MTL 103: Practice Sheet 1

1. If $S = \{(x, y) : -x+y \leq 4,\ 3x-2y \leq 9,\ x,\ y \geq 0\}$, solve graphically the following problems: (a) max x - 5y over S (b) max 6x + 4y + 18 over S

2. Solve max $F(x, y) = x + 10y$ over the solutions set of

$$\begin{aligned} &\min\ f(x,y) = y \\ &\text{s.t.}\quad -5x + 4y \leq 6 \\ &\qquad x + 2y \leq 10 \\ &\qquad 2x - y \leq 15 \\ &\qquad 2x + 10y \geq 15 \\ &\qquad x,\ y \geq 0 \end{aligned}$$

3. Solve max $z\ =\ \min\,(3x - 10,\ -5x + 5),\ \ 0 \leq x \leq 5.$

4. Suppose we want to show that all solutions of $x + y \leq 4;\ 2x - 3y \leq 6;\ x,\ y \geq 0$ also satisfy $x + 2y \leq 8$. Formulate this problem as an LP and verify your result.

5. Solve

$$\begin{aligned} &\max\ f(x,y) = 2x + y \\ &\text{s.t.}\ \ 0 \leq x \leq 2 \\ &\qquad x + y \leq 3 \\ &\qquad x + 2y \leq 5 \\ &\qquad y \geq 0 \end{aligned}$$

$$\begin{aligned} &\min\ f(x,y) = 5x + 2y \\ &\text{s.t.}\ \ x + 4y \geq 4 \\ &\qquad 5x + 2y \geq 10 \\ &\qquad x,\ y \geq 0 \end{aligned}$$

6. Consider the following LP

$$\begin{aligned} &\max\ \ z = x_1 - 4x_2 \\ &\ \text{s.t.}\ x_1 - x_2 \geq -4 \\ &4x_1 + 5x_2 \leq 45 \\ &5x_1 - 2x_2 \leq 20 \\ &x_1 \geq 0. \end{aligned}$$

Describe the range set of x_2 for which the LP (a) becomes unbounded; (b) infeasible.

7. For any non-zero cost vector $c = (c_1,\ c_2)$, can $x^* = (1,\ 3)$ be an optimal solution of the problem given below.

$$\begin{aligned} \max\quad & c_1x_1 + c_2x_2 \\ & 2x_1 + 3x_2 \leq 11 \\ & 3x_1 - 2x_2 \leq 9 \\ & x_1,\ x_2 \geq 0. \end{aligned}$$

Solve the LPP graphically when $c = (1,\ 5)$, and find the optimal value of the LP.

8. Show that if an LP has more than one optimal solution then it has infinitely many solutions.

9. Prove that the set of all convex combinations of a finite number of L.I. vectors is a convex set.

10. Find the extreme directions (if any) and extreme points of the set described by $\{(x_1, x_2) : 5x_1 + 3x_2 \geq 15,\ -x_1 + x_2 \leq 1,\ 5x_1 - 6x_2 \geq -30,\ x_1,\ x_2 \geq 0\}$.

11. Find the extreme points and extreme directions of the set

$$\begin{aligned} -3x_1 + x_2 &\leq -2 \\ -x_1 + x_2 &\leq 2 \\ -x_1 + 2x_2 &\leq 8 \\ x_1 \geq 0,\ x_2 &\geq 2 \end{aligned}$$

12. If $Ax = b,\ A : m \times n,\ \rho(A) = m$, has a solution which involves precisely m non-zero variables and if this solution is unique, then prove that it must be a basic solution.

13. Find the number of degenerate and non-degenerate basic feasible solutions for the system graphically

$$2x + 3y \leq 21;\ \ 3x - y \leq 15;\ \ x + y \geq 5;\ \ y \leq 5;\ \ x,\ y \geq 0.$$

14. Find all basic solutions of
(a) $x_1 + 2x_2 + 3x_3 + 4x_4 = 7,\ \ 2x_1 + x_2 + x_3 + 2x_4 = 3$
(b) $8x_1 + 6x_2 + 12x_3 + x_4 + x_5 = 6,\ \ 9x_1 + x_2 + 2x_3 + 6x_4 + 10x_5 = 11$