



# ELL100: INTRODUCTION TO ELECTRICAL ENGG.

## **3 Phase AC Power Circuits**

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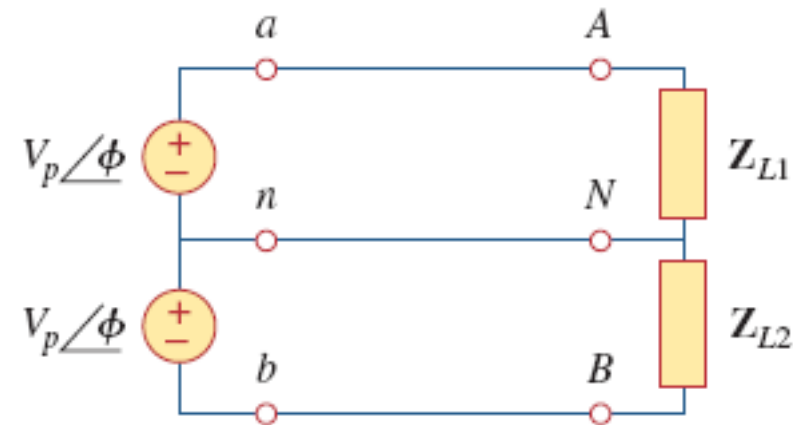
Department of Electrical Engineering, IITD

# Single Phase Circuits

- Single Phase AC Power: Power source (generator) and load connected via *a pair* of wires.
- Or Combination of sources which are ***in phase***.



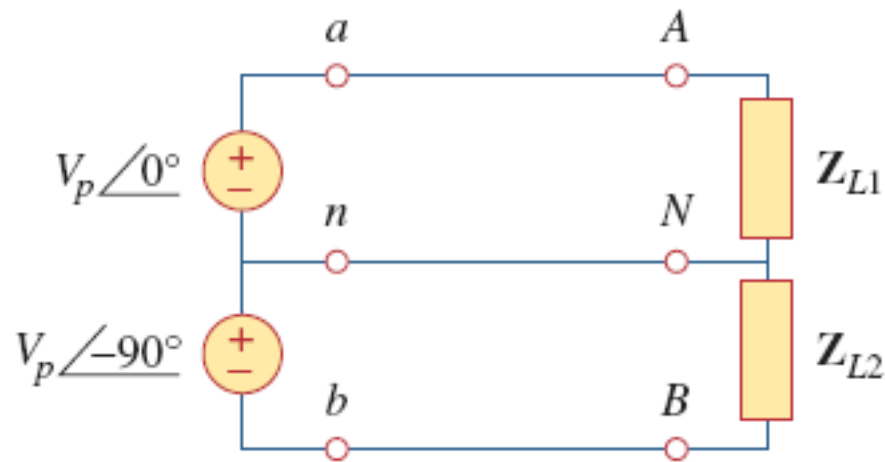
Single phase two wire type



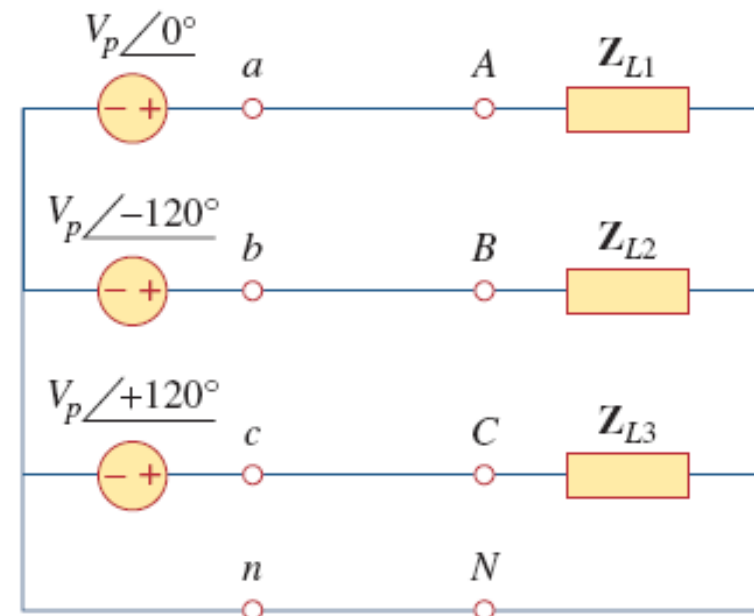
Single phase three wire type

# Polyphase Circuits

- Polyphase AC Power: Sources operate at **same frequency** but are **not in phase**
- Two phase system has a phase difference 90 degrees
- Three phase system has a phase difference of 120 degrees



Two phase three-wire system



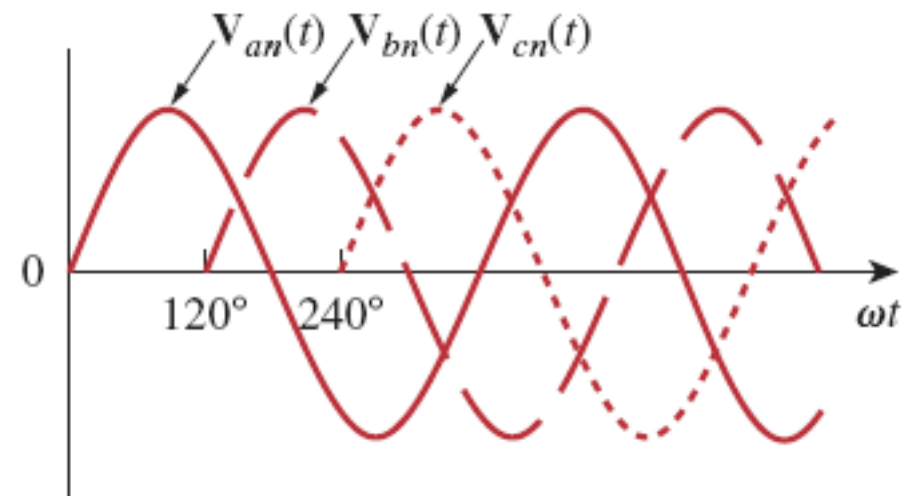
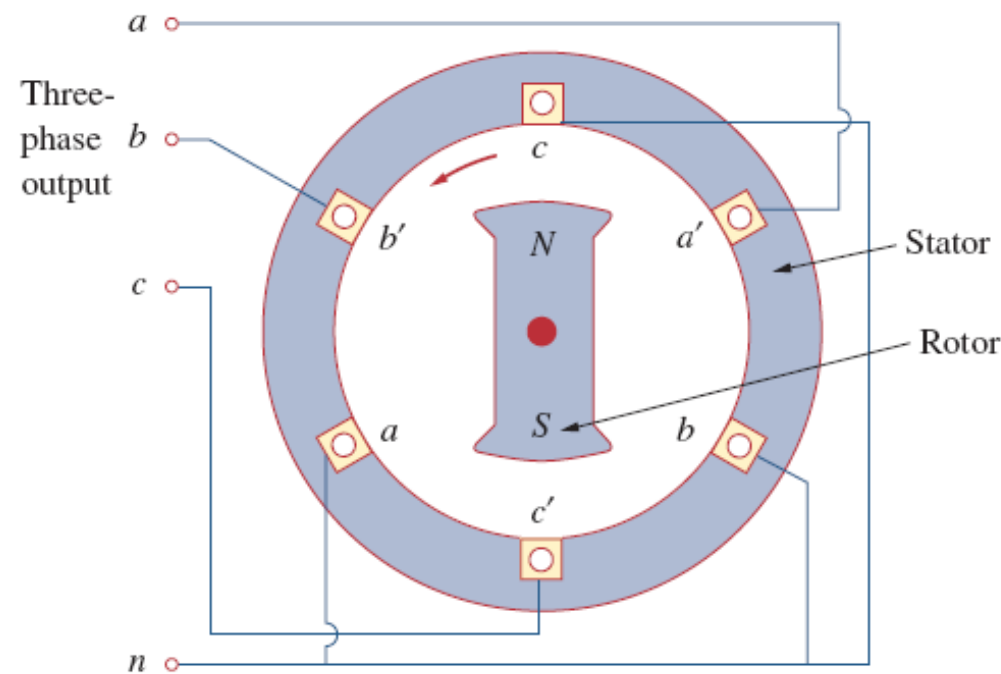
Three phase four-wire system

# Polyphase Circuits: Advantages

- One/Two phase inputs can be taken out from a three phase supply.
- Capable of delivering constant instantaneous power.
- More economical than single phase in power delivery. Less amount of wire required.
  - Increasing the number of phases, increases efficiency but also increases complexity of transmission.
  - A 'balance' is found when the number of phases is **three**.

# Balanced Three Phase Voltage

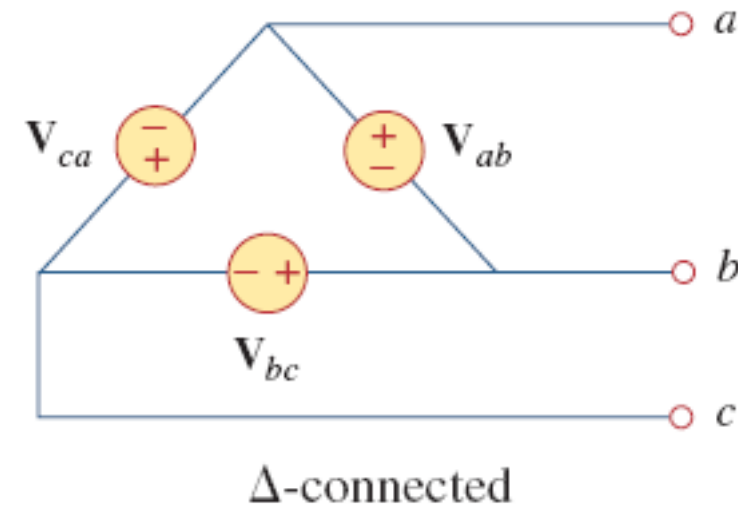
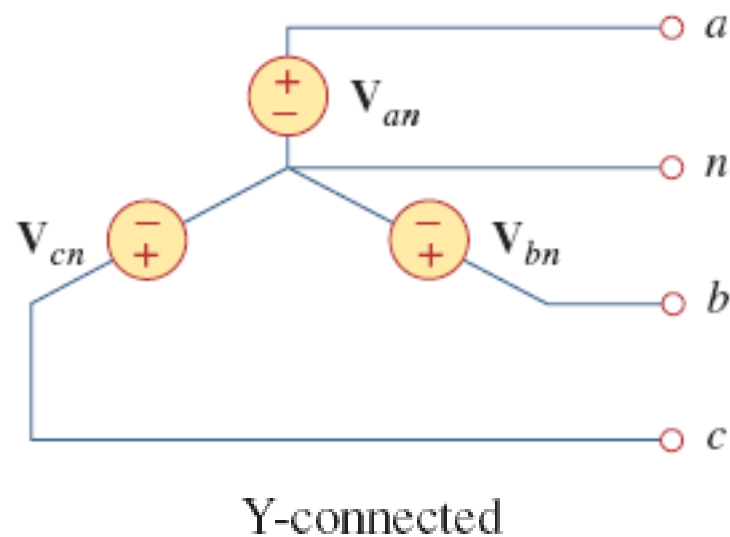
- A three phase supply is typically produced from a three phase generator whose stator coils are placed 120 deg apart.



- The three voltages are equal in magnitude but out of phase by 120 deg.

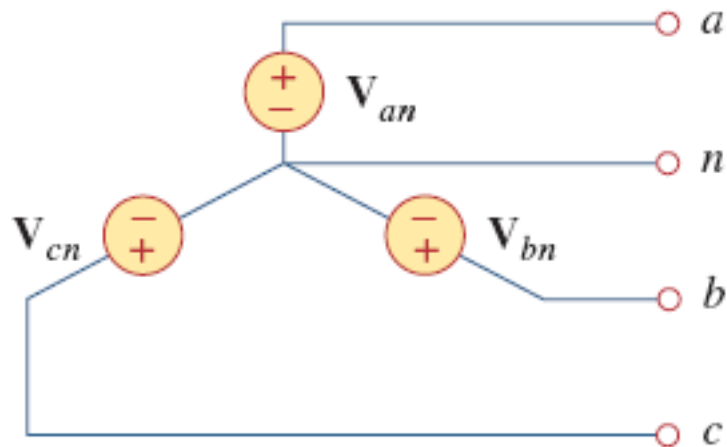
# Three Phase System

- Typical 3-phase system has 3 voltage sources connected to loads via 3 or 4 wires
- The sources (and the loads) can be either Delta-connected or Y-connected



# Balanced Three Phase Source

- Consider the Wye connected source

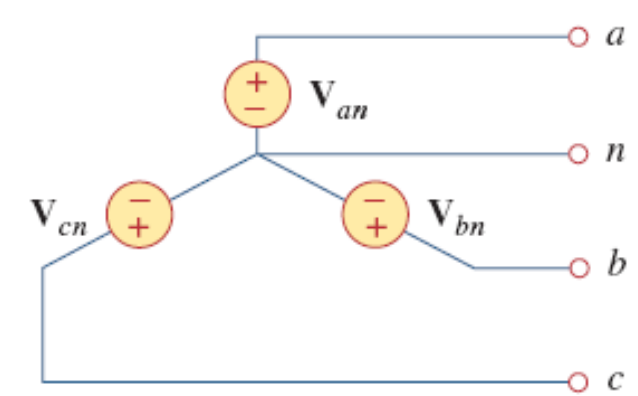


- Voltages  $V_{an}$ ,  $V_{bn}$ ,  $V_{cn}$  are voltages between the **lines** a, b, c and the **neutral line** n. They are called **phase voltages**.
- The voltages between the lines, i.e.,  $V_{ab}$ ,  $V_{bc}$ ,  $V_{ca}$  are called as **line voltages**.
- The source is said to be **balanced** if
  - Sources are of same amplitude and frequency and are out of phase with each other by 120 degrees.

$$V_{an} + V_{bn} + V_{cn} = 0$$

$$|V_{an}| = |V_{bn}| = |V_{cn}|$$

# Phase Sequence



- There are two possible ways in which a source can be balanced

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$

$$|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

$$\mathbf{V}_{an} = V_p \angle 0^\circ$$

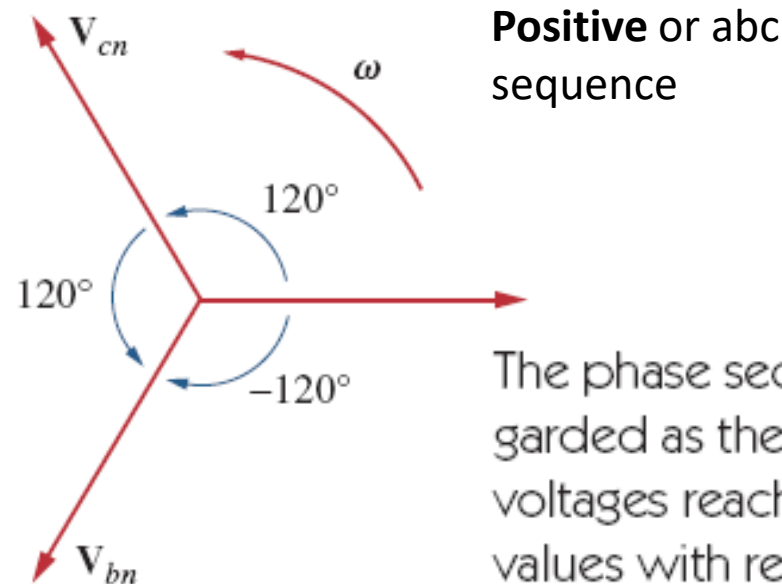
$$\mathbf{V}_{bn} = V_p \angle -120^\circ$$

$$\mathbf{V}_{cn} = V_p \angle -240^\circ = V_p \angle +120^\circ$$

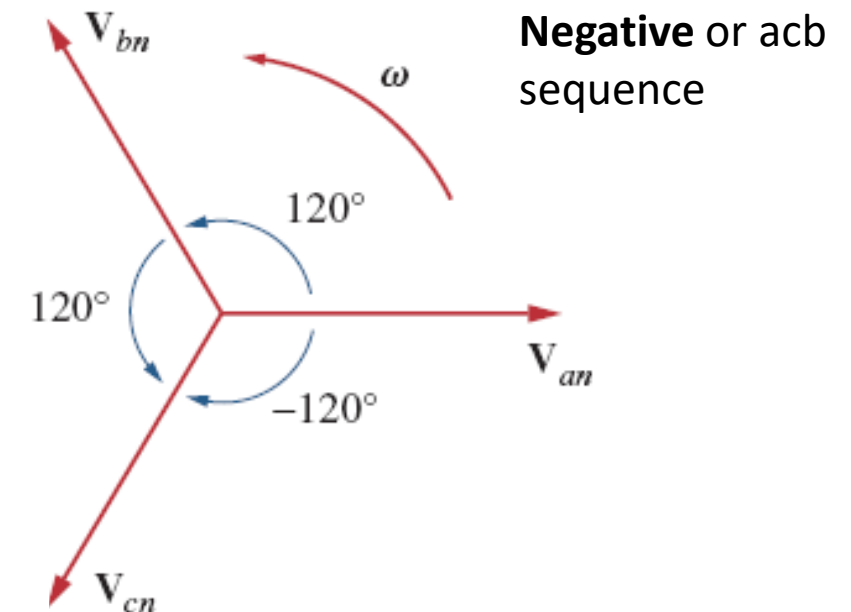
$$\mathbf{V}_{an} = V_p \angle 0^\circ$$

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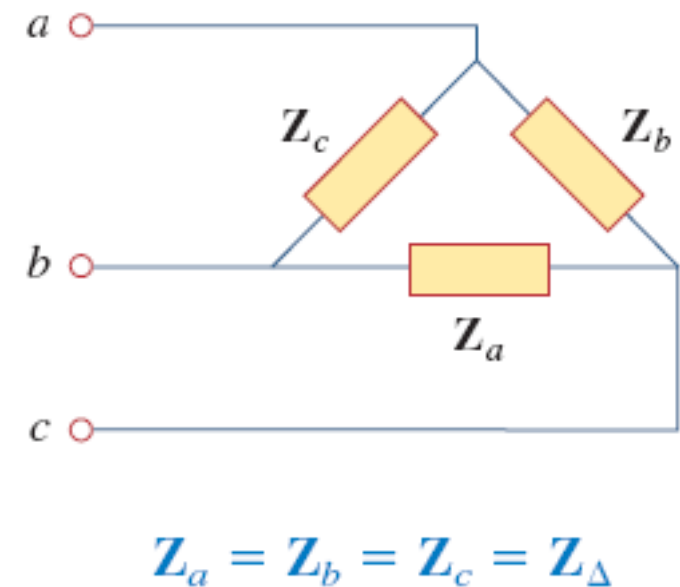
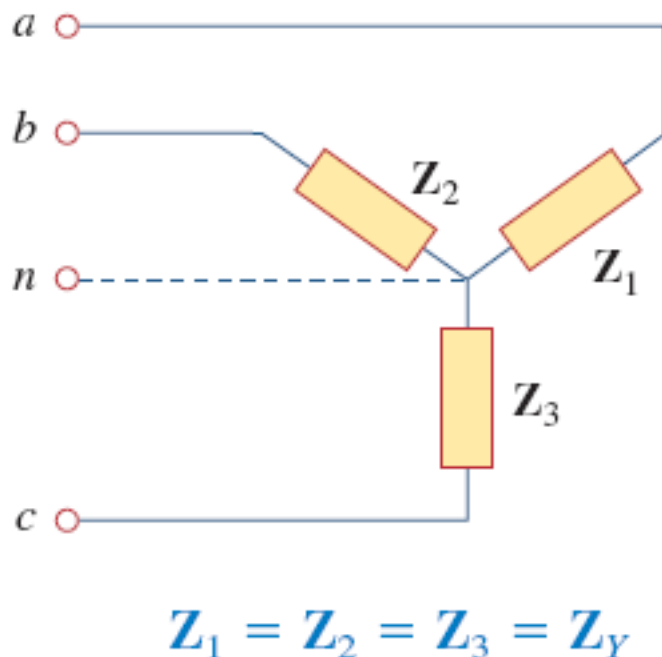
The phase sequence may also be regarded as the order in which the phase voltages reach their peak (or maximum) values with respect to time.





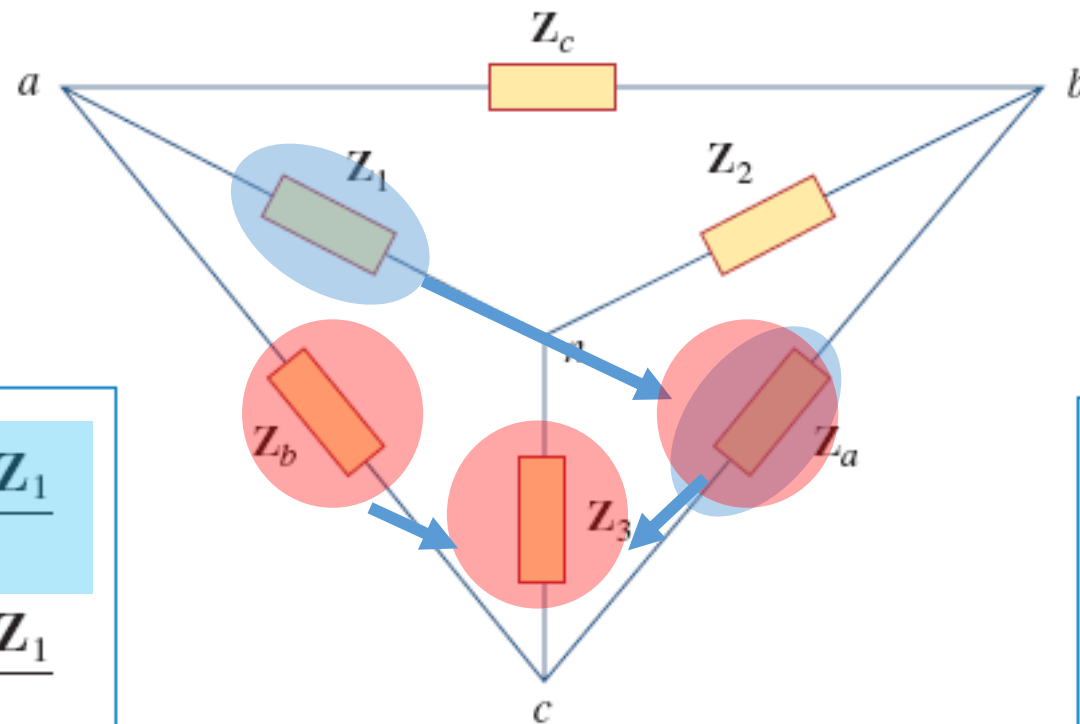
# Balanced Three Phase Load

- Similar to the source, the load can also be Delta or Wye connected.
- A balanced load is one in which the phase impedances are equal in magnitude and in phase.



# $Y \leftrightarrow \Delta$ Delta Conversion for the loads

- Recalling the Y-Delta conversions (in two port networks)



*Y- $\Delta$  Conversion:*

$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1}$$

$$Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2}$$

$$Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3}$$

*$\Delta$ -Y Conversion:*

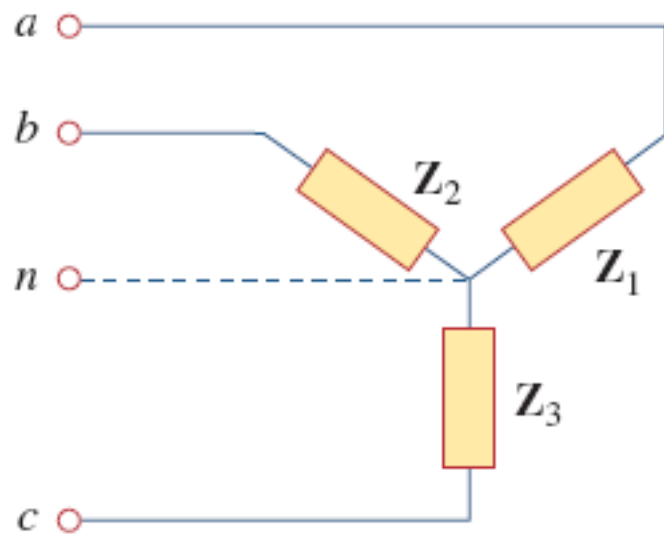
$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_c Z_a}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$

# Balanced Three Phase Load

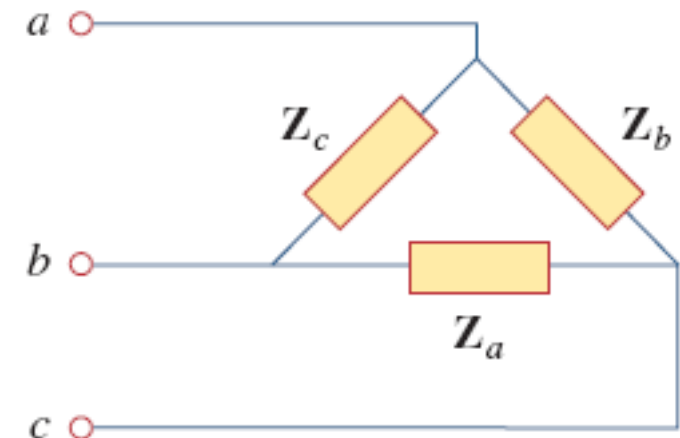
- Similar to the source, the load can also be Delta or Wye connected.
- A balanced load is one in which the phase impedances are equal in magnitude and in phase.



$$Z_1 = Z_2 = Z_3 = Z_Y$$

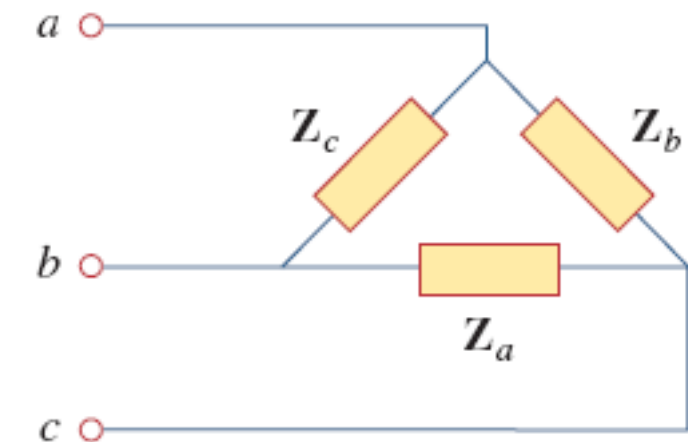
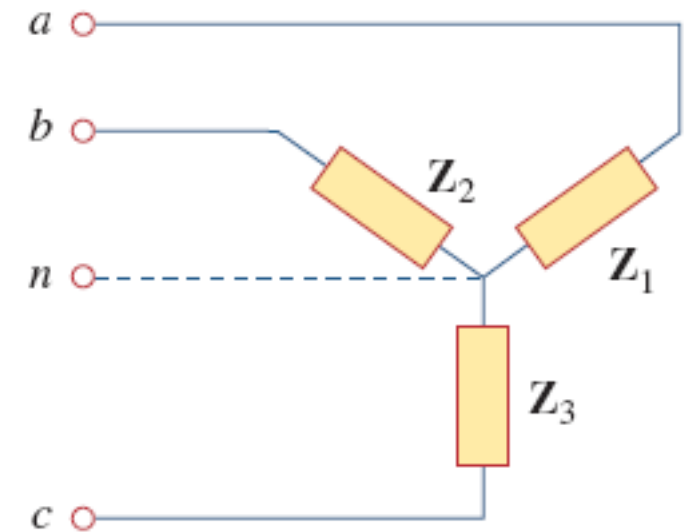
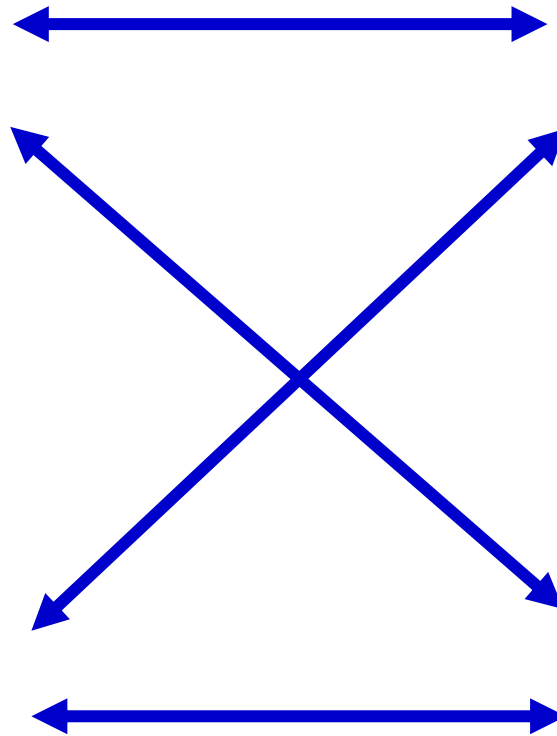
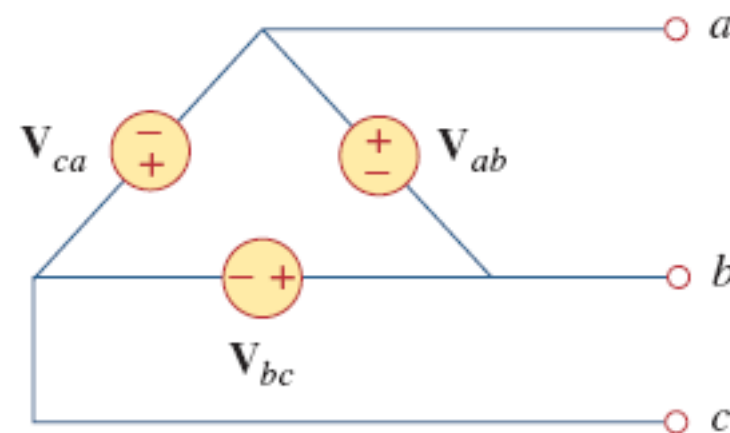
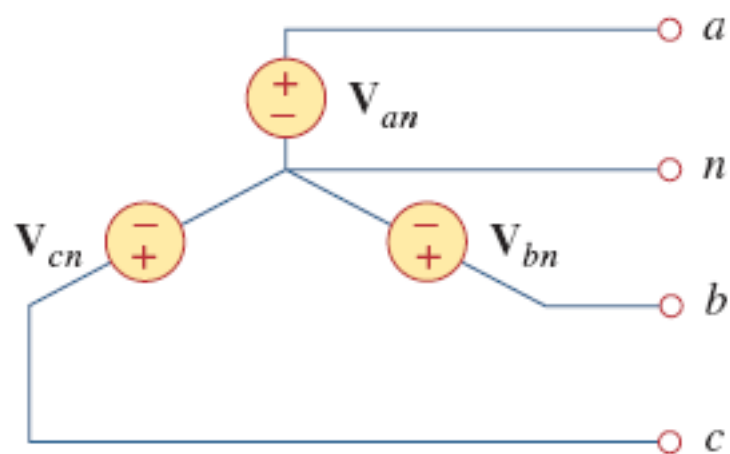
When load is balanced  
equivalent load is

$$Z_{\Delta} = 3Z_Y \quad \text{or} \quad Z_Y = \frac{1}{3} Z_{\Delta}$$



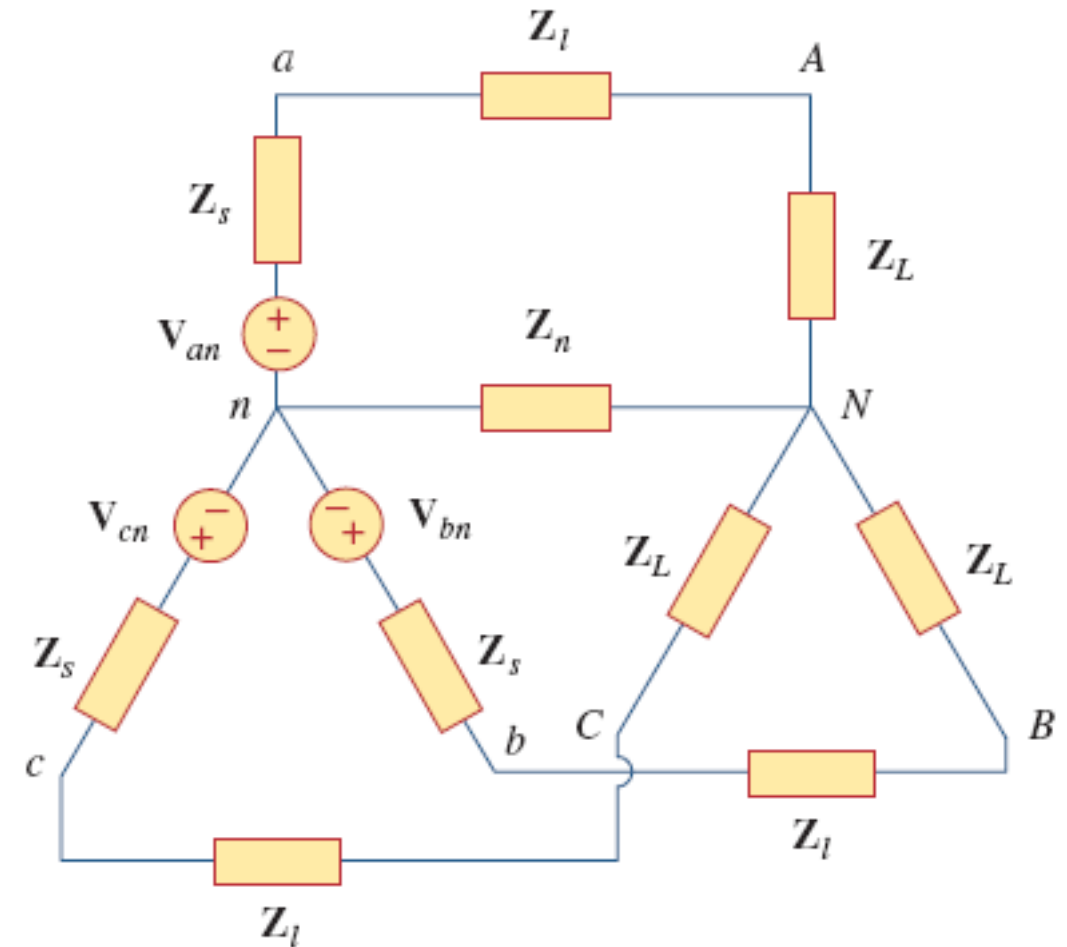
$$Z_a = Z_b = Z_c = Z_{\Delta}$$

# Configurations in 3 Phase Circuits



# Balanced Y-Y Connection

- A balanced Y-Y system is easy to analyze.
- Other configurations can be reduced to Y-Y configuration.
- Consider the 3 phase 4 wire Y-Y connection as shown.



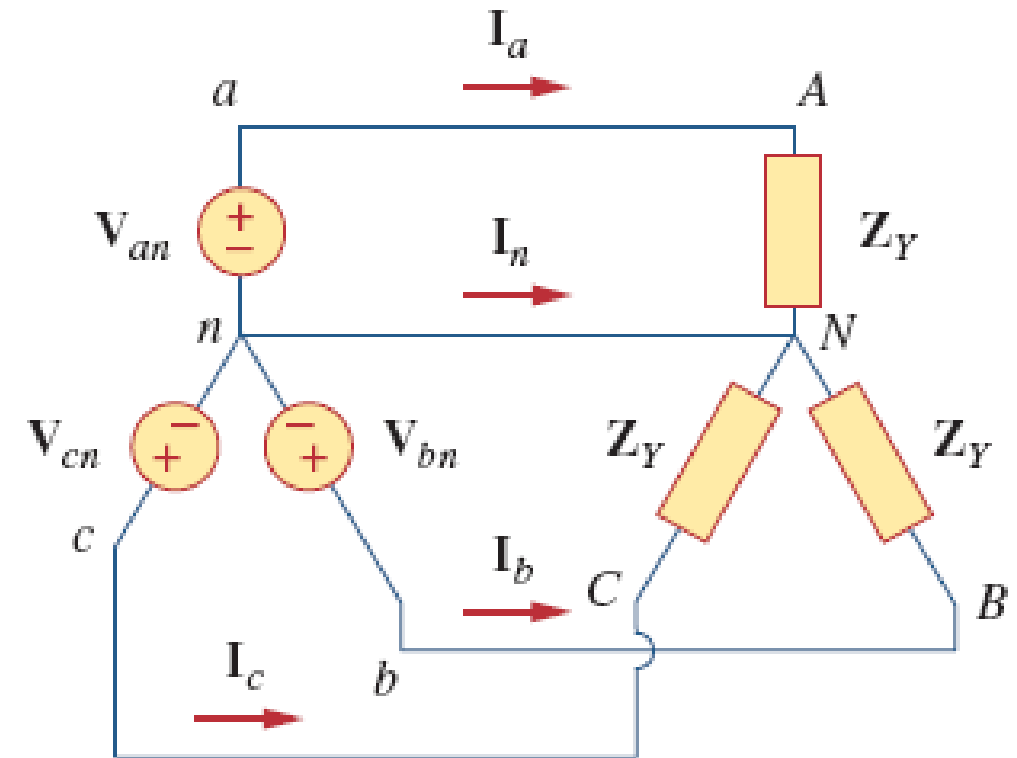
# Balanced Y-Y Connection

- Consider the 3 phase 4 wire Y-Y connection as shown.

$$Z_Y = Z_s + Z_\ell + Z_L$$

- The source *phase* voltages are

$$\begin{aligned} V_{an} &= V_p \angle 0^\circ \\ V_{bn} &= V_p \angle -120^\circ, \quad V_{cn} = V_p \angle +120^\circ \end{aligned}$$



# Balanced Y-Y Connection

- Consider the 3 phase 4 wire Y-Y connection as shown.

$$Z_Y = Z_s + Z_\ell + Z_L$$

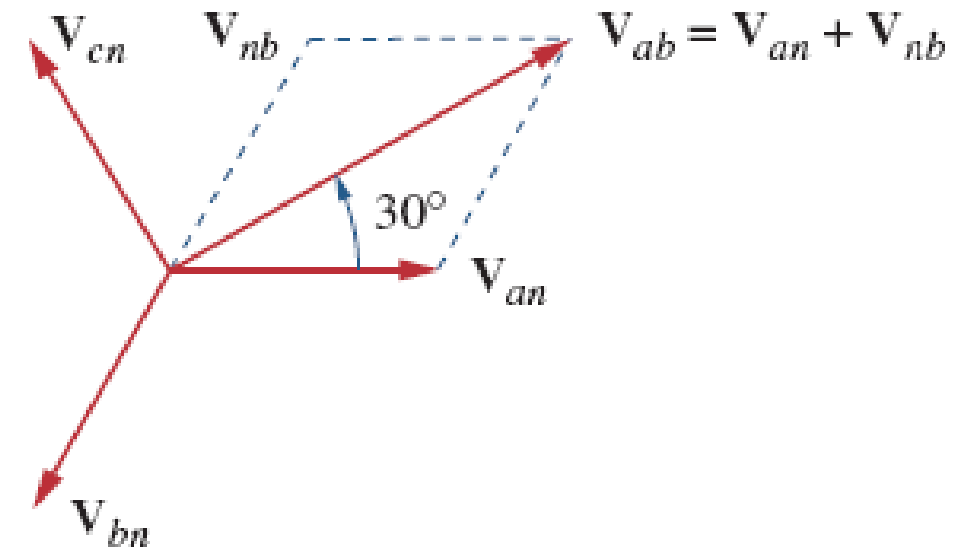
- The source *phase* voltages are

$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ, \quad V_{cn} = V_p \angle +120^\circ$$

- The line voltages can be found as

$$\begin{aligned} V_{ab} &= V_{an} + V_{nb} = V_{an} - V_{bn} = V_p \angle 0^\circ - V_p \angle -120^\circ \\ &= V_p \left( 1 + \frac{1}{2} + j\frac{\sqrt{3}}{2} \right) = \sqrt{3} V_p \angle 30^\circ \end{aligned}$$



$$V_{bc} = V_{bn} - V_{cn} = \sqrt{3} V_p \angle -90^\circ$$

$$V_{ca} = V_{cn} - V_{an} = \sqrt{3} V_p \angle -210^\circ$$

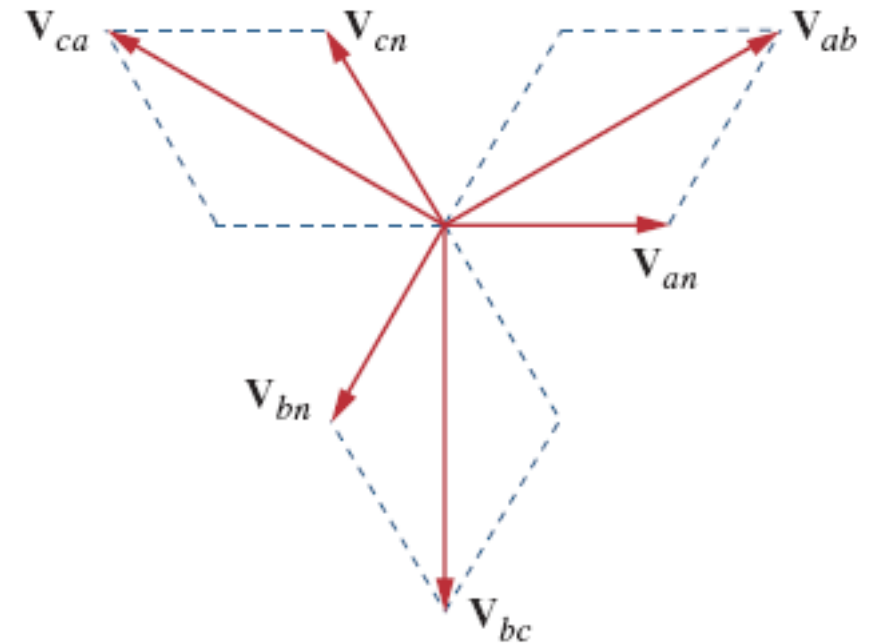
# Balanced Y-Y Connection

- Thus the *line voltages* are  $\sqrt{3}$  times the source phase voltages and **lead** them by **30 deg.** and are 120 deg. out of phase with one another in a **Y connected** source

$$V_p = |\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

$$V_L = |\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}|$$

$$V_L = \sqrt{3}V_p$$



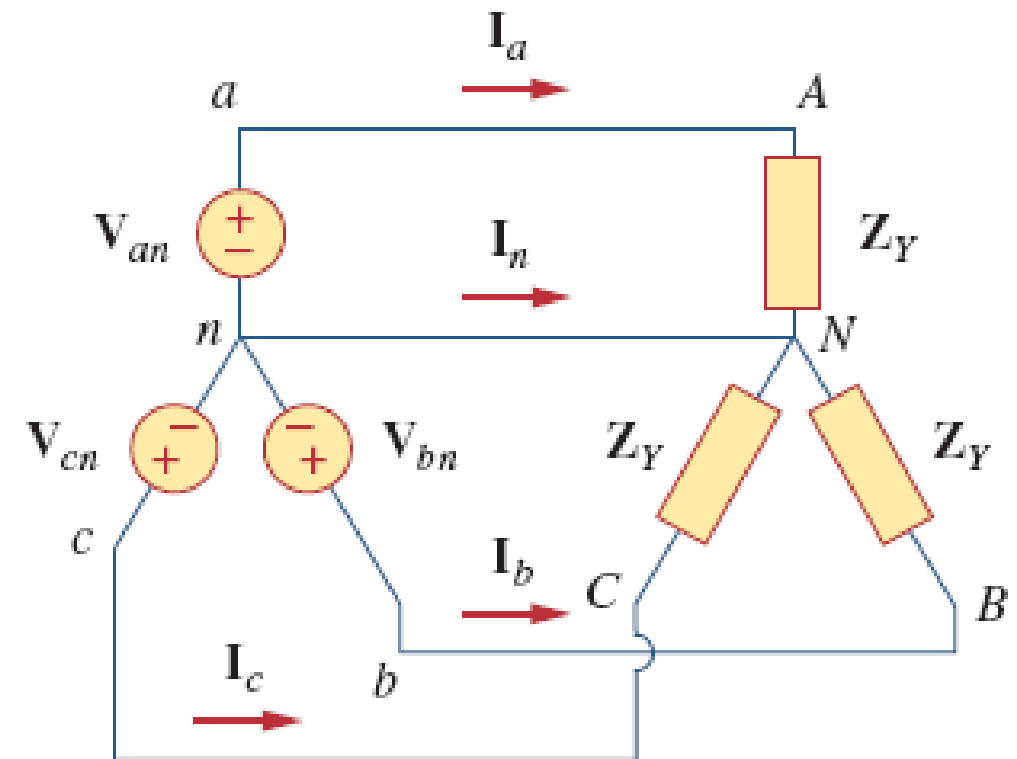


# Balanced Y-Y Connection

- In addition, because of balanced load

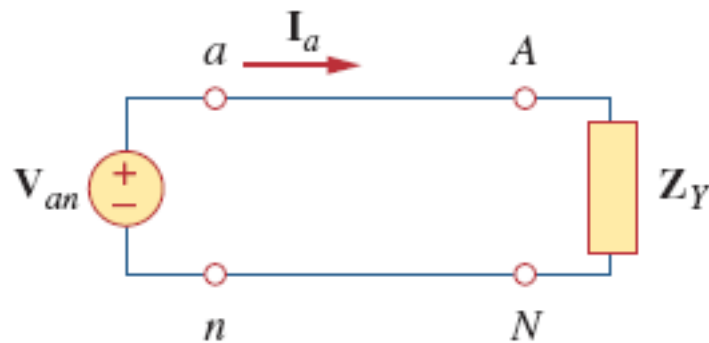
$$\begin{aligned} I_n &= -(I_a + I_b + I_c) \\ &= -\frac{1}{Z_Y}(V_{an} + V_{bn} + V_{cn}) \\ &= 0 \\ V_{nN} &= I_n Z_n = 0 \end{aligned}$$

- Voltage across neutral wire is zero.
- Earth is usually the neutral line.

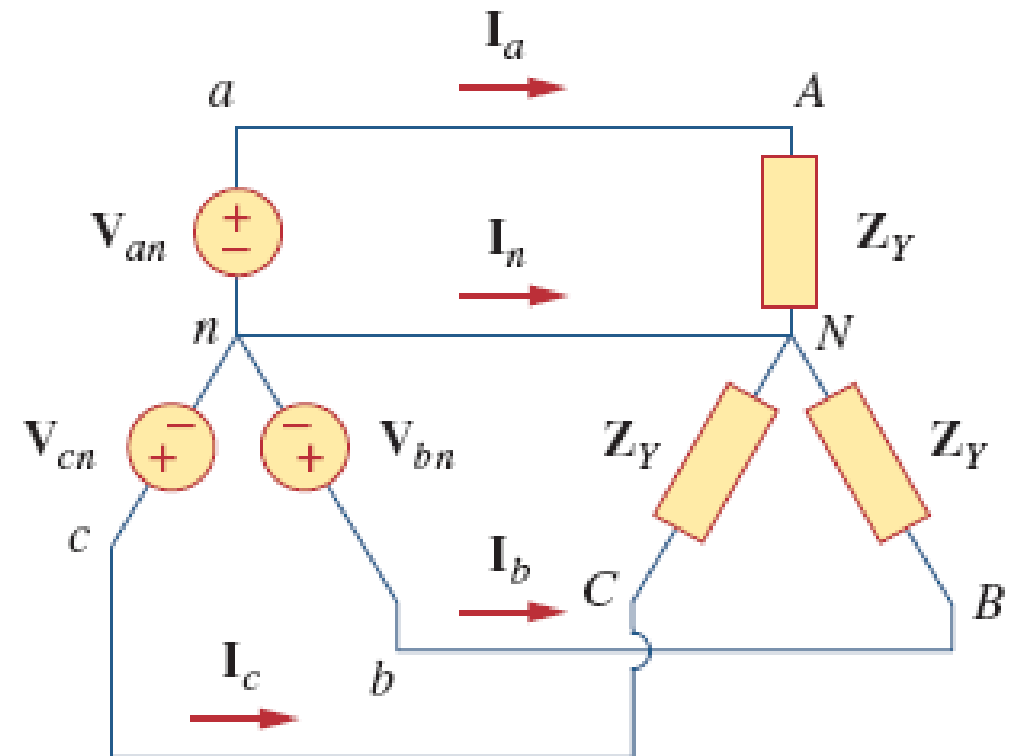


# Balanced Y-Y Connection

- The **line current** is the current in each line (A-a,B-b,C-c), the **phase current** is the current in each phase of load.
- **For Y-Y, both are equal.**
- In case of Y-Y, we can also do per phase analysis.



$$I_a = \frac{V_{an}}{Z_Y}$$



# Example 1: Balanced Y-Y Connection

- Find the line currents.

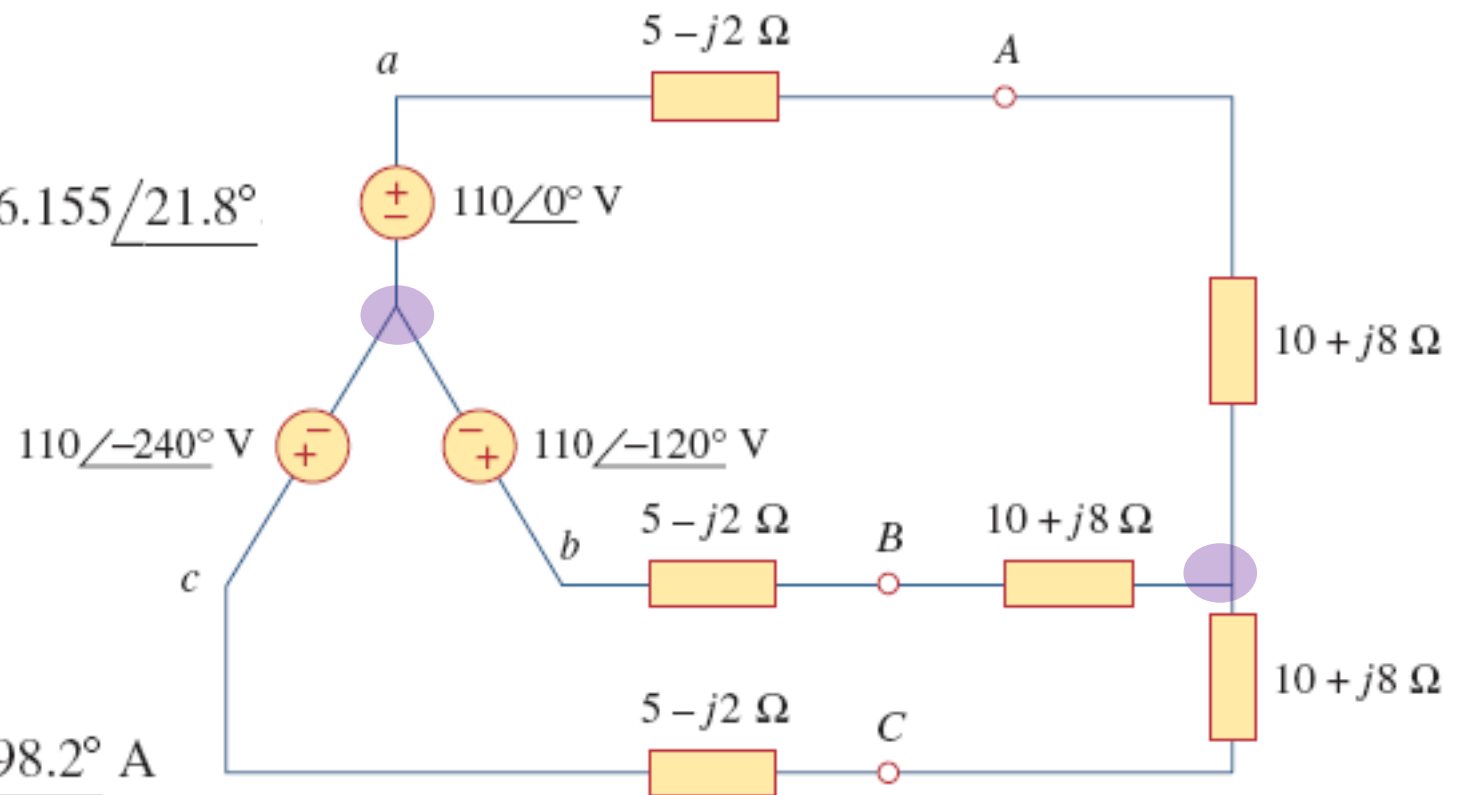
$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

$$\mathbf{Z}_Y = (5 - j2) + (10 + j8) = 15 + j6 = 16.155 \angle 21.8^\circ$$

$$\mathbf{I}_a = \frac{110 \angle 0^\circ}{16.155 \angle 21.8^\circ} = 6.81 \angle -21.8^\circ \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_a \angle -120^\circ = 6.81 \angle -141.8^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a \angle -240^\circ = 6.81 \angle -261.8^\circ \text{ A} = 6.81 \angle 98.2^\circ \text{ A}$$



# Power in Balanced 3-Phase Connections

- In a balanced three phase connection, with balanced source and balanced load ( $Z\angle\theta$ ) in each phase
- Consider a Y-connected load
- Let  $V_{ph}$  be rms value of the load phase voltage

$$\begin{aligned}v_{A,ph} &= \sqrt{2}V_{ph} \cos(\omega t) \\v_{B,ph} &= \sqrt{2}V_{ph} \cos(\omega t - 120^\circ) \\v_{C,ph} &= \sqrt{2}V_{ph} \cos(\omega t + 120^\circ)\end{aligned}$$

- Due to the load, phase currents would lag phase voltage by  $\theta$

$$\begin{aligned}i_{A,ph} &= \sqrt{2}I_{ph} \cos(\omega t - \theta) \\i_{B,ph} &= \sqrt{2}I_{ph} \cos(\omega t - 120^\circ - \theta) \\i_{C,ph} &= \sqrt{2}I_{ph} \cos(\omega t + 120^\circ - \theta) \\I_{ph} &= \frac{V_{ph}}{Z}\end{aligned}$$

# Power in 3 Balanced Phase Connections

- Now, the instantaneous power is :

$$\begin{aligned} p &= p_A + p_B + p_C \\ &= v_{A,ph} i_{A,ph} + v_{B,ph} i_{B,ph} + v_{C,ph} i_{C,ph} \\ &= 2V_{ph} I_{ph} [\cos(\omega t) \cos(\omega t - \theta) \\ &\quad + \cos(\omega t - 120^\circ) \cos(\omega t - 120^\circ - \theta) \\ &\quad + \cos(\omega t + 120^\circ) \cos(\omega t + 120^\circ - \theta)] \end{aligned}$$

- Using  $\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$

$$p = V_{ph} I_{ph} [3 \cos(\theta) + \cos(\alpha) + \cos(\alpha - 240^\circ) + \cos(\alpha + 240^\circ)]$$

where  $\alpha = 2\omega t - \theta$

# Power in 3 Balanced Phase Connections

- Instantaneous power is :

$$p = V_{ph} I_{ph} [3 \cos(\theta) + \cos(\alpha) + \cos(\alpha - 240^\circ) + \cos(\alpha + 240^\circ)]$$

where  $\alpha = 2\omega t - \theta$

$$p = V_{ph} I_{ph} [3 \cos(\theta) + \cos(\alpha) + \cos(\alpha) \cos(240^\circ) + \sin(\alpha) \sin(240^\circ) + \cos(\alpha) \cos(240^\circ) - \sin(\alpha) \sin(240^\circ)]$$

$$p = V_{ph} I_{ph} [3 \cos(\theta) + \cos(\alpha) + 2 \cos(\alpha)(-0.5)] = 3V_{ph} I_{ph} \cos(\theta)$$

- Total instantaneous power in a balanced 3-phase system is constant (Advantage 1)**

# Power in 3 Balanced Phase Connections

- Since, instantaneous power is constant and circuit is balanced, total power and average **per phase** power are

$$P = 3V_{ph}I_{ph} \cos(\theta) \qquad P_p = V_{ph}I_{ph} \cos(\theta)$$

- The corresponding reactive power, apparent power, and complex power would be

$$\begin{aligned} Q &= 3V_{ph}I_{ph} \sin(\theta) & Q_p &= V_{ph}I_{ph} \sin(\theta) \\ S &= 3V_{ph}I_{ph} & S_p &= V_{ph}I_{ph} \\ \mathbf{S} &= P + jQ = 3\mathbf{V}_{ph}\mathbf{I}_{ph}^* & \mathbf{S}_p &= P_p + jQ_p = \mathbf{V}_{ph}\mathbf{I}_{ph}^* \end{aligned}$$

# Example 2

- Find the total average power consumed by the load.

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{Z_Y}$$

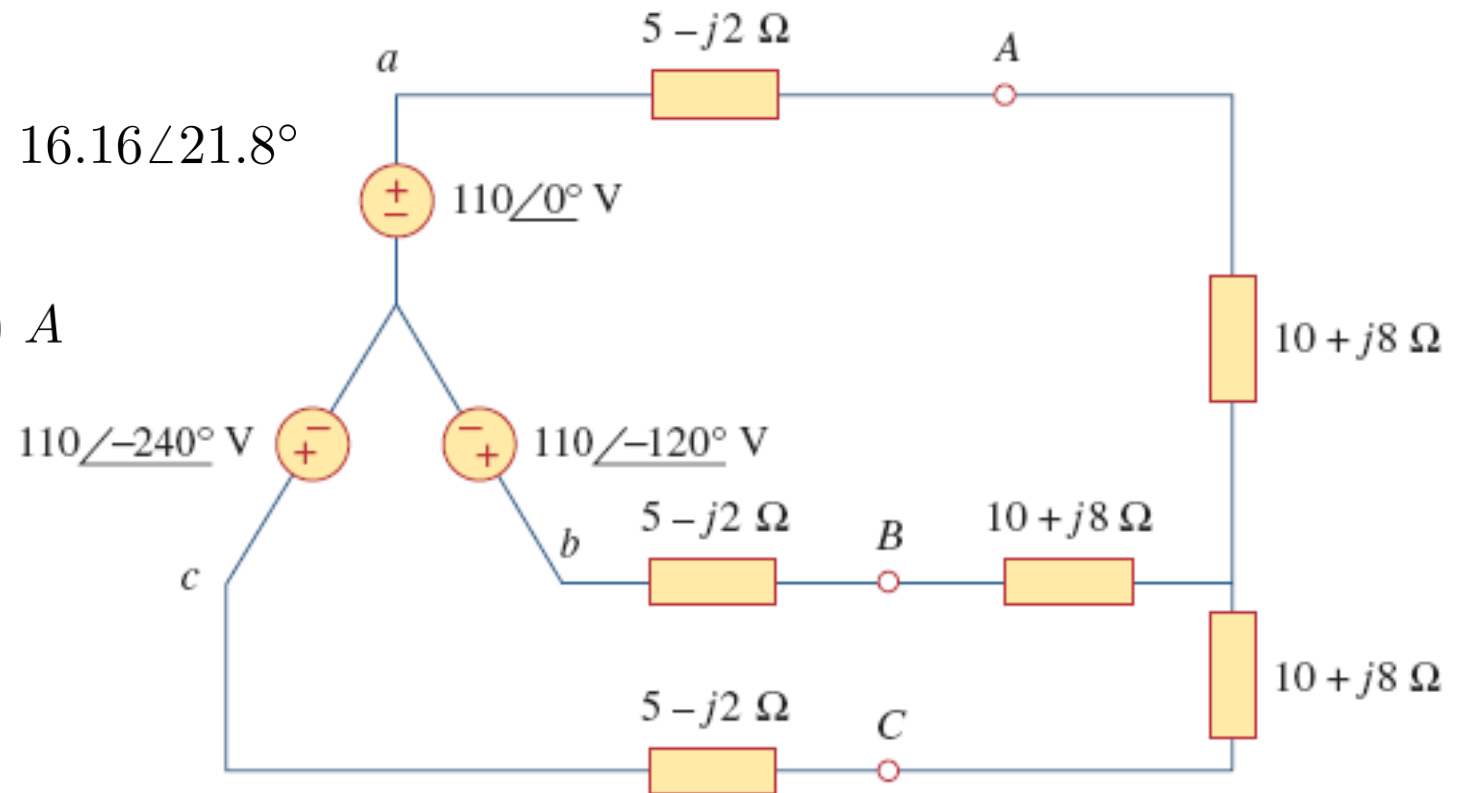
$$Z_Y = (5 - j2) + (10 + j8) = 15 + j6 = 16.16 \angle 21.8^\circ$$

$$\mathbf{I}_{ph} = \mathbf{I}_a = \frac{110 \angle 0^\circ}{16.15 \angle 21.8^\circ} = 6.81 \angle (-21.8^\circ) \text{ A}$$

$$\mathbf{V}_{ph} = \mathbf{I}_{ph} Z_L$$

$$P = 3 |\mathbf{V}_{ph}| |\mathbf{I}_{ph}| \cos(\angle Z_L) = 3 |\mathbf{I}_{ph}|^2 R_{ph}$$

$$P = 3 \times (6.81)^2 \times 15 = 2087 \text{ W}$$





# Polyphase Circuits: Advantages

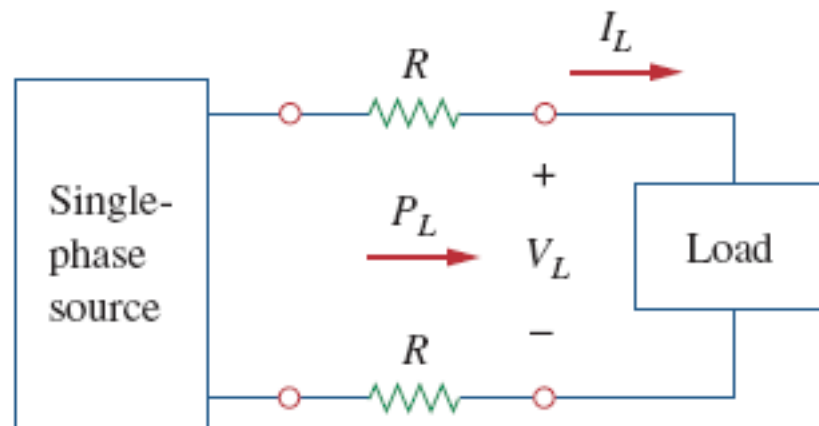
- Capable of delivering constant instantaneous power
- One/Two phase inputs can be taken out from a three phase supply.
- More economical than single phase in power delivery. Less amount of wire required.
  - Increasing the number of phases, increases efficiency but also increases complexity of transmission.
  - A 'balance' is found when the number of phases is **three**.

# Advantage 2: Economical Transmission

- Consider an amount of power  $P_L$  being transmitted at the same line voltage  $V_L$  using
  - Single phase supply
  - 3-phase balanced supply
- Power dissipation in transmission

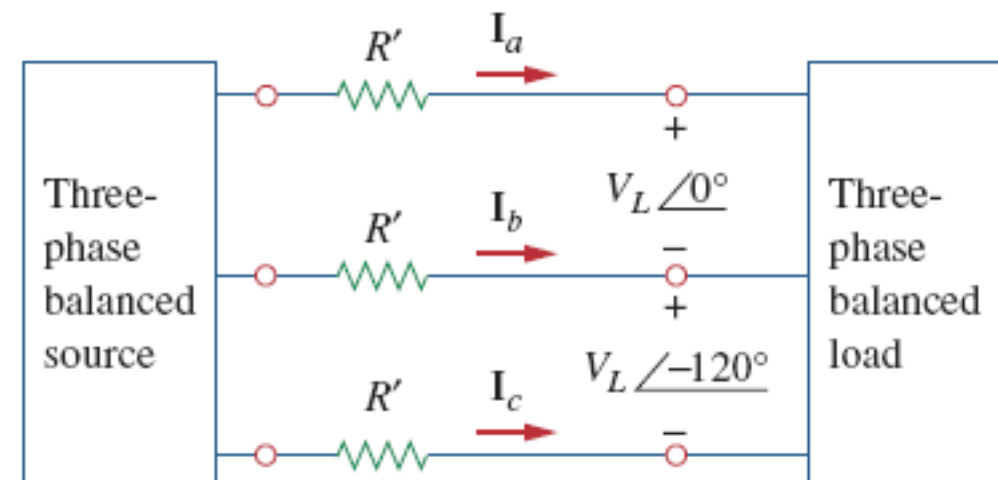
$$I_L = \frac{P_L}{V_L}$$

$$P_{loss} = 2I_L^2 R = 2R \frac{P_L^2}{V_L^2}$$



$$I'_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c| = P_L / (\sqrt{3}V_L)$$

$$P'_{loss} = 3(I'_L)^2 R' = 3R' \frac{P_L^2}{3V_L^2} = R' \frac{P_L^2}{V_L^2}$$



# Advantage 2: Economical Transmission

- Power dissipation in transmission

$$P_{loss} = 2I_L^2 R = 2R \frac{P_L^2}{V_L^2}$$

$$R = \frac{\rho l}{\pi r^2}$$

$$P'_{loss} = 3(I'_L)^2 R' = 3R' \frac{P_L^2}{3V_L^2} = R' \frac{P_L^2}{V_L^2}$$

$$R' = \frac{\rho l}{\pi (r')^2}$$

$$\frac{P_{loss}}{P'_{loss}} = \frac{2(r')^2}{r^2}$$

- If same amount of power loss is to be allowed  $r^2 = 2(r')^2$
- Ratio of material required for transmission wires

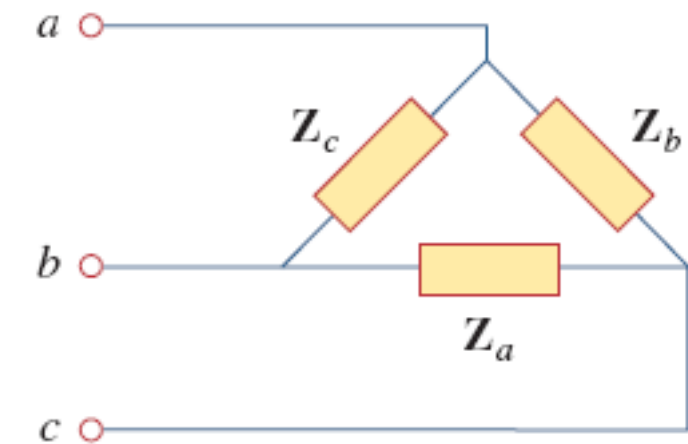
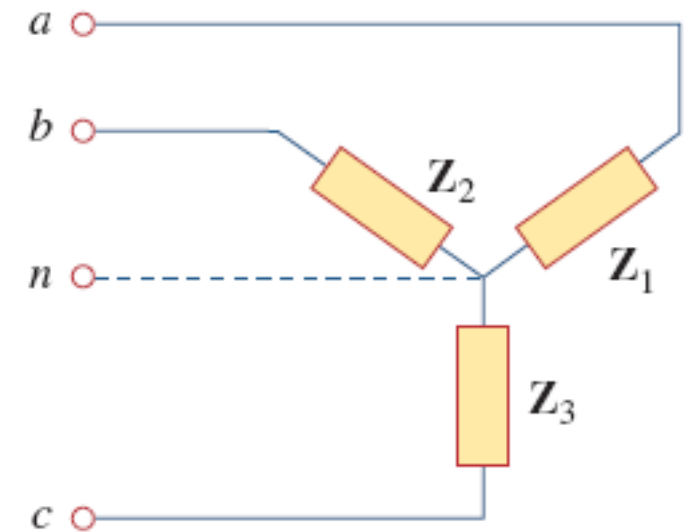
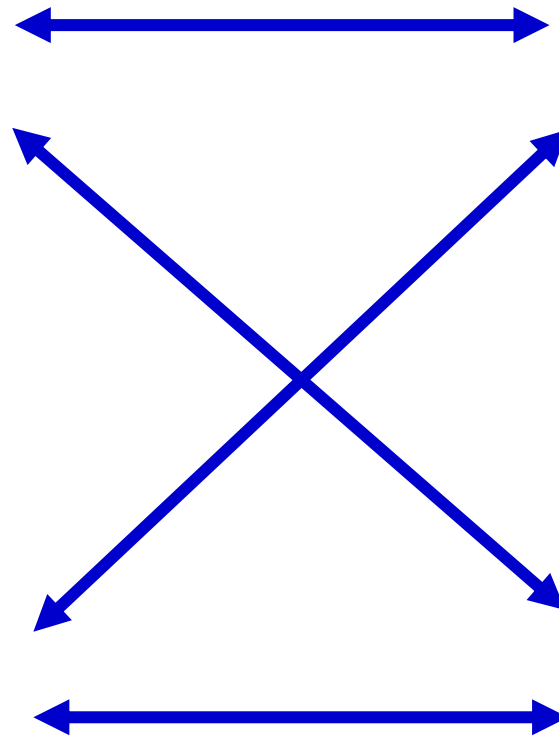
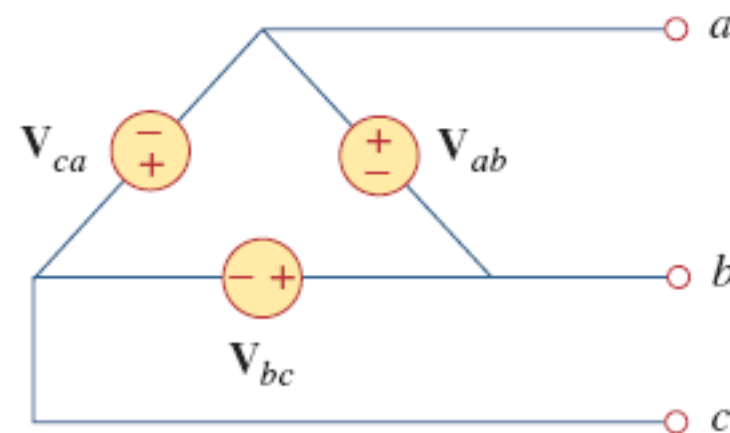
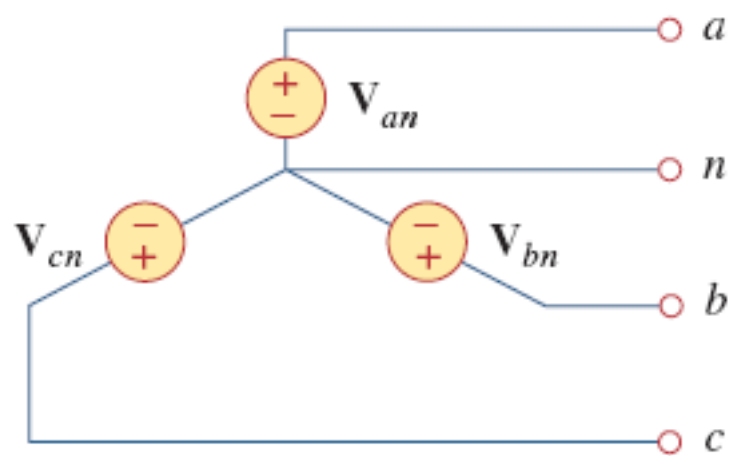
$$\begin{aligned} \frac{\text{material in single ph}}{\text{material in 3-ph}} &= \frac{2(\pi r^2 l)}{3(\pi r'^2 l)} \\ &= \frac{2}{3} \left( \frac{r}{r'} \right)^2 = \frac{2}{3} (2) = \frac{4}{3} \end{aligned}$$

In other words, a 3-phase supply can deliver power, and follow power loss constraints using only 75% of the material as an equivalent single phase supply.

# Polyphase Circuits: Advantages

- Capable of delivering constant instantaneous power.
- One/Two phase inputs can be taken out from a three phase supply.
- More economical than single phase in power delivery. Less amount of wire required.
  - Increasing the number of phases, increases efficiency but also increases complexity of transmission.
  - A 'balance' is found when the number of phases is **three**.

# Other Configurations



# Balanced Y- $\Delta$ Connection

- Source is Y Connected and load is Delta connected.

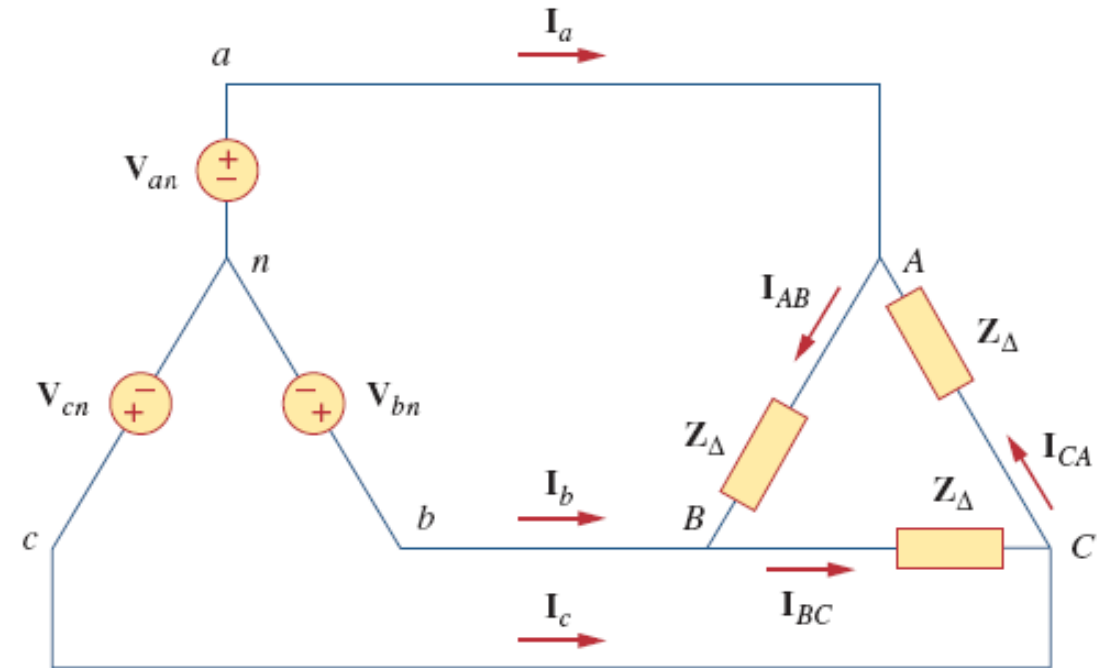
- Source Side:

- Phase Voltages

$$\begin{aligned} V_{an} &= V_p \angle 0^\circ \\ V_{bn} &= V_p \angle -120^\circ, \quad V_{cn} = V_p \angle +120^\circ \end{aligned}$$

- Line Voltages

$$\begin{aligned} V_{ab} &= \sqrt{3}V_p \angle 30^\circ = V_{AB}, & V_{bc} &= \sqrt{3}V_p \angle -90^\circ = V_{BC} \\ V_{ca} &= \sqrt{3}V_p \angle -150^\circ = V_{CA} \end{aligned}$$



Note that the line voltage is equal to the phase voltage of the Delta load

# Balanced Y- $\Delta$ Connection

- Source is Y Connected and load is Delta connected.

- Load Side:

- Phase currents

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}}, \quad I_{BC} = \frac{V_{BC}}{Z_{\Delta}}, \quad I_{CA} = \frac{V_{CA}}{Z_{\Delta}}$$

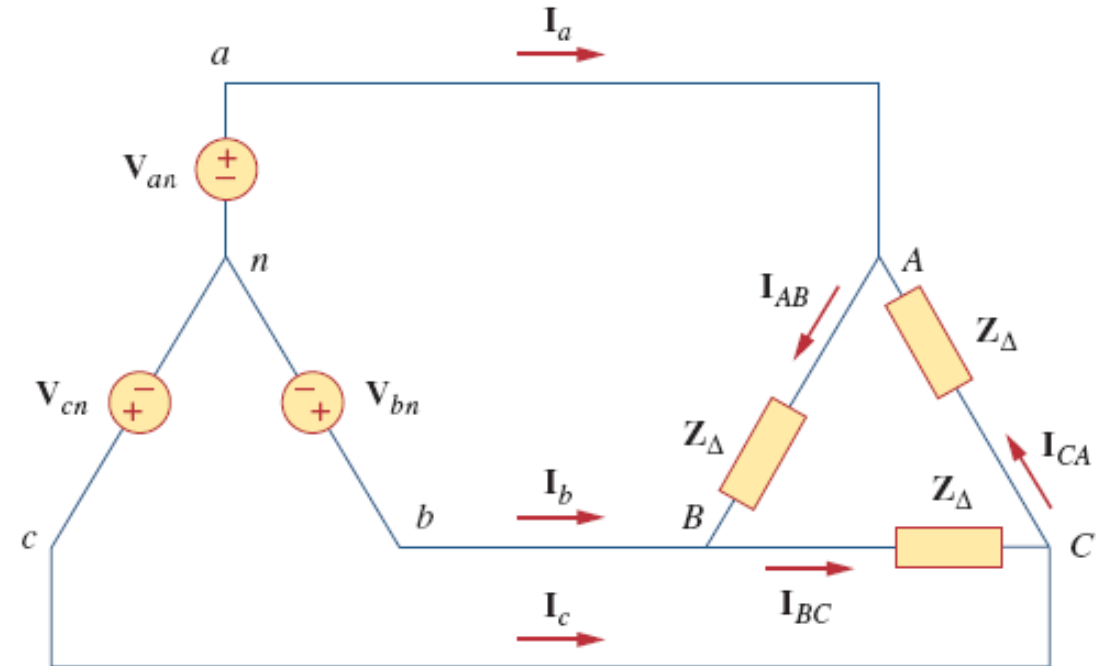
- Line Currents

$$I_a = I_{AB} - I_{CA}, \quad I_{CA} = I_{AB} \angle -240^\circ,$$

$$I_a = I_{AB} - I_{CA} = I_{AB}(1 - 1 \angle -240^\circ)$$

$$= I_{AB}(1 + 0.5 - j0.866) = I_{AB} \sqrt{3} \angle -30^\circ$$

$$I_L = \sqrt{3} I_p$$



Line Current is  $\sqrt{3}$  times the phase current and lags it by 30 deg. in a Delta connected load.

**Note :** The load can be replaced by an equivalent Y load  $Z_Y = Z_{\Delta}/3$ , and can be analyzed as a Y-Y config

## Example 3: Balanced Y- $\Delta$ Connection

- A balanced abc-sequence Y-connected source with  $V_{an} = 100\angle 10^\circ V$  is connected to a  $\Delta$  connected balanced load  $(8+j4)\Omega$  per phase. Calculate the phase and line currents.

- Converting the balanced  $\Delta$  load into equivalent Y load,

$$Z_Y = Z_\Delta / 3 = \frac{8+j4}{3} = 2.981\angle 26.6^\circ$$

- Thus, the line currents can be computed using single phase analysis of Y-Y connection

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{Z_Y} = \frac{100\angle 10^\circ}{2.98\angle 26.6^\circ} = 33.54\angle -16.6^\circ \text{ A}$$

$$\begin{aligned}\mathbf{I}_b &= \mathbf{I}_a \angle -120^\circ = 33.54\angle -136.6^\circ \\ \mathbf{I}_c &= 33.54\angle 103.4^\circ\end{aligned}$$

- The phase currents in balanced  $\Delta$  load are related to line currents as

$$\mathbf{I}_{AB} = \frac{\mathbf{I}_a}{\sqrt{3}} \angle 30^\circ = 19.36\angle 13.4^\circ$$

$$\begin{aligned}\mathbf{I}_{BC} &= \mathbf{I}_{AB} \angle -120^\circ = 19.36\angle -106.6^\circ \\ \mathbf{I}_{CA} &= 19.36\angle 133.4^\circ\end{aligned}$$



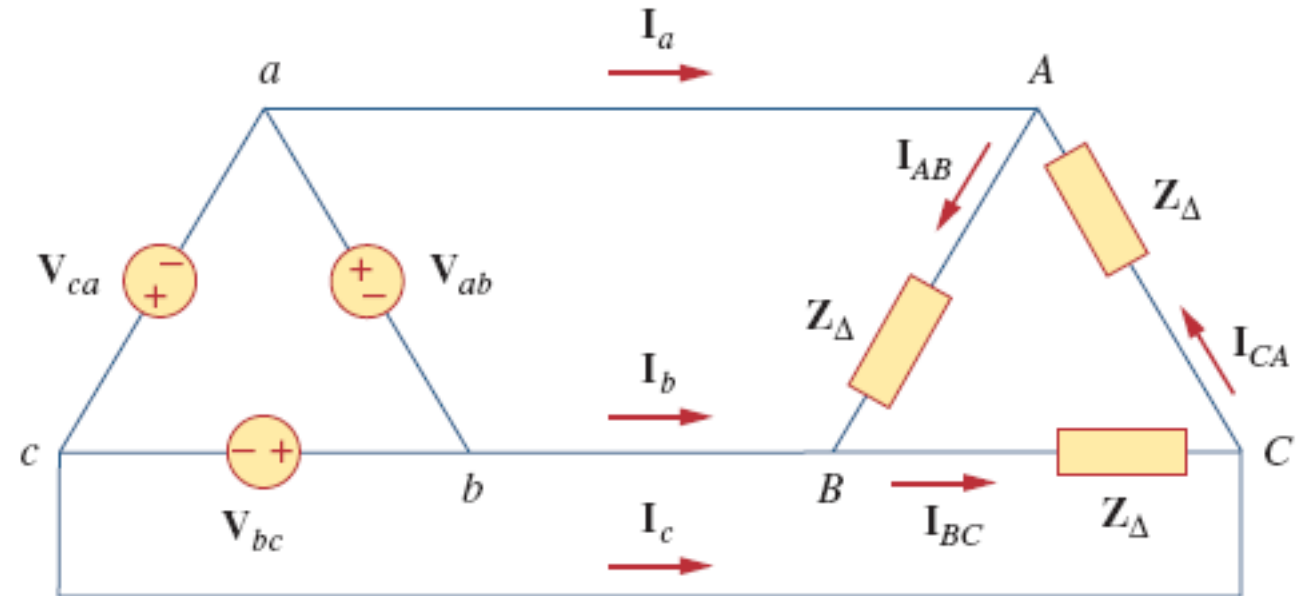
# Balanced $\Delta$ - $\Delta$ Connection

- Both source and load are Delta connected.
- Since both sides are Delta connected, Phase Voltages=Line Voltages

$$V_{ab} = V_{AB}, \quad V_{bc} = V_{BC}, \quad V_{ca} = V_{CA}$$

- Phase currents are

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}} = \frac{V_{ab}}{Z_{\Delta}}, \quad I_{BC} = \frac{V_{BC}}{Z_{\Delta}} = \frac{V_{bc}}{Z_{\Delta}}$$
$$I_{CA} = \frac{V_{CA}}{Z_{\Delta}} = \frac{V_{ca}}{Z_{\Delta}}$$

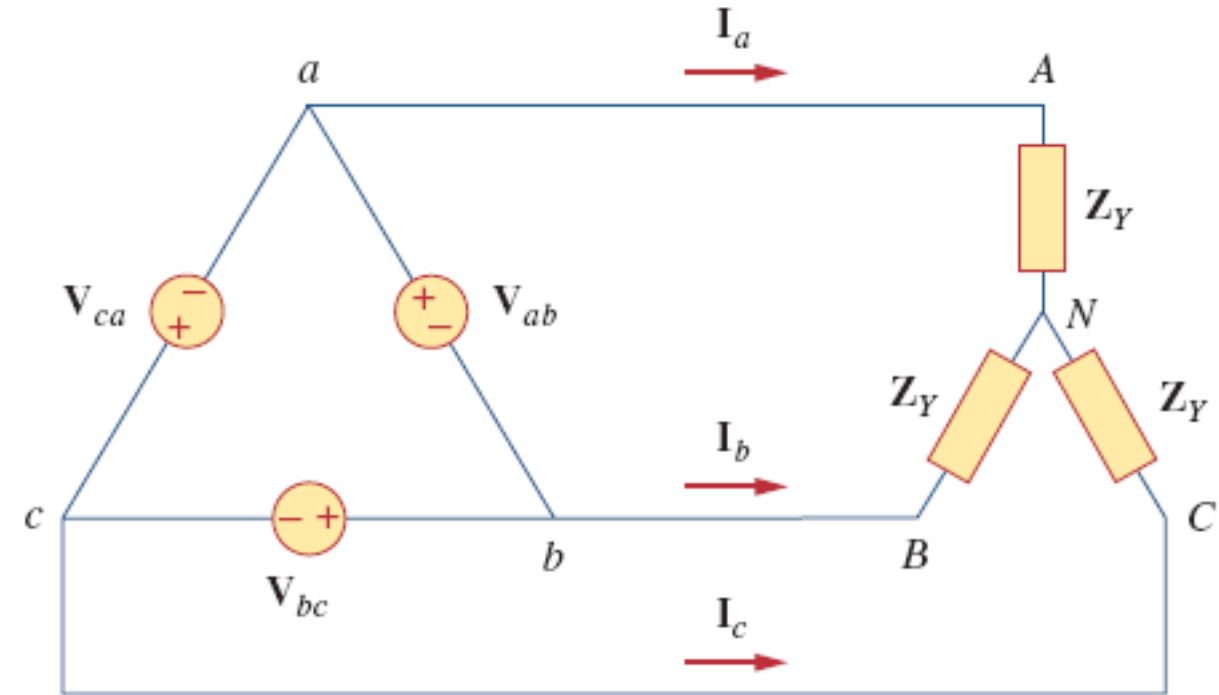


Line Current is related to phase current as (but lags it by 30 deg)

$$I_L = \sqrt{3}I_p$$

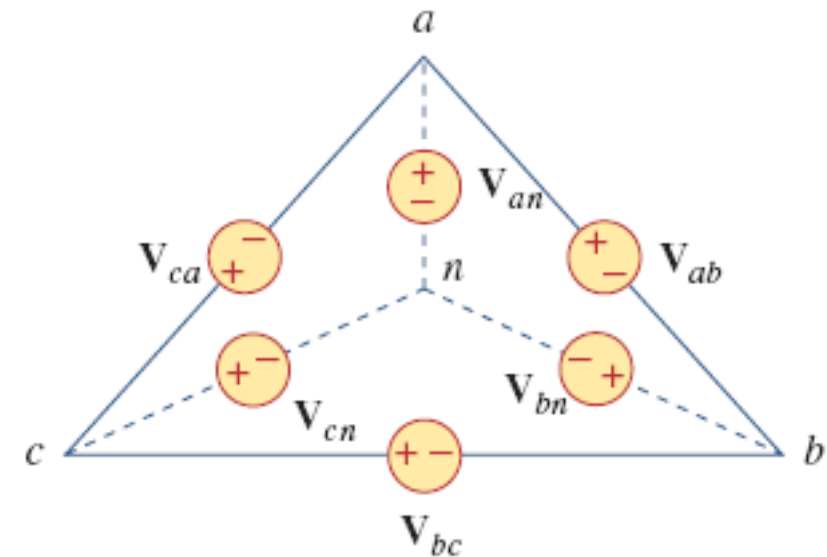
# Balanced $\Delta$ -Y Connection

- Source is Delta connected, Load is Y connected
- The line currents are equal to the phase currents (because of Y connected load).
- The line voltages
  - Are same as phase voltages of the source side.
  - But are  $\sqrt{3}$  times the load phase voltages and lead them by 30 degrees.



# $\Delta$ -Y Source Transformation

- Another possible way to analyze the Delta-Y connection
  - Transform Delta connected source to Y connected source
  - Analyze the Y-Y connection.
- Observing the phase voltage to line voltage relation in the Y-Y connection



$$\begin{aligned} V_{ab} &= V_p \angle 0^\circ, & V_{bc} &= V_p \angle -120^\circ \\ V_{ca} &= V_p \angle +120^\circ \end{aligned}$$



$$\begin{aligned} V_{an} &= \frac{V_p}{\sqrt{3}} \angle -30^\circ \\ V_{bn} &= \frac{V_p}{\sqrt{3}} \angle -150^\circ, & V_{cn} &= \frac{V_p}{\sqrt{3}} \angle +90^\circ \end{aligned}$$

# Example 4 - Balanced $\Delta$ -Y Connection

- A balanced abc-sequence Y-connected load with a phase impedance  $(40+j25) \Omega$  is supplied by a balanced, positive sequence,  $\Delta$ -connected source with line voltage of 210 V. Calculate the phase currents.

- The load impedance is

$$Z_Y = 40 + j25 = 47.17 \angle 32^\circ$$

- The source voltage is  $V_{ab} = 210 \angle 0^\circ$
- Transforming the Delta source, to Y source  $V_{an} = \frac{V_{ab}}{\sqrt{3}} \angle (-30^\circ) = 121.2 \angle (-30^\circ)$
- The phase currents are same as line currents in Y-connected load

$$I_a = \frac{V_{an}}{Z_Y} = 2.57 \angle (-62^\circ)$$

$$\begin{aligned} I_b &= I_a \angle -120^\circ = 2.57 \angle 178^\circ \\ I_c &= 2.57 \angle 58^\circ \end{aligned}$$

# Observations on Balanced Connections

- Line currents and line voltage interpretations are independent of the configuration of the source or load (Delta or Y)
- Phase current is current through each phase
  - It is **same** as line current in **Y** connected Source/Load,  $I_{AN} = I_a$
  - Phase current **leads** line current by **30 degrees** and **has**  $1/\sqrt{3}$  times the magnitude in **Delta** connected source/load,  $I_{AB,\Delta} = \frac{1}{\sqrt{3}} I_a \angle 30^\circ$
- Phase voltage is voltage across each phase
  - It is **same** as line voltage in **Delta** connected Source/Load,  $V_{AB,\Delta} = V_{ab}$
  - Phase voltage lags line voltage by 30 degrees and has  $1/\sqrt{3}$  times the magnitude in Y connected source/load,  $V_{AN} = \frac{1}{\sqrt{3}} V_{ab} \angle (-30^\circ)$

# Observations on Balanced Connections

- Notice that:
  - For Delta load:  $V_{ph} = V_L, I_{ph} = I_L/\sqrt{3}$
  - For Star or Y load:  $V_{ph} = V_L/\sqrt{3}, I_{ph} = I_L$
- Thus, total average power:  $P = \sqrt{3}V_L I_L \cos(\theta)$   
irrespective of the configuration of the balanced load/source