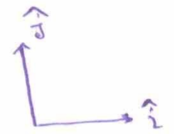
1.

Here our purpose is to know the existence of slip at the contact point or not.

Suppose A and B are the contact points of body 1 and body 2.

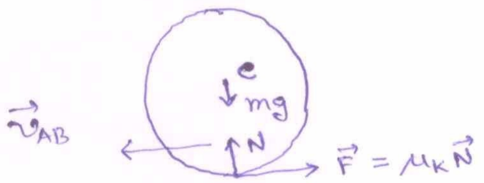


We shall consider $\hat{k}$ (+ve) if anticlockwise rotation and $\hat{k}$ (−ve) if clockwise rotation

$$\therefore \vec{v}_{AB}|_I = \vec{v}_A|_I - \vec{v}_B|_I$$
$$= \left[\vec{v}_C|_I + \vec{\omega} \times \vec{CA}\right] - \vec{v}_B|_I$$
$$= 2\hat{i} + (20\hat{k}) \times (-0.2\hat{j}) - (-\hat{i})$$
$$= 2\hat{i} - 4\hat{i} + \hat{i} = -\hat{i} = -\hat{i} \neq 0$$

$\therefore$ We have slip at the contact point.

$\therefore$ $\vec{v}_{AB}|_I$ acts opposite to $\hat{i}$ direction.



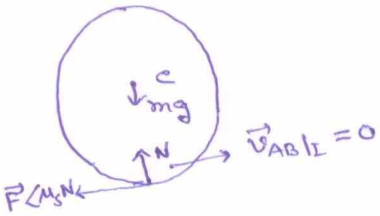
FBD

2.

Similarly,

$$\vec{v}_{AB}|_I = \vec{v}_A|_I - \vec{v}_B|_I$$
$$= \left[\vec{v}_C|_I + \vec{\omega} \times \vec{CA}\right] - \vec{v}_B|_I$$
$$= 3\hat{i} + (-10\hat{k}) \times (-0.2\hat{j}) - \hat{i}$$
$$= 3\hat{i} - 2\hat{i} - \hat{i} = 0$$

Note that $\vec{v}_{AB}|_I = 0$



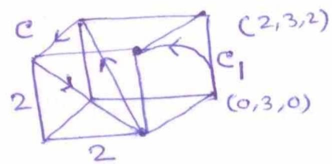
Now, we will check

$$\vec{a}_{AB}|_I = \vec{a}_A|_I - \vec{a}_B|_I$$
$$= \left[\vec{a}_C|_I + \dot{\vec{\omega}} \times \vec{r}_{AC} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{AC})\right] - \vec{a}_B|_I$$
$$= \hat{i} + (-6\hat{k}) \times (-0.2\hat{j}) + (-10\hat{k}) \times \left[-10\hat{k} \times -0.2\hat{j}\right] - (-0.2\hat{i})$$
$$= \hat{i} - 1.2\hat{i} + 20\hat{j} + 0.2\hat{i} = 20\hat{j} \neq 0$$

$\therefore \vec{a}_{AB}|_I \cdot \hat{e}_t = 20\hat{j} \cdot \hat{i} = 0$, which implies there exists no slip at the point of contact. In this case $\vec{F}$ has to determined from the equation of motion.. All we can say is $|\vec{F}| < \mu_s N$.

2.

We have $\vec{F} = (-2xy + yz)\hat{i} + (-x^2 + xz - z)\hat{j} + (xy - y)\hat{k}$. We know that if $\vec{F}$ is conservative, then we must have $\vec{\nabla} \times \vec{F} = 0$.



$$\therefore \vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (-2xy+yz) & (-x^2+xz-z) & (xy-y) \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}(xy-y) - \frac{\partial}{\partial z}(-x^2+xz-z)\right] + \hat{j}\left[\frac{\partial}{\partial z}(-2xy+yz) - \frac{\partial}{\partial x}(xy-y)\right] + \hat{k}\left[\frac{\partial}{\partial x}(-x^2+xz-z) - \frac{\partial}{\partial y}(-2xy+yz)\right]$$

$$= \hat{i}\left[(x-1)-(x-1)\right] + \hat{j}\left[y-y\right] + \hat{k}\left[-2x+z-(-2x+z)\right]$$

$$= 0$$

$\therefore$ The given force field is conservative.

As the force is conservative, then the work done along the closed path C will be zero. Suppose, there is a datum where $V(\vec{r}_0) = 0$, $V(0) = 0$ [Here $\vec{r}_0 = 0$]

$$V(\vec{r}) = -\int_0^{\vec{r}} \vec{F}\cdot d\vec{r} = -\int_0^x F_x\,dx - \int_0^y F_y\,dy - \int_0^z F_z\,dz$$

$$= -\int_0^x F_x(x,0,0)\,dx - \int_0^y F_y(x,y,0)\,dy - \int_0^z F_z(x,y,z)\,dz$$

$$= -\int_0^x 0\,dx - \int_0^y -x^2\,dy - \int_0^z (xy-y)\,dz$$

$$= x^2y - (xy-y)z$$

$$\therefore W_{1\to 2} = -\left[V(\vec{r}_2) - V(\vec{r}_1)\right] = -\left[V(2,3,2) - V(0,3,0)\right]$$

$$= -\left[4\cdot 3 - (2\cdot 3 - 3)\cdot 2 - 0\right] = -\left[12-6\right] = -6$$

(d)

The given force is $\vec{F} = (c_1 \phi^2 r + c_2)\hat{e}_r + \left(c_3 \frac{\phi^2}{r}\right)\hat{e}_\phi$, where

$r = R_2 + c_4(\phi - \phi_1)$ for portion CD

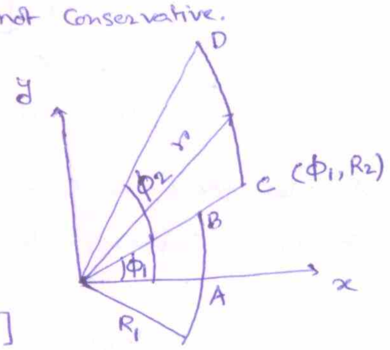
First, we shall check that the given force field is conservative or not

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \frac{1}{r}\hat{e}_r & \hat{e}_\phi & \frac{1}{r}\hat{e}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ F_r & F_\phi & F_z \end{vmatrix} = \frac{1}{r}\hat{e}_z\left[\frac{\partial}{\partial r}F_\phi - \frac{\partial}{\partial \phi}F_r\right]$$

$\because F_z = 0$ and F_r, F_ϕ are independent of z

$$= \frac{1}{r}\hat{e}_z\left[-c_3\frac{\phi^2}{r^2} - 2c_1\phi r\right] \neq 0$$

$\therefore$ The given force field is not conservative.



Now, we need to determine work done by the given force

First we calculate the work done for the path AB.

$$W_{AB} = \int_A^B \vec{F}\cdot d\vec{r} = \int_0^{R_1} F_r\, dr + \int_0^{\phi_1} F_\phi R_1\, d\phi \qquad [\text{Here } dr = 0 \text{ Radius is Constant}]$$

$$= \int_0^{\phi_1} c_3\frac{\phi^2}{R_1}\cdot R_1\, d\phi$$

$$= \frac{c_3\phi_1^3}{3}$$

Now we will calculate work done for the path BC

$$W_{BC} = \int_B^C \vec{F}\cdot d\vec{r} = \int_{R_1}^{R_2} F_r\, dr \qquad [\because \text{There is no change in angle } \therefore d\phi = 0]$$

$$= \int_{R_1}^{R_2} (c_1\phi_1^2 r + c_2)\, dr$$

$$= c_1\phi_1^2\frac{[R_2^2 - R_1^2]}{2} + c_2[R_2 - R_1]$$

Now we will calculate work done for the path CD.

$$W_{CD} = \int_C^D \vec{F}\cdot d\vec{r} = \int_{R_2}^{R_2 + c_4(\phi_2 - \phi_1)} F_r\, dr + \int_{\phi_1}^{\phi_2} F_\phi\, r\, d\phi$$

$dr = c_4\, d\phi$

$r = [R_2 + c_4(\phi - \phi_1)]$

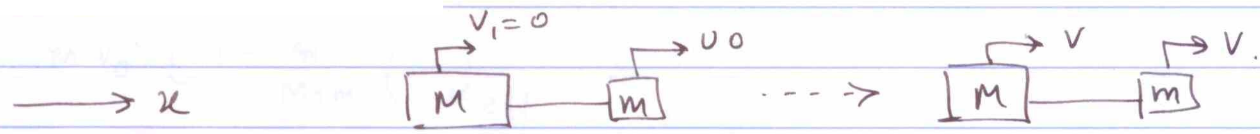
$$= \int_{\phi_1}^{\phi_2}\left[c_1\phi^2\{R_2 + c_4(\phi - \phi_1)\} + c_2\right]c_4\, d\phi + \int_{\phi_1}^{\phi_2}\frac{c_3\phi^2 r}{[R_2 + c_4(\phi - \phi_1)]}\, d\phi$$

$$= \int_{\phi_1}^{\phi_2}\left\{[c_1\phi^2 R_2 + c_1c_4\phi^2(\phi - \phi_1) + c_2 c_1\phi^2]c_4 + c_3\phi^2\right\}d\phi$$

$\therefore$ The total work done would be

$$W_{ABCD} = W_{AB} + W_{BC} + W_{CD}$$

3.



Assumptions : i) No external impulse during the speed up of M.

ii) Negligible resistance due to water and air

i) $\Rightarrow$ momentum conservation $\Rightarrow m v_0 = (m+M) V$

$$\therefore \quad V = \frac{m}{M+m} v_0 .$$

ii) $\Rightarrow$ change in kinetic energy = energy stored in the tow rope.

$$\therefore \quad \frac{1}{2} m v_0^2 - \frac{1}{2}(M+m) V^2 = \frac{1}{2} k_{eff} \, \delta_{max}^2$$

$$\Rightarrow \quad m v_0^2 \left[1 - \frac{m}{M+m} \right] = k_{eff} \, \delta_{max}^2 \qquad \text{——} \quad (A)$$

where, k_{eff} = effective spring constant of the rope

δ_{max} = maximum deflection

Data given $\rightarrow$ for a load of P^* $(= 1\,kN)$ the extension is eL $(L = \text{length})$

$$\Rightarrow \quad k_{eff} = \frac{P^*}{eL}$$

The maximum load in the rope is $P_{max} = k_{eff}\,\delta_{max}$

i.e. $P_{max} < P_0 \Rightarrow k_{eff}\,\delta_{max} \leq P_0$ for safe operation.

$$\Rightarrow \quad \delta_{max}^2 \leq \frac{P_0^2}{(k_{eff})^2} . \qquad \text{——} \quad (B)$$

From (A) and (B)

$$m V_0^2 \left[1 - \frac{m}{M+m} \right] \frac{1}{k e l t} \le \frac{p_0^2}{(k e l t)^2}$$

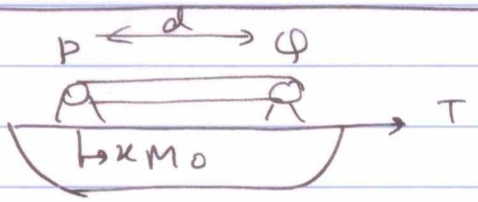
$$\therefore m V_0^2 \left[\frac{M}{m+M} \right] \le \frac{P_0^2}{k e l t} = \frac{P_0^2 \, e L}{P^*}$$

$$\therefore L \ge \frac{m V_0^2 M}{(m+M)} \frac{P^*}{e P_0^2}$$

In the text book $P^* = 1000$ N has been used.

4.

a)



Let the mass of the grain at P be m_P and that at Q be m_Q and let the mass of the rest of the barge be M_0, and let its centre of mass be at x_{c0}.

The location of the overall centre of mass is

$$x_c = \frac{m_P \times 0 + m_Q d + M_0 x_{c0}}{m_P + m_Q + M_0}$$

$$m_Q + m_P + M_0 = M = \text{constant}.$$

$$\therefore x_c = \frac{m_Q d + M_0 x_{c0}}{M}$$

$$\dot{x}_c = \frac{\dot{m}_Q d}{M} \Rightarrow M \ddot{x}_c = \ddot{m}_Q d$$

The rate of transport of mass is $m(t) \Rightarrow \dot{m}_P = \dot{m}(t)$

$\Rightarrow \ddot{m}_Q = \ddot{m}(t)$ (note the text book uses $m(t)$ in place of $\dot{m}(t)$)

$$\therefore M \ddot{x}_c = F_{ext} = \ddot{m}_Q d$$

$$\Rightarrow T = \ddot{m}_Q d = \ddot{m} d$$

b) If the mass flow is reversed the tension should be negative – not possible so the barge will start to move.