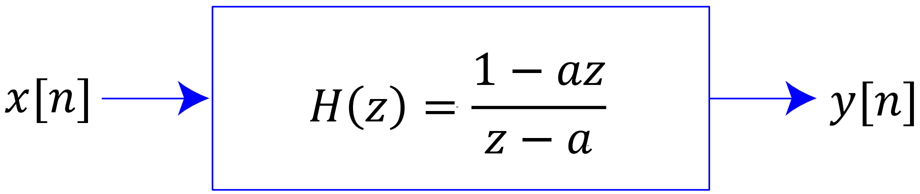
Extra class

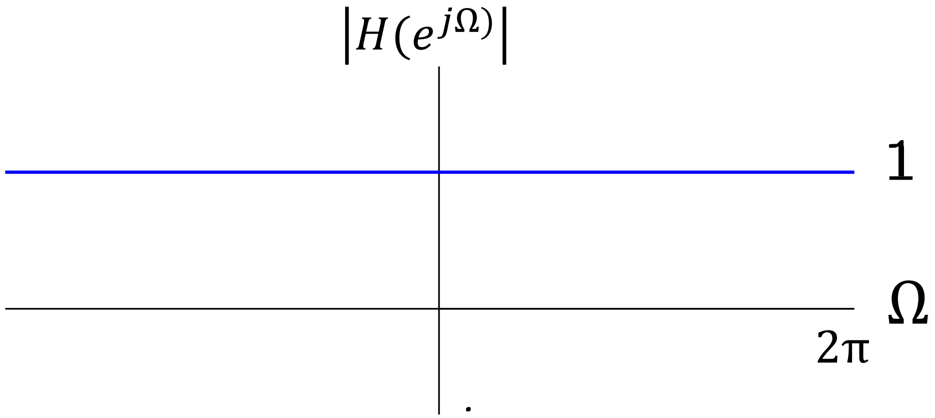
- March 19, 10 am onwards

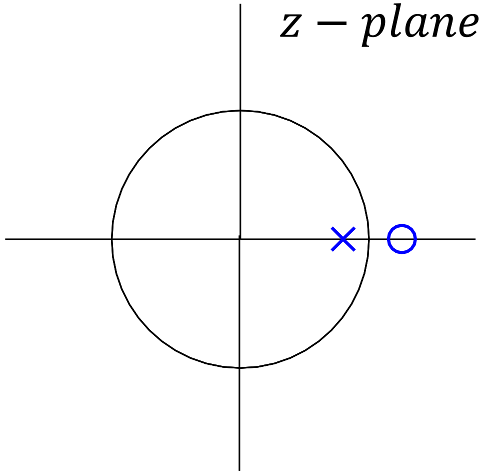
Discrete-Time Signals and Systems

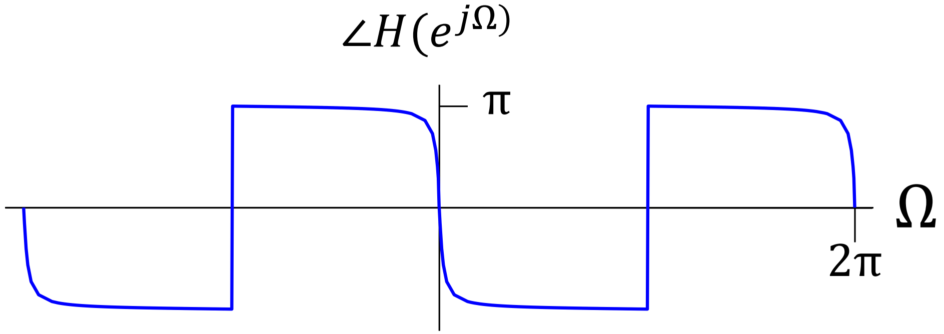
Lecture 26

Effect of all-pass filter









How does applications react to amplitude and phase distortions?

x(t) | All pass filters | y(t)

Effect of phase change on Music and Speech

- Music (without phase distortions)
- Music (with phase distortions)
- Speech (‘bat’ without phase distortions)
- Speech (‘bat’ with phase distortions)

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Ohm’s law: We hear only amplitude and frequency of sound waves

Effect of phase change on Music and Speech

- Complicated Speech (without phase distortions)
- Complicated Speech (with phase distortions)

Hello Class! Abhishek Dixit is an awesome teacher!

Effect of phase change on Music and Speech

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Effect of phase change on Music and Speech

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- Complicated Speech (with phase distortions)

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Effect of phase reversal on Music and Speech

- Complicated Speech (without phase distortions)
- Complicated Speech (with phase distortions)
- Complicated Speech (with phase reversal)

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Effect of phase reversal on Music and Speech

- Complicated Speech (without phase distortions)
- Complicated Speech (with phase distortions)
- Complicated Speech (with phase reversal)

Hello Class! Abhishek Dixit is an awesome teacher!

Effect of phase change on Music and Speech

- Music reversal

Effect of phase change on Music and Speech

- Music reversal

Effect of phase change on Music and Speech

- Music reversal

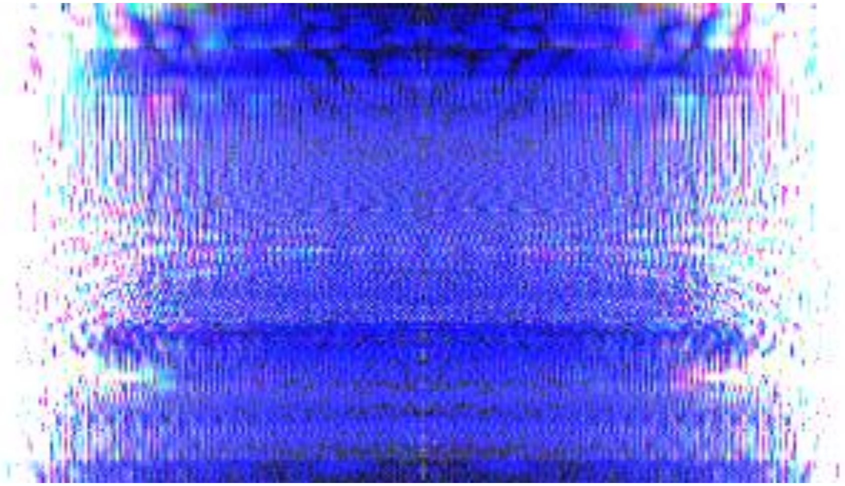
How are the phase of two signals related?

$$x[n] \leftrightarrow X(e^{j\Omega}) \qquad x[-n] \leftrightarrow X(e^{-j\Omega})$$

$$\cos[\Omega_o n + 15] \qquad \cos[-\Omega_o n + 15] = \cos[\Omega_o n - 15]$$

Phase gets flipped in sign

Effect of phase change on pictures



Effect of phase change on pictures

Amplitude profile is substituted from another picture

Phase profile is randomized

Effect of phase change on pictures

Complex conjugate

$$H(e^{j\Omega}) = \sum_{k=-\infty}^{\infty} h[k]e^{-j\Omega k}$$

$$H(e^{-j\Omega}) = \sum_{k=-\infty}^{\infty} h[k]e^{j\Omega k} = \overline{H(e^{j\Omega})}$$

Prove

$h[k]$ is real $\Rightarrow$

$$|H(e^{j\Omega_o})| \quad \text{is even}$$

$$\angle H(e^{j\Omega_o}) \quad \text{is odd}$$

Input & Output

$\cos[\Omega_o n]$ → LTI → ?

$\frac{1}{2}\{e^{j\Omega_o n} + e^{-j\Omega_o n}\}$ → LTI → $|H(e^{j\Omega_o})|\cos[\Omega_o n + \angle H(e^{j\Omega_o})]$

Discrete-time Fourier Transform

$$H(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\Omega n}$$

$$h[n] = ?$$

Deriving $h[n]$

$$H(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-jn\Omega}$$

$$H(e^{j\Omega})e^{jl\Omega} = \sum_{n=-\infty}^{\infty} h[n]e^{-jn\Omega}e^{jl\Omega}$$

$$\int_{0}^{2\pi} H(e^{j\Omega})e^{jl\Omega}d\Omega = \int_{0}^{2\pi} \sum_{n=-\infty}^{\infty} h[n]e^{-jn\Omega}e^{jl\Omega}d\Omega$$

Deriving $h[n]$

$$\int_0^{2\pi} H(e^{j\Omega})e^{jl\Omega}d\Omega = \sum_{n=-\infty}^{\infty} h[n] \int_0^{2\pi} e^{j(l-n)\Omega}d\Omega$$

$$\int_0^{2\pi} e^{j(l-n)\Omega}d\Omega = \begin{cases} 2\pi \textbf{ if } l = n \\ 0 \textbf{ if } l \neq n \end{cases}$$

$$\int_0^{2\pi} e^{j(l-n)\Omega}d\Omega = 2\pi\delta[l-n]$$

Deriving $h[n]$

$$\int_0^{2\pi} H(e^{j\Omega})e^{jl\Omega}d\Omega = \sum_{n=-\infty}^{\infty} h[n]2\pi\delta[l-n]$$

$$\int_0^{2\pi} H(e^{j\Omega})e^{jl\Omega}d\Omega = h[l]2\pi$$

$$h[n] = \frac{1}{2\pi}\int_0^{2\pi} H(e^{j\Omega})e^{jn\Omega}d\Omega$$

Discrete-time Fourier Transform

$$H(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} h[n] e^{-j\Omega n}$$

$$h[n] = \frac{1}{2\pi} \int_{0}^{2\pi} H(e^{j\Omega}) e^{jn\Omega} d\Omega$$

Discrete-time Fourier Transform

$$H(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\Omega n}$$

$$h[n] = \frac{1}{2\pi}\int_{0}^{2\pi} H(e^{j\Omega})e^{jn\Omega}d\Omega$$

These equations are transform equations if $\sum_{n=-\infty}^{\infty}|h[n]| < \infty$ or $\sum_{n=-\infty}^{\infty}|h[n]|^2 < \infty$

No Gibbs phenomenon or Dirichlet conditions here

Examples of FT pairs

- $\mathrm{F}\{\delta[n]\} = ?$

Examples of FT pairs

- F{δ[n]} $= ?$

$$H(e^{j\Omega}) = \sum_{k=-\infty}^{\infty} \delta[k] e^{-jk\Omega} = 1$$

Examples of FT pairs

- $x[n] = a^n u[n]$
- F$\{x[n]\} = ?$

Examples of FT pairs

- $x[n] = a^n u[n]$
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$$X(e^{j\Omega}) = \sum_{n=0}^{\infty} a^n e^{-j\Omega n} = \frac{1}{1 - ae^{-j\Omega}}$$

Examples of FT pairs

- $x[n] = \begin{cases} 1, |n| \leq N_1, \\ 0, |n| > N_1 \end{cases}$
- F$\{x[n]\} = ?$

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$$X(e^{j\Omega}) = \sum_{n=-N_1}^{N_1} e^{-j\Omega n} = \frac{e^{j\Omega N_1}\left(e^{-j\Omega(2N_1+1)} - 1\right)}{e^{-j\Omega} - 1}$$

$$X(e^{j\Omega}) = \frac{e^{j\Omega N_1}\left(-2je^{-j\Omega(N_1+\frac{1}{2})}\sin\left(\Omega\left(N_1 + \frac{1}{2}\right)\right)\right)}{-2je^{-j\Omega/2}\sin(\Omega/2)}$$

Examples of FT pairs

- $x[n] = \begin{cases} 1, |n| \leq N_1, \\ 0, |n| > N_1 \end{cases}$
- F$\{x[n]\} = ?$

$$X(e^{j\Omega}) = \sum_{n=-N_1}^{N_1} e^{-j\Omega n} = \frac{\left(\sin\left(\Omega\left(N_1 + \frac{1}{2}\right)\right)\right)}{\sin(\Omega/2)}$$

Examples of FT pairs

$$X(e^{j\Omega}) = \frac{\left(\sin\left(\Omega\left(N_1 + \frac{1}{2}\right)\right)\right)}{\sin(\Omega/2)}$$

Examples of FT pairs

- $X(e^{j\Omega}) = \delta(\Omega)$
- $x[n] = ?$

Examples of FT pairs

- $X(e^{j\Omega}) = \sum_{m \epsilon I} \delta(\Omega - m2\pi)$
- $x[n] = ?$

Examples of FT pairs

- $X(e^{j\Omega}) = \sum_{m \in I} \delta(\Omega - m2\pi)$
- $x[n] = ?$

$$x[n] = \frac{1}{2\pi}\int_0^{2\pi} X(e^{j\Omega})e^{jn\Omega}d\Omega = \frac{1}{2\pi}$$

Examples of FT pairs

- $X(e^{j\Omega}) = 2\pi \sum_{m \in I} \delta(\Omega - m2\pi)$
- $x[n] = 1$