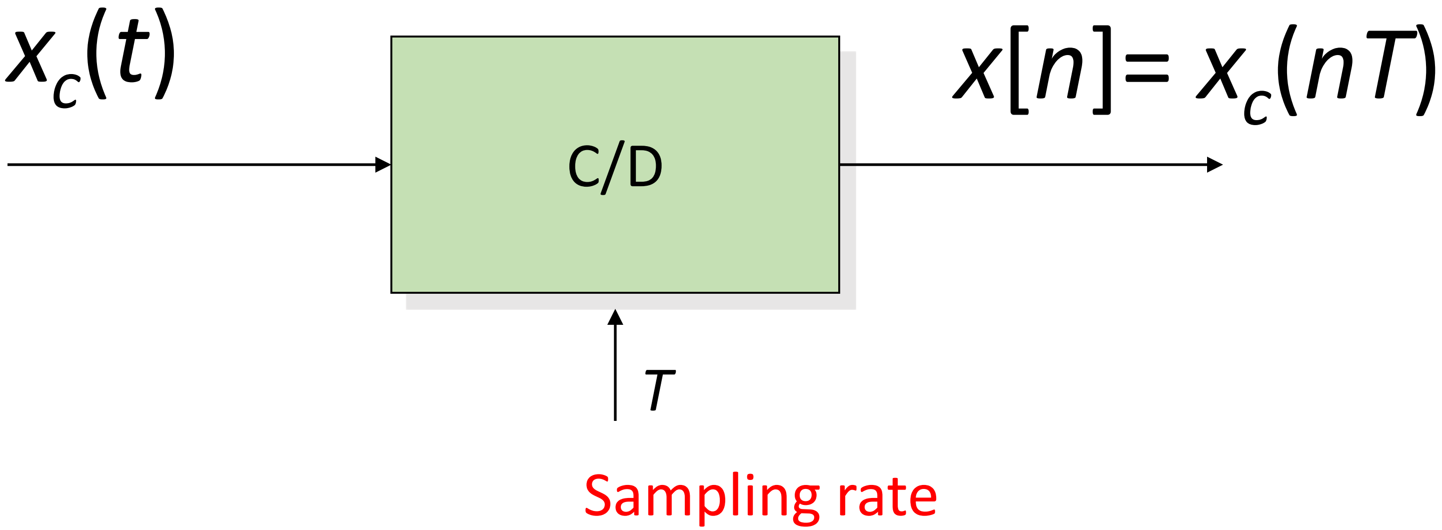
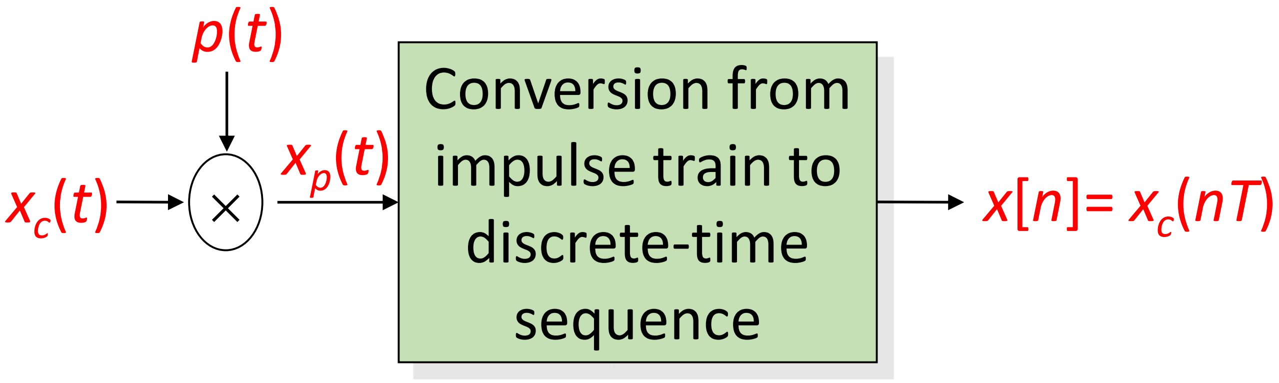
Sampling

Lecture 30

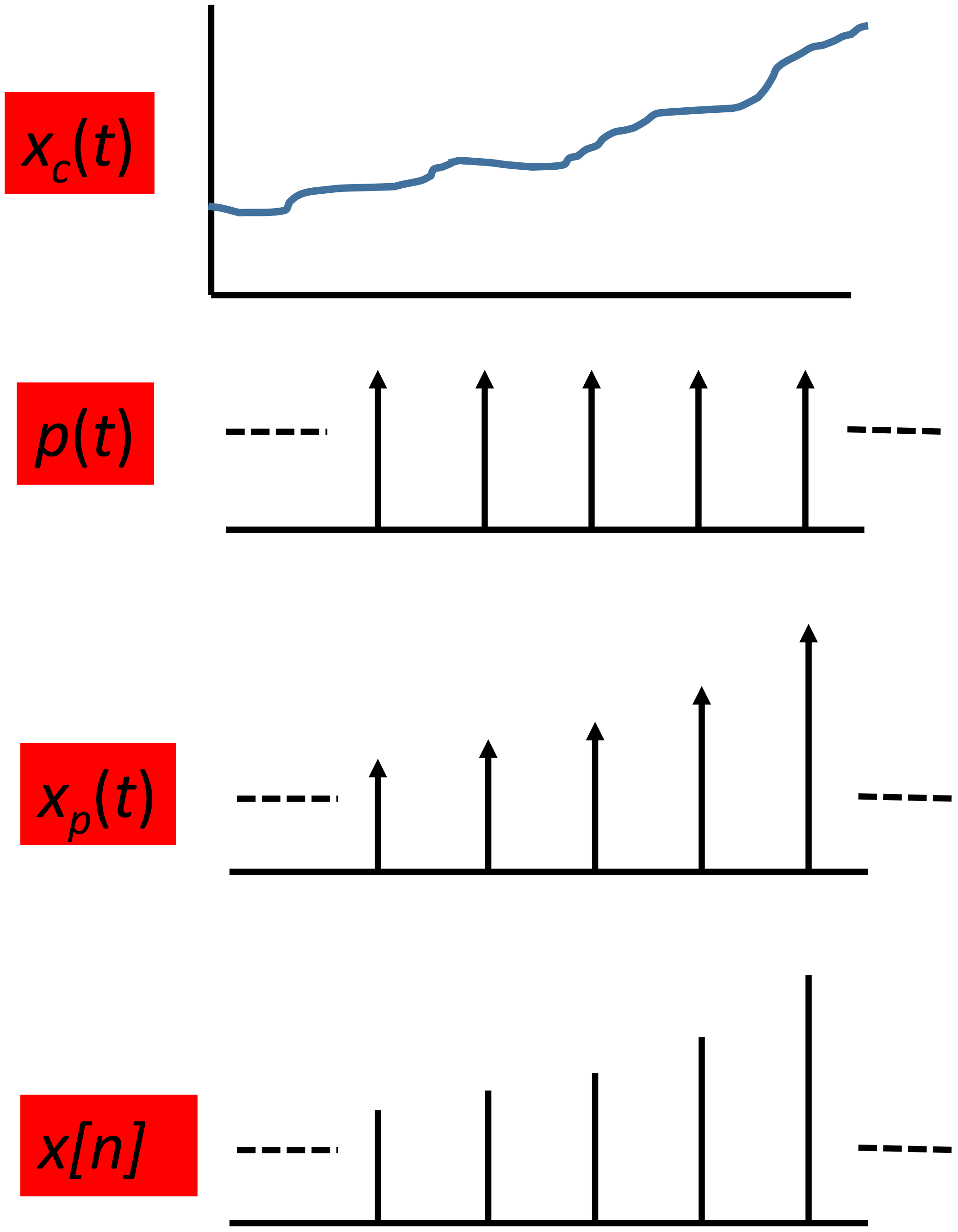
Continuous to Discrete-Time Signal Converter



C/D System



$p(t)$
$x_c(t)$
$x_p(t)$
Conversion from impulse train to discrete-time sequence
$x[n]= x_c(nT)$

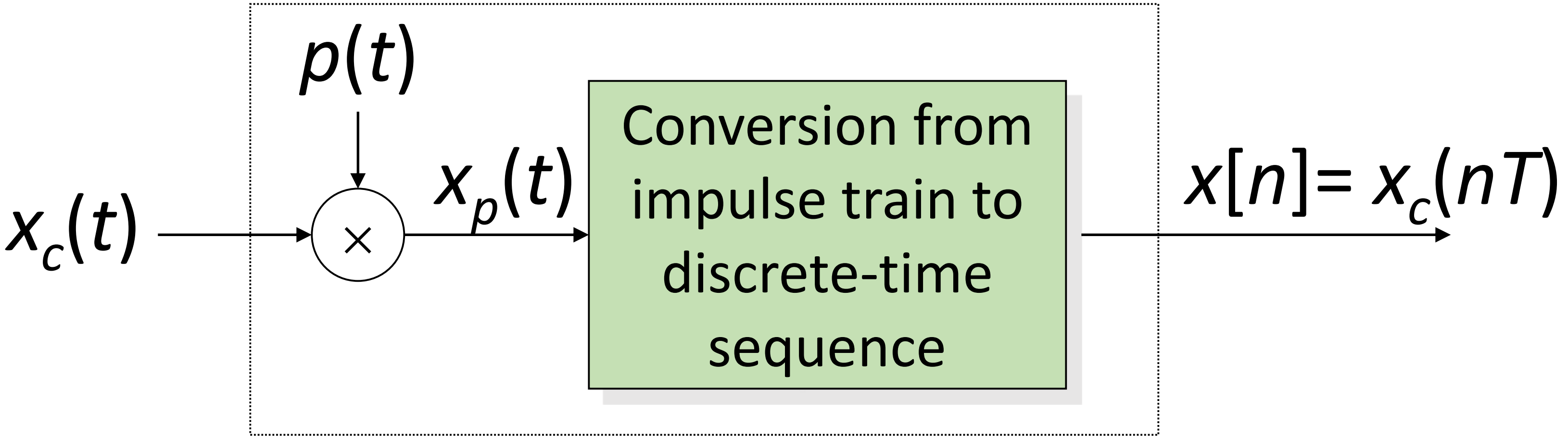


$x_c(t)$
$p(t)$
$x_p(t)$
$x[n]$

First focus

$$x_c(t) \longrightarrow \qquad x_p(t) \longrightarrow x_c(t)$$

C/D System



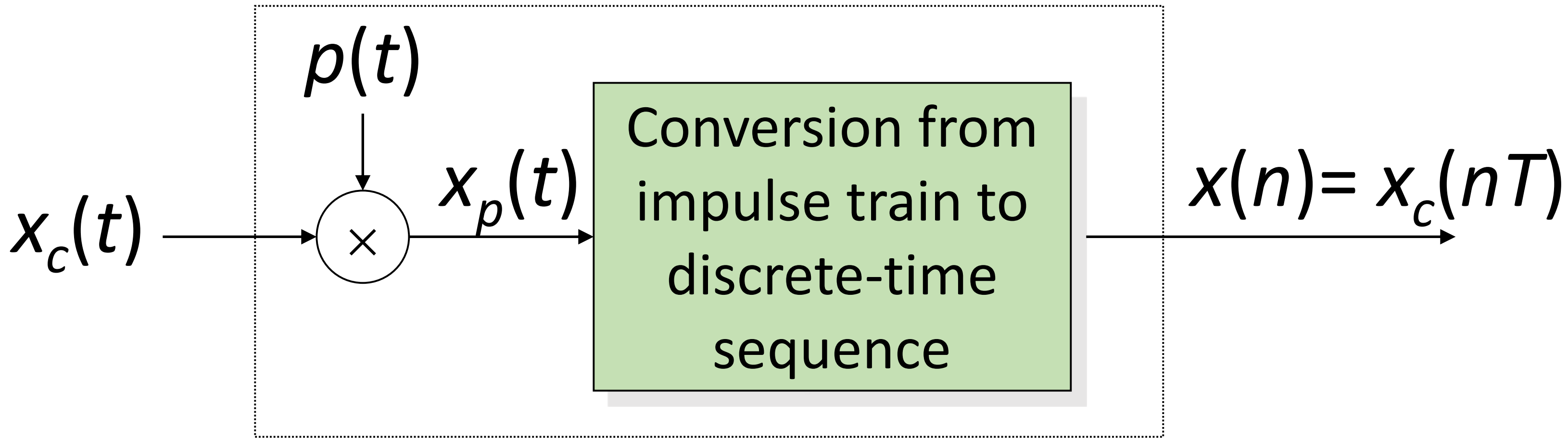
$$X_p(\omega) = \frac{1}{2\pi} X_c(\omega) * P(\omega)$$

$$P(\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s),\ \omega_s = \frac{2\pi}{T}$$

$$X_p(\omega) = \frac{1}{2\pi} X_c(\omega) * \left(\frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s) \right)$$

$$\begin{aligned} x_p(t) &= x_c(t)\, p(t) \\ &= x_c(t) \sum_{n=-\infty}^{n=\infty} \delta(t - nT) \\ &= \sum_{n=-\infty}^{n=\infty} x_c(nT)\, \delta(t - nT) \end{aligned}$$

C/D System



$$x_p(t) = x_c(t)\,p(t)$$

$$= x_c(t)\sum_{n=-\infty}^{n=\infty}\delta(t-nT)$$

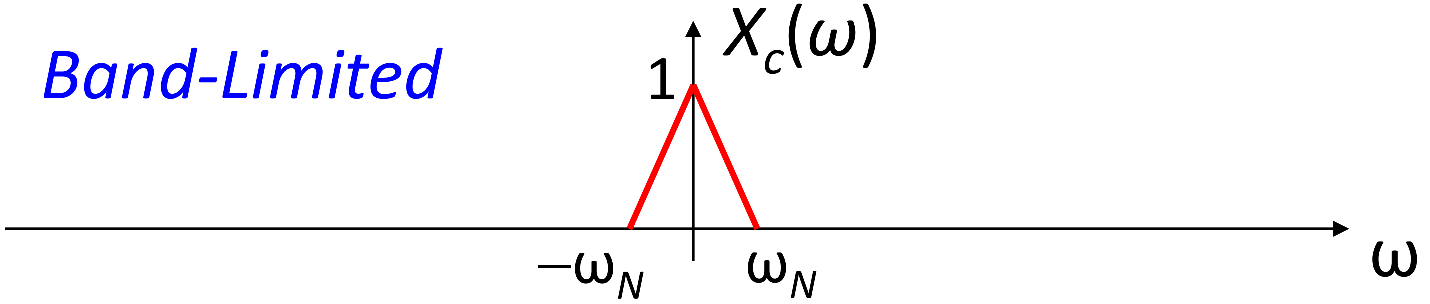
$$= \sum_{n=-\infty}^{n=\infty} x_c(nT)\,\delta(t-nT)$$

$$\boxed{X_p(\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(\omega - k\omega_s)}$$

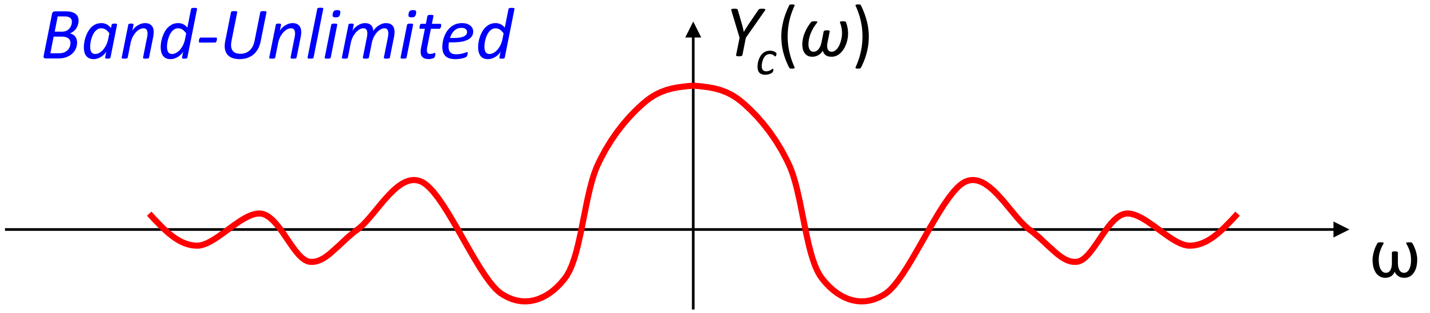
ω_s:
Sampling Frequency

Band-Limited Signals

Band-Limited

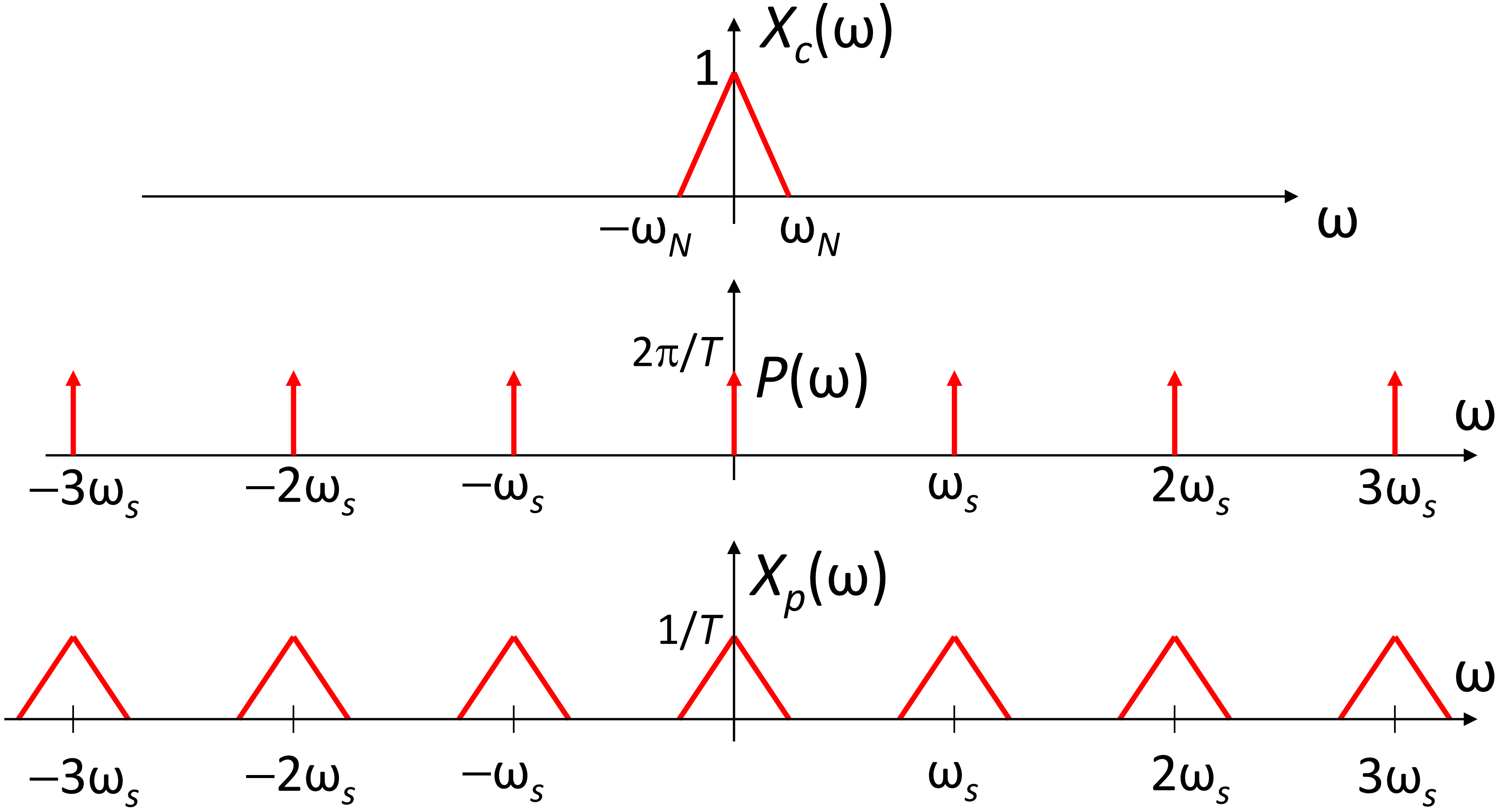


Band-Unlimited



Case 1: $\omega_s > 2\omega_N$

$$X_p(\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(\omega - k\omega_s), \quad \omega_s = \frac{2\pi}{T}$$

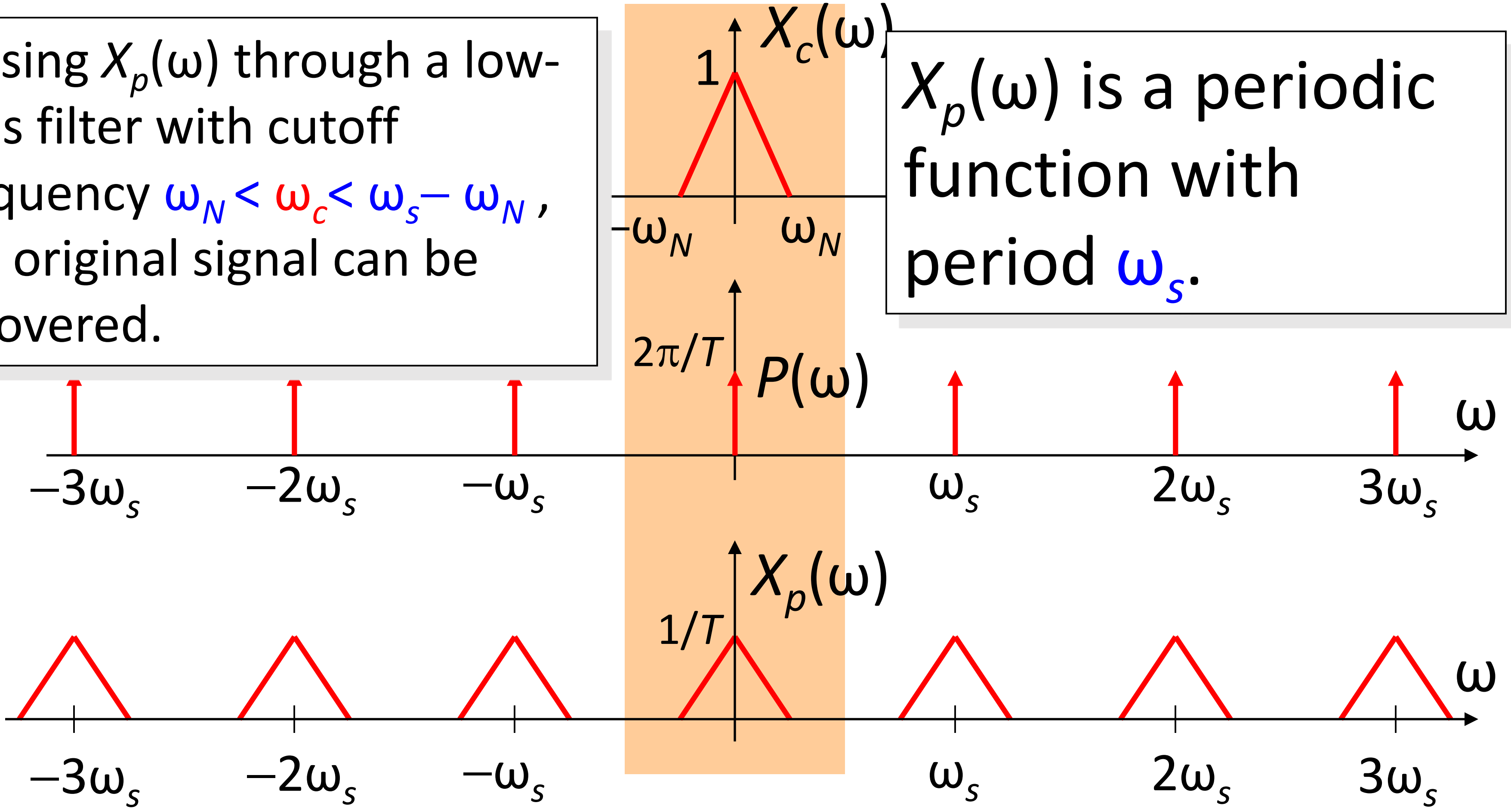


Case 1: $\omega_s > 2\omega_N$

$$X_p(\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(\omega - k\omega_s), \quad \omega_s = \frac{2\pi}{T}$$

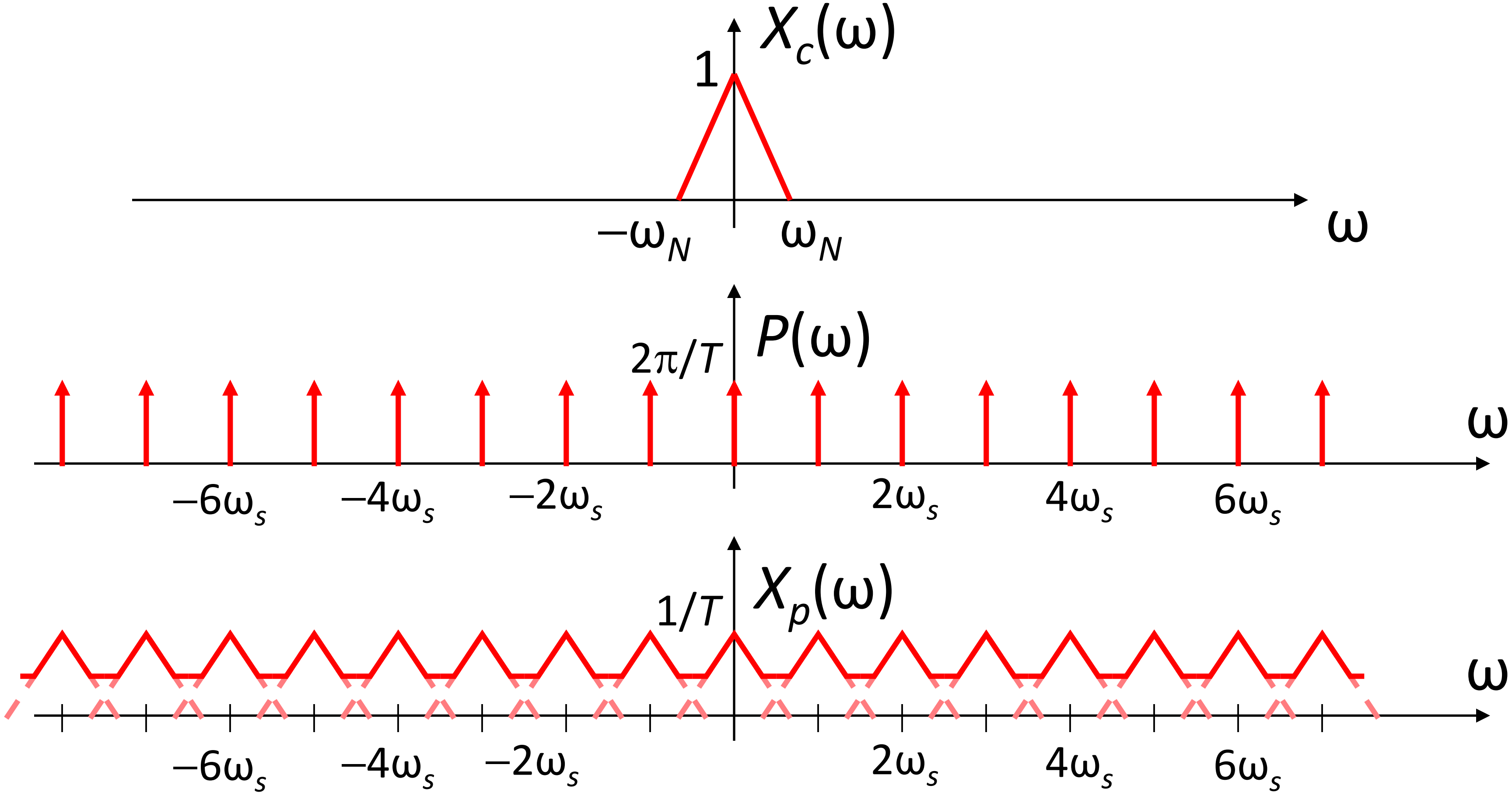
Passing $X_p(\omega)$ through a low-pass filter with cutoff frequency $\omega_N < \omega_c < \omega_s - \omega_N$, the original signal can be recovered.

$X_p(\omega)$ is a periodic function with period ω_s.



Case 2: $\omega_s < 2\omega_N$

$$X_p(\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(\omega - k\omega_s), \quad \omega_s = \frac{2\pi}{T}$$



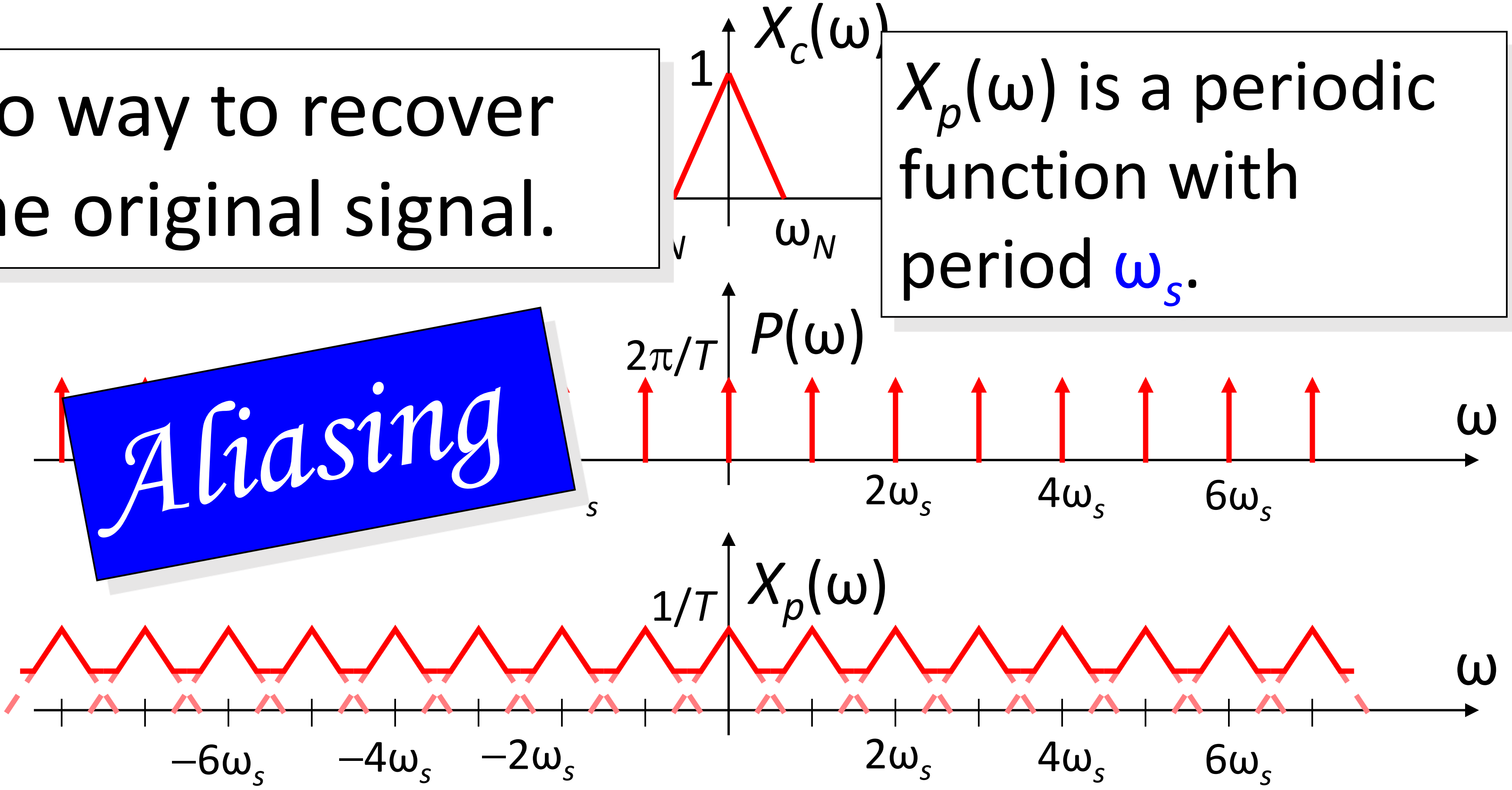
Case 2: $\omega_s < 2\omega_N$

$$X_p(\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(\omega - k\omega_s), \quad \omega_s = \frac{2\pi}{T}$$

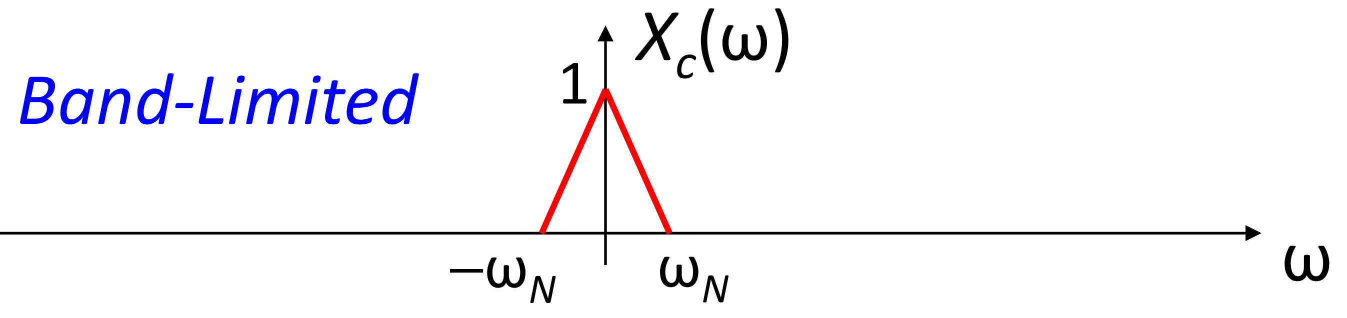
No way to recover the original signal.

$X_p(\omega)$ is a periodic function with period ω_s.

Aliasing



Nyquist Rate



Nyquist frequency (ω_N)

The highest frequency of a band-limited signal

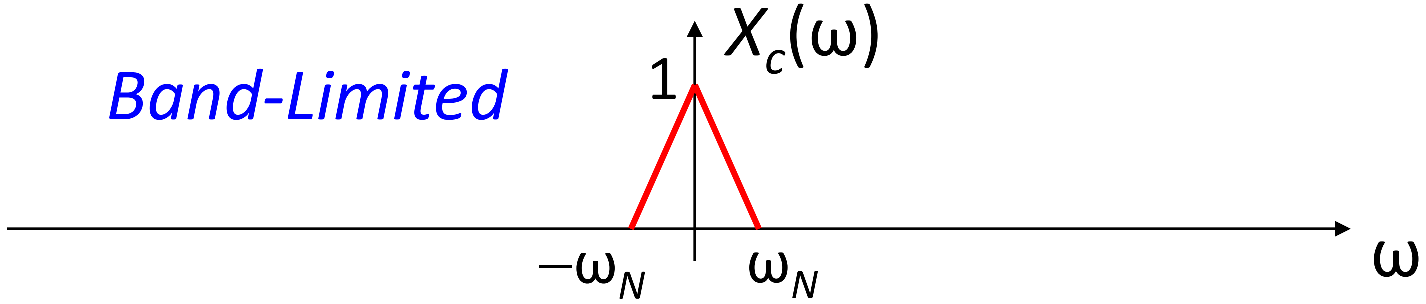
Nyquist rate = $2\omega_N$

Sampling Theorem

Aliasing ---

Nyquist Rate

Nyquist Sampling Theorem



$\omega_s > 2\omega_N$ ➡ Recoverable

$\omega_s < 2\omega_N$ ➡ Aliasing

Aliasing effect on Music

- Music sampled at 44.1 KHz
- Music sampled at 22 KHz
- Music sampled at 11 KHz
- Music sampled at 5.5 KHz
- Music sampled at 2.8 KHz

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Aliasing (demonstration for a sinusoidal input)

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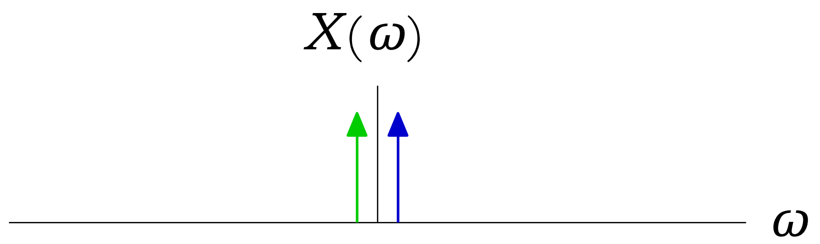
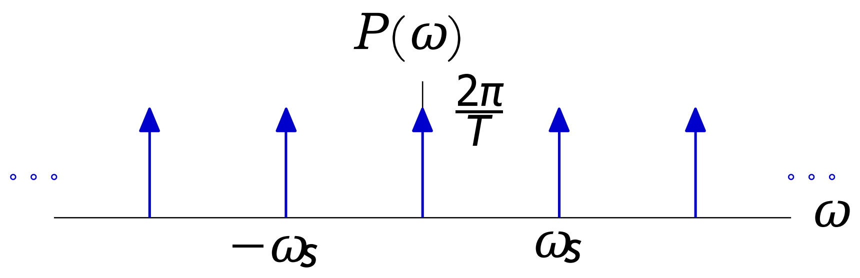
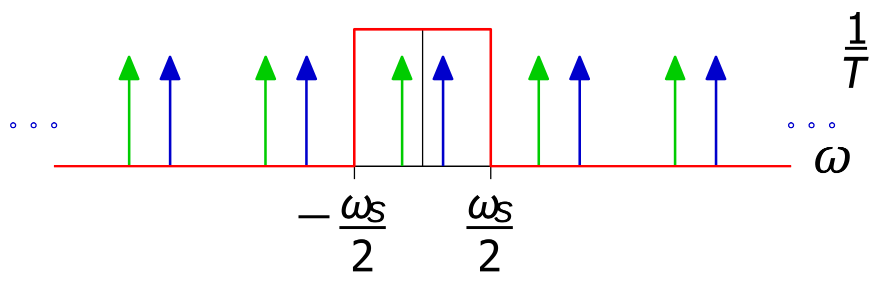
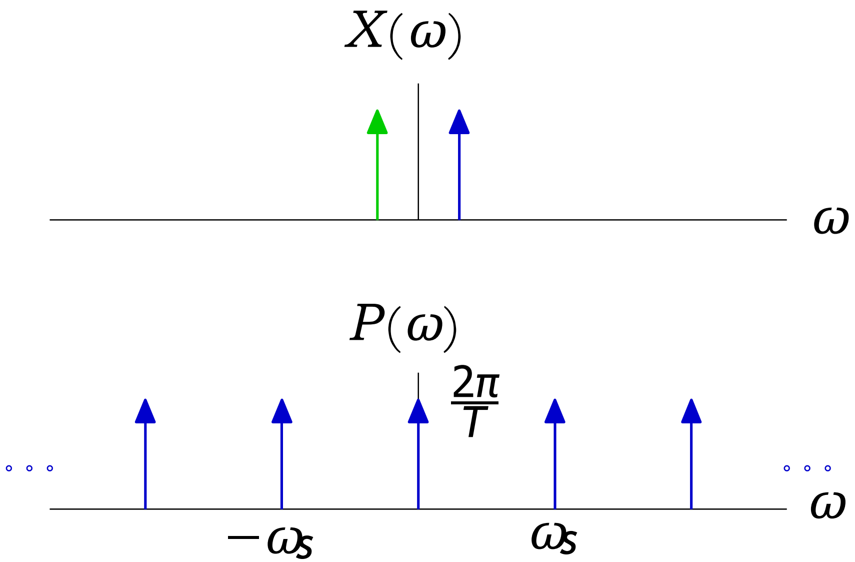




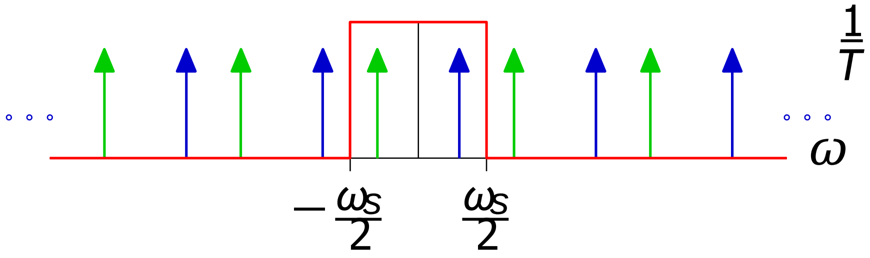
$$X_p(\omega) = \frac{1}{2\pi}\left(X(\cdot) * P(\cdot)\right)(\omega)$$



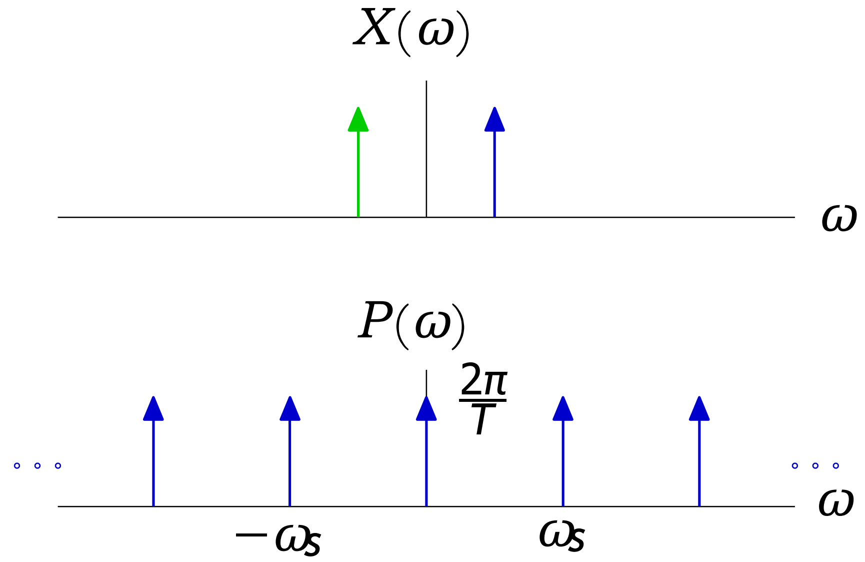
Aliasing (demonstration for a sinusoidal input)



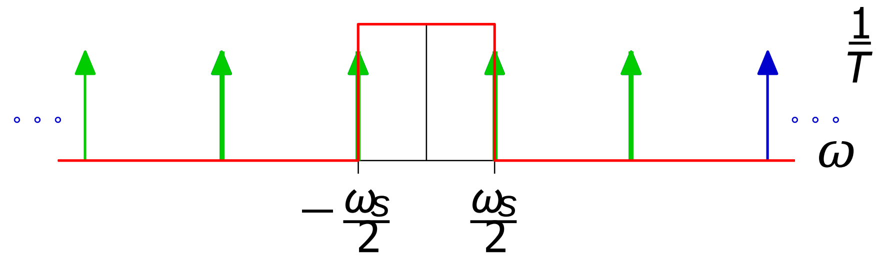
$$X_p(\omega) = \frac{1}{2\pi}\left(X(\cdot) * P(\cdot)\right)(\omega)$$



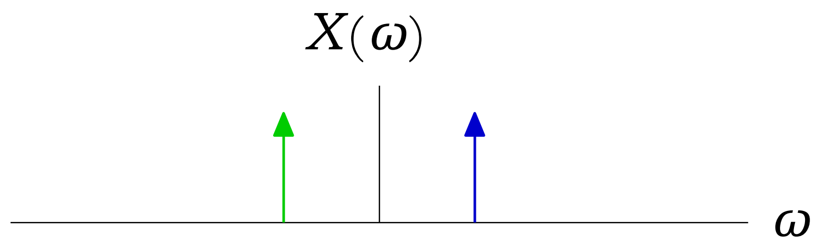
Aliasing (demonstration for a sinusoidal input)

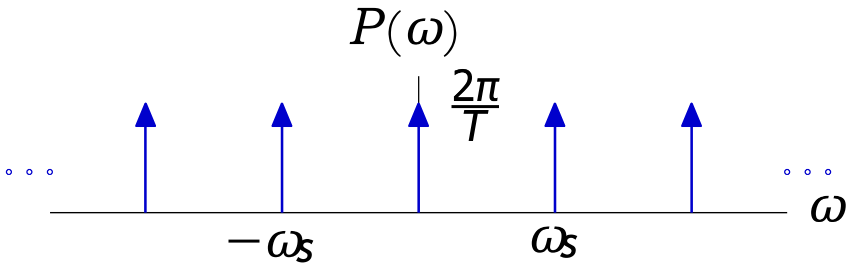


$$X_p(\omega) = \frac{1}{2\pi}(X(\cdot) * P(\cdot))(\omega)$$

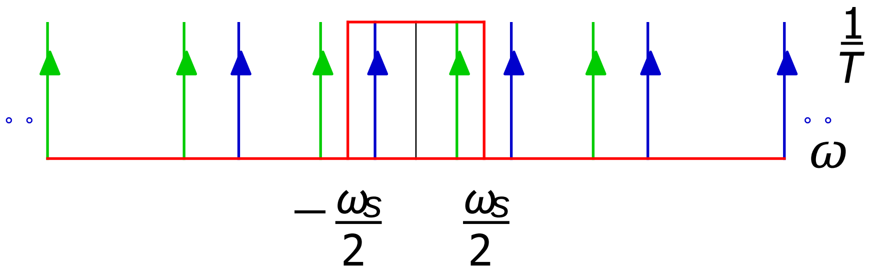


Aliasing (demonstration for a sinusoidal input)



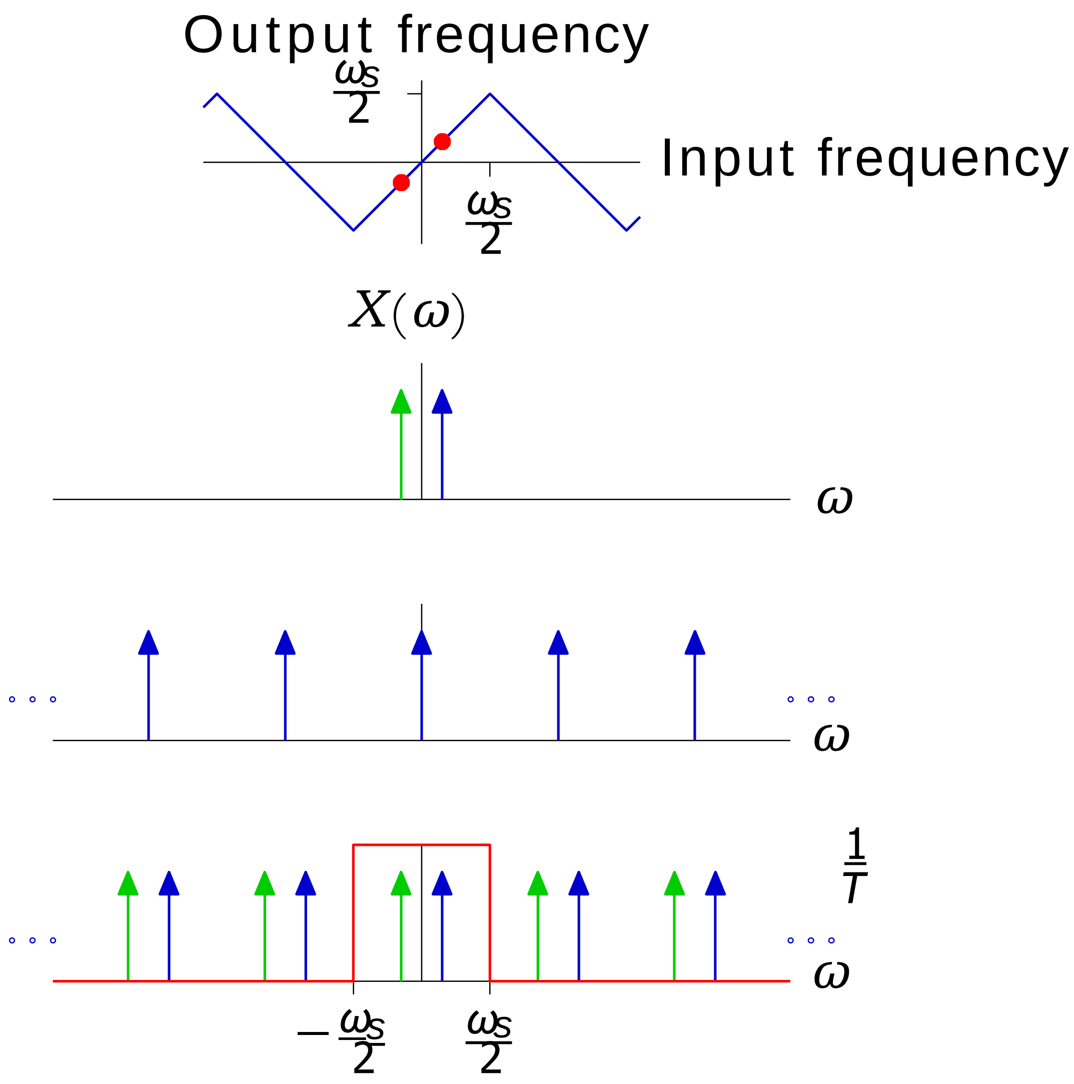


$$X_p(\omega) = \frac{1}{2\pi}(X(\cdot) * P(\cdot))(\omega)$$

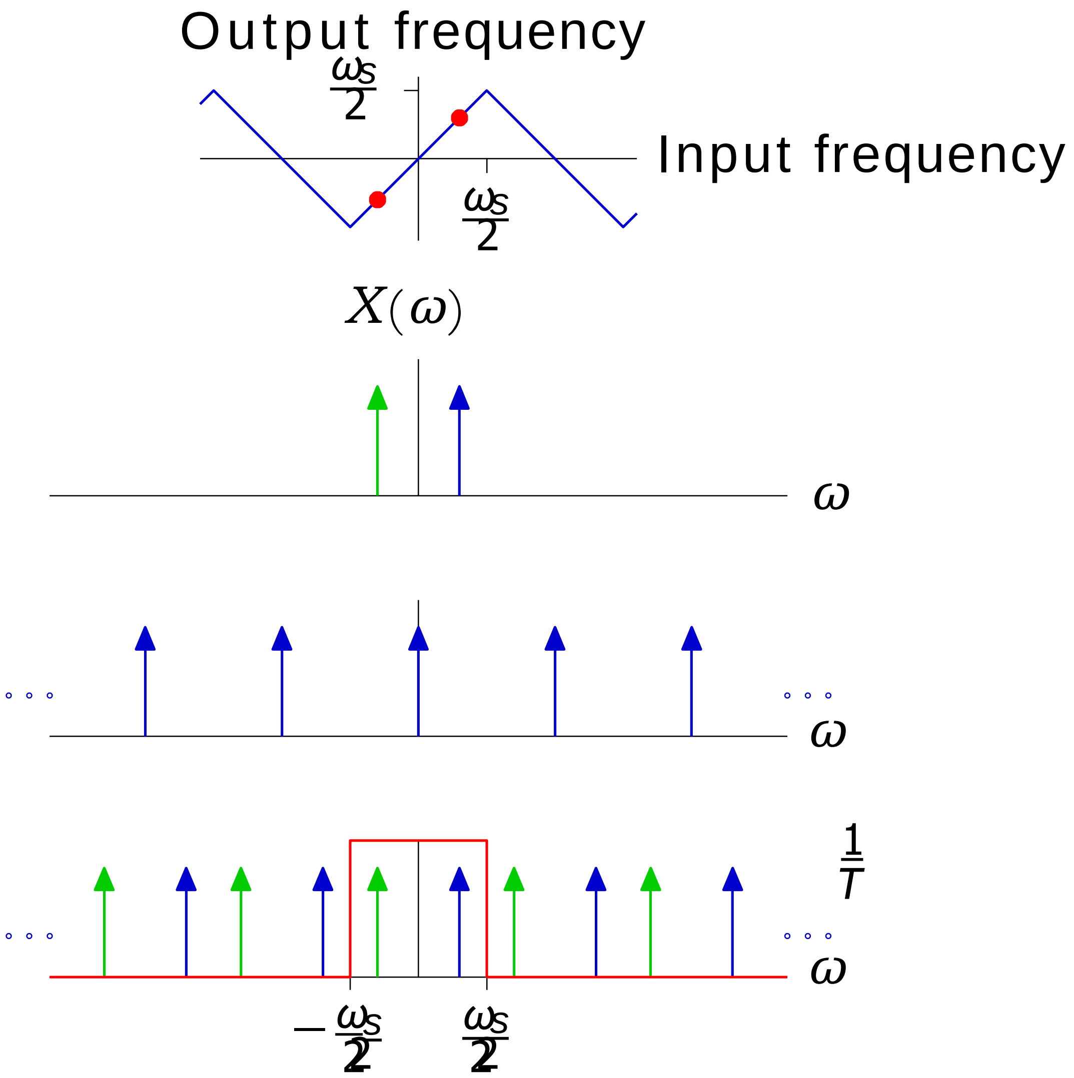


Aliasing: Frequency wrapping

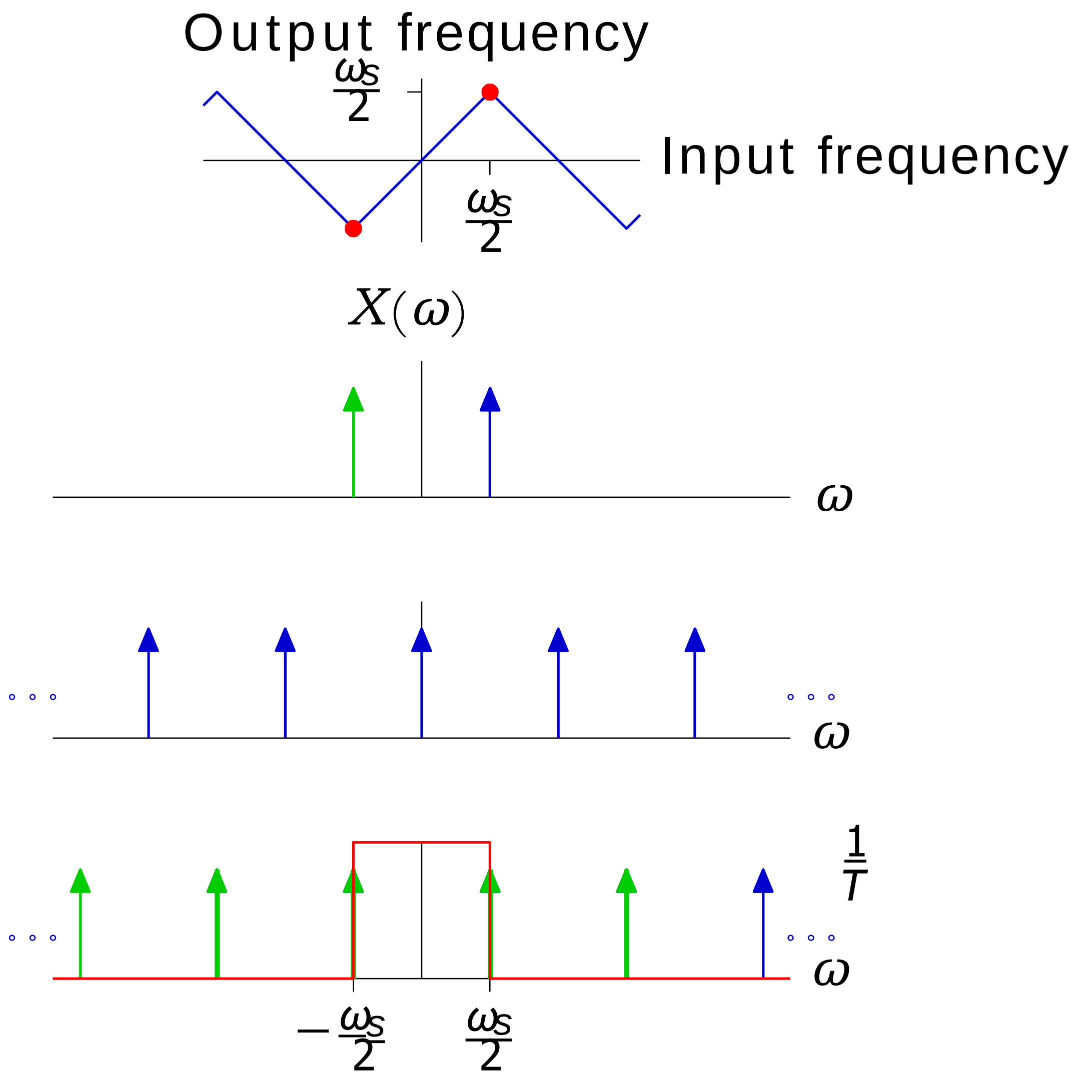
The effect of
aliasing
is to wrap
frequencies.



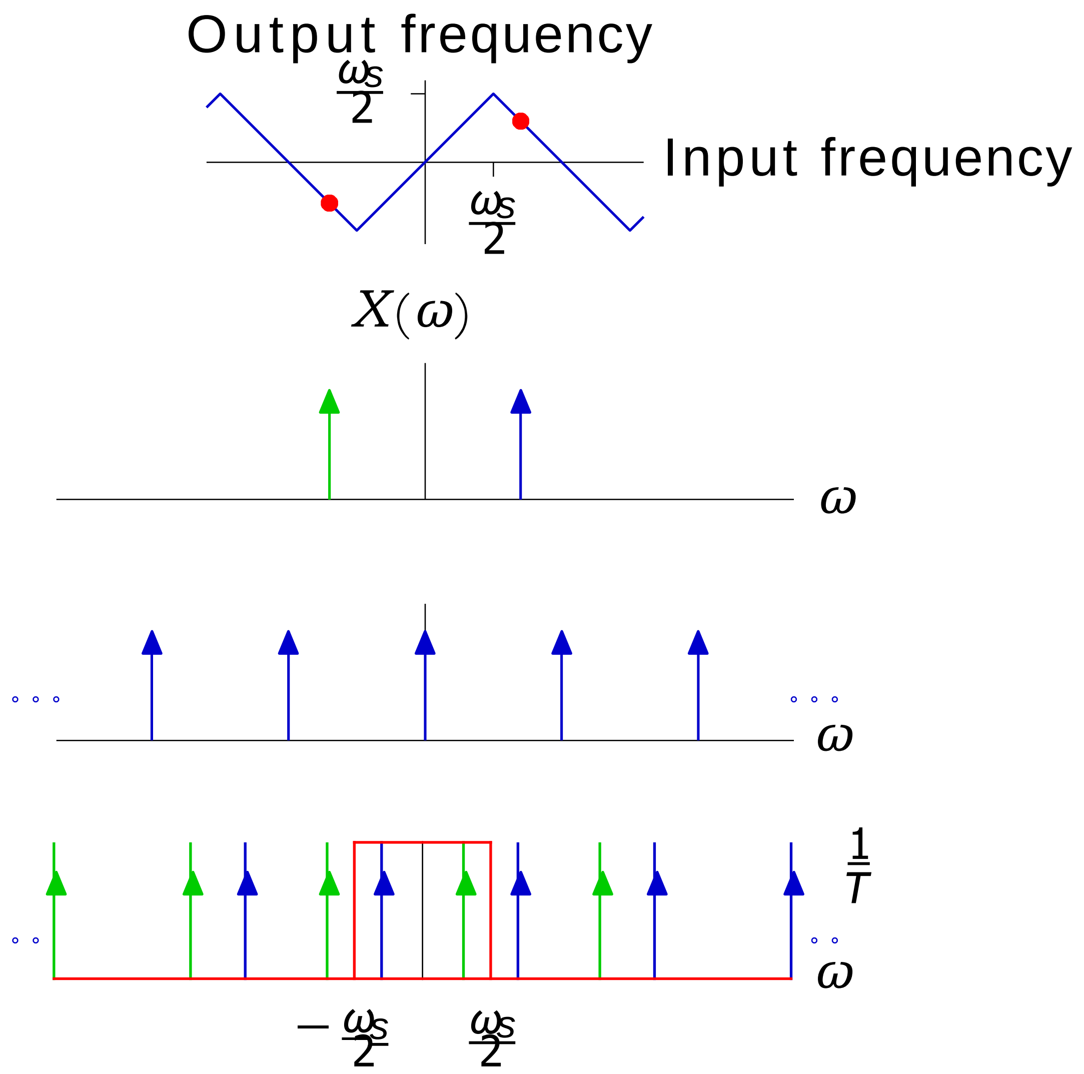
Aliasing: Frequency wrapping



Aliasing: Frequency wrapping



Aliasing: Frequency wrapping



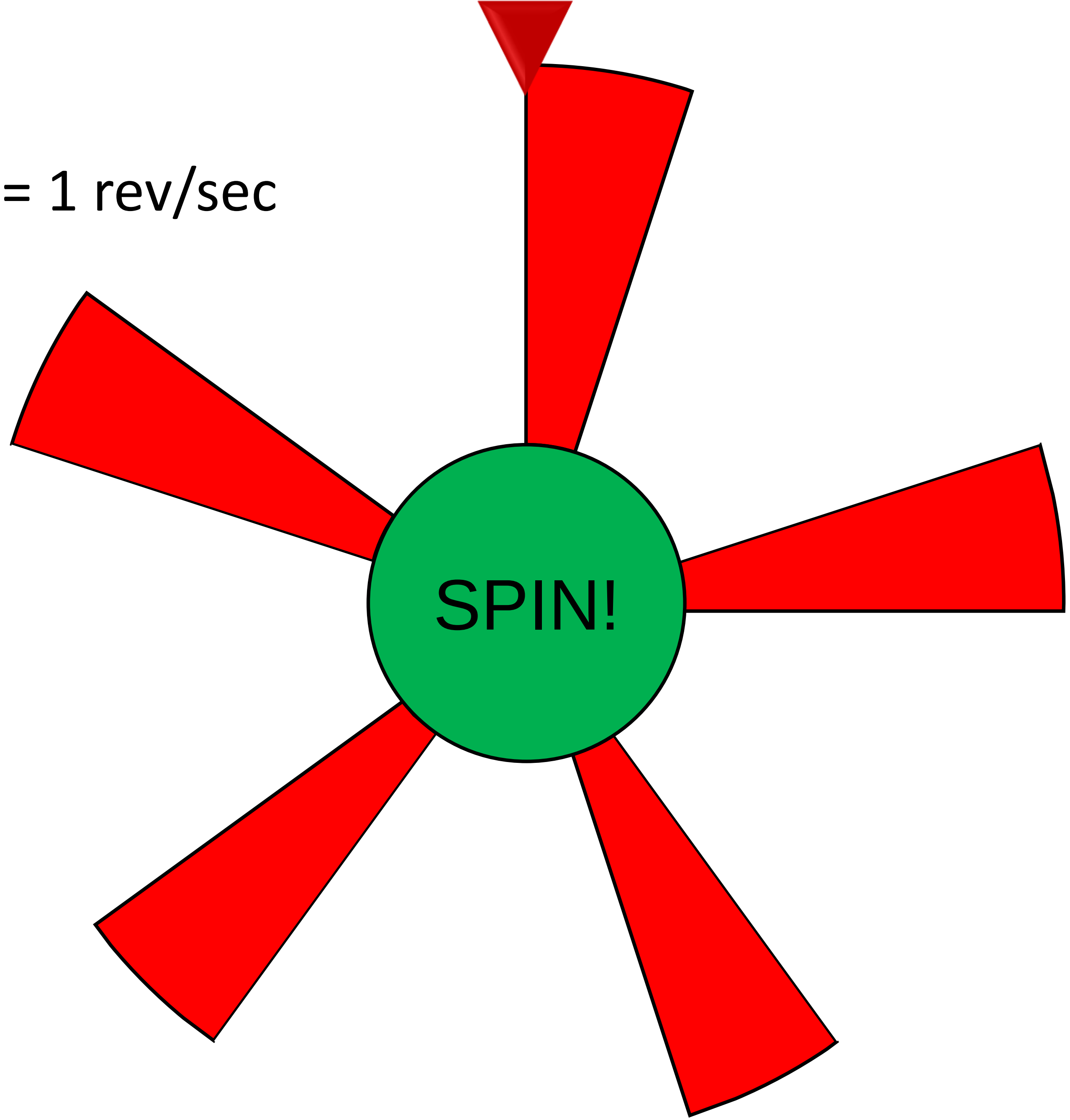
Think

A periodic signal with period 0.2 ms is sampled at 44 KHz. To what frequency, the 7 harmonic aliases?

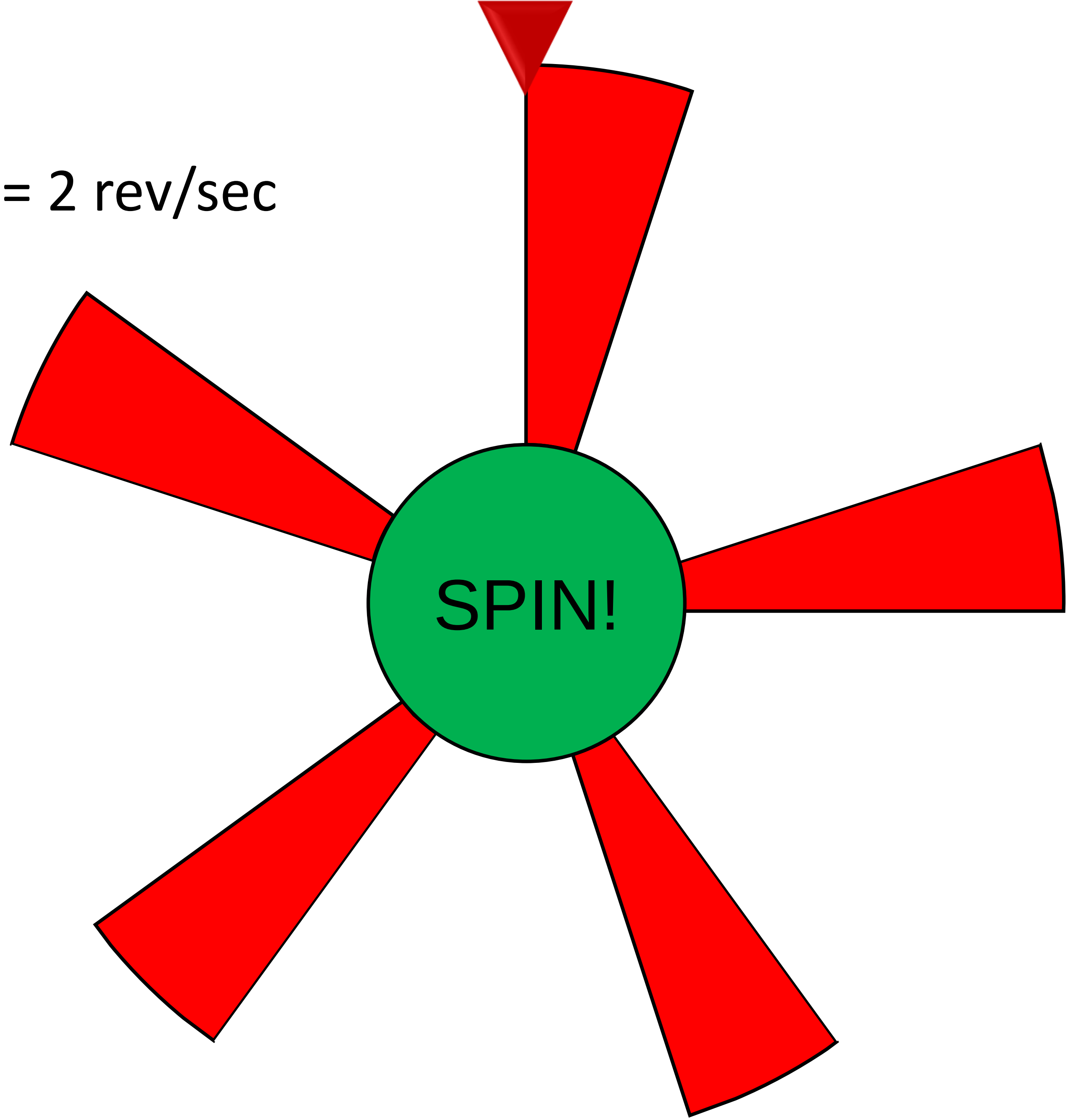
i) 9 KHz	**ii) 13 KHz**
iii) 35 KHz	iv) 19 KHz

$f_0 = 0$ KHz	$f_0 = 0$ KHz
$f_1 = 5$ KHz	$f_1 = 5$ KHz
$f_2 = 10$ KHz	$f_2 = 10$ KHz
$f_3 = 15$ KHz	$f_3 = 15$ KHz
$f_4 = 20$ KHz	$f_4 = 20$ KHz
$f_5 = 25$ KHz	$f_5 = 44\text{-}25$ KHz $= 19$ KHz
$f_6 = 30$ KHz	$f_6 = 44\text{-}30$ KHz $= 14$ KHz
$f_7 = 35$ KHz	$f_7 = 44\text{-}35$ KHz $= 9$ KHz
$f_n = 135$ KHz	$f_n = 135\text{-}44\times 3=3$ KHz

Angular velocity = 1 rev/sec

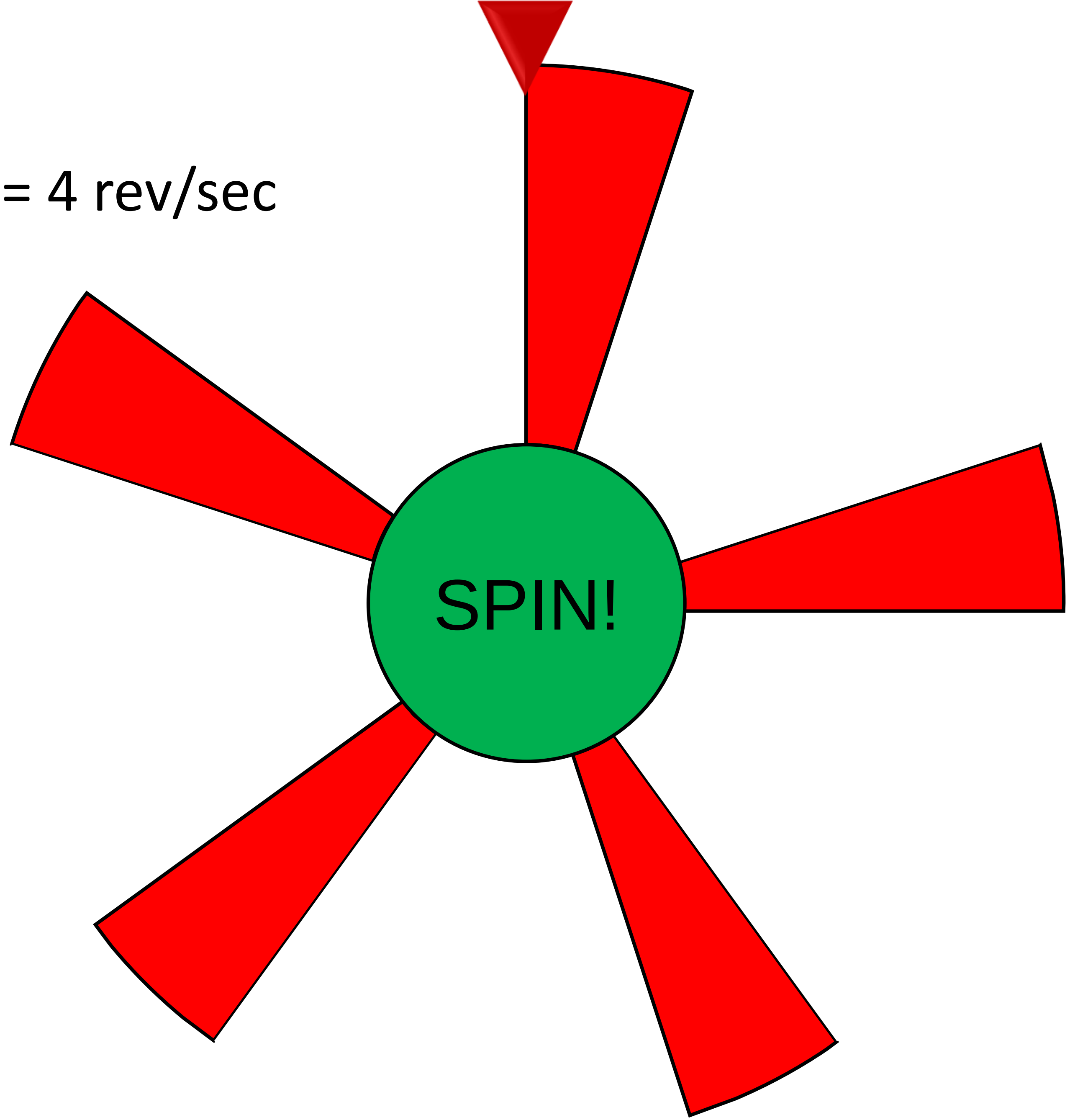


Angular velocity = 2 rev/sec

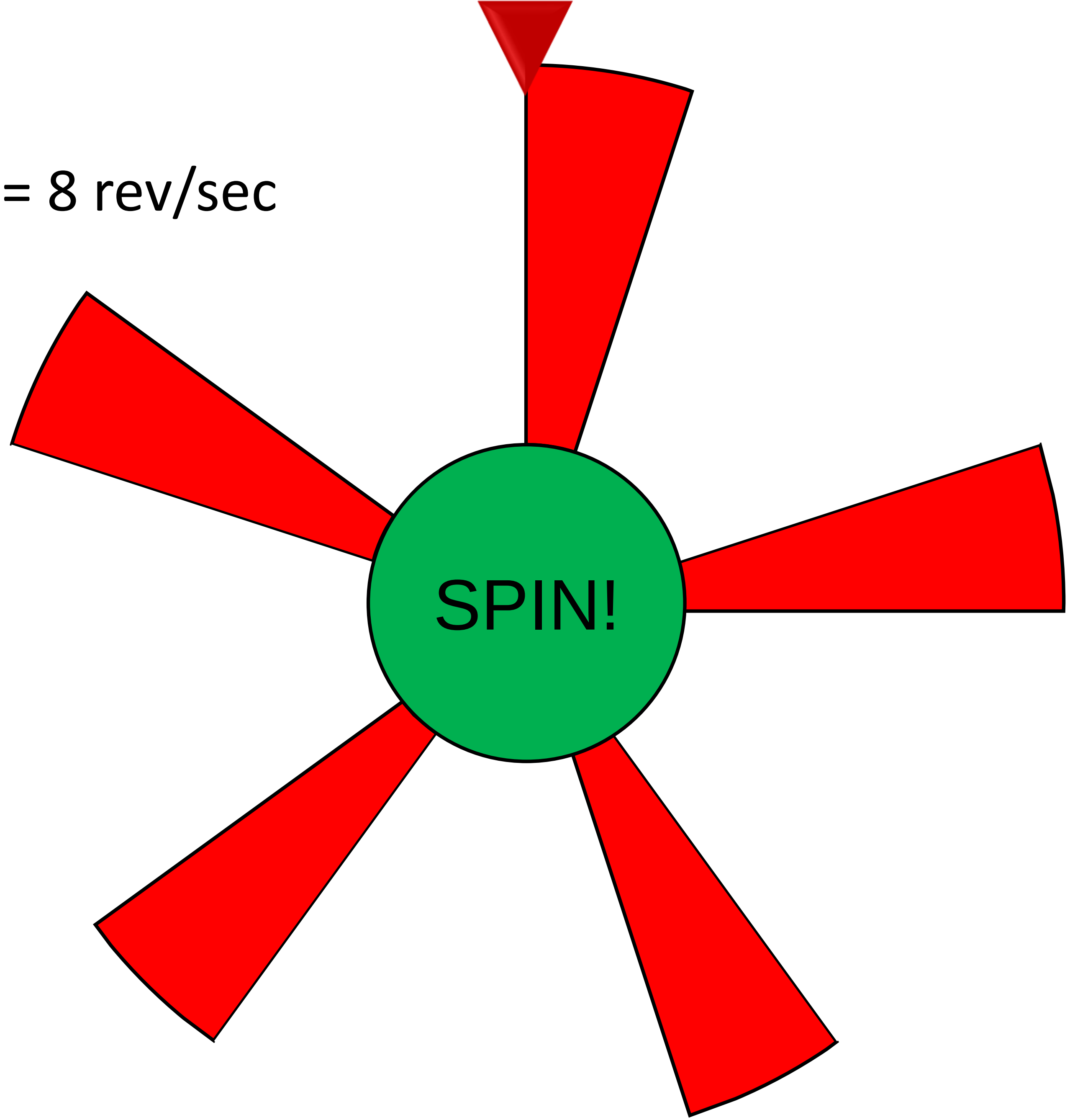


tekhnologic

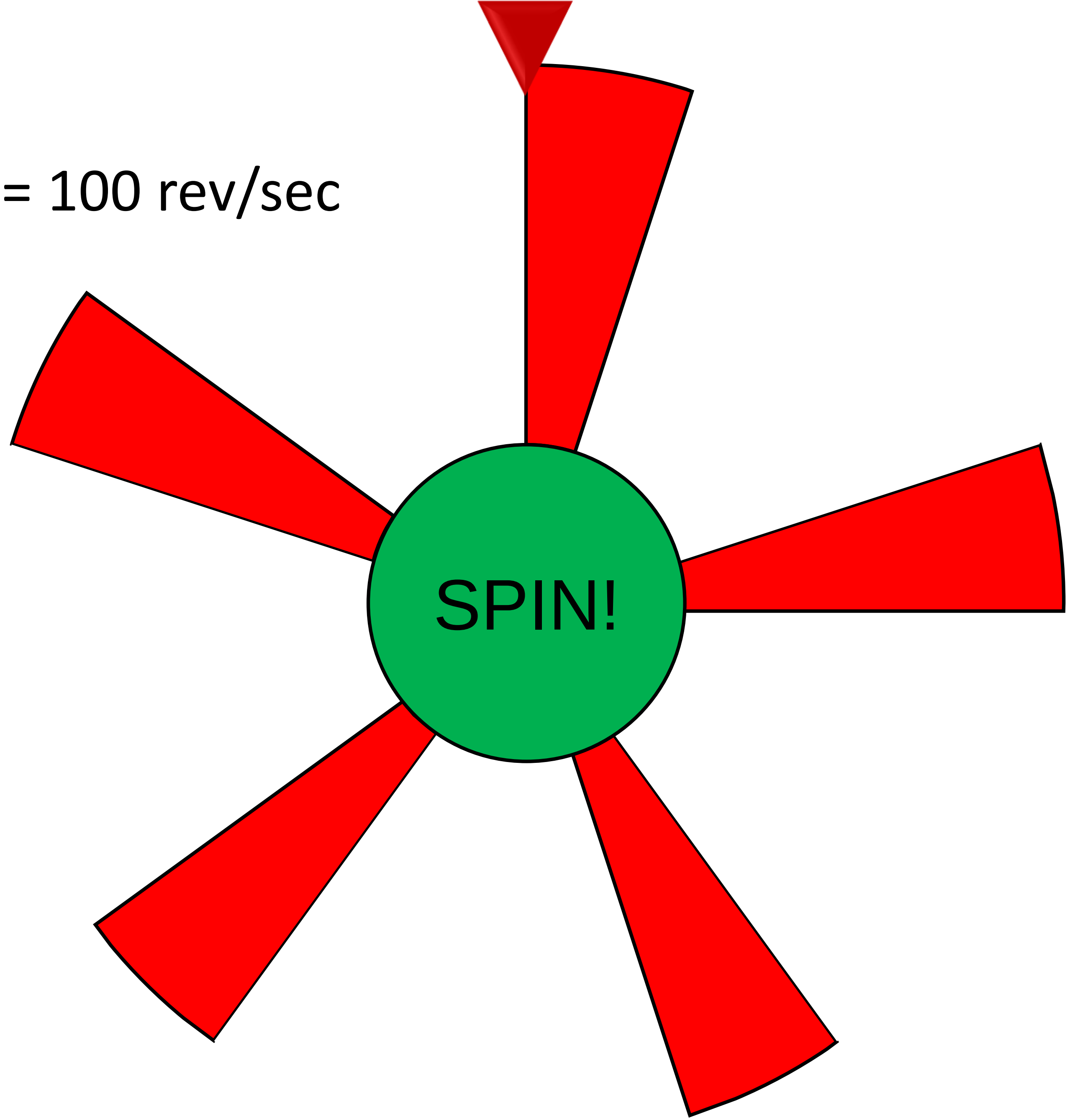
Angular velocity = 4 rev/sec



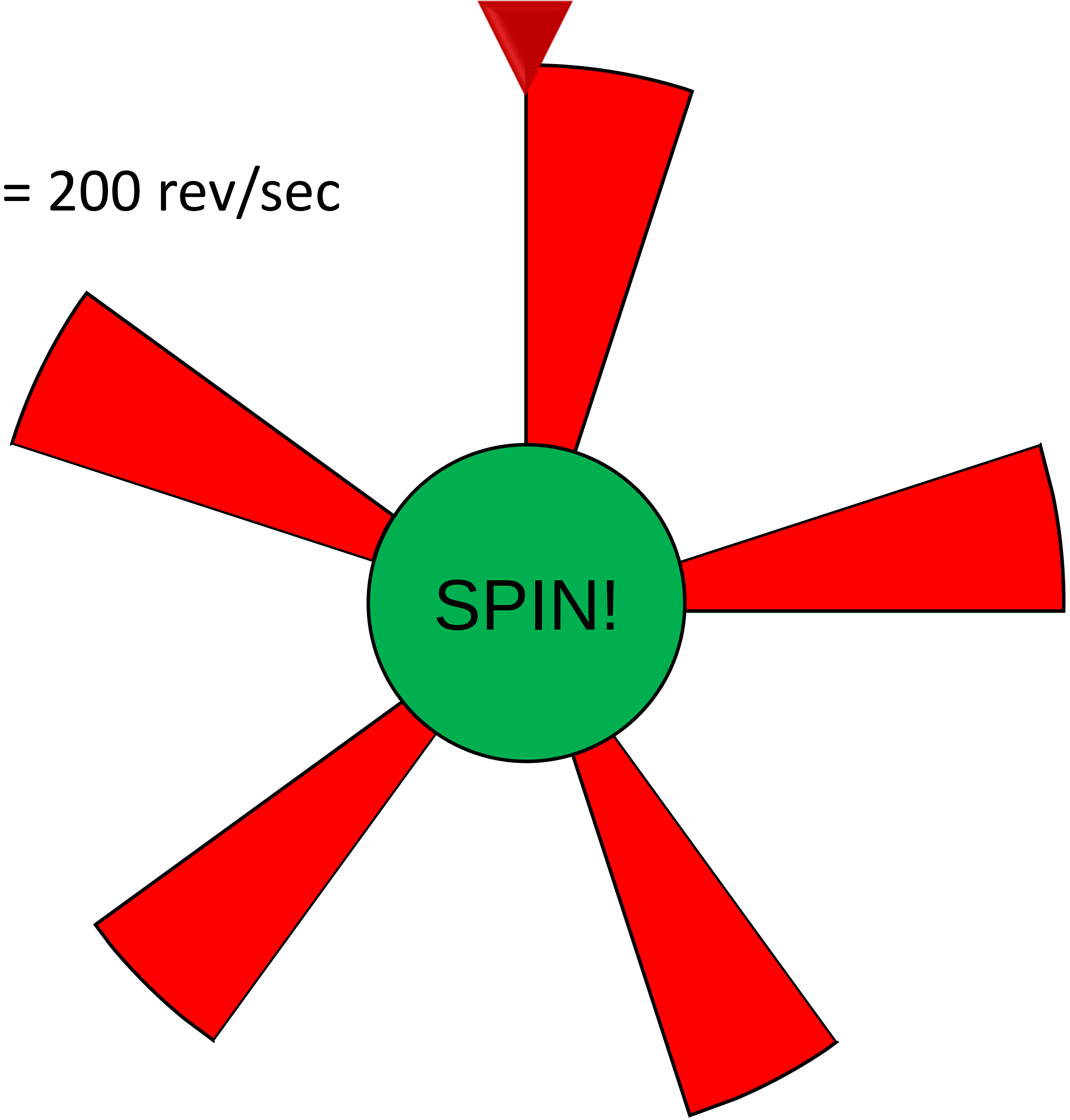
Angular velocity = 8 rev/sec



Angular velocity = 100 rev/sec

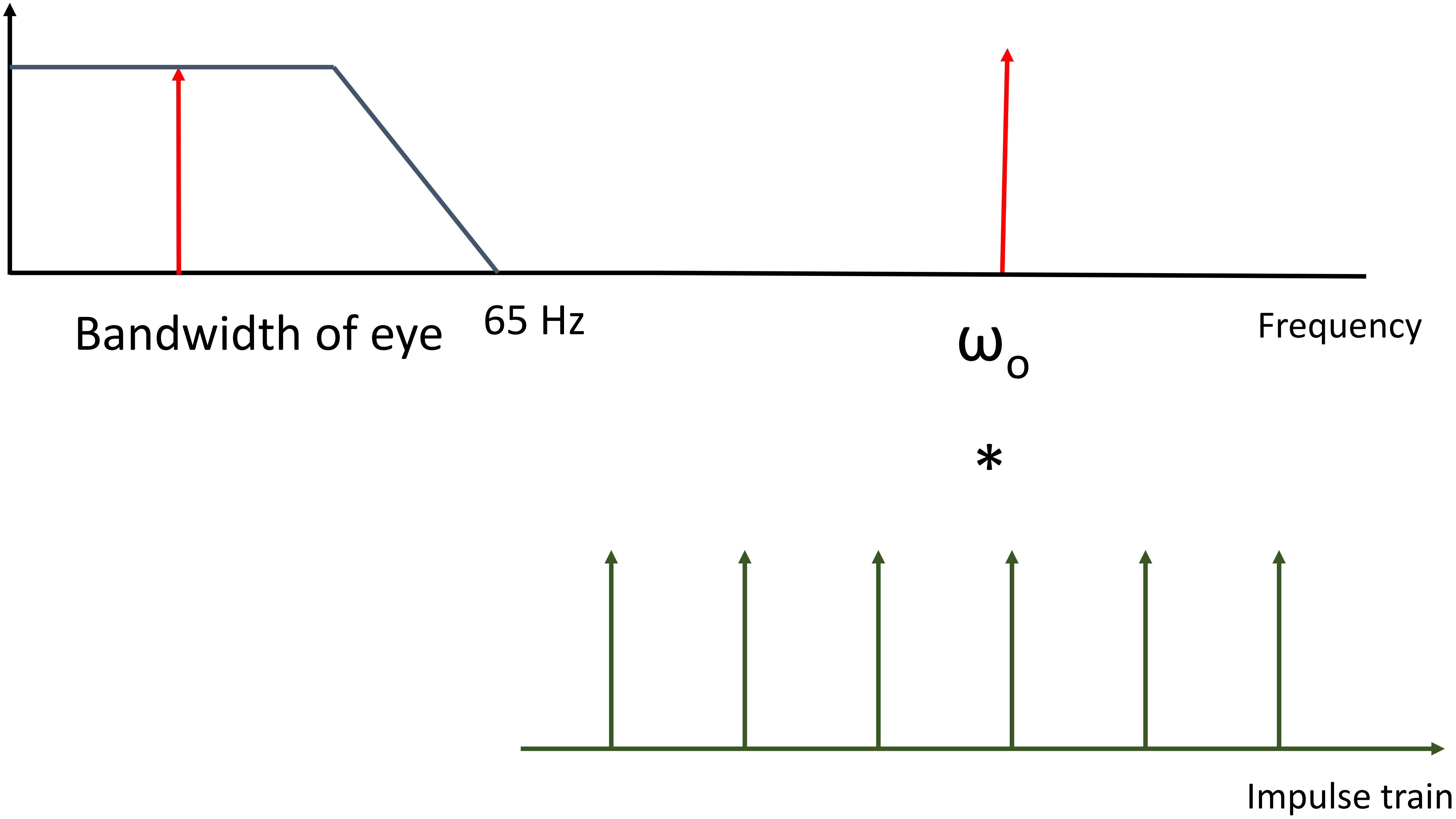


Angular velocity = 200 rev/sec

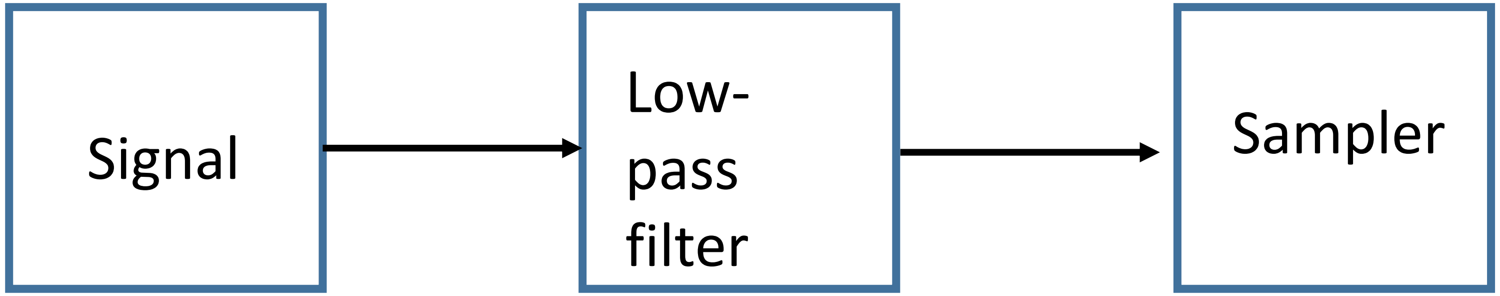


Stroboscope

Flashes light at a desired frequency


Bandwidth of eye
65 Hz
ω_o
Frequency
*
Impulse train

Anti-aliasing filter



Anti-aliasing filter

- Music sampled at 2.8 KHz (without anti-aliasing)

- Music sampled at 2.8 KHz (with anti-aliasing)

Anti-aliasing filter

- Music sampled at 2.8 KHz (without anti-aliasing)

- Music sampled at 2.8 KHz (with anti-aliasing)

Anti-aliasing filter

- Music sampled at 2.8 KHz (without anti-aliasing)

- Music sampled at 2.8 KHz (with anti-aliasing)