ELL101: INTRODUCTION TO ELECTRICAL ENG.

Basic Laws of Circuit Theory

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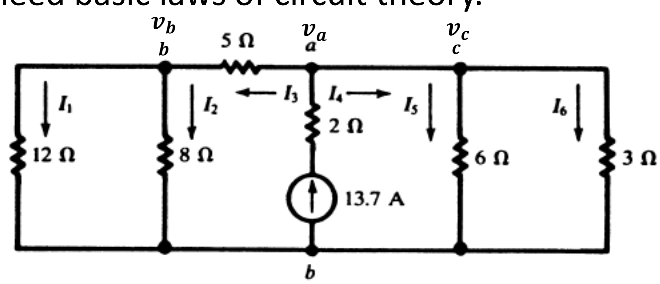
Basic Laws of Circuits

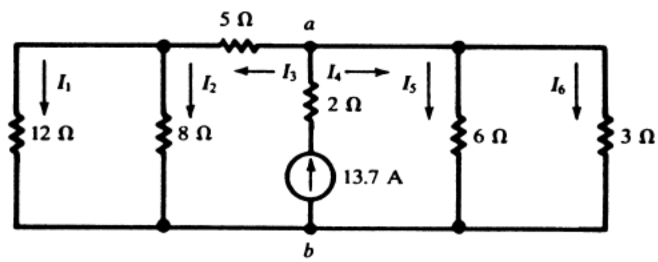
- Consider the electric circuit below.
- In this circuit there is a current source, resitances, nodes, branches, and loops.
- How to find the value of current in a particular branch or potential at a particular node.
- For that we need basic laws of circuit theory.



Source: M. Nahvi and J. A. Edminister, *SCHAUM's Outline: Electric Circuits*, McGRAWHILL Edu., 2018.

Sign Convention for Current

- The currents entering at a specific node can be taken as negative
- The currents leaving a specific node can be taken as positive
- Vice-versa is also fine.



Source: M. Nahvi and J. A. Edminister, *SCHAUM's Outline: Electric Circuits*, McGRAWHILL Edu., 2018.

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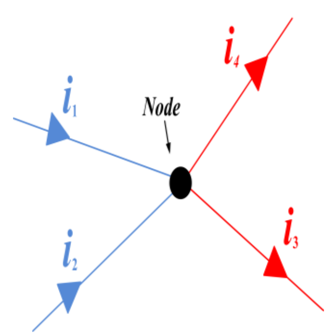
Kirchhoff's Current Law (KCL)

- The algebraic sum of the currents entering any node is zero
- A node is not a circuit element, and it certainly cannot store, destroy, or generate charge. Hence, the currents must sum to zero
- KCL is based on conservation of charge



Source: https://electricalacademia.com/basic-electrical/kirchhoffs-current-law-kcl-kirchhoffs-law/

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Illustration of KCL

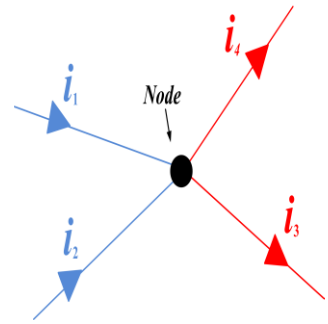
- Example node to illustrate the application of Kirchhoff's current law.
- Consider the node shown in the figure. The algebraic sum of the four currents entering the node must be zero:

$$(-i_1)+(-i_2)+(i_3)+(i_4)=0$$

- We might also wish to equate the sum of the currents having reference arrows directed into the node to the sum of those directed out of the node:

$$i_1+i_2=i_3+i_4$$

which simply states that the sum of the currents going in must equal the sum of the currents going out.



Source: https://electricalacademia.com/basic-electrical/kirchhoffs-current-law-kcl-kirchhoffs-law/

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Example of KCL

- Obtain the currents I_1 and I_2 for the network shown

Solution

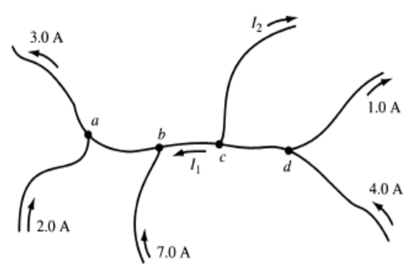
- *a* and *b* comprise one node.
- Applying KCL

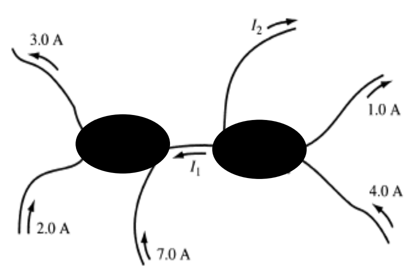
$$3-2-7-I_1=0$$

$$\Rightarrow I_1=-6A$$

$$I_1+I_2+1-4=0$$

$$\Rightarrow I_2=9A$$





Source: M. Nahvi and J. A. Edminister, *SCHAUM's Outline: Electric Circuits*, McGRAWHILL Edu., 2018.

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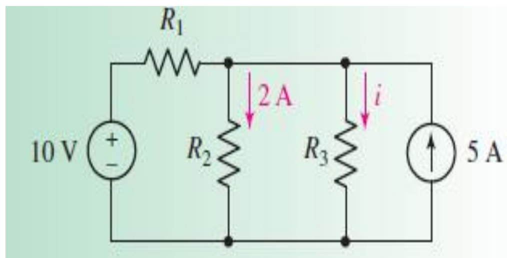
Example of KCL

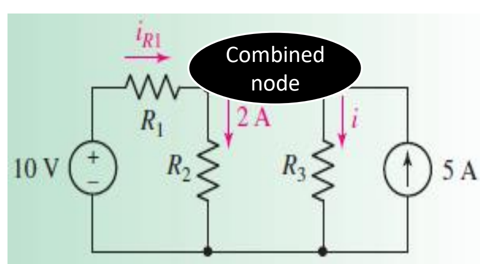
- For the circuit below, compute the current through resistor R_3 if it is known that the voltage source supplies a current of 3 A

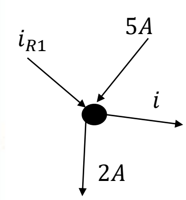


Source: W. H. Hayt et al., *Engineering Circuit Analysis*, McGrawHill, 2012

Solution

- The current through resistor R_3 , labeled as i on the circuit diagram
- If we label the current through R1, we may write a KCL equation at the top node of resistors R_2 and R_3.





Source: W. H. Hayt et al., *Engineering Circuit Analysis*, McGrawHill, 2012

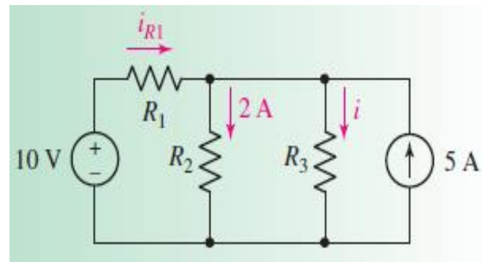
Summing the currents flowing into the node:

$(-i_{R_1}) + 2 + i - 5 = 0$

- We have one equation but two unknowns, which means we need to obtain an additional equation
- If we know the 10 V source is supplying 3 A comes in handy, then KCL shows us that this is also the current i_{R_1}

$$\Longrightarrow i_{R_1} = 3\text{A}$$

- Substituting, we find that $i = 6\text{A}$



Source: W. H. Hayt et al., *Engineering Circuit Analysis*, McGrawHill, 2012

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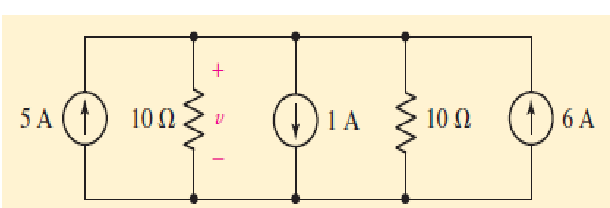
Problem Based on KCL with Current Sources

- Determine v in the circuit shown below:



Source: W. H. Hayt et al., *Engineering Circuit Analysis*, McGrawHill, 2012

- Applying KCL at the upper node of the resistor 10Ω, we get:

$$(-5) + \frac{v}{10} + 1 + \frac{v}{10} - 6 = 0$$

$$\Longrightarrow v = 50V$$

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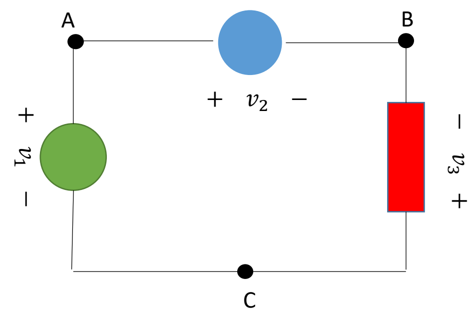
Kirchhoff's Voltage Law(KVL)

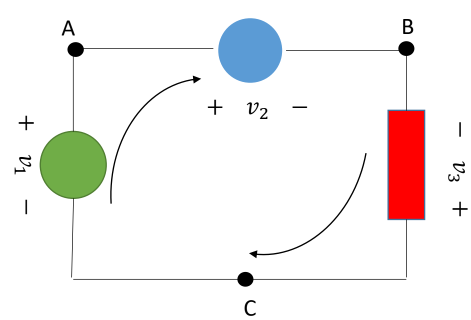
- The algebraic sum of the voltages around any closed path is zero
- KVL is based on conservation of energy



Sign Convention for KVL

- Moving around the closed path in a clockwise direction and writing down directly the voltage of each element first met
- It means if we first meet the (+) terminal of a voltage source, then (+V) is considered, and if we first meet the (-) terminal of a voltage source, then (-V) is considered
- Applying this to figure, we have:

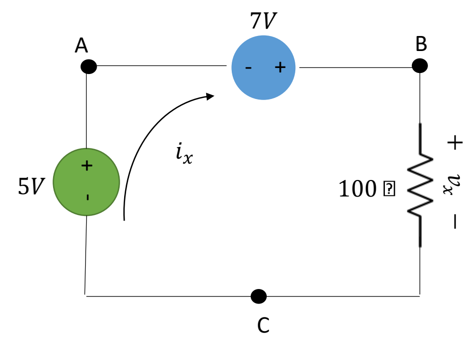
$$-v_1 + v_2 - v_3 = 0$$
$$\Longrightarrow v_2 = v_1 + v_3$$



Example of KVL

In the circuit, find v_x

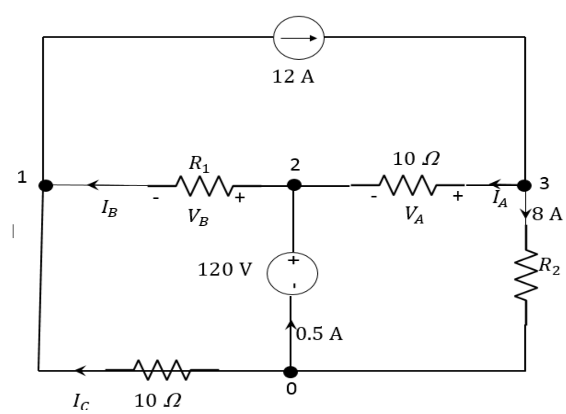
- We know the voltage across two of the three elements in the circuit
- Beginning with the bottom node of the 5 V source, we apply KVL clockwise around the loop:
- $-5 - 7 + v_x = 0$
- $\Longrightarrow v_x$=12V
- If we need to find i_x, we can invoke Ohm's law as:
- $i_x = \frac{v_x}{100} = \frac{12}{100} = 120$mA



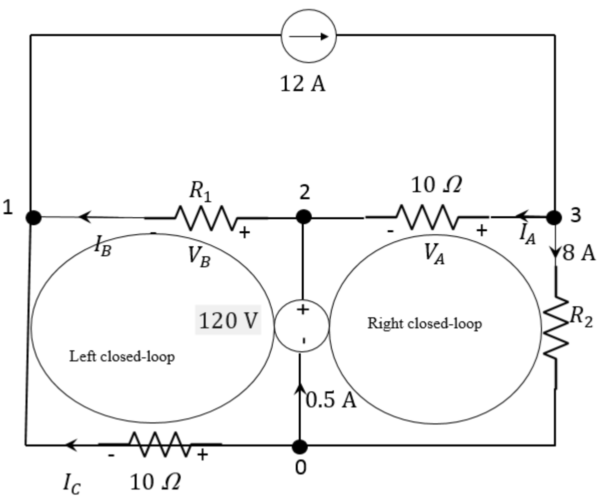
KVL-KCL Example

Calculate V_A, I_B, R_1, R_2 in the circuit in figure

- Applying KCL to node 3 gives
  $12 = I_A + 8$
  $I_A = 4A$
- From Ohm's law
  $V_A = 10I_A = 40V$
- Applying KCL to node 2 yields
  $I_B = I_A + 0.5$
  $I_B = 4.5A$
- At node 0, KCL shows that
  $8 = I_C + 0.5$
  $I_C = 7.5A$



- Applying KVL the closed-loop on the left of figure
- $V_B = (10 \times 7.5) + 120$
- $V_B = 195V$
- Hence, from Ohm's law
- $R_1 = \frac{V_B}{I_B} = \frac{195}{4.5} = 43.33\Omega$
- Applying KVL the closed-loop on the right of figure
- $120 + (10 \times I_A) = 8 \times R_2$
- $120 + 40 = 8 \times R_2$
- $R_2 = \frac{160}{8} = 20\Omega$



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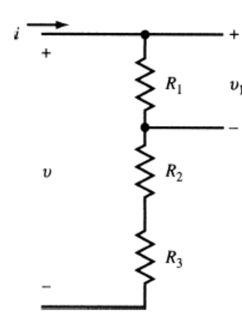
Voltage Division

- Since $v_1 = iR_1$
- From KVL: $i = \frac{v}{R_1+R_2+R_3}$
- $v_1 = \frac{R_1}{R_1+R_2+R_3} v$
- $v_2 = \frac{R_2}{R_1+R_2+R_3} v$
- $v_3 = \frac{R_3}{R_1+R_2+R_3} v$



Source: M. Nahvi and J. A. Edminister, *SCHAUM's Outline: Electric Circuits*, McGRAWHILL Edu., 2018.

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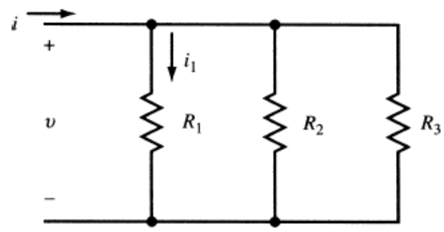
Current Division

- As $i_1 = \frac{v}{R_1}, i_2 = \frac{v}{R_2}, i_3 = \frac{v}{R_3}$
- From KCL: $i = i_1 + i_2 + i_3$
  $= \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3}$
- $\frac{i_1}{i} = \frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$
- $\frac{i_2}{i} = \frac{R_1 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$
- $\frac{i_3}{i} = \frac{R_1 R_2}{R_1 R_2 + R_2 R_3 + R_1 R_3}$
- If $R_3 = \infty$, R_3 is open circuit or not present, then

$\frac{i_1}{i} = \frac{R_2}{R_1 + R_2}$ and $\frac{i_2}{i} = \frac{R_1}{R_1 + R_2}$



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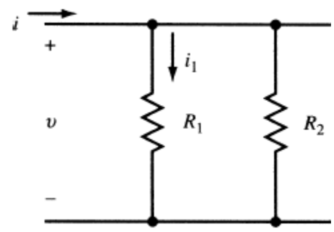

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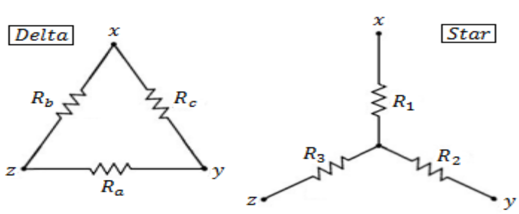
Start⇔Delta Conversion



$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \qquad R_1 = \frac{R_bR_c}{R_a + R_b + R_c}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \qquad R_2 = \frac{R_aR_c}{R_a + R_b + R_c}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \qquad R_3 = \frac{R_aR_b}{R_a + R_b + R_c}$$

Source:http://www.ambrsoft.com/CalcElectric/Star2Delta/Star2Delta.htm

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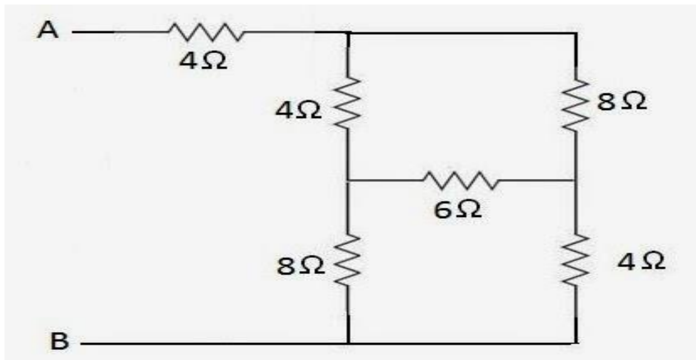
Find the equivalent resistance between A and B in the given network



Source: https://dcanalysis.blogspot.com/2014/04/solved-examples-problems-on-star-delta.html

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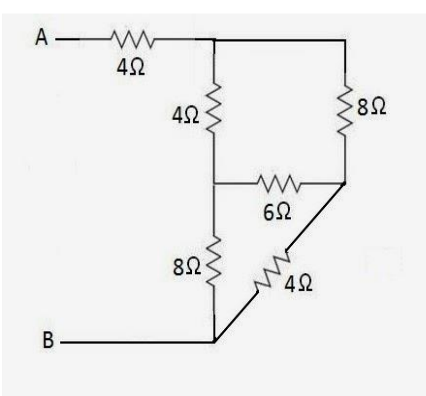
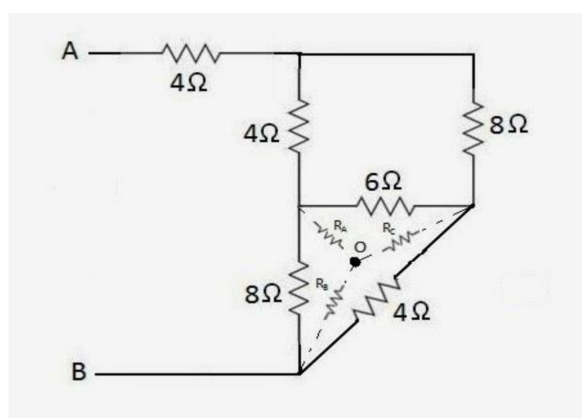
Apply Start⇔Delta Conversion





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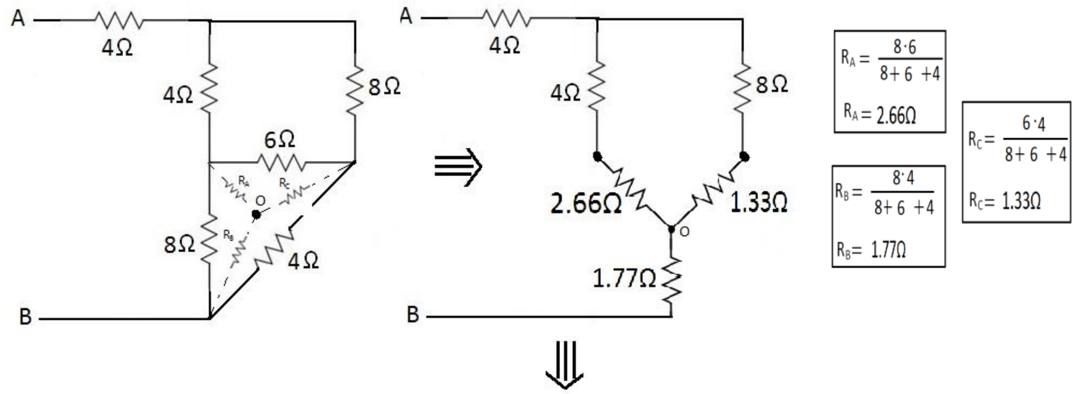
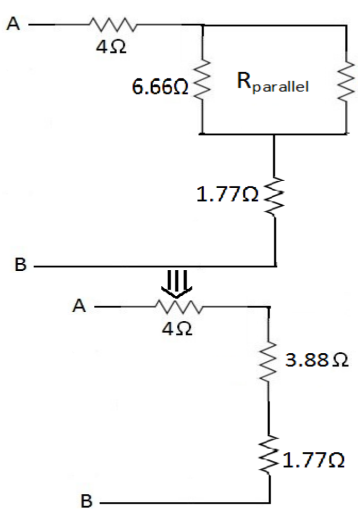
Delta to Star Conversio



Final Steps

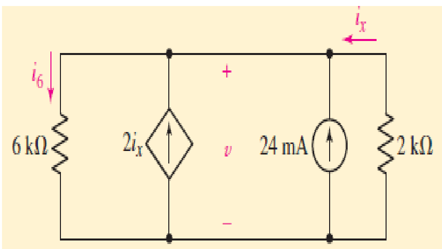


$$R_{parallel} = \frac{6.66 \text{ X } 9.33}{6.66+9.33} = 3.88\,\Omega$$

Req=9.65Ω

Problem Based on KCL with Dependent Current Source

- Determine the value of v?



Source: W. H. Hayt et al., *Engineering Circuit Analysis*, McGrawHill, 2012

- Applying KCL at the two upper nodes of the two current sources, we get:

$i_6 - 2\,i_x - 0.024 - i_x = 0$ (1)

- We next apply Ohm's law to each resistor:

$i_6 = \frac{v}{6000}$ and $i_x = \frac{-v}{2000}$ (2)

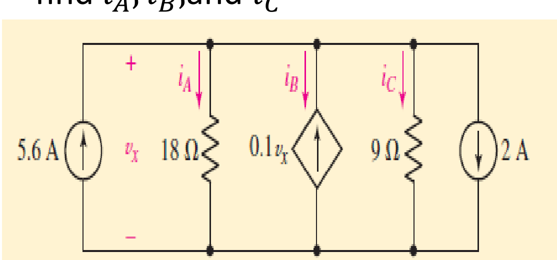
- Therefore, on substituing (2) into (1) we get,

$$\frac{v}{6000} - 2\left(\frac{-v}{2000}\right) - 0.024 - \left(\frac{-v}{2000}\right) = 0$$

and so $v = (600)(0.024) = 14.4\text{v}$

Problem Based on KCL with Dependent Current Source

- For the circuit shown below, find i_A, i_B,and i_C



Source: W. H. Hayt et al., *Engineering Circuit Analysis*, McGrawHill, 2012

- Let us first determine v_x
- Applying KCL , we get:

$(-5.6) + \frac{v_x}{18} - 0.1v_x + \frac{v_x}{9} + 2 = 0$

$$\Longrightarrow v_x = \frac{64.8}{1.2} = 54V$$

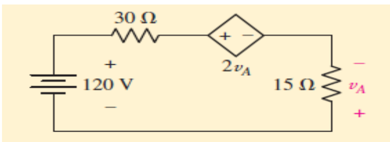
$$\therefore i_A = \frac{v_x}{18} = 3A,$$

$i_B = -0.1v_x = -5.4\text{A}$

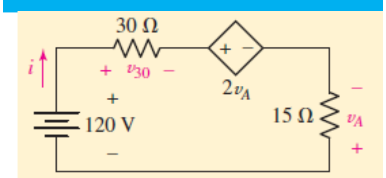
and $i_C = \frac{v_x}{9} = 6A$

Problem Based on KVL with Dependent Voltage Source

- Compute the voltage v_A for the circuit shown



- We first assign a reference direction for the current i and a reference polarity for the voltage v_{30} as shown below



Source: W. H. Hayt et al., *Engineering Circuit Analysis*, McGrawHill, 2012

- There is no need to assign a voltage to the 15Ω resistor, since the controlling voltage v_A for the dependent source is already available.
- This circuit contains a dependent voltage source, the value of which remains unknown until we determine v_A
- However, its algebraic value 2 v_A can be used in the same fashion as if a numerical value were available. Thus, applying KVL around the loop:

$-120 + v_{30} + 2\,v_A - v_A = 0$ (1)

- Using Ohm's law to introduce the known resistor values:

$v_{30} = 30i$ and $v_A = -15i$ (2)

- Note that the negative sign is required since i flows into the negative terminal of v_A
- Substituting (2)into (1) yields

$-120 + 30i - 30i + 15i = 0$ (3)

- and so we find that

$i = 8A$ (4)

- Therefore, $v_A = -120\text{V}$

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