Lecture 3 Fri 28 Jul 2023

(LP) $\longrightarrow$ S is its feasible set.

$S = \phi$

(LP) is unfeasible

$S \neq \phi$

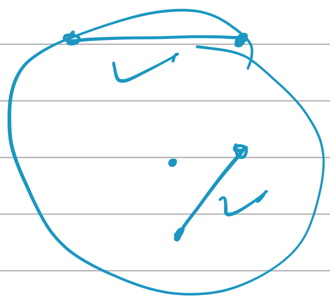
S is bounded

- unique sol^n of (LP)
- Infinite sol^n of (LP).

S is unbounded

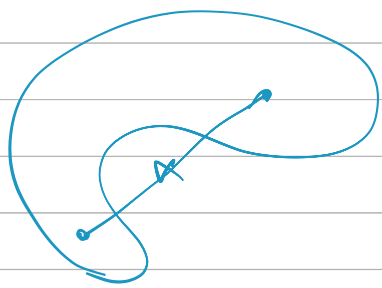
- unique sol^n of (LP)
- Infinite sol^n of (LP)
- (LP) is unbounded

* Convex Set : If $X \subseteq \mathbb{R}^n$ is a convex set if $x, y \in X$ and $\lambda \in [0,1]$, the line segment connecting x & y, i.e. $(1-\lambda)x + \lambda y \in X$.



Convex

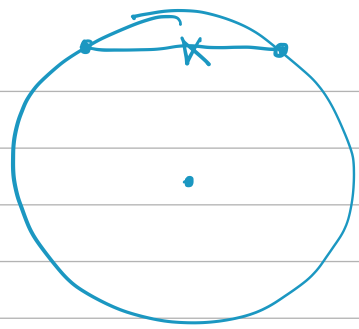
$x^2 + y^2 \leq 1$



Not Convex

$x^2 + y^2 = 1$

Not convex.



Empty set & singleton set are convex

Let $x, y \in S$, $\lambda \in [0,1]$

$\Rightarrow Ax \leq b \qquad x \geq 0$

$Ay \leq b \qquad y \geq 0$

$(1-\lambda) Ax \leq (1-\lambda) b$

$\lambda Ay \leq \lambda b$

$Ax - \lambda A (x-y) \leq b$

$A[x(1-\lambda) + \lambda y] \leq b$

$S = \{x \in \mathbb{R}^n ; Ax \leq b\}$

$\Rightarrow A(\underbrace{x(1-\lambda) + \lambda y}_{\in S.}) \leq b$

* Hyperplane

Given, $a \in \mathbb{R}^n$, $a = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$, $b \in \mathbb{R}$.

$\{x \in \mathbb{R}^n : a^T x \leq b\} \longrightarrow$ hyperplane or plane in n dim^n.

$$\sum_{i=1}^{n} a_i x_i = b$$

* Half Spheres

$X_1 = \{x \in \mathbb{R}^n : a^T x \leq b\}$

below & on the hyperplane.

$\leq \rightarrow$ Closed half space

$< \rightarrow$ open half space

$X_2 = \{x \in \mathbb{R}^n : a^T x \geq b\}$

Half Space $\rightarrow$ Always convex.

If $S = \{A_\lambda : \lambda \in \Lambda\}$ is a family of convex sets in $\mathbb{R}^n$, Λ is index set.

then, $X = \bigcap_{\lambda \in \Lambda} A_\lambda$ is a convex set.

If $X = \phi$, then convex set

Let $x, y \in X$ $\qquad \mu \in [0,1]$

$\Rightarrow x, y \in A_\lambda, \forall \lambda \in \Lambda$

$\Rightarrow (1-\mu)x + \mu y \in A_\lambda, \forall \lambda \in \Lambda.$

∵ A_j is given convex set.

* Union of convex set need not be a convex set

eg $X = \underbrace{\{(x,0) : x \in \mathbb{R}\}}_{\text{convex set}} \cup \underbrace{\{(0,y) : y \in \mathbb{R}\}}_{\text{convex set}}$

* Polyhedron

A set $X = \mathbb{R}^n$ formed by intersection of finite member of half-spaces is called a polyhedron.

Not convex.

* Bounded Polyhedron

↓

polytope

$$S = \{x \in \mathbb{R}^n : Ax \le b,\ x \ge 0\}$$

$$\left\{\begin{array}{l} a_{11}x_1 + \cdots + a_{1n}x_n \le b \\ \quad\vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n \le b \\ -x_1 \le 0 \\ \quad\ddots \\ -x_n \le 0 \end{array}\right.$$

$m+n$ inequalities

* All inequalities individually are half space.

* S is the intersection for all inequalities $\Rightarrow S$ is polyhedron
$\downarrow$
Intersection of $m+n$ half spaces.

$\Rightarrow S \rightarrow$ convex ($\because$ All polyhedron are convex).

Finite intersection of closed space is closed

$\Rightarrow S$ is closed polyhedron