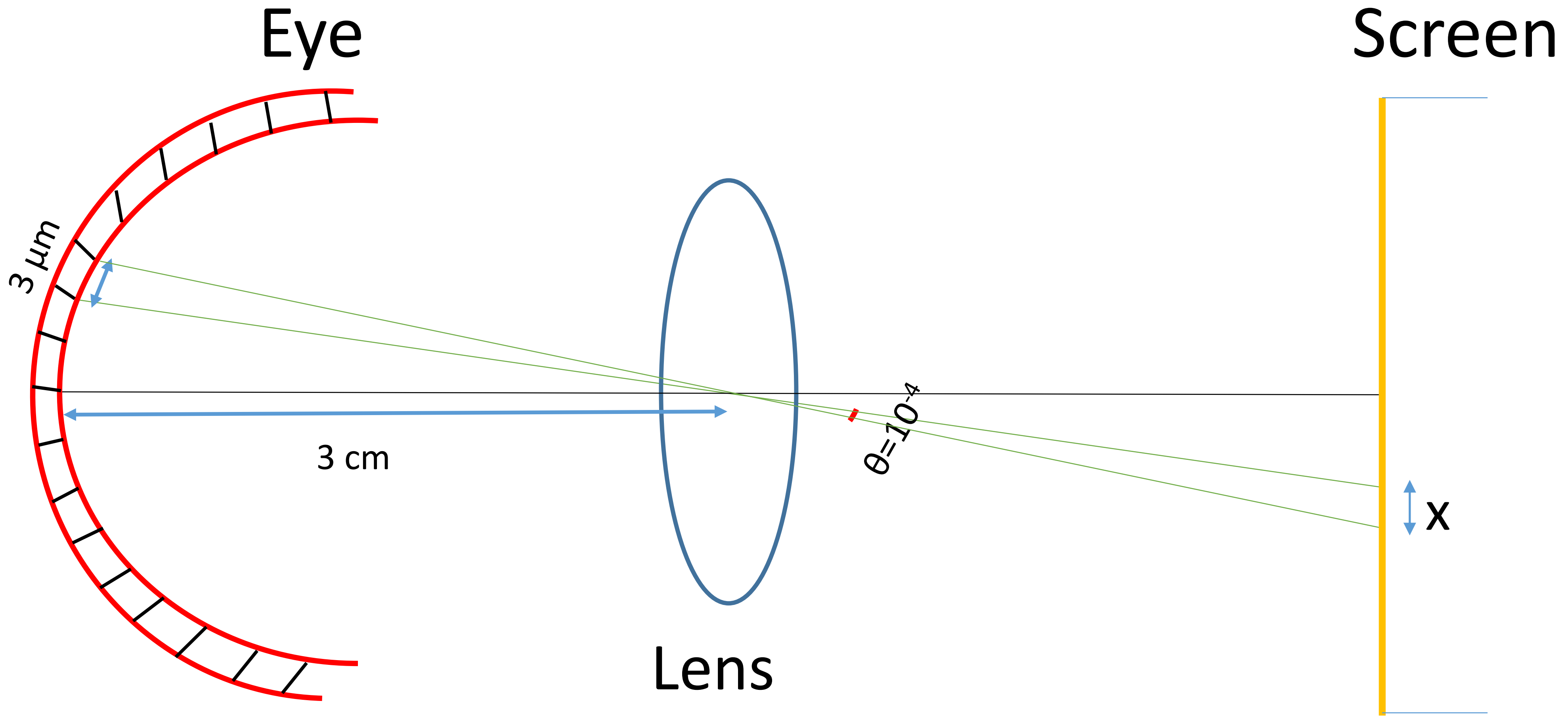
Discrete-Time Signals and Systems

Lecture 24

Why discrete systems ?

- In Discrete systems, time is discrete (storage)
- Discrete-time systems can be stored and processed digitally (using digital electronics)
- Digital electronics is inexpensive !!
- All modern systems are discrete-time systems

Is it that natural occurring signals are continuous-time?



How many pixels per inch (ppi) for a 5.5 inch mobile phone?

1. 50 pixels per inch
2. 500 pixels per inch
3. 5000 pixels per inch
4. 50000 pixels per inch

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3. 5000 pixels per inch
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$$\frac{x}{V_d} \approx \theta$$

$$ppi = \frac{1\,\mathbf{inch}}{x} = \frac{1\,\mathbf{inch}}{V_d \times \theta} = \frac{1 \times 2.5 \times 10^{-2}}{0.5 \times 10^{-4}} \approx 500$$

Mobile phone's ppi

Mobile Phones	PPI
Iphone X	498 ppi
Samsung galaxy S9 plus	568 ppi
LG G7	564 ppi
Huawei P20	429 ppi
Nokia 8	554 ppi
Google Pixel 2	537 ppi
Sony Xperia XZ3	537 ppi

Mobile phone’s ppi

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Nokia 8	554 ppi
Google Pixel 2	537 ppi
Sony Xperia XZ3	537 ppi
Sharp’s	800 ppi

What about ears?

Minimum sound pressure it can hear: 20 μPascal

Maximum sound pressure it can hear: $10^6 \times$ 20 μPascal

Frequencies that it can hear: 20 Hz to 20000 Hz

Audio bit depth ?

Audio bit depth ?

- Audio bit depth = $\log_2 10^6 = 6 \times \log_2 10 = 19\ \text{bits}$

Audio bit depth ?

- 16 bits
- 8 bits
- 4 bits

Audio bit depth = quantization error ?

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Discrete-time system characterization

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

$$x[n] = z^n \qquad\qquad y[n] = \sum_{k=-\infty}^{\infty} h[k]z^{n-k}$$

$$y[n] = z^n \sum_{k=-\infty}^{\infty} h[k]z^{-k} = z^n H(z) \qquad\qquad H(z) = \sum_{k=-\infty}^{\infty} h[k]z^{-k}$$

Discrete-time system characterization

$$H(z)=\sum_{k=-\infty}^{\infty} h[k]z^{-k}$$

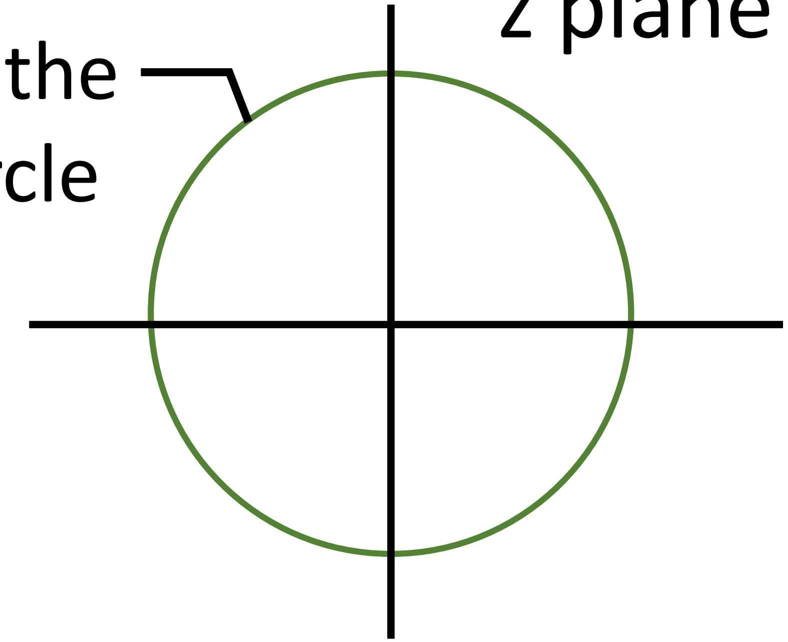
$$z = re^{j\Omega}$$

$$r = 1$$

$$z = e^{j\Omega}$$

$e^{j\Omega}$ lies on the unit circle

z plane



Discrete-time system characterization

$$H(e^{j\Omega}) = \sum_{k=-\infty}^{\infty} h[k]e^{-j\Omega k}$$

$$H(e^{j(\Omega+2\pi)}) = \sum_{k=-\infty}^{\infty} h[k]e^{-j(\Omega+2\pi)k}$$

$$= \sum_{k=-\infty}^{\infty} h[k]e^{-j\Omega k}e^{-j2\pi k} \qquad = H(e^{j\Omega})$$

Discrete-time system characterization

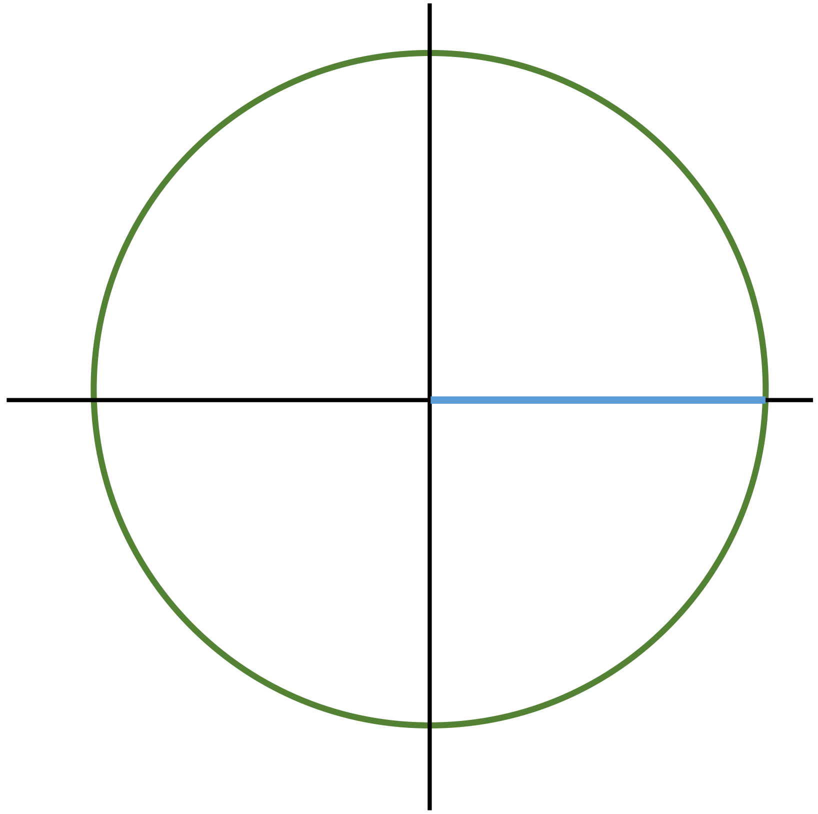
$H(e^{j\Omega})$ is periodic in 2π

$H(\omega)$ is not!!

Ω has units radians

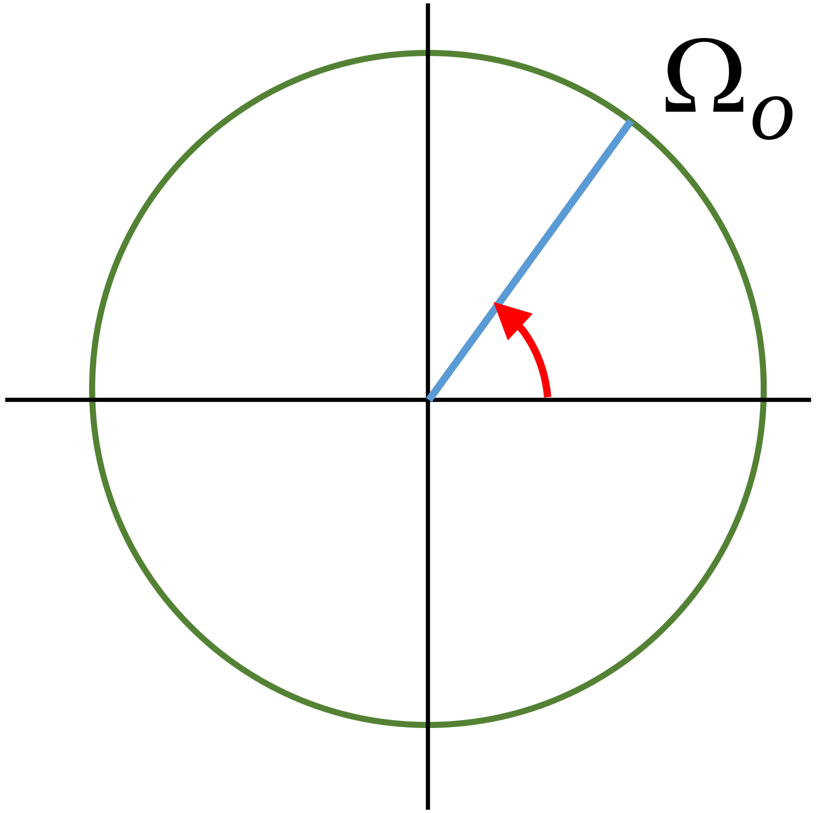
ω has units radians/sec

Discrete-time system characterization



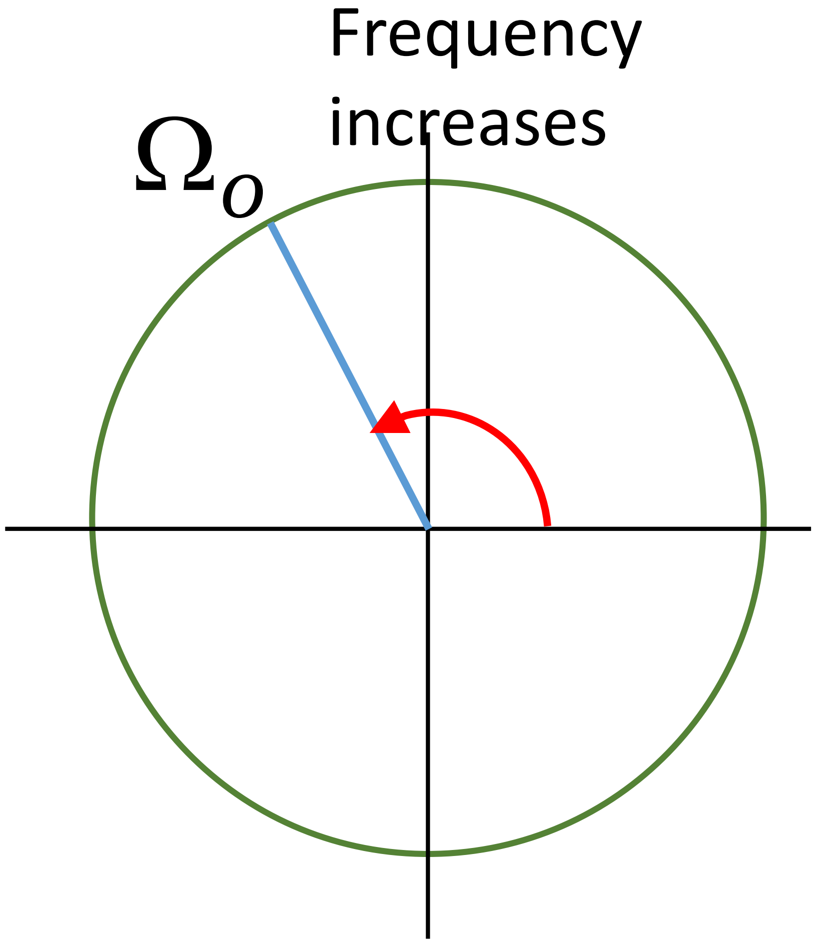
$\Omega_o = 0$ Zero frequency

Discrete-time system characterization



Frequency increases

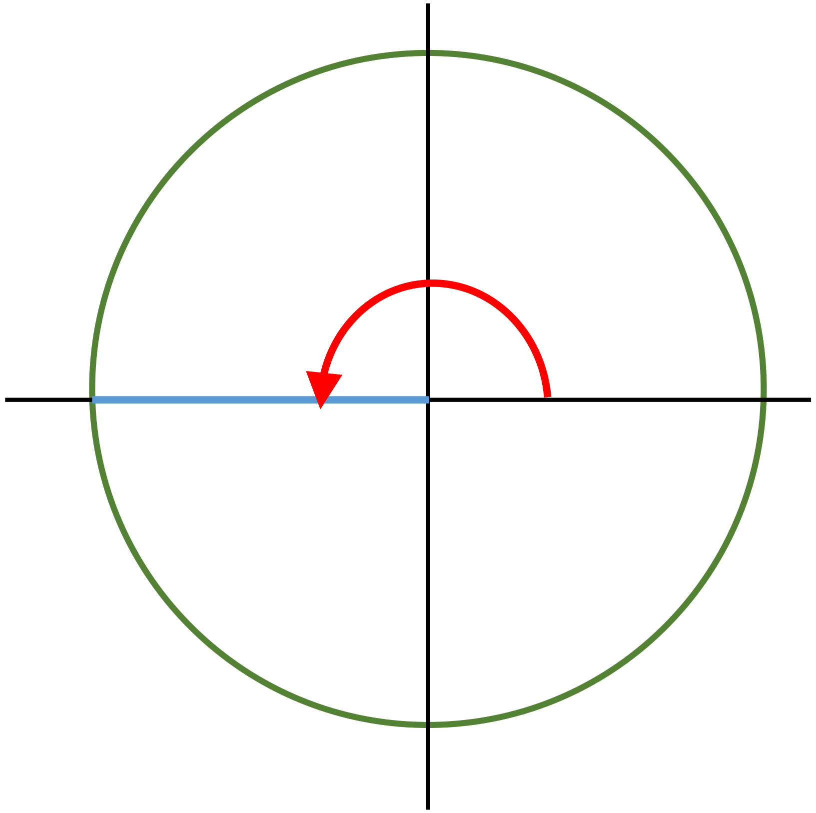
Discrete-time system characterization



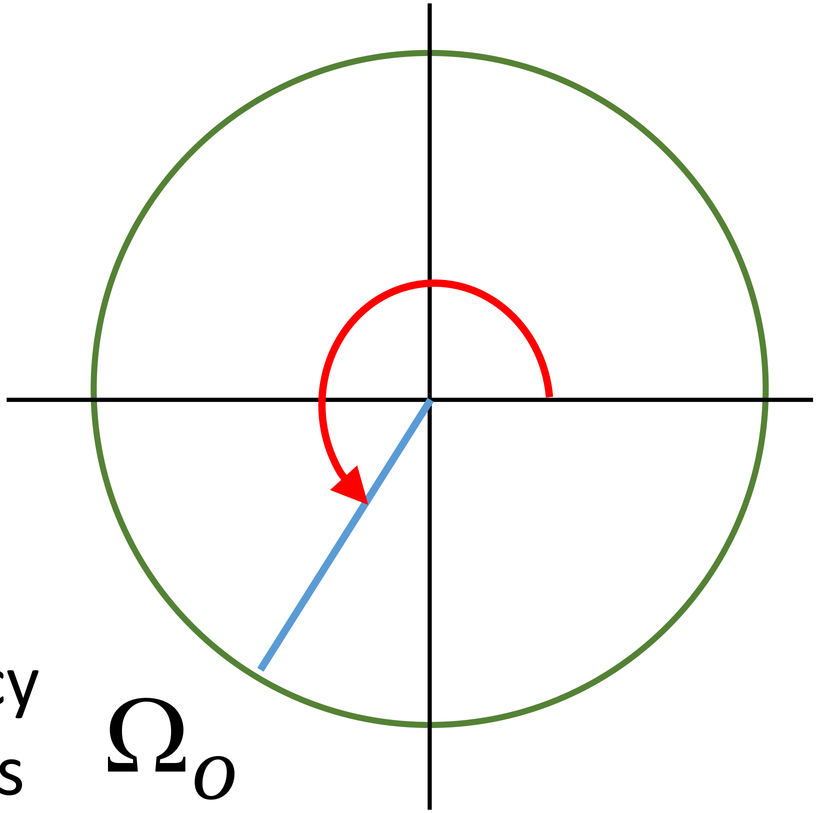
Discrete-time system characterization

Maximum frequency

$$\Omega_o = \pi$$

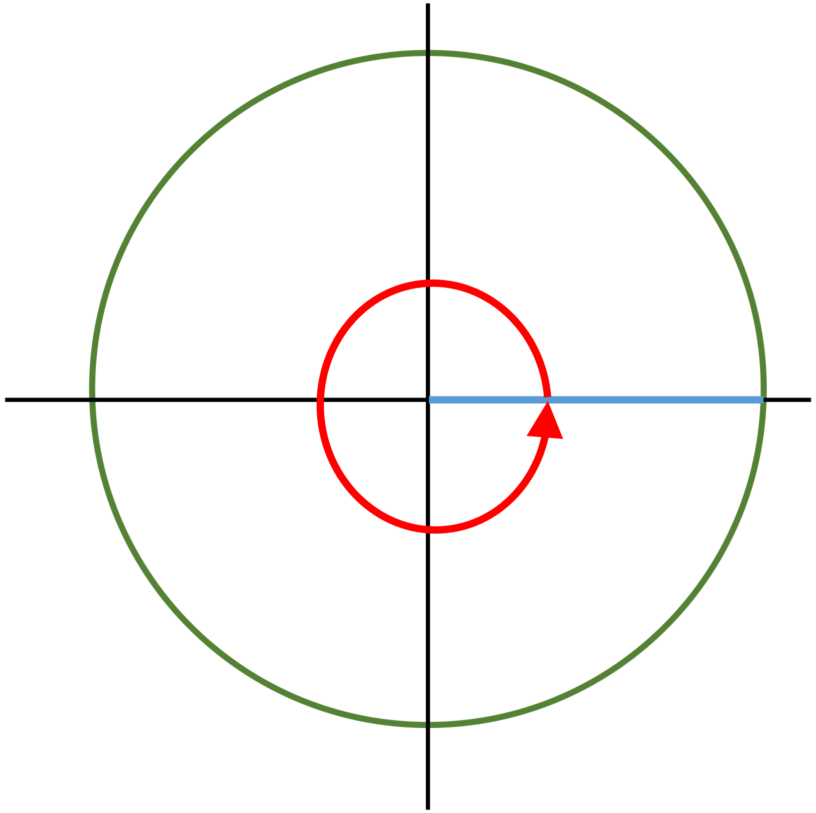


Discrete-time system characterization



Frequency decreases Ω_o

Discrete-time system characterization



$$\Omega_o = 2\pi$$

Frequency wraps

Arrange the following signals according to their frequency (lowest to highest).

$$x_1(t) = \cos(350t) \qquad x_2(t) = \cos(500t) \qquad x_3(t) = \cos(600t)$$

$$T = 0.01\,\textbf{seconds}$$

(a) $x_1[\mathrm{n}], x_2[\mathrm{n}], x_3[n]$

(b) $x_2[\mathrm{n}], x_1[\mathrm{n}], x_3[n]$

(c) $x_2[\mathrm{n}], x_1[\mathrm{n}], x_3[n]$

(d) $x_3[\mathrm{n}], x_2[\mathrm{n}], x_1[n]$

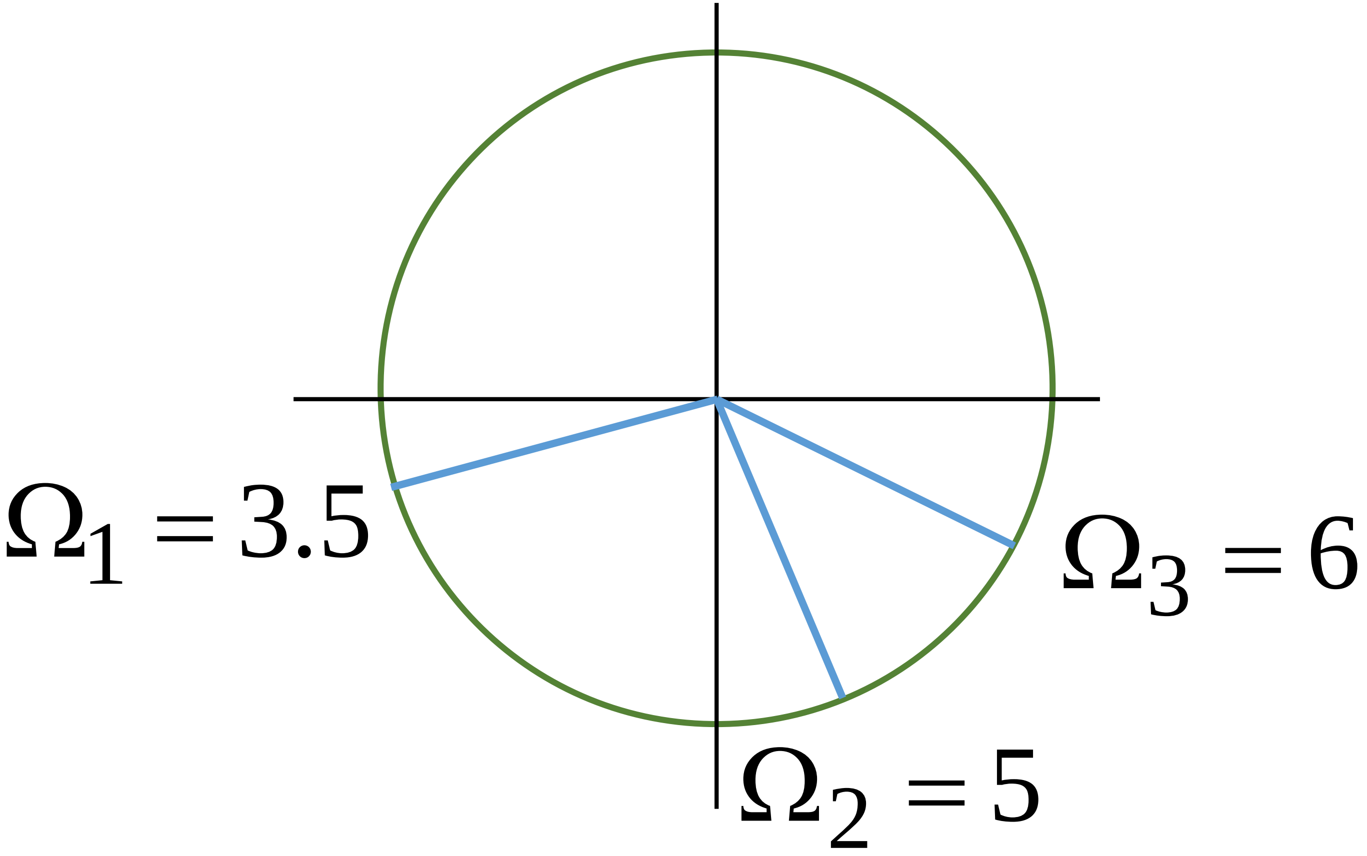
Arrange the following signals according to their frequency (lowest to highest).

$T = 0.01\,\mathbf{seconds}$

$x_1[nT] = \cos[3.5n]$

$x_2[nT] = \cos[5n]$

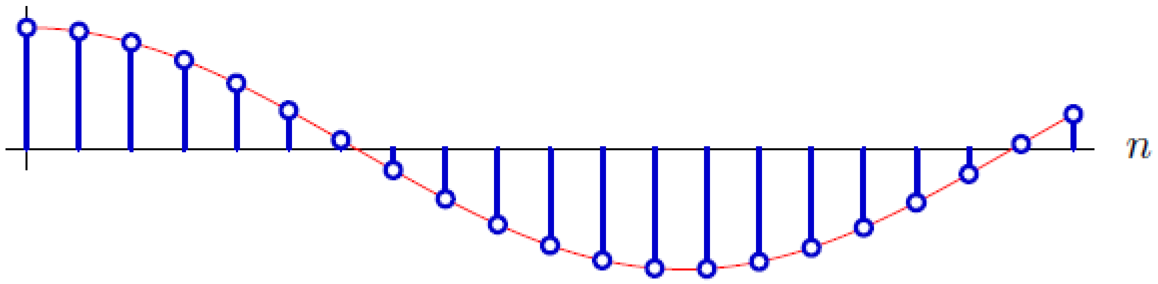
$x_3[nT] = \cos[6n]$



Frequency aliasing

$\Omega = 0.25$

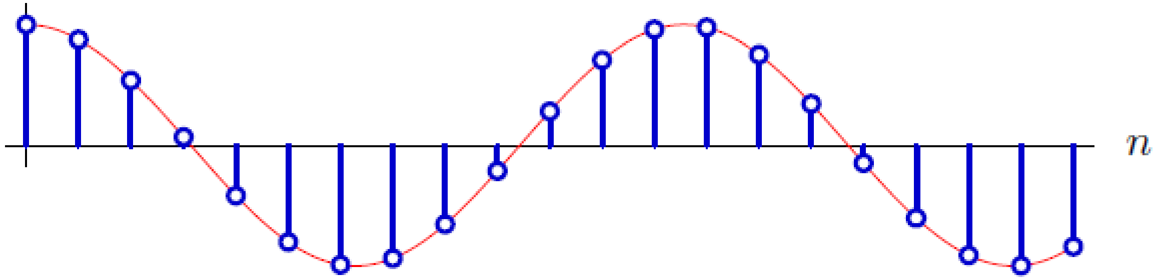
$x[n] = \cos(0.25n)$



Frequency aliasing

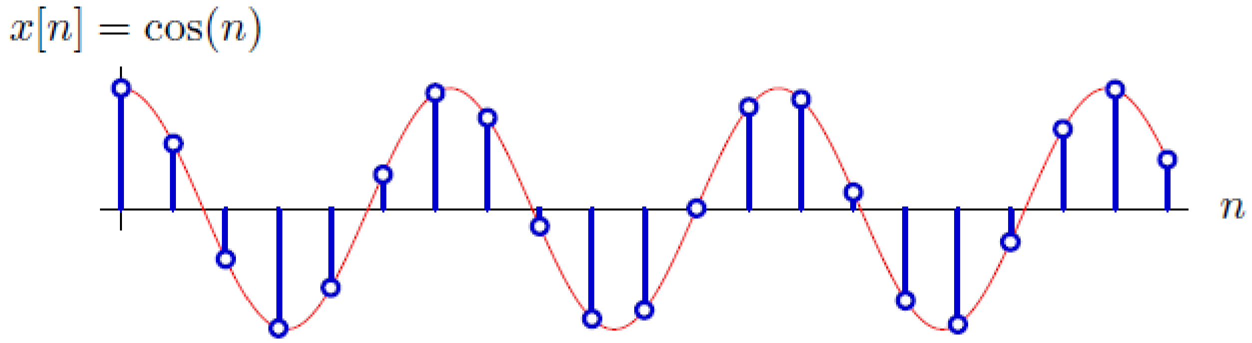
$\Omega = 0.5$

$x[n] = \cos(0.5n)$



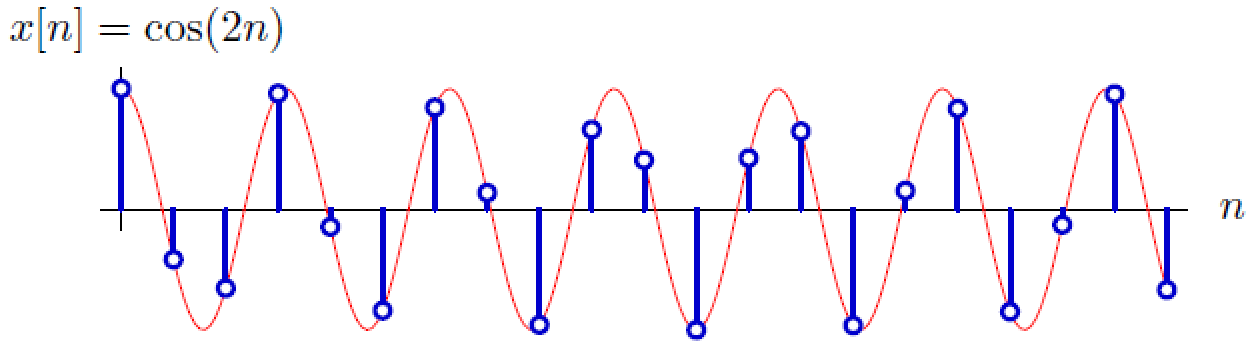
Frequency aliasing

$\Omega = 1$



Frequency aliasing

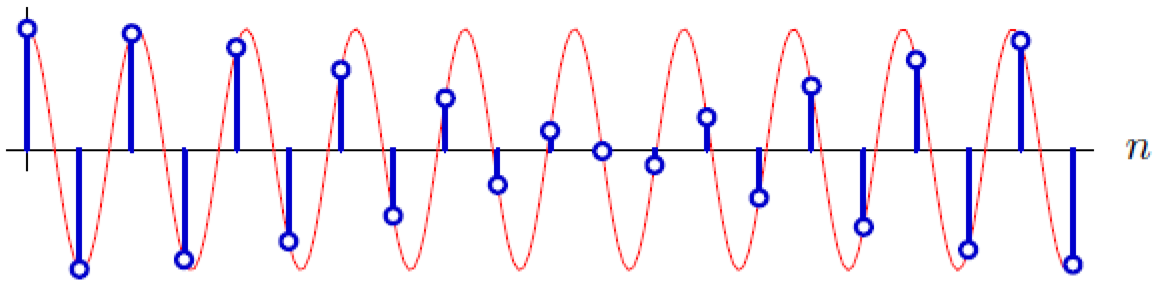
$\Omega = 2$



Frequency aliasing

$\Omega = 3$

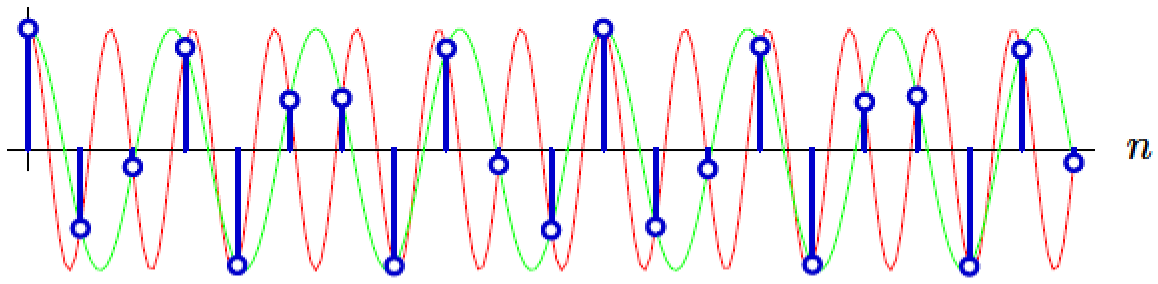
$x[n] = \cos(3n)$



Frequency aliasing

$\Omega = 4$

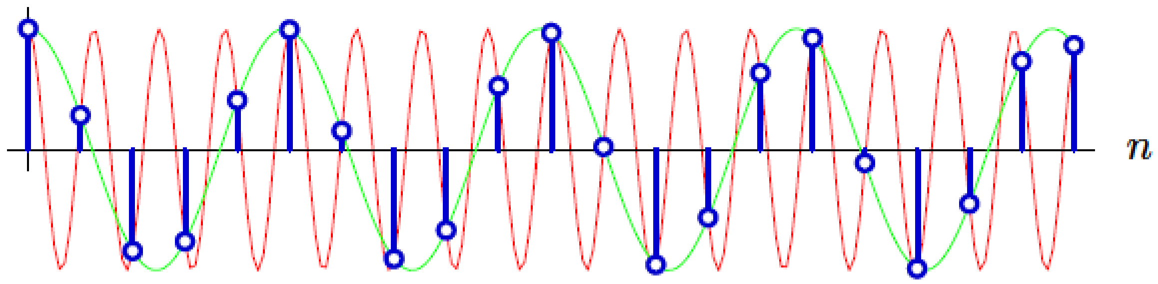
$$x[n] = \cos(4n) = \cos(2\pi - 4n) \approx \cos(2.283n)$$



Frequency aliasing

$\Omega = 5$

$$x[n] = \cos(5n) = \cos(2\pi - 5n) \approx \cos(1.283n)$$



Frequency aliasing

$\Omega = 6$

$$x[n] = \cos(6n) = \cos(2\pi - 6n) \approx \cos(0.283n)$$