Lecture 11
Signals and Systems (ELL205)

By Dr. Abhishek Dixit

Dept. of Electrical Engineering

IIT Delhi

Outline of the lecture

- Use *h(t)* to determine whether the system is:
  - Memoryless
  - Causal
  - Stable
  - Invertible
- Applications of *h(t)* to real-life scenarios
- System designing

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System properties

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Causal and linear system: Condition of initial rest

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Causal and linear system: Condition of initial rest

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Invertible: $h(t) * h_{inv}(t) = \delta(t)$

System properties

Functions	Linear	Causal	Condition of initial rest	ZIZO
$y[n] = 2x[n] + 3$				

System properties

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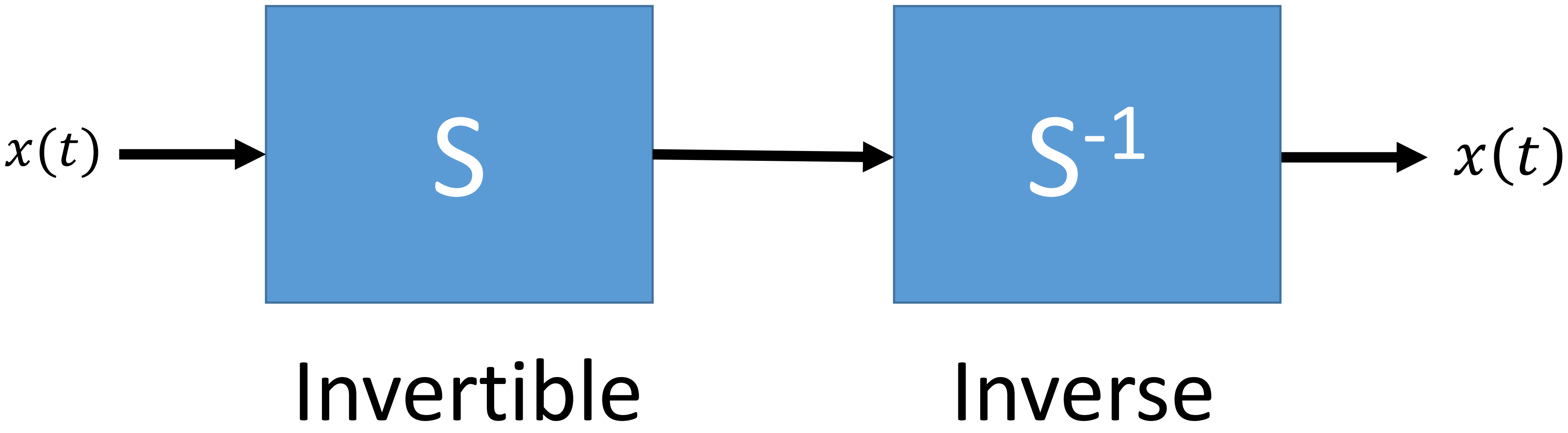
Linearity: It must satisfy ZIZO

Linearity and Causality: It must satisfy “Condition of initial rest.”

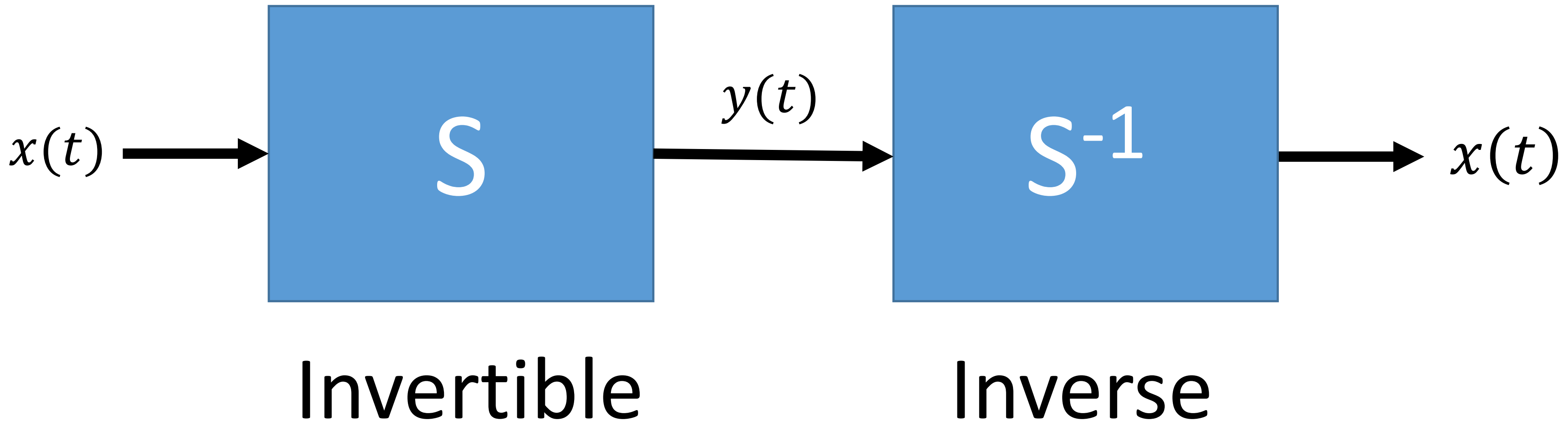
Linearity and it satisfies “condition of initial rest”: It must satisfy “Causality.” (Proof is in tutorials)

System properties



$$x(t) * h(t) * h_{inv}(t) = x(t)$$

System properties



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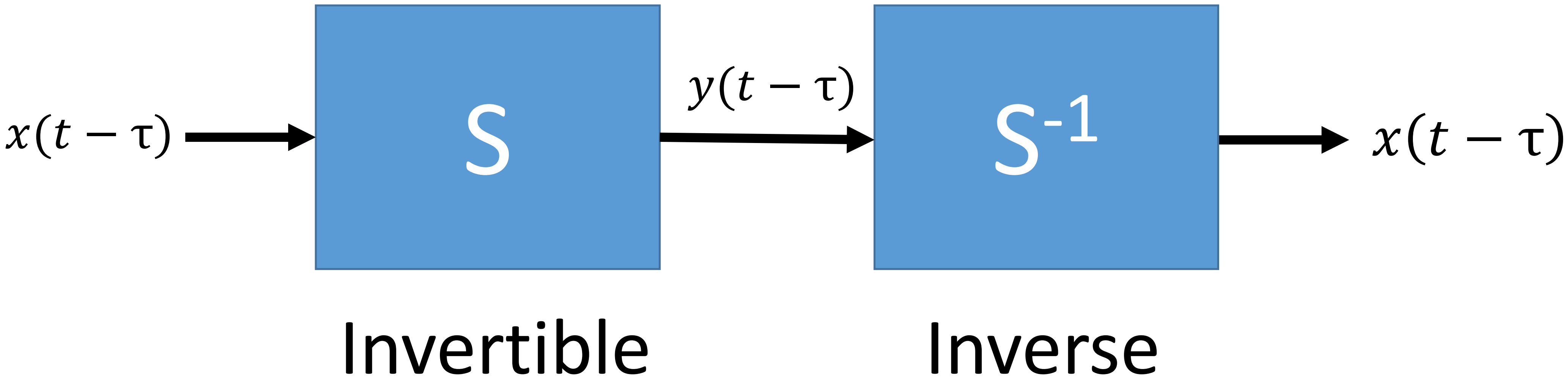

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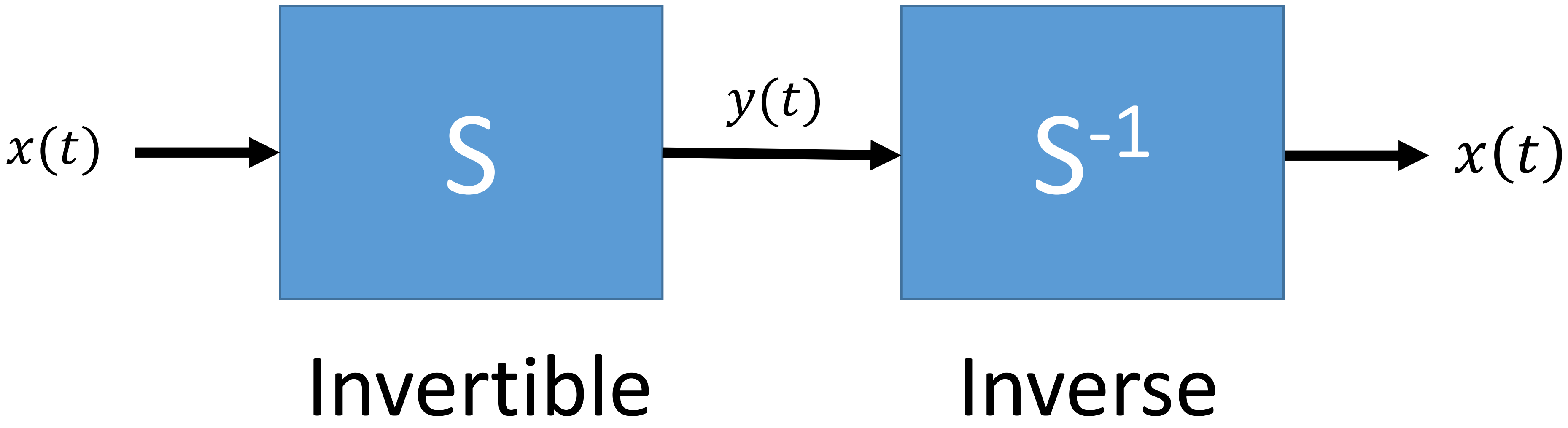

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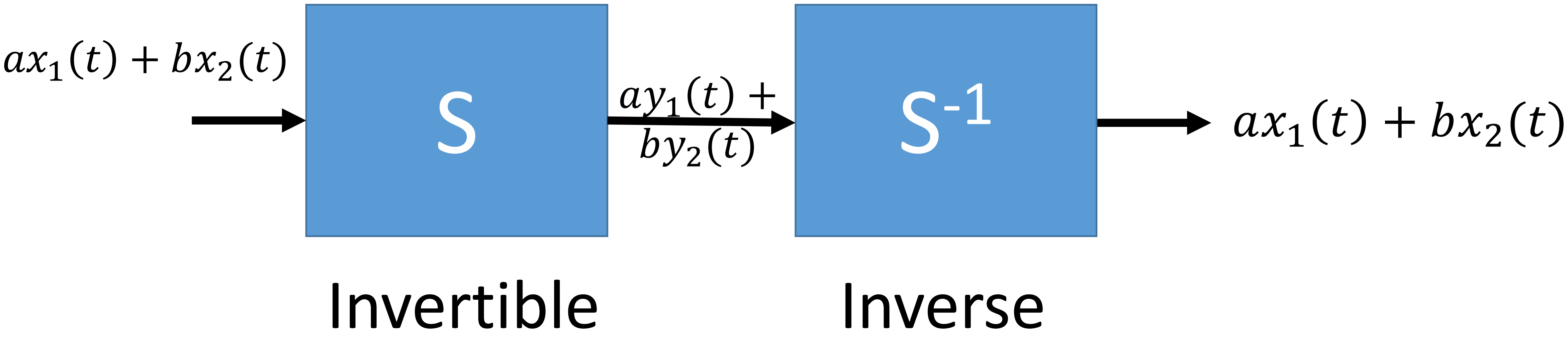
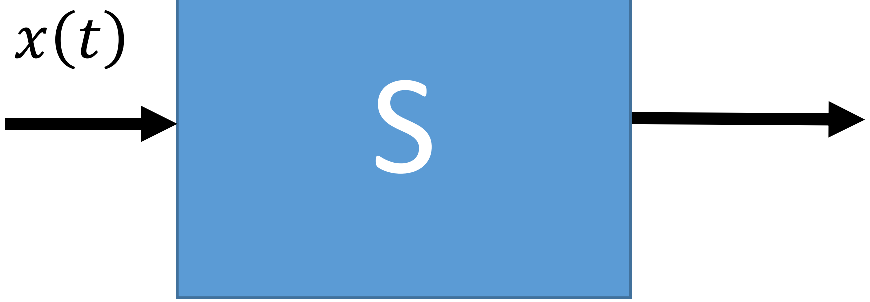


$$x(t) * h(t) * h_{inv}(t) = x(t)$$

System properties

$$x(t) = sin\omega t + C$$

$$y(t) = \frac{dx(t)}{dt}$$



LTI

System properties

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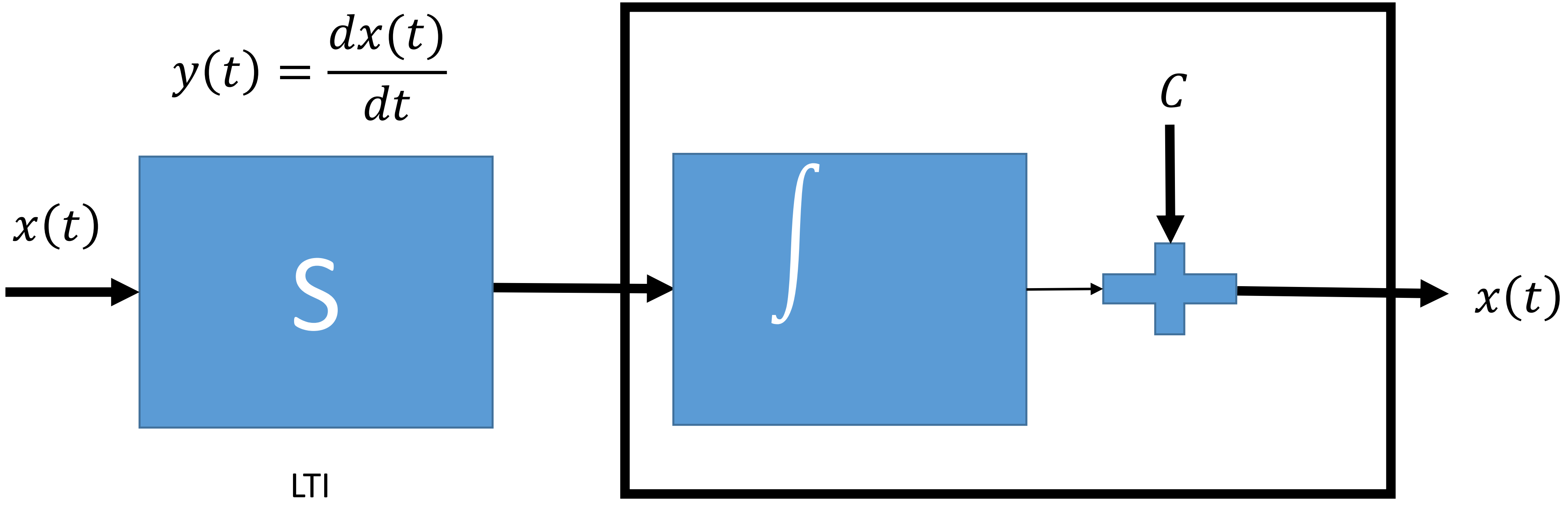
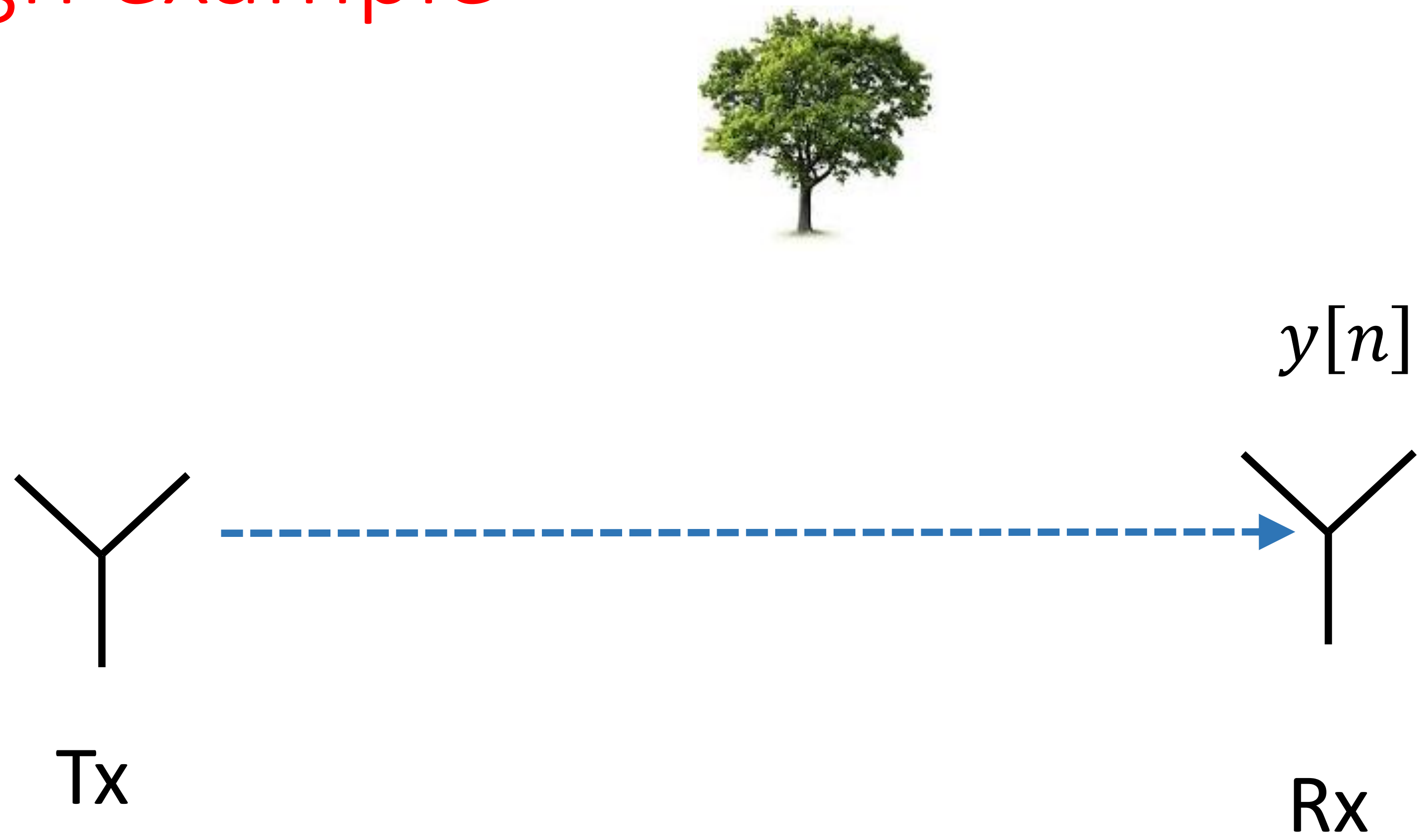
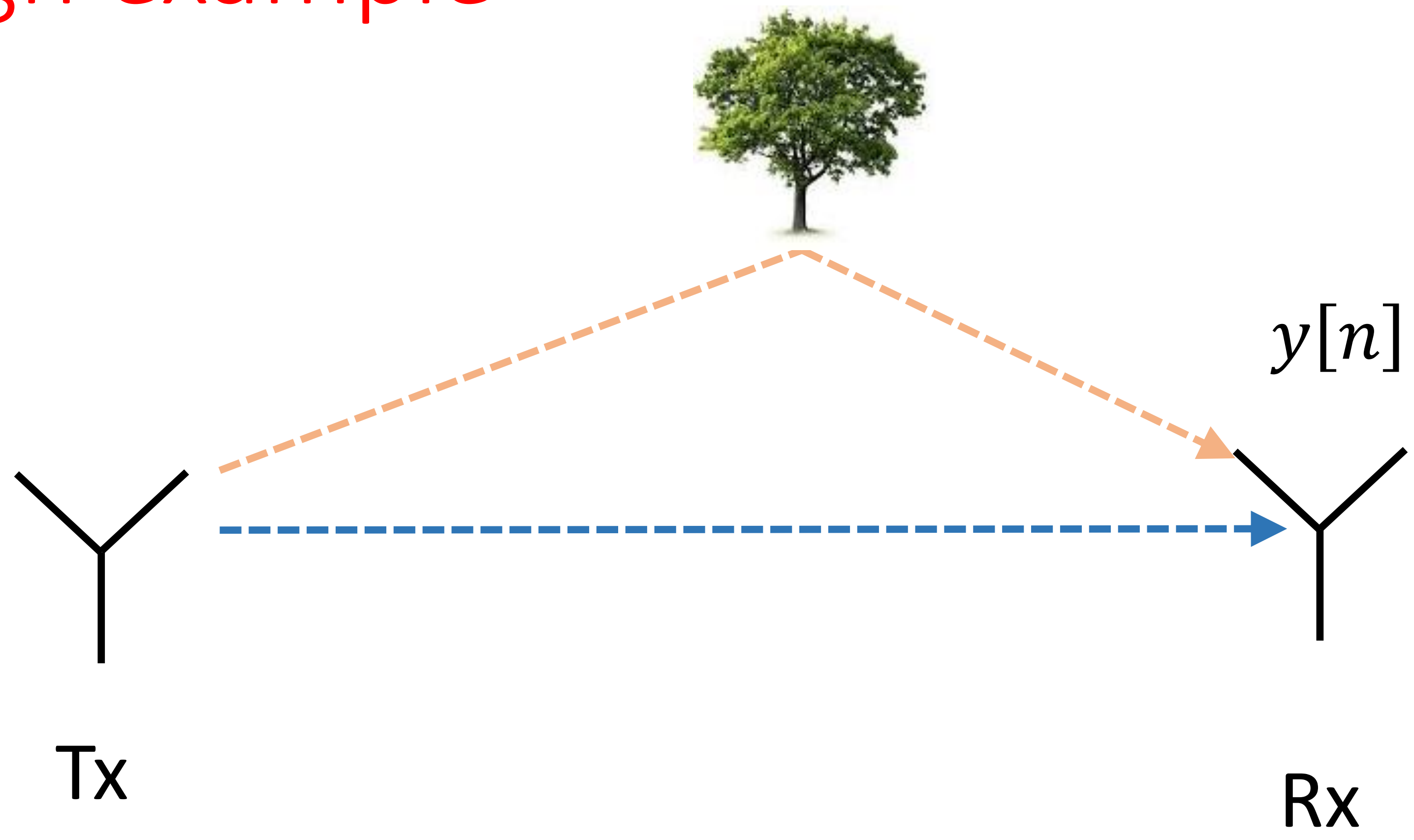


Design example



$$y[n] = x[n]$$

Design example



$$y[n] = x[n]+ax[n-1]$$

Multi-path fading

Design example

A causal system to inverse $y[n] = x[n] + ax[n-1]$ is:

(1) $\boldsymbol{h_{inv}[n] = (-a)^n u[n]}$	**(2)** $\boldsymbol{h_{inv}[n] = (a)^n u[n]}$
(3) $\boldsymbol{h_{inv}[n] = n(-a)^n u[n]}$	**(4)** $\boldsymbol{h_{inv}[n] = n(a)^n u[n]}$

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$$h_{inv}[n] * (\delta[n] + a\delta[n-1]) = \delta[n]$$

$$h_{inv}[n] + ah_{inv}[n-1] = \delta[n]$$

$$h_{inv}[0] = 1$$

$$h_{inv}[1] + ah_{inv}[0] = 0$$

$h_{inv}[1] = -a$ $\quad h_{inv}[2] = -ah_{inv}[1] = a^2$ $\quad h_{inv}[n] = (-a)^n u[n]$

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A causal system to inverse

$$y[n] = x[n] + e^{-\alpha}x[n-1]+e^{-2\alpha}x[n-2]+\ldots\ldots$$

is:

(1) $\boldsymbol{h_{inv}[n] = \delta[n] - e^{-\alpha}\delta[n-1]}$	(2) $\boldsymbol{h_{inv}[n] = \delta[n] - e^{\alpha}\delta[n-1]}$
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Examples of systems where *h(t)* is a natural metric of system description

1) Communication System
2) Optical System

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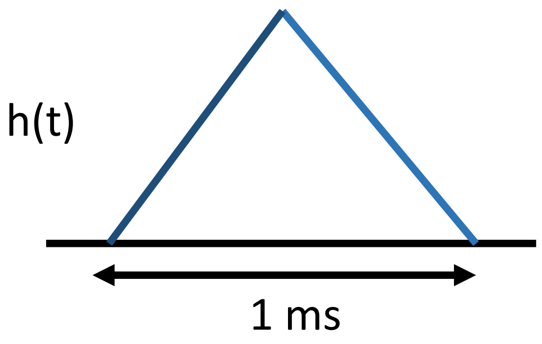
A twisted pair coax has an impulse response as shown:



The maximum bit rate is

1) 100 Kb/s	**2) 1 Mb/s**
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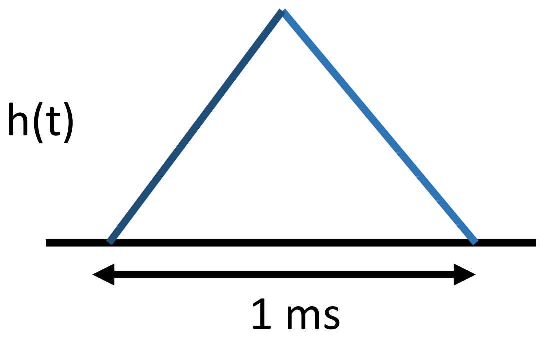

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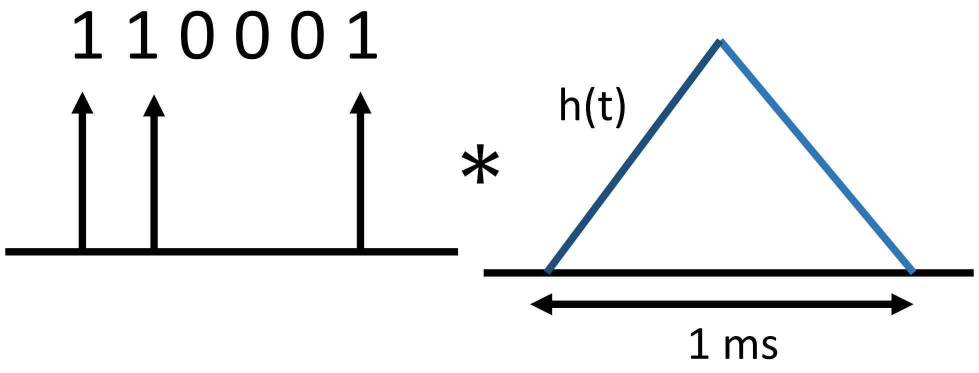
how fast the transmitter can spit out the bits 1100001100000 ?



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