ELL101 Tutorial 1 Questions (Magnetic Circuits)

Question: 1

A mild-steel ring having a cross-sectional area of 800 mm^2 and a mean circumference of 200 mm has a coil of 100 turns wound uniformly around it. Calculate:
(a) the reluctance of the ring;
(b) the current required to produce a flux of 1 mWb in the ring.
Use the following information : For mild-steel

Flux density (T)	Relative permeability (approx)
0.5	630
0.75	690
1	720
1.25	450
1.5	360

Solution

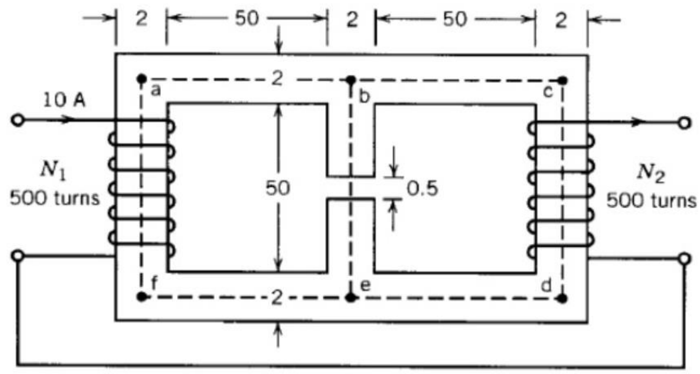
(a)
Flux density in the ring = $(1 \ast 10^{-3}\ Wb)/\ (800 \ast\ 10^{-6}\ mm^{-2})$ = 1.25T
From the table, the relative permeability of mild steel for a flux density of 1.25T is about 450.
Therefore, reluctance of ring = $(0.2)\ /\ (450 \times 4\pi \times 10^{-7}\ \times 8 \times 10^{-4})$ = **0.4433 * 10^6A/Wb**

(b)
$H\ =\ B/(\mu_r\mu_0)\ =\ 1.25/(450\ \ast\ 4\pi\ \ast\ 10^{-7})$ = 2210 A/m
And so, mmf = 2210 * 0.2 = 442A
Magnetising current = 442/100 = **4.42A**

Another Approach for (b):-
(Flux) x (Reluctance) = (Number of Turns) x (Current)
$(1\ \ast 10^{-3}\ Wb)\ \ast\ (\mathbf{0.4433}\ \ast\ 10^{6}\ \boldsymbol{A/Wb})\ =\ (100)\ \ast\ (i)$
Current (i) = **4.433 A**

Question: 2

In the magnetic circuit shown below, the relative permeability of the ferromagnetic material is 1200. Neglect magnetic leakage and fringing. All dimensions are in centimeters, and the magnetic material has a square cross sectional area. Determine the air gap flux, air gap flux density and magnetic field intensity in the air gap.



Solution

The mean magnetic paths are shown by dashed lines.

$$F_1 = N_1 I_1 = 500 \times 100 = 5000\ At$$
$$F_2 = N_2 I_2 = 500 \times 100 = 5000\ At$$
$$\mu_c = 1200\mu_0 = 1200 \times 4\pi \times 10^{-7}$$
$$\Re_{bafe} = \frac{l_{bafe}}{\mu_c A_c} = \frac{3 \times 52 \times 10^{-2}}{1200 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 2.58 \times 10^6\ At/Wb$$

From symmetry,

$$\Re_{bcde} = \Re_{bafe}$$
$$\Re_g = \frac{l_g}{\mu_0 A_g} = \frac{5 \times 10^{-3}}{4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 9.94 \times 10^6\ At/Wb$$
$$\Re_{be(core)} = \frac{l_{be(core)}}{\mu_c A_c} = \frac{51.5 \times 10^{-2}}{1200 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 0.82 \times 10^6\ At/Wb$$

The loop equations are

$$\phi_1(\Re_{bafe} + \Re_{be} + \Re_g) + \phi_2(\Re_{be} + \Re_g) = F_1$$
$$\phi_1(\Re_{be} + \Re_g) + \phi_2(\Re_{bcde} + \Re_{be} + \Re_g) = F_2$$

$$\phi_1(13.34 \times 10^6) + \phi_2(10.76 \times 10^6) = 5000$$
$$\phi_1(10.76 \times 10^6) + \phi_2(13.34 \times 10^6) = 5000$$

Solving the above equation and calculation air gap flux as

$$\phi_g = \phi_1 + \phi_2 = 4.134 \times 10^{-4}\, Wb$$

The air gap flux density is

$$B_g = \frac{\phi_g}{A_g} = \frac{4.134 \times 10^{-4}}{4 \times 10^{-4}} = 1.034\, T$$

The magnetic intensity in the air gap is

$$H_g = \frac{B_g}{\mu_0} = \frac{1.034}{4\pi \times 10^{-7}} = 0.822 \times 10^{6}\ At/m$$

Question: 3

A magnetic circuit comprises three parts in series, each of uniform cross-sectional area (c.s.a.) as mentioned below:

(a) a length of 80 mm and c.s.a. 50 mm^2 ,

(b) a length of 60 mm and c.s.a. 90 mm^2 ,

(c) an airgap of length 0.5 mm and c.s.a. 150 mm^2 .

A coil of 4000 turns is wound on part (b), and the flux density in the airgap is 0.30 T. Assuming that all the flux passes through the given circuit, and that the relative permeability μr is 1300, estimate the coil current to produce such a flux density.

Solution :

Flux in the air gap is $\phi = B_c A_c = 0.3 \times (1.5 \times 10^{-4}) = 0.45 \times 10^{-4} Wb$

MMF in part a

$$F_a = \phi S_a = \phi \frac{l_a}{\mu_o \mu_r A_a} = 0.45 \times 10^{-4} \frac{80 \times 10^{-3}}{(4\pi \times 10^{-7}) \times 1300 \times (50 \times 10^{-6})} = 44.1 AT$$

MMF in part b

$$F_b = \phi S_b = \phi \frac{l_b}{\mu_o \mu_r A_b} = 0.45 \times 10^{-4} \frac{60 \times 10^{-3}}{(4\pi \times 10^{-7}) \times 1300 \times (90 \times 10^{-6})} = 18.4 AT$$

MMF in part c

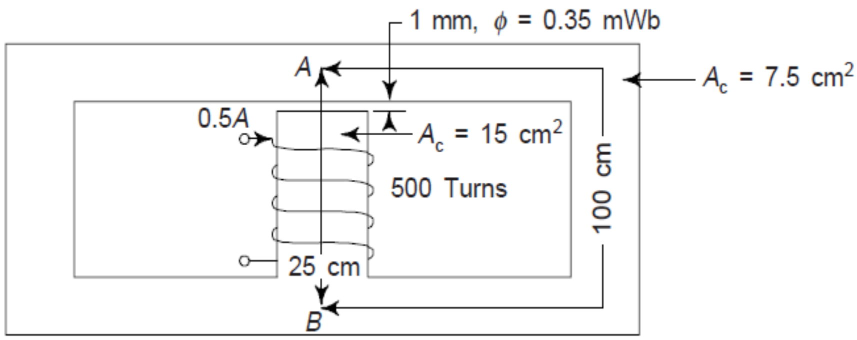
$$F_c = \phi S_c = \phi \frac{l_c}{\mu_o \mu_r A_c} = 0.45 \times 10^{-4} \frac{0.5 \times 10^{-3}}{(4\pi \times 10^{-7}) \times 1 \times (150 \times 10^{-6})} = 119.3 AT$$

Total MMF required $F = F_a + F_b + F_c = 181.8 AT$

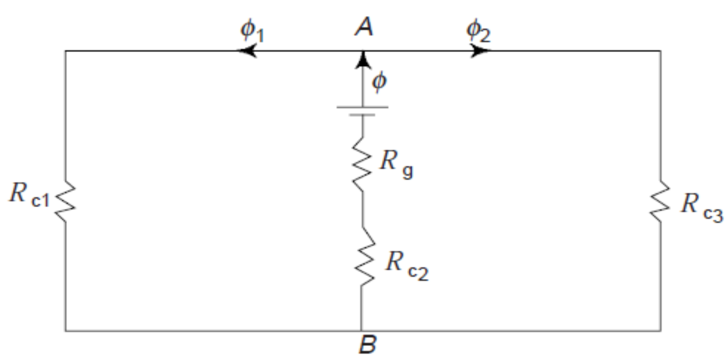
Required coil current $I = \frac{F}{N} = \frac{181.8}{4000} = 45.4 \times 10^{-3} A = 45.4 mA$

Question: 4

The magnetic circuit shown in Fig. has a coil of 500 turns wound on the central limb which has an air-gap of 1 mm. The magnetic path from A to B via each outer limb is 100 cm and via the central limb 25 cm (air-gap length excluded). The cross-sectional area of the central limb is 5 cm ¥ 3 cm and each outer limb is 2.5 cm 3 cm. A current of 0.5 A in the coil produces an air-gap flux of 0.35 mWb. Find the relative permeability of the medium.



Solution: Below Fig. gives the electric equivalent of the magnetic circuit.



$R_{c1} = R_{c3} = (100\times10^{-2})/\ (4\pi \times10^{-7}\times \mu_r \times7.5 \times10^{-4}) = (1061\times10^{9})/\ \mu_r$ AT/Wb

$$R_{c2} = (25\times10^{-2})/\ (4\pi\times10^{-7} \times\mu_r \times15\times10^{-4}) = (0.133\times10^{9})/\ \mu_r \text{ AT/Wb}$$

$$R_g = (1\times10^{-3})/\ (4\pi\times10^{-7} \times 15\times10^{-4}) = 0.5305\times10^{6} \text{ AT/Wb}$$

$$Ni=\Phi\ (R_{c1} \text{ II } R_{c3} + R_g + R_{c2})$$

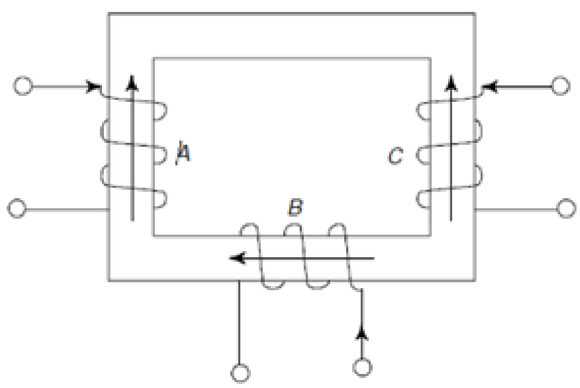
$$500\times 0.5 = 0.35\times10^{-3}\ \{(0.531\times10^{9})/\mu_r + (0.133\times10^{9})/\mu_r + 0.5305\times10^{6}\}$$

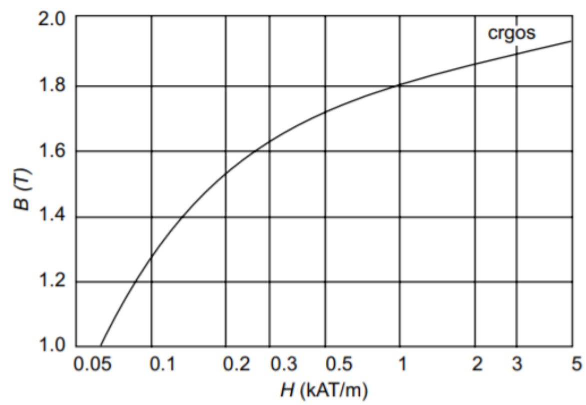
$$\mu_r = 3612$$

Question: 5

The core, made of cold-rolled silicon steel (B–H curve of Fig. (b)) is shown in Fig. (a). It has an uniform cross-section (not iron) of 5.9 cm2 and a mean length of 30 cm. Coils A, B and C carry 0.4, 0.8 and 1 A respectively in the directions shown. Coils A and B have 250 and 500 turns respectively.How many turns must coil C have to establish a flux of 1 mWb in the core?

(a)





(b)

Solution: $N_A i_A$=250×0.4 = 100 AT

$N_B i_B$=500× 0.8 = 400 AT

B= (1× 10^{-3})/ (5.9 ×10^{-4}) = 1.695 T

Corresponding *H* from *B-H* curve in is,

H=0.5 kAT/m = 500 AT/m

$(AT)_{net}$ = 500×30×10^{-2} = 150

= $(AT)_A + (AT)_B - (AT)_C$

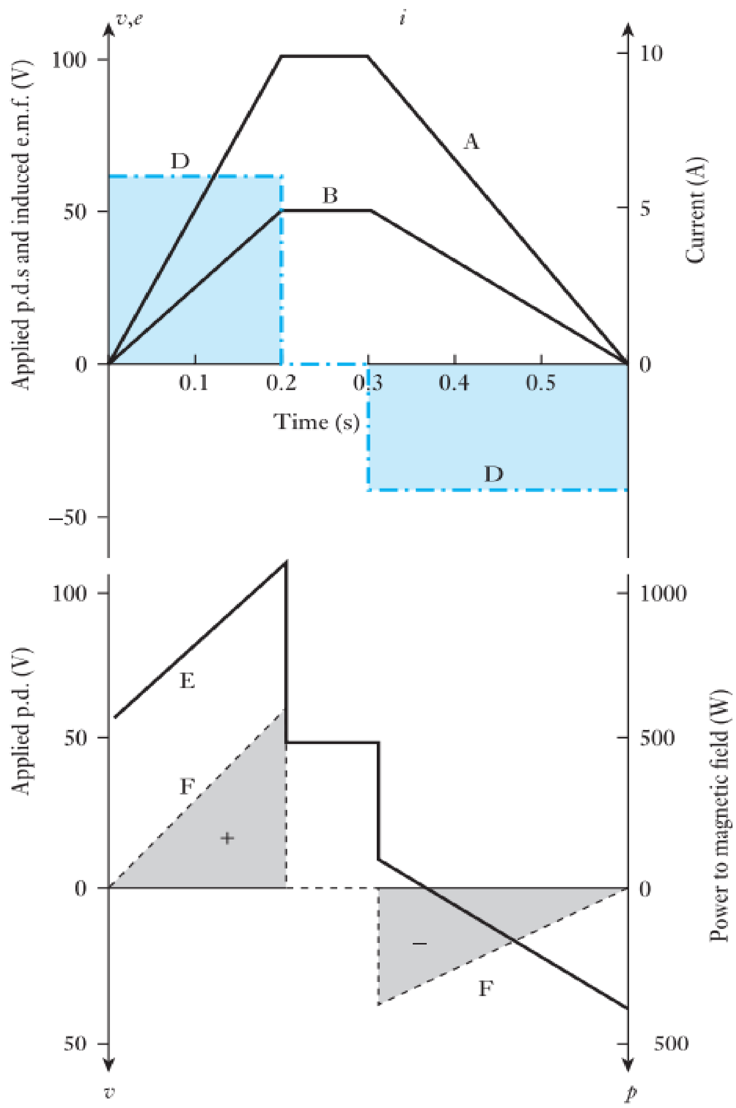
So, $(AT)_C$ = 100 + 400 -150 = 350

$N_c i_c$ = 350, therefore, N_c = 350

<u>Question: 6</u>

A coil has a resistance of 5 Ω and an inductance of 1.2 H. The current through the coil is increased uniformly from zero to 10 A in 0.2 s, maintained constant for 0.1 s and then reduced uniformly to zero in 0.3 s. Plot graphs representing the variation with time of: (a) the current; (b) the induced e.m.f.; (c) the potential differences across the resistance and the inductance; (d) the resultant applied voltage; (e) the power to and from the magnetic field. Assume the coil to be wound on a non-metallic core and calculate the energy absorbed and returned by the magnetic circuit.

Solution:



The variation of current is represented by graph A and since the p.d. across the resistance is proportional to the current, this p.d. increases from zero to (10 A × 5 Ω), namely 50 V, in 0.2 s, remains constant at 50 V for 0.1 s and then decreases to zero in 0.3 s, as represented by graph B.

During the first 0.2 s, the current is increasing at the rate of 10/0.2, namely 50 A/s, ∴ Corresponding induced e.m.f. = 50 × 1.2 = 60 V

During the following 0.1 s, the induced e.m.f. is zero, and during the last 0.3s, the current is decreasing at the rate of −10/0.3, namely −33.3 A/s, ∴ Corresponding induced e.m.f. = (−33.3 × 1.2) = −40 V The variation of the induced e.m.f. is represented by the uniformly dotted graph D.

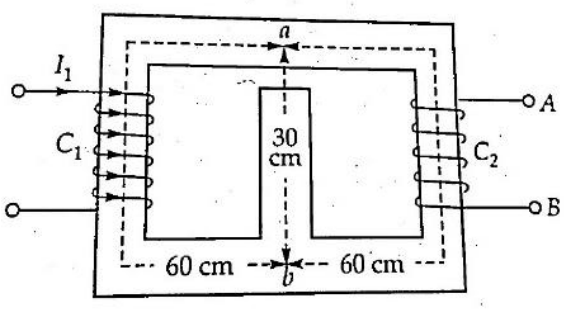
The resultant voltage applied to the coil is obtained by adding graphs B and D; thus the resultant voltage increases uniformly from 60 to 110 V during the first 0.2 s, remains constant at 50 V for the next 0.1 s and then changes uniformly from 10 to −40 V during the last 0.3 s, as shown by graph E.

The power supplied to the magnetic field increases uniformly from zero to (10 A × 60 V), namely 600 W, during the first 0.2 s. It is zero during the next 0.1 s. Immediately the current begins to decrease, energy is being returned from the magnetic field to the electric circuit, and the power decreases uniformly from (−40 V × 10 A), namely −400 W, to zero as represented by graph F.

The positive shaded area enclosed by graph F represents the energy (= 1/2× 600 × 0.2 = 60 J) absorbed by the magnetic field during the first 0.2 s; and the negative shaded area represents the energy (= 1/2× 400 × 0.3 = 60 J) returned from the magnetic field to the electric circuit during the last 0.3 s. The two areas are obviously equal in magnitude, i.e. all the energy supplied to the magnetic field is returned to the electric circuit.

Question 7:

For the magnetic circuit shown below, cross-sectional area of the core is 30cm2. $\mathcal{M}$ for the core is 4000. Coil C_1, having 200 turns, produces 500 ATs. Flux in the central limb is 10mWb from a to b. Coil C_2 has 100 turns. Calculate the current in the coil C_2 and its direction, also indicate polarities of the terminals A and B.



Solution:

Reluctance of each limb

$$Rl_1 = \frac{60 \times 10^{-2}}{4\pi \times 10^{-7} \times 4000 \times 30 \times 10^{-4}} = 3.98 \times 10^4 \text{ AT/Wb}$$

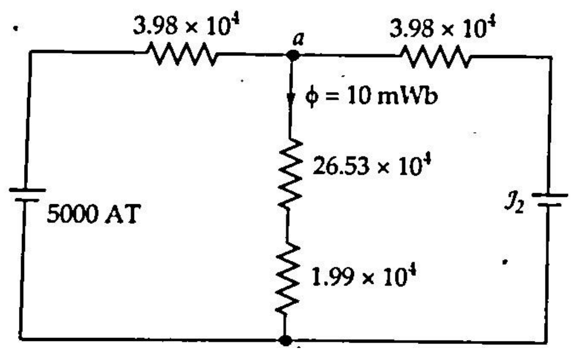
$$Rl_2 = \frac{30 \times 10^{-2}}{4\pi \times 10^{-7} \times 4000 \times 30 \times 10^{-4}} = 1.99 \times 10^4 \text{ AT/Wb}$$

$$Rl_g = \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 30 \times 10^{-4}} = 26.53 \times 10^4 \text{ AT/Wb}$$

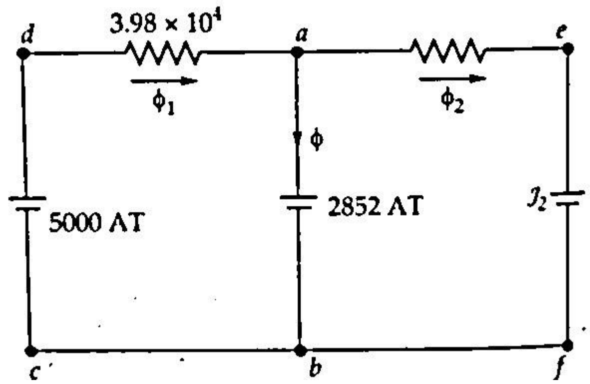
Electrical analog for magnetic circuit is shown below, where mmf produced by coil 2 is indicted by J_2 with upper polarity assumed positive

mmf across central limb=flux $\times$ reluctance of [air gap +central limb]

= 10 $\times$0.001[26.53+1.99]$\times$10000=2852 ATs



The circuit can now redrawn as



Now from this equivalent circuit,

$$-2852+5000-\phi_1 \times 3.98 \times 10000=0$$
$$\phi_1=53.97\text{mWb}$$

At node a gives flux ϕ_2 in branch ae as,

$$\phi+\phi_2=\phi_1$$
$$\phi_2=[53.97-10]\times 0.001=43.97\text{mWb}$$

For circuit efbae(upper polarity of J_2 is assumed positive) gives

$$-J_2+2852-3.98\times 10000\times 43.97\times 0.001=0$$
$$J_2=2852-1750=1102 \text{ ATs}$$

Current in coil C_2=1102/N_2

=1102/100 =11.02 A

As mmf J_2 has turned out to be positive ,assumed polarity of coil C_2 is correct, So current enters terminal A, passes through coil C_2 and leaves terminal B. Therefore, terminal A is positive and terminal B is negative.

Question: 8

(1). Determine the inductance and the magnetic stored energy for the structure of Fig. 5.0 The structure is identical to that of except for the air gap. (Assume the reluctance of the magnetic structure is negligible).

(2). Assume that the flux density in the air gap varies sinusoidally as B(t) = B0 sin(ωt) with frequency =60 Hz. Determine the induced voltage across the coil, e

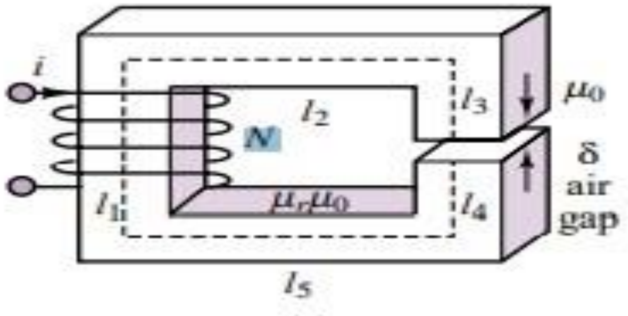


Fig 5.0

Solution :

Known Quantities: Relative permeability; number of coil turns; coil current; structure geometry; flux density in air gap.

Find: L; Wm; e.

Schematics, Diagrams, Circuits, and Given Data: $\mu r \rightarrow \infty$; N = 500 turns; i = 0.1 A. The air gap has lg = 0.002 m. B0 = 0.6 Wb/m2, Ag= 0.0001 m^2.

Assumptions: All magnetic flux is linked by the coil; the flux is confined to the magnetic core; the flux density is uniform. The reluctance of the magnetic structure is negligible.

Analysis: Part 1. To calculate the inductance of this magnetic structure, we use equation.

Part 1. To calculate the inductance of this magnetic structure, we use equation

$$L = \frac{N^2}{\mathcal{R}}$$

Thus, we need to first calculate the reluctance. Assuming that the reluctance of the structure is negligible, we have:

$$\mathcal{R}_{gap} = \frac{l_{gap}}{\mu_{gap} A_{gap}} = \frac{l_{gap}}{\mu_0 A_{gap}} = \frac{0.002}{4\pi \times 10^{-7} \times 0.0001} = 1.59 \times 10^7 \text{ A} \cdot \text{t/Wb}$$

and

$$L = \frac{N^2}{\mathcal{R}} = \frac{500^2}{1.59 \times 10^7} = 0.157 \text{ H}$$

Finally, we can calculate the stored magnetic energy as follows:

$$W_m = \frac{1}{2} L i^2 = \frac{1}{2} \times (0.157 \text{ H}) \times (0.1 \text{ A})^2 = 0.785 \times 10^{-3} \text{ J}$$

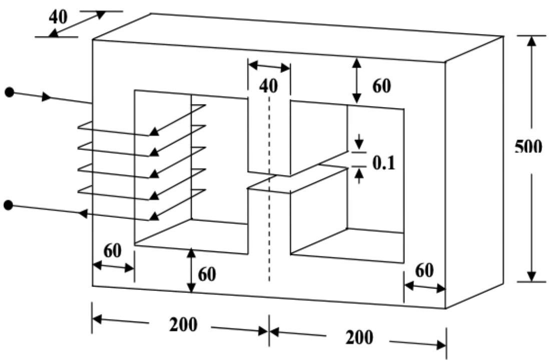
Part 2. To calculate the induced voltage due to a time-varying magnetic flux, we use equation

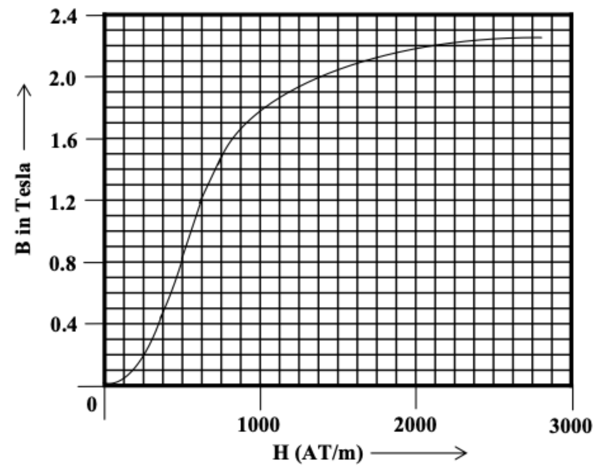
$$e = \frac{d\lambda}{dt} = N\frac{d\phi}{dt} = NA\frac{dB}{dt} = NAB_0\omega \cos(\omega t)$$
$$= 500 \times 0.0001 \times 0.6 \times 377 \cos(377t) = 11.31 \cos(377t) \text{ V}$$

Comments: The voltage induced across a coil in an electromagnetic transducer is a very important quantity called back electromotive force, or back emf.

Question: 9

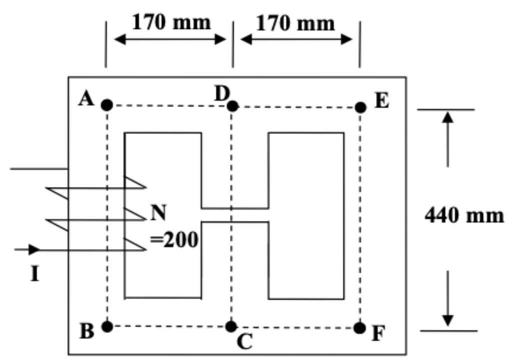
In the magnetic circuit detailed in the below figure with all dimensions in mm, calculate the required current to be passed in the coil having 200 turns in order to establish a flux of 1.28 mWb itn the air gap. Neglect the fringing effect and leakage flux. The B-H curve of the material is also given below. Permeability of air may be taken as $\mu 0 = 4\pi \times 10^{-7}$ H/m.



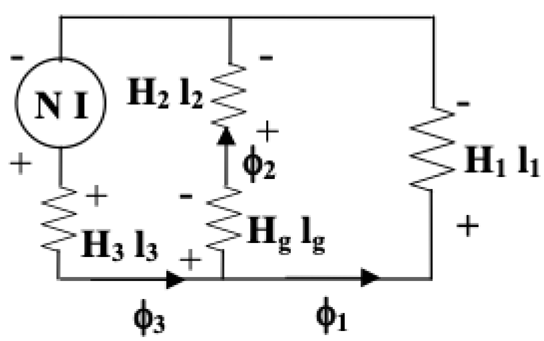


Solution

Circuit Showing mean lengths:



The equivalent electrical model with all the currents:



$\Phi g = \Phi 2 = 1.28\times10^{-3}$

Cross sectional area $A_2 = 16 * 10^{-4}\ m^2$

Flux density $B_g = B_2 = (1.28 \times 10^{-3})/(16 \times 10^{-4})$ T $= 0.8$ T

Finally, $H_g = \frac{B_g}{\mu_0} = \frac{0.8}{4\pi * 10^{-7}} AT/m$ = $63.66 \times 10^4 \; AT/m$

Mmf required for gap $H_g l_g = 63.66 \times 10^4 \times 1 \times 10^{-4} \; AT = 63.66 \; AT$

Now we must calculate the mmf required in the iron portion of the central limb as follows:

Flux density, B_2 = 0.8 T ∵ fringing & leakage neglected

Corresponding H from B-H curve, $H_2 \approx 500 \; AT/m$

Mean iron length, $l_2 = (440 - 0.1) \; mm \approx 0.44m$

mmf required for iron portion, $H_2 l_2 = 220 \; AT$

Total mmf required for iron & air gap = $(220 + 63.66) \; AT$

$$mmf_{CD} = 283.66 \; AT$$

We can see from the equivalent electrical circuit that mmf acting in path 1 is in parallel with mmf acting in path 2 and hence they are the same.

Now, we will calculate flux in path 1.

Mean length of path $l_1 = l_{DE} + l_{EF} + l_{FC}$ = 0.78m

$$H_1 = 283.66/0.78 = 363.67 \; A\,T\,/m$$

Corresponding B value from the B-H curve, $B_1 = 0.39T$

Flux, $\phi_1 = B_1 A_1 = 0.39 \times 24 \times 10^{-4} Wb = 0.94 \times 10^{-3} Wb$

Therefore, flux in path 3 $\phi_3 = \phi_1 + \phi_2 = 2.22 \times 10^{-3} \; Wb$

$$B_3 = \phi_3 / A_3 = 0.925T$$

Corresponding H from B-H curve ≈562.5AT/m

Also, $l_3 = 440 + 2 * 170 \; mm = 0.78m$

Total mmf for path 3 = $H_3 l_3 = 562.5 * 0.78 = 438.7AT$

For the coil, the mm supplied, $NI = 283.66 + 438.7\,A\,T = 722.36AT$

$$\Rightarrow I = 722.36/N = 722.36/200 = \mathbf{3.61A}$$