MODULE FIVE PROJECT

USING LINEAR PROGRAMMING MODELS FOR MAXIMIZATION OF PROFITS

Submitted By : **HARSHIT GAUR**
MASTER OF PROFESSIONAL STUDIES IN ANALYTICS
ALY 6050 : INTRODUCTION TO ENTERPRISE ANALYTICS
FEBRUARY 12, 2022

Submitted To : **PROF. RICHARD HE**

ABSTRACT

All of us have finite resources and time, and we should make the most of both. From using our time efficiently to solving supply chain issues for our companies, optimization is part of everything we do. One of the easiest ways to perform optimization is with Linear Programming (LP).

Linear Programming (LP) is a simple technique in which complex relationships are modelled using linear functions and then the optimum points are calculated. Using linear programming, one can maximize the use of productive resources. It also explains how a decision-maker can efficiently manage his productive factors by selecting and allocating them effectively. It improves the quality of the effective decisions made.

INTRODUCTION

With the medium of this project, we will be using *Linear Programming Models to the problems of Maximization of Profits*.

PROBLEM STATEMENT

One hardware company in the northern part of the country is exploring the prospect of opening a new distribution centre in the southeast. The company plans on renting a warehouse and an adjacent office to distribute its products to local dealers.

In the beginning, the company will focus on four of its main products: pressure washers, go karts, generators, and water pumps.

The company has imposed some constraints on few budgeting, space available kinds of resources to be able to meet with the demands and make use of the surplus resources at their hands.

The below table provides information about the ***Cost Price (CP)*** of the products the company is targeting for.

Item	Cost (in Dollars)
Pressure washer	335
Go-kart	380
Generator	415
(Case of 5 Water Pumps)	650

Table 1.1: The Cost Price (CP) of products in Dollars.

The below table provides information about the ***Selling Price (SP)*** of the products the company is targeting for.

Item	Selling Price (in Dollars)
Pressure washer	499.99
Go-kart	729.99
Generator	700.99
Water pump	269.99

Table 1.2: The Selling Price (SP) of products in Dollars.

The list of constraints the company has imposed for this warehouse :

- The purchasing monthly budget for the new location is $175,000.

- The second concern for the company is the space available in the warehouse. The warehouse has 80 shelves and each shelf measures as 30 ft long and 5 ft wide.
    - Pressure Washer is stored as 5 ft x 5 ft pallet.
    - Generator is stored as 5 ft x 5 ft pallet.
    - Go Kart is stored as 8 ft x 5 ft pallet.
    - 4 cases of Water Pumps are stored in 5 ft x 5 ft pallet.

- As a promotion for its brand products, the company's marketing department has decided to give atleast 25% of its warehouse to Pressure Washer and Go Karts.

- Also, they have decided to sell atleast twice as many Generators as Water Pumps.

As a consultant, I am going to use Linear Programming to provide the company the optimal solution of maximization of their profit and help with other decisions.

ANALYSIS

I - Mathematical Formulation of the given maximization problem.

The **Decision Variables** in the problem are assigned names as :

- PW - Number of units of Pressure Washers.
- GK - Number of units of Go-Karts
- GN - Number of units of Generators
- WP - Number of units of Water Pumps.

Decision Variables:
PW = Number of units of Pressure Washer
GK = Number of units of Go-Karts
GN = Number of units of Generators
WP = Number of units of Water Pump
Objective Function:

Figure 1.1: Decision Variables defined in the Excel Workbook.

The **Objective Function** for our problem :

- Maximize the Profit Earned by the company upon selling these products
- The table of Cost Price includes the information of cost of products to the company plus the transportation costs.
- *Profit = Selling Price - Cost Price*

Objective Function:	
Total Profit (Objective)	***Z = (SP - CP) PW + (SP - CP) GK + (SP - CP) GN + (SP WP - CP/5 WP)***
Equivalent to	***Z = 164.99 PW + 349.99 GK + 285.99 GN + 139.99 WP***
Constraints:	

Figure 1.2: Objective Function.

Mathematical Formulation:	
Maximize	***Z = (499.99 - 335) PW + (729.99 - 380) GK + (700.99 - 415) GN + (269.99 WP - 650/5 WP)***
Equivalent to	***Z = 164.99 PW + 349.99 GK + 285.99 GN + 139.99 WP***

Figure 1.3: Mathematical Formulation of the Objective Function.

The **Constraints (Subjected to Conditions)** for our problem are :

- The Budget available from the company to this warehouse is $175,000.
- The total space available in the warehouse is 80 shelves with 30 ft by 5 ft measurement of one shelf.
- Promotion 1 of atleast 25% of the warehouse for the company's brand products - Pressure Washer & Go Karts
- Promotion 2 of atleast twice of Generators to be sold as Water Pumps.

Constraints:	
Total Budget Available ≤ 175,000	*335 PW + 380 GK + 415 GN + 650 (WP / 5) ≤ 175,000*
*Total Space Available ≤ 80 * (5 * 30)*	*25 PW + 40 GK + 25 GN + 25 (WP / (4*5)) ≤ 80 * 150*
Promotion 1- PW + GK atleast 25% of inventory	*PW + GK ≥ (PW + GK + GN + WP) / 4*
Equivalent	*-3PW - 3GK + GN + WP ≤ 0*
Promotion 2- Atleast twice GN as WP	*2 WP ≤ GN*
Equivalent	*2 WP - GN ≤ 0*
Non-Negativity:	*PW, GK, GN, WP ≥ 0*

Figure 1.4: Constraints.

Subject To:	
Budget Available-	*335 PW + 380 GK + 415 GN + 650 (WP / 5) ≤ 175,000*
Space Available-	*25 PW + 40 GK + 25 GN + 25 (WP / (4*5)) ≤ 80 * 150*
Promotion 1-	*PW + GK ≥ (PW + GK + GN + WP) / 4*
Equivalent	*-3PW - 3GK + GN + WP ≤ 0*
Promotion 2-	*2 WP ≤ GN*
Non-Negativity:	*PW, GK, GN, WP ≥ 0*

Figure 1.5: Mathematical Formulation of the Constraints.

II - Linear Programming Setup in the Excel Workbook.

The Objective Parameters using the **Decision Variables** in the problem have been set up by computing the formula of *Selling Price - Cost Price*:

	PW	GK	GN	WP
Objective Parameters:	$ 164.99	$ 349.99	$ 285.99	$ 139.99

Figure 1.6: Objective Parameters using the Decision Variables defined in the Excel Workbook.

The Constraints in the problem have been set up:

Constraints:					Constraint LHS	Inequality	Constraint RHS
Budget Available-	$ 335.00	$ 380.00	$ 415.00	$ 130.00	0.00	≤	175000
Space Available-	25	40	25	1.25	0	≤	12000
Promotion 1-	-3	-3	1	1	0	≤	0
Promotion 2-	0	0	-1	2	0	≤	0

Figure 1.7: Constraints setup in the Solver function in the Excel Workbook.

III - Optimal Solution of the Maximization of Profit to the Company.

Using the Solver function, we have computed the Optimal Solution for the problem of Maximization of Profits to the company in this warehouse.

	Z = 164.99 PW + 349.99 GK + 285.99 GN + 139.99 WP
Objective Function:	**$1,38,975.89**

Figure 1.8: Objective Function with the Maximized Profit Value.

- The Maximized value of Profits (Monthly Optimal Profit Value) which the company can earn from the given information is **$138,975.89**

The Optimal Solution of this maximization problem is:

	PW	GK	GN	WP
	0.00	134.81	257.86	128.93
Objective Parameters:	$ 164.99	$ 349.99	$ 285.99	$ 139.99

Figure 1.9: Optimal Solution to this Maximization problem.

- The Optimal units of Pressure Washers for the warehouse is **0**.
- The Optimal units of Go Karts for the warehouse is around **134**.
- The Optimal units of Generators for the warehouse is around **257**.
- The Optimal units of Water Pumps for the warehouse is around **128**.
- This problem has been treated as Linear Programming problem and not as Integer Programming problem. We will treat the decimals by cumulating them in their respective categories in the following months.

The computed Constraint LHS and RHS with the unused resources with the help of Solver function:

Constraint LHS	Inequality	Constraint RHS	Unused Resources
175000.00	≤	175000	0.00
12000	≤	12000	0
-17.63540291	≤	0	17.63540291
0	≤	0	0

Figure 1.10: Optimal Solution to this Maximization problem.

- The whole budget of **$175,000** has been used.
- The whole space available of **12,000 sq. ft.** in the warehouse has been used.
- The combination of Pressure Washers & Go Karts take up 25% of the warehouse with around **17 unused** resources.

IV - Sensitivity Report for a Decision Variable with 0 as Optimal Value.

Using the Sensitivity Report provided by the Solver function of Excel, we can figure out the following points.

Microsoft Excel 16.57 Sensitivity Report
Worksheet: [ALY6050_Module5Project_GaurH.xlsx]Analysis 1 (MAIN)
Report Created: 13/02/22 10:18:19 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
E4	PW - Pressure Washer	0	-108.0642768	164.99	108.0642768	1E+30
F4	GK - Go Karts	134.8084544	0	349.99	205.6939024	68.16854167
G4	GN - Generators	257.8599736	0	285.99	86.10763158	131.7726563
H4	WP - Water Pumps	128.9299868	0	139.99	172.2152632	84.30090669

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
I10	Budget Available- Constraint LHS	175000	0.557031044	175000	1219.178082	61000
I11	Space Available- Constraint LHS	12000	3.457955086	12000	6421.052632	83.02238806
I12	Promotion 1- Constraint LHS	-17.63540291	0	0	1E+30	17.63540291
I13	Promotion 2- Constraint LHS	0	31.62676024	0	1032.804233	57.35767991

Figure 1.11: Sensitivity Report of the Analysis.

- The Final Value of Pressure Washer (PW) is 0 which means that no units of this product are stored in the warehouse .
- The **Reduced Cost of -$108.07** signifies that the Profit on this product is not sufficient for the company to endure its storage.
- To make the product be stored in the warehouse , we need to make changes to either the Cost Price or Selling Price. But, since the Cost Price cannot be changed in our case, the only option we're left with is **increase the Selling Price with the Allowable Increase.**
- The New Selling Price of a unit of Pressure Washer should be

  New Selling Price of Pressure Washer = $499.99 + 108.07

- This is the smallest (lowest) selling price which the company should go with for this product.
- The new Mathematical Formula for the Objective Function now becomes :

Mathematical Formulation:

Maximize ***Z = ((499.99 + 108.07) - 335) PW + (729.99 - 380) GK + (700.99 - 415) GN + (269.99 WP - 650/5 WP)***

Equivalent to ***Z = 273.06 PW + 349.99 GK + 285.99 GN + 139.99 WP***

Figure 1.12: New Mathematical Formula with Updated Selling Price of Pressure Washer.

Using the Solver function, we have computed the **New Optimal Solution** for the problem of Maximization of Profits to the company in this warehouse.

	Z = 273.06 PW + 349.99 GK + 285.99 GN + 139.99 WP
Objective Function:	**$1,38,978.03**

Figure 1.13: New Objective Function with the New Maximized Profit Value.

- The Maximized value of Profits (Monthly Optimal Profit Value) which the company can earn from the given information is **$138,978.03**

The New Optimal Solution of this maximization problem is:

	PW	GK	GN	WP
	373.47	0.00	103.93	51.97
Objective Parameters:	$ 273.06	$ 349.99	$ 285.99	$ 139.99

Figure 1.14: New Optimal Solution to this Maximization problem.

- The Optimal units of Pressure Washers for the warehouse is around **373**.
- The Optimal units of Go Karts for the warehouse is around **0**.
- The Optimal units of Generators for the warehouse is around **103**.
- The Optimal units of Water Pumps for the warehouse is around **51**.

The newly computed Constraint LHS and RHS with the unused resources with the help of Solver function:

Constraint LHS	Inequality	Constraint RHS	Unused Resources
175000.00	≤	175000	0.00
12000	≤	12000	0
-964.5013724	≤	0	964.5013724
0	≤	0	0

Figure 1.15: New Optimal Solution to this Maximization problem.

- The whole budget of **$175,000** has been used again in this case.
- The whole space available of **12,000 sq. ft.** in the warehouse has been used again in this case as well.
- The combination of Pressure Washers & Go Karts take up 25% of the warehouse with around **964 unused resources**.

V - Is Addition to the Budget of $175,000 Possible?

Using the Sensitivity Report provided by the Solver function of Excel, we can figure out that the ***addition of any amount to the Budget is possible*** in this problem.

Microsoft Excel 16.57 Sensitivity Report
Worksheet: [ALY6050_Module5Project_GaurH.xlsx]Analysis 1 (MAIN)
Report Created: 13/02/22 10:18:19 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
E4	PW - Pressure Washer	0	-108.0642768	164.99	108.0642768	1E+30
F4	GK - Go Karts	134.8084544	0	349.99	205.6939024	68.16854167
G4	GN - Generators	257.8599736	0	285.99	86.10763158	131.7726563
H4	WP - Water Pumps	128.9299868	0	139.99	172.2152632	84.30090669

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
I10	Budget Available- Constraint LHS	175000	0.557031044	175000	1219.178082	61000
I11	Space Available- Constraint LHS	12000	3.457955086	12000	6421.052632	83.02238806
I12	Promotion 1- Constraint LHS	-17.63540291	0	0	1E+30	17.63540291
I13	Promotion 2- Constraint LHS	0	31.62676024	0	1032.804233	57.35767991

Figure 1.16: Sensitivity Report of the Analysis marked with Budget Available constraint.

- The **Allowable Increase is $1,219.18** which means that we can make an addition to the budget available.

- The **Shadow Price is 0.56** which means that for an additional dollar added, the profit would go up by around **$0.56**

- The original Maximized Profit is **$138,975.89**
- The new Maximized Profit with the new budget available would become **$139,655.01**

Constraints:					Constraint LHS	Inequality	Constraint RHS	Unused Resources
Budget Available-	$ 335.00	$ 380.00	$ 415.00	$ 130.00	176219.18	≤	176219.18	0.00
Space Available-	25	40	25	1.25	12000	≤	12000	0
Promotion 1-	-3	-3	1	1	-2.84217E-14	≤	0	2.84217E-14
Promotion 2-	0	0	-1	2	0	≤	0	0

Figure 1.17: Budget Constraint Increase.

	Z = 273.06 PW + 349.99 GK + 285.99 GN + 139.99 WP
Objective Function:	**$1,39,655.01**

Figure 1.18: New Objective Function with the New Maximized Profit Value with Budget Constraint Increase.

VI - Should the company rent a Larger or Smaller Warehouse?

Using the Sensitivity Report provided by the Solver function of Excel, we can figure out that the ***company should rent a Larger Warehouse*** to increase their maximized profit because of ***the Allowable Increase in the constraint of 'Space Availability'***.

Microsoft Excel 16.57 Sensitivity Report
Worksheet: [ALY6050_Module5Project_GaurH.xlsx]Analysis 1 (MAIN)
Report Created: 13/02/22 10:18:19 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
E4	PW - Pressure Washer	0	-108.0642768	164.99	108.0642768	1E+30
F4	GK - Go Karts	134.8084544	0	349.99	205.6939024	68.16854167
G4	GN - Generators	257.8599736	0	285.99	86.10763158	131.7726563
H4	WP - Water Pumps	128.9299868	0	139.99	172.2152632	84.30090669

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
I10	Budget Available- Constraint LHS	175000	0.557031044	175000	1219.178082	61000
I11	Space Available- Constraint LHS	12000	3.457955086	12000	6421.052632	83.02238806
I12	Promotion 1- Constraint LHS	-17.63540291	0	0	1E+30	17.63540291
I13	Promotion 2- Constraint LHS	0	31.62676024	0	1032.804233	57.35767991

Figure 1.19: Sensitivity Report of the Analysis marked with Budget Available constraint.

- The **Allowable Increase is 6421.06 square feet** (~ 6422 sq. ft.) which means that we can make an addition to the space available in the warehouse if the company can rent a larger warehouse.

- The **Shadow Price is 3.46** which means that for an additional square feet of space added, the profit would go up by around **$3.46**

- The original Maximized Profit is **$138,975.89**
- The new Maximized Profit with the new space available (larger warehouse) would become **$161,179.61**

  $161,179.61 - $138,975.89 = $22,203.72
  $3.46 * (~ 6421.06) = $22,206.68

- In this case, the ideal size of a Larger Warehouse should be around ***18,421 square feet.***

	Z = 164.99 PW + 349.99 GK + 285.99 GN + 139.99 WP
Objective Function:	**$1,61,179.61**

Figure 1.20: New Objective Function with the New Maximized Profit Value with Space Constraint Increase.

CONCLUSION

We have used the Linear Programming (LP) model to compute/analyse the maximization of the Profits our company can earn in the given problem.

- With the usage of Linear Algebra in Linear Programming, we have computed the Optimal Values of Decision Variables used in this problem and maximized the Total Profits the company can earn using these decision variables along with the constraints they have.

- The Maximized value of Profits (Monthly Optimal Profit Value) which the company can earn from the given information is **$138,975.89**

- The Optimal units of Pressure Washers for the warehouse is **0**.
- The Optimal units of Go Karts for the warehouse is around **134**.
- The Optimal units of Generators for the warehouse is around **257**.
- The Optimal units of Water Pumps for the warehouse is around **128**.

- Using the Sensitivity Report, we found that the Selling Price is not up to the level using which the company can store them in the warehouse and earn profit from it.
- The **Reduced Cost of -$108.07** signifies that the Profit on this product is not sufficient for the company to endure its storage. ***The Selling Price of Pressure Washer should be increased by the Allowable Increase value.***

  New Selling Price of Pressure Washer = $499.99 + 108.07 = $608.48

- This will provide a new Maximized value of Profits (Monthly Optimal Profit Value) which the company can earn from the given information as **$138,978.03**
- The Optimal units of Pressure Washers, Go Karts, Generators, and Water Pumps for the warehouse will become around **373**, **0, 103**, **51** respectively.

- The company can increase their budget. The **Allowable Increase is $1,219.18** and the **Shadow Price is 0.56** which means that for an additional dollar added, the profit would go up by around **$0.56**
- **The maximized profit would be increased by $628.23.**
- In this case, the ideal budget should be around ***$176,219.18***

- The company can also rent a larger warehouse. The **Allowable Increase is 6421.06 square feet** (~ 6422 sq. ft.) and the **Shadow Price is 3.46**which means that for an additional square feet of space added, the profit would go up by around **$3.46**
- **The maximized profit would be increased by $22,203.72.**
- In this case, the ideal size of a Larger Warehouse should be around ***18,421 square feet.***

BIBLIOGRAPHY

1. Microsoft. (2021). *Excel functions (alphabetical)*. https://support.microsoft.com/en-us/office/excel-functions-alphabetical-b3944572-255d-4efb-bb96-c6d90033e188

2. ALY 6050 - Prof Richard He - *Lesson 5-1 — Linear Programming* (2022, February), https://northeastern.instructure.com/courses/97784/pages/lesson-5-1-linear-programming?module_item_id=6650802

3. ALY 6050 - Prof Richard He - *Lesson 5-2 — In-depth Linear Programming Models* (2022, February), https://northeastern.instructure.com/courses/97784/pages/lesson-5-2-in-depth-linear-programming-models?module_item_id=6650804

4. *Sensitivity Analysis Option Not Showing*. (2021). Sensitivity Analysis. https://www.excelforum.com/excel-programming-vba-macros/1052460-sensitivity-analysis-option-not-showing.html