Lecture 6 Fri 4 Aug 2023

$X \rightarrow$ polyhedron in $\mathbb{R}^n$

X is bounded $\iff D_X = \phi$

$$\bar{x} = \sum_{i=1}^{p} \lambda_i x^i + \sum_{j=1}^{s} \mu_j d^j$$

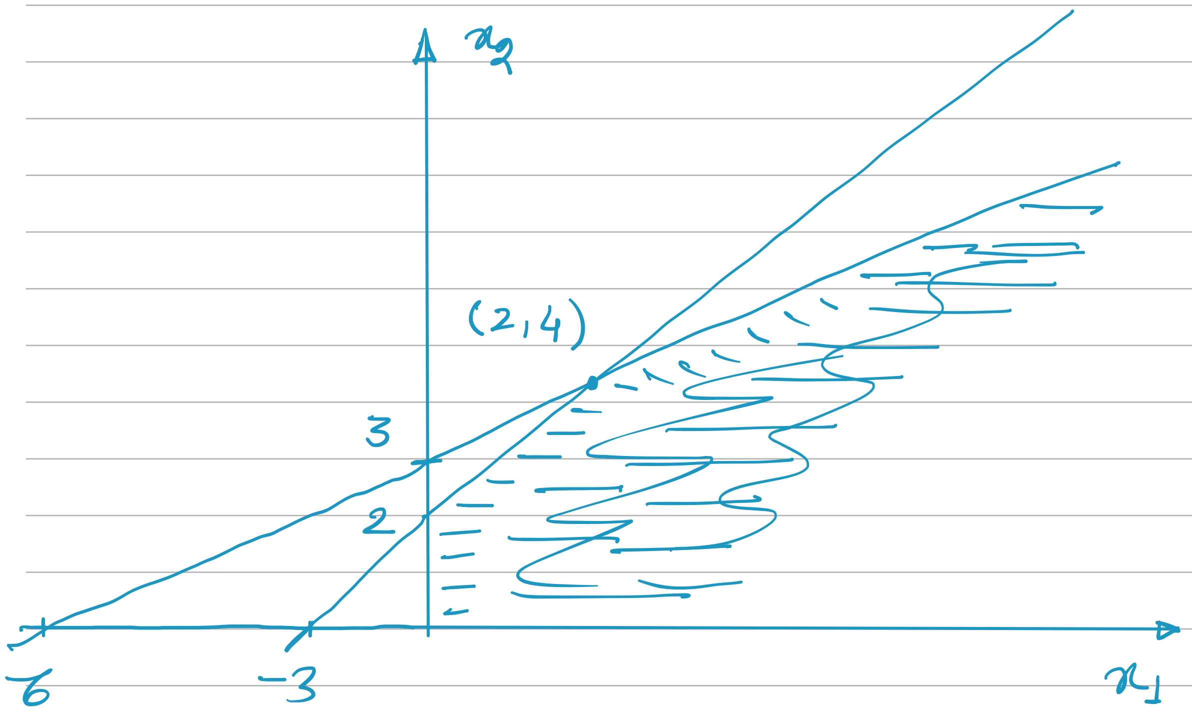
$\sum \lambda_i x^i \rightarrow$ EP of X.

$\sum \mu_j d^j \rightarrow$ ED of X

$\lambda_i, \mu_j \geq 0 \qquad \sum_{i=1}^{p} \lambda_i = 1$

$$\max\ c^T x$$
$$\text{s. to } \left.\begin{array}{l} Ax \leq b \\ x \geq 0 \end{array}\right\}$$

$$\max\ -x_1 + 3x_2$$
$$\text{s.t. } -x_1 + x_2 \leq 2$$
$$-x_1 + 2x_2 \leq 6$$
$$x_1, x_2 \geq 0$$



$D_S = \{ d \in \mathbb{R}^2 : Ad \leq 0 \ , \ d \geq 0 \ , \ d \neq 0 \}$

$$A = \begin{bmatrix} -1 & 1 \\ -1 & 2 \end{bmatrix}$$

$$-d_1 + d_2 \leq 0$$

$$d_2 \leq d_1$$

$$-d_1 + 2d_2 \leq 0$$

$$d_2 \leq \left(\frac{d_1}{2} \right)$$

$\underline{d_1, d_2 \geq 0}$

equivalent LP

$$\max \quad 0\lambda_1 + 6\lambda_2 + 10\lambda_3 + (-1)\mu_1 + (1)\mu_2$$

$$\lambda_1, \lambda_2, \lambda_3 \geq 0 \ ; \ \mu_1, \mu_2 \geq 0.$$

$$\boxed{\lambda_1 + \lambda_2 + \lambda_3 = 1}$$

unbounded by taking $\mu_2 \to \infty$.

$$\max \ -4x_1 + x_2$$

s.to. $\quad -x_1 + x_2 \leq 2$

$$-x_1 + 2x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

equivalent (LP)

$$\max \quad 0\lambda_1 + 2\lambda_2 - 4\lambda_3 - 4\mu_1 - 7\mu_2$$

$Ax \le b$ $\qquad \begin{bmatrix} A \\ -I \end{bmatrix} x \le \begin{bmatrix} b \\ 0 \end{bmatrix}$

$-x \le 0$

$n \times 1$ $\qquad (m+n) \times 1$

G

submatrix of $r \times n$. $\qquad \begin{bmatrix} A_{m\times n} \\ I_{n\times n} \end{bmatrix}$

$(m+n) \times n$

$Gx = g$

$r \times n \quad n \times 1 \quad = \quad r \times 1$

(1) If, full rank $= n$
then, G^{-1} exists & then,
x (unique sol^n) is an EP of S.

(2) Let rank $G < n$

$\Rightarrow Gy = 0$

has infinitely many sol^{n}s.

$\Rightarrow \exists\, d \neq 0 \quad Gd = 0$

then given any $\bar{x}$ of S, we can find $\varepsilon > 0$ small enough s.t.

$$x_1 = \bar{x} - \varepsilon d \in S$$

$$x_2 = \bar{x} + \varepsilon d \in S$$

$$\& \quad \bar{x} = \frac{x_1 + x_2}{2}$$

$$Gx_1 = G\bar{x} + \varepsilon Gd = G\bar{x} = g$$

$$Gx_2 = G\bar{x} + \varepsilon Gd = G\bar{x} = g$$

$$\begin{bmatrix} A \\ -I \end{bmatrix} \xrightarrow[\text{inequality}]{\text{remaining}} H\bar{x} \le rhs$$

G

$$Hx_1 = H\bar{x} - \varepsilon Hd < rhs$$

$$Hx_2 = H\bar{x} - \varepsilon Hd < rhs.$$