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Problem 0

Points:

Acknowledgements

(a) I did not work in a group.

(b) I did not consult without anyone my group members.

(c) I did not consult any non-class materials.

Problem 1

Points:

The minimum cut of a weighted graph is defined as the minimum sum of weights of edges that, when removed from the graph, divide the graph into two sets.

Algorithm 1: UniqueMinimumCut

Input : $G = (V, (E, \ell_e))$
Output: A unique cut is present or not

C = STCut(G)
$|C|$ = CapacityOfCut(C)
for $e_i \in C$ **do**
  capacity(e_i) $+ = 1$
  $|C_i| =$ STCut(C)
  if $|C| == |C_i|$ *and* $C \neq C_i$ **then**
    return "Min-cut is not unique"
  end if
end for
return "Min-cut is unique"

Conversely, if there is a different minimum cut C' in the original graph, there will be some $e_i \in C$ that is not in C', so increasing the capacity of that edge will not change the volume of C', thus $|C| = |C_i|$. In conclusion, the graph has a unique minimum cut iff $|C| < |Ci| \ \forall\ i$. The algorithm takes at most $n+1$ computing of minimum cuts, and therefore runs in **polynomial time**.

Problem 2

Points:

Problem 3

Points: