ELL101: INTRODUCTION TO ELECTRICAL ENG.

Nodal Analysis

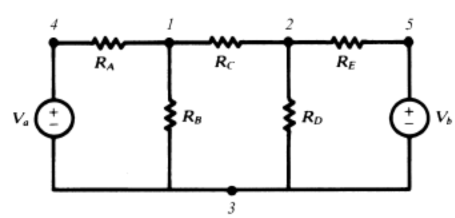
Course Instructors:
Manav Bhatnagar, Subashish Dutta, Debanjan Bhaumik, Harshan Jagadeesh
Department of Electrical Engineering, IITD

1

Nodal Analysis

Introduction

- Popularly known as Node Voltage Analysis
- It is a technique used to find the voltages at the nodes of the circuit
- It is based upon KCL
- It results in system of linear equations that can be solved for unknown voltages
- Number of node voltages = Number of nodes – 1



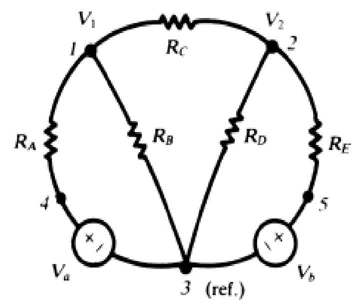
Source: M. Nahvi and J. A. Edminister, *SCHAUM's Outline: Electric Circuits*, McGRAWHILL Edu., 2018.

2

Nodal Analyis

How to Apply Nodal Analysis

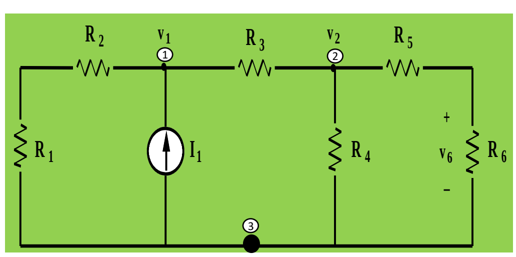
- Identify the nodes available in the circuit
- Identify the node that can be assigned as the reference node
- Assign the voltage to all the other nodes
- Apply the KCL at each node to get the equations
- Solve the equations to get the values of voltage



Source: M. Nahvi and J. A. Edminister, *SCHAUM's Outline: Electric Circuits*, McGRAWHILL Edu., 2018.

Nodal Analysis

Example 1: Use nodal analysis to form voltage relations for the given circuit



$v_3 = 0$

Step 1: The given circuit has three nodes

Step 2: The lowermost node has been taken as reference node and its voltage has been set zero

Step 3: All the other nodes had been assigned an unknown voltage

Nodal Analysis

Step 4: Apply KCL at node 1:

$$I_2 + I_3 = I_1$$

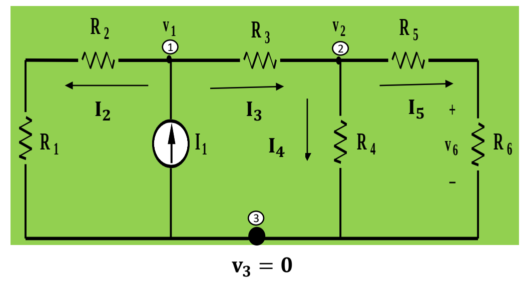
$$\Rightarrow \quad \frac{V_1}{R_1+R_2} + \frac{V_1-V_2}{R_3} = I_1 \qquad (1)$$

Step 5: Apply KCL at node 2:

$$I_4 + I_5 = I_3$$

$$\Rightarrow \quad \frac{V_2}{R_4} + \frac{V_2}{R_5+R_6} = \frac{V_1-V_2}{R_3} \qquad (2)$$

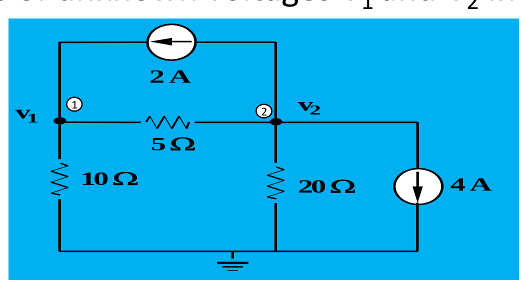
Solving the above equations we can get the value of Voltages V_1 and V_2



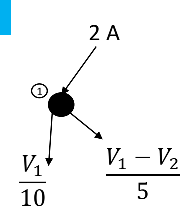
5

Nodal Anlysis

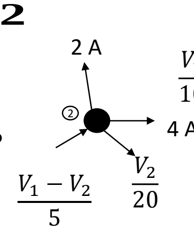
Example : Find the values of unknown voltages V_1 and V_2 in the given circuit



KCL at node 1 $\qquad \dfrac{V_1}{10} + \dfrac{V_1 - V_2}{5} = 2$



KCL at node 2 $\qquad \dfrac{V_2 - V_1}{5} + \dfrac{V_2}{20} = -6$



6

Node Analysis

Equation at node 1 can be simplified to

$$3V_1 - 2V_2 = 20 \qquad (1)$$

Equation at node 2 can be simplified to

$$5\,V_2 - 4V_1 = -120 \qquad (2)$$

Solving (1) and (2) we get the values of unknown voltages as

$\mathbf{V_1 = -20}$ and $\mathbf{V_2 = -40}$

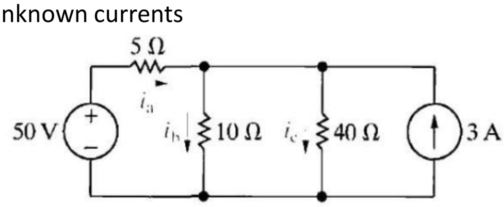
7

Nodal Analysis

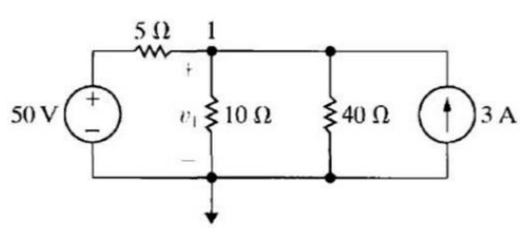
Various cases in Nodal Analysis

Case 1: Voltage source in the circuit (connected with the reference node)

Example: Use nodal analysis to find the value of unknown currents



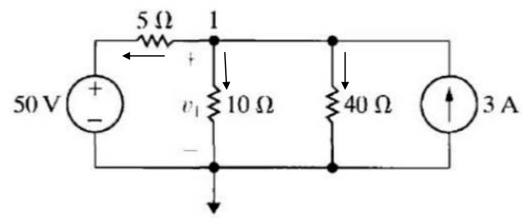
Assigning voltages to the different nodes



8

Nodal Analysis

Applying KCL at node 1



$$\frac{v_1 - 50}{5} + \frac{v_1}{10} + \frac{v_1}{40} - 3 = 0$$

Solving the above equation we get v_1 = 40V

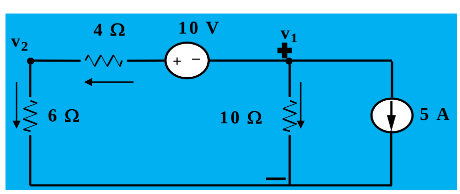
Now, using the above value of v_1 we can find i_a, i_b, and i_c as

i_a = - (v_1 -50) /5 = 2A ; i_b = v_1/ 10 = 4A ; i_c = v_1/40= 1A

Nodal Analysis

Case 2: When there is voltage source (not connected with reference node)

Example: Find the values of unknown voltages in the given circuits



KCL at V_1
$$\frac{V_1}{10} + \frac{V_1 + 10 - V_2}{4} = -5$$

KCL at V_2
$$\frac{V_2}{6} + \frac{V_2 - 10 - V_1}{4} = 0$$

Node Analysis

Equation at V_1 can be simplified to

$7V_1 – 5V_2 = -150$ (1)

Equation at V_2 can be simplied to

$5 V_2 – 3V_1 = 30$ (2)

Solving (1) and (2) we get the values of unknown voltages as

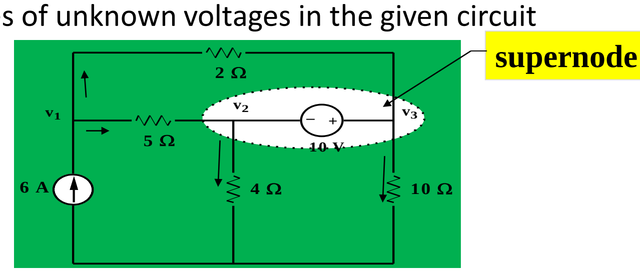
$\mathbf{V_1 = -30V}$ and $\mathbf{V_2 = -12V}$

11

Nodal Analysis

Case 3: Supernode (Voltage source between two nodes without any resistance)

Example: Find the values of unknown voltages in the given circuit



KCL at V_1
$$\frac{V_1 - V_2}{5} + \frac{V_1 - V_3}{2} = 6$$

KCL at Supernode
$$\frac{V_2 - V_1}{5} + \frac{V_2}{4} + \frac{V_3}{10} + \frac{V_3 - V_1}{2} = 0$$

12

Node Analysis

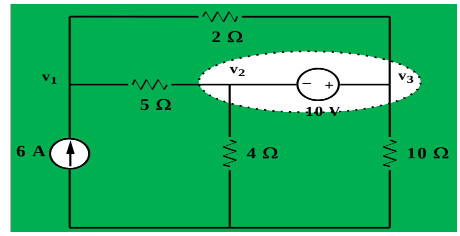
Equation at V_1 can be simplified to

$7V_1 - 2V_2 - 5V_3 = 60$ (1)

Equation at supernode can be simplified to

$9V_2 - 14V_1 + 12V_3 = 0$ (2)

$V_2 - V_3 = -10$ (3)



Solving (1), (2) and (3) we get the values of unknown voltages as

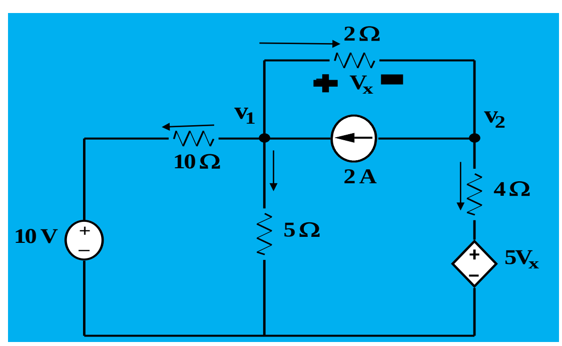
V_1 = 30V, V_2 = 14.29V and **V_3= 24.29V**

13

Nodal Analysis

Case 4: When dependent sources are present

Example: Find the unknown voltages



KCL at V_1 $\quad \dfrac{V_1 - 10}{10} + \dfrac{V_1}{5} + \dfrac{V_1 - V_2}{2} = 2$

KCL at V_2 $\quad \dfrac{V_2 - V_1}{2} + \dfrac{V_2 - 5V_x}{4} = -2$

14

Node Analysis

Equation at V_1 can be simplified to

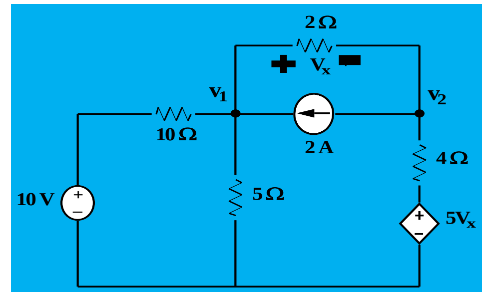
$$8V_1 - 5V_2 = 30 \qquad (1)$$

Equation at V_2 can be simplied to

$$3V_2 - 2V_1 - 5V_x = -8 \qquad (2)$$

Also,

$$V_x = V_1 - V_2 \qquad (3)$$



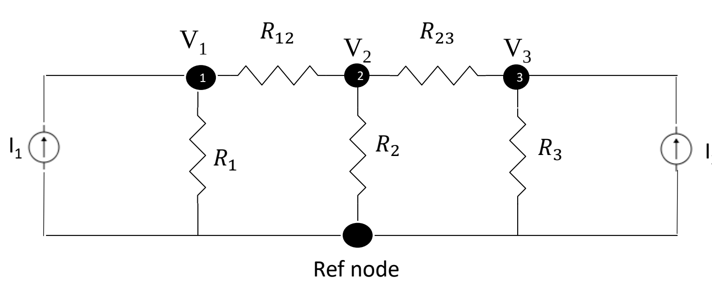
Solving (1), (2) and (3) we get the values of unknown voltages as

$\mathbf{V_1 = 6.9V}$ and $\mathbf{V_2 = 5.03V}$

15

Matrix Based Nodal Analysis

- Apply nodal analysis to find the values of V_1, V_2, V_3



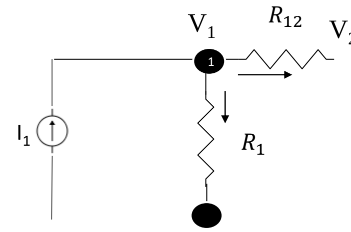
Need three equations to find the values of three node voltages.

16

KCL at Node 1

$$I_1 = \frac{V_1 - V_2}{R_{12}} + \frac{V_1}{R_1}$$

$$V_1\left(\frac{1}{R_{12}} + \frac{1}{R_1}\right) - \frac{V_2}{R_{12}} = I_1$$

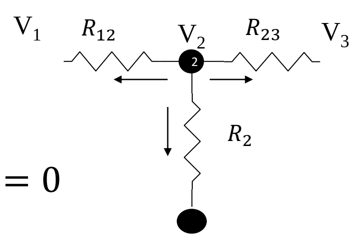


17

KCL at Node 2

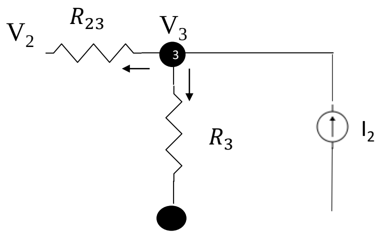
$$\frac{V_2 - V_1}{R_{12}} + \frac{V_2}{R_2} + \frac{V_2 - V_3}{R_{23}} = 0$$

$$-\frac{V_1}{R_{12}} + V_2\left(\frac{1}{R_{12}} + \frac{1}{R_2} + \frac{1}{R_{23}}\right) - \frac{V_3}{R_{23}} = 0$$



18

KCL at Node 3



$$\frac{V_3 - V_2}{R_{23}} + \frac{V_3}{R_3} = I_2$$

$$-\frac{V_2}{R_{23}} + V_3\left(\frac{1}{R_{23}} + \frac{1}{R_3}\right) = I_2$$

19

System of Equations

- Node 1:

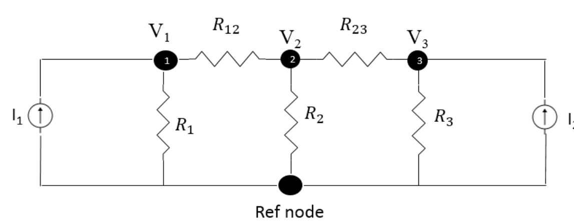
$$V_1\left(\frac{1}{R_{12}} + \frac{1}{R_1}\right) - \frac{V_2}{R_{12}} = I_1$$

- Node 2:

$$-\frac{V_1}{R_{12}} + V_2\left(\frac{1}{R_{12}} + \frac{1}{R_2} + \frac{1}{R_{23}}\right) - \frac{V_3}{R_{23}} = 0$$

- Node 3:

$$-\frac{V_2}{R_{23}} + V_3\left(\frac{1}{R_{23}} + \frac{1}{R_3}\right) = I_2$$



The left side of the equation:

- The node voltage is multiplied by the sum of *conductances* of all resistors connected to the node.
- Other node voltages are multiplied by the conductance of the resistor(s) connecting to the node and subtracted.

The right side of the equation:

- The right side of the equation is the sum of currents from sources entering the node.

20

Matrix Notation

- The three equations can be combined into a single matrix/vector equation

$$\begin{bmatrix} \left(\frac{1}{R_{12}}+\frac{1}{R_1}\right) & -\frac{1}{R_{12}} & 0 \\ -\frac{1}{R_{12}} & \left(\frac{1}{R_{12}}+\frac{1}{R_2}+\frac{1}{R_{23}}\right) & -\frac{1}{R_{23}} \\ 0 & -\frac{1}{R_{23}} & \left(\frac{1}{R_{23}}+\frac{1}{R_3}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

- The equation can be written in matrix-vector form as

$$\mathbf{AV} = \mathbf{I}$$

- The solution to the equation can be written as $\mathbf{V} = \mathbf{A}^{-1}\mathbf{I}$

21

Matrix Inversion

- Given the 2x2 matrix A

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

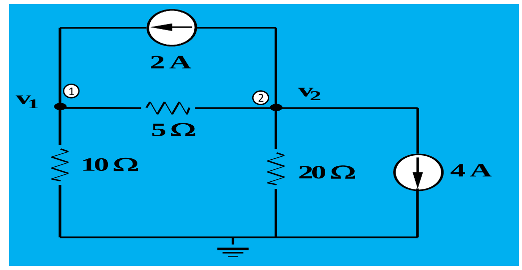
- The inverse of A is

$$\mathbf{A}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

22

Example : Find the values of unknown voltages V_1 and V_2 in the given circuit

$$\begin{bmatrix} \frac{1}{10}+\frac{1}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{20}+\frac{1}{5} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{3}{10} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{4} \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{1}{\frac{3}{40}-\frac{1}{25}} \begin{bmatrix} \frac{1}{4} & \frac{1}{5} \\ \frac{1}{5} & \frac{3}{10} \end{bmatrix} \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{200}{7} \begin{bmatrix} -\frac{7}{10} \\ -\frac{14}{10} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -20 \\ -40 \end{bmatrix}$$