DESE

ESL300 : Self-Organising Dynamical Systems

Minor I 17th Feb 2022

Time : 1 hour
M.M : 25

Note : Attempt all questions

Q.(1) For a logistic map answer the following questions :

(a) If $x^{(1)}$, $x^{(2)}$ are fixed points of first generation map $F_b(x)$, are they fixed points of higher generations i.e $F_b^2(x)$, $F_b^{(3)}(x)$, $F_b^{(4)}(x)$, ... $F_b^{(7)}(x)$? Prove your answer. (2)

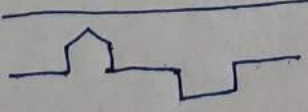
(b) If $x^{(1)}$, $x^{(2)}$ are fixed points of first generation and $x^{(3)}$, $x^{(4)}$ are fixed points exclusively of second generation $F_b^{(2)}(x)$, then find all the fixed points of $F_b^{(2)}(x)$ and $F_b(x)$. Prove your answer. Also find out in details the range of b for which the map behaves as one attractor system and period two attractor system. (4)

(c) If $x^{(3)}$, $x^{(4)}$ are fixed points exclusively of second generation $F_b^{(2)}(x)$, are they fixed points of $F_b^{(3)}(x)$, $F_b^{(4)}(x)$, $F_b^{(5)}(x)$? Give proof in support of your answer. (3)

Q.(2) Define fractals and fractal dimension. Calculate the fractal dimension in the following cases and justify your answer (give at least 2 steps of construction).

(a) Take a square, divide into 16 equal parts and remove all the corner parts. Construct the fractal structure by repeating this operation on remaining parts.

(b) Take a line and divide it into 5 equal parts. Apply the operation as given below in the figure and construct the fractal structure by continuing this process. (8)



– contd. –

Q. (3) The map of a dynamical system is given by $f(x) = 4x - 9x^2$
Calculate the following:

a) Fixed points of first generation and their nature
(Justify your answer).

b) Fixed points of second generation and their nature. (8)
(Justify your answer).

—