ELL100: INTRODUCTION TO ELECTRICAL ENGG.

Signal Waveforms

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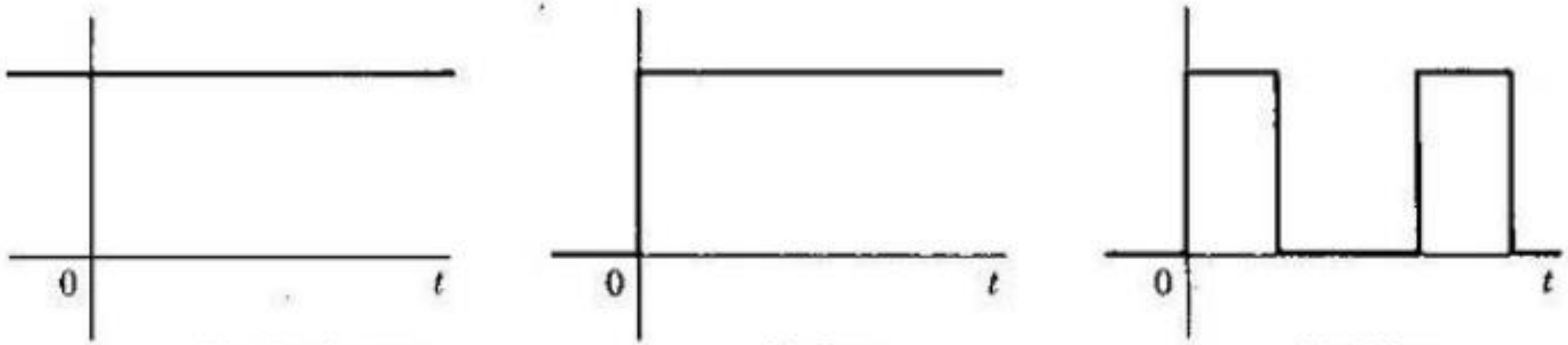
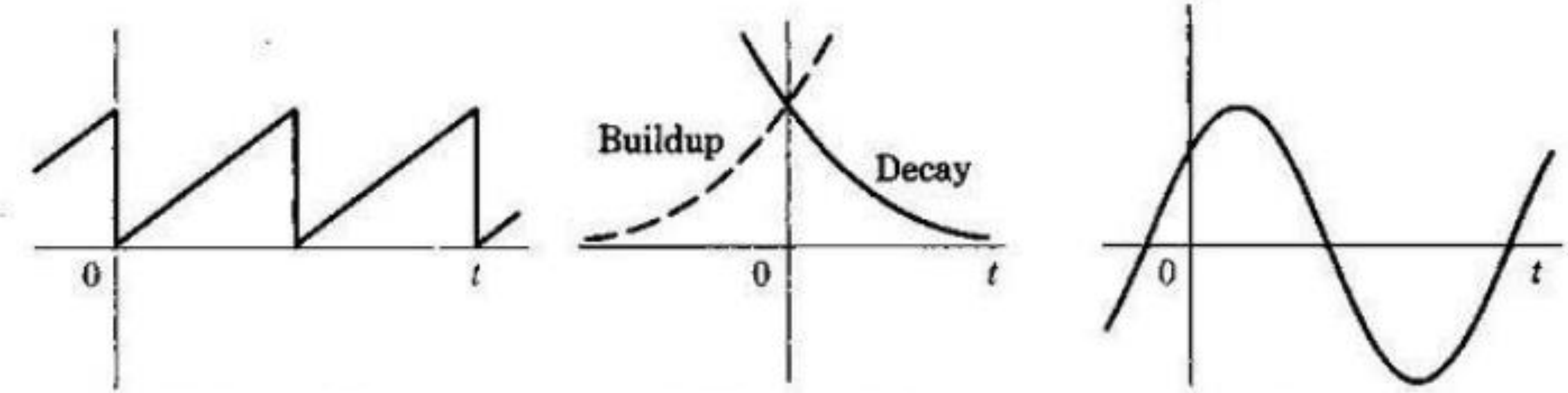
Signal Waveforms



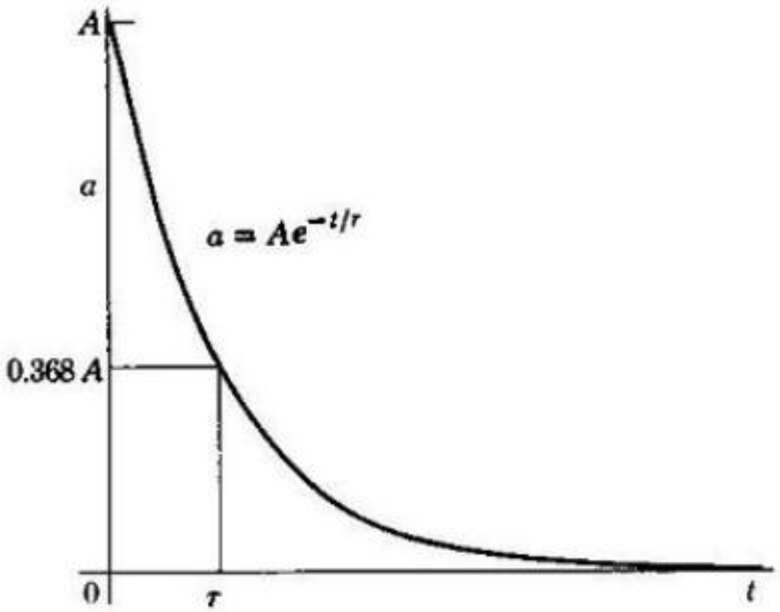
(a) Continuous (b) Step (c) Pulse

Common Signal Waveforms



(d) Sawtooth (e) Exponentials (f) Sinusoidal

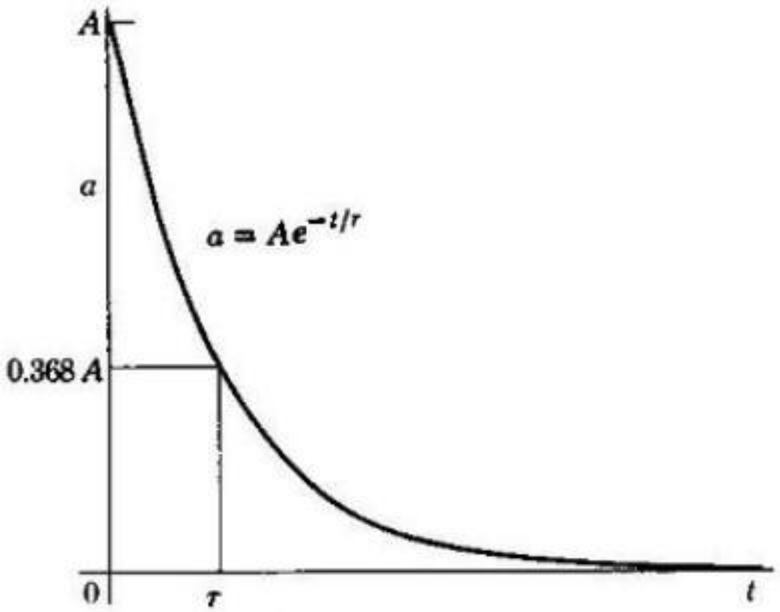
Exponential Signals



The decaying exponential.

$$a = A\, e^{-\frac{t}{\tau}}$$

Exponential Signals



The decaying exponential.

$$a = A\, e^{-\frac{t}{\tau}}$$

Time constant

When $t = \tau = Time\ Constant$

$$e^{-\frac{t}{\tau}} = e^{-1} = 0.368$$

Normalized Exponentials

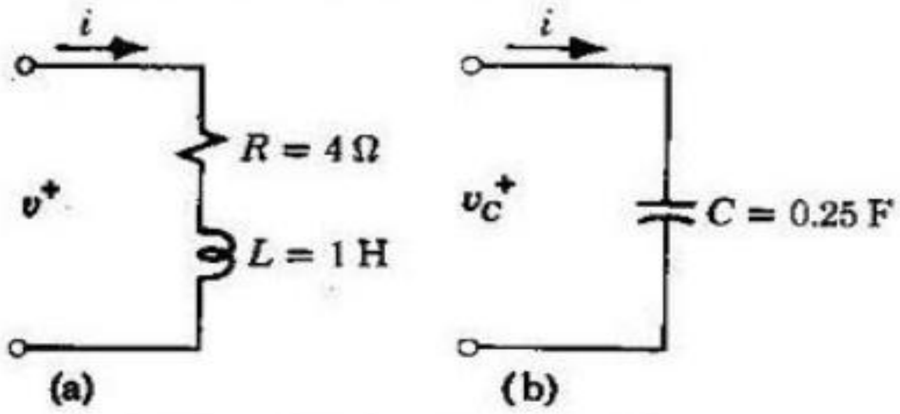
$$\frac{a}{A} = e^{-\frac{t}{\tau}}$$

For $t = 2\tau$, $a/A = 0.135$ and for $t = 5\tau$, $a/A = 0.0067$ i.e. practically negligible

Example 1

(a) Determine the voltage $v(t)$.

(b) If the current $i = 5\,e^{-2t}$ A flows in an initially uncharged 0.25-F capacitance ($v_C = 0$ at $t = 0$ in Fig. 3.5b), determine the voltage $v_C(t)$.



Response to exponentials.

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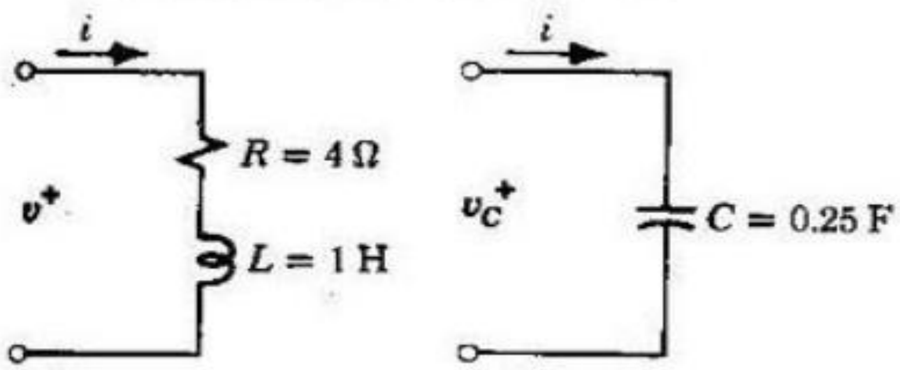

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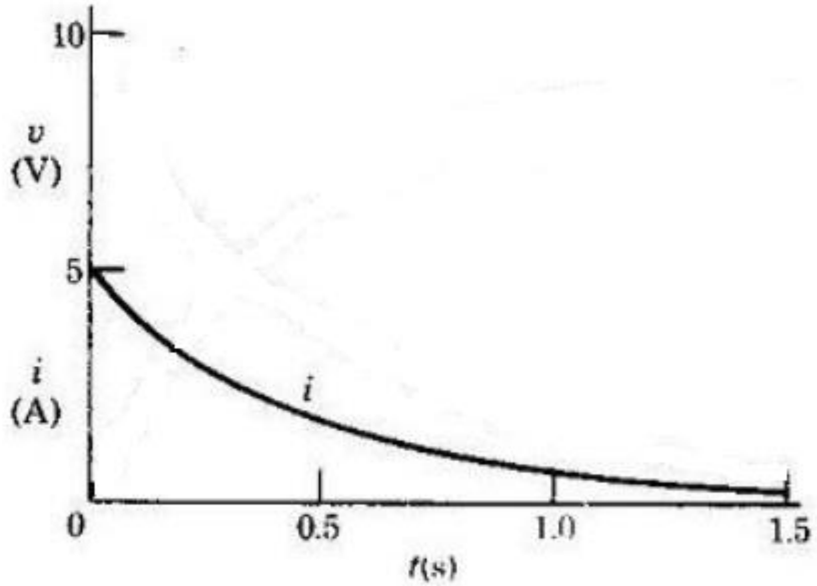

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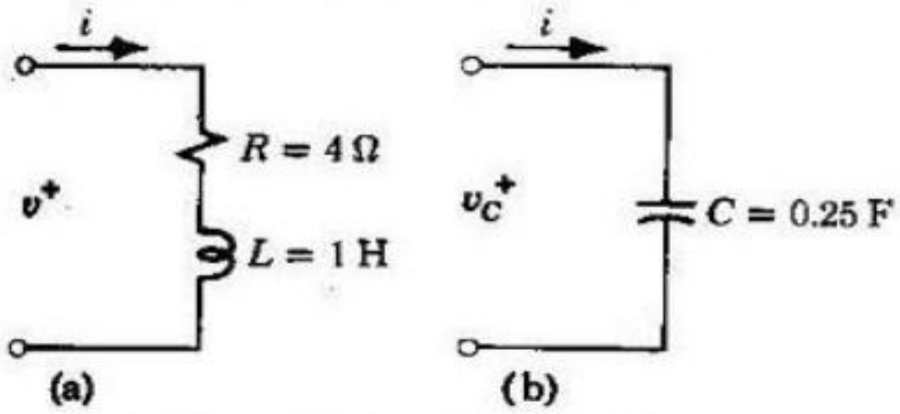
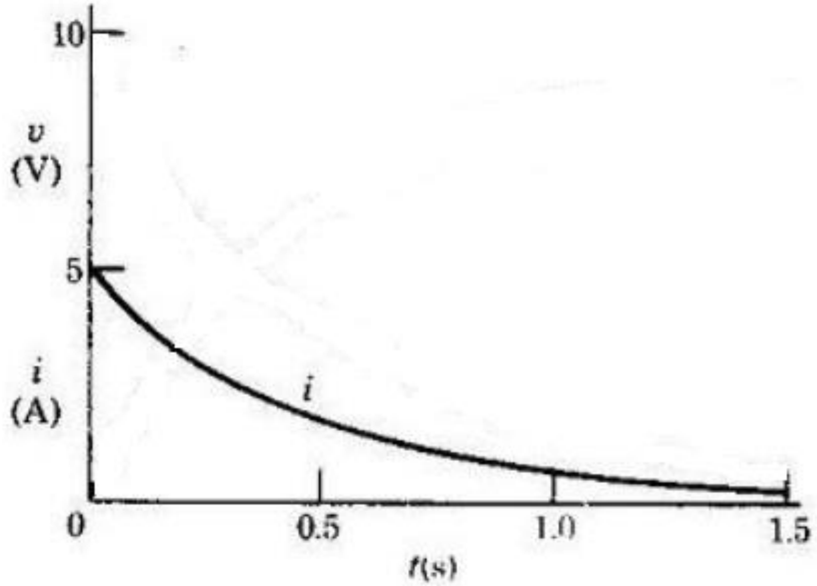


Response to exponentials.

Solution

Step 1: Calculate V_R

$$v_R = Ri = 4 \times 5\, e^{-2t} = 20\, e^{-2t} \text{ V}$$



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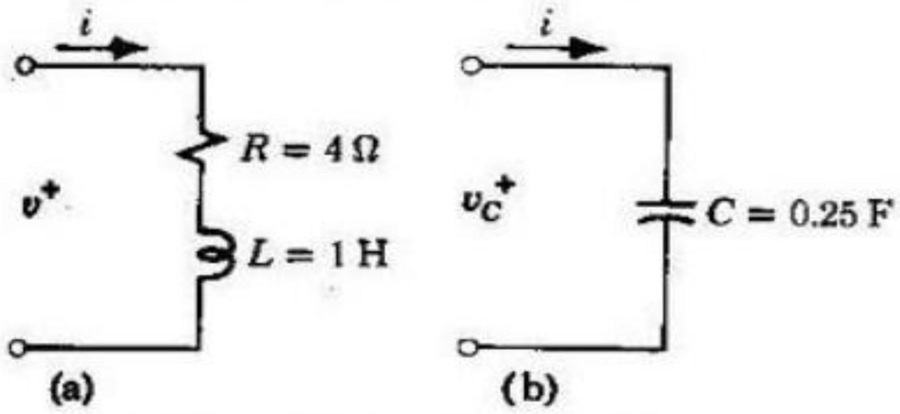

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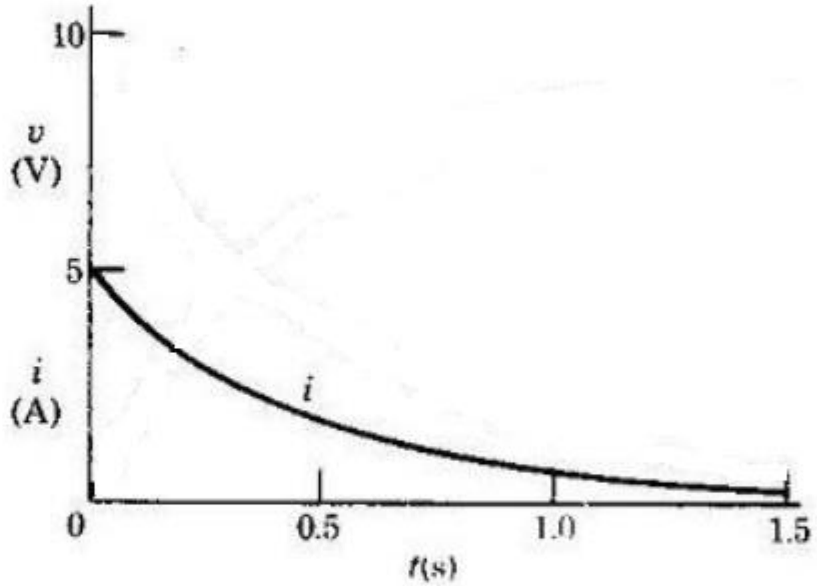
Solution

Step 1: Calculate V_R

$$v_R = Ri = 4 \times 5\,e^{-2t} = 20\,e^{-2t}\text{ V}$$

Step 2: Calculate V_L

$$v_L = L\frac{di}{dt} = 1(-2)5\,e^{-2t} = -10\,e^{-2t}\text{ V}$$



Example 1

Solution
Step 3: Calculate V_{RL}

$$v = v_R + v_L = 20\,e^{-2t} - 10\,e^{-2t} = +10\,e^{-2t}\ \text{V}$$

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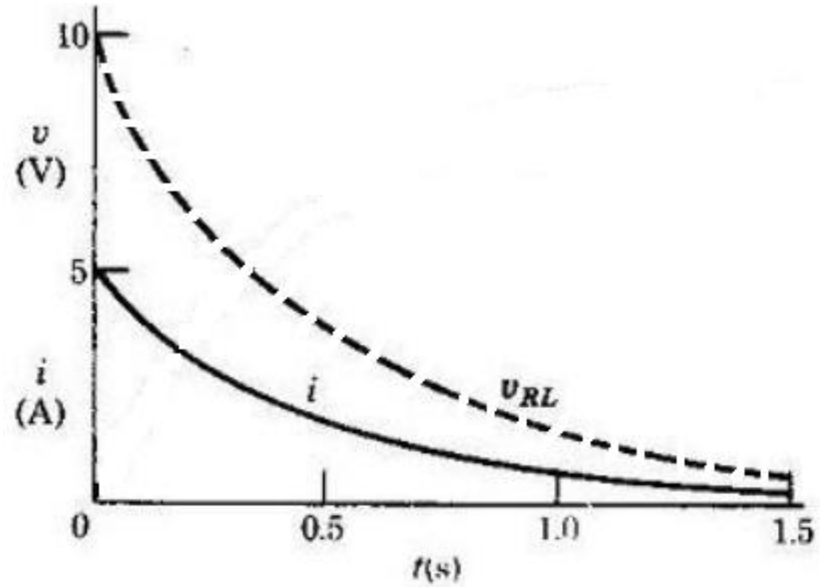

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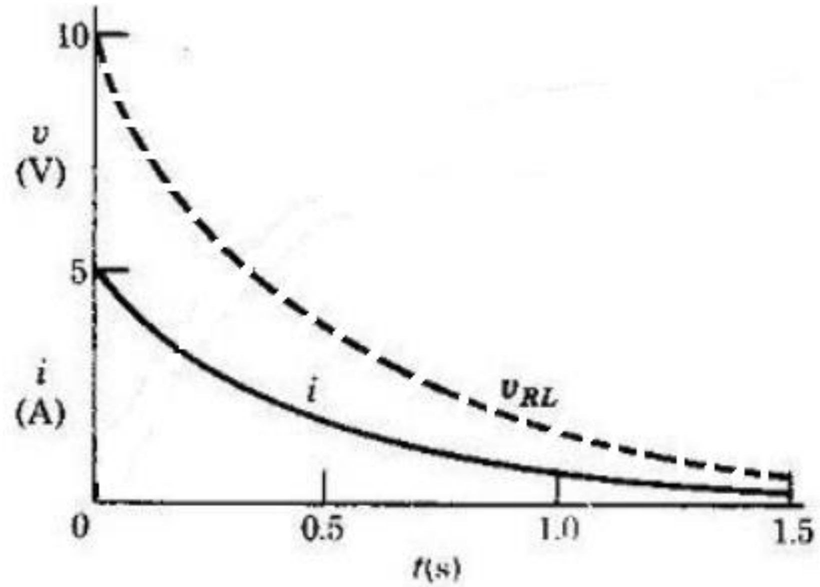


Step 4: Calculate V_C

$$v_C(t) = \frac{1}{C}\int_0^t i\,dt + V_O = \frac{1}{0.25}\int_0^t 5\,e^{-2t}\,dt + 0$$

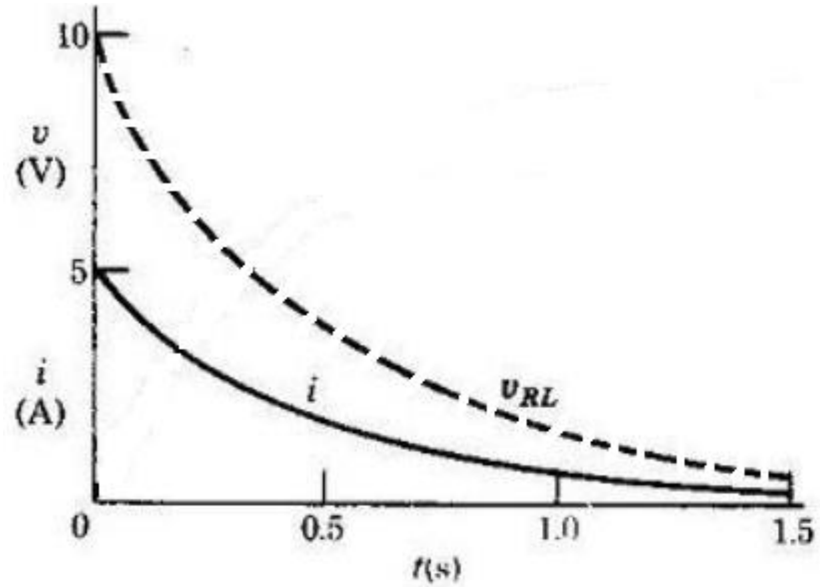
$$= \frac{5}{0.25(-2)}\,e^{-2t}\Big]_0^t = 10 - 10\,e^{-2t}\ \text{V}$$

Example 1

Solution

Step 3: Calculate V_{RL}

$$v = v_R + v_L = 20\,e^{-2t} - 10\,e^{-2t} = +10\,e^{-2t}\ \text{V}$$



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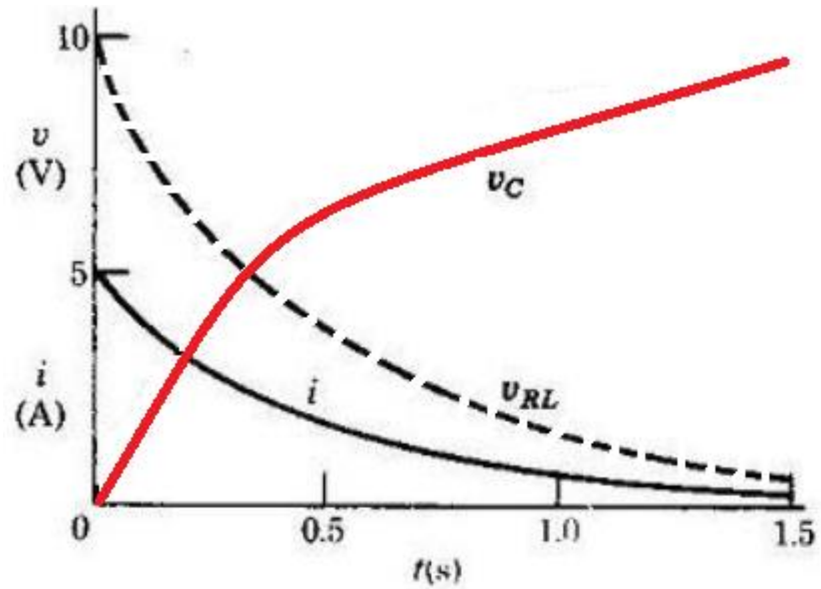
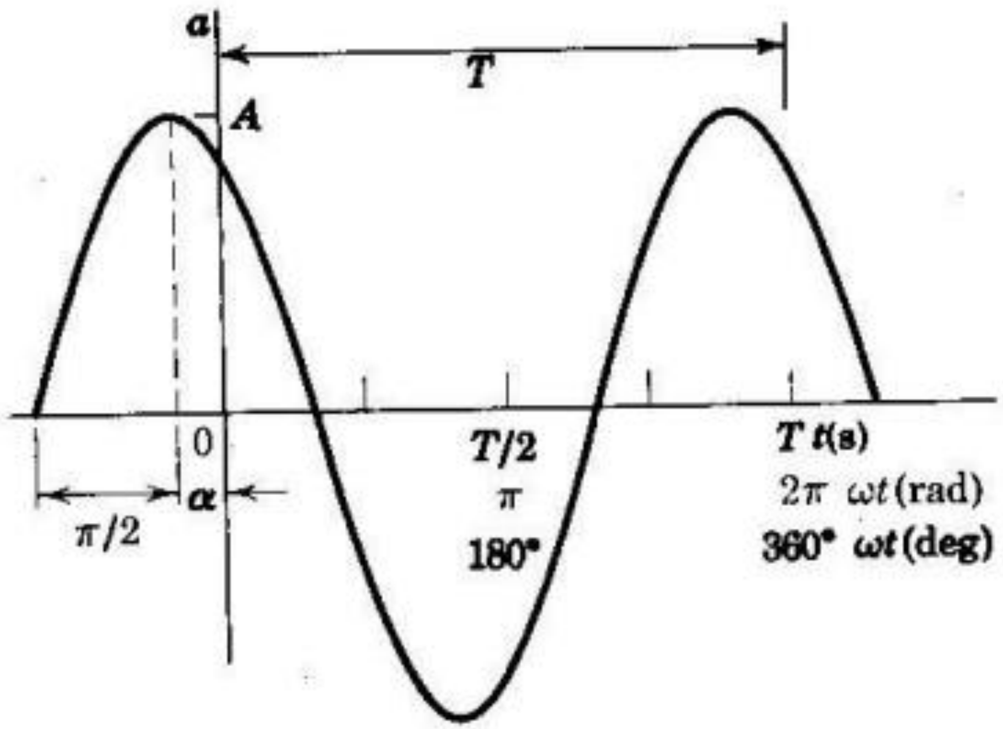


Sinusoids

Eg. Current in oscillating circuit, vibration of a string

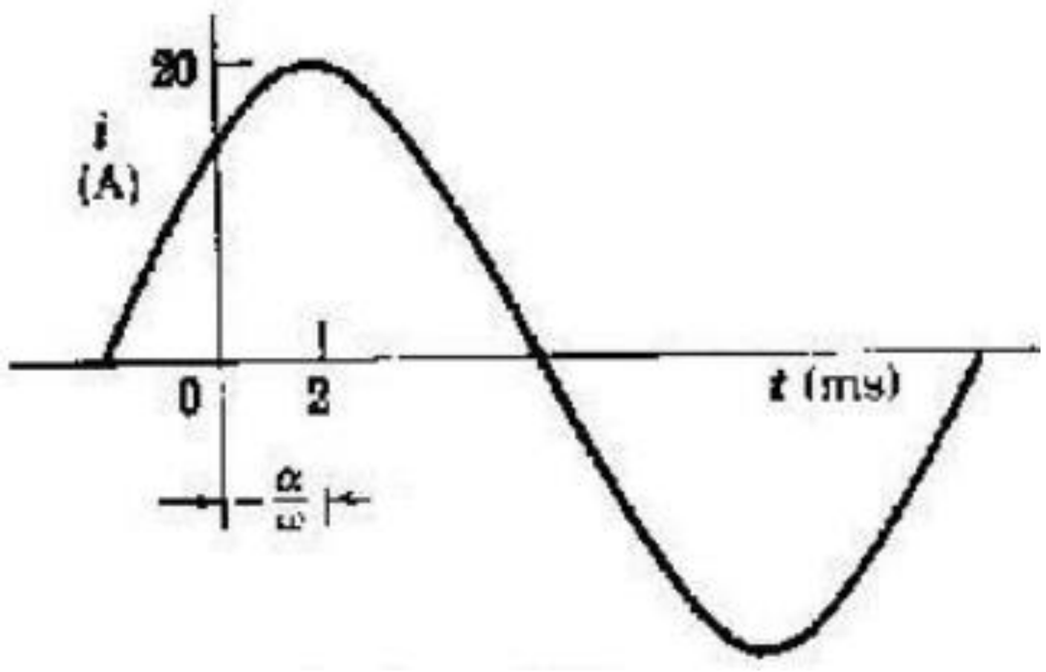
$$a = A\ \cos(\omega t + \alpha) \quad , \quad f = \frac{\omega}{2\pi}$$

$$a = A\ \sin(\omega t + \alpha + \frac{\pi}{2}) \quad , \quad T = \frac{1}{f}$$



Example 2

A sinusoidal current with a frequency of 60 Hz reaches a positive maximum of 20 A at t = 2 ms. Write the equation of current as a function of time.



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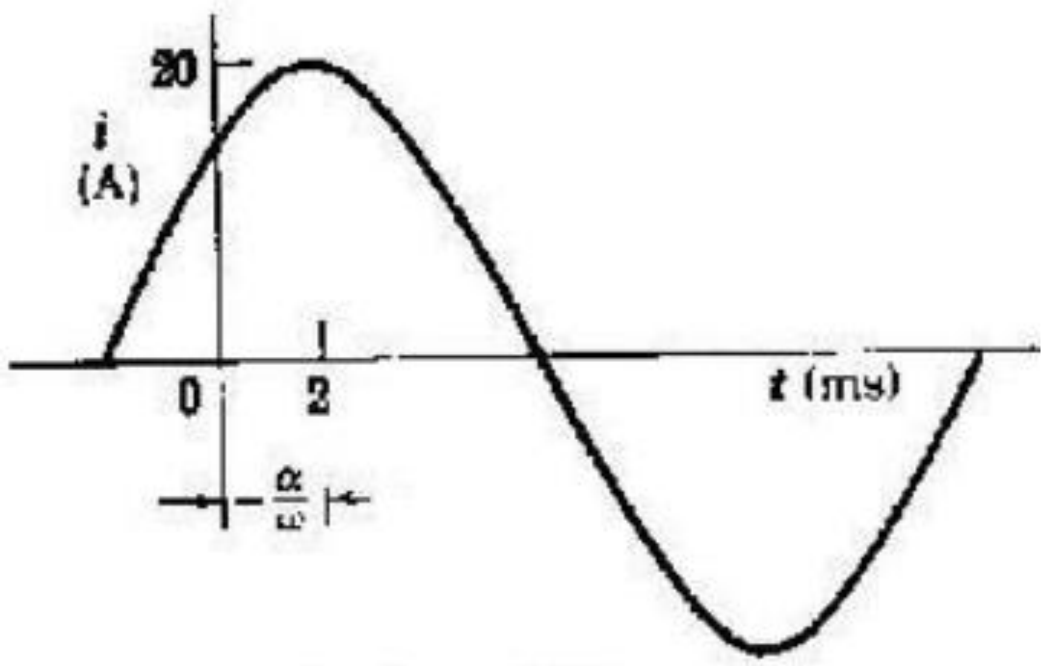

As per Cosine wave equation,

A = 20 A

$\omega = 2\pi f = 2\pi(60) = 377$ rad/s

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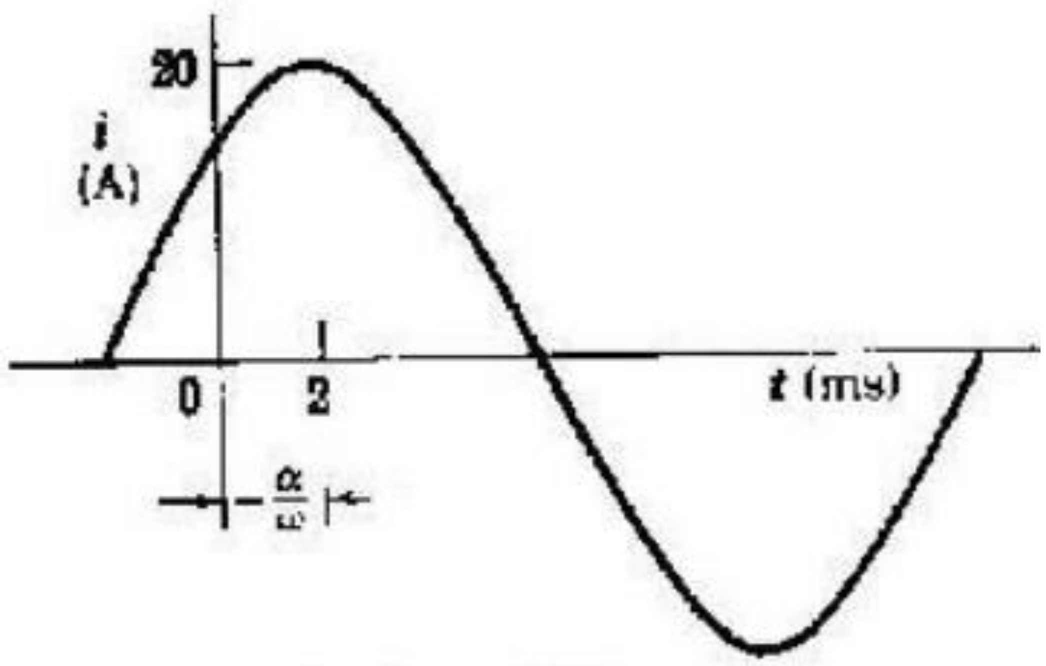

As per Cosine wave equation,

A = 20 A

$\omega = 2\pi f = 2\pi\ (60) = 377$ rad/s

The positive maximum is reached when $(\omega t + \alpha) = 0$ or any multiple of 2π. Letting $(\omega t + \alpha) = 0$,

$$\alpha = -\omega t$$
$$= -377 * 2 * 10^{-3}$$
$$= (-)\ 0.754 \text{ rad}$$

Or

$$\alpha = -0.754 * (360°/ 2\pi)$$
$$= -43.2\ °$$

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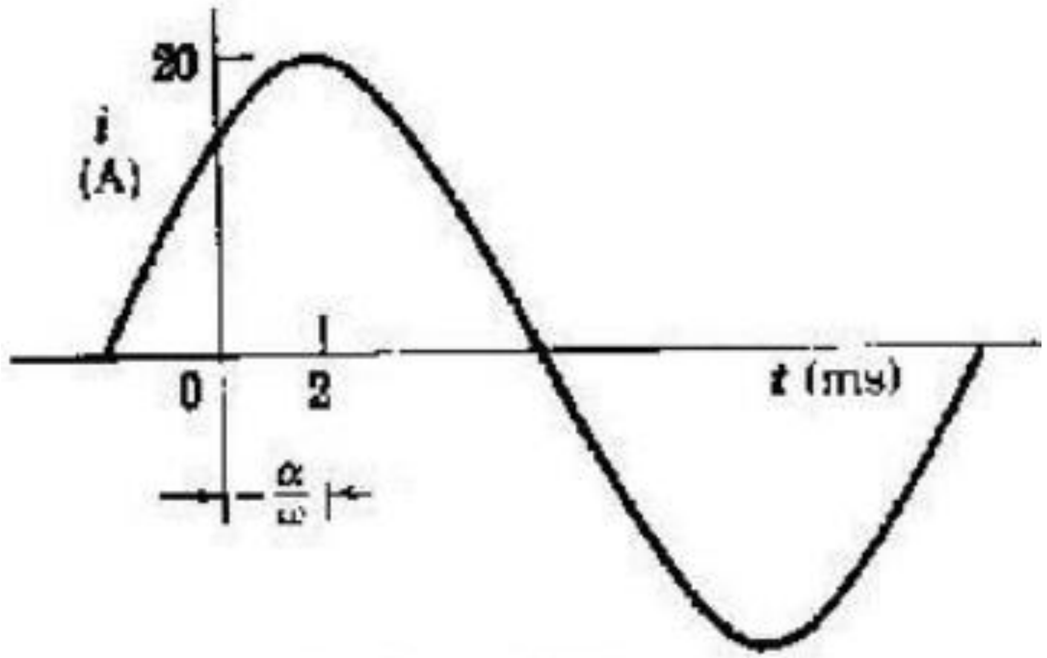
A Sinusoidal current with a frequency of 60 Hz reaches a positive maximum of 20 A at t = 2 ms. Write the equation of current as a function of time.

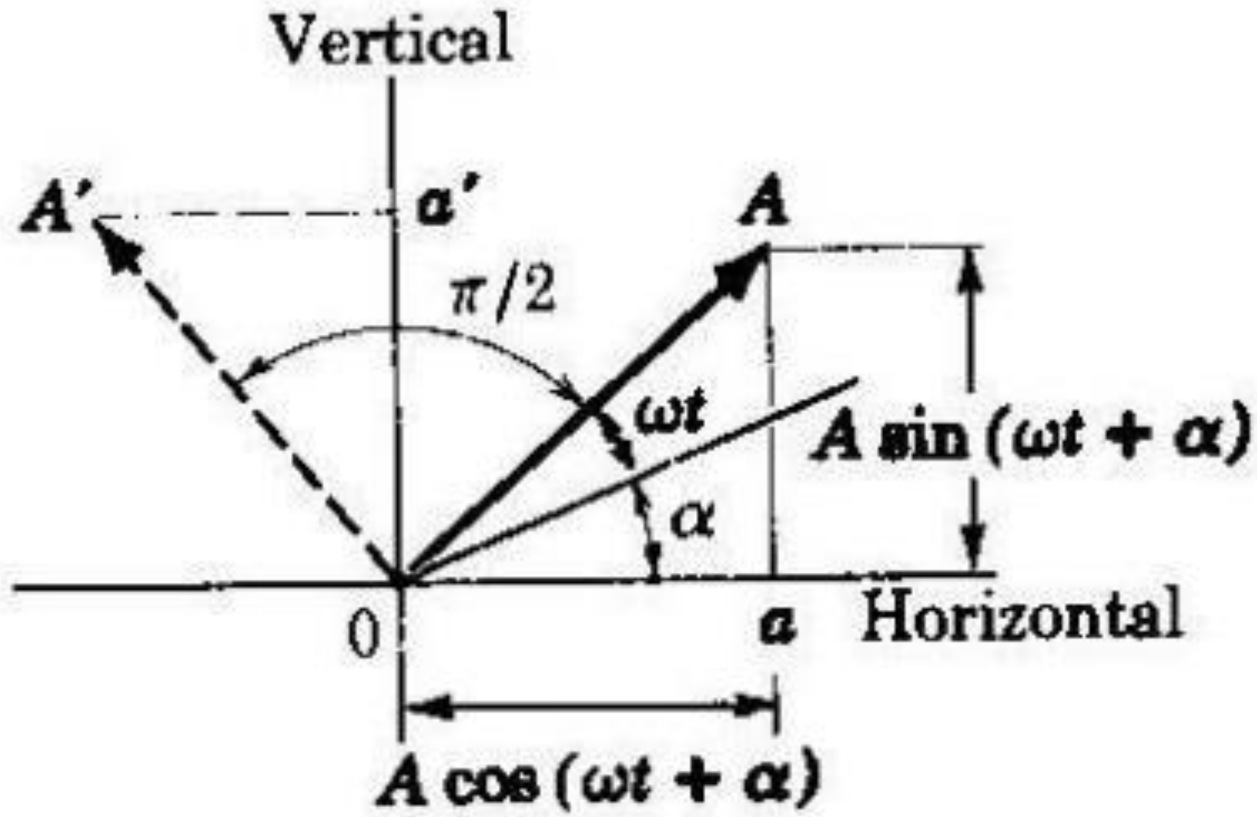


Therefore, the current equation is

$$i = 20\cos(377t - 43.2°)\ \text{A}$$

Note that the angle in parentheses is a convenient hybrid; to avoid confusion, the degree symbol is essential.

Projections of a Rotating Line

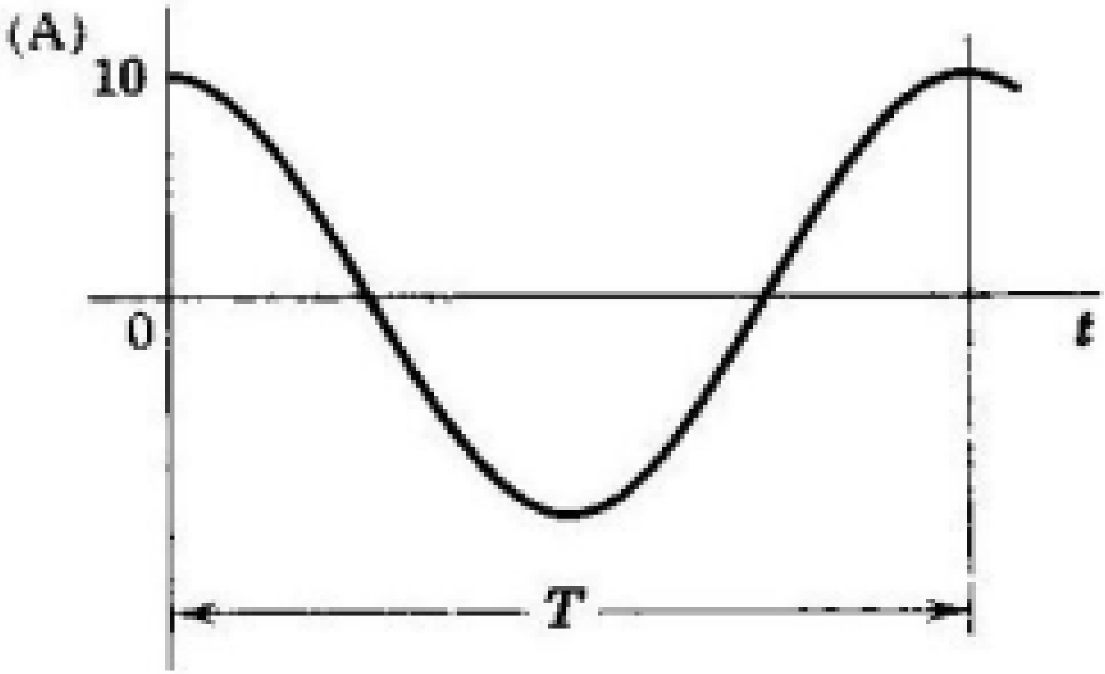


Horizontal projection : $a = A\cos(\omega t + \alpha)$

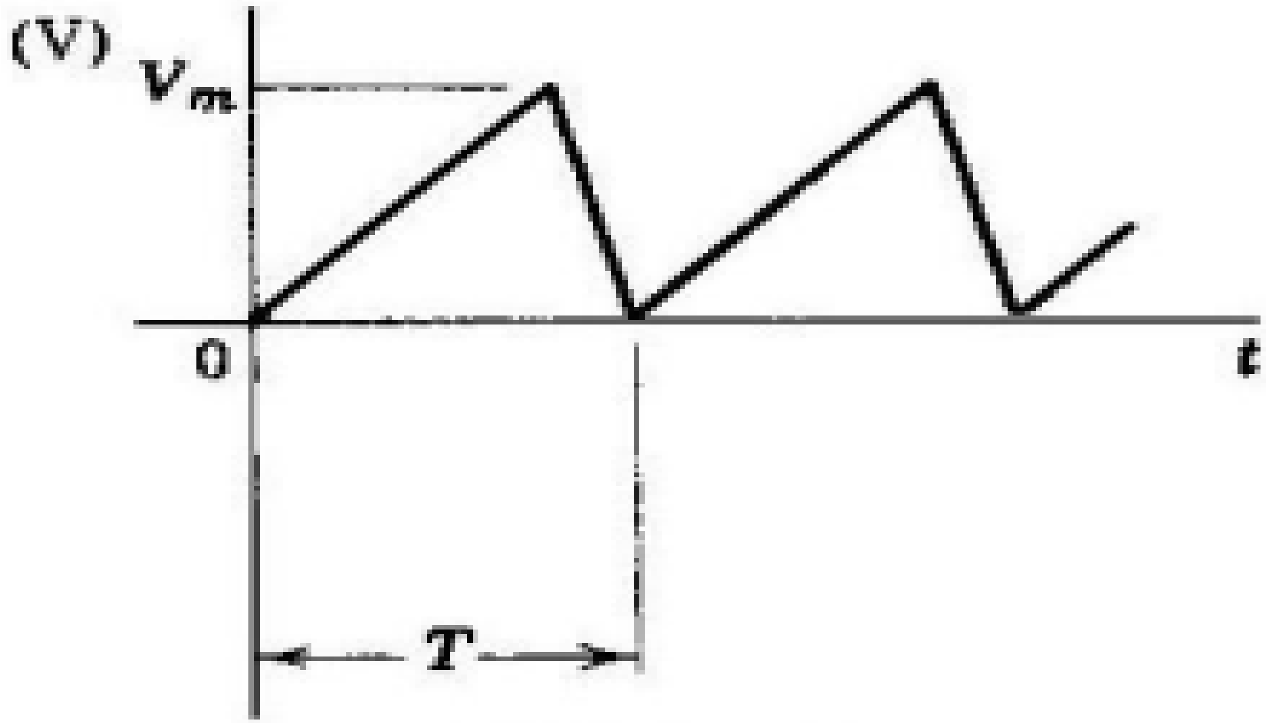
Vertical projection : $a' = A' \sin(\omega t + \alpha + \frac{\pi}{2})$

Periodic Waveforms

$$f(t + nT) = f(t)$$



(a) Sinusoidal



(b) Triangular

Periodic Waveforms

Average Value

The average value of a varying current i(t) over the period T is the steady value of current I_{av} that in the period T would transfer the same charge Q

Periodic Waveforms

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$$I_{av}T = Q = \int_{t}^{t+T} i(t)\ dt = \int_{0}^{T} i\ dt$$

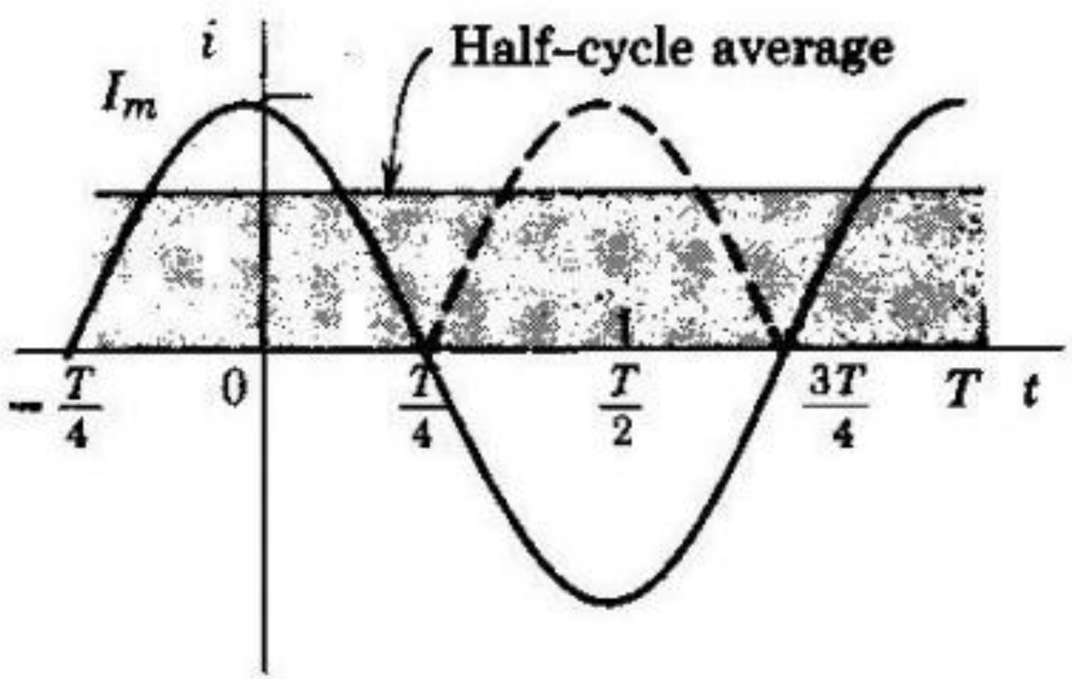
or

$$I_{av} = \frac{1}{T}\int_{0}^{T} i(t)\ dt$$

Similarly,

$$V_{av} = \frac{1}{T}\int_{0}^{T} v(t)\ dt$$

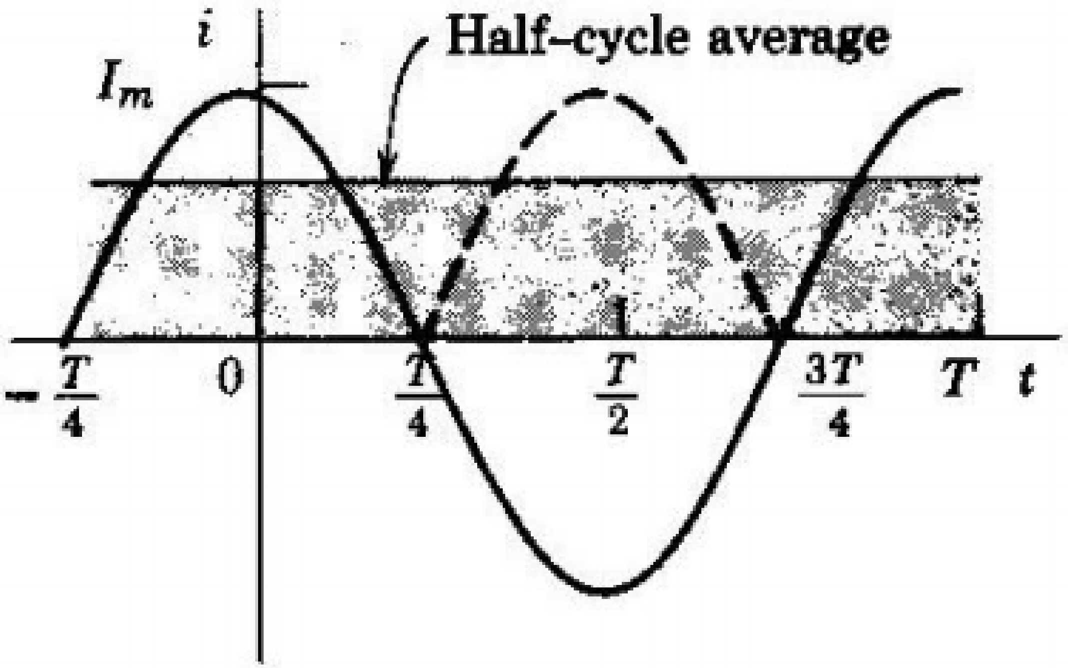
Half-Cycle Average



The half-cycle average.

Periodic Waveforms

Half cycle average



The half-cycle average.

$$I_{\text{half-cycle}} = \frac{1}{\frac{1}{2}T}\int_{-T/4}^{+T/4} I_m \cos\frac{2\pi t}{T}\,dt$$

$$= \frac{2I_m}{T}\left(\frac{T}{2\pi}\right)\sin\frac{2\pi t}{T}\Big]_{-T/4}^{+T/4}$$

$$= \frac{I_m}{\pi}\left[\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right]$$

$$= \frac{2}{\pi} I_m = 0.637 I_m$$

Effective Value

$$P_{av} = \frac{1}{T} \int_0^T p(t)\, dt$$

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For a resistance case,

$$P = \frac{1}{T}\int_0^T p\ dt = \frac{1}{T}\int_0^T i^2\ R\ dt = I_{eff}^2 R$$

where

$$I_{eff} = [\frac{1}{T}\int_0^T i^2\ dt]^{1/2} = I_{rms}$$

Effective Value

For a sinusoidal current

$$I_{eff}^2 = \frac{1}{T}\int_0^T I_m^2 \cos^2(\frac{2\pi t}{T})\ dt$$

$$= \frac{1}{T}\frac{I_m^2}{2}\int_0^T (1 + \cos(\frac{4\pi t}{T})\ dt = \frac{I_m^2}{2}$$

or $I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$