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$f : \mathbb{R}^n \to \mathbb{R}$

$$\Delta f(x) = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{pmatrix} = 0.$$

$$\max f(x) = c_1 x_1 + \cdots + c_n x_n = c^T x.$$

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$ Decision variable.

$$c = \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix} =$$ Const. vector.

$$x \in S \subsetneq \mathbb{R}^n.$$

$S \to$ can be described by linear inequalities by x.

$$S = \{ x \subseteq \mathbb{R}^n \quad Ax \leq b, \; x \geq 0 \}.$$

$$A_{m \times n} \, x_{n \times 1} \leq b_{m \times 1}$$

(LP)

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max $c^T x$.

Subject to.

$x \in S = \{x \in \mathbb{R}^n, Ax \le b, x \ge 0\}$.

max/min $c^T x = \langle c, x \rangle$

Non negative Restriction

S.t. $x \in S = \{x \in \mathbb{R}^n ; Ax \le b, x \ge 0\}$

where $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, $c \in \mathbb{R}^n$.

S: feasible set.

Const.

Singleton - unique soln of LP.

$m\,[Ax \le b]$ → Consistent ⇒ $S \ne \emptyset$ → more than 1.

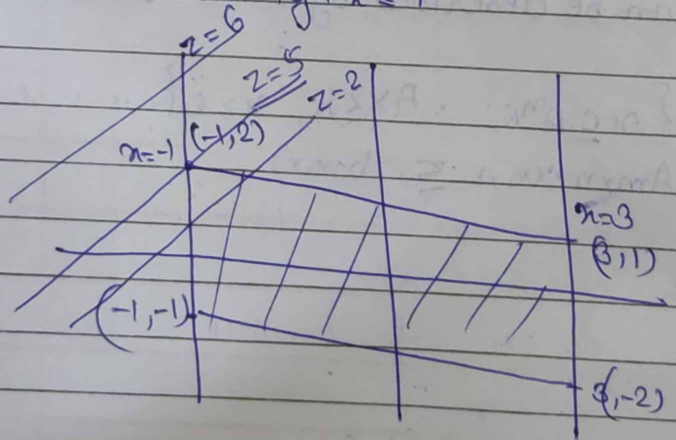
$m \times n$ $\quad x \ge 0$ → inconsistent ⇒ $S = \emptyset$ ⇒ LP is infeasible.

$n \times 1$

max $\quad -x + 2y$.

S.to $\quad -1 \le x \le 3$

$-5 \le 4y + x \le 7$



S: closed Bounded polyhedran.

$f(x,y) = -x + 2y = z$

let $z = c$ ⇝ change parameter

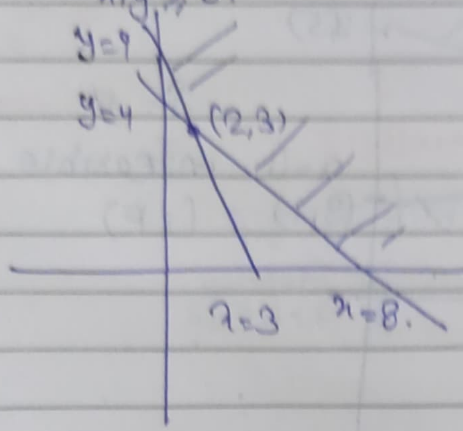
$-x + 2y = c$.

Ex min $2x+3y$

s.to $3x+2y \geq 24$

$3x+y \geq 9$

$x, y \geq 0$.



level curve $2x+3y=c$.

S: unbounded polyhedran.

$\{(x_n, y_n)\}$ in S.

$z_n = 2x_n + 3y_n \to \infty$ as $n \to \infty$.

Cases.

1) $S = \emptyset$ infeasible.

2) $S \neq \emptyset$ & $\exists$ a seq (x_k) in S.

where $x_k \in \mathbb{R}^n$ s $f(x_k) = z_k \to \infty$ for max

or $f(x_k) \to -\infty$ for min

problem is unbounded.

③. The LP is feasible, bounded and solvable.

1) unique Sol^n.

2) multiple Sol^n.

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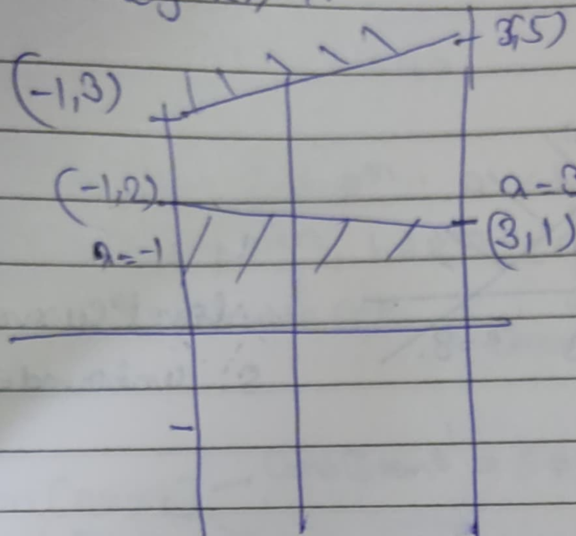
eg max $-x+2y$

$-1 \le x \le 3$

$4y+x \le 7$

$2y-x \ge 7.$



$S = \phi$

x=3 . infeasible (LP)

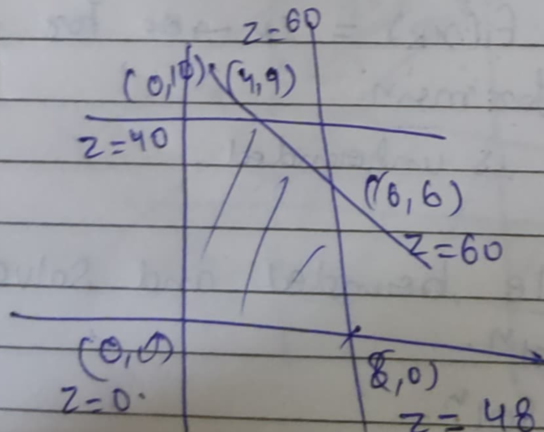
eg max $+6x+4y$

$x+4y \le 40$

$3x+2y \le 30$

$3x+y \le 24$

$x, y \ge 0.$



multiple soln of type

$(1-\alpha)(4,9) + \alpha(6,6)$ have $\underline{\underline{z=60}}$

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LP → S is a feasible set.

$S = \emptyset$ → LP is infeasible

$S \neq \emptyset$ → S is Bounded, S is unbounded

S is Bounded → unique Soln of LP; Infinite Soln of LP

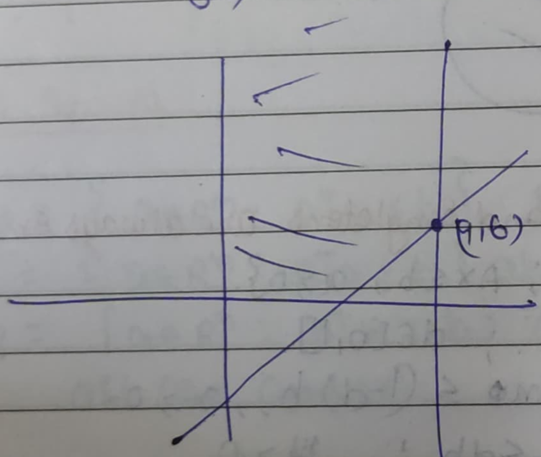
S is unbounded → unique Soln; Infinite Soln; No Soln. (is unbounded)

Q max $z = 6x - 2y \rightarrow (4,6)$ Soln.

S.to $2x - y \leq 2$

$0 \leq x \leq 4$

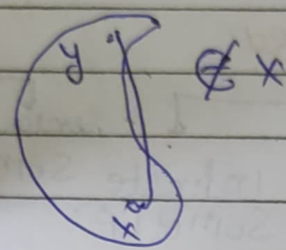
$y \geq 0$



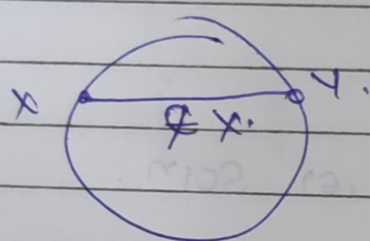
Convex Sets

If $x \in R^n$ is a convex set if $x, y \in X$ and $d \in [0,1]$ the line segment connecting x & y i.e.

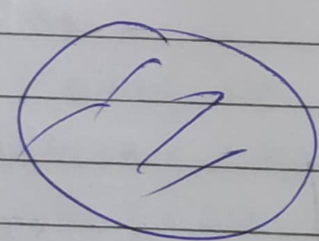
$(1-d)x + dy \in X$.



Not Convex.



$X: \{(x,y) : x^2+y^2=1\}$
(Not interior)



Convex.

★ Empty sets and Singletons are always empty.

Proof $S = \{x \in R^n ; Ax \le b, x \ge 0\}$.

Let $(x, y) \in S$; $d \in [0,1]$

$(1-d) Ax \le (1-d) b$, $x \ge 0$.

$d\,Ay \le db$, $y \ge 0$

$A((1-d)x + dy) \le b$ & $(1-d)x + dy \ge 0$.

$\Rightarrow \in S$.

S is a convex set.

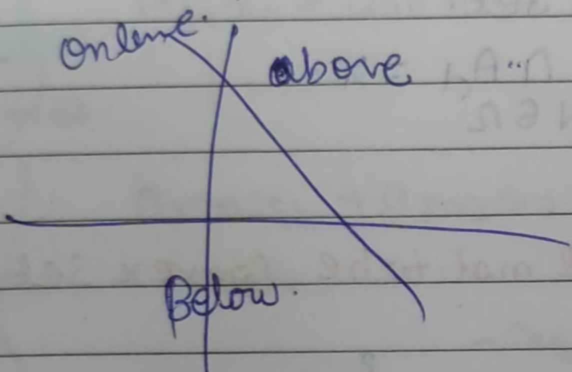
Hyperplane

Given $a \in R^n$.

$$a = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \quad b \in R$$

$\{x \in R^n : a^T x = b\} \rightarrow$ hyperplane in n dimensio.

$$\sum_{i=1}^{n} a_i x_i = b.$$



Half Spaces

$x_1 = \{x \in R^n ; A^T x \le b\}$ Below or on plane.
$x_2 = \{x \in R^n : A^T x \ge b\}$ above or on plane.
$x_3 = \{x \in R^n : A^T x = b\}$ on plane

all are Convex.

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Theorem: If $S = \{A_\alpha : \alpha \in \Omega\}$ a family of convex sets in $\mathbb{R}^n$; Ω is an index set then $X = \bigcap_{\alpha \in \Omega} A_\alpha$ is a convex set.

Proof: If $X = \emptyset$ or singleton - convex.

Let $x, y \in X$ $\mu \in [0,1]$.

$\Rightarrow x, y \in A_\alpha, \forall \alpha \in \Omega$

$\Rightarrow (1-\mu)x + \mu y \in A_\alpha \quad \forall \alpha \in \Omega.$

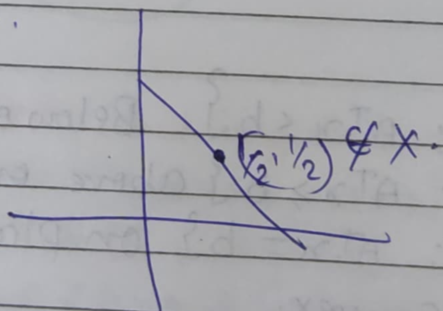
A_α is a convex set.

$\Rightarrow (1-\mu)x + \mu y \in \bigcap_{\alpha \in \Omega} A_\alpha = X.$

★ Union of convex set need not to be convex set.

$\{(x,0) : x \in \mathbb{R}\} \cup \{(0,y) : y \in \mathbb{R}\}.$

Convex.



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polyhedron.

A set of $X \subseteq R^n$ formed by intersection of finite No. of half spaces is called as polyhedron.

A bounded polyhedron - polytope.

$S = \{ x \in R^n ; Ax \leq b ; x \geq 0 \}$.

m+n inequalities / halfspaces.

$$\left\{ \begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m \\ -x_1 \leq 0 \\ -x_2 \leq 0 \\ \vdots \\ -x_n \leq 0 \end{array} \right.$$

S is intersection of (m+n) half spaces.

$\Rightarrow$ S is a closed polyhedron.