PLAGIARISM COMPARISON SCAN REPORT

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COL 202 Homework 2Harshit Mawandia, Tanish Gupta, Ashish ChoudharyProblem 1.Proof by Contradiction.Assumption Initially lets assume that there exists no edgee2E2nE1such thatF (VE1[feg)is also an acyclicgraph.Proof It is a property of acyclic graphF (VE)thatjVj 2jEj. (following basic convention that V are thevertices and E are the edges connecting them)ClaimsForF (VE1[feg)not being acyclicClaim 1@e2E2s.t. it joins 2 vertices in V in which 1 vertex was not already connected withE1to a vertex alreadyconnected withE1.Claim 2@e2E2s.t. it joins 2 vertices in V which were already not connected withE1.ConclusionWith these 2 claims we can say thatfvertices in E1g fvertices in E2g. So by using the formulajVj 2jEj, we get2jE1j 2jE2jWhich implies thatjE1j jE2j, that contradictsjE1jltjE2j. This contradiction occurs because our assumption waswrong.Hence, proved.Problem 2.Proof by Induction on the number of vertices.Base Case When the number of vertices is 2, sayv1v2. There can be only 1 edge. So, moving fromv1tov2andback forms a closed graph with all edges traversed exactly twice.Induction Hypothesis Let the claim be true forjVjn1.Induction step Now lets add another vertex say,vnto the graph, and lets say it is connected to certain othervertices. Since by the induction hypothesis our claim is true for the graph withn1vertices, we just have to showthat even if we add another vertex, we can still form a closed path traversing all edges twice.Lets say thatvnis attached tokother vertices, wherek lt n, let those vertices befv1v2vkg. Now, there alreadyexists a closed path which traverses then1vertices, and a closed path can begin and end at any vertex visited in it.So lets say it ends onv1. Now, fromv1we move tovnrst, then from there we move to and fro to all other vertices inthe setfv2vkgback tovnand then nally back tov1.We can see that this again forms a closed path which traverses all the new edges twice too, and the older edges werealready traversed by our Induction Hypothesis.Therefore, our claim is true by P.M.I.8 jVj 2Hence, ProvedNow for the second claim that every connected graphG (VE)has a closed walk of length2jVj2which visitsevery vertex inVat least once.We will prove this by induction.Base Case When the number of vertices is 2, sayv1v2. There can be only 1 edge. So, moving fromv1tov2andback forms a closed graph which visits every vertex of length2which is2jVj2 sincejVj 2.Induction Hypothesis Let the claim be true forjVjn1.Induction Step Now lets add another vertex say,vnto the graph, and lets say it is connected to certain othervertices. Since by the induction hypothesis our claim is true for the graph withn1vertices, we just have to showthat even if we add another vertex, we can still form a closed path of length2jVj2, which visits every vertex.Lets say thatvnis connected toviwherei lt n. Now we know thatviis already a part of a closed path that satisesour claim by Induction Hypothesis. We also know that we can assume the closed path to end atvi. Now we traversefromvitovnand then back toviafter traversing the older closed path containingn1vertices.We can see that this again forms a closed path which satises our claim since earlier the length of path was2(n1)2, and we add 2 more steps to it, so the length becomes,2(n)2, which satises the claim thatlength 2jVj2.Therefore, our claim is true by P.M.I.8 jVj 2Hence, ProvedProblem 3.Lemma A regular bipartite graph always has a perfect matching.Let us take 2 sets sayX andYbe the parts of the bipartite graph.Now lets take a subsetSof the setX, and the set of the pointsP(S)to which the points inSare connected.We know thatjSj jP(S)jbecause every matching with a vertex inShas a vertex inP(S)but not vice-versa.Thus we can say that ifdis the degree of each vertex, then cardinality of the edge set ofS,djSjwould denitely beless than or equal to the cardinality of the edge set ofP(S).Now, usingHall0sTheorem, we can say that there exists a matching, say,M, which matches every vertex inX.Because the graph isregularjXjjYj. Therefore, the matchingMwould beperfect.Now let's prove that the edge set of every bipartite regular graph can be partitioned intoperfectmatchings byinduction on the degrees of the vertices,d.Base Case Whend 0, Edge set has no elements, so there are 0 perfect matchings. Whend 1, the edge set isperfectly matched because all vertices will be the end point of exactly one edge.Induction Hypothesis Assume that our claim holds for a graph wheredn1.Induction Step Let's consider a graphG (VE), with degreedn. SinceGis bipartite and regular, it will haveaperfectmatching, sayM.Consider another graphG0 (VEnM)G0is also a regular bipartite graph, but with degreed lt n(As every vertex will have the same number of edges(1)inM).AsG0satieses our induction hypothesis,EnMcan be partitioned into perfect matchings. ThereforeGtoo can bepartitioned into perfect matchings, sinceEis has all elements perfectly matched (fEnM[Mgall elements in this setis perfectly matched)So by P.M.I, our claim is true8d0.Hence ProvedProblem 4.Claim The number of perfect matchings in a complete graph on k vertices is given by N(k) 8ltk(k2)2k2if k is even0otherwise (i.e. if k is odd)A)Proof We will prove the claim by construction.Consider a graph having n vertices. Clearly, if n is odd, then number of perfect matchings is zero. (This isstraightforward from the denition of perfect matchings, since every vertex needs to be matched to exactly one vertex,the total number of vertices has to be even).So, consider the case when n is even. Let n 2m, for some natural number m.Now, our goal is to divide these n vertices into m pairs, such that every vertex belongs to exactly one pair. Call thismatching M.Claim M is a perfect matching.Proof The proof is easy to see. Since

every vertex of the graph[1]

is included in a pair, and every vertex is includedexactly once, by the denition of perfect matching, M is a perfect matching.So, the number of ways to divide 2m objects into m groups such that each group has exactly 2 elements is given byBm)m2mwhich proves the claim.Problem 5.Claim The number of

ways in which[2]

n passengers can occupy m seats following social distancing norms is given by N(nm)(n2m2)(n3m2)Proof Seat m passengers on random m seats out of the given n seats.Letx1x2x3xm1be the number of vacant seats between passenger 1 and 2, passenger 2 and 3, passenger 3 and 4, ... , passenger m -1 and m respectively.According to the conditions given in the question, we needxi28i2[1m1.Also, lety1y2be the number of vacant seats to the left of passenger 1 and to the right of passenger m respectively.Clearly,y10andy20.Now, since m seats are occupied by m passengers, the number of vacant seats is n - m.So,x1x2xm1y1y2nmwherexi2yi0Substitutezixi2, or in other words,xizi2. So, the equation becomes,z1z2zm12(m1)y1y2nmz1z2z3zm1y1y2n3m2wherezi0yi0The problem has now reduced to a similar problem as discussed in class. We have to arrange n - 3m 2 identicalballs in m bins. The number of solutions for this, using formula discussed in class is n3m2m11m1n2m2m1

HomeWork2 SolutionsTushar Gurjal Arin Kedia Rishabh VermaProblem 1LetF1 (VE1)andF2 (VE2)be any two acyclic graphs (a.k.a. forests) on the same vertex set Vsuch thatjE1jjE2j xssremoved xssremoved xssremoved xssremoved xssremoved xssremoved jE1jjVE1j xssremoved xssremoved xssremoved pathpv1v2vnasp1 walk,wv1v2vn1p1o. xssremoved xssremoved So,jEjjVj1. xssremoved regular,jV1jjV2j. xssremoved xssremoved fBk) (xssremoved1.Ifk 2N f0gthen that means the number of vertices will either be fractional which is not possible or it willodd, in which perfect matching does not exists.So, we are only left to prove whenk2Nf0g.Now we will use induction on k to proove our claim.Base CaseWhenk 1, we only have one perfect matching which is the edge connecting both vertices.Induction HypothesisWe assume our claim holds true for somekn1.Induction StepWe consider a graph of 2nvertices.We take any vertex, sayv1, from the graph.It will only be counted as 1 as

every vertex of the graph[1]

should be matched in a perfect matching.Then we have 2n1 edges which matchv1to distinct vertices.We arbitrarily choose an edge.Now, we are left with 2n2 unmatched vertices.By our induction hypothesis, we know total possible perfect matching for a graph of 2n2 vertices will befBn2).Also since we arbitraly chose an edge from the set of 2n1 edges and for each edge we havefBn2) possible perfectmatchings, wecansaythetotalnumberofperfectmatchingsofouroriginalgraphwillbefBn) Bn1)fBn2)(By the multiplicative rule of combinatorics).So, by theprinciple of mathematical induction, we have our claim true,8k2Nf0g.Hence Proved.4Problem 5Consider a Delhi Metro train consisting ofnseats numbered1ncarryingm(distinct) passengers.The government rules for physical distancing prevent passengers from standing in the compartmentduring their journey. Moreover, they must leave a gap of at least two seats between themselvesif a seatkis occupied, seatsk2,k1,k 1,k 2must remain vacant. Find an expression forthe number of

ways in which[2]

the passengers can occupy seats while following the physical distanc-ing norms, and prove your answer. Again, your expression must involve only a constant numberof applications of only the following mathematical operators addition, subtraction, multiplication,division, exponentiation, and factorial.SolutionWe claim,f(nm) (n2m2)(n3m2)Heref(nm) is the number of ways to arrangempassengers innseats such that the conditions in question aresatised.ProofWe disregard the number on seats such that the seats become identical.We arrange the passengers inmseats disregarding any conditions.That can be done bym ways.Now we put 2 seats between every pair of adjecent passengers.Now we are left withn3m2 seats.We try to put them in the arrangement we have obtained so that we get the total of n seats.We consider the leftover seats as balls and the positions where they have to be put, i.e., before rst passenger, afterlast passenger or between 2 adjecent passengers, as bins.And here we havem1 bins.Due to presence of passengers these bins have become distinct and since we have erased the number on seats, theballs here are identical.From class, we have the number of ways to arrangenidentical balls inmdistinct bins asnm1m1.So, to put the seats in our arrangement, this can be done inn2m2mways.Now we renumber the seats.So, our total answer becomesmn2m2m, which is the same asf(nm).Hence Proved.5