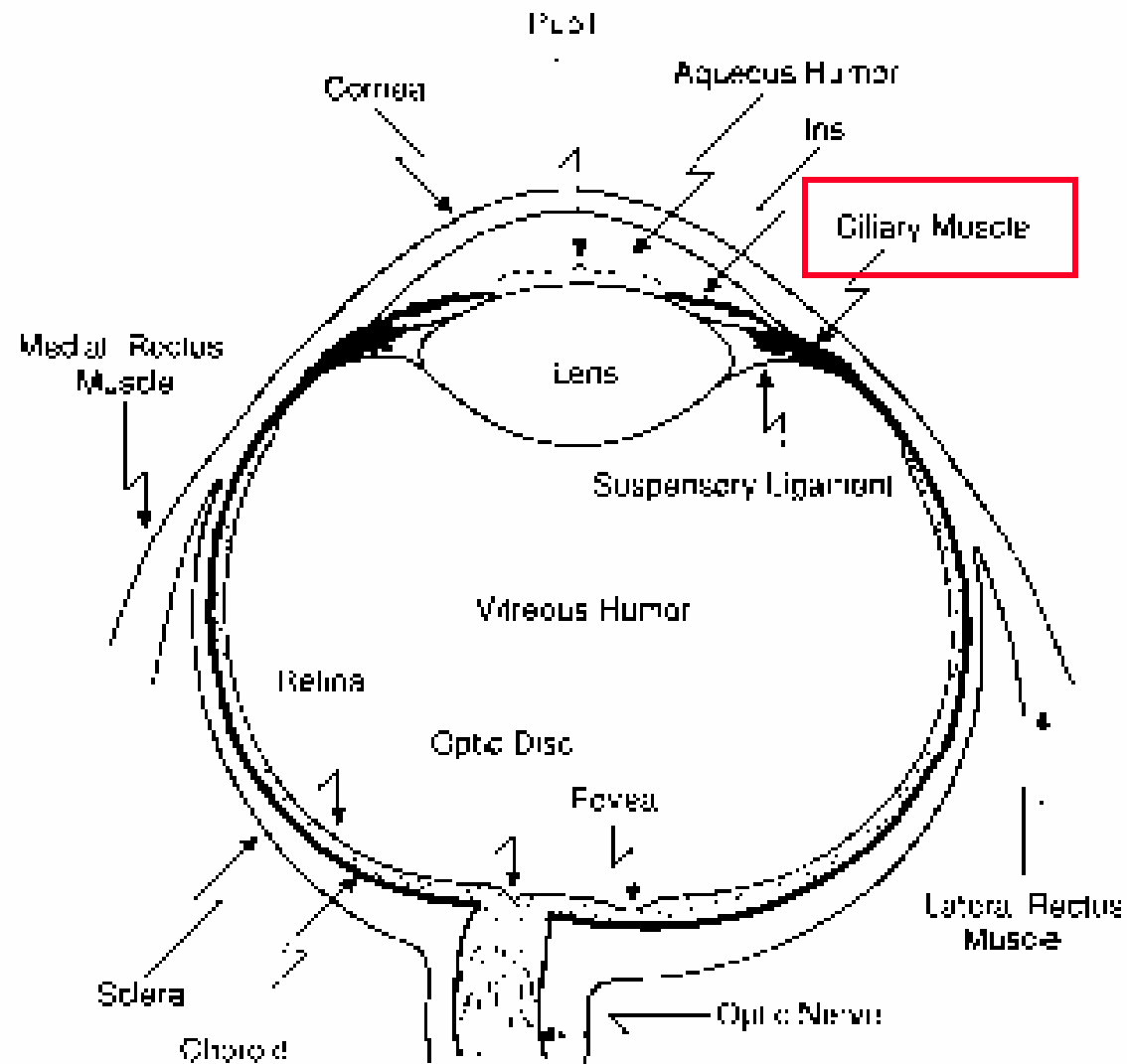
Accommodation
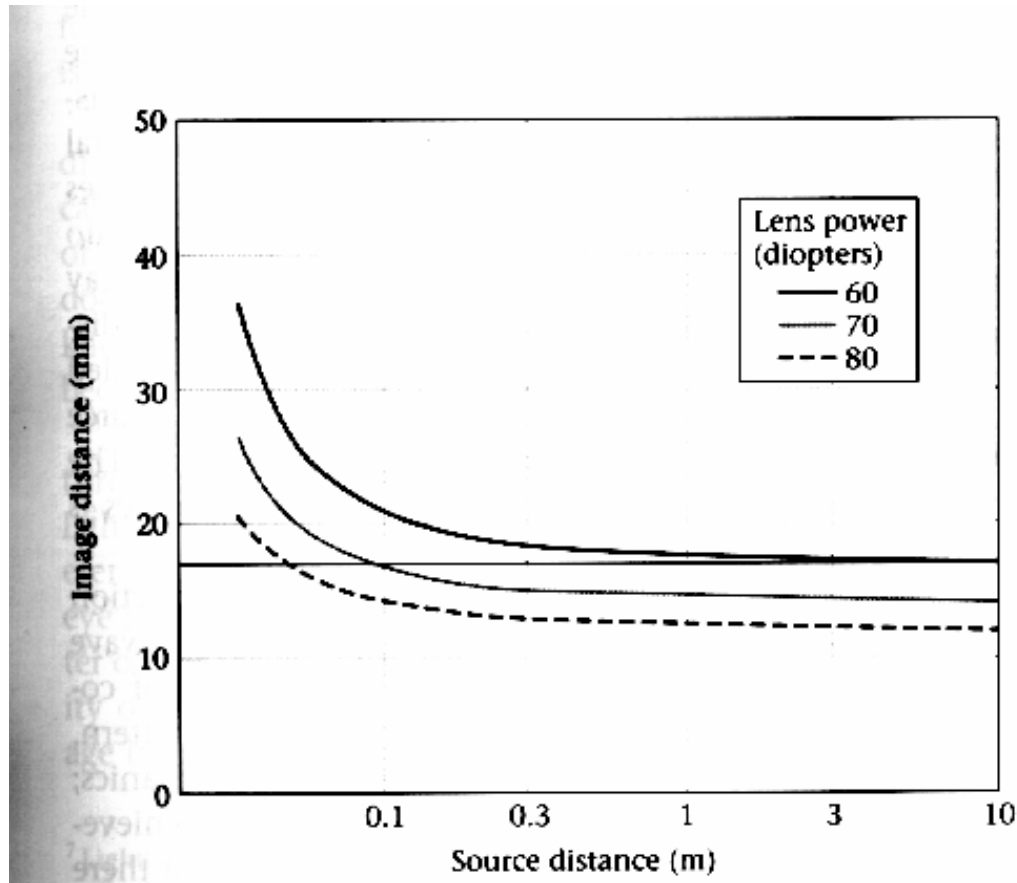
2.16 DEPTH OF FIELD OF THE HUMAN EYE. Image distance is shown as a function of source distance. The solid horizontal line shows the distance of the retina from the lens center. A lens power of 60 diopters brings distant objects into focus, but not nearby objects; to bring nearby objects into focus the power of the lens must increase. The depth of field (the distance over which objects will continue to be in reasonable focus) can be estimated from the slope of the curve.

Sources at > 1 meter are imaged at same distance
Sources closer than 1 m are imaged at different distances

Accommodation

❖ Physical cameras - mechanically change the distance between the lens and the image plane
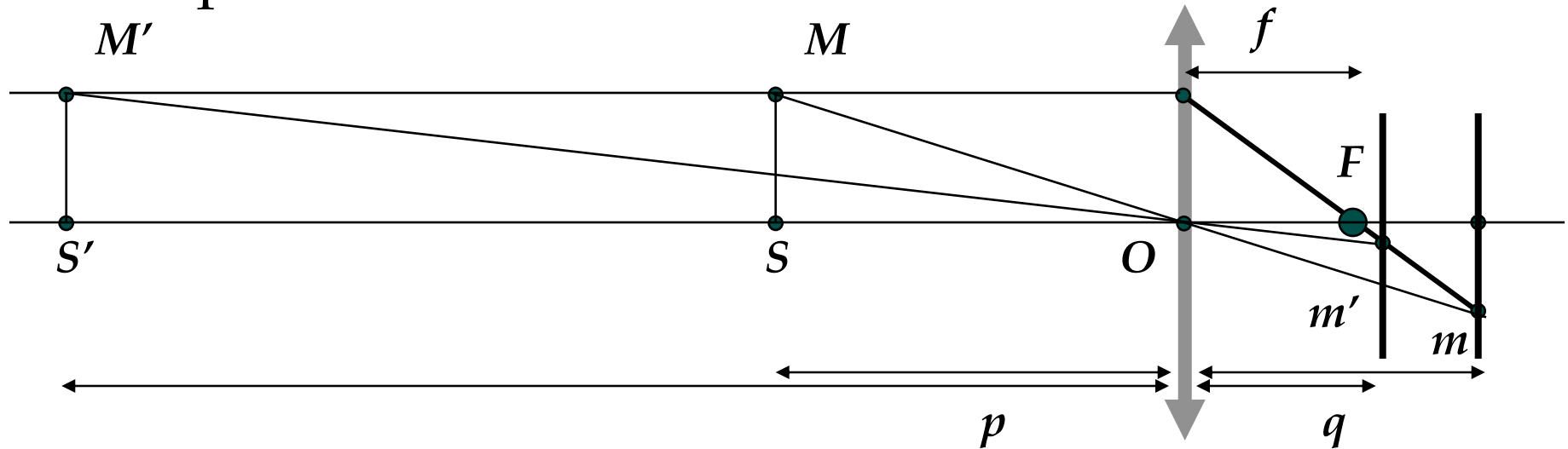
Pixel Brightness and Scene Brightness

Irradiance, E

❖ Light power per unit area (watts per square meter) incident on a surface.

❖ If surface tilts away from light, same amount of light strikes bigger surface (less irradiance)(no foreshortening)

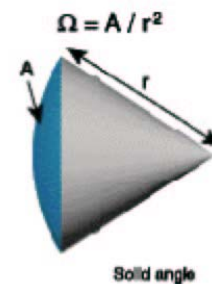❖ E times pixel area times exposure time -> pixel intensity
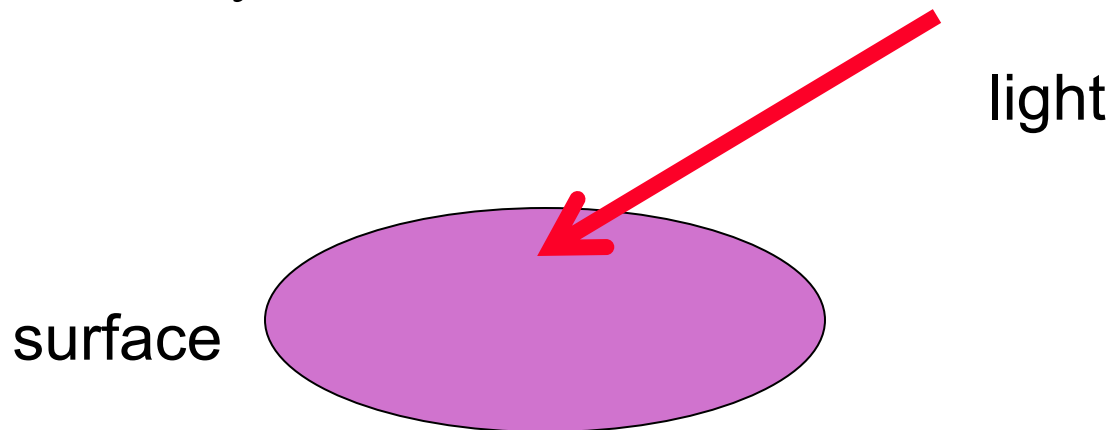
light

surface

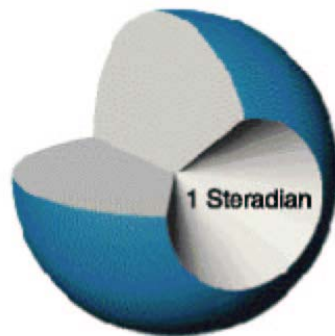# Radiance, L

❖ Amount of light radiated from a surface into a given solid angle per unit area (watts per square meter per steradian).

❖ Note: the area is the foreshortened area, as seen from the direction that the light is being emitted.
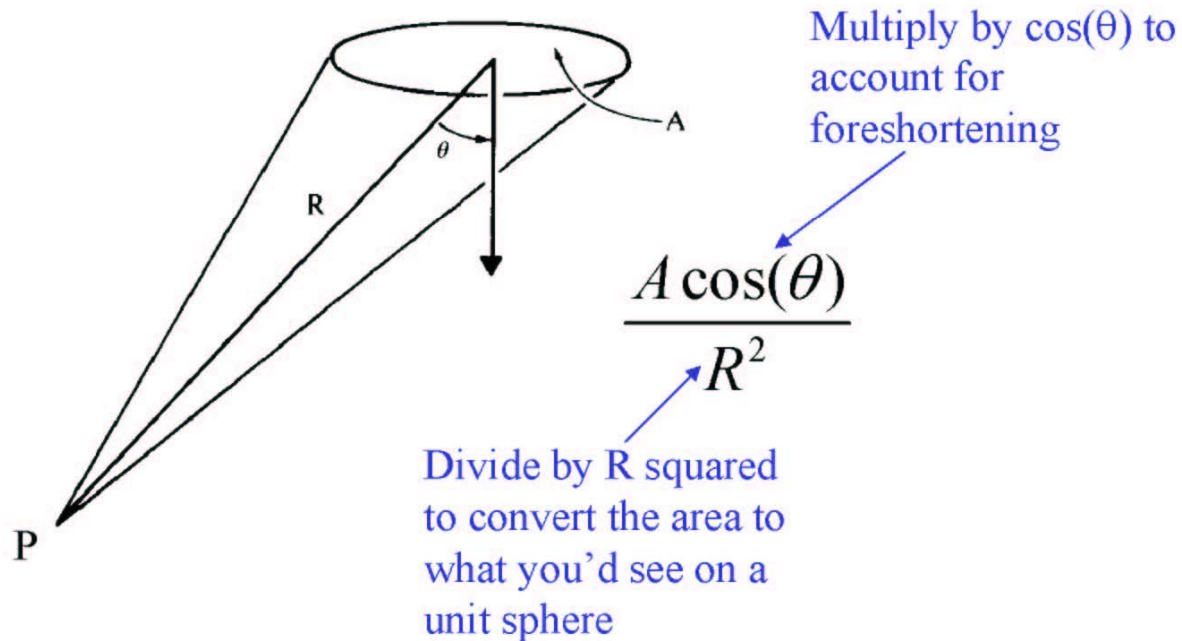
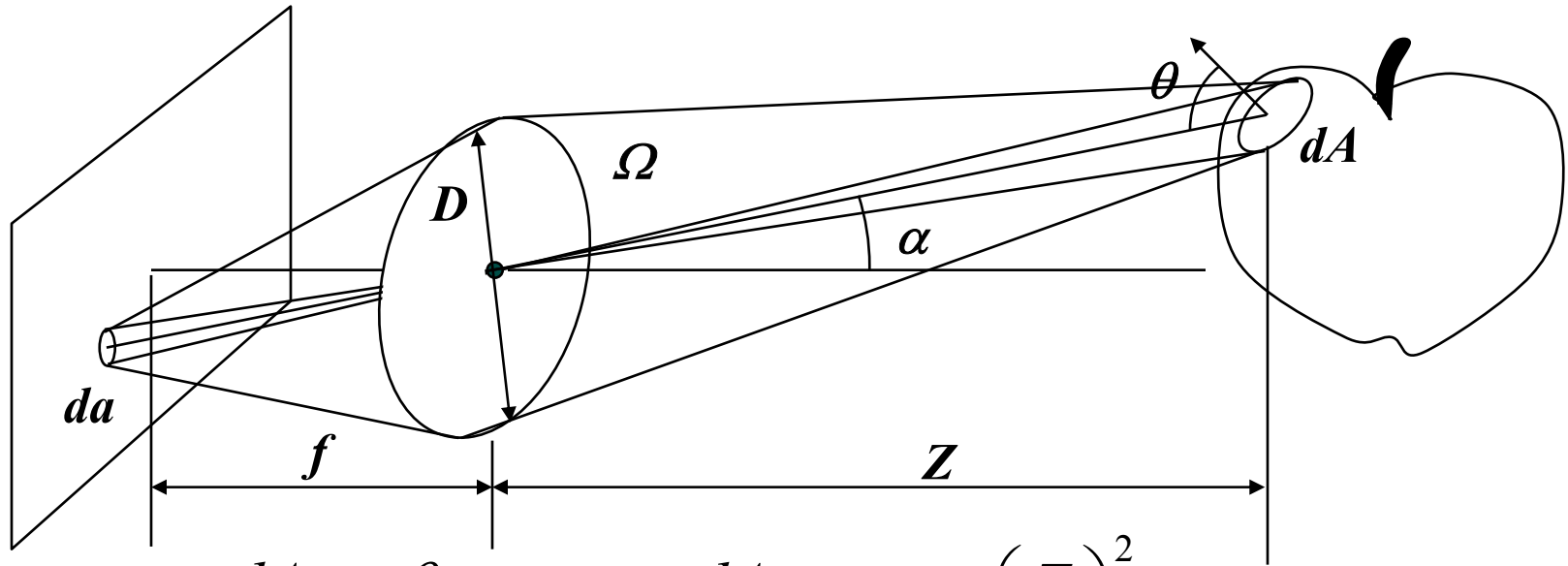❖ **Brightness corresponds roughly to radiance**

light

surface
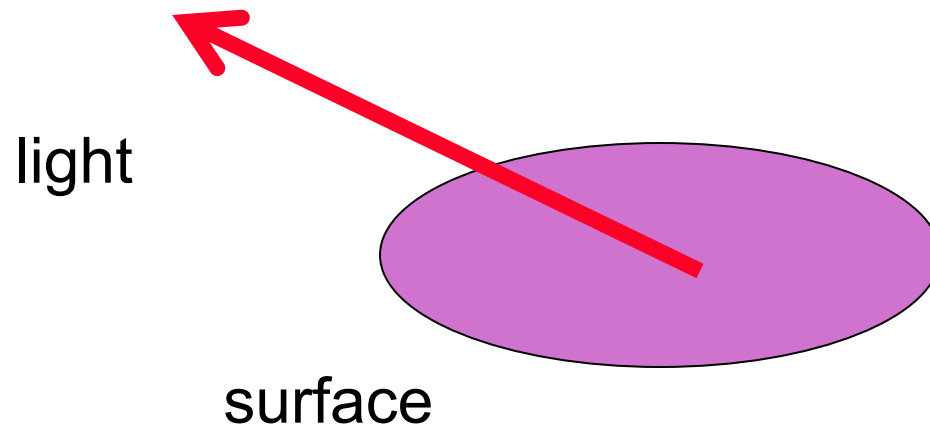
Solid angle

❖ The solid angle subtended by a cone of rays is the area of a unit sphere (centered at the cone origin) intersected by the cone

❖ A hemisphere cover 2π sterradians

$$\Omega = A / r^2$$

Solid angle

What's the solid angle subtended by this patch, area A, seen from P?

Multiply by $\cos(\theta)$ to account for foreshortening

$$\frac{A\cos(\theta)}{R^2}$$

Divide by R squared to convert the area to what you'd see on a unit sphere

Pixel Brightness and Scene Brightness

$$\frac{da\cos\alpha}{(f/\cos\alpha)^2} = \frac{dA\cos\theta}{(Z/\cos\alpha)^2} \implies \frac{dA}{da} = \frac{\cos\alpha}{\cos\theta}\left(\frac{Z}{f}\right)^2$$

$$dP = LdA\,\Omega\cos\theta \implies dP = LdA\frac{\pi}{4}\left(\frac{D}{Z}\right)^2\cos^3\alpha\cos\theta$$

$$E = \frac{dP}{da} = L\frac{dA}{da}\frac{\pi}{4}\left(\frac{D}{Z}\right)^2\cos^3\alpha\,\cos\theta \implies E = \frac{\pi}{4}\left(\frac{D}{f}\right)^2\cos^4\alpha\;L$$

Image Irradiance and Scene Radiance

$$E = \frac{\pi}{4} \left(\frac{D}{f} \right)^2 \cos^4 \alpha \; L$$

❖ Image irradiance *E* is proportional to scene radiance

❖ ***Brighter scene points produce brighter pixels***

❖ Image irradiance is proportional to inverse of square of f-number (*f/D*), is larger for small f-number

The Human Eye

❖ Limitations of human vision
  – the image is upside-down!
  – high resolution vision only in the fovea
    ◆ only one small fovea in man
    ◆ other animals (birds, cheetas) have different foveal organizations
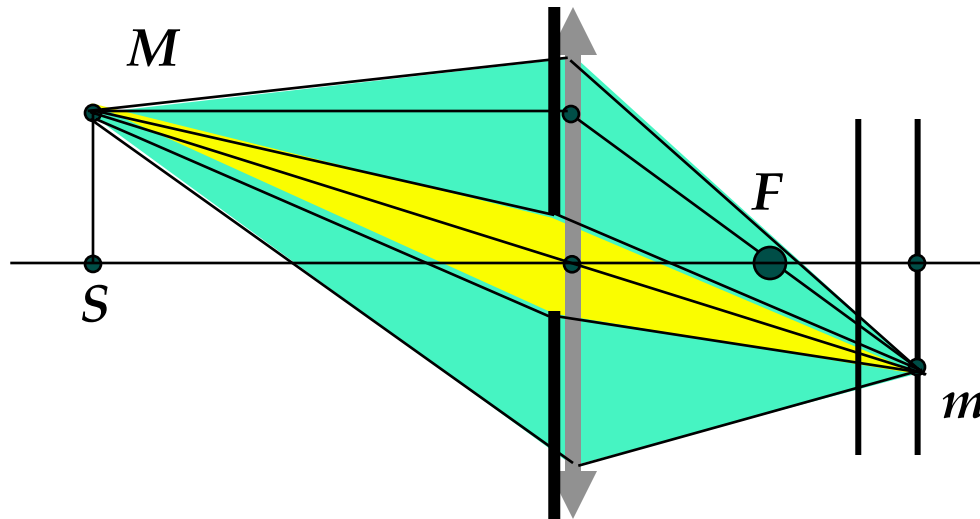 – blind spot
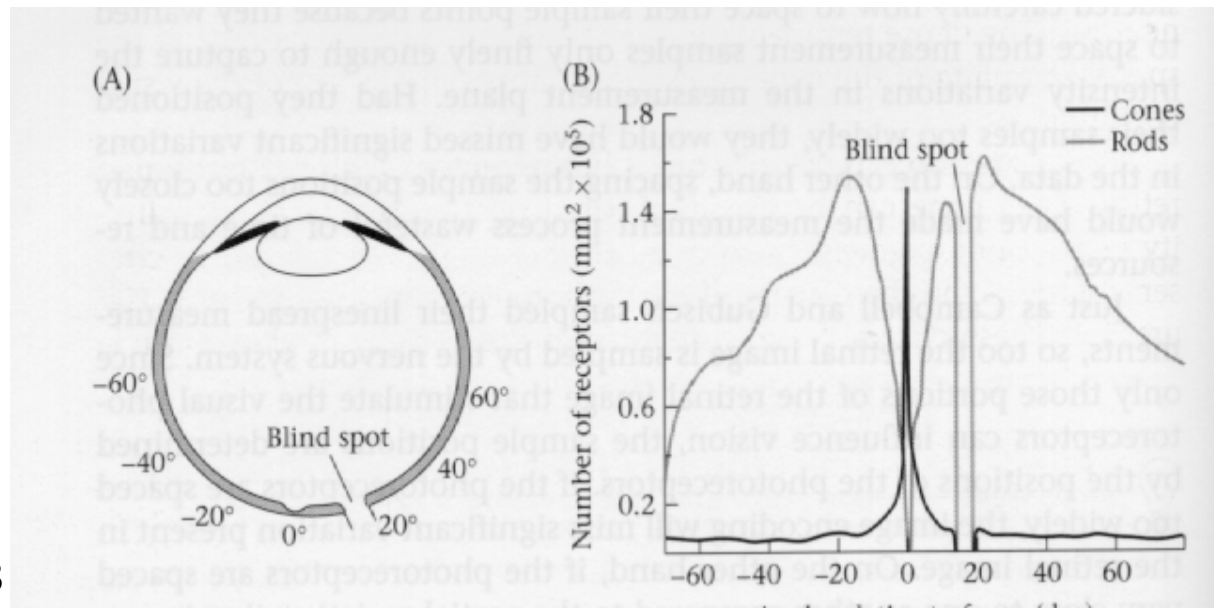
Blind Spot

❖   Close left eye

❖ Look steadily at white cross

❖ Move head slowly toward and away from figure

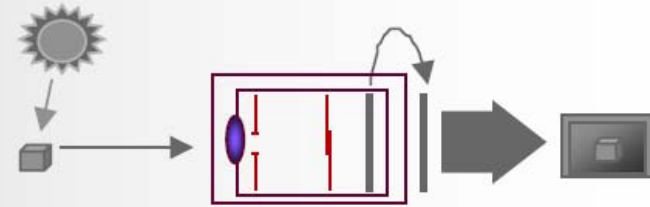❖ At a particular head position, the white disk disappears completely from view.
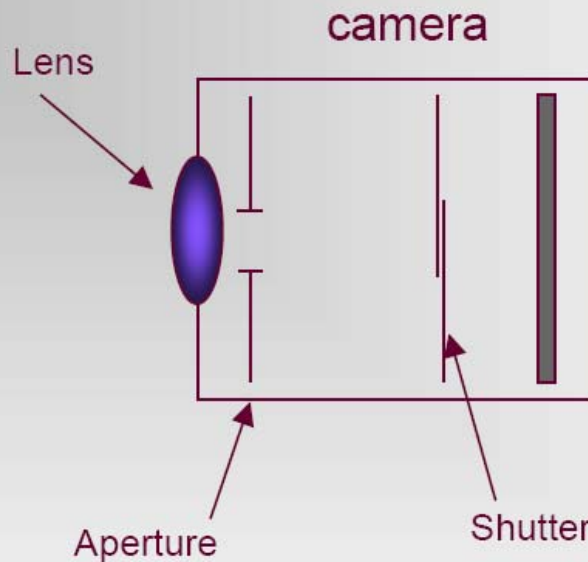
Depth of Field and f-number
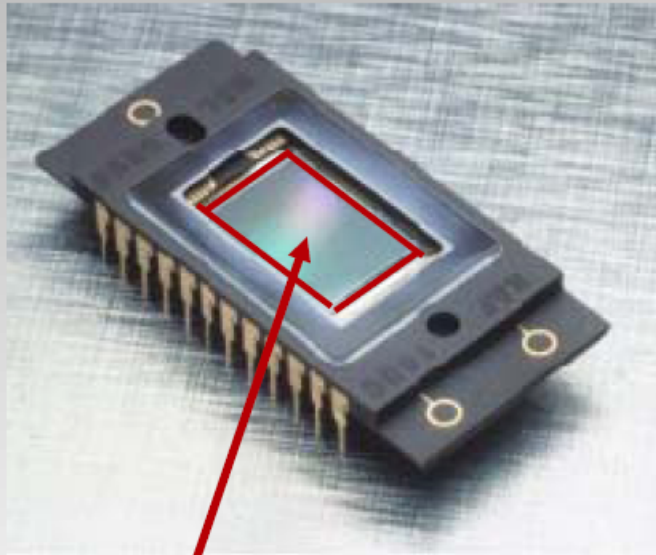
❖ Depth of field is smaller for small f-number

Exposure

- Exposure is defined as the total amount of light falling on the film.
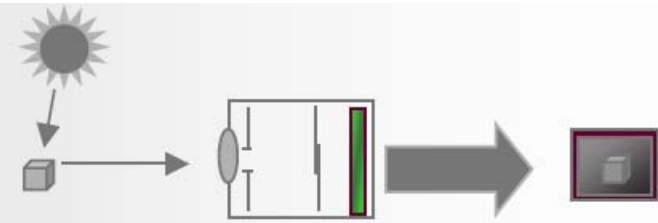- Exposure = Illuminance * Time

Charge Coupled Device (CCD)

Light Sensitive Area

- CCD replaces AgX film

- Based on *silicon chip*

- Disadvantages vs. AgX:
  - Difficulty/cost of CCD manufacture; large arrays are VERY expensive
  - "Young" technology; rapidly changing

Image formation - 32

Basic structure of CCD

Divided into small elements called pixels (*picture elements*).

Rows

Image Capture Area

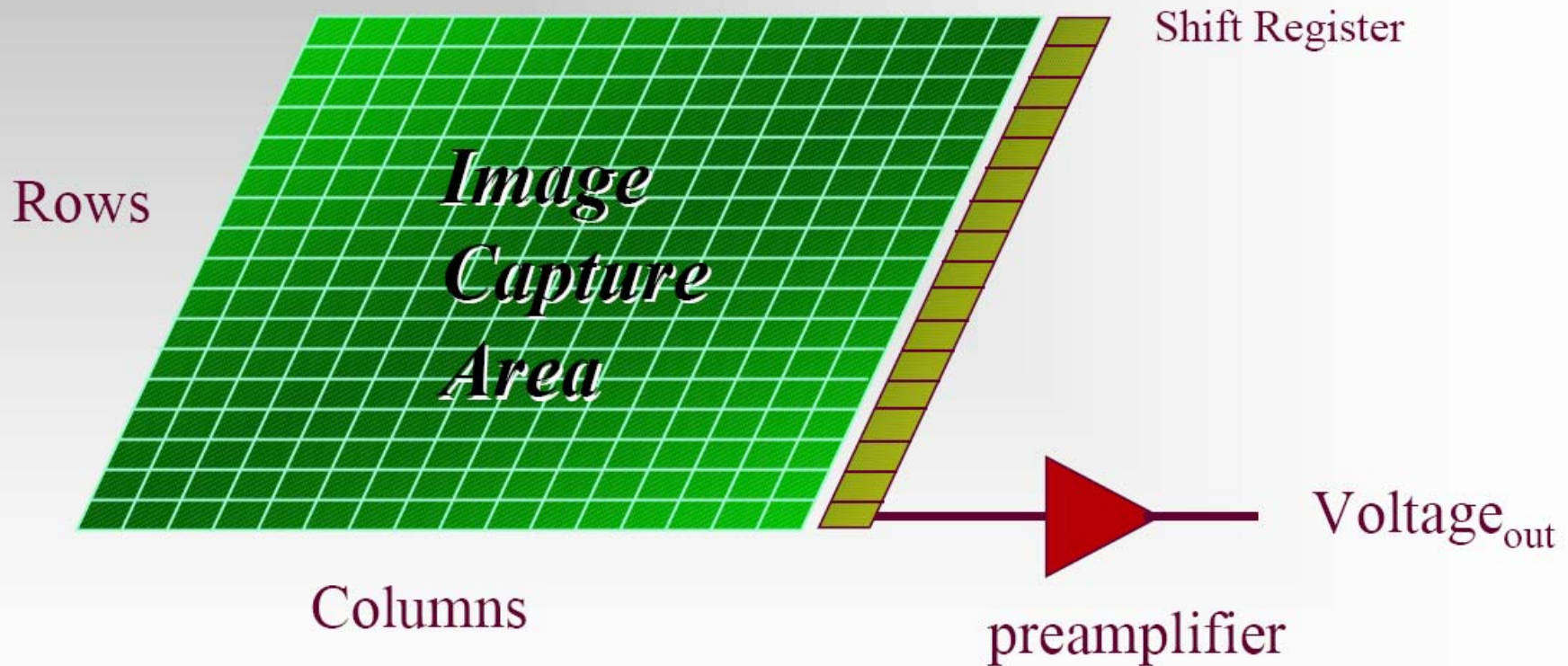Shift Register

Columns

Voltage~out~

preamplifier

Image is quantized

❖ Pixel sizes are at certain specified locations
  – Picture elements are called "pixels"
  – Image becomes "digital"
  – In the past with photo-film, the conversion was done via scanners
  – Now images are directly digital

❖ Pixel values are mapped to some range
  – 8 bits means 0-255
  – 12 bits means 0-4095, etc.