Electromagnetic Waves in Plasmas

PYL101: Electromagnetics and Quantum Mechanics

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References

Fundamentals of Plasma Physics by J.A. Bittencourt

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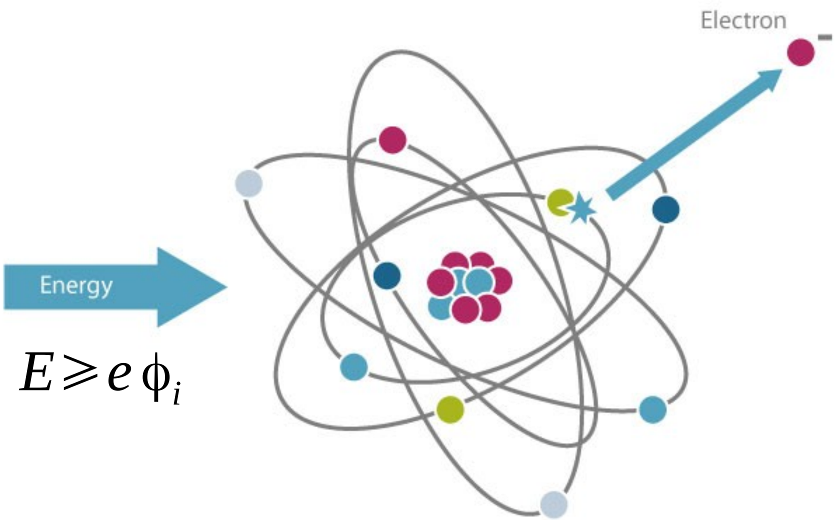
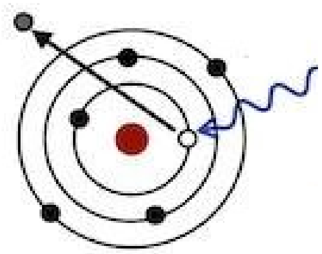
Ionization



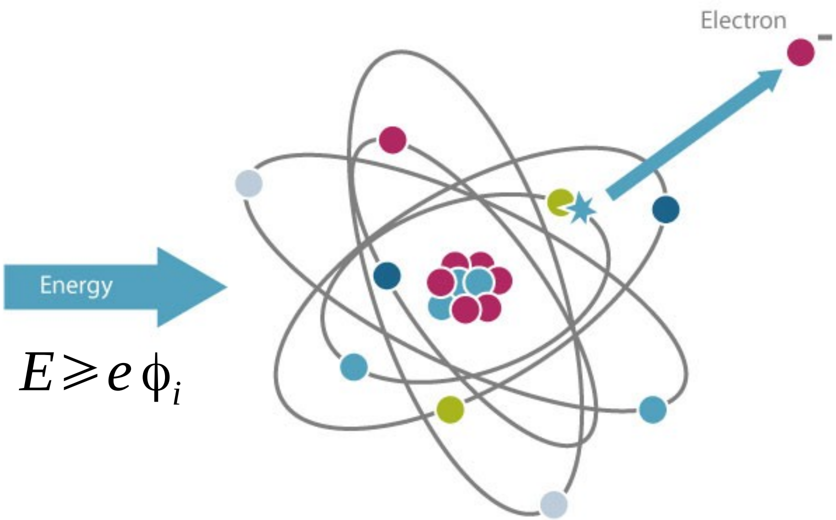
$$E \geqslant e\phi_i$$



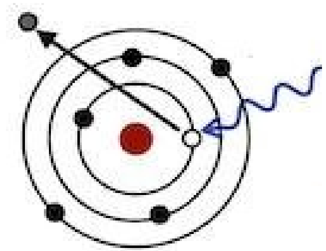
$$h\nu \geqslant e\phi_i$$

Photo-ionization

Ionization

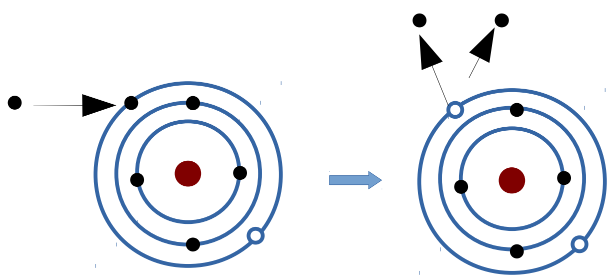


$E \geq e\phi_i$



$h\nu \geq e\phi_i$

Photo-ionization



Electron impact ionization

For H: $e\phi_i = 13.6\,eV$

$$\boxed{E = \frac{3}{2} k_B T}$$

$$k_B = 1.38 \times 10^{-23}\,J/K$$

$$\Rightarrow \quad 1\,eV = 11600\,K$$

Saha Ionization Formula

There is also a possibility of an electron to recombine with an ion thus becoming a neutral atom.

In thermal equilibrium, the degree of ionization is given by:

$$\boxed{\frac{n_i}{n_n} = 2.4 \times 10^{21} \frac{T^{3/2}}{n_i} e^{-e\phi_i / k_B T}}$$

where

$$n_i - density\ of\ ionized\ atoms\ (m^{-3})$$
$$n_n - density\ of\ neutral\ atoms\ (m^{-3})$$
$$T - gas\ temperature\ (K)$$
$$e - electronic\ charge\ (1.6 \times 10^{-19}\ Coulomb)$$
$$\phi_i - ionization\ potential\ (eV)$$
$$k_B - Boltzmann\ constant$$

Saha Ionization Formula

For air

$$n_n = 3 \times 10^{25}\, m^{-3}$$
$$k_B = 1.38 \times 10^{-23}\, J/K\,;\, T = 300\, K$$
$$\phi_i = 14.5\, eV\, (Nitrogen)$$

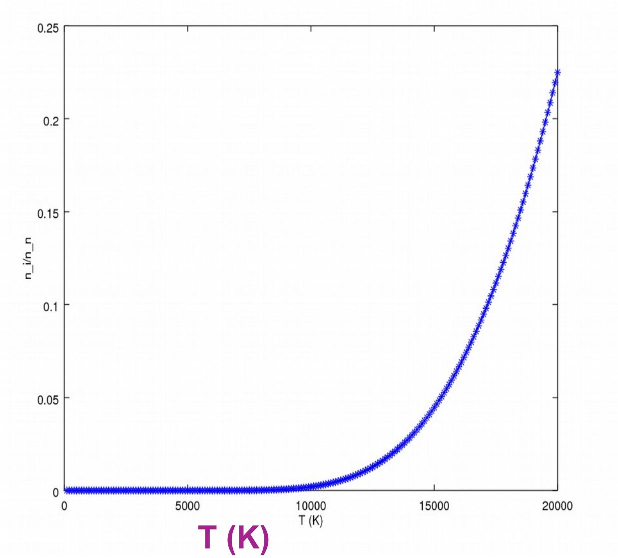
Saha Ionization Formula

For air

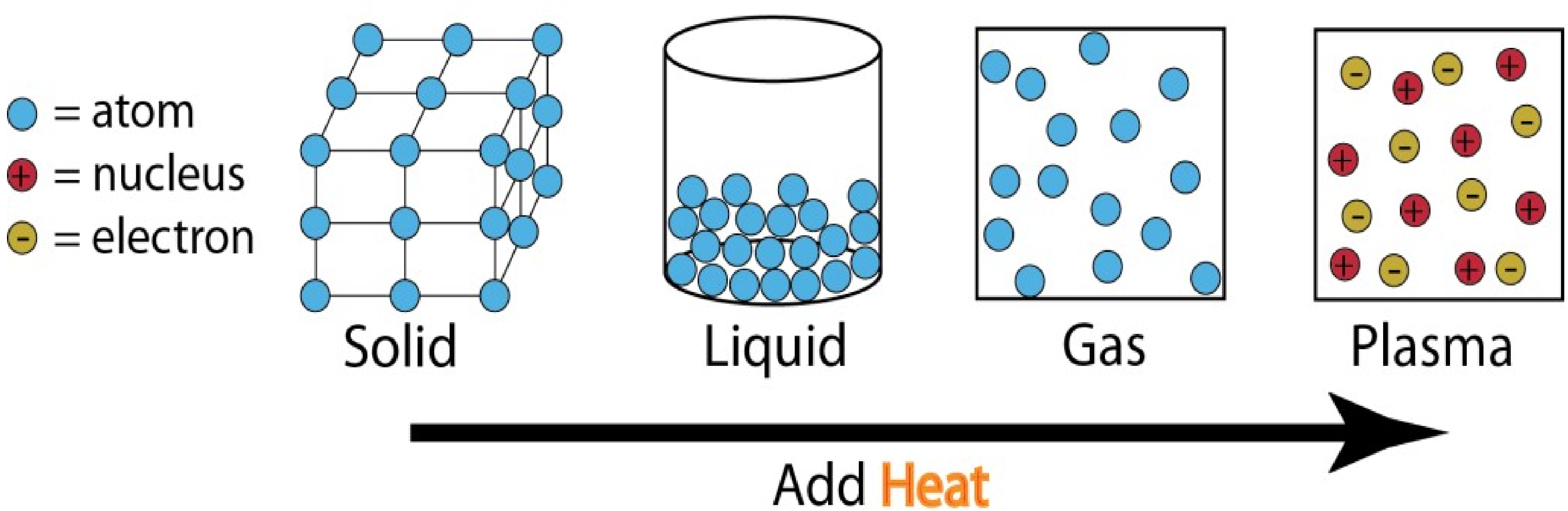
$$n_n = 3 \times 10^{25} \, m^{-3}$$
$$k_B = 1.38 \times 10^{-23} \, J/K \, ; T = 300 \, K$$
$$\phi_i = 14.5 \, eV \, (Nitrogen)$$

$$n_i \approx 10^{-97} \, m^{-3}$$
$$n_i / n_n \approx 10^{-122}$$

n_i / n_n



Plasma : The fourth state of matter

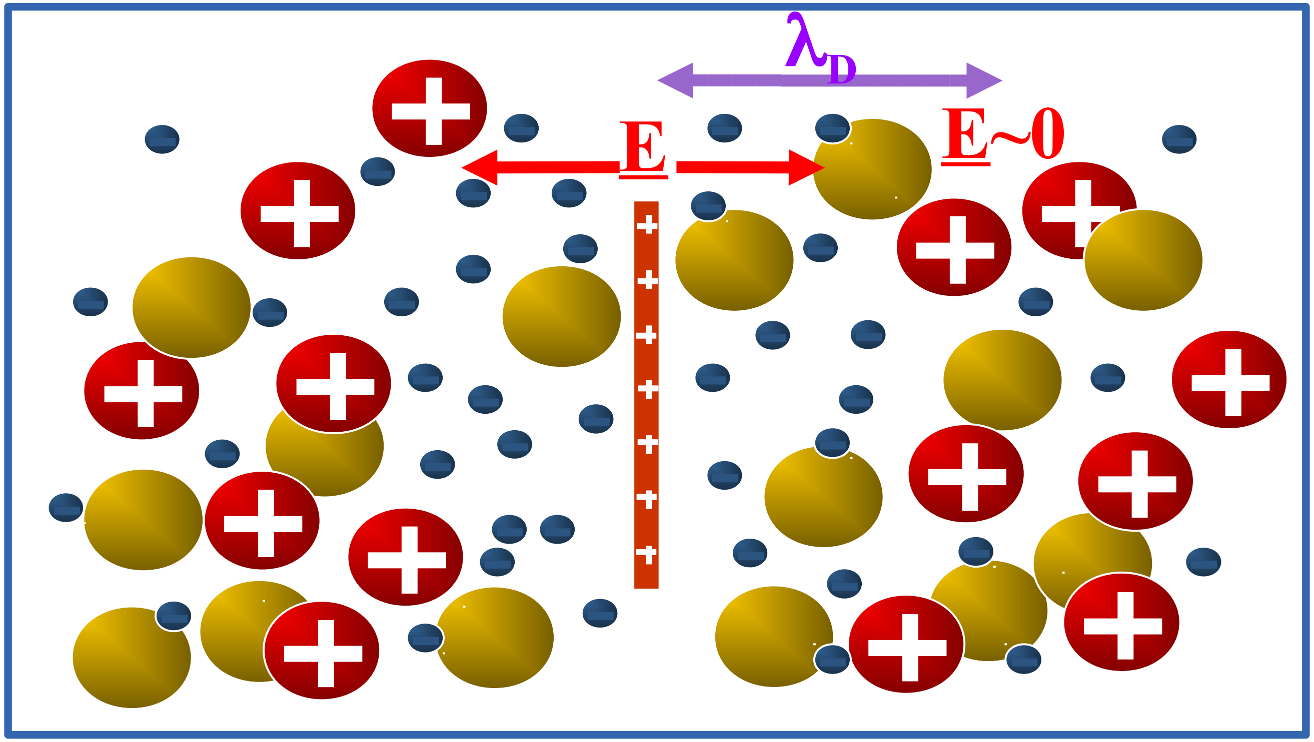


	273K		373K		10^4K		10^5K		10^7K		~10^{12}K	
ICE	→	WATER	→	STEAM	→	H and O atoms	→	H^+, e^- and O^+	→	$H^+ + e^-$ $O^{+8} + 8e^-$	→	QGP
					1eV		13.6eV		870eV		100MeV	

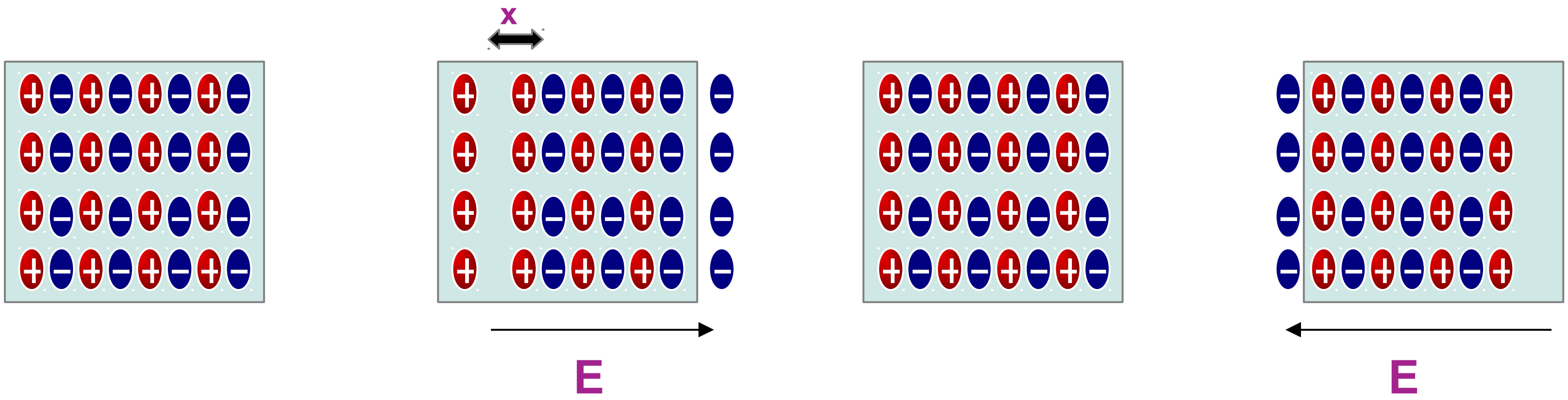
Debye Screening

The screened potential : $\phi(x)=\phi_0 * \exp(-x/\lambda_D)$

where $\lambda_D=(kT/4\pi n_0 e^2)^{1/2}$ is called Debye length.



Plasma Oscillations



Plasma frequency

$$m\ddot{x} = \left(\frac{nex}{\epsilon_0}\right) * -e$$

$$\Rightarrow \quad \ddot{x} + \frac{ne^2}{\epsilon_0 m} x = 0$$

$$\Rightarrow \quad \ddot{x} + \omega_p^2 x = 0$$

where $\boxed{\omega_p = \left(\frac{ne^2}{\epsilon_0 m}\right)^{1/2}}$ is called the electron plasma frequency.

Plasma Conditions

A plasma is quasi-neutral:

$$\lambda_D << L$$

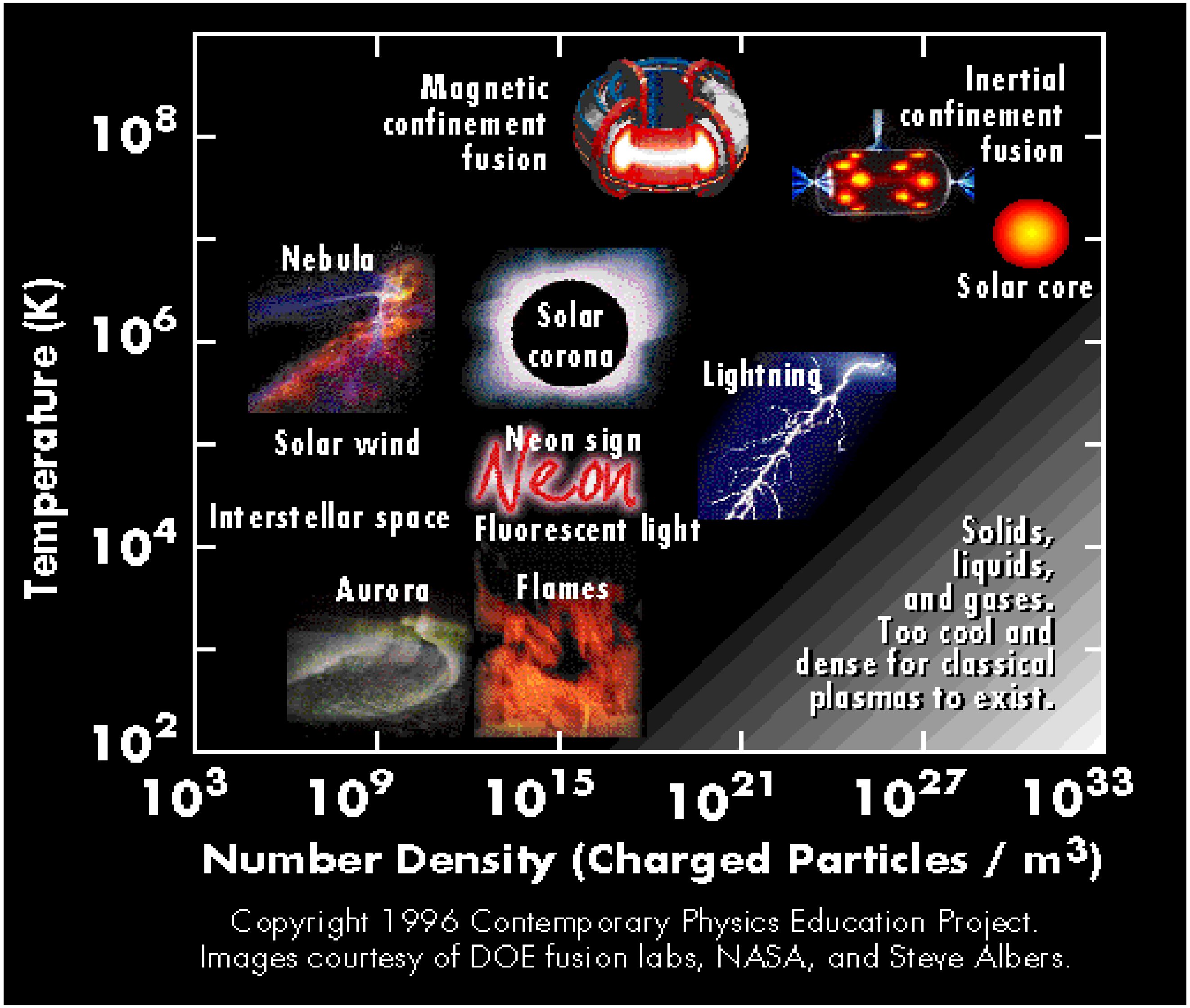
It exhibits "collective dynamics"

$$N_D = n\,(4\pi/3)\,\lambda_D^{\,3} >> 1$$

$$\nu_c < \omega_p$$

Plasma is a quasi-neutral collection of charged particles (and neutrals) which exhibits collective behaviour.

Plasma Abundance



RF Plasma Conductivity

Consider a plasma subjected to a uniform oscillating RF field

$$\boldsymbol{E}(t)=\boldsymbol{E}_{\mathbf{0}}e^{-i\omega t}$$

Plasma electrons will oscillate at the same frequency

$$\boldsymbol{v}(t)=\boldsymbol{v}_{\mathbf{0}}e^{-i\omega t}$$

Eq. of motion for plasma electrons

$$m\left(\frac{\partial \boldsymbol{v}}{\partial t}+\cancelto{0}{(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}}\right)=-e\boldsymbol{E}-m\boldsymbol{v}\nu$$

$$\frac{\partial \boldsymbol{v}}{\partial t}+\boldsymbol{v}\nu=-\frac{e\boldsymbol{E}}{m}$$

$$multiplying\, by\quad e^{\nu v}\, then\, integrating\, w.r.t.time$$

$$\boldsymbol{v}e^{\nu t}=-\frac{e}{m}\int_0^t \boldsymbol{E}_{\mathbf{0}}e^{-i\omega t'}e^{\nu t'}dt'+C$$

RF Plasma Conductivity

$$\mathbf{v} e^{\nu t} = -\frac{e}{m}\int_0^t \mathbf{E_0} e^{-i\omega t'} e^{\nu t'} dt' + \mathbf{C}$$

$$= -\frac{e\mathbf{E_0} e^{-i\omega t}}{m(\nu - i\omega)} e^{\nu t} + \mathbf{C}$$

$$\mathbf{v} = -\frac{e\mathbf{E_0} e^{-i\omega t}}{m(\nu - i\omega)} + \mathbf{C} e^{-\nu t}$$

At t=0, **v**=0 so $\mathbf{C} = -\frac{e\mathbf{E_0}}{m(\nu - i\omega)}$

$$\boxed{\mathbf{v}(t) = -\frac{e\mathbf{E_0}}{m(\nu - i\omega)}\left[e^{-i\omega t} - e^{-\nu t}\right]}$$

This describes the drift velocity of plasma electrons in a unifrom RF field.

RF Plasma Conductivity

$$\boxed{\mathbf{v}(t) = -\frac{e\,\mathbf{E}_0}{m(\nu - i\,\omega)}\left[e^{-i\,\omega t} - e^{-\nu t}\right]}$$

For times larger than the collision time, $e^{-\nu t} \rightarrow 0$

$$\mathbf{v}(t) = -\frac{e\,\mathbf{E}}{m(\nu - i\,\omega)}$$

The current

$$\mathbf{J} = -ne\,\mathbf{v}(t) = \frac{ne^2\,\mathbf{E}}{m(\nu - i\,\omega)} = \sigma\mathbf{E}$$

$$\boxed{\sigma = \frac{ne^2}{m(\nu - i\,\omega)}}$$ is the rf plasma conductivity.

Effective Plasma Permittivity

$$\boldsymbol{E}(\boldsymbol{r},t)=\boldsymbol{E}_{\boldsymbol{0}}(\boldsymbol{r})e^{-i\omega t}$$

$$\begin{aligned}
\nabla\times\boldsymbol{H} &= \boldsymbol{J}_f + \frac{\partial \boldsymbol{D}}{\partial t} \\
&= \sigma\boldsymbol{E} - i\omega\epsilon_0\boldsymbol{E} \\
&= -i\omega\epsilon_0\left(1+\frac{i\sigma}{\omega\epsilon_0}\right)\boldsymbol{E}
\end{aligned}$$

$$\boxed{\nabla\times\boldsymbol{H} = -i\omega\epsilon_0\epsilon_{eff}\boldsymbol{E}}$$

where $\boxed{\epsilon_{eff} = 1 + \frac{i\sigma}{\omega\epsilon_0}}$

Effective Plasma Permittivity

$$\textit{For a dielectric} \qquad \nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} = -i\,\omega\,\epsilon_0\,\epsilon_r\,\boldsymbol{E}$$

$$\textit{For a metal} \qquad \nabla \times \boldsymbol{H} = -i\,\omega\,\epsilon_0\,\epsilon_{eff}\,\boldsymbol{E} \quad \textit{with} \quad \epsilon_{eff} = \epsilon_L + \frac{i\,\sigma}{\omega\,\epsilon_0}$$

$$\textit{For a plasma} \qquad \nabla \times \boldsymbol{H} = -i\,\omega\,\epsilon_0\,\epsilon_{eff}\,\boldsymbol{E} \quad \textit{with} \quad \epsilon_{eff} = 1 + \frac{i\,\sigma}{\omega\,\epsilon_0}$$

Effective Plasma Permittivity

$$\textit{For a dielectric} \qquad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} = -i\,\omega\,\epsilon_0\,\epsilon_r\,\mathbf{E}$$

$$\textit{For a metal} \qquad \nabla \times \mathbf{H} = -i\,\omega\,\epsilon_0\,\epsilon_{eff}\,\mathbf{E} \quad with \quad \epsilon_{eff} = \epsilon_L + \frac{i\,\sigma}{\omega\,\epsilon_0}$$

$$\textit{For a plasma} \qquad \nabla \times \mathbf{H} = -i\,\omega\,\epsilon_0\,\epsilon_{eff}\,\mathbf{E} \quad with \quad \epsilon_{eff} = 1 + \frac{i\,\sigma}{\omega\,\epsilon_0}$$

- Both Metal and plasma have a complex dielectric constant.

Effective Plasma Permittivity

$$\begin{aligned} \nabla \cdot \boldsymbol{B} &= 0 \\ \nabla \times \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t}; \quad \boldsymbol{B} = \mu_0 \boldsymbol{H} \end{aligned}$$

$$Gauss's\,law \quad \nabla \cdot \boldsymbol{D} = \rho_f$$

$$For\,dielectrics \quad \boldsymbol{D} = \epsilon_0 \epsilon_r \boldsymbol{E}; \;\; \rho_f = 0; \;\; \nabla \cdot \left(\epsilon_0 \epsilon_r \boldsymbol{E} \right) = 0$$

Effective Plasma Permittivity

$$\begin{aligned}\nabla\cdot\boldsymbol{B} &= 0\\ \nabla\times\boldsymbol{E} &= -\frac{\partial\boldsymbol{B}}{\partial t}; \quad \boldsymbol{B}=\mu_0\boldsymbol{H}\end{aligned}$$

$$Gauss's\,law \qquad \nabla\cdot\boldsymbol{D} = \rho_f$$

$$For\,dielectrics \qquad \boldsymbol{D} = \epsilon_0\,\epsilon_r\,\boldsymbol{E};\ \rho_f=0;\ \nabla\cdot(\epsilon_0\,\epsilon_r\,\boldsymbol{E}) = 0$$

$$For\,plasmas \qquad \begin{aligned}\frac{\partial\rho_f}{\partial t} + \nabla\cdot\boldsymbol{J}_f &= 0\\ -i\,\omega\rho_f + \nabla\cdot(\sigma\boldsymbol{E}) &= 0\end{aligned}$$

$$\begin{aligned}\nabla\cdot\boldsymbol{D} &= \rho_f\\ \nabla\cdot(\epsilon_0\boldsymbol{E}) &= \frac{1}{i\,\omega}\nabla\cdot(\sigma\boldsymbol{E})\end{aligned} \quad \Rightarrow \quad \begin{aligned}\nabla\cdot\left(\epsilon_0\left(1+\frac{i\,\sigma}{\omega\,\epsilon_0}\right)\boldsymbol{E}\right) &= 0\\ \boxed{\nabla\cdot(\epsilon_0\,\epsilon_{eff}\,\boldsymbol{E}) = 0}\end{aligned}$$

Effective Plasma Permittivity

$$\textit{Now RF conductivity of plasma} \quad \sigma = -\frac{ne^2}{im(\omega + i\nu)}$$

So effective permittivity of plasma

$$\epsilon_{eff} = 1 - \frac{ne^2}{\omega^2 \epsilon_0 m \left(1 + \frac{i\nu}{\omega}\right)}$$

$$\boxed{\epsilon_{eff} = 1 - \frac{\omega_p^2}{\omega^2 \left(1 + \frac{i\nu}{\omega}\right)} < 1}$$

$$\textit{where } \omega_p = \sqrt{\frac{ne^2}{\epsilon_0 m}} \textit{ is the plasma frequency.}$$

EM Waves in Plasmas

- Consider a plane wave traveling inside a plasma medium

$$\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E_0} e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$$

$$\textit{So we can substitute} \quad \nabla \rightarrow ik\,;\, \partial/\partial t \rightarrow -i\,\omega$$

Now from the Maxwell's curl equations

$$\begin{aligned}
\nabla\times\boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t} \\
\rightarrow\ \boldsymbol{k}\times\boldsymbol{E} &= +\omega\mu_0 \boldsymbol{H} \\
\nabla\times\boldsymbol{H} &= \boldsymbol{J}_f + \frac{\partial \boldsymbol{D}}{\partial t} = -i\,\omega\,\epsilon_0\,\epsilon_{eff}\,\boldsymbol{E} \\
\rightarrow\ \boldsymbol{k}\times\boldsymbol{H} &= -\omega\,\epsilon_0\,\epsilon_{eff}\,\boldsymbol{E}
\end{aligned}$$

EM Waves in Plasmas

Now taking curl of one of the curl equation

$$\begin{aligned} \boldsymbol{k} \times \boldsymbol{E} &= +\omega \mu_0 \boldsymbol{H} \\ \boldsymbol{k} \times \boldsymbol{H} &= -\omega \epsilon_0 \epsilon_{eff} \boldsymbol{E} \end{aligned}$$

We obtain

$$\boldsymbol{k} \times (\boldsymbol{k} \times \boldsymbol{E}) = +\omega \mu_0 \boldsymbol{k} \times \boldsymbol{H}$$

$$\boxed{\rightarrow \quad \boldsymbol{k} \times (\boldsymbol{k} \times \boldsymbol{E}) = -\omega^2 \mu_0 \epsilon_0 \epsilon_{eff} \boldsymbol{E}}$$

EM Waves in Plasmas

Now taking curl of one of the curl equation

$$\begin{aligned} \boldsymbol{k}\times\boldsymbol{E} &= +\omega\mu_0\boldsymbol{H} \\ \boldsymbol{k}\times\boldsymbol{H} &= -\omega\epsilon_0\epsilon_{eff}\boldsymbol{E} \end{aligned}$$

We obtain

$$\boldsymbol{k}\times(\boldsymbol{k}\times\boldsymbol{E}) = +\omega\mu_0\boldsymbol{k}\times\boldsymbol{H}$$

$$\boxed{\rightarrow\ \boldsymbol{k}\times(\boldsymbol{k}\times\boldsymbol{E}) = -\omega^2\mu_0\epsilon_0\epsilon_{eff}\boldsymbol{E}}$$

Now if we take a dot product with wave vector **k**,
> left hand side will vanish
> so right hand side should also vanish, i.e.

$$\epsilon_{eff}(\boldsymbol{k}\cdot\boldsymbol{E})=0$$

$$\Rightarrow\ \boxed{\text{Either}\quad \epsilon_{eff}=0 \quad \text{OR} \quad \boldsymbol{k}\cdot\boldsymbol{E}=0}$$

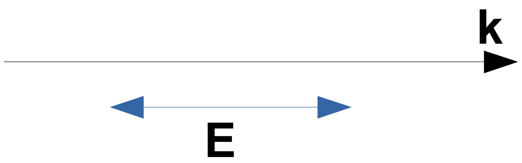
EM Waves in Plasmas

$(i)\ \epsilon_{eff}=0:$

$$\boldsymbol{k}\times(\boldsymbol{k}\times\boldsymbol{E})=-\omega^2\mu_0\,\epsilon_0\,\epsilon_{eff}\,\boldsymbol{E} \;=\; 0$$

$$\boldsymbol{k}\,(\boldsymbol{k}\cdot\boldsymbol{E})-k^2\,\boldsymbol{E} \;=\; 0$$

$$\Rightarrow \quad \boldsymbol{k} \;\|\; \boldsymbol{E}$$



Also $$\boldsymbol{k}\times\boldsymbol{E} \;=\; \omega\mu_0\,\boldsymbol{H} \;=\; 0$$

$$\Rightarrow \quad \boldsymbol{H} \;=\; 0$$

- This represents a longitudinal electrostatic wave.
- Electric field oscillates along the propagation direction.
- Electron density also oscillates along propagation direction.

EM Waves in Plasmas

$(i)\, \underline{\mathbf{k}\cdot\mathbf{E}=0}:$

$$\mathbf{k}\times(\mathbf{k}\times\mathbf{E}) = -\omega^2\mu_0\epsilon_0\epsilon_{eff}\mathbf{E}$$

$$\mathbf{k}(\mathbf{k}\cdot\mathbf{E}) - k^2\mathbf{E} = -\omega^2\mu_0\epsilon_0\epsilon_{eff}\mathbf{E}$$

$$-k^2\mathbf{E} = \omega^2\mu_0\epsilon_0\epsilon_{eff}\mathbf{E}$$

$$\Rightarrow\quad k^2 = \omega^2\mu_0\epsilon_0\epsilon_{eff}$$

$$\Rightarrow\quad \boxed{k^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2\left(1 + \frac{i\,\nu}{\omega}\right)}\right)}$$

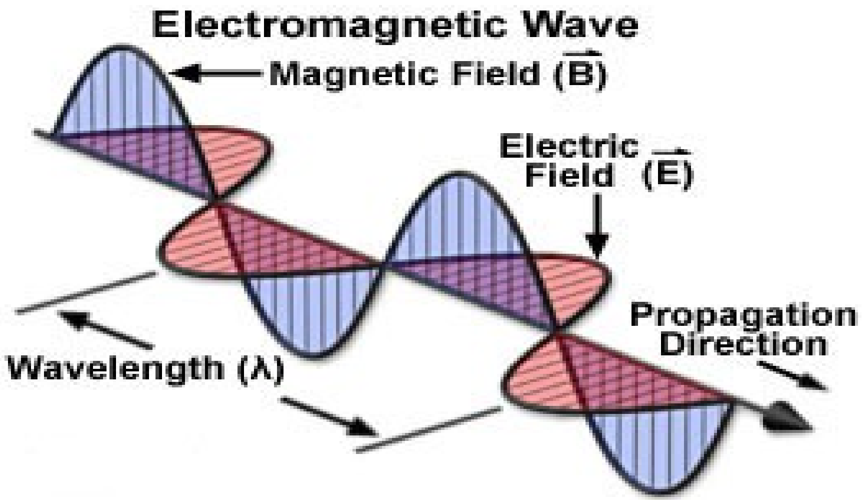
- This is the dispersion relation of an EM wave in a plasma.

EM Waves in Plasmas

- Now for nonzero ϵ_{eff}

$$\begin{aligned} \boldsymbol{k}\times\boldsymbol{E} &= +\omega\mu_0\boldsymbol{H} \\ \boldsymbol{k}\times\boldsymbol{H} &= -\omega\epsilon_0\epsilon_{eff}\boldsymbol{E} \end{aligned}$$

> **k** and **E** are perpendicular to **H**
> **k** and **H** are perpendicular to **E**



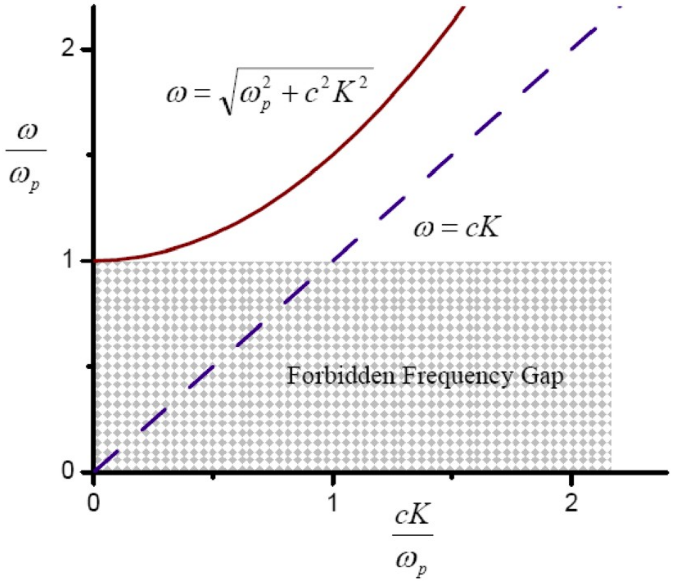
EM Waves in Plasmas: Dispersion Relation

$$\boxed{k^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2\left(1 + \frac{i\,\nu}{\omega}\right)}\right)}$$

For $\nu \ll \omega$ $\quad k^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right) = \frac{\omega^2}{c^2} - \frac{\omega_p^2}{c^2}$

Dispersion Relations becomes

$$\boxed{\omega^2 = \omega_p^2 + k^2 c^2}$$



Underdense and Overdense Plasmas

$$\omega^2 = \omega_p^2 + k^2 c^2$$

For $\boldsymbol{\omega > \omega_p}$

Refractive index = c/v$_p$ $= ck/\omega = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} < 1$ ⇒ $v_p > c$

- Plasma is called underdense plasma and wave propagation is possible.

For $\boldsymbol{\omega < \omega_p}$

$$k = \frac{\omega}{c}\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2} \text{ becomes imaginary}$$

- Plasma is called overdense plasma and wave can not propagate inside the plasma.

EM Waves in Plasmas: Skin depth

$$\boxed{\omega^2 = \omega_p^2 + k^2 c^2}$$

$$\textbf{For } \omega < \omega_p \qquad k = \frac{\omega}{c}\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2} = i\,\alpha$$

$$\textit{where} \quad \alpha = \frac{\omega}{c}\left(\frac{\omega_p^2}{\omega^2} - 1\right)^{1/2}$$

For one dimensonal wave propagation

$$\boldsymbol{E}(z,t) = \boldsymbol{E}_{\mathbf{0}} e^{i(kz - \omega t)} = \boldsymbol{E}_{\mathbf{0}} e^{-\alpha z} e^{-i\omega t}$$

One can define the collision-less skin depth ($\nu \approx 0$)

$$\boxed{\delta = \frac{1}{\alpha} = \frac{c}{\sqrt{\omega_p^2 - \omega^2}} \approx \frac{c}{\omega_p}}$$

EM Waves in Plasmas: Skin depth

$$\boldsymbol{For\ \nu \neq 0 \ll \omega_p}$$

$$k = \frac{\omega}{c}\left(1 - \frac{\omega_p^2}{\omega^2\left(1+i\,\nu/\omega\right)}\right)^{1/2} = k_r + ik_i$$

$$where\quad k_r = \frac{\omega}{c}\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2};\quad k_i = \frac{\nu}{2c}\frac{\omega_p^2/\omega^2}{\left(1 - \omega_p^2/\omega^2\right)^{1/2}}$$

For one dimensonal wave propagation

$$\boldsymbol{E}(z,t) = \boldsymbol{E}_0 e^{i(kz - \omega t)} = \boldsymbol{E}_0 e^{-k_i z} e^{i(k_r z - \omega t)}$$

k_i *dominates when* $\omega \approx \omega_p$ → *the wave is heavily attenuated*

EM Waves in Plasmas: Skin depth

Now for $\omega \ll \nu$

$$k^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2\left(1 + \frac{i\,\nu}{\omega}\right)}\right) \approx \frac{\omega^2}{c^2}\frac{\omega_p^2}{\omega\,\nu}i$$

$$k = \frac{\omega_p}{c}\left(\frac{\omega}{\nu}\right)^{1/2}\frac{1+i}{\sqrt{2}} = k_r + ik_i$$

And skin depth:

$$\boxed{\delta = \frac{1}{k_i} = \frac{c}{\omega_p}\left(\frac{2\,\nu}{\omega}\right)^{1/2} \Rightarrow \delta \propto \frac{1}{\omega^{1/2}}}$$

So for very low frequencies skin depth would be quite large.

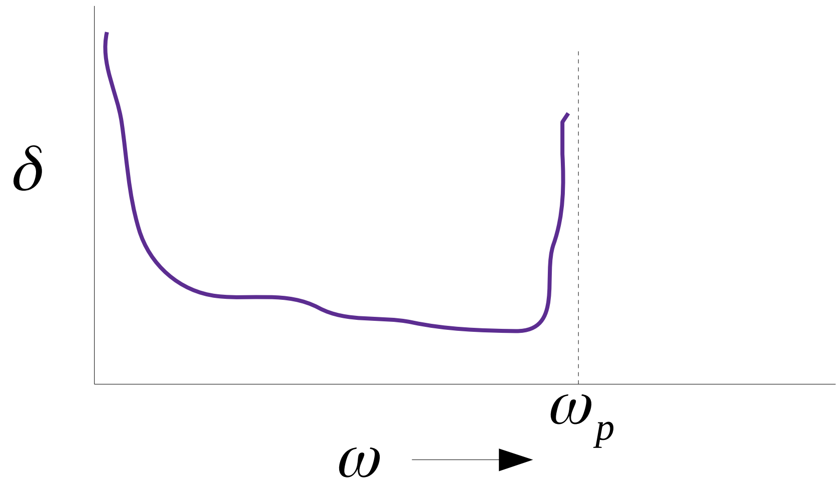
EM Waves in Plasmas: Skin depth

Now for $\omega \ll \nu$

$$\delta = \frac{c}{\omega_p}\left(\frac{2\nu}{\omega}\right)^{1/2}$$

whereas for $\omega \gg \nu$

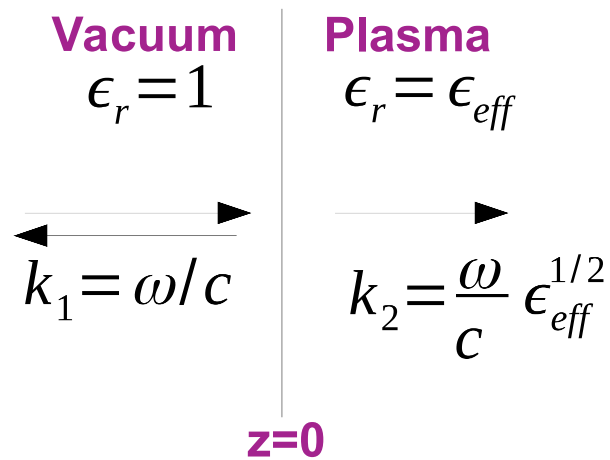
$$\delta = \frac{c}{\sqrt{\omega_p^2 - \omega^2}}$$



Reflection from Plasmas

Incident, reflected and transmitted waves

$$
\begin{aligned}
\boldsymbol{E_I}(z,t) &= \hat{x}\, E_{0I}\, e^{i(k_1 z - \omega t)} \\
\boldsymbol{B_I}(z,t) &= \hat{y}\, \frac{E_{0I}}{c}\, e^{i(k_1 z - \omega t)} \\
\boldsymbol{E_R}(z,t) &= \hat{x}\, E_{0R}\, e^{i(-k_1 z - \omega t)} \\
\boldsymbol{B_R}(z,t) &= -\hat{y}\, \frac{E_{0R}}{c}\, e^{i(-k_1 z - \omega t)} \\
\boldsymbol{E_T}(z,t) &= \hat{x}\, E_{0T}\, e^{i(k_2 z - \omega t)} \\
\boldsymbol{B_T}(z,t) &= \hat{y}\, \frac{k E_{0T}}{\omega}\, e^{i(k_2 z - \omega t)}
\end{aligned}
$$



Applying boundary conditions at z=0

Continuity of E_x: $E_{Ix} + E_{Rx} = E_{Tx}$

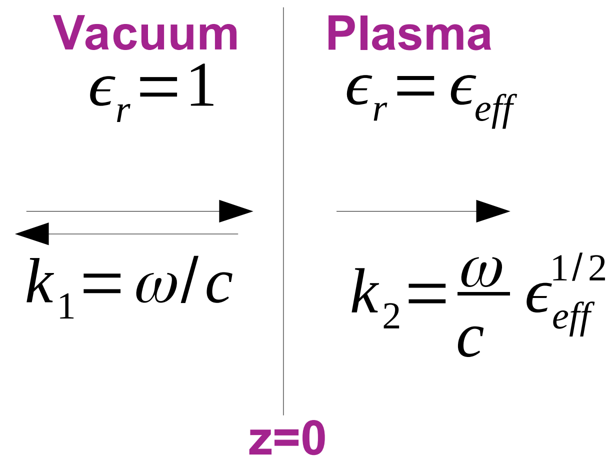
Continuity of H_y: $H_{Iy} + H_{Ry} = H_{Ty}$

Reflection from Plasmas

Applying boundary conditions at z=0

Continuity of E_x: $$E_{Ix} + E_{Rx} = E_{Tx}$$

Continuity of H_y: $$H_{Iy} + H_{Ry} = H_{Ty}$$



$$E_{I0} + E_{R0} = E_{T0}$$

$$E_{I0} - E_{R0} = \frac{ck}{\omega} E_{T0}$$

Solving these equations one obtains

$$\boxed{E_{R0} = \frac{1-\eta}{1+\eta} E_{I0}; \quad E_{T0} = \frac{2}{1+\eta} E_{I0} \quad with \quad \eta = \frac{ck}{\omega}}$$

Reflection from Plasmas

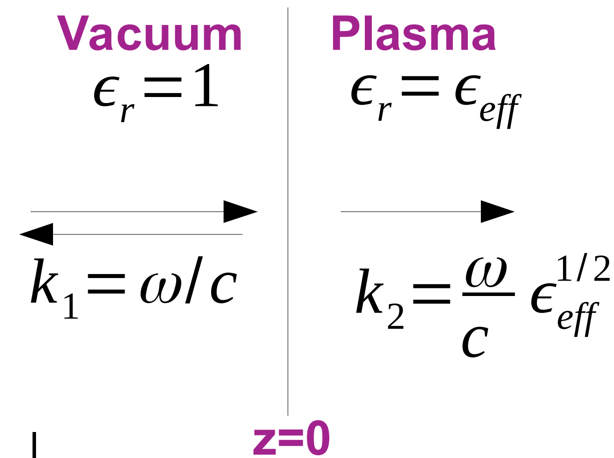
$$E_{R0}=\frac{1-\eta}{1+\eta}E_{I0};\quad E_{T0}=\frac{2}{1+\eta}E_{I0}\quad with\quad \eta=\frac{ck}{\omega}$$

$$For\quad \omega>\omega_p\quad ,\eta\ <\ 1$$

> E_{R0}/E_{I0} < 1 but positive

> No phase change on reflection.

$$For\quad \omega<\omega_p\quad ,\quad \eta\ =\ i\alpha'$$



$$E_{R0}=\frac{1-i\alpha'}{1+i\alpha'}E_{I0}=\frac{(1-\alpha^2)-i(2\alpha)}{(1+\alpha^2)}E_{I0};\quad \frac{|E_{R0}|}{|E_{I0}|}=1$$

> All the incident energy gets reflected but phase changes on reflection.

Evanescent field: $\boldsymbol{E}_{\boldsymbol{T}}=\hat{x}\dfrac{2E_{I0}}{(1+i\alpha')}e^{-\frac{\omega}{c}\alpha' z}e^{-i\omega t}$