MODULE THREE PROJECT

FORECASTING A TIME SERIES

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ALY 6050 : INTRODUCTION TO ENTERPRISE ANALYTICS
JANUARY 31, 2022

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ABSTRACT

A **time series** is simply a series of data points ordered according to time. Time is generally the independent variable and the goal is to forecast the future.

Few examples of time series :

- Time series of weather conditions
- Time series of height of ocean tides
- Time series of patient health monitoring, like in Electrocardiogram (ECG)

In **time series analysis**, statistical methods are used to analyse time series data and extract meaningful statistics and characteristics about the data. Time series analysis helps identify trends, cycles, and seasonal variations to helps or aids forecast future events.

A time series can be decomposed into 4 aspects or components :

1. ***TREND*** :
   The increasing or decreasing behaviour of the time series over time which might not be linear every time. The trend might change its direction when time changes.

2. ***SEASONAL*** :
   The repeating cycles or patterns of behaviour in time series over time. These patterns are regular behaviours of the series (example- every month, quarter, year).

3. ***CYCLE*** :
   This component exists when the data is exhibiting rises and falls which are not of fixed periods. It is not similar to seasonal as the average length of cycles is longer than it.

4. ***IRREGULAR OR RANDOM*** :
   It is a stationary process where the data shows random variances because of sudden downs and ups in the series. It is a causal effect due to the irregularities present in and around the time series.

INTRODUCTION

In this project, we will perform *Forecasting of a Time Series* and use linear algebra to perform single/simple or multiple regression.

PROBLEM STATEMENT

We have the historic stock prices of Apple Incorporation (AAPL) and Honeywell International Incorporation (HON) for a total time period of one year which consists of 252 days of market.

In this project, we will be performing and analysing the followings :

- Short Term Forecasting with Error Analysis using Exponential Smoothing and Adjusted Exponential Smoothing in *Part 1*
- Long Term forecasting with Error Analysis using Weighted Moving Averages and Linear Trend in *Part 2*
- Simple Regression with Residual Analysis and Error Analysis in *Part 3*

A few records of historic stock price data of both the incorporations is shown below.

Date	Period	AAPL (Apple Inc) / $	AAPL (Apple Inc) / Volume	HON (Honeywell Inc) / $	HON (Honeywell Inc) Volume
08/11/19	1	63.95	69986400	177.03	1636800
11/11/19	2	64.46	81821200	176.66	1594600
12/11/19	3	64.40	87388800	177.81	1816900
13/11/19	4	65.02	102734400	177.75	1855200
14/11/19	5	64.57	89182800	176.38	2208300
15/11/19	6	65.34	100206400	178.44	3242700
18/11/19	7	65.67	86703200	176.53	2405800
19/11/19	8	65.47	76167200	176.84	2673200
20/11/19	9	64.70	106234400	173.99	4190700
21/11/19	10	64.41	121395200	173.31	3135100
22/11/19	11	64.36	65325200	173.57	1852200
25/11/19	12	65.49	84020400	173.29	2657500
26/11/19	13	64.97	105207600	175.27	4578900
27/11/19	14	65.85	65235600	176.15	1044800
29/11/19	15	65.70	46617600	175.30	1633100
02/12/19	16	64.94	94487200	171.14	3053500
03/12/19	17	63.78	114430400	169.41	3775500

Part 1: Short Term Forecasting

I (a) - Line Plot of the AAPL time series for detecting behaviours in it.

- The line plot of time series of Apple Incorporations (AAPL) shows almost an **upward trend** of the stock prices from 8th November 2019 to 8th October 2020.
- The trend changed its direction and decreased around the months of February, March but again changed its direction and started moving in upward direction from then onwards.
- This could be a result of a seasonal behaviour of the time series but since we do not have the data for a significant time period which could describe this behaviour, we cannot ascertain the seasonal behaviour happening here.
- The time period of this time series is not very large or significant enough for us to interpret the cyclical behaviour of the data in the series.

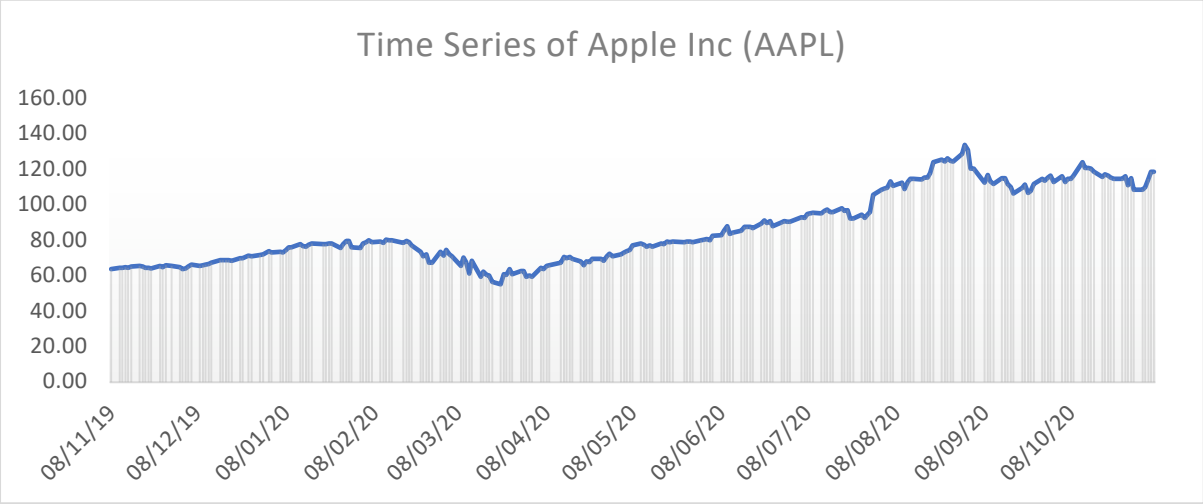

Figure 1.1: Time Series of Apple Incorporations (AAPL).

I (b) - Line Plot of the HON time series for detecting behaviours in it.

- The line plot of time series of Honeywell International Incorporations (HON) shows a **mixed trend of steady, upward and downward directions** of the stock prices from 8th November 2019 to 8th October 2020.
- The trend remains kind of steady in the initial months; changed its direction and decreased around the months of February, March but again changed its direction and started rising slowly in upward direction from then onwards with a little bit of down falls in between.
- This could be a result of a seasonal behaviour of the time series but since we do not have the data for a significant time period which could describe this behaviour, we cannot ascertain the seasonal behaviour happening here.

- The time period of this time series is not very large or significant enough for us to interpret the cyclical behaviour of the data in the series.

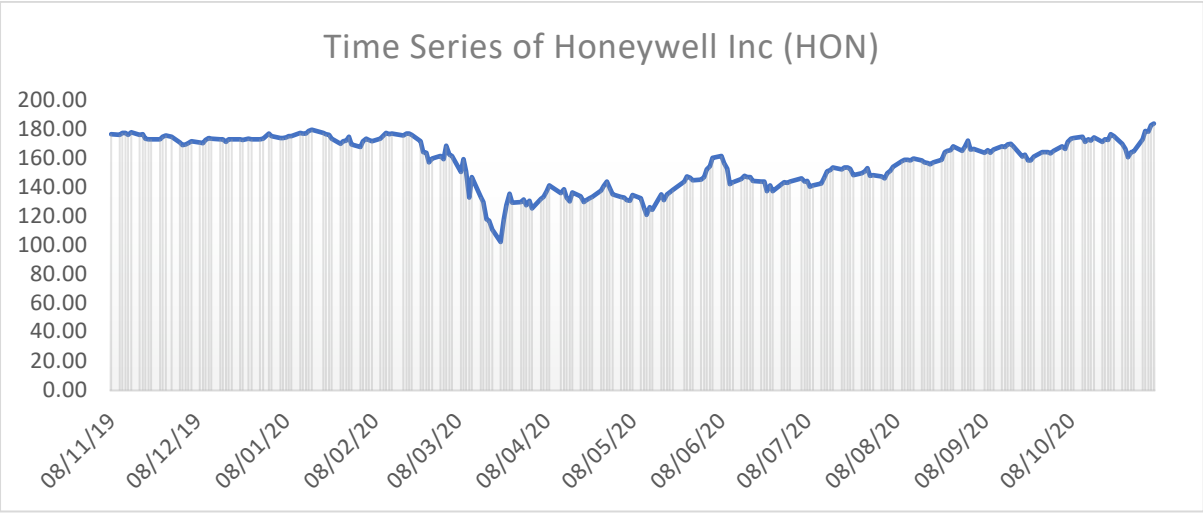

Figure 1.2: Time Series of Honeywell International Incorporations (HON).

II (a) - Exponential Smoothing for Short-Term forecasting of the 253rd period in AAPL time series along with Mean Absolute Percentage Error (MAPE).

Exponential Smoothing Method

- The formula for calculating Exponential Smoothing Forecasted values from the observed values is presented below.
- **α** is a *smoothing parameter of the level* which lies between 0 to 1, both exclusive.
- The first forecasted value is always the first observed value. The second and following forecasted values are calculated using the below formula which utilizes the previously forecasted value in its computation.

Mean Absolute Percentage Error (MAPE)

- The formula for calculating Mean Absolute Percentage Error (MAPE) uses each observed and its corresponding forecasted values to take a mean from them.

$$F_1 = A_1,$$
$$F_t = \alpha A_{t-1} + (1-\alpha) F_{t-1},$$
$$\text{for } t = 2, 3, \ldots, n$$
$$0 < \alpha < 1$$

Figure 1.3: Formula for calculating Exponential Smoothing Forecasted values.

$$MAPE = \frac{\sum_i \left|\frac{Ai-Fi}{Ai}\right|^2}{n} \times 100\%$$

Mean Absolute Percentage Error

Figure 1.4: Formula for calculating Mean Absolute Percentage Error (MAPE) values.

Exponential Smoothing (α = 0.18)	Mean Absolute Percentage Error (MAPE)		Exponential Smoothing (α = 0.33)	Mean Absolute Percentage Error (MAPE)
117.61	0.017817		117.56	0.017368
117.24	0.020872		116.90	0.017880
116.81	0.017027		116.22	0.011892
116.46	0.000483		115.77	0.005432
116.45	0.048977		115.98	0.044745
115.47	0.002999		114.34	0.006819
115.40	0.061948		114.60	0.054503
114.19	0.051667		112.64	0.037375
113.18	0.026605		111.30	0.009541
112.65	0.018275		110.95	0.033093
113.03	0.048749		112.21	0.055689
114.08	0.038882		114.39	0.036217
114.91			**115.81**	
	3.47834			**2.52755**

Figure 1.5: Exponential Smoothing and MAPE values with **α = 0.18 and 0.33**.

Exponential Smoothing (α = 0.5)	Mean Absolute Percentage Error (MAPE)		Exponential Smoothing (α = 0.8)	Mean Absolute Percentage Error (MAPE)
117.10	0.013385		116.76	0.010488
116.32	0.012905		115.79	0.008282
115.58	0.006365		115.03	0.001569
115.22	0.010153		114.89	0.012984
115.81	0.043238		116.10	0.045838
113.41	0.014880		112.03	0.026887
114.26	0.051461		114.50	0.053646
111.47	0.026579		109.84	0.011566
110.03	0.002033		108.83	0.012843
110.14	0.040211		109.97	0.041702
112.44	0.053693		113.79	0.042332
115.63	0.025740		117.82	0.007339
117.16			**118.52**	
	2.13744			**1.95424**

Figure 1.6: Exponential Smoothing and MAPE values with **α = 0.5 and 0.8**.

1. The Exponential Smoothing values for AAPL time series corresponding to different alpha values are -
    1. **114.91 (α = 0.18)**
    2. **115.81 (α = 0.33)**
    3. **117.16 (α = 0.5)**
    4. **118.52 (α = 0.8)**

2. The Mean Absolute Percentage Error (MAPE) values for AAPL time series corresponding to different alpha values are -
    1. **3.47834 (α = 0.18)**
    2. **2.52755 (α = 0.33)**
    3. **2.13744 (α = 0.5)**
    4. **1.95424 (α = 0.8)**

3. Since, the time series is an yearlong series with 252 market days records, the alpha value **(α) of 0.8** gives out the **best forecasted value of the 253rd record** in the series because ***it is more responsive to the recent OBSERVED values, i.e., more responsive to most recent values and the mean absolute percentage error (MAPE) is also significantly low than other values computed using different alpha values***.

II (b) - Exponential Smoothing for Short-Term forecasting of the 253rd period in HON time series along with Mean Absolute Percentage Error (MAPE).

Exponential Smoothing (α = 0.18)	Mean Absolute Percentage Error (MAPE)		Exponential Smoothing (α = 0.33)	Mean Absolute Percentage Error (MAPE)
172.09	0.004540		172.95	0.000451
172.23	0.026144		172.92	0.022210
173.06	0.014136		174.22	0.007529
173.51	0.019600		174.65	0.026353
172.90	0.036911		173.17	0.038529
171.80	0.066003		171.05	0.061395
169.88	0.032092		167.79	0.019376
168.93	0.024138		166.74	0.010833
168.21	0.031076		166.15	0.042986
169.19	0.055934		168.61	0.059149
170.99	0.044266		172.11	0.038019
172.42	0.059276		174.35	0.048709
174.37	0.053718		177.30	0.037832
176.15			**179.60**	
	2.90556			**2.38936**

Figure 1.7: Exponential Smoothing and MAPE values with **α = 0.18 and 0.33**.

Exponential Smoothing (α = 0.5)	Mean Absolute Percentage Error (MAPE)		Exponential Smoothing (α = 0.8)	Mean Absolute Percentage Error (MAPE)
173.00	0.000726		173.04	0.000982
172.93	0.022150		172.90	0.022313
174.89	0.003695		176.06	0.002967
175.22	0.029651		175.64	0.032169
172.69	0.035639		171.26	0.027075
169.72	0.053124		167.65	0.040289
165.44	0.005108		162.46	0.013010
165.02	0.000427		164.17	0.004718
164.99	0.049679		164.79	0.050779
169.30	0.055312		171.85	0.041087
174.25	0.026025		177.74	0.006554
176.58	0.036546		178.68	0.025123
179.93	0.023547		182.36	0.010370
182.10			**183.89**	
	2.08573			**1.86917**

Figure 1.8: Exponential Smoothing and MAPE values with **α = 0.5 and 0.8**.

1. The Exponential Smoothing values for HON time series corresponding to different alpha values are -
    1. **176.15 (α = 0.18)**
    2. **179.60 (α = 0.33)**
    3. **182.10 (α = 0.5)**
    4. **183.89 (α = 0.8)**

2. The Mean Absolute Percentage Error (MAPE) values for AAPL time series corresponding to different alpha values are -
    1. **2.90556 (α = 0.18)**
    2. **2.38936 (α = 0.33)**
    3. **2.08573 (α = 0.5)**
    4. **1.86917 (α = 0.8)**

3. Since, the time series is an yearlong series with 252 market days records, the alpha value **(α) of 0.8** gives out the **best forecasted value of the 253rd record** in the series because ***it is more responsive to the recent OBSERVED values, i.e., more responsive to most recent values and the mean absolute percentage error (MAPE) is also significantly low than other values computed using different alpha values***.

III (a) - Adjusted Exponential Smoothing for Short-Term forecasting of the 253rd period in AAPL time series along with Mean Absolute Percentage Deviation (MAPD).

Adjusted Exponential Smoothing Method

- The formula for calculating Adjusted Exponential Smoothing Forecasted values from the observed values is presented below.
- **α** is a *smoothing parameter for the level* which lies between 0 to 1, both exclusive.
- **β** is a *smoothing parameter for the trend* which lies between 0 to 1, both exclusive.
- The first forecasted value is always the first observed value. The second and following forecasted values are calculated using the below formula which utilizes the previously forecasted value.

Mean Absolute Percentage Error (MAPE)

- The formula for calculating Mean Absolute Percentage Deviation (MAPD) uses each observed and its corresponding forecasted values to take a mean from them.

$$AF_t = F_t + T_t,$$

$$T_t = \beta\,(F_t - F_{t-1}) + (1 - \beta)\,T_{t-1}$$

Figure 1.9: Formula for calculating Adjusted Exponential Smoothing Forecasted values.

$$MAPD = \frac{\sum_i |A_i - F_i|}{\sum_i A_i} \times 100\%$$

Mean Absolute Percentage Deviation

Figure 1.10: Formula for calculating Mean Absolute Percentage Deviation (MAPD) values.

Trend Parameter (β = 0.18)	Adjusted Exponential Smoothing (β = 0.18)	Mean Absolute Percentage Deviation (MAPD)		Trend Parameter (β = 0.33)	Adjusted Exponential Smoothing (β = 0.33)	Mean Absolute Percentage Deviation (MAPD)
-0.31	116.01	1.172710		-0.55	115.77	0.927626
-0.40	115.19	0.335354		-0.62	114.97	0.115031
-0.39	114.83	1.571468		-0.53	114.68	1.715162
-0.19	115.61	4.606245		-0.16	115.65	4.637445
-0.63	112.77	2.347834		-0.90	112.51	2.613755
-0.34	113.93	5.255803		-0.32	113.94	5.271509
-0.83	110.64	2.057548		-1.14	110.33	1.748321
-0.95	109.07	1.175505		-1.24	108.79	1.462577
-0.74	109.40	5.352987		-0.79	109.34	5.407077
-0.13	112.32	6.509651		0.23	112.68	6.149931
0.53	116.17	2.520679		1.21	116.84	1.848120
0.73	**117.90**			1.31	**118.48**	
		1.98936				**1.96160**

Figure 1.11: Adjusted Exponential Smoothing and MAPD values with **β = 0.18 and 0.33**.

Trend Parameter (β = 0.66)	Adjusted Exponential Smoothing (β = 0.66)	Mean Absolute Percentage Deviation (MAPD)		Trend Parameter (β = 0.88)	Adjusted Exponential Smoothing (β = 0.88)	Mean Absolute Percentage Deviation (MAPD)
-0.68	115.64	0.800768		-0.73	115.59	0.750398
-0.72	114.86	0.010327		-0.74	114.84	0.008856
-0.49	114.73	1.668096		-0.41	114.81	1.592253
0.22	116.03	5.024454		0.47	116.28	5.270539
-1.51	111.90	3.220560		-2.06	111.35	3.768432
0.05	114.32	5.645088		0.51	114.77	6.099437
-1.83	109.64	1.058480		-2.40	109.07	0.486242
-1.57	108.45	1.797873		-1.56	108.47	1.781943
-0.46	109.68	5.075406		-0.09	110.05	4.702614
1.37	113.81	5.014185		2.02	114.46	4.360432
2.57	118.20	0.485208		3.05	118.68	0.005450
1.88	**119.04**			1.71	**118.87**	
		1.93966				**1.95020**

Figure 1.12: Adjusted Exponential Smoothing and MAPD values with **β = 0.66 and 0.88**.

1. The Adjusted Exponential Smoothing values for AAPL time series corresponding to different alpha values are -
    1. **117.90 (β = 0.18)**
    2. **118.48 (β = 0.33)**
    3. **119.04 (β = 0.66)**
    4. **118.87 (β = 0.88)**

2. The Mean Absolute Percentage Deviation (MAPD) values for AAPL time series corresponding to different alpha values are -
    1. **1.98936 (β = 0.18)**
    2. **1.96160 (β = 0.33)**
    3. **1.93966 (β = 0.66)**
    4. **1.95020 (β = 0.88)**

3. Since, the time series is an yearlong series with 252 market days records, the alpha value **(α) of 0.5 and (β) of 0.66** gives out the **best forecasted value of the 253rd record** in the series because ***it is more responsive to the recent Trends and not to the older trends and the mean absolute percentage deviation (MAPD) is also significantly low than other values computed using different alpha values***.

III (b) - Adjusted Exponential Smoothing for Short-Term forecasting of the 253rd Period in HON time series along with Mean Absolute Percentage Deviation (MAPD).

Trend Parameter (β = 0.18)	Adjusted Exponential Smoothing (β = 0.18)	Mean Absolute Percentage Deviation (MAPD)		Trend Parameter (β = 0.33)	Adjusted Exponential Smoothing (β = 0.33)	Mean Absolute Percentage Deviation (MAPD)
0.53	175.42	0.123184		0.65	175.54	0.004592
0.49	175.70	5.530889		0.54	175.76	5.590354
-0.12	172.58	5.826432		-0.47	172.23	5.475228
-0.69	169.03	7.874004		-1.29	168.43	7.267546
-1.41	164.03	0.565366		-2.28	163.16	1.438821
-1.21	163.81	1.138567		-1.67	163.35	1.595638
-0.97	164.01	9.599002		-1.13	163.86	9.752653
0.08	169.38	9.829279		0.67	169.97	9.244968
1.06	175.31	3.598453		2.08	176.34	2.573466
1.31	177.89	5.386275		2.16	178.75	4.534388
1.72	181.65	2.619786		2.55	182.49	1.784182
1.81	**183.91**			2.43	**184.53**	
		1.87568				**1.82772**

Figure 1.13: Adjusted Exponential Smoothing and MAPD values with **β = 0.18 and 0.33**.

Trend Parameter (β = 0.66)	Adjusted Exponential Smoothing (β = 0.66)	Mean Absolute Percentage Deviation (MAPD)		Trend Parameter (β = 0.88)	Adjusted Exponential Smoothing (β = 0.88)	Mean Absolute Percentage Deviation (MAPD)
-0.05	172.89	3.962852		-0.04	172.89	3.957556
1.28	176.17	0.628573		1.72	176.61	1.070140
0.65	175.86	5.693973		0.49	175.71	5.537328
-1.44	171.25	4.498181		-2.16	170.53	3.781736
-2.45	167.27	6.109094		-2.87	166.85	5.687234
-3.66	161.78	2.818351		-4.11	161.33	3.271219
-1.52	163.50	1.451151		-0.86	164.16	0.792980
-0.54	164.44	9.165357		-0.13	164.85	8.759383
2.66	171.96	7.250007		3.78	173.08	6.133643
4.18	178.43	0.479889		4.81	179.07	0.158711
2.96	179.54	3.741602		2.63	179.21	4.071577
3.22	183.15	1.123473		3.26	183.19	1.076708
2.53	**184.63**			2.30	**184.40**	
		1.74661				**1.72567**

Figure 1.14: Adjusted Exponential Smoothing and MAPD values with **β = 0.66 and 0.88**.

1. The Adjusted Exponential Smoothing values for AAPL time series corresponding to different alpha values are -
    1. **183.91 (β = 0.18)**
    2. **184.53 (β = 0.33)**
    3. **184.63 (β = 0.66)**
    4. **184.40 (β = 0.88)**

2. The Mean Absolute Percentage Deviation (MAPD) values for AAPL time series corresponding to different alpha values are -
    1. **1.87568 (β = 0.18)**
    2. **1.82772 (β = 0.33)**
    3. **1.74661 (β = 0.66)**
    4. **1.72567 (β = 0.88)**

3. Since, the time series is an yearlong series with 252 market days records, the alpha value **(α) of 0.5 and (β) of 0.88** gives out the **best forecasted value of the 253rd record** in the series because ***it is more responsive to the recent Trends and not to the older trends and the mean absolute percentage deviation (MAPD) is also significantly low than other values computed using different alpha values***.

Part 2: Long Term Forecasting

I - Weighted Moving Averages using different 4 weights & Trend Based Forecasting.

- The weight 0.4 is used for the most recent period; the weight 0.3 for the period before the most recent; the weight 0.2 for 3 periods before; and 0.1 for 4 periods before.

- Later, the observed values are used from the period 101 as base for a linear trend in order to forecast periods 101 to 257.

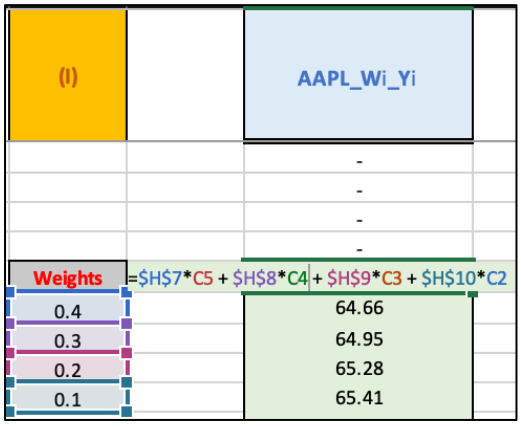

Figure 2.1: Weighted Moving Averages formula for AAPL.

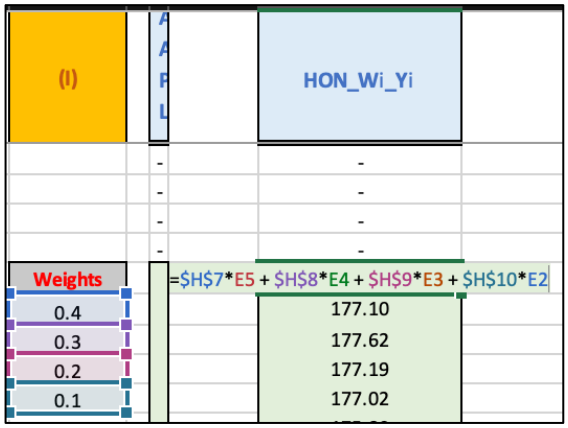

Figure 2.2: Weighted Moving Averages formula for HON.

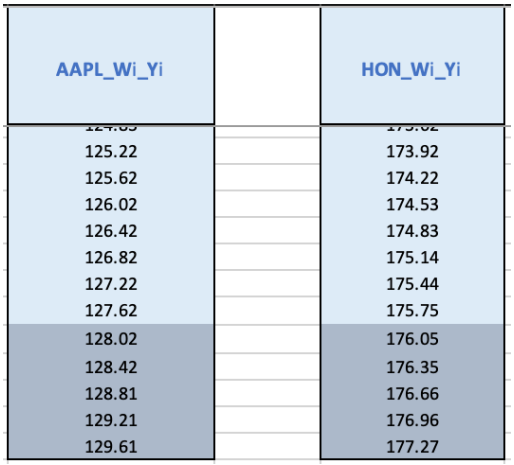

Figure 2.3: Trend-Based Forecasted values from 101 to 257.

- The Long-Term forecasted values for Apple Incorporation (AAPL) for the periods 253 to 257 came out to be **128.02, 128.42, 128.81, 129.21, 129.61**.

- The Long-Term forecasted values for Honeywell International Incorporation (HON) for the periods 253 to 257 came out to be **176.05, 176.35, 176.66, 176.96, 177.27.**

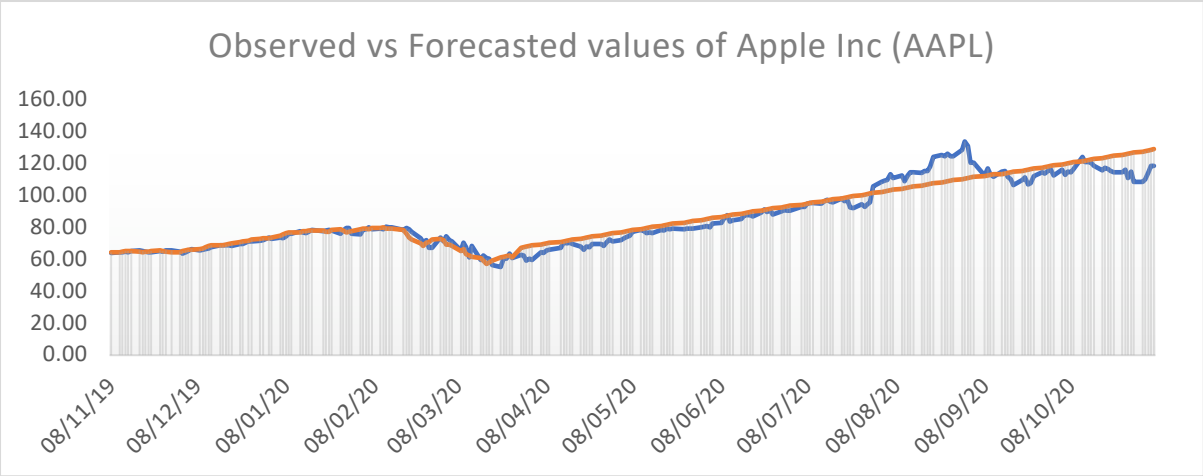

Figure 2.4: Observed vs Forecasted values of Apple Incorporations (AAPL).

- The observed and forecasted (based on weighted moving averages and trend-based) values follow each other in a similar fashion and close to each other.
- The weighted moving average values keep up with the observed values very closely in such a way that they follow the pattern which observed values are making.
- The trend-based forecasted values followed a straight line corresponding to the trend made by the observed values.

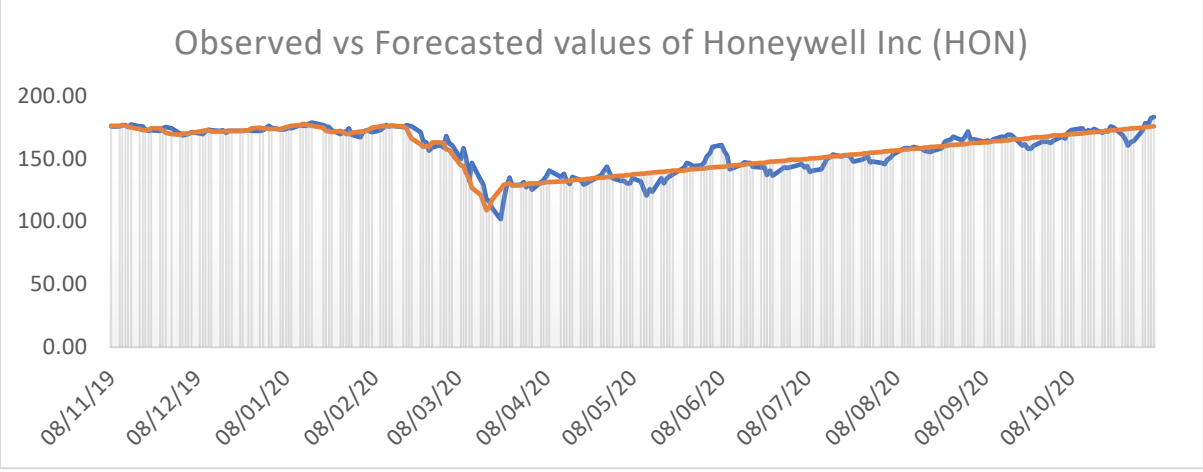

Figure 2.5: Observed vs Forecasted values of Honeywell International Incorporations (HON).

- The observed and forecasted (based on weighted moving averages and trend-based) values follow each other in a similar fashion and very close to each other.
- The weighted moving average values keep up with the observed values very closely in such a way that they follow the pattern which observed values are making.
- The trend-based forecasted values followed a straight line corresponding to the trend made by the observed values.

II - Mean Absolute Percentage Error (MAPE) for AAPL and HON.

AAPL - Mean Absolute Percentage Error (MAPE)		HON - Mean Absolute Percentage Error (MAPE)
0.124464		0.077285
0.087756		0.056619
0.155976		0.056222
0.160606		0.005288
0.146674		0.024427
0.105161		0.021090
0.070637		0.042771
0.075215		0.046262
3.80485		**2.74627**

Figure 2.6: Observed vs Forecasted values of Honeywell International Incorporations (HON).

- The Mean Absolute Percentage Error (MAPE) values for AAPL and HON between the observed and forecasted values come out to be **3.8049** and **2.7462** respectively.

- **The MAPE value of weighted moving averages and trend-based forecasted values of AAPL (3.80485) is significantly higher than all the MAPE values of forecasted values computed using Exponential Smoothing (example - 1.95424 , 3.47834, 2.52755, 2.13744).**

- **The MAPE value of weighted moving averages and trend-based forecasted values of HON (2. 74627) is near most of the MAPE values of forecasted values computed using Exponential Smoothing (example - 2.90556, 2.38936, 2.08573, 1.86917).**
  In fact, it is lower than the MAPE value computed using alpha level = 0.18

- For both of the stocks, the Exponential Smoothing method has yielded better and accurate forecast results.

Part 3: Regression

I - Application of Simple Regression for forecasting values

We're going to apply simple regression to predict the values for the period of 1 to 257 of both the stock prices time series. The charts of Predicted Y values of AAPL and HON are below along with the linear algebra equations in them.

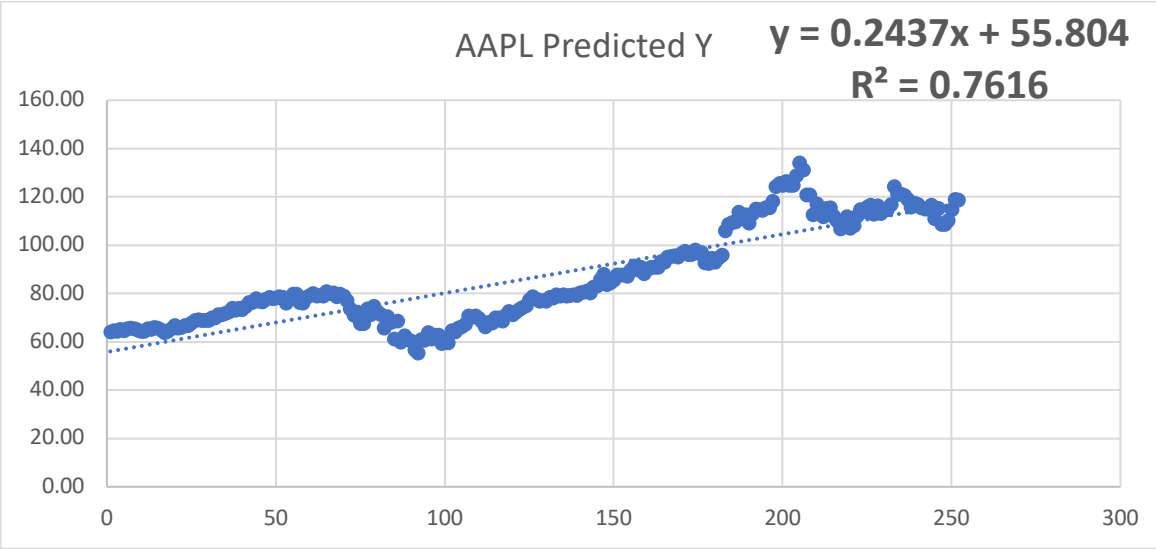

Figure 3.1: Predicted Y values of AAPL and linear equation with slope and intercept.

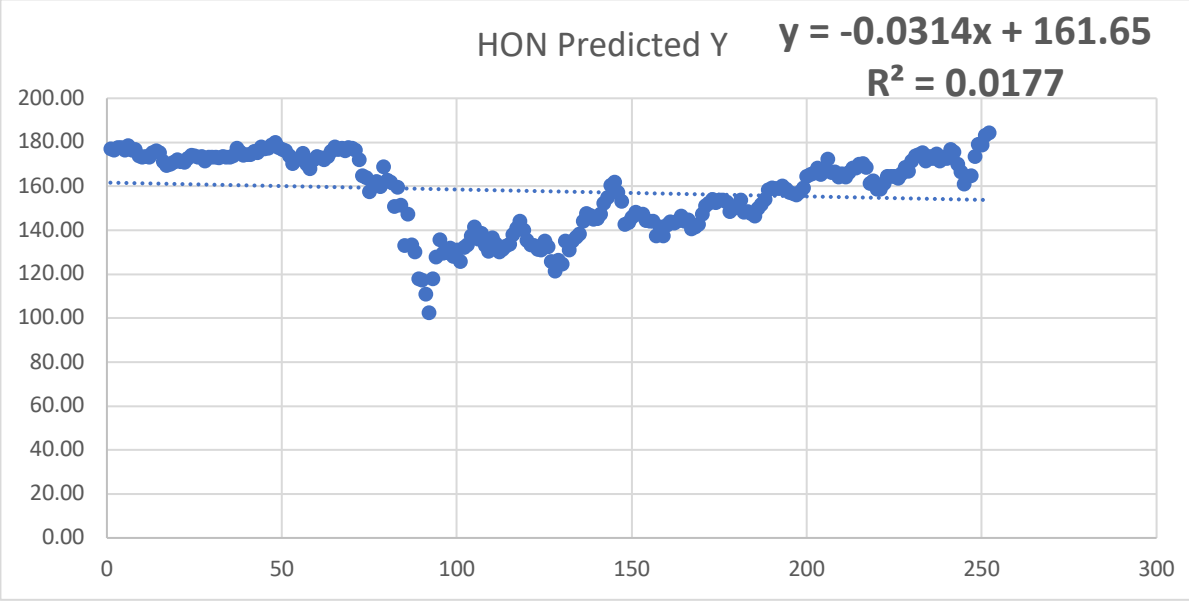

Figure 3.2: Predicted Y values of HON and linear equation with slope and intercept.

AAPL_Predicted _Y	AAPL_Residual (Observed - Predicted)	AAPL_Sort_Residual	AAPL_Standardize	AAPL_Rank	AAPL_Cummulative_Area	AAPL_Standard_Z_Value
56.05	7.91	-22.93	-2.31	1	0.0020	-2.881
56.29	8.17	-21.49	-2.16	2	0.0060	-2.515
56.54	7.87	-20.93	-2.11	3	0.0099	-2.329
56.78	8.24	-20.56	-2.07	4	0.0139	-2.200
57.02	7.55	-19.82	-1.99	5	0.0179	-2.100
57.27	8.07	-18.21	-1.83	6	0.0218	-2.017
57.51	8.16	-18.15	-1.83	7	0.0258	-1.947
57.75	7.71	-17.63	-1.77	8	0.0298	-1.884

Figure 3.2: Residuals, Sorted Res, Standardized, Ranks, Cumulative Area, Standard Z values of AAPL

HON_Predicted _Y	HON_Residual (Observed - Predicted)	HON_Sort_Residual	HON_Standardize	HON_Rank	HON_Cummulative_Area	HON_Standard_Z_Value
161.62	15.41	-56.24	-3.30	1	0.0020	-2.881
161.59	15.07	-47.74	-2.80	2	0.0060	-2.515
161.56	16.26	-41.40	-2.43	3	0.0099	-2.329
161.52	16.23	-40.79	-2.39	4	0.0139	-2.200
161.49	14.89	-40.76	-2.39	5	0.0179	-2.100
161.46	16.98	-36.25	-2.13	6	0.0218	-2.017
161.43	15.10	-32.86	-1.93	7	0.0258	-1.947

Figure 3.3: Residuals, Sorted Res, Standardized, Ranks, Cumulative Area, Standard Z values of HON.

AAPL_Predicted _Y	AAPL_MAPE
116.25	0.0706
116.49	0.0566
116.73	0.0173
116.98	0.0156
117.22	0.0124
	10.56076225

Figure 3.4: MAPE value of AAPL

HON_Predicted _Y	HON_MAPE
153.87	0.1137
153.83	0.1416
153.80	0.1403
153.77	0.1610
153.74	0.1657
	9.7586

Figure 3.5: MAPE value of HON

	Apple Inc (AAPL)	Honeywell Inc (HON)
SLOPE	0.2437	-0.0314
INTERCEPT	55.804	161.650
CORRELATION, R	0.8727	-0.1330
DETERMINATION, R2	0.7616	0.0177
MEAN OF RESIDUAL	0.00	0.00
S.D. OF RESIDUAL	9.94	17.05

Figure 3.6: Slope, Intercept, Correlation, Determination values of AAPL & HON

- The MAPE value of forecasted AAPL using Simple Regression is **10.5607** which is significantly higher than the MAPE values of AAPL forecasted values using Exponential Smoothing and a mix of weighted moving averages - Trend based forecasted values.

- The MAPE value of forecasted HON using Simple Regression is **9.7586** which is significantly higher than the MAPE values of HON forecasted values using Exponential Smoothing and a mix of weighted moving averages - Trend based forecasted values.

II - Properties of Residual Values

Homoscedasticity

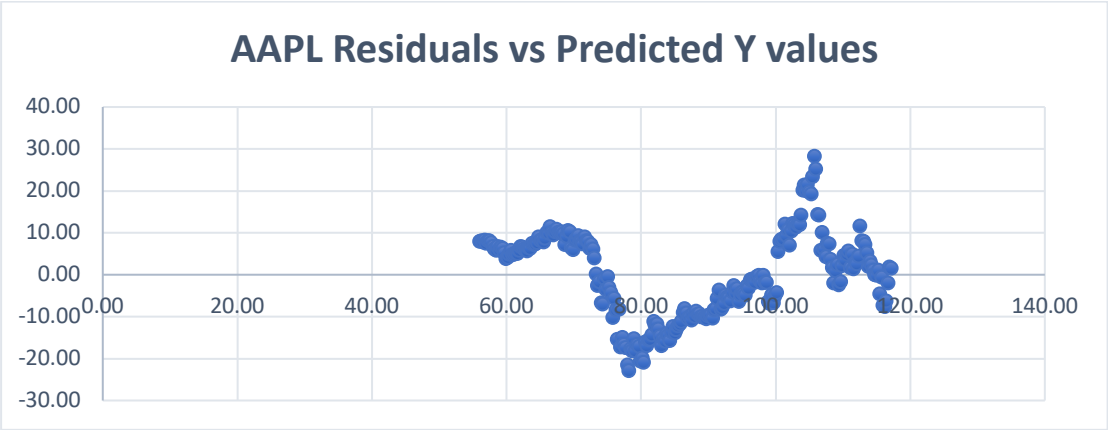

Figure 3.7: Residuals vs Predicted Y values of AAPL.

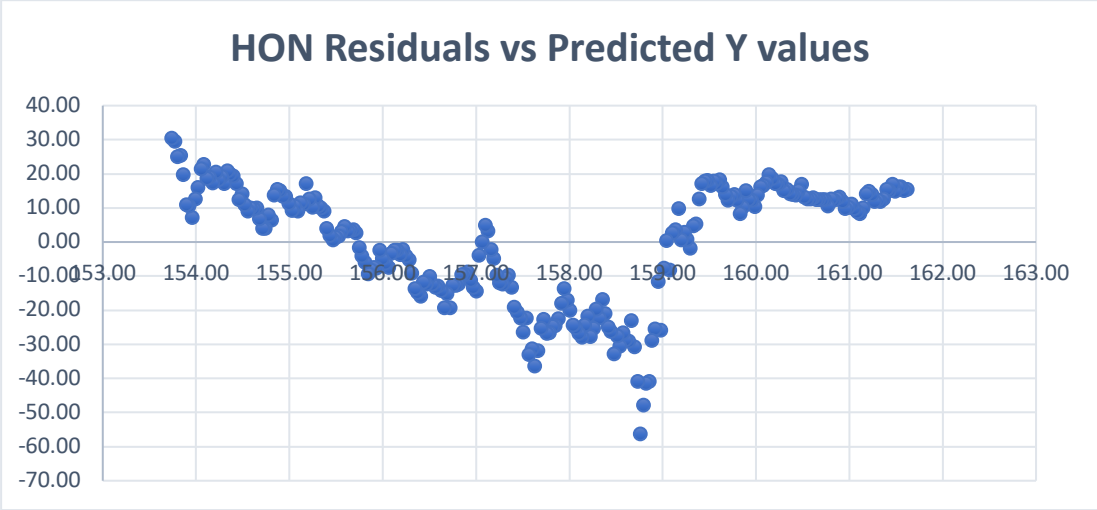

Figure 3.8: Residuals vs Predicted Y values of HON.

- The Residuals vs Predicted Y values for the forecasted values of both stocks have been plotted above to show the homodescasticity of the residuals.

- *The charts show that the residuals of both the series **are not homoscedastic** in nature as they are distributed and the variance of these residuals does not seem to be constant.*

Independence

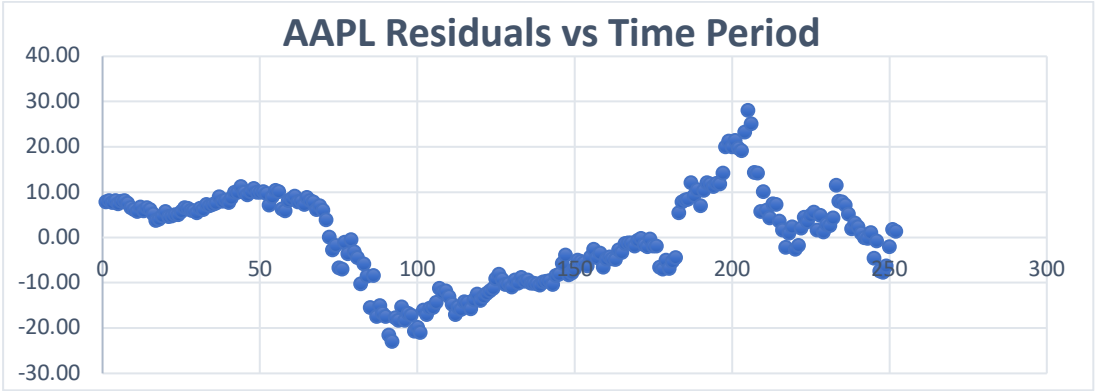

Figure 3.9: Residuals vs Time Period values of AAPL.

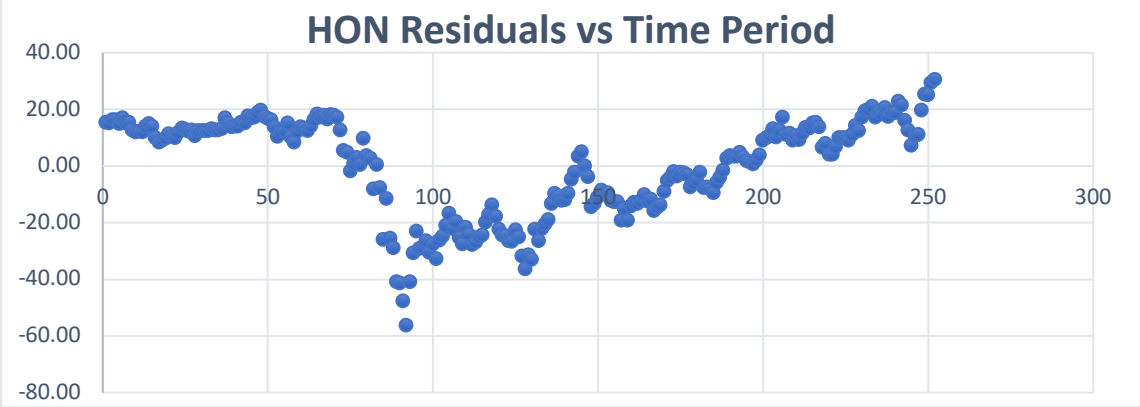

Figure 3.10: Residuals vs Time Period values of HON.

- The Residuals vs Time Period values for the forecasted values have been plotted above to show the independence. *The charts show that the residuals of both the series are* ***Not-Independent*** *in nature as they show relationship between the residuals and the variables. They seem to have a trend in themselves.*

Normal Probability Plot of the Residuals

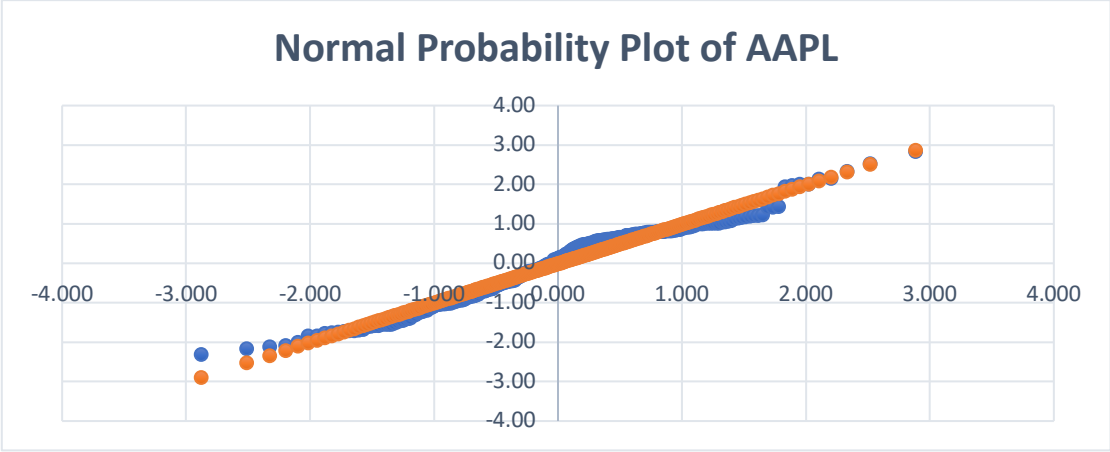

Figure 3.11: Normal probability plot of AAPL.

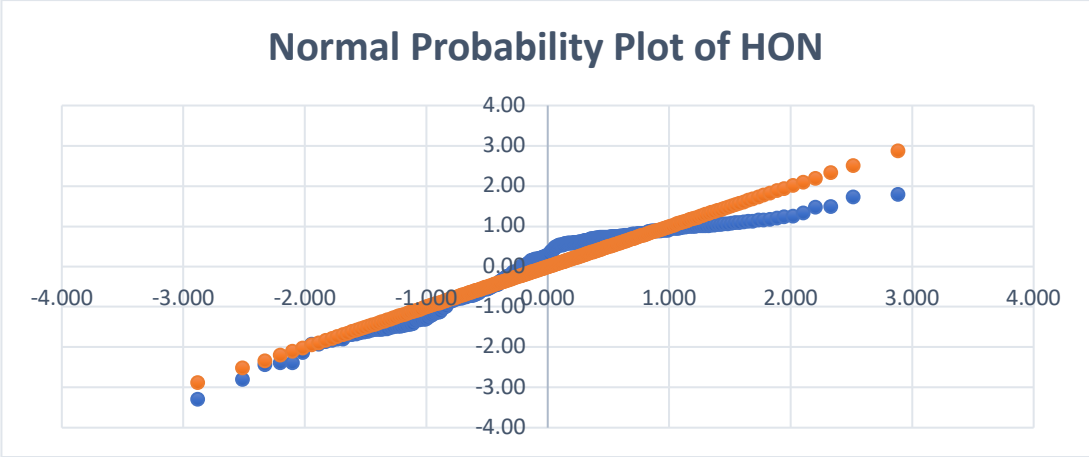

Figure 3.12: Residuals vs Time Period values of HON.

- *The charts show that the residuals of both the series are not normal as they do not follow the observed values very closely.*

Chi-Squared Test (Goodness of Fit)

We can figure out the goodness of fit using Chi-Squared test which will give p-values.

Chi-Squared Test Statistic	70.25
Chi-Squared P-value	0.000000004039
Degree of Freedom	15

Figure 3.13: Goodness of Fit Test statistics of AAPL.

Chi-Squared Test Statistic	147.33
Chi-Squared P-value	0.00000000000000000000008163
Degree of Freedom	15

Figure 3.14: Goodness of Fit Test statistics of HOM.

1. The NULL Hypothesis,
   H_0 : Residuals are normally distributed for AAPL

2. The ALTERNATE Hypothesis,
   H_1 : Residuals are not normally distributed for AAPL

3. Alpha value = 0.05 (95% confidence).

4. The P-value of the series comes out to be
   a. **0.0000004039 for AAPL**
   b. **0.00000000000008163 for HOM**

5. The P-values emerges out to be less than 0.05 in both of the cases which means we can reject the null hypothesis. We can say that there are sufficient evidences that the residuals are not normally distributed for AAPL and HOM.

Part 4: Portfolio

In my portfolio based on the observations made above, I would select 45% of Honeywell International Incorporations (HON) stocks and the rest 55% for Apple Incorporations (AAPL).

The reason behind this logic is that the even though Error estimates in AAPL stocks seem to be significantly higher than the error estimates in HON stocks, but the stock prices of AAPL are significantly low as compared to those in HON.

This would help me balance out the uncertainty in the HON because of their high prices.

The growth rate of AAPL also is very high than the growth rate of HON

CONCLUSION

We have performed Forecasting of the time series of Apple Incorporations (AAPL) and Honeywell International Incorporations (HON).

- The forecasted values of AAPL and HOM using Exponential Smoothing method give out the better and accurate predicted values than any other methods.
- The Short-term and Long-term forecasting was performed and values were predicted amongst which the Exponential Smoothing method gave out the better results.
- Simple regression as performed to predict the values and both the residual series were checked for homoscedasticity, independence, normally distributed.
- Both of the series are NOT homoscedastic, independent, normally distributed.
- The Chi-Squared test was also applied to verify normally distributed.
- In my portfolio based on the observations made above, I would select 45% of Honeywell International Incorporations (HON) stocks and the rest 55% for Apple Incorporations (AAPL). The reason behind this logic is that the even though Error estimates in AAPL stocks seem to be significantly higher than the error estimates in HON stocks, but the stock prices of AAPL are significantly low as compared to those in HON
- The growth rate of AAPL also is very high than the growth rate of HON

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