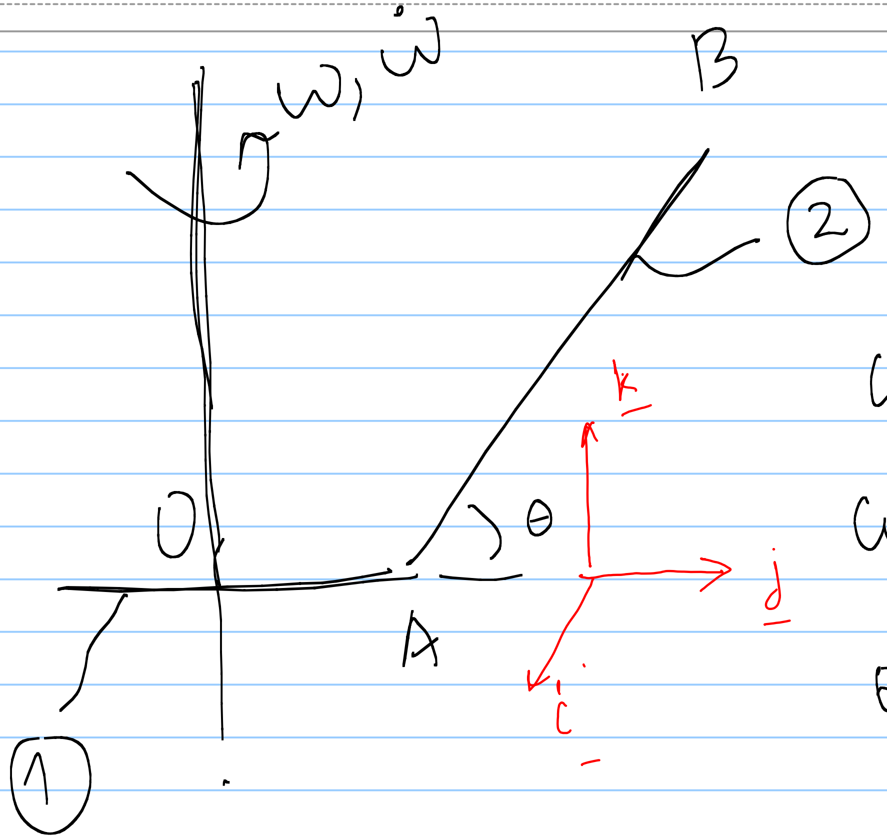
Note Title 15-03-2021

Examples

1.18 pg 81-83.



$\omega_1, \dot{\omega}_1$ ✓

$\omega_{2|1}, \dot{\omega}_{2|1}$ ✓

θ ✓ $\underline{r}_{AO} = \overrightarrow{OA}$ ✓

$\underline{r}_{BA}$ ✓

$\underline{v}_{B|F} = \underline{v}_B$? $\underline{a}_{B|F} = \underline{a}_B$

$$\underline{\omega}_{2|F} = \underline{\omega}_2 = \underline{\omega}_{1|F} + \underline{\omega}_{2|1} \qquad (\underline{\omega}_1 = \underline{\omega})$$

$$= \underline{\omega}_1 + \underline{\omega}_{2|1}$$

$$\dot{\underline{\omega}}_2 = \dot{\underline{\omega}}_{2|F} = \dot{\omega}_{1|F} + \dot{\omega}_{2|1} + \underline{\omega}_1 \times \underline{\omega}_{2|1}$$

$$= \dot{\underline{\omega}}_1 + \dot{\omega}_{2|1} + \underline{\omega}_1 \times \underline{\omega}_{2|1} \qquad (\dot{\underline{\omega}}_1 = \dot{\underline{\omega}})$$

$$\underline{v}_B = \underline{v}_A + \underline{\omega}_2 \times \underline{r}_{BA}$$

$$= \cancel{\underline{v}_0}^{\,0} + \underline{\omega}_1 \times \underline{r}_{A0} + \underline{\omega}_2 \times \underline{r}_{BA}$$

$\underline{\omega}_1 = \omega_1 \underline{k}$, $\dot{\underline{\omega}}_1 = \dot{\omega}_1 \underline{k}$; $\underline{r}_{A0} = r \underline{j}$

$$\dot{\underline{\omega}}_2 = \dot{\underline{\omega}}_1 + \underline{\omega}_1 \times \underline{\omega}_{2|1}$$

$$\underline{\omega}_{2|1} = \omega_{2|1}\, \underline{i}$$

$$\dot{\underline{\omega}}_{2|1} = \dot{\omega}_{2|1}\, \underline{i}$$

$$= \dot{\omega}_1 \underline{k} + \omega_1 \underline{k} \times (\omega_{2|1}\, \underline{i})$$

$$= \dot{\omega}_1 \underline{k} + \omega_1 \omega_{2|1}\, \underline{j} + \dot{\omega}_{2|1}\, \underline{k}$$

$$\underline{v}_B = \underline{v}_A + \underline{\omega}_2 \times \underline{r}_{BA} = \cancel{\underline{v}_{O}}^{0} + \underline{\omega}_1 \times \underline{r}_{AO} + \underline{\omega}_2 \times \underline{r}_{BA}$$

$$\omega_1 \underline{k} \times r\underline{j} + (\omega_1 \underline{k} + \omega_{2|1}\, \underline{i}) \times (AB(\cos\theta\, \underline{j} + \sin\theta\, \underline{k}))$$

$$= \omega_1 r(-\underline{i}) + \omega_1 AB\cos\theta(-\underline{i}) + \omega_{2|1} AB\cos\theta\, \underline{k}$$

$$+ \omega_{2|1} AB\sin\theta\, (-\underline{j})$$

$$-(\omega_1 r + \omega_1 AB\cos\theta)\, \underline{i} - \omega_{2|1} AB\sin\theta(\underline{j}) +$$

$\omega_2 \|_1 \; AB \cos\theta \; \underline{k}$

$$\underline{a}_B = \underline{a}_A + \dot{\underline{\omega}}_2 \times \underline{r}_{BA} + \underline{\omega}_2 \times (\underline{\omega}_2 \times \underline{r}_{BA})$$

$$= \dot{\underline{\omega}}_1 \times \underline{r}_{AO} + \underline{\omega}_1 \times (\underline{\omega}_1 \times \underline{r}_{AO}) + \dot{\omega}_2 \big|^{1} \times \underline{r}_{BA} + \underline{\omega}_2 \times (\underline{\omega}_2 \times \underline{r}_{BA})$$

$$\dot{\underline{\omega}}_1 = \underline{k}\, \dot{\omega}_1$$

Simplify:

Suppose B has a motion relative to 2 eg. slider

eqns change slightly

sliding velocity

$$\underline{v}_B = \underline{v}_A + \underline{\omega}_2 \times \underline{r}_{BA} + \underline{v}_{B|2}$$

$$\underline{a}_B = \underline{a}_A + \dot{\underline{\omega}}_2 \times \underline{r}_{BA} + \underline{\omega}_2 \times (\underline{\omega}_2 \times \underline{r}_{BA}) + \underline{a}_{B|2}$$

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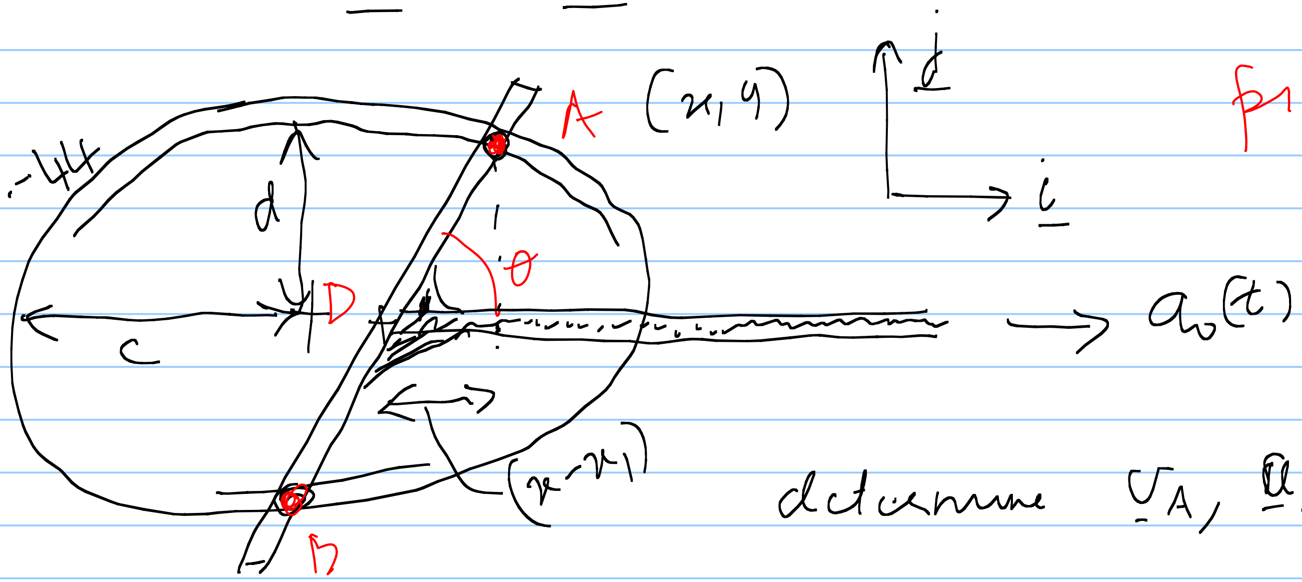


pins move in both slots

c, d: maj & min axes

;

determine $\underline{v}_A$, $\underline{a}_A$, ($\underline{v}_B$; $\underline{a}_B$ very similar)

all other quantities can be determined from this

method 1

$x, y \rightarrow$ coordinates of the pin

$x_1 \rightarrow$ position of D

$$\frac{x^2}{c^2} + \frac{y^2}{d^2} = 1 \quad ; \quad y = (x - x_1)\tan\theta$$

$$\frac{2x\dot{x}}{c^2} + \frac{2y\dot{y}}{d^2} = 0 \quad (a)$$

$$\dot{y} = (\dot{x} - \dot{x}_1)\tan\theta \quad (b)$$

$\ddot{x}_1 = a_0$

$\dot{x}_1 = a_0 t + \dot{x}_1(0)$

$= a_0 t$

$x_1 = \frac{1}{2}a_0 t^2 + x_0$

Solve for $\dot{x}, \dot{y}$ from (a), (b)

$$\underline{v}_A = \dot{x}\,\underline{i} + \dot{y}\,\underline{j}$$

Similarly for v_B

Differentiate again

$$\frac{2}{c^2}(x\ddot{x} + \dot{x}^2) + \frac{2}{d^2}(y\ddot{y} + \dot{y}^2) = 0$$

$$\ddot{y} = (\ddot{x} - \ddot{x}_1)\tan\theta$$

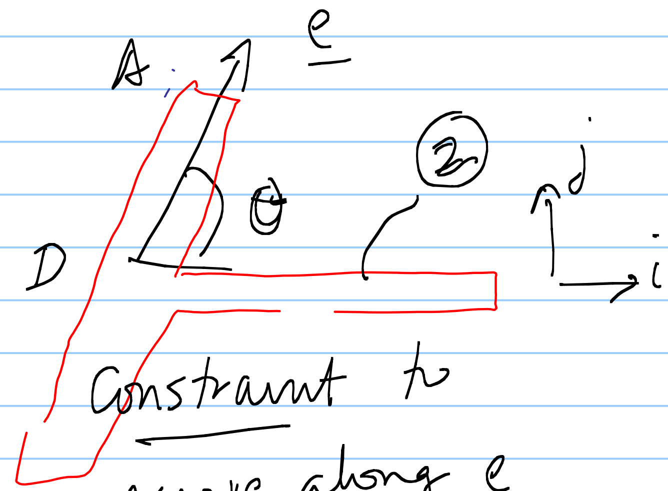
Solve to get $\ddot{x}, \ddot{y}$

$$\ddot{x}\,\underline{i} + \ddot{y}\,\underline{j} = \underline{a}_A$$

method 2

$$\underline{v}_A = \underline{v}_D + \underline{v}_{AD} \quad (v_{A/2})$$

$$= \underline{v}_D + v\,\underline{e}$$



Constraint to move along $\underline{e}$

$$= \cos\theta\,\underline{i} + \sin\theta\,\underline{j}$$

$$\left(\underline{v}_A = \underline{v}_D + \omega_1 \times r_{AD} + \underline{v}_{AD} \right)$$

0 frame $1 \to$ translating

$$\dot{x}\,\underline{i} + \dot{y}\,\underline{j} = a_0 t\,\underline{i} + v\left(\cos\theta\,\underline{i} + \sin\theta\,\underline{j}\right)$$

$$\dot{x} = a_0 t + v\cos\theta$$
$$\dot{y} = v\sin\theta$$

} 2 comps eqns

$$\Downarrow$$

$$\frac{2x\dot{x}}{c^2} + \frac{2y\dot{y}}{d^2} = 0 \Rightarrow v$$

$$\Rightarrow \dot{x}, \dot{y}$$

Acceleration

$$\underline{a}_A = \underline{a}_D + \overset{0}{\dot{\omega}} \times r_{AD} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{AD})^{\,0}$$
$$+ 2\omega \times \underbrace{v_{A/2}}_{0} + \underline{a}_{A/2}$$

$$= a_D + \underline{a}_e$$

$$\ddot{x}\,\underline{i} + \ddot{y}\,\underline{j} = a_0\,\underline{i} + a(\cos\theta\,\underline{i} + \sin\theta\,\underline{j})$$

$$\& \qquad 2\left[\frac{(\dot{x})^2 + x\ddot{x}}{c^2}\right] + 2\left[\frac{\dot{y}^2 + y\ddot{y}}{d^2}\right] = 0$$

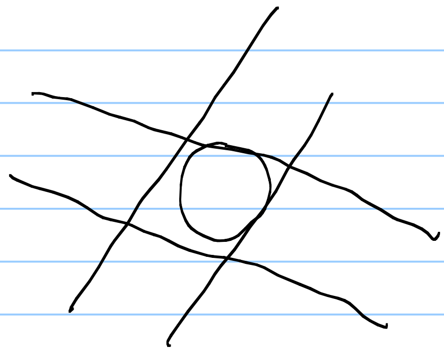
$$\Big\} \quad a,\ \ddot{x},\ \ddot{y}$$

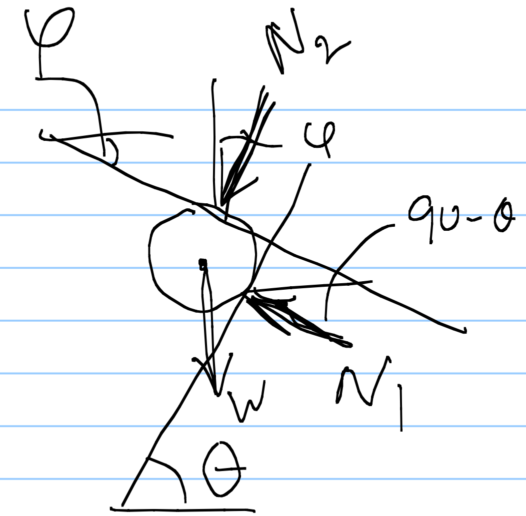
Similarly $v_B, \underline{a_B} \Rightarrow \quad v_{BA},\ a_{BA}$.

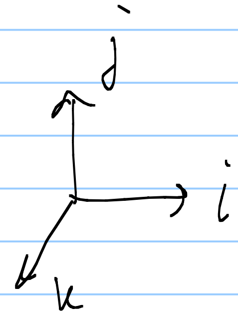
Forces on the pins

$$-\tan\varphi = \frac{dy}{dx}$$

↑ ellipse







$$\left.\begin{aligned} m\ddot{x} &= -N_2\sin\varphi - N_1\sin\theta \\ m\ddot{y} - mg &= -N_2\cos\varphi + N_1\cos\theta \end{aligned}\right\} \underline{N_1, N_2}$$

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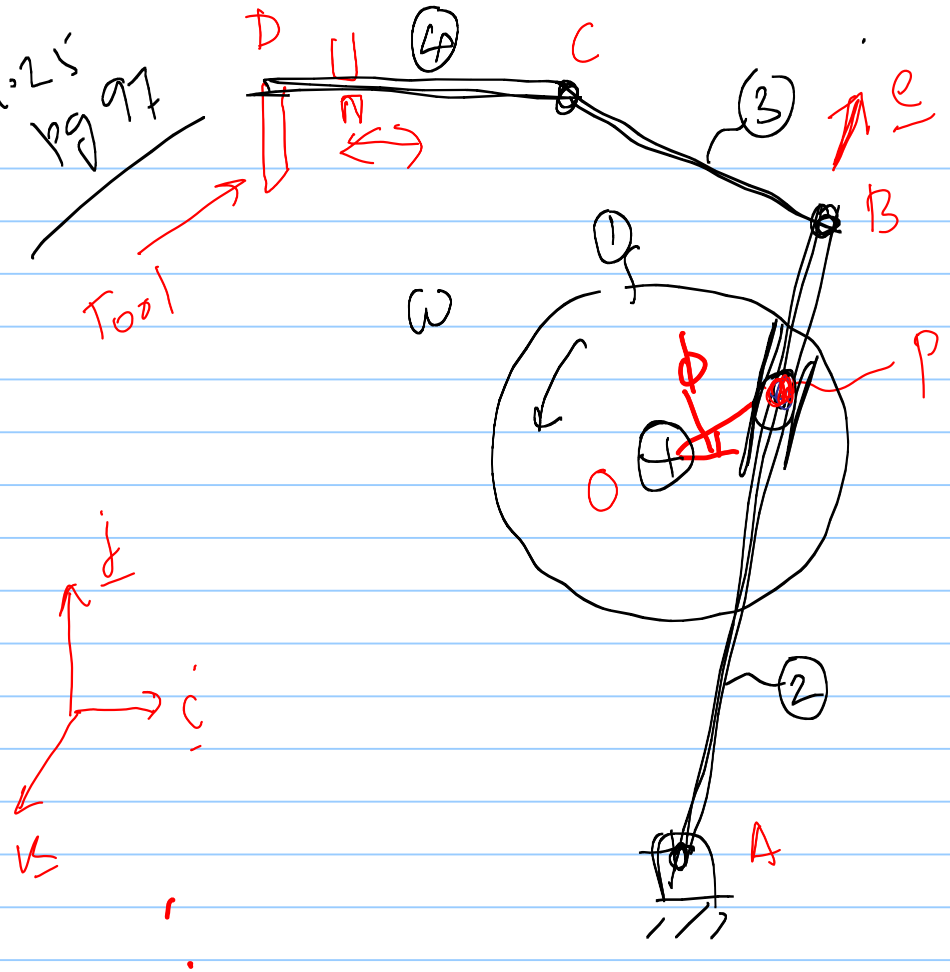

quick return mechanism

p is fixed to 1
free to slide on 2

2 oscillates about A

D is a tool

ω_1 ✓ $\dot{\omega}_1$ ✓

$\omega_2, \dot{\omega}_2 \to ?$

$v_{p|2} = v\underline{e}$; v?

$a_{p|2} = a\underline{e}$; a?

$\underline{v}_D = -v_D \underline{i}, \quad a_D = -a_D \underline{i}$

Eg: P: Motion described starting from two points O & A

Velocity: $\underline{v}_P = \cancel{\underline{v}_O}^{0} + \underline{\omega}_1 \times \underline{r}_{PO}$ $\qquad \underline{r}_{PO}$ ✓

$$\underline{v}_P = \underline{v}_A + \underline{\omega}_2 \times \underline{r}_{PA} + \underline{v}_{P|2}$$

$$= 0 + \underline{\omega}_2 \times \underline{r}_{PA} + v\underline{e}$$

v, ω_2 unknowns

$$\underline{v}_P = \underline{\omega}_1 \times \underline{r}_{PO} = \underline{\omega}_2 \times \underline{r}_{PA} + v\underline{e}$$

2 Scalar eqns for components along $\underline{i}$ & $\underline{j}$

$$\omega_1 \underline{k} \times OP(\cos\phi\, \underline{i} + \sin\phi\, \underline{j})$$

$$= \omega_2 \underline{k} \times (e_x \underline{i} + e_y \underline{j})\, AP + v(e_x \underline{i} + e_y \underline{j})$$

2 eqns balancing $\underset{\sim}{i}$ & $\underset{\sim}{j}$ components separately

$\Rightarrow$ v, ω_2

acceleration:

$$\underline{a}_P = \cancel{\underline{a}_0}^{\,0} + \cancel{\dot{\underline{\omega}}_1}^{\,0} \times \underline{r}_{P0} + \underline{\omega}_1 \times (\underline{\omega}_1 \times \underline{r}_{P0})$$

ve

$$= \cancel{\underline{a}_A}^{\,0} + \dot{\underline{\omega}}_2 \times \underline{r}_{PA} + \underline{\omega}_2 \times (\underline{\omega}_2 \times \underline{r}_{PA}) + 2\,\underline{\omega}_2 \times \underline{v}_{P|2}$$

$$+ \underline{a}_{P|2} \leftarrow a\underline{e}$$

Balance components: 2 eqns for $\dot{\omega}_2$ & a $\Big\}$ $\dot{\underline{\omega}}_2$, $\underline{a}$

$v_B = \underline{v}_A + \underline{\omega}_2 \times \underline{r}_{BA}$; $\underline{a}_B = \underline{a}_A + \dot{\underline{\omega}}_2 \times \underline{r}_{BA} + \underline{\omega}_2 \times (\underline{\omega}_2 \times r_{BA})$

$$\underline{v}_D = v_D \underline{i} = \underline{\omega}_3 \times r_{DB} + \underline{v}_B$$

2 Scalar eqns for v_D and ω_3

$\Rightarrow v_D$ & ω_3

$$\underline{a}_D = \underline{a}_B + \dot{\omega}_3 \times r_{DB} + \underline{\omega}_3 \times (\underline{\omega}_3 \times \underline{r}_{DB}) = a_D \underline{i}$$

$\Rightarrow a_D$ $\underline{\text{and } \dot{\omega}_3}$