Note Title 01-04-2021

Lecture 9

Impulse-momentum relations

Impulse $\underline{I}$ of a force $\underline{F}$ & its angular impulse $\underline{I}_{ang\,A}$ about a point A for time interval (t_1, t_2) are defined by

$$\underline{I}(t_1, t_2) = \int_{t_1}^{t_2} F(t)\,dt$$

$$\underline{I}_{ang\,A}(t_1, t_2) = \int_{t_1}^{t_2} \underline{M}_A(t)\,dt$$

Instantaneous impulse act at t_1 if

$$\underline{I}(t_1) = \lim_{t_1 \to t_2} \int_{t_1}^{t_2} F(t)\, dt \neq 0$$

$$\underline{I}_{ang\,A}(t_1) = \lim_{t_1 \to t_2} \int_{t_1}^{t_2} \underline{M}_A(t)\, dt \neq 0$$

Eulers 1st axiom $\Rightarrow$

$$I(t_1, t_2) = \int_{t_1}^{t_2} \underline{F}\, dt = \int_{t_1}^{t_2} \dot{\underline{p}}\, dt = \int_{\underline{p}(t_1)}^{\underline{p}(t_2)} d\underline{p}$$

$$\Rightarrow I(t_1, t_2) = p(t_2) - p(t_1) = \Delta \underline{p} = m \Delta \underline{v}_c = \sum_i m_i \Delta v_{ci}$$

Impulse of external force $\underline{F}$ equals the change in momentum

Inst. Impulse $\Rightarrow$

$$\underline{I}(t_1) = \Delta \underline{p} = \underline{p}(t_1^+) - \underline{p}(t_1^-) = m \, \Delta \underline{v}_c = \sum m_i \, \Delta v_{c_i}$$

all changes are instantaneous (there is no change in position)

if $I(t_1, t_2) = 0 \Rightarrow$ conservation of momentum

$$p(t_2) = p(t_1) \; ; \; v_c(t_2) = v_c(t_1) \; ; \; \sum m_i v_{c_i}(t_2) = \sum m_i v_{c_i}(t_1)$$

If a particular component of impulse is zero, then the corresponding component of momentum is conserved

Angular impulse - moment of momentum relations

All wrt Inertial frame I $\qquad \underline{a}_A \equiv \underline{a}_A|_I$ etc

If A is a point such that $\underline{a}_A = 0$ or $A \equiv C$ or $\underline{a}_A \parallel \underline{AC}$

$$\Rightarrow \underline{M}_A = \dot{\underline{H}}_A$$

$$\underline{I}_{ang\,A}(t_1, t_2) = \int_{t_1}^{t_2} \underline{M}_A \, dt = \int_{\underline{H}_A(t_1)}^{\underline{H}_A(t_2)} d\underline{H}_A$$

$$= \underline{H}_A(t_2) - \underline{H}_A(t_1) = \Delta \underline{H}_A$$

external angular impulse about A equals the change in moment of momentum about A

- if $\underline{I}_{ang\,A}(t_1, t_2) = 0 \Rightarrow$ conservation of moment of momentum about A $\underline{H}_A(t_2) = \underline{H}_A(t_1)$

- if $\underline{M}_A = \underline{0}$ then $\underline{H}_A(t) = \underline{H}_{A0}$

If A is fixed in I; if components of $\underline{I}_{ang\,A}(t_1, t_2)$ along a fixed direction is zero then H_A along that direction is conserved

- If inst ang impulse is considered, no change in configuration; but a change in $\underline{H}_A$ or $\underline{H}_C$ or $\underline{H}_O$

A ! such that $\underline{a}_A = 0$, $a_A \parallel \underline{CA}$ or $\underline{CA} = 0$

O – fixed in I

$$\underline{I}_{ang\,C}(t_1) = \Delta \underline{H}_C, \qquad \underline{I}_{ang\,O}(t_1) = \Delta \underline{H}_O$$

$$\underline{I}_{ang\,A}(t_1) = \Delta \underline{H}_A$$

But $\underline{I}_{ang\,B}(t_1) \neq \Delta \underline{H}_B$ for general B

Work energy relation for centre of mass C

$$\underline{F} \cdot \underline{v}_{C|I} = m\underline{a}_{C|I} \cdot \underline{v}_{C|I} = \frac{d}{dt}\left[\frac{1}{2} m\, \underline{v}_{C|I} \cdot \underline{v}_{C|I}\right]\Big|_I$$

$$= \frac{d}{dt}\left[\frac{1}{2} m v_{C|I}^2\right]\Big|_I$$

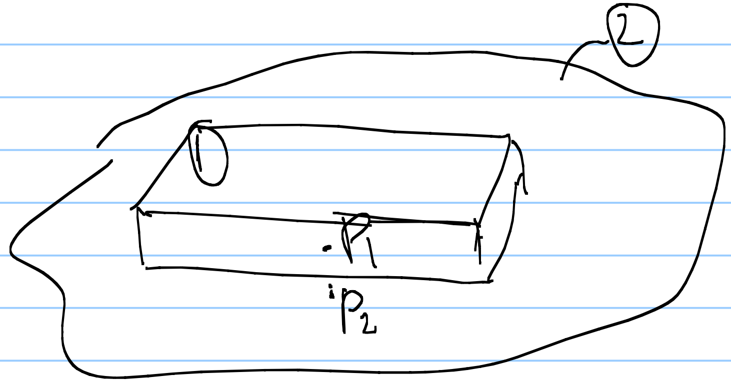
$$\Rightarrow \quad \dot{T}_C = \dot{W}^* \qquad \left(T_{C_2} - T_{C_1} = W^*_{1-2} \right)$$

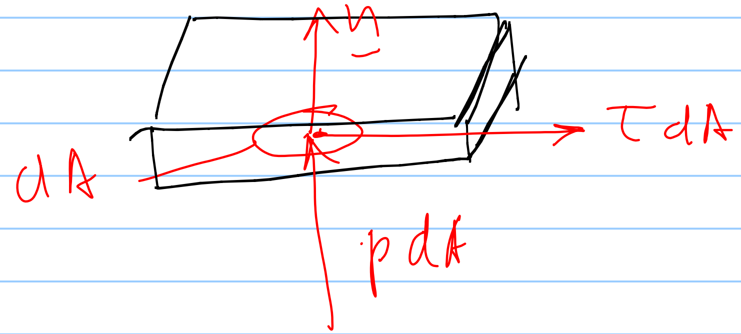
$\dot{W}^* = \underline{F} \cdot \underline{v}_C$ = rate of work done by forces as if acting at C $[\neq \dot{W}$ in general$]$

$T_C = \frac{1}{2} m v_{C|I}^2$ – kinetic energy as if all mass was concentrated at c [$\nexists T$ in general]

Axiom of Coulomb friction

$P_1 - P_2$ in contact





Normal force $\rightarrow$ $p\, dA$

tangential force $\rightarrow$ $\tau\, dA$

Coulomb's axiom of friction

for no slip bet'n (1 & 2) P_1 & P_2 $\boxed{\tau < \mu_s p}$

direction & magnitude of τ are determined from the eqns of motion.

for impending slip bet'n P_1 and P_2

$\tau = \mu_s p$ $\underline{\tau}$ directed opposite to the component of $\underline{a}_{P_1P_2}$ in the tangent plane $\perp$ $\underset{\sim}{n}$

for slip bet'n P_1 and P_2,

$\tau = \mu_k p$ directed opposite to $\underline{v}_{P_1P_2}$

μ_s — static coeff of friction

μ_k — dynamic coeff of friction.

If the contact area is a plane then $N = \int p \, dA$

$F = \int \tau \, dA$

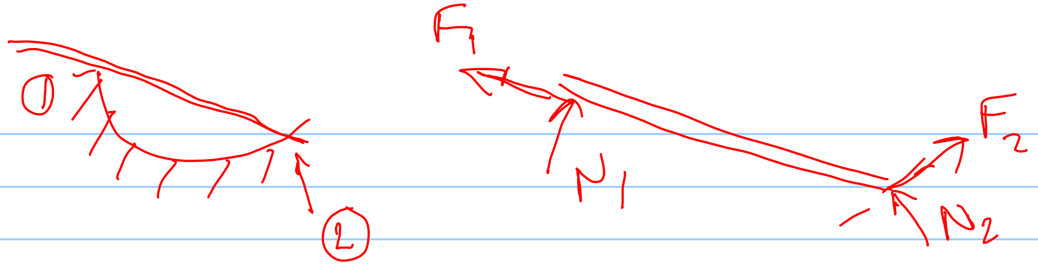
For no slip $F < \mu_s N$

Imp slip $F = \mu_s N$

Slip $F = \mu_k N$

(X)

Discrete point of contact? (X) are valid



$$\underline{F} \longrightarrow \parallel \underline{e}_t$$

$$\underline{N} \longrightarrow n_1 \underline{e}_n + n_2 \underline{e}_b$$

Free body diagrams

To apply Euler's axioms

① The system should be well identified and its sketch drawn in isolation from its surroundings

② External forces and moments exerted by the surroundings on the system should be drawn on the sketch

Such a diagram is called the Free body diagram (FBD)

Internal forces & internal moments should not be shown on FBD.

Equations of equilibrium

If a system is in equilibrium, then for its every part,

$$\underline{F} = 0, \quad \underline{M}_A = 0$$

Proof If O is fixed point in I

$$\underline{v}_p|_I = 0, \quad \forall t$$

$$\underline{p}|_I = \int \underline{v}_P|_I \, dm = 0$$

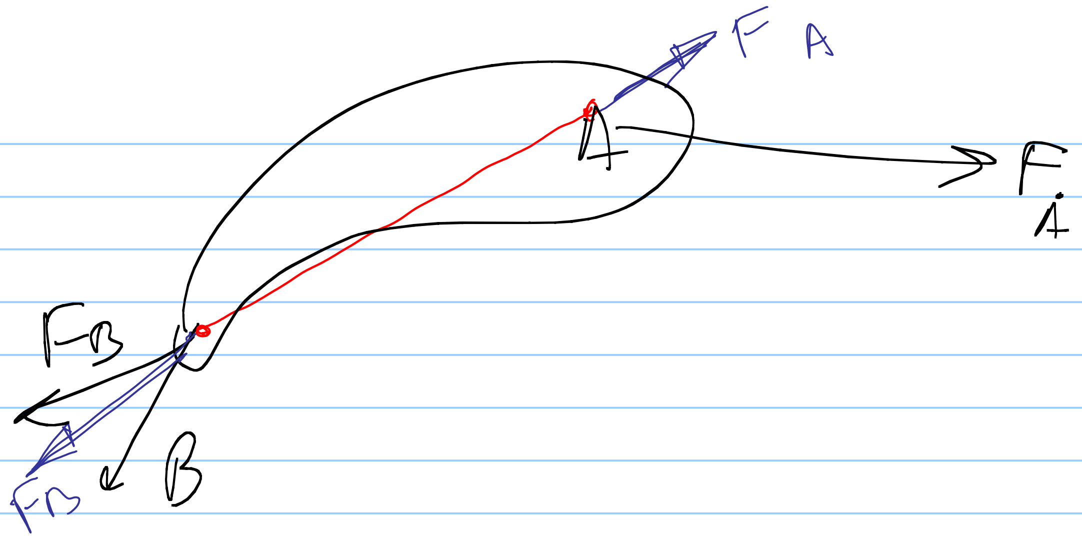
$$\underline{H}_O|_I = \int \underline{r}_{PO}|_I \times \underline{v}_{P}|_I \, dm = 0 \qquad \forall t$$

Euler's Axioms $\Rightarrow$ $\underline{F} = \dot{\underline{p}}|_I = 0$

$$\underline{M}_O = \dot{\underline{H}}_O|_I = 0 \implies \underline{M}_A = \underbrace{\underline{M}_O}_{=0} + \underline{r}_{OA} \times \underline{F}$$

Necessary but not sufficient for equilibrium.

<u>Two force member</u> A member under the action of two forces <u>only</u> (no moments)



For equilibrium

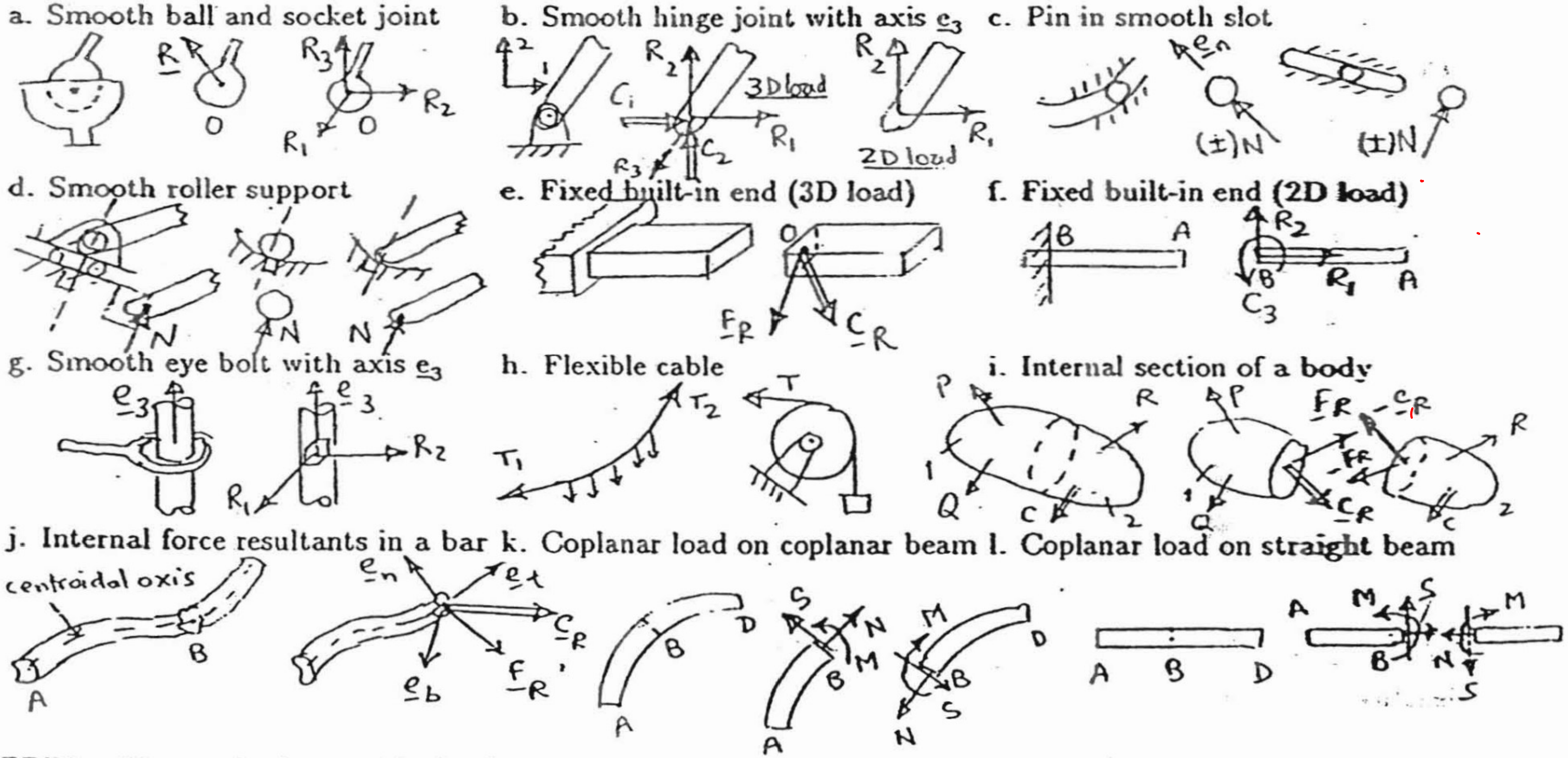
$$\underline{F}_A + \underline{F}_B = 0$$

abt A $\quad r_{BA} \times F_B = 0 \Rightarrow F_B$ is thru A along AB

abt B $\quad \underline{r}_{AB} \times \underline{F}_A = 0$ $\quad F_A$ is through B along AB

$$F_A = -F_B$$

Forces have equal magnitudes, opposite directions and act along the line joining their points of application



FBD's of front wheel assembly (m_1), rear wheel assembly (m_2), chassis (m_3), and the complete road-roller with driving torque M on the rear wheels, assuming no slip, are given in Fig. 2.19: