Note Title 01-03-2021

APL 100

Lecture 1

Classical Mechanics $v < \frac{c}{10}$ ($c \to$ speed of light)

Newtons Laws ? eg. point of application of force in 2nd law etc.

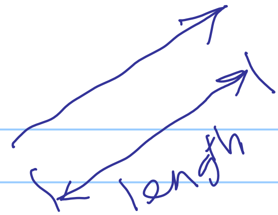
$\Rightarrow$ Euler's Axioms

Preliminary concepts.

Vectors

Scalar $\to$ an entity that depends only on its magnitude but is independent of the coordinate system

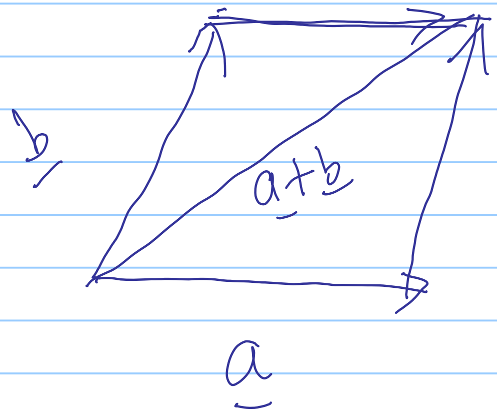
Vector: Directed line segment



$\vec{a} \equiv \underline{a}$

magnitude & direction important

Follow the parallelogram addition law

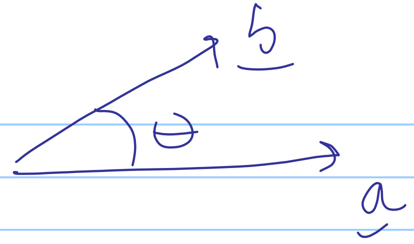


$\vec{a}+\vec{b} \rightarrow$ given by the diagonal of the parallelogram

some entities with direction & magnitude, but do not obey this law are <u>not</u> vectors eg. angular rotation

Various operations addition (as above)

$\bar{a} \cdot \bar{b} = |\bar{a}|\,|\bar{b}| \cos\theta$



$\underline{a} \times \underline{b} = |\underline{a}|\,|\underline{b}| \sin\theta$

$\underline{i}, \underline{j}, \underline{k}$ unit vectors in x, y, z directions

then $\underline{a} \times \underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$

$\underline{a} = a_x \underline{i} + a_y \underline{j} + a_z \underline{k}$

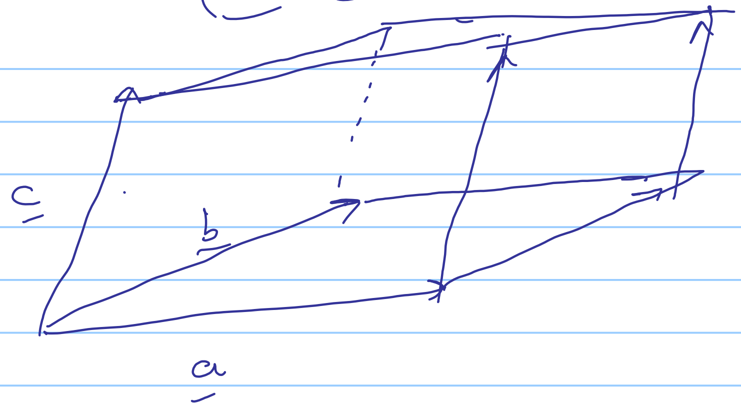
$\underline{b} = b_x \underline{i} + b_y \underline{j} + b_z \underline{k}$

Can define vector triple product

& scalar triple product

Scalar triple product $\underline{a} \cdot (\underline{b} \times \underline{c}) = (\underline{a} \times \underline{b}) \cdot \underline{c}$

$$= \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$



= volume of the parallelepiped.

<u>Vector triple product</u>

$$(\underline{a} \times \underline{b}) \times \underline{c} \neq \underline{a} \times (\underline{b} \times \underline{c})$$

$$\underline{a} \times (\underline{b} \times \underline{c}) = (\underline{a} \cdot \underline{c})\, \underline{b} - (\underline{a} \cdot \underline{b})\, \underline{c}$$

$$(\underline{a} \times \underline{b}) \times \underline{c} = (\underline{a} \cdot \underline{c})\, \underline{b} - (\underline{b} \cdot \underline{c})\, \underline{a}$$

$\underline{i}$, $\underline{j}$, $\underline{k}$ are unit vectors in the Cartesian coordinate system. Basis vectors Mutually orthogonal.

$$\underline{A}(t) = a_x(t)\,\underline{i} + a_y(t)\,\underline{j} + a_z(t)\,\underline{k}$$

arbitrary vector $\underline{A}$

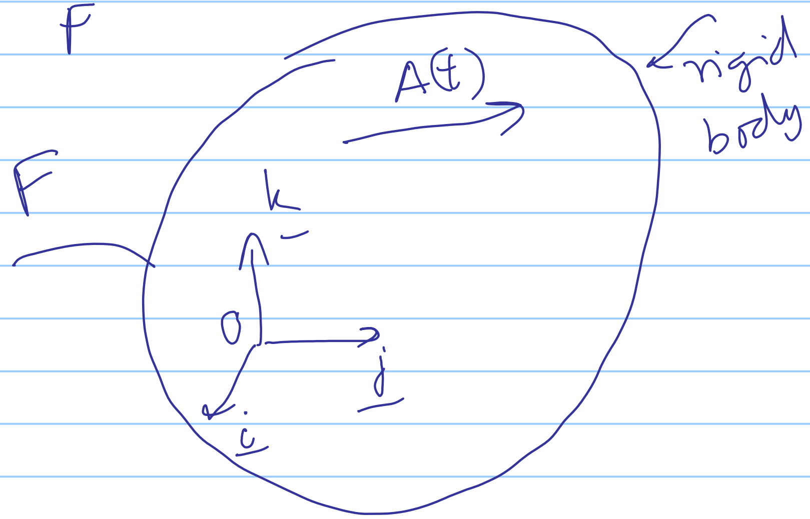
Time derivative?

Depends of the <u>Frame of reference</u>

Frame of reference / Reference frame is a set of points in 3D having <u>invariant</u> distances between them, with Euclidean geometry being valid.

Can attach the reference frame to any rigid body.

Embed rectangular / cartesian coordinates can be embedded in the frame $\to$ denoted by F



$$\dot{A}(t)\Big|_F = \frac{dA(t)}{dt}\Big|_F = \dot{a}_x(t)\,\underline{i} + \dot{a}_y(t)\,\underline{j} + \dot{a}_z(t)\,\underline{k}$$

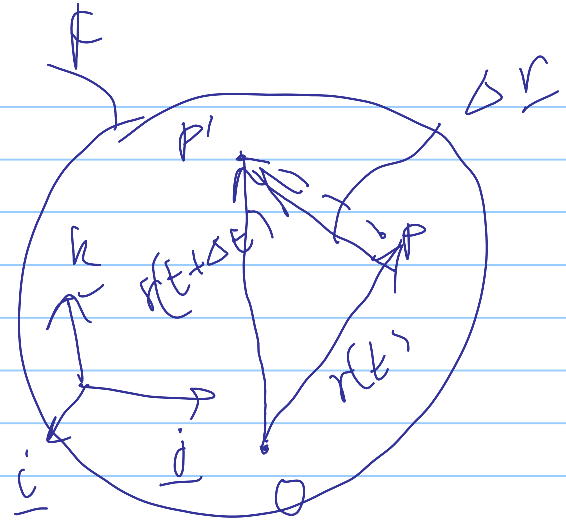
These depend on F but not on the choice of the origin O or its orientation.

Kinematics of a point

$P - P' \rightarrow$ locations of the point in F at times t & $t+\Delta t$

positions of P and P' are given wrt the origin O by vectors

$\overrightarrow{OP}$ & $\overline{OP'}$ (Position Vectors)



$\underline{r}(t) = \underline{r}_{PO} = \overline{OP}$

$\underline{r}(t+\Delta t) = \underline{r}_{P'O} = \overline{OP'}$

Change in position vector is the displacement

$$\Delta \underline{r} = \underline{r}(t+\Delta t) - \underline{r}(t)$$

$$= \underline{r}_{p'0} - \underline{r}_{p0}$$

Velocity of P $\rightarrow$ limiting value of this rate of change:

$$\underline{v}(t)\big|_F = \lim_{\Delta t \to 0} \frac{PP'}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta \underline{r}}{\Delta t} = \dot{\underline{r}}\big|_F$$

$$\underline{v}\big|_F = \dot{\underline{r}}\big|_F$$

Illy

$$\underline{a}\big|_F = \ddot{\underline{r}}\big|_F = \dot{\underline{v}}\big|_F$$

$\underline{v}|_F$ & $\underline{a}|_F$ depend on the frame of reference F but not the choice of origin or orientation of axes.

Can integrate the above expressions:

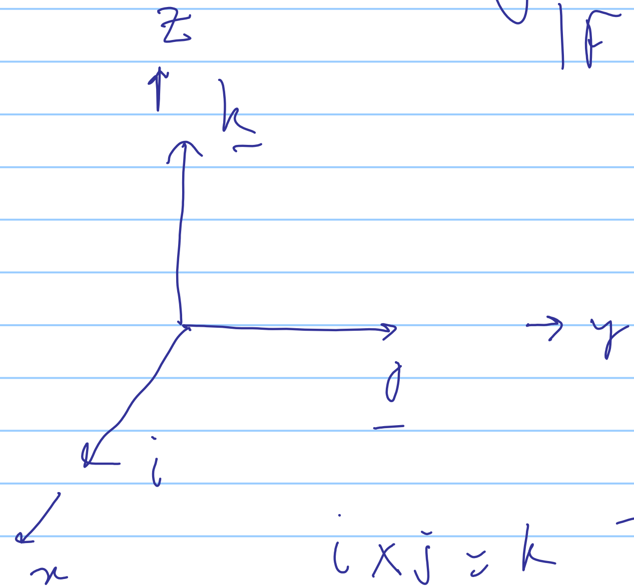
$$\underline{r}(t) = \underline{r}(0) + \int_0^t \underline{v}(t)\,dt$$

$$\underline{v}(t) = \underline{v}(0) + \int_0^t \underline{a}(t)\,dt$$

$(\ )|_F$ is not written (understood)

$\underline{v}|_F \equiv \underline{v}$

Cartesian Coordinates



$$\underline{r} = x(t)\,\underline{i} + y(t)\,\underline{j} + z(t)\,\underline{k}$$

$$\underline{v} = \frac{d\underline{r}}{dt} = \dot{x}(t)\,\underline{i} + \dot{y}(t)\,\underline{j} + \dot{z}(t)\,\underline{k}$$

$$\underline{a} = \frac{d\underline{v}}{dt} = \ddot{x}(t)\,\underline{i} + \ddot{y}(t)\,\underline{j} + \ddot{z}(t)\,\underline{k}$$

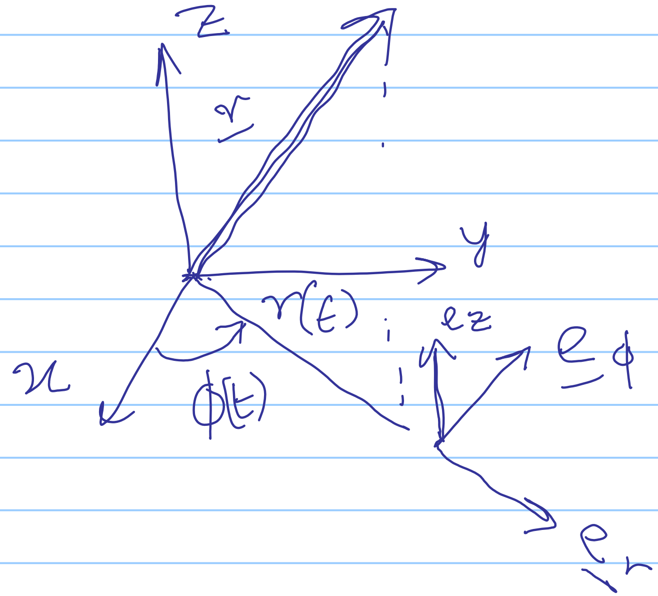
$$\left.\begin{aligned} \underline{i} \times \underline{j} &= \underline{k} \\ \underline{j} \times \underline{k} &= \underline{i} \\ \underline{k} \times \underline{i} &= \underline{j} \end{aligned}\right\} \text{ordered triplet}$$

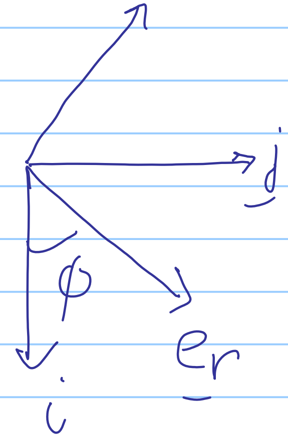
$v_x = \dot{x}, \quad v_y = \dot{y}, \quad v_z = \dot{z}$

$a_x = \ddot{x} = \dot{v}_x, \quad a_y = \ddot{y} = \dot{v}_y, \quad a_z = \ddot{z} = \dot{v}_z.$

$\underline{e}_z = \underline{k}$

<u>Cyl. polar coordinates</u>





$\underline{e}_r \times \underline{e}_\phi = \underline{e}_z$

$\underline{e}_\phi \times \underline{e}_z = \underline{e}_r$

$\underline{e}_z \times \underline{e}_r = \underline{e}_\phi$

$\underline{e}_r = \cos\phi\, \underline{i} + \sin\phi\, \underline{j}$

$\underline{e}_\phi = -\sin\phi\, \underline{i} + \cos\phi\, \underline{j}$

$\underline{e}_z = \underline{k}$

$\phi = \phi(t)$; $\quad \underline{e}_r, \underline{e}_\phi$ are <u>not</u> <u>constant</u> !

$$\overset{\circ}{\underline{e}}_r \qquad \left(\overset{\circ}{(\;)}\Big|_F \text{ is implied !} \right)$$

$$\overset{\circ}{\underline{e}}_r = \left(-\sin\phi\, \underline{i} + \cos\phi\, \underline{j}\right)\dot{\phi} = \dot{\phi}\, \underline{e}_\phi$$

$$\overset{\circ}{\underline{e}}_\phi = \left(-\cos\phi\, \underline{i} - \sin\phi\, \underline{j}\right)\dot{\phi} = -\dot{\phi}\, \underline{e}_r$$

$$\overset{\circ}{\underline{e}}_z = 0$$

$$\Rightarrow \quad \underline{r}(t) = r\,\underline{e}_r + z\,\underline{e}_z$$

$$\underline{v}(t) = \overset{\circ}{\underline{r}}(t) = \dot{r}\,\underline{e}_r + r\,\overset{\circ}{\underline{e}}_r + \dot{z}\,\underline{e}_z + z\,\cancel{\overset{\circ}{\underline{e}}_z}^{\,0}$$

$$= \dot{r}\, e_r + r\dot{\phi}\, e_\phi + \dot{z}\, e_z$$

$$a(t) = \dot{v}(t) = \ddot{r}\, \underline{e}_r + \dot{r}\, \underline{\dot{e}}_r + \dot{r}\dot{\phi}\, e_\phi + r\ddot{\phi}\, e_\phi$$

$$+ r\dot{\phi}\, \underline{\dot{e}}_\phi + \ddot{z}\, \underline{e}_z + 0$$

$$= (\ddot{r} - \dot{\phi}^2 r)\, \underline{e}_r + (r\ddot{\phi} + 2\dot{\phi}\dot{r})\, \underline{e}_\phi + \ddot{z}\, e_z$$

$$\underbrace{a_r \neq \dot{v}_r \,; \quad a_\phi \neq \dot{v}_\phi}_{} \qquad a_z = \dot{v}_z \checkmark$$

!