## MTL103: Tut Sheet 3

1. Construct the simplex table for the basic feasible solution (3, 0) for the LPP

$$\begin{aligned} \text{Max} \quad & x_1 + x_2 \\ \text{subject to} \quad & -x_1 + x_2 \le 2 \\ & 2x_1 + 3x_2 \ge 6 \\ & x_1 + 2x_2 \le 8 \\ & 2x_1 - x_2 \le 6 \\ & x_1, x_2 \ge 0 \end{aligned}$$

2. Solve:

(i) Min $z = 2x_1 + x_2$

Subject to $3x_1 + x_2 = 2$

$4x_1 + 3x_2 \ge 6$

$x_1 + 2x_2 \le 3$

$x_1, x_2 \ge 0$

(ii) Max $z = 3x_1 + 2x_2 + 5x_3$

Subject to $x_1 + 2x_2 + x_3 \le 430$

$3x_1 + 2x_3 \ge 460$

$x_1 + 4x_2 \le 420$

x1, x2, x3 ≥ 0

3. Use Simplex method to verify that the following problem has an unbounded solution

$$\begin{aligned} \text{Max } z = {} & x_1 + 2x_2 \\ \text{Subject to} \quad & -2x_1 + x_2 + x_3 \le 2 \\ & -x_1 + x_2 + x_3 \le 1 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

From final tableau, construct a feasible solution for giving objective value 2000.

4. Solve the problem. Is the solution unique? If not, find the other three solutions.

$$\begin{aligned} \text{Max} \quad & 6x_1 + 4x_2 \\ \text{Subject to} \quad & 2x_1 + 3x_2 \le 30 \\ & 3x_1 + 2x_2 \le 24 \\ & x_1 + x_2 \ge 3 \\ & x_1, x_2 \ge 0 \end{aligned}$$

5. Use simplex method to minimize $-3x_1 + x_2$ over the triangle with vertices (-1, 0), (2, 0) and (0, 1). Verify your answer graphically.

6. The following is the current simplex tableau of a LPP in the maximization form:

| $X_b$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | |
|---|---|---|---|---|---|---|---|---|
| c | 1 | 0 | 0 | 0 | a | 3 | 2 | |
| f | 0 | 1 | 0 | 0 | 2 | d | -1 | |
| 1 | 0 | 0 | 1 | 0 | -1 | 0 | g | |
| 3 | 0 | 0 | 0 | 1 | 4 | -1 | 6 | |
| z = 5 | 0 | 0 | 0 | 0 | 3 | -4 | 1 | : $z_j - c_j$ |

Determine the conditions on a a, c, d, f, g so that the current tableau and the updated tableau represent respectively

(i) Non-degenerate and degenerate bfs's.

(ii) Non-degenerate and non-degenerate bfs's.

(iii) Degenerate and non-degenerate bfs's.

(iv) Degenerate and degenerate bfs's.

7. Use simplex method to check the consistency of the following system

$$-6x_1 + x_2 + x_3 \le 5$$
$$-2x_1 + 2x_2 + 3x_3 \ge 3$$
$$2x_1 - 4x_3 = 1$$
$$x_1,\ x_2,\ x_3 \ge 0$$

8. Use simplex to determine the solution of the system of linear equations:

$$x_1 - x_3 + 4x_4 = 3$$
$$2x_1 - x_2 \quad = 3$$
$$3x_1 - 2x_2 - x_4 = 1$$
$$x_1, x_2, x_3, x_4 \ge 0$$

9. Solve the following systems of equations by two phase technique

$$5x_1 + x_2 - 3x_3 = 1 \qquad\qquad 5x_1 + 3x_2 + x_3 = 2$$
$$4x_1 + x_2 + 6x_3 = 9 \qquad\qquad 4x_1 - x_2 + 6x_3 = 0$$
$$-3x_1 + x_2 + 7x_3 = 3 \qquad\qquad 9x_1 + 7x_2 + 4x_3 = 3$$
$$x_1,\ x_3 \ge 0$$

10. Use Simplex method to find the inverse of the following matrices:

(i) $\begin{bmatrix} 5 & 1 & 7 \\ 3 & 4 & 8 \\ 2 & 6 & 8 \end{bmatrix}$ (ii) $\begin{bmatrix} 4 & 1 & 2 \\ 0 & 1 & 0 \\ 8 & 4 & 5 \end{bmatrix}$

11. Consider the matrix B= ($b_1$, $b_2$, $b_3$), whose inverse is

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix}.$$

Find inverse of matrix B* = ($b_1$, $b_2$, a), where a = $(1, 1, 1)^t$.

12. Solve the following LP by simplex. Identify B and $B^{-1}$ at each iteration.

$$\text{Max} \quad 3x_1 + 2x_2 + x_3$$
$$\text{Subject to} \quad 2x_1 - 3x_2 + 2x_3 \le 3$$
$$-x_1 + x_2 + x_3 \le 5$$
$$x_1,\ x_2,\ x_3 \ge 0$$

13. Show that in a given Simplex table if for certain non basic variable $x_j$, $z_j - c_j = 0$ and for that some $y_{ij} > 0$ then given LPP has infinitely many optimal solutions.