# FLUID SLOSHERS

Benjamin Musci, Harshith Gowrachari, Peter Kalmar, Yi Ting Char

Young ERCOFTAC Montestigliano Spring School

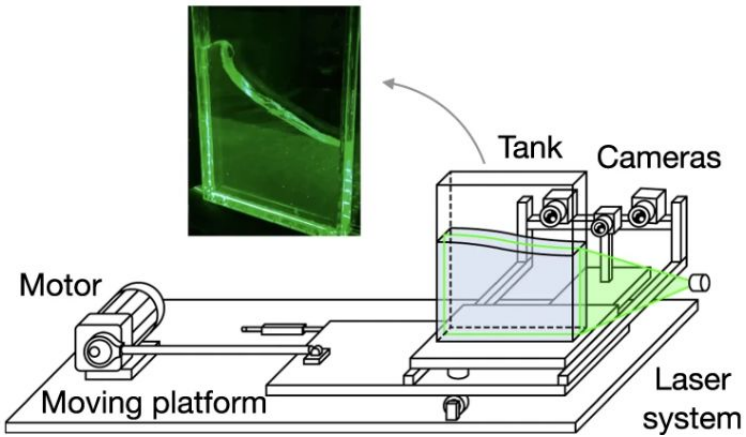
**Data-Driven Model Reduction for Dynamical Systems**

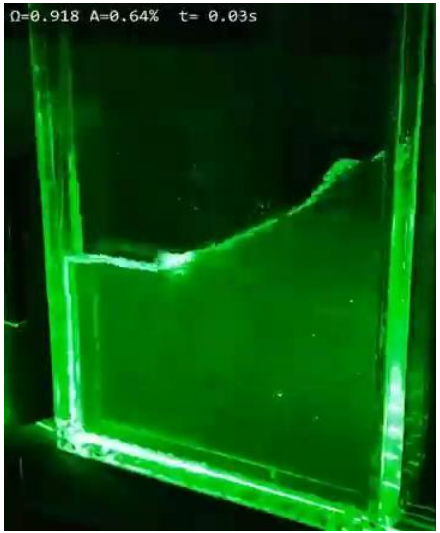
Prof.George Haller, ETH Zurich, Switzerland

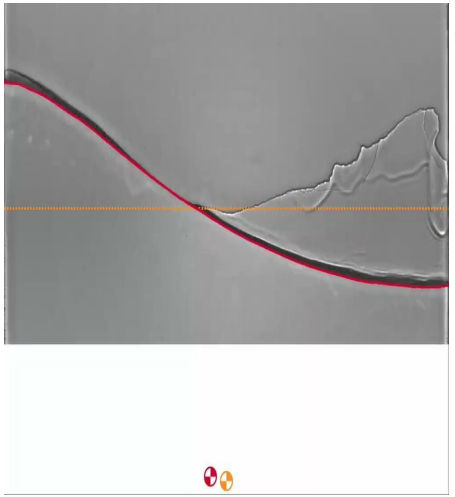
13th - 19th April 2025

Montestigliano, Italy

# Recap: Fluid Sloshing



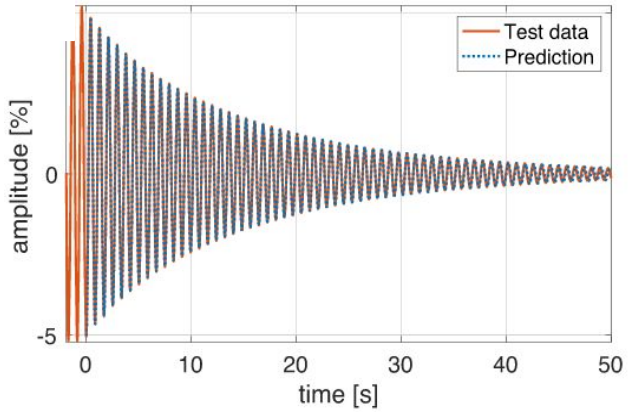
**Observable:** Horizontal position of fluid center of mass.

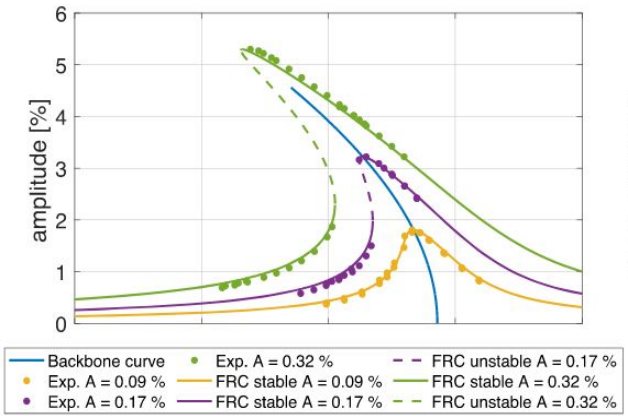
Cenedese, Axås, Bäuerlein, Avila & H., **Data-Driven Modeling and Prediction of Non-Linearizable Dynamics via Spectral Submanifolds,** *Nature Comm.* [2022]

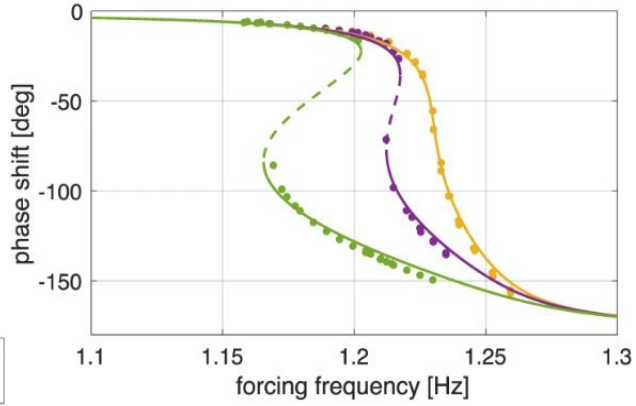
# Recap: Fluid Sloshing

**SSM with cubic extended normal form:** $\dot{\rho} = -0.063179\rho - 0.041214\rho^3, \quad \dot{\theta} = 7.8144 - 1.5506\rho^2.$

**NMTE = 1.88%**



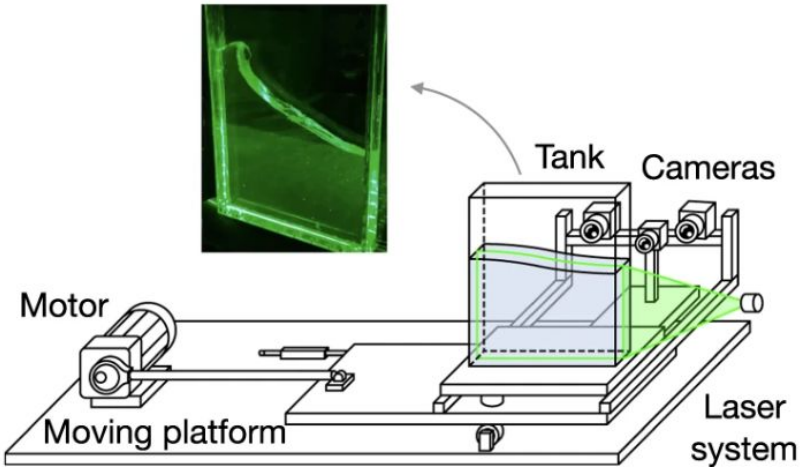



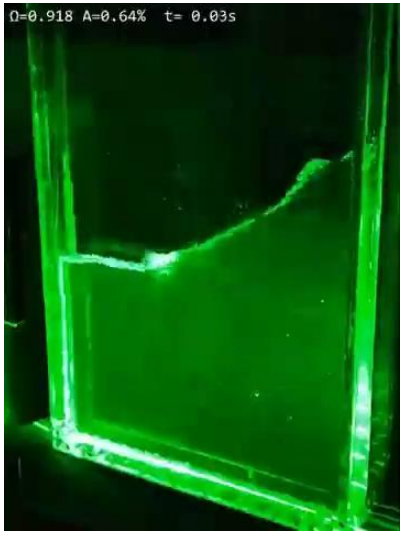



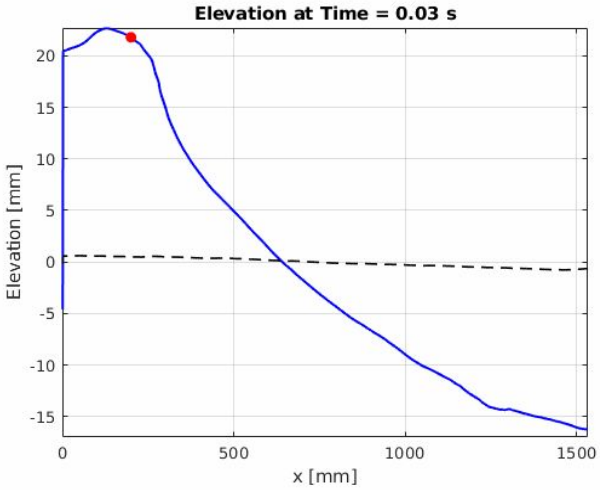

**Observable:** Horizontal position of fluid center of mass.

Cenedese, Axås, Bäuerlein, Avila & H., **Data-Driven Modeling and Prediction of Non-Linearizable Dynamics via Spectral Submanifolds,** *Nature Comm.* [2022]

# Our observables

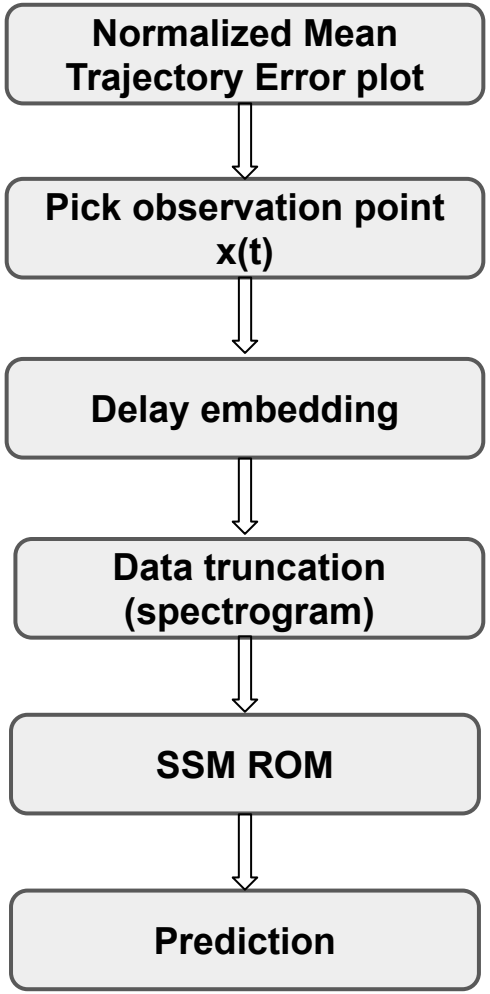


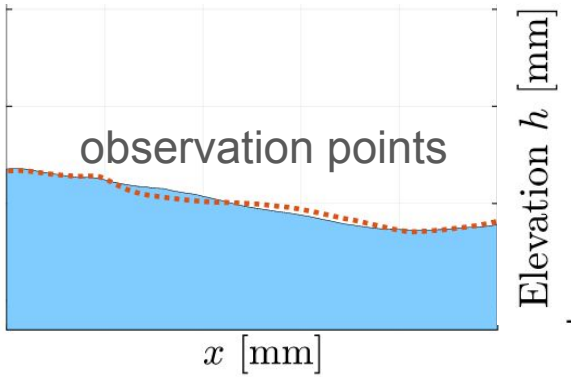




**Observable:** Elevation of the interface at **x** observation point.
(Two experiments: 1 for training and 1 for testing.)

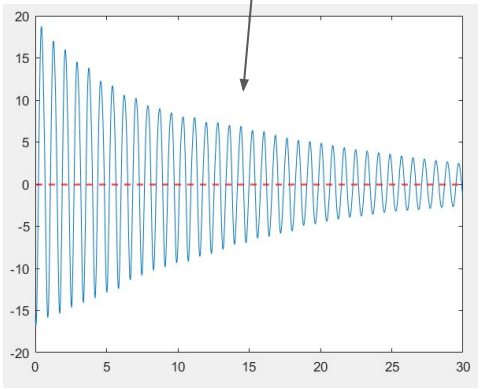
# Workflow:

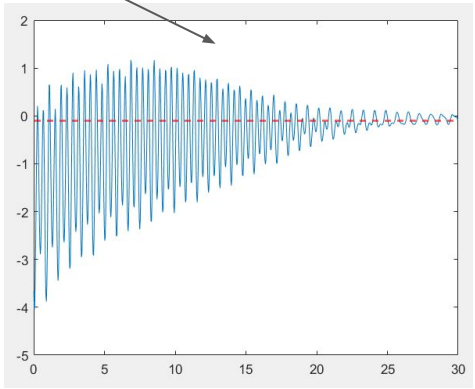

Normalized Mean Trajectory Error plot
Pick observation point x(t)
Delay embedding
Data truncation (spectrogram)
SSM ROM
Prediction

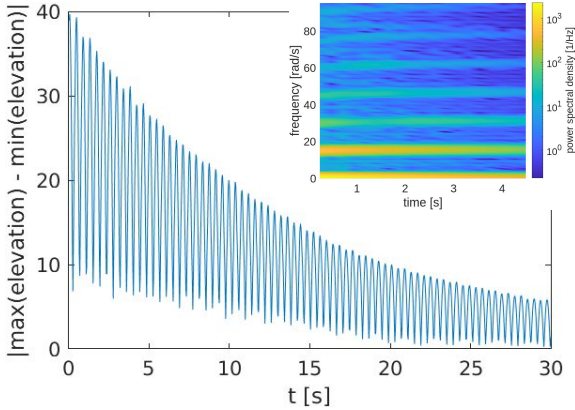

observation points
Elevation $h$ [mm]
$x$ [mm]

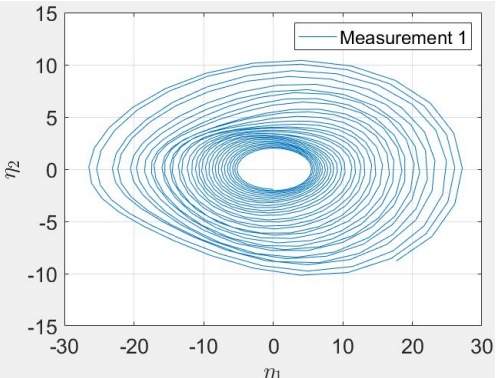
# Observable selection

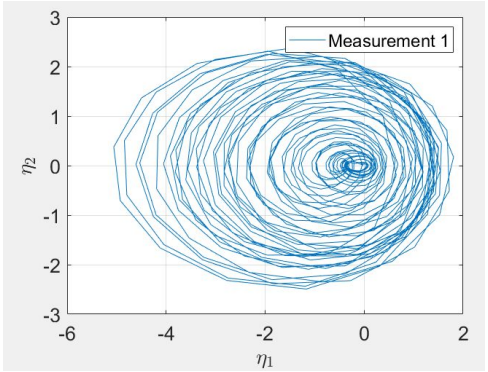
Interface end points, mid point, max amplitude, or derivatives of these

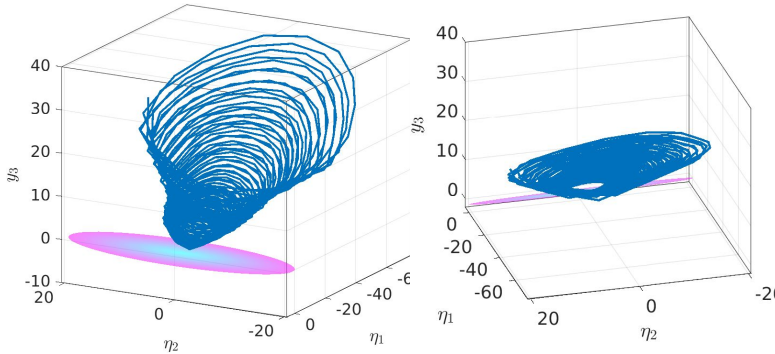




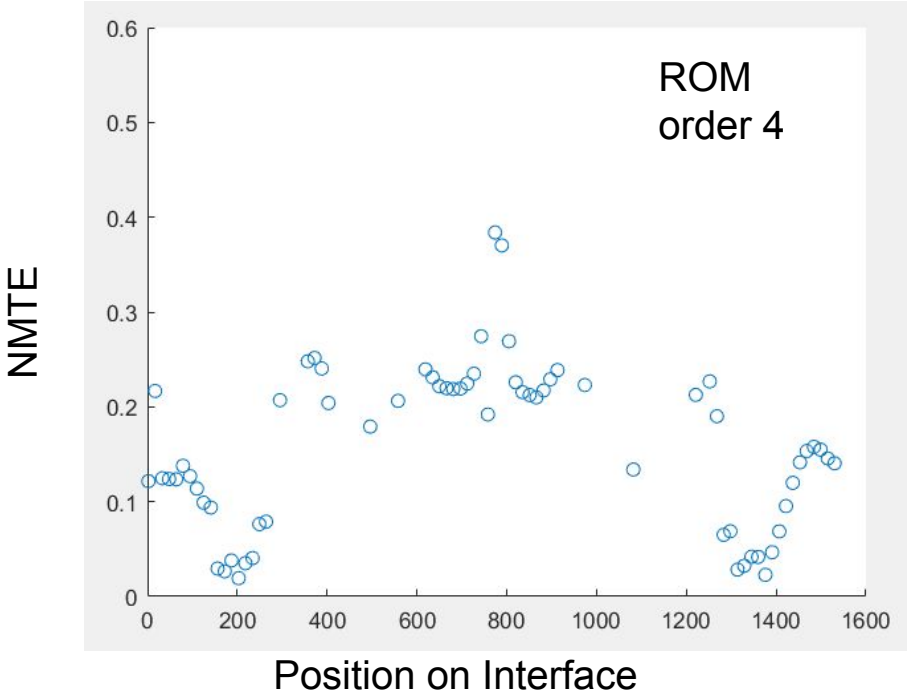


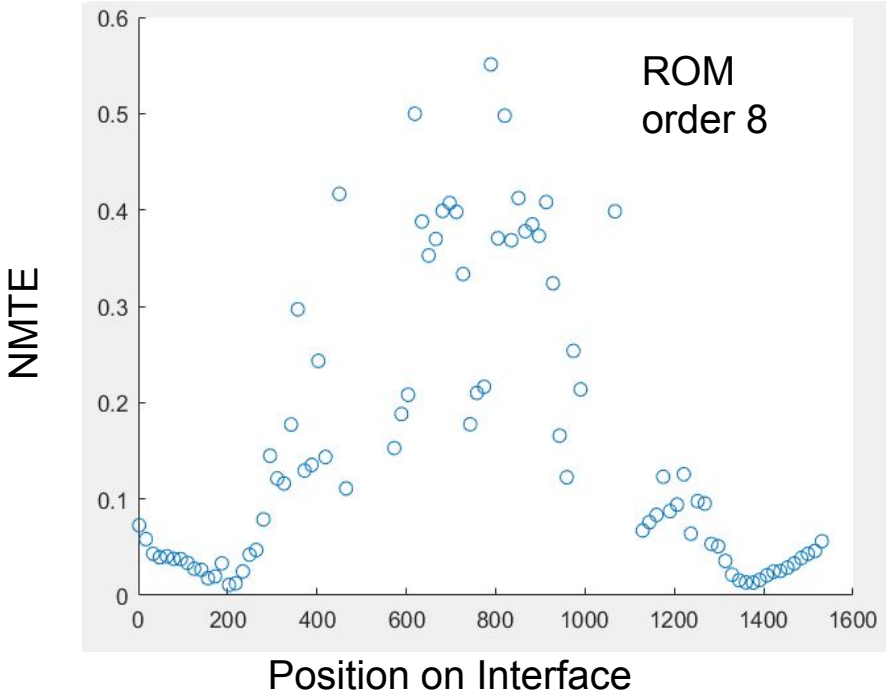



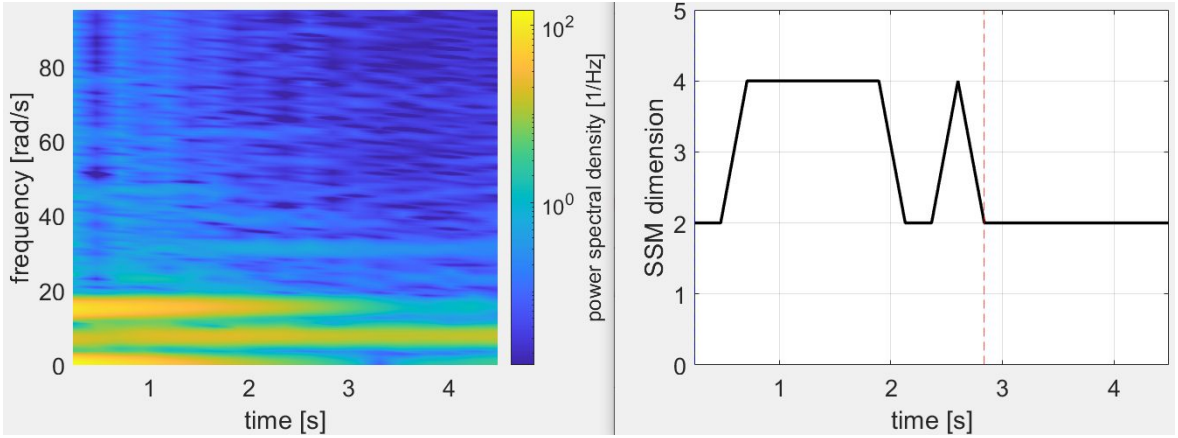



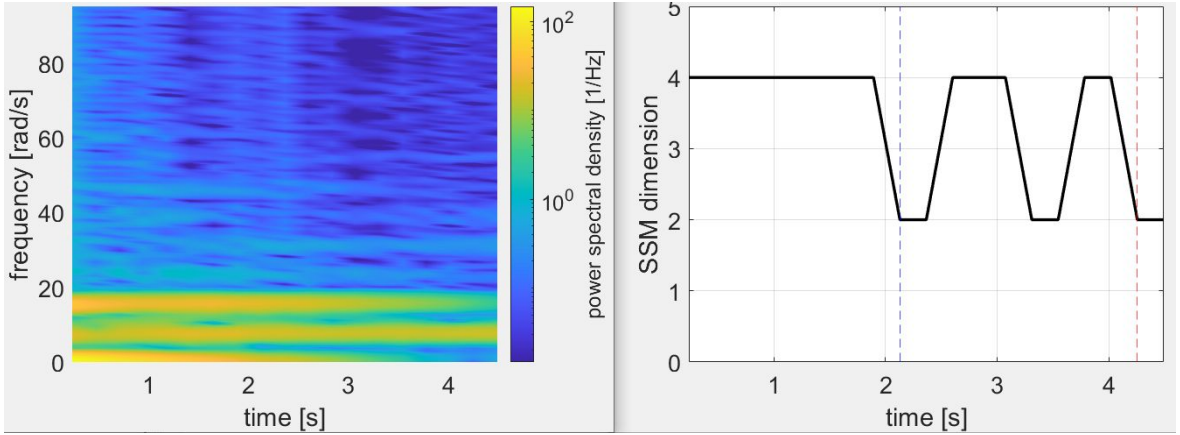
# Ability to find appropriate SSM depends on interface position used as observable



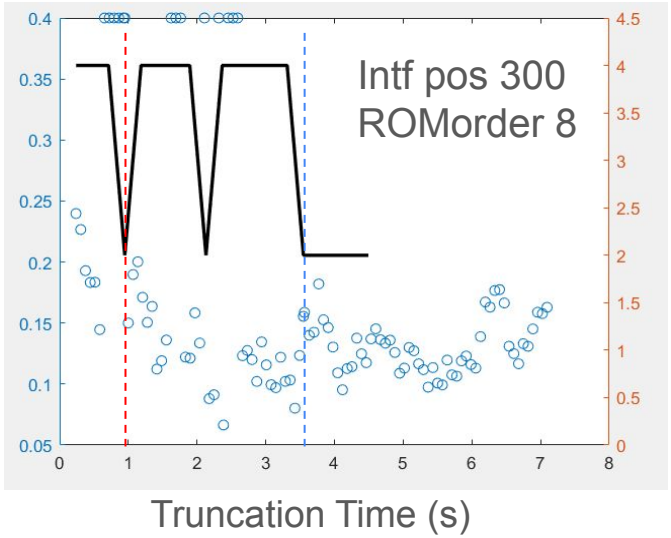
# Variation in data truncation time

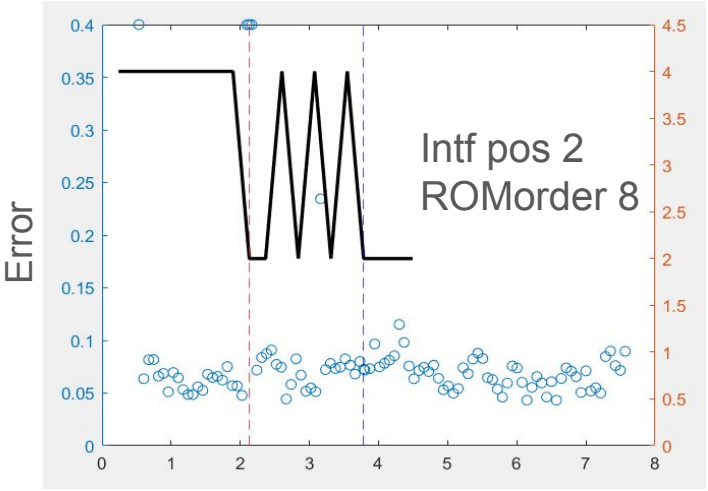


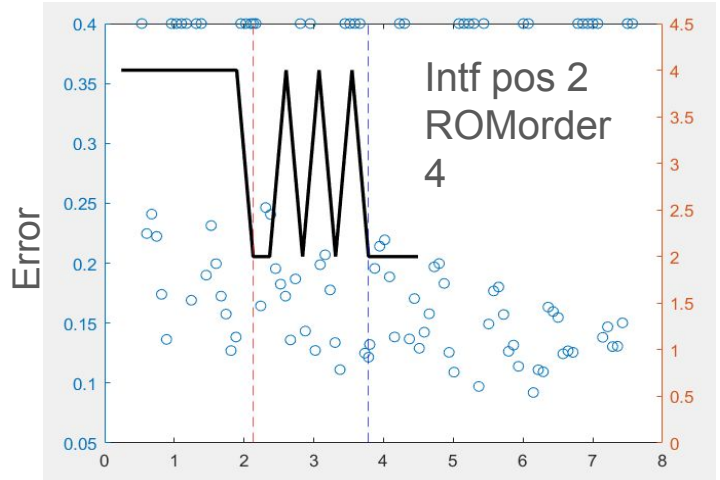

Modified SSM process to delay truncation time in cases of persistent higher modes

# SSM selection can demonstrate high sensitivity to truncation time

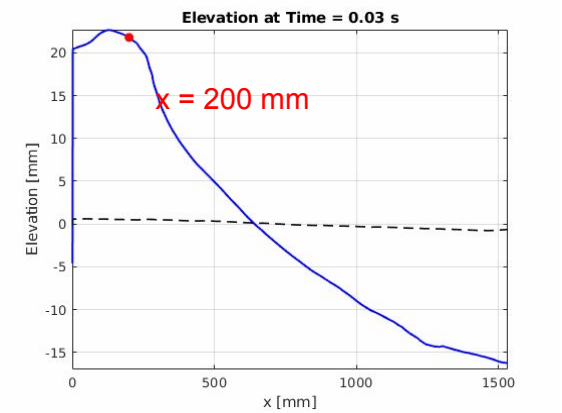


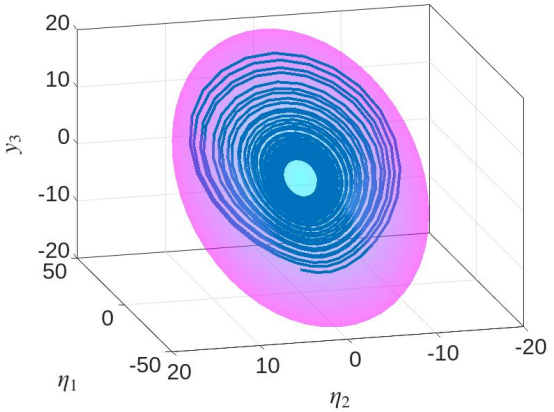

Sensitivity to truncation time depends to varying degree on interface position and ROMorder

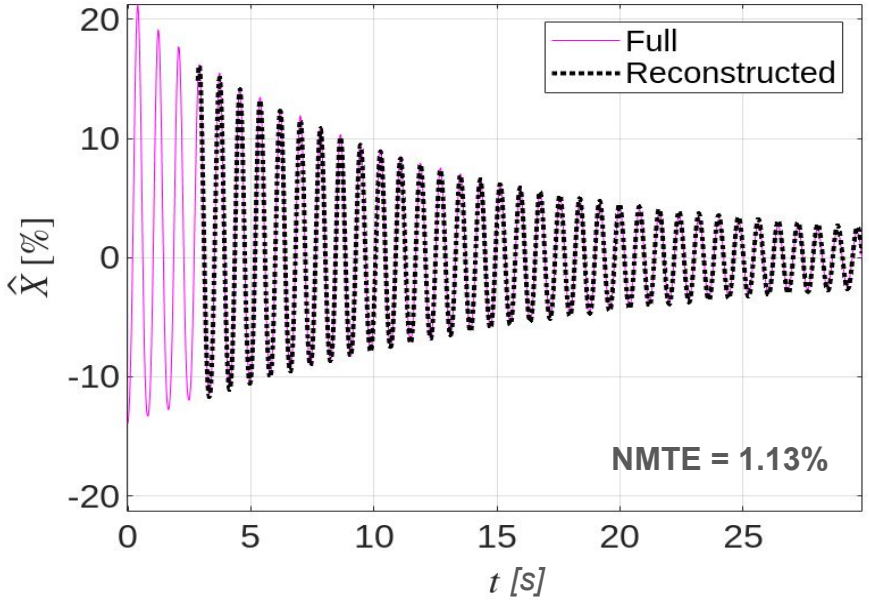


# Predictions: Unforced




**SSM**:
2-D
1st order



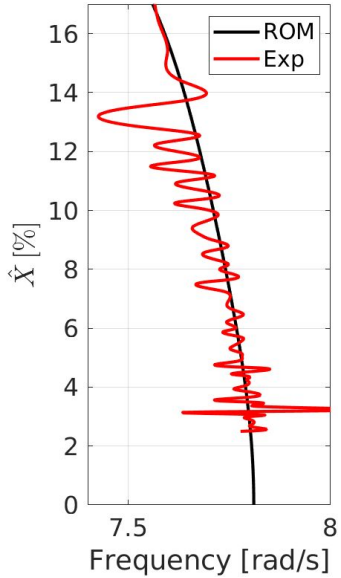



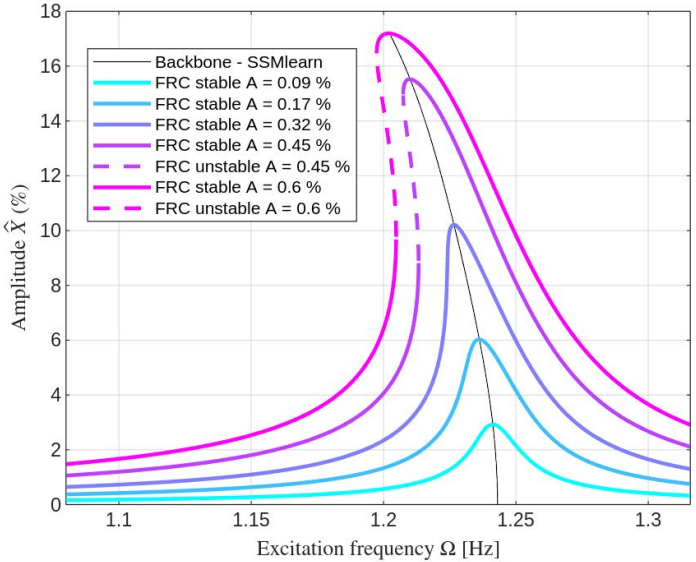

**ROM**: 7th order

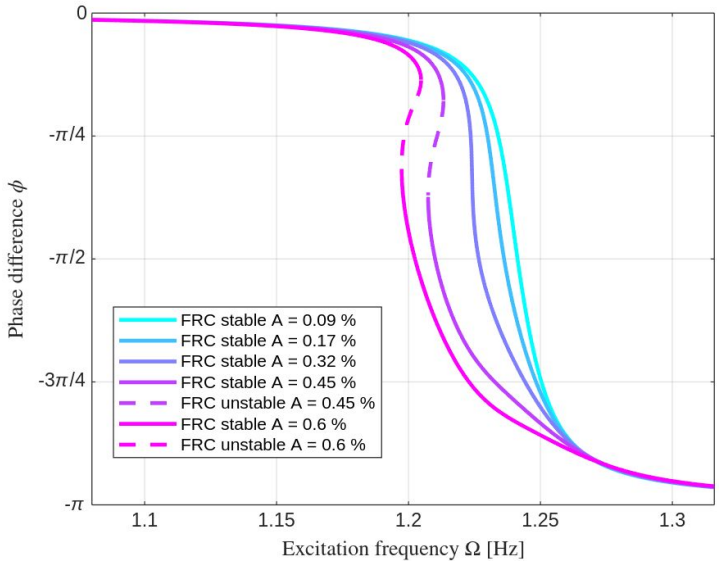
$$\dot{\rho} = -0.4862\rho - 0.5651 + 5.885\rho^5 - 21.19\rho^7$$

$$\dot{\theta} = 7.810 - 1.400\rho^2 + 1.884\rho^4 - 6.226\rho^6$$

# Predictions: Backbone curve & Forced response




Experiment backbone curve estimated using the PFF (Peak Finding and Fitting)* method

*M. Jin, W. Chen, M.R.W. Brake, H. Song, I**dentification of Instantaneous Frequency and Damping From Transient Decay Data**, *J. Vib. Acoust.* [2020]

# Conclusions

Axial locations +/- 200 mm from walls provide accurate predictions:

- Reproduced decay trajectory
- Validated backbone curve

Predictions are sensitive to data truncation (time)

# Future Work

Investigate more physically meaningful observables

Controlling damping with external forcing