Harshit Manandhar 2020CS10348

1) And can be written as :

$$ite \stackrel{df}{=} \lambda x\, y\, z\, [(x\, y\, z)]$$

$$And \stackrel{df}{=} (ite\ x\ L\ False)$$

$$L := (ite\ y\ True\ False)$$

$$or \stackrel{df}{=} (ite\ x\ True\ L)$$

$$L := (ite\ y\ True\ False)$$

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2) Suppose we take x = True, y = False

↳ $\lambda x[\lambda y[x]]$ ↳ $\lambda x[\lambda y[y]]$

And = (ite $\lambda x[\lambda y[x]]$ (ite ($\lambda x[\lambda y[y]]$ $\lambda x[\lambda y[x]]$) $\lambda x[\lambda y[y]]$) $\lambda x[\lambda y[x]]$)

→ 1 (ite $\lambda x[\lambda y[x]]$ ($\lambda x[\lambda y[y]]$) $\lambda x[\lambda y[y]]$)

→ 1.1 $\lambda x[\lambda y[y]]$ = False

But in Or → 1.1 ite ($\lambda x[\lambda y[x]]$ True L)

= $\lambda x[\lambda y[x]]$ = True

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Similarly for x = True y = True

And gate : $\lambda_x[\lambda_y[x]]$ = True

Or gate : $\lambda_x[\lambda_y[x]]$ = True

for x = False, y = False

And gate : $\lambda_x[\lambda_y[y]]$ = False

Or gate = $\lambda_x[\lambda_y[y]]$ = False

3) $a \wedge (a \vee b)$

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3) $a \wedge (a \vee b)$

= first evaluating $a \vee b$

$\rightarrow_{1/B}$ d_{ab} [Ite a (Ite (Ite a True (Ite b True False)) True False) False

$\rightarrow_{1/B}$ d_{ab} [Ite a = True

$\rightarrow_{1/B}$ d_{ab} [Ite True (Ite (True True False) False)

$\rightarrow_{1/B}$ d_{ab} [Ite True True False]

$\rightarrow_{1/n}$ True = a

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If a = False

$\rightarrow_{\eta/\beta}$ $\lambda a\, b$ [If a False (...) False]

$\rightarrow_{\eta/\beta}$ False → a

∴ Both cases we get a ⟹

∴ answer: a

Hence Proved

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