PYL101: Electromagnetics and Quantum Mechanics

Quantum Mechanics: Wave-particle duality, de-Broglie waves, quantum mechanical operators, Schrödinger equation, wave function, statistical interpretation, superposition principle, continuity equation for probability density, stationary states, bound states, free-particle solution, 1-D infinite potential well, expectation values and uncertainty relations, 1-D finite potential well, and quantum mechanical tunneling and alpha-decay.

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Reference books:

(1) Nouredine Zettili, Quantum Mechanics – concepts and applications, Wiley Publications, 2nd edition.

(2) Griffiths, D.J., Introduction to Quantum Mechanics, 2nd edition, Pearson, 2015.

Birth of Quantum Mechanics: particle aspect of radiation (blackbody radiation, photoelectric effect, Compton effect), wave aspect of particle (de Broglie's hypothesis, Davisson-Germer experiment), wave-particle duality, double-slit experiment	5
Quantum Mechanical Wave Function: wave function, representation of wave function, Schrödinger equation, probability density, statistical interpretation, superposition principle, continuity equation.	4
Quantum Mechanical Operators: observables and operators, linear operators, eigenvalues and eigen vectors of operators, Hermitian operators, product of operators, expectation values and uncertainty relations	4
Time-Independent Schrodinger Equation: stationary states, free particle solution, bound states.	3
One Dimensional Problems: 1-D infinite potential well, 1-D finite potential well, and quantum mechanical tunneling and alpha-decay.	5

Tutorials on Mon-Fri as before.

Keep checking moodle and impartus for updates on Tut sheets and lecture materials.

Lecture 1

Quantum Mechanics and its Origin

- It is the theory that describes the physics of matter at the microscopic level
- It is the modern physics which helps in understanding atoms, molecules, solids, optics, thermodynamics, statistical mechanics, semiconductors, lasers, superconductors, plasma, devices and so on

Energy and matter are quantized

Quantum Mechanics and its Origin

Questions:

- What is radiation and matter, and how does the interaction between the two take place
- Is Newtonian mechanics and/or continuum theories applicable at microscopic level
- How much do we know about atoms, molecules, nuclei, etc. and why are they the way they are (stability)
- What if these material bodies are escalated to very high speeds, comparable to speed of light

Quantum Mechanics and its Origin

Classical physics, Maxwell's waves and electromagnetism, and thermodynamics:

- These were doing well to understand most of the action and processes happening around us until a few observations in the 19th and early 20th century struck which could not be explained using existing theories.

Quantum Mechanics and its Origin

Development of Quantum mechanics

- It started taking place in bits and pieces since then until it was properly formulated by Heisenberg (1925), Schrodinger (1926), and Dirac (1928).

Quantum Mechanics and its Origin

The after story:

- At microscopic level, it is difficult to ascertain only wave nature to radiation and only particle nature to matter. (which otherwise were exclusive in classical physics)
- Both are both. It is the measurement and its type which results into one type of nature.

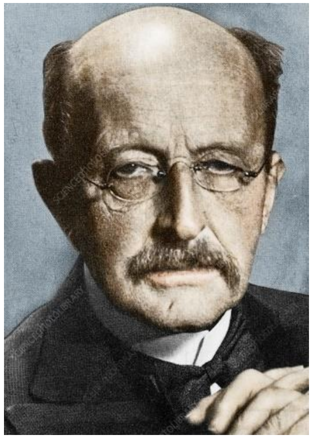
Indeterministic nature of microscopic objects/systems

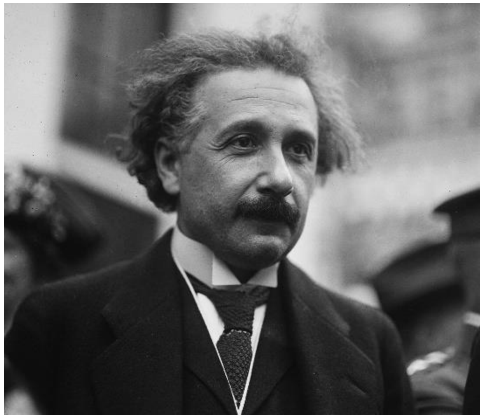
All measurement outcomes are probabilistic

Development of quantum mechanics

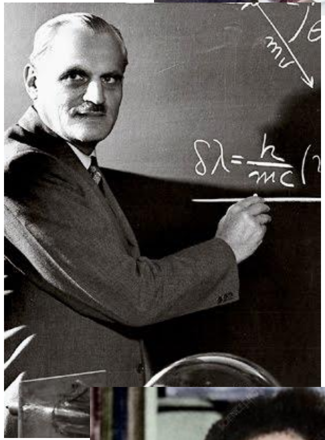
Major contributions:

- Max Planck (1900): concept of quantum of energy, Planck's constant 'h'; exchange of energy or the interaction between matter and radiation is in terms of discrete or quantized amounts, integral multiples of 'hv'
- Einstein (1905): light is made of discrete bits of energy or tiny particles called photons





Development of quantum mechanics



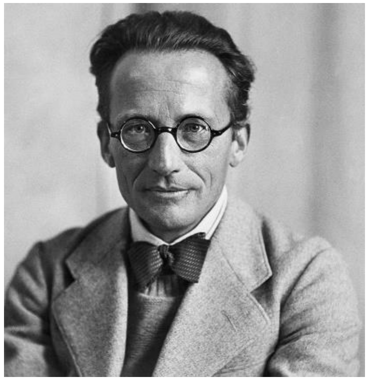
Major contributions:

- Neils Bohr (1913): hydrogen atom model to satisfactorily explain the stability of atoms and atomic spectra
- Compton (1923): particle nature of X-rays, momentum ‘h/λ’
- De Broglie (1923): wave-particle duality; wave-like nature of material particles and particle-like nature of electromagnetic radiation

Development of quantum mechanics

Major contributions:

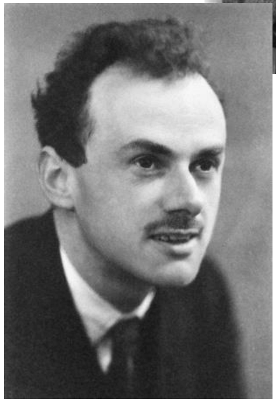
- Heisenberg (1925): matrix mechanics by describing dynamical quantities as matrices
- Schrodinger (1926): wave mechanics, wave function to describe the state of a system, Schrodinger's wave equation (use Hamiltonian for energy operator) in the form of a differential equation whose solutions provide the Eigen energy spectrum and Eigen states



Development of quantum mechanics

Major contributions:

- Max Born (1927): probabilistic interpretation of the wave function, square moduli of wave functions as the probability densities
- Davisson and Germer (1927): wave-like nature of electrons, electron scattering from solids
- PAM Dirac (1928): combined Einstein's special relativity and quantum mechanics (Heisenberg and Schrodinger) to formulate Dirac's theory of relativistic quantum mechanics



Development of quantum mechanics
(it followed from experiments)

- **Black body radiation**
- **Photoelectric effect**
- **Compton effect (X-ray scattering by electrons)**
- **Pair production (Anderson, 1932: photons from cosmic rays disappear by creating an electron and a positron)**
- **Davisson and Germer experiment of electron diffraction from solids**
- **Thomson and others' experiments on electron diffraction from thin films**
- **Two-slit interference experiments with electrons**

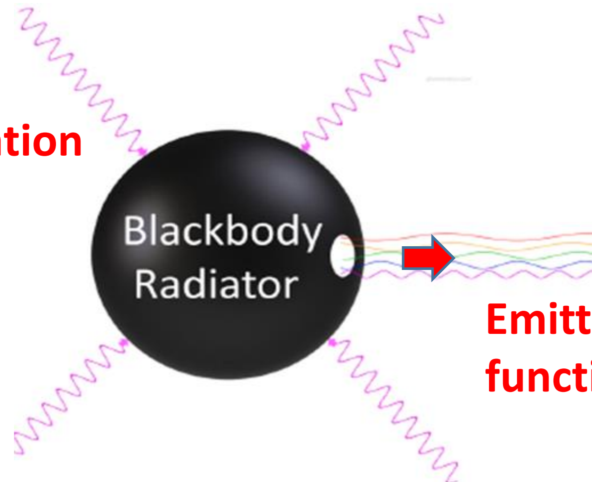
Particle aspects of radiation (Planck, Einstein)

Wave aspects of matter particles (de Broglie)

Black body radiation

- An object that absorbs all radiation falling on it, (appears as black under reflection when illuminated)
- In thermal equilibrium with its surroundings, it radiates as much energy as it absorbs

All incident radiation is absorbed

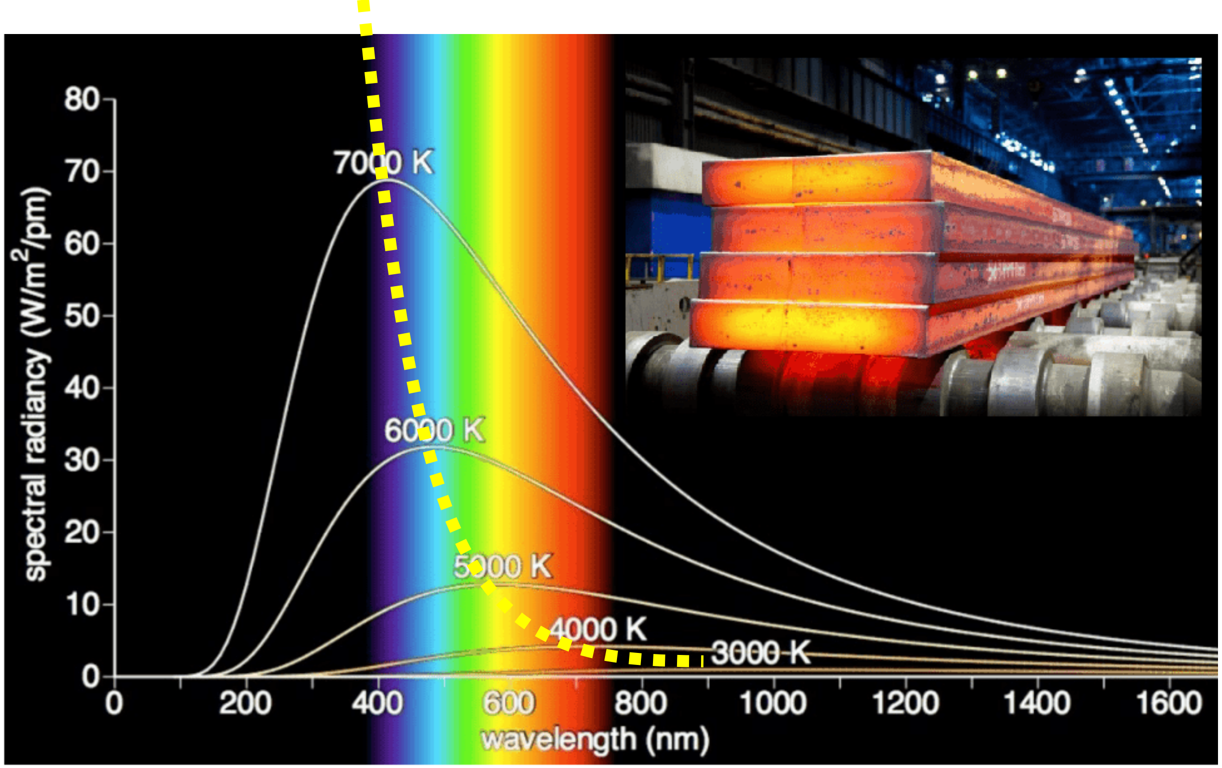


Emitted radiation a function of body T

Stefan – Boltzmann, 1879: Experimental observation

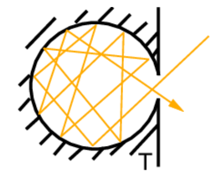
- Total intensity or the power per unit surface area,
  (Stefan-Boltzmann constant, σ = 5.67 x 10^{-8} $Wm^{-2}K^{-4}$)

$$P = \sigma T^4$$



$$\lambda_{peak} T = 2.898 \times 10^{-3}\ mK$$

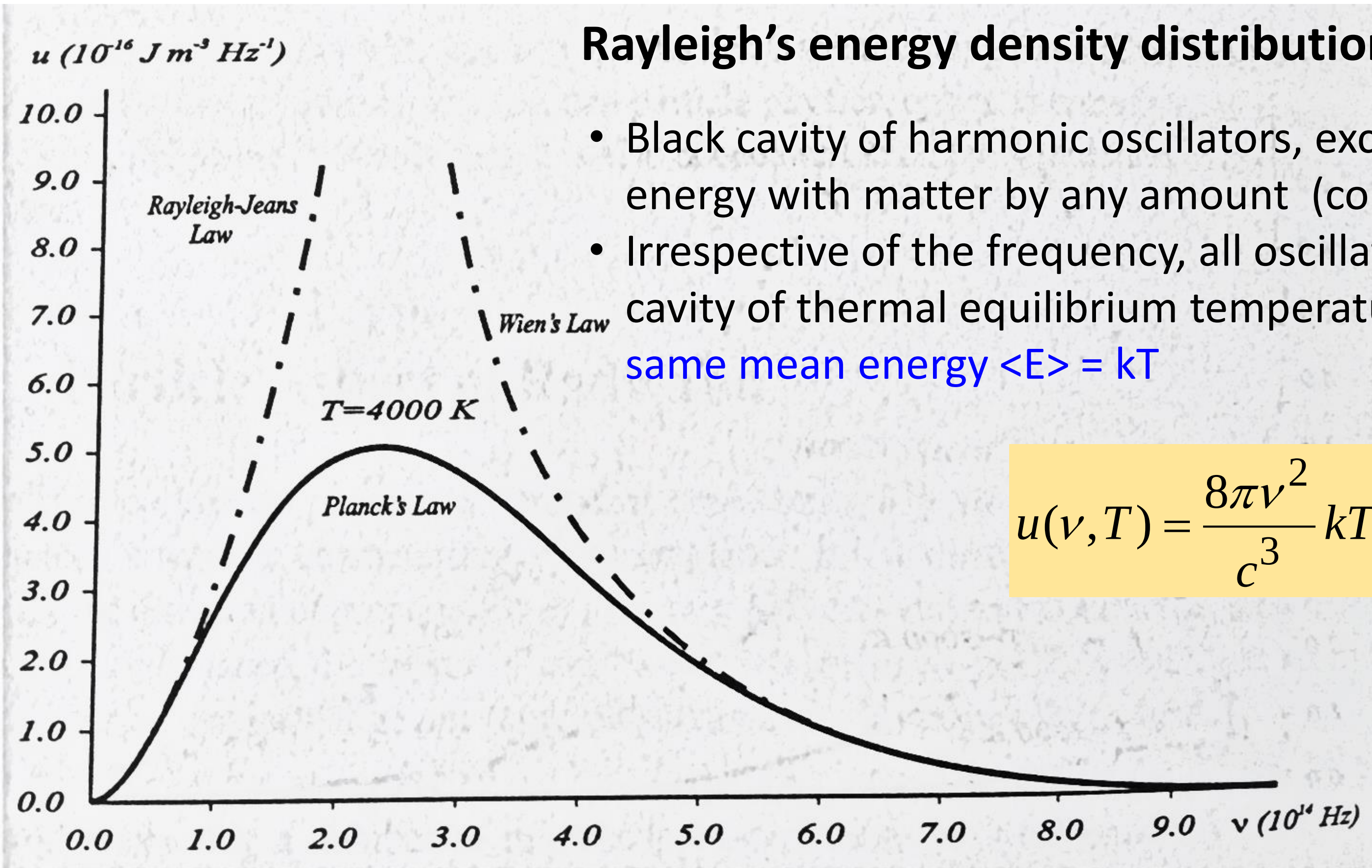
Black cavity at T



Wien's energy density distribution (1894)

- Empirical formula based on thermodynamic arguments

$$u(\nu,T) = A\nu^3 e^{-\beta\nu/T}$$



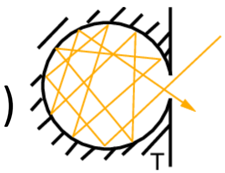
Rayleigh's energy density distribution (1900)

- Black cavity of harmonic oscillators, exchange of energy with matter by any amount (continuum)
- Irrespective of the frequency, all oscillators inside the cavity of thermal equilibrium temperature T, have the same mean energy <E> = kT

$$u(\nu,T) = \frac{8\pi\nu^2}{c^3}kT$$

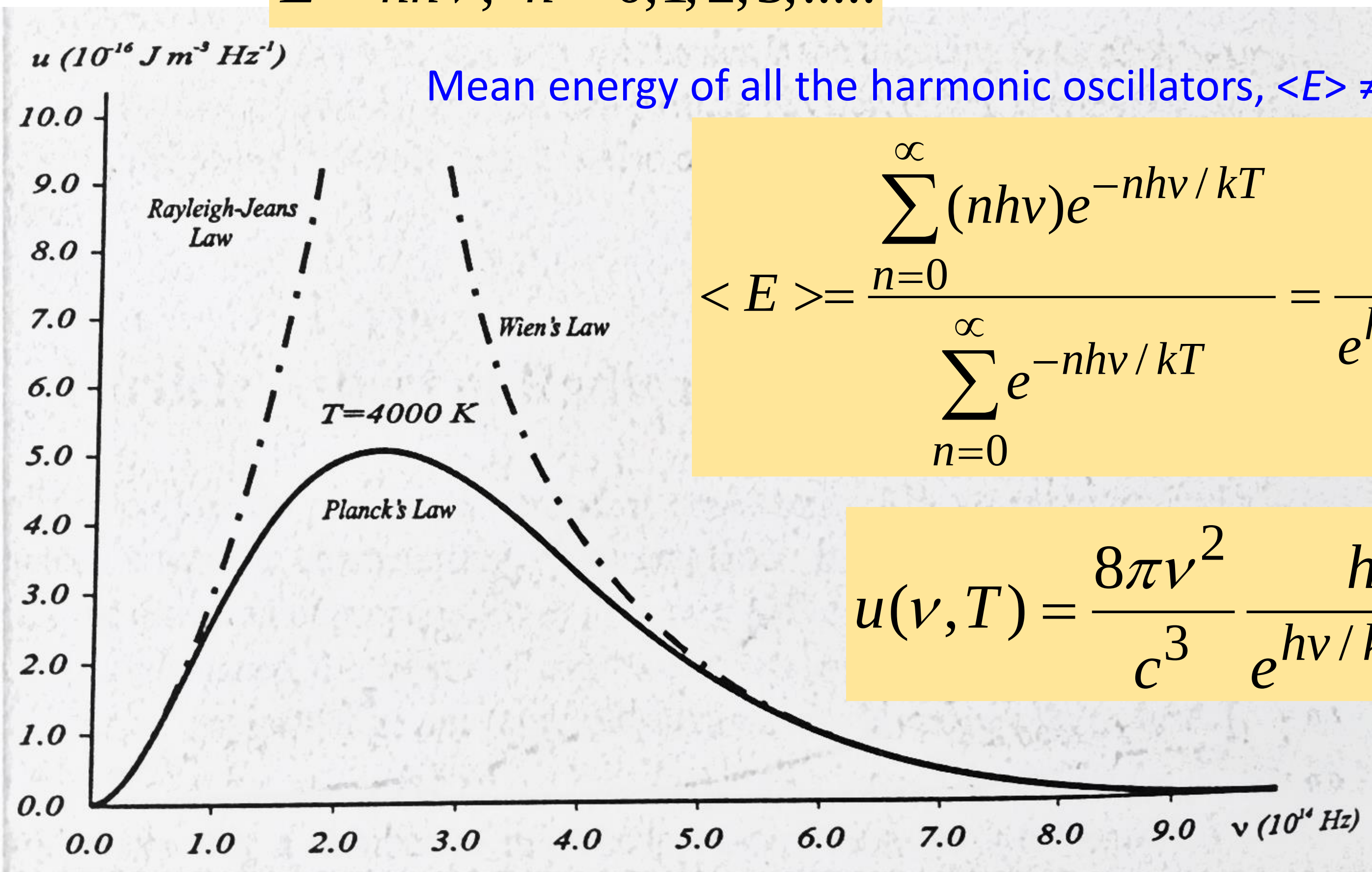
Planck's energy density distribution (1900)

Black cavity at T



- Energy exchange between radiation and matter is discrete (quanta)
- Energy emitted by the oscillators is integer multiple of 'hν'

$$E = nh\nu; \;\; n = 0, 1, 2, 3, ...$$



Mean energy of all the harmonic oscillators, <*E*> ≠ *kT*

$$< E >= \frac{\sum_{n=0}^{\infty} (nh\nu) e^{-nh\nu / kT}}{\sum_{n=0}^{\infty} e^{-nh\nu / kT}} = \frac{h\nu}{e^{h\nu / kT} - 1}$$

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu / kT} - 1}$$

Hands on

- **Convert Planck's formula in terms of λ**
- **Calculate total energy by integrating the energy density (from ν = 0 to ∞) and compare with Stefan-Boltzmann's relation**
- **Find the low frequency limit of the Planck's formula**
- **Find the high frequency limit of the Planck's formula**
- **Obtain numeric value of Planck's constant from fitting**

  $h = 6.626 \times 10^{-34}$ Joules.Sec