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Problem 1

Proof by contradiction: assume that $L = \{w : w \text{ has balanced parentheses } \}$ is regular.

Then by the Pumping Lemma, there is some pumping length n, such that any word w in L of length at least n can be split into $w = xyz$ satisfying the following conditions:

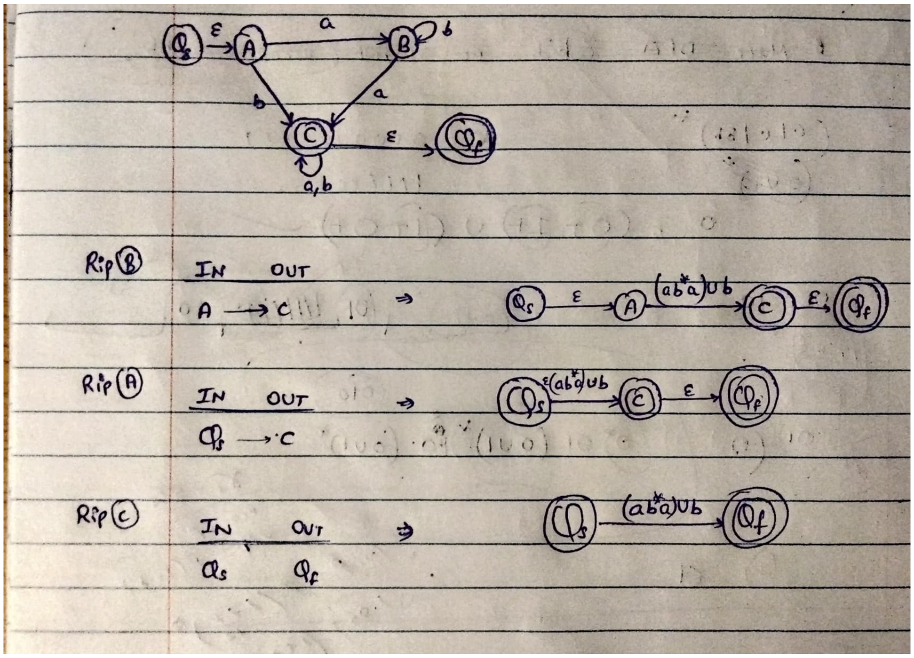
- $|xy| \leq n$,
- $|y| > 0$,
- $xy^i z \in L$ for all $i > 0$.

We let w be the word with n left parentheses, followed by n right parentheses. So, $w = (^n)^n$. Clearly, w has balanced parentheses, so $w \in L$.

Thus, since $|w| \geq n$, by the pumping lemma we must be able to write $w = xyyz$ with $|xy| \leq n$, $|y| \geq 1$, and such that $xy^i z \in L$ for all $i \geq 0$. However, since w starts with n ('s, y must consist entirely of one or more ('s. Then by the first condition, we know that y consists only of left parentheses. By the second condition, we know that y is nonempty. So the string $xyyz$ must have more left parentheses than right parentheses.

Therefore, for any $i > 1$, $xy^i z \notin L$ since it has more ('s than)'s. This is a contradiction, so L is not regular.

Problem 2



Problem 3

Proof: We will use the Pumping Lemma for regular languages to prove that L is not regular. The Pumping Lemma states that for any regular language L, there exists a constant p (the pumping length) such that any string s in L with length at least p can be split into three parts, xyz, satisfying the following conditions:

1. For each $i \geq 0$, $xy^iz \in L$.
2. $|y| > 0$ (y is non-empty).
3. $|xy| \leq p$.

Now, assume for contradiction that the language L is regular. Consider the string $s = 0^p + 1^p = 2^p$ in L. According to the Pumping Lemma, we can write s as xyz such that the conditions are satisfied. Let $s = xyz$ where $|xy| \leq p$ and $|y| > 0$.

Since $|xy| \leq p$, the string y can only consist of 0s or 1s (because the other characters in Σ are '+', '=', and digits 2 to 9).

Now, consider the pumped string $s' = xy^2z$. Since y is non-empty, the pumped string will have more 0s or 1s than the original string. Now, let's analyze s' in terms of the given language L. The equation "$a+b=c$" implies that the number of occurrences of the digit '0' should be equal to the number of occurrences of the digit '1'. However, pumping y will disturb this balance, leading to a string that does not satisfy the equation in L.

Therefore, $s' = xy^2z$ is not in L, which contradicts the Pumping Lemma. As a result, our initial assumption that L is regular is false.

Hence, we can conclude that the language L is not regular.

Problem 4