Lecture 20
Signals and Systems (ELL205)

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Lecture 20: Farewell to Fourier Series and Introduction to Fourier Transforms

Two-dimensional Fourier Series

$$x(t_1, t_2) = x(t_1 + T_1, t_2 + T_2) \qquad \text{for all } t_1, t_2$$

$$x(t_1, t_2) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} a_{mn}\, e^{j(m\omega_1 t_1 + n\omega_2 t_2)}$$

$$a_{mn} = \frac{1}{T_1 T_2} \int_{T_2} \int_{T_1} x(t_1, t_2) e^{-j(m\omega_1 t_1 + n\omega_2 t_2)}\, dt_1 dt_2$$

Chessboard

$$x(t_1, t_2) = x(t_1 + T_1, t_2 + T_2) \text{ for all } t_1, t_2$$

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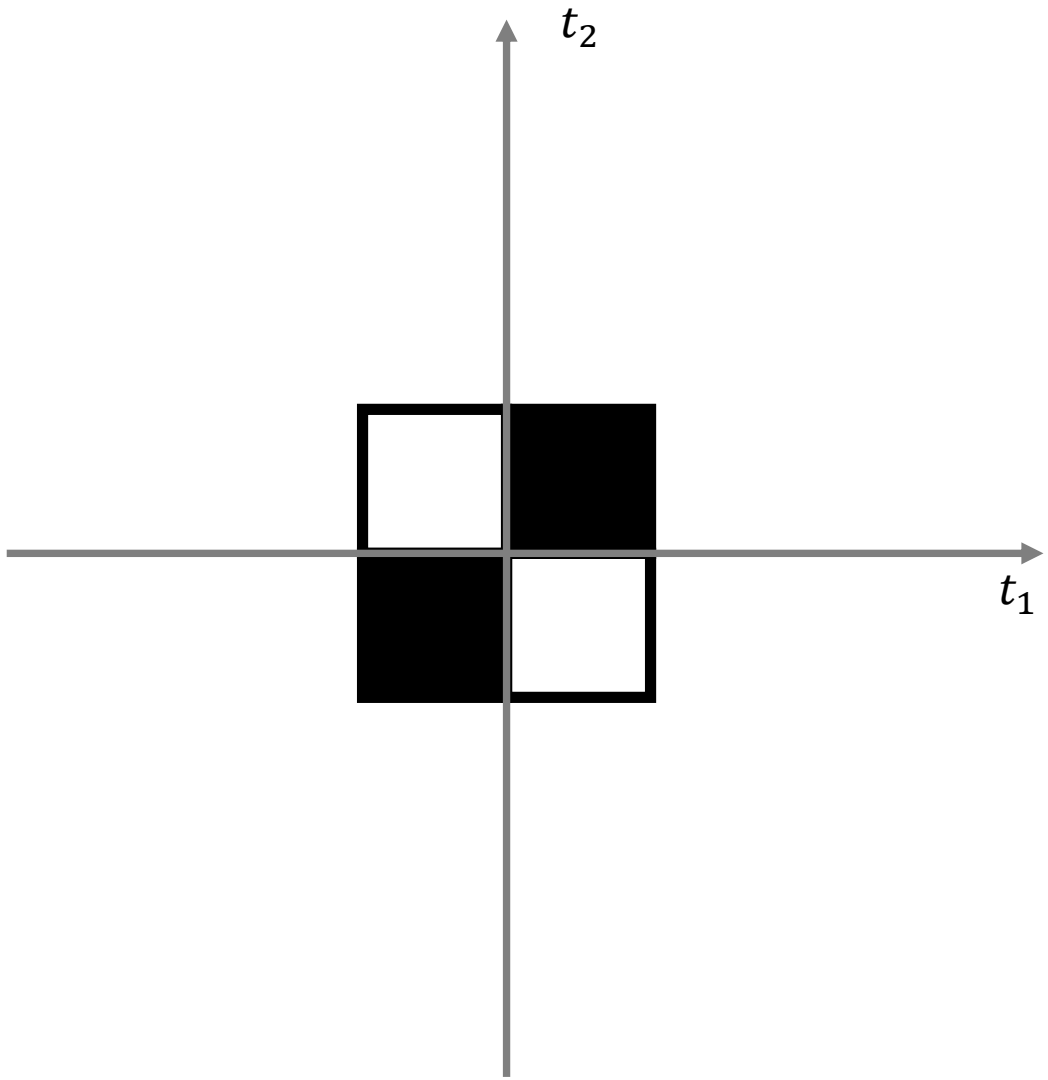
$$a_{mn} = ?$$



Chessboard

$$x(t_1, t_2) = x(t_1 + T_1, t_2 + T_2) \text{ for all } t_1, t_2$$

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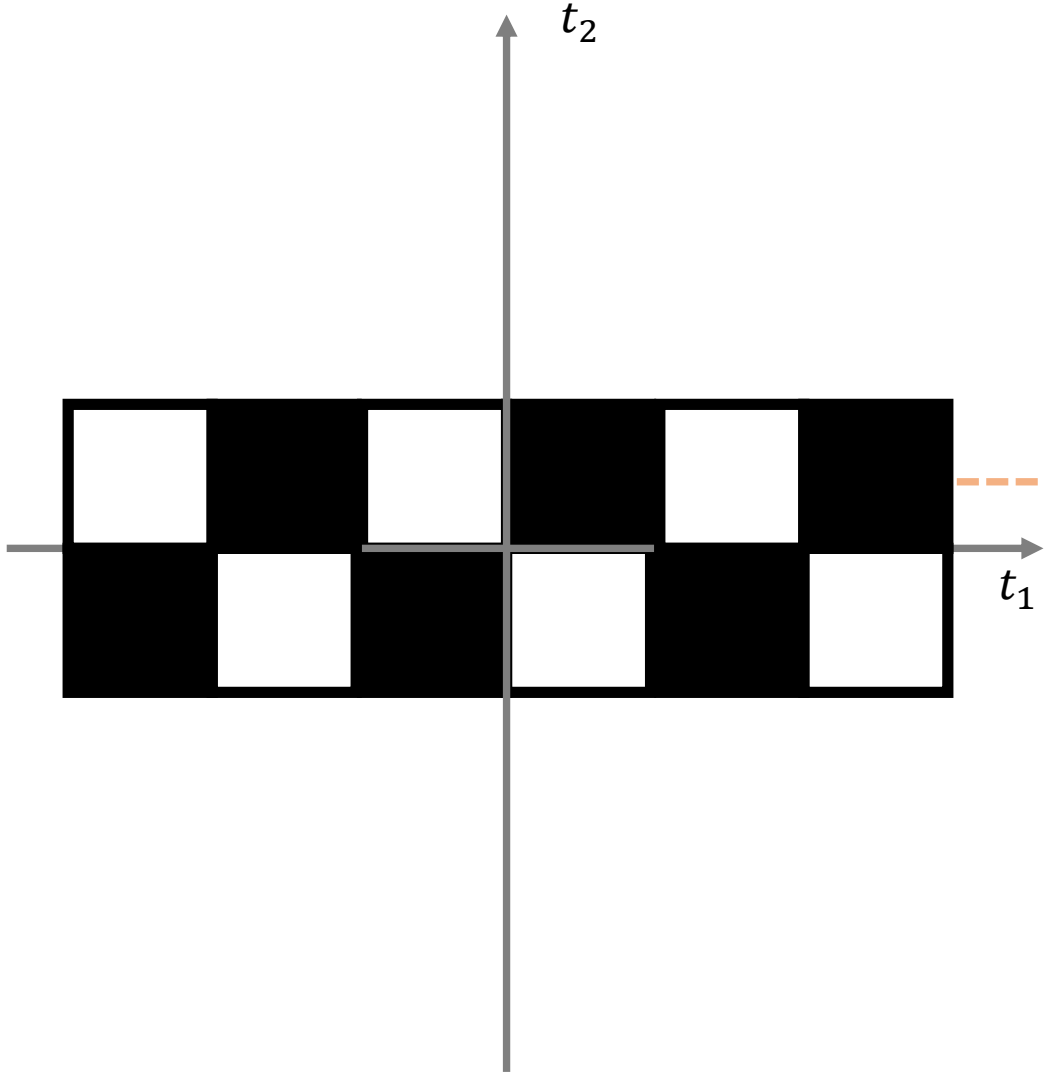

Chessboard

$$x(t_1, t_2) = x(t_1 + T_1, t_2 + T_2) \text{ for all } t_1, t_2$$

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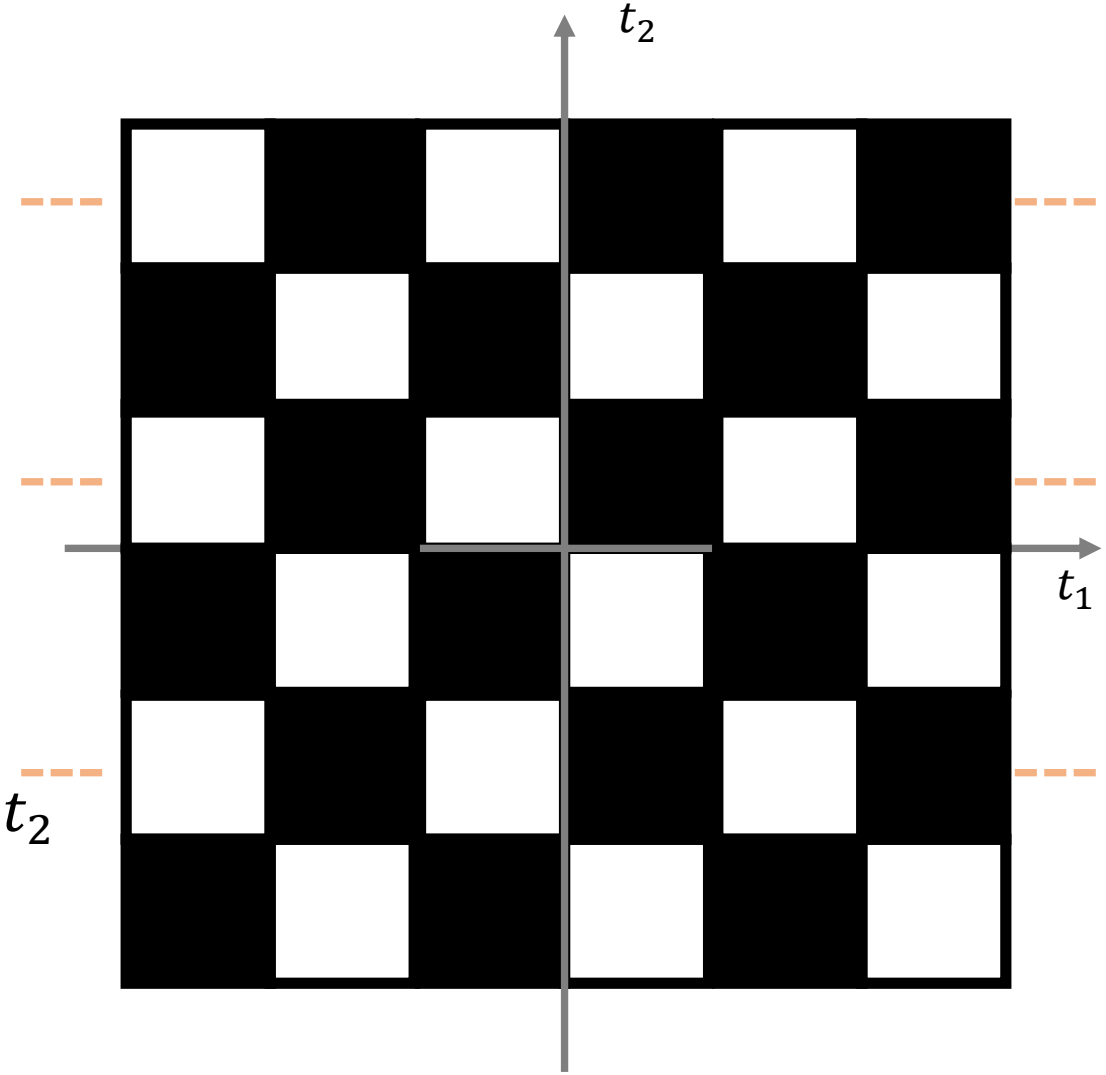
$$a_{mn} = ?$$

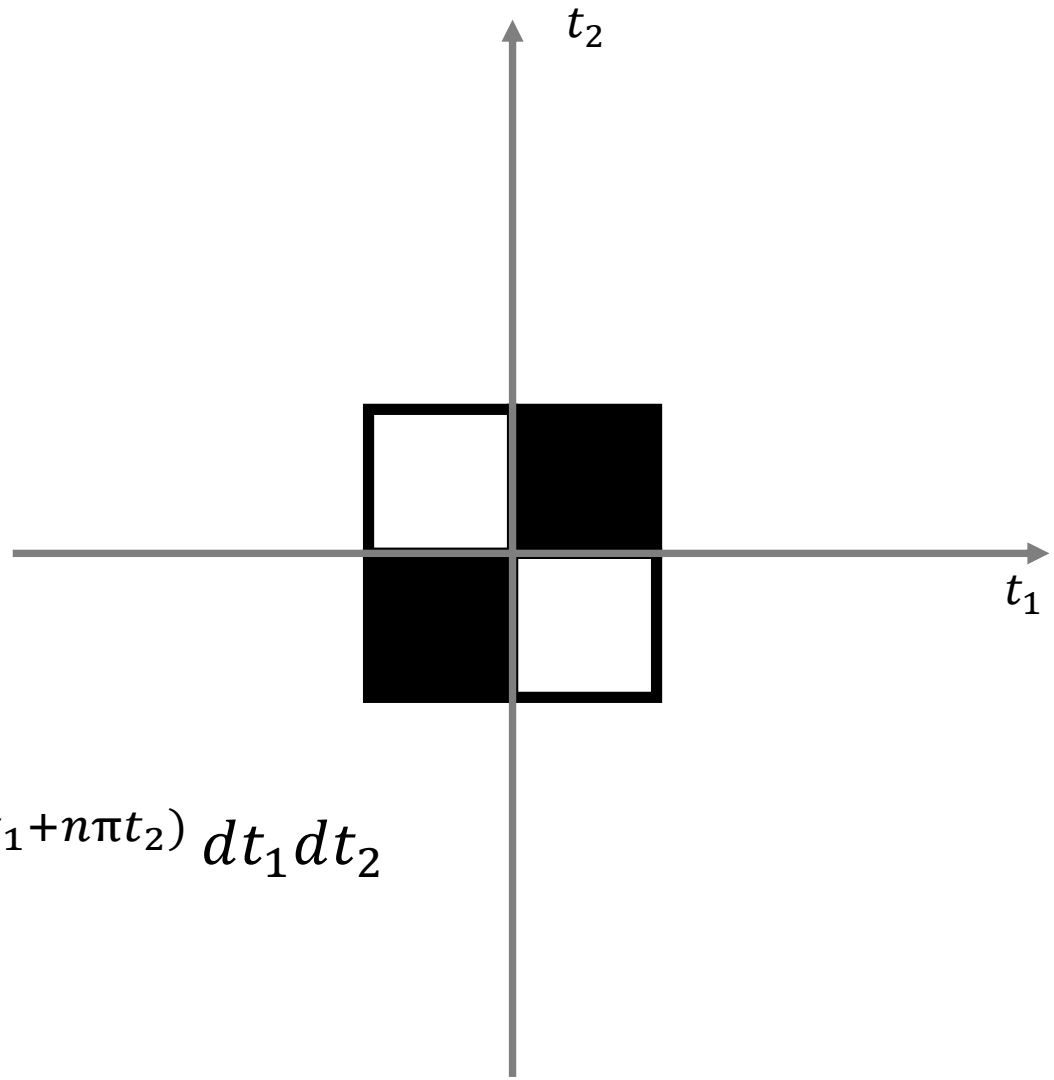


Chessboard

$x(t_1, t_2) = 0$ if white

$x(t_1, t_2) = 1$ if black

$T_1 = 2, T_2 = 2$



$$x(t_1, t_2) = u(t_2)u(t_1) + u(-t_2)u(-t_1)$$

$$a_{mn} = \frac{1}{T_1 T_2} \int_{-1}^{1} \int_{-1}^{1} (u(t_2)u(t_1) + u(-t_2)u(-t_1)) e^{-j(m\pi t_1 + n\pi t_2)} \, dt_1 dt_2$$

Chessboard

$$a_{mn} = \frac{1}{T_1 T_2} \int_{-1}^{1} \int_{-1}^{1} (u(t_2)u(t_1) + u(-t_2)u(-t_1)) e^{-j(m\pi t_1 + n\pi t_2)}\, dt_1 dt_2$$

$$a_{mn} = \frac{1}{4} \int_{-1}^{1} e^{-jn\pi t_2} \{ \int_{-1}^{1} (u(t_2)u(t_1) + u(-t_2)u(-t_1)) e^{-jm\pi t_1}\, dt_1 \} dt_2$$

$$a_{mn} = \frac{1}{4} \int_{-1}^{1} e^{-jn\pi t_2} \{ u(t_2) \int_{-1}^{1} u(t_1) e^{-jm\pi t_1} dt_1 + u(-t_2) \int_{-1}^{1} u(-t_1) e^{-jm\pi t_1} dt_1 \} dt_2$$

$$a_{mn} = \frac{1}{4} \int_{-1}^{1} e^{-jn\pi t_2} \{ u(t_2) \int_{0}^{1} e^{-jm\pi t_1} dt_1 + u(-t_2) \int_{-1}^{0} e^{-jm\pi t_1} dt_1 \} dt_2$$

$$a_{mn} = \frac{1}{4} \int_{-1}^{1} e^{-jn\pi t_2} \{ u(t_2) \frac{(e^{-jm\pi} - 1)}{-jm\pi} + u(-t_2) \frac{(1 - e^{jm\pi})}{-jm\pi} \} dt_2$$

Chessboard

$$a_{mn} = \frac{1}{4}\int_{-1}^{1} e^{-jn\pi t_2}\{u(t_2)\frac{(e^{-jm\pi}-1)}{-jm\pi} + u(-t_2)\frac{(1-e^{jm\pi})}{-jm\pi}\}dt_2$$

$$a_{mn} = \frac{1}{4jm\pi}\int_{-1}^{1} e^{-jn\pi t_2}\{u(t_2)\frac{(1-e^{-jm\pi})}{1} - u(-t_2)\frac{(1-e^{jm\pi})}{1}\}dt_2$$

$$a_{mn} = \frac{(1-e^{-jm\pi})}{4jm\pi}\int_{-1}^{1} e^{-jn\pi t_2}\{u(t_2) - u(-t_2)\}dt_2 = \frac{-(1-e^{-jm\pi})(1-e^{-jn\pi})}{2mn\pi^2}$$

$$a_{mn} = \begin{cases} \frac{-2}{mn\pi^2} & m,n\ are\ odd \\ 0 & otherwise \end{cases}$$

Chladni Figure

https://www.youtube.com/watch?v=wvJAgrUBF4w

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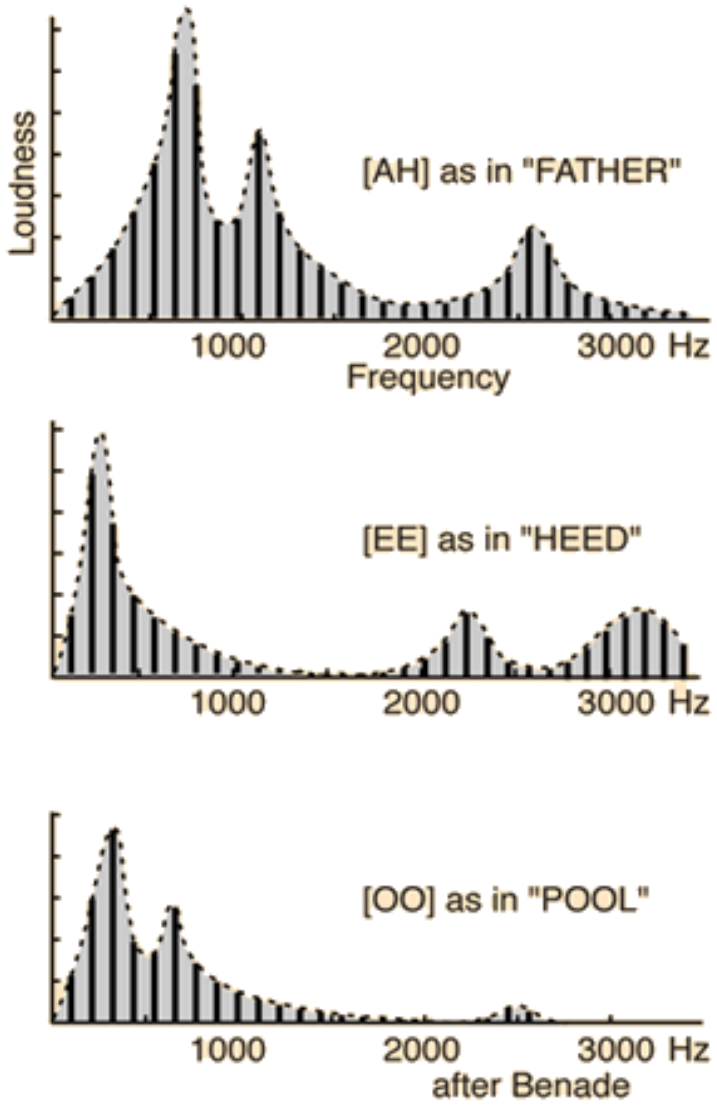
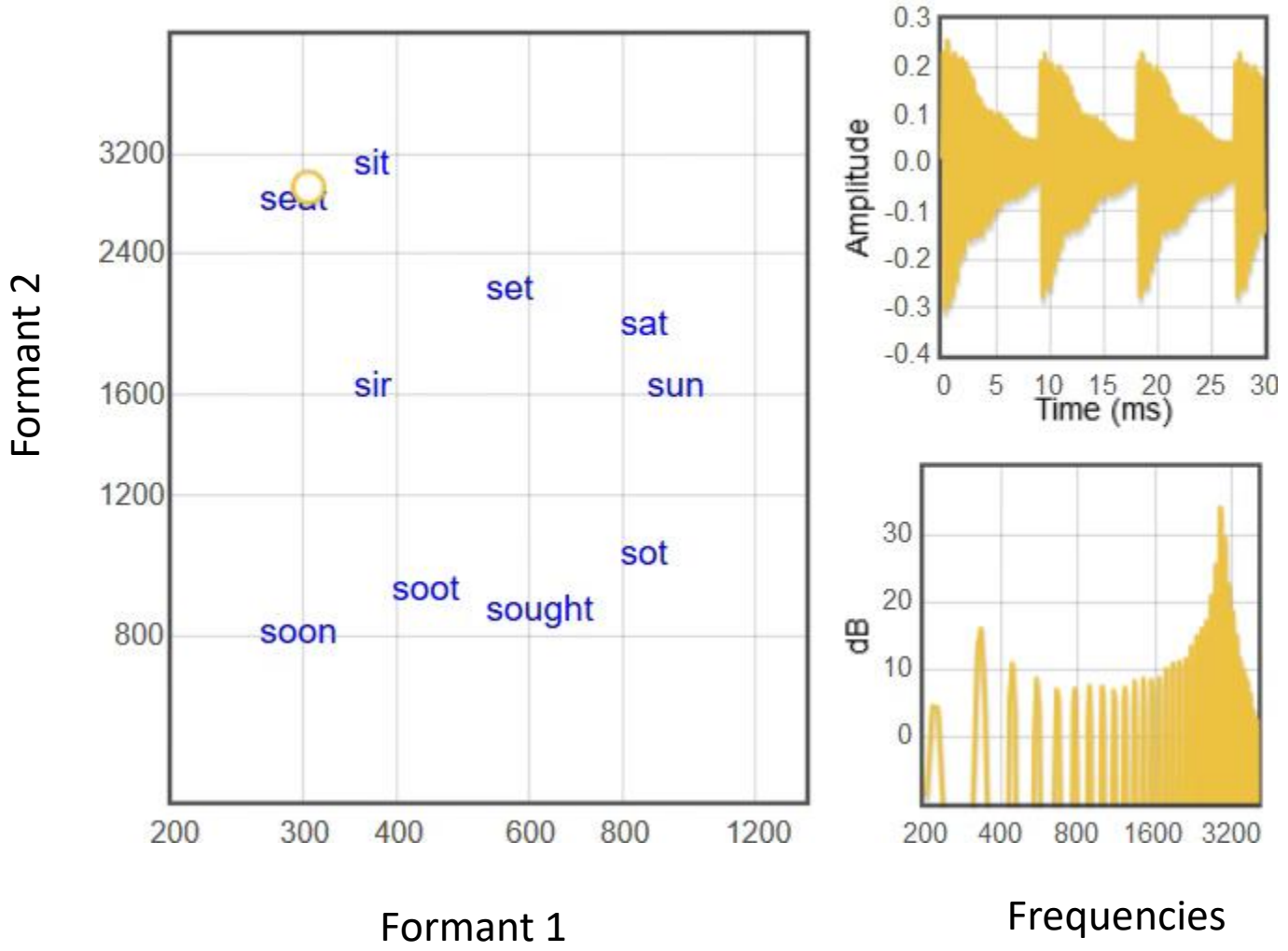
Applications of Fourier Series to Speech Synthesis

Fourier Series of Vowels

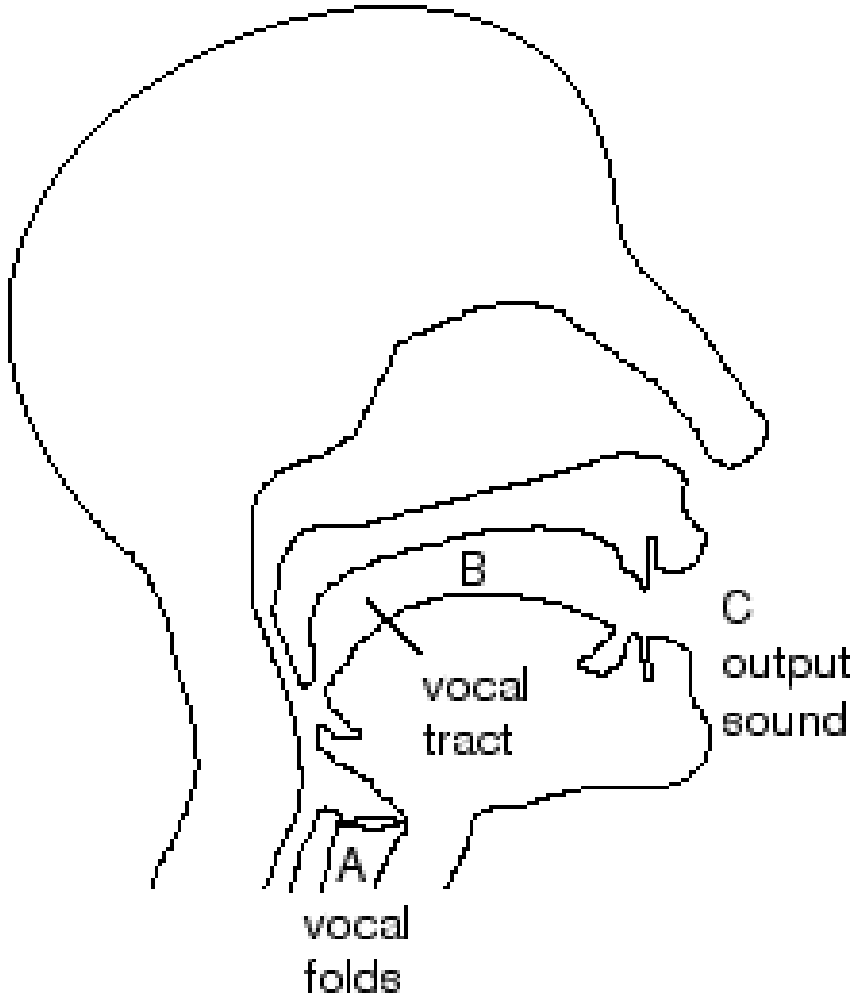


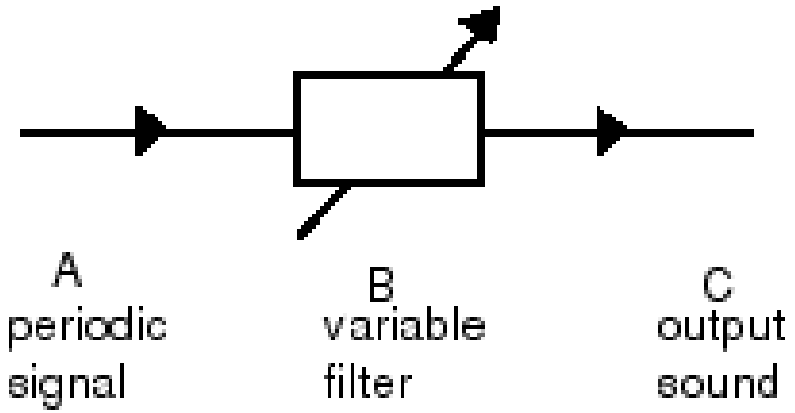
Item	F1	F2	Duration (ms)
heed	292	2352	177
hid	285	2410	126
head	668	1863	179
had	652	1877	132
hud	695	1235	152
hard	818	1182	174
herd	524	1389	169
hawed	568	866	181
hod	460	875	176
whod	289	813	220
hood	296	935	163

Understanding speech

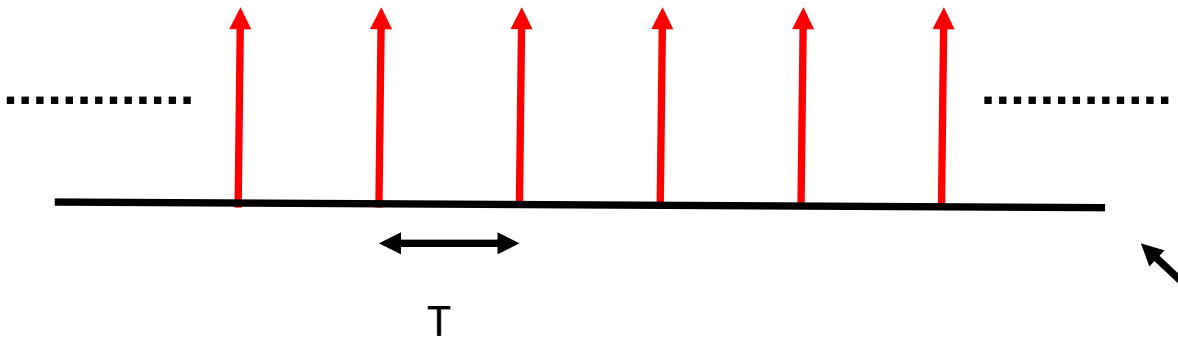


Source-filter idea

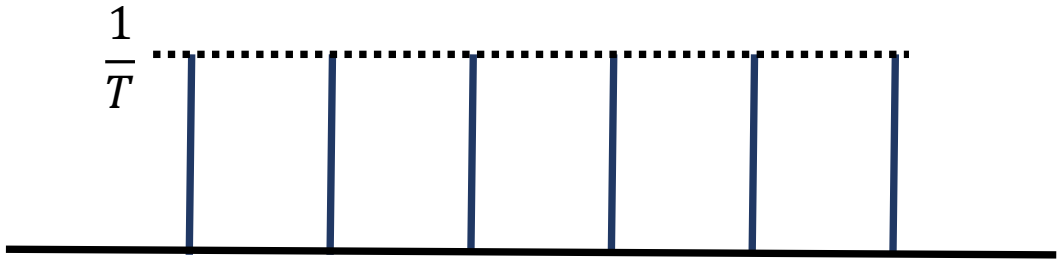




Source



$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} \delta(t) e^{-jk\omega_o t} dt = \frac{1}{T}$$



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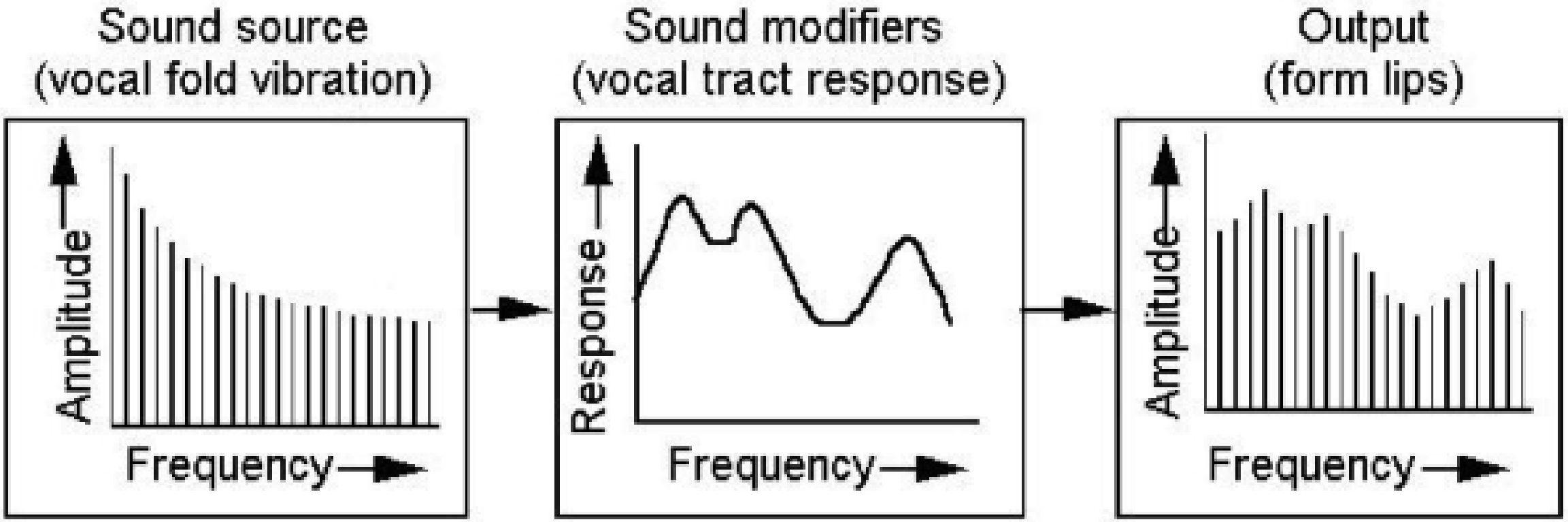

Understanding anomalies

Sopranos effect

Sopranos effect

Individual notes at varying scales

"La"

"Lore"

"Loo"

"Ler"

"Lee"

Low scale: "La, Lore, Loo, Ler, Lee"

High scale: "La, Lore, Loo, Ler, Lee"

http://newt.phys.unsw.edu.au/jw/soprane.html

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