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**Problem 0**

<b>Points:</b>
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**Acknowledgements**

- (a) I worked with Yug Jarodiya.
- (b) I did not consult with anyone in my group members.
- (c) I did not consult any non-class materials.

**Problem 1**

<b>Points:</b>
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**Problem 2****Points:**

$$\text{maximize} \Rightarrow 2x_1 + 7x_2 + x_3$$

$$\text{subject to} \Rightarrow x_1 - x_3 = 7$$

$$3x_1 + x_2 \geq 24$$

$$x_2 \geq 0$$

$$x_3 \leq 0$$

Here,  $(x_1 - x_3 = 7)$  will undergo equality to inequality which in turn make the constraints as  $(x_1 - x_3 \leq 7)$  and  $(x_1 - x_3 \geq 7)$ .

Now,  $(x_1 - x_3 \geq 7)$ ,  $(3x_1 + x_2 \geq 24)$  and  $(x_3 \leq 0)$  has the wrong inequality direction so we will multiple both sides by  $-1$  to change the inequality which results into  $(x_3 - x_1 \leq -7)$ ,  $(-3x_1 - x_2 \leq -24)$ , and  $(-x_3 \geq 0)$  (non-negative constraint).

Here, non-negative constraint of  $x_1$  is missing. So,  $x_1 \geq 0$ ,  $x_1 \leq 0$ . We want to combine these two constraints in a way that we can enforce both these conditions together. We will rewrite  $x_1 = x_1^+ - x_1^-$  in all the above equations.

Final answer:

$$\text{maximize} \Rightarrow 2(x_1^+ - x_1^-) + 7x_2 + x_3$$

$$\text{subject to} \Rightarrow (x_1^+ - x_1^-) - x_3 \leq 7$$

$$x_3 - (x_1^+ - x_1^-) \leq -7$$

$$(-3)(x_1^+ - x_1^-) - x_2 \leq -24$$

$$x_2 \geq 0$$

$$(-x_3) \geq 0$$

$$x_1^+ \geq 0$$

$$x_1^- \geq 0$$

According to Standard form 1:

$$\text{maximize} \Rightarrow [2 \quad -2 \quad 7 \quad 1] \times \begin{bmatrix} x_1^+ \\ x_1^- \\ x_2 \\ x_3 \end{bmatrix}$$

$$\text{Subject to} \Rightarrow \begin{pmatrix} 1 & -1 & 0 & -1 \\ -1 & 1 & 0 & 1 \\ -3 & 3 & -1 & 0 \end{pmatrix} \times \begin{pmatrix} x_1^+ \\ x_1^- \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 7 \\ -7 \\ -24 \end{pmatrix}$$

$$x_1^+, x_1^-, x_2, -x_3 \geq 0$$