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Problem 0 | **Points:** |

Acknowledgements

(a) I worked with Yug Jarodiya.

(b) I did not consult with anyone in my group members.

(c) I did not consult any non-class materials.

Problem 1

Points:

(a) Objective Function:

Let x_{ij} be a variable representing whether client i is served by server j. So, $x_{ij} = 0$ or 1. The objective function is to minimize the total service cost, which is the sum of the service costs over all clients for the optimal solution. It is represented as:

$$\text{maximize} \sum_{i \in C} \sum_{j \in S} c_{ij} x_{ij}$$

.

Constraints:

- Each client must be served by at least one server, which can be represented as:

$$\sum_{j \in S} (x_{ij} \geq 1) \forall i \in C$$

.

- At most, k servers can be functional at a time, which can be represented as:

$$\sum_{j \in S} (x_{ij} \leq k) \forall i \in C$$

.

- The variable x_{ij} can only take the values 0 or 1, which can be represented as:

$$x_{ij} \in \{0,1\} \forall i \in C, j \in S$$

.

(b) Objective Function:

Let y_i be continous variable representing the amount of service cost that must be provided to client i. The objective function is to maximize the total amount of service cost that can be provided to the clients, which can be represented as:

$$\text{maximize} \sum_{i \in C} y_i$$

Constraints:

- The amount of service cost provided to each client must be less than or equal to the service cost of the corresponding server, which can be represented as:

$$y_i \leq c_{ij} \forall i \in C, j \in S$$

- The total amount of service cost provided to all clients must be less than or equal to the total amount of service cost that can be provided by the k servers, which can be represented as:

$$(\sum_{i \in C} y_i) \leq (\sum_{j \in S} c_{ij}) \forall i \in C, j \in S$$

- The continous variable y_i must be non-negative which can be represented as:

$$y_i \geq 0 \forall i \in C$$

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Problem 2 | Points: |

maximize $\Rightarrow 2x_1 + 7x_2 + x_3$

subject to $\Rightarrow x_1 - x_3 = 7$

$$3x_1 + x_2 \geq 24$$

$$x_2 \geq 0$$

$$x_3 \leq 0$$

Here, $(x_1 - x_3 = 7)$ will undergo equality to inequality which in turn make the constraints as $(x_1 - x_3 \leq 7)$ and $(x_1 - x_3 \geq 7)$.

Now, $(x_1 - x_3 \geq 7)$, $(3x_1 + x_2 \geq 24)$ and $(x_3 \leq 0)$ has the wrong inequality direction so we will multiple both sides by -1 to change the inequality which results into $(x_3 - x_1 \leq -7)$, $(-3x_1 - x_2 \leq -24)$, and $(-x_3 \geq 0)$ (non-negative constraint).

Here, non-negative constraint of x_1 is missing. So, $x_1 \geq 0, x_1 \leq 0$. We want to combine these two constraints in a way that we can enforce both these conditions together. We will rewrite $x_1 = x_1^+ - x_1^-$ in all the above equations.

Final answer:

maximize $\Rightarrow 2(x_1^+ - x_1^-) + 7x_2 + x_3$

subject to $\Rightarrow (x_1^+ - x_1^-) - x_3 \leq 7$

$$x_3 - (x_1^+ - x_1^-) \leq -7$$

$$(-3)(x_1^+ - x_1^-) - x_2 \leq -24$$

$$x_2 \geq 0$$

$$(-x_3) \geq 0$$

$$x_1^+ \geq 0$$

$$x_1^- \geq 0$$

According to Standard form 1:

$$\text{maximize} \Rightarrow \begin{bmatrix} 2 & -2 & 7 & 1 \end{bmatrix} \times \begin{bmatrix} x_1^+ \\ x_1^- \\ x_2 \\ x_3 \end{bmatrix}$$

$$\text{Subject to} \Rightarrow \begin{pmatrix} 1 & -1 & 0 & -1 \\ -1 & 1 & 0 & 1 \\ -3 & 3 & -1 & 0 \end{pmatrix} \times \begin{pmatrix} x_1^+ \\ x_1^- \\ x_2 \\ x_3 \end{pmatrix} \leq \begin{pmatrix} 7 \\ -7 \\ -24 \end{pmatrix}$$

$$x_1^+, x_1^-, x_2, -x_3 \geq 0$$