MODULE SIX FINAL PROJECT

OPTIMIZATION - TRANSSHIPMENT PROBLEM AND RISK MINIMIZING PROBLEM

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ABSTRACT

We all have finite resources and time, and it's important to use both rightly. From solving supply chain problems in our companies to using our time efficiently, optimization is a vital part of everything we do. **Quadratic Programming** is a way for optimizing a multivariable quadratic function that might or might not be linearly constrained. An example of a real-world quadratic program would be optimizing a manufacturer's costs or optimizing a company's portfolio with multivariable quadratic functions. A quadratic program can be applied to a variety of real-world optimization problems.

In mathematical optimization, **Quadratic Programming (QP)** is a method to solve problems involving quadratic functions by optimizing (minimizing or maximizing) a multivariate quadratic function under linear constraints on the variables.

$$
\begin{aligned}
\textbf{Minimize} \quad & \frac{1}{2}\mathbf{x}^{\mathrm{T}} A\mathbf{x} + \mathbf{x}^{\mathrm{T}}\mathbf{c} \\
\textbf{Subject to} \quad & A\mathbf{x} \leq \mathbf{b} \\
& \mathbf{x} \geq 0
\end{aligned}
$$

Figure 1.1: Mathematical Equation of Quadratic Programming

INTRODUCTION

With the medium of this project, we will be *Optimizing* two different problems - *Trans-Shipment problem and Risk Minimization problem.* We will use the implementation of Quadratic Programming (QP) to solve these problems.

PROBLEM STATEMENT

PART 1 - Rockhill Shipping & Transport Company

One hardware company in the northern part of the country is exploring the prospect of opening a new distribution centre in the southeast. The company plans on renting a warehouse and an adjacent office to distribute its products to local dealers.

Allen is the manager of the Rocky Hill Shipping & Transport Company's South Atlantic region. He is negotiating a new shipping contract with a company that manufactures chemicals for industrial use, a company called Chimotoxic. Allen is very concerned about this proposal arrangement. If Chimotoxic waste leaks, the contaminated chemicals can be hazardous to humans and the environment. The company wants Rockhill to pick up and transport waste products from its six plants to three waste disposal sites. There are also some communities in the regions where the plants are located which prohibit the shipment of hazardous materials through their boundaries. It is therefore important not only to handle shipments carefully and to transport them at reduced speeds, but it may also be necessary to travel in circuitous routes in some cases.

This table shows Allen's estimate of how much a barrel of waste would cost to ship from each of the six plants to each of the three waste disposal sites.

	Waste Proposal Site		
Plant:	Orangeburg	Florence	Macon
Denver	$12	$15	$17
Morganton	14	9	10
Morrisville	13	20	11
Pineville	17	16	19
Rockhill	7	14	12
Statesville	22	16	18

Table 1.1: The Waste Site Transport Cost Estimate.

Waste products generated by the plants each week are as follows.

Plant:	Waste per Week (bbl)
Denver	45
Morganton	26
Morrisville	42
Pineville	53
Rockhill	29
Statesville	38

Table 1.2: The waste generated by each proposal site weekly.

A maximum of 67, 89, and 108 barrels per week can be disposed of at the waste disposal sites in Orangeburg, Florence, and Macon, respectively.

Waste Disposal Site:	Max Capacity (bbl/week)
Orangeburg	67
Florence	89
Macon	108

Table 1.3: The waste disposal site accommodation capacity.

As a consultant, I am going to use Quadratic Programming (QP) to minimize the Total Transportation Cost to be beared by the Rockhill Transport company.

ANALYSIS

I - Minimization of the Normal Transportation problem.

The **Decision Variables** in the problem are represented by the matrix in the following figure :

- X_{DO} - Number of units of Waste Barrels from Denver to Orangeburg.
- X_{DF} - Number of units of Waste Barrels from Denver to Florence.
- X_{MO} - Number of units of Waste Barrels from Morganton to Orangeburg.
- X_{MF} - Number of units of Waste Barrels from Morganton to Florence.
- X_{MVF} - Number of units of Waste Barrels from Morrisville to Florence.
- X_{MVM} - Number of units of Waste Barrels from Morrisville to Macon.
- X_{RM} - Number of units of Waste Barrels from Rockhill to Macon.
- X_{SO} - Number of units of Waste Barrels from Statesville to Orangeburg.
- **We have a total of *6 x 3 = 18 pairs*.**

	Waste Proposal Site		
Plant:	*Orangeburg*	*Florence*	*Macon*
Denver	X_{DO}	X_{DF}	X_{DM}
Morganton	X_{MO}	X_{MF}	X_{MM}
Morrisville	X_{MVO}	X_{MVF}	X_{MVM}
Pineville	X_{PO}	X_{PF}	X_{PM}
Rockhill	X_{RO}	X_{RF}	X_{RM}
Statesville	X_{SO}	X_{SF}	X_{SM}

Figure 1.2: Decision Variables for Plants and Waste Disposal Site defined in the Workbook.

The **Objective Function (Minimization)** in our problem :

- Minimize the Total Transportation Cost to be beared by the Rockhill Transport company for this service and proposal.
- C_{ij} - Cost to transport one unit of waste barrel from *i source to j destination*.
- X_{ij} - Number of units of waste barrel transported from *i source to j destination*.

Minimize Z = $\sum C_{ij} * X_{ij}$

where
i = Plant,
j = Waste Disposal Site

Figure 1.3: Objective Function.

The **Constraints (Subjected to Conditions)** in our problem are :

- The Waste generating plants have a pre-defined quantity of waste generated by them on a weekly basis.
- The plant located at Denver generates 45 bbl waste weekly, and likewise do the others.
- The maximum capacity of waste any Waste Disposal Site can accommodate is also provided in the problem statement.
- Orangeburg (Waste Disposal Site) can accommodate a maximum of 67 barrels of waste per week.
- We apply **Equality** to the plants (sources) constraints.
- We apply **Inequality** to the waste disposal sites (destinations) constraints.
- A non-negativity constraint needs to be added.

Subject To:				
Plants (Sources)	**Denver**	$X_{DO} + X_{DF} + X_{DM}$	=	45
	Morganton	$X_{MO} + X_{MF} + X_{MM}$	=	26
	Morrisville	$X_{MVO} + X_{MVF} + X_{MVM}$	=	42
	Pineville	$X_{PO} + X_{PF} + X_{PM}$	=	53
	Rockhill	$X_{RO} + X_{RF} + X_{RM}$	=	29
	Statesville	$X_{SO} + X_{SF} + X_{SM}$	=	38
Waste Disposal Sites (Destination)	**Orangeburg**	$X_{DO} + X_{MO} + X_{MO} + X_{PO} + X_{RO} + X_{SO}$	≤	67
	Florence	$X_{DF} + X_{MF} + X_{MF} + X_{PF} + X_{RF} + X_{SF}$	≤	89
	Macon	$X_{DM} + X_{MM} + X_{MM} + X_{PM} + X_{RM} + X_{SM}$	≤	108
	Non-Negativity	X_{ij}	≥	0

Figure 1.4: Constraints.

The Optimization Model matrix for our problem is as follows. It contains the Decision Variables within the matrix along with its Constraints present outside the matrix shaded in yellow colour with equalities/inequalities.

Decision Variable		Waste Disposal Sites (Destinations)					
		Orangeburg	**Florence**	**Macon**	*Shipped From:*		*Waste Generation:*
Plants (Sources)	**Denver**				0	=	45
	Morganton				0	=	26
	Morrisville				0	=	42
	Pineville				0	=	53
	Rockhill				0	=	29
	Statesville				0	=	38
	Shipped To:	0	0	0			Total Waste Generation 233
		≤	≤	≤			
	Disposal:	67	89	108	Total Waste Disposal 264		

Figure 1.5: Optimization Model matrix with decision variables and constraints.

The constraint for one of the Plants (Sources) - Denver - in this problem is as follows. The total waste generated from this plant is 45 barrels per week.

	Orangeburg	Florence	Macon	*Shipped From:*		*Waste Generation:*
Denver				0	=	45

Figure 1.6: Constraint for one plant - Denver.

The constraint for one of the Waste Disposal Site (Destination) - Orangeburg - in this problem is as follows. The maximum capacity of waste to be accommodated in this plant is 67 barrels per week.

	Orangeburg
Denver	
Morganton	
Morrisville	
Pineville	
Rockhill	
Statesville	
Shipped To:	0
	≤
Disposal:	67

Figure 1.7: Constraint for one waste disposal site - Orangeburg.

I - Optimal Solution of the Minimization of Transportation Cost to the Company.

Using the Solver function, we have computed the Optimal Solution for the problem of Minimization of the Transportation Cost to the Rockhill company in order to provide a proposal to the Chimotoxic company.

Total Cost Z (Minimized)	
$	2,960.00

Figure 1.8: Objective Function with the Minimized Transportation Cost Value.

- The Minimized value of the Transportation Cost (Weekly Optimized Transport Cost) which the company will bear from the given information is **$2,960.00**

- This means that waste barrels will be transported using the optimized routes with lowest costs.

The Optimal Solution of this Minimization problem is:

		Waste Disposal Sites (Destinations)		
		Orangeburg	Florence	Macon
Plants (Sources)	Denver	38	7	0
	Morganton	0	0	26
	Morrisville	0	0	42
	Pineville	0	53	0
	Rockhill	29	0	0
	Statesville	0	29	9

Figure 1.9: Optimal Solution to this Minimization problem.

- The optimal number of units of waste barrels to be sent from the plant (Denver) is
  - **38 to Orangeburg**
  - **7 to Florence**
- The optimal number of units of waste barrels to be sent from the plant (Morganton) is
  - **26 to Macon**
- The optimal number of units of waste barrels to be sent from the plant (Morrisville) is
  - **42 to Macon**
  - **7 to Florence**
- The optimal number of units of waste barrels to be sent from the plant (Pineville) is
  - **53 to Florence**
- The optimal number of units of waste barrels to be sent from the plant (Rockhill) is
  - **29 to Orangeburg**
- The optimal number of units of waste barrels to be sent from the plant (Statesville) is
  - **29 to Florence**
  - **9 to Macon**

The Optimization Matrix model has been computed as follow. The constraints have been fulfilled successfully for both the Waste Generating Plants and Waste Disposal Sites.

Decision Variable							
		Waste Disposal Sites (Destinations)					
		Orangeburg	Florence	Macon	*Shipped From:*		***Waste Generation:***
Plants (Sources)	Denver	38	7	0	45	=	45
	Morganton	0	0	26	26	=	26
	Morrisville	0	0	42	42	=	42
	Pineville	0	53	0	53	=	53
	Rockhill	29	0	0	29	=	29
	Statesville	0	29	9	38	=	38
	Shipped To:	67	89	77			Total Waste Generation
		≤	≤	≤			233
	Disposal:	67	89	108	Total Waste Disposal	264	

Figure 1.10: Optimal Solution to this Minimization Transportation problem.

II - Optimization for Trans-Shipment Problem

Furthermore, Allen is also considering another option. The plants and the waste disposal sites may be used as intermediate shipping points in addition to transporting waste directly from each of the six plants to one of the three waste disposal sites. The truck would drop off the load at a plant or a disposal site, where another truck would pick it up and transport it to the final destination.

Additionally, Rockhill would not incur any handling costs for the plants and disposal sites since Chimotoxic will handle them all. In other words, the only cost Rockhill will incur is transportation. Thus, Allen is interested in being able to make an informed decision as to whether it may be more cost-effective to drop and pick up loads at intermediate points rather than ship them directly to the destination.

According to Allen, shipping is estimated to cost between each of the six plants at the following rate:

	Plant					
Plant:	Denver	Morganton	Morrisville	Pineville	Rockhill	Statesville
Denver	$---	$3	$4	$9	$5	$4
Morganton	6	---	7	6	9	4
Morrisville	5	7	---	3	4	9
Pineville	5	4	3	---	3	11
Rockhill	5	9	5	3	---	14
Statesville	4	7	11	12	8	---

Figure 1.11: Shipping Cost per barrel of waste from each plant to another plant.

As a rough estimate, the shipping costs for each waste disposal site are as follows::

	Waste Proposal Site		
Waste Disposal Site:	Orangeburg	Florence	Macon
Orangeburg	$---	$12	$10
Florence	12	---	15
Macon	10	15	---

Figure 1.12: Shipping Cost per barrel of waste from each waste disposal site to another disposal site.

- The transportation cost of per unit of barrel of waste from Denver to Morganton is $3.
- The transportation cost of per unit of barrel of waste from Pineville to Rockhill is also $3.
- Similarly, information about the cost from each plant to another is provided.
- The transportation cost of per unit of barrel of waste from Orangeburg to Florence (Waste Disposal Sites) is $12.
- The transportation cost of per unit of barrel of waste from Macon to Florence (Waste Disposal Sites) is $15.
- Similarly, information about the cost from each waste disposal site to another is provided.

The Model for the variation (Trans Shipment) in our problem is as follows. It contains the Cost Values from each node to another node present.

Cost		Plants & Waste Disposal Sites (Nodes)								
		Denver	Morganton	Morrisville	Pineville	Rockhill	Statesville	Orangeburg	Florence	Macon
Plants & Waste Disposal Sites (Sources)	Denver	4000	3	4	9	5	4	12	15	17
	Morganton	6	4000	7	6	9	4	14	9	10
	Morrisville	5	7	4000	3	4	9	13	20	11
	Pineville	5	4	3	4000	3	11	17	16	19
	Rockhill	5	9	5	3	4000	14	7	14	12
	Statesville	4	7	11	12	8	4000	22	16	18
	Orangeburg	4000	4000	4000	4000	4000	4000	4000	12	10
	Florence	4000	4000	4000	4000	4000	4000	12	4000	15
	Macon	4000	4000	4000	4000	4000	4000	10	15	4000

Figure 1.13: Model with cost values with plants and disposal sites as intermediaries.

- A value of $4000 is taken in those cells where no transport cost value is provided in order to satiate the need of a value for Excel's Solver function and also not to interfere with the results.

The Optimization Model matrix for this trans-shipment problem is as follows. It contains the Decision Variables within the matrix along with its Constraints present outside the matrix shaded with equalities/inequalities and transient values which is used to compute the difference of total inwards and total outwards traffic.

Decision Variable		Plants & Waste Disposal Sites (Nodes)												
		Denver	Morganton	Morrisville	Pineville	Rockhill	Statesville	Orangeburg	Florence	Macon	*Transient:*		*Waste Generation:*	***Outward - Inward (Retaining Supply):***
Plants & Waste Disposal Sites (Sources)	Denver										0	=	45	0
	Morganton										0	=	26	0
	Morrisville										0	=	42	0
	Pineville										0	=	53	0
	Rockhill										0	=	29	0
	Statesville										0	=	38	0
	Orangeburg										0	≤	67	
	Florence										0	≤	89	
	Macon										0	≤	108	
	Shipped To:	0	0	0	0	0	0	0	0	0			Total Waste Generation	
		=	=	=	=	=	=	≤	≤	≤			233	
	Disposal:	45	26	42	53	29	38	67	89	108	Total Waste Disposal	264		
	Outward - Inward (Retaining Supply):							0	0	0				

Figure 1.13: Model with decision variables with plants and disposal sites as intermediaries.

The constraint for one of the Plants (Sources) - Denver - in this problem is as follows. The difference of ***INWARD Supply and OUTWARD Supply*** should be equal to 45 barrels per week which is the total waste generated from this plant.

n Variable	Denver	Morganton	Morrisville	Pineville	Rockhill	Statesville	Orangeburg	Florence	Macon	*Transient:*		*Waste Generation:*	***Outward - Inward (Retaining Supply):***
Denver										0	=	45	0

Figure 1.14: Constraint for one plant - Denver.

The constraint for one of the Waste Disposal Site (Destination) - Orangeburg - in this problem is as follows. The difference of ***INWARD Supply and OUTWARD Supply*** should be at most or equal to 67 barrels per week which is the maximum waste accommodated in this waste disposal site.

	Orangeburg
Denver	
Morganton	
Morrisville	
Pineville	
Rockhill	
Statesville	
Orangeburg	
Florence	
Macon	
Shipped To:	0
	≤
Disposal:	67
Outward - Inward (Retaining Supply):	0

Figure 1.15: Constraint for one waste disposal site - Orangeburg

Using the Solver function, we have computed the Optimal Solution for the problem of Minimization of the Trans-Shipment Cost for the Rockhill company in order to provide a proposal to the Chimotoxic company with all its plants and waste disposal sites working as intermediary nodes.

Total Cost Z (Minimized)	
$	2,657.00

Figure 1.16: Objective Function with the Minimized Trans-Shipment Cost Value.

- The Minimized value of the Trans-Shipment Cost (Weekly Optimized Total Trans-Shipment Cost) which the company will bear from the given information is **$2,657.00**

The Optimal Solution of this Trans-Shipment Minimization problem is:

Decision Variable		Plants & Waste Disposal Sites (Nodes)								
		Denver	Morganton	Morrisville	Pineville	Rockhill	Statesville	Orangeburg	Florence	Macon
Plants & Waste Disposal Sites (Sources)	Denver	0	45	0	0	0	0	0	0	0
	Morganton	0	0	0	0	0	0	0	51	35
	Morrisville	0	0	0	0	0	0	0	0	42
	Pineville	0	15	0	0	38	0	0	0	0
	Rockhill	0	0	0	0	0	0	67	0	0
	Statesville	0	0	0	0	0	0	0	38	0
	Orangeburg	0	0	0	0	0	0	0	0	0
	Florence	0	0	0	0	0	0	0	0	0
	Macon	0	0	0	0	0	0	0	0	0

Figure 1.17: Optimal Solution to this Trans-Shipment Minimization problem.

- The optimal number of units of waste barrels to be sent from the plant (Denver) is
  - **45 to Morganton**

- The optimal number of units of waste barrels to be sent from the plant (Morganton) is
  - **51 to Florence**
  - **35 to Macon**
- The optimal number of units of waste barrels to be sent from the plant (Morrisville) is
  - **42 to Macon**
- The optimal number of units of waste barrels to be sent from the plant (Pineville) is
  - **15 to Morganton**
  - **38 to Rockhill**
- The optimal number of units of waste barrels to be sent from the plant (Rockhill) is
  - **67 to Orangeburg**
- The optimal number of units of waste barrels to be sent from the plant (Statesville) is
  - **38 to Florence**

The Optimization Matrix model has been computed as follow. The constraints have been fulfilled successfully for both the Waste Generating Plants and Waste Disposal Sites.

Decision Variable		Plants & Waste Disposal Sites (Nodes)												
		Denver	Morganton	Morrisville	Pineville	Rockhill	Statesville	Orangeburg	Florence	Macon	*Transient:*		*Waste Generation:*	*Outward - Inward (Retaining Supply):*
Plants & Waste Disposal Sites (Sources)	Denver	0	45	0	0	0	0	0	0	0	45	=	45	45
	Morganton	0	0	0	0	0	0	0	51	35	86	=	26	26
	Morrisville	0	0	0	0	0	0	0	0	42	42	=	42	42
	Pineville	0	15	0	0	38	0	0	0	0	53	=	53	53
	Rockhill	0	0	0	0	0	0	67	0	0	67	=	29	29
	Statesville	0	0	0	0	0	0	0	38	0	38	=	38	38
	Orangeburg	0	0	0	0	0	0	0	0	0	0	≤	67	
	Florence	0	0	0	0	0	0	0	0	0	0	≤	89	
	Macon	0	0	0	0	0	0	0	0	0	0	≤	108	
	Shipped To:	0	60	0	0	38	0	67	89	77			Total Waste Generation	
		=	=	=	=	=	=	≤	≤	≤			233	
	Disposal:	45	26	42	53	29	38	67	89	108	Total Waste Disposal	264		
	Outward - Inward (Retaining Supply):							67	89	77				

Figure 1.18: Optimal Solution to this Trans-Shipment Minimization problem.

- The intermediary nodes (plants and waste disposal sites) are used to transport barrels of waste in between each other and then finally to their destinations.
- Transient is also kept in mind to compute the difference of inward and outward supply.
- The barrels of waste reach their destinations using different routes with optimal and lowest costs.

The Minimized Total Cost for this trans-shipment problem is $2,657.00 which is lower than its transportation minimized total cost of $2,900.00. Therefore, Allen should use this model and propose the same to the company.

III - If Chimotoxic agrees to increase the capacity of their waste disposal sites by 5 barrels per week.

The constraints of destinations would change with addition of 5 barrels per week to each of them.

Decision Variable		Waste Disposal Sites (Destinations)					
		Orangeburg	Florence	Macon	Shipped From:		Waste Generation:
Plants (Sources)	Denver	43	2	0	45	=	45
	Morganton	0	1	25	26	=	26
	Morrisville	0	0	42	42	=	42
	Pineville	0	53	0	53	=	53
	Rockhill	29	0	0	29	=	29
	Statesville	0	38	0	38	=	38
	Shipped To:	72	94	67			Total Waste Generation
		≤	≤	≤			233
	Disposal:	72	94	113	Total Waste Disposal	279	

Total Cost Z (Minimized)	
$ 2,926.00	

Figure 1.19: New Optimal Solution to this Minimization Transportation problem.

- The Minimized value of the Transportation Cost (Weekly Optimized Transport Cost) which the company will bear from the given information is **$2,926.00**

- ***The total cost has been reduced with $36 with the increase of 5 barrels per week in capacity of disposal sites in transportation problem.***

Decision Variable		Plants & Waste Disposal Sites (Nodes)												
		Denver	Morganton	Morrisville	Pineville	Rockhill	Statesville	Orangeburg	Florence	Macon	Transient:		Waste Generation:	Outward - Inward (Retaining Supply):
Plants & Waste Disposal Sites (Sources)	Denver	0	45	0	0	0	0	0	0	0	45	=	45	45
	Morganton	0	0	0	0	0	0	0	56	15	71	=	26	26
	Morrisville	0	0	0	0	0	0	0	0	52	52	=	42	42
	Pineville	0	0	10	0	43	0	0	0	0	53	=	53	53
	Rockhill	0	0	0	0	0	0	72	0	0	72	=	29	29
	Statesville	0	0	0	0	0	0	0	38	0	38	=	38	38
	Orangeburg	0	0	0	0	0	0	0	0	0	0	≤	72	
	Florence	0	0	0	0	0	0	0	0	0	0	≤	94	
	Macon	0	0	0	0	0	0	0	0	0	0	≤	113	
	Shipped To:	0	45	10	0	43	0	72	94	67			Total Waste Generation	
		=	=	=	=	=	=	≤	≤	≤			233	
	Disposal:	45	26	42	53	29	38	72	94	113	Total Waste Disposal	279		
	Outward - Inward (Retaining Supply):							72	94	67				

Total Cost Z (Minimized)	
$ 2,632.00	

Figure 1.20: New Optimal Solution to this Minimization Trans-Shipment problem.

- The Minimized value of the Trans-Shipment Cost (Weekly Optimized Transport Cost) which the company will bear from the given information is **$2,632.00**

- ***The total cost has been reduced with $25 with the increase of 5 barrels per week in capacity of disposal sites in trans-shipment problem.***

PART 2 - Investment Allocations (Portfolio Optimization)

This investor has chosen to invest in the following asset types. A historical analysis has been used to estimate the return on each type of asset.

	Expected Returns
Bonds	7%
High tech stocks	12%
Foreign stocks	10%
Call options	14%
Put options	14%
Gold	9%

Figure 2.1: The Investment Stocks and their Expected Returns in Percentage.

	Expected Returns
Bonds	**0.07**
High tech stocks	**0.12**
Foreign stocks	**0.10**
Call options	**0.14**
Put options	**0.14**
Gold	**0.09**

Figure 2.2: The Investment Stocks and their Expected Returns in Proportional Decimals.

A covariance matrix shows the correlation between assets' returns. A diagonal entry is the variance of an asset and a non-diagonal entry is a correlation between any two assets

	Bonds	High tech stocks	Foreign stocks	Call options	Put options	Gold
Bonds	0.001	0.0003	-0.0003	0.00035	-0.00035	0.0003
High tech stocks		0.009	0.0004	0.0016	-0.0016	0.0006
Foreign stocks			0.008	0.0015	-0.0055	-0.0007
Call options				0.012	-0.0005	0.0008
Put options					0.012	-0.0008
Gold						0.005

Figure 2.3: The Co-Variance Matrix between Assets' Returns.

- The following list shows the ***VARIANCES*** of the assets.
    - Bonds - 0.001
    - High Tech Stocks - 0.009
    - Foreign Stocks - 0.008
    - Call Options - 0.012
    - Put Options - 0.012
    - Gold - 0.005

- The following list provides the ***CO-VARIANCES*** between the assets.
    - Bonds - High Tech Stocks : 0.0003
    - Bonds - Foreign Stocks : -0.0003
    - Bonds - Call Options : 0.00035
    - Bonds - Put Options : -0.00035
    - Bonds - Gold : 0.0003
    - High Tech Stocks - Foreign Stocks : 0.0004
    - High Tech Stocks - Call Options: 0.0014
    - High Tech Stocks - Put Options : -0.0016
    - High Tech Stocks - Gold : 0.0006
    - Foreign Stocks - Call Options: 0.0015
    - Foreign Stocks - Put Options : -0.0055
    - Foreign Stocks - Gold : -0.0007
    - Call Options - Put Options : -0.0005
    - Call Options - Gold : 0.0008
    - Put Options - Gold : -0.0008

- The mathematical formula for Portfolio Variance is

$$\sigma_p^2 = \sum_{i=1}^{k} w_i^2 \sigma_i^2 + 2 \sum_{i=1}^{k} \sum_{j>i} w_i w_j \sigma_{ij}$$

- For example, mathematical formula for Two assets and Three assets are :

- Two assets:

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \sigma_{12}$$

- Three assets:

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_3^2 \sigma_3^2 + 2 w_1 w_2 \sigma_{12} + 2 w_1 w_3 \sigma_{13} + 2 w_2 w_3 \sigma_{23}$$

The weight elements belonging to the variances and co-variances of the assets (decision variables) present in the formula for calculating the Portfolio Variance have been calculated and is present below in the figure.

	Variance - Covariance
X1^2	**0.001**
X2^2	**0.009**
X3^2	**0.008**
X4^2	**0.012**
X5^2	**0.012**
X6^2	**0.005**
2*X1*X2	**0.0006**
2*X1*X3	**-0.0006**
2*X1*X4	**0.0007**
2*X1*X5	**-0.0007**
2*X1*X6	**0.0006**
2*X2*X3	**0.0008**
2*X2*X4	**0.0032**
2*X2*X5	**-0.0032**
2*X2*X6	**0.0012**
2*X3*X4	**0.003**
2*X3*X5	**-0.011**
2*X3*X6	**-0.0014**
2*X4*X5	**-0.001**
2*X4*X6	**0.0016**
2*X5*X6	**-0.0016**

Figure 2.4: Calculation of the variances and co-variances weight elements in the Portfolio variance formula.

The **Decision Variables** have been defined in the solution. These decision variables are the weights used to compute the Expected Return Percentage.

		Weights (Decision Variables)
Bonds	X1	
High tech stocks	X2	
Foreign stocks	X3	
Call options	X4	
Put options	X5	
Gold	X6	

Figure 2.5: Assets' nomenclature given in the solution and Decision Variables.

These decision variables are multiplied with the Expected Returns (stated in the problem) to calculate the Return Percentage.

Return		
12.00%	≥	12.00%

Figure 2.6: Return percentage variable.

The **Objective Function** is a sum product of the calculated weight elements (of variances and co-variances) and the variances calculated using the Solver function.

		Variance - Covariance
X1^2	0	**0.001**
X2^2	0.02664314	**0.009**
X3^2	0.080182104	**0.008**
X4^2	0.013365017	**0.012**
X5^2	0.108783601	**0.012**
X6^2	0.011702378	**0.005**
2*X1*X2	0	**0.0006**
2*X1*X3	0	**-0.0006**
2*X1*X4	0	**0.0007**
2*X1*X5	0	**-0.0007**
2*X1*X6	0	**0.0006**
2*X2*X3	0.046220158	**0.0008**
2*X2*X4	0.018870242	**0.0032**
2*X2*X5	0.053836203	**-0.0032**
2*X2*X6	0.017657522	**0.0012**
2*X3*X4	0.032735839	**0.003**
2*X3*X5	0.093394314	**-0.011**
2*X3*X6	0.03063203	**-0.0014**
2*X4*X5	0.038129971	**-0.001**
2*X4*X6	0.012506098	**0.0016**
2*X5*X6	0.035679502	**-0.0016**
	VARIANCE	**0.001304592**

Figure 2.7: Objective Function along with elements variances calculation.

The **Decision Variables (Weights)** have been calculated as well using the Solver function.

The **Constraints** of this problem are as follows :

- The Sum of Investment Proportions should be equal to 1.
- The return percentage computed using the sum product of the expected returns and the weights should be greater than or equal to the static percentage we're using for our calculations.

		Weights (Decision Variables)		
Bonds	X1	0		
High tech stocks	X2	0.16322025		
Foreign stocks	X3	0.283161237		
Call options	X4	0.115609902		
Put options	X5	0.329825707		
Gold	X6	0.108182904		
	Total	1	=	1

Figure 2.8: Decision variables computed in this problem.

- The computed Weight of Bonds is **0**
- The computed Weight of High Tech Stocks is **0.16322025**
- The computed Weight of Foreign Stocks is **0.283161237**
- The computed Weight of Call Options is **0.115609902**
- The computed Weight of Put Options is **0.32985707**
- The computed Weight of Gold is **0.108182904**

The Portfolio Return and its Minimized Risk :

Portfolio Return (e)	Portfolio Variance (r)
9%	**0.045%**
10%	**0.054%**
11%	**0.082%**
11.50%	**0.103%**
12%	**0.130%**
13%	**0.249%**
14%	**0.603%**
15%	**Out of Feasibility Region**
16%	**Out of Feasibility Region**

Figure 2.9: Portfolio Return & Variance for different return percentages.

- Portfolio Return at **9%** is with Minimised Risk (Portfolio Variance) at **0.045%**
- Portfolio Return at **10%** is with Minimised Risk (Portfolio Variance) at **0.054%**
- Portfolio Return at **11%** is with Minimised Risk (Portfolio Variance) at **0.082%**
- Portfolio Return at **11.50%** is with Minimised Risk (Portfolio Variance) at **0.103%**
- Portfolio Return at **12%** is with Minimised Risk (Portfolio Variance) at **0.130%**
- Portfolio Return at **13%** is with Minimised Risk (Portfolio Variance) at **0.249%**
- Portfolio Return at **14%** is with Minimised Risk (Portfolio Variance) at **0.603%**
- At 15% and 16% portfolio return, there are no feasible variances.

Optimal Portfolio's Investment of Assets (Percentage Wise) -

- **Bonds - 0%**
- **High Tech Stocks - 16.3%**
- **Foreign Stocks - 28.31%**
- **Call Options - 11.56%**
- **Put Options - 32.98%**
- **Gold - 10.82%**

Optimal Portfolio's Investment of Assets (Capital Wise) -

Optimized Results			
Assets	**Optimal Investment Proportion**	**Optimal Investment %**	**Investment Amount (from $10,000)**
Bonds	0	**0.00%**	**$ -**
High tech stocks	0.16322025	**16.32%**	**$ 1,632.20**
Foreign stocks	0.283161237	**28.32%**	**$ 2,831.61**
Call options	0.115609902	**11.56%**	**$ 1,156.10**
Put options	0.329825707	**32.98%**	**$ 3,298.26**
Gold	0.108182904	**10.82%**	**$ 1,081.83**

Figure 2.10: Investment Portfolio Results.

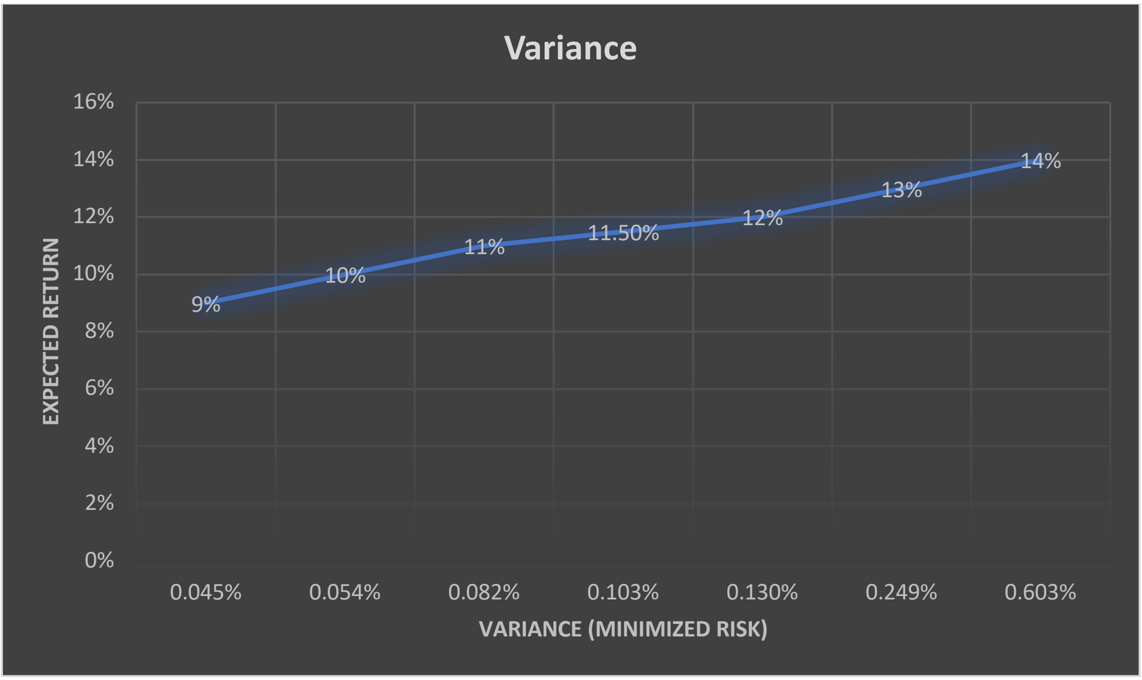

Figure 2.10: Plot for Portfolio Return & Variance for different return percentages.

- The plot signifies that Expected Return and Variance (minimized risk) are directly proportional to each other.
- As the Expected Return increase, so does the Variance (minimized risk).
- There is a linear relationship between these 2 attributes.
- The increase in return will increase the risk factor of the portfolio.

CONCLUSION

We have used the Quadratic Programming (QP) model to compute/analyse the Minimization of the Transportation Costs to be beared by the Rockhill Transport Company.

- With the usage of Quadratic Programming, we have computed the Optimal Values of Decision Variables used in this problem and minimized the Total Transportations Cost the company will need to bear using these decision variables along with the constraints they have.

- In the Direct Shipment (Transportation Problem), the Minimized value of the Transportation Cost (Weekly Optimized Transport Cost) which the company will bear from the given information is
$2,960.00

- The optimal number of units of waste barrels to be sent from the plant (Denver) is **38 to Orangeburg**, **7 to Florence.**
- The optimal number of units of waste barrels to be sent from the plant (Morganton) is **26 to Macon.**
- The optimal number of units of waste barrels to be sent from the plant (Morrisville) is **42 to Macon**, **7 to Florence.**
- The optimal number of units of waste barrels to be sent from the plant (Pineville) is **53 to Florence.**
- The optimal number of units of waste barrels to be sent from the plant (Rockhill) is **29 to Orangeburg**
- The optimal number of units of waste barrels to be sent from the plant (Statesville) is **29 to Florence**, **9 to Macon.**

- For the second problem of Trans-Shipment where deliveries are made amongst sources and destinations with no barriers of transport within sources, within destinations, and from sources to destinations, but no transport from destinations to sources. The difference of ***INWARD Supply and OUTWARD Supply*** should be taken into account for proper calculations.

- The Minimized value of the Trans-Shipment Cost (Weekly Optimized Total Trans-Shipment Cost) which the company will bear from the given information is
$2,657.00

- The optimal number of units of waste barrels to be sent from the plant (Denver) is **45 to Morganton.**
- The optimal number of units of waste barrels to be sent from the plant (Morganton) is **51 to Florence**, **35 to Macon**
- The optimal number of units of waste barrels to be sent from the plant (Morrisville) is **42 to Macon**

- The optimal number of units of waste barrels to be sent from the plant (Pineville) is **15 to Morganton**, **38 to Rockhill.**
- The optimal number of units of waste barrels to be sent from the plant (Rockhill) is **67 to Orangeburg.**
- The optimal number of units of waste barrels to be sent from the plant (Statesville) is **38 to Florence.**

- The difference between the minimized Total Transportation Cost between Direct Shipment (transportation) and Trans-Shipment is **$2,900 - $2,657 = $243**
- This means that for the manager at Rockhill Company, the trans-shipment method is more cost effective from the 2.

- The total cost gets reduced by **$36** with *the increase of 5 barrels per week* in capacity of disposal sites in transportation problem (Direct Shipment problem) as the total minimized transportation cost becomes **$2,926**
- The total cost gets reduced by **$25** with *the increase of 5 barrels per week* in capacity of disposal sites in Trans-Shipment problem as the total minimized transportation cost becomes **$2,632**

- For the 2nd part of Portfolio Optimization, Portfolio Return at **9%** is with Minimised Risk (Portfolio Variance) at **0.045%**
- Portfolio Return at **10%** is with Minimised Risk (Portfolio Variance) at **0.054%**
- Portfolio Return at **11%** is with Minimised Risk (Portfolio Variance) at **0.082%**
- Portfolio Return at **11.50%** is with Minimised Risk (Portfolio Variance) at **0.103%**
- Portfolio Return at **12%** is with Minimised Risk (Portfolio Variance) at **0.130%**
- Portfolio Return at **13%** is with Minimised Risk (Portfolio Variance) at **0.249%**
- Portfolio Return at **14%** is with Minimised Risk (Portfolio Variance) at **0.603%**
- At 15% and 16% portfolio return, there are no feasible variances.

- **Optimal Portfolio's Investment of Assets (Percentage wise) -**
  - **Bonds - 0%**
  - **High Tech Stocks - 16.3%**
  - **Foreign Stocks - 28.31%**
  - **Call Options - 11.56%**
  - **Put Options - 32.98%**
  - **Gold - 10.82%**

- **Optimal Portfolio's Investment of Assets (Capital wise) -**

Bonds	**$0**
High Tech Stocks	**$1,603.20**
Foreign Stocks	**$2,831.61**
Call Options	**$1,156.10**
Put Options	**$3,298.26**
Gold	**$1,082.83**

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