HWD (Basic Math for ML)

a)

1) $A + B$

$$\begin{bmatrix} 2 & -4 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 4 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 6 & -5 \\ 3 & 3 \end{bmatrix}$$

2) $3B$

$$3 \begin{bmatrix} 4 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 12 & -3 \\ 6 & 0 \end{bmatrix}$$

3) AC

$$\begin{bmatrix} 2 & -4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 2\times4 - 4\times3 \\ 1\times4 + 3\times3 \end{bmatrix}$$

2×2 $\quad$ 2×1

$$= \begin{bmatrix} -4 \\ 13 \end{bmatrix}$$

4) $$\begin{vmatrix} 2 & -4 \\ 1 & 3 \end{vmatrix} = ad - bc$$

$$= 6 + 4 = 10.$$

5) $\begin{bmatrix} 2 & -4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} -3 & 2 & 0 \\ 1 & -1 & -2 \end{bmatrix} = \begin{bmatrix} 2\times-3+-4\times1 & 2\times2+4 & 0+8 \\ -3+3 & 2-3 & 0-6 \end{bmatrix}$

2×2 $\quad$ 2×3

$= \begin{bmatrix} -10 & 8+8 \\ 0+-1 & -6 \end{bmatrix}$

6) AD

$\begin{bmatrix} 2 & -4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & 1 \end{bmatrix}$ $= \begin{bmatrix} \text{Undefined} \end{bmatrix}$

2×2 $\quad$ 1×2

not equal

7) B+D

$\begin{bmatrix} 4 & -1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 1 \end{bmatrix}$ Undefined

Both matrices should be same (r x c)

8) B − 2A

$\begin{bmatrix} 4 & -1 \\ 2 & 0 \end{bmatrix} - 2\begin{bmatrix} 2 & -4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 7 \\ 0 & -6 \end{bmatrix}$

b) $$A = \begin{bmatrix} 1 & 6 & 5 \\ 2 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix}$$

$$|A| = 1\begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} - 6\begin{vmatrix} 2 & 1 \\ 0 & 4 \end{vmatrix} + 5\begin{vmatrix} 2 & 3 \\ 0 & 2 \end{vmatrix}$$

$$= 12 + 2 - 6(8) + 5(-4)$$

$$= -18$$

$$A^T = \begin{bmatrix} 1 & 2 & 0 \\ 6 & 3 & 2 \\ 5 & 1 & 4 \end{bmatrix}$$

$$|A| = 1(3 \cdot 4 - 2 \cdot 1) - 2(6 \cdot 4 - 5 \cdot 2) + 0$$

$$= 10 - 28$$

$$= -18$$

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} & -\begin{vmatrix} 6 & 2 \\ 5 & 4 \end{vmatrix} & \begin{vmatrix} 6 & 3 \\ 5 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & 0 \\ 1 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 5 & 4 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 5 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & 0 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 6 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 6 & 3 \end{vmatrix} \end{bmatrix}$$

$$= \frac{1}{-18}\begin{bmatrix} 10 & -14 & -9 \\ -4 & 4 & +9 \\ 4 & -2 & -9 \end{bmatrix} = \begin{bmatrix} -5/9 & +7/9 & 1/2 \\ +2/9 & -2/9 & -1/2 \\ -2/9 & +1/9 & 1/2 \end{bmatrix}$$

$$(A^T \cdot A)^{-1} \cdot A^T = A^{-1} \cdot (A^T)^{-1} \cdot A^T$$

$$= A^{-1} \cdot I \qquad A \cdot A^{-1} = I$$

$$= A^{-1}$$

$$= \begin{bmatrix} -5/9 & 7/9 & 1/2 \\ 2/9 & -2/9 & -1/2 \\ -2/9 & 1/9 & 1/2 \end{bmatrix}$$

c) $2x + 3y - z = 10 \quad -①$

$x + 0 - 3z = 6 \quad -②$

$5x - 2y + 0 = 13 \quad -③$

① × 3 $\quad 6x + 9y - 3z = 30$

$x + 0 - 3z = 6$

$5x + 9y = 24$

$5x + 9y = 24$

$-5x + 2y = -13$

$11y = 11$

$y = 1 \quad x = 3 \quad z = -1$

$(3, 1, -1)$

d) $f(x,y) = x3y + ex$

$\frac{\partial}{\partial x} f(x,y) = 3y + e$

$\frac{\partial}{\partial y} f(x,y) = 3x + 0$

$f(x,y) = xe2x + 3y$

$\frac{\partial}{\partial x} f(x,y) = 4xe + 0$

$\frac{\partial}{\partial y} f(x,y) = 0 + 3$

e) $z = x^{-2} y^6 - 4x$

$x = u^2 v$

$y = v - 3u$

$\frac{\partial z}{\partial u} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial u}$

$$\frac{\partial f}{\partial x} = -2y^6x^{-3} - 4 \qquad \frac{\partial f}{\partial y} = 6y^5x^{-2}$$

$$\frac{\partial x}{\partial u} = 2uv \qquad \frac{\partial y}{\partial u} = -3$$

$$\frac{\partial z}{\partial u} = (-2y^6x^{-3} - 4)\,2uv + 6y^5x^{-2} \cdot (-3)$$