$Add \stackrel{df}{=} \lambda m\, n\, f\, x\, [((m\, f)\ (n\, f\, x))]$

where $m = \lambda f x (f^m\ x) = \beta (f\ (f \ldots \underbrace{(f\ x) \ldots))}_{f \text{ applied } m \text{ times}}$.

similarly $n = \lambda f x (f^n\ x)$

$(\lambda m\, n\, f\, x\ [(((m\, f)\ (n\, f\, x)))]\ \ \lambda f x\, (f^m\, x)\ \ \lambda f x\, (f^n\, x))$

$\xrightarrow[\beta]{1}$

$\xrightarrow[\beta]{2} \lambda f x\, [\lambda x (f^m\ x)\ \ (f^n\ x)]$

$\xrightarrow[\beta]{1} \lambda f x\, [(f^m\ f^n\ x)]$

$\xrightarrow[\beta]{1} \lambda f x\, [(f^{m+n}\ x)]$

which represents $m + n$

$Plus \stackrel{df}{=} \lambda g\, u\, v\, [ITE\ (IsZero\ u)\ v\ (g\ (Pred\, u)\ (Succ\ v))]$

$Sum \stackrel{df}{=} (Y\ Plus) \xrightarrow[\beta]{1} (Plus\ (Y\ Plus))$

$\xrightarrow[\beta]{1} \lambda u\, v\, [ITE\ (IsZero\ u)\ v\ ((Y\ Plus)\ (Pred\, u)\ (Succ\ v))]$

Lets consider Curry's paradoxical combinator

we can see that is $u \neq 0 = \lambda f x[x]$
we again can reduce (Y Plus) to (Plus Y Plus)

therefore →

$$((Y\ Plus)\ (Pred\ u)\ (succ\ v))$$

$$\rightarrow (Plus\ (Y\ Plus)\ (u-1)\ (v+1))$$

this will go on u times
till u becomes 0 and $v \rightarrow v+u$

therefore Sum m n will also give
same answer as Add m n $= \lambda f x[(f^{m+n}\ x)]$

Hence proved.