

COL 352 Introduction to Automata and Theory of Computation

Nikhil Balaji

Bharti 420
Indian Institute of Technology, Delhi
nbalaji@cse.iitd.ac.in

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Lecture 29: Rice's Theorem

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Undecidability is the not an exception - it is the rule!

Rice's theorem: A systematic way of proving undecidability of languages.

Property P

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We say that a property P is trivial if either $\mathcal{L}_P = \emptyset$ or \mathcal{L}_P is the set of all the Turing recognizable languages.

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Therefore, P is not trivial.

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$$3. \mathcal{L}_P = \left\{ M \mid \begin{array}{l} M \text{ is a TM and } L(M) \text{ is accepted by} \\ \text{a TM that has even number of states} \end{array} \right\}.$$

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Therefore, P is in fact all Turing recognizable languages.

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