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**Problem 0**

**Points:**

**Acknowledgements**

(a) I worked with Yug Jarodiya.

(b) I did not consult with anyone in my group members.

(c) I did not consult any non-class materials.

**Problem 1**

**Points:**

(a) Objective Function:

Let $x_{ij}$ be a variable representing whether client $i$ is served by server $j$. So, $x_{ij} = 0$ or 1. The objective function is to minimize the total service cost, which is the sum of the service costs over all clients for the optimal solution. It is represented as:

$$\text{maximize} \sum_{i \in C} \sum_{j \in S} c_{ij} x_{ij}$$

.

Constraints:

- Each client must be served by at least one server, which can be represented as:

$$\sum_{j \in S} (x_{ij} \geq 1) \forall i \in C$$

.

- At most, $k$ servers can be functional at a time, which can be represented as:

$$\sum_{j \in S} (x_{ij} \leq k) \forall i \in C$$

.

- The variable $x_{ij}$ can only take the values 0 or 1, which can be represented as:

$$0 \leq x_{ij} \leq 1 \forall i \in C, j \in S$$

.

(b) Objective Function:

Let $y_i$ be continous variable representing the amount of service cost that must be provided to client $i$. The objective function is to maximize the total amount of service cost that can be provided to the clients, which can be represented as:

$$\text{maximize} \sum_{i \in C} y_i$$

Constraints:

- The amount of service cost provided to each client must eb less than or equal to the service cost of the corresponding server, which can be represented as:

$$y_i \leq c_{ij} \forall i \in C, j \in S$$

- The total amount of service cost provided to all clients must be less than or equal to the total amount of service cost that can be provided by the $k$ servers, which can be represented as:

$$(\sum_{i \in C} y_i) \leq (\sum_{j \in S} c_{ij}) \forall i \in C, j \in S$$

- The continous variable $y_i$ must be non-negative which can be represented as:

$$y_i \geq 0 \forall i \in C$$

**Problem 2**

**Points:**

maximize $\Rightarrow 2x_1 + 7x_2 + x_3$

subject to $\Rightarrow x_1 - x_3 = 7$

$3x_1 + x_2 \geq 24$

$x_2 \geq 0$

$x_3 \leq 0$

Here, $(x_1 - x_3 = 7)$ will undergo equality to inequality which in turn make the constraints as $(x_1 - x_3 \leq 7)$ and $(x_1 - x_3 \geq 7)$.

Now, $(x_1 - x_3 \geq 7)$, $(3x_1 + x_2 \geq 24)$ and $(x_3 \leq 0)$ has the wrong inequality direction so we will multiple both sides by $-1$ to change the inequality which results into $(x_3 - x_1 \leq -7)$, $(-3x_1 - x_2 \leq -24)$, and $(-x_3 \geq 0)$ (non-negative constraint).

Here, non-negative constraint of $x_1$ is missing. So, $x_1 \geq 0, x_1 \leq 0$. We want to combine these two constraints in a way that we can enforce both these conditions together. We will rewrite $x_1 = x_1^+ - x_1^-$ in all the above equations.

<u>Final answer:</u>

maximize $\Rightarrow 2(x_1^+ - x_1^-) + 7x_2 + x_3$

subject to $\Rightarrow (x_1^+ - x_1^-) - x_3 \leq 7$

$x_3 - (x_1^+ - x_1^-) \leq -7$

$(-3)(x_1^+ - x_1^-) - x_2 \leq -24$

$x_2 \geq 0$

$(-x_3) \geq 0$

$x_1^+ \geq 0$

$x_1^- \geq 0$

According to Standard form 1:

maximize $\Rightarrow \begin{bmatrix} 2 & -2 & 7 & 1 \end{bmatrix} \times \begin{bmatrix} x_1^+ \\ x_1^- \\ x_2 \\ x_3 \end{bmatrix}$

Subject to $\Rightarrow \begin{pmatrix} 1 & -1 & 0 & -1 \\ -1 & 1 & 0 & 1 \\ -3 & 3 & -1 & 0 \end{pmatrix} \times \begin{pmatrix} x_1^+ \\ x_1^- \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 7 \\ -7 \\ -24 \end{pmatrix}$

$x_1^+, x_1^-, x_2, -x_3 \geq 0$