ASSIGNMENT - 1

Ques 1:- 1. Given,

$A = \{1, 2, 3, 4\}$ $B = \{2, 4, 3, 1\}$

Since, $\forall x \in A \; \exists x \in B$.

Hence, both set A and B are equal.

2. $A = \{1, 3, 5, 7\}$ $n = 4$

Subsets of $A = 2^n = 2^4 = 16$

$\{\}, \{1\}, \{3\}, \{5\}, \{7\}, \{1,3\}, \{1,5\}, \{1,7\}, \{3,5\}, \{3,7\}, \{5,7\}$,
$\{1,3,5\}, \{1,3,7\}, \{1,5,7\}, \{3,5,7\}, \{1,3,5,7\}$.

3. $A = \{1, 2, 3, 4, 5 \ldots\}$

$A = \{x \in N \mid x \geq 1\}$

4. Given, $A = \{1, 3, 5, 7, 9, 11\}$ $B = \{1, 2, 3, 13\}$

$A - B = \{5, 7, 9, 11\}$ $B - A = \{2, 13\}$

5. Given, $A = \{1, 3, 5\}$ $B = \{2, 4, 6\}$ $C = \{1, 5, 7\}$

$B \cup C = \{1, 2, 4, 5, 6, 7\}$

$A \cup (B \cup C) = \{1, 2, 3, 4, 5, 6, 7\}$

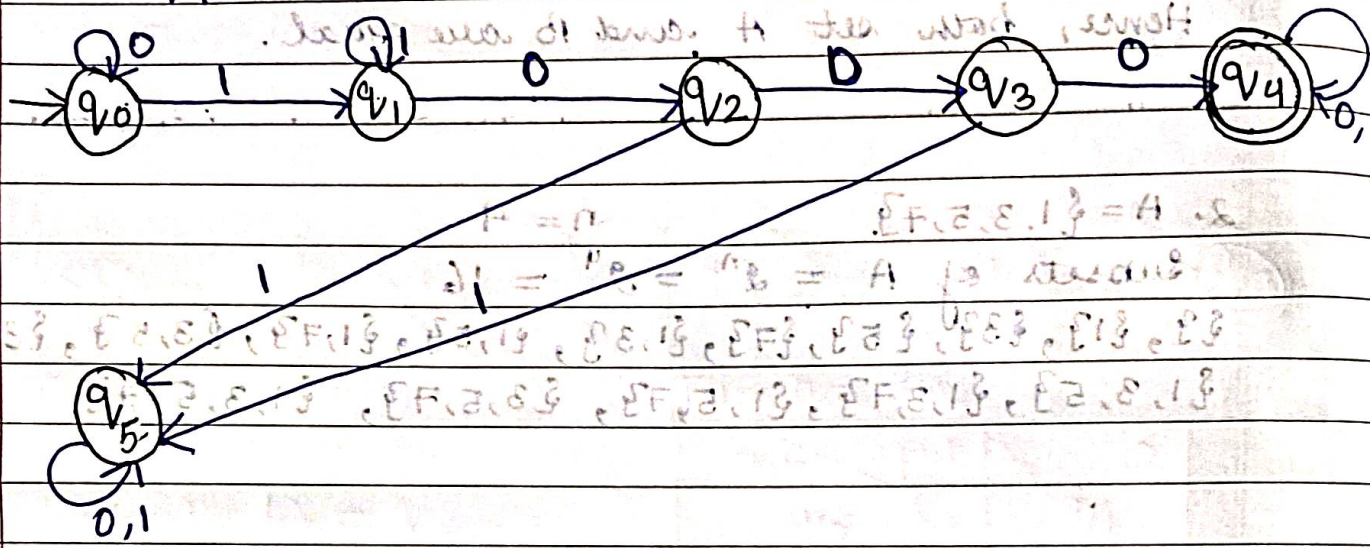
Ques3= 1. $L = \{1000, 0010, \ldots\}$

$Q = q_0, q_1, q_2, q_3, q_4, q_5$

$\Sigma = \{a, b\}$

$q_0 = q_0$

$F = q_4$



Transition function:-

$\delta: (q_0, 0) = q_0$ $\quad \delta: (q_5, 0) = q_5$

$\delta: (q_0, 1) = q_1$ $\quad \delta: (q_5, 1) = q_5$

$\delta: (q_1, 0) = q_2$

$\delta: (q_1, 1) = q_1$

$\delta: (q_2, 0) = q_3$

$\delta: (q_2, 1) = q_d$

$\delta: (q_3, 0) = q_4$

$\delta: (q_3, 1) = q_d$

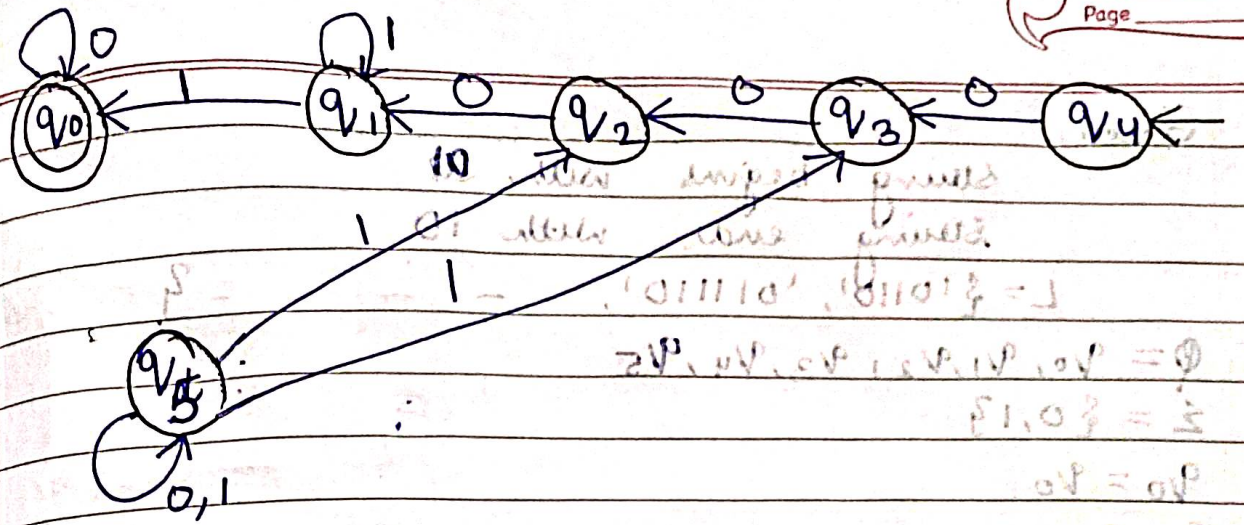
$\delta: (q_4, 0) = q_4$

$\delta: (q_4, 1) = q_4$

Transition Table:-

States	Input 0	Input 1
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_3	q_d
q_3	q_4	q_d
q_4	q_4	q_4
q_5	q_5	q_5

2.

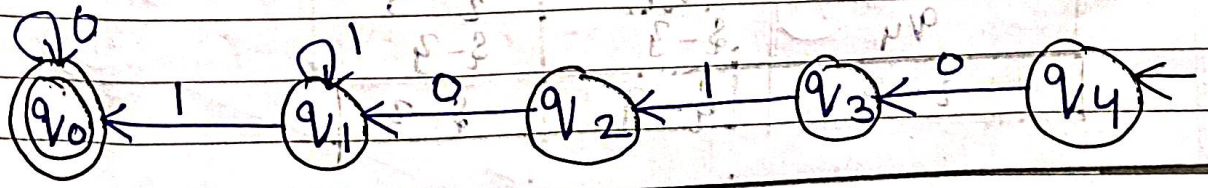


3. Yes, it is a valid finite automation because it has:-
→ finite number of states i.e. 6
→ initial state q_4
→ final state q_0

It is NFA (nondeterministic finite automata) because from q_5 there are 3 possible transitions on 1. In DFA we have at most one transition from each symbol.

4. Step1 – Remove unreachable states → q_5 (no incoming edge, only self loop.

Step2 – (Diagram)



Ques4 Given,

String begins with 01

String ends with 10

$L = \{'01110', '011110', \ldots\}$

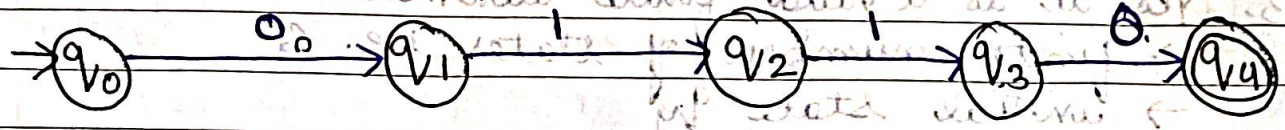
NFA

$Q = q_0, q_1, q_2, q_3, q_4$

$\Sigma = \{0, 1\}$

$q_0 = q_0$

$F = q_4$



Transition function :-

$\delta : (q_0, 0) = q_1$

$\delta : (q_1, 1) = q_2$

$\delta : (q_2, 1) = q_3$

$\delta : (q_3, 0) = q_4$

Transition Table

State \ Input	0	1
q_0	q_1	–
q_1	–	q_2
q_2	–	q_3
q_3	q_4	–
q_4	–	–

DFA

State \ Input	0	1
q_0	q_1	$\{-\}$
q_1	$\{-\}$	q_2
q_2	$\{-\}$	q_3
q_3	q_4	$\{-\}$
q_4	$\{-\}$	$\{-\}$
$\{-\}$	$\{-\}$	$\{-\}$

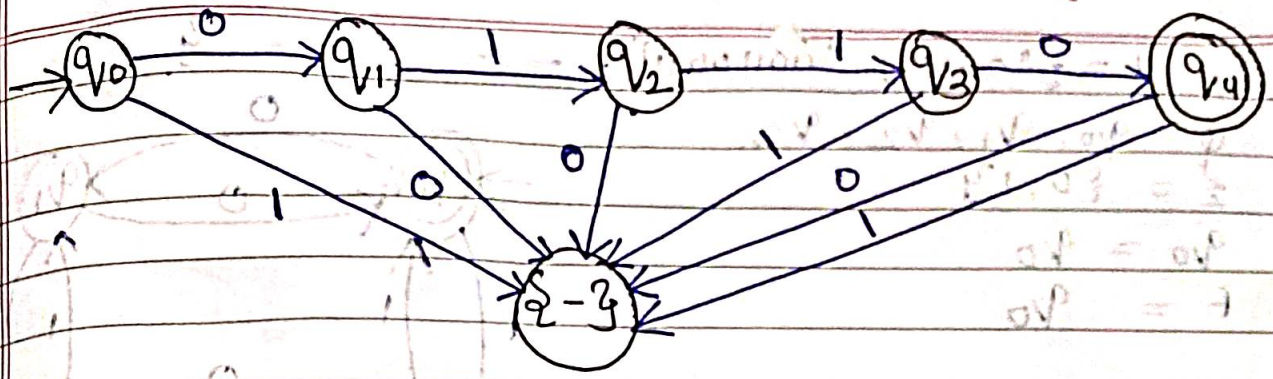
⇐ Transition Table

$Q = q_0, q_1, q_2, q_3, q_4$

$\Sigma = \{0, 1\}$

$F = q_4$

$q_0 = q_0$.



Transition Function:-

$\delta:(q_0,0) = q_1$
$\delta:(q_0,1) = \xi-\zeta$
$\delta:(q_1,0) = \xi-\zeta$
$\delta:(q_1,1) = q_2$
$\delta:(q_4,0) = \xi-\zeta.$
$\delta:(q_2,0) = \xi-\zeta$
$\delta:(q_2,1) = q_3$
$\delta:(q_3,0) = q_4$
$\delta:(q_3,1) = \xi-\zeta$
$\delta:(q_4,1) = \xi-\zeta$

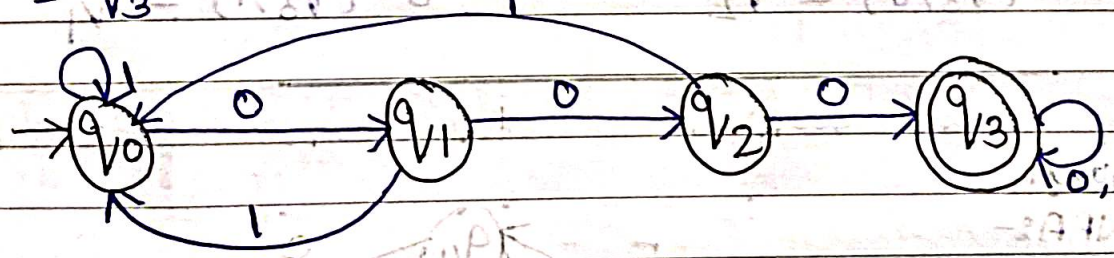
Ques5= Given, $L = \{000, 1000, 10001, \ldots\ldots\zeta\}$

$Q = q_0, q_1, q_2, q_3$

$\Sigma = \{0,1\}$

$q_0 = q_0$

$F = q_3$



Transition function:-

$\delta:(q_0,0) = q_1$
$\delta:(q_0,1) = q_1$
$\delta:(q_1,0) = q_2$
$\delta:(q_1,1) = q_0$
$\delta:(q_2,0) = q_3$
$\delta:(q_2,1) = q_0$
$\delta:(q_3,0) = q_3$
$\delta:(q_3,1) = q_3.$

Transition Table:-

States \ Input	0	1
q_0	q_1	q_1
q_1	q_2	q_0
q_2	q_3	q_0
q_3	q_3	q_3

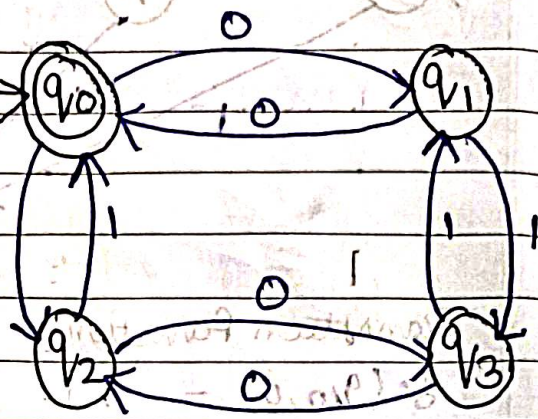
Ques 6= $L = \{"0011", "00110011", \ldots\}$

$Q = q_0, q_1, q_2, q_3.$

$\Sigma = \{0, 1\}$

$q_0 = q_0$

$F = q_0$



Transition Table:-

State \ Input	0	1
q_0	q_1	q_2
q_1	q_0	q_3
q_2	q_3	q_0
q_3	q_2	q_1

Transition function:-

$\delta : (q_0, 0) = q_1$

$\delta : (q_1, 0) = q_0$

$\delta : (q_2, 0) = q_3$

$\delta : (q_3, 0) = q_2$

$\delta : (q_0, 1) = q_2$

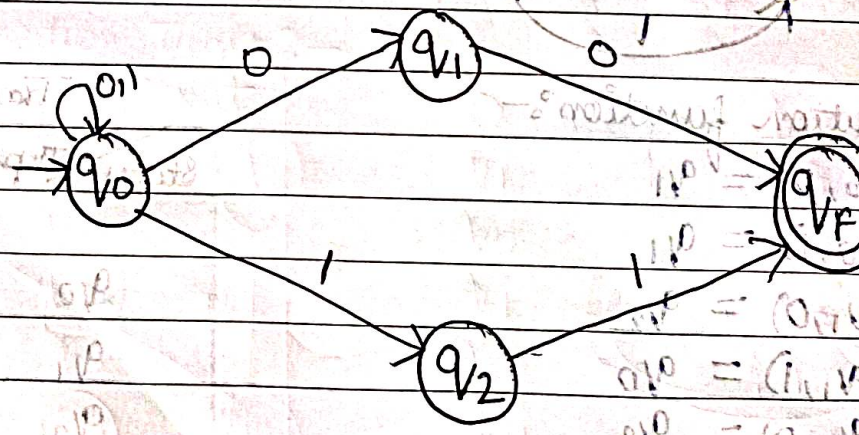
$\delta : (q_1, 1) = q_3$

$\delta : (q_2, 1) = q_0$

$\delta : (q_3, 1) = q_1$

Ques 7= Given,

NFA:-

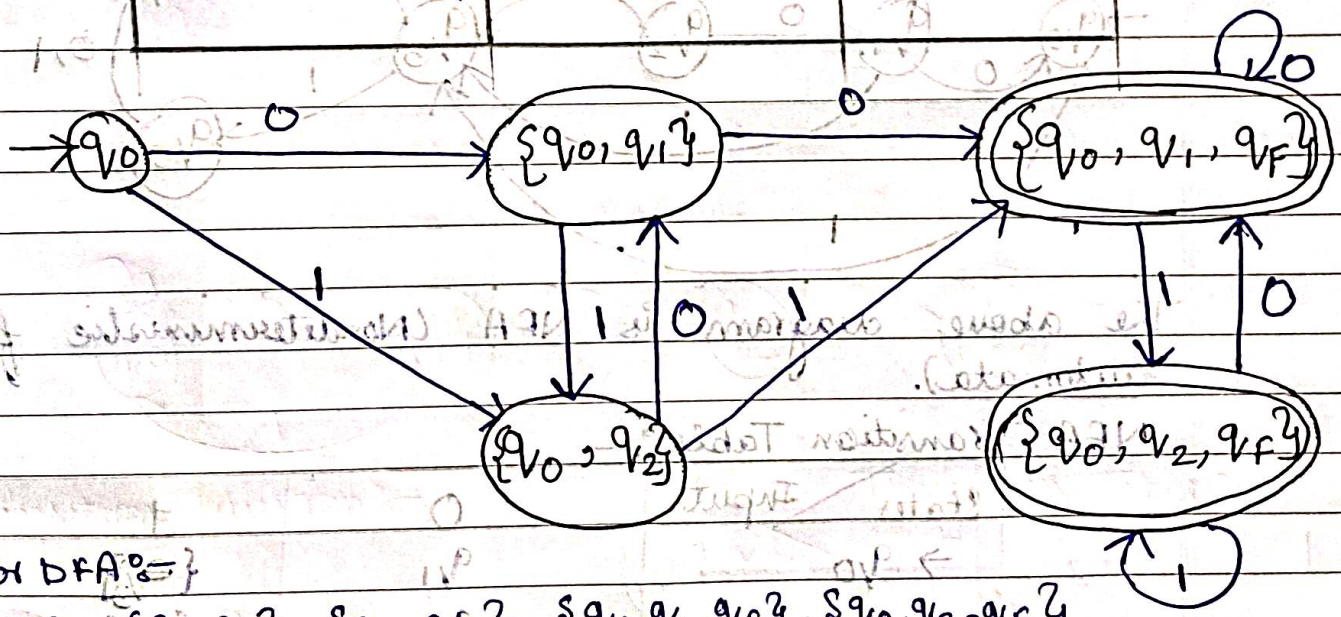


NFA transition table :-

States \ Input	0	1
q_0	$\{q_0, q_1\}$	$\{q_0, q_2\}$
q_1	q_F	-
q_2	-	q_F
q_F	-	-

DFA transition table :-

states \ Input	0	1
q_0	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_F\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_0, q_1, q_F\}$
$\{q_0, q_1, q_F\}$	$\{q_0, q_1, q_F\}$	$\{q_0, q_2, q_F\}$
$\{q_0, q_2, q_F\}$	$\{q_0, q_1, q_F\}$	$\{q_0, q_2, q_F\}$



For DFA :-

$Q = q_0, \{q_0, q_1\}, \{q_0, q_2\}, \{q_0, q_1, q_F\}, \{q_0, q_2, q_F\}$

$\Sigma = \{0, 1\}$

$q_0 = q_0$

$f = \{q_0, q_1, q_F\}, \{q_0, q_2, q_F\}$.

Transition function :-

$\delta : (q_0, 0) = \{q_0, q_1\}$

$\delta : (q_0, 1) = \{q_0, q_2\}$

$\delta : (\{q_0, q_1\}, 0) = \{q_0, q_1, q_F\}$

$\delta : (\{q_0, q_1\}, 1) = \{q_0, q_2\}$

$\delta:(\{q_0,q_2\},0)=\{q_0,q_1\}$ $\delta:(\{q_0,q_2\},1)=\{q_0,q_1,q_F\}$

$\delta:(\{q_0,q_1,q_F\},0)=\{q_0,q_1,q_F\}$ $\delta:(\{q_0,q_1,q_F\},1)=\{q_0,q_2,q_F\}$

$\delta:(\{q_0,q_2,q_F\},0)=\{q_0,q_2,q_F\}$ $\delta:(\{q_0,q_2,q_F\},1)=\{q_0,q_2,q_F\}$

Ques 8 = Step 1 – Remove the unreachable states.

Step 2 – Create the transition table of given DFA

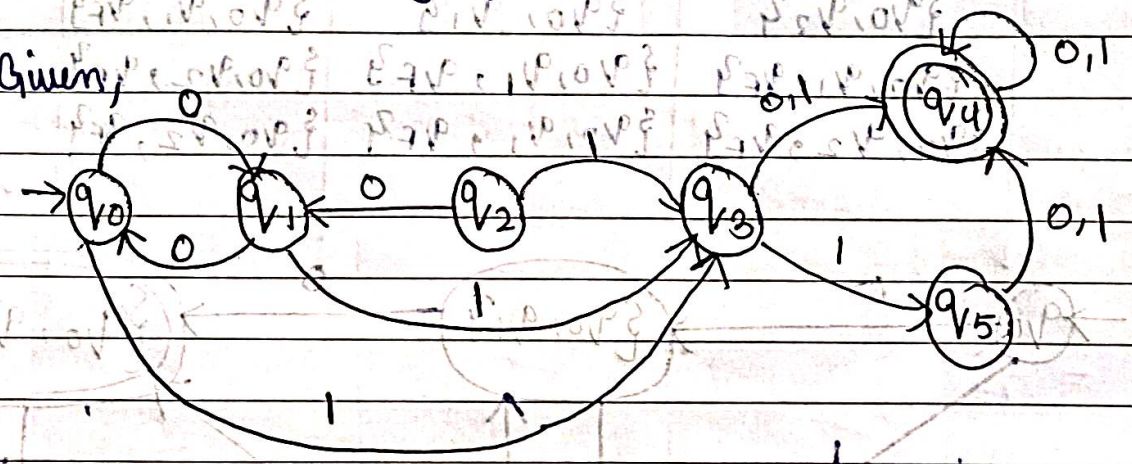
Step 3 – Create the transition table for non-final states

Step 4 – Create the transition table for final states

Step 5 – Remove duplicate rows from both the tables

Step 6 :– Re-draw DFA with transition table formed after combining final and non-final states table.

Given,



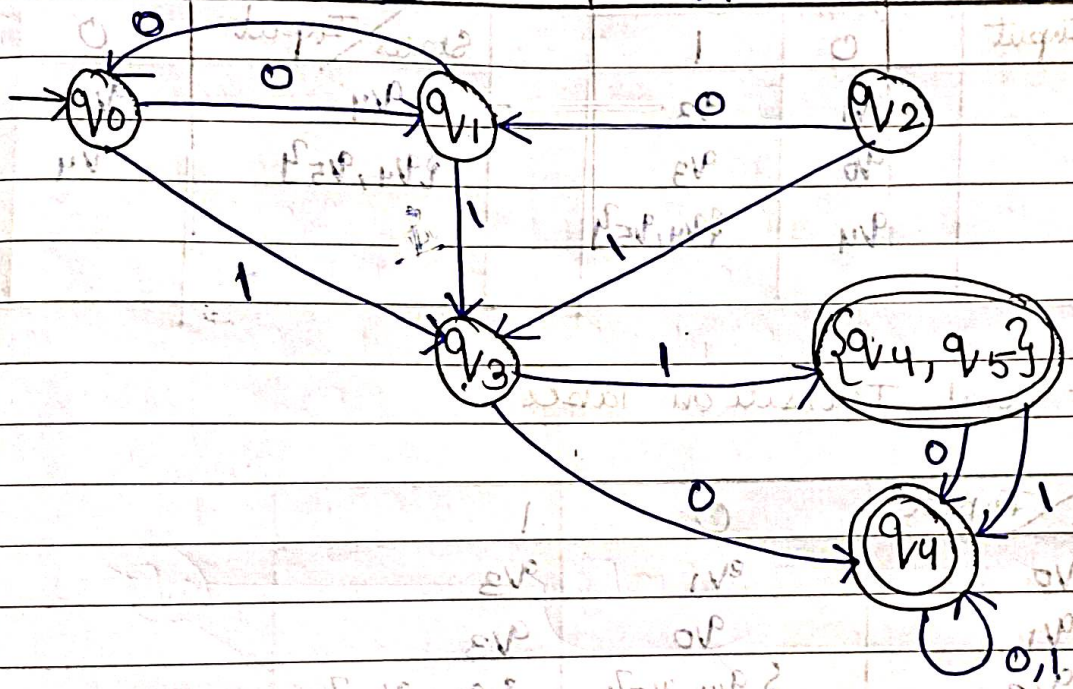
The above diagram is NFA (Nondeterministic finite automata).

NFA Transition Table :–

States \ Input	0	1
→ q_0	q_1	q_3
q_1	q_0	q_3
q_2	q_1	q_3
q_3	q_4	$\{q_4, q_5\}$
(*q_4)	q_4	q_4
q_5	q_4	q_4

DFA Transition Table:-

States \ Input	0	1
→ q_0	q_1	q_3
q_1	q_0	q_3
q_3	q_4	$\{q_4, q_5\}$
q_4	q_4	q_4
$\{q_4, q_5\}$	$\{q_4\}$	q_4
q_2	q_1	q_3



Step 1 – Remove unreachable states
q_2 is unreachable so we remove it

Step 2 – DFA transition table.

States \ Input	0	1
q_0	q_1	q_3
q_1	q_0	q_3
q_3	q_4	$\{q_4, q_5\}$
q_4	q_4	q_4
$\{q_4, q_5\}$	q_4	q_4

Step 3 – Remove duplicate rows. q_4 and $\{q_4, q_5\}$ are duplicate rows so we replace q_4 with $\{q_4, q_5\}$ from final states table.

Step 3 :- Transition Table (Non-final states)

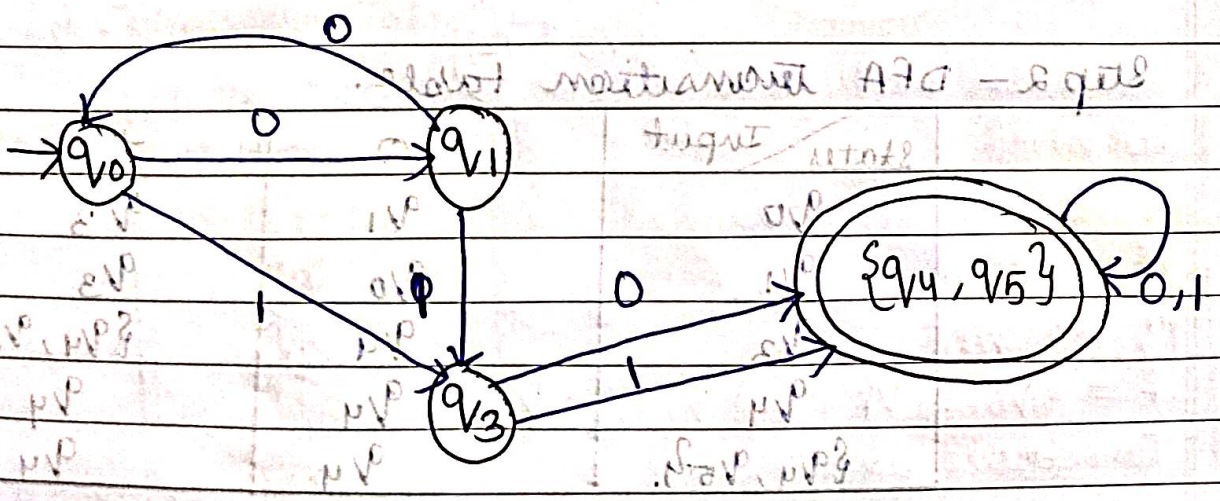
State / Input	0	1
q_0	q_1	q_3
q_1	q_0	q_3
q_3	q_4	$\{q_4, q_5\}$

Step 4 – Transition Table (Final State)

States / Input	0	1
q_4	q_4	q_4
$\{q_4, q_5\}$	q_4	q_4

Step 6 – Final Transition Table.

State / Input	0	1
q_0	q_1	q_3
q_1	q_0	q_3
q_3	$\{q_4, q_5\}$	$\{q_4, q_5\}$
$\{q_4, q_5\}$	$\{q_4, q_5\}$	$\{q_4, q_5\}$



Minimised DFA

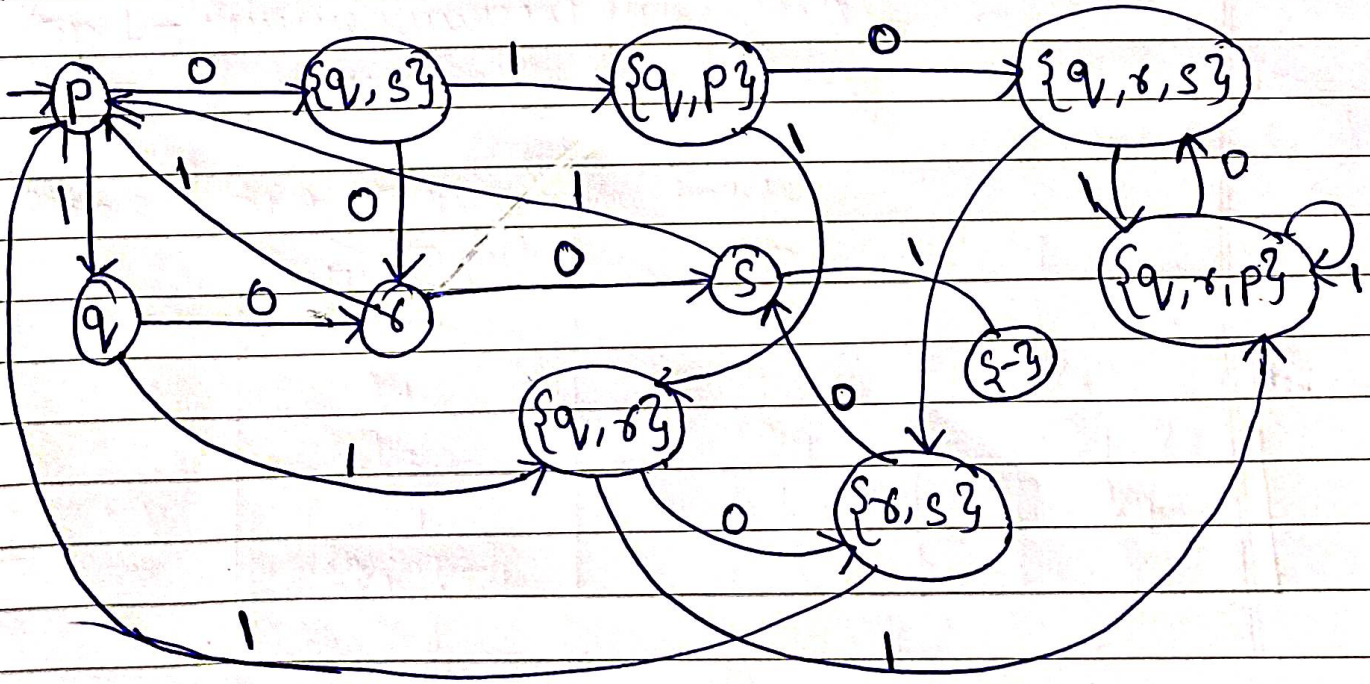
Question=

δ	0	1
p	{q, s}	{q}
q	{r}	{q, r}
r	{s}	{p}
s	–	{p}

⇐ Given, NFA transition table

DFA transition table :–

δ	0	1
p	{q, s}	{q}
q	{r}	{q, r}
{q, s}	{r}	{q, p}
r	{s}	{p}
{q, r}	{r, s}	{q, r, p}
{q, p}	{r, q, s}	{q, r}
s	{–}	{p}
{r, s}	{s}	{p}
{q, r, p}	{r, s, q}	{q, r, p}
{r, q, s}	{s, r}	{p, q, r}
{–}	{–}	{–}



DFA

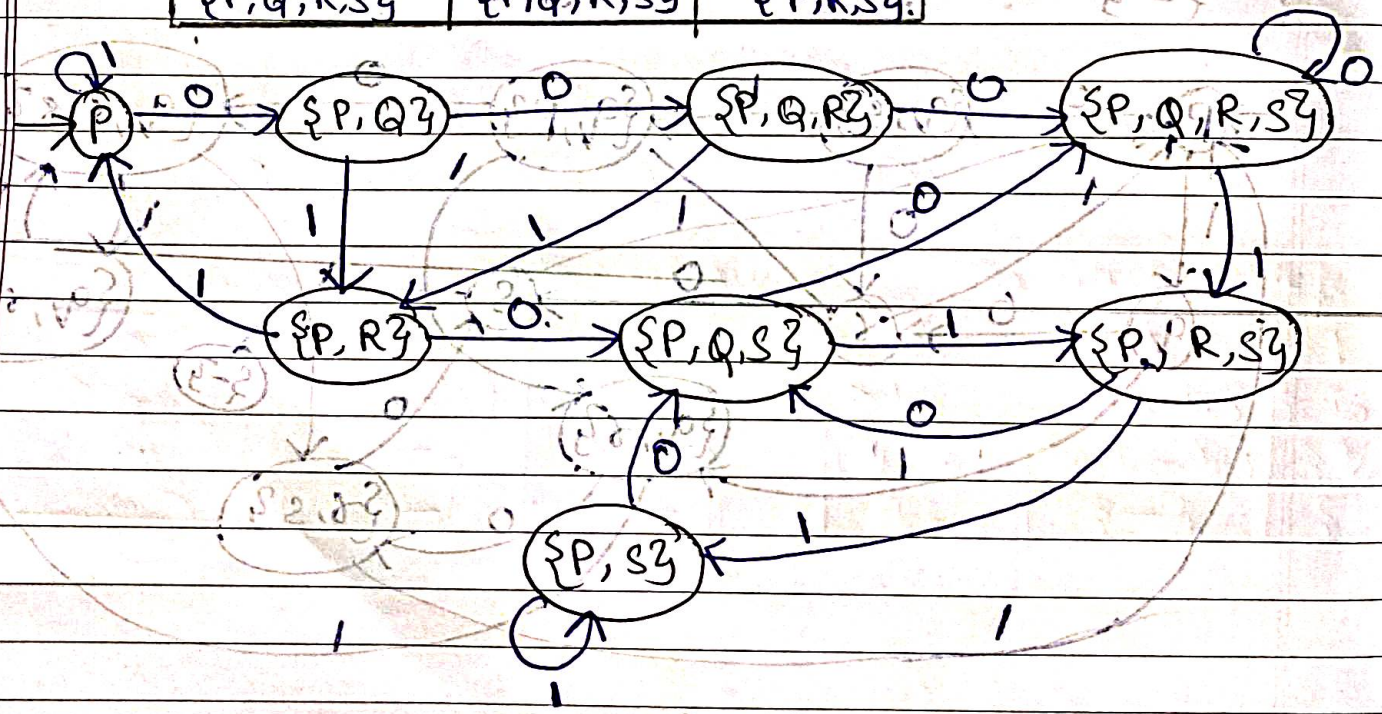
Ques 11 = Given,

NFA Transition Table :-

δ	0	1
P	{P,Q}	P
Q	R	R
R	S	—
S	S	—

DFA Transition Table :-

δ	0	1
P	{P,Q}	P
{P,Q}	{P,Q,R}	{P,R}
{P,R}	{P,Q,S}	{P}
{P,Q,R}	{P,Q,R,S}	{P,R}
{P,Q,S}	{P,Q,R,S}	{P,R,S}
{P,R,S}	{P,Q,S}	{P,S}
{P,S}	{P,Q,S}	{P,S}
{P,Q,R,S}	{P,Q,R,S}	{P,R,S}



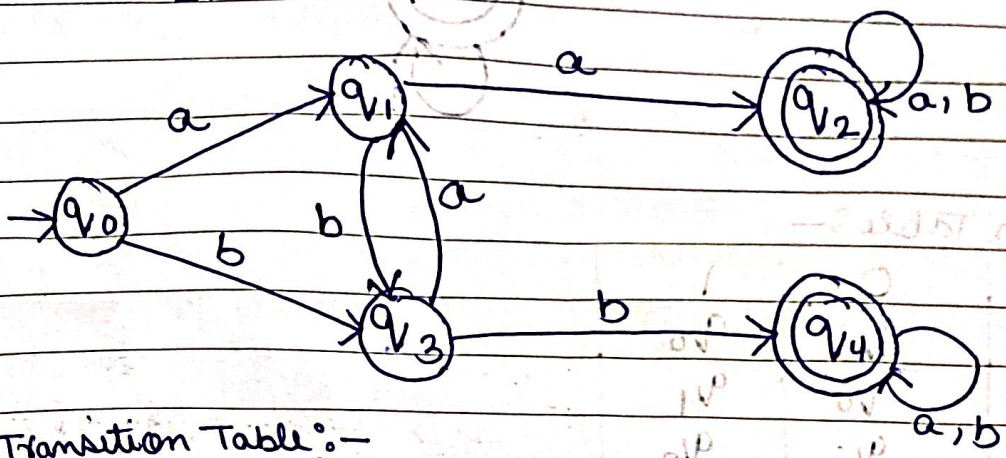
Ques13= $L = \{"aa", "bb", - - - \}$

$Q = q_0, q_1, q_2, q_3, q_4$

$\Sigma = \{a, b\}$

$q_0 = q_0$

$F = q_2, q_4$



Transition Table :-

δ	a	b.
q_0	q_1	q_2
q_1	q_2	q_3
q_2	q_2	q_2
q_3	q_1	q_4
q_4	q_4	q_4

Transition functions :-

$\delta : (q_0, a) = q_1$

$\delta : (q_0, b) = q_2$

$\delta : (q_1, a) = q_2$

$\delta : (q_1, b) = q_3$

$\delta : (q_2, a) = q_2$

$\delta : (q_2, b) = q_2$

$\delta : (q_3, a) = q_1$

$\delta : (q_3, b) = q_4$

$\delta : (q_4, a) = q_4$

$\delta : (q_4, b) = q_4.$

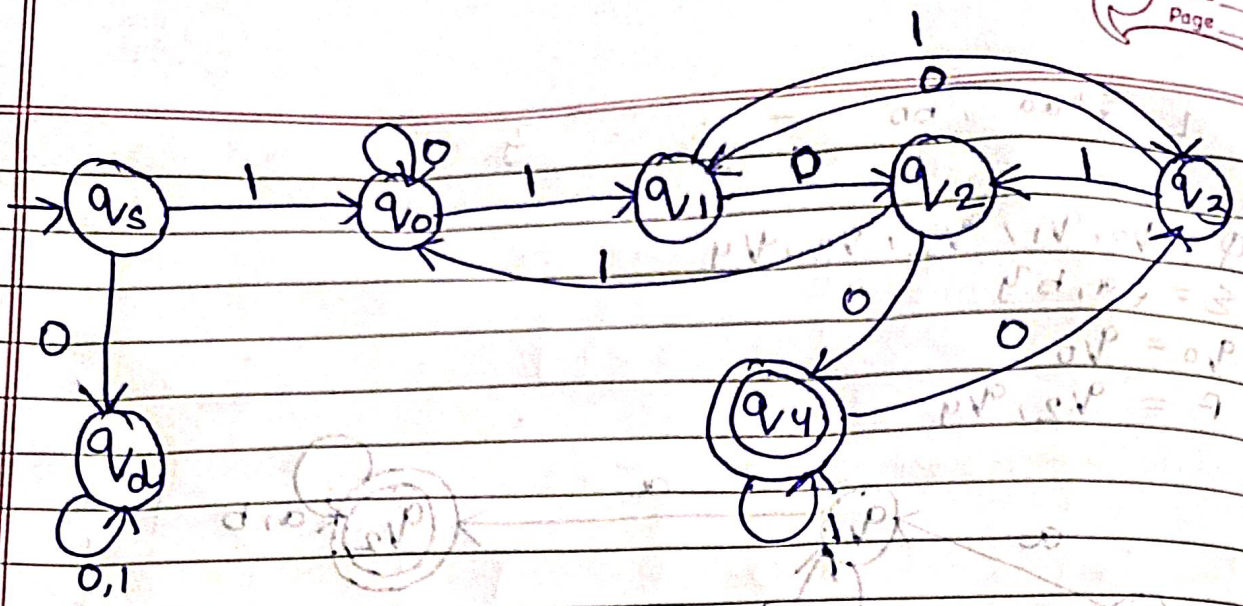
Ques14= Given, $L = \{1010, 10100, - - - \}$

$Q = q_s, q_0, q_1, q_2, q_3, q_4, q_d$

$\Sigma = \{1, 0\}$

$q_0 = q_0$

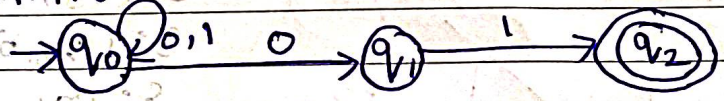
$f = q_4$



Transition Table :-

δ	0	1
→q_s	q_d	q_0
q_0	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2
(*)q_4	q_3	q_4
q_d	q_d	q_d

Ques 15= Given NFA :-



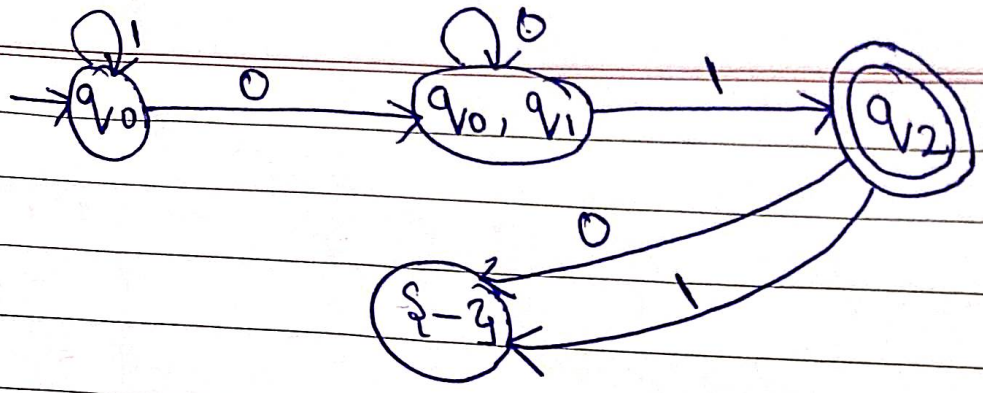
Transition table for NFA :

δ	0	1
→q_0	$\{q_0, q_1\}$	$\{q_0\}$
q_1	—	$\{q_2\}$
(*)q_2	—	—

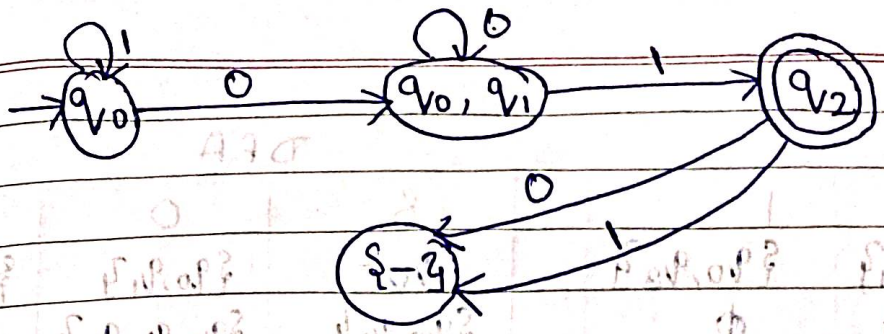
⇒

Transition table for DFA

δ	0	1
→q_0	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_2\}$
(*)q_2	$\{-\}$	$\{-\}$

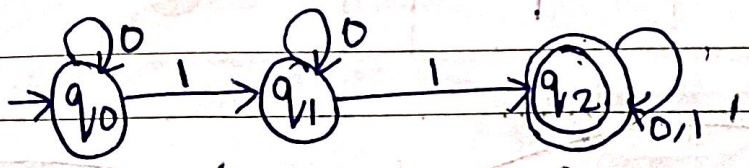


DFA for given NFA.



DFA for given NFA.

Ques 19= Given,



checking for 101101

for 1 :- $q_0 \xrightarrow{1} q_1$

for 0 : $q_1 \xrightarrow{0} q_1$

for 1 : $q_1 \xrightarrow{1} q_2$

for 1 : $q_2 \xrightarrow{1} q_2$

for 0 : $q_2 \xrightarrow{0} q_2$

for 1 : $q_2 \xrightarrow{1} q_2$

Hence, the given automata is acceptable for given string.

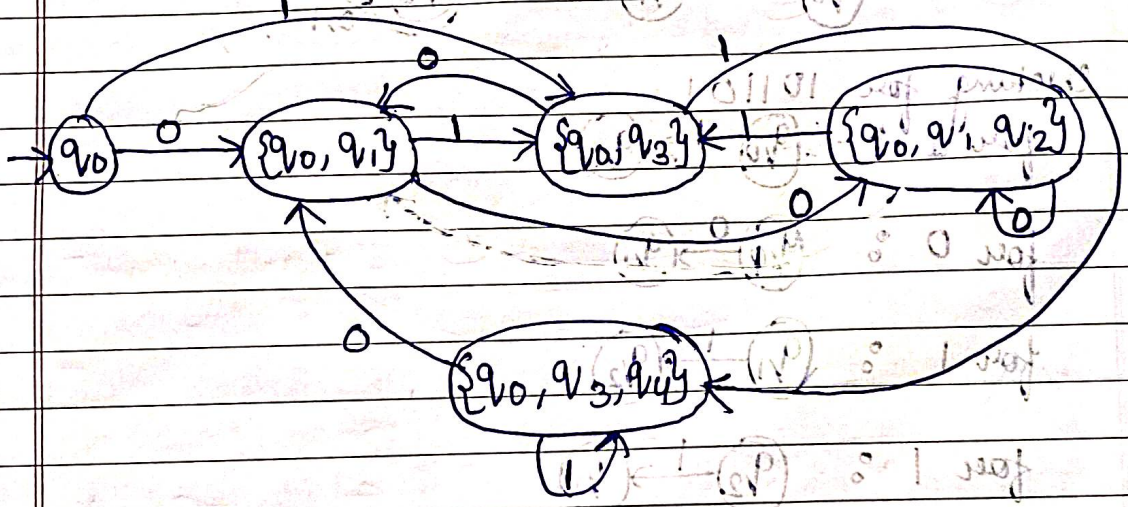
Ques 20- Given,

NFA

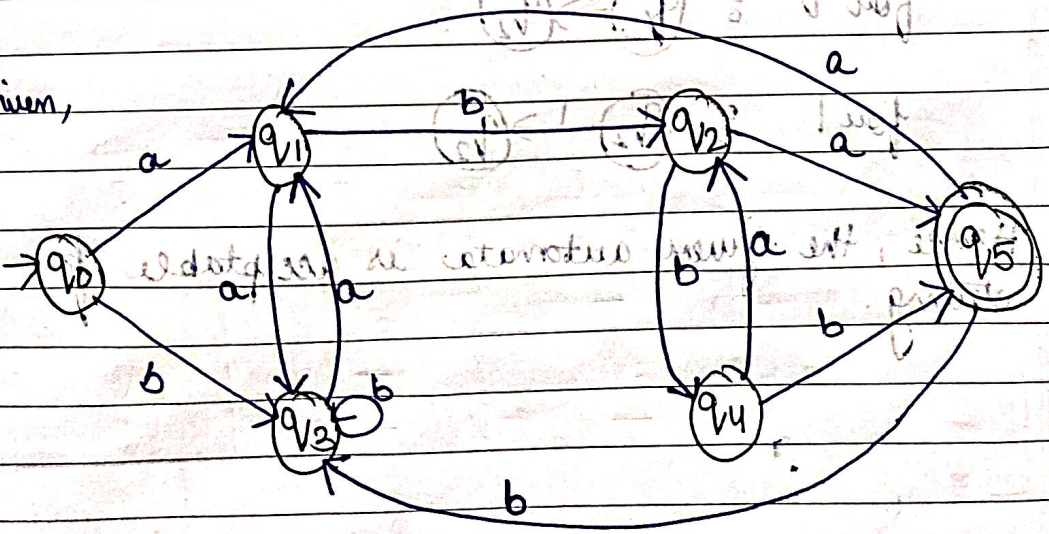
δ	0	1
q_0	$\{q_0, q_1\}$	$\{q_0, q_3\}$
q_1	$\{q_2\}$	ϕ
q_2	ϕ	ϕ
q_3	ϕ	$\{q_4\}$
q_4	ϕ	ϕ

DFA

δ	0	1
q_0	$\{q_0, q_1\}$	$\{q_0, q_3\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_3\}$
$\{q_0, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_3, q_4\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_3\}$
$\{q_0, q_3, q_4\}$	$\{q_0, q_1\}$	$\{q_0, q_3, q_4\}$



Ques 23= Given,



Step 1 – Remove unreachable states
There are no unreachable states

Step 2 – Transition Table for given DFA

δ	a	b
→q_0	q_1	q_3
q_1	q_3	q_2
q_2	q_5	q_4
q_3	q_1	q_3
q_4	q_2	q_5
(q_5)	q_1	q_3

Step 3 – Non-final state transition table.

δ	a	b
→q_0	q_1	q_3
q_1	q_3	q_2
q_2	q_5	q_4
q_3	q_1	q_3
q_4	q_2	q_5

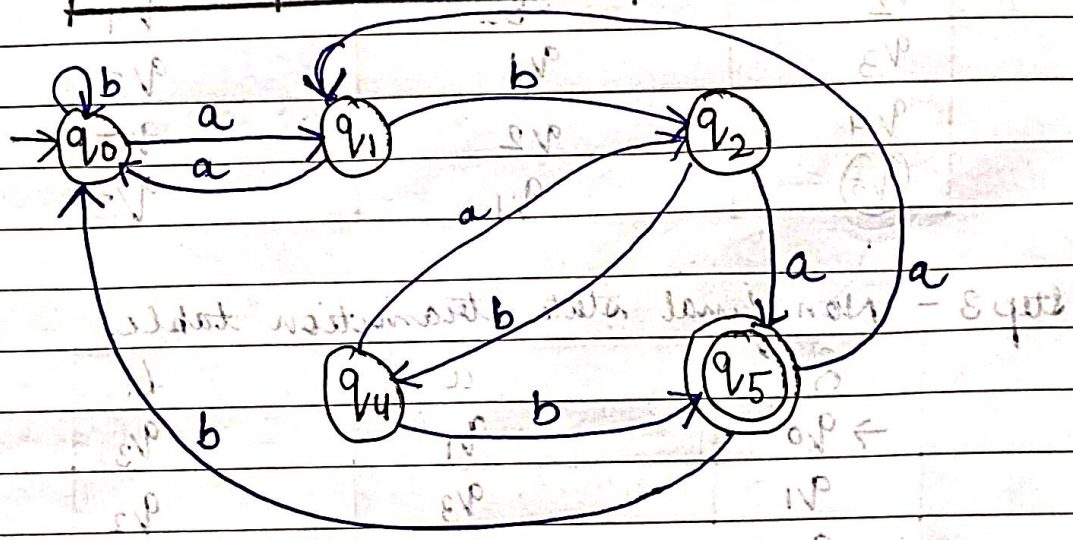
Step 4 – Final state transition table.

δ	a	b
q_5	q_1	q_3

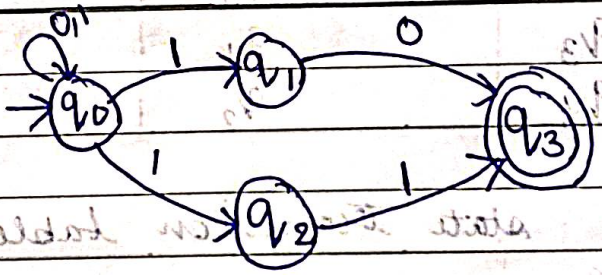
Step 5 – Remove duplicate rows.
q_0, q_3 are duplicate so we replace q_3 with q_0 and remove q_3 row.

Step 6- Final transition table

δ	a	b
→q_0	q_1	q_0
q_1	q_0	q_2
q_2	q_5	q_4
q_4	q_2	q_5
(q_5)	q_1	q_0



Ques 22- NFA :-

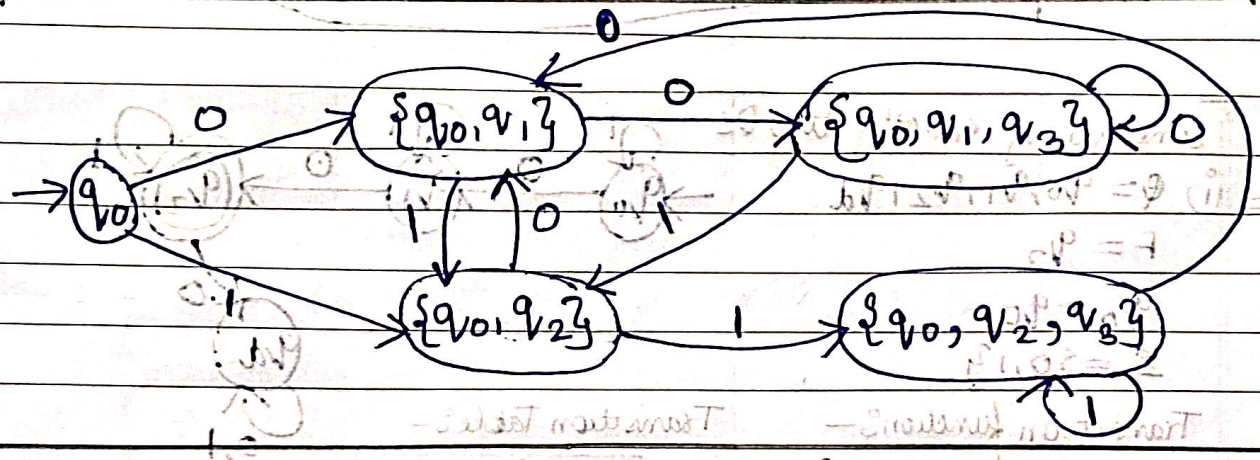


Transition Table :-

δ	0	1
→q_0	q_1	$\{q_0, q_2\}$
q_1	q_3	—
q_2	—	q_3
q_3	—	—

Transition Table for DFA

δ	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_3\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_0, q_2, q_3\}$
$(\{q_0, q_1, q_3\})$	$\{q_0, q_1, q_3\}$	$\{q_0, q_2\}$
$(\{q_0, q_2, q_3\})$	$\{q_0, q_1\}$	$\{q_0, q_2, q_3\}$

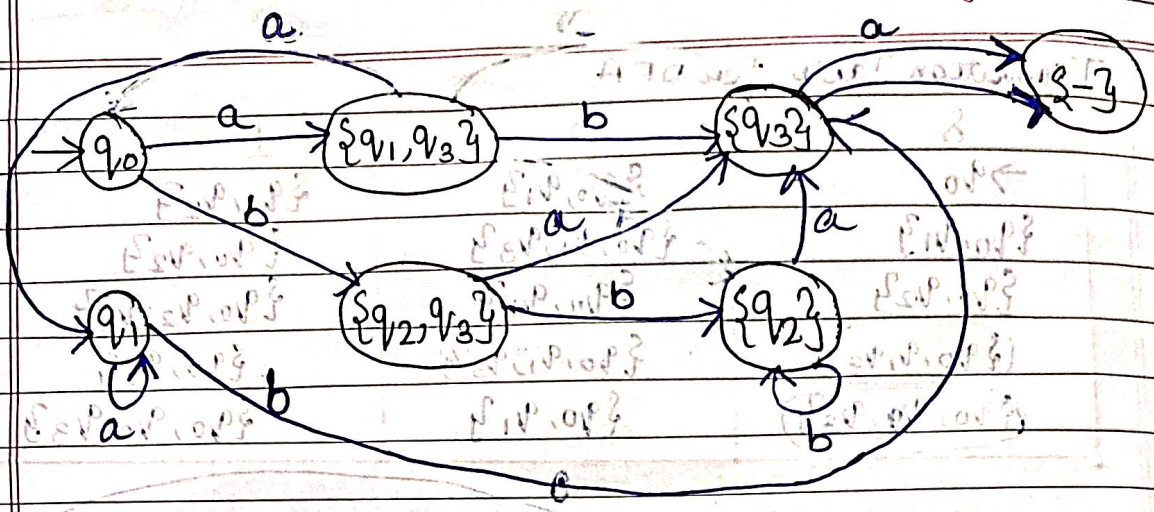


δ	a	b
$\rightarrow q_0$	$\{q_1, q_3\}$	$\{q_2, q_3\}$
q_1	q_1	q_3
q_2	q_3	q_2
(q_3)	—	—

⇐ NFA to table

Now, the DFA table.

δ	a	b
$\rightarrow\{q_0\}$	$\{q_1, q_3\}$	$\{q_2, q_3\}$
$\{q_1, q_3\}$	$\{q_1\}$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_3\}$	$\{q_2\}$
$\{q_3\}$	$\{-\}$	$\{-\}$
$\{q_2\}$	$\{q_3\}$	$\{q_2\}$
$\{q_1\}$	$\{q_1\}$	$\{q_3\}$



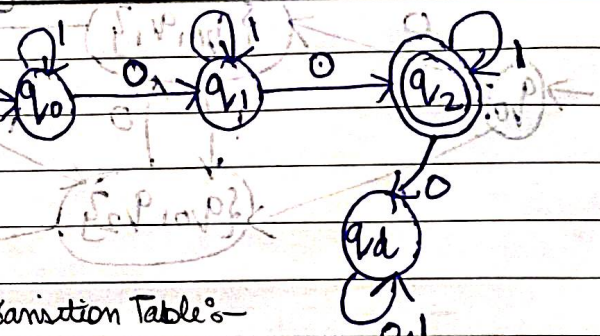
Ques25= containing exactly two 0's

(i) $Q = q_0, q_1, q_2, q_d$

$F = q_2$

$q_0 = q_0$

$\Sigma = \{0, 1\}$



Transition functions:-

$\delta : (q_0, 0) = q_1$ $\delta : (q_0, 1) = q_0$

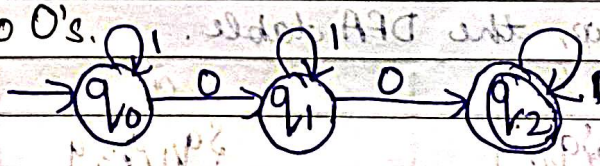
$\delta : (q_1, 0) = q_2$ $\delta : (q_1, 1) = q_1$

$\delta : (q_2, 0) = q_d$ $\delta : (q_2, 1) = q_2$

Transition Table:-

δ	0	1
$\rightarrow q_0$	q_1	q_0
q_1	q_2	q_1
q_2	q_d	q_2

(ii) containing at least two 0's.



$Q = q_0, q_1, q_2$

$F = q_2$

$q_0 = q_0$

$\Sigma = \{0, 1\}$

Transition functions:-

$\delta : (q_0, 0) = q_1$ $\delta : (q_0, 1) = q_0$

$\delta : (q_1, 0) = q_2$ $\delta : (q_1, 1) = q_1$

$\delta : (q_2, 0) = q_2$ $\delta : (q_2, 1) = q_2$

Transition table:-

δ	0	1
$\rightarrow q_0$	q_1	q_0
q_1	q_2	q_1
q_2	q_2	q_2