

# Trajectory planning issues in cuspidal commercial robots

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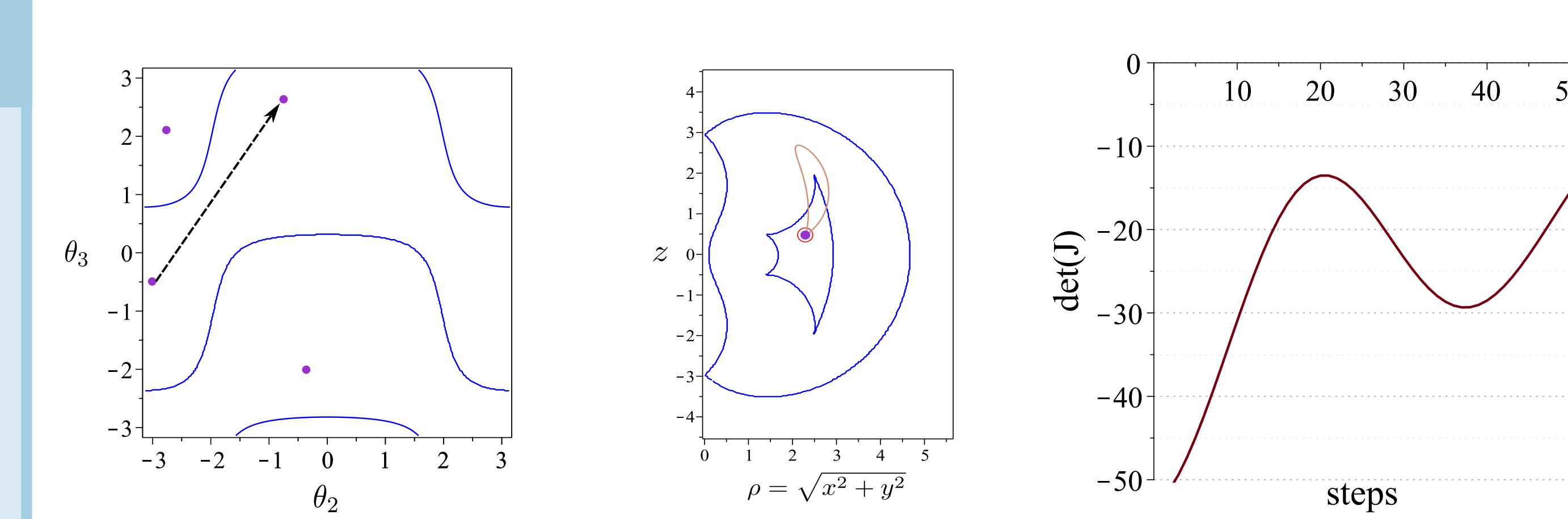
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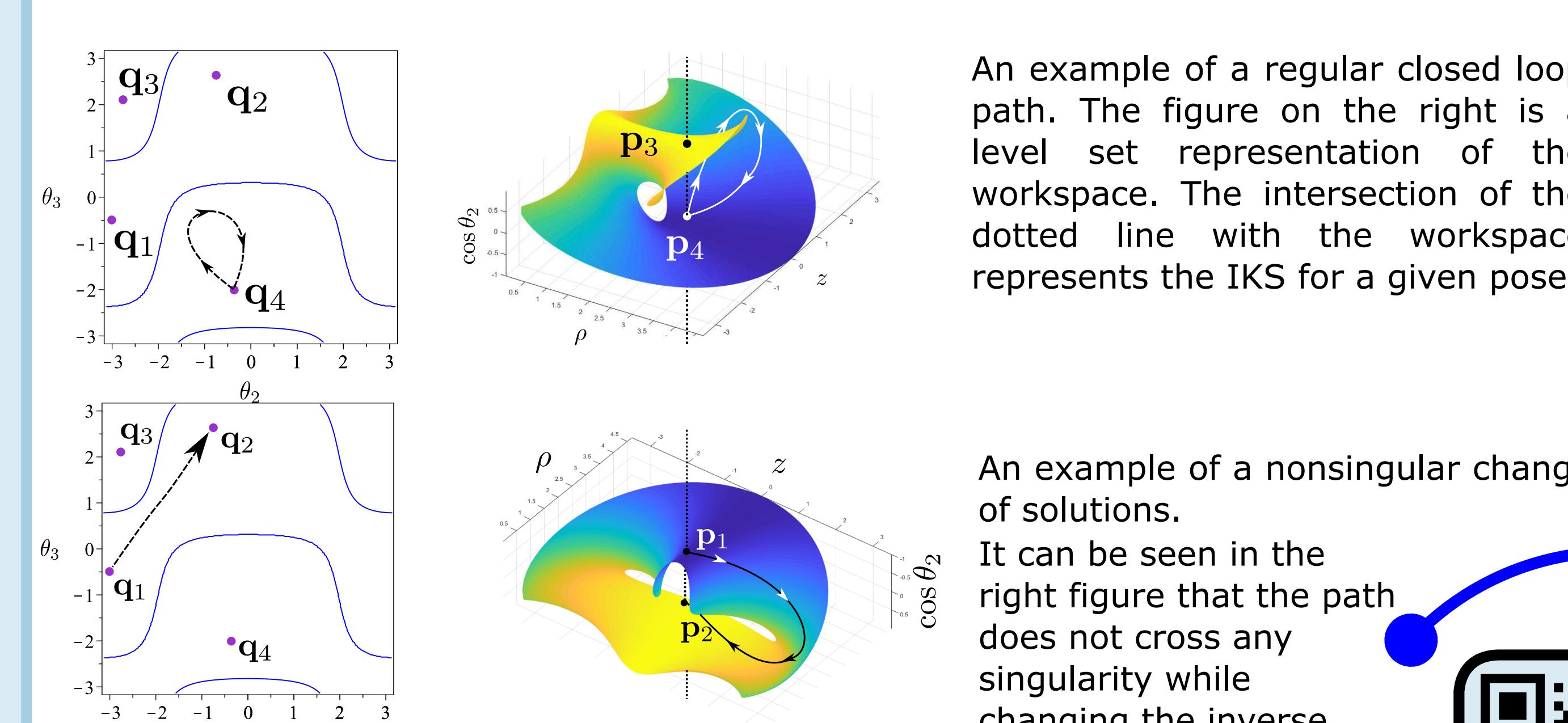
## Introduction

A cuspidal robot can travel from one inverse kinematic solution to another without crossing a singularity. Identifying an aspect related to one unique inverse kinematic solution is not possible in cuspidal robots. The issues related to motion planning with cuspidal robots are related to the inherent property arising from the geometric design of the robot. In this work, we analyze JACO robot (gen 2, non-spherical wrist), a 6R serial arm by Kinova Robotics, which is deployed in various applications, and is cuspidal in nature. The workspace of this robot has multiple regions with varying number of IKS. We highlight the issues with choosing the initial solution of the path in cuspidal robots, and its consequence is illustrated with an example path in the workspace of the JACO robot. Implementing numerical tools to obtain the next IKS in path planning may result in catastrophic failure for cuspidal robots, and this begs the question: is your robot cuspidal?



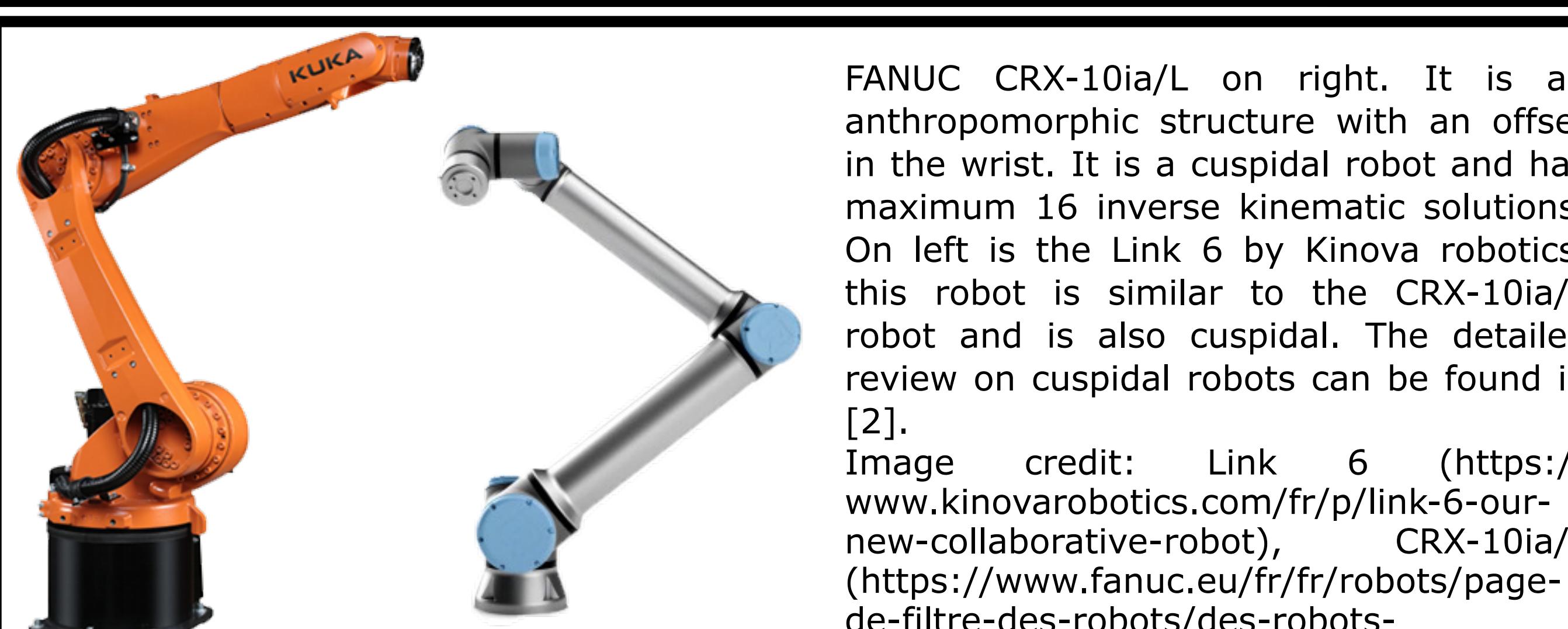
Example of nonsingular change of solutions in the joint space (left) as well as the workspace (center). The rightmost figure shows the progress of determinant value on the path. The determinant value not crossing zero is the proof for nonsingular change of solutions.

Robot parameters:  $d = [0, 1, 0]$ ,  $a = [1, 2, 3/2]$ ,  $\alpha = [-\pi/2, \pi/2, 0]$ , path in joint space : from  $(-3, -0.5)$  to  $(-0.742, 2.628)$ .



An example of a regular closed loop path. The figure on the right is a level set representation of the workspace. The intersection of the dotted line with the workspace represents the IKS for a given pose.

An example of a nonsingular change of solutions. It can be seen in the right figure that the path does not cross any singularity while changing the inverse kinematic solutions.



KUKA KR10 on left. It is a conventional wrist-partitioned robot. It is known that this robot is a non-cuspidal robot. On right, is the UR10 by Universal robots, it has a simplified geometry with three consecutive parallel axes, and is a non-cuspidal robot.

Image credits: KUKA KR10(<https://www.kuka.com>), UR 10 (<https://www.universal-robots.com>)

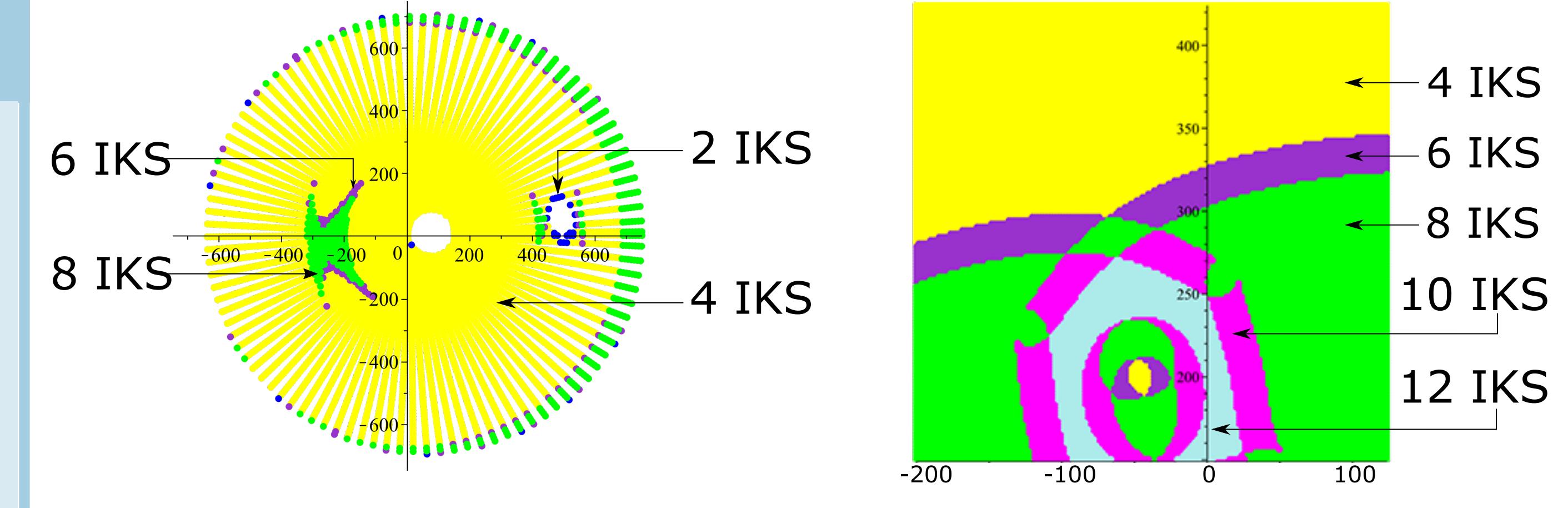
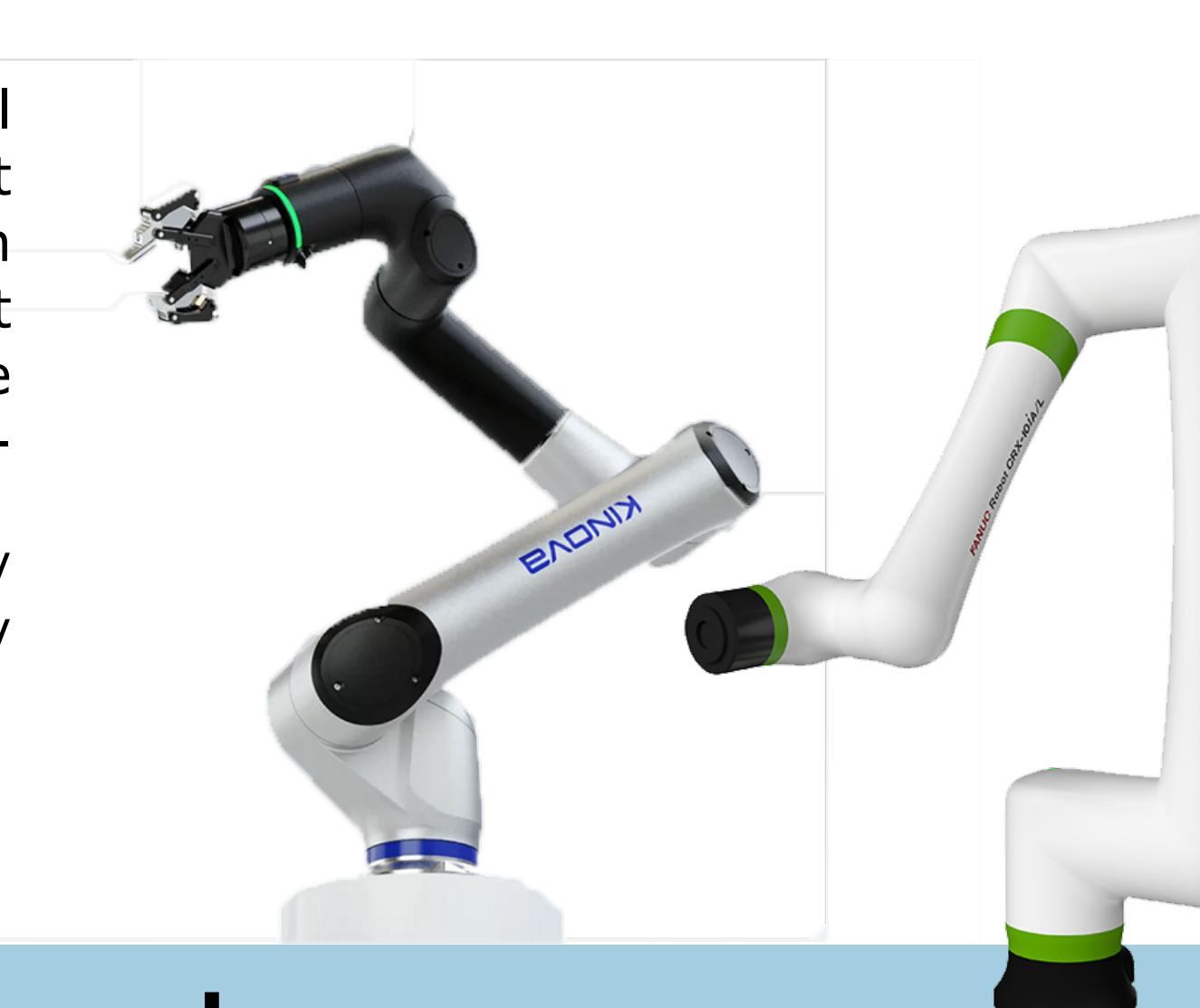


Figure 4: Regions with different number of inverse kinematic solutions in 2D slice (xy-plane) in the workspace of JACO Gen2 robot. The fixed orientation as a quaternion ( $h$ ) and fixed z-coordinate(mm) for the slices are: Slice 1:  $h = 0.2565 + 0.033 i + 0.812 j + 0.582 k$ ,  $z = 560.56$  Slice 2:  $h = 0.984 + 0.004 i + 0.103 j + 0.147 k$ ,  $z = 257.94$

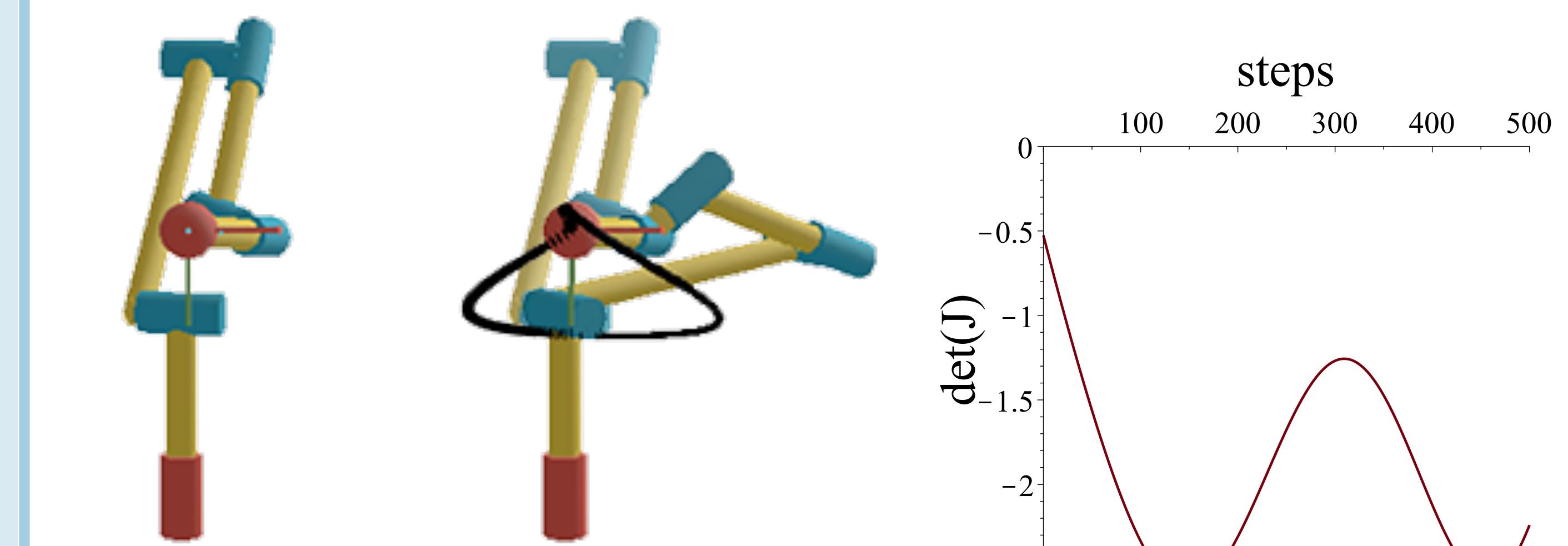


Figure 5: An example of a nonsingular change of solutions in JACO Gen 2 robot. The change of IKS is clear from the first two subfigures show the robot reaching the same pose in a different configuration. The third subfigure shows the plot for determinant of the Jacobian along the path.

## Acknowledgement

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## Conclusions

Being aware of the cuspidal property of a robot, the choice of initial IKS may not be a problem if the complete trajectory to be followed condition for the path are known prior to execution. It is of great importance that such robots are strictly used in environments with pre-planned trajectories. If deployed in collaborative areas (as is the present case for the JACO robot), such robots are bound to error and can lead to unexpected behavior. Many existing industrial robots like FANUC CRX series and Kinova Link 6 are cuspidal robots and thus it is important to consider the cuspidal property in the path planning of such robots. The impact of joint limits and collision constraints on the cuspidal nature will be studied in the future.

## References

- [1] C. Gosselin and H. Liu, Polynomial Inverse Kinematic Solution of the Jaco Robot, in International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, vol. Volume 5B: 38th Mechanisms and Robotics Conference, 08 2014.
- [2] P. Wenger and D. Chablat, A review of cuspidal serial and parallel manipulators, in ASME Journal of Mechanisms and Robotics, 2022

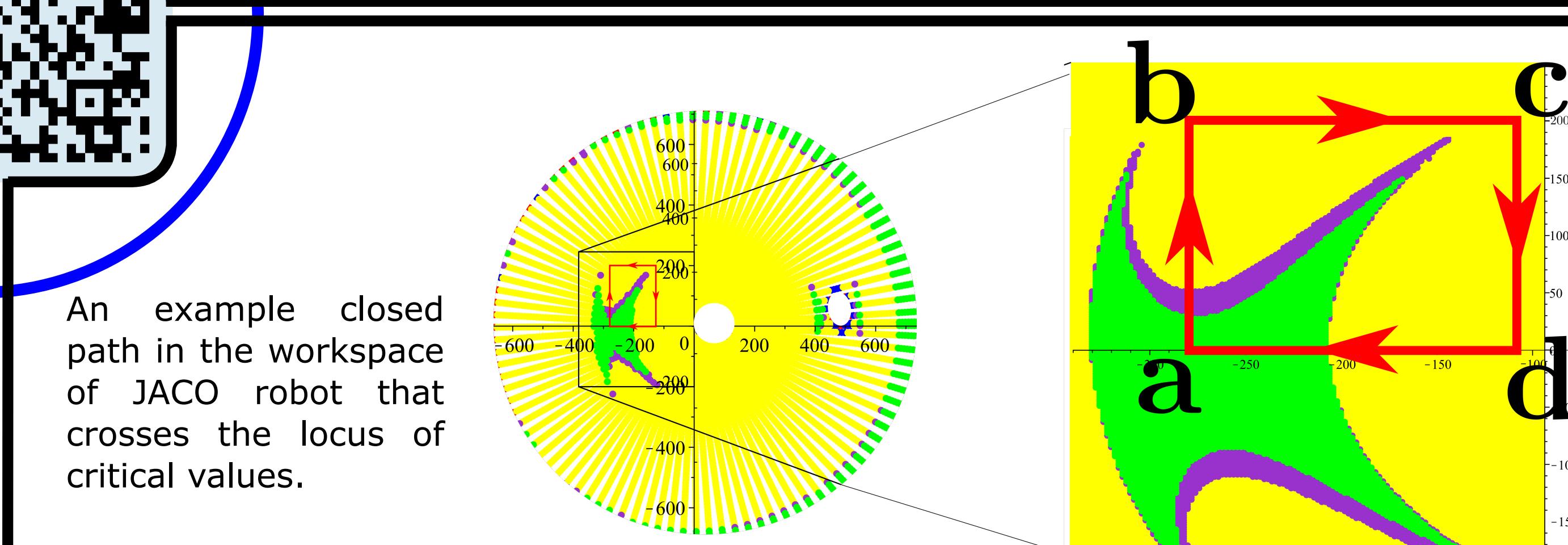
## Commercial cuspidal robot

JACO Gen2 (non-spherical wrist) is a cuspidal robot!

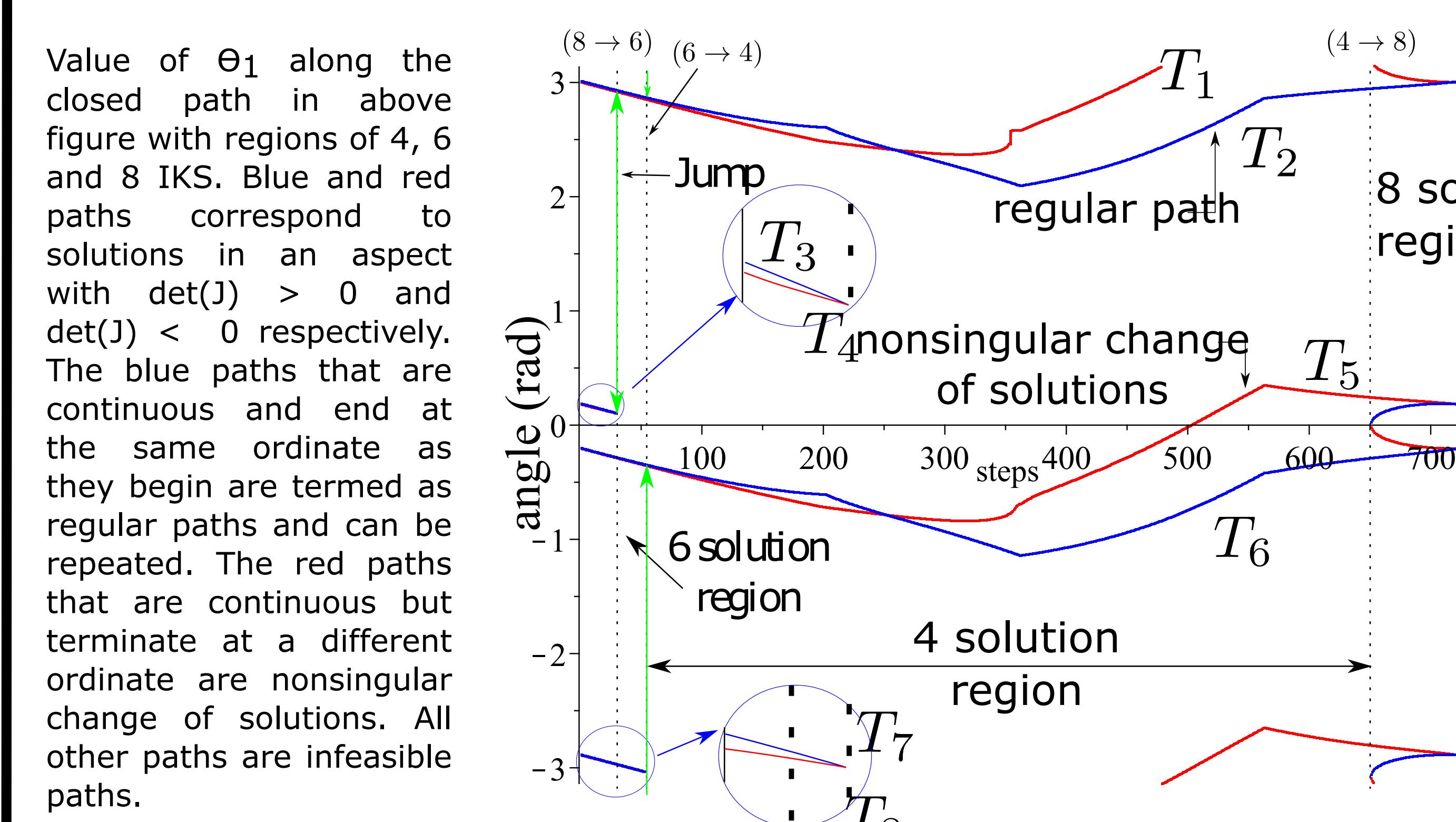
- Maximum 12 inverse kinematic solutions are found [1]
- The 12 solutions lie in 2 aspects, i.e, 6 IKS can be connected without crossing the singularity
- Many connected regions with varying number of IKS are present in the workspace.



JACO robot (gen 2, non-spherical wrist) from Kinova robotics. The robot is used in various commercial applications. The JACO robot was mainly designed for rehabilitation and thus is employed near persons with disability and interacts with them in a few cases.  
Image credit: <https://www.fsg.ulaval.ca/recherche/lab-en-ligne/equipements/robot-jaco-de-kinova>



An example closed path in the workspace of JACO robot that crosses the locus of critical values.



Value of  $\theta_1$  along the closed path in above figure with regions of 4, 6 and 8 IKS. Blue and red paths correspond to solutions in an aspect with  $\det(J) > 0$  and  $\det(J) < 0$  respectively. The blue paths that are continuous and end at the same ordinate as they begin are termed as regular paths and can be repeated. The red paths that are continuous but terminate at a different ordinate are nonsingular change of solutions. All other paths are infeasible paths.

## Trajectory planning issues

It is known that when a workspace boundary associated with a singularity is crossed, at least two solutions disappear (resp. appear) if the robot moves toward a region with less IKS (resp. with more IKS). In classical path planning algorithms, a change of solution only occurs when  $\det(J)$  changes sign. Thus, an end-effector path can be declared infeasible if  $\det(J)$  changes sign. In the case of cuspidal robots, a jump to another IKS in the same aspect can be experienced without being detected, and this jump results into going off the planned end-effector path. Closed-loop paths that do not result in a change of IKS can be repeated but this is not the case for the paths corresponding to the nonsingular change of solutions. The dependency of path feasibility and repeatability on the initial IKS creates several challenges in planning trajectories of 6R cuspidal robots. A dedicated path planning framework is necessary to perform operations in the task space of a cuspidal robot.