

<p>Name: Harshit Jain Access ID: hmj5262 Recitation: 8</p>

Problem 0

Points:

Acknowledgements

- (a) I did not work in a group.
- (b) I did not consult without anyone my group members.
- (c) I did not consult any non-class materials.

Problem 1**Points:****Compare Growth Rates**

- (a) $n^{1.5} = \Omega(n^{1.3}) \quad \because n^{1.3} < n^{1.5}$
- (b) $2^{n-1} = \Theta(2^n) \quad \because 2^{n-1} = \frac{2^n}{2}$ and constants (in this case, $\frac{1}{2}$) does not matter
- (c) $n^{1.3 \log n} = \Omega(n^{1.5}) \quad \because$ asymptotically $n^{1.3 \log n} > n^{1.5}$
- (d) $3^n = \Omega(n \cdot 2^n) \quad \because$ asymptotically $3^n > n \cdot 2^n$
- (e) $(\log n)^{100} = O(n^{0.1}) \quad \because$ power of $\log n$ will only weaken the infinity of $\log n$ and $n^{0.1} > (\log n)^{100}$
- (f) $n = \Omega((\log n)^{\log(\log n)}) \quad \because$ infinity of $(\log n)^{\log(\log n)} >$ infinity of n
- (g) $2^n = \Omega(n!)$ \because infinity of exponential function is greater than the infinity of factorial function
- (h) $\log(e^n) = O(n \cdot \log n) \quad \because \log(e^n) = n$ and we know that $n < n \log n$
- (i) $n + \log n = \Theta(n + (\log n)^2) \quad \because$ dominating term is n and powers of $\log n$ will not affect infinities
- (j) $5n + \sqrt{n} = \Omega(\log n + n) \quad \because$ comparable functions are \sqrt{n} and $\log n$; $\sqrt{n} > \log n$

Problem 2

Points:

Tribonacci Numbers

- (a) We proceed by induction on the variable i . Let $P(i)$ holds the property for i . $P(i)$ be the statement that $R(i) \geq 3^{i/2} \forall i \geq 2$.

Base Case:

$$R(0) = 1, R(1) = 2, R(2) = 3 \text{ (Given)}$$

$$\text{When } i = 0, R(0) \geq 3^{0/2} \text{ [True]}$$

$$\text{When } i = 1, R(1) \geq 3^{1/2} \text{ [True]}$$

$$\text{When } i = 2, R(2) \geq 3^{2/2} \text{ [True]}$$

$P(1)$ is True. Therefore, base case is proved.

Inductive Hypothesis ($i = k$):

Let k be any arbitrary natural number and $k > 2$ and we assume that $P(k)$ is True.

That means, $R(k) \geq 3^{k/2}$.

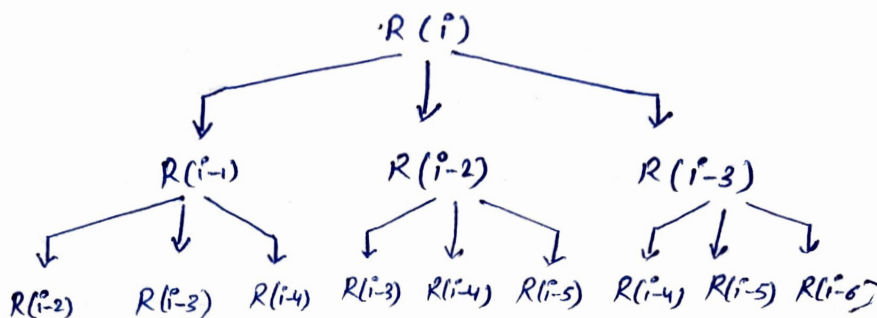
Inductive Step ($i = k + 1$):

We must show that $P(k+1)$ is True. That means, $R(k+1) \geq 3^{(k+1)/2}$.

Given equation: $R(i) = R(i-1) + R(i-2) + R(i-3)$.

$$\begin{aligned} R(k+1) &= R(k) + R(k-1) + R(k-2) \\ &\Rightarrow (\geq 3^{k/2}) + (\geq 3^{(k-1)/2}) + (\geq 3^{(k-2)/2}) \\ &\Rightarrow \geq 3^{k/2}(1 + 3^{-1/2} + 3^{-1}) \\ &\Rightarrow \geq 3^{k/2}(1.9) \\ &\Rightarrow \geq 3^{k/2}(\geq 3^{1/2}) \\ &\Rightarrow \geq 3^{(k+1)/2} \end{aligned}$$

Hence, $P(k+1)$ is proved. \square



(b)

Problem 3

Points:

Big Oh Definitions