Machine Learning

Assignment 9: Markov Process

Code:

MarkovProcess.java

```java
import java.util.HashMap;
import java.util.Map;
import java.util.Scanner;

public class MarkovProcess {

    public static void main(String[] args) {

        // Define the states
        String[] states = {"rainy", "cloudy", "sunny", "happy", "sad"};

        // Get user input for the transition probabilities
        Scanner scanner = new Scanner(System.in);
        Map<String, Map<String, Double>> transitionProbabilities = new HashMap<>();
        for (String state : states) {
            System.out.println("Enter the transition probabilities from " + state + ":");
            Map<String, Double> probabilities = new HashMap<>();
            for (String nextState : states) {
                System.out.print("  " + nextState + ": ");
                double probability = scanner.nextDouble();
                scanner.nextLine(); // consume newline character
                probabilities.put(nextState, probability);
            }
            transitionProbabilities.put(state, probabilities);
        }

        // Get user input for the observations
        System.out.print("Enter the observations, separated by spaces: ");
        String input = scanner.nextLine();
        String[] observations = input.trim().split( regex: "\\s+");

        // Compute the probability of the observations
        double probability = 1.0;
        String currentState = states[0];
        for (int i = 0; i < observations.length; i += 2) {
            String observation = observations[i];
            String nextState = observations[i + 1];
            double transitionProbability = transitionProbabilities.get(currentState).get(nextState);
            double observationProbability = transitionProbabilities.get(nextState).get(observation);
            probability *= transitionProbability * observationProbability;
            currentState = nextState;
        }

        System.out.println("Probability of observations: " + probability);
    }

}
```

Output:

```
"C:\Program Files\Java\jdk-19\bin\java.exe" "-javaagent:C:\Program Files\JetBra
Enter the transition probabilities from rainy:
  rainy: 0.5
  cloudy: 0.3
  sunny: 0.2
  happy: 0.1
  sad: 0.9
Enter the transition probabilities from cloudy:
  rainy: 0.4
  cloudy: 0.2
  sunny: 0.4
  happy: 0.4
  sad: 0.6
Enter the transition probabilities from sunny:
  rainy: 0.0
  cloudy: 0.3
  sunny: 0.7
  happy: 0.8
  sad: 0.2
Enter the transition probabilities from happy:
  rainy: 0.0
  cloudy: 0.0
  sunny: 0.0
  happy: 0.0
  sad: 0.0
Enter the transition probabilities from sad:
  rainy: 0.0
  cloudy: 0.0
  sunny: 0.0
  happy: 0.0
  sad: 0.0
Enter the observations, separated by spaces: sunny happy cloudy happy sunny sad
Probability of observations: 0.0039
```

Notes

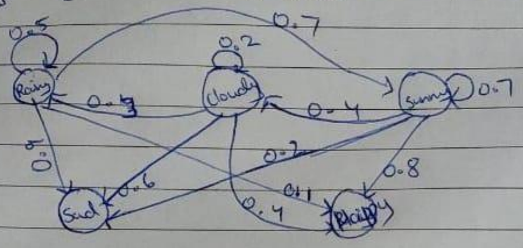
Markov Process

→ TPM (Transition Probability Matrix)

→ Row x column $\xrightarrow{\text{square}}$ sum of each row = 1

$$\underset{\text{initial values}}{TPM =} \overset{\text{future values}}{\begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}}$$

$$P_{ij} = \left[\frac{\text{Present state at } j|t=1}{\text{initial state at } i|t=0}\right]$$

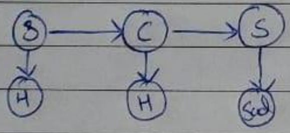


① Observation. Fin the probability of the scenario

1) Sunny → Happy 2) Cloudy → Happy 3) Sunny → Sad

$$\begin{array}{c} \\ \text{Rainy} \\ \text{Cloudy} \\ \text{Sunny} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Rainy} & \text{Cloudy} & \text{Sunny} \end{array} \\ \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.4 & 0.2 & 0.4 \\ 0 & 0.3 & 0.7 \end{bmatrix} \end{array} \quad \begin{array}{c} \begin{array}{cc} \text{Happy} & \text{Sad} \end{array} \\ \begin{bmatrix} 0.1 & 0.9 \\ 0.4 & 0.6 \\ 0.8 & 0.2 \end{bmatrix} \end{array}$$

(TPM) (Emmission matrix)



P(H-H-Sad, Sunny-Cloud-Sunny)

= P(Sunny) P(Happy | Sunny) P(Cloudy | Sunny) P(Happy | Cloudy) P(Sunny | Cloudy) P(Sad / Sunny)

= P(Sunny) × 0.8 × 0.3 × 0.4 × 0.4 × 0.2

= P(Sunny) × 0.00768

Initial probability formulas:

1) $\pi P = \pi$ $\Rightarrow (\pi_1 \ \pi_2 \ \pi_3) \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.4 & 0.2 & 0.4 \\ 0 & 0.3 & 0.7 \end{bmatrix} = (\pi_1 \ \pi_2 \ \pi_3)$

2) $\pi_1 + \pi_2 + \pi_3 = 1$

$= 0.5\pi_1 + 0.4\pi_2 + 0.0\pi_3 = \pi_1 \rightarrow (1)$

$0.3\pi_1 + 0.2\pi_2 + 0.3\pi_3 = \pi_2 \rightarrow (2)$

$0.2\pi_1 + 0.4\pi_2 + 0.7\pi_3 = \pi_3 \rightarrow (3)$

Replacing eqn (3) with $\pi_1 + \pi_2 + \pi_3 = 1$

$-0.5\pi_1 + 0.4\pi_2 + 0\pi_3 = 0 \rightarrow (1)$

$0.3\pi_1 - 0.8\pi_2 + 0.3\pi_3 = 0 \rightarrow (2)$

$\pi_1 + \pi_2 + \pi_3 = 1 \rightarrow (3)$

Substituting eqn(1) & eqn(2)

$-0.5\pi_1 + 0.4\pi_2 + 0\pi_3 = 0$

$-0.3\pi_1 + 0.8\pi_2 - 0.3\pi_3 = 0$

$-0.8\pi_1 + 0.12\pi_2 - 0.3\pi_3 = 0$

$\pi_1 = -0.12\pi_2 + 0.3\pi_3 + 0.8$

(2)&(3) $\rightarrow$ $0.3\pi_1 - 0.8\pi_2 + 0.3\pi_3 = 0$

$\pi_1 + \pi_2 + \pi_3 = 1$

$\pi_1 = 0.2182, \ \pi_2 = 0.2727, \ \pi_3 = 0.509$

$$[\pi_1 \ \pi_2 \ \pi_3] = \begin{array}{ccc} P(\text{Rainy}) & P(\text{cloudy}) & P(\text{sunny}) \\ \end{array}$$
$$= [0.2182 \quad 0.2727 \quad 0.509]$$

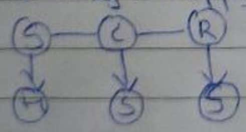
$\therefore P(\text{Sunny}) = 0.509$

$= 0.509 \times 0.00768$

$= 0.0039$

$\therefore$ Probability of given scenario $= 0.0039$

(ii) Sunny → Happy Cloudy → Sad Rainy → Sad



P(H-S-S, Sunny-Cloudy-Rainy)

= P(Sunny) P(Happy | Sunny) P(Cloudy | Sunny) P(Sad | cloudy) P(Rainy | Cloudy) P(Sad | Rainy)

$= 0.509 \times 0.8 \times 0.3 \times 0.6 \times 0.4 \times 0.9$

$= 0.0264$

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