ELL101: INTRODUCTION TO ELECTRICAL ENG.

Mesh Analysis

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MESH ANALYSIS

Introduction

- Popularly known as Mesh Current Analysis
- It is a technique used to find the currents circulating around a loop or mesh with in any closed path of a circuit
- It is based upon KVL
- It results in system of linear equations that can be solved for unknown currents
- Number of mesh currents = Number of Branches – Number of nodes + 1

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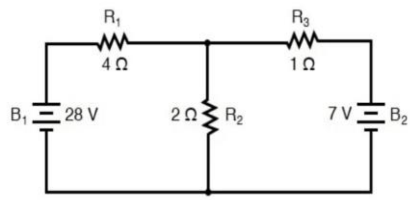
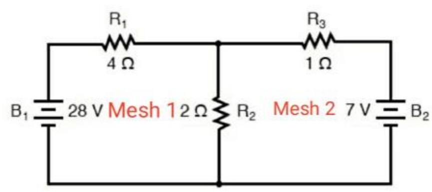
Mesh Analysis

Example: Apply the Mesh Analysis to get the values of current for the given circuit



- Step 1: Identify the number of meshes in the circuit



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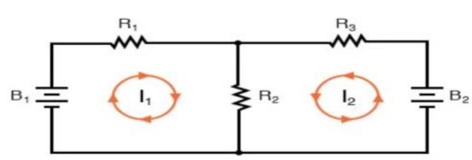
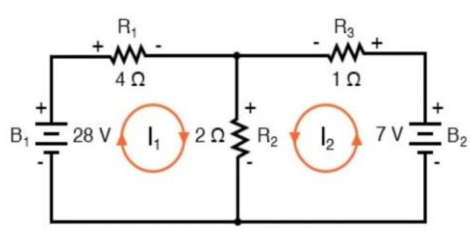
Mesh Analyis

- Step 2: Assign the current value to each mesh



- Step 3: Apply the KVL to form the linear equations



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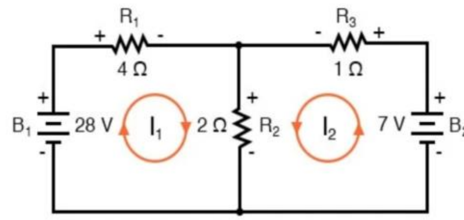
Mesh Analyis

Applying KVL at both the loops we get

Mesh 1: $4I_1 + 2(I_1+I_2) = 28$

Mesh 2: $2(I_1+I_2) + I_2 = 7$



- Step 4: Solving the equation to get the values of current

Mesh 1: $6I_1 + 2I_2 = 28$

Mesh 2: $2I_1 + 3I_2 = 7$

Answer: $I_1 = 5A$; $I_2 = -1A$

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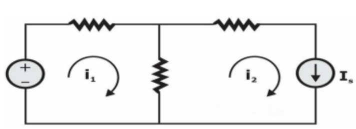
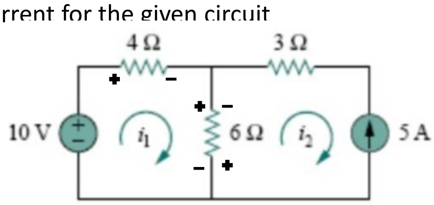
Mesh Analyis

Various Cases in Mesh Analysis

Case 1: When current source exist in one mesh only



Example: Find the current for the given circuit



Mesh 1: $4i_1 + 6(i_1 - i_2) = 10$

Mesh 2: $i_2 = -5A$

Using the above equations we get the value of i_1 as: $\boldsymbol{i_1} = \mathbf{-2A}$

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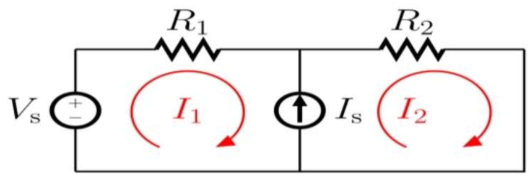
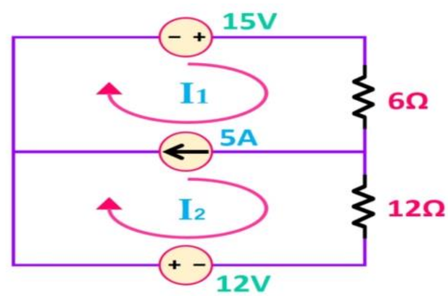
Mesh Analysis

Case 2: Supermesh



Example: Find the currents for the given circuit



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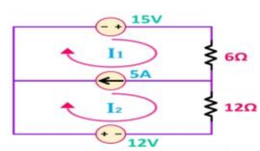
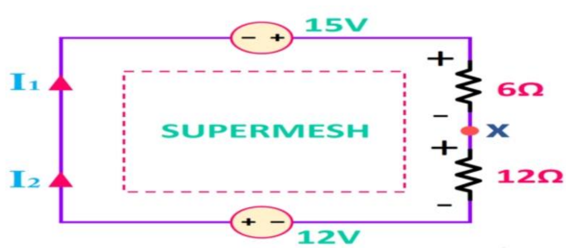
Mesh Analyis

Solution:

Step 1: Apply the supermesh





Step2: Apply KVL in supermesh to form the linear equation

Supermesh: $6I_1 + 12\ I_2 = 27$

Equation at Current source: $I_1 - I_2 = 5$

Solving both the equation we get the values as: $I_1 = 29/6$, $I_2 = -1/6$

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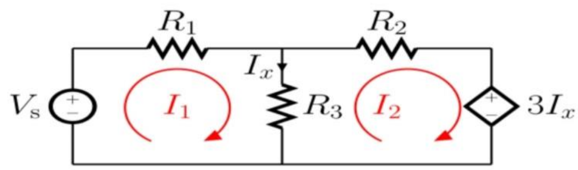
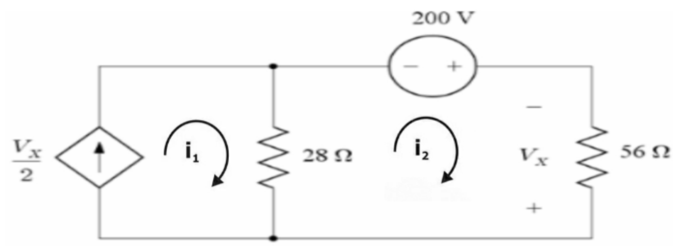
Mesh Analysis

Case 3: When Dependent source is there in the mesh



Example: Find the currents in the given circuit



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Mesh Analysis

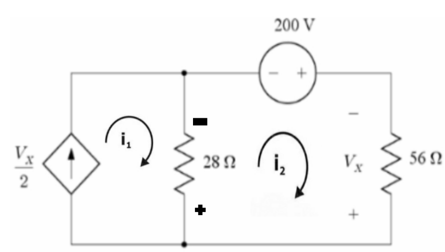
Solution: Apply Ohm' law to find the value of V_x in terms of I_2

$V_x = -56\ I_2$ (1)

Apply KVL in both the mesh to form the equation

$I_1 = V_x/2$ (2)

$56\ I_2 + 28(I_2 - I_1) = 200$ (3)



Solving these equations we get the value of I_1 and I_2 as: I_1= -200/31, I_2= 50/217

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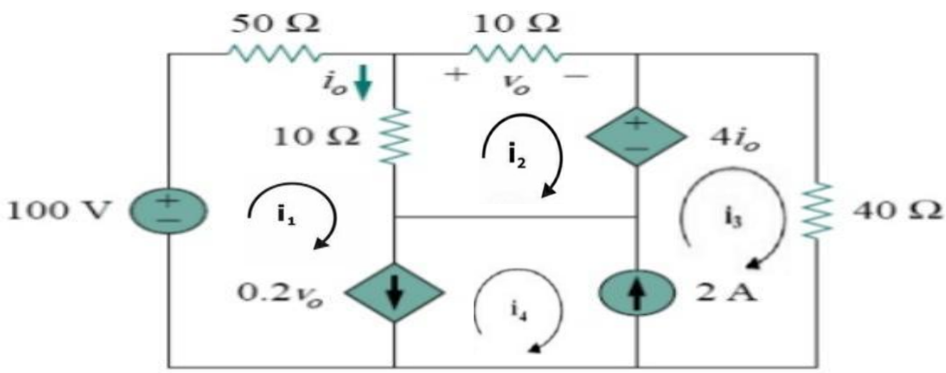
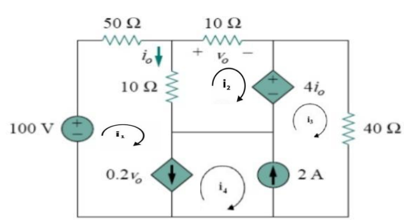
Mesh Analysis

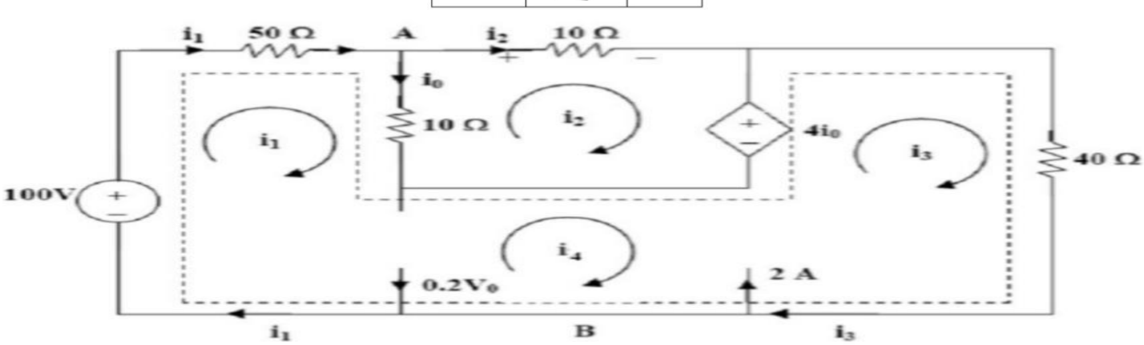
Problem: Find the value of i_0 in the given circuit



Mesh Analysis

Solution: Apply the supermesh method to reduce the circuit as





Mesh Analysis

Applying KVL in mesh 2:

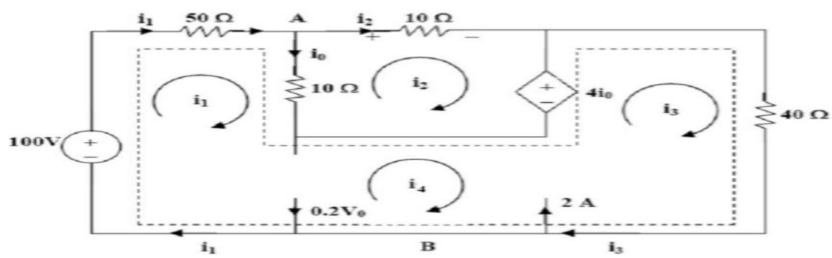
$-10i_1 + 20i_2 + 4i_0 = 0$ (1)

At point A using KCL we get:

$i_0 = i_1 - i_2$ (2)

Using (1) and (2) we get:

$i_1 = (16/6)\, i_2$ (3)



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Mesh Analysis

In Supermesh

$i_1 = (16/6)\, i_2$

$50\, i_1 + 10(i_1 - i_2) - 4i_0 + 40i_3 = 100$ (4)

Using $i_0 = i_1 - i_2$ (4) can be simplified to

$28\, i_1 - 3i_2 + 20i_3 = 50$ (5)

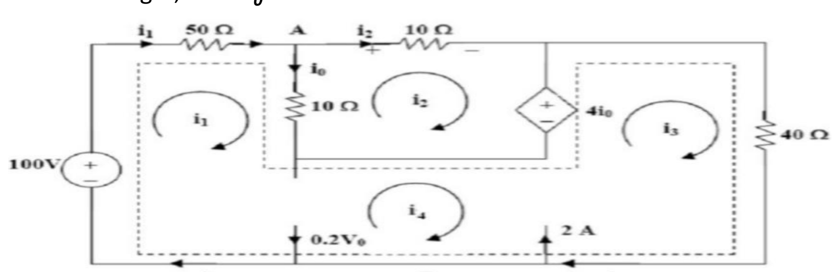
Also, $i_3 - i_4 = 2$ and $-i_4 + i_1 = 0.2\, v_0$

But, $v_0 = 10i_2$ which gives us

$i_3 = 2 + (2/3)\, i_2$ (6)

Solving, (1) - (6) we get $i_2 = 0.11764$ A

We get, $\boldsymbol{i_0 = 0.196}$ **A**



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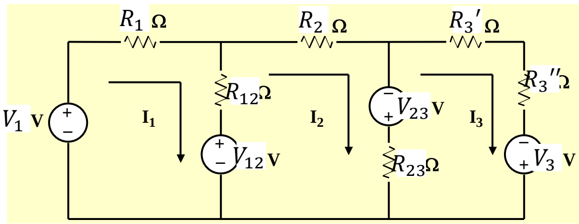
Matrix Analysis



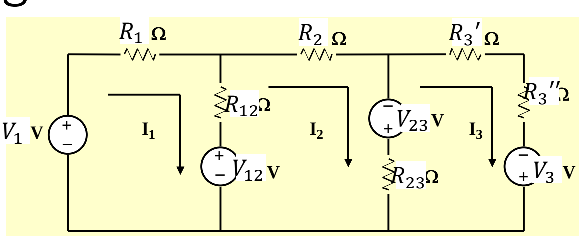
- Apply KVL:

Mesh 1: $R_1 I_1 + R_{12}(I_1 - I_2) = V_1 - V_{12}$

Mesh 2: $R_{12}(I_2 - I_1) + R_2\, I_2 + R_{23}(I_2 - I_3) = V_{12} + V_{23}$

Mesh 3: $R_{23}(I_3 - I_2) + R_3'\, I_3 + R_3''\, I_3 = V_3 - V_{23}$

Simplifying



Mesh 1: $(R_1 + R_{12})I_1 - R_{12}I_2 + 0I_3 = V_1 - V_{12}$

Mesh 2: $-R_{12}I_1 + (R_{12} + R_2 + R_{23})I_2 - R_{23}\, I_3 = V_{12} + V_{23}$

Mesh 3: $0I_1 - R_{23}I_2 + (R_3' + R_3'' + R_{23})\, I_3 = V_3 - V_{23}$

Matrix Form:

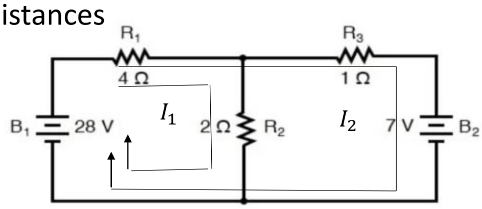
$$\begin{bmatrix} (R_1 + R_{12}) & -R_{12} & 0 \\ -R_{12} & (R_{12} + R_2 + R_{23}) & -R_{23} \\ 0 & -R_{23} & (R_3' + R_3'' + R_{23}) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 - V_{12} \\ V_{12} + V_{23} \\ V_3 - V_{23} \end{bmatrix}$$

$$\boldsymbol{RI = V}$$

$$\rightarrow \boldsymbol{I = R^{-1}V}$$

Loop Analysis

Find currents in all resistances



Apply KVL:
Loop 1: $R_1(I_1 + I_2) + R_2 I_1 = B_1$
Loop 2: $R_1(I_1 + I_2) + R_3 I_2 = B_1 - B_2$

Simplifying:
Loop 1: $(R_1 + R_2)I_1 + R_1 I_2 = B_1$
Loop 2: $R_1 I_1 + (R_1 + R_3)I_2 = B_1 - B_2$

Solving:

$$I_1 = \frac{R_3 B_1 + R_1 B_2}{(R_1 + R_2)(R_1 + R_3) - R_1^2}$$

$$I_2 = \frac{(R_1 + R_2)B_2 - R_2 B_1}{R_1^2 - (R_1 + R_2)(R_1 + R_3)}$$

Solution:
$I_1 = 4A$
$I_2 = 1A$

Currents:
$R_1 \rightarrow I_1 + I_2 = 5\ A$
$R_2 \rightarrow I_1 = 4\ A$
$R_3 \rightarrow I_2 = 1\ A$

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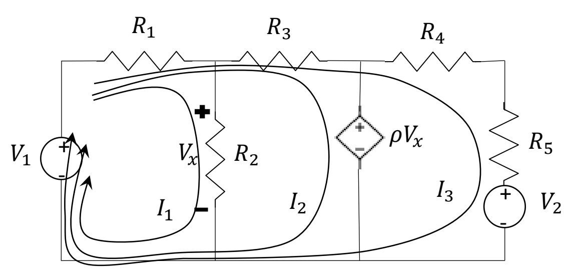
Applicability



Applying KVL:
Loop 1: $R_1(I_1 + I_2 + I_3) + R_2 I_1 = V_1$
Loop 2: $R_1(I_1 + I_2 + I_3) + R_3(I_2 + I_3) = V_1 - \rho V_x$
Loop 3: $R_1(I_1 + I_2 + I_3) + R_3(I_2 + I_3) + (R_4 + R_5)I_3 = V_1 + V_2$

Note that $V_x = R_1 I_1$

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