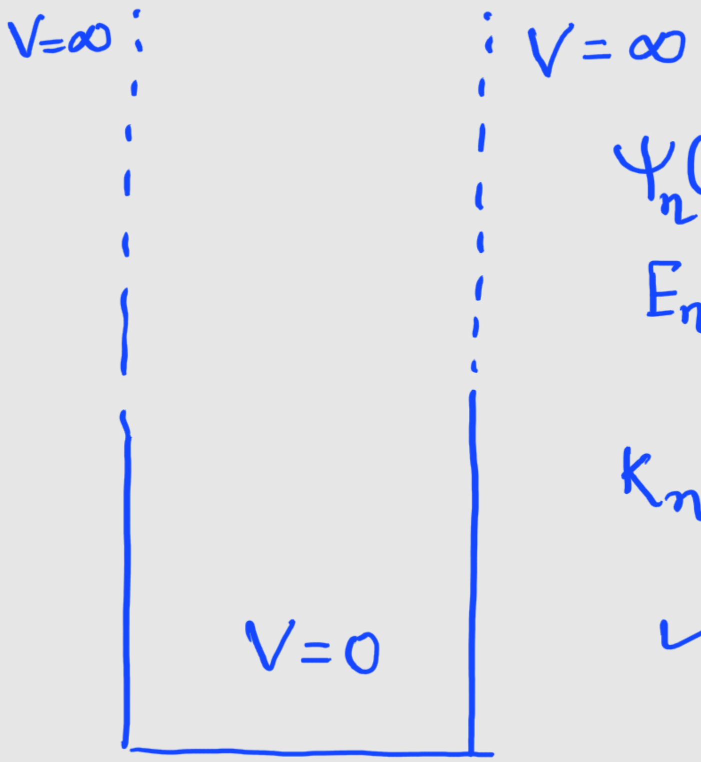
# Infinite square well




$\psi_n(x)$

$$E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$$

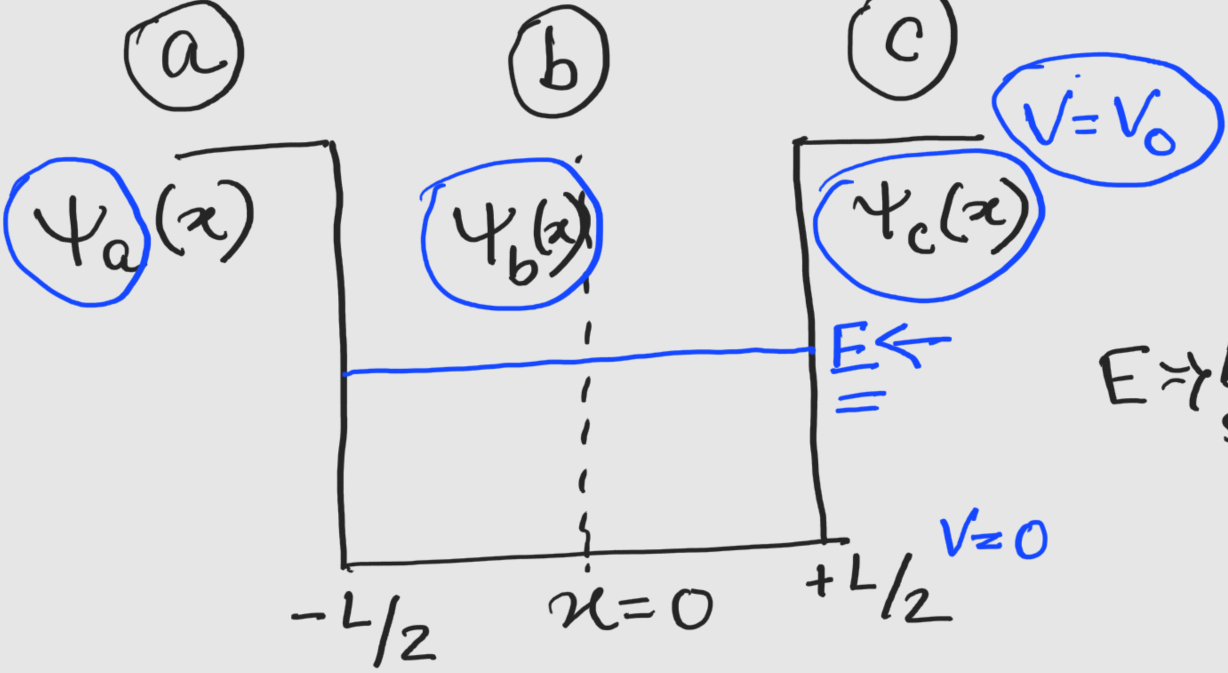
$$k_n = \sqrt{\frac{2mE_n}{\hbar^2}}$$

inaccurate soln

## Finite square well potential


here??

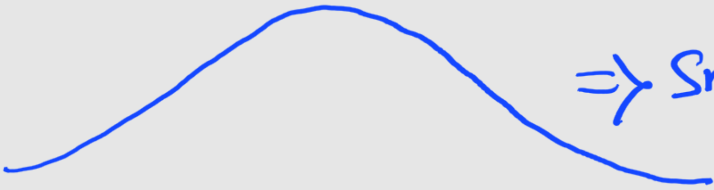
$$\boxed{\frac{d^2\psi(x)}{dx^2} = \frac{2m(V-E)}{\hbar^2}\psi(x)}$$




E ⇒ bound state

# Boundary Cond$^n$:-

$\Rightarrow$ discontinuous ✗



$\Rightarrow$ Smooth ✓

1. $\psi(x)$ is to be continuous
2. $\psi'(x)$ is to be continuous

:) $\Rightarrow$ ✓ E larger V ✓

$$\underline{\underline{V - E}} = -ve$$

V $-$ E $\Rightarrow -ve$

$$\frac{d^2\psi}{dx^2} = -k^2\psi$$

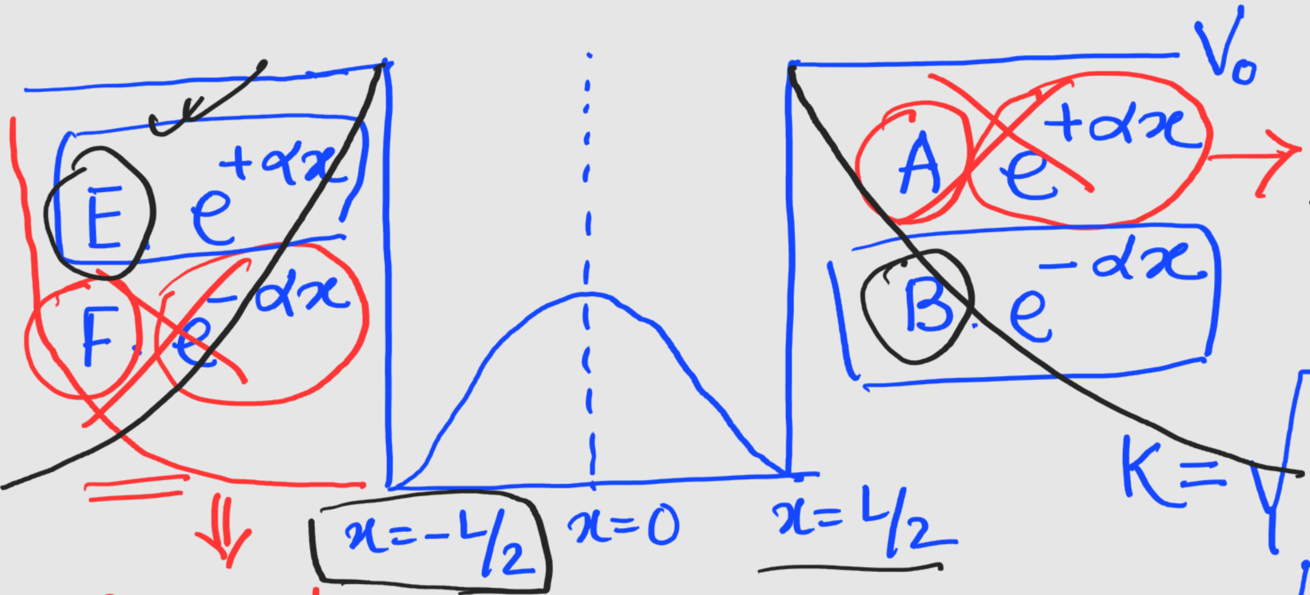
$\searrow$

$\sin kx, \cos kx$ ✓

:) $\Rightarrow$ $V = V_0$ ; $\underline{\underline{E < V_0}}$

$$V - E = +ve \qquad \frac{d^2\psi}{dx^2} = +\alpha^2\psi$$

$$\underline{\underline{e^{-\alpha x}, e^{\alpha x}}} \checkmark$$




$$K=\sqrt{\frac{2mE}{\hbar^2}}$$

Completely unbounded → blows up

$$\alpha=\sqrt{\frac{2m(V-E)}{\hbar^2}}$$

$C\,\mathrm{Sin}(Kx)$

$\boxed{D\,\mathrm{Cos}(Kx)}$

⇒ Symmetry theory of Quantum mechanics

Symmetric $V(x)$ → even, odd

but not Combination of both

Boundary Cond$^n$:

$\boxed{E=B}$

$$E\cdot e^{\alpha(-L/2)}=D\cdot \mathrm{Cos}\,K(-L/2)$$

$$\boxed{E\,e^{-\alpha L/2}=D\,\mathrm{Cos}\,KL/2} \quad \cdots\cdots (1)$$

$$\boxed{\alpha E\cdot e^{-\alpha L/2}=KD\,\mathrm{Sin}\,KL/2} \quad \cdots (2)$$

$$K=\sqrt{\frac{2mE}{\hbar^2}}\ ;\quad \alpha=\sqrt{\frac{2m(V_0-E)}{\hbar^2}}$$

$$\boxed{K^2+\alpha^2=\frac{2mV_0}{\hbar^2}}$$

$$②/① \Rightarrow \boxed{\alpha = K\tan\left(\frac{KL}{2}\right)}$$

$\alpha, K \Rightarrow A, B, C, D, E, F \rightarrow \psi(x)$.

$\boxed{E=?}$ $E_1, E_2, E_3, \ldots$

$$① \quad \alpha^2+K^2=\frac{2mV_0}{\hbar^2}$$

$$② \quad \alpha = K\tan\frac{KL}{2}$$

$\alpha \rightarrow \frac{1}{L}$ (unit)

$K \rightarrow \frac{1}{L}$ (unit)

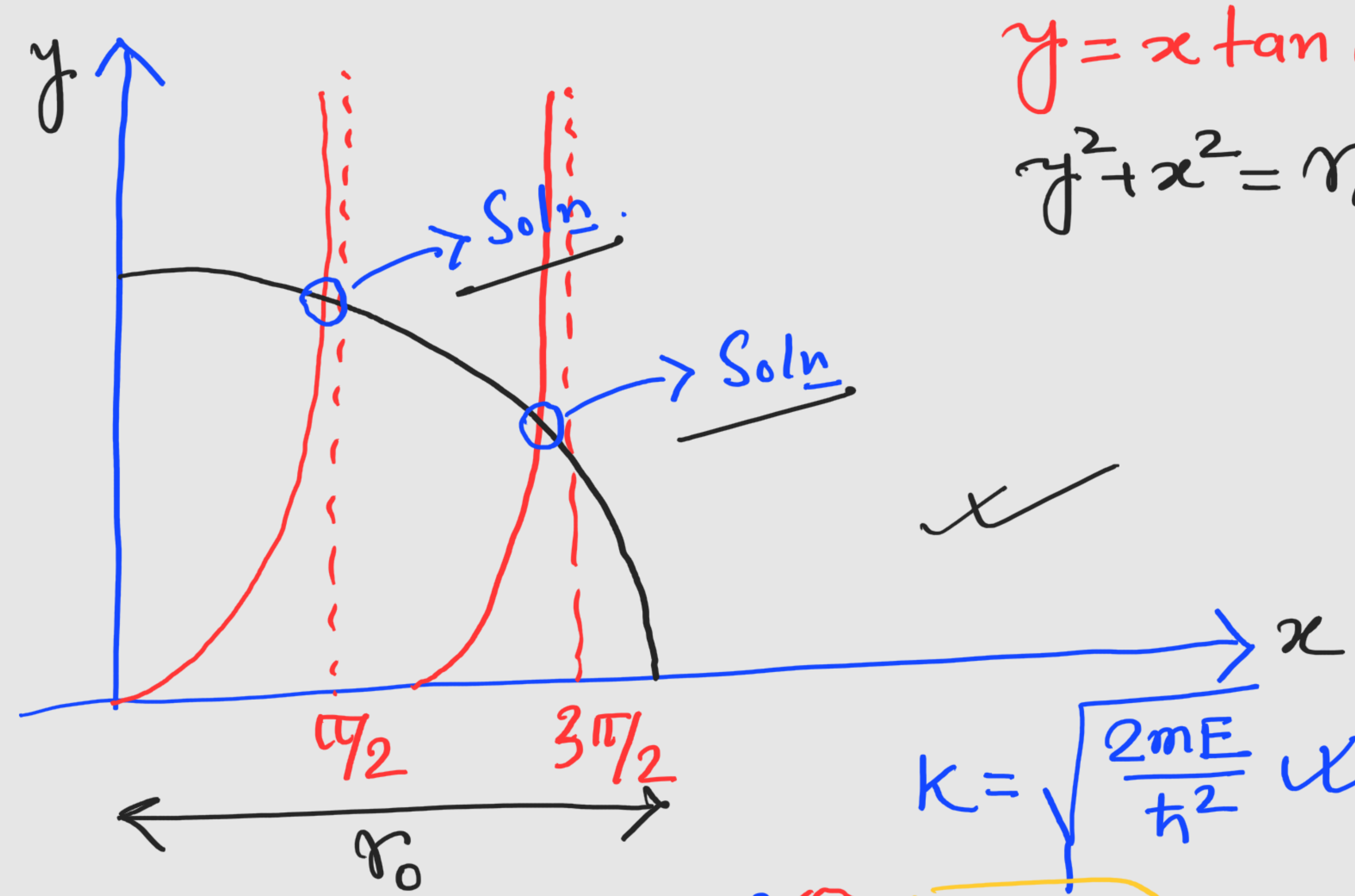
$\alpha L \rightarrow$ unit free $\rightarrow KL$

$$\frac{\alpha L}{2} = \frac{KL}{2}\tan\frac{KL}{2}$$

$$\boxed{y = x\tan x}$$

$$\frac{\alpha^2L^2}{4}+\frac{K^2L^2}{4} = \frac{\cancel{2}mV_0}{\hbar^2}\cdot\frac{L^2}{\cancel{4}} = \frac{mV_0L^2}{2\hbar^2}$$

$$y^2+x^2=\boxed{r_0^2} \qquad r_0=\sqrt[2]{\frac{mV_0L^2}{2\hbar^2}}$$




$y = x\tan(x)$

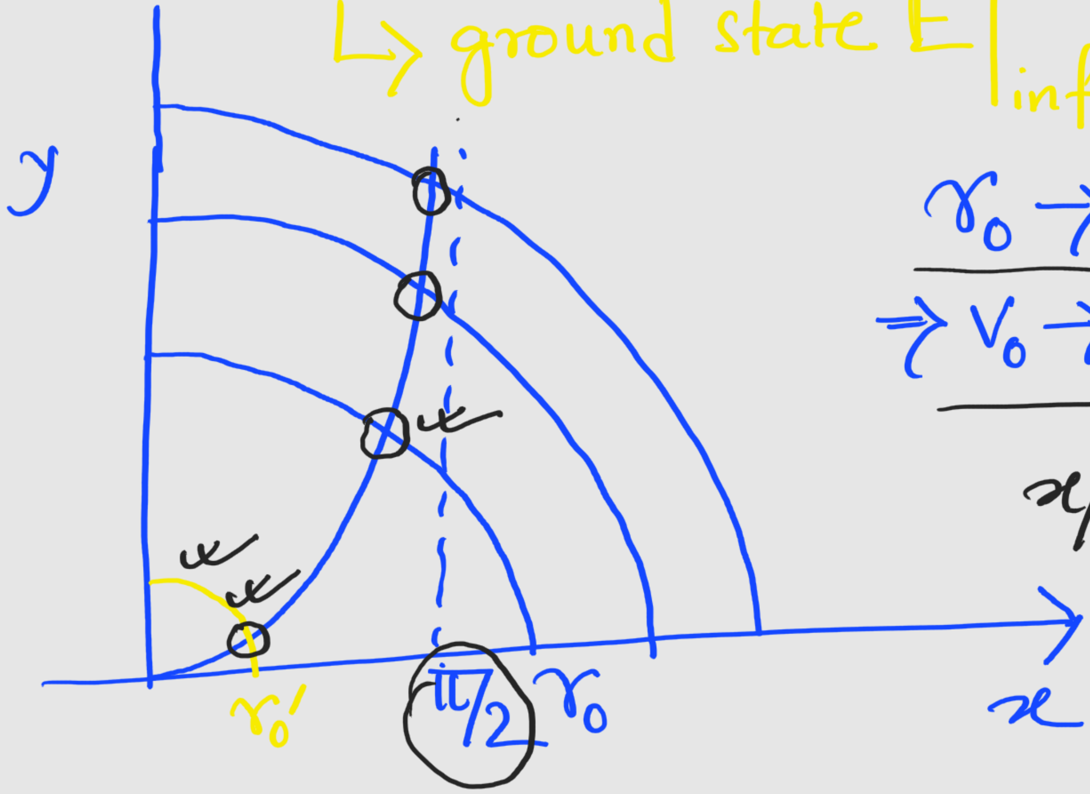
$y^2 + x^2 = r_0^2$

$K = \sqrt{\frac{2mE}{\hbar^2}}$

$$E = \frac{\hbar^2}{2m} \times \frac{4}{L^2} \cdot \frac{K^2 L^2}{4^2} \cdot \frac{(\pi/2)^2}{(\pi/2)^2}$$

$$E = \boxed{\frac{\hbar^2}{2m}\frac{\pi^2}{L^2}} \left(\frac{x}{\pi/2}\right)^2$$

↳ ground state $E\Big|_{\text{infinite sq. well pot.}}$




$r_0 \to$ up to infinity

$\Rightarrow V_0 \to$ infinity.

$x/\pi/2 \Rightarrow 1.$

$$E = \frac{\hbar^2}{2m}\frac{\pi^2}{L^2}.$$


$r_0 \to$ Very small

$V_0 \to$ Very small

$\Rightarrow$ One Sol$\underline{n}$.