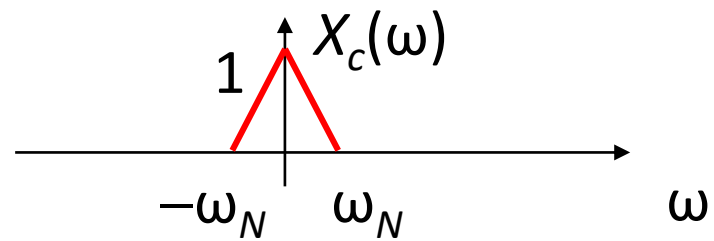


Sampling

Lecture 31

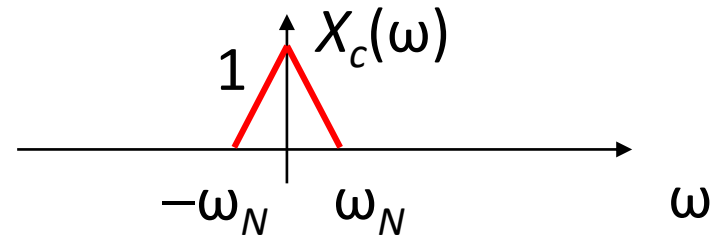
Sampling

$x_c(t)$

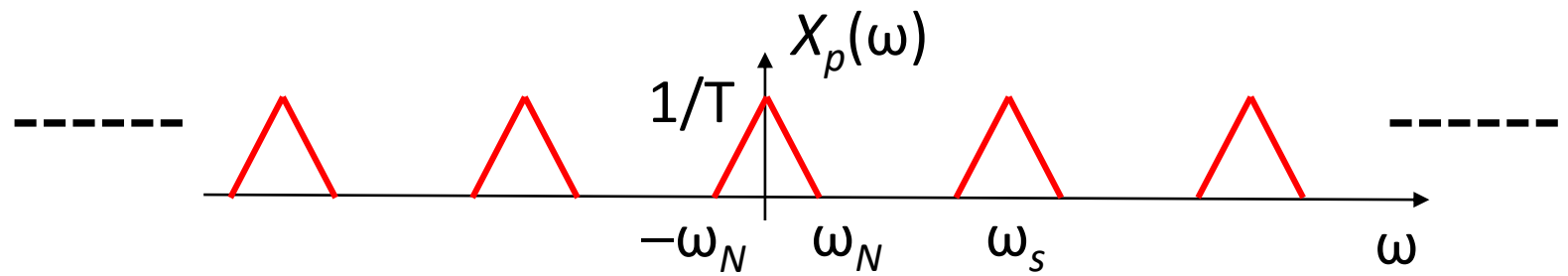


Sampling

$x_c(t)$

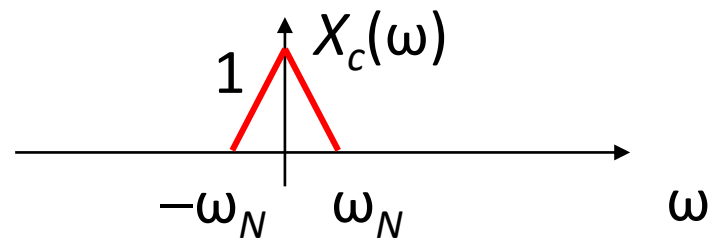


$x_p(t)$

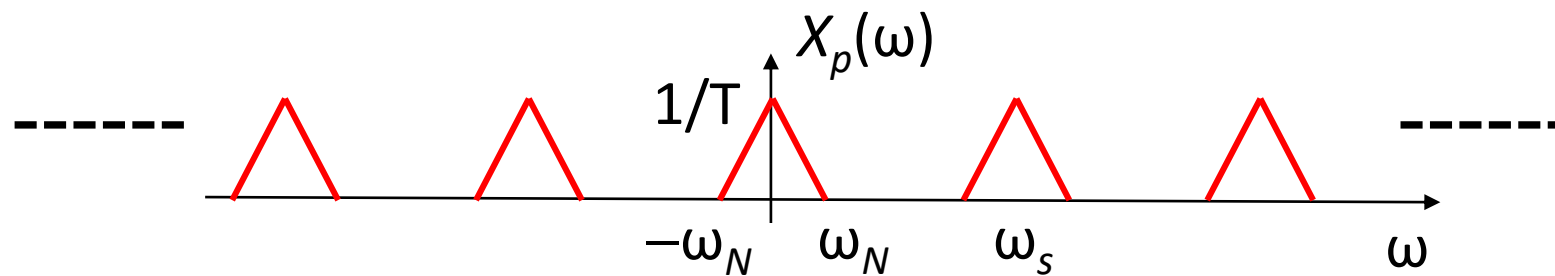


Sampling

$x_c(t)$

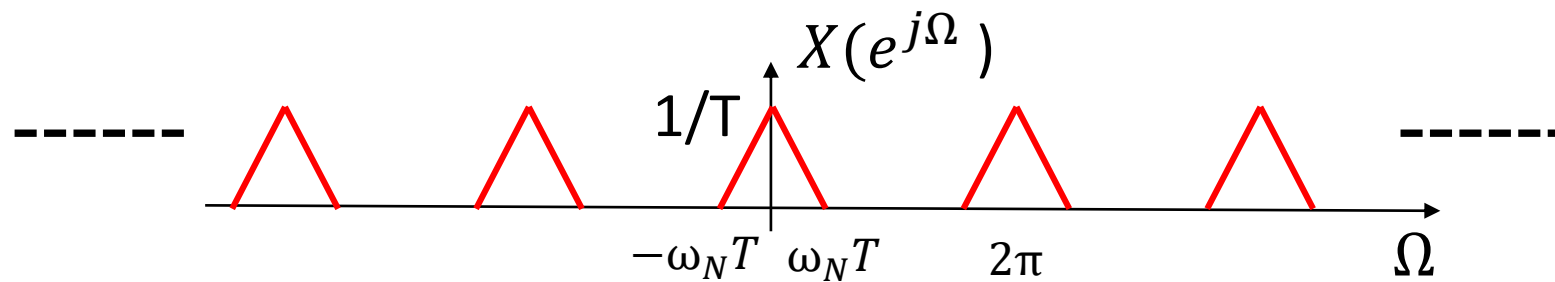


$x_p(t)$

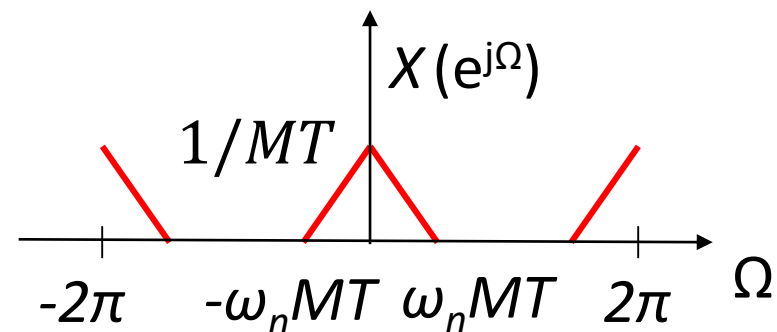
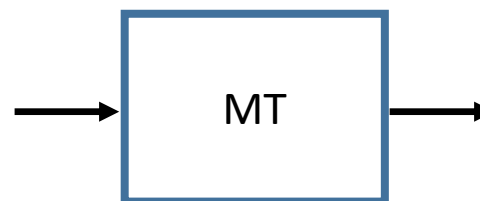
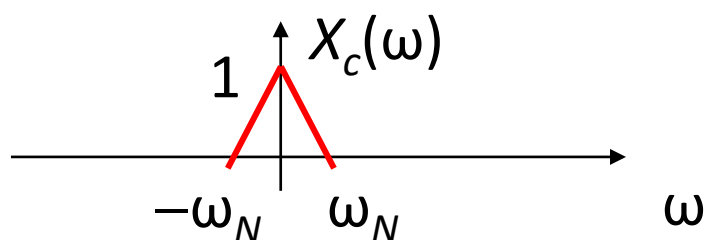
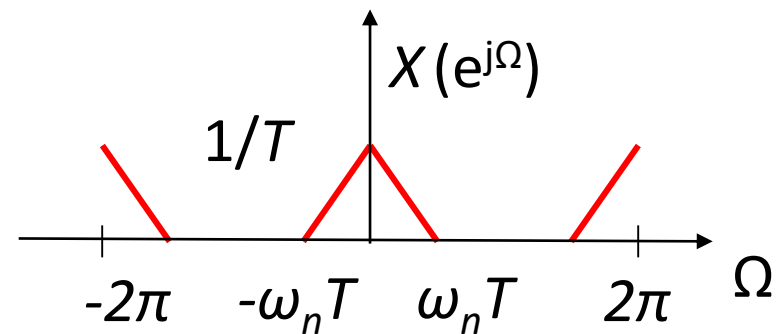
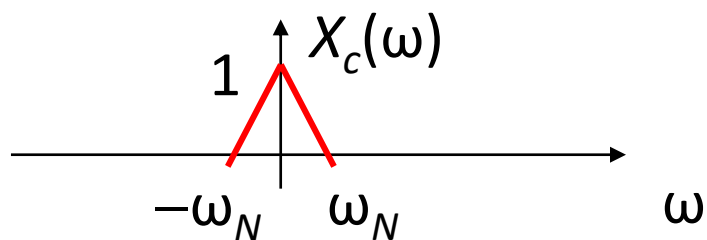


$x[n]$

$\Omega = \omega T$



Down-sampling



Decimation

$$x_1[n] = x[nM]$$

$$X_1(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[nM]e^{-j\Omega n}$$

Decimation

$$x_1[n] = x[nM]$$

$$X_1(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[nM] e^{-j\Omega n}$$

$$y[n] = x[n] \sum_k \delta[n - kM] \quad X_1(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} y[nM] e^{-j\Omega n}$$

Let $nM = v$

$$X_1(e^{j\Omega}) = \sum_{v=-\infty}^{\infty} y[v] e^{-j\Omega v/M} = Y(e^{j\Omega/M})$$

Decimation

$$x_1[n] = x[nM]$$

$$X_1(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[nM] e^{-j\Omega n}$$

$$y[n] = x[n] \sum_k \delta[n - kM] \qquad Y(e^{j\Omega}) = \frac{1}{2\pi} \left(X(e^{j\Omega}) \circledast \frac{2\pi}{M} \sum_k \delta\left(\Omega - k \frac{2\pi}{M}\right) \right)$$

$$Y(e^{j\Omega}) = \frac{1}{M} \left(\sum_k X\left(e^{j\left(\Omega - k \frac{2\pi}{M}\right)}\right) \right)$$

Decimation

$$x_1[n] = x[nM]$$

$$X_1(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[nM]e^{-j\Omega n}$$

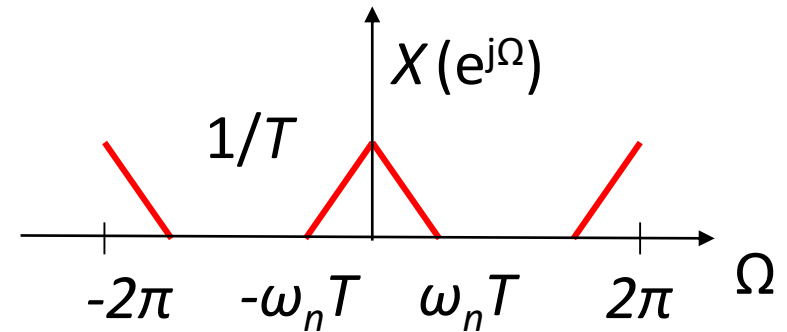
$$Y(e^{j\Omega}) = \frac{1}{M} \left(\sum_k X \left(e^{j\left(\Omega - k\frac{2\pi}{M}\right)} \right) \right)$$

$$X_1(e^{j\Omega}) = Y(e^{j\Omega/M}) = \frac{1}{M} \left(\sum_k X \left(e^{j\left(\frac{\Omega - k2\pi}{M}\right)} \right) \right)$$

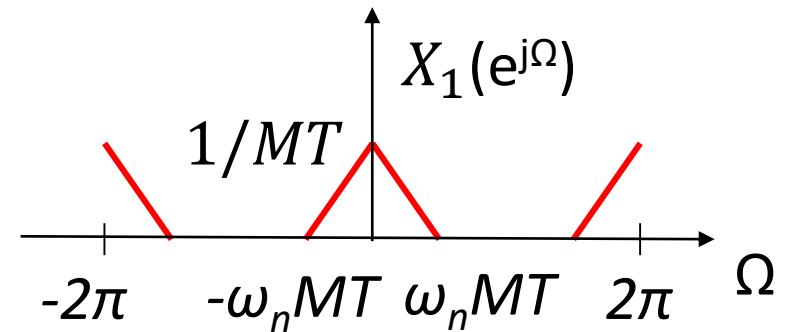
Decimation as down-sampling

$$x_1[n] = x[nM]$$

$x[n]$



$x[nM]$



Upsampling

$$x_M[n] = \begin{cases} x\left[\frac{n}{M}\right] & \text{if } \frac{n}{M} \text{ is an integer} \\ 0 & \text{Otherwise} \end{cases}$$

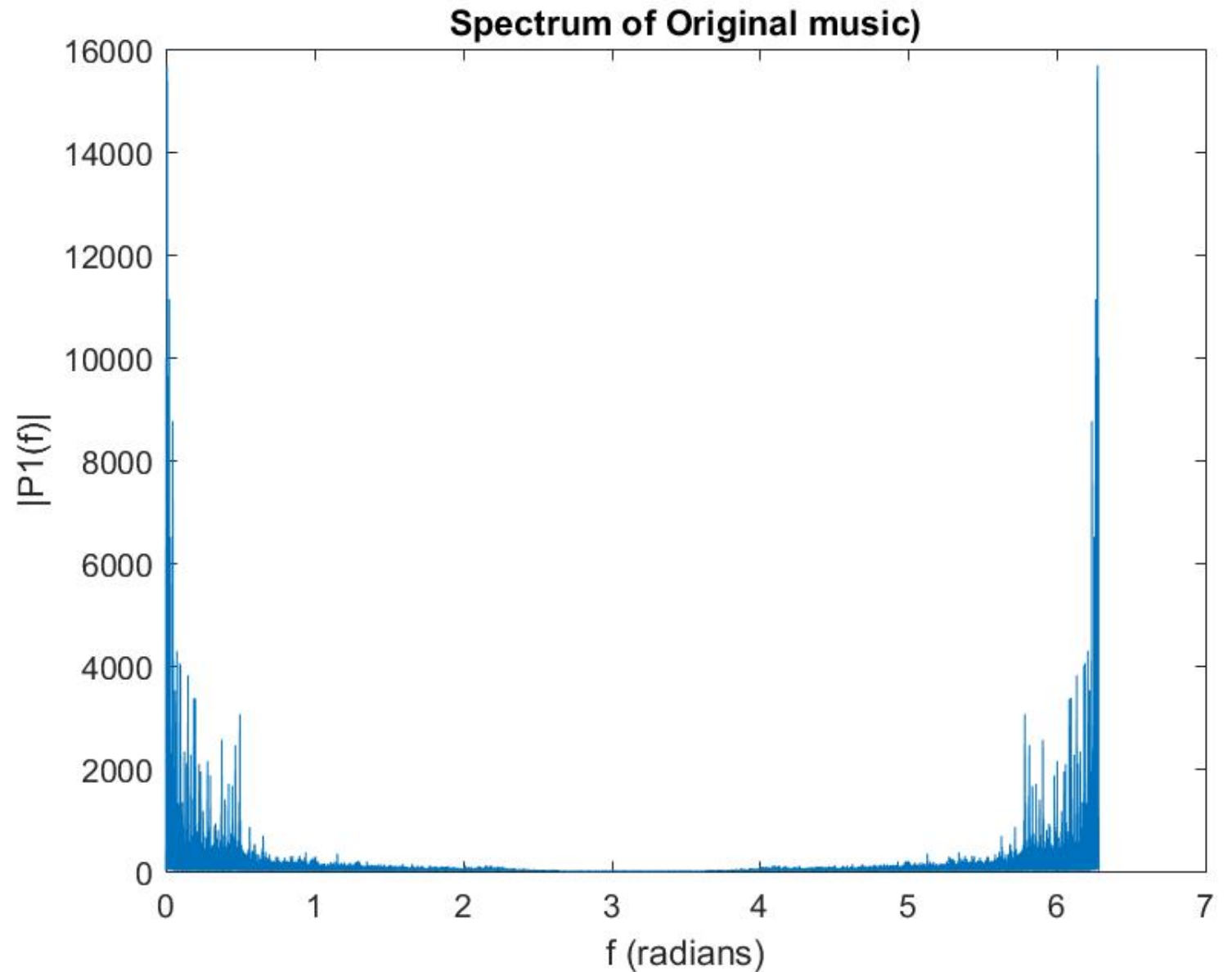
$$X_1(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x_M[n] e^{-j\Omega n}$$

Let $n/M = v$

$$X_1(e^{j\Omega}) = \sum_{v=-\infty}^{\infty} x[v] e^{-j\Omega M v} = X(e^{j\Omega M})$$

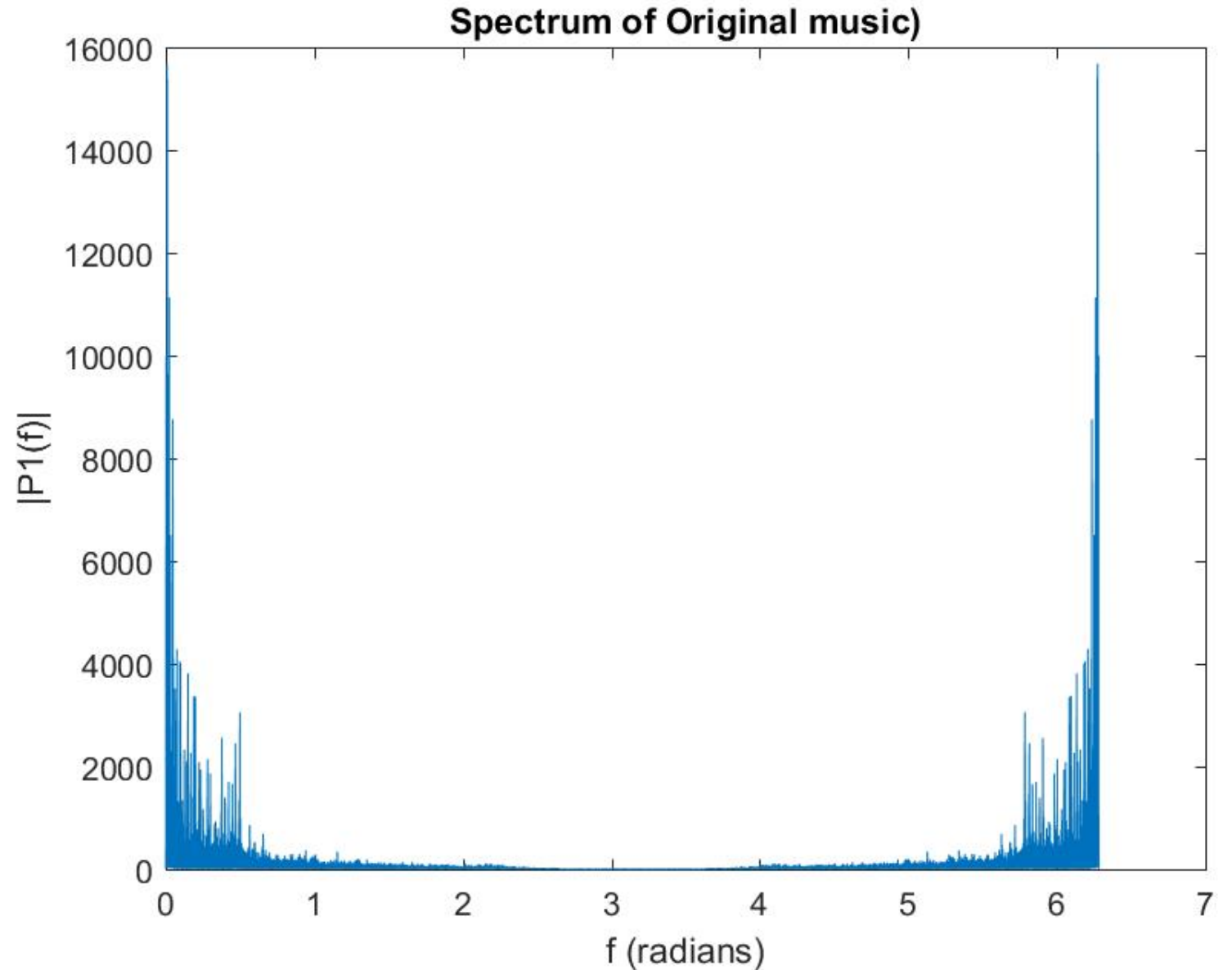
Decimation (music)

Original music file



Decimation (music)

Original music file

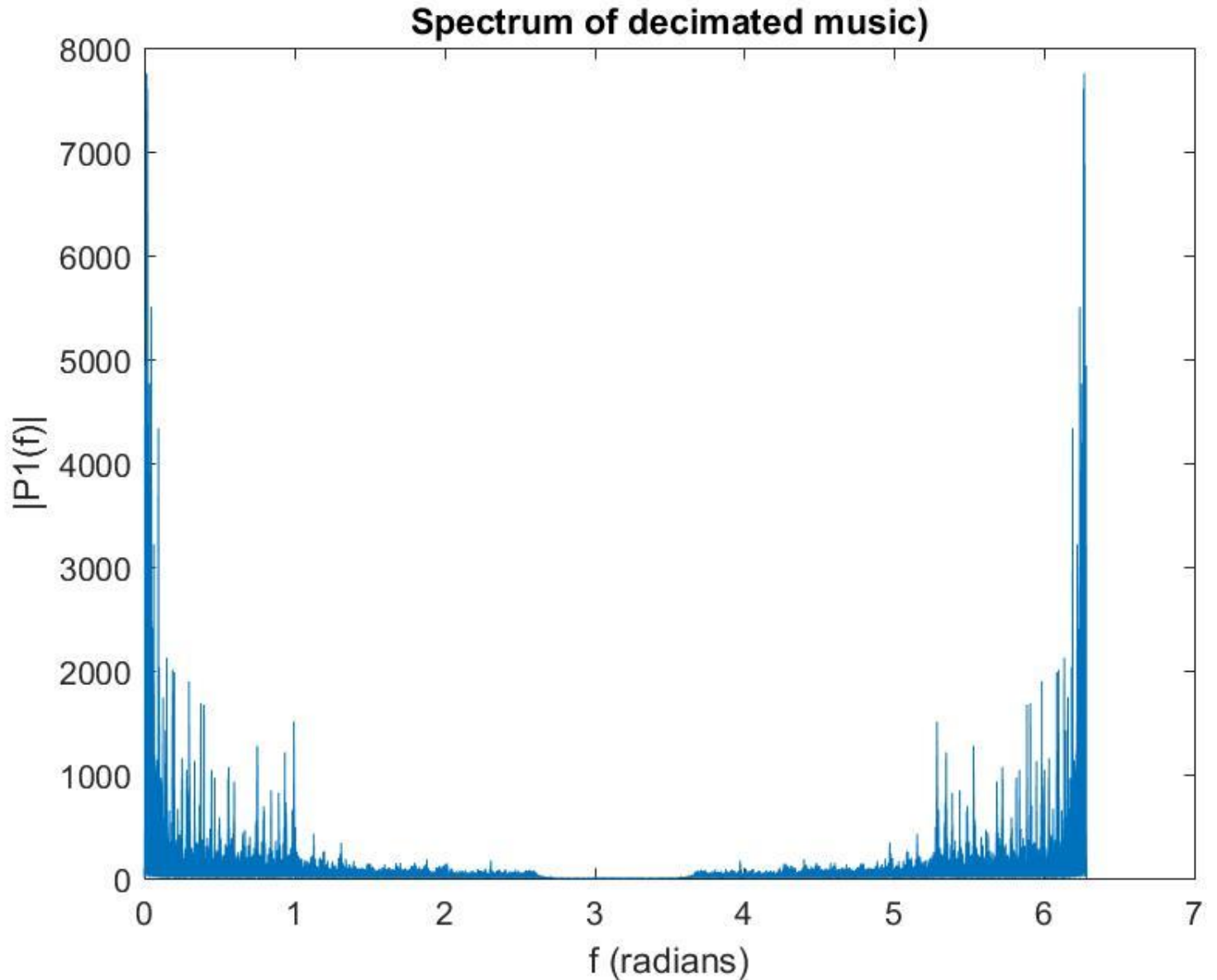


Decimation (/2: music)

Downsampled

Upsampled

Upsampled and filtered

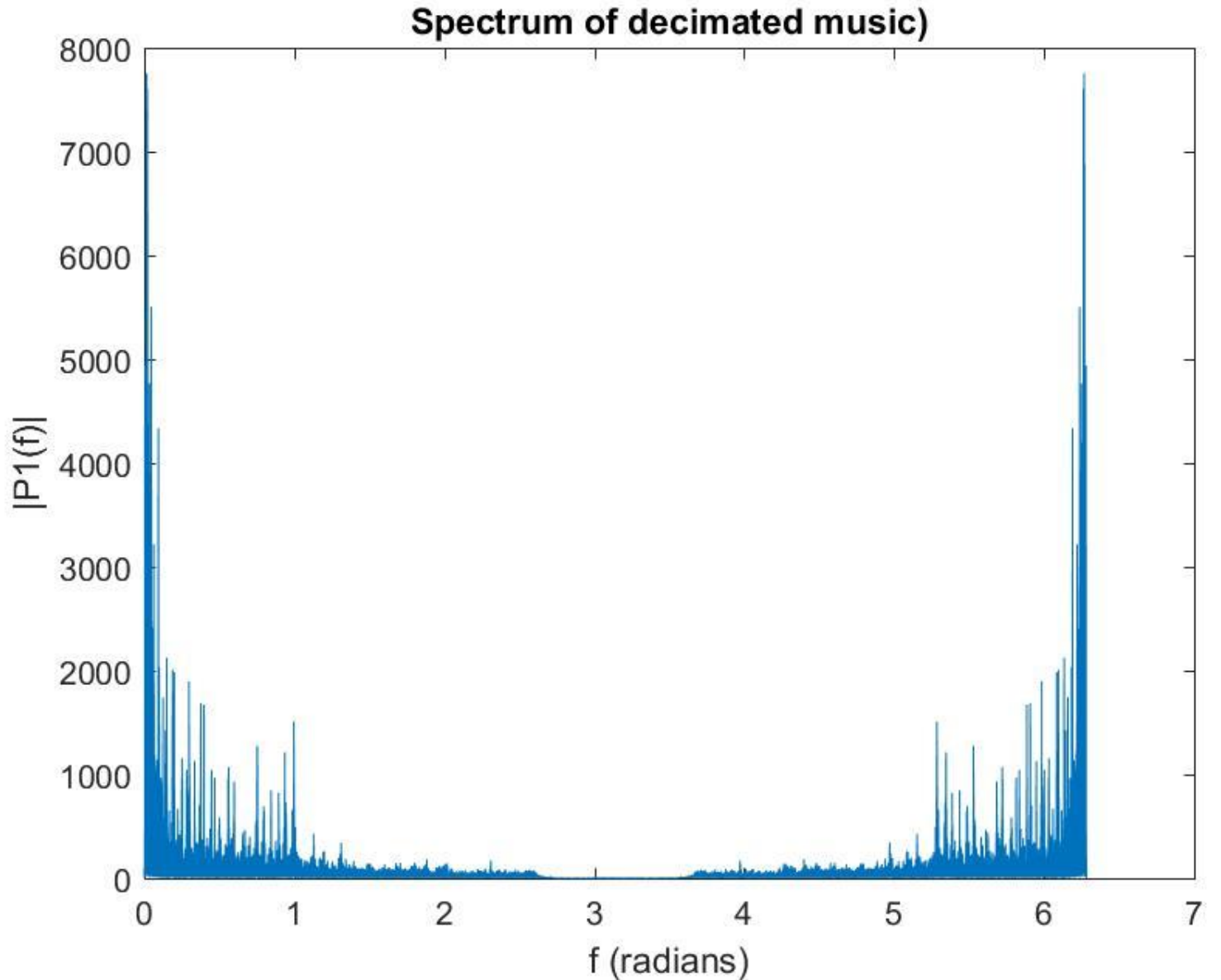


Decimation (/2: music)

Downsampled

Upsampled

Upsampled and filtered

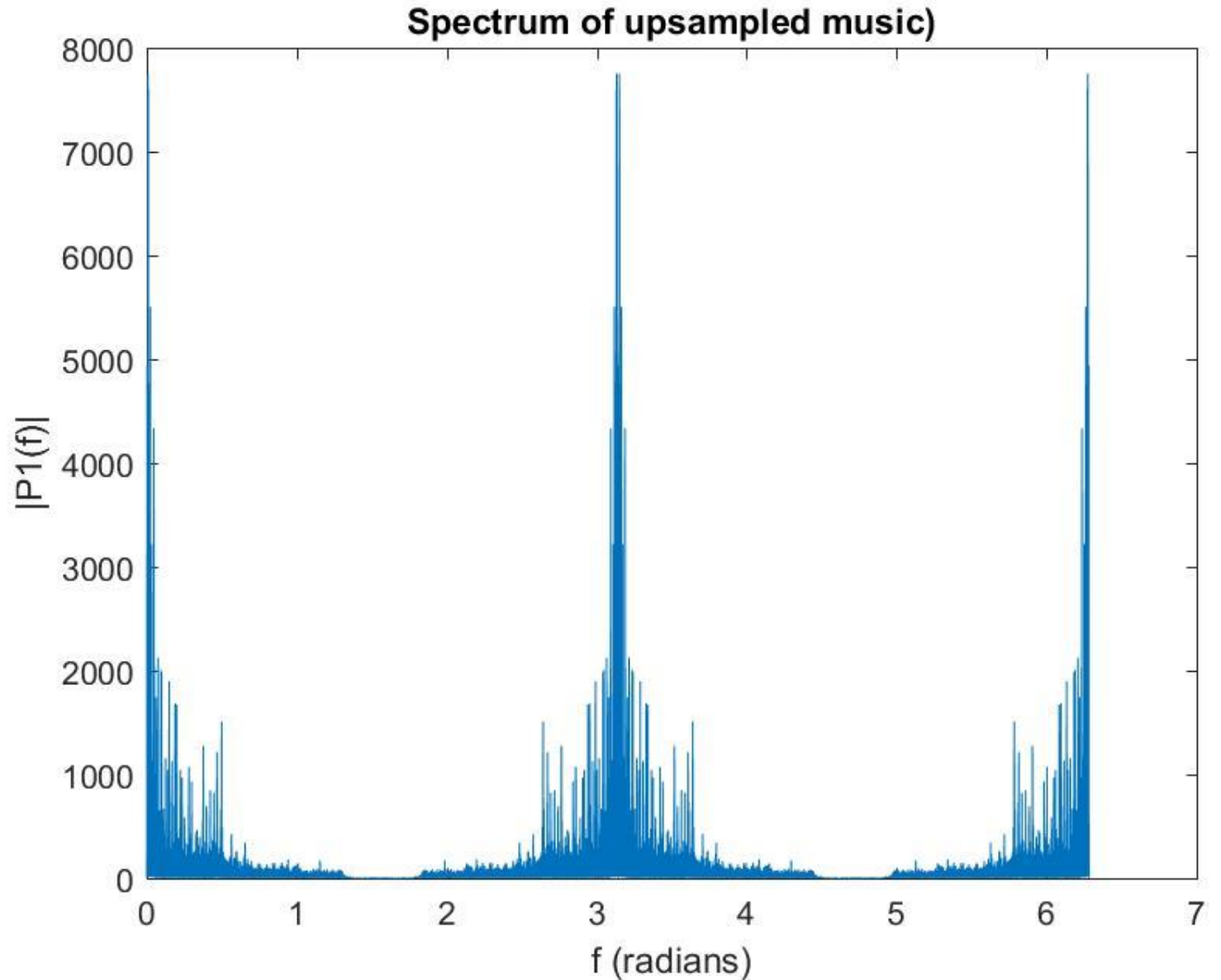


Decimation (/2: music)

Downsampled

Upsampled

Upsampled and filtered

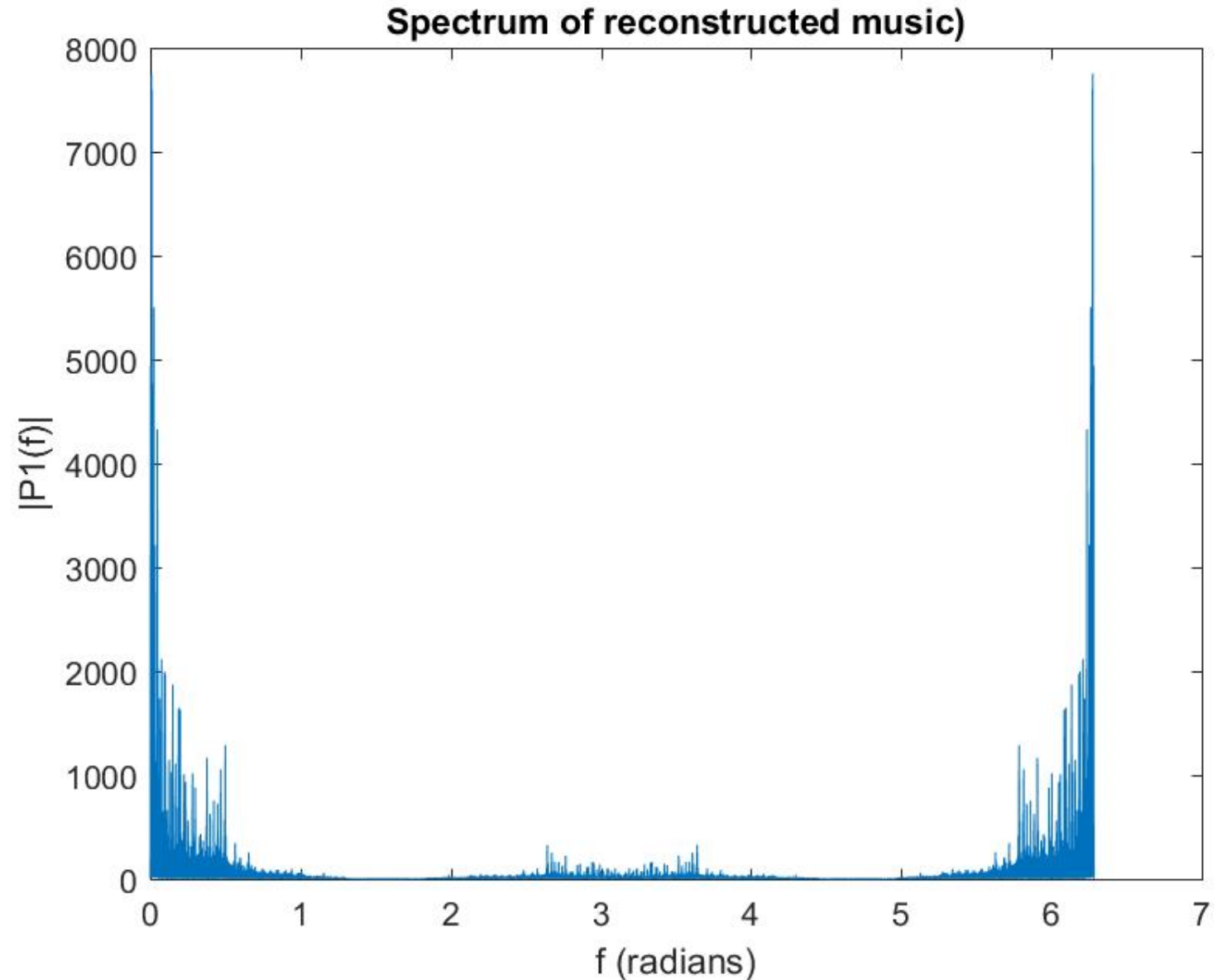


Decimation (/2: music)

Downsampled

Upsampled

Upsampled and filtered

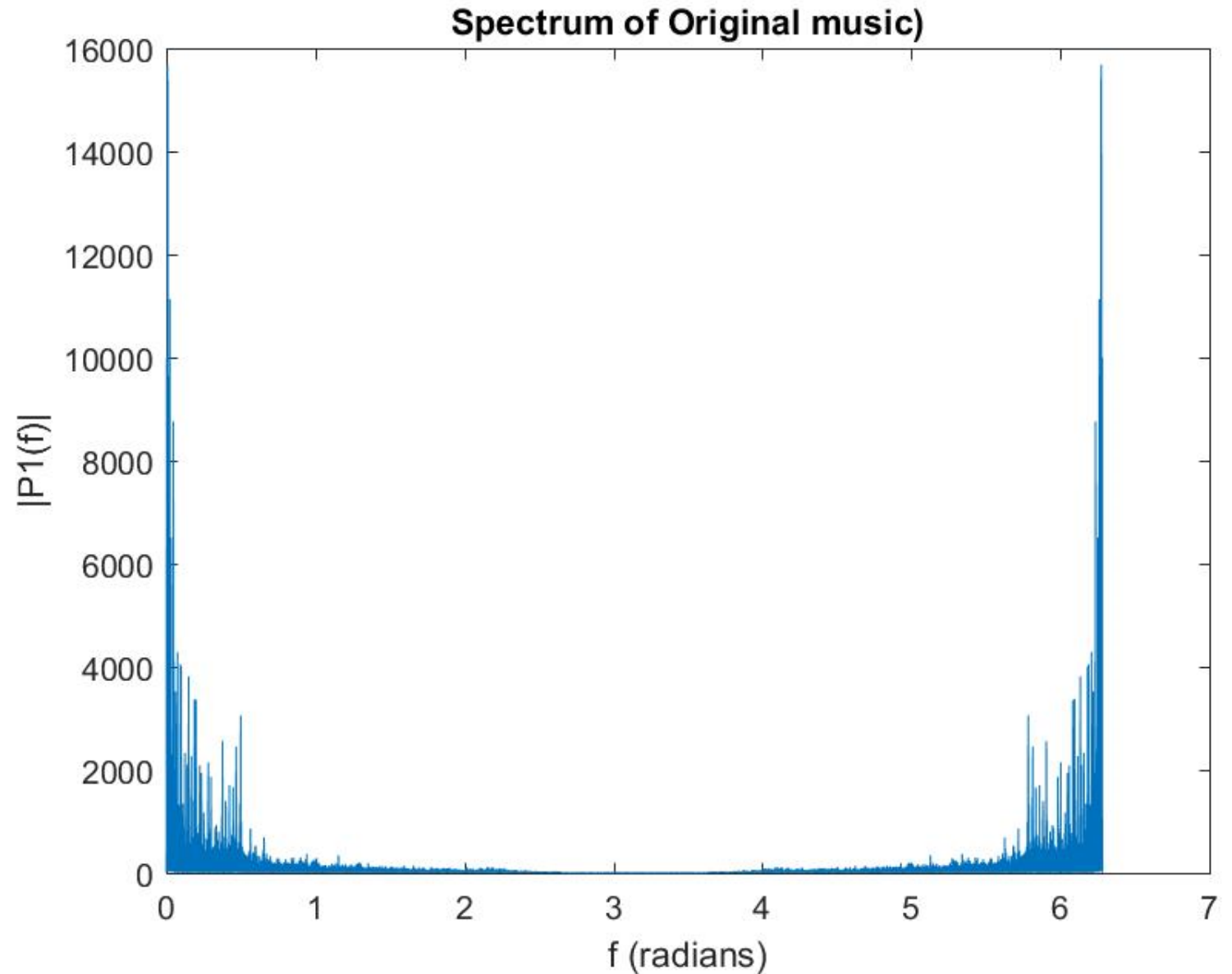


Decimation (/4: music)

Downsampled

Upsampled

Upsampled and filtered

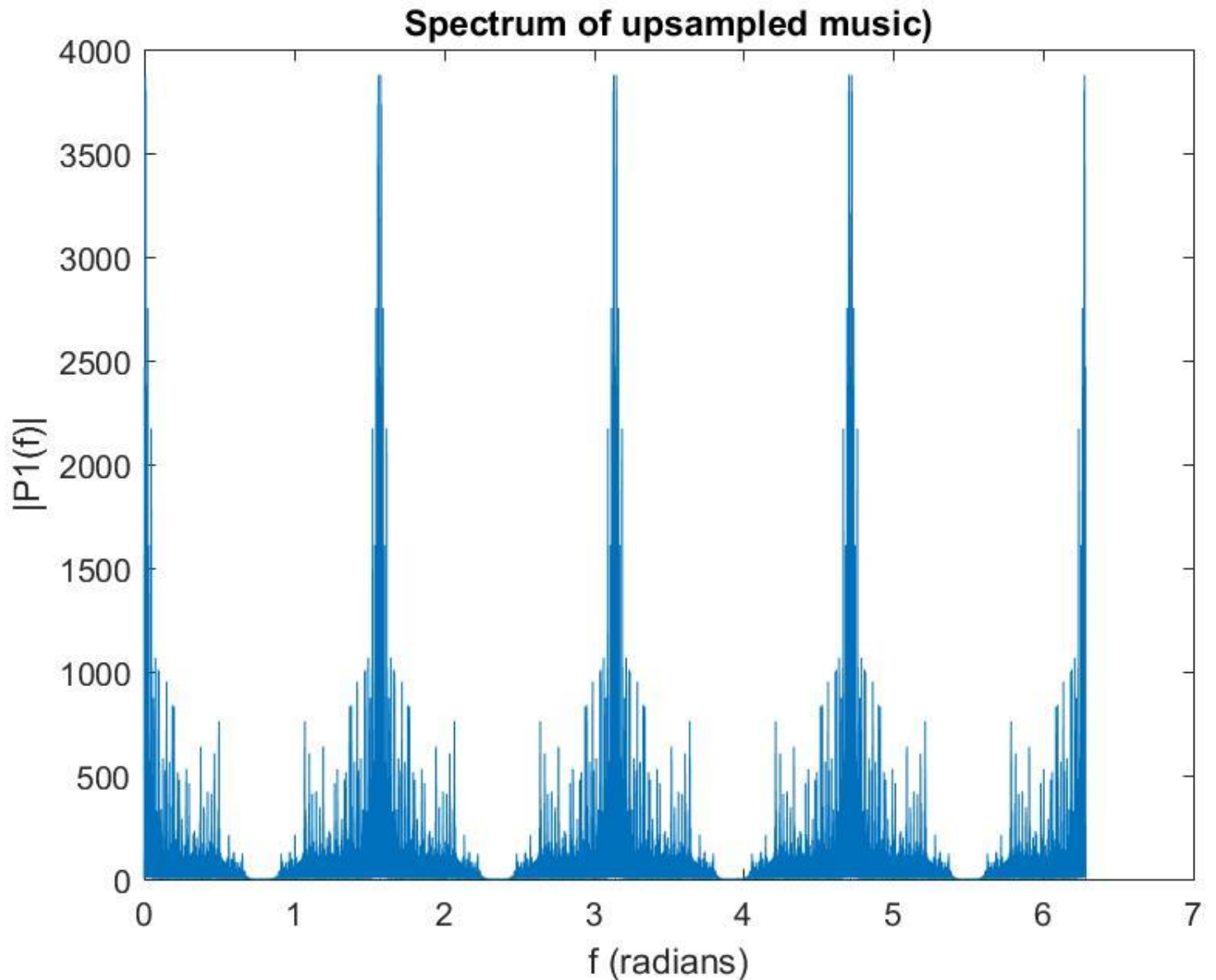


Decimation (/4: music)

Downsampled

Upsampled

Upsampled and filtered

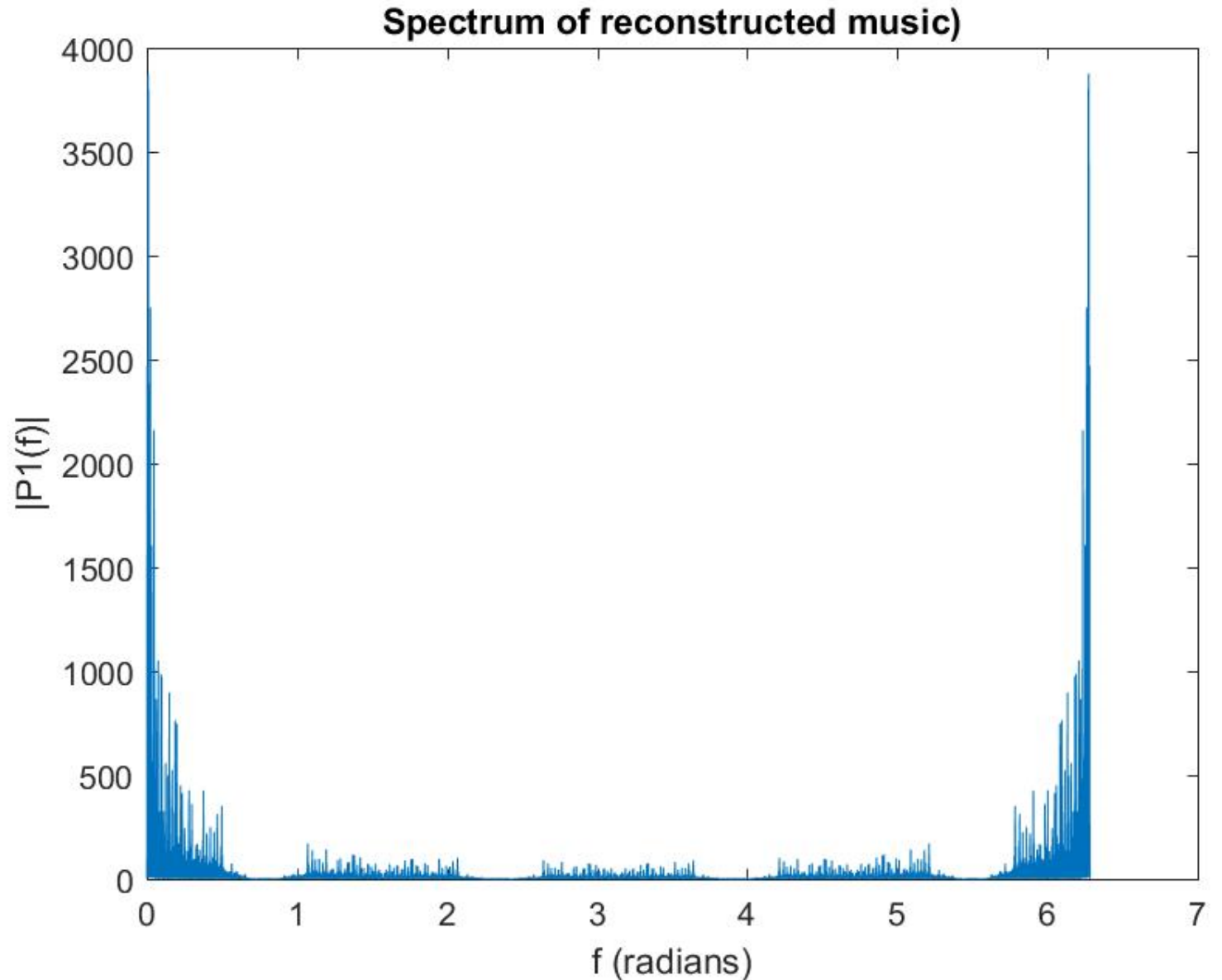


Decimation (/4: music)

Downsampled

Upsampled

Upsampled and filtered



audioinfo(filename)

Filename: 'C:\Users\Abhishek\Desktop\original.wav'

CompressionMethod: 'Uncompressed'

NumChannels: 2

SampleRate: 44100

TotalSamples: 1764000

Duration: 40

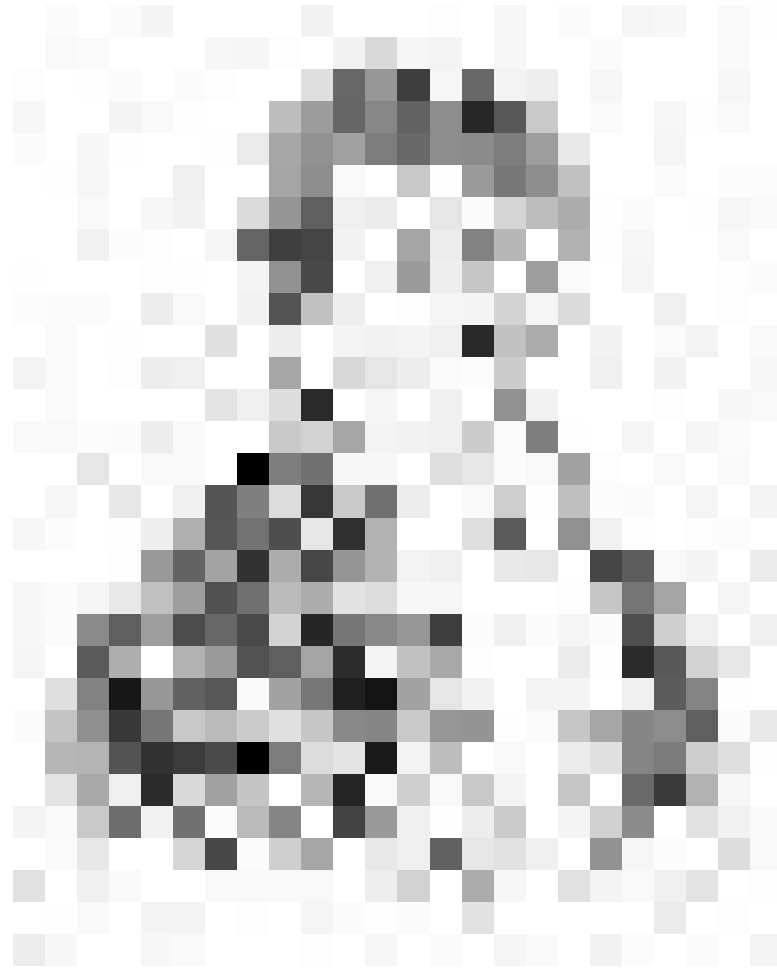
Title: []

Comment: []

Artist: []

BitsPerSample: 16

Decimation (pictures): Idea of progressive refinement



M= 16

Decimation (pictures): Idea of progressive refinement



$M=8$

Decimation (pictures): Idea of progressive refinement



$M=4$

Decimation (pictures): Idea of progressive refinement



$M=2$

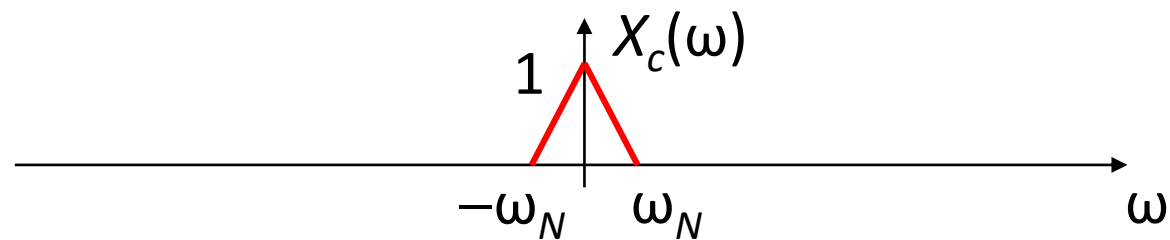
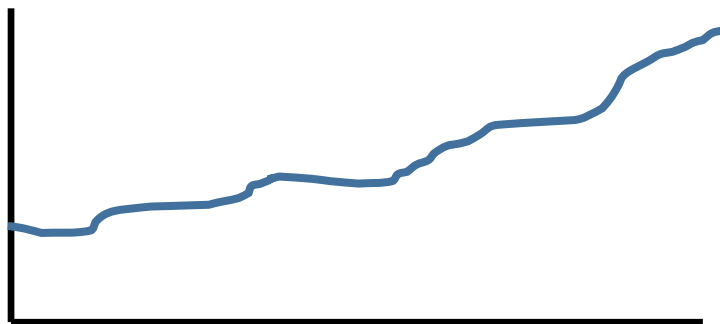
Decimation (pictures): Idea of progressive refinement



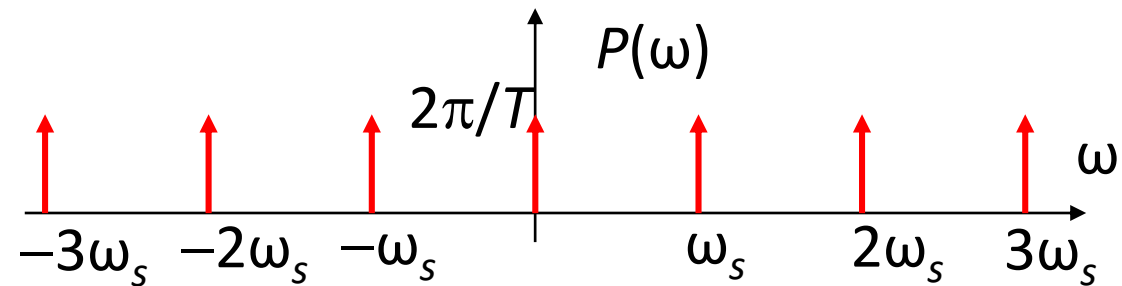
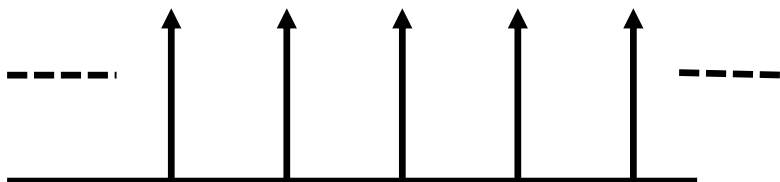
$M=1$

Interpolation

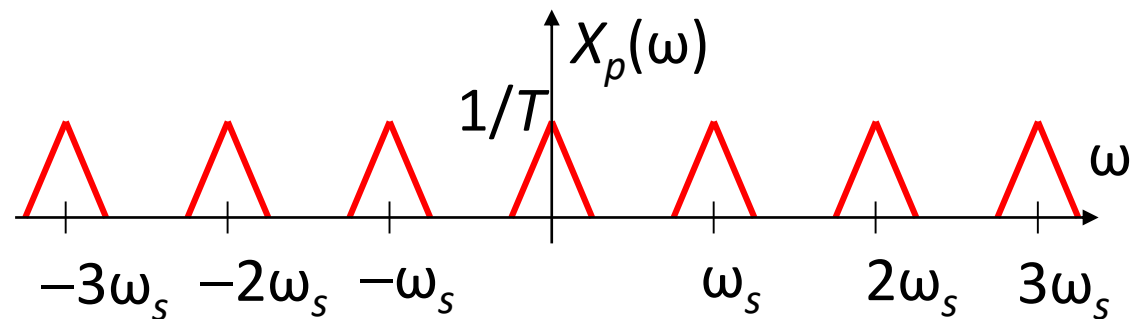
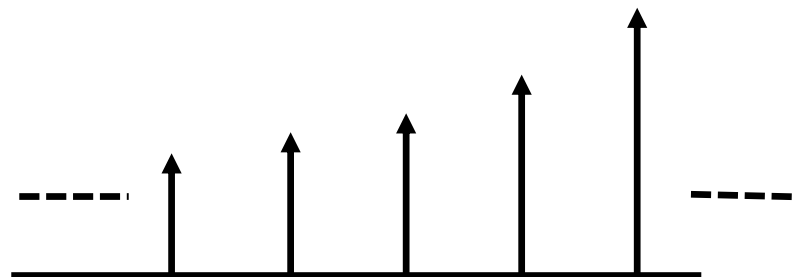
$x_c(t)$



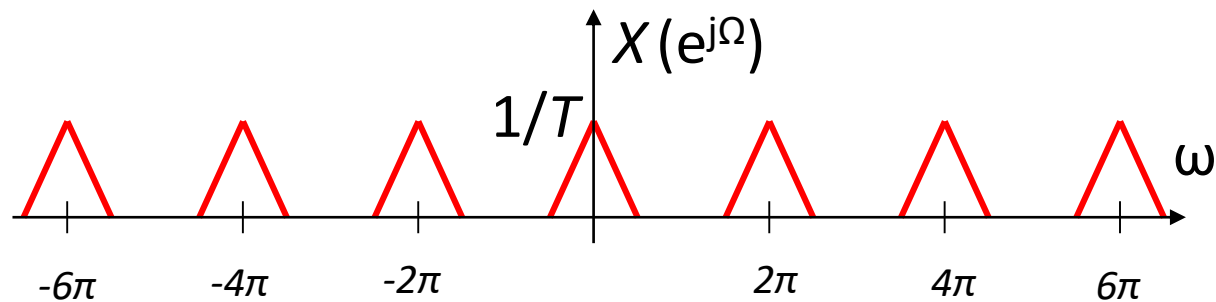
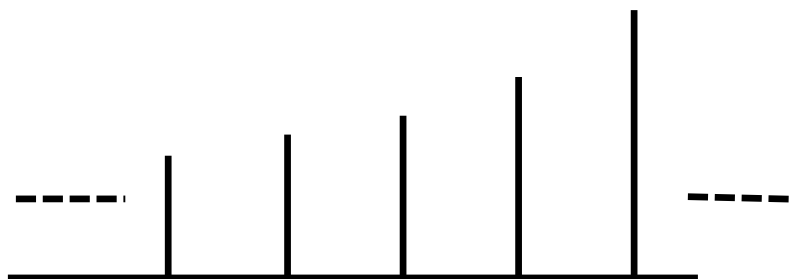
$p(t)$



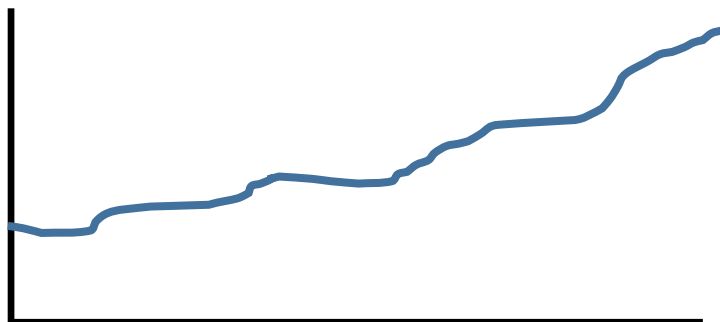
$x_p(t)$



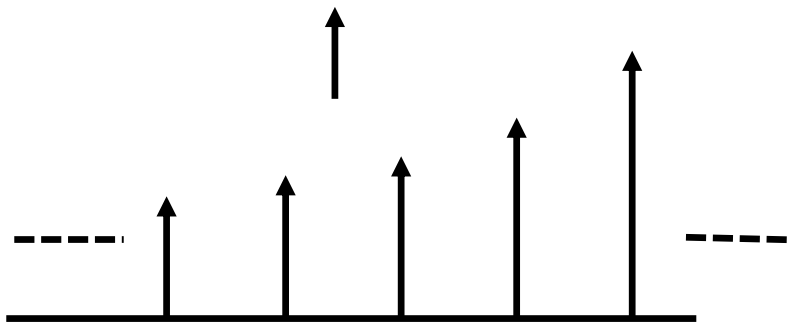
$x[n]$



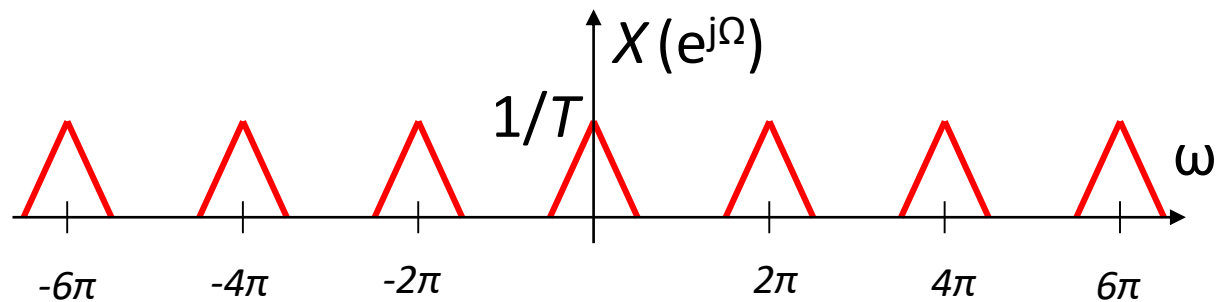
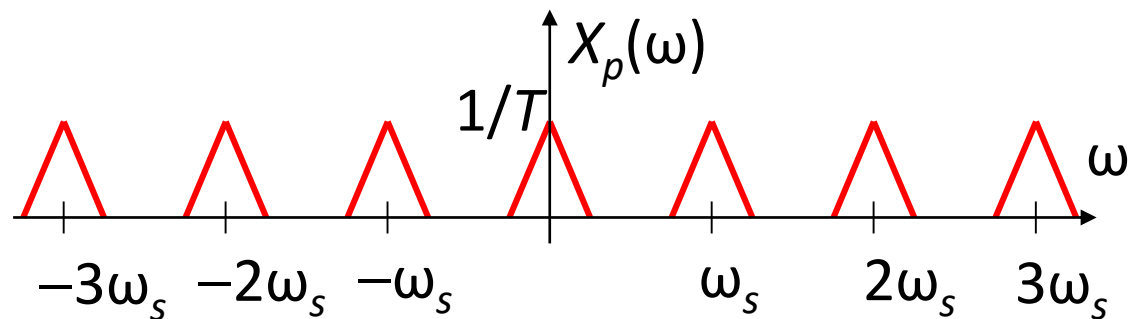
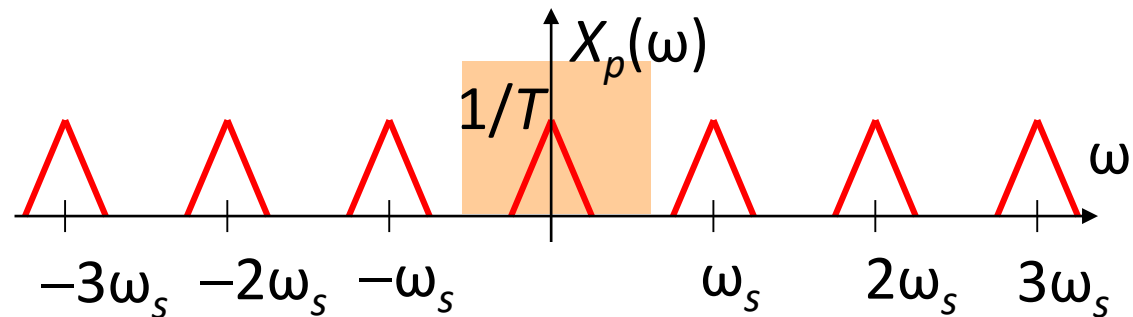
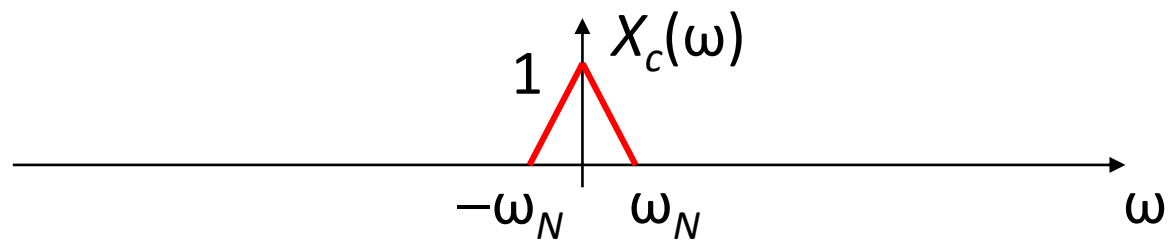
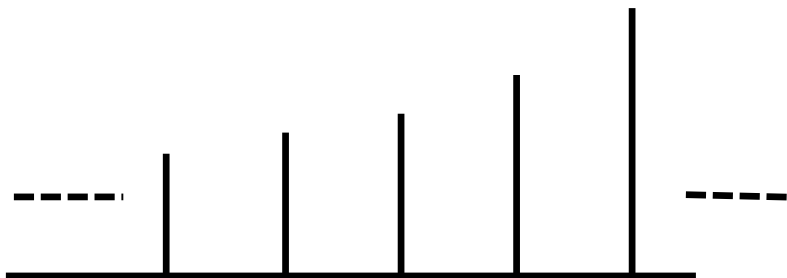
$x_c(t)$



$x_p(t)$

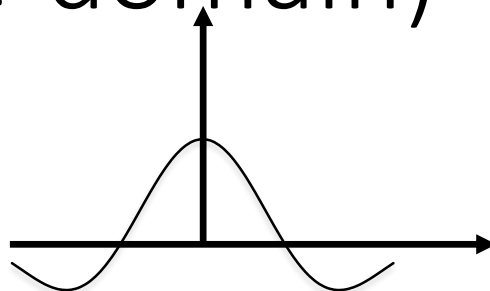
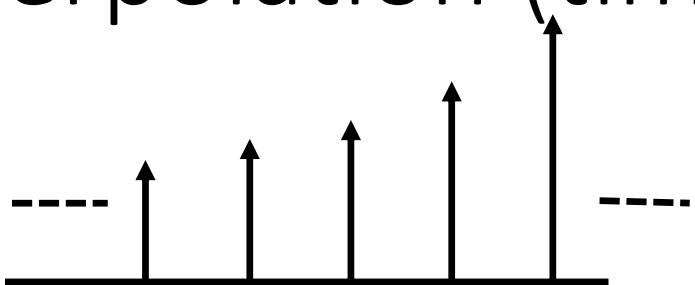


$x[n]$

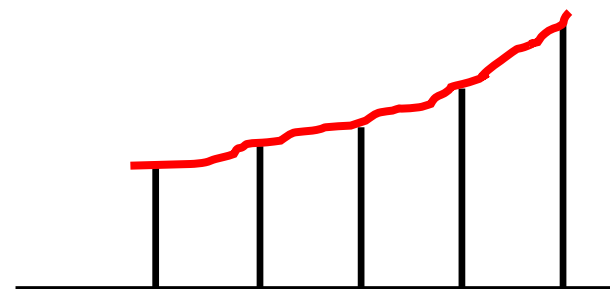


Interpolation (time-domain)

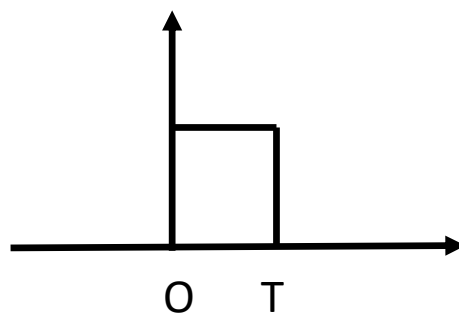
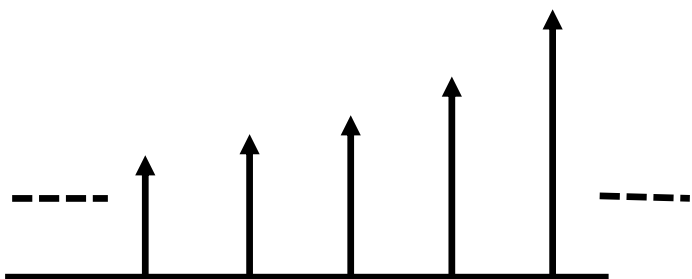
$x_p(t)$



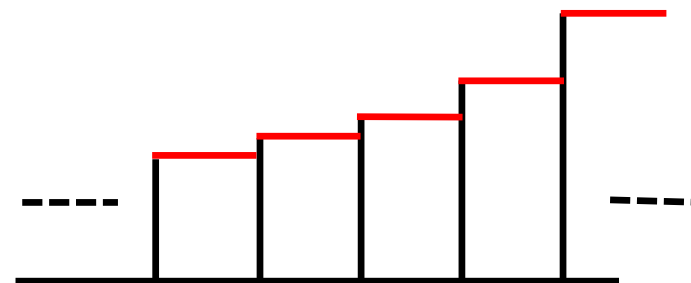
$x_c(t)$



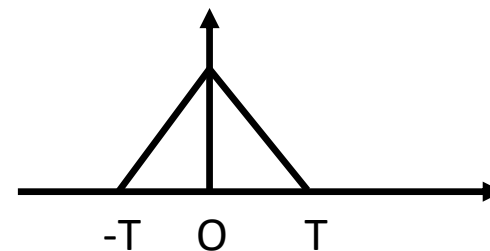
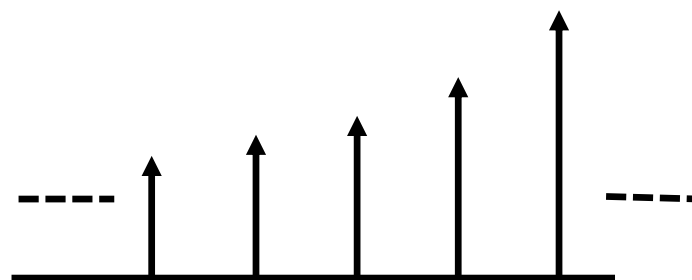
$x_p(t)$



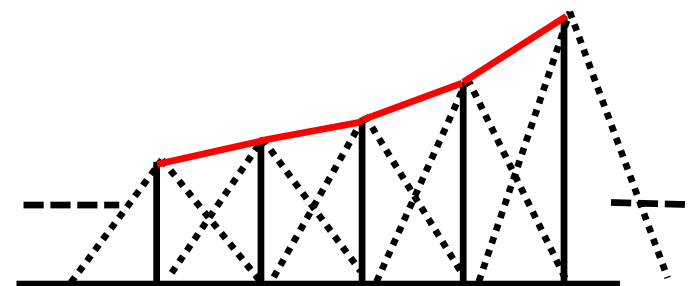
$x_c(t)$



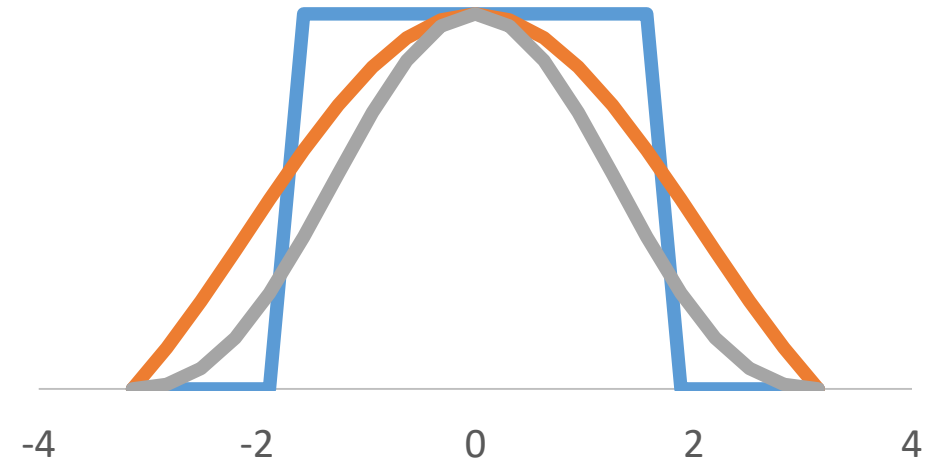
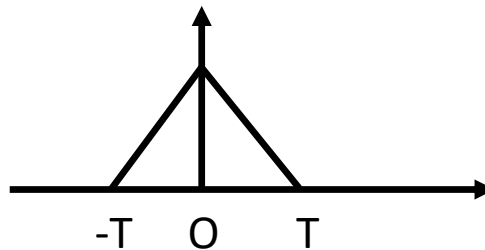
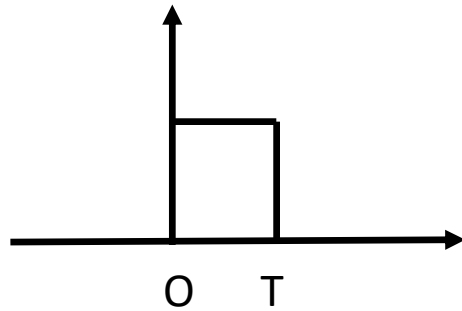
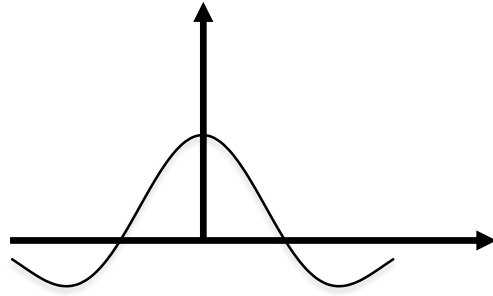
$x_p(t)$



$x_p(t)$



Time domain vs Frequency domain





Sampling period = 2 times



Sampling period = 4 times



Sampling period = 8 times



Effect of Interpolation



Effect of Interpolation



Anti-aliased



Linear



ZOH

Effect of Interpolation



Effect of Interpolation



Effect of Interpolation



ZOH



Linear



Anti-aliased



Linear



ZOH



Anti-aliased



Linear

Music

- Music (linear interpolation)
- Music (ZOH)

Music

- Music (linear interpolation)



- Music (ZOH)

Music

- Music (linear interpolation)

- Music (ZOH)

