PYL101
(Electromagnetic Waves and Quantum Mechanics)
Tutorial Sheet 4 (L7-L8)

(1) A particle of mass m is confined in a box of unit length in one dimension. It is described by the wavefunction $\psi(x) = \sqrt{\frac{8}{5}} \sin \pi x \,(1 + \cos \pi x)$, for $0 \leq x \leq 1$ and zero outside this interval. Find the expectation value of energy.

(2) The ground state energy of a particle of mass m in an infinite square well potential of width a is E. If the width of the well is reduced to $a/2$, find the new ground state energy.

(3) The state of a particle of mass m in a one-dimensional infinite potential well in the interval 0 to L is given by a normalized wave function $\psi(x) = \sqrt{\frac{2}{L}}\left(\frac{3}{5} \sin\left(\frac{2\pi x}{L}\right) + \frac{4}{5} \sin\left(\frac{4\pi x}{L}\right)\right)$. If its energy is measured, what are the possible outcomes. Also, calculate the average value of energy.

(4) A free particle of mass m moves along the x-direction at $t = 0$. The state of the particle is given by, $\psi(x, 0) = \frac{1}{(2\pi\alpha)^{\frac{1}{4}}} \exp\left[-\frac{x^2}{4\alpha^2} + ix\right]$, where α is a real constant. Find the expectation value of momentum in this state.

(5) A wavefunction represents the normalized state of a free particle,
$$\psi(x, 0) = N \; e^{\frac{-x^2}{2a^2} + ik_0 x}$$
Find the normalization constant N.