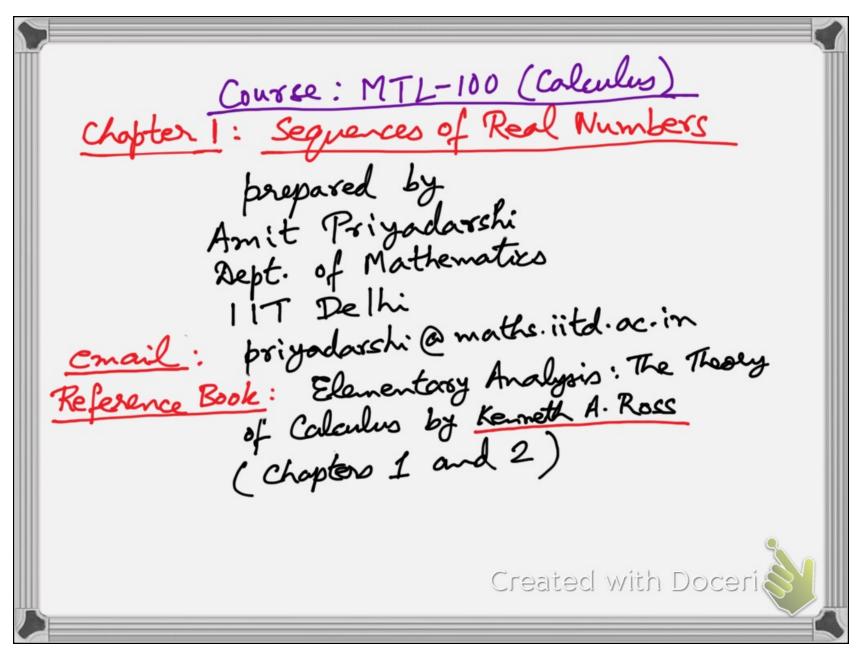
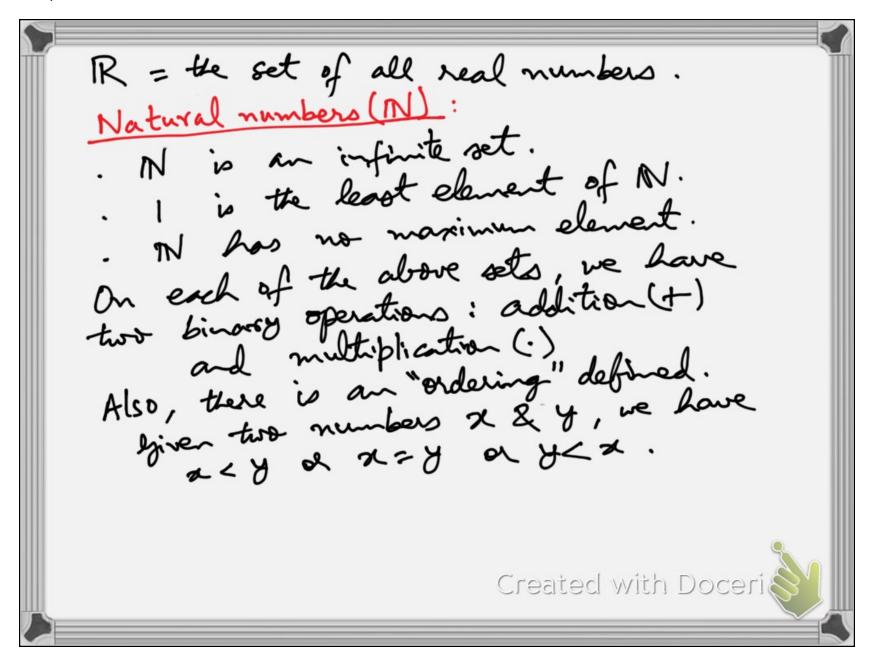
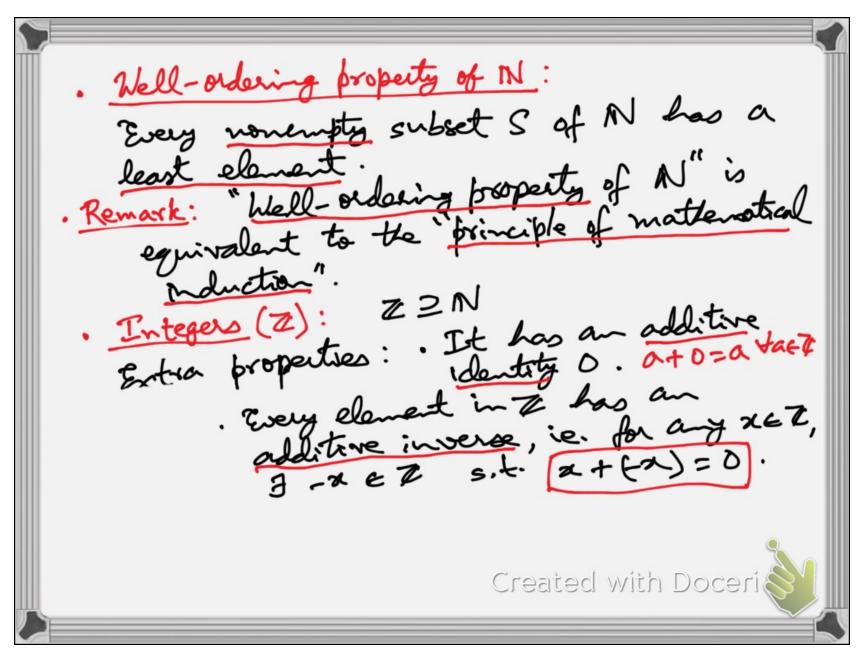
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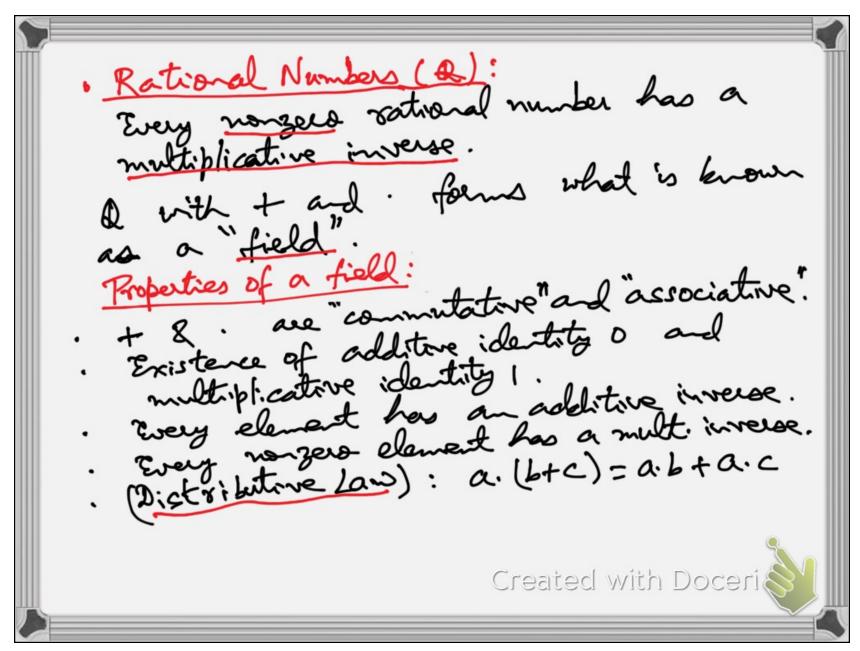


## Lecture 1: A brief introduction to real numbers We will assume that the students are amiliae with natural numbers, integers IN = the set of all natural numbers = {1,2,3,4,...} Z = the set of all integers = 2-,-3,-2,-1,0,1,2,3,...2 Created with Doceri





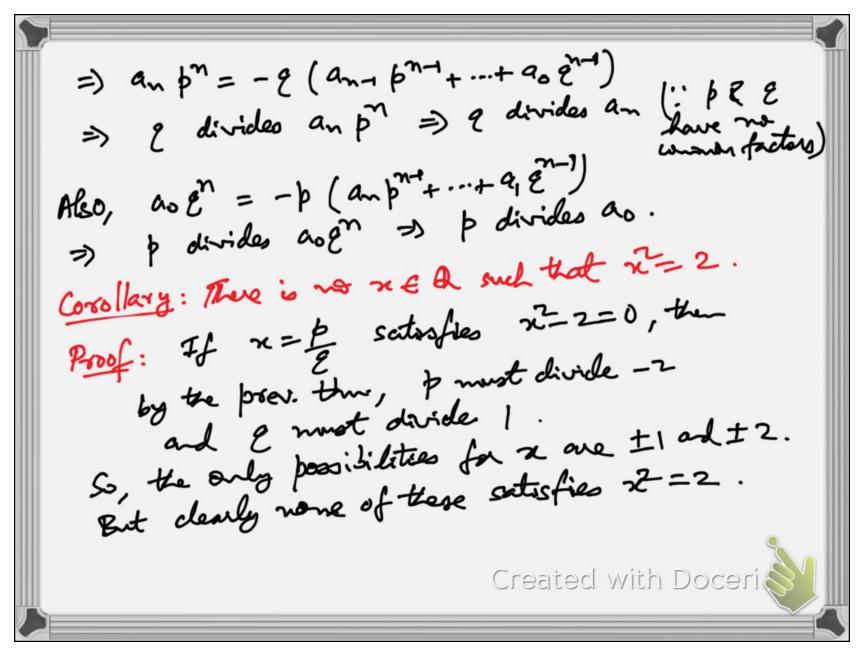
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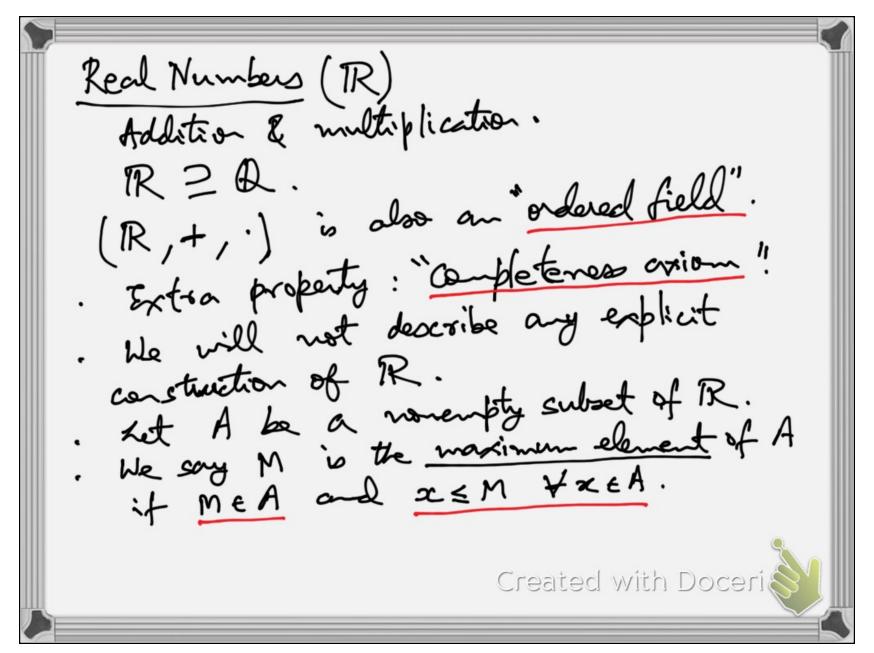


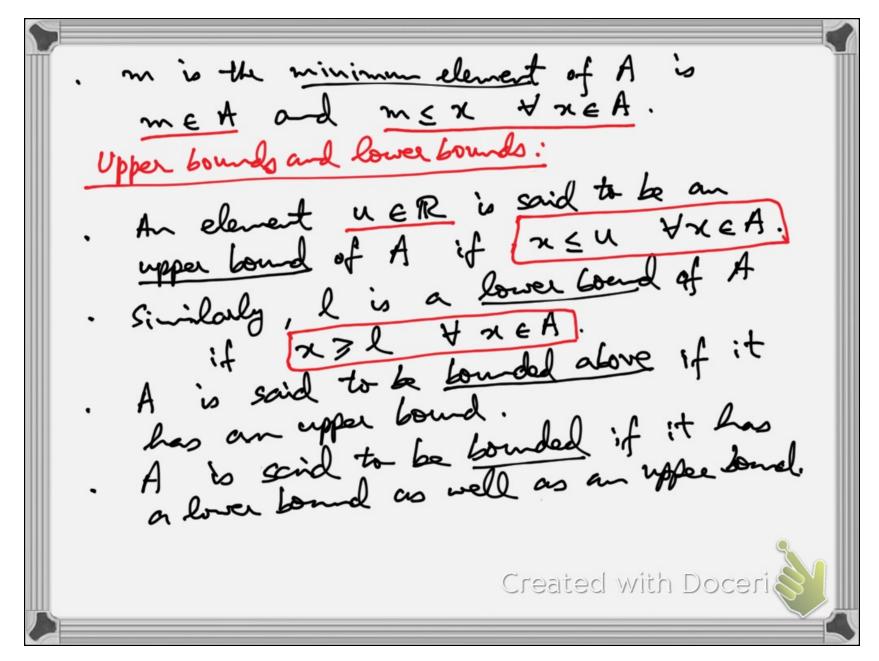
This order < on Q soctisfies the usual properties. a is an "ordered field" with respect to the addition, multiplication and ordering . Shortcomings: a has "gaps" in some sense. For example, there is no of ED such that  $8^2 = 2$ Created with Doceria

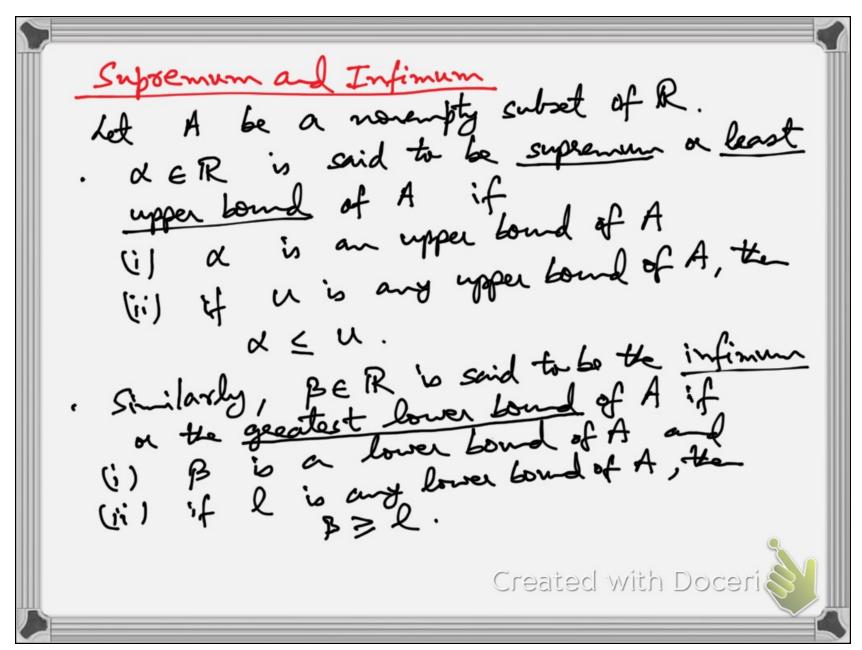
· Rational root theorem: Consider the polynomial equation: anx"+an-12"+ ... + a, x+a=0, where nen, ao, a, ..., an EZ, ao = 0, an = 0. Suppose that a restrand number & satisfies this equation where P& & have no common factors. Then & must divide as and 9 must divide an. Proof: he have an (=) + an (=) + ... + a, (=) + a = 0 =) an pm + an-1 pm-1 2 + ... + a, pen+ ave=0 Created with Doceria

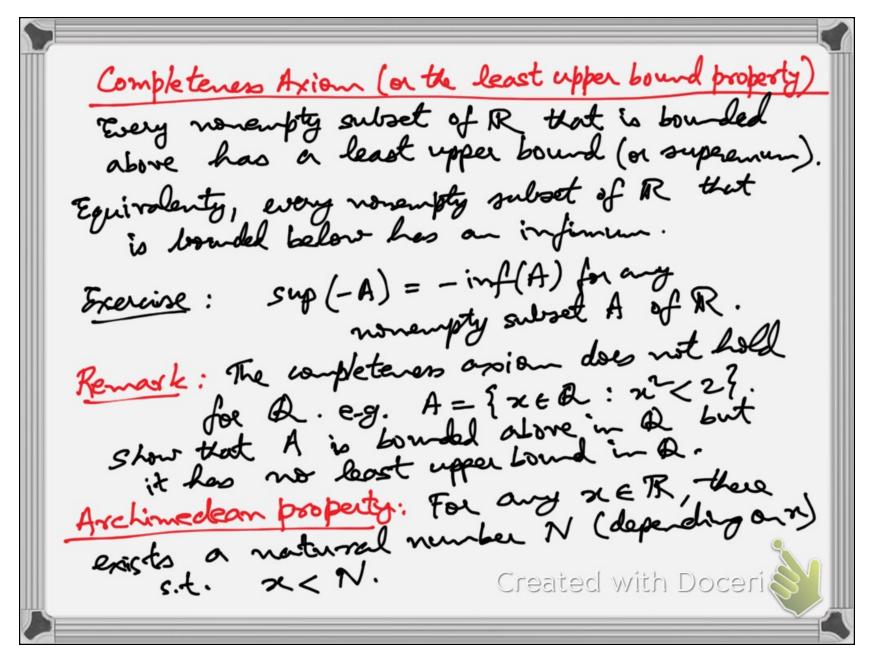
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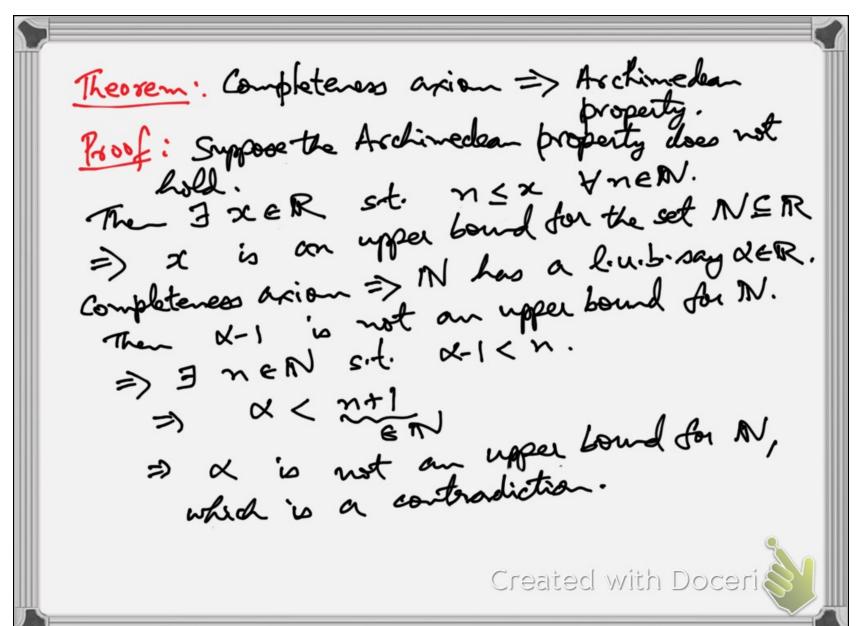




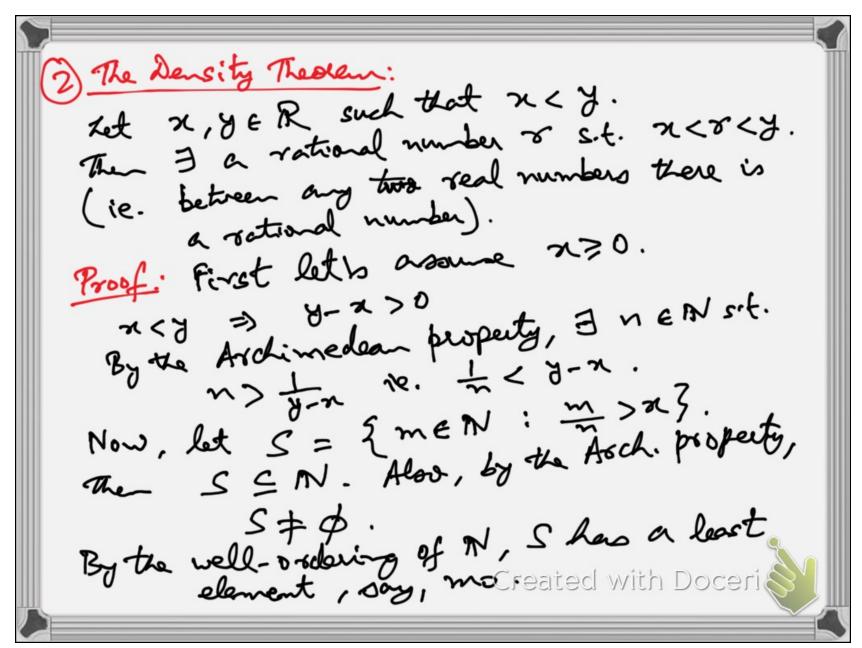








Remark: The converse of the poerious theorem is not true. a satisfies the Archimedean property (why?) but not the completeness axiom. Consequences of the Archimedean people ) Let S = { \( \dagger : neN \)?. Then inf(s) = 0 Proof: Suce =>0 + NEW, Sis bounded below. Let  $x = \inf(s)$ . Then  $x \ge 0$  (; 0 is a lower If x>0, then by the Archimedean property, ヨnen st. かっよ コ ペンからら =) & is not a lower bond for S, which is a contradiction. -e,  $\alpha = 0$ Created with Doceria



Then, mo > x (; moes)
and mo-1 < x (If mo > 2, then
and mo-1 < x (If mo > 2, then
mo-1 & x, mo-1 & N If wo=1, the x>0= mo-1 mo-1 < x => mo < x++ < x+(y-x)=> in a < mo < y. Hence done. If x<0, by the Arch. prop., I neNet. w>-x Also, x< y =>0< x+n < y+n. i. By the above proof., I a rational no. or set. 2+n < o < y+n ラ スくがかくか Also, 8-n E Q. Remee we are dire. Exercise: Let x, y & R st. x < y. Show that I am irrational number 2 s.t. x< Z< y. Proof: Suce x < y, 2-52 < 4-52 By the density theorem, 3 real st. x-52 < x < y-52 ラ 又くがなくな Let Z = 8+52. Then x<2<8 Also, z is an irrational number. Created with Doceria

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