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Recitation: 8

Problem 0

Points:

Acknowledgements

(a) I did not work in a group.

(b) I did not consult without anyone my group members.

(c) I did not consult any non-class materials.

Problem 1

Points:

Compare Growth Rates

(a) $n^{1.5} = \Omega(n^{1.3}) \ \because n^{1.3} < n^{1.5}$

(b) $2^{n-1} = \Theta(2^n) \ \because 2^{n-1} = \frac{2^n}{2}$ and constants (in this case, $\frac{1}{2}$) does not matter

(c) $n^{1.3logn} = \Omega(n^{1.5}) \ \because$ asymptotically $n^{1.3logn} > n^{1.5}$

(d) $3^n = \Omega(n \cdot 2^n) \ \because$ asymptotically $3^n > n \cdot 2^n$

(e) $(logn)^{100} = O(n^{0.1}) \ \because$ power of $logn$ will only weaken the infinity of $logn$ and $n^{0.1} > (logn)^{100}$

(f) $n = O((logn)^{log(logn)}) \ \because$ infinity of $(logn)^{log(logn)} >$ infinity of n

(g) $2^n = O(n!) \ \because$ infinity of factorial function is greater than the infinity of exponential function

(h) $log(e^n) = O(n \cdot logn) \ \because log(e^n) = n$ and we know that $n < nlogn$

(i) $n + logn = \Theta(n + (logn)^2) \ \because$ dominating term is n and powers of $logn$ will not affect infinities

(j) $5n + \sqrt{n} = \Theta(logn + n) \ \because$ both $logn$ and $\sqrt{n}$ grows slower than n. So, n term will dominate

Problem 2

Points:

Tribonaci Numbers

(a) We proceed by induction on the variable i. Let $P(i)$ holds the property for i. $P(i)$ be the statement that $R(i) \geq 3^{i/2}\ \forall\ i \geq 2$.

Base Case:

$R(0) = 1$, $R(1) = 2$, $R(2) = 3$ (Given)

When $i = 0$, $R(0) \geq 3^{0/2}$ [True]

When $i = 1$, $R(1) \geq 3^{1/2}$ [True]

When $i = 2$, $R(2) \geq 3^{2/2}$ [True]

$P(1)$ is True. Therefore, base case is proved.

Inductive Hypothesis ($i = k$):

Let k be any arbitrary natural number and $k > 2$ and we assume that $P(k)$ is True.

That means, $R(k) \geq 3^{k/2}$.

Inductive Step ($i = k+1$):

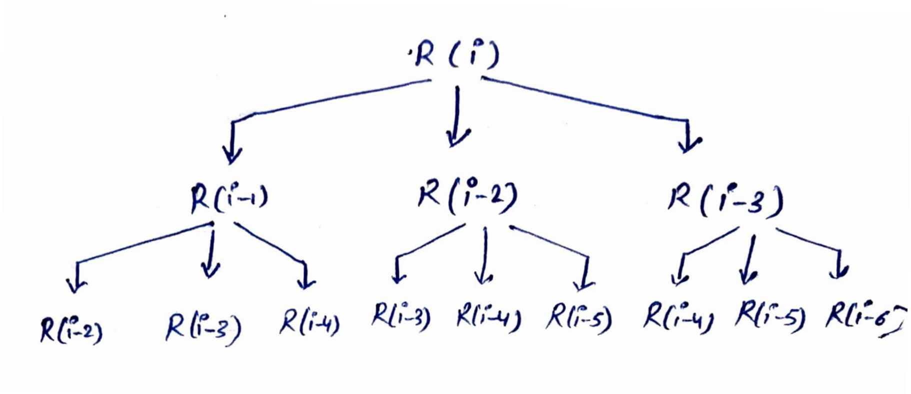
We must show that $P(k+1)$ is True. That means, $R(k+1) \geq 3^{(k+1)/2}$.

Given equation: $R(i) = R(i-1) + R(i-2) + R(i-3)$.

$$
\begin{aligned}
R(k+1) &= R(k) + R(k-1) + R(k-2) \\
&\Rightarrow (\geq 3^{k/2}) + (\geq 3^{(k-1)/2}) + (\geq 3^{(k-2)/2}) \\
&\Rightarrow \geq 3^{k/2}(1 + 3^{-1/2} + 3^{-1}) \\
&\Rightarrow \geq 3^{k/2}(1.9) \\
&\Rightarrow \geq 3^{k/2}(\geq 3^{1/2}) \\
&\Rightarrow \geq 3^{(k+1)/2}
\end{aligned}
$$

Hence, $P(k+1)$ is proved. □

(b)



Problem 3

Points:

Big Oh Definitions

Definition 1: It says that, after a certain point $(n = n_0)$, $f(n) < g(n)$. As Big-Oh gives the strict upper bound, then $g(n) \geq f(n)$ and this is possible iff c_1 is a positive constant i.e. $c_1 > 0$ and provided that n_0 cannot be a negative value.
So, if $f(n) \leq c_1 \cdot g(n)$ where $c_1 > 0$, then $f(n) = O_1(g(n))$ where $n > n_0$ and $n_0 > 0$.

Definition 2: It says that, all the Big-Oh conditions should be satisfied i.e. $n_0 > 0$ and c_2 should be positive value i.e. $c_2 > 0$.
So, if $f(n) \leq c_2 \cdot g(n)$ where $c_2 > 0$, then $f(n) = O_2(g(n))$ where $n > n_0$ and $n_0 > 0$.

From above, $f(n) = O_1(g(n)) = O_2(g(n))$ for all g.