DNA Group Assignment - 3

Team AGDP

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1.

Consider

$$R(A_1, A_2, ...A_n)$$

to be a relation with these dependencies:

$$A_1 \rightarrow A_2A_3...A_n(i=1)$$

$$A_2A_3 \rightarrow A_4A_5...A_nA_1(i=2)$$

$$A_4A_5A_6 \rightarrow A_7A_8...A_nA_1A_2A_3(i=3)$$

$$A_{\frac{(i)(i-1)}{2}+1} + A_{\frac{(i)(i-1)}{2}+2} + ... + A_{\frac{(i)(i-1)}{2}+i} \rightarrow A_{\frac{(i)(i-1)}{2}+1+i} + A_{\frac{(i)(i-1)}{2}+i+2}...A_nA_i...A_{\frac{(i)(i-1)}{2}}$$

1.1

In order to ensure that the functional dependencies are possible:

Let m be the number of functional dependencies.

Therefore, the set of functional dependencies is possible only when

$$n = 1 + 2 + 3 + 4 + ... + k$$

$$n = \frac{(k)(k+1)}{2}, k \geq 3$$

For $k = 1$, n = 1, the first FD will not exist. Similarly, for $k = 2, n = 3$

1.2

Given the number of functional dependencies k, and the total number of attributes n,

$$n = \frac{(k)(k+1)}{2}$$

Every attribute in relation to R is a prime attribute (An attribute that is a part of one of the candidate keys is known as a prime attribute.).

Taking A_1 as the primary key, in every functional dependency, the LHS is a candidate key. As each one of them uniquely identifies a tuple in the relation.

∴ The total number of keys for R is equal to k i.e the total number of Functional dependencies and the keys will be -

$$A_1, A_2A_3, A_4A_5A_6, A_7A_8A_9A_{10}...A_{\frac{(i)(i-1)}{2}+1} + A_{\frac{(i)(i-1)}{2}+2} + ... + A_{\frac{(i)(i-1)}{2}+i}$$

1.3

- **1NF** : A relation is in first normal form if every attribute in that relation is singled valued attribute. Assuming all the attributes in R to be simple, single-valued and indivisible, R will be in first normal form.
- **2NF :** A relation must be in first normal form and relation must not contain any partial dependency i.e., no non-prime attribute (attributes which are not part of any candidate key) is fully dependent on the primary key. Since, all the attributes in relation R are **prime attributes,** thus, R is in second normal form.
- **3NF :** A relation is in third normal form, if there is no transitive dependency for non-prime attributes as well as it is in second normal form. Since R has no non-prime attributes, no non- prime attribute of R is transitively dependent on the primary key. Thus, R is in the third normal form.
- **BCNF :** BCNF is an extension of 3NF, A relation R is in BCNF, if P $\rightarrow$ Q is a trivial functional dependency and P is a superkey for R. R is in BCNF since every functional dependency (X $\rightarrow$ A) defined in relation R has X as a key.

1.4

The given set of FD F = {

$$A_1 \rightarrow A_2A_3...A_n(i=1)$$

$$A_2A_3 \rightarrow A_4A_5...A_nA_1(i=2)$$

$$A_4A_5A_6 \rightarrow A_7A_8...A_nA_1A_2A_3(i=3)$$

$A_{\frac{(i)(i-1)}{2}+1} + A_{\frac{(i)(i-1)}{2}+2} + ... + A_{\frac{(i)(i-1)}{2}+i} \to A_{\frac{(i)(i-1)}{2}+1+i} + A_{\frac{(i)(i-1)}{2}+i+2} ... A_n A_i ... A_{\frac{(i)(i-1)}{2}}, (i \geq 3)$

}, F has a total of k FDs where $n = \frac{(k)(k+1)}{2}$.

A minimal cover of a set of Functional Dependencies, F is a minimal set of functional dependencies F_{min} that is equivalent to F. There can be many minimal covers for a set of functional dependencies F. In other words, a set of FDs F is minimum if F has as few FDs as any equivalent set of FDs.

Step 1 - For all functional dependencies, the right-hand side must be a single attribute.

$A_1 \to A_i, 2 \leq i \leq n$, $A_2 A_3 \to A_i, i = 1 \cup [4, n]$

$$A_{\frac{(i)(i-1)}{2}+1} + A_{\frac{(i)(i-1)}{2}+2} + ... + A_{\frac{(i)(i-1)}{2}+i} \to A_i \;\forall\; 1 \leq i \leq \frac{(i-1)(i)}{2}$$

After this step, F has a total of around k(n-1) FDs.

Step 2 - Removing redundant functional dependencies we get

$$\begin{gathered} G = A_1 \to A_i, \forall 2 \leq i \leq n \\ A_2 A_3 \to A_1 \\ A_4 A_5 A_6 \to A_1 ... \\ A_{(i-1)/(i)+1} A_{(i-1)(i)+2} ... A_{(i-1)/(i)+i} \to A_1 \end{gathered}$$

G has a minimal value on the LHS of every FD, thus, G is the minimal cover of F.

We proved in 1.3 that R is in a BCNF under F, since G is a subset of F, R is also a BCNF under G.

2

Given that $R(A_1, A_2, ..., A_n)$ be a relation R with functional dependencies as follows:

$$A_i \to A_j \;\forall\; 1 \leq i > j \leq n$$

and

$$A_i \to A_j \;\forall\; 1 \leq i > j \leq n$$

2.1

For every $1 \leq i \leq n$, $A_i \to A_j \forall j \neq i, 1 \leq j \leq n$, thus every attribute uniquely identifies every other attribute in the relation. Thus, there are n keys. Every attribute in the relation R is a prime attribute as well as a key and the keys are $(A_1, A_2, ..., A_n)$.

2.2

- **1NF** : A relation is in first normal form if every attribute in that relation is singled valued attribute. Assuming all the attributes in R to be simple, single-valued and indivisible, R will be in first normal form.
- **2NF :** A relation must be in first normal form and relation must not contain any partial dependency i.e., no non-prime attribute (attributes which are not part of any candidate key) is fully dependent on the primary key. Since, all the attributes in relation R are **prime attributes,** thus, R is in second normal form.
- **3NF :** A relation is in third normal form, if there is no transitive dependency for non-prime attributes as well as it is in second normal form. Since R has no non-prime attributes, no non- prime attribute of R is transitively dependent on the primary key. Thus, R is in the third normal form.
- **BCNF :** BCNF is an extension of 3NF, A relation R is in BCNF, if P $\rightarrow$ Q is a trivial functional dependency and P is a superkey for R. R is in BCNF since every functional dependency (X $\rightarrow$ A) defined in relation R has X as a key.

2.3

The given set of functional dependencies F = for every $1 \leq i \leq n$, $A_i \rightarrow A_j \forall j \neq i, 1 \leq j \leq n$.

There are a total of $n(n-1)$ FDs in F.

A minimal cover of a set of Functional Dependencies, F is a minimal set of functional dependencies F_{min} that is equivalent to F. There can be many minimal covers for a set of functional dependencies F. In other words, a set of FDs F is minimum if F has as few FDs as any equivalent set of FDs.

Step 1 - RHS of all FDs should be a single attribute, which is already the case in given F.

Step 2 - Eliminate redundant functional dependencies, $X \rightarrow Y$ if $(F)+ = (F - (X \rightarrow Y))$, where $(F)+$ is closure of functional dependency set.

For example, $A_1 \rightarrow A_2$, $A_1 \rightarrow A_3$, $A_2 \rightarrow A_3$, $A_2 \rightarrow A_3$, $A_3 \rightarrow A_1$, $A_3 \rightarrow A_2$, has multiple redundant FDs and can be reduced to a combination of cyclic FDs $A_1 \rightarrow A_2$, $A_2 \rightarrow A_3$, $A_3 \rightarrow A_1$.

Similarly F can be reduced to F_{min} = { $A_1 \rightarrow A_2$, $A_2 \rightarrow A_3$, $A_3 \rightarrow A_4$, ..., $A_n \rightarrow A_1$

F_{min} combination of cyclic FDs and satisfies minimal cover

$F_{min} =$ { $A_i \rightarrow A_{i+1} \; \forall \; 1 \leq i \leq n-1$ } + { $A_n \rightarrow A_1$}.

As we implied in 2.2, that R is in BCNF under F, since F_{min} is a subset of F, R is also a BCNF under F_{min}.