PYL101: Electromagnetic waves and Quantum Mechanics

Lecture 2

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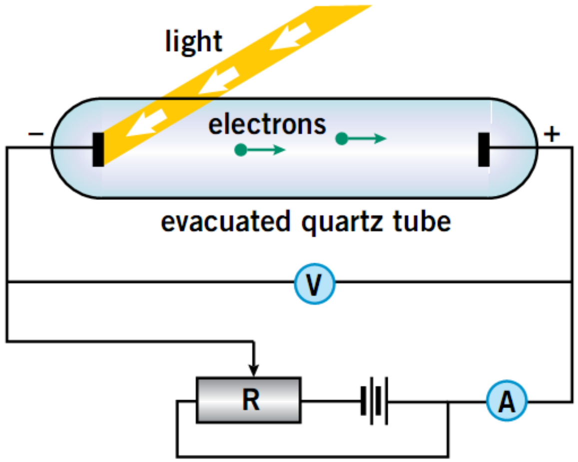
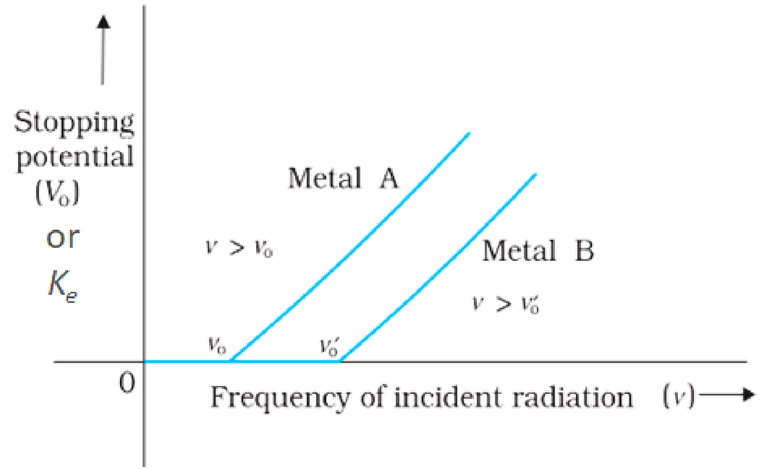
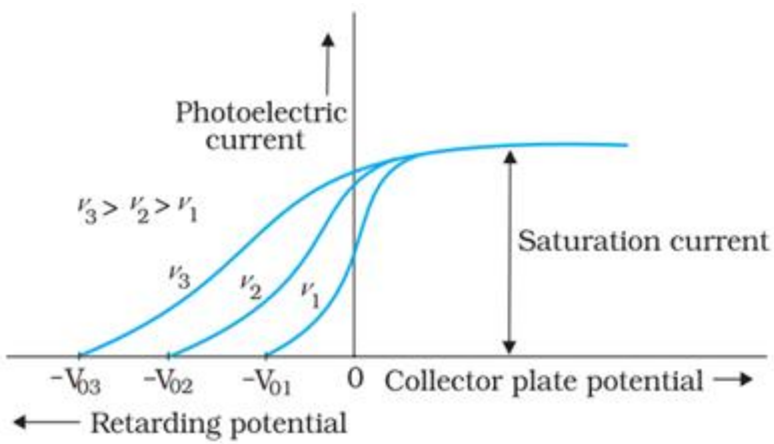
Photoelectric effect

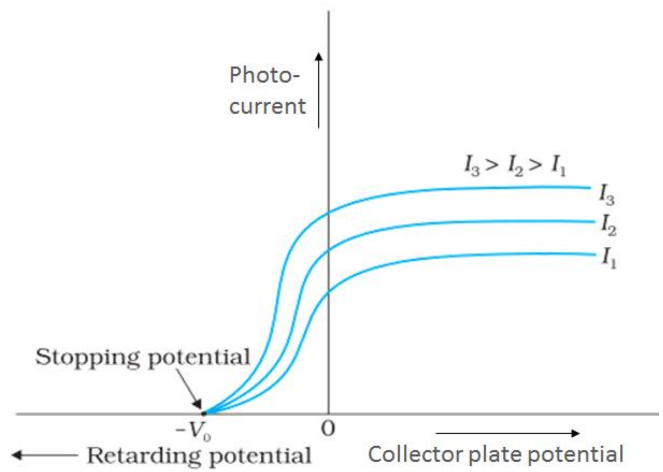
Hertz (1887): experiment

- Electrons are ejected from metal surfaces when irradiated with appropriate light







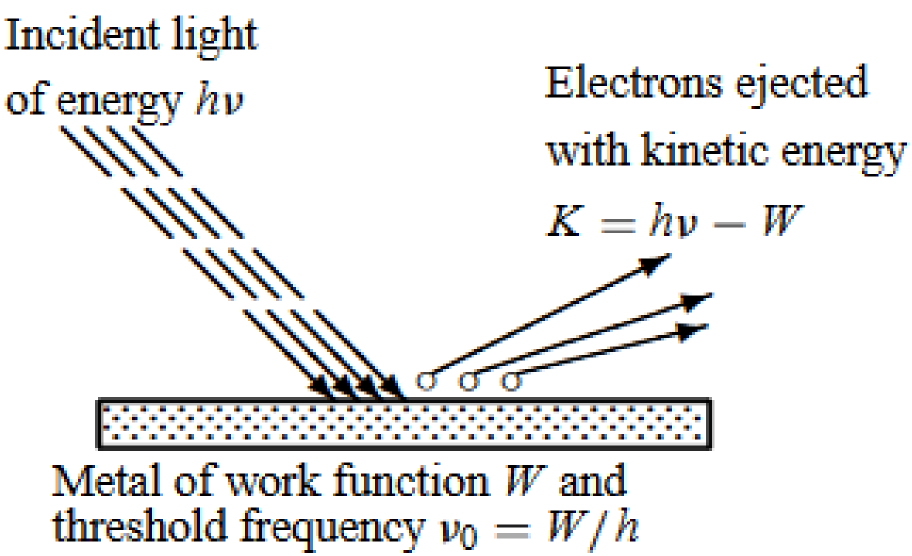


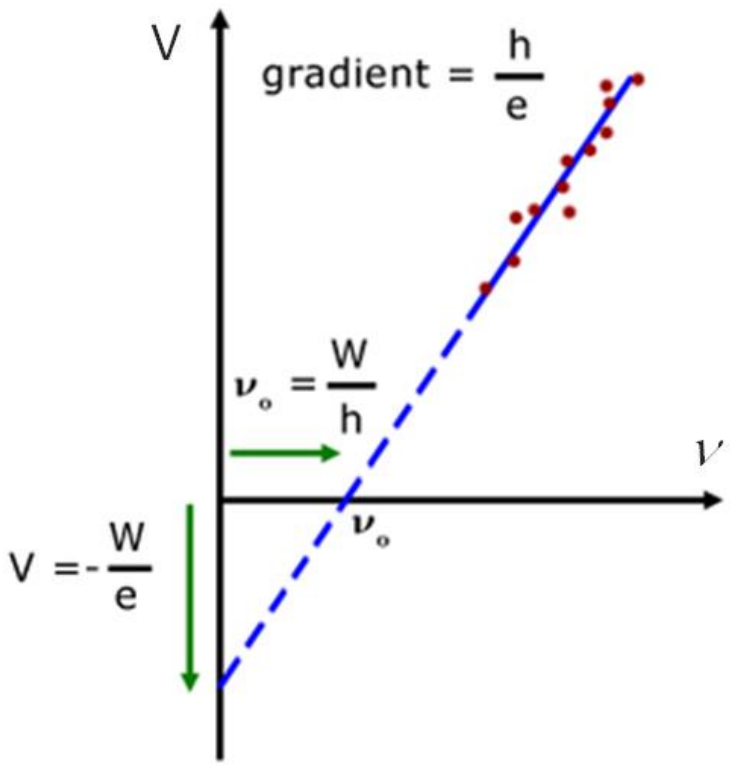
Einstein (1905): theoretical interpretation

- Light is made of particles called photon in terms of which the energy is quantized

$$K = h\nu - W = h(\nu - \nu_0)$$

$$K = \frac{1}{2}mu^2 = e(V - V_0)$$





h = 6.626 x 10^{-34} Joules.Sec

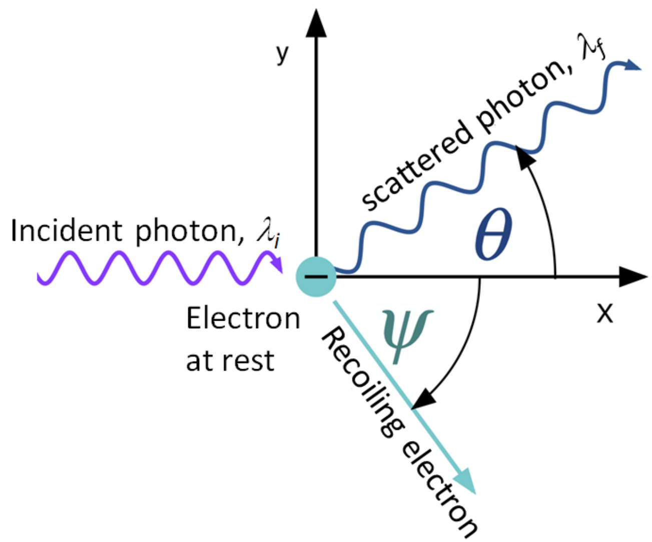
Compton effect

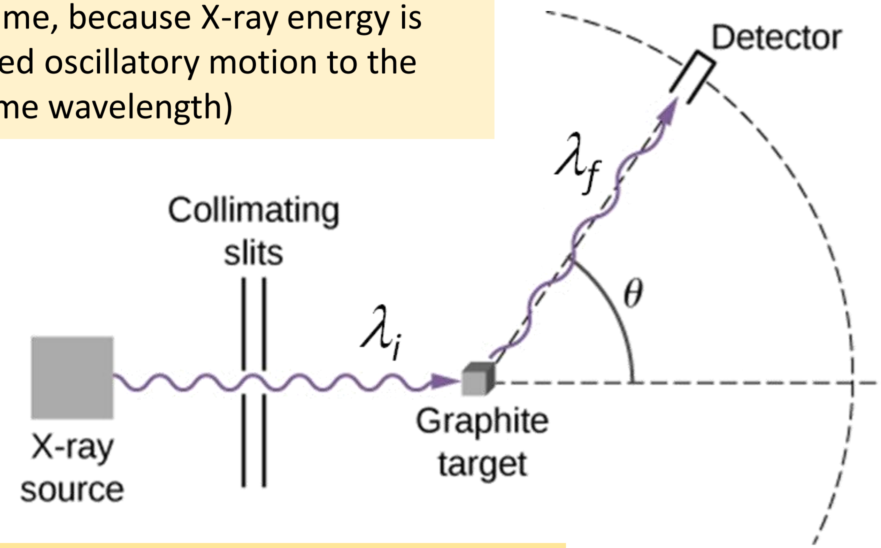
Compton (1923): experiment

- First direct evidence of particle nature of electromagnetic radiation
- Scattering of X-rays by free electrons

$\lambda_f > \lambda_i$

(Classically these should have been same, because X-ray energy is too large and it would have just induced oscillatory motion to the electron which would then radiate same wavelength)





$$\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_e c}(1 - \cos\theta)$$

Compton wavelength = 2.426×10^{-12} m

- Energy and momentum of X-ray photon: $E = h\nu = \dfrac{hc}{\lambda}, \quad p = \dfrac{h}{\lambda} = \dfrac{h\nu}{c},$
- Elastic scattering of X-ray photon from free electron at rest
- Energy and momentum conservation

Linear momentum: $p_i = p_f + p_e$ (Vector)

$$p_e{}^2 = (p_i - p_f)^2 = p_i{}^2 + p_f{}^2 - 2 p_i p_f \cos\theta$$
$$= \frac{h^2}{c^2}\left(\nu_i{}^2 + \nu_f{}^2 - 2\nu_i \nu_f \cos\theta\right)$$

Electron rest mass energy: $E_0 = m_e c^2,$

Recoiling electron energy: $E_e = \sqrt{p_e{}^2 c^2 + (m_e c^2)^2}$

- De Broglie wavelength: $\Lambda = h/p_e$
- Energy-momentum: $E = p_e c$

Apply energy conservation:

$$h\nu_i + m_e c^2 = h\nu_f + h\sqrt{\nu_i^2 + \nu_f^2 - 2\nu_i \nu_f \cos\theta + \frac{m_e^2 c^4}{h^2}}$$

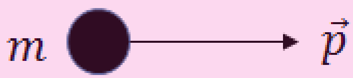
Simplify this to get

$$\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_e c}(1 - \cos\theta)$$

- **Compare Compton scattered wavelength shifts for X-rays (1 nm) and visible radiation (500 nm) from an electron at rest and a nucleus (*M* = $10^4 m_e$) at rest.**
- **You should imagine the size differences in the two particles and the two radiations considered.**
- **In which case the scattering is significant?**

De Broglie hypothesis of matter waves

Waves: wavelength λ and wave vector $\vec{k}$



Particles: energy E and momentum $\vec{p}$

- The way radiation has dual wave-particle nature, all material particles also should display dual wave-particle behavior.
- Each material particle of momentum $\vec{p}$ behaves as a group of matter waves having wavelengths λ and wave vector $\vec{k}$

$$\lambda = \frac{h}{p}, \qquad \vec{k} = \frac{\vec{p}}{\hbar}$$

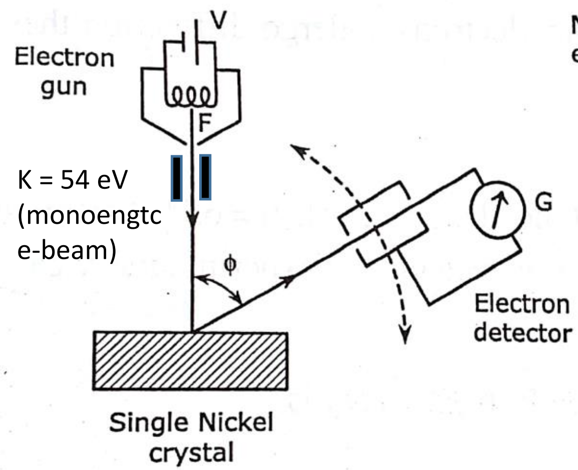
Niels Bohr used this in his H-atom model which accurately described the absorption spectrum and hence the atomic structure.

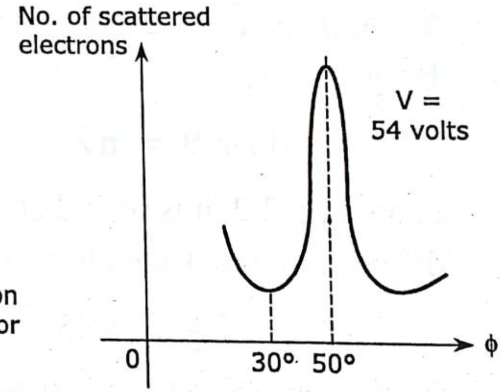
Examples:

- De Broglie wavelength of an electron (mass $m_e = 9.1 \times 10^{-31}$ kg) moving with speed $u = 10^6$ m/s will be ~0.7 nm. The size of the wave is right in the atomic scale.
- On the other hand, the de Broglie wavelength of a classical body of mass M ($M = m_e \times 10^{31}$ kg) moving with the same speed as above will be ~ 0.7×10^{-40} m. This is insignificant at the atomic length scale. What does this mean?

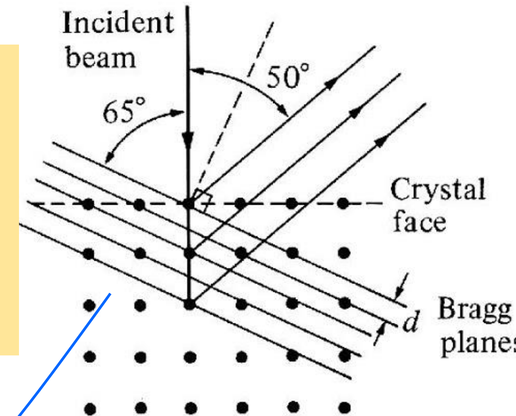
Davisson Germer experiment

- Electron diffraction from solids (proves wave nature of electrons like X-rays)





Equivalence with Bragg's X-ray diffraction from crystals



Verify this for constructive interference

Bragg's law $n\lambda = 2d\, sin\, 65$

Only at $\phi = 50^\circ$, electron count was maximum, this should correspond to $n = 1$

Using d = 0.091 nm, $\lambda = 2 \times d \times sin\, 65 = 2 \times 0.091 \times 0.906 = 0.165\ nm$

De Broglie wavelength of the electrons at 54 eV, $\lambda = \dfrac{h}{\sqrt{2m_e K}} = 0.167\, nm$

Plane waves $\psi(\vec{r},t) = Ae^{i(\vec{k}.\vec{r} - \omega t)} = Ae^{i(\vec{p}.\vec{r} - Et)/\hbar}$

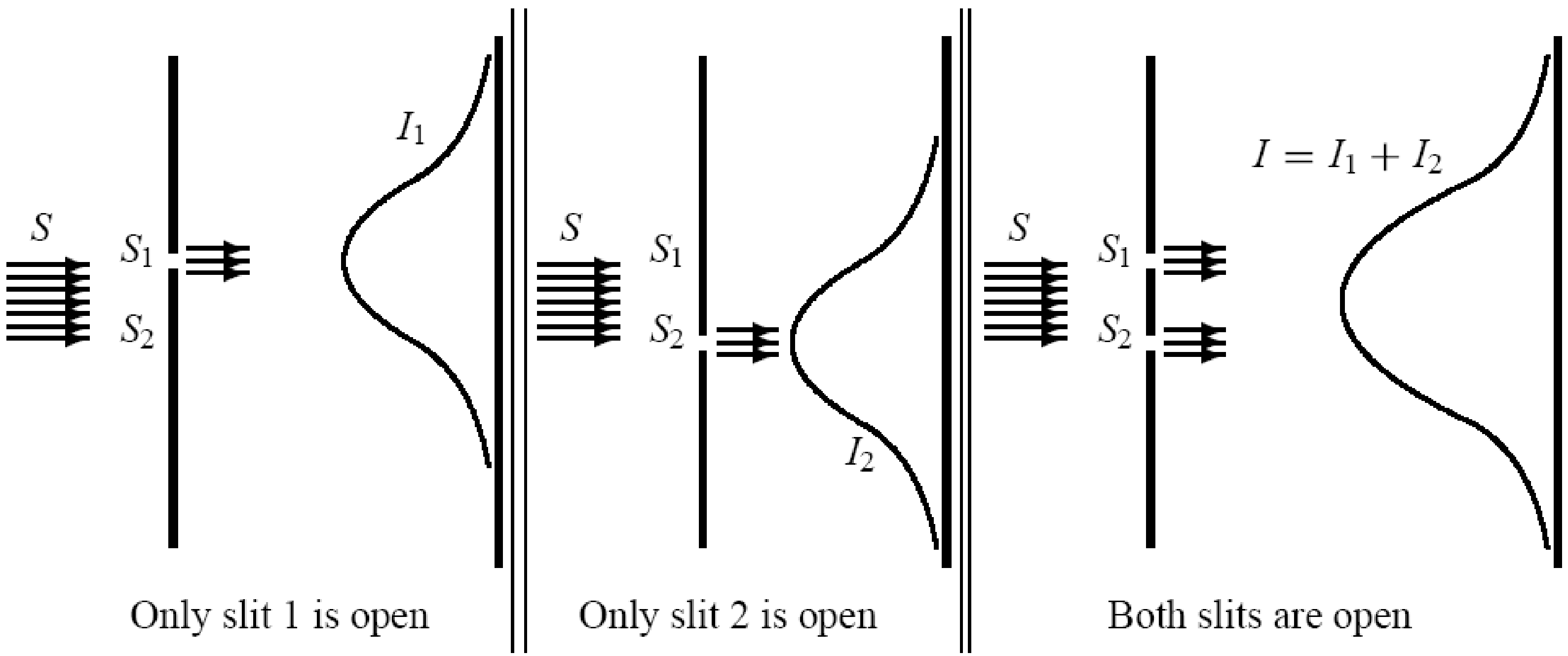
Dual wave-particle description at microscopic level

Double slit experiments with particles

Waves: Amplitude and phase	Particles: quantum (microscopic) or classical
(Amplitude add)	(Counts/ticks/intensities add)

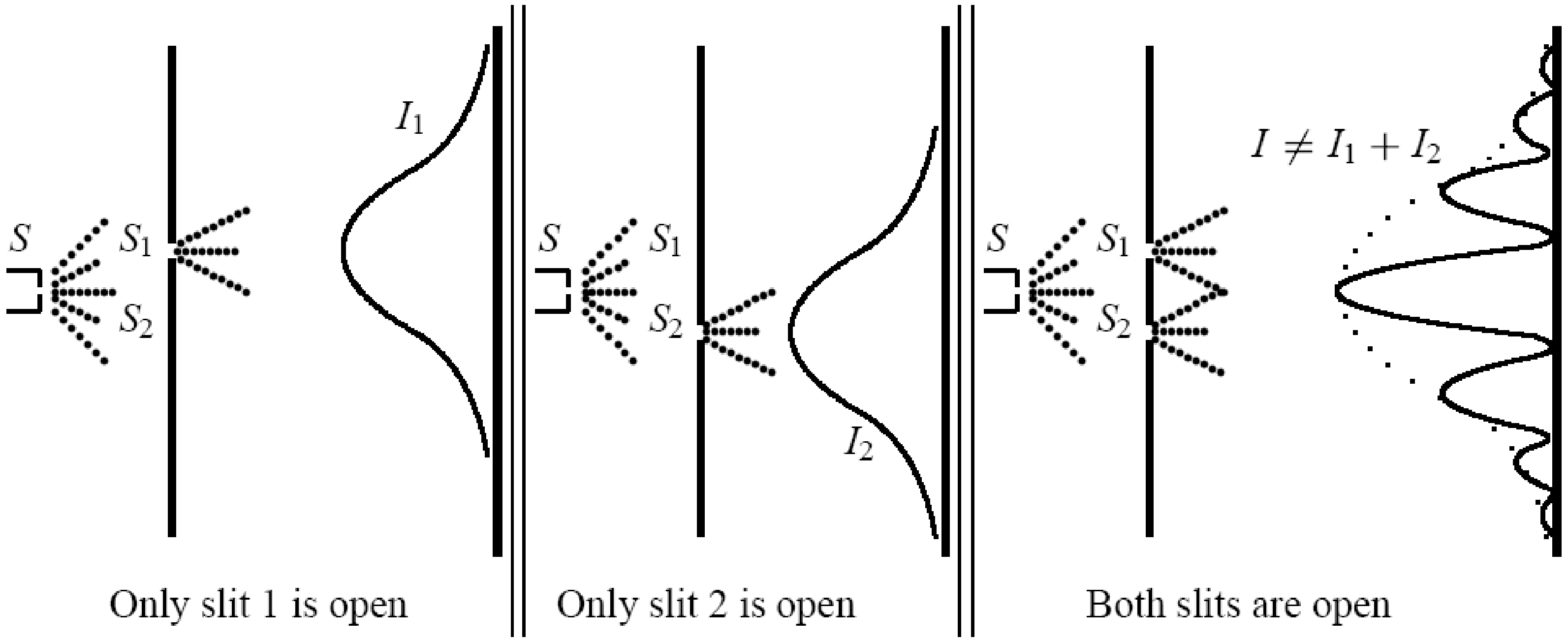
We will see that amplitudes add for quantum particles.

1. Two-slit experiment with classical (macroscopic) particles



Double slit experiments with particles

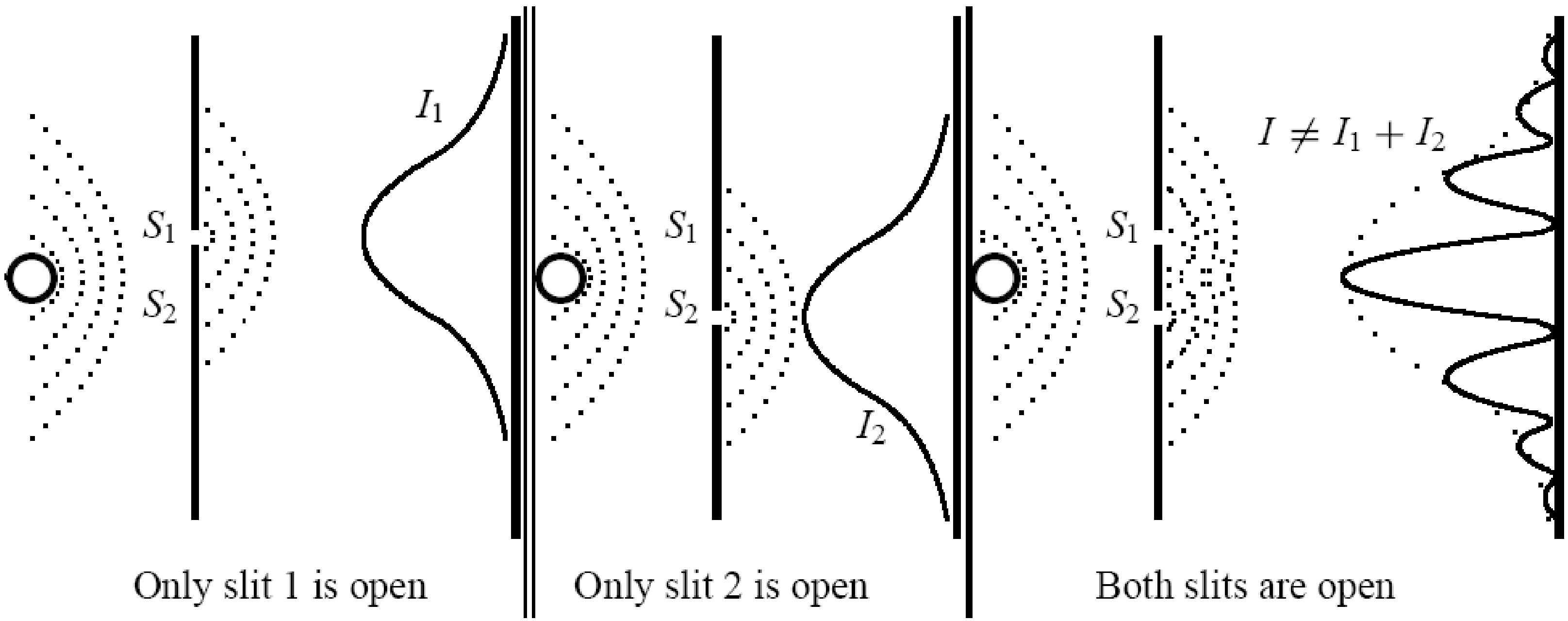
2. Two-slit experiment with quantum (microscopic) particles



This is wave-like behavior!

Double slit experiments with particles

3. Two-slit experiment with waves

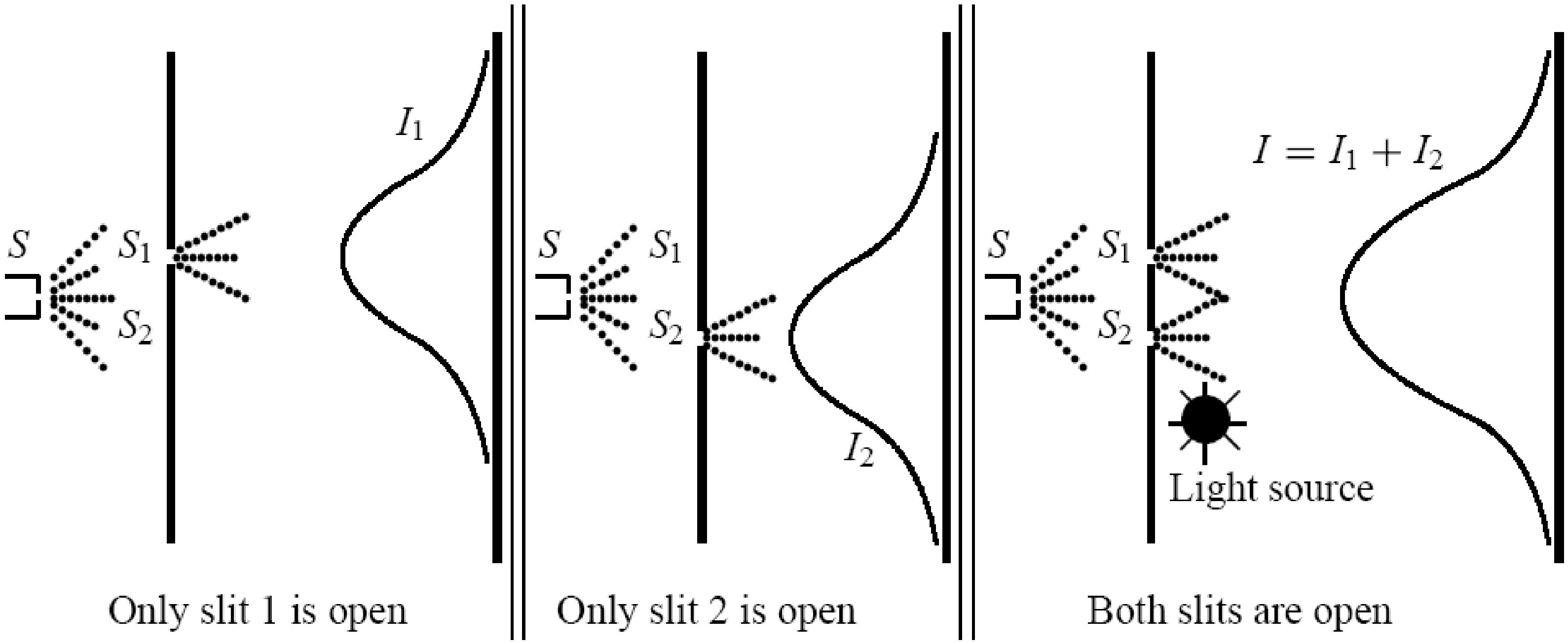


$$I = \left|\psi_1 + \psi_2\right|^2 = \left|\psi_1\right|^2 + \left|\psi_2\right|^2 + (\psi_1^*\psi_2 + \psi_1\psi_2^*)$$

$$= I_1 + I_2 + 2\,Re(\psi_1^*\psi_2) = I_1 + I_2 + 2\sqrt{I_1 I_2}\,cos\,\delta$$

Double slit experiments with particles

4. Two-slit experiment with quantum (microscopic) particles



Classical particle-like results when you watch the electrons pass through the slits

Conclusions

- Act of measurement disturbs the outcome of an experiment with microscopic particles.

  Quantum mechanical principle:
  - **measurements interfere with the states of microscopic objects**
  - **microphysical world is indeterministic**
  - **this led to Heisenberg's uncertainty principle**
- These either behave like waves or particles but not both at once.
  - **They are neither pure particles nor pure waves but both (think them as complimentary not exclusive)**

Conclusions

- Dual wave-particle behavior at microscopic level is enough to be described by plane waves, the quantum mechanical wave function to describe state of a microscopic system,

$$\psi(\vec{r},t) = Ae^{i(\vec{k}.\vec{r}-\omega t)} = Ae^{i(\vec{p}.\vec{r}-Et)/\hbar}$$

Therefore, principle of linear superposition can be applied

$$\psi(\vec{r},t) = a_1\psi_1(\vec{r},t) + a_2\psi_2(\vec{r},t) + \ldots$$

- Since waves are not localized in space, so a probabilistic feature has to be associated with the wave function. **(Max Born, 1927)**