PYL 101

Electromagnetic waves and **Quantum Mechanics**

Tutorial Sheet 1 (L1-L2)

Q.1 Whether wave- or the particle-nature was used by Niels Bohr to describe the hydrogen atom spectrum? Explain.

Solution.1 Niels Bohr used dual nature of particle to describe the hydrogen atom spectrum. He applied Planck's quantum hypothesis to Rutherford’s atomic mode.

Postulates

- Electrons can move only in those orbits for which angular momentum of an electron is an integral multiple of $h/2\pi$

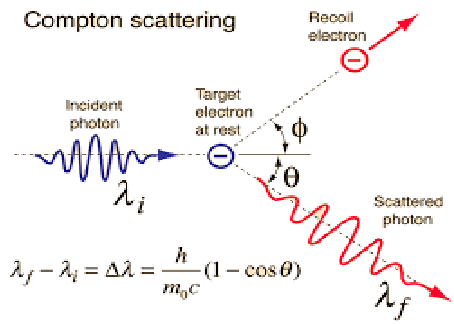
$$\boldsymbol{m_e v r = n \frac{h}{2\pi}}$$

- Emission or absorption of radiation by the atom take place when an electron jumps from one permitted orbit to another.

$$\boldsymbol{hv = E_i - E_f}$$

Q.2 Which experiment showed particle nature of X-rays? Draw clear schematic.

Solution:2 Compton Effect showed particle nature of X-rays



Q.3 What is the de Broglie wavelength of an electron at rest, moving with speed $c/10$, and $c/2$? c is speed of light in free space.

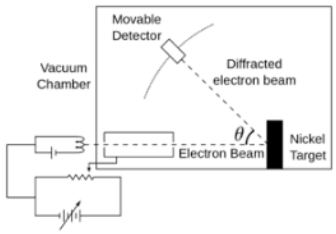
Solution:3 De Broglie wavelength $\lambda = \frac{h}{m_e v}$

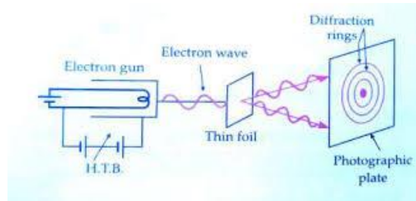
- For v = 0, wavelength associated only with moving particles. So, De Broglie wavelength does not exist
- For $v = \frac{c}{10}, \frac{c}{2}, c$ Calculated by $\lambda = \frac{h}{m_e v}$

Q.4 Which experiment showed wave nature of electrons? Draw clear schematic.

Solution:4 Experiments which show wave nature(diffraction) of electron

- ❖ Davission and Germer experiment
- ❖ G.P. Thomson's experiment





Q.5 In the pair production process, one γ-photon disappears. Explain.

Solution:5 In pair production process photon get disappeared as it is converted into Electron(e^-) and Positron(e^+)

Q.6 Like pair production process, can there exist pair annihilation process? What would be the conditions and the outcomes?

Solution:6 Yes, pair annihilation process exists. $e^- + e^+ = \gamma + \gamma$

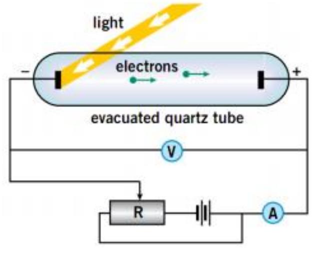
Conditions

- ❖ $\gamma = 0.51 MeV +$ half kinetic energy of particle relative to their center of mass.
- ❖ Momentum & Energy must be conserved.
- ❖ Can occur in vacuum.

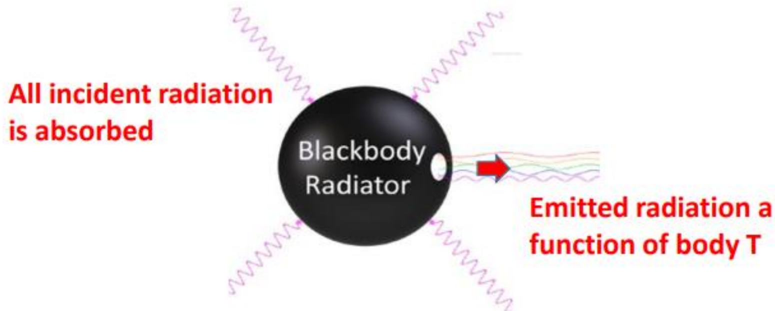
Q.7 Name any two experiments that can give an estimate of the Planck's constant. Explain.

Solution:7 Planck's constant can be calculated with experiments

- Photoelectric Experiment



- Blackbody Radiation



Q.8 Two black bodies of masses M_1 = 10 kg and M_2 = 1000 kg are maintained at constant temperature of 1000 K. Which one would give brighter (more intense) radiation out and why?

Solution:8 Blackbody radiation independent of mass, depends on temperature only. Both masses emit same intense radiation.

Q.9 How did Planck correct Rayleigh-Jean's model to give correct description of black body radiation?

Solution:9 Rayleigh-Jean assumed that standing wave can emit any amount of **energy (continuum)** with matter

$$u = \frac{8\pi v^2 kT}{c^3}$$

Planck's consider that energy exchanged b/w radiation and matter is **discrete**

$$u = \frac{8\pi v^2 kT}{c^3}\frac{1}{e^{\frac{hv}{kT}} - 1}$$

For $hv \ll kT$ (low frequency) reduce to Rayleigh-Jean

Zettili Pg:7

Q.10 Hertz prepared an experiment on two metal surfaces exposed to the same type of light. In one case the photocurrent in the external circuit was double than the other. What is the reason?

Solution:10 Photocurrent depends on number of photoelectron which is proportional to intensity of incident light i.e. **Photocurrent is proportional to intensity.** Thus, circuit having double photocurrent is irradiated by double light intensity.

Q.11 In the Compton's experiment with an electron and a proton both initially at rest, what would be the wavelength shift in the two cases found using a photodetector placed at an arbitrary angle θ from the initial direction of the photons?

Solution:11

$$\Delta\lambda_e = \frac{h}{m_e c}(1 - \cos\theta) = 4.857 \times 10^{-12} \sin^2(\theta/2)\ m$$

$$\Delta\lambda_p = \frac{h}{m_p c}(1 - \cos\theta) = 2.64 \times 10^{-15} \sin^2(\theta/2)\ m$$

Q.12 Compare Compton scattered wavelength shifts for X-rays (1 nm) and visible radiation (500 nm) from an electron (mass m_e) at rest and a nucleus ($M = 10^4 m_e$) at rest.

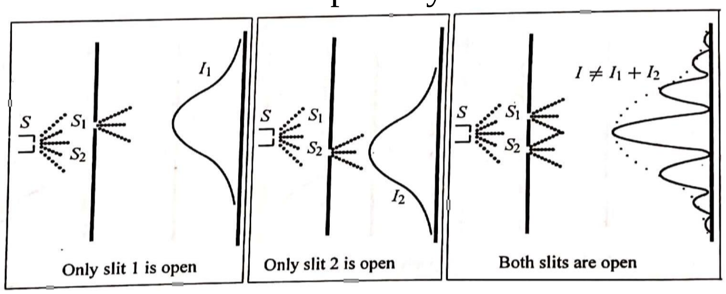
Solution:12

$$\Delta\lambda_e = \frac{h}{m_e c}(1 - \cos\theta) = 4.857 \times 10^{-3} \sin^2(\theta/2)\ nm$$

$$\Delta\lambda_{10^4} = \frac{h}{10^4 m_e c}(1 - \cos\theta) = 2.64 \times 10^{-7} \sin^2(\theta/2)\ nm$$

❖ $\Delta\lambda_e$ can be easily detected by X-rays of 1nm

Q.13 Double slit experiment with a single electron source, i.e., one electron at a time, is conducted. Draw the expected intensity pattern on the screen kept away from the slits at distance D. Where does the maximum lie?



Given in: Zettili

Q.14 Consider the formula for energy density from Planck's distribution law for black body as given below where all symbols have usual meaning,

$$u(v,T) = \frac{8\pi v^2 kT}{c^3}\frac{1}{e^{\frac{hv}{kT}} - 1}$$

Proceed to take the low frequency limit of this. How does it compare with the Rayleigh-Jean's law of black body radiation?

Solution:14 At low frequency limit $hv \ll kT$

Therefore $e^{\frac{hv}{kT}} = 1 + \frac{hv}{kT} + \cdots$

Substituting in above equation , which reduce to

$$u(v,T) = \frac{8\pi v^2 kT}{c^3}$$

Hence from Planck's distribution law for black body in the low frequency limit, we get Rayleigh-Jean's law

Q.15 In the above problem, proceed to take the high-frequency limit of this. How does it compare with the Wein's displacement law of black body radiation?

Solution:15

For high frequency limit

$$u(\lambda,T) = u_{max} \Rightarrow \frac{du(\lambda,T)}{d\lambda} = 0 \Rightarrow \frac{\lambda kT}{hc} = constent$$

$$\Rightarrow \lambda\,\alpha \frac{1}{T}$$

This is Wein's displacement law of black body radiation

Q.16 In Hertz's experiment, two metal surfaces were exposed to same type of light. The stopping potential for one case was found to be double of the other. At a certain voltage V on the anode, what would be the ratio between the photocurrents in the two cases?

Solution:16 Kinetic energy proportional to stopping potential

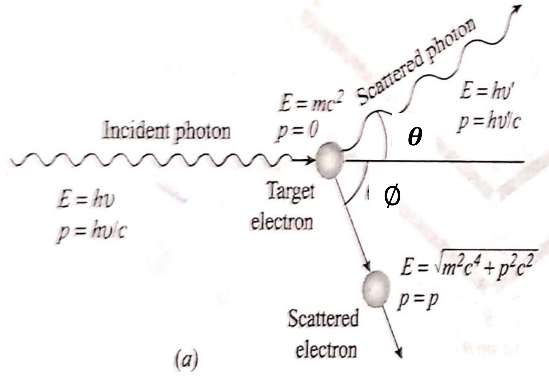
$$\frac{1}{2}mv^2 = eV \rightarrow \quad v = \sqrt{\frac{2eV}{m}} \rightarrow v\,\alpha\,\sqrt{V}$$

$$I = AJ = Anev \rightarrow I\,\alpha\,\sqrt{V}$$

$$\boldsymbol{\frac{I_1}{I_2} = \sqrt{2}}$$

Q.17 Consider the above Compton's experiment again and derive the famous equation given below,

$$\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_e c}(1-\cos\theta)$$



- **Moment conservation**

$\boxed{\frac{hv}{c} + 0 = \frac{hv'}{c}\cos\theta + p\cos\emptyset}$ ⇒ in photon direction

$\boxed{0 = \frac{hv'}{c}\sin\theta + p\sin\emptyset}$ ⇒ in photon perpendicular direction

- **Multiplying by c and rearranging**

$\boxed{pc\cos\emptyset = hv - hv'\cos\theta}$ $\boxed{pc\sin\emptyset = hv'\sin\theta}$

- **Squaring and adding**

$\boxed{p^2c^2 = (hv)^2 - 2(hv)(hv')cos\theta + (hv')^2}$

- **Total energy of particle**

$\boxed{E = KE + mc^2}$

$\boxed{E = \sqrt{m^2c^4 + p^2c^2}}$

$\boxed{p^2c^2 = (hv)^2 - 2(hv)(hv') + (hv')^2 + 2mc^2(hv - hv')}$

- **Kinetic energy**

$\boxed{KE = hv - hv'}$

$\boxed{2mc^2(hv - hv') = 2(hv)(hv')(1-\cos\theta)}$

$$\boxed{\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)}$$

Given in: Arthur Beiser

Q.18 In Electron diffraction from a nickel single crystal target, the angle corresponding to intensity maximum on a detector got reduced by 5 degrees when the electron source was changed to a new one. What is the ratio of energies of the electrons in the two cases?

Solution:18

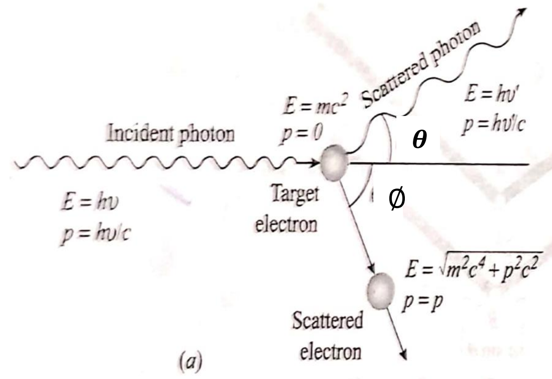
$$\lambda_e = 2d \sin\theta$$
$$\lambda_{new} = 2d \sin(\theta - 5)$$
$$E_{ratio} = \frac{\lambda_{new}}{\lambda_e} = \frac{\sin(\theta-5)}{\sin(\theta)}$$

Q.19 In Davisson Germer experiment on electron diffraction from a single crystal nickel target, maximum intensity was found on the detector at angle theta = 50 degrees from the incident direction. Given that the interplanar distance in nickel is 0.091 nm, what would be the energy of the electrons in the incident beam?

Solution:19

$$\mathrm{n}\lambda_e = 2d \sin\theta$$
$$E = \frac{hc}{\lambda_e} = \frac{hc}{2d \sin\theta} = 1.4267 \times 10^{-15}\mathrm{J}$$

Q.20 In Compton's experiment with X-rays and a free electron initially at rest, wavelength shift of 1 nm is detected on the detector placed at an angle θ. Calculate the direction in which the recoiling electron flies off.



Solution:20

- **Moment conservation**

$$\boxed{0 = \frac{h\nu'}{c}\sin\theta + p\,sin\,\emptyset} \qquad \boxed{\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\theta + p\cos\emptyset}$$

$$\boxed{\cos\theta + \sin\theta\cot\emptyset = \frac{\nu}{\nu'}}$$

$$\boxed{\cos\theta - 1 + \sin\theta\cot\emptyset = \left(\frac{\nu}{\nu'} - 1\right)\frac{\nu}{\nu} = \left(\frac{\nu - \nu'}{\nu\nu'}\right)\nu}$$

$$\boxed{2mc^2(h\nu - h\nu') = 2(h\nu)(h\nu')(1 - \cos\theta)}$$

$$\boxed{\cos\theta - 1 + \sin\theta\cot\emptyset = \nu\left[\frac{h}{mc^2}(1 - \cos\theta)\right]}$$

$$\boxed{\sin\theta\cot\emptyset = \left(1 + \frac{h\nu}{mc^2}\right)(1 - \cos\theta)} \Rightarrow \boxed{\cot\emptyset = \left(1 + \frac{h\nu}{mc^2}\right)\frac{(1 - \cos\theta)}{\sin\theta}} \Rightarrow \boxed{\cot\emptyset = \left(1 + \frac{h\nu}{mc^2}\right)\tan\theta/2}$$