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Q1)

a) i) CDF of x and Y are same

ii) Characteris function of x and Y are same

iii) ~~$X(\omega) = Y(\omega) \ \forall \ \omega \in \Omega$~~

b) ii) $Cov(X,Y) = 0$

c) iii) $Q((-\infty, n)) = P\{X \le x\}$

iv) $Q: R \ [0,1]$

d) i) X_1 and X_2 are independent

ii) $g(x_1)$ and $g(x_2)$ are independent

e) i) $P\{X \le 1\} = \dfrac{1}{2}$

ii) $P\{X = 1\} = \dfrac{1}{6}$

Q2) a)

$$\int_0^1 x(\alpha + \beta x^2)\,dx = \frac{3}{7}$$

$$\left[\frac{\alpha x^2}{2} + \frac{\beta x^4}{4}\right]_0^1 = \frac{3}{7}$$

$$\frac{\alpha}{2} + \frac{\beta}{4} = \frac{3}{7} \qquad\qquad —①$$

$$\int_0^1 \alpha x + \beta x^2\,dx = 1$$

$$\left[\alpha x + \frac{\beta x^3}{3}\right]_0^1 = 1$$

$$\alpha + \frac{\beta}{3} = 1 \qquad\qquad —②$$

$$\frac{1}{2} - \frac{\beta}{6} + \frac{\beta}{4} = \frac{3}{7}$$

$$\frac{1}{2} + \frac{\beta}{12} = \frac{3}{7}$$

$$\frac{\beta}{12} = -\frac{1}{14}$$

$$\beta = -\frac{6}{7}$$

$$\alpha = 1 + \frac{6\cdot 2}{2\cdot 7}$$

$$= \frac{9}{7}$$

$$\alpha = \frac{9}{7}, \quad \beta = -\frac{6}{7}$$

b)

$$\int_{-\infty}^{\infty} x f(x) \, dx \quad \Longrightarrow \quad 3 \qquad \sum_{k} x_k P_x(x_k) = 3$$

$$\int_{\infty}^{\infty} e^{\cdot t_k} f(x) \, dx = (\alpha + \beta e^{7t}) \quad 7\alpha + \beta + 0$$

$$\beta \quad \frac{3}{7}$$

$$\sum_{k} e^{t x_k} P_x(x_k) = \alpha + \beta e^{7t}$$

$$\frac{3}{7} \times e^{7t} * \frac{4}{7}$$

$$\boxed{\alpha = \frac{4}{7} \qquad \beta = \frac{3}{7}}$$

$$PMF, \quad P\{x = k\} = \begin{cases} 3/7 & ; \quad k = 7 \\ \frac{4}{7} & ; \quad k = 0 \end{cases}$$

c) $E[x] = \dfrac{50^2 + 30^2 + 20^2 + 48^2}{148} \approx 41.243$

$E[y] = \dfrac{50 + 30 + 20 + 48}{4} = 37$

d) ⊘ Pmf

$$P\langle x \mid Y = 3\rangle = \begin{cases} \dfrac{1}{2} & \text{if } x = 0 \\[2mm] \dfrac{1}{2} & \text{if } x = 3 \end{cases}$$

e) $P(X_1 + X_2 = k)$

$$\text{Pmf}(X_1 + X_2) = \begin{cases} \dfrac{1}{4} & k = 0 \\[2mm] \dfrac{1}{12} + \dfrac{1}{6} = \dfrac{\theta 3}{4} = \dfrac{1}{2} & k = 1 \\[2mm] \dfrac{1}{36} + \dfrac{1}{9} + \dfrac{2}{18} = \dfrac{1}{4} & k = 2 \end{cases}$$

Q3) i)

$$f(x) = \begin{cases} 4e^{-4x} & \text{if } (x > 0) \\ 0 & \text{if } x \le 0 \end{cases}$$

$$u = x + y \qquad v = x - y$$

$$g(x, y) = (x+y, \; x-y) \qquad x = \frac{U+V}{2} \qquad y = \frac{U-V}{2}$$

$$h(x, y) = \left(\frac{x+y}{2}, \; \frac{x-y}{2} \right)$$

$$J = -\frac{1}{2} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} \Rightarrow |J| = \frac{1}{2}$$

$$f(u, v) = \frac{1}{2} \cdot 4 \times 4 \times e^{-4(x+y)}$$

Q3) 2)

find probability when $y > x$

$P(x, y)$

$= \{x > y\}$

$$= \int_0^5 \int_x^6 f(x) \cdot f(y) \, dy \, dx$$

$$= \int_0^5 \int_x^6 \frac{18}{5} \times \frac{1}{6} \, dy \, dx$$

$$= \int_0^5 \left[\frac{y}{30} \right]_x^6 \, dx$$

$$= \int_0^5 \frac{6 - x}{30} \, dx$$

$$= \left[\frac{x}{5} - \frac{x^2}{60} \right]_0^5$$

$$= 1 - \frac{25}{60} \, \frac{5}{12}$$

$$= \frac{7}{12}$$

Q3) ③

$$f(x) = \begin{cases} 1/5 & \text{if } 0 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

$$U = \frac{x}{1+x}$$

$$U = x(1-u)$$

$$x = \frac{U}{1-u} = x$$

$$f(u) = \begin{cases} 1/5 & \text{if } 0 \le \frac{u}{1-u} \le 5 \\ 0 & \text{else } 0 \end{cases}$$

$$= \begin{cases} 1/5 & \text{if } 0 \le u \le 5 - 5u \\ 0 & \text{else } 0 \end{cases} \quad \text{given } [u < 1]$$

$$= \begin{cases} 1/5 & \text{if } 0 \le u \le \frac{5}{6} \\ 0 & \text{else } 0 \end{cases}$$

if $u > 1$

$$= \begin{cases} 1/5 & \text{if } 0 \ge u \ge \frac{5}{6} \\ 0 \end{cases}$$

$$= 0$$

$$\therefore f(u) = \begin{cases} 1/5 & \text{if } 0 \le u \le 5/6 \\ 0 & \text{else } 0 \end{cases}$$