Lecture 9
Signals and Systems (ELL205)

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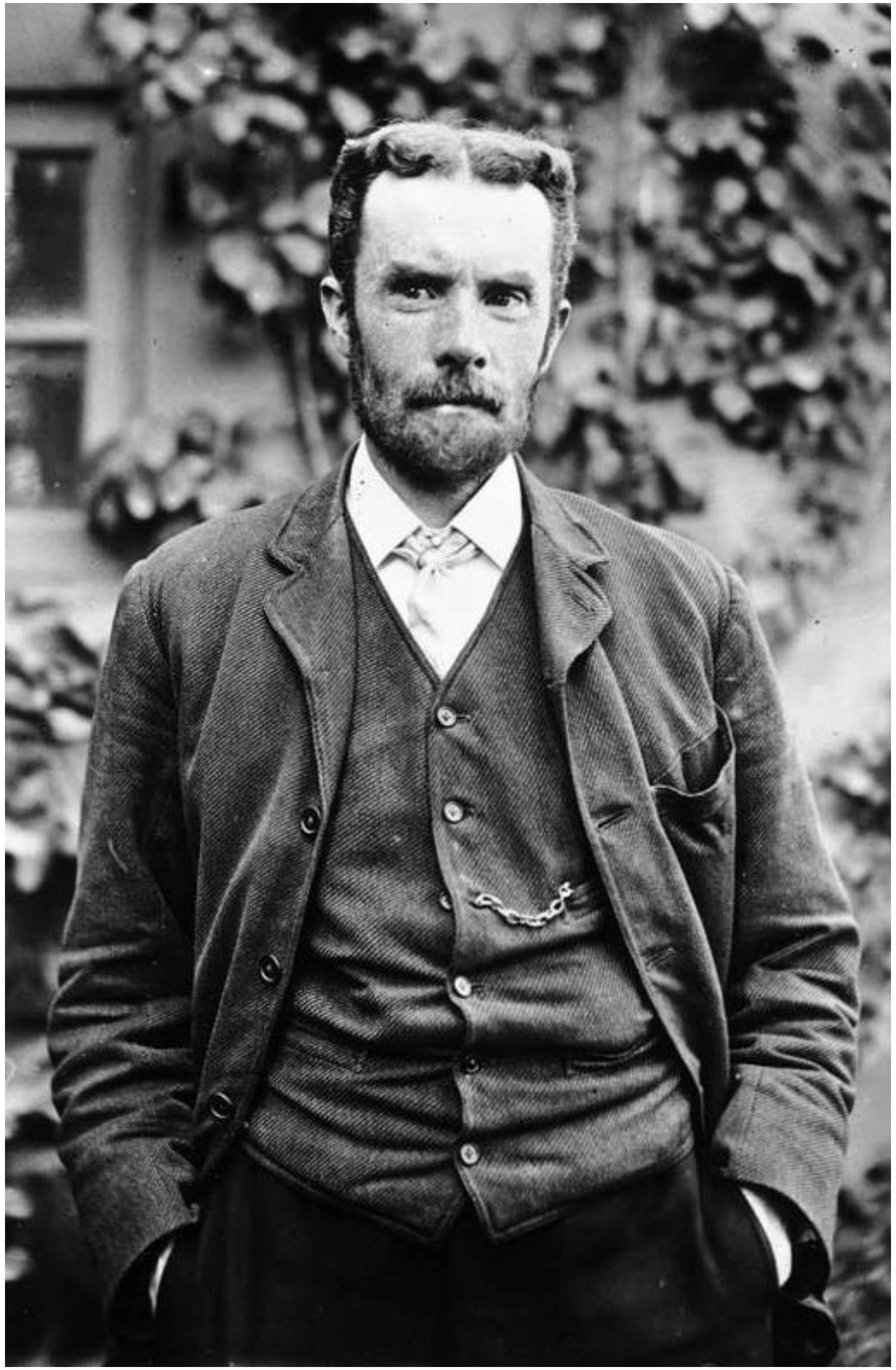
Oliver Heaviside

(18 May 1850 – 3 February 1925).

He invented mathematical techniques for the solution of differential equations (equivalent to Laplace transforms).

He quoted "I do not refuse my dinner simply because I do not understand the process of digestion"

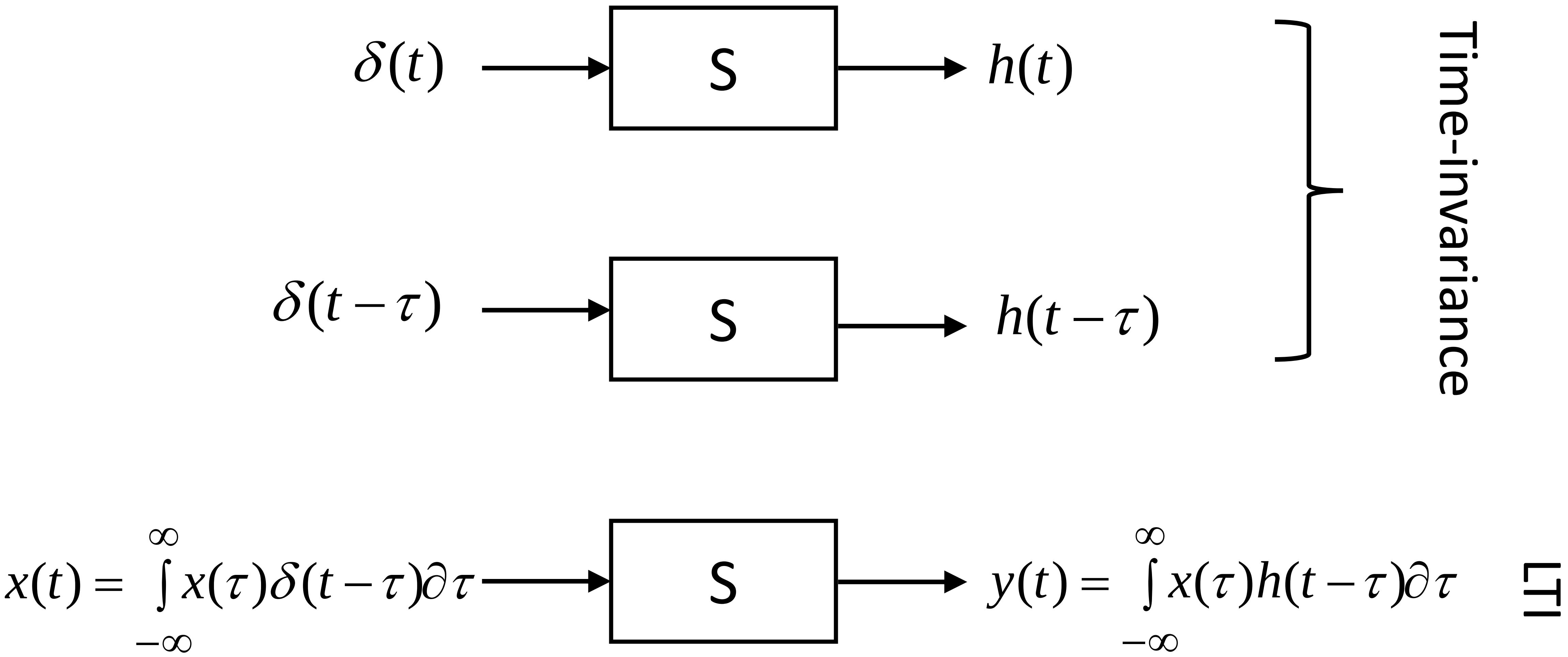
He was a polymath. He was electrical engineer, mathematician, physicist, a great musician, and could speak professionally more than 8 languages.



$$\left.\begin{array}{l} \delta[n] \longrightarrow \boxed{S} \longrightarrow h[n] \\[2em] \delta[n-n_o] \longrightarrow \boxed{S} \longrightarrow h[n-n_o] \end{array}\right\} \text{Time-invariance}$$

Linearity & Time Invariance

$$x[n]=\sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \longrightarrow \boxed{S} \longrightarrow y[n]=\sum_{k=-\infty}^{\infty} x[k]h[n-k]$$


$\delta(t)$
S
$h(t)$
$\delta(t-\tau)$
S
$h(t-\tau)$
Time-invariance
$x(t)=\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\partial\tau$
S
$y(t)=\int_{-\infty}^{\infty} x(\tau)h(t-\tau)\partial\tau$
LTI

Convolution in DT

$$x[n] = a^n u[n] \qquad h[n] = a^n u[n]$$

$$y[n] = ?$$

1 $y[n] = (n+1)a^n u[n]$

2 $y[n] = na^n u[n]$

3 $y[n] = (n+1)a^n$

4 $y[n] = na^n$

Convolution in DT

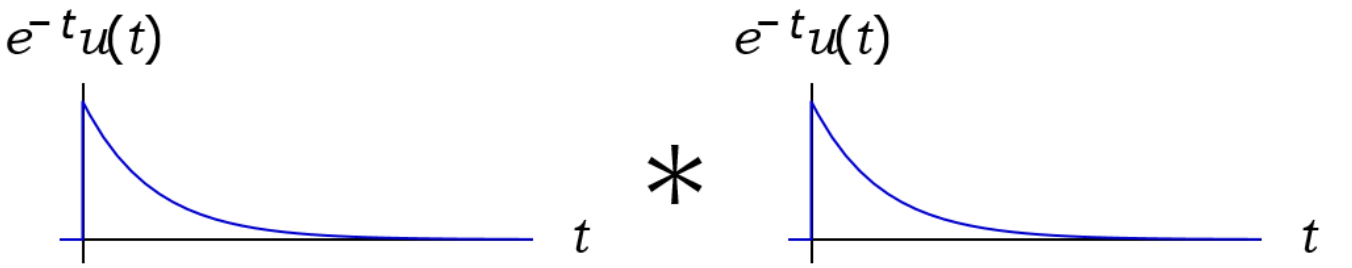
$$\mathrm{x}[n] = a^n u[n] \qquad \mathrm{h}[n] = a^n u[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \qquad y[n] = \sum_{k=-\infty}^{\infty} a^k u[k] a^{n-k} u[n-k]$$

$$y[n] = a^n u[n] \sum_{k=0}^{n} u[k]u[n-k] \qquad y[n] = a^n u[n] \sum_{k=0}^{n} 1$$

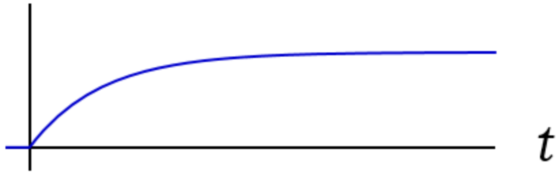
$$y[n] = (n+1)a^n u[n]$$

Which plot shows the results of the convolution below?

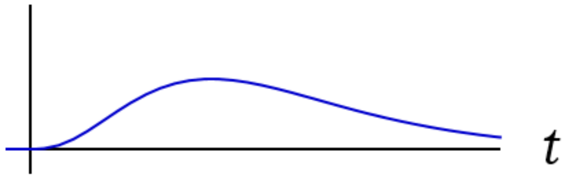


1.

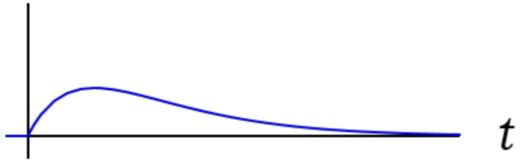
2.



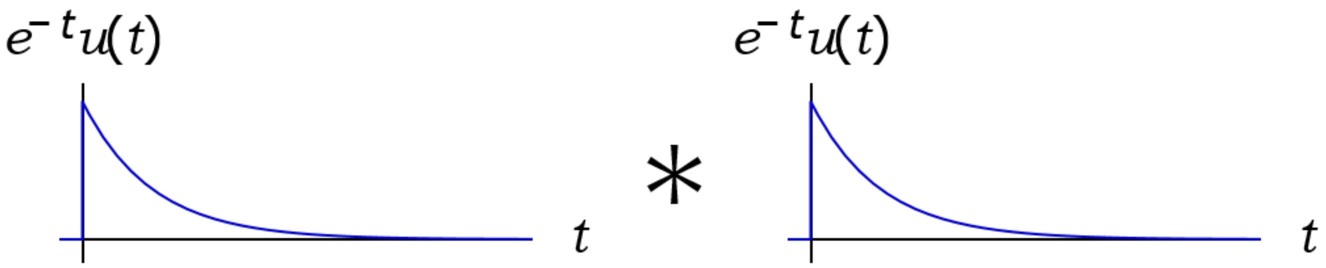
3.



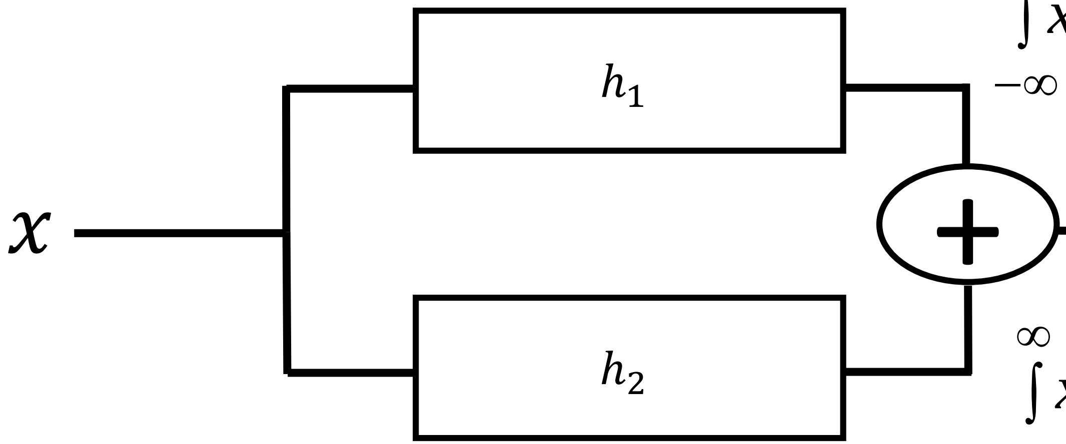
4.



Which plot shows the results of the convolution below?



Properties of Convolution

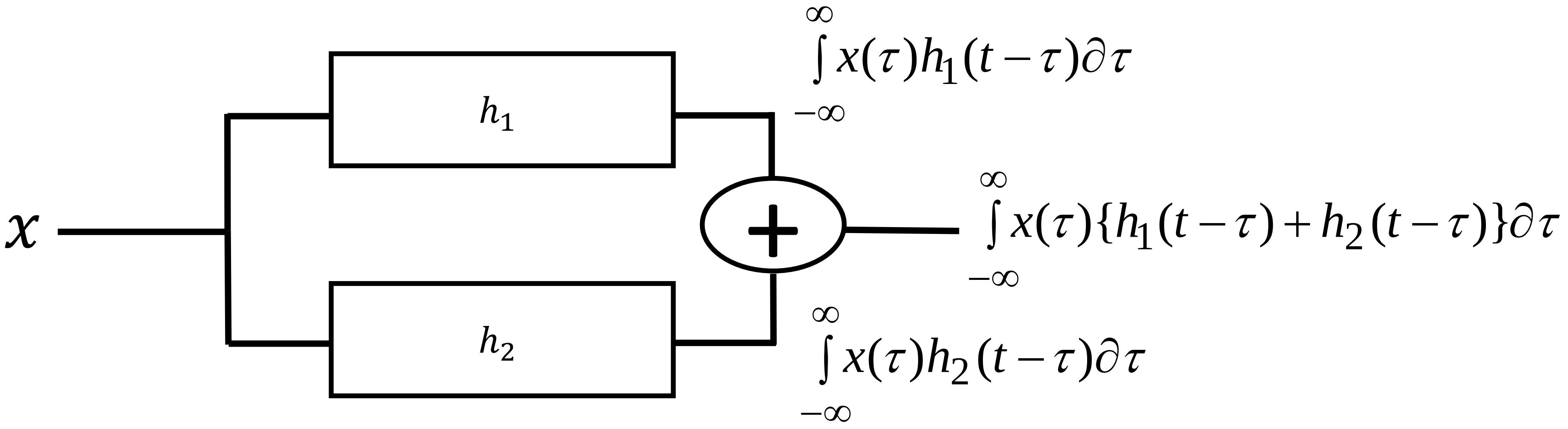


$$\int_{-\infty}^{\infty} x(\tau)h_1(t-\tau)\partial\tau$$

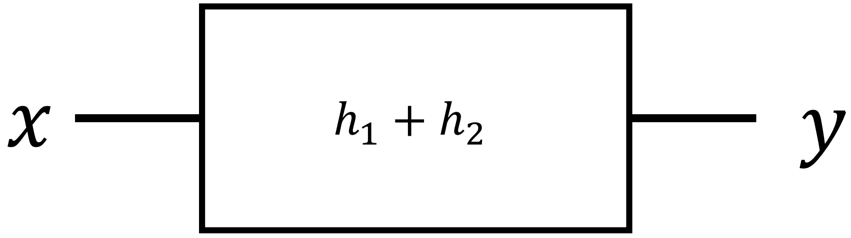
$$\int_{-\infty}^{\infty} x(\tau)\{h_1(t-\tau)+h_2(t-\tau)\}\partial\tau$$

$$\int_{-\infty}^{\infty} x(\tau)h_2(t-\tau)\partial\tau$$

Properties of Convolution

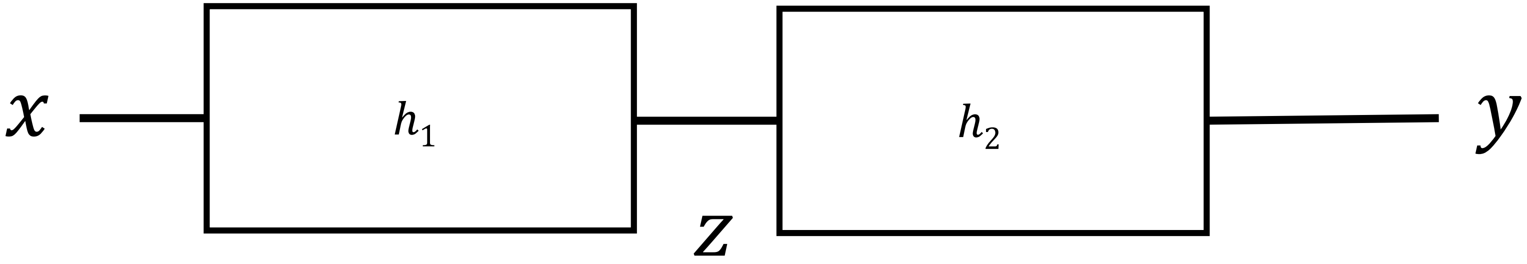


$$x * h_1 + x * h_2$$
$$= x * (h_1 + h_2)$$



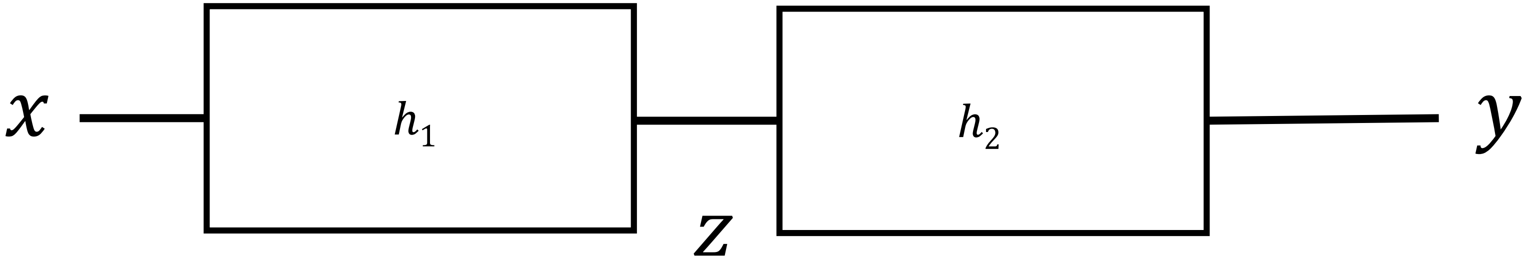
Distributivity = Parallel connection of system can be replaced by an equivalent system with impulse response $h_1 + h_2 + h_3$.....

Properties of Convolution



$$y(t) = \int_{-\infty}^{\infty} z(\tau) h_2(t-\tau)\partial\tau = \int_{-\infty}^{\infty} \{ \int_{-\infty}^{\infty} x(\upsilon) h_1(\tau-\upsilon)\partial\upsilon \} h_2(t-\tau)\partial\tau$$

Properties of Convolution



$$y(t) = \int_{-\infty}^{\infty} z(\tau)h_2(t-\tau)\partial\tau = \int_{-\infty}^{\infty} \{ \int_{-\infty}^{\infty} x(\upsilon)h_1(\tau-\upsilon)\partial\upsilon\} h_2(t-\tau)\partial\tau$$

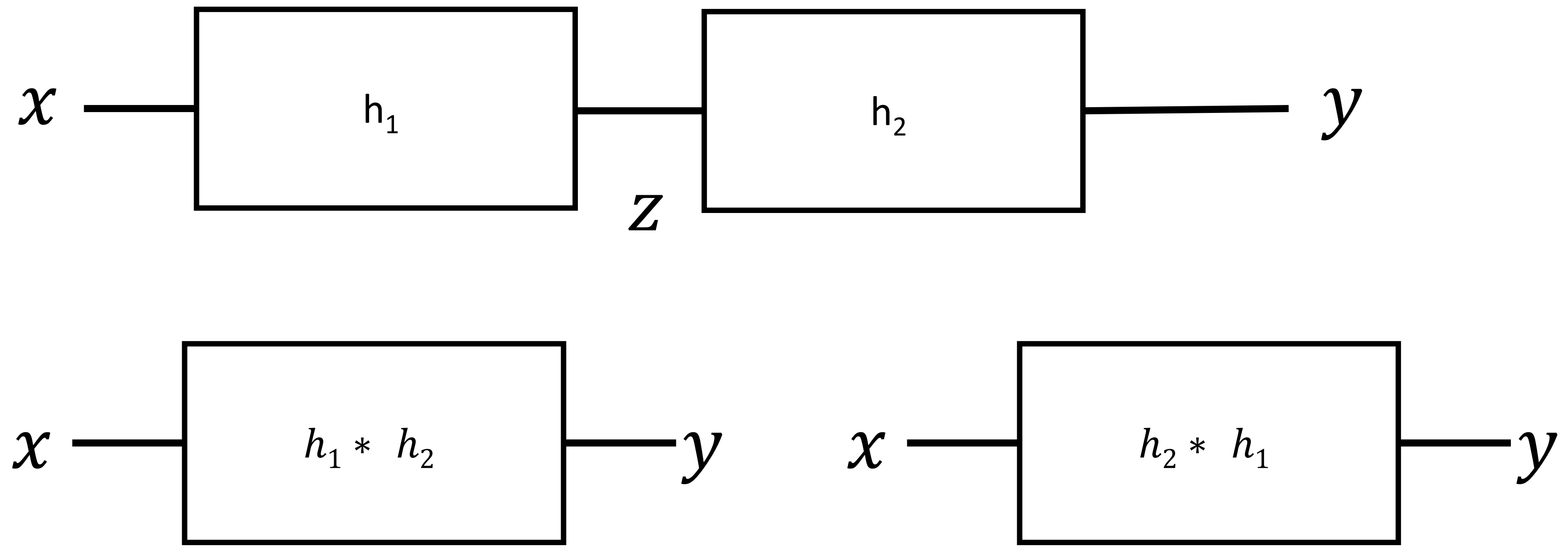
Assuming $\eta = \tau - \upsilon$

$$y(t) = \int_{-\infty}^{\infty} x(\upsilon)\{ \int_{-\infty}^{\infty} h_1(\eta)h_2(t-\upsilon-\eta)\partial\eta\}\partial\upsilon = x*(h_1*h_2)$$

$$\begin{aligned} y &= z * h_2 = (x * h_1) * h_2 \\ &= x * (h_1 * h_2) \end{aligned}$$

Convolution operation is associative !!

Properties of Convolution

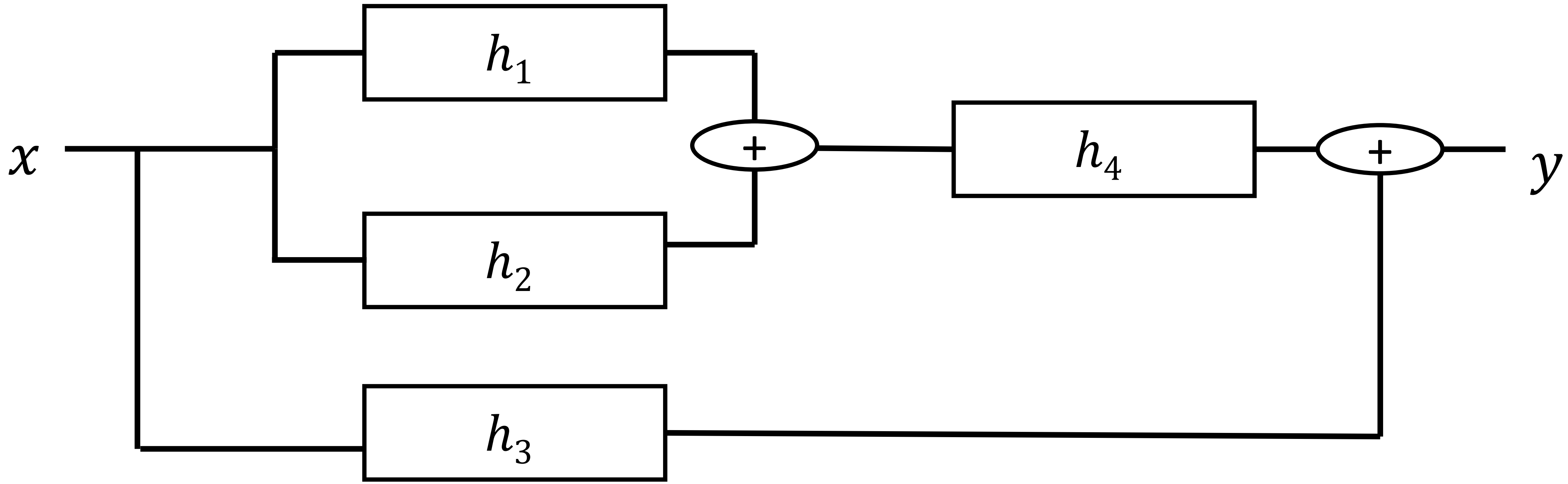


Convolution operation is commutative !!

Associativity = Cascade connection of system can be replaced by an equivalent system with impulse response $h_1 * h_2 * h_3$.....

Commutativity = Order of system is irrelevant

Question



1) $(\boldsymbol{h_1} + \boldsymbol{h_2}) * \boldsymbol{h_4} + \boldsymbol{h_3}$

2) $(\boldsymbol{h_1} + \boldsymbol{h_2}) * \boldsymbol{h_3} + \boldsymbol{h_4}$

3) $((\boldsymbol{h_1} * \boldsymbol{h_2}) + \boldsymbol{h_4}) * \boldsymbol{h_3}$

4) $((\boldsymbol{h_1} * \boldsymbol{h_2}) + \boldsymbol{h_3}) * \boldsymbol{h_4}$