$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial \Psi(x,t)}{\partial x^2} + V(x,t)\,\Psi(x,t) = i\hbar\frac{\partial \Psi(x,t)}{\partial t}.}$$

$$\Psi(x,t) = \psi(x)\, e^{-iEt/\hbar}.$$

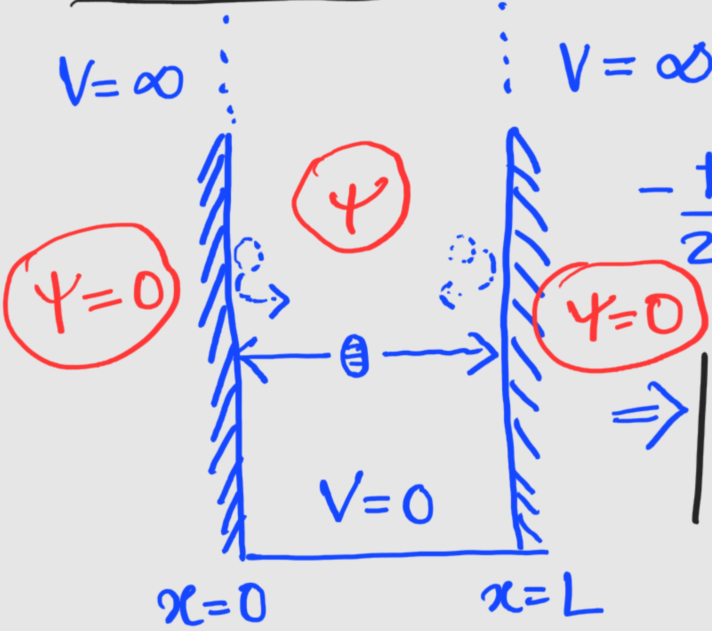
$$\Downarrow$$

$$\boxed{-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\,\psi(x).}$$

└> function of x only.

$$\boxed{\Psi(x,t)}$$

:)=(> Particle in infinite square rectangular Potential well:—



$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + \underbrace{V(x)}_{=\,0}\psi(x) = E\psi(x)$$

$$\Rightarrow \frac{d^2\psi(x)}{dx^2} + \frac{2mE}{\hbar^2}\psi(x) = 0$$

$$K = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\Rightarrow \frac{d^2\psi(x)}{dx^2} + K^2\psi(x) = 0 \quad \ldots\ldots (1)$$

$\psi(x) = A \sin kx + B \cos kx$ $A, B = \text{Const.}$

Boundary Conditions:

1) $\psi = 0$ at $x = 0$

$$0 = A \underbrace{\sin k(0)}_{=0} + \underbrace{B \cos k(0)}_{=B}$$

$$\Rightarrow \boxed{B = 0}$$

$$\boxed{\psi(x) = A \sin kx} \Rightarrow A = ?$$

2) $\psi = 0$ at $x = L$

$$0 = A \sin k(L)$$

$KL = n\pi$ $n = 1, 2, 3, 4, \ldots$

$$\sin(-x) = - \sin x.$$

$$k_n = \frac{n\pi}{L}$$

$$\frac{2Em}{\hbar^2} = \frac{n^2\pi^2}{L^2} \Rightarrow \boxed{E_n = \frac{\hbar^2 n^2 \pi^2}{2mL^2}}$$

$A = ?$

$$\Psi(x,t) = A \sin \frac{n\pi}{L} x \cdot e^{-iEt/\hbar}$$

$$\int_{x=-\infty}^{x=\infty} \Psi^*(x,t)\,\Psi(x,t)\,dx = 1$$

$$\int_0^L \underbrace{A \sin\frac{n\pi}{L}x\, \cancel{e^{iEt/\hbar}}}_{\Psi^*(x,t)} \cdot \underbrace{A \sin\frac{n\pi}{L}x\, \cancel{e^{-iEt/\hbar}}}_{\Psi(x,t)} \cdot dx = 1$$

$$\Rightarrow A^2 \int_0^L \sin^2\frac{n\pi}{L}x\,dx = 1.$$

$\sin^2\theta$

$\cos 2\theta = 2\sin^2\theta - 1$

$\Rightarrow \sin^2\theta = \frac{1}{2}(1 + \cancel{\cos 2\theta})$

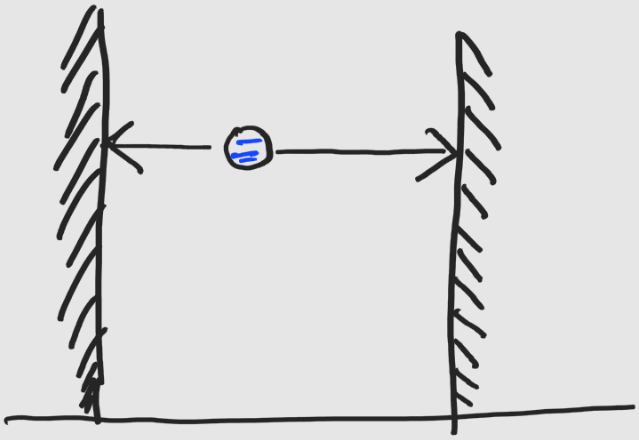
$$\Rightarrow A^2 \frac{1}{2} L = 1.$$

$$\Rightarrow \boxed{A = \sqrt{\frac{2}{L}}}$$

$$\Psi(x) = \sqrt{\frac{2}{L}} \cdot \sin\frac{n\pi}{L}x.$$

$$\Psi(x,t) = \sqrt{\frac{2}{L}} \sin\frac{n\pi}{L}x \cdot e^{-iEt/\hbar}.$$

$n = 1, 2, 3, 4, \ldots\ldots$



$n=1$

$$\psi(x)=\sqrt{\frac{2}{L}}\cdot\sin\frac{\pi}{L}x.$$

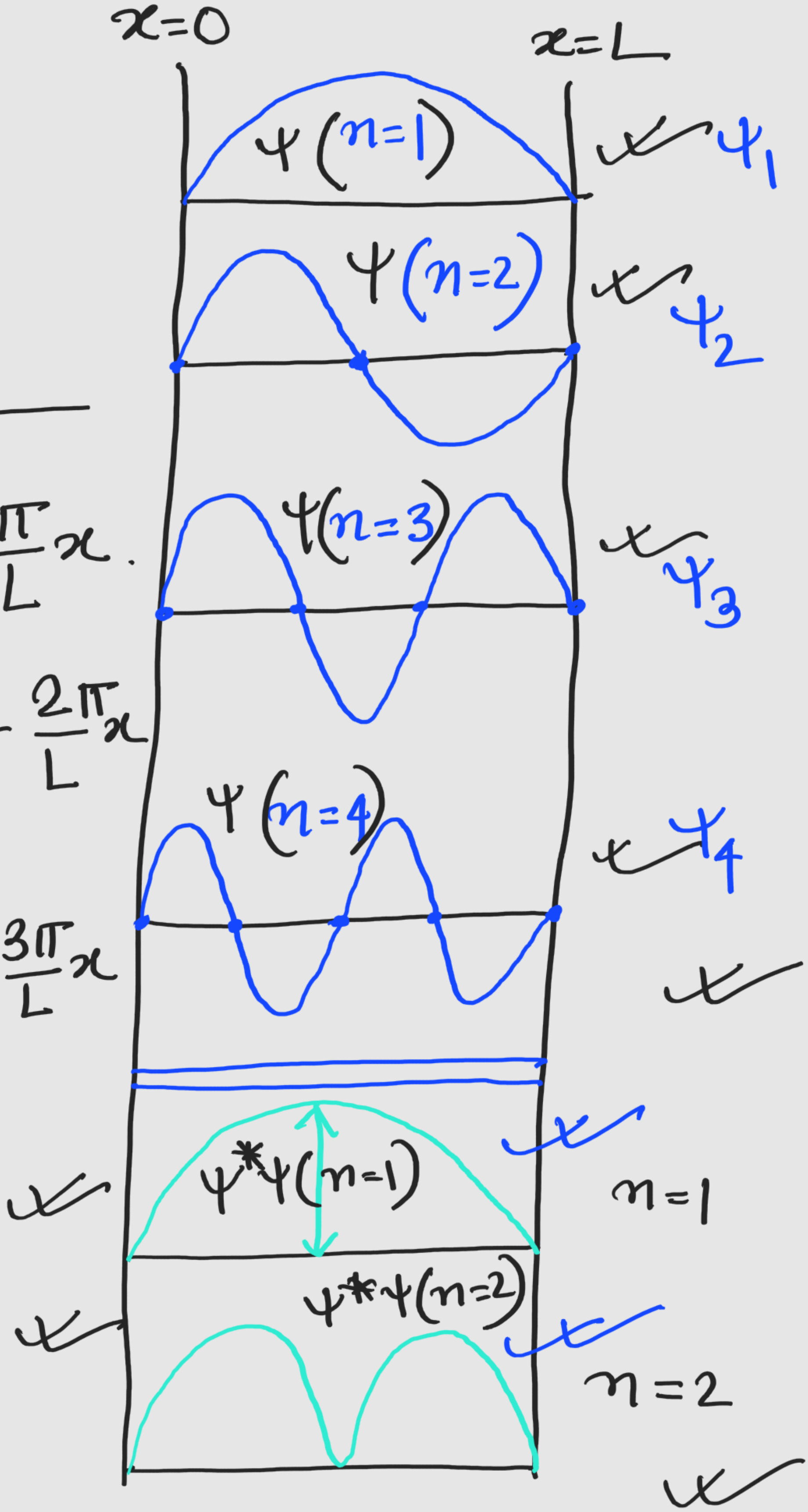
$n=2$

$$\psi(x)=\sqrt{\frac{2}{L}}\cdot\sin\frac{2\pi}{L}x$$

$n=3$

$$\psi(x)=\sqrt{\frac{2}{L}}\cdot\sin\frac{3\pi}{L}x$$

$\psi^*\psi$



Energy Levels :

$$K^2 = \frac{2mE}{\hbar^2} \; ; \; K = \frac{n\pi}{L}$$

$$\Rightarrow \boxed{E_n = \frac{\hbar^2}{2m} \cdot \frac{n^2\pi^2}{L^2}}$$

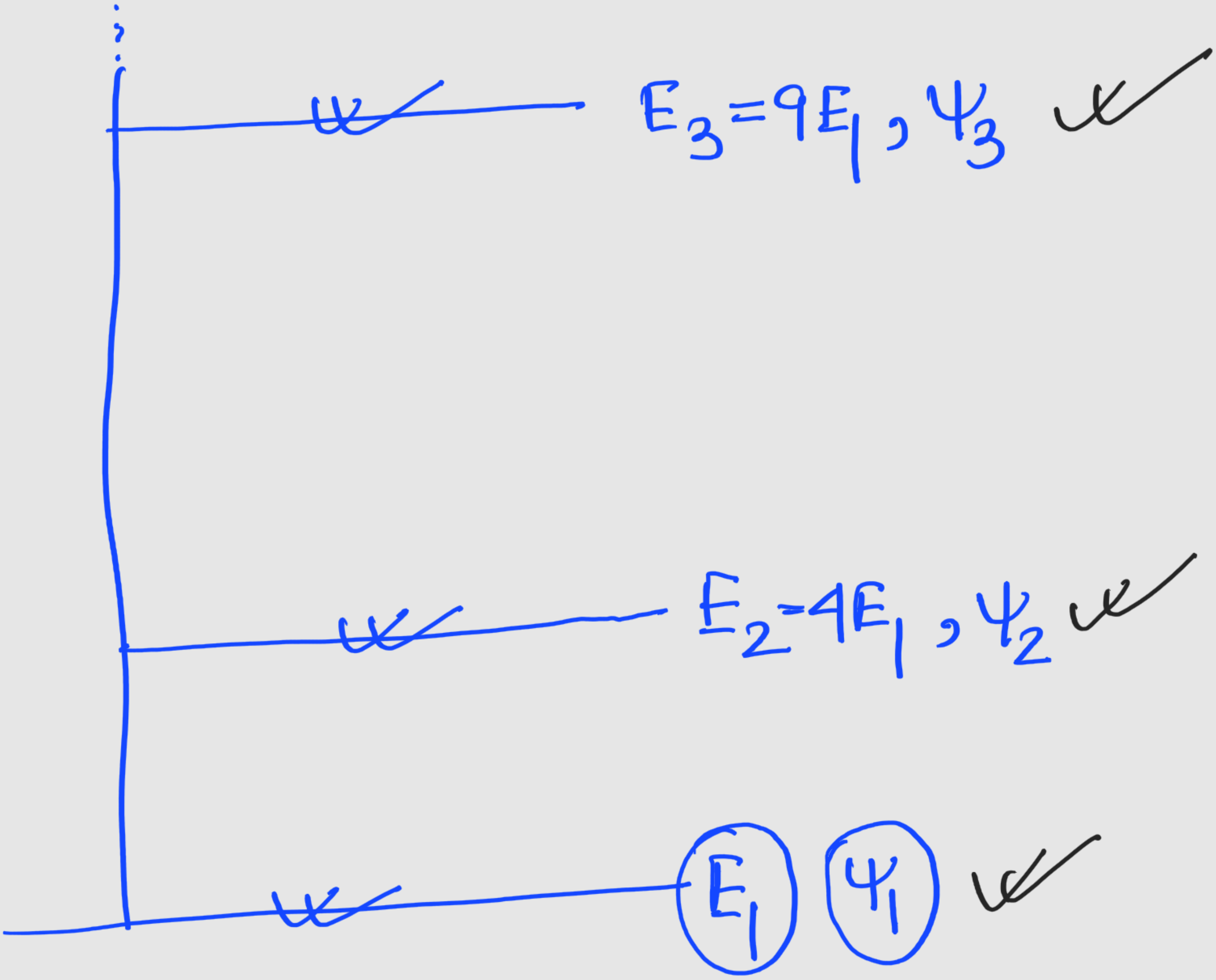
$\Rightarrow$ discrete Energy levels.

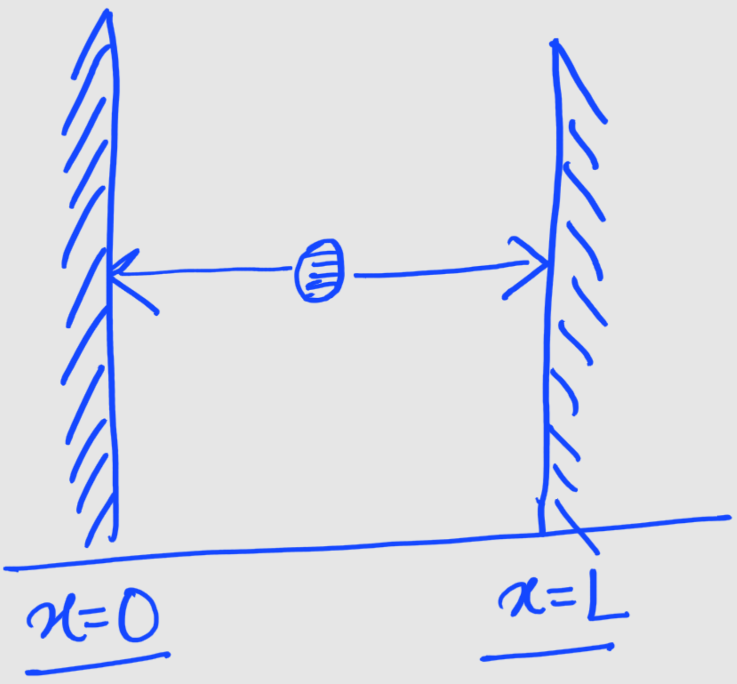
$n = \underline{1}$ $\Rightarrow$ ground state energy.

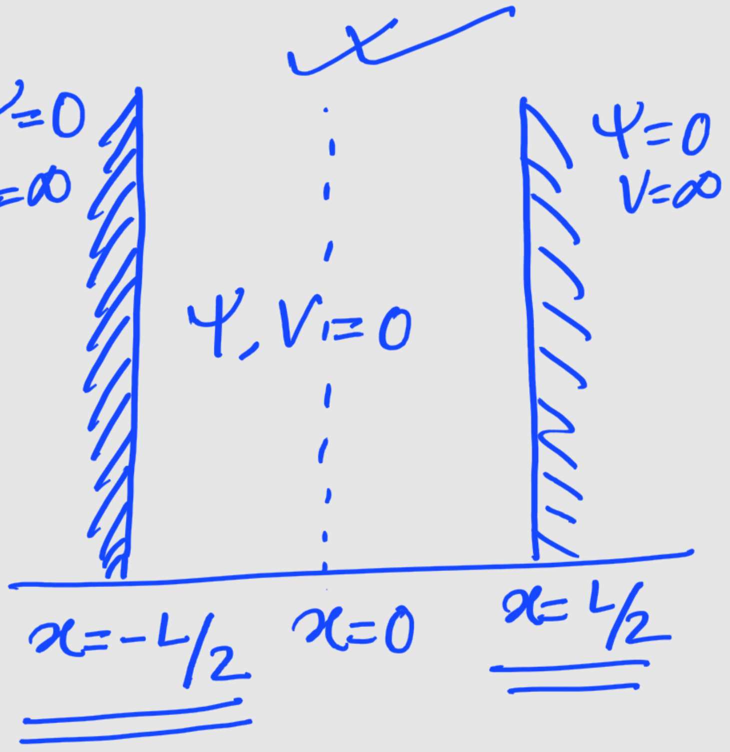
$$E_1 = \frac{\pi^2\hbar^2}{2mL^2}.$$

$$n=2 \Rightarrow E_2 = 4 \cdot \frac{\pi^2\hbar^2}{2mL^2}.$$

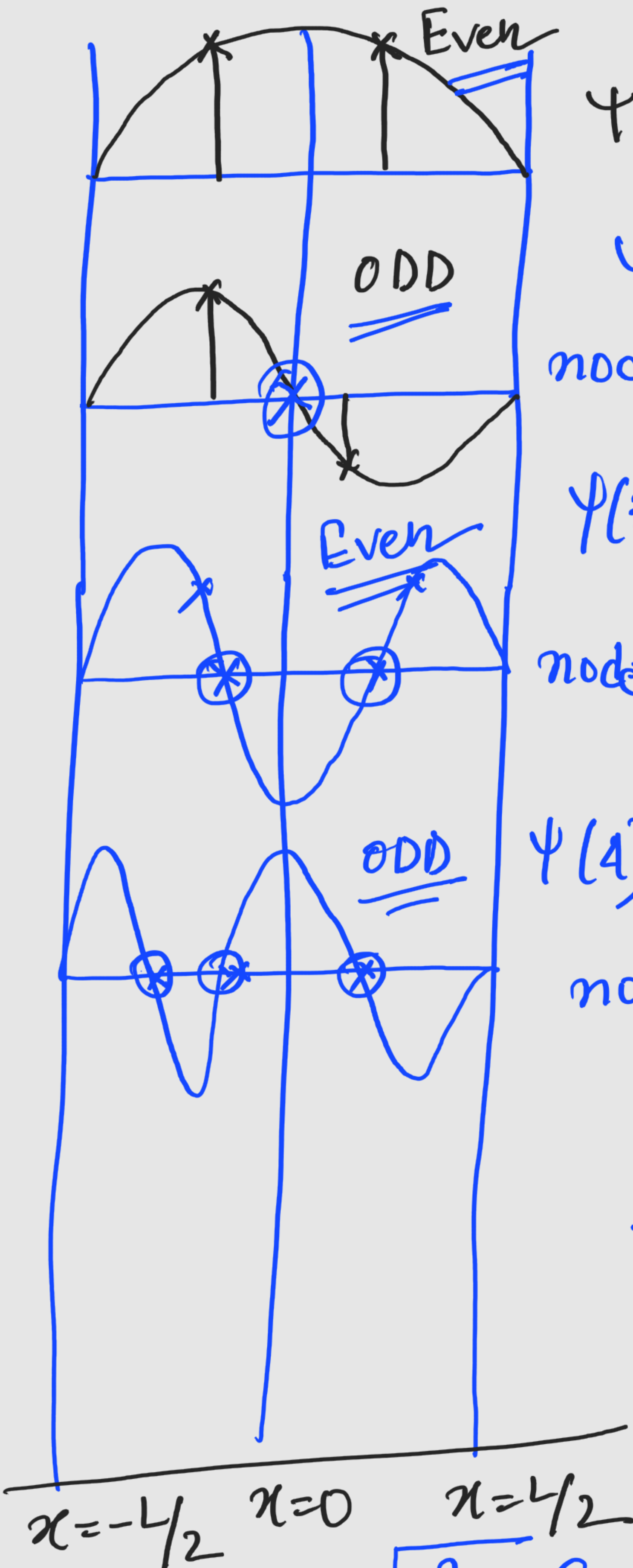
$$n=3 \Rightarrow E_3 = 9 \cdot \frac{\pi^2\hbar^2}{2mL^2}.$$


$E_3 = 9E_1, \psi_3$
$E_2 = 4E_1, \psi_2$
E_1 ψ_1


$x=0$
$x=L$


$\psi=0$
$V=\infty$
$\psi, V=0$
$\psi=0$
$V=\infty$
$x=-L/2$
$x=0$
$x=L/2$

Hometask



$\psi(1) \leftarrow$ node $=0$

$\psi(2)$ $\quad \psi(-x) = -\psi(x)$

node $=1$. $\Rightarrow$ ODD

$\psi(3)$ $\quad \psi(-x) = \psi(x)$

node $=2$ $\quad \Rightarrow$ EVEN

$\psi(4)$

node $=3$

$$\int \psi_j \psi_i \, dx = 0 \;\; \forall i \neq j$$

$\Rightarrow$ orthogonal.

$$\Psi(x,t) = \sqrt{\frac{2}{L}} \, \sin \frac{n\pi}{L} x \, . \, e^{\frac{i\pi^2 n^2 \hbar}{2mL^2} t}$$

$$\Psi(x,t) = \sum_{n=0}^{1} C_n \sqrt{\frac{2}{L}} \, \sin \frac{n\pi}{L} x \, . \, e^{\frac{i\pi^2 n^2 \hbar}{2mL^2} t}$$