Harshit Mawandia

2020 CS 10348

GRP - 4

Q1)

a) Photo electric effect:

A certain threshold frequency is required for the current to be generated. Also there is no time lag between illumination and release of electrons. Current depends on intensity of the ~~electrons~~ photons

~~b) Quantum Mechanical effects on carrier distri~~

b) Width of depletion region in the p-n junction:

The depletion layer that has formed around the p-N junction is calculated quantum mechanically. ~~For example~~ The p-N predicts deviates by orders of magnitude from results from different doping concentration.

c) Quantum Tunelling:

The wavefunction can propagate even through a potential barrier higher than the total energy. Therefore, some electrons ~~were~~ not having enough energy may also cross the potential barrier to reach conduction band.

d) $E = h\nu$

Energy of photons are integral multiple of $h\nu$ where ν is the frequency of the wave. These packets of energy are called 'photons'.

Q2) From uncertainity principle
For ground state of H atom:

$$\Delta x . \Delta p \sim \hbar$$

maximum uncertainity in position is r :-

$$\Delta x \rightarrow r$$

$$r \Delta p \sim \hbar$$

$$\Delta p \sim \frac{\hbar}{r}$$

Min value of momentum cannot be less than the uncertainity

$$p \sim \frac{\hbar}{r}$$

Now electron proton energy -

$$E(r) = \frac{p^2}{2m_e} - \frac{e^2}{4\pi\epsilon_0 r} = \frac{\hbar^2}{2m_e r^2} - \frac{e^2}{4\pi\epsilon_0 r} \quad \text{--- (1)}$$

At the ground state, energy must be minimum

$$\frac{dE(r)}{dr} = 0 \quad \text{at } r = r_1 \Rightarrow \quad -\frac{\hbar^2}{m_e r^3} + \frac{e^2}{4\pi\epsilon_0 r^2} = 0$$

~~After solving this~~ $\boxed{\cancel{r_0 = 0.53\text{ nm}}}$

~~Now energy at this r_0 = -13.6eV~~

$$\boxed{\frac{\hbar^2}{m_e} \times \frac{4\pi\epsilon_0}{e^2} = r}$$

p = From eq^n 1

$$p = \left(\left(E(r) + \frac{e^2}{4\pi\epsilon_0 r}\right) \times 2m_e\right)^{1/2}$$

a)

$$\lambda_{\text{de Broglie}} = \frac{h}{p}$$

$r_2 = 2r_1$ (given)

$E_2 = E_1/4$ (given)

$$\therefore p_1 = \left(\left(E_1 + \frac{e^2}{4\pi\epsilon_0 r_1}\right)\times 2m_e\right)^{1/2}$$

$$\therefore p_2 \left(\left(\frac{E_1}{4} + \frac{e^2}{2\times 4\pi\epsilon_0 r_1}\right)\times 2m_e\right)^{1/2}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{p_1}{p_2} = \left(\frac{E_1 + \frac{e^2}{4\pi\epsilon_0 r_1}}{\frac{E_1}{4} + \frac{e^2}{2\times 4\pi\epsilon_0 r_1}}\right)^{1/2}$$

Now $r_1 = \frac{\hbar^2}{m_e} \times \frac{4\pi\epsilon_0}{e^2}$

$$E_1 = \frac{e^4 m_e}{32\hbar^2\times\pi^2\epsilon_0^2} - \frac{e^2}{4\pi\epsilon_0} \times \frac{m_e}{\hbar^2\times 4\pi\epsilon_0}$$

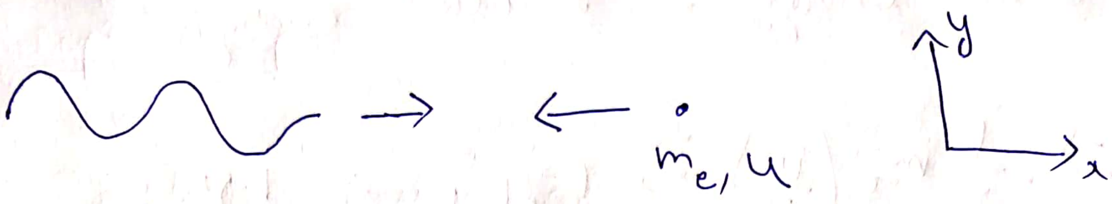
$$= \frac{-e^4 m_e}{32\hbar^2\times\pi^2\epsilon_0^2} = \boxed{\frac{-e^2}{8\pi\epsilon_0 r_1}}$$

$$\frac{\lambda_2}{\lambda_1} = \left(\frac{\frac{-e^2}{8\pi\epsilon_0 r_1} + \frac{e^2}{4\pi\epsilon_0 r_1}}{\frac{-e^2}{4\times 8\pi\epsilon_0 r_1} + \frac{e^2}{8\pi\epsilon_0 r_1}}\right)^{1/2} = \left(\frac{-\frac{1}{2}+1}{-\frac{1}{8}+\frac{1}{2}}\right)^{1/2}$$

$$= \left(\frac{1/2}{3/8}\right)^{1/2} = (4/3)^{1/2}$$

Q3)

a)



Since before and after collision is in negative x direction and the photon was also moving in +ve x direction.

Let the momentum of photon be $= p_{photon}\hat{i}$

where p is +ve

by momentum conservation:

$$-m_e u \hat{i} + p_{photon}\hat{i} = p_{final}\hat{i} - m_e\frac{u}{2}\hat{i}$$

$$p_{final} = \left(-m_e\frac{u}{2} + p_{photon}\right)\hat{i}$$

since $p_{phon} << m_e\frac{u}{2}$

direction will be along negative x axis.

b) Energy of photon:

$$\text{Initial energy} = \underbrace{h\nu_i}_{E_{photon}} + \underbrace{\frac{1}{2}m_e u_0^2}_{E_{e^-}}$$

since collision is elastic

energy lost = 0

Final energy = Initial energy

$$\Rightarrow \frac{1}{2} m_e \left(\frac{u}{2}\right)^2 + E_{photo} = h\nu_i + \frac{1}{2} m_e u^2$$

$$\boxed{E_{photon} = h\nu_i + \frac{3}{8} m_e u^2}$$

8)

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Q4) a) Since ϕ is a quantum mechanical wave function it must be normalised

$\therefore$ $\langle \phi | \phi \rangle = 1$

Since ψ_i are orthonormal $\psi_i^* \psi_j = 0 \;\forall\; i \neq j$

$$\therefore \langle \phi | \phi \rangle = A^2 \langle \psi_1 | \psi_1 \rangle + \frac{1}{5} \langle \psi_2 | \psi_2 \rangle - \frac{1}{7} \langle \psi_3 | \psi_3 \rangle = 1$$

$$\Rightarrow A^2 + \frac{1}{5} + \frac{1}{7} = 1 \qquad (\langle \psi_i | \psi_i \rangle = 1$$

(they are normalised))

$$\Rightarrow A^2 = 1 - \frac{12}{35} = \frac{23}{35}$$

$$\Rightarrow A = \sqrt{\frac{23}{35}}$$

b) $\langle \hat{O} \rangle = \langle \phi | \hat{O} | \phi \rangle$

$$\hat{O} | \phi \rangle = 2\sqrt{\frac{33}{35}}\,\psi_1 + \frac{5}{\sqrt{5}}\,\psi_2 + i \times \frac{10}{\sqrt{7}}\,\psi_3$$

$$\langle \hat{O} \rangle = \langle \phi | \hat{O} | \phi \rangle \qquad (\text{since } \langle \phi | \phi \rangle = 1)$$

$$= 2 \times \frac{23}{35} + \frac{5}{5} + \frac{10}{7}$$

$$= \frac{46}{35} + \frac{35}{35} + \frac{50}{35}$$

$$= \cancel{\frac{51}{35} \approx 1.457} = \frac{131}{35} \approx 3.742$$

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c) $\langle \psi_2 | \hat{O} | \psi_2 \rangle = \langle \psi_2 | 5\psi_2 \rangle$

$= 5 \langle \psi_2 | \psi_2 \rangle$

$= 5$

Q5) Partile is confined to move between $-\frac{a}{2} \le x \le \frac{a}{2}$

By 1-D shrodinger equation

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} E(\psi) = 0$$

gives $\psi(x) = A\cos kx + B\sin kx$

where $k = \sqrt{\frac{2mE}{\hbar^2}}$

a) The boundary conditions are

$\psi\left(\frac{a}{2}\right) = 0$ & $\psi\left(-\frac{a}{2}\right) = 0$

Which gives $A\cos\left(\frac{ak}{2}\right) = 0$

$B\sin\left(\frac{ak}{2}\right) = 0$

b) $\psi_n(x) = \sqrt{\frac{2}{a}}\cos\left(\frac{n\pi x}{a}\right)$ $n = 1, 3, 5$

and

$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$, $n = 2, 4, 6$

$\sqrt{\frac{2}{a}}$ is obtained after normalisation

$\therefore$ Ground state function $(n=1)$

$= \sqrt{\frac{2}{a}}\cos\left(\frac{\pi x}{a}\right)$

1st excited state wave function $(n=2)$ is

$\sqrt{\frac{2}{a}}\sin\left(\frac{2\pi x}{a}\right)$

c) $$P = \int_{-a/4}^{a/4} \frac{2}{a} \sin\left(\frac{2\pi x}{a}\right) \sin\left(\frac{2\pi x}{a}\right) dx$$

$$= \frac{2}{a} \int_{-a/4}^{a/4} \frac{1}{2} \left(1 - \cos\left(\frac{4\pi x}{a}\right)\right) dx$$

$$= \frac{1}{a} \left[x - \frac{a}{4\pi} \sin\left(\frac{4\pi x}{a}\right)\right]_{-a/4}^{a/4}$$

$$= \frac{1}{2} \times \frac{a}{2} = \frac{1}{2} \quad \text{Ans}$$

d) $$k = \sqrt{\frac{2mE}{\hbar^2}} \Rightarrow E = \frac{k^2\hbar^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

$$\therefore \Delta E = E_2 - E_1$$

$$= \frac{3}{2} \frac{\pi^2\hbar^2}{ma^2} = \frac{3}{8} \frac{\pi^2\hbar^2}{\cancel{8}m_e a^2}$$

$$= 0.45 \times 10^{-19} \text{ J}$$

$$= 0.28 \text{ eV}$$

6) a) ~~For~~ ~~B~~ We know that

$$\frac{p^2}{2m} = E$$

for particles having $p = 2\hbar k$

$$E = \frac{2\hbar^2 k^2}{m}$$

By time evaluation of wave function:

$$\psi(x,t) = \psi(x,0)\, e^{-\frac{i}{\hbar}\frac{2\hbar^2k^2}{m}t}$$

$$\psi(x,t) = \sqrt{\frac{\alpha}{\pi}}\, e^{-\frac{\alpha x^2}{2}}\, e^{-\frac{i}{\hbar}\frac{2\hbar^2k^2}{m}t}$$

Probability of function is independent of time

as $$\psi^*\psi = \sqrt{\frac{\alpha}{\pi}}\, e^{-\frac{\alpha x^2}{2}}\, \cancel{e^{-\frac{i}{\hbar}\frac{2\hbar^2k^2}{m}t}} \cdot \sqrt{\frac{\alpha}{\pi}}\, e^{-\frac{\alpha x^2}{2}}\, \cancel{e^{\frac{i}{\hbar}\frac{2\hbar^2k^2}{m}t}}$$

$$\boxed{\text{propability density} = \frac{\alpha}{\pi}\, e^{-\alpha x^2}}$$

b) $$\langle E\rangle = \frac{\langle\psi|\hat{H}|\psi\rangle}{\langle\psi|\psi\rangle}$$

$$= \frac{\int_{-\infty}^{\infty}\psi^* H \psi\, dx}{\int_{-\infty}^{\infty}\psi^*\psi\, dx} = \frac{\int_{-\infty}^{\infty}\sqrt{\frac{\alpha}{\pi}}\, e^{-\frac{\alpha x^2}{2}} \cdot -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} e^{-\frac{\alpha x^2}{2}}\, dx}{\int_{-\infty}^{\infty}\sqrt{\frac{\alpha}{\pi}}\, e^{-\frac{\alpha x^2}{2}}\sqrt{\frac{\alpha}{\pi}}\, e^{-\frac{\alpha x^2}{2}}\, dx}$$

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$$= \frac{\int_{-\infty}^{\infty} \cancel{\frac{\alpha}{\pi}} e^{-\frac{\alpha x^2}{2}} \cdot -\frac{h^2}{2m}\left(-\alpha+\alpha x^2\right) e^{-\frac{\alpha x^2}{2}} dx}{\int_{-\infty}^{\infty} \cancel{\frac{\alpha}{\pi}} e^{-\alpha x^2} dx}$$

$$= \frac{\int_{-\infty}^{\infty} e^{-\alpha x^2} \left(\frac{-h^2}{2m}\right)\left(\alpha x^2 - \alpha\right) dx}{\int_{-\infty}^{\infty} e^{-\alpha x^2} dx}$$

$$= \frac{-\frac{h^2}{2m}\left(\frac{\alpha}{2}\sqrt{\frac{\pi}{\alpha}} - \alpha\sqrt{\frac{\pi}{\alpha}}\right)}{\sqrt{\pi/\alpha}} = \frac{\hbar^2}{2m}\cdot\frac{\alpha}{2}$$

$$\langle E \rangle = \frac{\hbar^2 \alpha}{4m}$$

For value of α we can normalise ψ

$$\langle \psi | \psi \rangle = 1 = \int_{-\infty}^{\infty} \frac{\alpha}{\pi} e^{-\alpha x^2} dx$$

$$\Rightarrow \frac{\alpha}{\pi} \times \sqrt{\pi/\alpha} = 1$$

$$\Rightarrow \alpha = \pi$$

$$\therefore \langle E \rangle = \frac{\hbar^2 \pi}{4m}$$

Q 7)

a) i) $V_0 \to \infty$ for $x < 0$

$\therefore \psi(x) = 0$ for $x < 0$

For $x > 0$ for bound state sol^n

$-V_0 < E < 0$ is given by

$$\frac{d^2(\psi_1)}{dx^2} + k_1^2 \psi(x) = 0 \quad (0 < x < a) \quad -①$$

an

$$\frac{d^2}{dx^2}\psi_2(x) - k_2^2 \psi_2(x) = 0 \quad (a < x) \quad -②$$

where $k_1^2 = \frac{2m(V_0 + E)}{h^2}$ & $k_2^2 = \frac{-2mE}{h^2}$

The sol^n of eq^{①} is of the type

$$\psi_1(x) = A \sin k_1 x + B \cos k_1 x$$

Boundary condition

$$\psi_1(0) = 0 \Rightarrow B = 0$$

$$\psi_1(x) = A \sin k_1 x$$

for ψ_2

$D = 0$ since $e^{k_2 x}$ is unbounded

$\psi_2 = Ce^{-k_2 x}$

$$\psi_x = \begin{cases} 0 & 0 > x \\ A \sin k_1 x & 0 < x < a \\ C e^{-k_2 x} & a < x \end{cases}$$

$\psi_1(a) = \psi_2(a)$

$A \sin k_1 a = Ce^{-k_2 a}$ — (3)

$\psi_1'(a) = \psi_2'(a)$

$A k_1 \cos k_1 a = -C k_2 e^{-k_2 a}$ — (4)

$\frac{(4)}{(3)} \Rightarrow k_1 \cot k_1 a = -k_2 a$

Also, $(k_1 a)^2 + (k_2 a)^2 = r_0^2$

$r_0 = \frac{\sqrt{2mV}\, a}{\hbar}$

ii) For case II. $0 < E < V_2$

similarly we get

$\psi_1(x) = A e^{k_1 x}$ $(x < -a)$

$\psi_3(x) = D e^{-k_1 x}$ $(x > a)$

also $k_2 \cot(k_2 a) = -k_1$

and $k_2 \tan(k_2 a) = k_1$

These eq^n can be solved graphically

$-\alpha_n \cot \alpha_n = \sqrt{R^2 - \alpha_n^2}$ (for odd cases)

$\alpha_n \tan \alpha_n = \sqrt{R^2 - \alpha_n^2}$ (for even cases)

where $R^2 = \dfrac{2ma^2V_0}{\hbar^2}$

We always get atleast 1 bound state no matter what for $V_0 > 0$