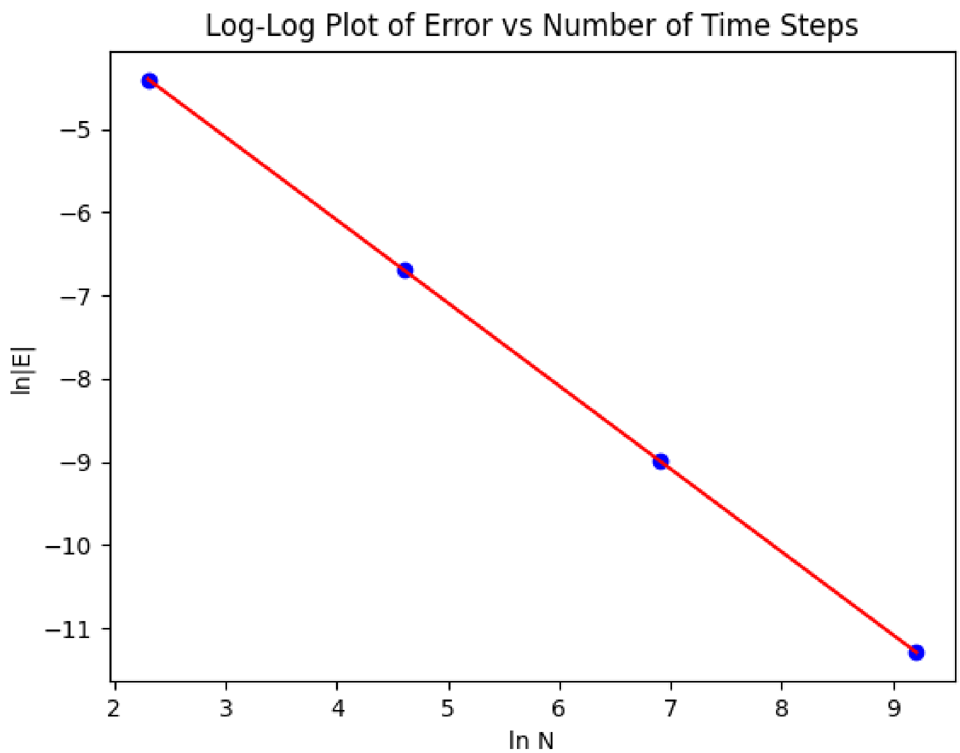
(a) Prices using Binomial Tree method are displayed in the table in (c) part.

(b) Price using Black-Scholes: 0.5224453276436325

(c) Table:

Number of Timesteps	Binomial Tree Solution	$\|E\|$
10	0.510182	0.0122635
100	0.521203	0.0012422
1000	0.522321	0.0001244
10000	0.522433	0.0000124

(d) The least squares line is: $\ln|E| = -0.9981 \ln N + -2.0992$



(e) The convergence rate of the Binomial Tree method is: 0.9981

Python Code:

```
# Import libraries
import numpy as np
from scipy.stats import norm
import matplotlib.pyplot as plt
import math

# Parameters
K = 10                                   # Strike price of option
r = 0.02                                 # Constant risk-free interest rate
sigma = 0.25                             # Constant volatility of the stock price
T = 0.25                                 # Time to maturity
S0 = 10                                  # Current stock price

# Function to calculate u and d for the binomial tree
def calculate_u_d(sigma, T, N):
  dt = T / N
  u = math.exp(sigma * math.sqrt(dt))               # Compute u
  d = 1 / u                              # Compute d
  return u, d

# Function to calculate option price using binomial tree method
def calculate_price_binomial_tree(N):
  dt = T / N
  u, d = calculate_u_d(sigma, T, N)
  p = (math.exp(r * dt) - d) / (u - d)              # Probability of up movement
  stock_price = [0] * (N + 1)                       # Initialize the stock price at maturity
  option_value = [0] * (N + 1)                      # Initialize the option values at maturity
```

```
    # Loop through each time step
    for i in range(N + 1):
        stock_price[i] = S0 * (u ** (N - i)) * (d ** i)        # Calculate stock price at maturity
        option_value[i] = max(stock_price[i] - K, 0)        # Calculate option value at maturity

    # Calculate option values at earlier time steps using backward recursion
    for j in range(N - 1, -1, -1):
        for i in range(j + 1):
            option_value[i] = math.exp(-r * dt) * (p * option_value[i] + (1 - p) * option_value[i + 1])

    # Return option value at time 0
    return option_value[0]

# Function to calculate option price using Black-Scholes formula
def calculate_price_black_scholes():
    d1 = (math.log(S0 / K) + (r + sigma ** 2 / 2) * T) / (sigma * math.sqrt(T))
    d2 = d1 - sigma * math.sqrt(T)
    price = S0 * norm.cdf(d1) - K * math.exp(-r * T) * norm.cdf(d2)    # Black-Scholes formula
    return price

if __name__ == "__main__":

    # (b) Calculate the option price using the Black-Scholes formula
    price_black_scholes = calculate_price_black_scholes()

    # (a) Calculate option prices using binomial tree method for number of time steps N = 10, 100,
1000, and 10000
    prices_binomial_tree = []
    errors = []
```

```
for N in [10, 100, 1000, 10000]:
    price = calculate_price_binomial_tree(N)
    prices_binomial_tree.append(price)
    error = abs(price - price_black_scholes)
    errors.append(error)

# (c) Print the table and Black-Scholes price
print("Price using Black-Scholes: ", price_black_scholes)
print("N\tBinomial Tree\t|E|")
for i in range(len(prices_binomial_tree)):
    print(f"{[10, 100, 1000, 10000][i]}\t{prices_binomial_tree[i]:.6f}\t{errors[i]:.7f}")

# (d) Create log-log plot of ln|E| vs ln N
ln_N = [math.log(N) for N in [10, 100, 1000, 10000]]        # Calculate ln(N) for each value of N
ln_E = [math.log(E) for E in errors]                        # Calculate ln(errors) for each error value
plt.plot(ln_N, ln_E, 'bo')                                  # Plot ln|E| vs ln N with blue circles
plt.xlabel('ln N')                                          # Set x-axis label as 'ln N'
plt.ylabel('ln|E|')                                         # Set y-axis label as 'ln|E|'
plt.title('Log-Log Plot of Error vs Number of Time Steps')  # Set plot title

# (d) Perform linear regression to find slope and intercept of the least squares line
slope, intercept = np.polyfit(ln_N, ln_E, 1)

# (d) Plot the least squares line on the plot with red color
plt.plot(ln_N, slope*np.array(ln_N) + intercept, 'r')

# (d) Print the equation of the least squares line
print(f"The least squares line is: ln|E| = {slope:.4f} lnN + {intercept:.4f}")
```

```
# (e) Calculate the convergence rate (-A) from the slope
convergence_rate = -slope

# (e) Print the convergence rate
print(f"The convergence rate of the Binomial Tree method is: {convergence_rate:.4f}")

# Display the plot
plt.show()
```