Lecture 14
Signals and Systems (ELL205)

By Dr. Abhishek Dixit

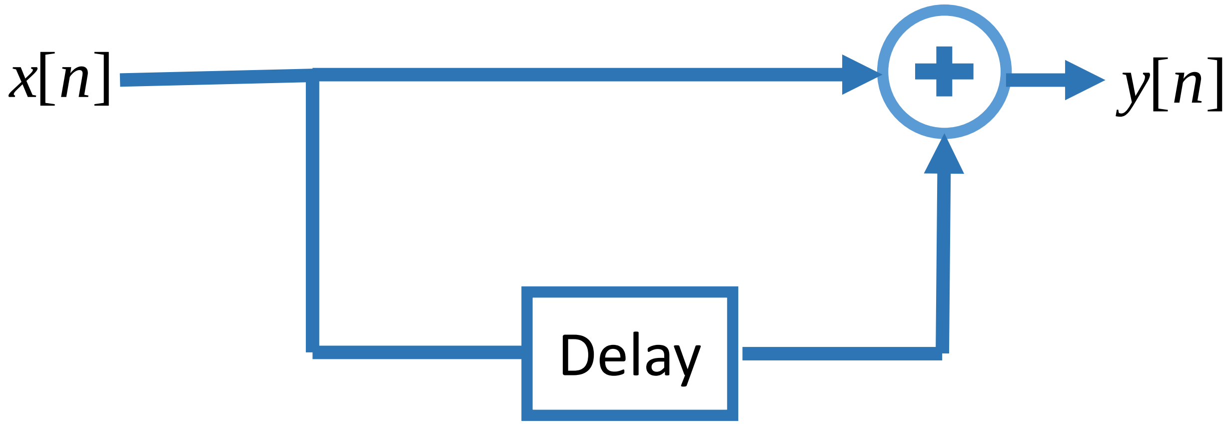
Dept. of Electrical Engineering

IIT Delhi

Outline of the lecture

- System designing

Basic DT system



Basic characteristics:

Linear (if delay starts at rest)

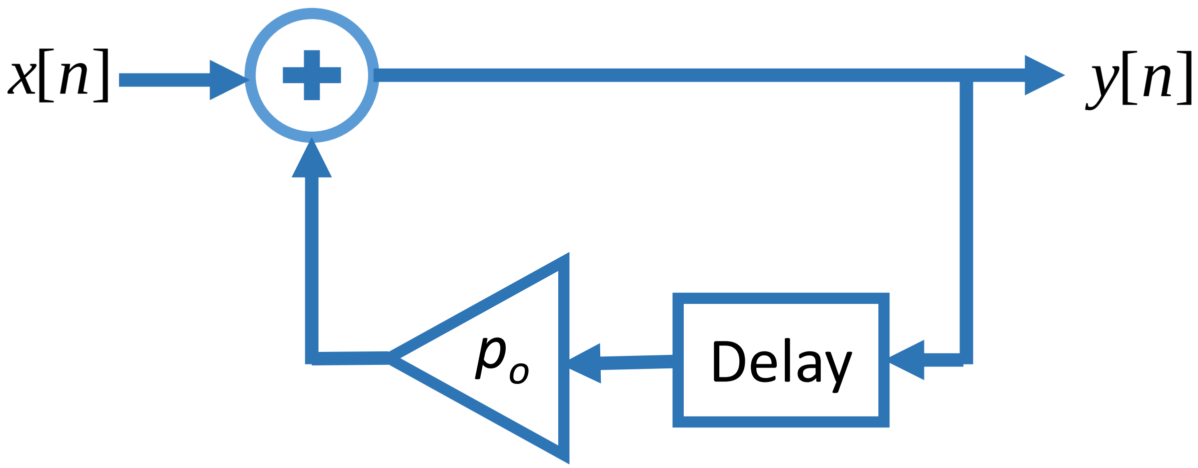
Time-Invariant

Causal

Recipe system

FIR system

Basic DT system



Basic characteristics:

Linear

Time-Invariant

Causal

Constraint/feedback system

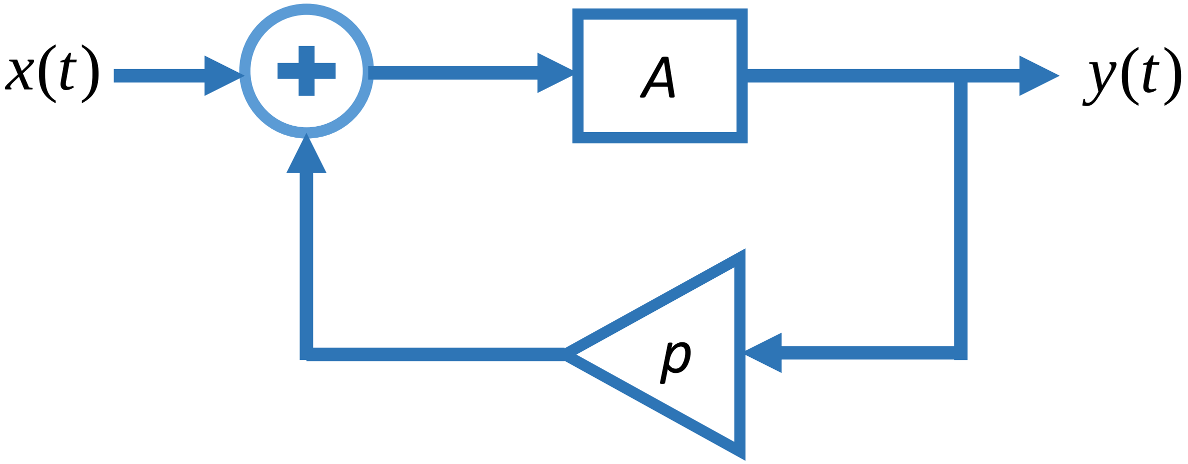
IIR system

Different approaches

1) Graphical method
2) Step-by-step method
3) Guess method
4) Polynomial approach

CT system (Graphical)

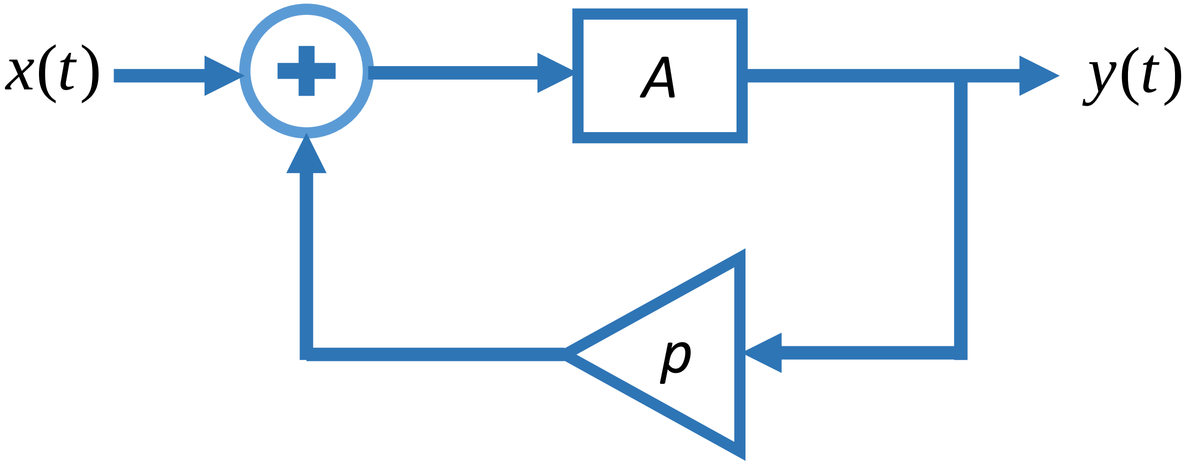
Basic CT system



$$h(t) = e^{pt}u(t)$$

CT system (Graphical)

Basic CT system



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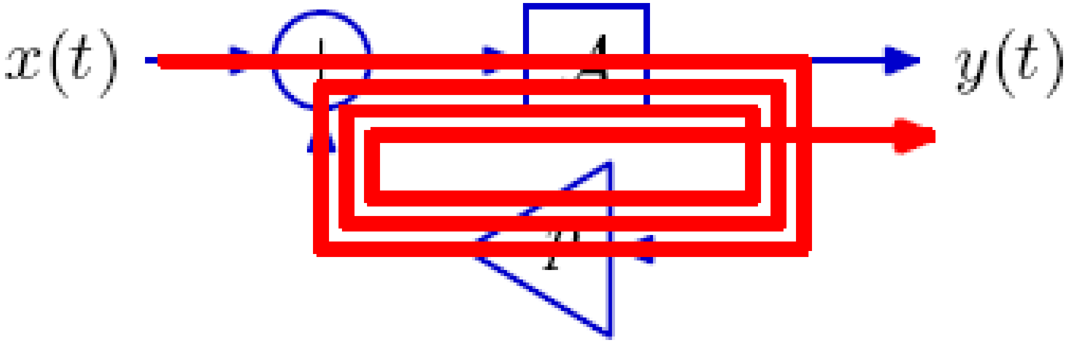
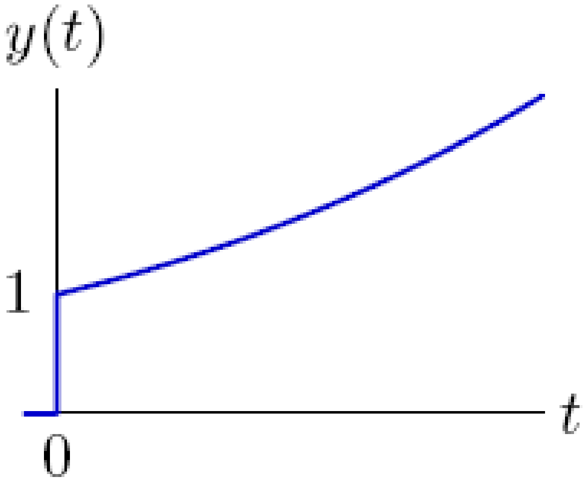


$$h(t)=\left(1 + pt + \frac{1}{2}p^2t^2 + \frac{1}{6}p^3t^3 + \right.$$



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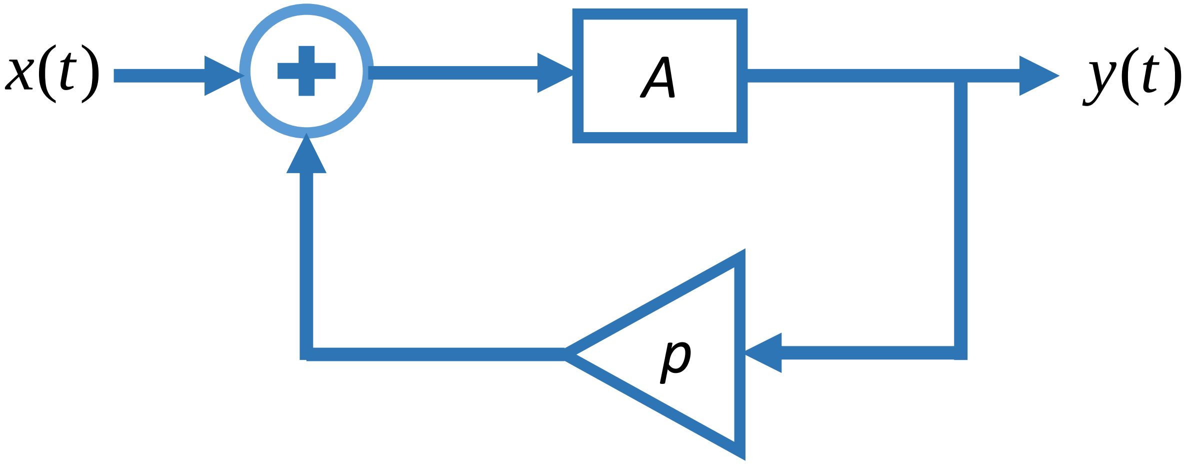
Basic CT system



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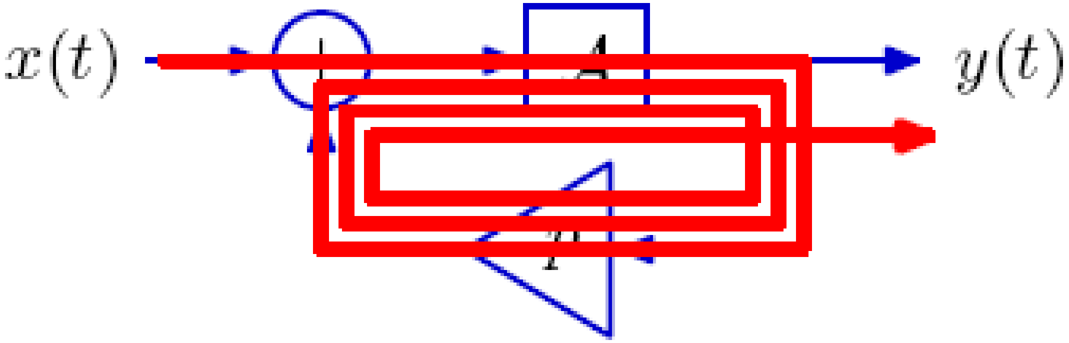
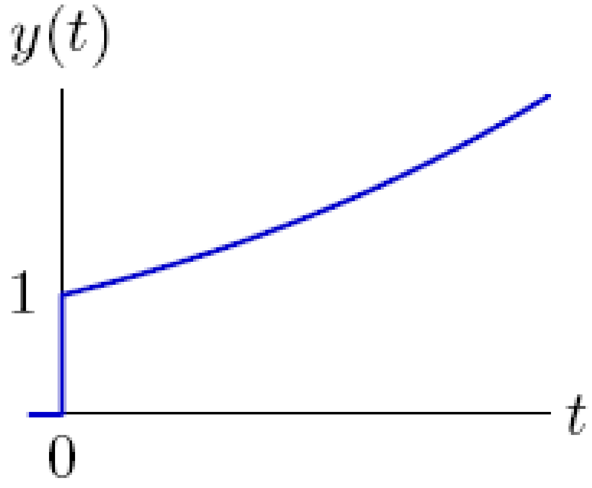
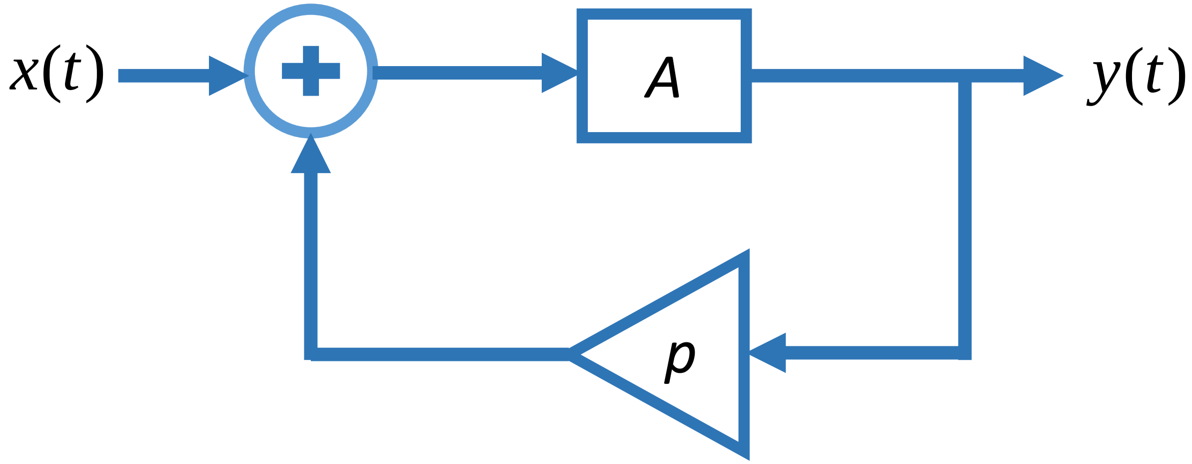


$h(t)=e^{pt}u(t)$



CT system (Guess)

Basic CT system



$$h(t) = e^{pt}u(t)$$

$$\dot{y}(t) = x(t) + py(t)$$

By Guess

$$y(t) = Ce^{st}u(t)$$

Substituting

$$Ce^{st}\delta(t) + sCe^{st}u(t) = \delta(t) + pCe^{st}u(t)$$

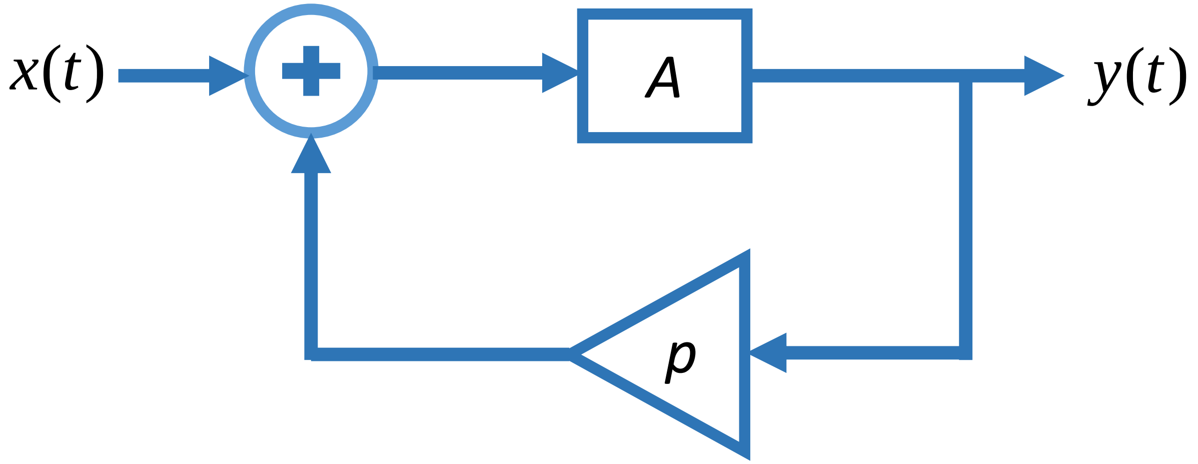
Comparing

$$C = 1 \; \& \; s = p$$

$$y(t) = e^{pt}u(t)$$

CT system (Polynomial)

Basic CT system



$$h(t) = e^{pt}u(t)$$

$$\dot{y}(t) = x(t) + py(t)$$

$$y(t) = \int x(t) + p\int y(t)$$

Operator $\quad Y = AX + pAY$

$$\frac{Y}{X} = \frac{A}{1 - pA}$$

$$h(t) = (1 + pA + p^2A^2 + \cdots)A\delta(t)$$

$$h(t) = (1 + pA + p^2A^2 + \cdots)u(t)$$

$$h(t) = \left(1 + pt + \frac{p^2t^2}{2} + \cdots\right)u(t)$$

$$h(t) = e^{pt}u(t)$$

Why does polynomials work even for CT system?

Fourier series/Fourier Transforms

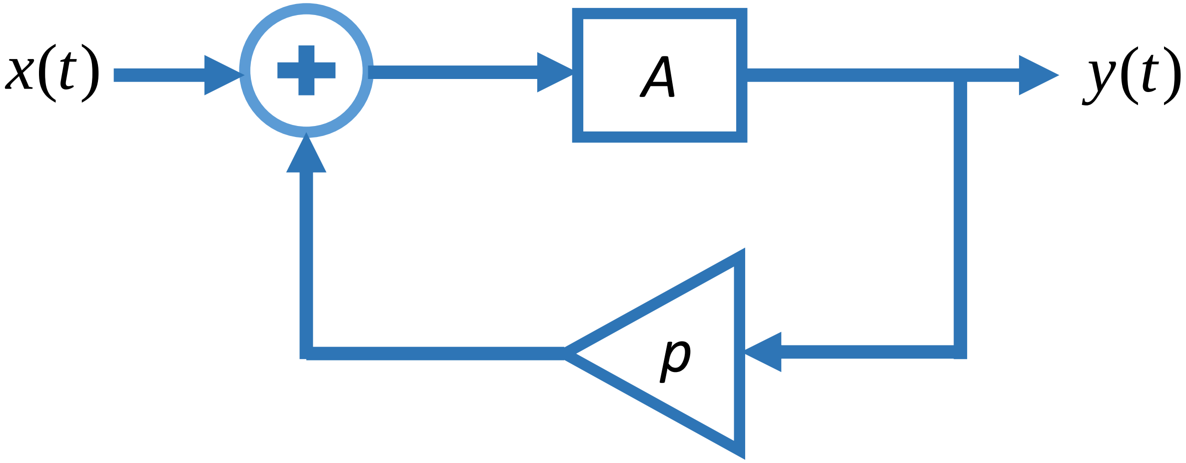
$$x(t) = \sum_k a_k e^{jk\omega_o t}$$

$$x(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \cdots$$

$$x(t) = b_0 + b_1 A + b_2 A^2 + b_3 A^3 + \cdots$$

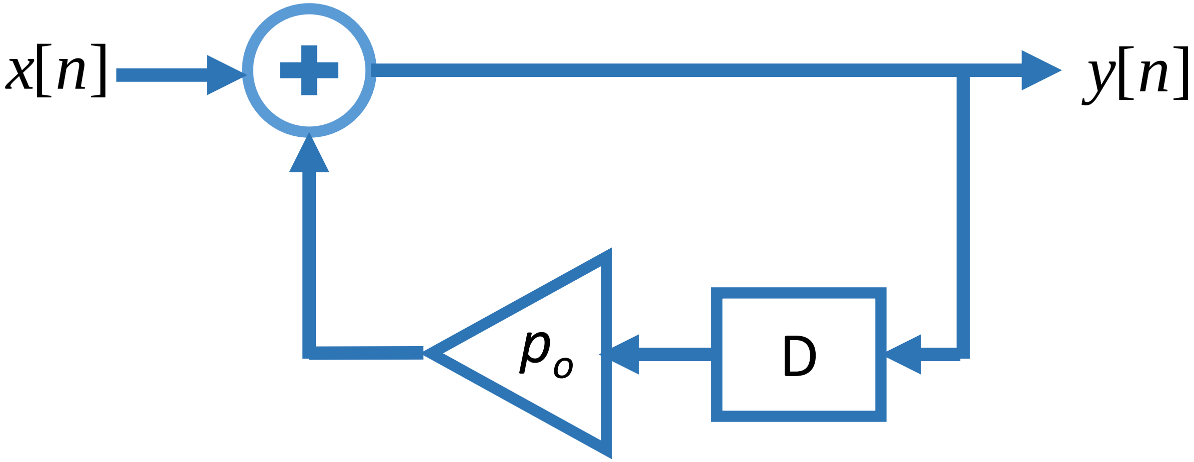
CT and DT system

Basic CT system



$$h(t) = e^{pt}u(t)$$

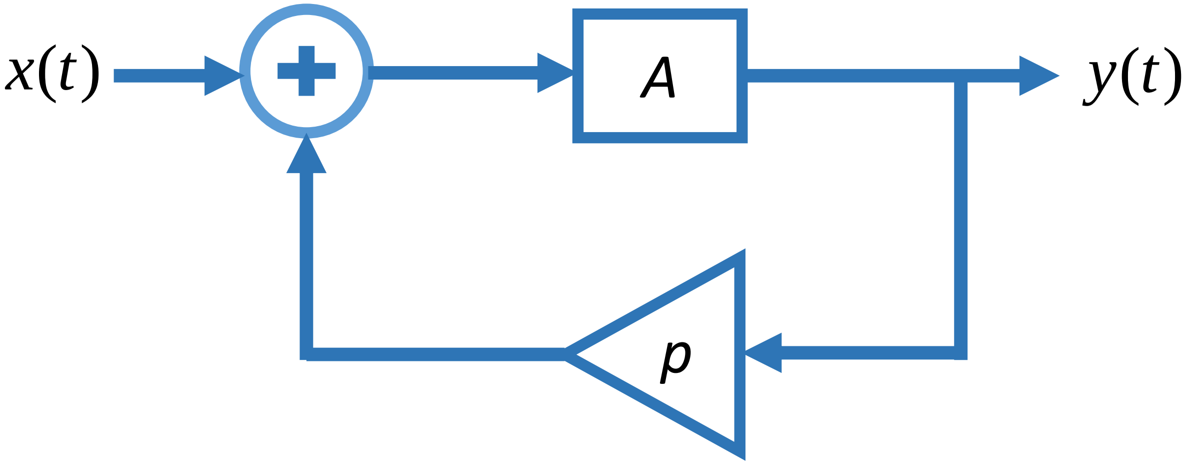
Basic DT system



$$h[n] = {p_o}^n u[n]$$

Impulse-invariance

Basic CT system



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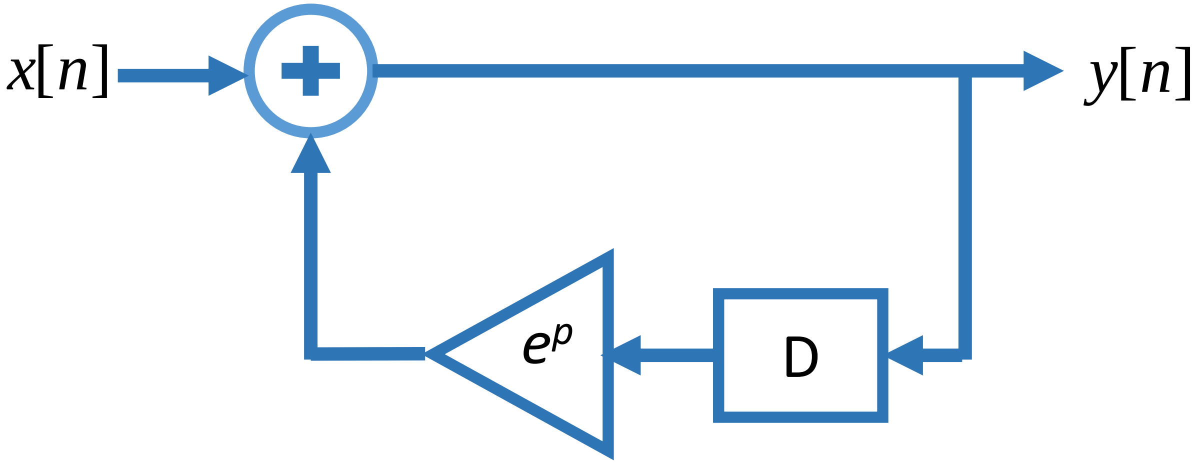


$$h(t) = e^{pt}u(t)$$

$$h[n] = p_o{}^n u[n] = e^{pn}u[n]$$

Towards Fourier Series

Love of Sinusoid

Outline of the lecture

- Why sinusoids?
- How an LTI system react to sinusoids?
- How do we break signal as sums of sinusoids?

Outline of the lecture

- Why sinusoids?
  - Psychoacoustics
  - Images
  - Signal generation
- How an LTI system react to sinusoids?
- How do we break signal as sums of sinusoids?

Outline of the lecture

- Why sinusoids?

    Psychoacoustics

    Images

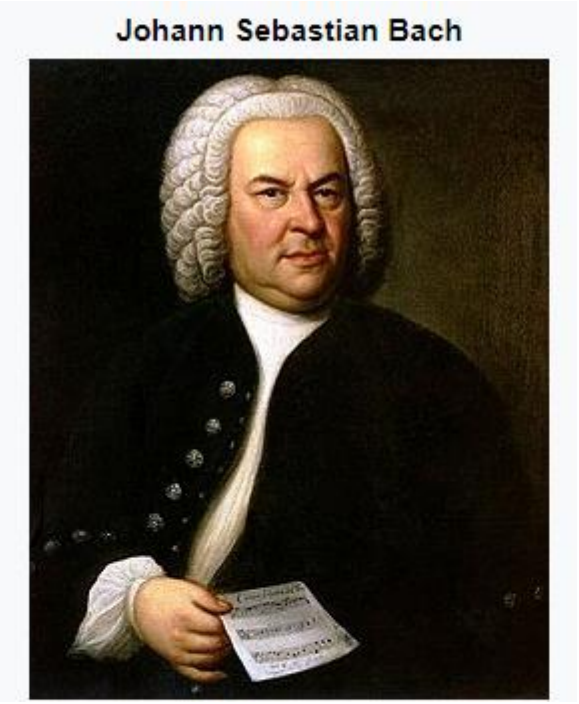
    Signal generation

- How an LTI system react to sinusoids?
- How do we break signal as sums of sinusoids?

Psychoacoustics

- Study of how human ear hears

Western Music



J. S. Bach, 1685-1750

A	880
G#	831
G	784
F♯	740
F	698
E	659
D♯	622
D	587
C♯	554
C	523
B	494
A#	466
A	440

440 to 880 octave (musical term for factor two)

There are twelve notes in one octave

$$494 \times \sqrt[12]{2}$$
$$466 \times \sqrt[12]{2}$$
$$440 \times \sqrt[12]{2}$$

Western Music

Johann Sebastian Bach

J. S. Bach, 1685-1750

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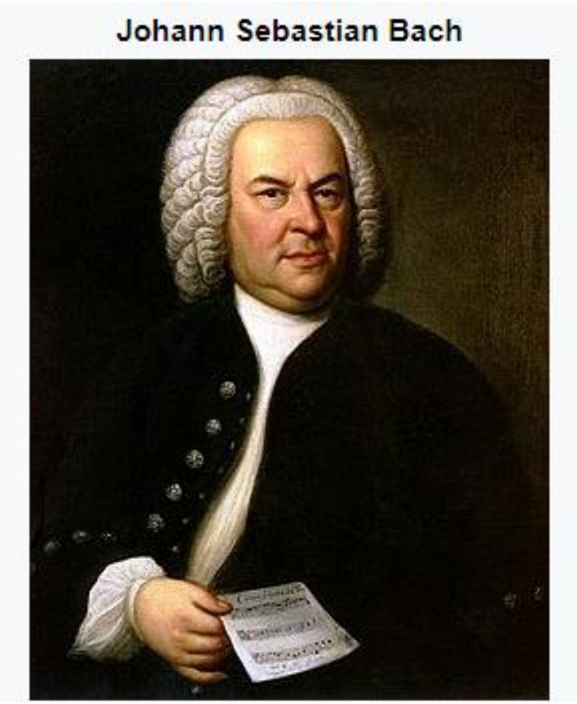

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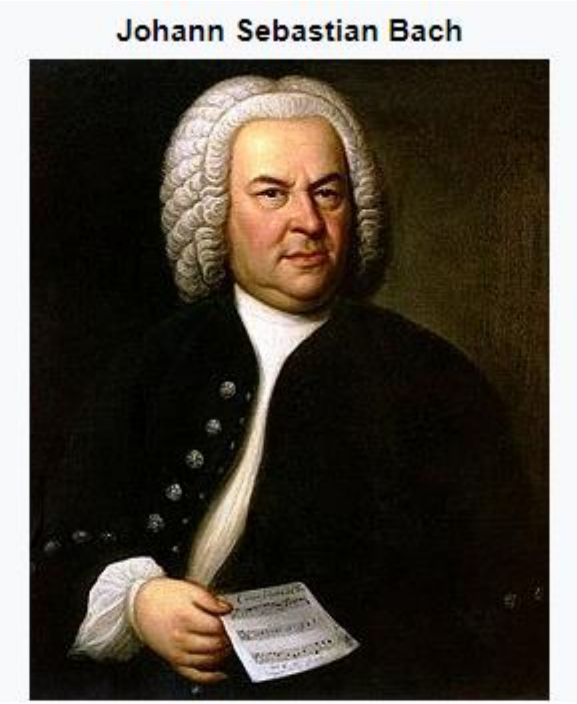

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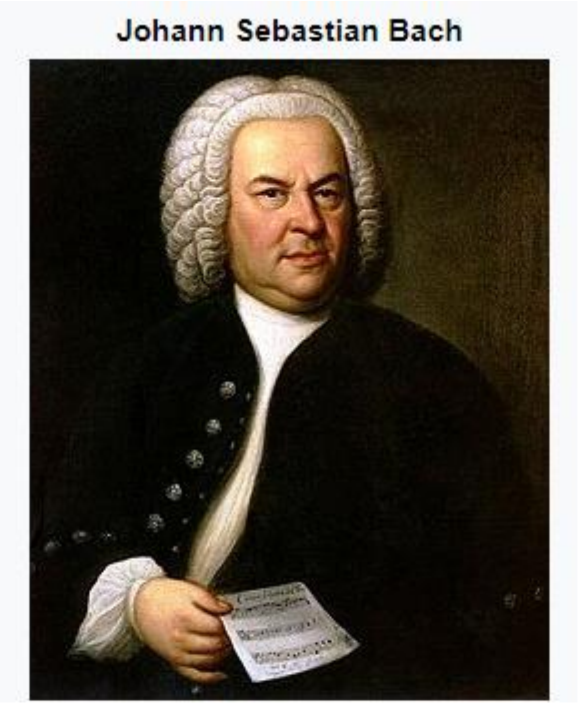

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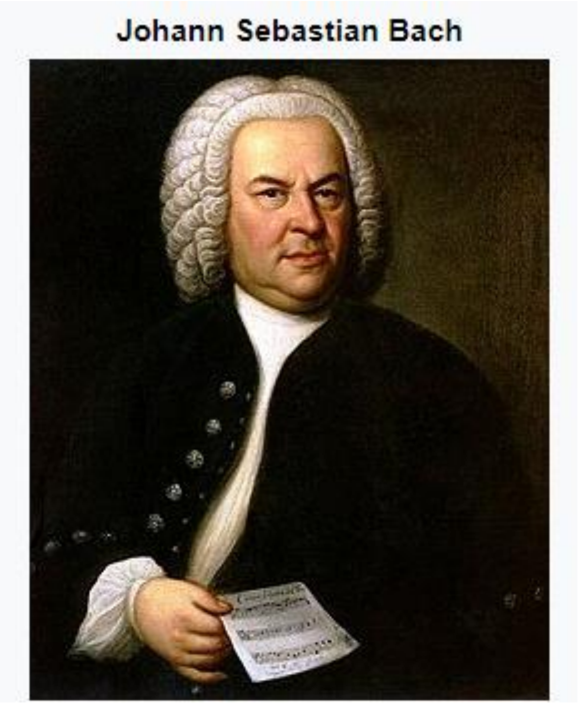

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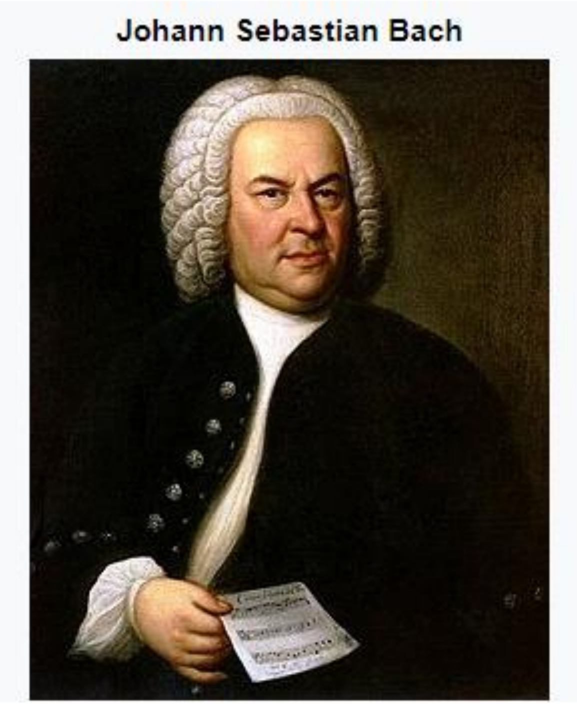

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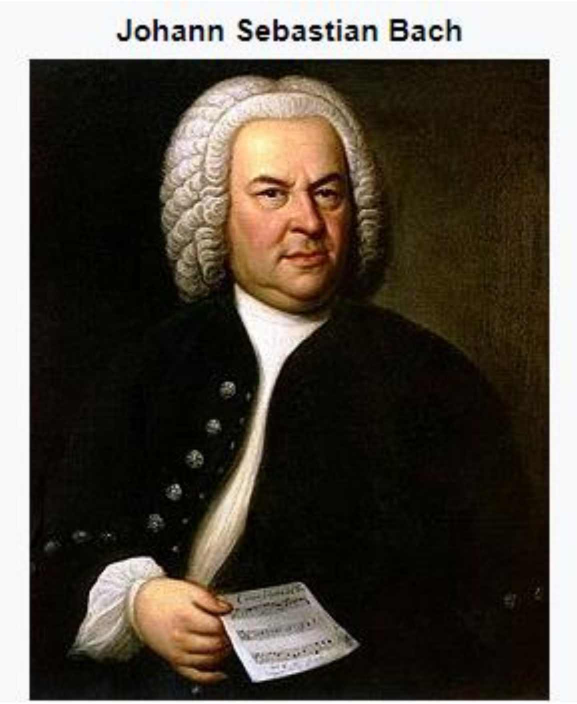

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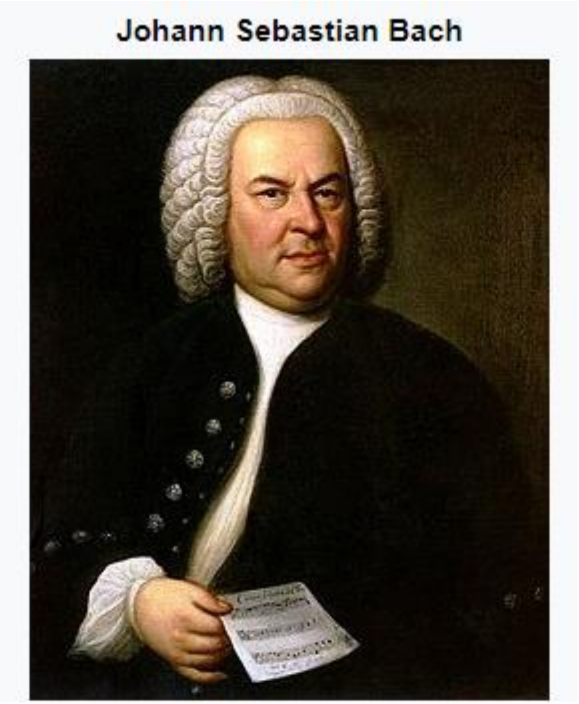

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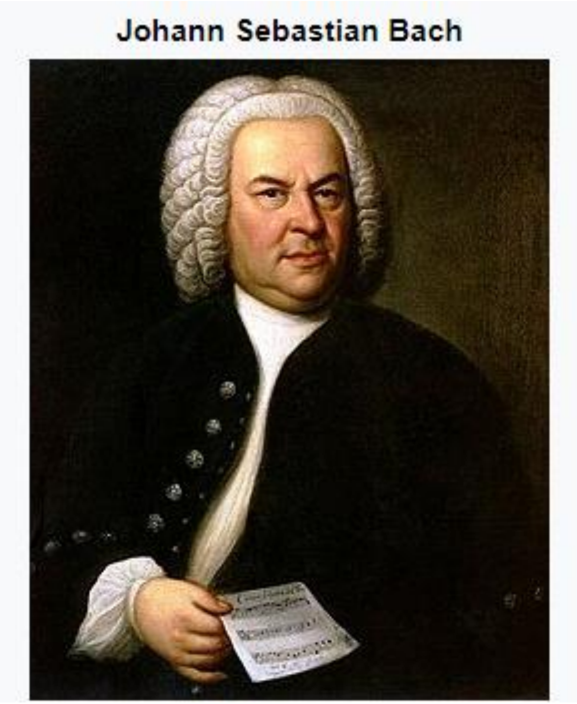

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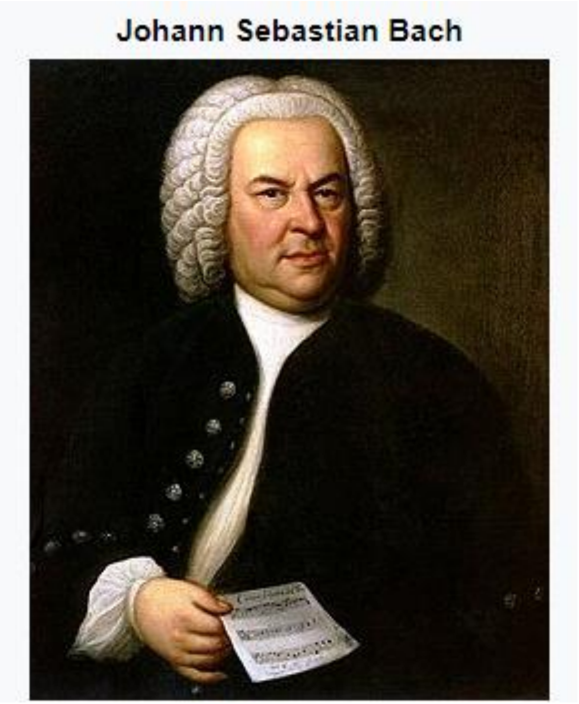

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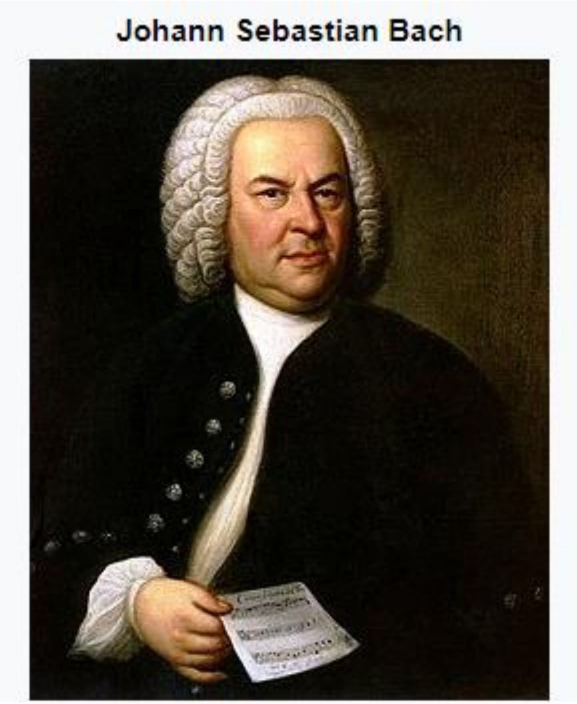

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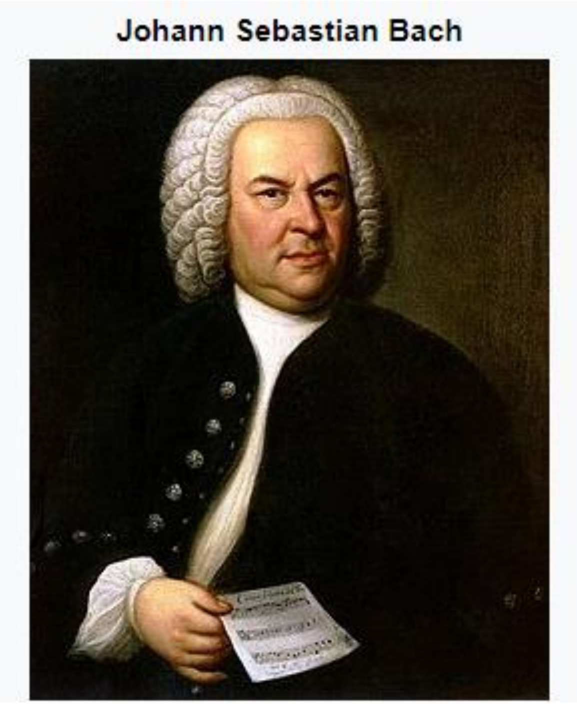

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Combination of Tones to Music

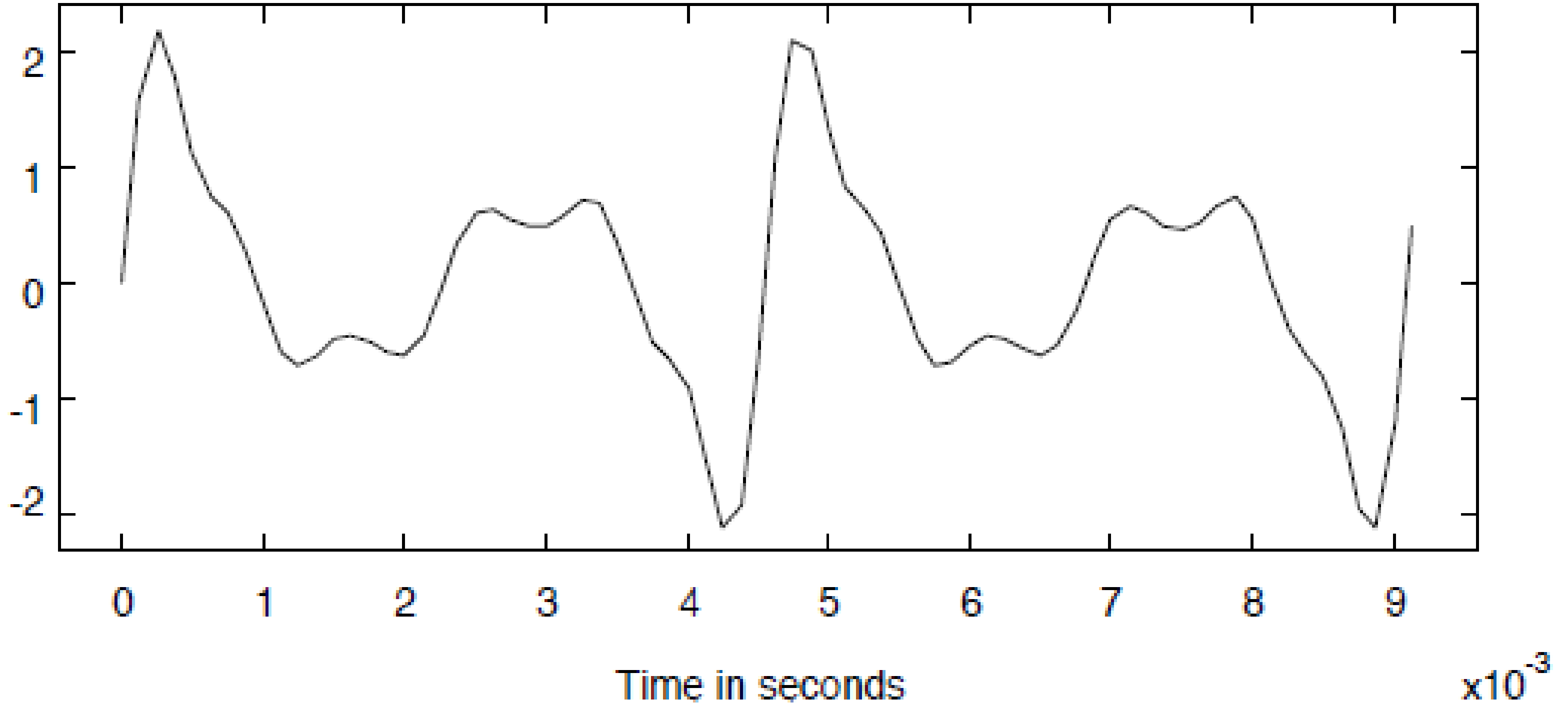
```
song = {'A' 'A' 'E' 'E' 'F#' 'F#' 'E' 'E' 'D' 'D' 'C#'
'C#' 'B' 'B' 'A' 'A'};
```

Combination of Tones to Music

```
song = {'A' 'A' 'E' 'E' 'F#' 'F#' 'E' 'E' 'D' 'D' 'C#'
'C#' 'B' 'B' 'A' 'A'};
```

Timbre

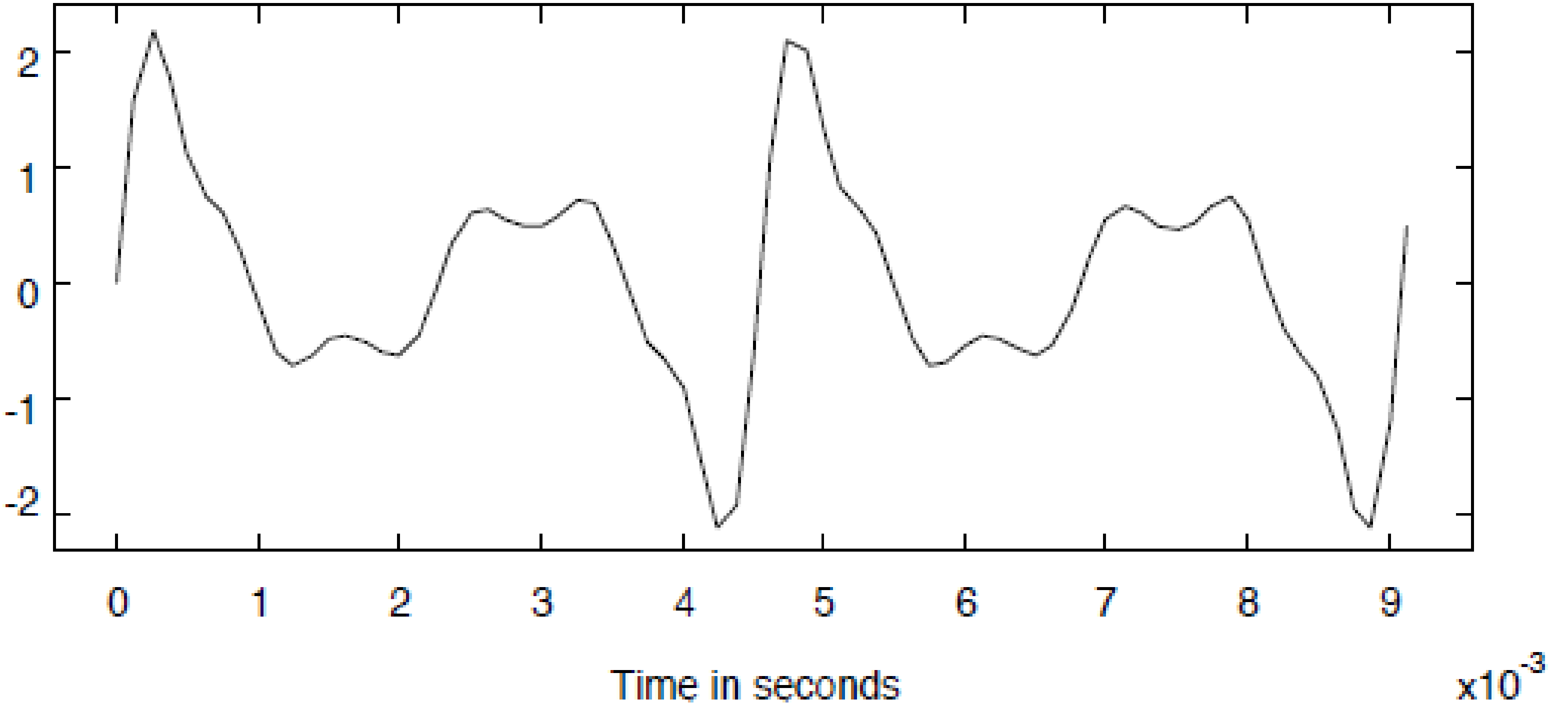
Note A: 220 Hz,
ω = 1382 rad/sec



$$x(t) = \sin(1382t) + r_1\sin(2 \times 1382t) + r_2\sin(3 \times 1382t) + \ldots\ldots.$$

Timbre

Note A: 220 Hz,
ω = 1382 rad/sec



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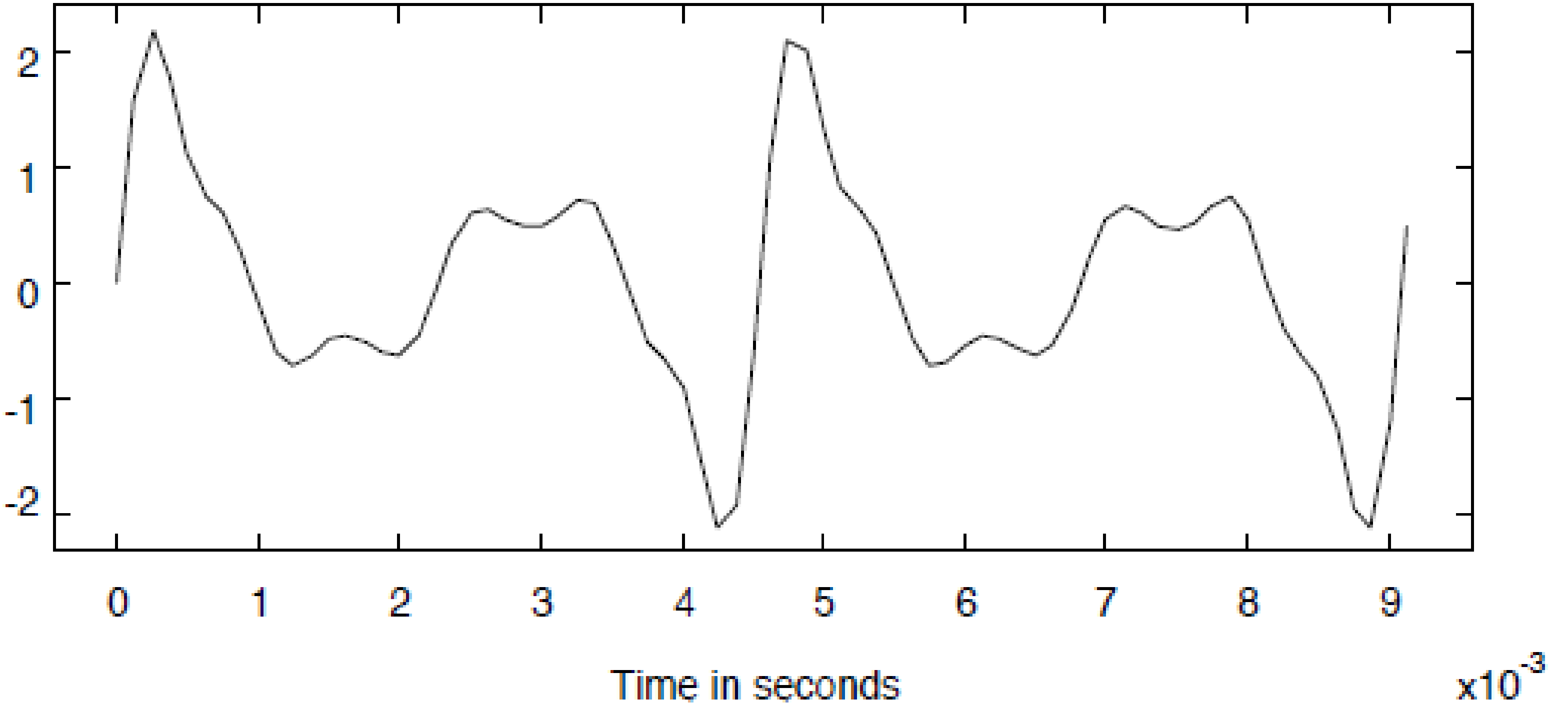


$$x(t) = \sin(1382t)$$

Ludwig van Beethoven

- 17 December 1770 – 26 March 1826

(56 years)

- German composer and a pianist
- His best-known compositions include

9 symphonies, 5 piano concertos,

1 violin concerto, 32 piano sonatas, 16 string quartets, his great Mass the *Missa solemnis*, and one opera, *Fidelio*.

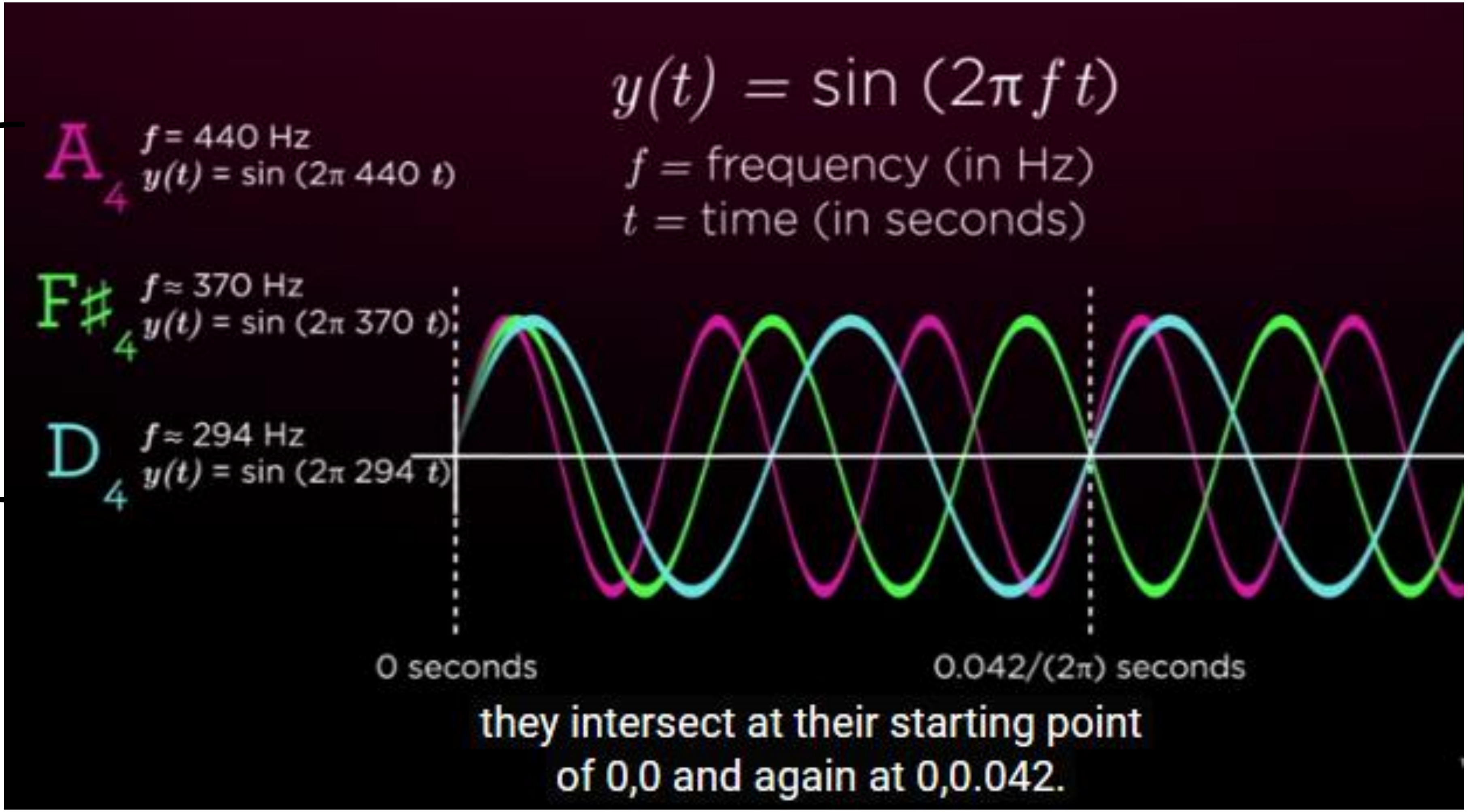
- He was deaf !!

Ludwig van Beethoven

Portrait by Joseph Karl Stieler, 1820

Beethoven (moonlight sonata)

$$220 \times 2^{n/12}$$



Moonlight Sonata

Moonlight Sonata

Arbitary Sonata

Moonlight Sonata

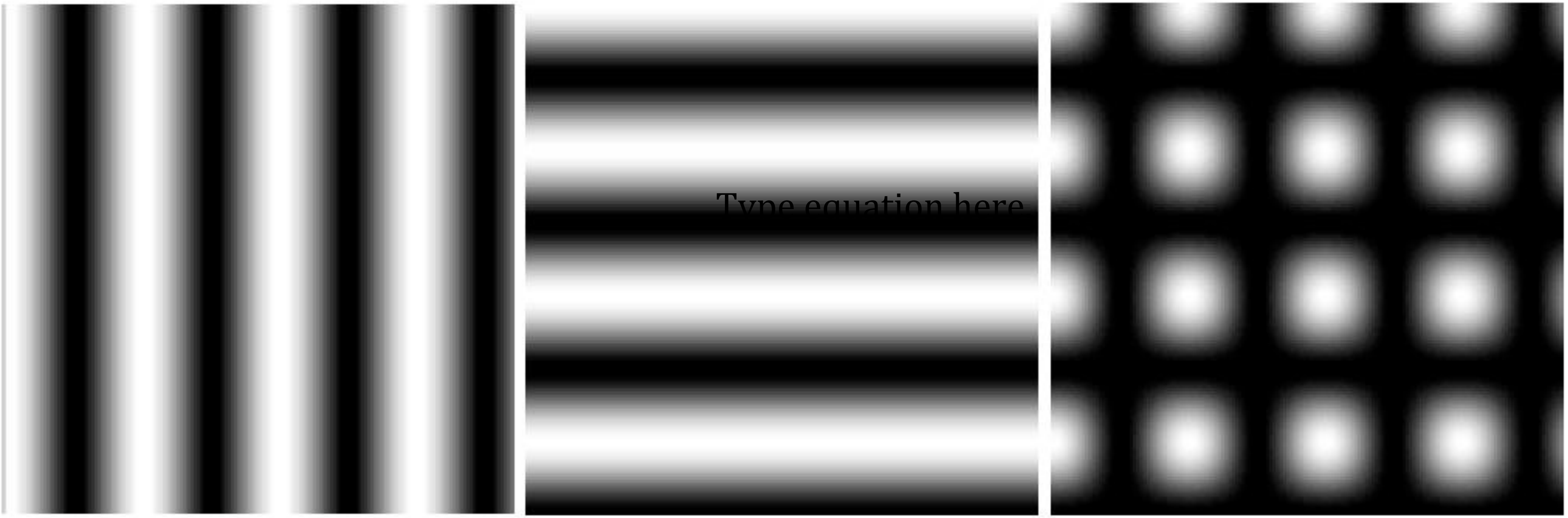
Moonlight Sonata

Arbitary Sonata

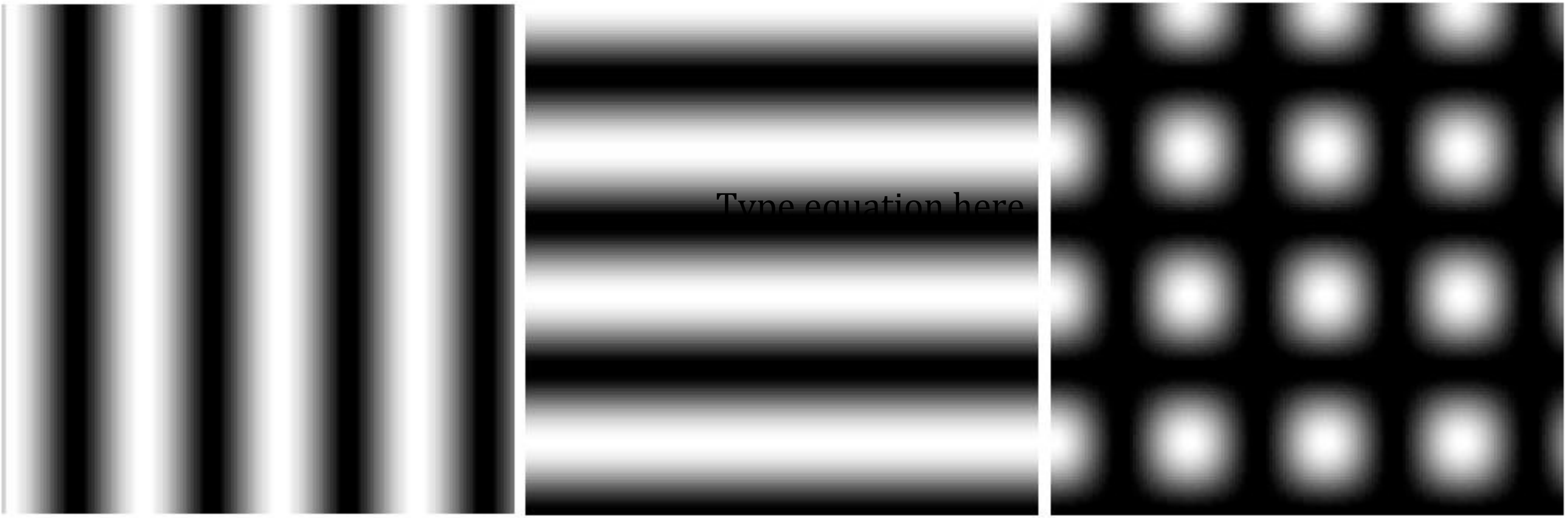
Outline of the lecture

- Applications
  - Psychoacoustics
  - Images
  - Signal generation

Spatial frequencies

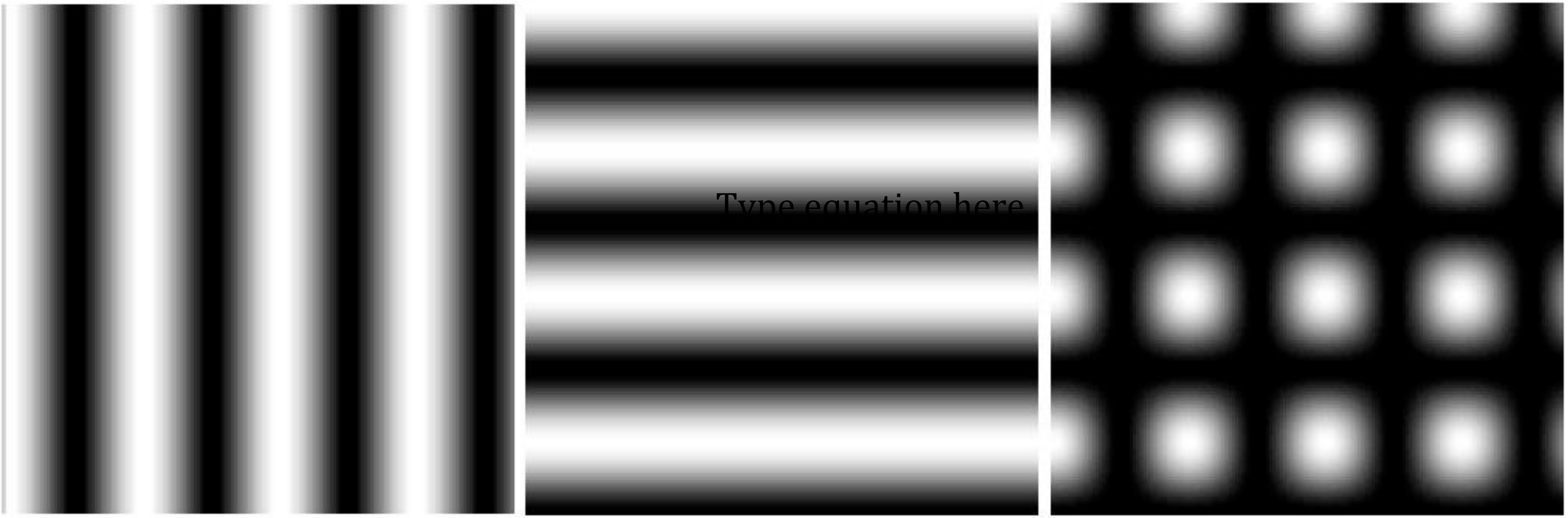


Spatial frequencies



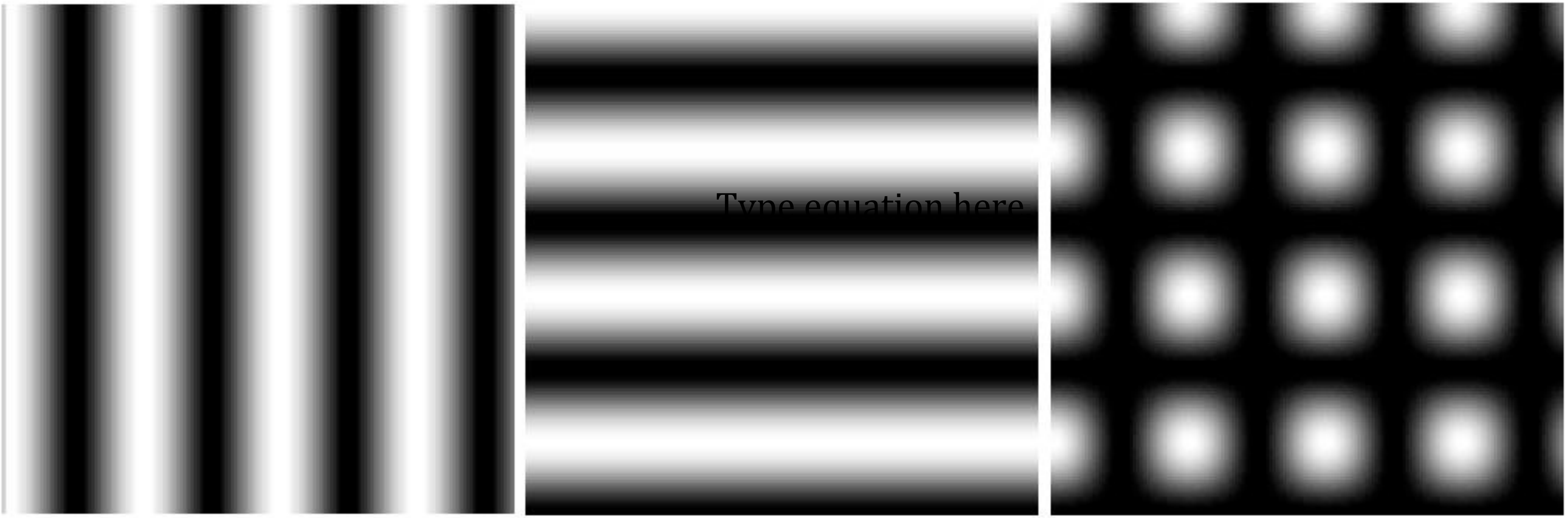
$$\mathrm{Image}(\mathrm{x}, \mathrm{y}) = \sin(\frac{2\pi x}{H})$$

Spatial frequencies



$$\mathrm{Image}(\mathrm{x},\mathrm{y}) = \sin\left(\frac{2\pi x}{H}\right) \quad \mathrm{Image}(\mathrm{x},\mathrm{y}) = \sin\left(\frac{2\pi y}{V}\right)$$

Spatial frequencies



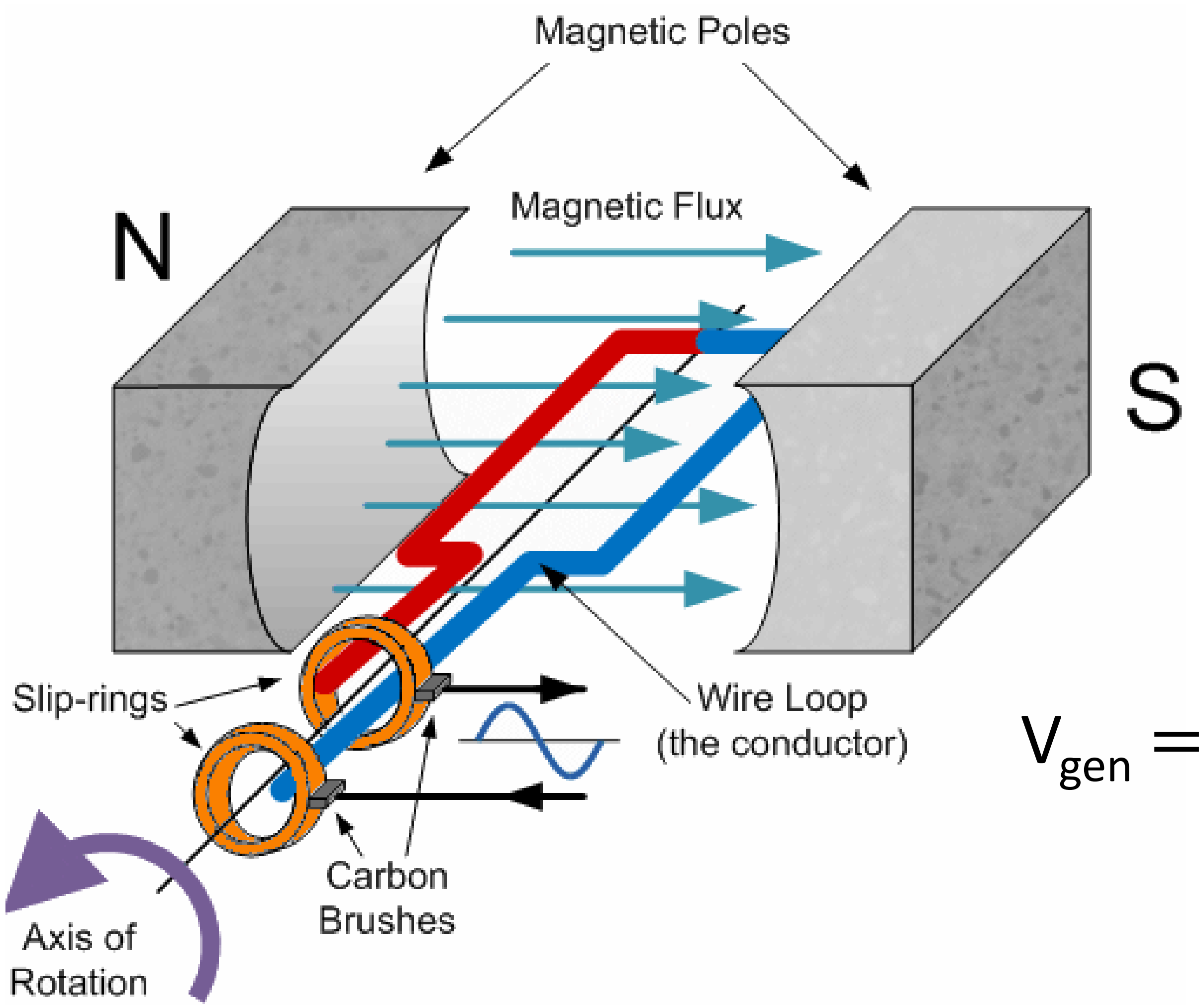
$$\mathrm{Image}(x, y) = \sin\left(\frac{2\pi x}{H}\right) \quad \mathrm{Image}(x, y) = \sin\left(\frac{2\pi y}{V}\right)$$

$$\mathrm{Image}(x, y) = \sin\left(\frac{2\pi x}{H}\right) \times \sin\left(\frac{2\pi y}{V}\right)$$

Outline of the lecture

- Applications
    - Psychoacoustics
    - Images
    - Signal generation

Sinusoidal source generator

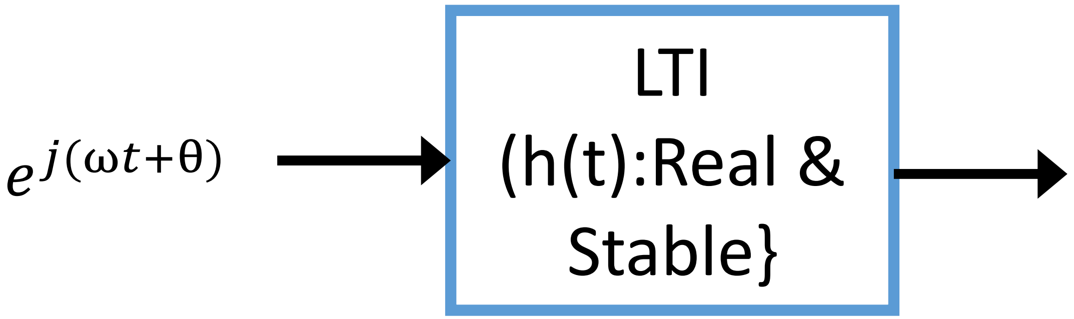


$$V_{gen} = -BA\omega sin\omega t$$

Outline of the lecture

- Why sinusoids?
- How an LTI system react to sinusoids?
- How do we break signal as sums of sinusoids?

Output of an LTI system to Complex Exponentials



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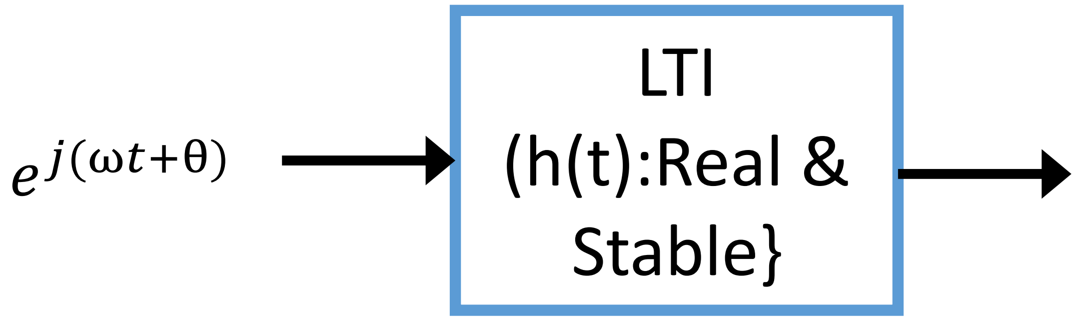


$$\int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

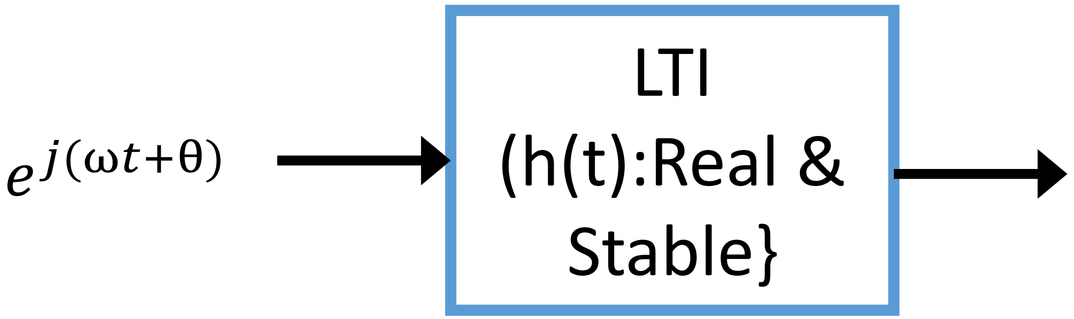
$$y(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) e^{j(\omega(t-\tau)+\theta)} d\tau$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) e^{j(\omega t+\theta)} e^{-j\omega\tau} d\tau$$

$$y(t) = e^{j(\omega t+\theta)} \int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau$$

Output of an LTI system to Complex Exponentials



$$\int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty$$

$$y(t) = e^{j(\omega t+\theta)} \int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau$$

$$H(\omega) = \int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau$$

$$y(t) = e^{j(\omega t+\theta)} H(\omega)$$

Eigen function   Eigen value

Output of an LTI system to Complex Exponentials

$e^{j(\omega t+\theta)}$ $\longrightarrow$ $\boxed{\text{LTI (h(t):Real \& Stable\}}}$ $\longrightarrow$

$$y(t) = e^{j(\omega t+\theta)}H(\omega)$$

$$\int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty$$

$$H(\omega) = \int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau$$

Frequency response

Output of an LTI system to Complex Exponentials

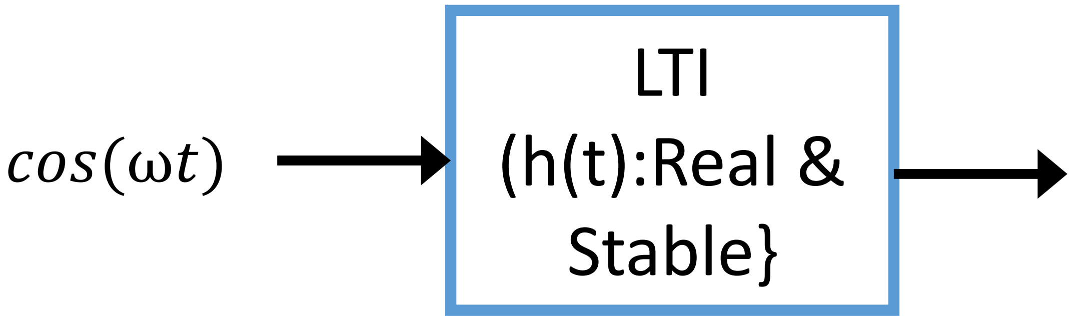
$$e^{-j(\omega t+\theta)} \longrightarrow \boxed{\begin{array}{c}\text{LTI}\\ \text{(h(t):Real \&}\\ \text{Stable\}}\end{array}} \longrightarrow \quad y(t) = e^{-j(\omega t+\theta)}H(-\omega)$$

$$\int_{-\infty}^{\infty} |h(\tau)|d\tau < \infty$$

$$H(-\omega) = \int_{-\infty}^{\infty} h(\tau)e^{j\omega\tau}d\tau$$

Frequency response

Output of an LTI system to Complex Exponentials



$$\int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty$$