Name: Harshit Jain
User ID: hmj5262

Problem 1

1. Let's suppose $\sqrt{13}$ is a rational number. Then we can write it $\sqrt{13} = \frac{a}{b}$ where a, b are whole numbers, b not zero.

   We additionally assume that this $\frac{a}{b}$ is simplified to lowest terms, since that can obviously be done with any fraction. Notice that in order for $\frac{a}{b}$ to be in simplest terms, both of a and b cannot be even. One or both must be odd. Otherwise, we could simplify $\frac{a}{b}$ further.

   From the equality $\sqrt{13} = \frac{a}{b}$ it follows that $13 = \frac{a^2}{b^2}$, or $a^2 = 13b^2$. Since 13 is prime and a^2 is a multiple of 13, then a is multiple of 13.

   If we substitute $a = 13k$ into the original equation $\sqrt{13} = \frac{a}{b}$, we get:

   $\Rightarrow (13k)^2 = 13b^2$

   $\Rightarrow b^2 = 13k^2$

   Since 13 is prime and b^2 is a multiple of 13 then b is multiple of 13.

   We now have a contradiction since a and b must have no common factors (except 1) but we have proved that if $\frac{a}{b}$ exits then a and b must have common factor 13.

   So $\frac{a}{b}$ can not exist and the square root of 13 is irrational.

2. Yes, we can prove that square root of any prime number is irrational.

   Let's suppose $\sqrt{p}$ is a rational number, where p is any prime number. Let $\sqrt{p} = \frac{m}{n}$ where $m, n \in N$. and m and n have no factors in common.

   Now $p = \frac{m^2}{n^2}$, or $m^2 = pn^2$.

   Since p is prime and m^2 is a multiple of p then m is multiple of p.

   If we substitute $m = pk$ into the original equation $\sqrt{p} = \frac{m}{n}$, we get:

   $\Rightarrow (pk)^2 = pn^2$

   $\Rightarrow n^2 = pk^2$

   Since p is prime and n^2 is a multiple of p then n is multiple of p.

   We now have a contradiction since m and n must have no common factors (except 1) but we have proved that if $\frac{m}{n}$ exits then m and n must have common factor p.

   So $\frac{m}{n}$ can not exist and the square root of any prime is irrational.

Problem 2

1.