Lecture 28

Mechanical properties of Materials- Part 2
True stress and true strain

Robert Hooke

Isaac Newton

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Quiz-4 : 2nd April 2022 (Saturday)

Offline mode: “Fun Quiz”

- Syllabus: whole syllabus (including Prof. Sangeeta Santra’s part)
- Duration: 2 hrs (Suggested timings: 10 am-12 pm or 3 pm-5 pm)
- Group activity: homogeneously distributed groups will be formed to participate in the quiz
- Marks: same weightage as earlier quizzes conducted
- Content (or questions) will be fun-based activities covering the topics taught in the lectures.
- Venue and final timing details will be shared soon.

Recap…

1. Engineering Stress and Strain
2. Tensile test
3. Defined mechanical properties from tensile test
4. Offset yield stress

Why do we need to study Tensile test?

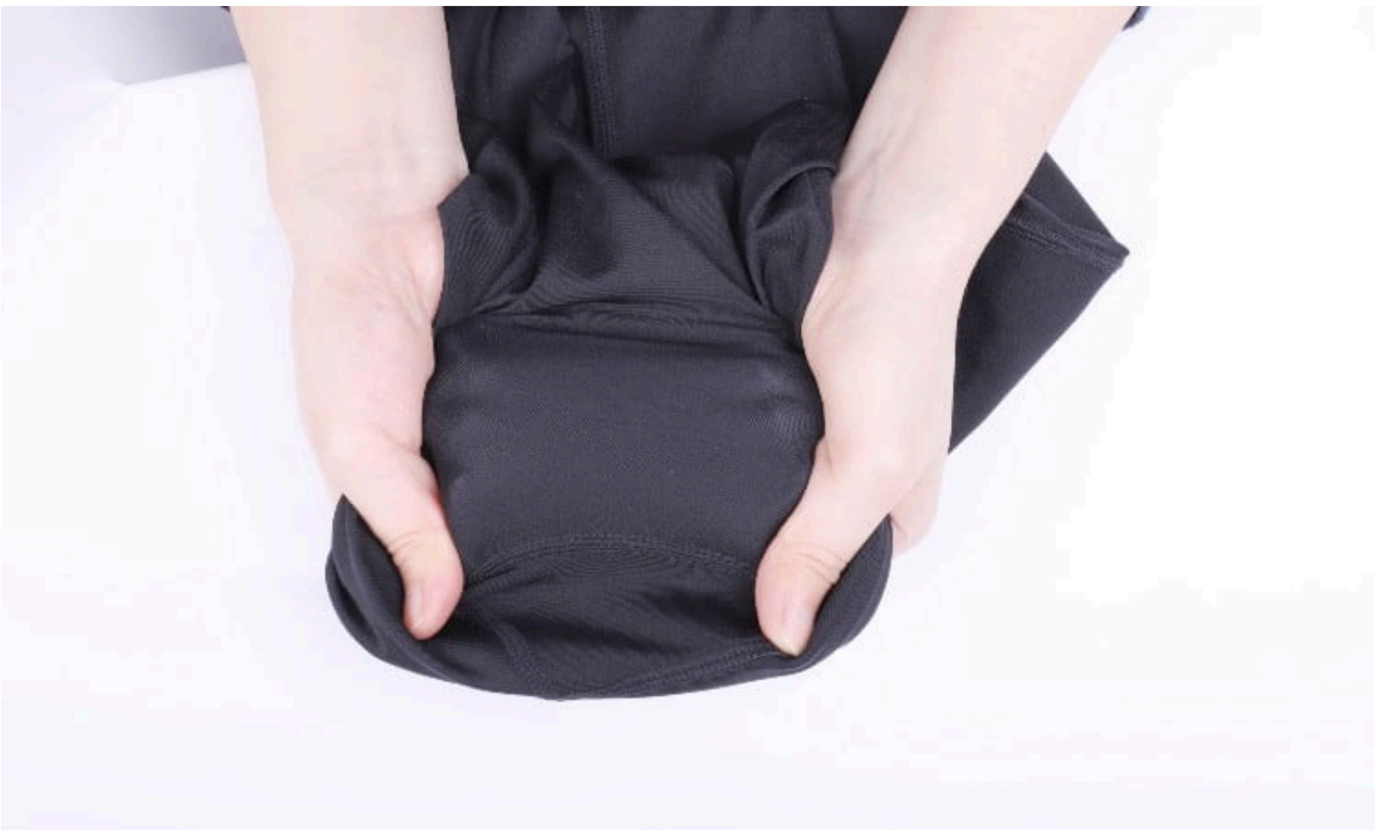
Why do we need to understand the strength of materials?

Biscuit

Stiff but weak

Steel

Stiff and strong



Nylon

Flexible and strong

Bones

Stiff, strong BUT brittle !!

FRACTURE!!

- STIFF
- STRONG
- FLEXIBLE
- WEAK
- BRITTLE

Why do we need to study Tensile test?

Why do we need to understand the failure of materials?

We want some materials to bend

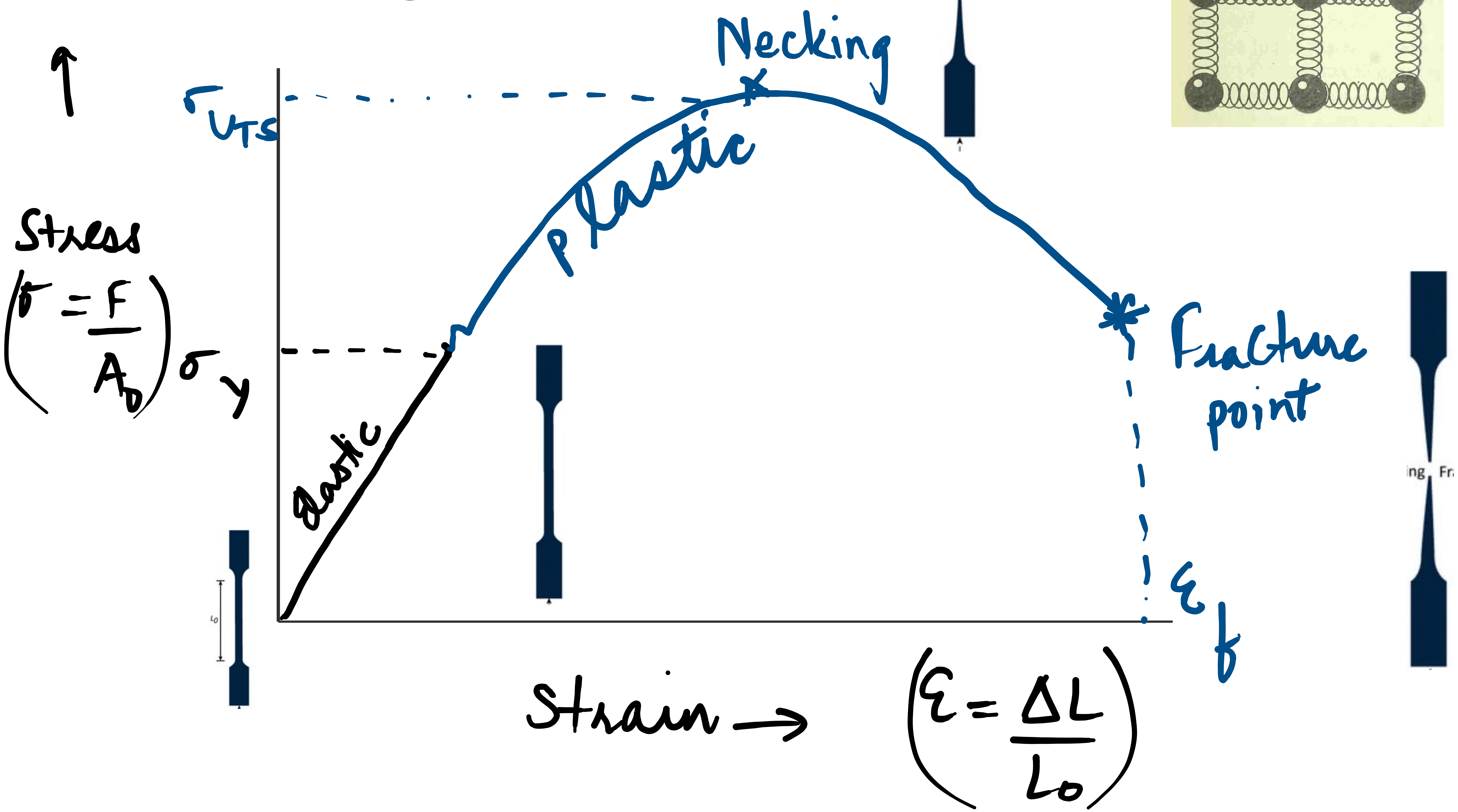
We want some materials to break

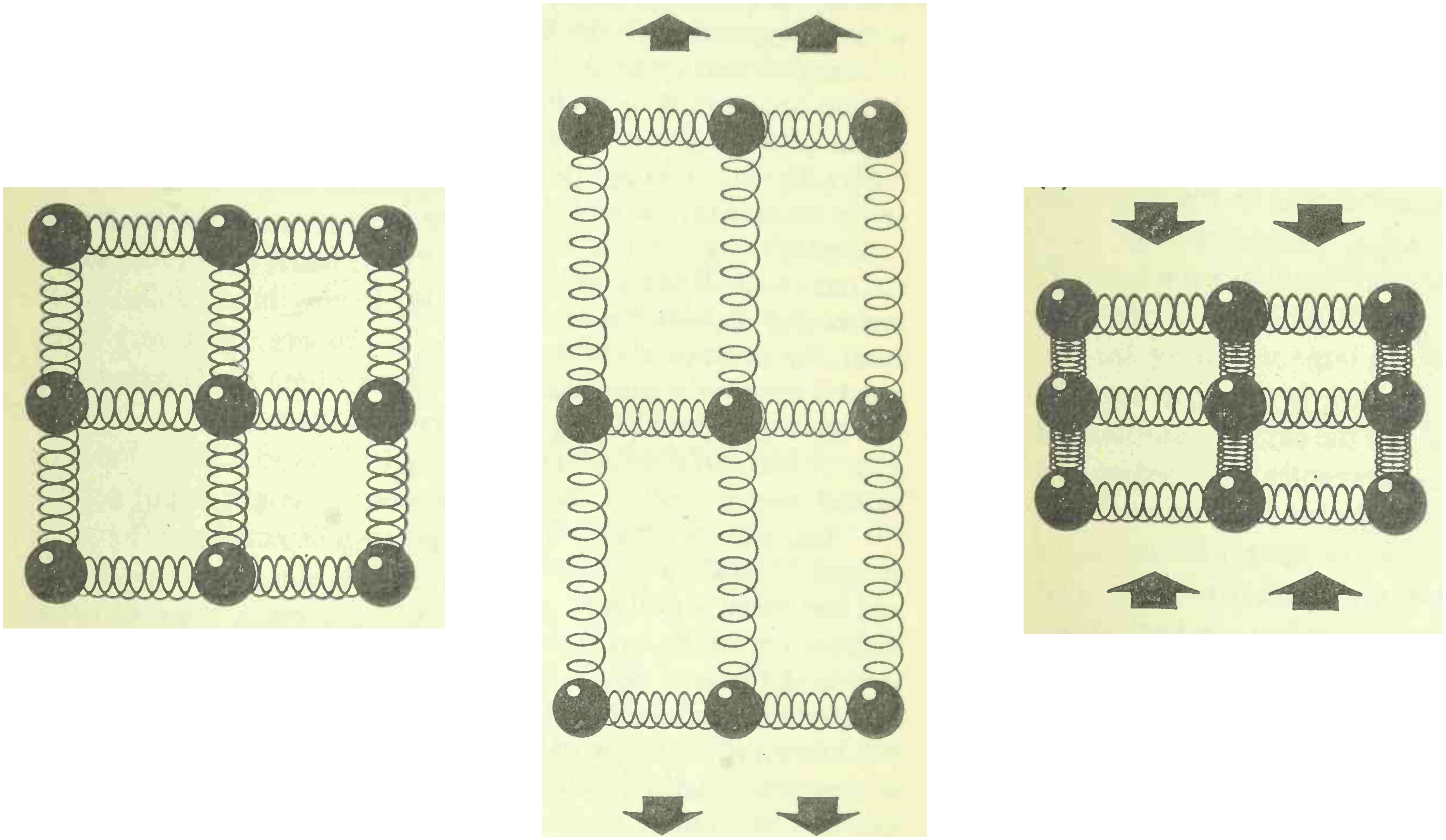
We want some materials NOT to fail !!

Designing materials with tailored properties!

Stress-strain curve

Load-Elongation curve





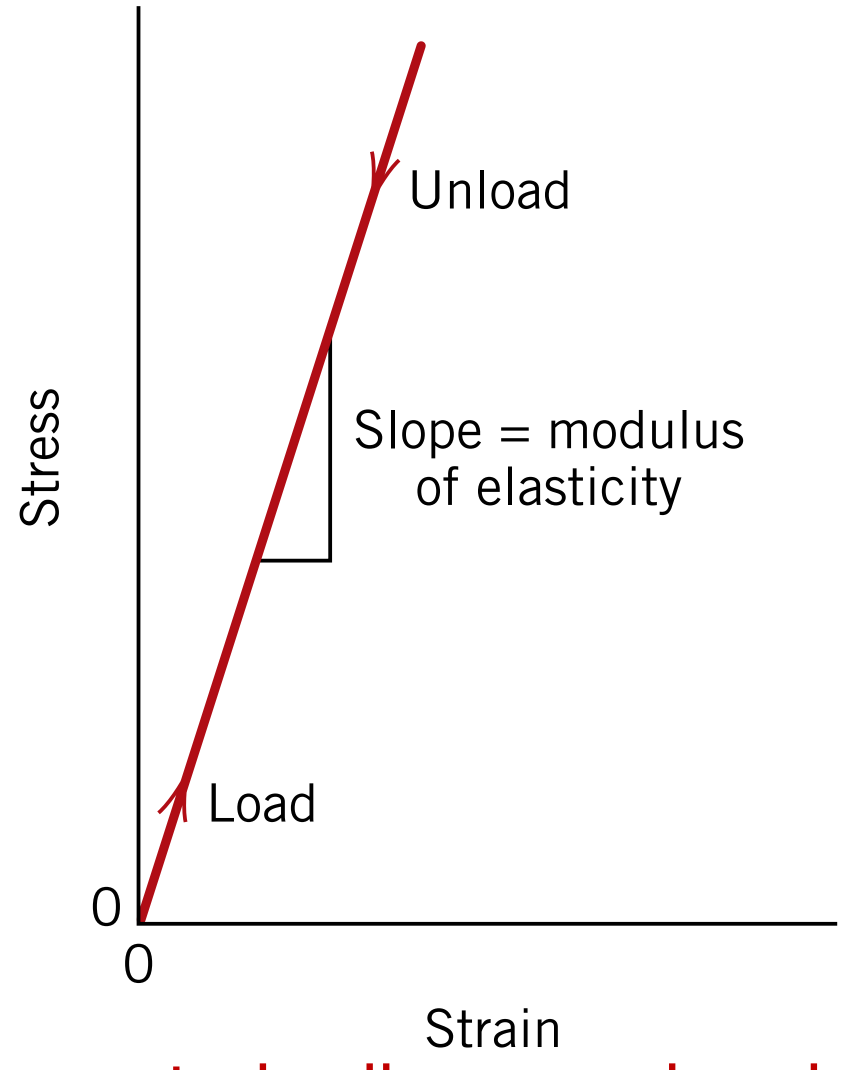
Elastic deformation

Deformation in which stress and strain are linearly proportional

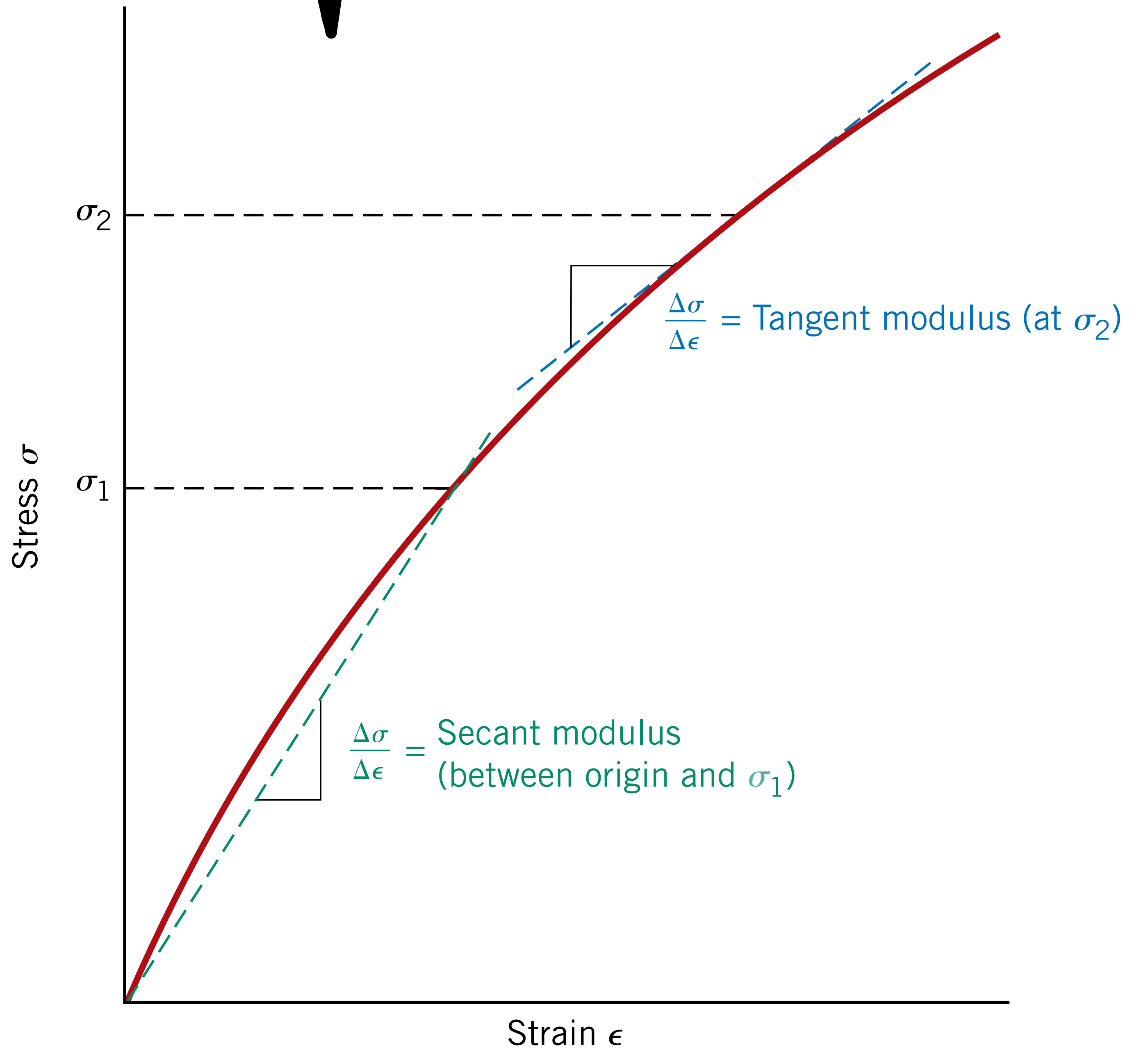
- Hooke's law:

$(F = k \cdot \Delta x) \Rightarrow \sigma = Y \varepsilon$, where Y = Young's modulus or modulus of elasticity

- For most typical metals the magnitude of this modulus ranges between 45 GPa (6.5×10^6 psi), for magnesium, and 407 GPa (59×10^6 psi), for tungsten.
- The greater the modulus, the stiffer the material, or the smaller the elastic strain that results from the application of a given stress.
- Elastic deformation is *non-permanent*: when the applied load is released, the piece returns to its original shape
- Young's modulus is a characteristic of each substance due to its chemical nature.



stress–strain diagram showing linear elastic deformation for loading and unloading cycles



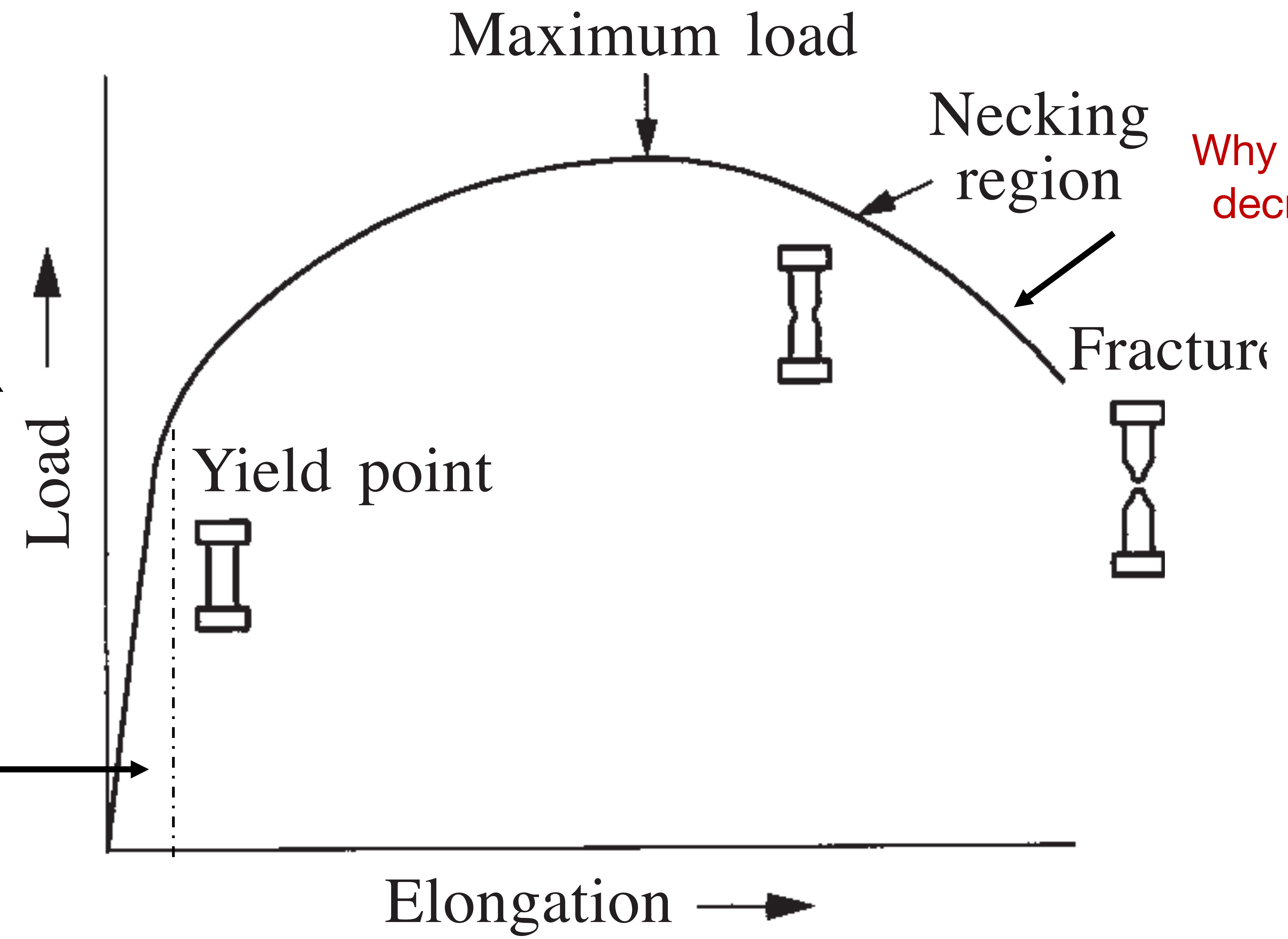
tangent or *secant modulus* for gray cast iron, concrete, and many polymers

Some obvious questions from stress-strain curve?

What if the elastic regime is non-linear? How to find Young's modulus?

What if elastic-to-plastic deformation is smooth? How to find the yield point?

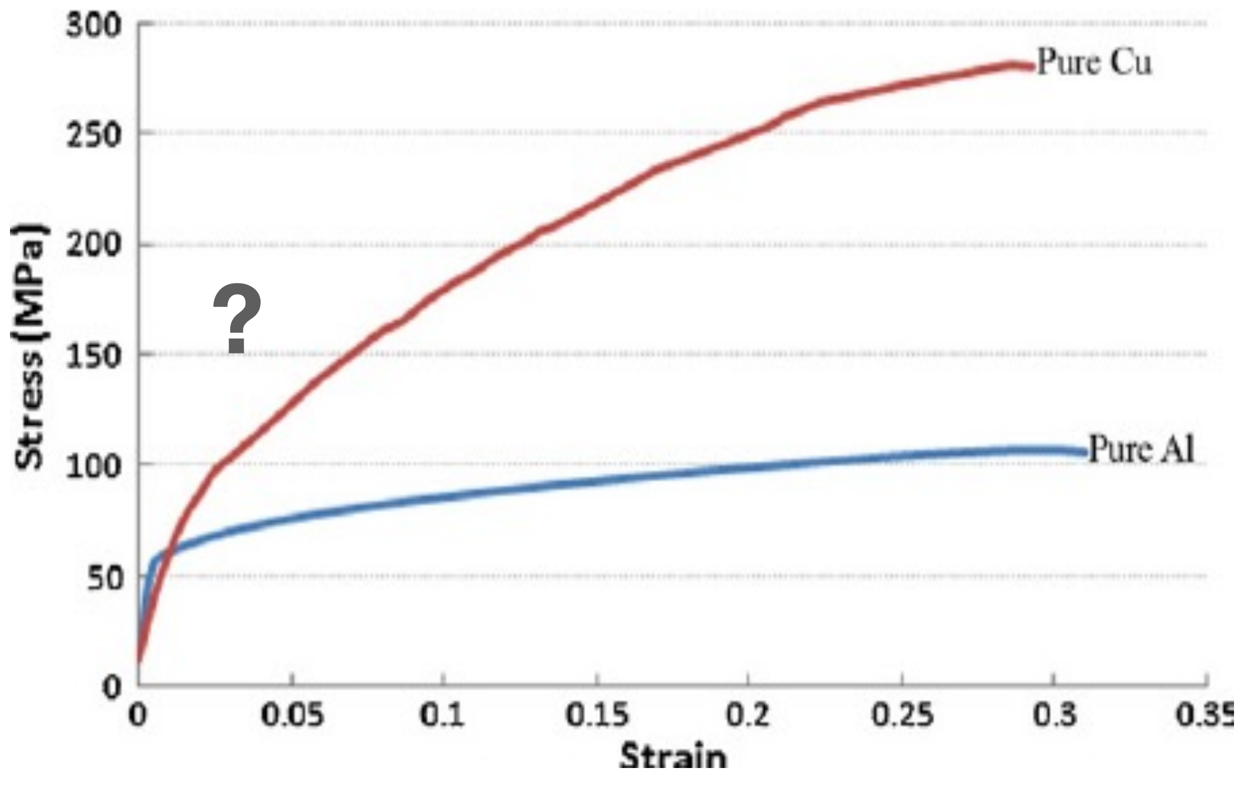
What property does this area provide?

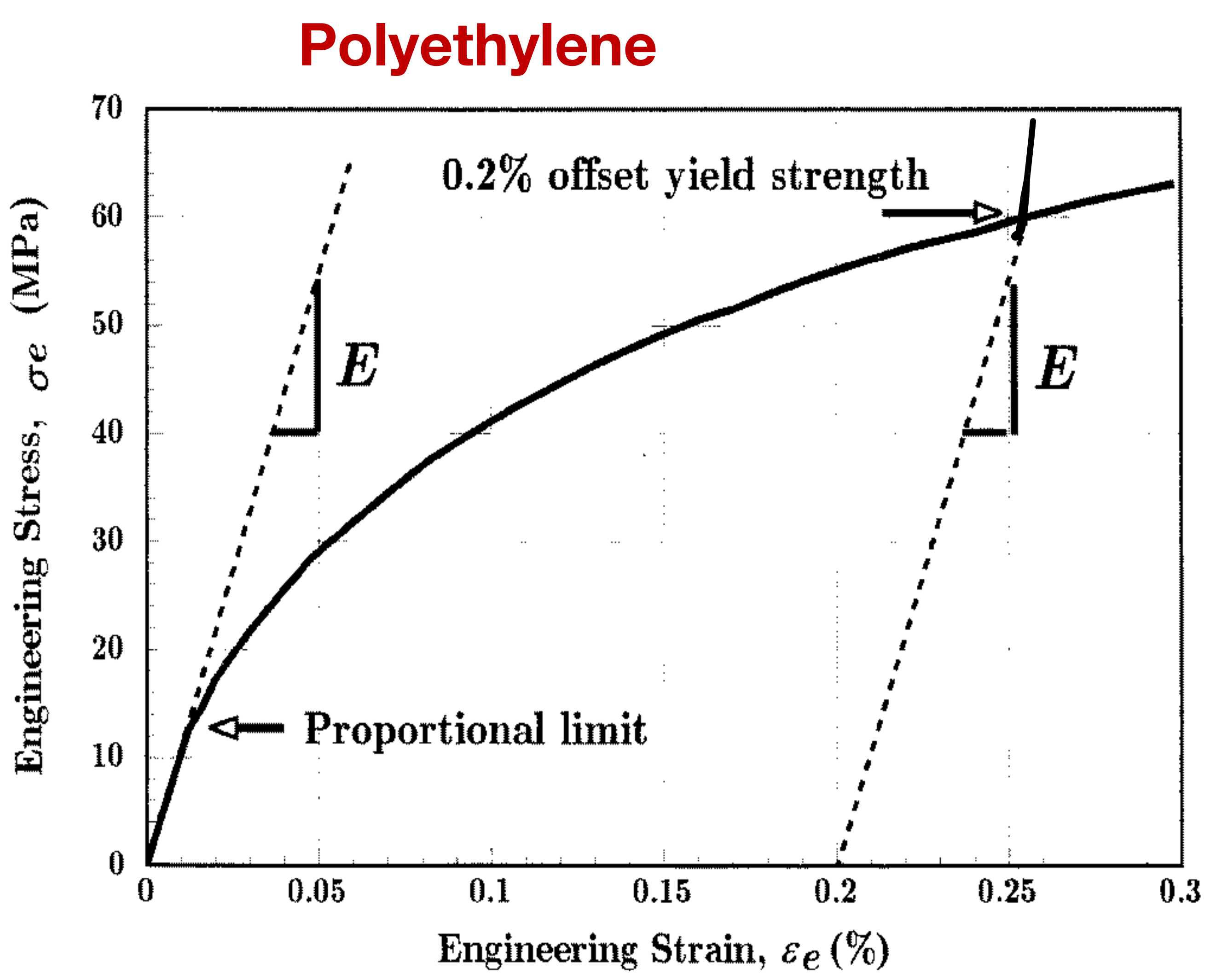


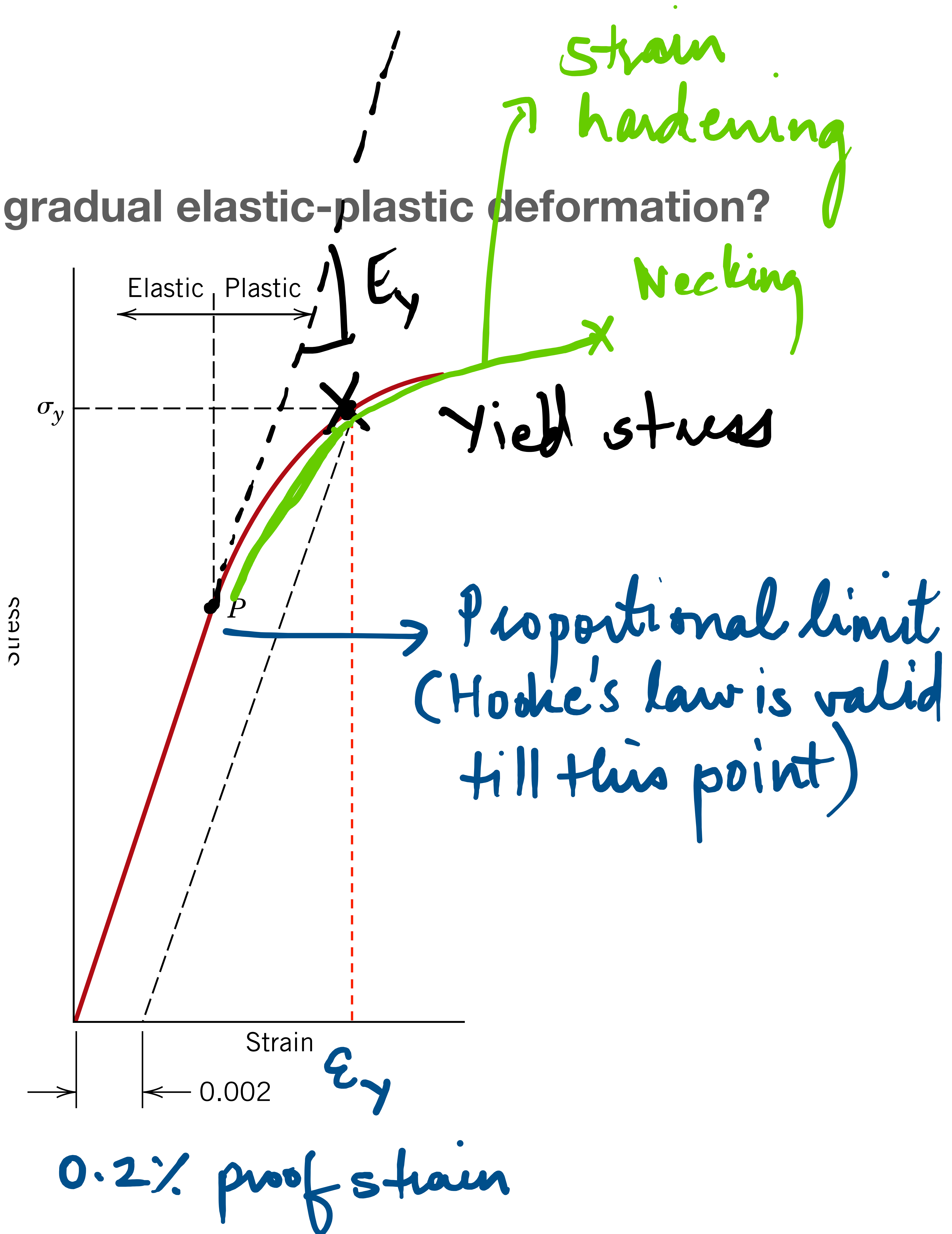
Why the Engineering stress decreases after necking?

Tensile Properties: Yield stress

Offset yield stress: How to determine yield stress for gradual elastic-plastic deformation?





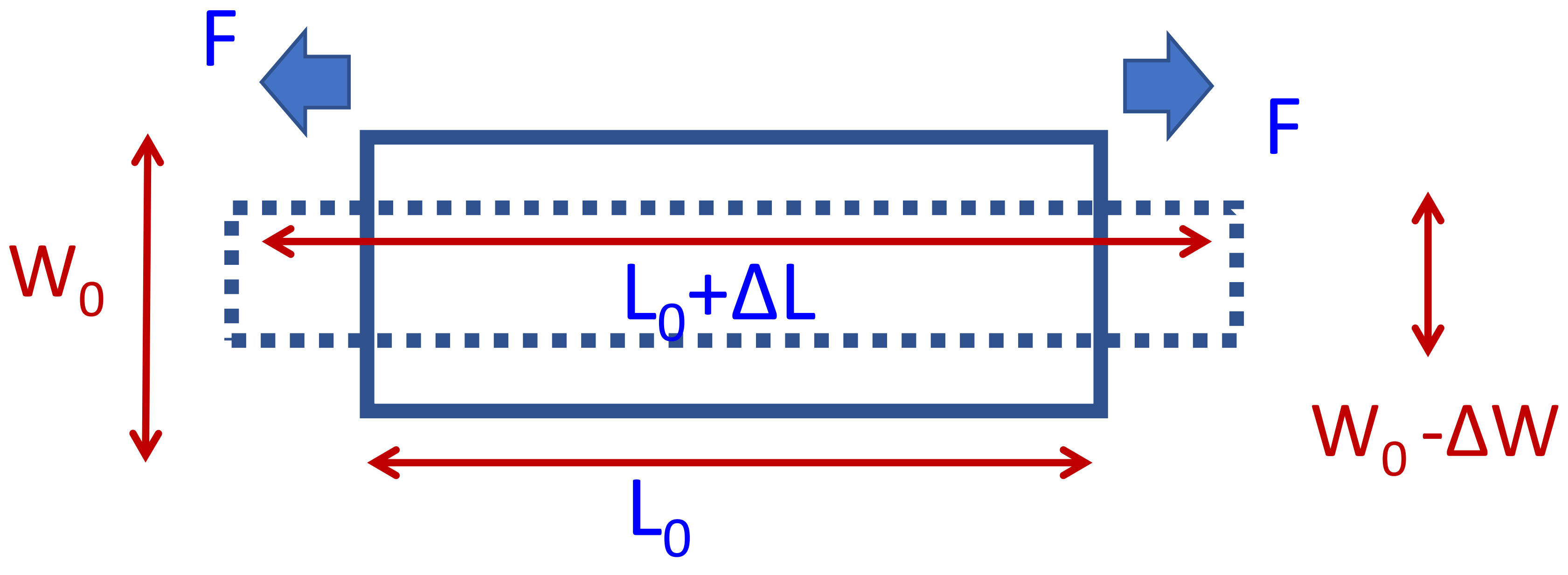


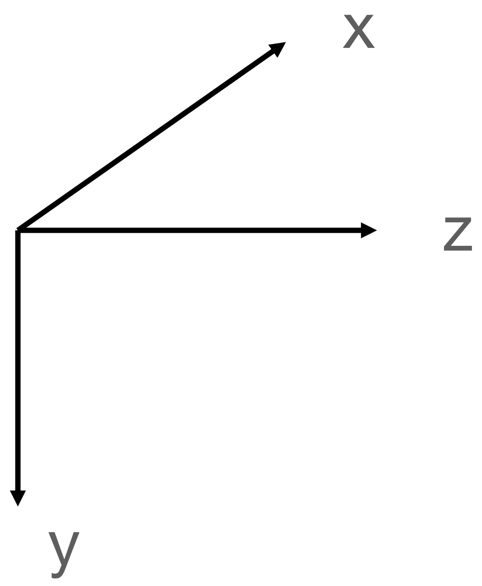
- **Proportional Limit:** initial departure from linearity of the stress–strain curve
- A straight line is constructed parallel to the elastic portion of the stress–strain curve at some specified strain offset, usually 0.002.
- Slope of this constructed straight line gives the Young' modulus

Elastic properties of materials

Poisson's ratio: What happens to the cross-section area on elongation?

Poisson's ratio measures the deformation in the material in a direction perpendicular to the direction of the applied force





$$\varepsilon_{longitudnal} = \frac{\Delta L}{L_0}$$

$$\varepsilon_{lateral} = \frac{-\Delta W}{W_0}$$

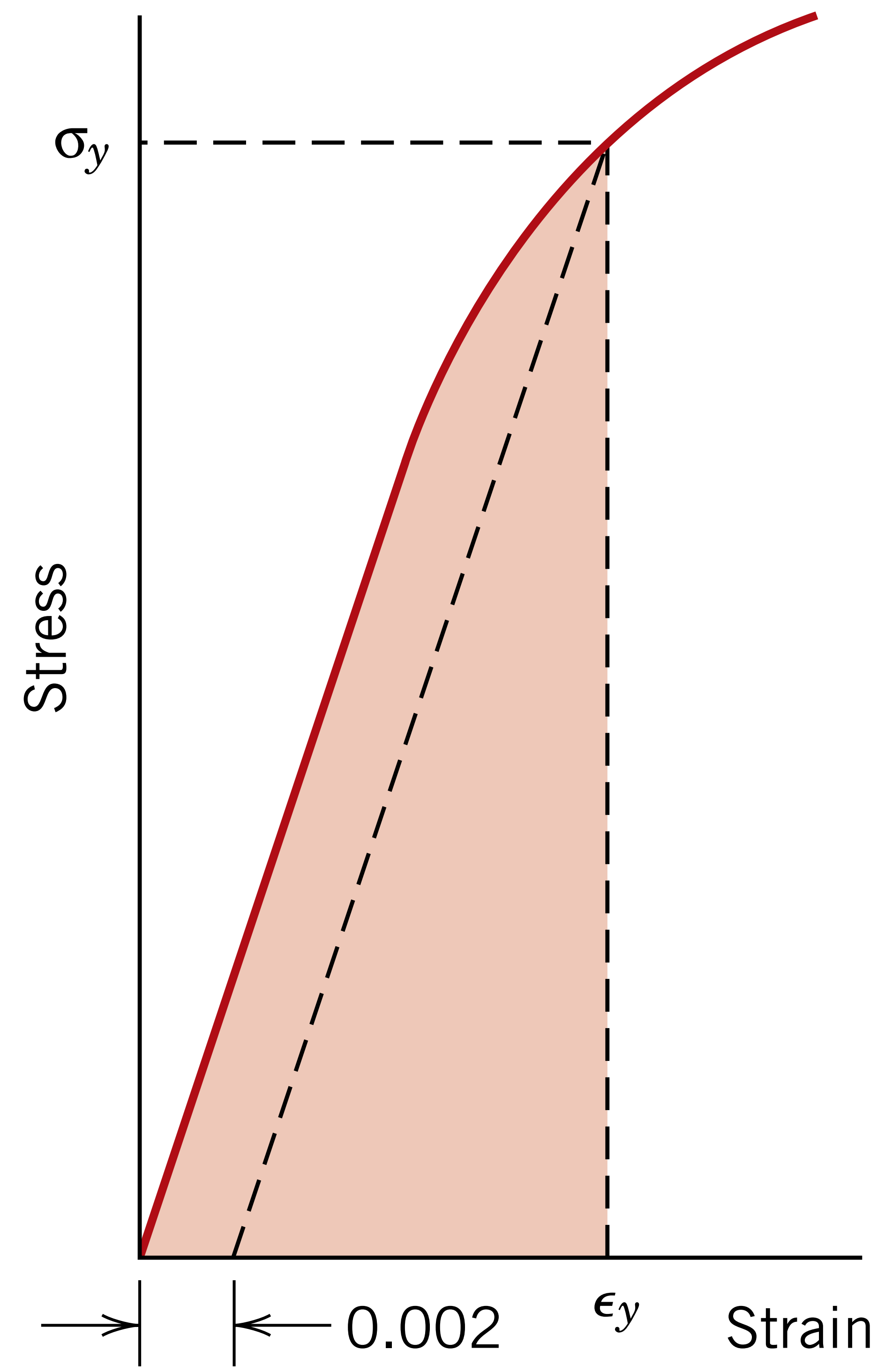
$$\upsilon = -\left[\frac{\frac{-\Delta W}{W_0}}{\frac{\Delta L}{L_0}}\right] = -\left[\frac{\varepsilon_{lateral}}{\varepsilon_{longitudnal}}\right]$$

$$\nu = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\epsilon_y}{\epsilon_z}$$

- If the applied stress is uniaxial (only in the *z* direction), and the material is isotropic, then $\varepsilon_x = \varepsilon_y$ (x and y are lateral and z is longitudinal direction).
- It is the ratio of lateral strain to longitudinal strain
- The negative sign is so that the ratio is always positive
- ε_x or ε_y *and* ε_z *will always be of opposite sign.*
- A high Poisson's ratio denotes that **the material exhibits large elastic deformation, even when exposed to small amounts of strain**.
- Metals: ~0.33, ceramics: ~0.25 and polymers: ~0.40

Resilience

What property does the area under stress-strain give before yielding point?



- Resilience: the capacity of a material to absorb energy when it is deformed elastically and then, upon unloading, to have this energy recovered.
- Modulus of resilience: strain energy per unit volume required to stress a material from an unloaded state up to the point of yielding
- *Modulus of resilience is the area under the stress-strain curve taken to yielding.*
- Resilience is energy absorbed per unit volume upto elastic limit.

$$U_r = \int_0^{\epsilon_y} \sigma \, d\epsilon$$

Assuming linear elastic region,

$$U_r = \frac{1}{2}\sigma_y \varepsilon_y$$

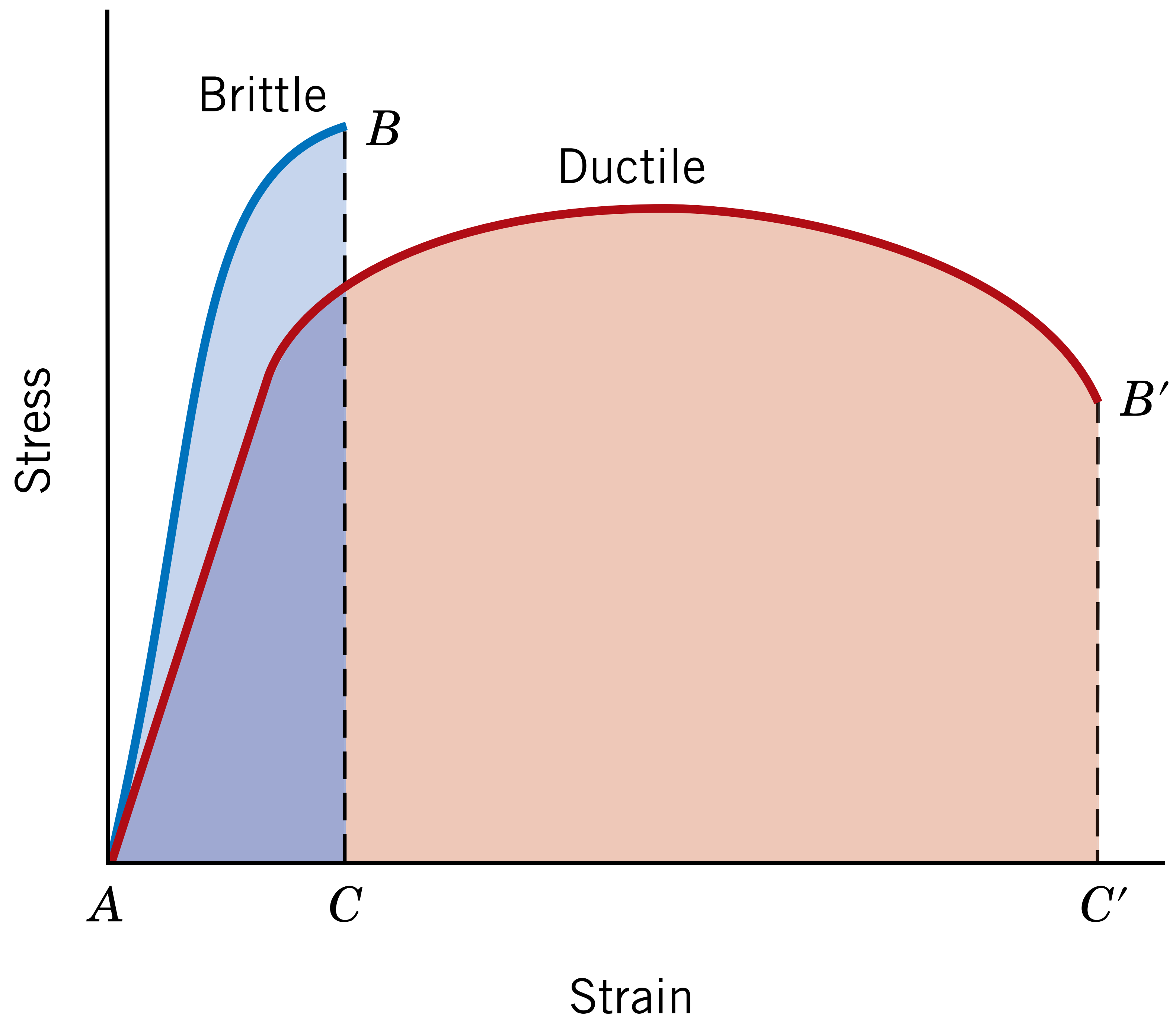
σ_y = yield stress
ε_y = yield strain

Incorporating Hooke's law : $\sigma = E\varepsilon$ (E: Young's modulus)

$$\Rightarrow U_r = \frac{1}{2}\sigma_y\left(\frac{\sigma_y}{E}\right) = \frac{\sigma_y^2}{2E}$$

$\therefore$ Resilient materials have high yield strengths & low elastic modulus.
(SPRING)

Ductility and Brittle



Ductility in design:

- Degree of allowable deformation
- Degree to which structure deforms plastically before fracture.

- **Ductility**: A measure of the degree of plastic deformation that has been sustained at fracture.
- **Brittle**: A material that experiences very little or no plastic deformation upon fracture. Brittle materials are *approximately* considered to be those having a fracture strain of less than about 5%.
- Ductility may be expressed quantitatively as either *percent elongation* or *percent reduction in area.*

$$\%EL = \left(\frac{L_f - L_0}{L_0}\right) \times 100$$

%EL = % of plastic strain at fracture

L_f = fracture length

L_0 = original gauge length

$$\%RA = \left(\frac{A_0 - A_f}{A_0}\right) \times 100$$

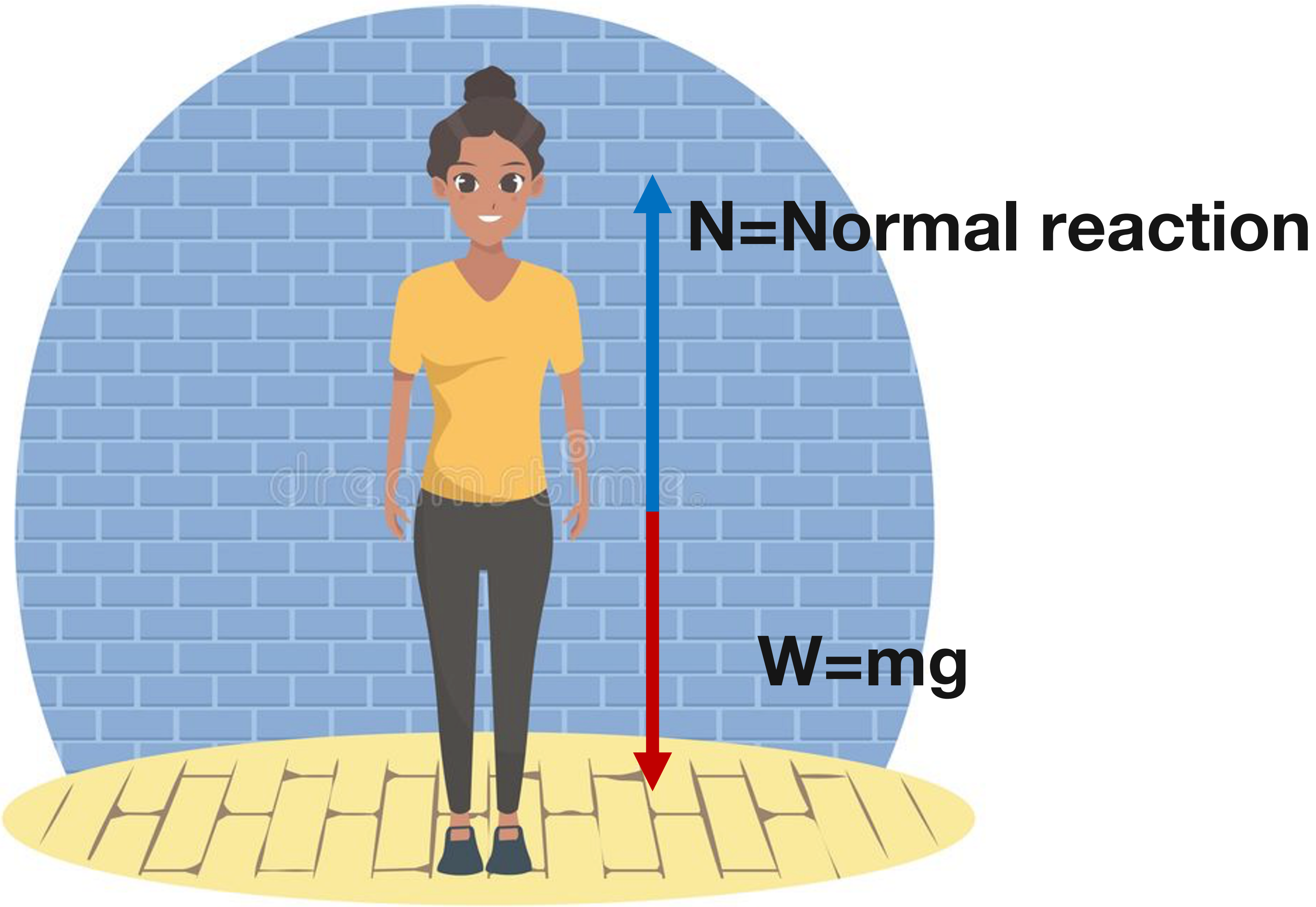
%RA = % reduction in area, A_0 = original cross-section area

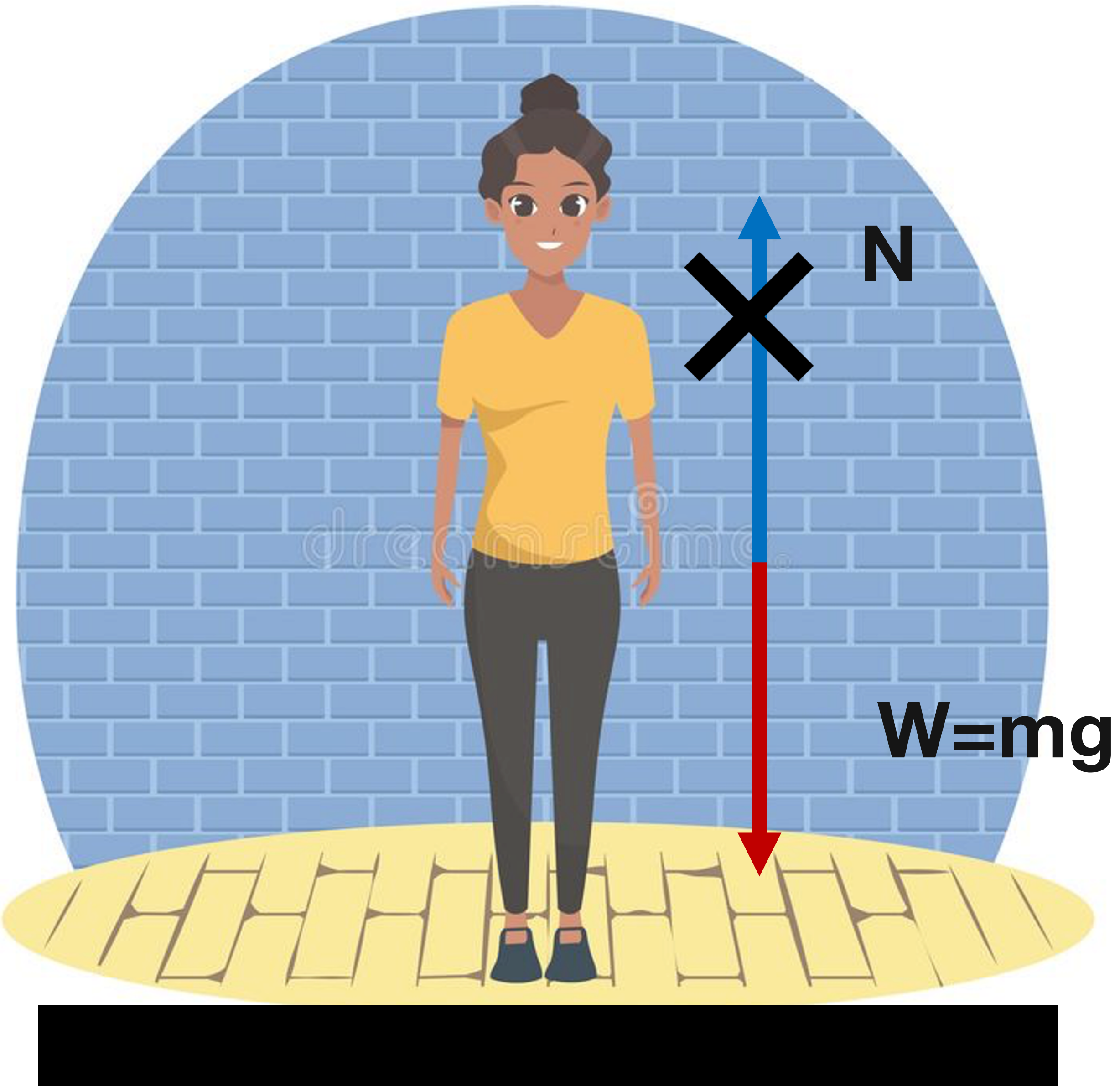
A_f = cross-sectional area at fracture

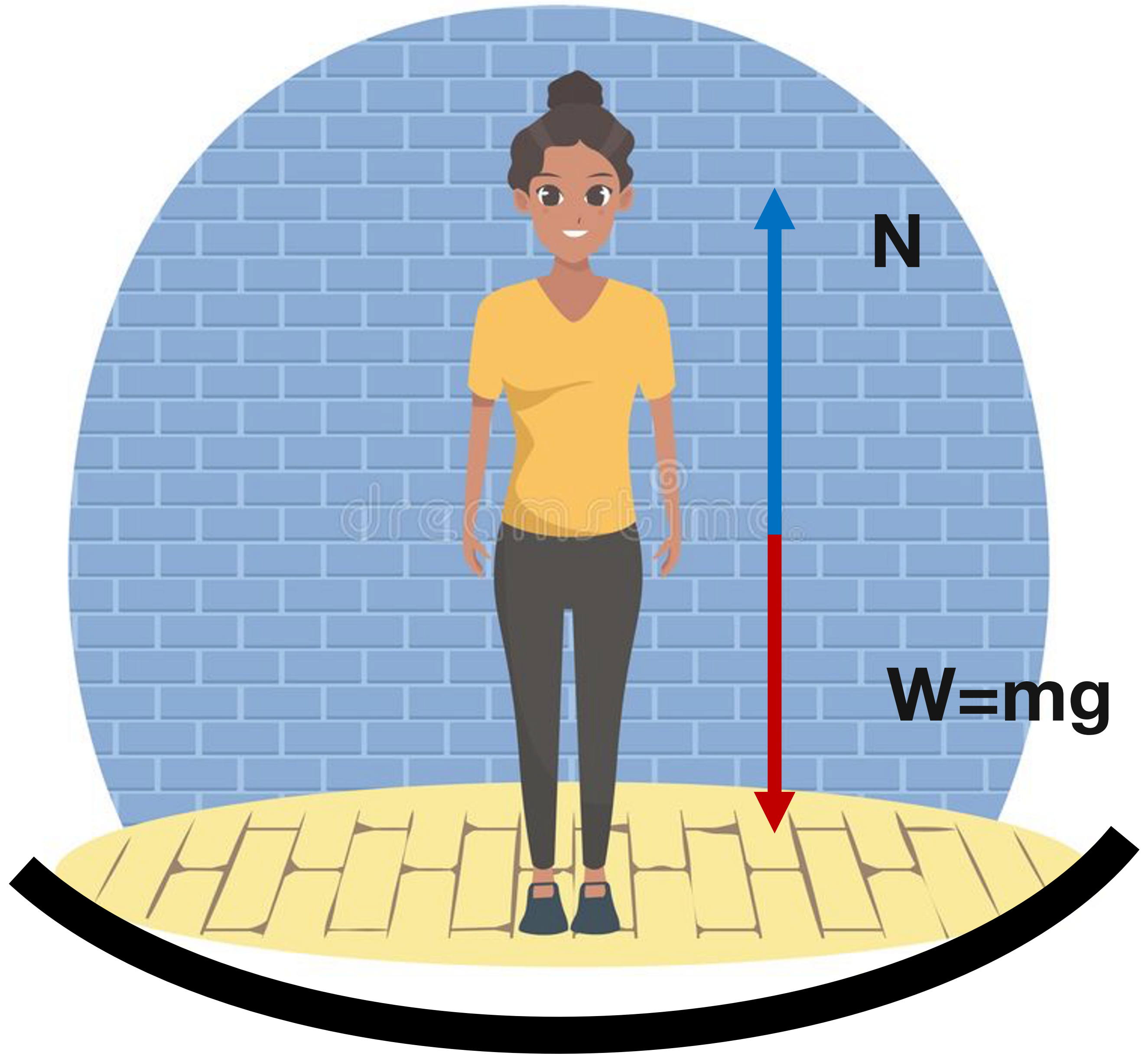
So now let us see why we don’t fall through the floor?

Unless you are sitting on a missile!

Is floor rigid or deformable?







Floor is elastically deformable

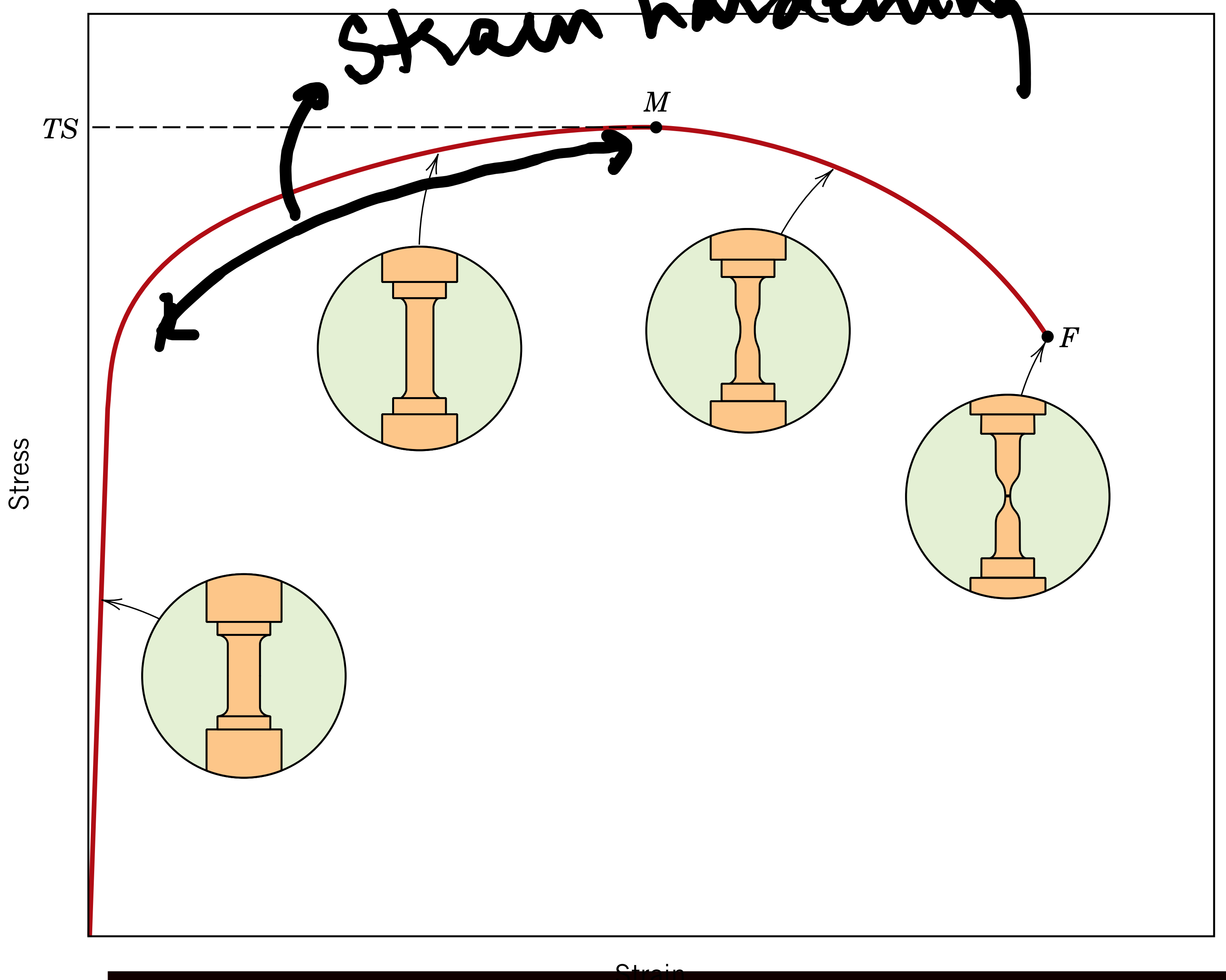
Hooke’s law

Under the weight of the person, the floor deforms and in order to come back to original state, it creates the normal force

True stress

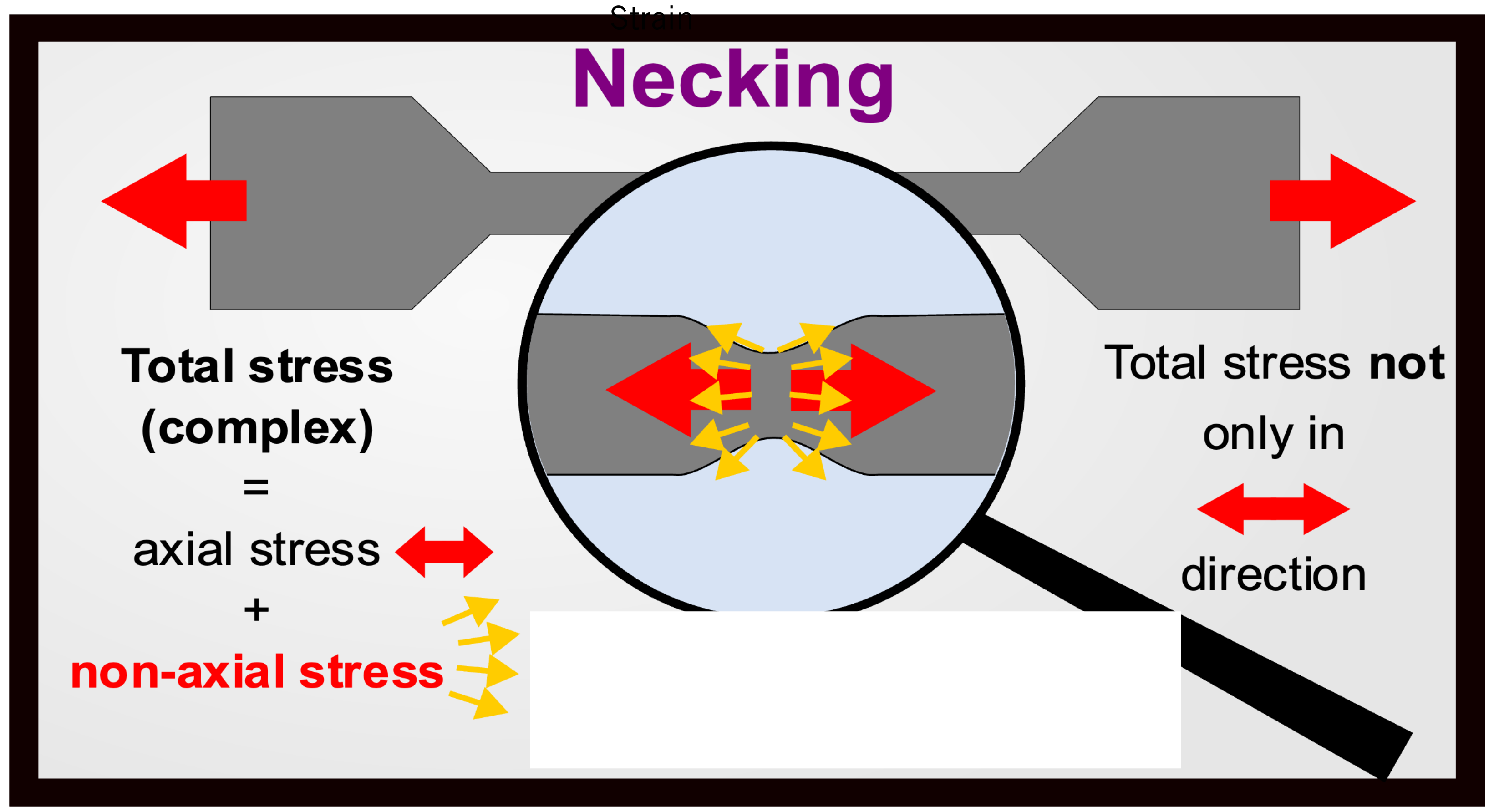
Engineering stress does not consider the instantaneous change in the cross-section area

strain hardening

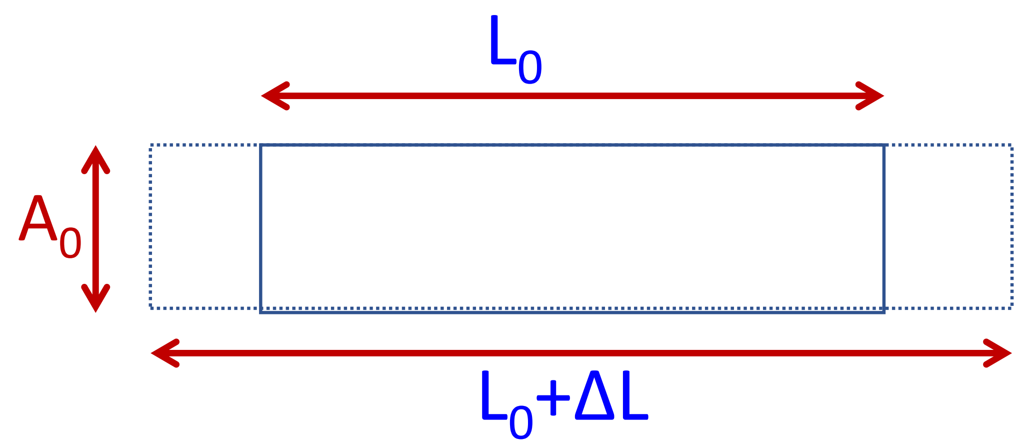


- Once the maximum in the engineering curve has been reached, the localized deformation at this site cannot be compensated by further strain hardening, so the cross-section area is reduced further. This increases the local stress even more, which accelerates the deformation further. This localized and increasing deformation soon leads to a “neck” in the gage length of the specimen.

- Material at necking point experiences stress which is no longer uniaxial. The Engineering stress does not take into account the reduction in cross-sectional area at necking.

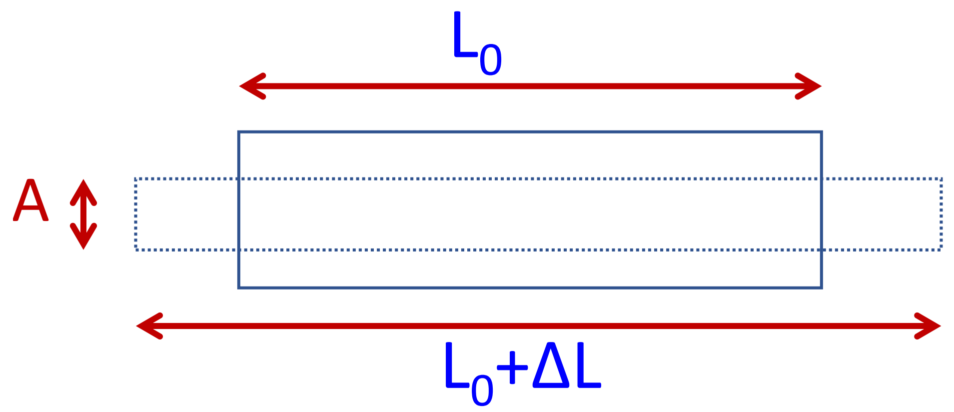


Engineering stress, $\sigma_E = \frac{F}{A_0}$



True stress, $\sigma_T = \frac{F}{A}$

Instantaneous area



Relationship between Engineering stress and True stress

$$\sigma_T = \frac{F}{A}, \quad \sigma_E = \frac{F}{A_0}$$

Assuming volume is constant,

$$A \cdot L = A_0 L_0 \Rightarrow \frac{A_0}{A} = \frac{L}{L_0}$$

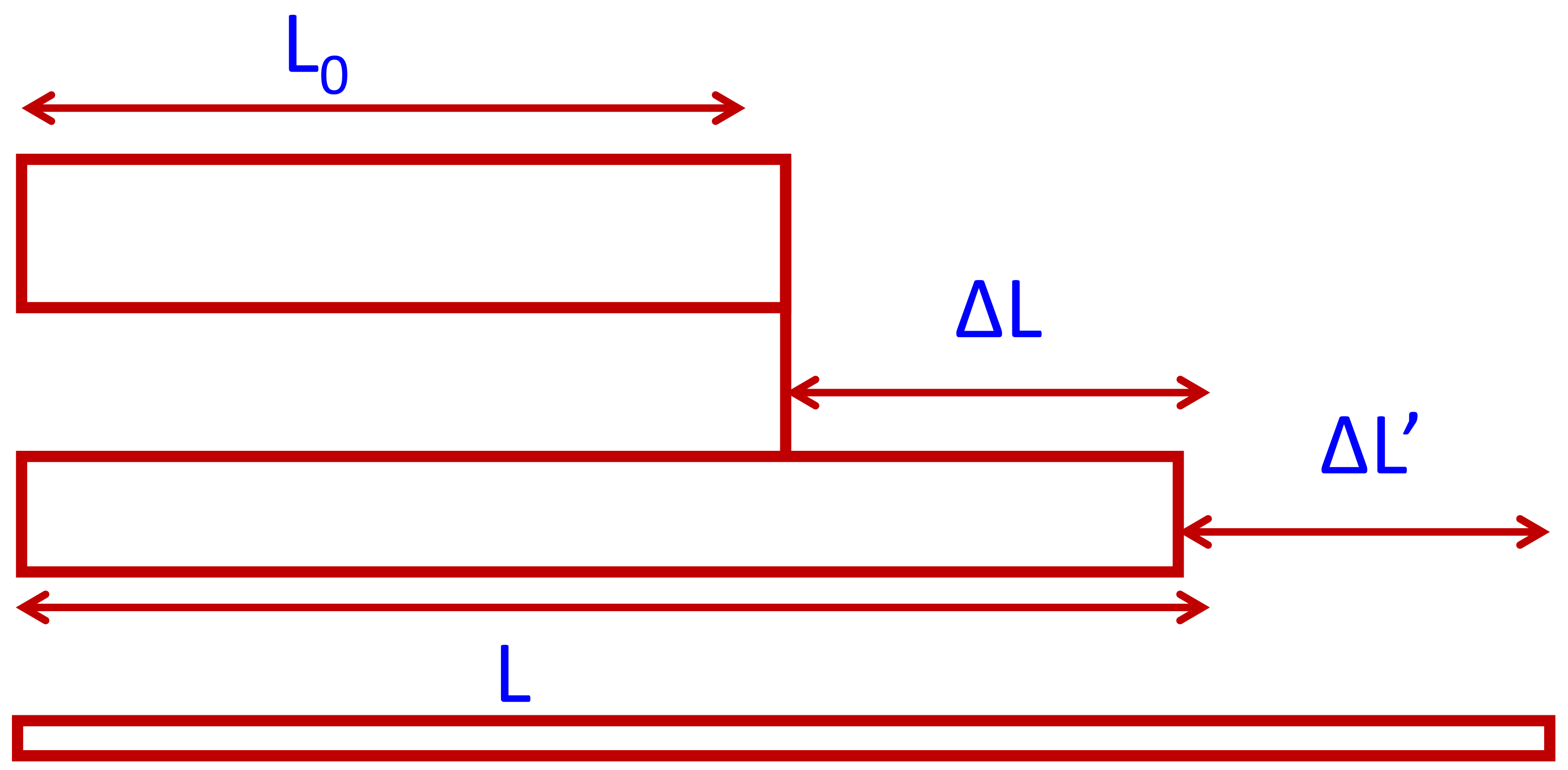
$$\Rightarrow \sigma_T = \frac{F}{A_0} \cdot \left(\frac{A_0}{A}\right) = \frac{F}{A_0} \cdot \left(\frac{L}{L_0}\right) = \frac{F}{A_0} \cdot \frac{(L_0 + \Delta L)}{L_0} = \frac{F}{A_0} \cdot \left(1 + \frac{\Delta L}{L_0}\right)$$

$$= \frac{F}{A_0} \cdot (1 + \varepsilon_E) = \sigma_E (1 + \varepsilon_E)$$

$$\therefore \boxed{\sigma_T = \sigma_E (1 + \varepsilon_E)}$$

True strain

Engineering strain does not consider the instantaneous change in the elongation



$$\varepsilon_E = \frac{\Delta L}{L_0},$$

True incremental strain,

$$d\varepsilon_T = \frac{dL}{L}$$

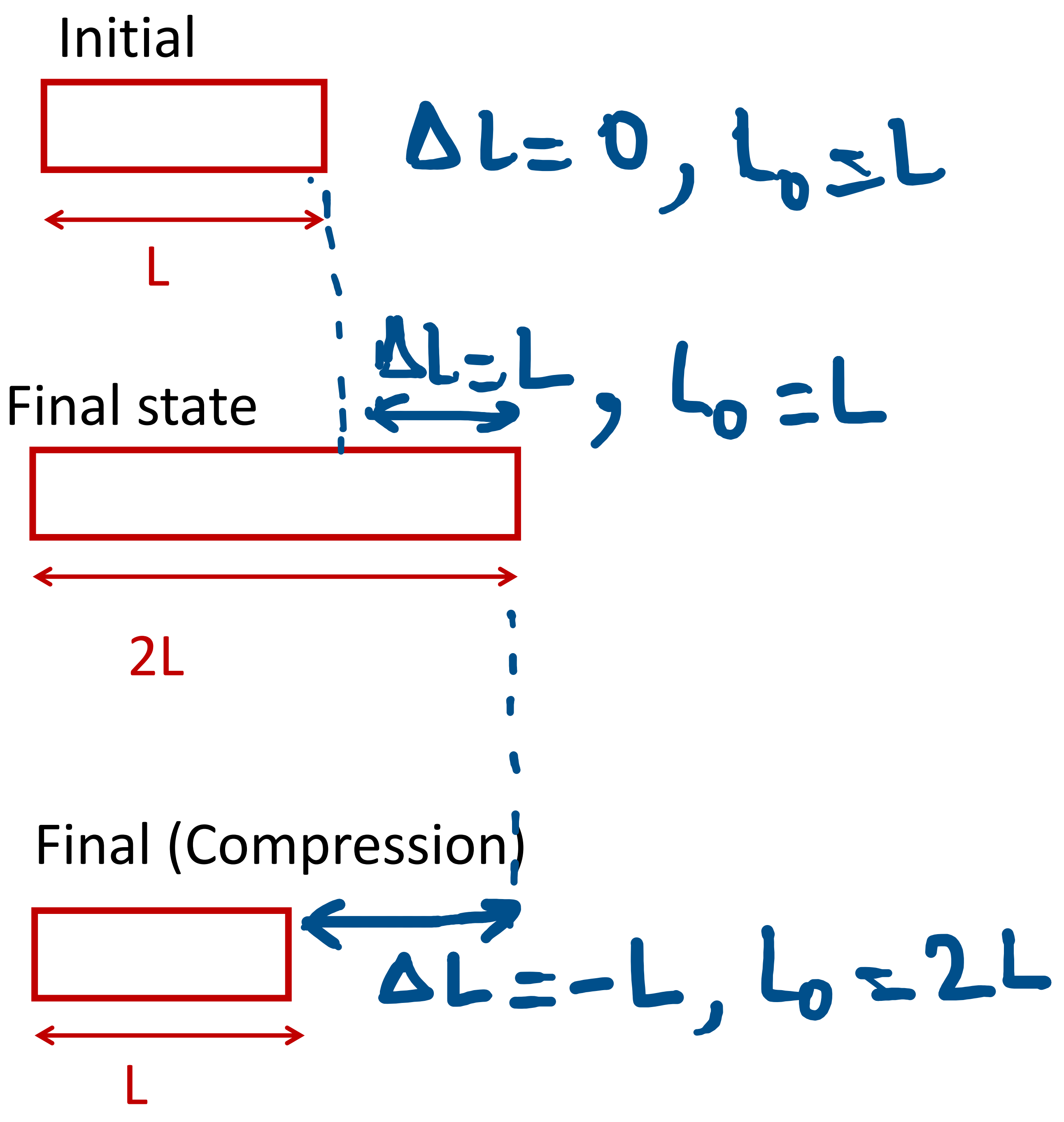
$$\varepsilon_T = \int_0^{\varepsilon_T} d\varepsilon_T = \int_{L_0}^{L} \frac{dL}{L} = \ln \frac{L}{L_0} = \ln\left(\frac{L_0 + \Delta L}{L_0}\right)$$

$$= \ln(1 + \varepsilon_E)$$

$$\therefore \boxed{\varepsilon_T = \ln(1 + \varepsilon_E)}$$

Implications

True strain is the real strain: for practical purposes



Engineering strain	True strain
$\varepsilon_E = \frac{\Delta L}{L_0} = 0$	$\varepsilon_T = \ln \frac{L}{L_0} = 0$
$\varepsilon_E = \frac{\Delta L}{L_0} = \frac{L}{L} = 1$	$\varepsilon_T = \ln \frac{2L}{L} = \ln 2 = 0.69$
$\varepsilon_E = \frac{\Delta L}{L_0} = \frac{-L}{2L} = -0.5$	$\varepsilon_T = \ln \frac{L}{2L} = -\ln 2 = -0.69$
$\text{Net } \varepsilon_E = 0.5$	$\text{Net } \varepsilon_T = 0$