ELL100: INTRODUCTION TO ELECTRICAL ENGG.

Impedance Concepts

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$$Z = (R + jX)\ \Omega$$
$$R = \text{Resistance}$$
$$X = \text{Reactance}$$

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- Impedance (Z) is a complex number; units same as resistance (Ohms). Thus, has both magnitude and phase.
- For exponential/sinusoidal functions Z is defined as: ratio of complex representation of the voltage to complex representation of current.

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- For Capacitor $Z_C = \frac{v}{i} = \frac{v}{C\frac{dv}{dt}} = \frac{v}{Csv} = \frac{1}{sC}\ \Omega$

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Z_C(0) &= \frac{1}{Cs}|_{s=0} = \infty \quad \text{(Capacitance is open circuit to DC voltage)}
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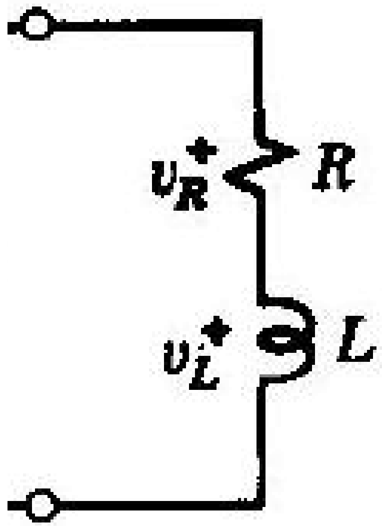
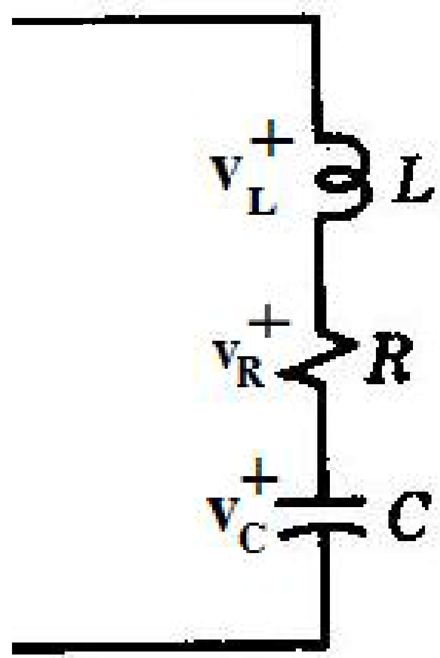
Equivalent Impedance

- Impedance can be combined in series or parallel, just like resistances





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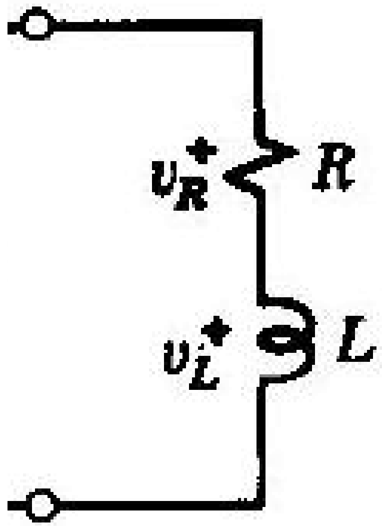
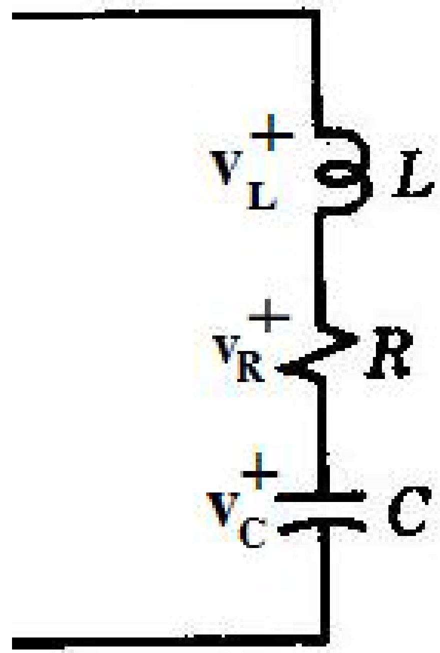


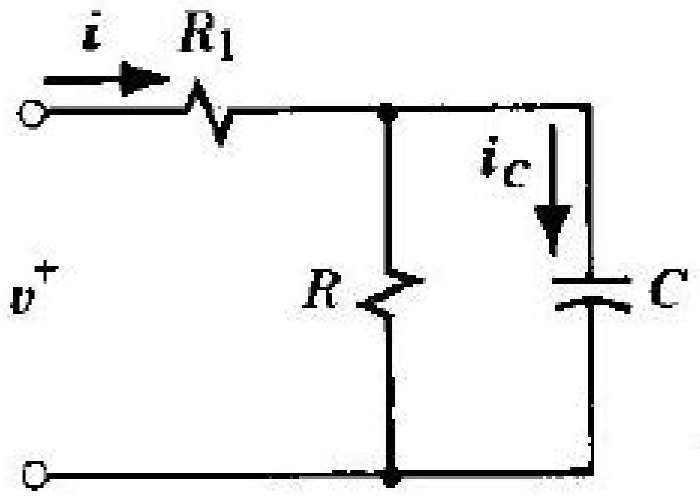
$$Z(s) = R + sL$$



$$Z(s) = R + sL + \frac{1}{sC}$$

Example

$R_1 = 2\Omega$, $C = 0.25F$ and $R = 4\Omega$. Using impedance concept, find the current i and i_c for $v = 6\ e^{-2\ t}V$



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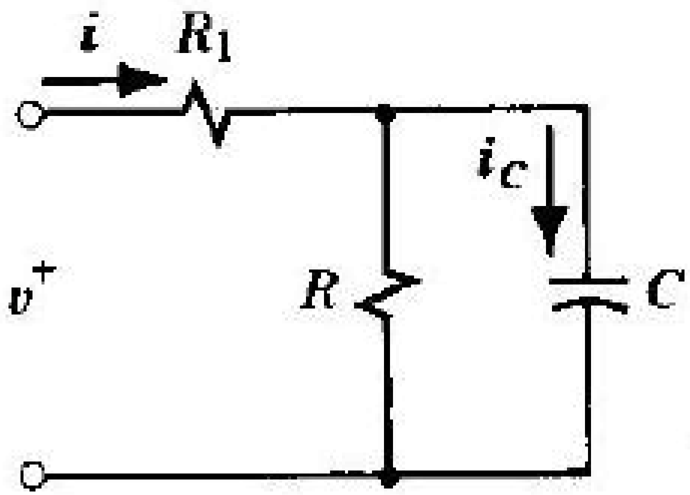

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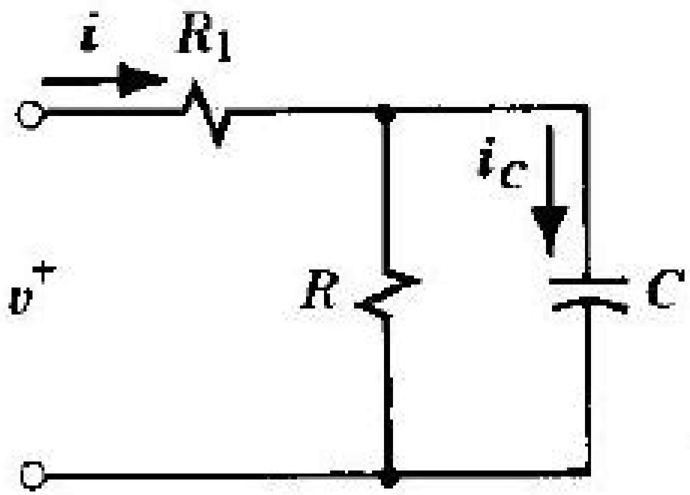

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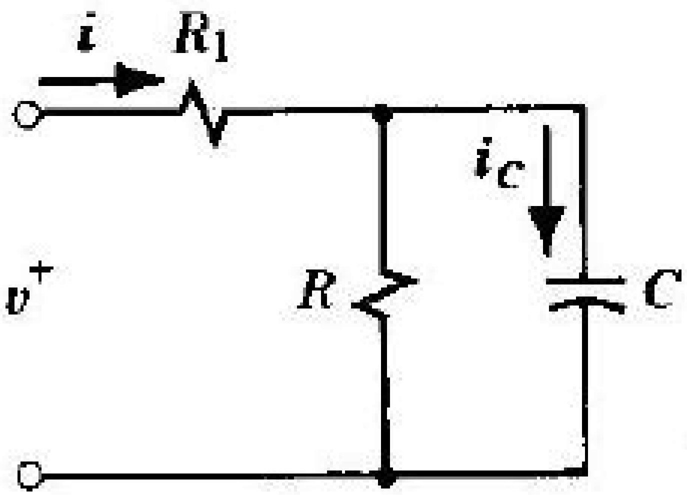

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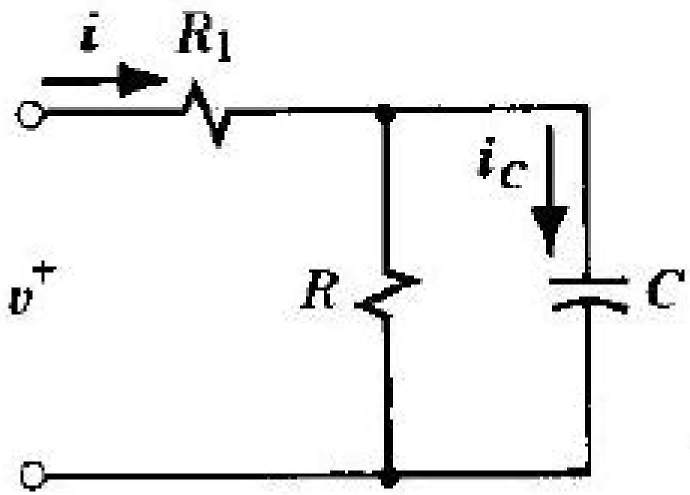

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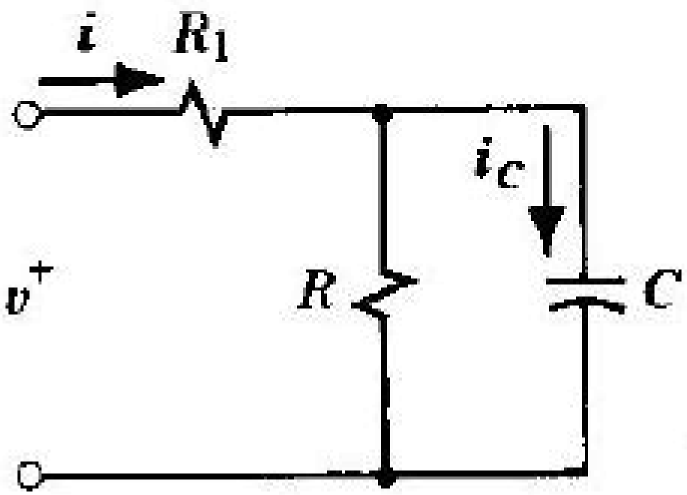

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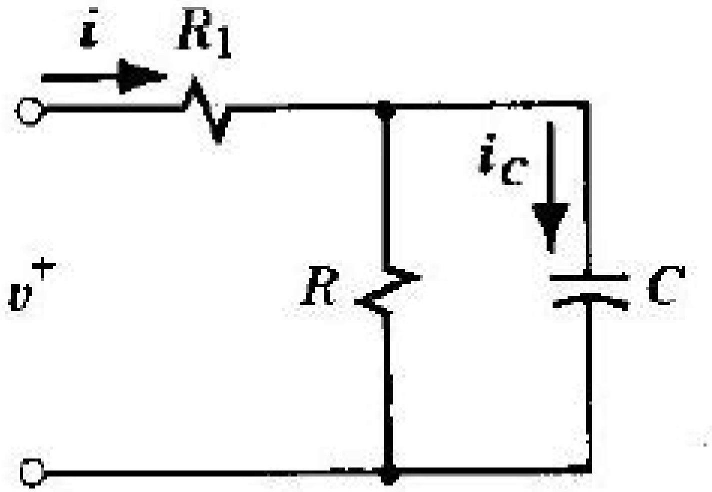
Using current divider and impedance concepts,

$$i_C = \frac{\mathcal{Z}_R.i}{\mathcal{Z}_R+\mathcal{Z}_C} = \frac{4.i}{4-2} = -6\ e^{-2t}\ A$$

General Impedance or Impulse Function

- Typical impedance functions are of type



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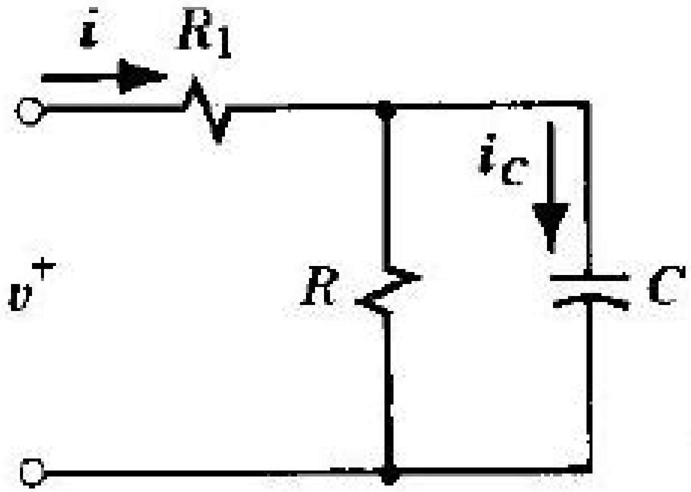

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Z(s) &= R_1 + \frac{R(1/sC)}{R + (1/sC)} \\
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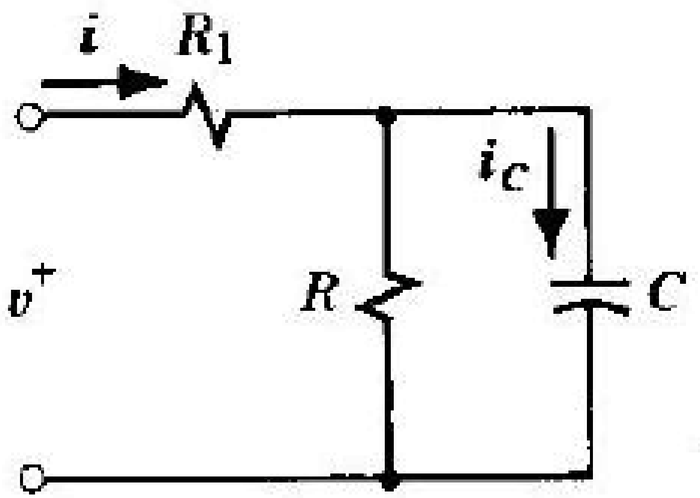

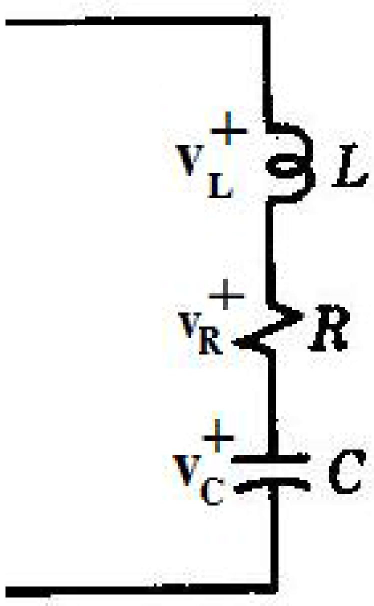

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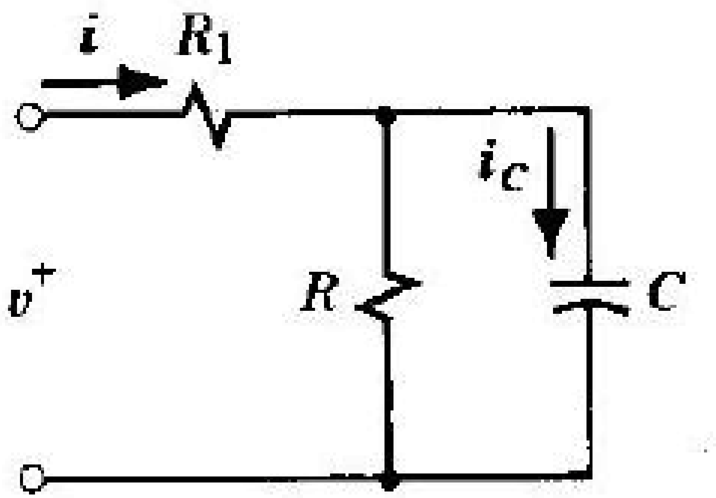

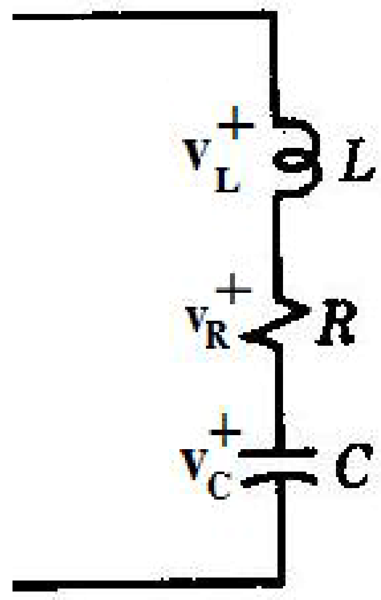

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- Here, the z_i's are called zeros as $Z(z_i)=0$,
- The p_i's are called poles.

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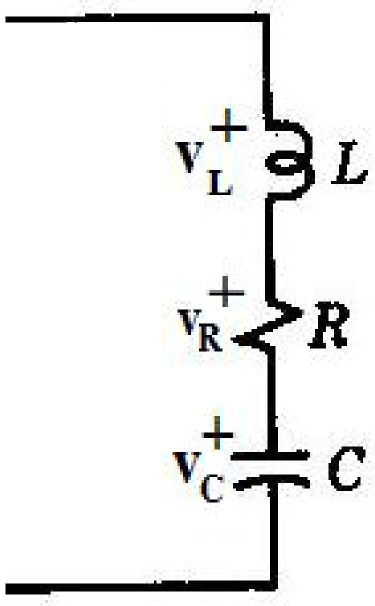

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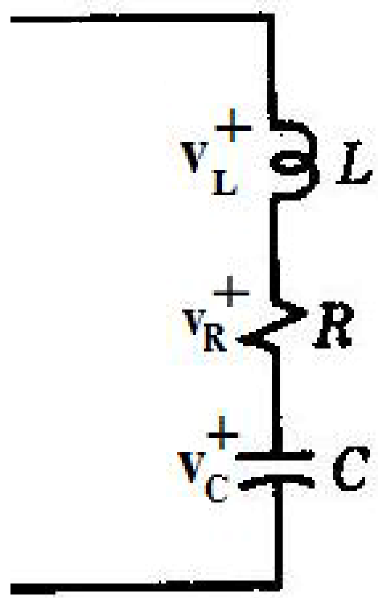

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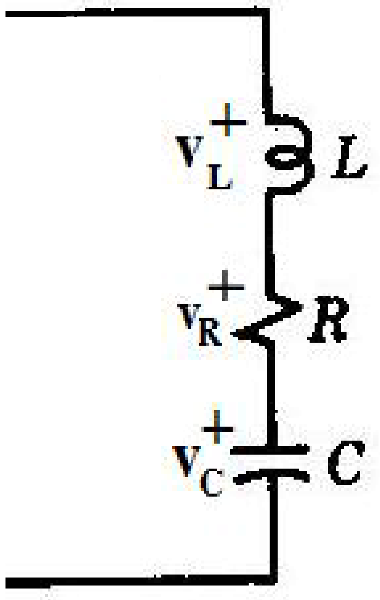

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v_c(t) &= \frac{1}{C}\int i dt = \frac{1}{Cs} i(t) \\
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