Theorem[section] Problem Lemma[section] [theorem]Corollary
Example[section] [problem]Definition

Question 1.6.24

AI25BTECH11009 — Dasu Harshith Kumar

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Find the values of *k* if the points

$$A(2,3), \quad B(4,k), \quad C(6,-3)$$

are collinear.

Write position vectors:

$$A = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \qquad B = \begin{pmatrix} 4 \\ k \end{pmatrix}, \qquad C = \begin{pmatrix} 6 \\ -3 \end{pmatrix}.$$

Direction vectors:

$$B - A = \begin{pmatrix} 2 \\ k - 3 \end{pmatrix}, \qquad C - A = \begin{pmatrix} 4 \\ -6 \end{pmatrix}.$$

Construct the matrix:

$$(B - A, \ C - A) = M = \begin{pmatrix} 2 & 4 \\ k - 3 & -6 \end{pmatrix}.$$

$$M \xrightarrow{R_1 \leftarrow \frac{1}{2}R_1} \begin{pmatrix} 1 & 2 \\ k-3 & -6 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - (k-3)R_1} \begin{pmatrix} 1 & 2 \\ 0 & -2k \end{pmatrix}.$$
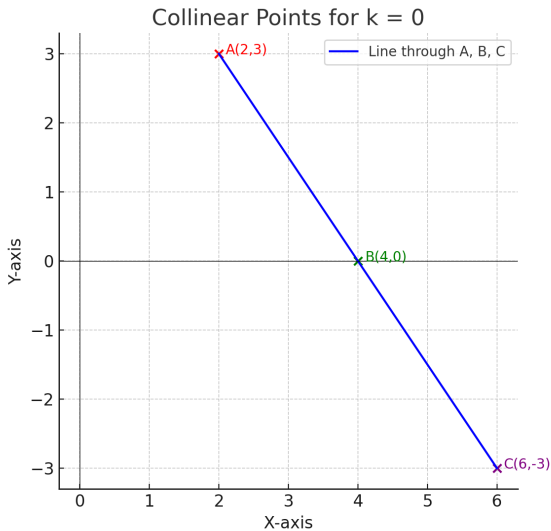
For collinearity, rank must be 1:

$$-2k = 0 \implies k = 0.$$

$\therefore$ The points are collinear when $k = 0$.

Conclusion & verification

Result: $k = 0$.

Collinear Points for k = 0

Python code

```python
1  import numpy as np
2  import matplotlib.pyplot as plt
3
4  A = np.array([2.0, 3.0])
5  C = np.array([6.0, -3.0])
6  k = 0.0
7  B = np.array([4.0, k])
8
9  t = np.linspace(-0.2, 1.2, 100)
10 line = A + np.outer(t, (C - A))
11
12 plt.figure(figsize=(6,4))
13 plt.plot(line[:,0], line[:,1], 'k--')
14 plt.plot([A[0], B[0], C[0]],
15          [A[1], B[1], C[1]], 'ro')
16
17 plt.text(A[0]+0.08, A[1]+0.08, 'A')
18 plt.text(B[0]+0.08, B[1]+0.08, 'B')
19 plt.text(C[0]+0.08, C[1]+0.08, 'C')
```

C code

```c
#include <stdio.h>

int main(void) {
    double k = 0.0;
    printf("Solution: k = %.0f\n", k);
    printf("B = (4, %.0f) makes A,B,C collinear.\n", k);
    return 0;
}
```