Lecture 8
Signals and Systems (ELL205)

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Outline of the Lecture

System Properties

1. Memoryless
2. Causal
3. Invertible
4. Stable
5. Time invariant
6. Linear
7. Incrementally Linear

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Linearity

A system is said to be linear iff it satisfies superposition (additivity & homogeneity)

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Additivity

$$x_1(t) \longrightarrow y_1(t)$$

$$x_2(t) \longrightarrow y_2(t)$$

$$x_1(t) + x_2(t) \longrightarrow y_1(t) + y_2(t)$$

Linearity

A system is said to be linear iff it satisfies superposition (additivity & homogeneity)

Homogeneity

$$x_1(t) \longrightarrow y_1(t)$$

$$ax_1(t) \longrightarrow ay_1(t)$$

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Homogeneity

$$x_1(t) \longrightarrow y_1(t)$$

$$ax_1(t) \longrightarrow ay_1(t)$$

If $a = 0$

$$0 \longrightarrow 0 \quad \text{ZIZO}$$

Linearity

Example of an additive system which does not satisfy homogeneity.

$$y(t) = \overline{x(t)}$$

Linearity

Example of a system which satisfy homogeneity but not additivity.

$$y(t) = \frac{x^2(t)}{x(t-1)}$$

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Incrementally Linear

An incrementally Linear system is

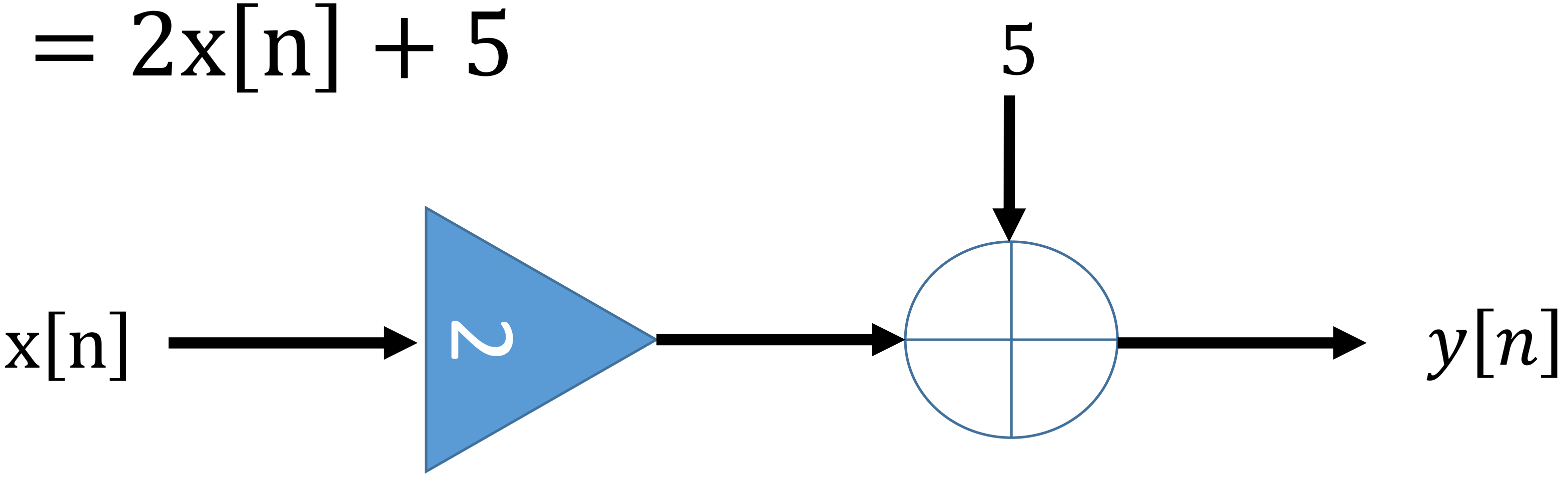
Linear + ZIR

Incrementally Linear

An incrementally Linear system is

Linear + ZIR

$$y[n] = 2\mathrm{x}[\mathrm{n}] + 5$$



Linear + ZIR

Outline of the Lecture

Introduction to Convolution

IEEE Spectrum, March 1991, p. 60: Convolution has driven many electrical engineering undergraduates to contemplate theology either for salvation or as an alternative career.

Convolution: its bark is worse than its bite!

System properties

- Memoryless
- Invertibility
- Causality
- Stablity
- Linearity
- Time invariance

LTI (Linear Time Invariant)

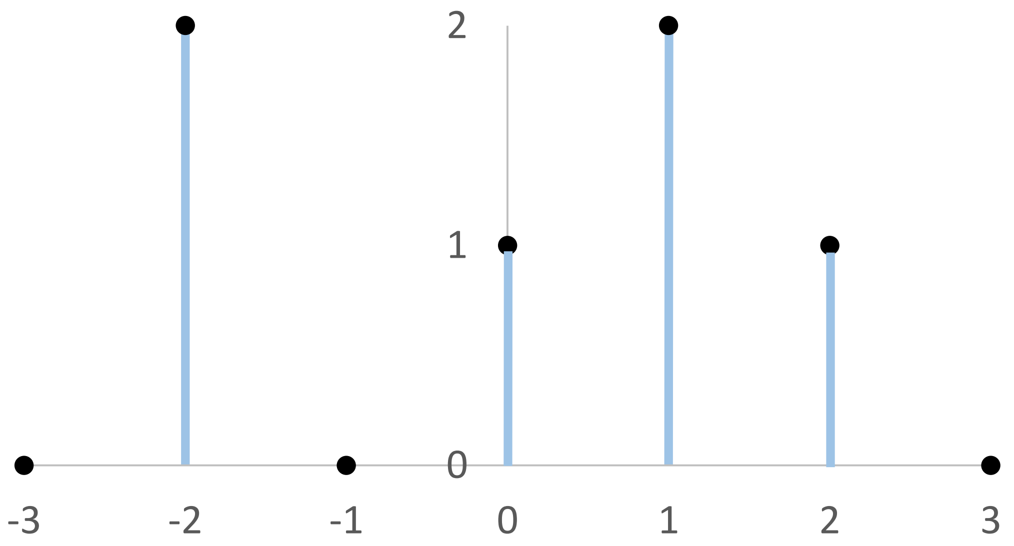
Decomposition of a signal

- Is there exists a better way of finding output of an LTI system for a given input ?
  - Decompose an input signal into a linear combination of basic signals
  - Response due to sum of inputs = Sum of response due to each inputs

Decomposition of a signal

- Is there exists a better way of finding output of an LTI system for a given input ?
  - Decompose an input signal into a linear combination of basic signals
  - Response due to sum of inputs = Sum of response due to each inputs
- Two kinds of basic signals:
  - Delayed impulses $\longleftrightarrow$ Convolution
  - Complex exponentials $\longleftrightarrow$ Fourier Series

How a signal can be decomposed into a linear combination of delayed impulses?

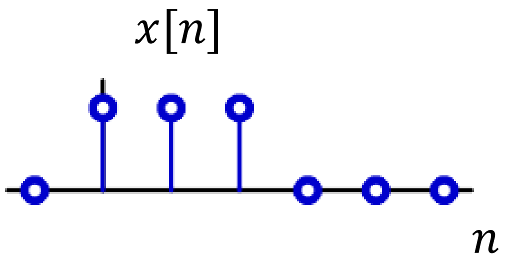


$$0\times\delta[n+3]+2\times\delta[n+2]+0\times\delta[n+1]+$$
$$1\times\delta[n]+2\times\delta[n-1]+1\times\delta[n-2]+0\times\delta[n-3]$$

$$x[n]=\sum_{k=-\infty}^{\infty}\underbrace{x[k]}_{\text{Weights}}\underbrace{\delta[n-k]}_{\text{Delayed impulses}}$$

How decomposition helps?

Break input into additive parts and sum the responses to the parts.



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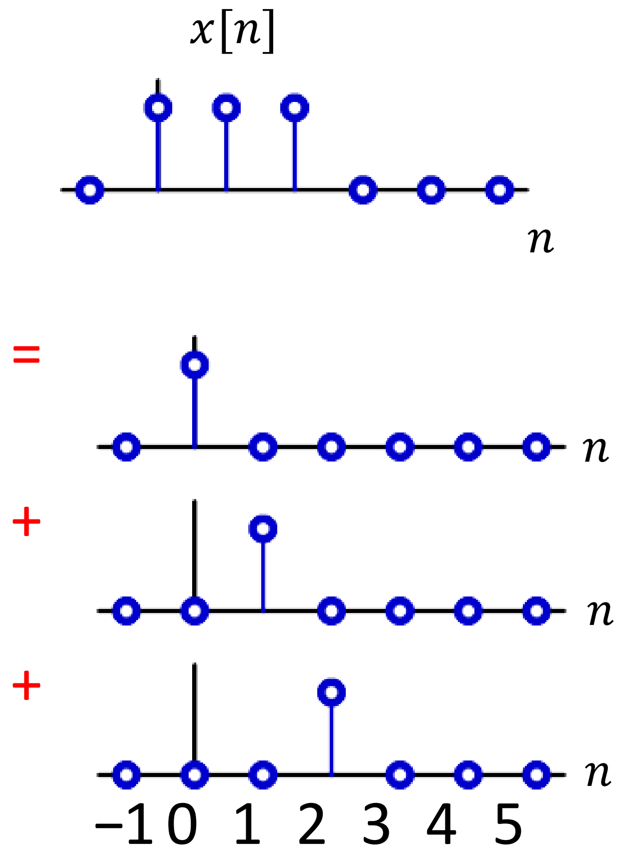

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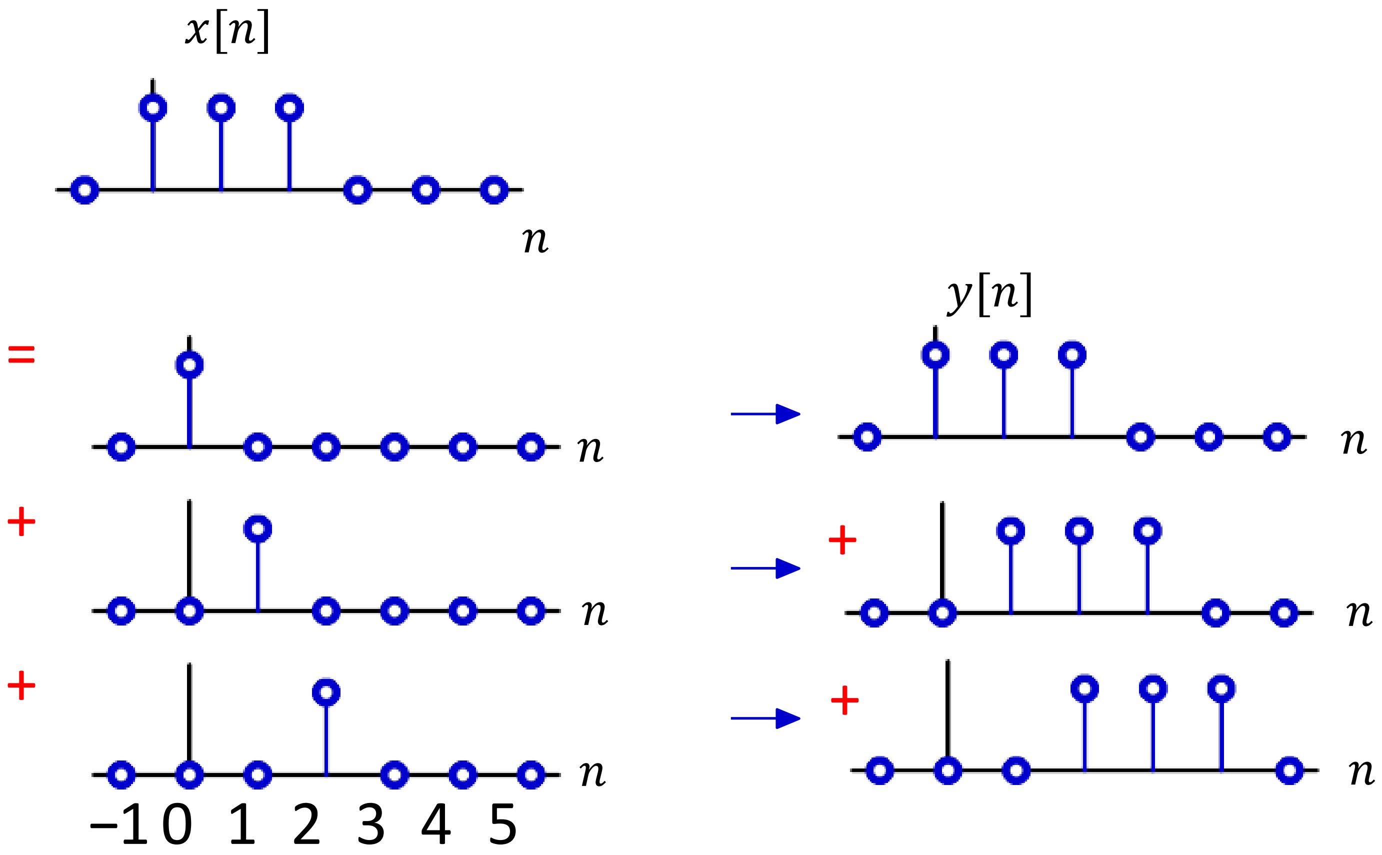
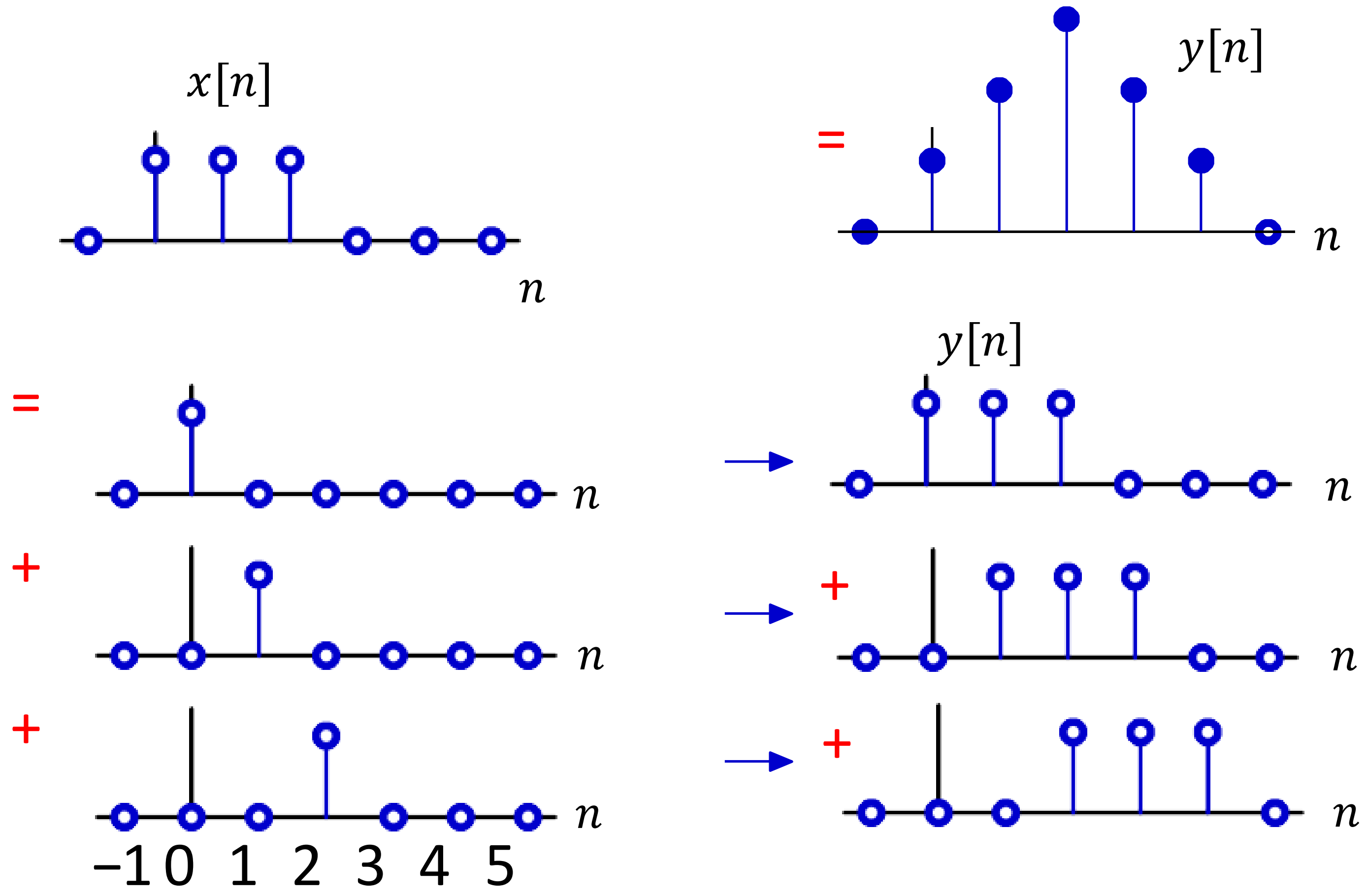


How decomposition helps?

Break input into additive parts and sum the responses to the parts.



$$\left.\begin{array}{l} \delta[n] \longrightarrow \boxed{S} \longrightarrow h[n] \\[2em] \delta[n-n_o] \longrightarrow \boxed{S} \longrightarrow h[n-n_o] \end{array}\right\} \text{Time-invariance}$$

Linearity & Time Invariance

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \longrightarrow \boxed{S} \longrightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$x[n] \longrightarrow \boxed{\;S\;} \longrightarrow y[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

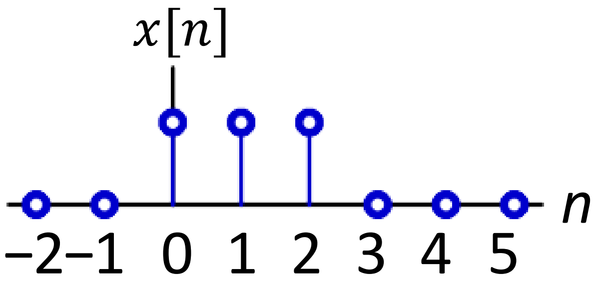
- This operation is referred to as **convolution**
- Representing a system by a signal
- Very powerful if you know the response of a system to any impulse, you can find response of the system to any input

Notation:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[n] * h[n] \; = (x*h)[n]$$

Convolution operation

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



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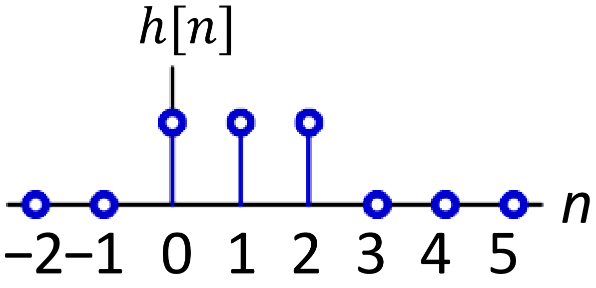
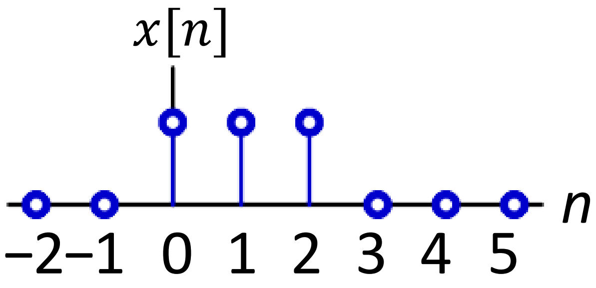


Convolution operation

$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[0-k]$$



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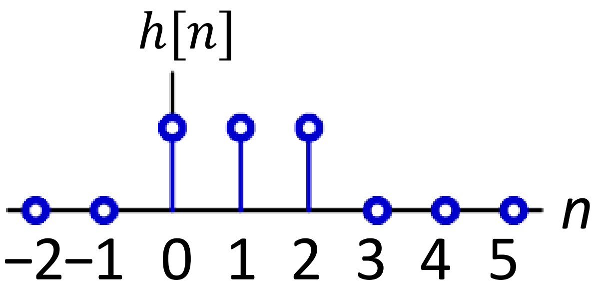

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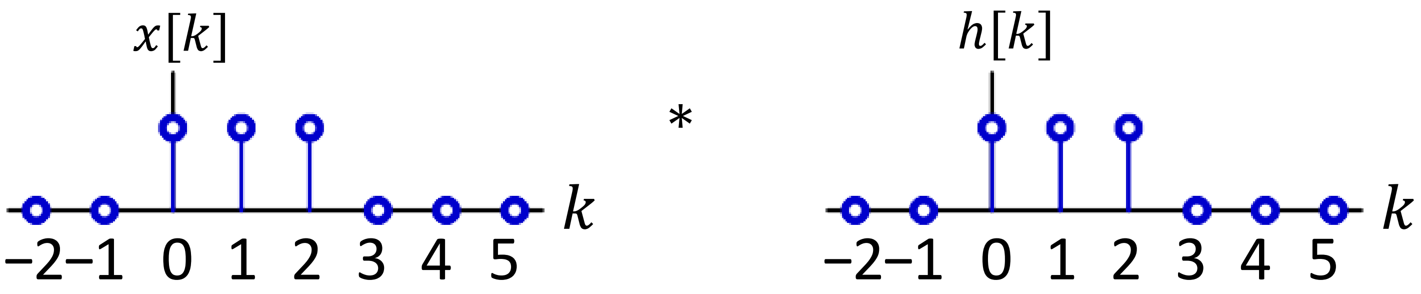

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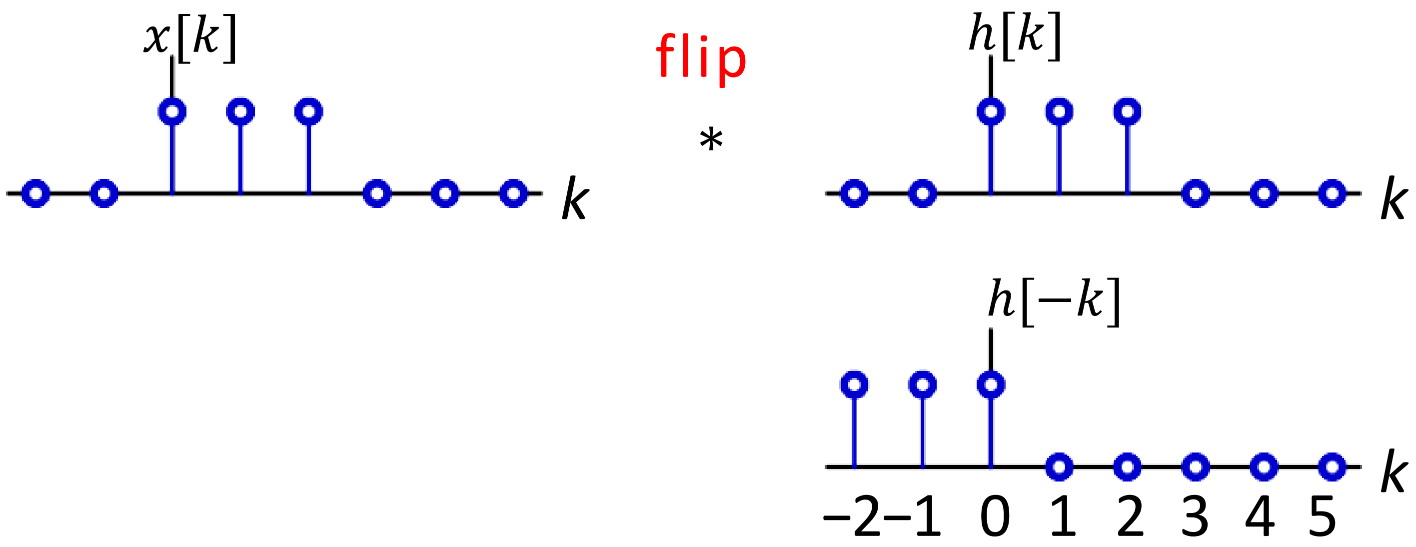

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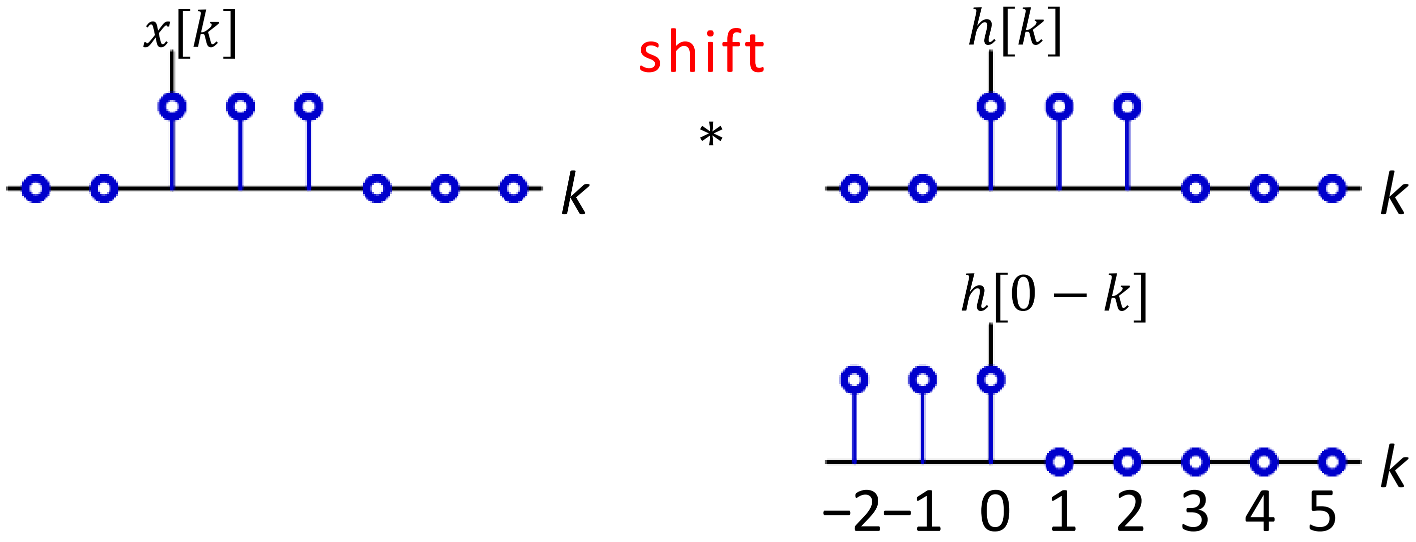

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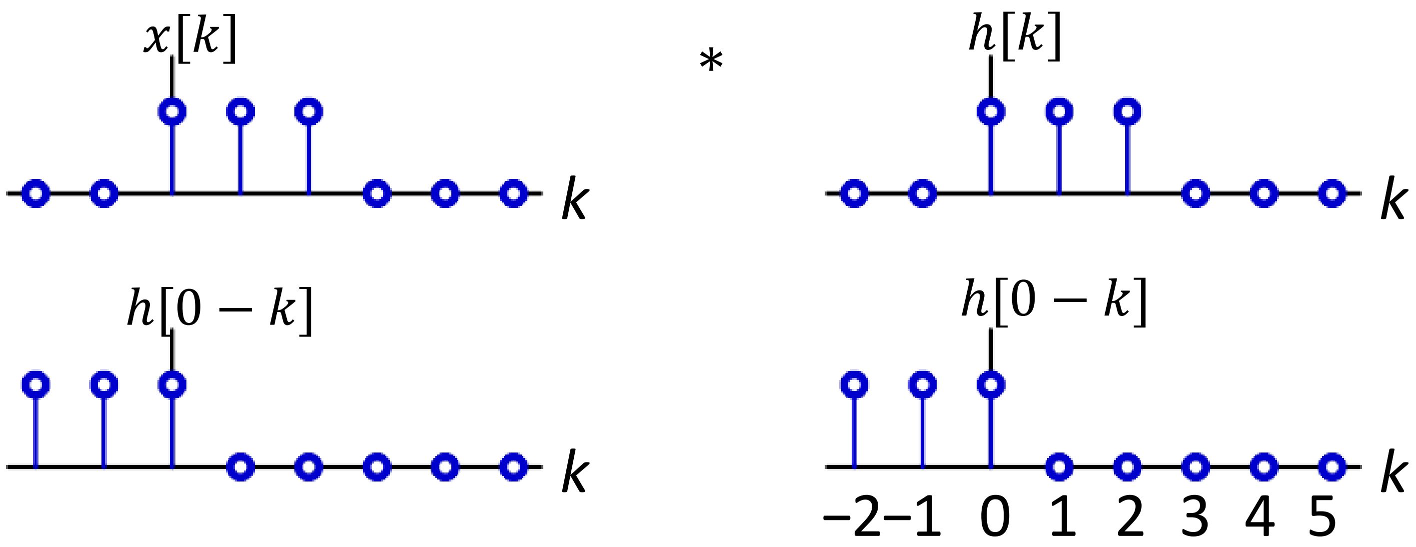

Convolution operation

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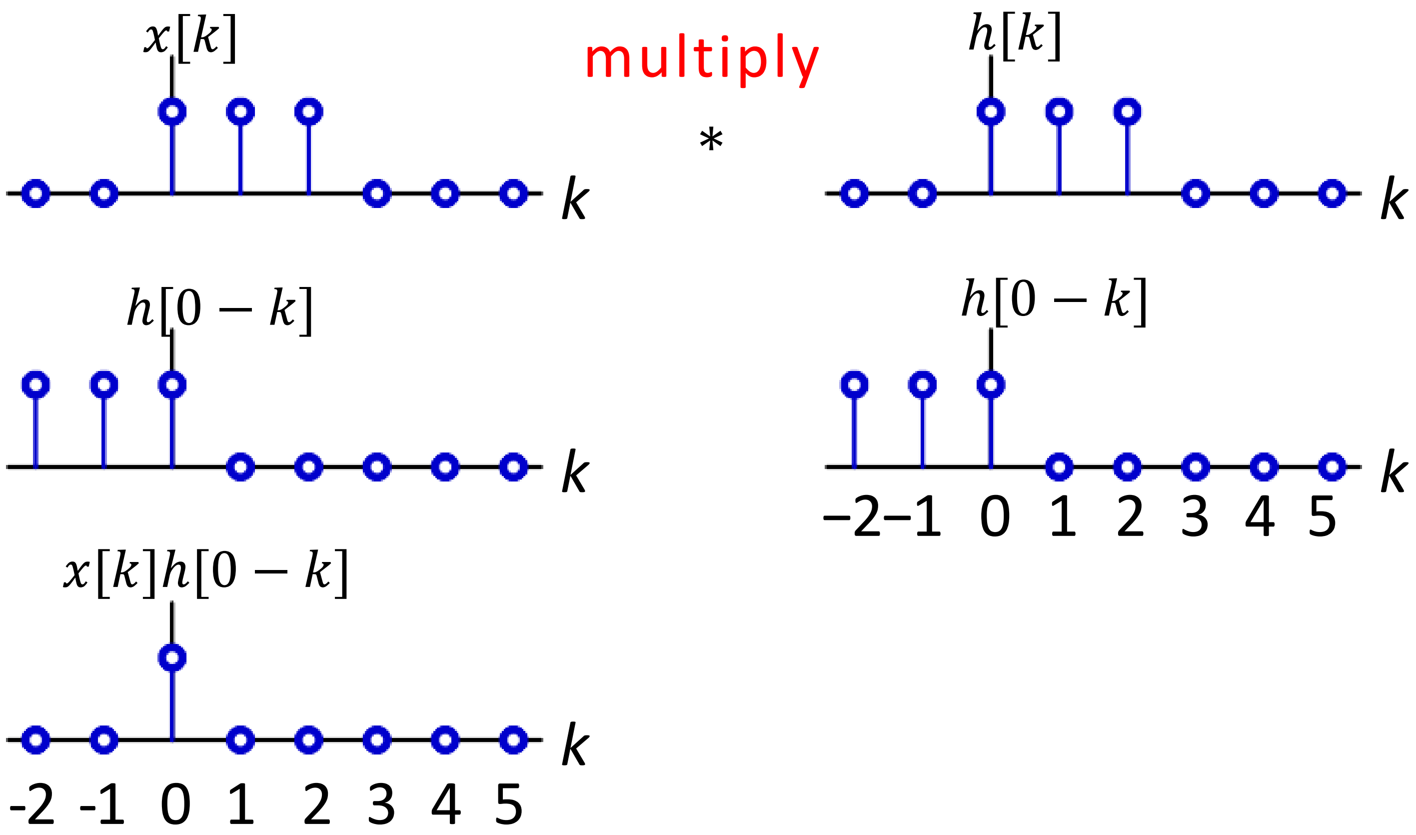

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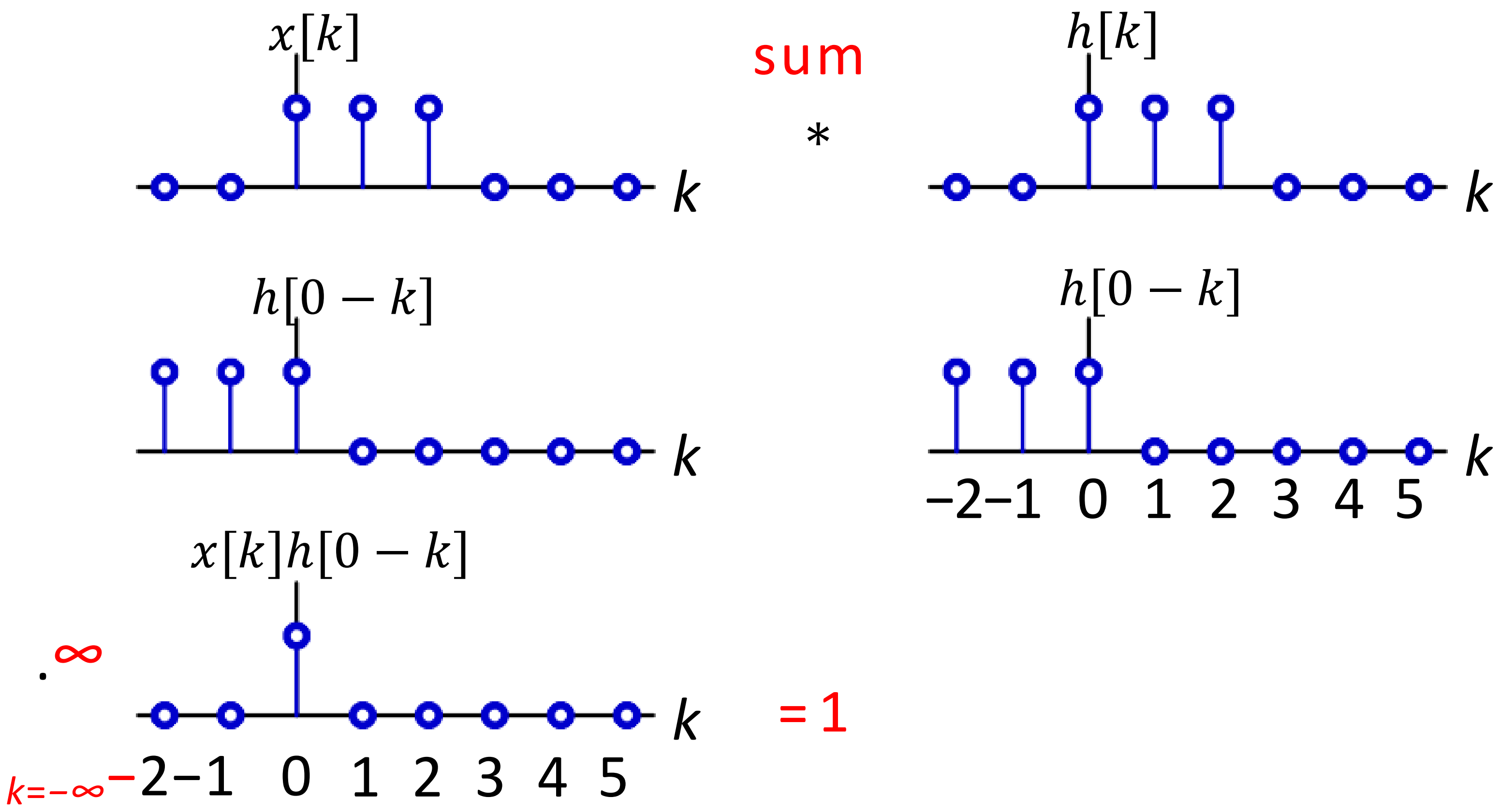
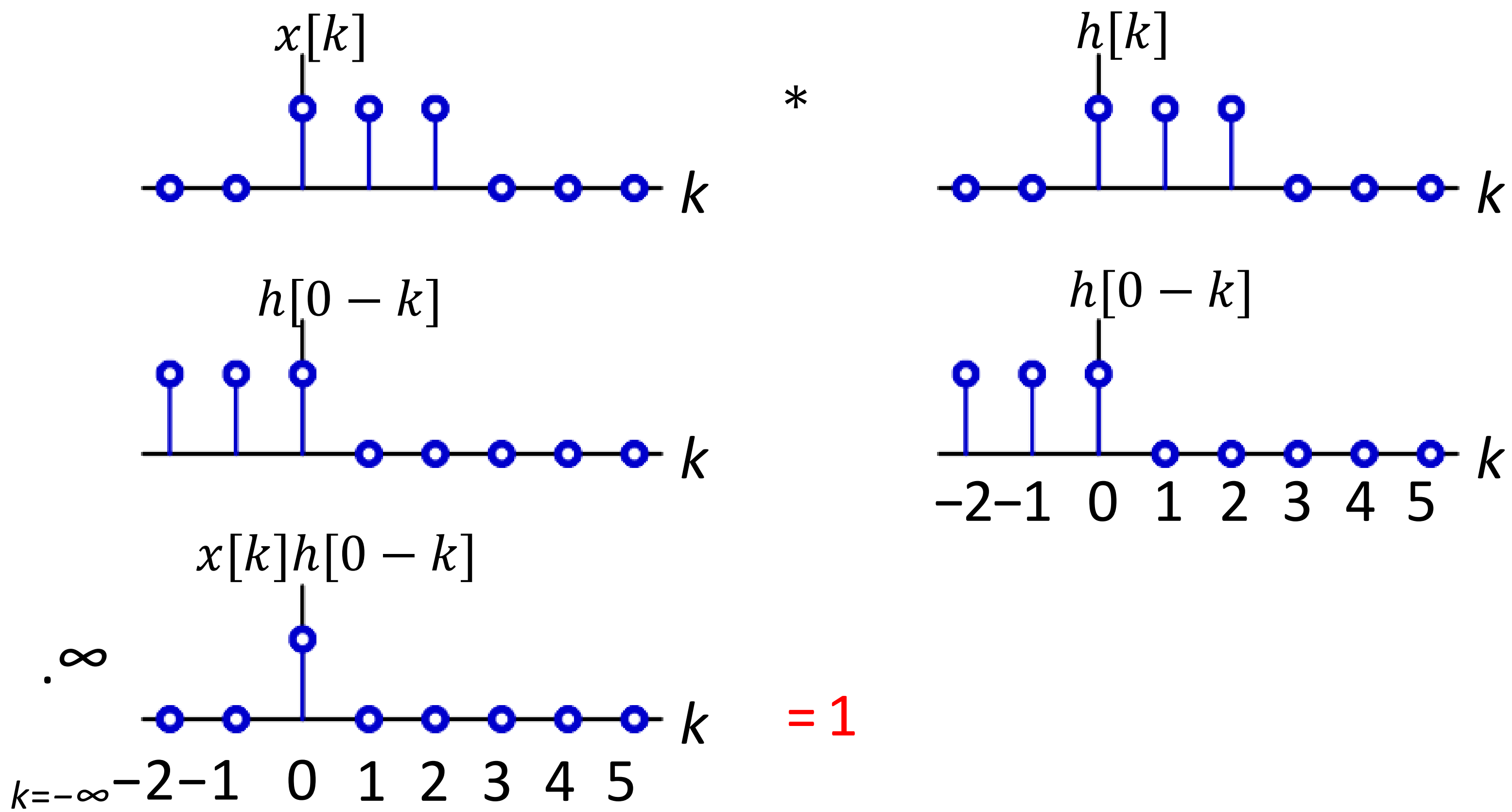


Convolution operation

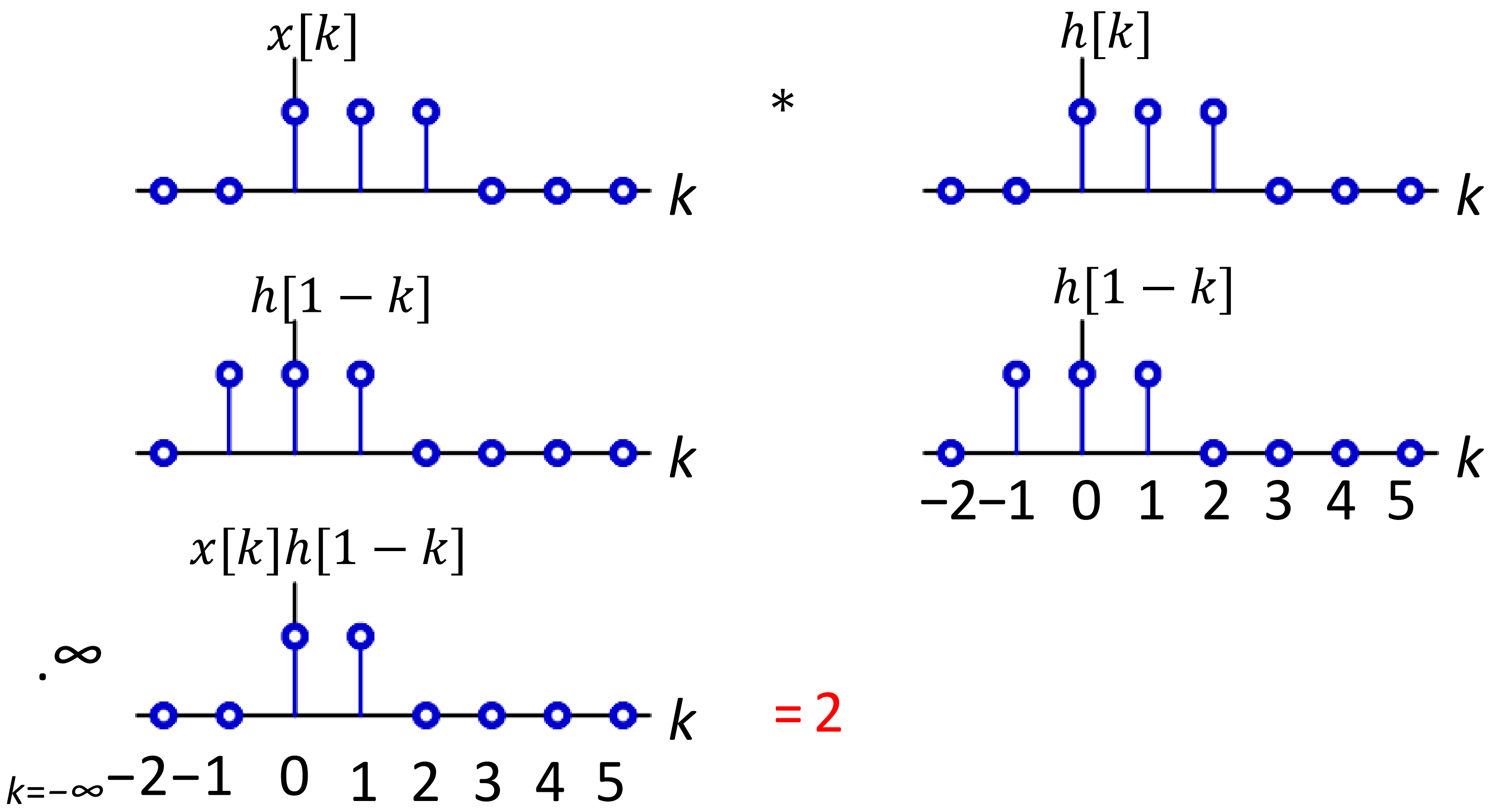
$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[0-k]$$



= 1

Convolution operation

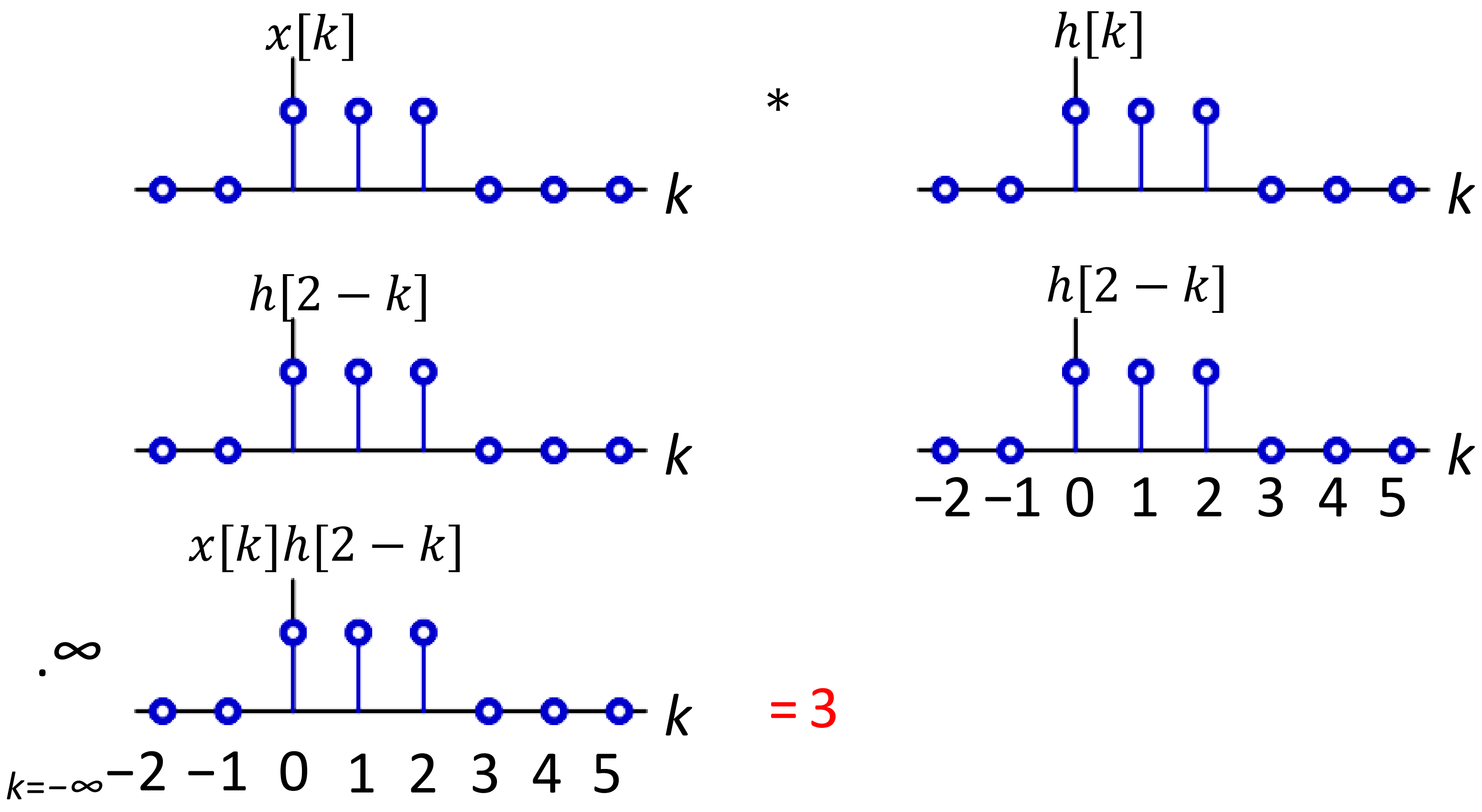
$$y[\textcolor{red}{1}] = \sum_{k=-\infty}^{\infty} x[k]h[1-k]$$



= 2

Convolution operation

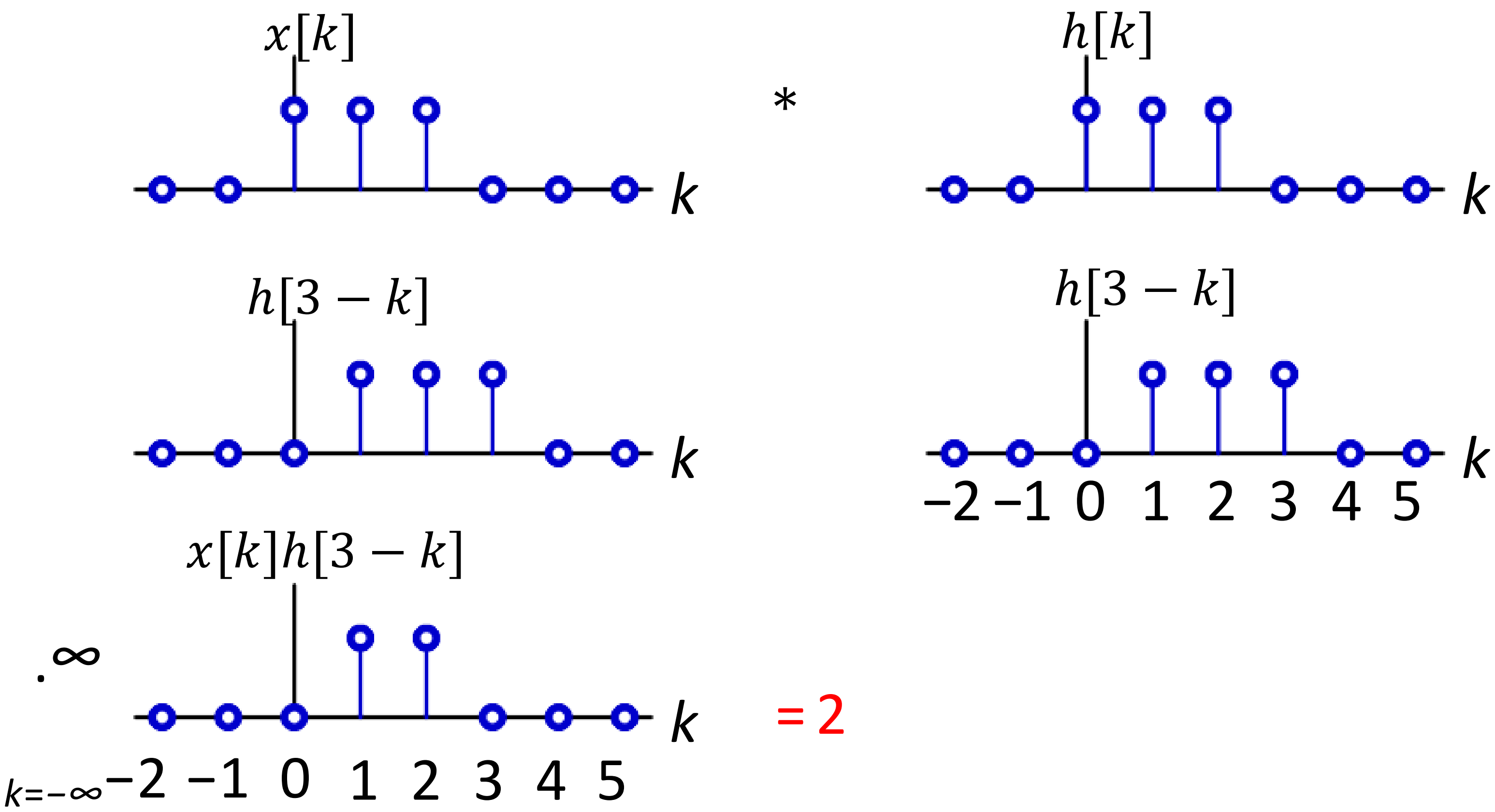
$$y[2] = \sum_{k=-\infty}^{\infty} x[k]h[2-k]$$



= 3

Convolution operation

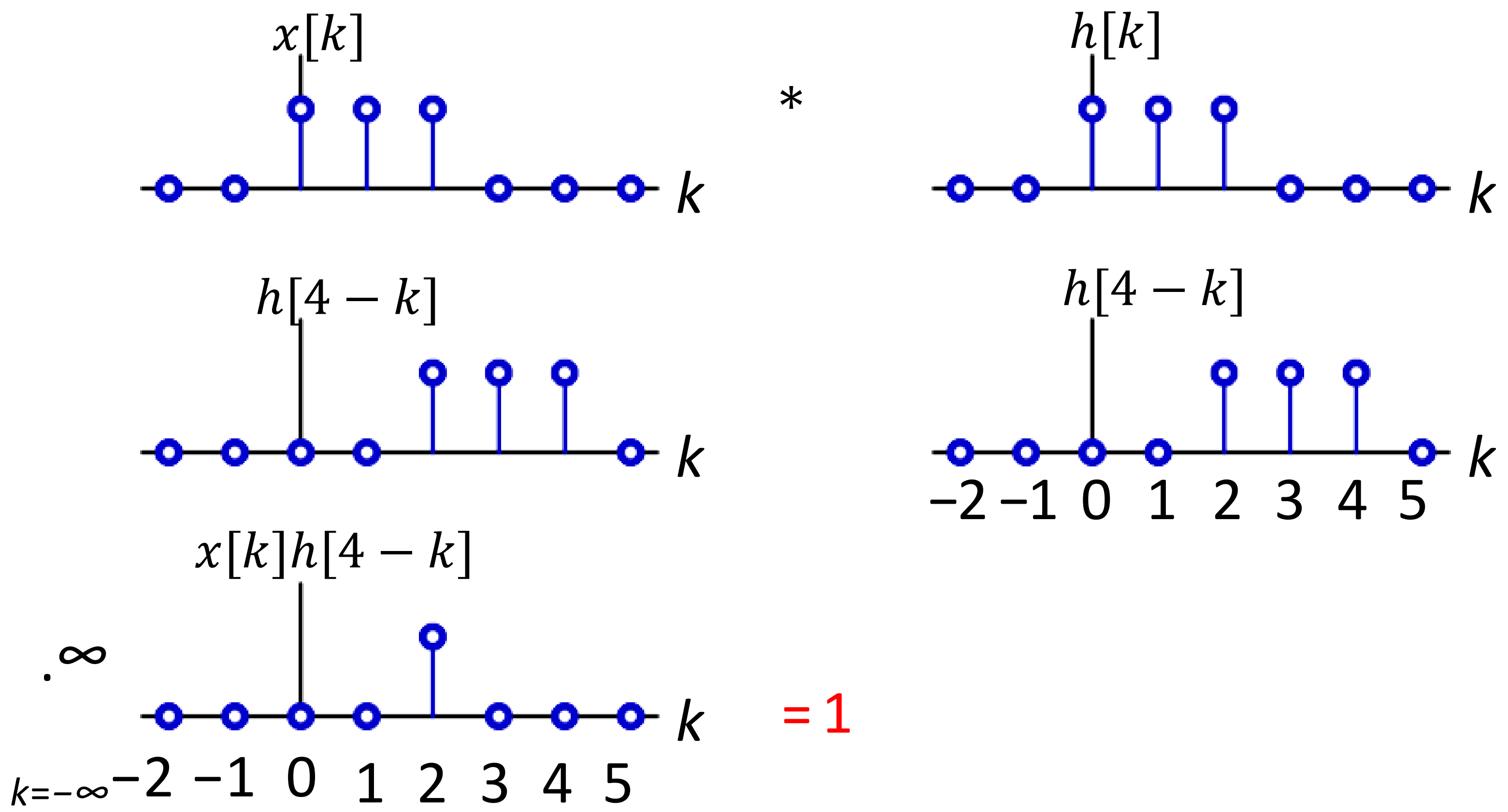
$$y[3] = \sum_{k=-\infty}^{\infty} x[k]h[3-k]$$



$= 2$

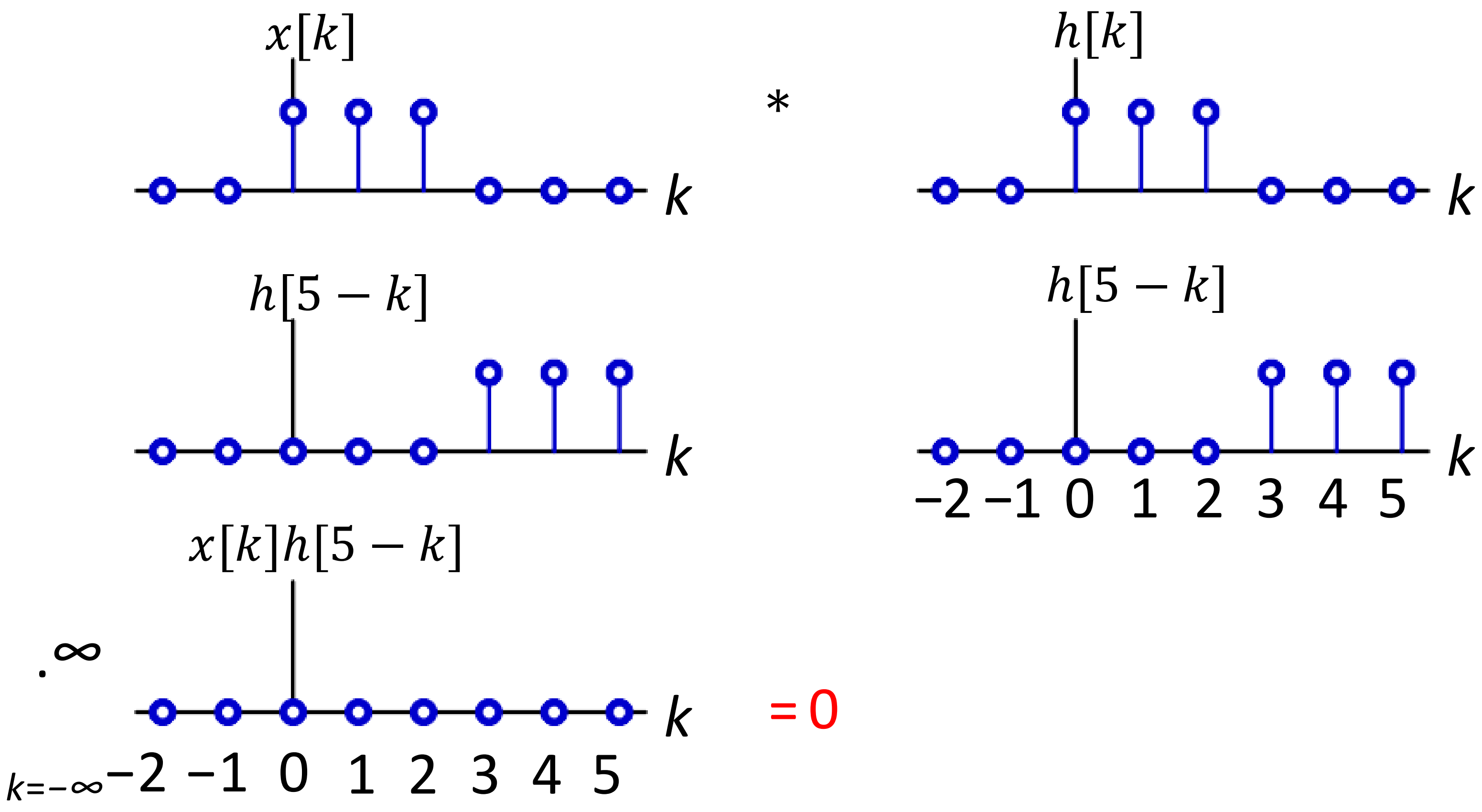
Convolution operation

$$y[4] = \sum_{k=-\infty}^{\infty} x[k]h[4-k]$$



Convolution operation

$$y[5] = \sum_{k=-\infty}^{\infty} x[k]h[5-k]$$



$= 0$

Summary of DT convolution

We can represent an LTI system by a single signal.

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

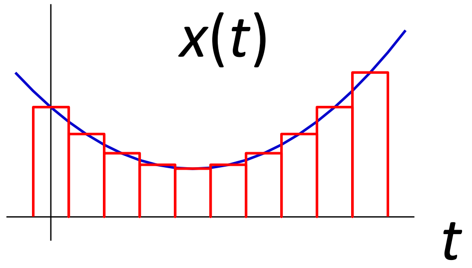
Unit-impulse response $h[n]$ is a complete description of an LTI system.

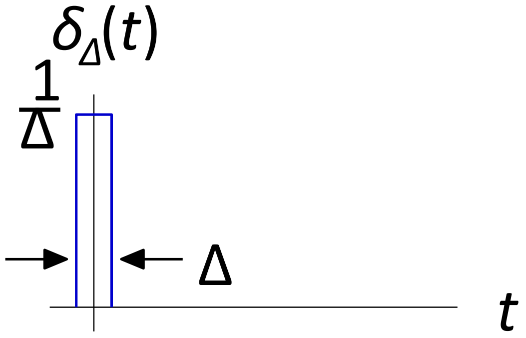
Given $h[n]$, we can compute the response $y[n]$ to any arbitrary input signal $x[n]$:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[n] * h[n] = (x*h)[n]$$

CT convolution

The same applies to CT signals



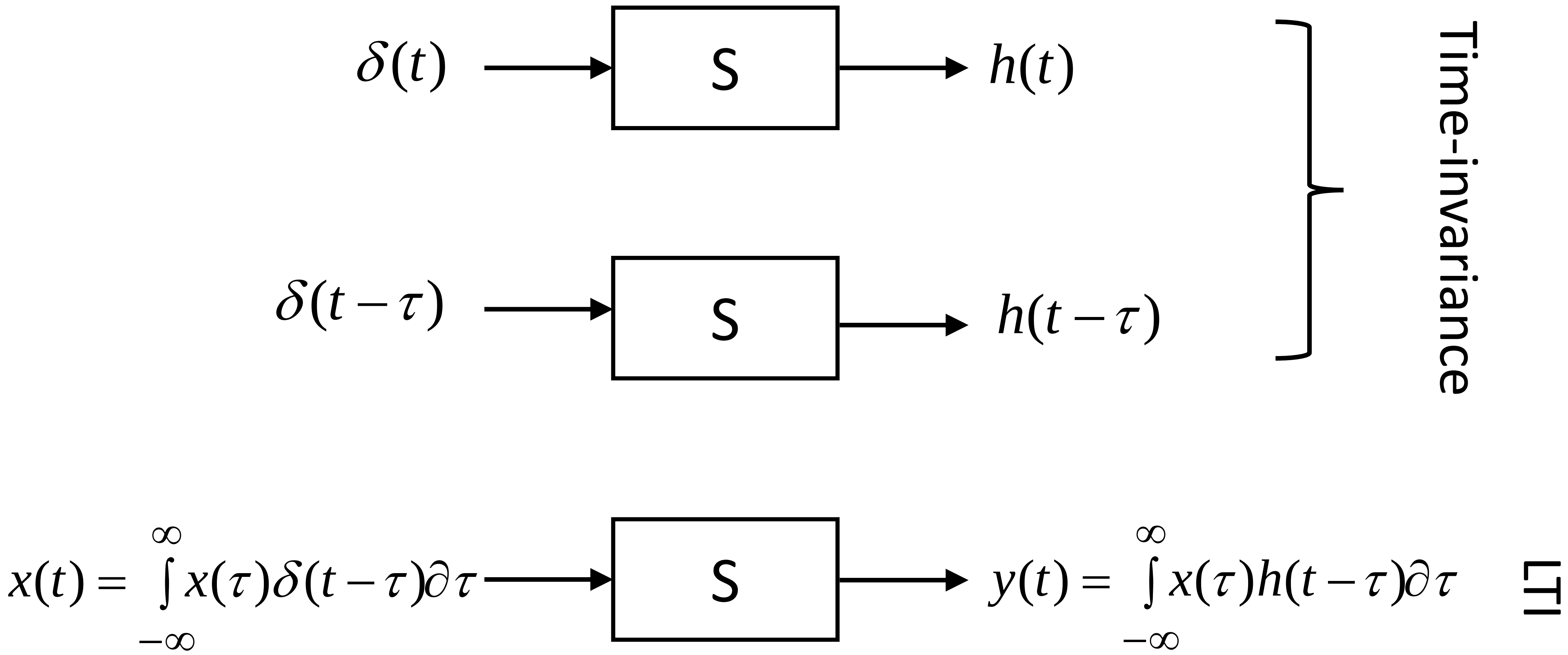


$$x(t) \cong x(0)\delta_{\Delta}(t)\Delta + x(\Delta)\delta_{\Delta}(t-\Delta)\Delta + x(2\Delta)\delta_{\Delta}(t-2\Delta)\Delta + \ldots\ldots$$

$$x(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{\infty} x(k\Delta)\delta_{\Delta}(t-k\Delta)\Delta$$

As $\Delta \to 0$, $k\Delta \to \tau$, $\Delta \to d\tau$, and $\delta_{\Delta}(t) \to \delta(t)$:

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\partial\tau$$


$\delta(t)$
S
$h(t)$
$\delta(t-\tau)$
S
$h(t-\tau)$
Time-invariance
$x(t)=\int_{-\infty}^{\infty}x(\tau)\delta(t-\tau)\partial\tau$
S
$y(t)=\int_{-\infty}^{\infty}x(\tau)h(t-\tau)\partial\tau$
LTI

Comparison of DT vs. CT

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\partial\tau$$

$$y[n] = x[n] * h[n]$$

$$y(t) = x(t) * h(t)$$

$$y[n] = (x * h)[n]$$

$$y(t) = (x * h)(t)$$