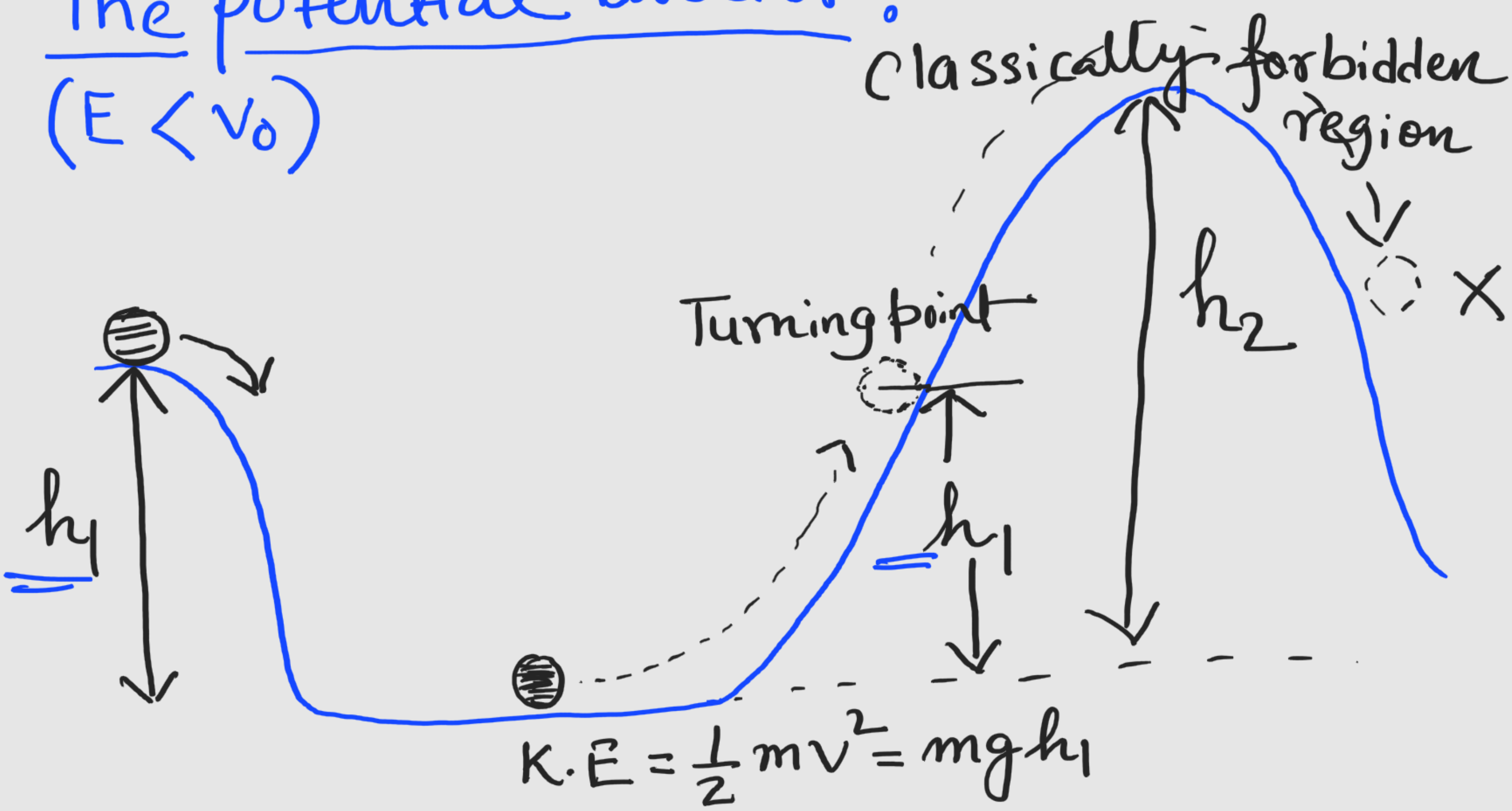
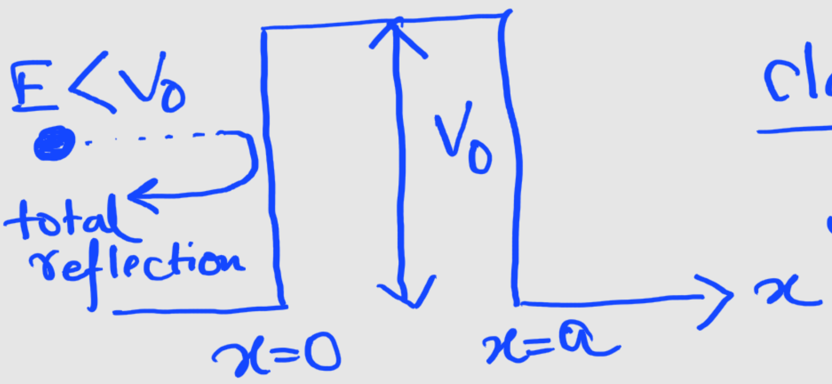
The potential barrier :-

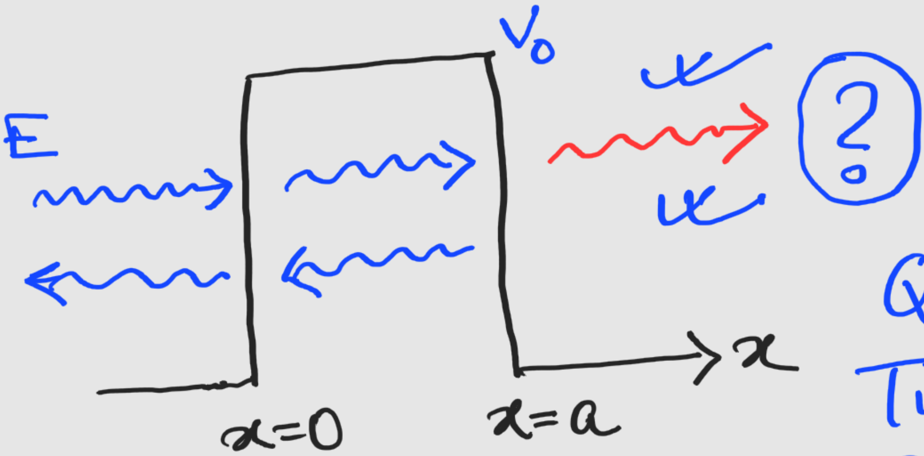
$(E < V_0)$



$h_2 > h_1$



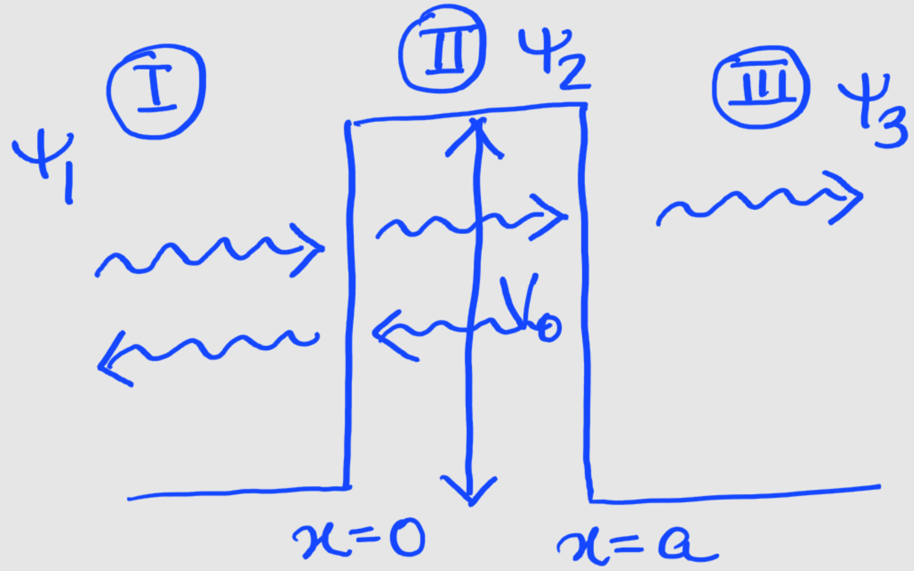
Classical Physics



$E < V_0$

Quantum Tunneling Effect.

Very small chance but finite depending on the value of E, V_0.



I region

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_1}{dx^2} = E\psi_1 \Rightarrow \frac{d^2\psi_1}{dx^2} + \frac{2mE}{\hbar^2}\psi_1 = 0$$

$$\Rightarrow \frac{d^2\psi_1}{dx^2} + k_1^2\psi_1 = 0 \qquad k_1 = \sqrt{\frac{2mE}{\hbar^2}}.$$

$$\psi_1 = \underline{A.e}^{ik_1x} + \underline{Be}^{-ik_1x}.$$

II region

$$\frac{d^2\psi_2}{dx^2} + \frac{2m}{\hbar^2}.\underline{(E-V_0)}\psi_2 = 0.$$

$E-V_0 = -\text{ve}$ because $E < V_0$.

$$k_2' = \sqrt{\frac{2m}{\hbar^2}(E-V_0)} = \sqrt{(-1).\frac{2m}{\hbar^2}.(V_0-E)}$$

$$= i.k_2.$$

$$\Rightarrow \frac{d^2\psi_2}{dx^2} - K_2^2\,\psi_2 = 0. \qquad K_2 = \sqrt{\frac{2m}{\hbar^2}(V_0 - E)}.$$

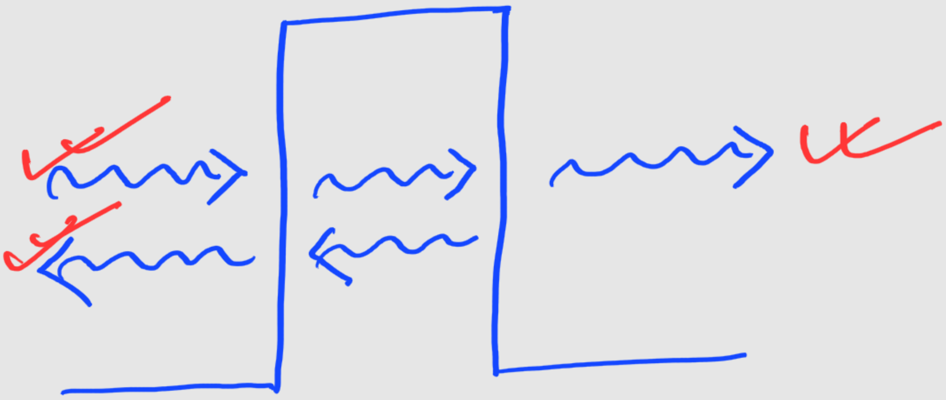
$$\psi_2 = \underline{C\cdot e^{K_2 x}} + \underline{D\cdot e^{-K_2 x}}.$$

III region

$$\frac{d^2\psi_3}{dx^2} + \frac{2m}{\hbar^2}E\,\psi_3 = 0.$$

$$\frac{d^2\psi_3}{dx^2} + k_1^2\,\psi_3 = 0.$$

$$\boxed{\psi_3 = F\cdot e^{ik_1 x}}\ \boxed{+\ \underset{\substack{\| \\ 0}}{G}\cdot e^{-ik_1 x}}\ \times$$



$E < V_0$

A ⇒ arbitrary Const.

B, C, D, F → depend on barrier height, barrier width & E.

Reflection Co-eff^n

$$R = \frac{k_1 |B|^2}{k_1 |A|^2}.$$

k_1 = angular wave no.

$$\left. \begin{array}{l} p = h/\lambda \\ k = 2\pi/\lambda \end{array} \right]$$

Transmission Co-eff^n

$$T = \frac{k_1 |F|^2}{k_1 |A|^2}$$

$$|A|^2 = A^* A$$

Boundary Cond^n

$\psi, \frac{d\psi}{dx} \rightarrow$ to be continuous.

1) at $x=0$ $\quad \psi_1 \Big|_{x=0} = \psi_2 \Big|_{x=0}$

$$\Rightarrow \boxed{A + B = C + D} \quad \text{- - - - -} \quad (1)$$

2) $\left.\frac{d\psi_1}{dx}\right|_{x=0} = \left.\frac{d\psi_2}{dx}\right|_{x=0}.$

$$\boxed{ik_1 (A - B) = k_2 (C - D)} \quad \cdots\cdots \quad (2)$$

3) $\psi_2 \Big|_{x=a} = \psi_3 \Big|_{x=a}$

$$\boxed{C \cdot e^{k_2 a} + D \cdot e^{-k_2 a} = F \cdot e^{ika}} \quad \text{-} \cdots \quad (3)$$

4) $$\left.\frac{d\Psi_2}{dx}\right|_{x=a} = \left.\frac{d\Psi_3}{dx}\right|_{x=a}$$

$$\boxed{k_2\left(Ce^{k_2 a} - De^{-k_2 a}\right) = ik_1 . F . e^{ik_1 a}} \quad \ldots (4)$$

By Solving:

$$\lambda = a\sqrt{\frac{2mV_0}{\hbar^2}} \quad ; \quad \epsilon = \frac{E}{V_0}$$

$$R = \frac{T}{4\epsilon . (1-\epsilon)} . \sinh^2\left(\lambda\sqrt{1-\epsilon}\right)$$

$$T = \left[1 + \frac{1}{4\epsilon(\epsilon-1)} . \sinh^2\left(\lambda\sqrt{1-\epsilon}\right)\right]^{-1}$$

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