

<p>Name: Harshit Jain Access ID: hmj5262 Recitation: 8</p>

Problem 0

Points:

Acknowledgements

- (a) I worked with Yug Jarodiya.
- (b) I did not consult with anyone in my group members.
- (c) I did not consult any non-class materials.

Problem 1**Points:****(a) Objective Function:**

Let x_{ij} be a variable representing whether client i is served by server j . So, $x_{ij} = 0$ or 1 . The objective function is to minimize the total service cost, which is the sum of the service costs over all clients for the optimal solution. It is represented as:

$$\text{minimize } \sum_{i \in C} \sum_{j \in S} c_{ij} x_{ij}$$

.

Constraints:

- Each client must be served by at least one server, which can be represented as:

$$\sum_{j \in S} (x_{ij} \geq 1) \forall i \in C$$

.

- At most, k servers can be functional at a time, which can be represented as:

$$\sum_{j \in S} (x_{ij} \leq k) \forall i \in C$$

.

- The variable x_{ij} can only take the values 0 or 1, which can be represented as:

$$x_{ij} \in \{0, 1\} \forall i \in C, j \in S$$

.

(b) Objective Function:

Let y_i be continuous variable representing the amount of service cost that must be provided to client i . The objective function is to maximize the total amount of service cost that can be provided to the clients, which can be represented as:

$$\text{maximize } \sum_{i \in C} y_i$$

Constraints:

- The amount of service cost provided to each client must be less than or equal to the service cost of the corresponding server, which can be represented as:

$$y_i \leq c_{ij} \forall i \in C, j \in S$$

- The total amount of service cost provided to all clients must be less than or equal to the total amount of service cost that can be provided by the k servers, which can be represented as:

$$(\sum_{i \in C} y_i) \leq (\sum_{j \in S} c_{ij}) \forall i \in C, j \in S$$

- The continuous variable y_i must be non-negative which can be represented as:

$$y_i \geq 0 \forall i \in C$$

Problem 2**Points:**

$$\text{maximize} \Rightarrow 2x_1 + 7x_2 + x_3$$

$$\text{subject to} \Rightarrow x_1 - x_3 = 7$$

$$3x_1 + x_2 \geq 24$$

$$x_2 \geq 0$$

$$x_3 \leq 0$$

Here, $(x_1 - x_3 = 7)$ will undergo equality to inequality which in turn make the constraints as $(x_1 - x_3 \leq 7)$ and $(x_1 - x_3 \geq 7)$.

Now, $(x_1 - x_3 \geq 7)$, $(3x_1 + x_2 \geq 24)$ and $(x_3 \leq 0)$ has the wrong inequality direction so we will multiple both sides by -1 to change the inequality which results into $(x_3 - x_1 \leq -7)$, $(-3x_1 - x_2 \leq -24)$, and $(-x_3 \geq 0)$ (non-negative constraint).

Here, non-negative constraint of x_1 is missing. So, $x_1 \geq 0$, $x_1 \leq 0$. We want to combine these two constraints in a way that we can enforce both these conditions together. We will rewrite $x_1 = x_1^+ - x_1^-$ in all the above equations.

Final answer:

$$\text{maximize} \Rightarrow 2(x_1^+ - x_1^-) + 7x_2 + x_3$$

$$\text{subject to} \Rightarrow (x_1^+ - x_1^-) - x_3 \leq 7$$

$$x_3 - (x_1^+ - x_1^-) \leq -7$$

$$(-3)(x_1^+ - x_1^-) - x_2 \leq -24$$

$$x_2 \geq 0$$

$$(-x_3) \geq 0$$

$$x_1^+ \geq 0$$

$$x_1^- \geq 0$$

According to Standard form 1:

$$\text{maximize} \Rightarrow [2 \quad -2 \quad 7 \quad 1] \times \begin{bmatrix} x_1^+ \\ x_1^- \\ x_2 \\ x_3 \end{bmatrix}$$

$$\text{Subject to} \Rightarrow \begin{pmatrix} 1 & -1 & 0 & -1 \\ -1 & 1 & 0 & 1 \\ -3 & 3 & -1 & 0 \end{pmatrix} \times \begin{pmatrix} x_1^+ \\ x_1^- \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 7 \\ -7 \\ -24 \end{pmatrix}$$

$$x_1^+, x_1^-, x_2, -x_3 \geq 0$$