Lecture 17 Tue 29th Aug 2023

Cycling (Klee & Minty)

$$\max \left(\frac{3}{4}\right) x_4 - 20x_5 + \left(\frac{1}{2}\right) x_6 - 6x_7$$

S.to.

$$x_1 + \frac{1}{4} x_4 - 8x_5 - x_6 + 9x_7 = 0$$

$$x_2 + \frac{1}{2} x_4 - 12x_5 - \frac{1}{2} x_6 + 3x_7 = 0$$

$x_i \geq 0 \quad \forall i$ $\qquad x_3 + x_6 = 1$

Initial Basis

$\{x_1, x_2, x_3\} \rightarrow B = I_{3\times 3}$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = x_B = b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \rightarrow \text{Degenerate BFS.}$$

$c_j \rightarrow$			0	0	0	3/4	-20	1/2	-6
c_B	v_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6	y_7
0	x_1	0	1	0	0	1/4	-8	-1	4
0	x_2	0	0	1	0	1/2	-12	-1/2	3
0	x_3	1	0	0	1	0	0	1	0
$z_j - c_j \rightarrow$			0	0	0	-3/4	20	-1/2	+6

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cycling occurs in this case.

(1) Largest coefficient Rule

1) entering variable as the one with largest coefficient in the objection func^n $z_j - c_j < 0$.

2) leaving variable is one having smallest index.

* cycling

$$\max z = 10x_1 - 57x_2 - 9x_3 - 24x_4.$$

s.t. $\left(\frac{1}{2}\right)x_1 - \left(\frac{11}{5}\right)x_2 - \left(\frac{5}{2}\right)x_3 + 9x_4 + x_5 = 0$

$$\frac{1}{2}x_1 - \frac{3}{2}x_2 - \frac{1}{2}x_3 + x_4 + x_6 = 0$$

$x_i \geq 0 \ \forall i$. $\qquad x_4 + x_7 = 1$

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counter example to above rule

(2) Bland's Rule (Robert G. Bland)

1) Among all variables with $z_j - c_j < 0$, choose the one with smallest index to enter

2) leaving variable is the one (in case of ties) having lowest index row from top of table