lecture 14 Tue 22 Aug, 2023

$z = c^T x$, current BFS $:= (x_B, 0)$

$$z_{current} = c_B^T x_B = \sum_{i=1}^{n} c_{Bi} x_{Bi}$$

$[d_1, -, d_r, -, d_m]$ $\quad$ $[d_1, -, d_{r-1}, a_j, d_{r+1}, -, d_m]$

$B \rightarrow \hat{B}$; new $\hat{z}$

$$\text{New} :- \hat{z} = \hat{c}_B^T \hat{x}_B = \sum_{i=1}^{m} \hat{c}_{Bi} \hat{x}_{Bi}$$

$$= \sum_{\substack{i=1 \\ i \neq r}}^{m} \hat{c}_{Bi} \hat{x}_{Bi} + \hat{c}_{Br} \hat{x}_{Br}$$

$$= \sum_{\substack{i=1 \\ i \neq r}}^{m} c_{Bi} \left(x_{Bi} - \frac{y_{ij}}{y_{rj}} x_{Br} \right) + c_j \frac{x_{Br}}{y_{rj}}$$

c_j : cost coefficient in the objective func^n corresponding to variable x_j attach with column a_j in A.

$$= \sum_{i=1}^{m} c_{Bi} \left(x_{Bi} - \frac{y_{ij} x_{Br}}{y_{rj}} \right) + c_j \frac{x_{Br}}{y_{rj}}$$

(including $i = r$ in the summ^n will not have any effect $\because$ its value is zero

$$= \sum_{i=1}^{m} c_{Bi} x_{Bi} + \frac{x_{Br}}{y_{rj}} \left(c_j - \sum_{i=1}^{m} c_{Bi} y_{ij} \right)$$

New $\hat{z} = z_{current} + \frac{x_{Br}}{y_{rj}} (c_j - z_j)$

using a notation

$$z_j = \sum_{i=1}^{m} c_{Bi}\, y_{ij}$$

$$= c_B^T y_j$$

For strict improvement with objective values (in sense of maximization)

New $\hat{z} > z_{current}$

iff $\frac{x_{Br}}{y_{rj}} (c_j - z_j) > 0 \quad \forall j$

$y_{rj} > 0$

(Note index r is fixed by min ratio earlier).

* Under non degeneracy in the current BFS, strict improvement (max) in objective fn is possible iff we choose the index j s.t.

$$z_j - c_j < 0.$$

In case of degenaracy then it is possible that in the minimum ratio we may have $x_{Br} = 0$ then

$$z_{current} = \hat{z}_{new}$$

there is no improvement in obj. func^n., but the good thing is then $\hat{z}_{new}$ is not decreasing. So, this needs more explanation

look for any j for which $z_j - c_j < 0$. Then, proceed in the algorithm as there is a chance of improvement in iteration to come

If no such j exists, that means,

$$z_j - c_j \geq 0 \quad \forall j \;,\; 1 \leq j \leq n.$$