COL351: Analysis and Design of Algorithms

Tutorial Sheet - 4

September 1, 2022

Question 1 You are given a server and a collection of n jobs numbered 1 to n, wherein the i^{th} job has a start time s_i and a termination t_i. Further, for $i \in [1, n]$, scheduling job i on the server generates a revenue R_i. Your task is to find a subset $\mathcal{I}$ of non-overlapping jobs for which $\sum_{i \in \mathcal{I}} R_i$ is maximized.

Question 2 Let X, Y be two sequences of length respectively m, n. Design an $O(m + n)$ space algorithm to compute the longest common subsequence of X, Y in time that is polynomial in m, n.

Question 3 You are given a checker-board that has 4 rows and n columns, and with an integer written in each square. You are also given a set of $2n$ pebbles, and your task is to place some or all of these pebbles on the checker-board (each pebble can be placed on exactly one square) so as to maximize the sum of the integers in the squares that are covered by pebbles. There is one constraint: for a placement of pebbles to be legal, no two of them can be on horizontally or vertically adjacent squares (diagonal adjacency is fine). Give an $O(n)$ time algorithm to find an optimal placement of the pebbles.

Question 4 For a pair of strings $X = (x_1, \ldots, x_n), Y = (y_1, \ldots, y_m)$, a string $Z = (z_1, \ldots, z_{m+n})$ is said to to be an interleaving of X and Y if the indices of Z can be partitioned into two sets I, J respectively of size $n,\ m$, such that restriction of Z to I gives X and restriction of Z to J gives Y. For example, if $X = (1010)$ and $Y = (0011)$, then $Z = (10001101)$ is an interleaving because the restriction of Z to odd indices gives X, and restriction of Z to even indices gives Y. Provide an $O(mn)$ time algorithm to verify whether Z is an interleaving of X and Y.

Question 5 A town has n residents labeled $1, \ldots, n$ living on a straight road. It has been decided to open k Covid-test centers along this road. The goal is to minimize the sum total of distance that all the residents need to travel to get to their nearest testing center. As input, you are given $n,\ k$, and an array A of size n, where $A[i]$ is the location of resident i. You can assume A has integer entries and is sorted in non-decreasing order. Your task is to compute a k-sized integer array C of locations such that the following sum is minimized:

$$\sum_{i=1}^{n} d_i, \text{ where, } d_i = \min_{j \in \{1,\ldots,k\}} |A[i] - C[j]|.$$