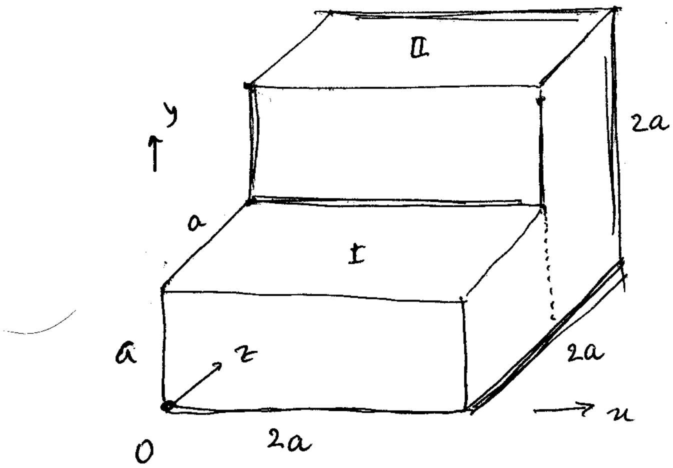
1a



Fix co ordinates to O.

Composition of two cuboids; as shown.

I : $\underline{r}_{c_I}$ at $\left(a, \frac{a}{2}, \frac{a}{2} \right)$

II : $\underline{r}_{c_{II}}$ at $\left(a, a, 3\frac{a}{2} \right)$

Masses: I : $m_I = \rho 2a^3$

II : $m_{II} = \rho \cdot 4a^3$

$$\underline{r}_c = \frac{\Sigma m_i \underline{r}_{ci}}{\Sigma m_i} \; ; \qquad x_c = \frac{\Sigma m_i x_{ci}}{\Sigma m_i} \quad \text{etc}$$

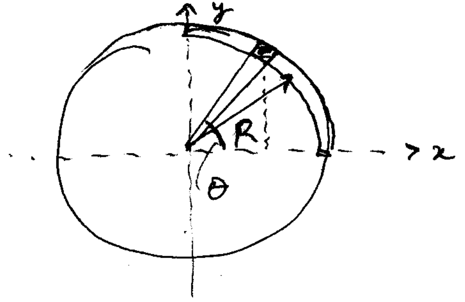
$$x_c = \frac{a \times 2 + a \times 4}{6} = a$$

$$y_c = \left(\frac{a}{2} \cdot 2 + a \times 4 \right) / 6 = \frac{5a}{6}$$

$$z_c = \left(\frac{a}{2} \cdot 2 + \frac{3a}{2} \times 4 \right) / 6 = \frac{7a}{6}$$

$\underline{r}_c$ at $\left(a, \frac{5a}{6}, \frac{7a}{6} \right)$

1c



Consider the element shown.

$x_c(\theta) = R\cos\theta$; $x_c = \int x_c(\theta)\,dm \Big/ \int dm$

$y_c(\theta) = R\sin\theta$ $\quad y_c = \int y_c(\theta)\,dm \,/\, \int dm$

$dm = \rho R\,d\theta$; $\int dm = \int_0^{\pi/2} dm = \rho \frac{\pi}{2} R$

$$x_c = \int_0^{\pi/2} R\cos\theta \;\rho R\,d\theta \Big/ \int dm = \frac{\rho R^2 \sin\theta \big|_0^{\pi/2}}{\rho\frac{\pi R}{2}}$$

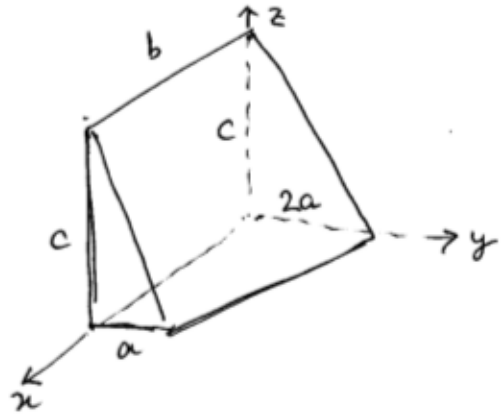
$$= \frac{2R}{\pi}$$

$$y_c = \int_0^{\pi/2} R\sin\theta \;\rho R\,d\theta \Big/ \left[\rho\frac{\pi R}{2}\right]$$

$$= \rho R^2(-\cos\theta)\big|_0^{\pi/2} \Big/ \rho\frac{\pi R}{2} = \frac{2R}{\pi}$$

ta

2b



consider triangular elements of thickness dx in yz plane.

At distance x; $z_c(x) = c/3$

$$y_c(x) = \frac{1}{3}(2a - x/b\, a) = (2 - \frac{x}{b})\frac{a}{3}$$

$$dm = \rho \frac{1}{2} c \times (2a - \frac{x}{b} a) dx = \frac{\rho}{2} ac (2 - \frac{x}{b}) dx$$

$$\int dm = \frac{\rho ac}{2} \int_0^b (2 - x/b) = \frac{\rho ac}{2} \left[2x - \frac{x^2}{2b}\right]_0^b = \frac{3}{4}\rho acb$$

$$\rightarrow \quad x_c = \frac{\int x_c(x)\, dm}{\int dm} = \frac{\int \frac{\rho ac}{2}(2x - \frac{x^2}{b}) dx}{\frac{3}{4}\rho acb}$$

$$= \frac{\cancel{\rho ac}}{2}\left[x^2 - x^3/3b\right]\Big|_0^b \Big/ 3\cancel{\rho acb}$$

$$= 4b/9$$

$$\rightarrow \quad y_c = \int y_c(x)\, dm \Big/ \int dm = \int (2 - \frac{x}{b})\frac{a}{3}\, \frac{\rho ac}{2}(2 - \frac{x}{b}) dx$$

$$= \frac{\rho a^2 c}{6} \int_0^b (4 - \frac{4x}{b} + \frac{x^2}{b^2}) dx \Big/ \int dm = \frac{\rho a^2 c}{6}\left[4x - \frac{4x^2}{2b} + \frac{x^3}{3b^2}\right]\Big|_0^b \Big/ \int dm$$

$$= \frac{\rho a^2 c}{6} \cdot (4b - 2b + \frac{b}{3}) \Big/ \tfrac{3}{4}\rho acb = \frac{7}{6}\rho a^2 cb \times \frac{4}{9} \cdot \frac{1}{\rho acb} = \frac{14}{27}a$$

$$\rightarrow z_c = \int z_c(x)\, dm \Big/ \int dm = \int \frac{c}{3} \cdot dm \Big/ \int dm = \frac{c}{3}.$$

centroid is at $\left(\frac{4b}{9}, \frac{14a}{27}, \frac{c}{3}\right)$