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Problem 1

Proof by reduction: SAT $\leq_p$ DOUBLE-SAT

We will define the function $f(\phi) = \psi$ such that: $\psi = \phi \wedge (x \vee \bar{x})$ where x is a variable that does not appear in ϕ.

If n is the length of ϕ, it will take $O(n)$ time to find a variable name that is not in ϕ and constant time to append "$\wedge(x \vee \bar{x})$" to ϕ. Thus this reduction can be computed in polynomial time.

If $\phi \in$ SAT, then $\psi \in$ DOUBLE-SAT. If $\phi \in$ SAT, then we know that the left side of ψ is satisfiable. We can then set our new variable x to True, which will satisfy ψ. Alternatively, we can then set our new variable x to False, which will satisfy ψ. Thus there are at least 2 different satisfying assignments and $\psi \in$ DOUBLE-SAT.

If $\phi \notin$ SAT, then $\psi \notin$ DOUBLE-SAT. If $\phi \notin$ SAT, then there is no way to satisfy the left side of ψ. Because of the $\wedge$ operator, this leaves us with no way to satisfy ψ overall. If ψ cannot be satisfied then it certainly cannot have 2 satisfying assignments and $\psi \notin$ DOUBLE-SAT.

Thus, DOUBLE-SAT is in NP-HARD. Since it is in NP as well, **DOUBLE-SAT is NP-Complete**.

Problem 2

First, we show that the set-partition problem belongs to NP. Given the set S, our certificate is a set A which is a solution to the problem. The verification algorithm checks that $A \subseteq S$ and that $\sum_{x \in A} x = \sum_{x \in S \setminus A} x$. Clearly, this can be done in polynomial time.

To show that the problem is NP-complete, we reduce from SUBSET-SUM. Let (S,t) be an instance of SUBSET-SUM. The problem is to determine whether there is a subset $A \subseteq S$ such that $t = \sum_{x \in A} x$. We construct an instance S_0 of the set-partition problem by setting $S_0 = S \cup \{r\}$ where $r = 2t - \sum_{x \in S} x$. Clearly, this reduction can be done in polynomial time.

Now it remains to show that there is a subset of S whose sum is t if and only if S_0 can be partitioned into two distinct subsets of equal weight. First, suppose that $A \subseteq S$ and the sum of elements in A is t. But now we have $\sum_{x \in S_0 \setminus A} x = \sum_{x \in S \setminus A} x + r = 2t - \sum_{x \in A} x = 2t - t = t = \sum_{x \in A} x$. Thus, the partition into A and $S_0 \setminus A$ is a solution to the set-partitioning problem.

Now, suppose that there exists $A \in S_0$ such that $\sum_{x \in A} x = \sum_{x \in S \setminus A} x$. Without loss of generality, assume that $r \notin A$. But now we have $2t = \sum_{x \in S} x + r = \sum_{x \in S_0} x = \sum_{x \in A} x + \sum_{x \in S \setminus A} x = 2 \sum_{x \in A} x$. Thus, $\sum_{x \in A} x = t$ and A is a solution to the subset-sum problem.

Problem 3

Problem 4