Lecture 10
Signals and Systems (ELL205)

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Outline of the lecture

- Use $h(t)$ to determine whether the system is:
  - Memoryless
  - Causal
  - Stable
  - Invertible
- Applications of $h(t)$ to real-life scenarios
- System designing

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Memoryless

A memoryless system output depends only on the current input.

The output of the system is given by:

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$$

Thus, memoryless implies $h(t-\tau)$ is non-zero only at $t = \tau$

$\Rightarrow$ $h(t) = k\delta(t)$ For CT system

$h[n] = k\delta[n]$ For DT system

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Causal

A causal system output depends only on previous or current input.

The output of the system is given by:

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$$

Thus, Causality implies $h(t-\tau)$ is zero for $t-\tau < 0$

$$\Rightarrow \quad h(t) = 0 \quad t < 0 \qquad \text{For CT system}$$

$$h[n] = 0 \quad \mathrm{n} < 0 \qquad \text{For DT system}$$

Causality and Linearity

A causal and linear system satisfies condition of initial rest, that is,

$$\text{if } x(t) = 0 \text{ for } t < t_o$$

$$\text{then } \mathrm{y}(t) = 0 \text{ for } t < t_o$$

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Stability

Starting from def.:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Stability

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$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Since convolution is commutative:

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

Stability

Bound on output:

$$|y[n]| = \left| \sum_{k=-\infty}^{\infty} h[k]x[n-k] \right|$$

$$|y[n]| \leq \sum_{k=-\infty}^{\infty} |h[k]x[n-k]|$$

$$|y[n]| \leq \sum_{k=-\infty}^{\infty} |h[k]||x[n-k]|$$

Stability

Bound on output:

$$|y[n]| \leq \sum_{k=-\infty}^{\infty} |h[k]||x[n-k]|$$

Let $|x[n-k]| \leq M_x$

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Let $|x[n-k]| \leq M_x$

$$|y[n]| \leq \sum_{k=-\infty}^{\infty} |h[k]|M_x$$

Stability

Bound on output:

$$|y[n]| \leq \sum_{k=-\infty}^{\infty} |h[k]||x[n-k]|$$

Let $|x[n-k]| \leq M_x$

$$|y[n]| \leq M_x \sum_{k=-\infty}^{\infty} |h[k]|$$

Stability

Bound on output:

$$|y[n]| \leq \sum_{k=-\infty}^{\infty} |h[k]||x[n-k]|$$

Let $|x[n-k]| \leq M_x$

$$|y[n]| \leq M_x \sum_{k=-\infty}^{\infty} |h[k]|$$

$$\sum_{k=-\infty}^{\infty} |h[k]| \leq M_h \qquad |y[n]| \leq M_x M_h$$

Stability

$$\sum_{k=-\infty}^{\infty} |h[k]| \leq M_h \qquad |y[n]| \leq M_x M_h$$

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty \qquad |y[n]| < \infty$$

Sufficient condition for stability is that, h[n] is absolutely summable, that is, $\sum_{k=-\infty}^{\infty} |h[k]| < \infty$

Stability

Is this (absolutely summable, that is, $\sum_{k=-\infty}^{\infty} |h[k]| < \infty$) also a necessary condition for stability?

Stability

Is this (absolutely summable, that is, $\sum_{k=-\infty}^{\infty}|h[k]| < \infty$) also a necessary condition for stability?

Assuming input $x[-n] = \begin{cases} \frac{\overline{h[n]}}{|h[n]|} & |h[n]| \neq 0 \\ 0 & |h[n]| = 0 \end{cases}$

Stability

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Stability

Assuming input $x[-n] = \begin{cases} \frac{\overline{h[n]}}{|h[n]|} & |h[n]| \neq 0 \\ 0 & |h[n]| = 0 \end{cases}$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

$$y[0] = \sum_{k=-\infty}^{\infty} h[k]x[-k]$$

Stability

Assuming input $x[-n] = \begin{cases} \dfrac{\overline{h[n]}}{|h[n]|} & |h[n]| \neq 0 \\ 0 & |h[n]| = 0 \end{cases}$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

$$y[0] = \sum_{k=-\infty}^{\infty} h[k]x[-k] = \sum_{k=-\infty}^{\infty} h[k]\frac{\overline{h[k]}}{|h[k]|}$$

Stability

Assuming input $x[-n] = \begin{cases} \frac{\overline{h[n]}}{|h[n]|} & |h[n]| \neq 0 \\ 0 & |h[n]| = 0 \end{cases}$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

$$y[0] = \sum_{k=-\infty}^{\infty} h[k]x[-k] = \sum_{k=-\infty}^{\infty} h[k]\frac{\overline{h[k]}}{|h[k]|} = \sum_{k=-\infty}^{\infty} |h[k]| = \infty$$

Stability Summary

$\sum_{k=-\infty}^{\infty}|h[k]| < \infty$ (that is, absolute summability) is both a necessary and sufficient condition for stability

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Similarly, $\int_{-\infty}^{\infty} |h(t)| dt < \infty$ (that is, absolute integrability) is both a necessary and sufficient condition for stability (to be proved).

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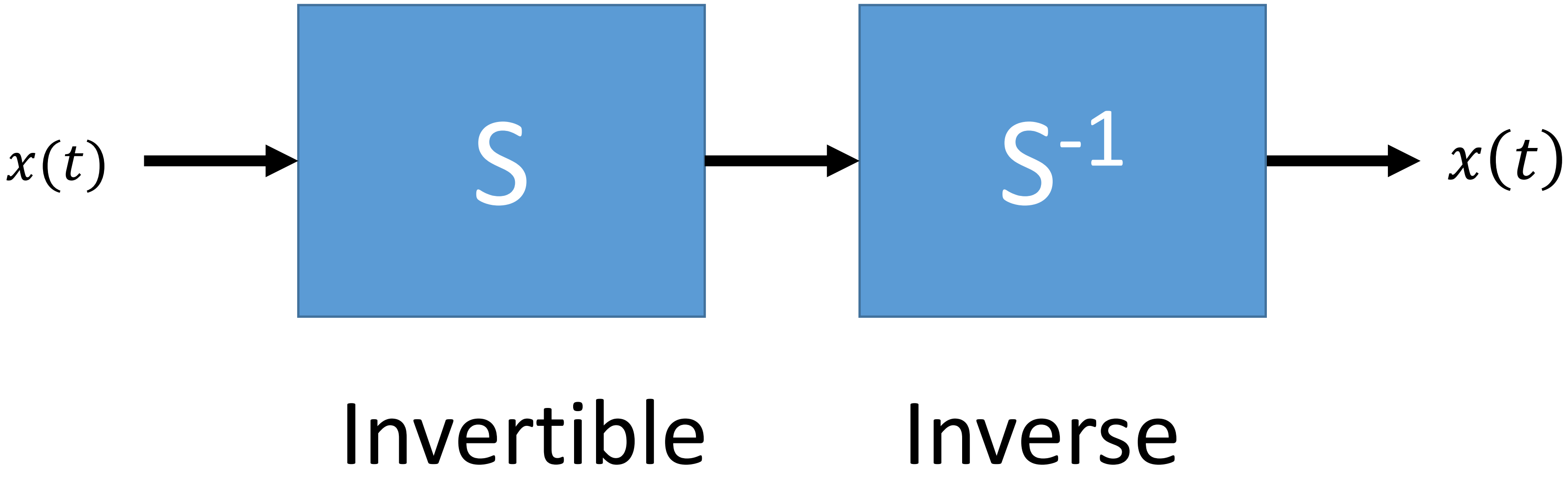
Invertible

A system is said to be invertible if distinct inputs lead to distinct outputs.



$$x(t) * h(t) * h_{inv}(t) = x(t)$$

Invertible

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$$x(t) * h(t) * h_{inv}(t) = x(t)$$

$$h(t) * h_{inv}(t) = ?$$

$$x(t) * \delta(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau = x(t)$$

Invertible

A system is said to be invertible if distinct inputs lead to distinct outputs.

$$x(t) * h(t) * h_{inv}(t) = x(t)$$

$$h(t) * h_{inv}(t) = \delta(t)$$

$$x(t) * \delta(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau = x(t)$$

Summary of Invertible

The relation between invertible and inverse system

$$h(t) * h_{inv}(t) = \delta(t)$$

$$h[n] * h_{inv}[n] = \delta[n]$$