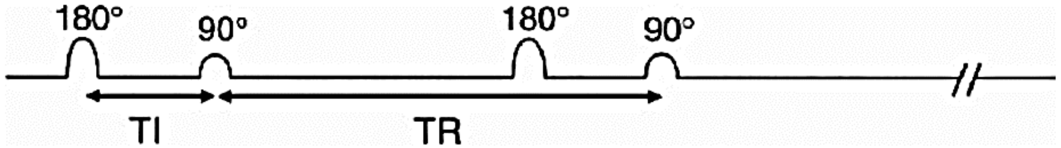
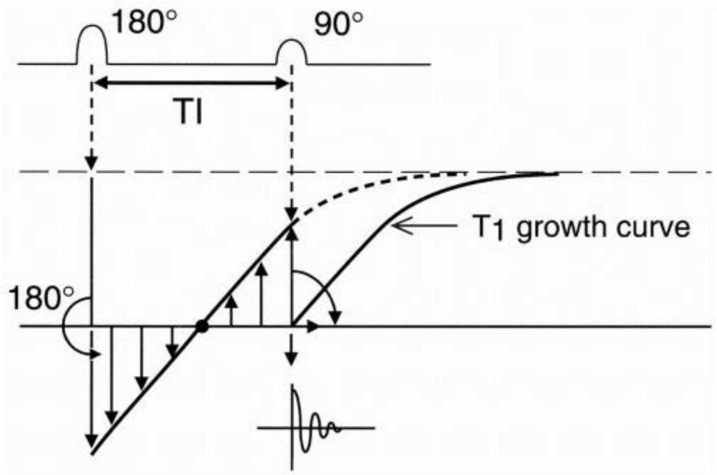
HW #2 (Matlab Programming)

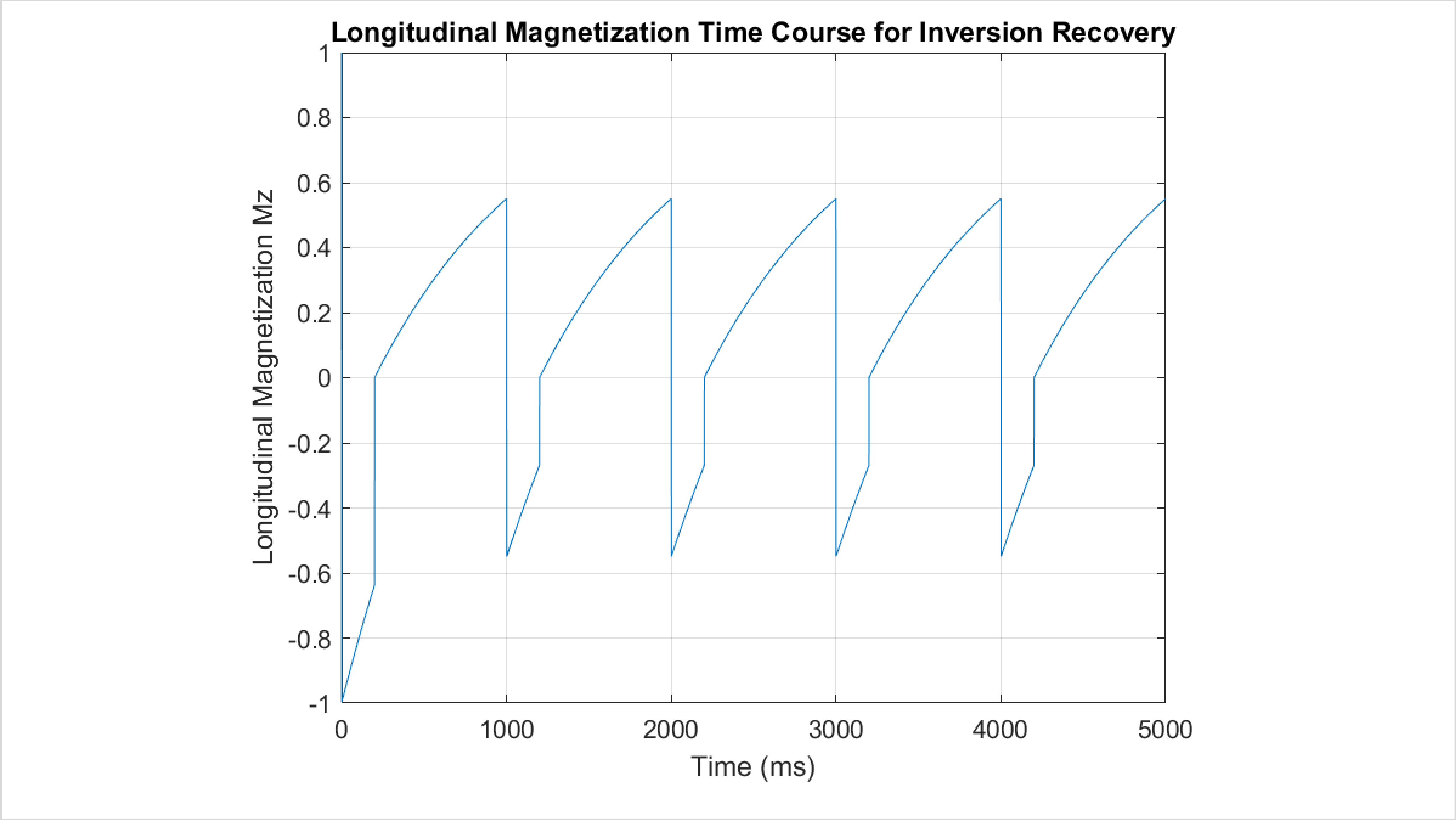
Part I: Inversion Recovery (TI = 200 ms and TR = 1000 ms)



1. For a tissue with T1 = 1000 ms and T2 = 100 ms, plot the time course of longitudinal magnetization up to five repetitions of the above inversion recovery pulse sequence.



After the 180° pulse, the longitudinal magnetization vector is flipped 180° and starts to recover from a value that is the negative of its initial maximal value.



Longitudinal Magnetization Time Course for Inversion Recovery
Longitudinal Magnetization Mz
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
1000
2000
3000
4000
5000
Time (ms)

Code:

```
% Parameters
T1 = 1000; % ms
T2 = 100; % ms
TI = 200; % ms
TR = 1000; % ms
num_reps = 5;
time_vector = 0:1:5000; % Complete time vector

% Initialize magnetization vector
M1z(1) = 1; % Longitudinal magnetization
```

```
%%
k = 0;
while k <= 4
for n = 2:1:201
Mz = -M1z(1000 * k + 1);
M1 = T1relaxation(0, 0, Mz, 1, n - 1, T1);
M1z(n + 1000 * k) = M1(3, 1);
end
for n = 2:1:801
M1 = T1relaxation(0, 0, 0, 1, n - 1, T1);
M1z(n + 200 + 1000 * k) = M1(3, 1);
end
k = k + 1;
end
% Plotting the longitudinal magnetization
figure;
plot(time_vector, M1z); % Corrected the variable name here
xlabel('Time (ms)');
ylabel('Longitudinal Magnetization Mz');
title('Longitudinal Magnetization Time Course for Inversion Recovery');
grid;
```

Observation:

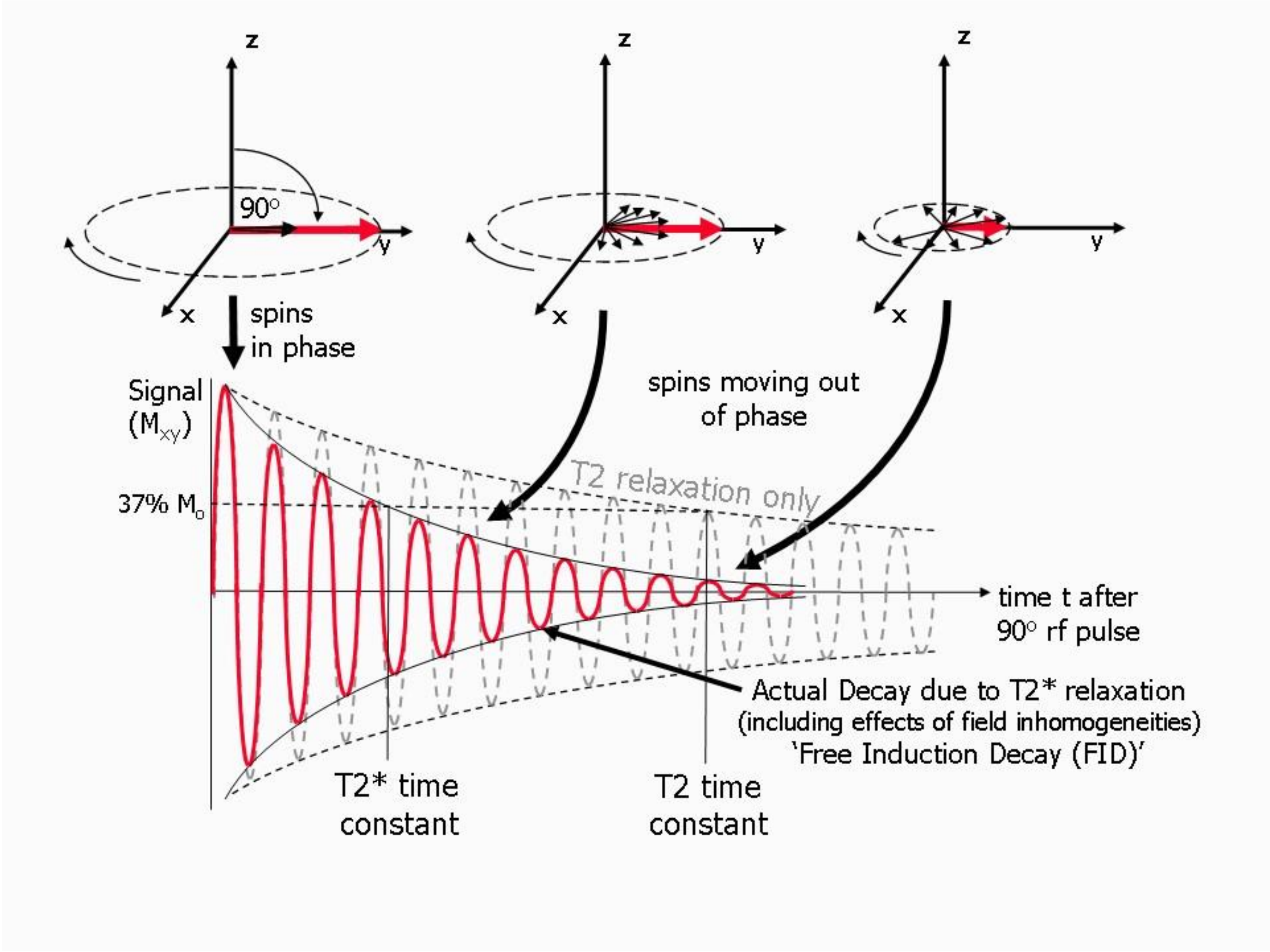
We then allow the magnetization vector to recover along a T1 growth curve. As it recovers, it gets smaller and smaller in the $-z$ direction until it goes to zero, and then starts growing in the +z direction, ultimately recovering to the original longitudinal magnetization.

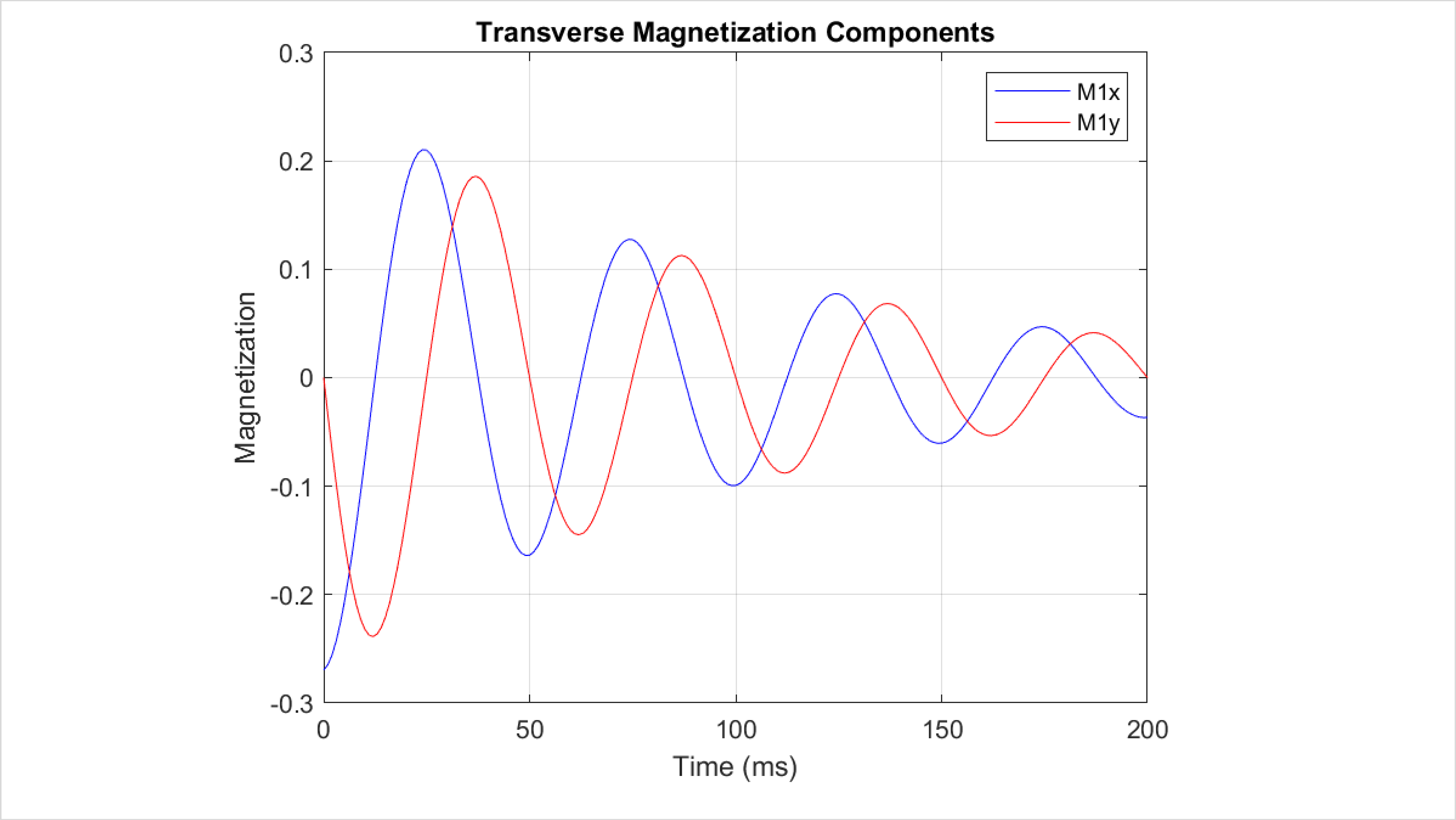
After a time TI, we apply a 90° pulse. This then flips the longitudinal magnetization into the x-y plane. The amount of magnetization flipped into the x-y plane will depend on the amount of longitudinal magnetization that has recovered during time TI after the original 180° RF pulse.
We measure this flipped magnetization.

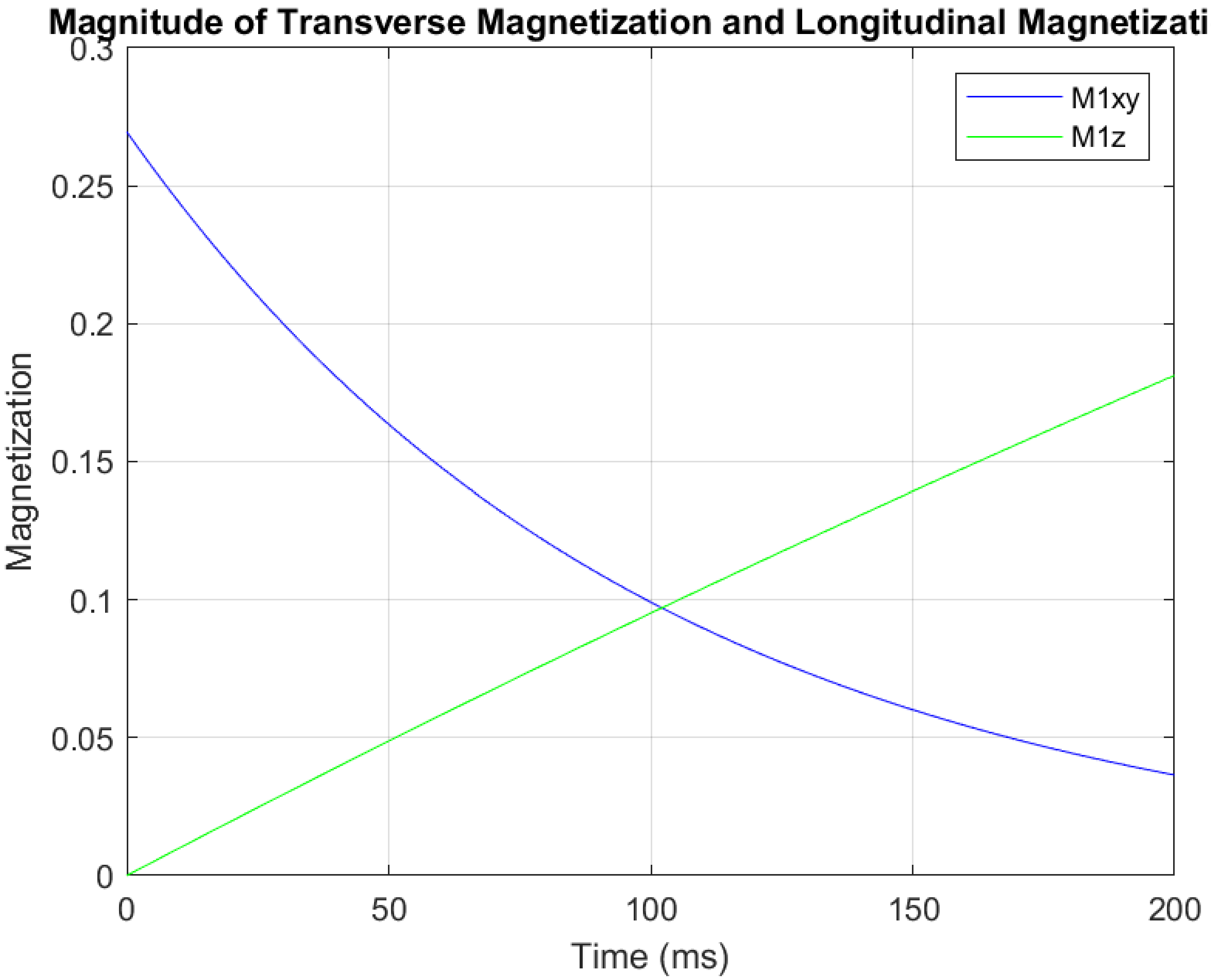
Therefore, at this point we get an FID proportional to the longitudinal magnetization flipped into the x-y plane. Also, at this point, we begin the regrowth of the longitudinal magnetization.

Recall that for a typical T1 recovery curve, the formula for the exponential growth of the curve is$1 - e^{-t/T1}$
However, when the magnetization starts to recover from $-M0$ instead of zero, the formula for recovery is $1 - 2e^{-t/T1}$

2. With the same parameters as given in (1), plot the time course of FID in the steady state for 200 ms. Assume a single spin with its resonance frequency of 20 Hz.




Transverse Magnetization Components
M1x
M1y
Magnetization
Time (ms)
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
50
100
150
200



Magnitude of Transverse Magnetization and Longitudinal Magnetization
M1xy
M1z
Magnetization
Time (ms)
0
0.05
0.1
0.15
0.2
0.25
0.3
0
50
100
150
200

Code:

```
% Parameters
T1 = 1000; % ms
T2 = 100; % ms
f = 20; % Hz
M0 = 1; % Initial transverse magnetization


% Time vector for FID (0 to 200 ms)
Time = 0:1:200; % ms


% Functions (Assuming T1relaxation and M_time functions are defined)
Minitial = T1relaxation(0, 0, 0, 1, 800, T1); % Relaxation over 800 ms
Minvert = -Minitial(3, 1); % Invert the longitudinal magnetization
M = T1relaxation(0, 0, Minvert, 1, 200, T1); % Relaxation over additional 200 ms
Mextract= M(3, 1); % Longitudinal component at 1200 ms


Ms = [Mextract; 0; 0]; % Magnetization vector just before the 90-degree pulse
```

```
% Calculate the magnetization evolution
M1 = M_time(Time, T1, T2, M0, Ms, f); % Assuming M_time is a function that computes magnetization over time
M1x = M1(1, :);
M1y = M1(2, :);
M1z = M1(3, :);
M1xy = sqrt(M1x.^2 + M1y.^2);
% Plotting the magnetization components
figure;
plot(Time, M1x); grid on; hold on;
plot(Time, M1y); grid on;
xlabel('Time (ms)');
ylabel('Magnetization');
legend('M1x', 'M1y');
title('Transverse Magnetization Components');
figure;
plot(Time, M1xy); grid on; hold on;
plot(Time, M1z); grid on;
xlabel('Time (ms)');
ylabel('Magnetization');
legend('M1xy', 'M1z');
title('Magnitude of Transverse Magnetization and Longitudinal Magnetization');
```

3. If we want to null the tissue in (1), what TI do we need? Derive first with your hand and prove your answer with Matlab simulation.

We want to find the TI at which Mz=0.

$$Mz = M0(1 - 2e^{-\frac{TI}{T1}})$$

$$0 = -1\ (1 - 2e^{-\frac{TI}{T1}}) \longrightarrow e^{-\frac{TI}{T1}} = 1/2$$

$$-\frac{TI}{T1} = \ln(1/2) \longrightarrow TI = T1\ln(2)$$

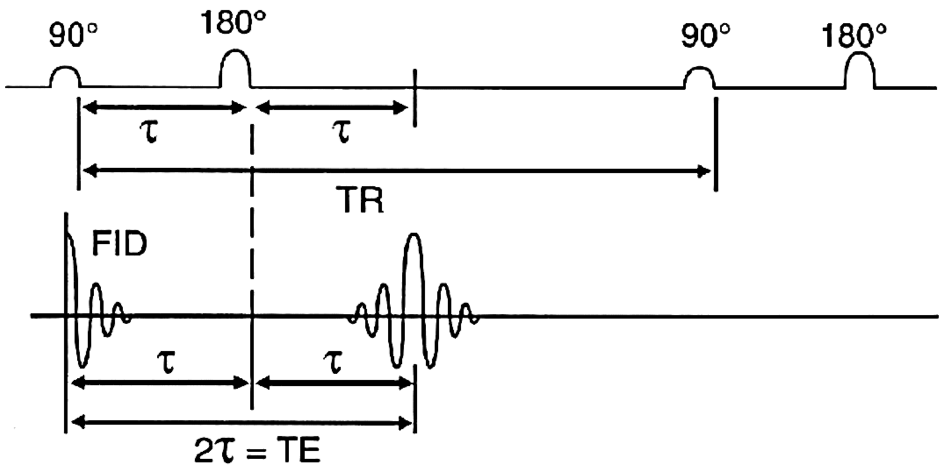
```
%%
T1=1000;
T1_NULL=log(2)*T1;
Mz=-1;
M1=T1relaxation(0,0,Mz,1,T1_NULL,T1);
M1z=M1(3,1)
```

```
%%
T1=1000;
T1_NULL=log(2)*T1;
Mz=-1;
M1=T1relaxation(0,0,Mz,1,T1_NULL,T1);
M1z=M1(3,1)
```

Command Window

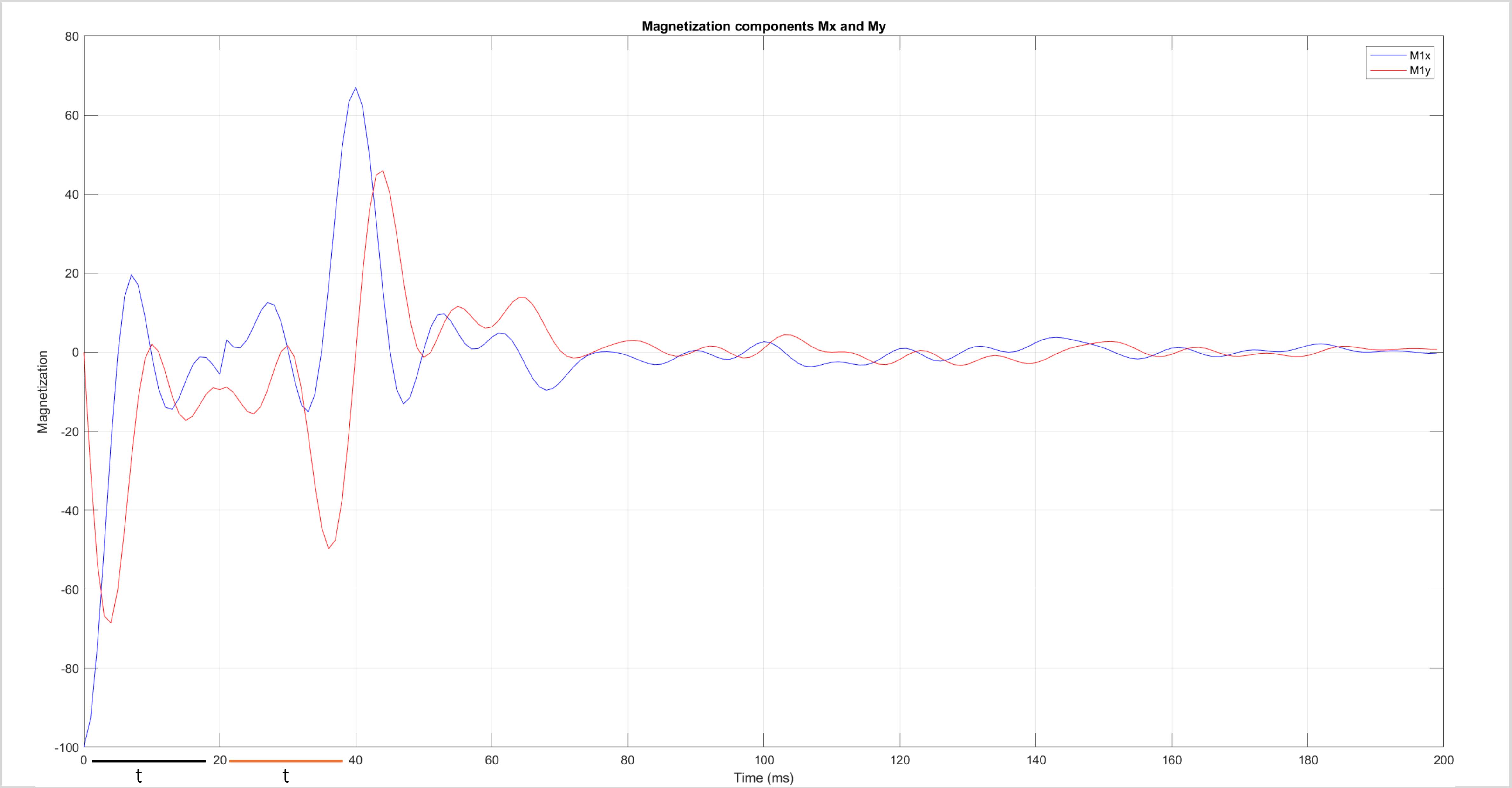
```
M1z =

     0

>>
```

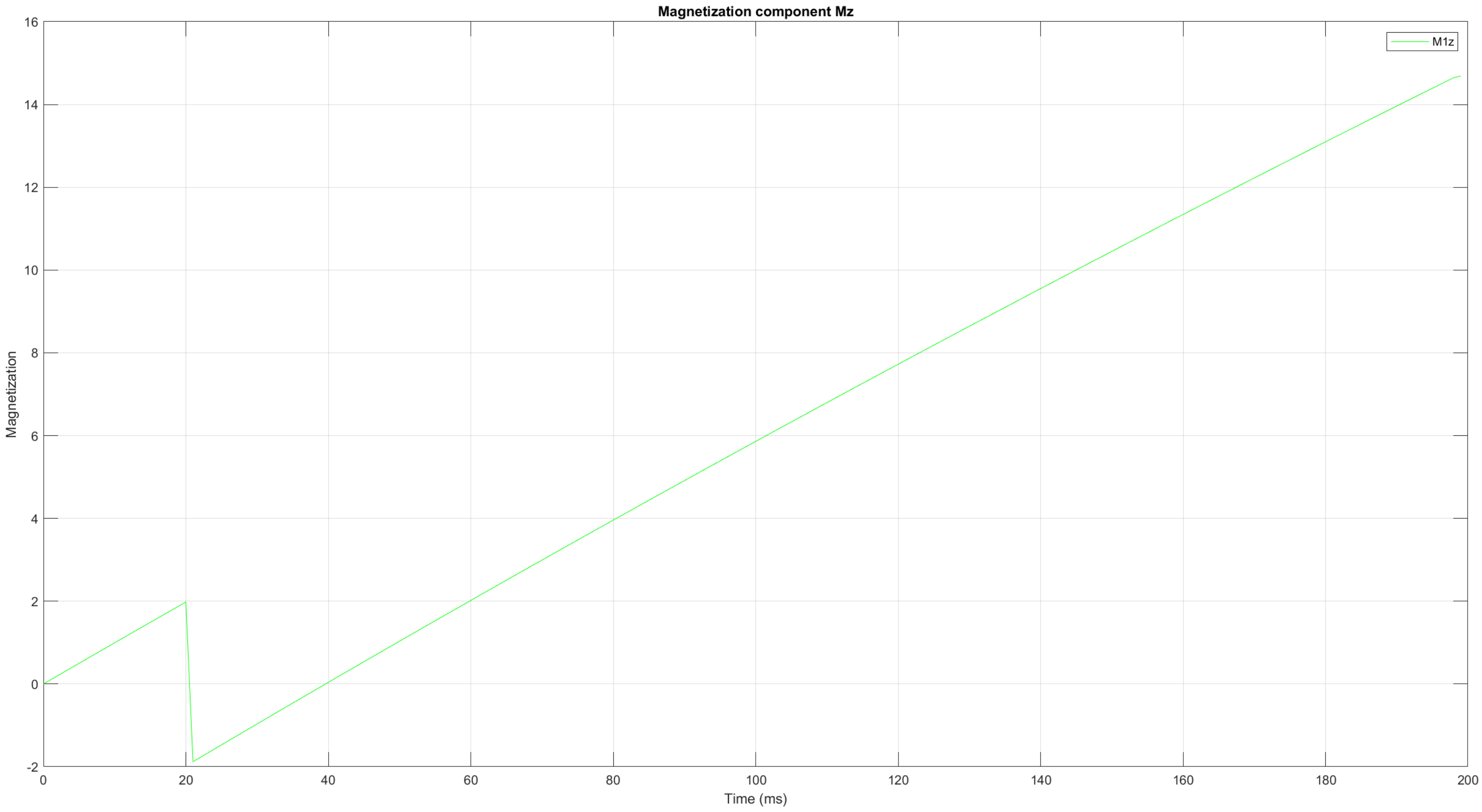
Part II: Spin Echo (TR = 200 ms and TE = 30 ms)



- Assume that there are 100 spins following a uniform frequency distribution between 0 and 100 Hz.

1. For the tissue in (1), plot the time courses of Mx, My, and Mz for a one cycle of TR.


Magnetization components Mx and My
M1x
M1y
80
60
40
20
0
-20
-40
-60
-80
-100
Magnetization
0
20
40
60
80
100
120
140
160
180
200
Time (ms)
t
t



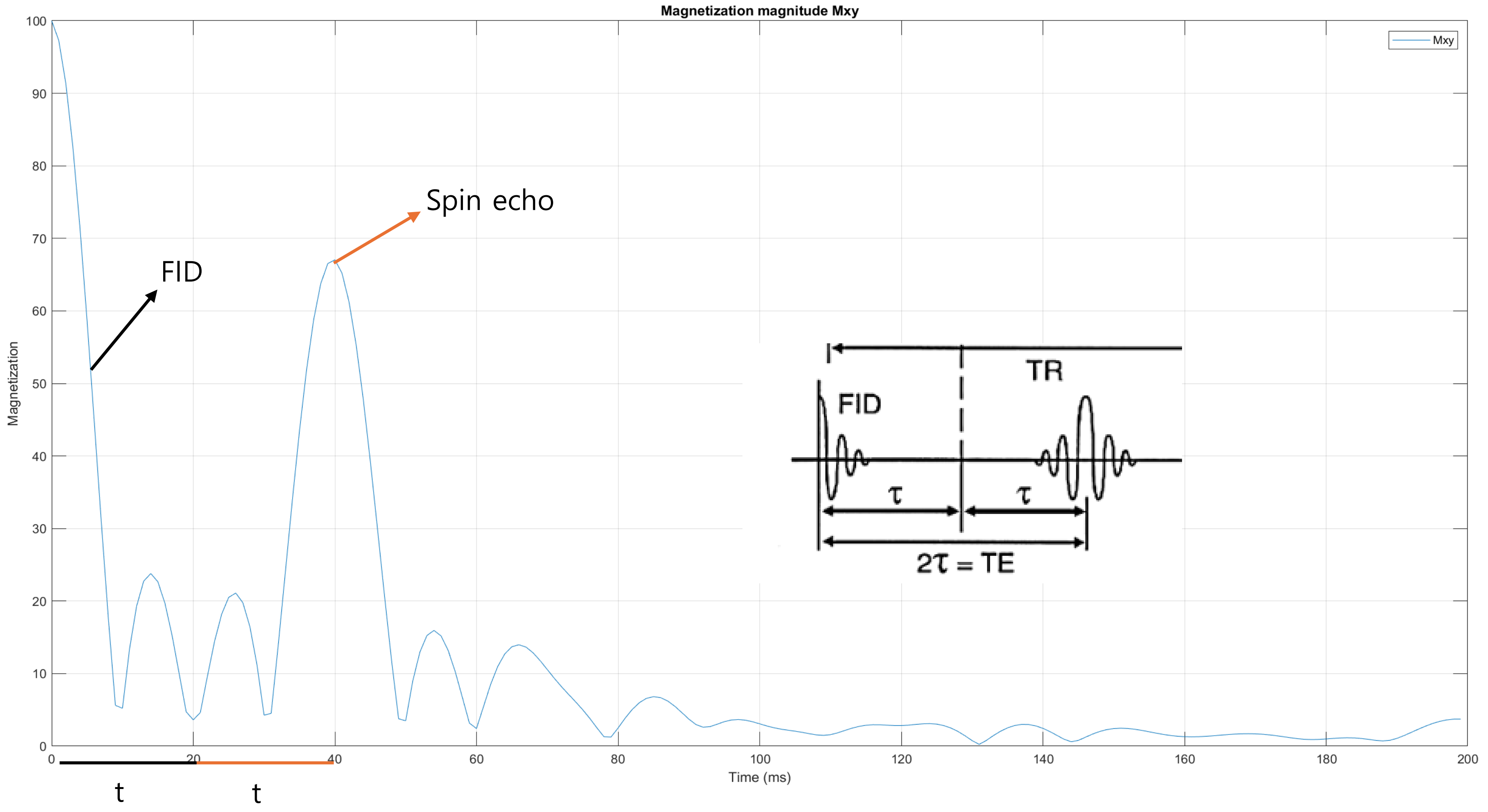
Magnetization component Mz
M1z
Magnetization
Time (ms)
16
14
12
10
8
6
4
2
0
-2
0
20
40
60
80
100
120
140
160
180
200

Code:

```
T1=1000;
T2=100;
M0=1;
M=[-1;0;0];
for i = 1:1:100
freq=randi([0,100],1,100);
end
t = 0:1:200;% TR
t1 = 0:1:15; % t
t2 = 0:1:185; % TR-t
%% During t
M1s = zeros(3, length(t1));
M_bp = []; % M before pulse
for i = 1:size(freq,2)
f = freq(i);
M1 = M_time(t1, T1, T2, M0, M, f);
M_bp = [M_bp; (M1(:,(length(t1))))'];
M1s = M1s + M1;
end
%%
M_ap = [ -M_bp(:,1) M_bp(:,2) -M_bp(:,3) ]; % M after pulse
%%
```

```
%% During TR-t
M2s = zeros(3, length(t2));
M_bp_2 = [];
% M before 2nd 90 degree pulse
numFreq = size(freq, 2);
M_bp_2 = [];
M2s = zeros(3, 186);
for i = 1:numFreq
f = freq(i);
M_ap_1 = M_ap(i, :)';
M2 = M_time(t2, T1, T2, M0, M_ap_1, f);
M_bp_2 = [M_bp_2; M2(:, end)']; % M before the 2nd 90 degree pulse
M2s = M2s + M2;
end
%%
MTxs = [M1s(1,:) M2s(1,2:length(M2s))]; % M2xs(185)*(exp(-1/100))];
MTys = [M1s(2,:) M2s(2,2:length(M2s))]; % M2ys(185)*(exp(-1/100))];
MTzs = [M1s(3,:) M2s(3,2:length(M2s))]; % M2zs(185)*(exp(-1/1000))+100*(1-exp(-1/1000))];
data = [t;MTxs;MTys;MTzs];
figure,
subplot(1,2,1);
plot(t, MTxs,'b');
hold on;
plot(t, MTys,'r');
xlabel('Time (ms)');
ylabel('Magnetization');
legend('M1x', 'M1y');
figure;
plot(t, MTzs,'g');
xlabel('Time (ms)');
ylabel('Magnetization');
legend('M1z');
```

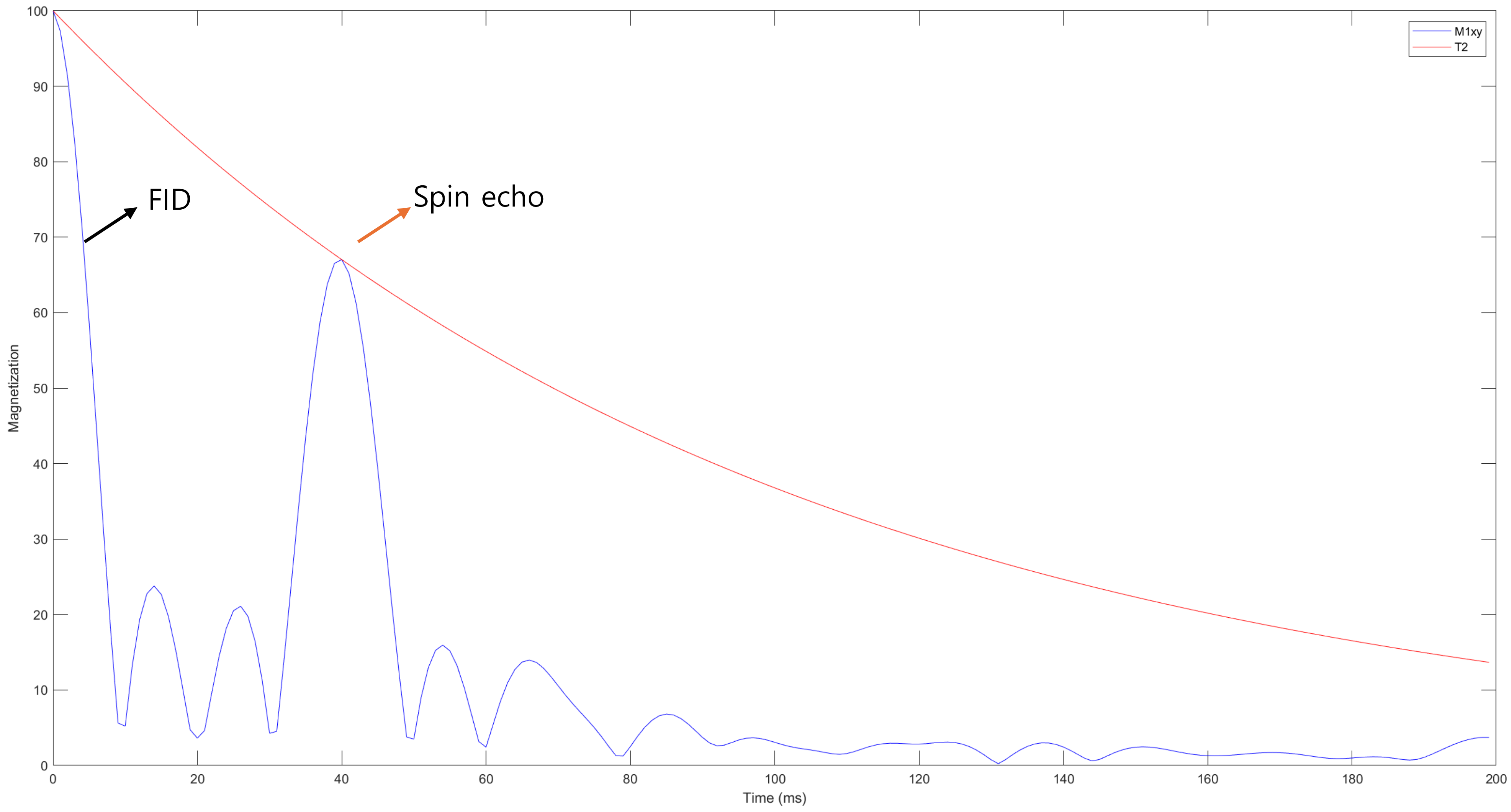
5. Set Mxy = Mx+jMy, and plot the time course of its magnitude. Mark FID and spin-echo in your plot.



Code:

```
% Calculate the Euclidean norm (or magnitude) of vectors
represented by their components in two dimensions.
Mxys = sqrt(Mxs.^2 + Mys.^2);

  % Plotting Mxy (magnitude)
  figure;
  plot(t, Mxys); grid on;
  xlabel('Time (ms)');
  ylabel('Magnetization');
  legend('Mxy');
  title('Magnetization magnitude Mxy');
```

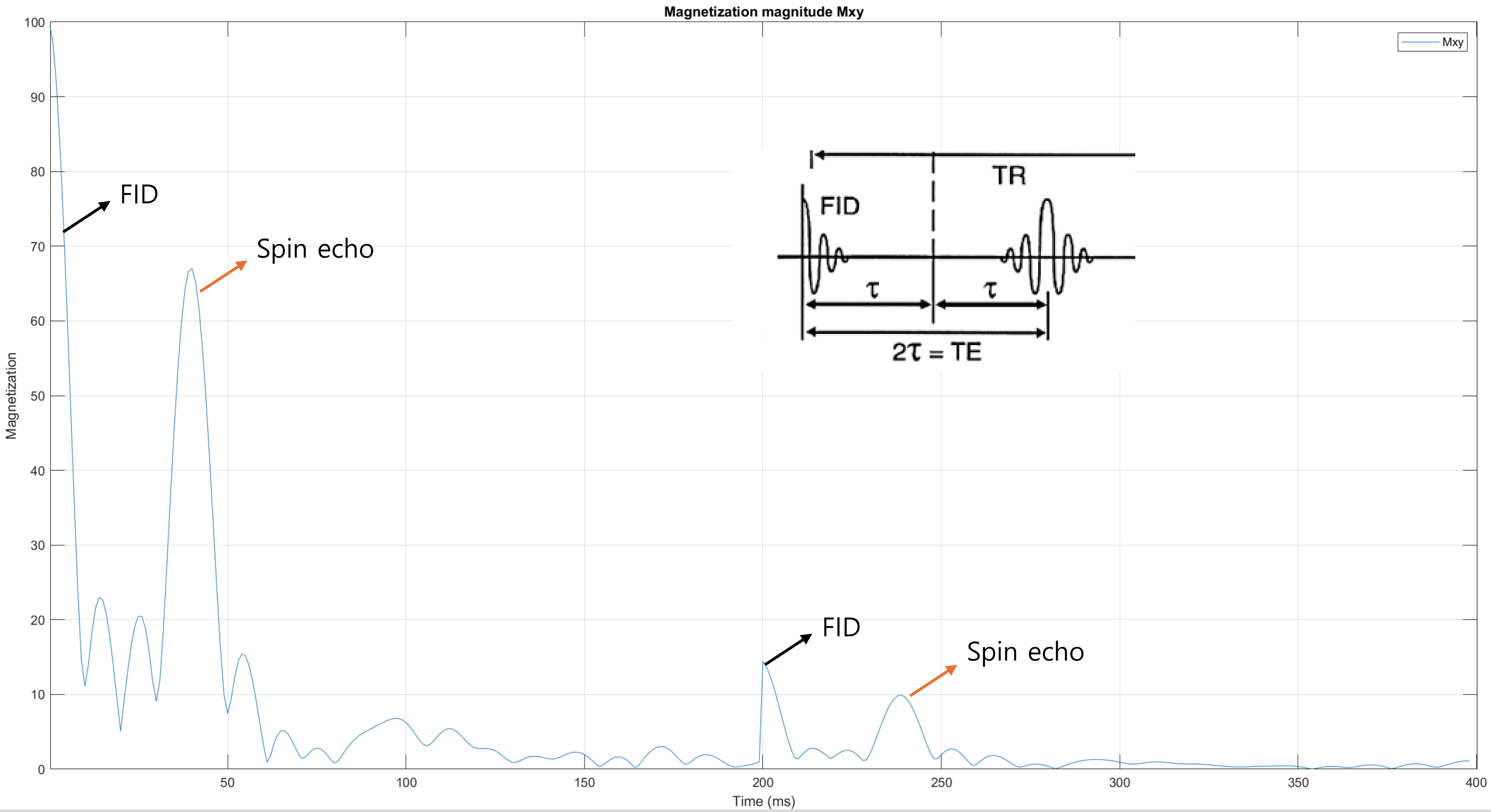
6. Verify in your result above that the peaks of FID and spin-echo are related to T2.

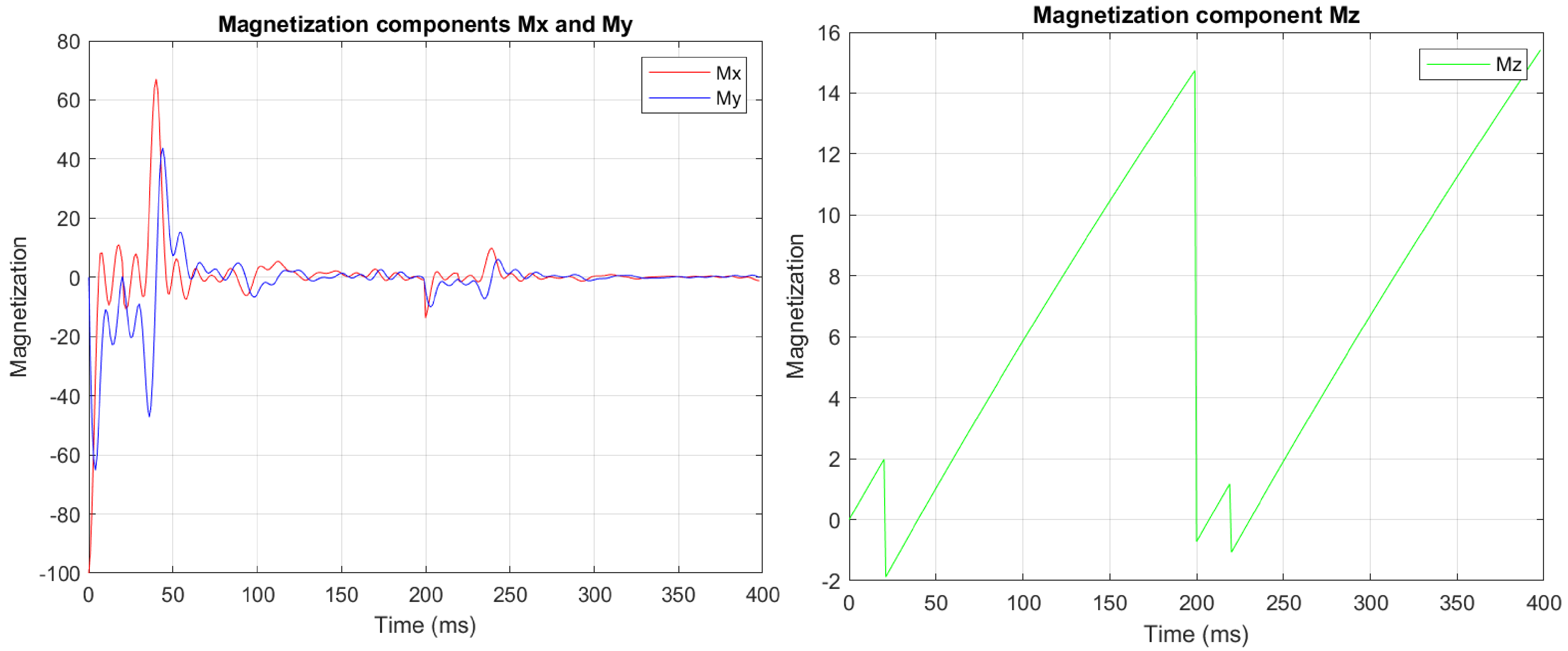


Code:

```
Mxys = (Mxs.^2+Mys.^2).^(1/2);
T2relax = 100*exp(-t/T2);

figure,

plot(t, Mxys,'b');
hold on;
plot(t, T2relax,'r');xlabel('Time (ms)');
ylabel('Magnetization');legend('M1xy', 'T2');
```

7. Now, apply another pair of 90-180 RF pulses, and plot the resultant |Mxy| along time. Compare signal levels between the first and second spin echoes.




Magnetization components Mx and My
Mx
My
Magnetization
Time (ms)
80
60
40
20
0
-20
-40
-60
-80
-100
0
50
100
150
200
250
300
350
400
Magnetization component Mz
Mz
Magnetization
Time (ms)
16
14
12
10
8
6
4
2
0
-2
0
50
100
150
200
250
300
350
400

```
%% 2nd 90-degree RF pulse about y-axis %%

M_ap_2 = [M_bp_2(:,3).*(-1) M_bp_2(:,2) M_bp_2(:,1)];
% M after the 2nd 90-degree pulse, 100x3

%% During ti3 %%
M3s = zeros(3, length(ti3)); % 3x17
M_bp_3 = [];
% M before the 2nd 180-degree pulse, 100x3

for i = 1:size(freq,2)
f = freq(i);
M_ap_2_single = M_ap_2(i,:)'; % 1x3
M3 = M_time(ti3, T1, T2, M0, M_ap_2_single, f);
% 3x17
M_bp_3 = [M_bp_3; (M3(:,(length(ti3))))']; % 100x3
M3s = M3s + M3;
end
%% 2nd 180 degree RF pulse about y-axis %%
M_ap_3 = [M_bp_3(:,1).*(-1) M_bp_3(:,2) M_bp_3(:,3).*(-1)];
% M after the 2nd 180 degree pulse, 100x3
%% During ti4 %%
M4s = zeros(3, length(ti4)); % 3x186
M_bp_4 = []; % M before 3rd 90 degree pulse
for i = 1:size(freq,2)
f = freq(i);
M_ap_3_single = M_ap_3(i,:)'; % 1x3
M4 = M_time(ti4, T1, T2, M0, M_ap_3_single, f);
% 3x186
M_bp_4 = [M_bp_4; (M4(:,(length(ti4))))']; % 100x3
M4s = M4s + M4;
end
```

```
%% Plot %%
MTxs = [M1s(1,:) M2s(1,2:length(M2s)) M3s(1,2:length(M3s)) M4s(1,2:length(M4s))];
MTys = [M1s(2,:) M2s(2,2:length(M2s)) M3s(2,2:length(M3s)) M4s(2,2:length(M4s))];
MTzs = [M1s(3,:) M2s(3,2:length(M2s)) M3s(3,2:length(M3s)) M4s(3,2:length(M4s))];
MTxys = (MTxs.^2+MTys.^2).^(1/2);
% 1x400
data = [ti;MTxs;MTys;MTzs;MTxys];
Q = [MTxs;MTys;MTzs];

figure,
plot(ti, MTxs,'r');
hold on;
plot(ti, MTys,'b');
legend('My','MX')
xlabel('Time (ms)');
ylabel('Magnetization')
figure;
plot(ti, MTzs,'g');
legend('Mz')
% subplot(2,2,3);
% plot(ti, MTxys);
figure;
plot(ti, MTxys);xlabel('Time (ms)');
ylabel('Magnetization');legend('M1xy');
```

8. How would you increase the level of the second spin echo signal? Prove with simulations. Discuss pros and cons of your strategy.

Optimize Repetition Time (TR):Increase the TR to allow for complete relaxation of the longitudinal magnetization between successive RF pulses. This ensures that the magnetization is fully recovered before the next excitation, leading to higher signal intensity.

Pros:

1. Enhanced Signal Intensity: By optimizing the TR, allows sufficient time for the longitudinal magnetization to recover between successive RF pulses. This results in increased signal intensity, particularly for the second spin echo.

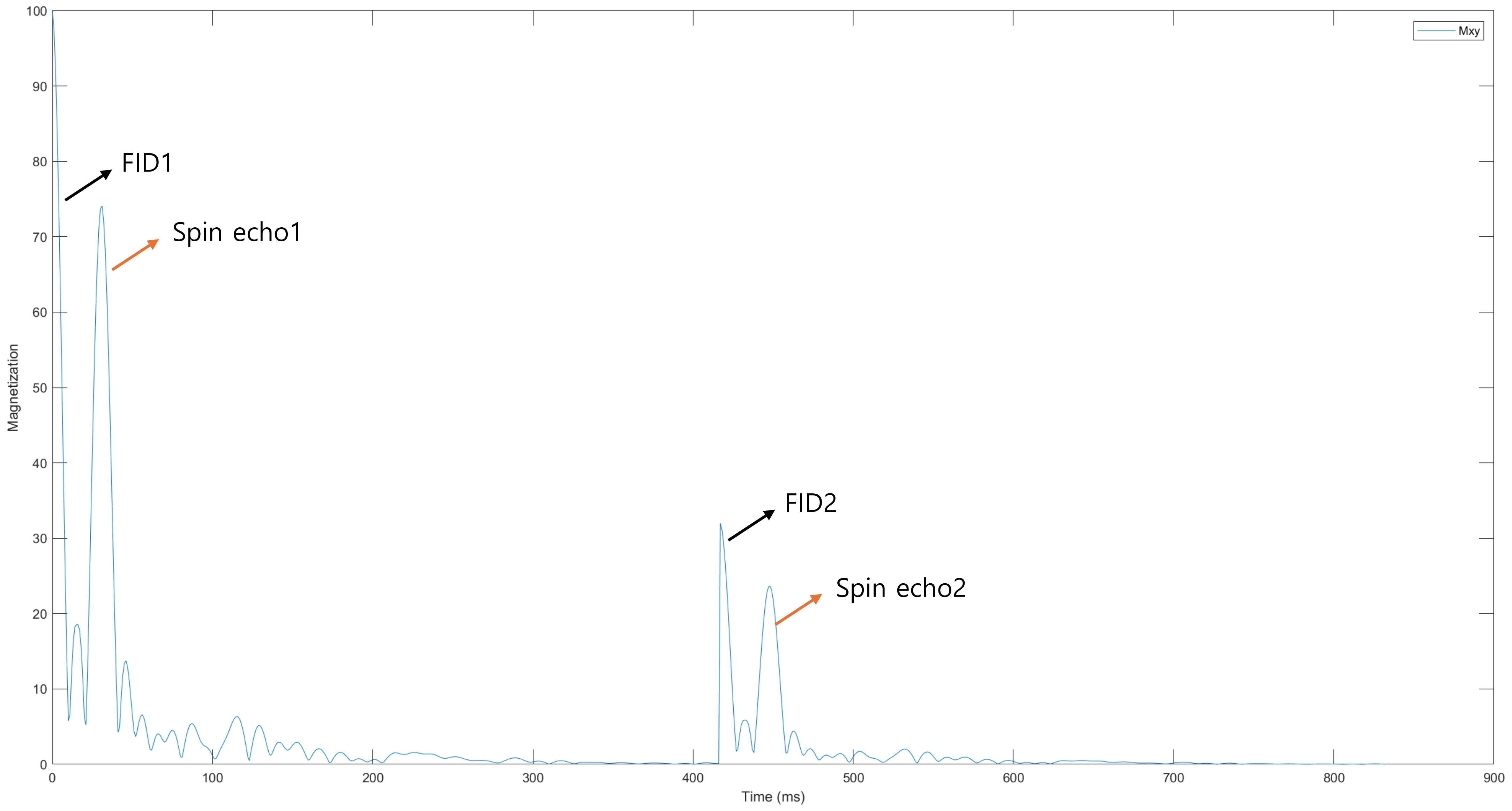
2. Improved Image Contrast: Increased signal intensity of the second spin echo can improve image contrast, making it easier to distinguish between different tissue types or pathological conditions.

3. Reduced Artefacts: Adequate relaxation between RF pulses reduces artifacts such as ghosting and blurring in the MRI images, resulting in clearer and more accurate diagnostic information.

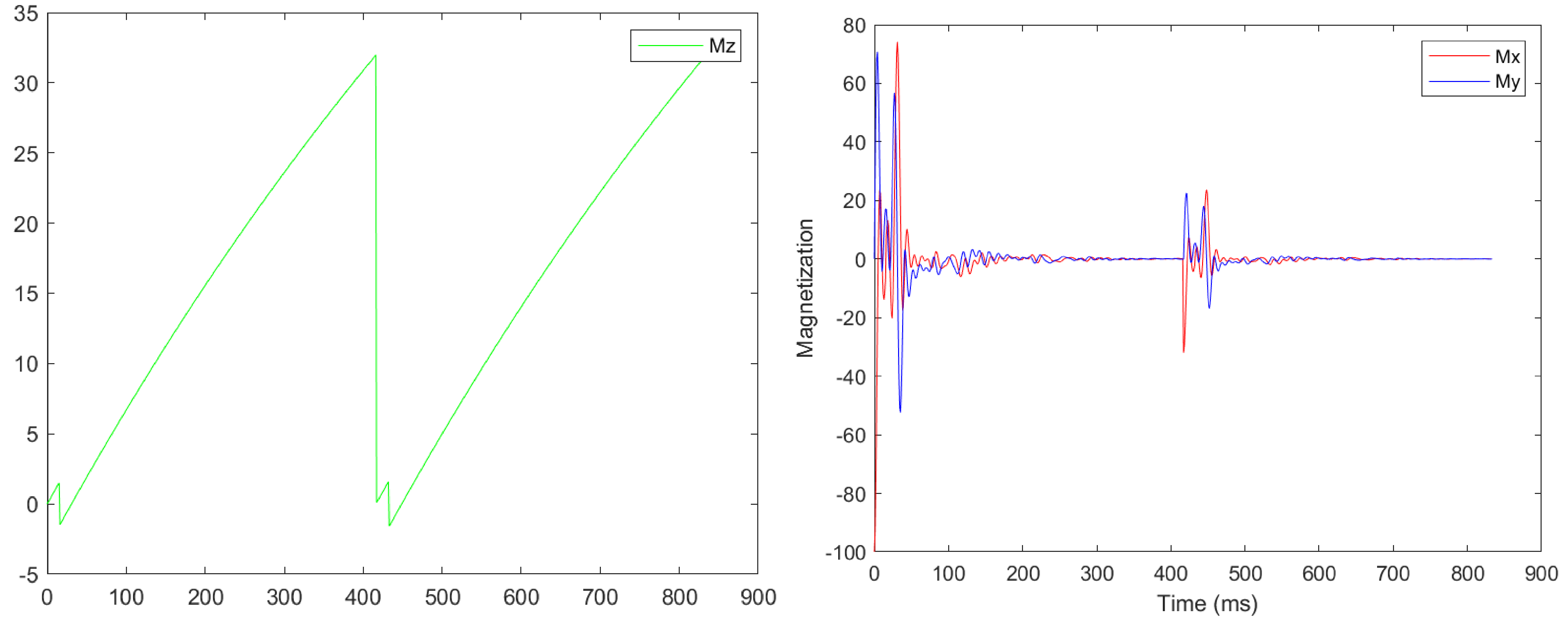
Cons:

1. Prolonged Scan Time: Optimal TR values may require longer scan times, especially when using longer relaxation times (T1) or smaller flip angles. This can increase patient discomfort and reduce overall throughput in clinical settings.

2. Trade-offs with Spatial Resolution: Prolonged TR may limit the achievable spatial resolution in MRI images, especially in sequences where shorter TR values are necessary to capture rapid physiological processes or dynamic contrast changes.



Mxy
FID1
Spin echo1
FID2
Spin echo2
Magnetization
Time (ms)
100
90
80
70
60
50
40
30
20
10
0
0
100
200
300
400
500
600
700
800
900


Mz
Mx
My
Magnetization
Time (ms)
35
30
25
20
15
10
5
0
-5
0
100
200
300
400
500
600
700
800
900
80
60
40
20
0
-20
-40
-60
-80
-100

```
T1 = 1000;
T2 = 100;
M0 = 1;
M = [-1; 0; 0];


% Generate frequency
freq = randi([0, 100], 1, 100);


% Time intervals
ti = 0:1:833; % Adjusted total length to match the intervals below


ti1 = 0:1:15; % Length 16
ti2 = 0:1:400; % Length 251
ti3 = 0:1:15; % Length 16
ti4 = 0:1:400; % Length 251
```

```
%% Plot %%
MTxs = [M1s(1, :) M2s(1, :) M3s(1, :) M4s(1, :)];
MTys = [M1s(2, :) M2s(2, :) M3s(2, :) M4s(2, :)];
MTzs = [M1s(3, :) M2s(3, :) M3s(3, :) M4s(3, :)];
MTxys = sqrt(MTxs.^2 + MTys.^2);

figure,
plot(ti, MTxs, 'r');
hold on;
plot(ti, MTys, 'b');
legend('Mx', 'My')
xlabel('Time (ms)');
ylabel('Magnetization');

figure;
plot(ti, MTzs, 'g');
legend('Mz');

figure;
plot(ti, MTxys);
xlabel('Time (ms)');
ylabel('Magnetization');
legend('Mxy');
```