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1) $|a_{n+2} - a_{n+1}| \leq \frac{1}{8}|a_{n+1}^2 - a_n^2|$

for $n \geq 1$

$$\left|\frac{a_{n+2} - a_{n+1}}{a_{n+1} - a_n}\right| \leq \frac{1}{8}|a_{n+1} + a_n|$$

$$\leq \frac{|a_{n+1}| + |a_n|}{8}$$

$$< \frac{4}{8} = \frac{1}{2}$$

and $-2 < a_n < 2$

Therefore it is a cauchy sequence

hence we can say then $\{a_n\}$ converges.

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2.a) $a_n = \frac{1+(-1)^n}{n^{1/n}}$ $n \geq 1$

$\lim_{n \to \infty}$ at $n \to \infty$

$n^{1/n} \to 1$

for ~~even~~

$a_{2n} \to 2$ for $n \to \infty$

$a_{2n+1} \to 0$ for $n \to \infty$

$\lim_{n \to \infty} \inf a_n = 0$

$\lim_{n \to \infty} \sup a_n = 2$

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3) $a_n = \left(\frac{n}{3} - \left[\frac{n}{3}\right]\right)\left(1 + \frac{1}{n}\right)$

$1 + \frac{1}{n} \leq 2$ for all $n \geq 1$

$0 \leq \left(\frac{n}{3} - \left[\frac{n}{3}\right]\right) < 1$ for all n

$\therefore$ therefore as $n \longrightarrow \infty$

$1 + \frac{1}{n} \longrightarrow 1$ as

$\frac{1}{n} \longrightarrow 0$

and $\left(\frac{n}{3} - \left[\frac{n}{3}\right]\right)$ still lies between $[0, 1)$

therefore $\lim\limits_{n \to \infty} \inf a_n =$

$\lim\inf (0 \times 1) = 0$

and $\lim \sup a_n = 1 \times 1 = 1$

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4) $$\sum_{n\geq 1}^{\infty} \frac{e^n}{e^{2n}+1} = \frac{1}{e^n+e^{-n}}$$

$e^n + e^{-n} > n^2$ for all $n \geq 1$

Since $n > \log n$ for all $n \geq 1$

$\therefore e^n + e^{-n} > n^2$ for all $n \geq 1$

$\therefore \frac{1}{e^n+e^{-n}} < \frac{1}{n^2}$ for all $n \geq 1$

Since $\frac{1}{n^2}$ is convergent therefore $\frac{1}{e^n+e^{-n}}$ is convergent therefore $\frac{e^n}{e^{2n}+1}$ is convergent

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5) $\sum_{n=3}^{\infty} \frac{1}{n\log(\log(n))}$ ~~is convergent if~~

$= \sum_{n=3}^{\infty} a_n$

$\sum_{n=3}^{\infty} a_n$ is convergent if and only if

$\sum 2^n a_{2^n}$ is convergent

$\frac{\cancel{2^n}\ 1}{\cancel{2^n}\log(\log 2^n)} > \cancel{1}$

$\log\log(2^n) < \log(2^n)$ for $n \geq 2$

$\log 2^n < 2^n$ for $n \geq 2$

So $\frac{1}{\log(\log(2^n))} > \frac{1}{\log 2^n} > \frac{1}{2^n}$

for even $n \geq 2$

since $\frac{1}{2^n}$ is not convergent

Therefore $\frac{1}{\log(\log(2^n))}$ is also not

convergent ~~for all~~ by comparison