1. The Be atom has an electronic structure of $1s^2 2s^2$. Although the Be atom has a full 2s energy level, solid Be is a metal. Why?

2. In copper, the Fermi energy of conduction electrons is 7.0 eV. What is the speed of conduction electrons around this energy?

3. Given the width of an energy band is typically around 10 eV, calculate the following, in per cm^3 and per eV units:

a. The density of states at the center of the band
b. The number of states per unit volume within a small energy range *kT* about the center.
c. The density of states at *kT* above the bottom of the band.
d. The number of states per unit volume within a small energy range of *kT* and *2kT* from the bottom of the band

4. Total Number of States in a Band

a. Based on the overlap of atomic orbitals to form the electron wavefunction in the crystal, how many states should there be in a band?

b. Consider the density of states function

$$g(E) = \left(8\pi 2^{\frac{1}{2}}\right)\left(\frac{m_e}{h^2}\right)^{\frac{3}{2}} E^{1/2}$$

By integrating g(E), estimate the total number of states in a band per unit volume, and compare this with the atomic concentration for silver. For silver, we have E_{FO} = 5.5 eV and Φ = 4.5 eV. (Note that "state" means a distinct wavefunction, including spin.)

5. Calculate the Fermi energy E_{FO} at 0 K for copper and estimate the average speed of conduction electrons in Cu. The density of Cu is 8.96 g cm^{-3} and the relative atomic mass (atomic weight) is 63.5.

6. Consider a thermocouple pair from Al and Cu which have Fermi energies and x as in Table 4.3 given below. Estimate the emf available from this thermocouple if one junction is held at 0 ^{0}C and the other at 100 ^{0}C.

Table 4.3 Seebeck coefficients of selected metals (from various sources)

Metal	*S* at 0 °C (μV K^{-1})	*S* at 27 °C (μV K^{-1})	E_F (eV)	*x*
Al	−1.6	−1.8	11.6	2.78
Au	+1.79	+1.94	5.5	−1.48
Cu	+1.70	+1.84	7.0	−1.79
K		−12.5	2.0	3.8
Li	+14		4.7	−9.7
Mg	−1.3		7.1	1.38
Na		−5	3.1	2.2
Pd	−9.00	−9.99		
Pt	−4.45	−5.28		

7. Consider a two-dimensional electron gas in which the electrons are restricted to move freely within a square area a^2 in the x-y plane. Show that the density of states g(E) is constant (independent of energy).

8. The Fermi energy of electrons in copper at room temperature is 7.0 eV. The electron drift mobility in copper, from Hall Effect measurements, is 33 $cm^2 V^{-1} s^{-1}$.

a. What is the speed v_F of conduction electrons with energies around E_F in copper? By how many times is this larger than the average thermal speed $v_{thermal}$ of electrons, if they behaved like an ideal gas (Maxwell-Boltzmann statistics)? Why is v_F much larger than $v_{thermal}$?

b. What is the De Broglie wavelength of these electrons? Will the electrons get diffracted by the lattice planes in copper, given that interplanar separation in Cu = 2.09 Å? (Solution guide: Diffraction of waves occurs when $2d\sin\theta = \lambda$, which is the Bragg condition. Find the relationship between λ and *d* that results in $\sin\theta > 1$ and hence no diffraction.)

c. Calculate the mean free path of electrons at E_F and comment.

9. Temperature dependence on Fermi level

a. Given that the Fermi energy for Cu is 7.0 eV at absolute zero, calculate the E_F at 300 K. What is the percentage change in E_F and what is your conclusion?

b. Given the Fermi energy for Cu at absolute zero, calculate the average energy and mean speed per conduction electron at absolute zero and 300 K, and comment.

10. Consider a thermocouple pair that consists of gold and aluminum. One junction is at 100 ^{0}C and the other is at 0 ^{0}C. A voltmeter (with a very large input resistance) is inserted into the aluminum wire. Use the properties of Au and Al in Table below to estimate the emf registered by the voltmeter and identify the positive end.

Table

	Au	**Al**
Atomic Mass, M_{at}, g/mol	197	27.0
Density, g cm^{-3}	19.3	2.7
Conduction electrons per atom	1	3
x (Mott-Jones index)	-1.48	2.78

11. Consider Cu and Ni with their density of states as schematically sketched in Figure given below. Both have overlapping 3*d* and 4*s* bands, but the 3*d* band is very narrow compared to the 4*s* band. In the case of Cu the band is full, whereas in Ni, it is only partially filled.

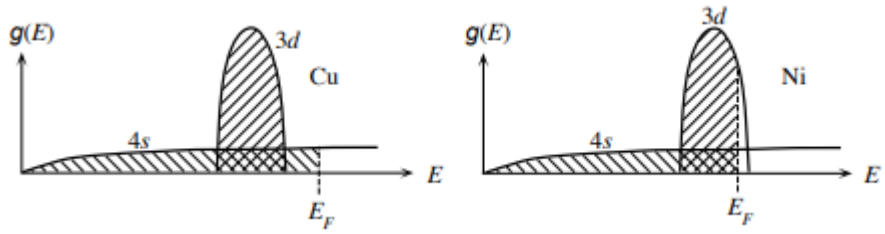
a. In Cu, do the electrons in the 3*d* band contribute to electrical conduction? Explain.

b. In Ni, do electrons in both bands contribute to conduction? Explain.

c. Do electrons have the same effective mass in the two bands? Explain.

d. Can an electron in the 4*s* band with energy around *EF* become scattered into the 3*d* band as a result of a scattering process? Consider both metals.

e. Scattering of electrons from the 4*s* band to the 3*d* band and vice versa can be viewed as an additional scattering process. How would you expect the resistivity of Ni to compare with that of Cu, even though Ni has 2 valence electrons and nearly the same density as Cu? In which case would you expect a stronger temperature dependence for the resistivity?



12. Consider the model one-dimensional monoatomic chain of N atoms, equally spaced with separation a, and each with the same mass m. The force constant coupling each atom to its nearest-neighbours is K. We know that normal mode vibrational frequency ω(k) of a mode with wavevector k for this model is:

$$m\omega^2 = 4Ksin^2\left(\frac{ka}{2}\right) \qquad \left[\left(-\frac{\pi}{a}\right) \leq k \leq \left(\frac{\pi}{a}\right)\right]$$

a. Derive an expression for the group velocity v_g as a function of k.

b. Using the results of part a, evaluate very small values of k (k->0). Briefly discuss the physical significance of this low k group velocity.

c. Using the results of part a, evaluate v_g for k at the Brillouin Zone boundary [k=π/a]. Briefly discuss the physical significance of this Brillouin Zone boundary group velocity. Specifically, what we say about propagation of longitudinal waves in this lattice at frequency ω(k=π/a)?