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Q6) $E(k) = E_0 - \alpha - 2\beta \cos ka$

a) when $E(k)$ is max ; $\frac{dE}{dk} = 0$; $\frac{d^2E}{dk^2} < 0$

$$\frac{dE}{dk} = 2\beta a \sin ka \quad , \quad \frac{d^2E}{dk^2} = 2\beta a^2 \cos ka$$

$$ka = (2n+1)\pi \quad ; \quad n = 0, 1, 2, 3 \ldots$$

$$\rightarrow \quad k = \frac{(2n+1)\pi}{a}$$

smallest k for max energy $= \frac{\pi}{a}$

(b) $$\frac{d^2E}{dk^2} = 2\beta a^2 \cos (ka)$$

We know ; $E = \frac{\hbar^2 k^2}{2m^*} \Rightarrow m^* = \hbar^2 \left(\frac{d^2E}{dk^2}\right)^{-1}$

At the bottom of first Brillouin zone.

$k = 0$

$$\frac{d^2E}{dk^2} = 2\beta a^2 \rightarrow m^* = \frac{\hbar^2}{2\beta a^2}$$

At the top of first Brillouin zone

$k = \pi/a$; $\frac{d^2E}{dk^2} = -2\beta a^2 \Rightarrow m^*_{(k=\pi/a)} = \frac{-\hbar^2}{2\beta a^2}$