

## Tutorial 3:

Q4.

```
1 VISITED[v] ← True;
2 HIGH-POINT[v] ← LEVEL[v];
3 foreach w ∈ N(v) do
4   if (VISITED[w] = False) then
5     Set PARENT[w] ← v and LEVEL[w] = 1 + LEVEL[v];
6     Invoke DFS(w);
7     HIGH-POINT[v] ← min{HIGH-POINT[w], HIGH-POINT[v]};
8     if HIGH-POINT[w] ≥ LEVEL[v] then
9       IS-CUT-VERTEX[v] ← True;
10    end
11  else if (w ≠ PARENT[v]) then
12    HIGH-POINT[v] ← min{LEVEL[w], HIGH-POINT[v]};
13  end
14 end
```

**Procedure DFS(v)**

```
1 Let (v1, ..., vn) be any ordering of vertices of G;
2 for i = 1 to n do
3   VISITED[vi] ← False and IS-CUT-VERTEX[vi] ← False;
4 end
5 for i = 1 to n do
6   if (VISITED[vi] = False) then
7     LEVEL[vi] ← 0;
8     Invoke DFS(vi);
9     if (vi has one child) then IS-CUT-VERTEX[vi] ← False;
10  end
11 end
```

**Procedure Compute-Cut-vertices(G)**

## Tutorial 4

### Q5.

Solution: We first define the concept of *median*.

**Definition:** Given  $K$  real numbers  $x_1, \dots, x_L$ , the *median* of  $x_1, \dots, x_L$  is a real number  $y$  satisfying  $\sum_{i=1}^L |x_i - y|$  is minimized.

**Fact** (exercise, try on simple examples): If real numbers  $x_1, \dots, x_L$  are sorted in non-decreasing order then  $y = x_{\lfloor L/2 \rfloor}$  and  $y = x_{\lceil L/2 \rceil}$  are *medians* of  $x_1, \dots, x_L$ .

Make a 2D table  $T$  of size  $n \times k$ , where for  $i \in [1, n]$  and  $j \in [1, k]$ , the value  $T[i, j]$  denotes the minimum sum total distance traveled for the sub-problem with first  $i$  residents if  $j$  testing centers are to be opened.

Note that <sup>if</sup> the  $j$  center positions were already decided, then the residents can be partitioned into  $j$  intervals such that for the first interval first covid-test center is closest, for the second interval second covid-test center is closest, and so on. Based upon this observation, the recursive approach to build table  $T$  is as follows:

$$T(i, j) = \min_{\alpha \in [1, i]} \left( T(\alpha - 1, j - 1) + \sum_{r=\alpha}^i \left| A[r] - A\left[\left\lceil \frac{\alpha+i}{2} \right\rceil\right] \right| \right)$$

In the above formulation for having  $j$  centers for first  $i$  resident, we are trying all possibilities for the last interval, that is, corresponding to  $j^{th}$  test center. For the optimal choice of  $\alpha$ , the median of resident locations  $A[\alpha], \dots, A[i]$ , that is,  $A\left[\left\lceil \frac{\alpha+i}{2} \right\rceil\right]$  is the optimal position for  $j^{th}$  test center.