Lecture 12
Signals and Systems (ELL205)

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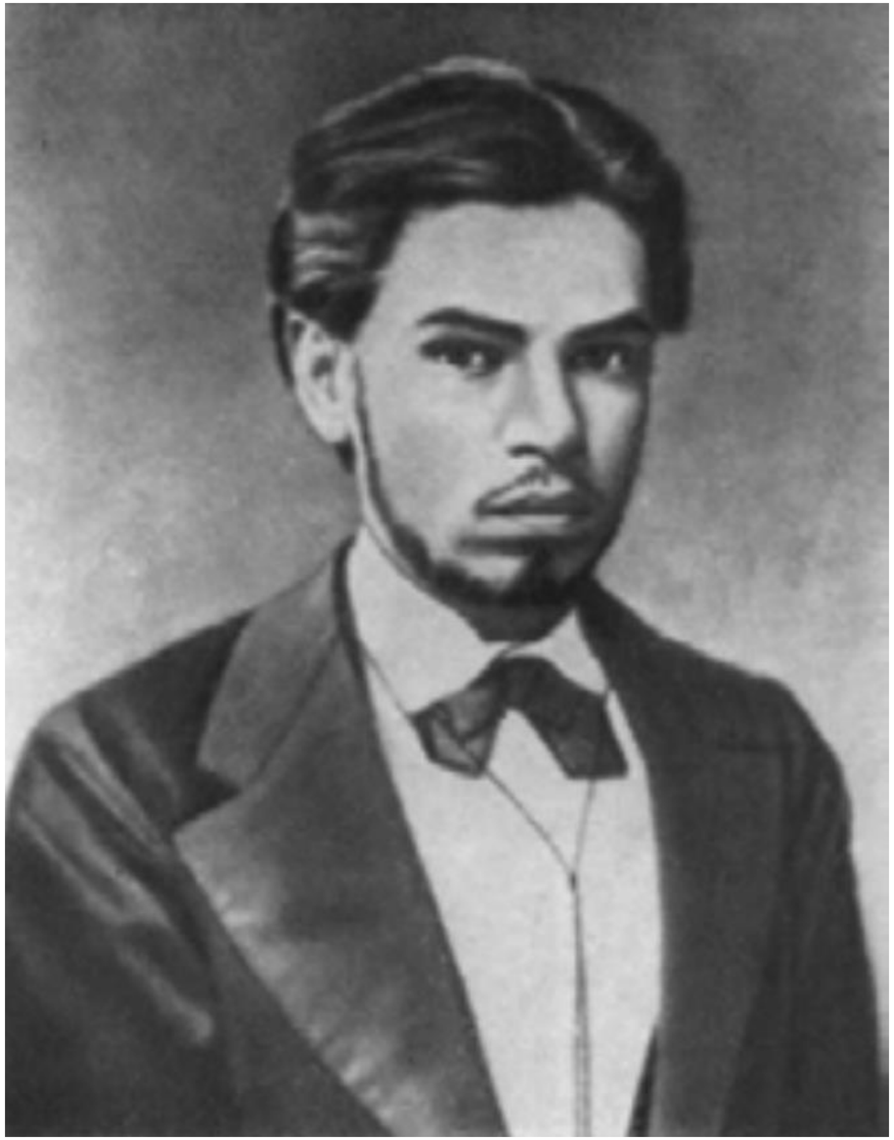
Aleksandr Lyapunov (1857 – 1918)

- Russian mathematician, regarded as Father of modern theory of stability.
- Student of Chebyshev and he also collaborated with Markov.
  - Bifurcation problem : Is it true that the ellipsoid is transformed at a critical velocity into new equilibrium forms?
- The period marked turmoil in Russia but he only talked Maths and science to everyone
- Extremely reserved and hard working (gave many lectures without sleeping).
- He was too found of his wife; shot himself when his wife died of tuberculosis.



Outline of the lecture

- Applications of $h(t)$ to real-life scenarios
- System designing

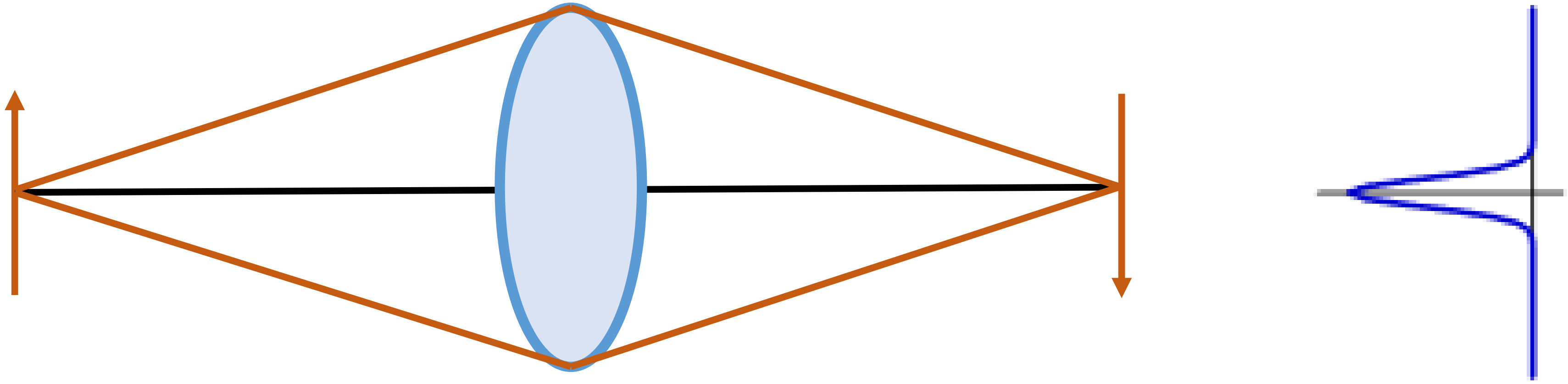
Examples of systems where *h(t)* is a natural metric of system description

1) Communication System
2) Optical System

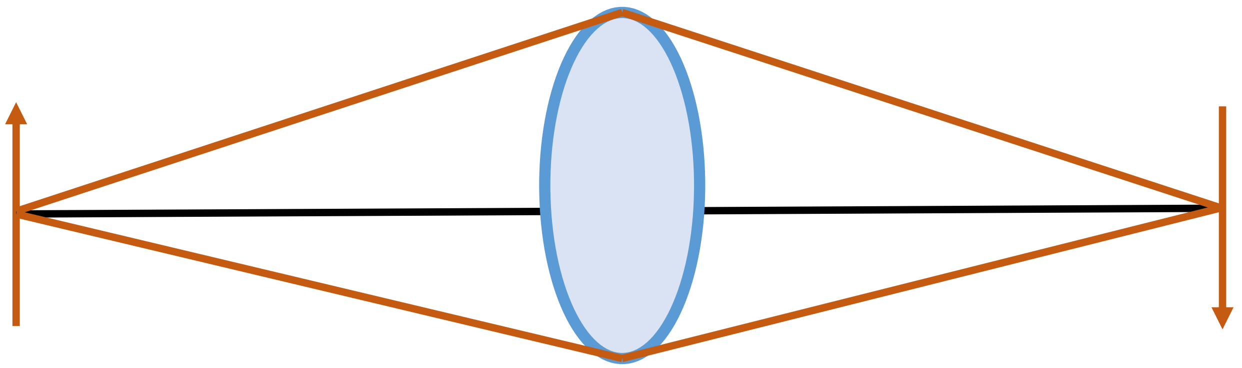
Examples of systems where *h(t)* is a natural metric of system description

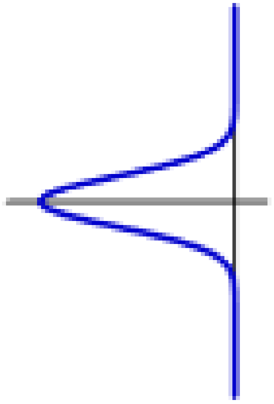
1) Communication System
2) Optical System

Optical system



Optical system



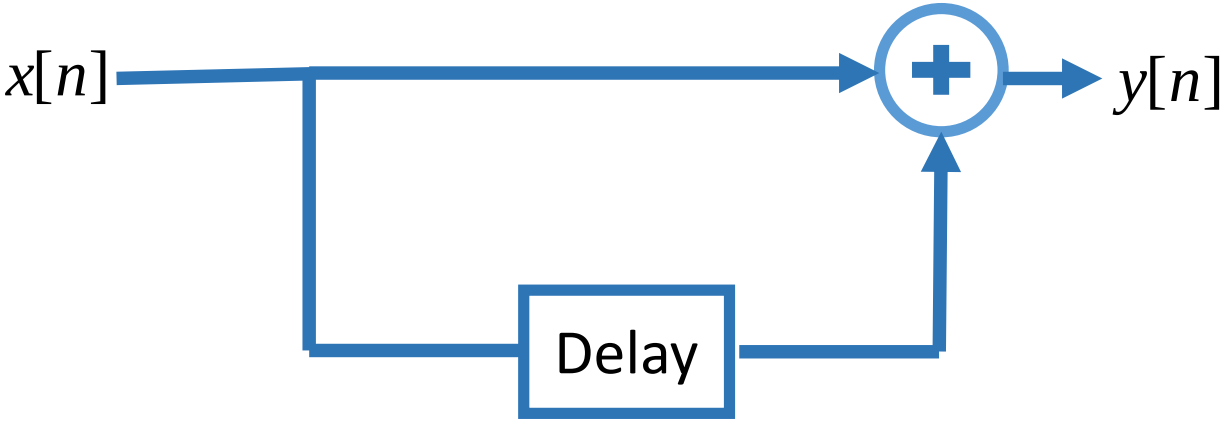


Point spread function

Outline of the lecture

- Applications of *h(t)* to real-life scenarios
- System designing

Basic DT system



Basic characteristics:

Linear (if delay starts at rest)

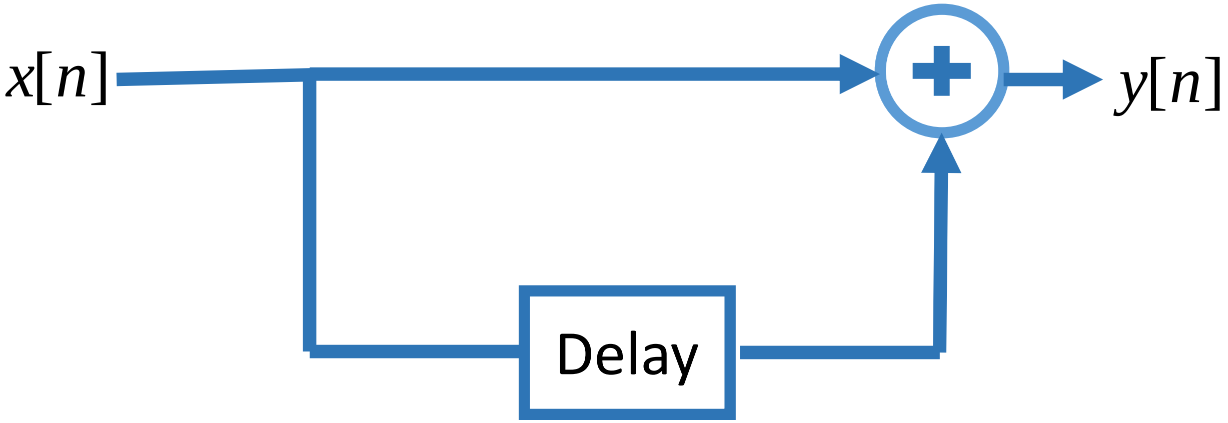
Time-Invariant

Causal

Recipe system

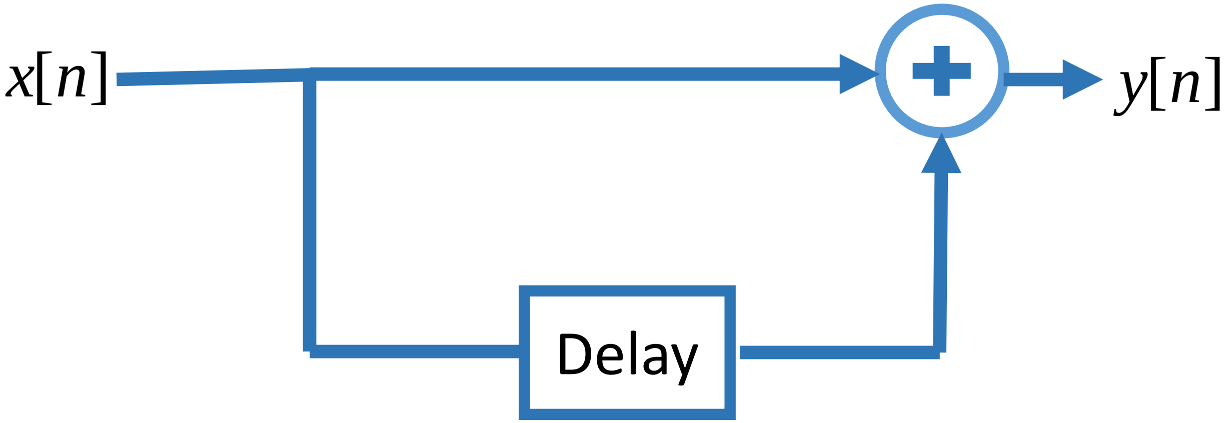
FIR system

Find the impulse response



1) $\boldsymbol{h}[\boldsymbol{n}] = \delta[\boldsymbol{n}] + \delta[\boldsymbol{n} - \mathbf{1}]$	**2)** $\boldsymbol{h}[\boldsymbol{n}] = \delta[\boldsymbol{n}] - \delta[\boldsymbol{n} - \mathbf{1}]$
3) $\boldsymbol{h}[\boldsymbol{n}] = \delta[\boldsymbol{n}] + \delta[\boldsymbol{n} + \mathbf{1}]$	4) $\boldsymbol{h}[\boldsymbol{n}] = \delta[\boldsymbol{n}] - \delta[\boldsymbol{n} + \mathbf{1}]$

Find the impulse response



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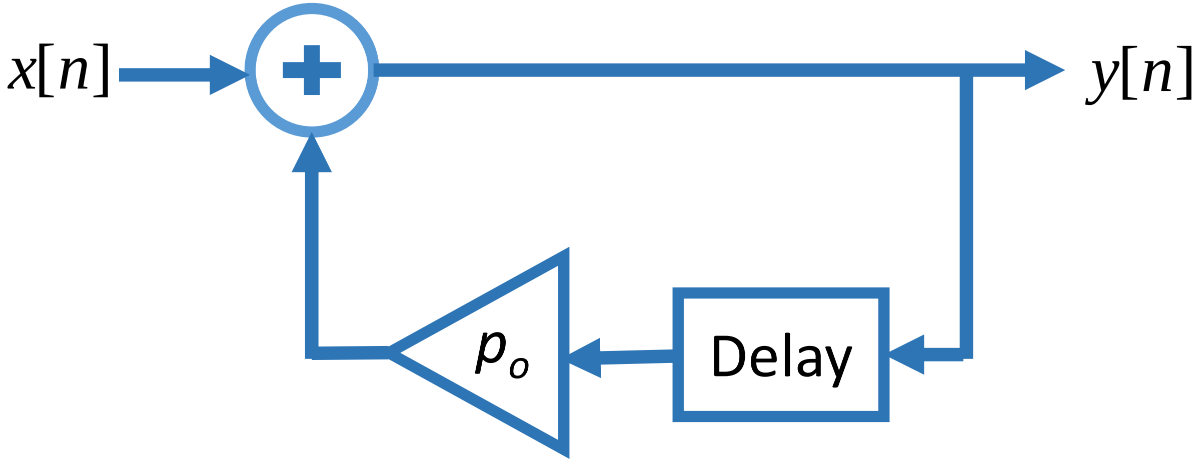
Basic DT system



Basic characteristics:

Linear

Time-Invariant

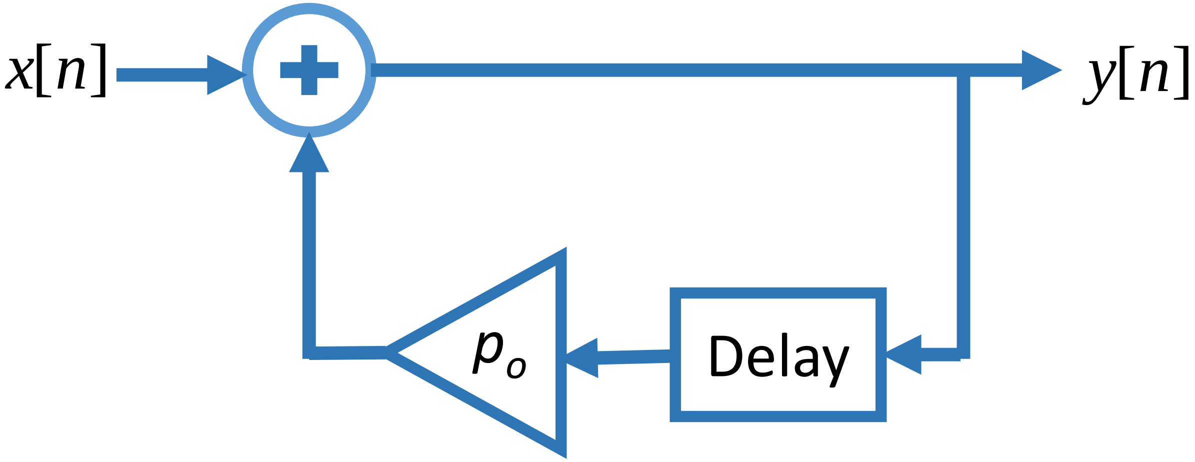
Causal

Constraint/feedback system

IIR system

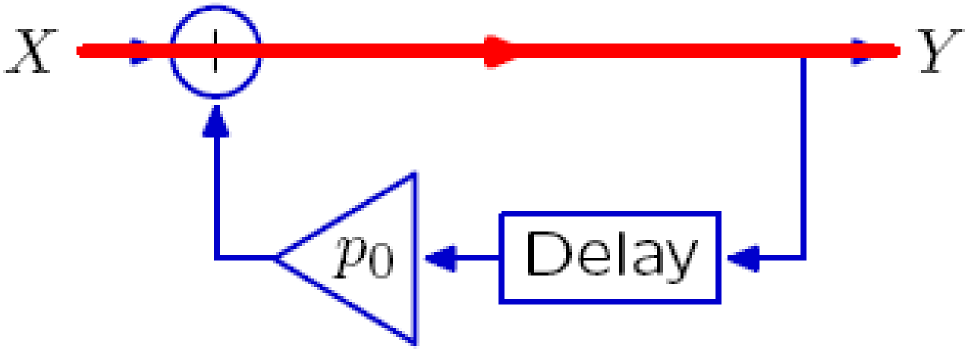
Basic DT system

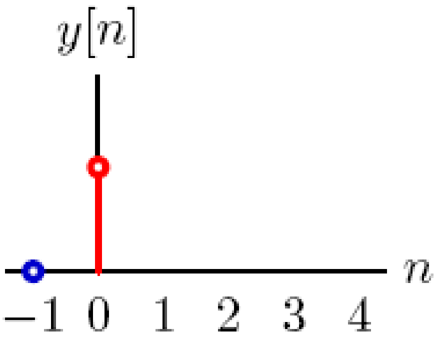
$$\boldsymbol{h[n]} = ?$$



Basic DT system (Graphical method)

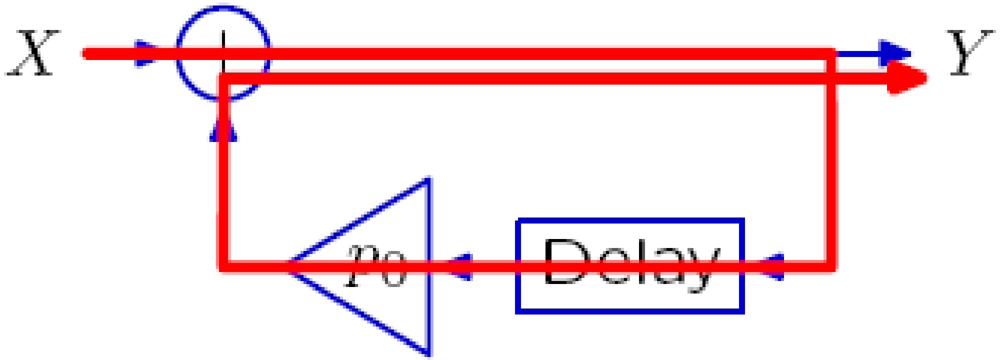
$$\boldsymbol{h}[\boldsymbol{n}] = \delta[\boldsymbol{n}]+..$$

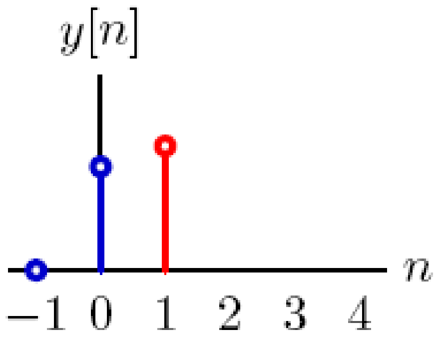




Basic DT system (Graphical method)

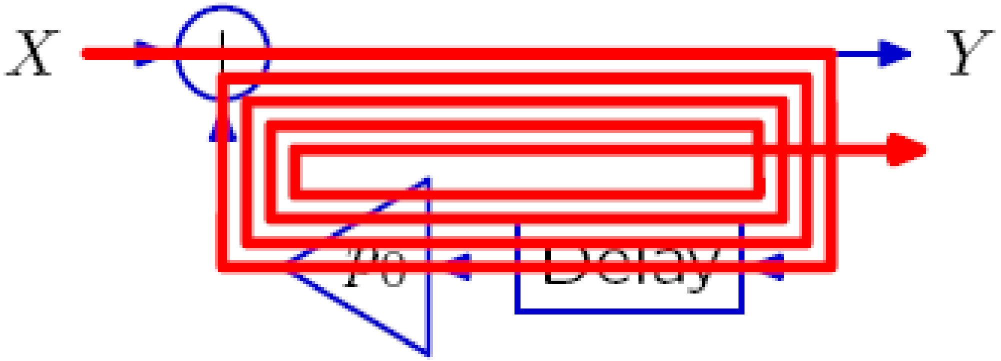
$$\boldsymbol{h}[\boldsymbol{n}] = \delta[\boldsymbol{n}] + p_o\delta[\boldsymbol{n} - \mathbf{1}] + ..$$

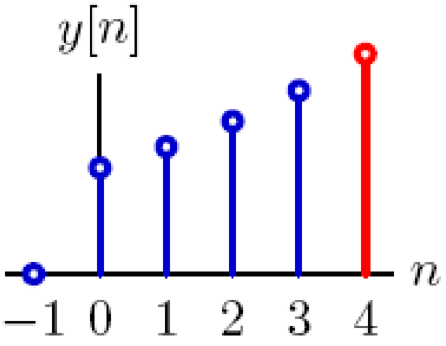




Basic DT system (Graphical method)

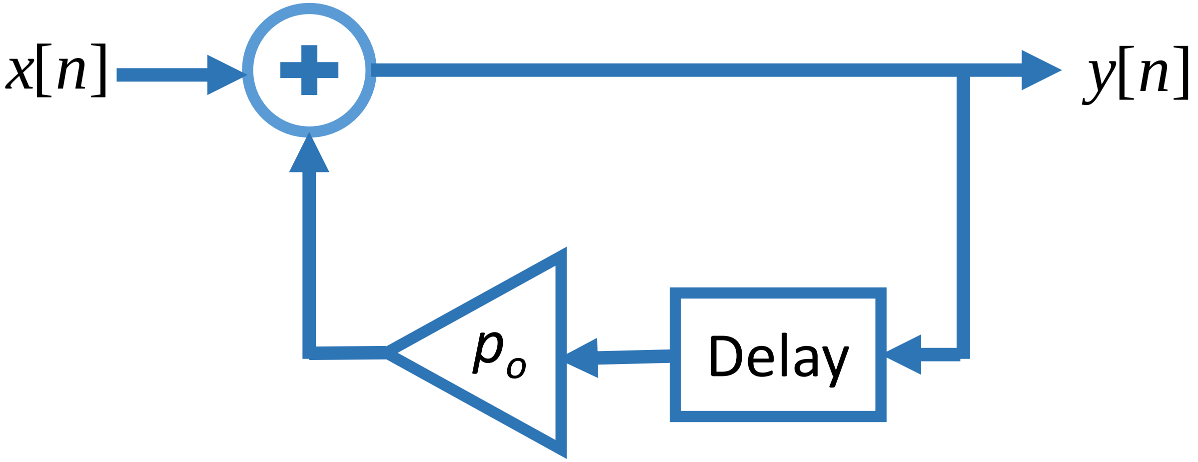
$$\boldsymbol{h[n] = (p_o)^n u[n]}$$





Basic DT system (Step-by-step)

$$\boldsymbol{h[n]} = ?$$



$$y[n] = x[n] + p_o y[n-1]$$

$$\mathrm{h}[n] = \delta[n] + p_o h[n-1]$$

$$\mathrm{h}[n] = 0 \qquad n < 0$$

$$\mathrm{h}[0] = 1$$

$$\mathrm{h}[1] = p_o h[0] = p_o$$

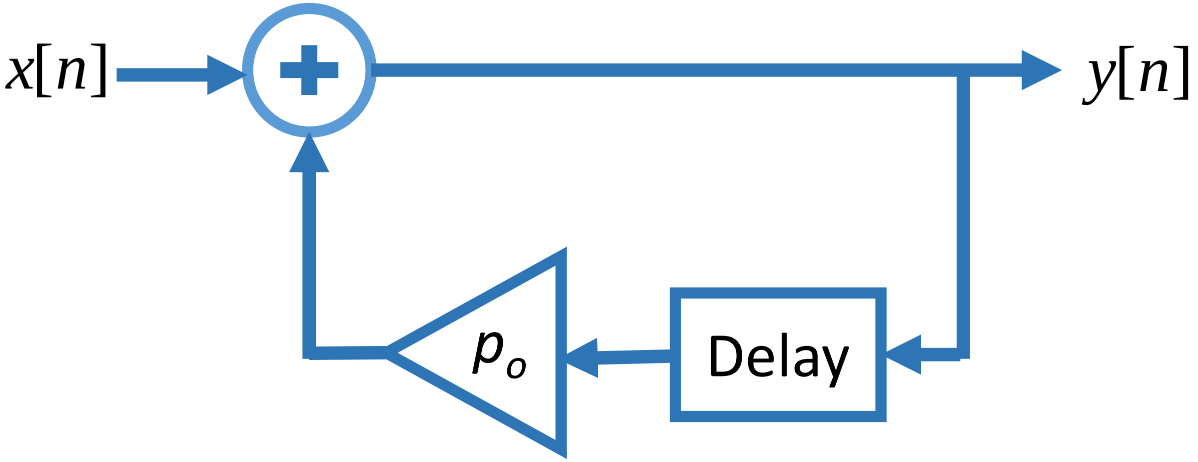
$$\mathrm{h}[2] = p_o h[1] = p_o{}^2$$

⋮

$$\mathrm{h}[n] = p_o h[n-1] = p_o{}^{\mathrm{n}}$$

Basic DT system (Guess-method)

$$\boldsymbol{h[n]} = ?$$



$$y[n] = x[n] + p_o y[n-1]$$

$$\mathrm{h}[n] = Az^n u[n]$$

$$Az^n u[n]$$
$$= \delta[n] + p_o(Az^{n-1}u[n-1])$$

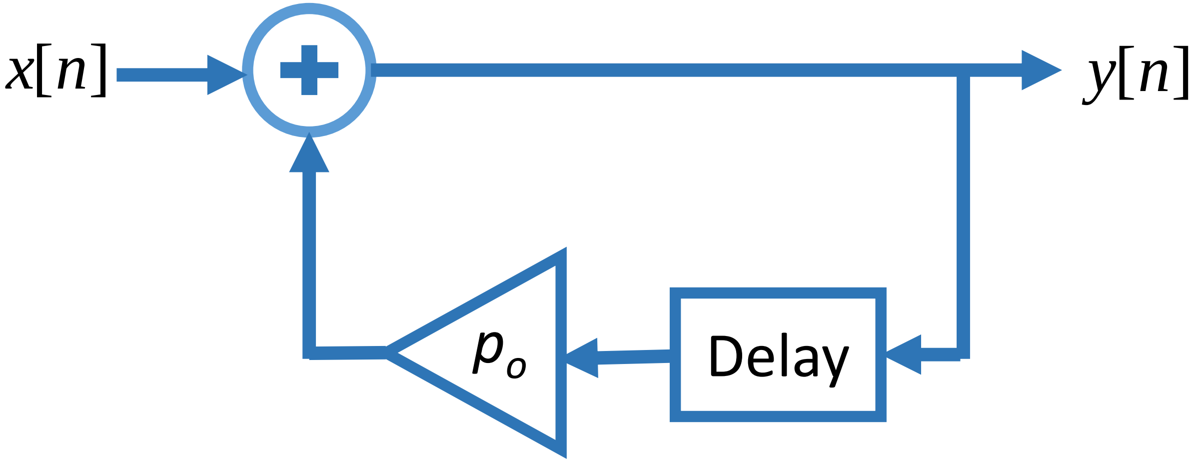
$$n = 0\ A = 1$$

$$n = 1\ z = p_0$$

$$\mathrm{h}[n] = {p_0}^n u[n]$$

Basic DT system (Polynomials)

$$\boldsymbol{h[n]} = ?$$



$$y[n] = x[n] + p_o y[n-1]$$

$$Y = X + p_o R Y$$

$$Y(1 - p_o R) = X$$

$$\frac{Y}{X} = \frac{1}{1 - p_o R}$$

Synthetic Division

$$1 - p_o R \,\overline{\big)\; 1 \qquad\qquad\qquad\qquad\qquad\qquad}$$

Synthetic Division

$$\begin{array}{r l} & 1 \\ 1 - p_o R \,\big) & \overline{1 \qquad\qquad\qquad\qquad\qquad\qquad} \end{array}$$

Synthetic Division

$$\begin{array}{r|l} & 1 \\ \cline{2-2} 1 - p_oR & 1 \\ & 1 - p_oR \\ \cline{2-2} & \quad p_oR \end{array}$$

Synthetic Division

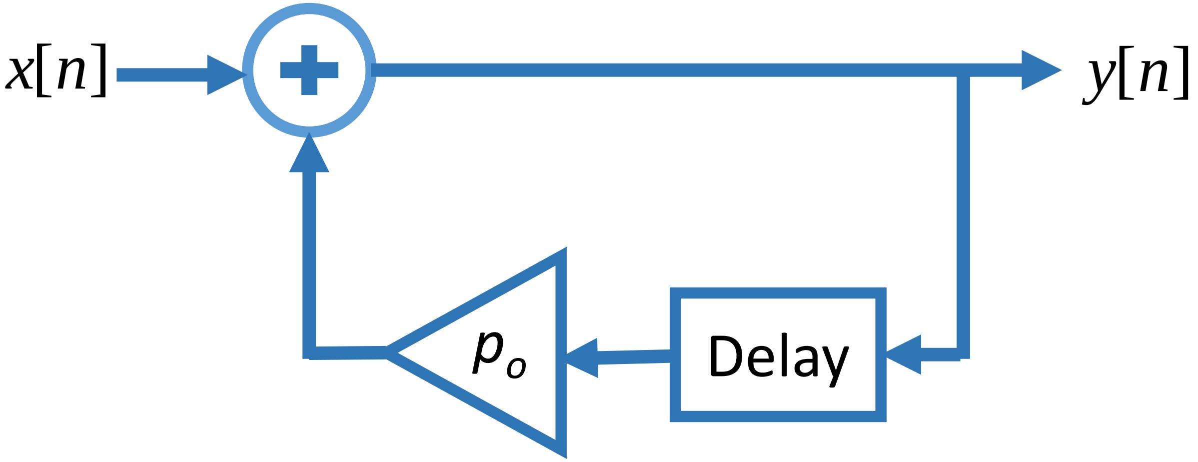
$$\begin{array}{r|l}
 & 1+p_oR \\ \cline{2-2}
1-p_oR & 1 \\
 & 1-p_oR \\ \cline{2-2}
 & \quad p_oR \\
 & \quad p_oR-p_o{}^2R^2 \\ \cline{2-2}
 & \qquad p_o{}^2R^2
\end{array}$$

Synthetic Division

$$
\begin{array}{r|l}
 & 1+p_oR + p_o{}^2R^2 \\ \cline{2-2}
1 - p_oR & 1 \\
 & 1 - p_oR \\ \cline{2-2}
 & p_oR \\
 & p_oR - p_o{}^2R^2 \\ \cline{2-2}
 & p_o{}^2R^2
\end{array}
$$

Basic DT system (Polynomials)

$$\boldsymbol{h[n]} = ?$$



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$$Y = X + p_o R Y$$

$$Y(1 - p_o R) = X$$

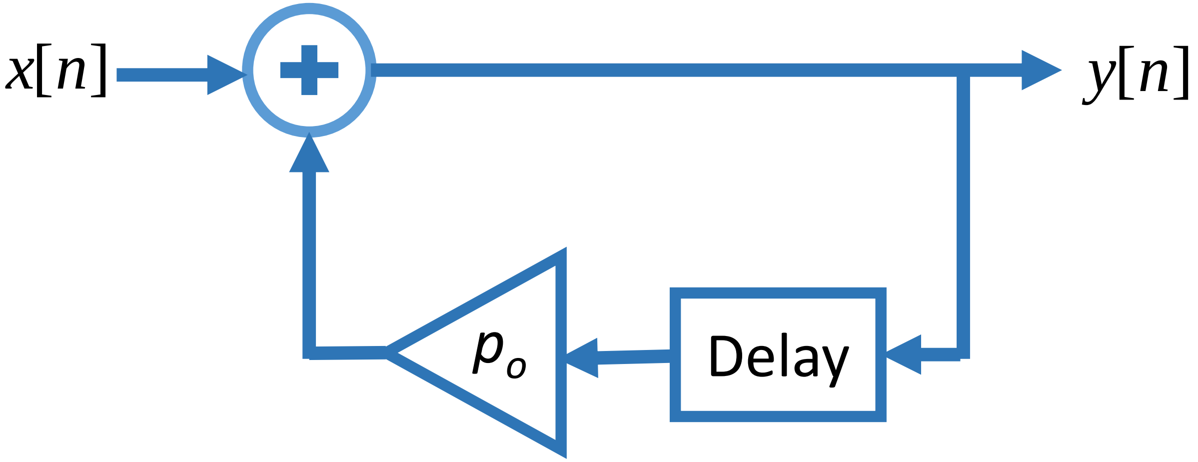
$$\frac{Y}{X} = \frac{1}{1 - p_o R}$$

$$\frac{Y}{X} = (1 + p_o R + {p_o}^2 R^2 + \cdots)$$

$$\mathrm{h[n]} = (1 + p_o R + {p_o}^2 R^2 + \cdots)\delta[n]$$

Basic DT system (Polynomials)

$$\boldsymbol{h}[\boldsymbol{n}] = ?$$



$$y[n] = x[n] + p_o y[n-1]$$

$$\mathrm{h}[\mathrm{n}] = (1 + p_o R + {p_o}^2 R^2 + \cdots)\delta[n]$$

$$\mathrm{h}[\mathrm{n}] = \delta[n] \; + po\delta[n-1] \; + {p_o}^2\delta[n-2] \; + \cdots$$

$$\mathrm{h}[n] = {p_0}^n u[n]$$