1.6.24

AI25BTECH11009 — Dasu Harshith Kumar

Question 1.6.24

Find the values of k if the points

$$A(2,3), \quad B(4,k), \quad C(6,-3)$$

are collinear.

Solution

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 4 \\ k \end{pmatrix}, \qquad \mathbf{C} = \begin{pmatrix} 6 \\ -3 \end{pmatrix} \tag{0.1}$$

Form the direction vectors:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2 \\ k-3 \end{pmatrix}, \qquad \mathbf{C} - \mathbf{A} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}. \tag{0.2}$$

Construct the matrix (columns are the direction vectors):

$$(\mathbf{B} - \mathbf{A},\ \mathbf{C} - \mathbf{A}) = M = \begin{pmatrix} 2 & 4 \\ k-3 & -6 \end{pmatrix}.$$

Row reduction to get echelon form(showing main steps):

$$M \xrightarrow{R_1 \leftarrow \frac{1}{2}R_1} \begin{pmatrix} 1 & 2 \\ k-3 & -6 \end{pmatrix}, \tag{0.3}$$

$$\xrightarrow{R_2 \leftarrow R_2 - (k-3)R_1} \begin{pmatrix} 1 & 2 \\ 0 & -2k \end{pmatrix}. \tag{0.4}$$

For collinearity $\operatorname{rank}(M) = 1$, so the second row must be zero:

$$-2k = 0 \quad \Longrightarrow \quad k = 0.$$

Thus $k = 0$ and the points are collinear for $B = (4, 0)$.

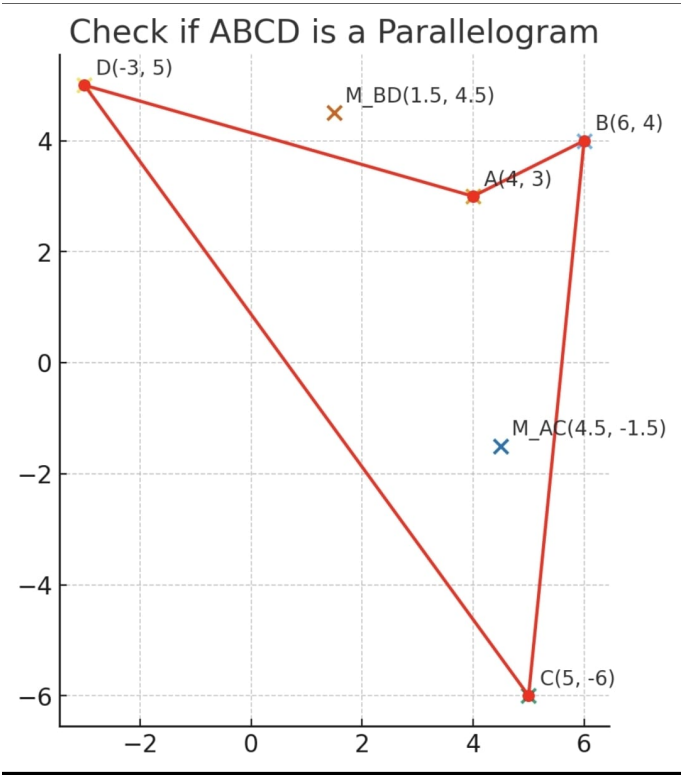

Fig. 0.1: Verification plot