Harshit Mawandia 2020CS10348

Q1) $a_n = \frac{(-1)^n(n+5)}{n}$

$a_{2n} = \left(\frac{n+5}{n}\right)$ (even terms)

$a_{2n+1} = -\frac{(n+5)}{n}$ (odd terms)

$\lim_{n\to\infty} a_{2n} = \lim_{n\to\infty}\left(1+\frac{5}{n}\right) = 1$

$\left(\text{as } \frac{1}{n} \to 0 \text{ as } n\to\infty\right)$

$\lim_{n\to\infty} a_{2n+1} = \lim_{n\to\infty} -\left(1+\frac{5}{n}\right) = -1$

(similarly as a_{2n})

So as $n\to\infty$ $-1 \le a_n \le 1$

since a_n is either -1 or 1

therefore $\lim_{n\to\infty} \inf a_n = -1$

$\lim_{n\to\infty} \sup a_n = 1$

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Q2) ~~We can~~

$x_1 = 1$ $\qquad x_{n+1} = 3 + \frac{1}{x_n}$

We can see that x_n is always positive

$x_{n+1} - x_n = \cancel{(3-3)} + \frac{1}{x_n} - \frac{1}{x_{n-1}}$

$x_{n+1} - x_n = \frac{x_{n-1} - x_n}{\underline{x_{n-1} x_n}}$

always positive and greater than 3

~~If x_{n+1}~~

~~as~~ $x_n > 3$ ~~therefore~~ for $\forall n \geq 1$

therefore $3 < x_{n+1} \leq 4 \quad \forall n > 1$

$\frac{x_{n+1} - x_n}{x_n - x_{n-1}} = \frac{-1}{\underline{x_{n-1} x_n}}$

always positive and greater than 3

$\left| \frac{x_{n+1} - x_n}{x_n - x_{n-1}} \right| < 1$

since $\left(0 > \frac{-1}{x_{n-1} x_n} > -1 \right)$

Therefore sequence is cauchy

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$$\lim_{n\to\infty} x_{n+1} = L$$

$$= 3 + \frac{1}{\lim_{n\to\infty} x_n}$$ $\lim_{n\to\infty} x_n = \lim_{n\to\infty}(x_{n+1}) = L$

$$L = 3 + \frac{1}{L}$$

$$L^2 - 3L - 1 = 0$$

$$\therefore \frac{3 \pm \sqrt{9+4}}{2} = \frac{3 \pm \sqrt{13}}{2}$$

We know that L is positive and $L > 3$

$$\therefore L = \frac{3+\sqrt{13}}{2}$$

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$Q_3 =$ $x_n = (2^n + 3^n)^{1/n}$

$$\lim_{n\to\infty} a_n^{1/n} = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

$$\lim_{n\to\infty} (2^n + 3^n)^{1/n} = \lim_{n\to\infty} \frac{2^{n+1} + 3^{n+1}}{2^n + 3^n}$$

$$= \underbrace{\lim \frac{2^{n+1}}{2^n + 3^n}}_{a} + \underbrace{\lim \frac{3^{n+1}}{2^n + 3^n}}_{b}$$

$$a = \lim_{n\to\infty} \frac{2 \times 2^n}{2^n + 3^n} = \frac{2}{1 + \left(\frac{3}{2}\right)^n}$$

$$\lim_{n\to\infty} \left(\frac{3}{2}\right)^n \longrightarrow \infty \Rightarrow \frac{2}{\infty} = 0$$

$$b = \lim_{n\to\infty} \frac{3^{n+1}}{2^n + 3^n} = \lim_{n\to\infty} \frac{3}{\left(\frac{2}{3}\right)^n + 1}$$

$$\lim_{n\to\infty} \left(\frac{2}{3}\right)^n \longrightarrow 0 \quad \left(\frac{2}{3} < 1\right) \Rightarrow \lim_{n\to\infty} \frac{3}{0+1}$$

$$= 3$$

$$\therefore \lim_{n\to\infty} a_n^{1/n} = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 0 + 3 = 3$$

$= \lim_{n\to\infty} x_n$ (Hence x_n converges)

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Q4) $a_n = \sqrt{n^4+1} - \sqrt{n^4-1} \times \dfrac{\sqrt{n^4+1}+\sqrt{n^4-1}}{\sqrt{n^4+1}+\sqrt{n^4-1}}$

$= \dfrac{n^4+1-(n^4-1)}{\sqrt{n^4+1}+\sqrt{n^4-1}} = \dfrac{2}{\sqrt{n^4+1}+\sqrt{n^4-1}}$

lets assume $b_n = \dfrac{1}{n^2}$

If $\lim_{n\to\infty} \dfrac{a_n}{b_n}$ exists, and the lim is non zero then b_n is convergent $\Longleftrightarrow$ a_n is convergent

$\dfrac{a_n}{b_n} = \dfrac{2 \quad n^2}{\sqrt{n^4+1}+\sqrt{n^4-1}} = \dfrac{2}{\sqrt{1+\dfrac{1}{n^2}}+\sqrt{1-\dfrac{1}{n^2}}}$

$\lim_{n\to\infty} \dfrac{a_n}{b_n} = \dfrac{2}{\sqrt{1}+\sqrt{1}} = 1$ (since $\dfrac{1}{n^2} \to 0$ as $n \to \infty$)

since the lim exists and is not equal to zero and we know $\sum_{n=1}^{\infty} \dfrac{1}{n^2} = \sum_{n=1}^{\infty} b_n$ is convergent we can say then $\sum_{n=1}^{\infty} a_n$ is convergent

Hence Proved