

1.6.24

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Question: Find the values of k if the points $A(2,3)$, $B(4,k)$, $C(6,-3)$ are collinear.

Solution:

We check collinearity using the vector method.

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 4-2 \\ k-3 \end{pmatrix} = \begin{pmatrix} 2 \\ k-3 \end{pmatrix}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 6-2 \\ -3-3 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$$

Now form the matrix:

$$\mathbf{M} = (\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^T = \begin{pmatrix} 2 & 4 \\ k-3 & -6 \end{pmatrix}$$

For collinearity, $\text{rank}(\mathbf{M}) = 1$. That means the determinant of \mathbf{M} must be zero:

$$\det \begin{pmatrix} 2 & 4 \\ k-3 & -6 \end{pmatrix} = -12 - 4(k-3) = 0$$

$$-12 - 4k + 12 = -4k = 0 \quad \Rightarrow \quad k = 0$$

$\therefore A(2, 3), B(4, 0), C(6, -3)$ are collinear for $k = 0$.

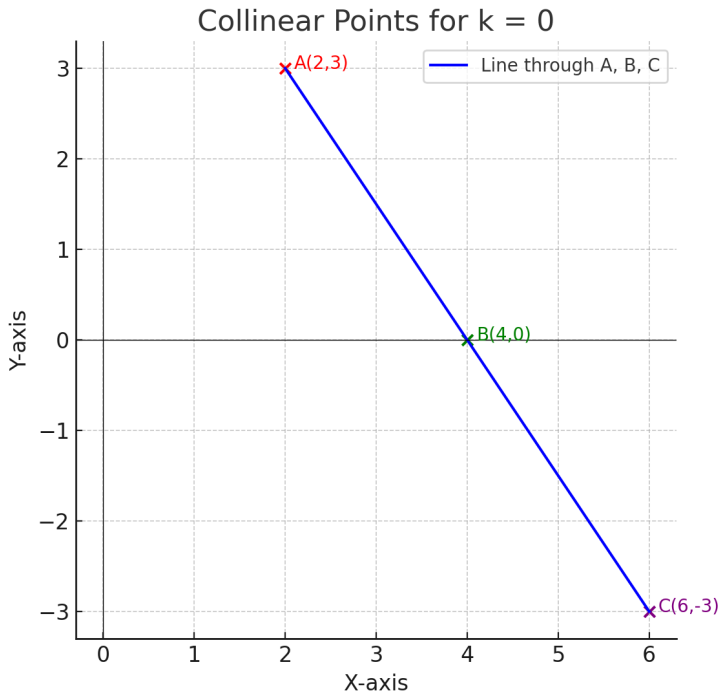


Fig. 0.1: Graph showing collinear points A, B, C for $k = 0$