Name: Harshit Jain Access ID: hmj5262

Recitation: 8

Problem 0 Points:

Acknowledgements

- (a) I did not work in a group.
- (b) I did not consult without anyone my group members.
- (c) I did not consult any non-class materials.

Problem 1 Points:

The minimum cut of a weighted graph is defined as the minimum sum of weights of edges that, when removed from the graph, divide the graph into two sets.

```
Algorithm 1: UniqueMinimumCut
  Input : G = (V, (E, \ell_e))
  Output: A unique cut is present or not
1 C = STCut(G)
_{2} |C| = CapacityOfCut(C)
3 for e_i \in C do
      capacity(e_i) += 1
      |C_i| = STCut(C)
5
      if |C| == |C_i| and C \neq C_i then
6
          return "Min-cut is not unique"
7
      end if
8
9 end for
10 return "Min-cut is unique"
```

Conversely, if there is a different minimum cut C' in the original graph, there will be some $e_i \in C$ that is not in C', so increasing the capacity of that edge will not change the volume of C', thus $|C| = |C_i|$. In conclusion, the graph has a unique minimum cut iff $|C| < |C_i| \forall i$. The algorithm takes at most m+1 computing of minimum cuts, and therefore runs in polynomial time.