Lecture 13 Mon 21 Aug 2023

* Simplex Method

1) $b \geq 0$ 2) $x \geq 0$

3) Convert all inequalities into equal

Slack Variables ≥ 0
with cost coefficients set 0 with each of them in objective func^n

$Ax = b, \quad x \geq 0.$

Identity submatrix should be present in A.

$$\underset{m \times n}{\underset{\downarrow}{Ax}} \qquad \left[[I_{m \times m}] \right]$$

This ease out the choice of initial BFS to start the algo.

$\therefore$ We can take basis matrix $B = I_{m \times m}$

$A = [B : R]$ set $x_R = 0$

$$B x_B = b$$
$$\Rightarrow x_B = b \geq 0$$ used the assumption 1)

$\therefore$ x_B is the BFS.

If I is already a submatrix of A, then we can start the algorithm
Else if not, we find out which columns of I are missing. We force these columns into the system by using additional ≥ 0 variables called artificial variables.

In order to get rid of these variables quickly, we modify the objective func^n

Suppose we aim

$$\max z = c^T x - M x_a$$

s.t. $Ax \pm x_a = b$

$x, x_a \geq 0$

x_a: artificial variable

of $x_a \leq m$

by attaining a high -ve value in the obj. func^n, $M >>> 0$.

Result: $\max z = c^T x$ (we have converted it into eq^{n}s = equation forms)

s.to. $x \in S$
(added slack + artificial variable).

$S = \{Ax = b, x \geq 0\}$

equivalent to $x \in S = \{x \in \mathbb{R}^n, Ax \leq b, x \geq 0\}$

$$c = (\underbrace{c_1, \ldots, c_k}_{\text{original}}, \underbrace{0, \ldots, 0}_{\text{slack}}, \underbrace{-M, \ldots, -M}_{\text{artificial}})$$

let $x = \begin{pmatrix} x_B \\ 0 \end{pmatrix}$ is the current BFS @ some of iter^n of algo.

let $B = [d_1, d_2, \ldots, d_m]_{m \times m}$ where each d_i is some column of A matrix.

Let $a_j \in A$ (column of A) which is not in B
i.e. $a_j \notin B$.

$a_j : m \times 1 \Rightarrow a_j \in \mathbb{R}^m$.

& $\{d_1, d_2, -, d_m\}$ is a basis of $\mathbb{R}^m$.
$m \times 1$

We can write

$$a_j = \sum_{i=1}^{m} y_{ij}\, d_i, \text{ for some scalars}$$
(unique combination)

Assumption: At least one $y_{rj} > 0$ in the combination $1 \le r \le n$.

$$\Rightarrow d_r = \sum_{\substack{i \neq r \\ i=1}}^{n} \frac{-y_{ij}}{y_{rj}} d_i + \frac{1}{y_{rj}} a_j$$

substitute this d_r in the eq^n

$$B x_B = b.$$

$$\Rightarrow d_1 x_{B_1} + - + d_m x_{B_m} = b.$$

$$\sum_{i=1}^{m} d_i x_{B_i} = b.$$

$$\Rightarrow \sum_{\substack{i=1 \\ i \neq r}}^{m} d_i x_{B_i} + \left(\sum_{i=1} \frac{-y_{ij}}{y_{rj}} d_i + \frac{a_j}{y_{rj}} \right) x_{B_i} = b.$$

$$\Rightarrow \sum_{\substack{i=1 \\ i \neq r}}^{m} \left(x_{B_i} - \frac{y_{ij}}{y_{rj}} x_{B_r} \right) + \frac{x_{B_r}}{y_{rj}} a_j = b.$$

$$\Rightarrow \sum_{\substack{i=1 \\ i \neq r}}^{m} \hat{x}_{B_i} d_i + \hat{x}_{B_r} a_j = b.$$

$\Rightarrow$ B is changed as a_j enters into B and d_r leaves B.

So, new matrix,

$$B = (d_1, -, d_{r-1}, a_j, d_{r+1}, -, d_m)_{m \times m}$$