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Recitation: 8

Problem 0　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　| **Points:** |

Acknowledgements

(a) I did not work in a group.

(b) I did not consult without anyone my group members.

(c) I did not consult any non-class materials.

Problem 1

<div style="border:1px solid black; display:inline-block; padding:4px;">**Points:**</div>

Analyze Running Time

1. **Runtime:** $\theta(n^2)$

    When $i = 1$, then $j = 1$. Here, while loop runs for $\frac{n-1}{5}$ times.

    When $i = 2$, then $j = 2$. Here, while loop runs for $\frac{n-2}{5}$ times.

    $\vdots$

    When $i = (n-5)$, then $j = (n-5)$. Here, while loop runs for 1 time.

    $\vdots$

    When $i = (n-1)$, then $j = (n-1)$. Here, while loop runs for 1 time.

    Total time: $\frac{n-1}{5} + \frac{n-2}{5} + \ldots + \frac{5}{5} + 1 + 1 + 1 + 1 + 1 \Rightarrow \theta(n^2)$

2. **Runtime:** $\theta(n^2)$

    When $i = 1$, then while loop runs from 4 to n. Time $= (n-4)$

    When $i = 2$, then while loop runs from 8 to n. Time $= (n-8)$

    $\vdots$

    When $i = \frac{n}{4}$, then while loop will run 1 time. Time $= 1$

    For loop will run for total of n times.

    $\Rightarrow \theta(n^2)$

3. **Runtime:** $\theta(n log n)$

    The main while loop will run for $\theta(log n)$ time as i is reduced to its half after every iteration.

    The inner for loop will run for $\frac{n}{2} + \frac{n}{2^2} + \frac{n}{2^3} + \ldots + 1 = \theta(n)$

    So, total time $\Rightarrow \theta(n log n)$

Problem 2

Points:

Polynomials and Horner's rule

   (a)

   (b)

   (c)

Problem 3   <u>**Points:**</u>

Solving recurrences

(a) Height $= \log_2 n$

Branching factor $= 2$

The size of the sub-problems at depth $k = \frac{n}{2^k}$

Number of sub-problems at depth $k = 2^k$

Total work done $=$

$$\sum_{k=0}^{\log_2 n} (2^k) \cdot \left(\frac{n}{2^k}\right)^{\frac{1}{2}} \Rightarrow \sqrt{n} \cdot \sum_{k=0}^{\log_2 n} (2^{\frac{k}{2}}) \Rightarrow \sqrt{n} \cdot \theta(\sqrt{n}) \Rightarrow \theta(n)$$

(Note that, $\sum_{k=0}^{\log_2 n} (2^{\frac{k}{2}})$ is a geometric series $= 2^{\frac{0}{2}} + 2^{\frac{1}{2}} + \cdots + 2^{\frac{\log_2 n}{2}} = \theta(\sqrt{n})$)

(b) Height $= \log_3 n$

Branching factor $= 2$

The size of the sub-problems at depth $k = \frac{n}{3^k}$

Number of sub-problems at depth $k = 2^k$

Total work done $=$

$$\sum_{k=0}^{\log_3 n} (2^k) \cdot \left(\frac{n}{3^k}\right)^0 \Rightarrow \sum_{k=0}^{\log_3 n} (2^k) \Rightarrow 2^0 + 2^1 + 2^3 + \cdots + 2^{\log_3 n} \Rightarrow \theta(n^{\log_2 3})$$

(c) Height $= \log_4 n$

Branching factor $= 5$

The size of the sub-problems at depth $k = \frac{n}{4^k}$

Number of sub-problems at depth $k = 5^k$

Total work done $=$

$$\sum_{k=0}^{\log_4 n} (5^k) \cdot \left(\frac{n}{4^k}\right)^1 \Rightarrow n \cdot \sum_{k=0}^{\log_4 n} \left(\left(\frac{5}{4}\right)^k\right) \Rightarrow n \cdot \left(\left(\frac{5}{4}\right)^0 + \left(\frac{5}{4}\right)^1 + \cdots + \left(\frac{5}{4}\right)^{\log_4 n}\right) \Rightarrow n \cdot \theta\left(\frac{5^{\log_4 n}}{n}\right) \Rightarrow \theta(n^{\log_5 4})$$

(d) Height $= \log_7 n$

Branching factor $= 7$

The size of the sub-problems at depth $k = \frac{n}{7^k}$

Number of sub-problems at depth $k = 7^k$

Total work done $=$

$$\sum_{k=0}^{\log_7 n} (7^k) \cdot \left(\frac{n}{7^k}\right)^1 \Rightarrow n \cdot \sum_{k=0}^{\log_7 n} 1 \Rightarrow n \cdot \log n \Rightarrow \theta(n \log n)$$

(e) Height $= \log_3 n$

Branching factor $= 9$

The size of the sub-problems at depth $k = \frac{n}{3^k}$

Number of sub-problems at depth $k = 9^k$

Total work done $=$

$$\sum_{k=0}^{\log_3 n} (9^k) \cdot (\frac{n}{3^k})^2 \Rightarrow n^2 \cdot \sum_{k=0}^{\log_3 n} 1 \Rightarrow n^2 \cdot \log n \Rightarrow \theta(n^2 \log n)$$