Lecture 5
Signals and Systems (ELL205)

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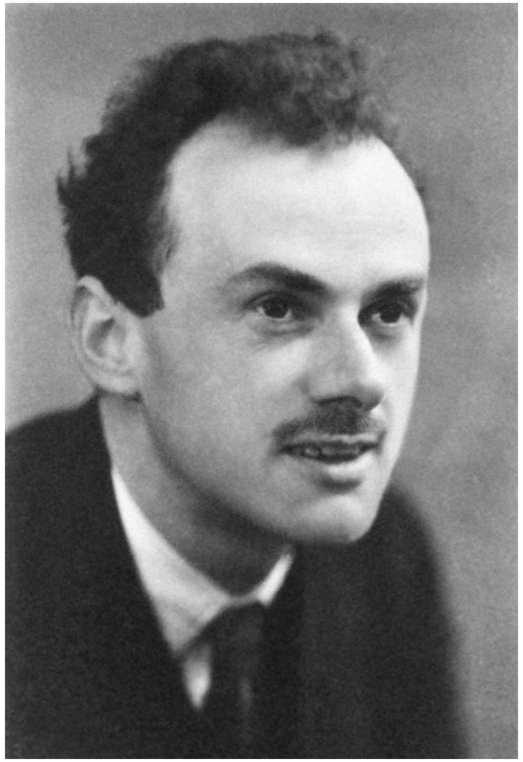
Paul Dirac (1902-1984)

- Won the Nobel prize in Physics in 1933 for his contributions to quantum filed theory.
- The Dirac equation:

$$\left[i\hbar A^{\mu}\gamma^{\mu}_{(a)}\partial_{\mu} - m_0 c\right]\psi = 0$$

  It was awarded as the most beautiful mathematical equation.
- Workaholic and quite a quiet person:
  - He concentrated solely on his research, and stopped only on Sunday when he took long strolls alone.
  - His colleagues in Cambridge jokingly defined a unit called a "dirac", which was one word per hour.
  - He met young Richard Feynman at a conference, he said after a long silence, "I have an equation. Do you have one too?“
- Had troubles with Einstein.
  - Einstein said, “I don't understand Dirac at all.”
- Among his students was Homi Bhabha.
- Read more about him from the book: The strangest man.



Outline

- Signal transformations
- Classifications of Signals
- Basic Signals
    - Exponential
    - Unit-step
    - Unit-impulse

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Unit-step

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Unit-step

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \quad \text{or} \quad u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

Unit-step

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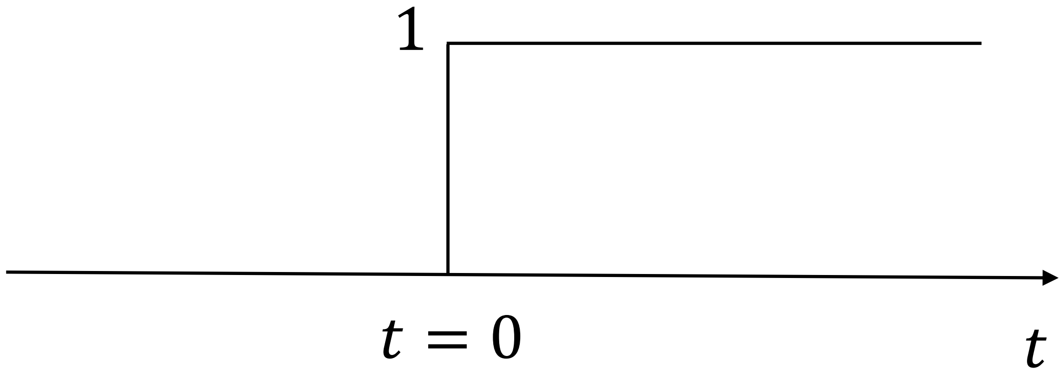

At $t = 0$, it is not continuous.

Function equivalence

CT systems are implemented using:

Adders $+$

Integrators $\boxed{\int}$

Function equivalence

$$f(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \qquad g(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

$$f(t) - g(t) \longrightarrow \boxed{\int} \longrightarrow$$

Integrator

Function equivalence

$$f(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \qquad\qquad g(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

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Integrator

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$$f(t) - g(t) \longrightarrow \boxed{\int} \longrightarrow 0$$

Integrator

An Integrator sees both "$f(t)$" and "$g(t)$" as equivalent.

Function equivalence

Two functions are said to be “equivalent” if they are different only at a countable number of points.

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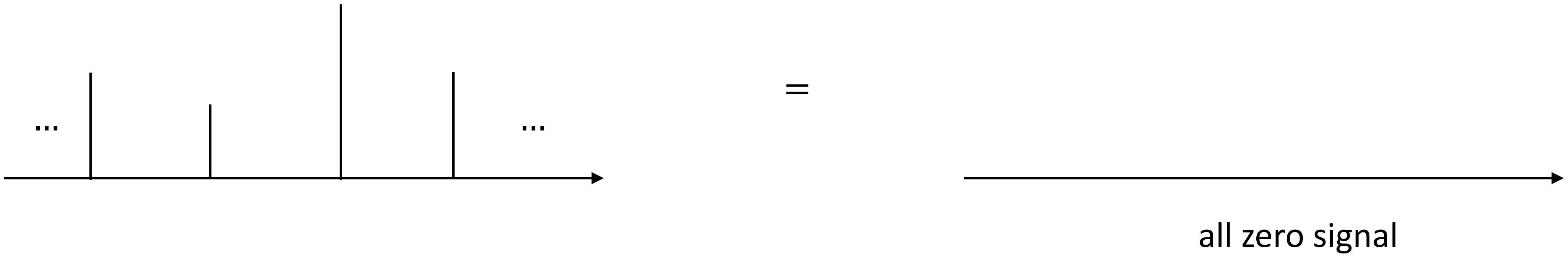
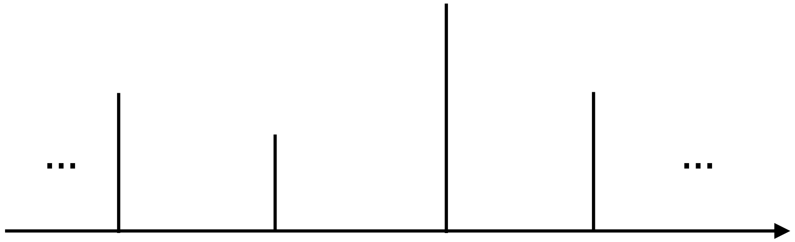


Function equivalence

Two functions are said to be “equivalent” if they are different only at a countable number of points.

Examples of equivalence class 0

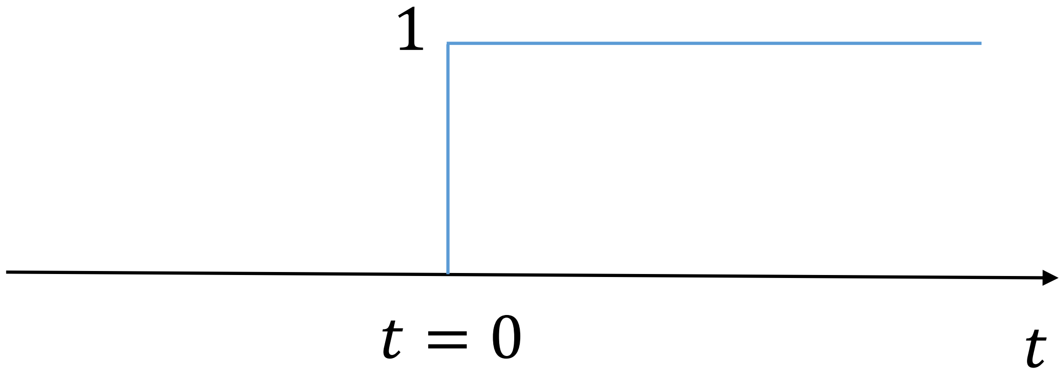


$$a(t) = \begin{cases} 1 & t = rational\ number \\ 0 & t \neq rational\ number \end{cases}$$

all zero signal

Unit-step

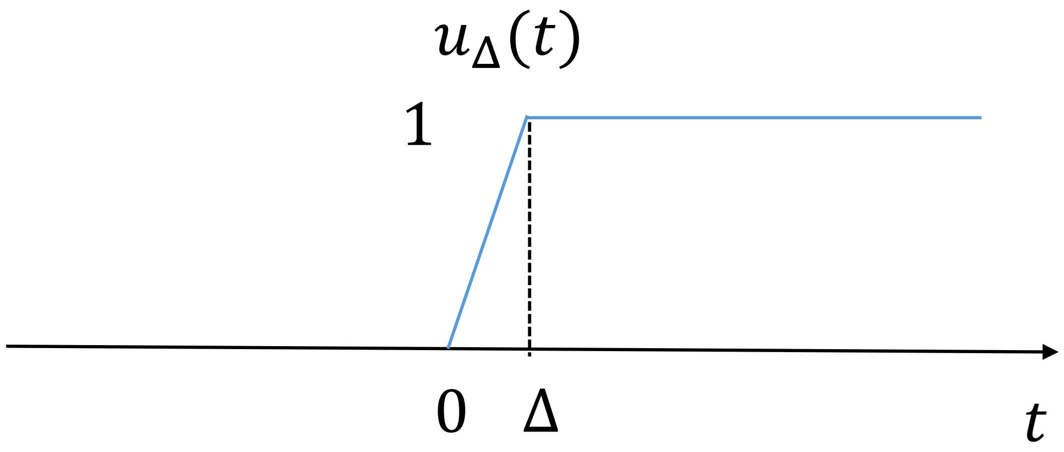
$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \quad \text{or} \quad u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$



At $t = 0$, it is not continuous.

Unit-step as a limiting function

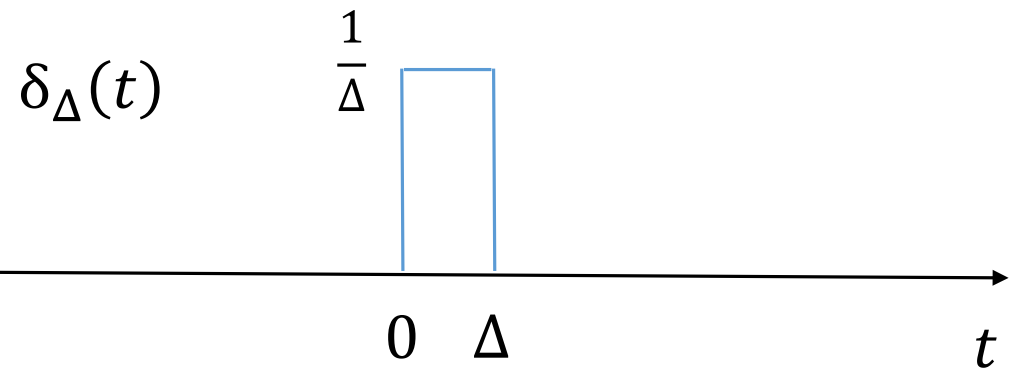
$$u(t) = \lim_{\Delta \to 0} u_\Delta(t)$$



At $t = 0$ and $t = \Delta$, it has slope discontinuity.

Unit-impulse

$$\delta_\Delta(t) = \frac{du_\Delta(t)}{dt}$$

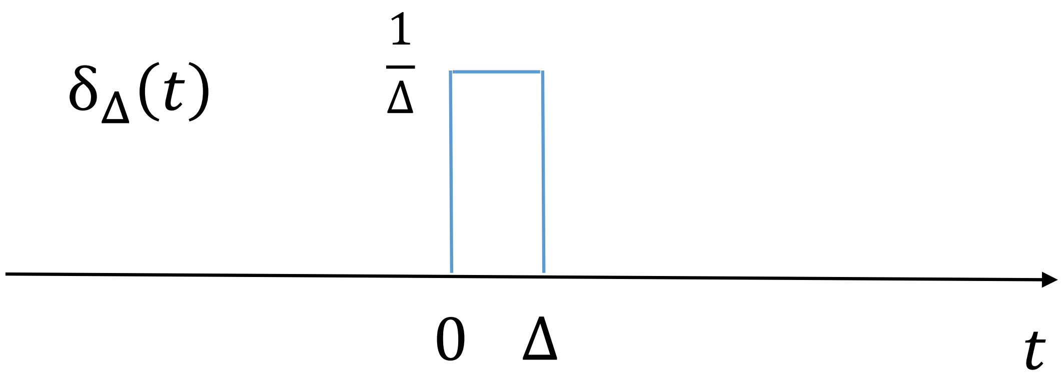


At $t = 0$ and $t = \Delta$, it is discontinuous and ambiguous (but we do not worry about countable points).

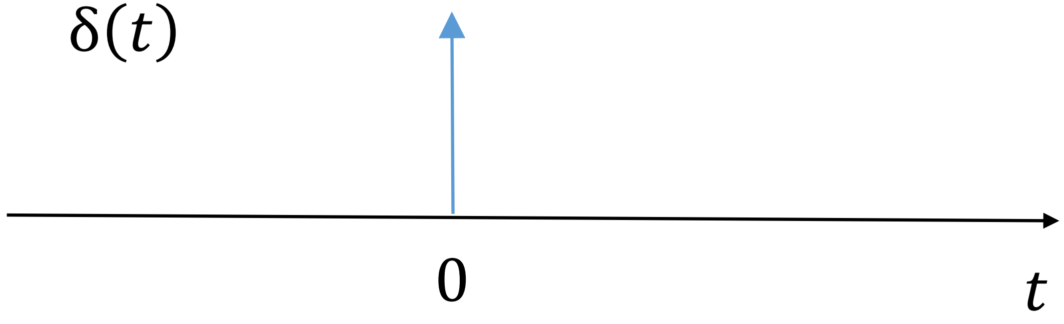
Unit-impulse

$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$$

$\delta_\Delta(t)$



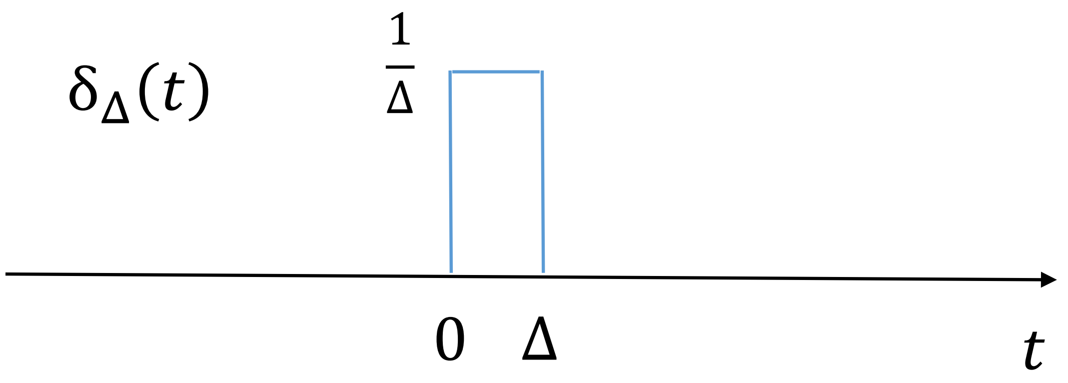
$\delta(t)$

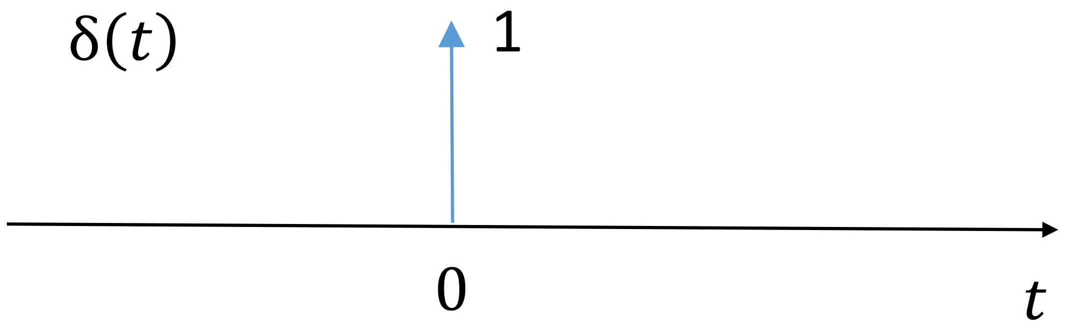


Unit-impulse

$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$$

Known as Dirac delta





Unit-impulse

Von Neumann:

The method of Dirac in no way satisfies mathematical rigor. It is a mathematical ‘fiction’.

Unit-impulse

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The method of Dirac in no way satisfies mathematical rigor. It is a mathematical ‘fiction’.

Laurent Schwartz:

Do not treat this as a function, treat it as a distribution.

How do we define unit-impulse ?

How do we define unit-impulse ?

How do we define any function $f(t)$?

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Amplitude vs. t

How do we define unit-impulse ?

How do we define any function $f(t)$?

Amplitude vs. t

How do we that for $\delta(t)$?

We do not, because we could not.

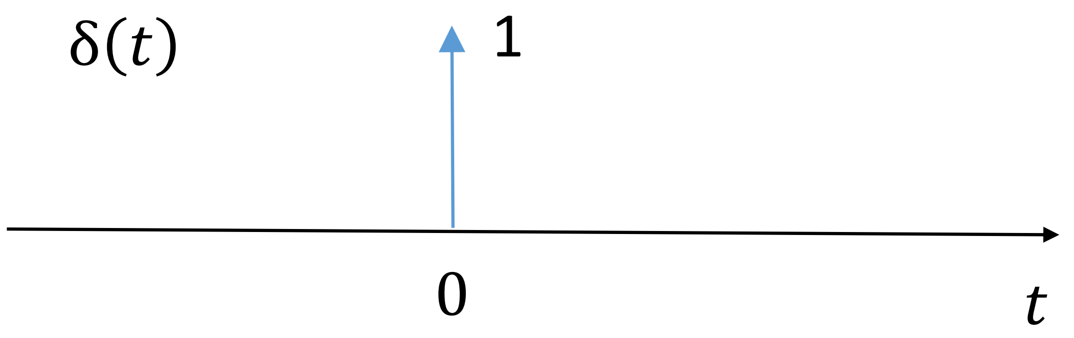
Rather, we define it in terms of its properties.

Generalized functions.

How do we define unit-impulse ?

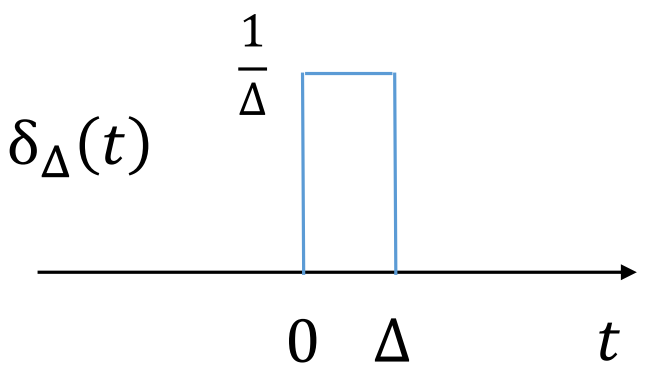
Properties of $\delta(t)$

- $\delta(t) = 0$ for $t \neq 0$
- $\int_{-\infty}^{\infty} \delta(t) dt = 1$



Unit-impulse

$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$$



Unit-impulse

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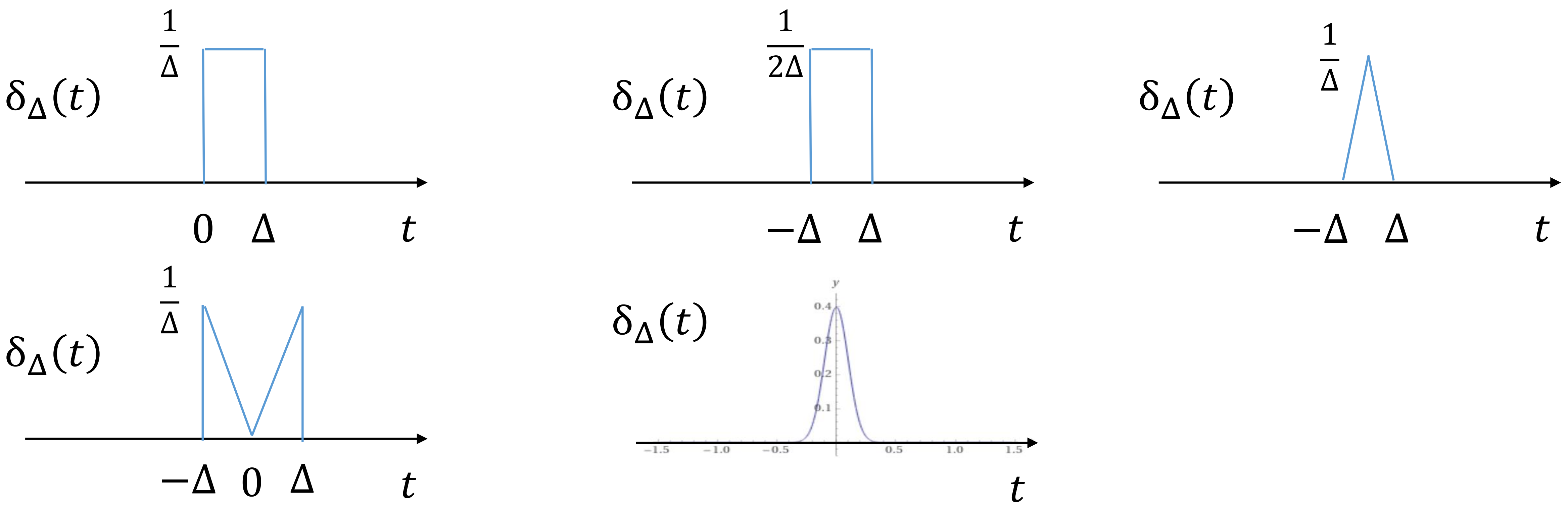

Unit-impulse

Evaluate $\int_{-\infty}^{t} \delta(\tau) d\tau = f(t)$

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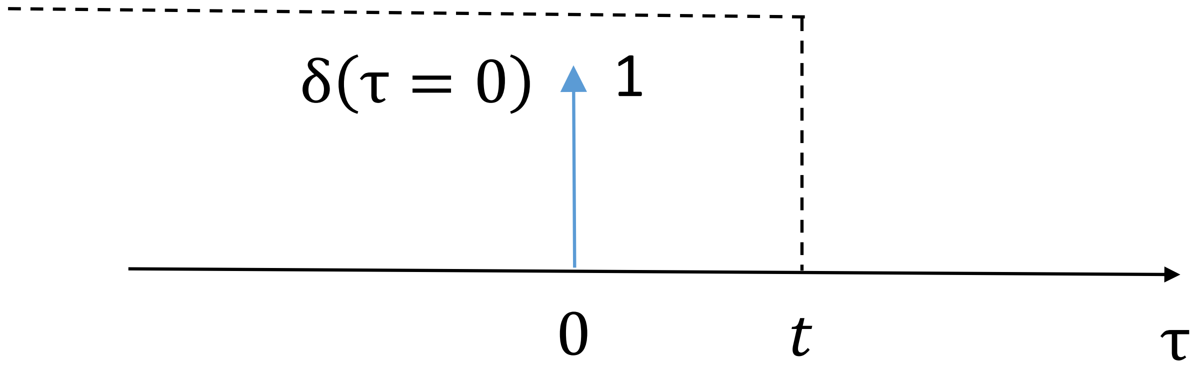
Evaluate $\int_{-\infty}^{t} \delta(\tau)d\tau = f(t)$

running integration

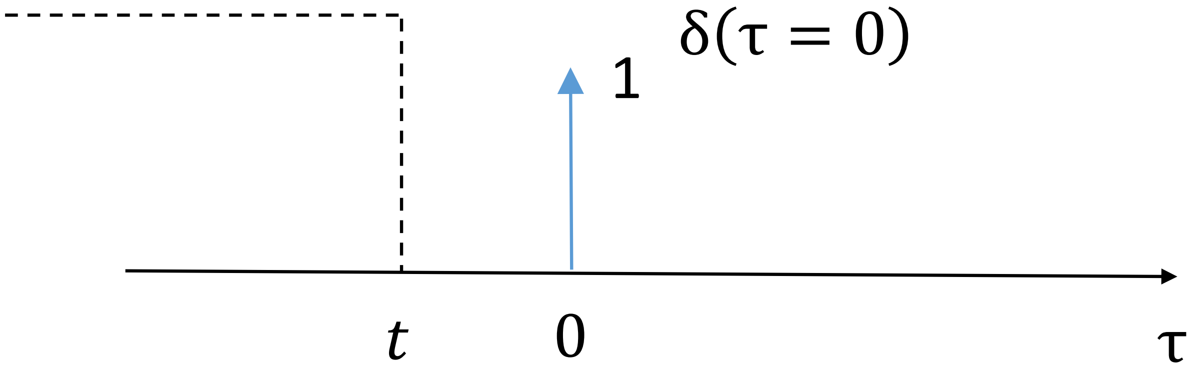


$f(t) = 1 \;\; t > 0$

Unit-impulse

Evaluate $\int_{-\infty}^{t} \delta(\tau)d\tau = f(t)$

running integration



$f(t) = 1 \;\; t > 0$

$f(t) = 0 \;\; t < 0$

Unit-impulse

$$\int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$$

Unit-impulse

$$\int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$$

$$\delta(t) = \frac{du(t)}{dt}$$

Heaviside definition

Unit-impulse

$$\int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$$

$$\delta(t) = \frac{du(t)}{dt}$$

Heaviside definition

$$\int_{0}^{\infty} \delta(t - \sigma)d\sigma = u(t)$$

Unit-impulse

$$x(t)\delta(t) = ?$$

$$\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau = ?$$

Unit-impulse

$$x(t)\delta(t) = x(0)\delta(t)$$

$$\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau = \int_{-\infty}^{\infty} x(t)\delta(t-\tau)d\tau = x(t)\int_{-\infty}^{\infty} \delta(t-\tau)d\tau = x(t)$$

Unit-impulse

$$x(t)\delta(t) = x(0)\delta(t)$$

$$\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau = \int_{-\infty}^{\infty} x(t)\delta(t-\tau)d\tau = x(t)\int_{-\infty}^{\infty} \delta(t-\tau)d\tau = x(t)$$

$x(t)$ is integration of weighted delayed impulses.

Unit-impulse

$$x(t)\sum_{k=-\infty}^{\infty}\delta(t-kT)=x(0)\delta(t)+x(T)\delta(t-T)+x(2T)\delta(t-2T)+\cdots$$

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- Signal transformations
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- Basic Signals (DT)
  - Exponential
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Unit-impulse

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

Unit-sample or Kronecker delta

Unit-step

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

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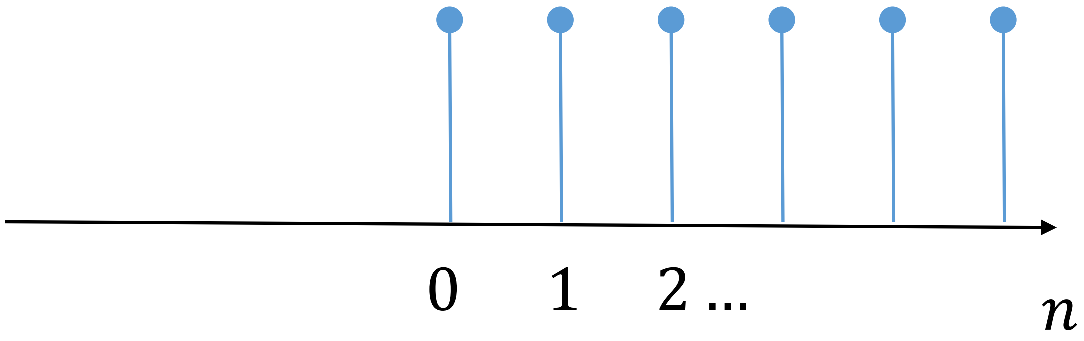
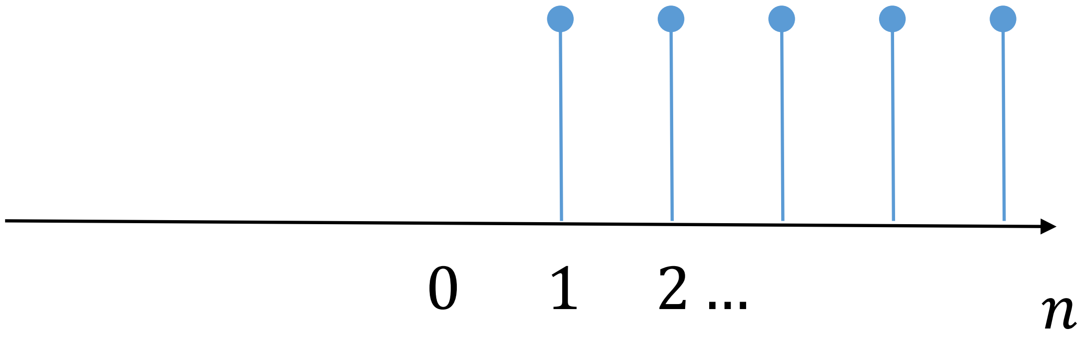
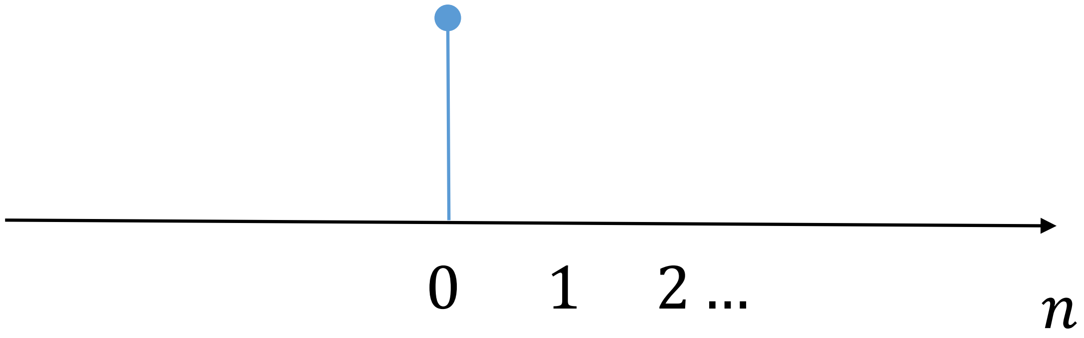


$$u[n-1] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



$$\delta[n] = u[n] - u[n-1]$$



Unit-step

$$\delta[n] = u[n] - u[n-1]$$

$$\delta(t) = \frac{du(t)}{dt}$$

$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau$$

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

$$u(t) = \int_{0}^{\infty} \delta(t-\sigma)d\sigma$$

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Exponential

$Ce^{\alpha n}$

C is 1, and $\mathrm{e}^{\alpha} = a$

a^n

$|a| = 1$, Oscillatory

$|a| < 1$, Decreasing

$|a| = 1$, Increasing

Exponential

$Ce^{\alpha n}$

C is 1, and α is purely imaginary ($j\Omega_0$)

$$e^{j(\Omega_0+2m\pi)n} = e^{j\Omega_0 n} e^{j2mn\pi} = e^{j\Omega_0 n}$$

Signal with frequency Ω_0 and signal with frequency $\Omega_0 + 2m\pi$ are the same.

Frequency wrapping.

Exponential

Why frequency wrapping ?

$e^{j\Omega_0 n}$ Ω_0 is not frequency but it is phase

Thus, for DT complex exponentials, it is enough to consider:
$0 \leq \Omega_0 \leq 2\pi$

In fact, it is enough to consider $0 \leq \Omega_0 \leq \pi$

{baseband frequencies}