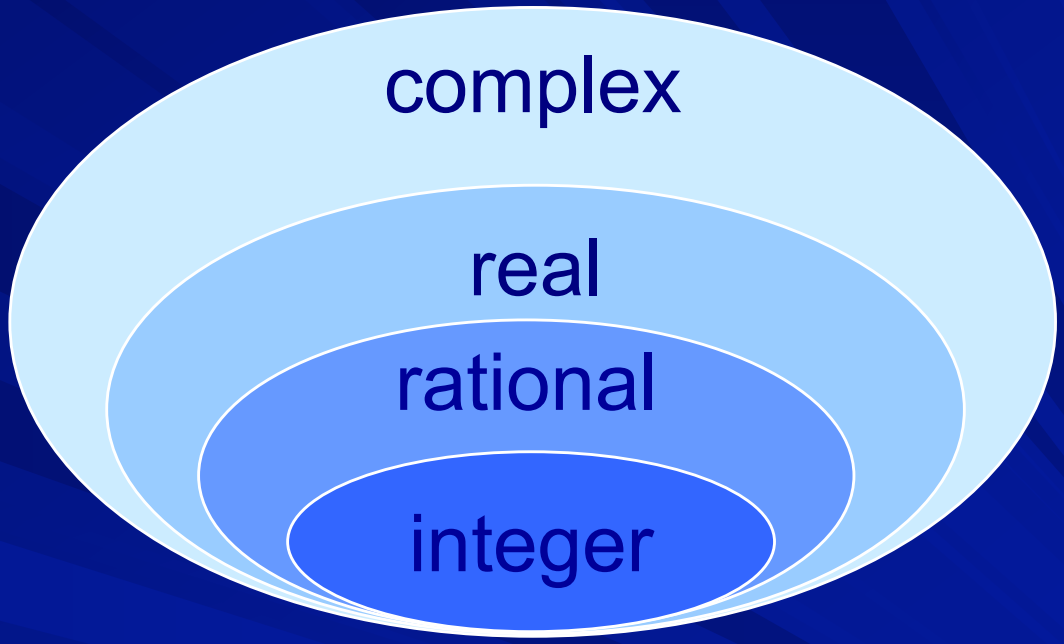
COL216
Computer Architecture

Concluding Remarks:
Floating point numbers and arithmetic
31st March 2022

Need to go beyond integers

- integer 7
- rational 5/8
- real $\sqrt{3}$
- complex 2 - 3 i



Extremely large and small values:

- distance pluto - sun = $5.9\ 10^{12}$ m
- mass of electron = 9.1×10^{-28} gm

Representing fractions

- Integer pairs (for rational numbers)

5	8	= 5/8

- Strings with explicit decimal point

-	2	4	7	.	0	9

- Implicit point at a fixed position

0	1	0	0	1	1	0	1	0	1	1	0	0	0	1	0	1	1

implicit point

- Floating point

  fraction x base $^{\text{power}}$

Numbers with binary point

$101.11 = 1x2^2 + 0x2^1 + 1x2^0 + 1x2^{-1} + 1x2^{-2}$

$= 4 + 1 + 0.5 + 0.25 = 5.75_{10}$

$0.6 = 0.1001100110011001100110011001.....$

$.6 \times 2 = 1 + .2$

$.2 \times 2 = 0 + .4$

$.4 \times 2 = 0 + .8$

$.8 \times 2 = 1 + .6$

FP numbers with base = 2

$(-1)^S \times F \times 2^E$

S = sign

F = fraction (fixed point number)
usually called mantissa or significand

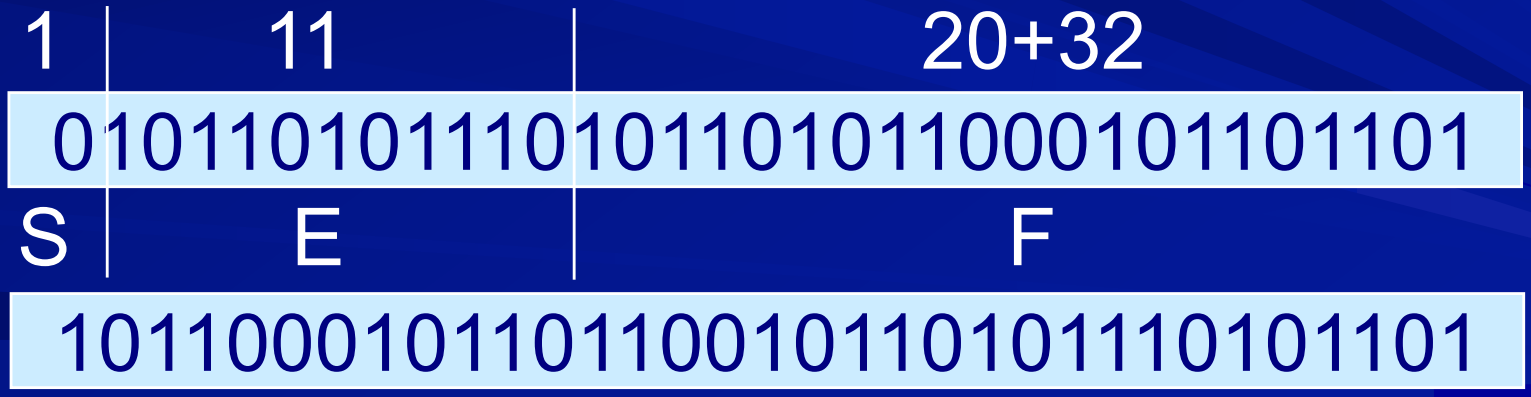
E = exponent (positive or negative integer)

- How to divide a word into S, F and E?
- How to represent S, F and E?

IEEE 754 standard

- Single precision numbers

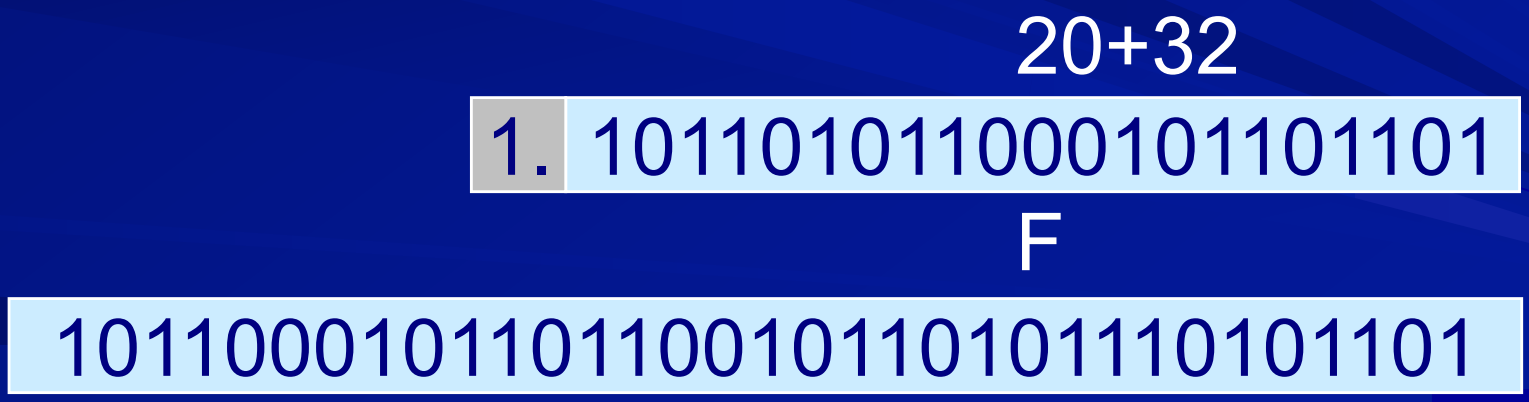
- Double precision numbers



Representing F in IEEE 754

- Single precision numbers

- Double precision numbers



Value range for F

- Single precision numbers

$1 \le F \le 2 - 2^{-23}$ or $1 \le F < 2$

- Double precision numbers

$1 \le F \le 2 - 2^{-52}$ or $1 \le F < 2$

These are “normalized”.

Representing E in IEEE 754

- Single precision numbers

  8

  10110101 — 54

  E — (+ bias 127)

- Double precision numbers

  11

  10110101110 — 431

  E — (+ bias 1023)

Value range for E

- Single precision numbers

  $-126 \leq E \leq 127$

  (all 0's and all 1's have special meanings)
- Double precision numbers

  $-1022 \leq E \leq 1023$

  (all 0's and all 1's have special meanings)

Representing zero

- All bits of F are zero, no explicit “1” to the left

  (not a normalized value)
- E can have any value, we choose to have all bits zero
- Sign bit is also zero

- Exactly same as integer zero!

Testing and comparing

- Test for zero, negative and positive for FP numbers is same as that for integers
- Magnitude comparison of FP numbers is same as magnitude comparison for integers

  Why?

  Because exponent is in biased notation and is located to the left of mantissa

Overflow and underflow

largest positive/negative number (SP) =
$\pm(2 - 2^{-23}) \times 2^{127} \cong \pm 2 \times 10^{38}$
smallest positive/negative number (SP) =
$\pm 1 \times 2^{-126} \cong \pm 2 \times 10^{-38}$

largest positive/negative number (DP) =
$\pm(2 - 2^{-52}) \times 2^{1023} \cong \pm 2 \times 10^{308}$
smallest positive/negative number (DP) =
$\pm 1 \times 2^{-1022} \cong \pm 2 \times 10^{-308}$

Floating point operations

- Add/subtract

$[(-1)^{S1} \times F1 \times 2^{E1}] \pm [(-1)^{S2} \times F2 \times 2^{E2}]$

suppose E1 > E2, then we can write it as

$[(-1)^{S1} \times F1 \times 2^{E1}] \pm [(-1)^{S2} \times F2' \times 2^{E1}]$

where $F2' = F2 / 2^{E1-E2}$,

The result is

$(-1)^{S1} \times (F1 \pm F2') \times 2^{E1}$

It may need to be normalized

FP Add/Sub Unit