BFS: $\quad x_1 + x_2 \geq 1$

$\qquad -x_1 + 6x_2 \leq 3$

$\qquad x_1 \leq 2 \qquad\qquad x_1, x_2 \geq 0.$

$x_1 + x_2 - x_3 = 1$

$-x_1 + 6x_2 + x_4 = 3$

$x_1 + x_5 = 2$

$x_i \geq 0 \quad \forall i$

$A : 3 \times 5$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ -1 & 6 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{matrix} x_3 & x_4 & x_5 \\ \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

$x_1 = x_2 = 0$

$x_3 = -1$

$x_4 = 3$

$x_5 = 2$

(Not a BFS

Not soln)

$$B = \begin{matrix} x_1 & x_4 & x_5 \\ \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{matrix}$$

$x_2, x_3 = 0$

$x_1 = 1$

$x_4 = 4$

$x_5 = 1$

$$B = \begin{matrix} x_1 & x_2 & x_3 \\ \begin{bmatrix} 1 & 1 & -1 \\ -1 & 6 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{matrix}$$

$x_4 = x_5 = 0$

$x_1 = 2$

$x_2 = 5/6$

$x_3 = 2 + 5/6 - 1 = 11/6$

$$B = \begin{matrix} x_1 & x_2 & x_4 \\ \begin{bmatrix} 1 & 1 & 0 \\ -1 & 6 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{matrix}$$

$x_3 = 0, \quad x_5 = 0.$

$x_1 = 2 \quad , \quad x_2 = -1$

$\qquad\qquad \downarrow$

(Not a soln)

$(3/9, A_1)$ $(2, 5/6)$

$(1/3, 0)$ $(2, 0)$
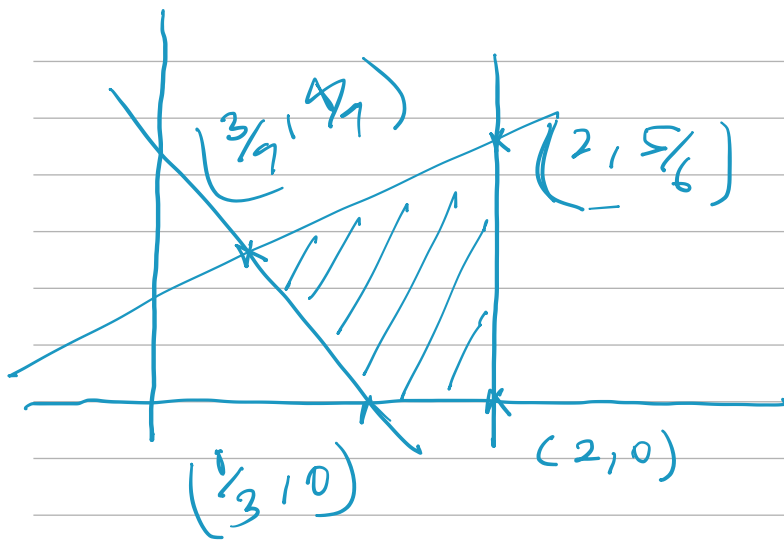
Consider, $\quad x_1 - x_2 \geq 0$

$x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0$

$(2, 2)$

$(0, 0)$

$(4, 0)$
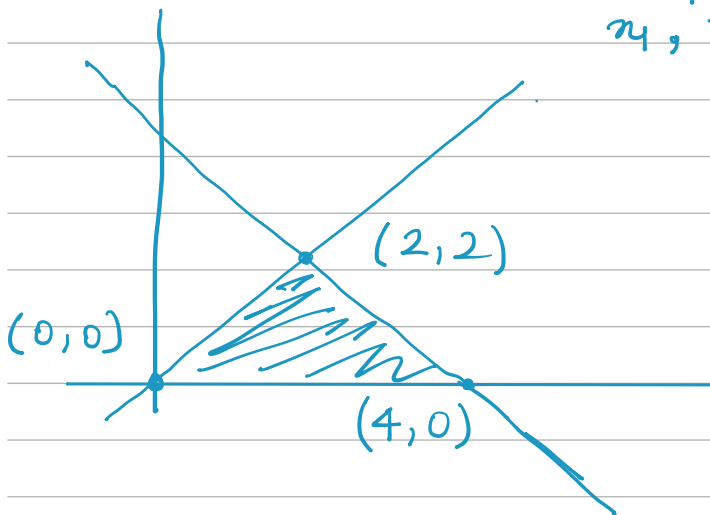
$$\begin{cases} x_1 - x_2 - x_3 = 0 \\ x_1 + x_2 + x_4 = 4 \\ \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$$

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$\underset{(A)}{\uparrow} \quad _{2 \times 4}$

rank $A = 2$

max # of BFS $^4C_2 = 6$

$\left.\begin{array}{c} m = 2 \\ n = 4 \\ m < n \end{array}\right\}$

$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = B \qquad x_1 = x_2 = 2 \qquad x_3 = x_4 = 0$

$\underset{x_1 \quad x_3}{\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}} = B \qquad \begin{array}{l} x_2 = x_4 = 0 \\ x_1 = 4, \quad x_3 = 4 \end{array}$

$$\begin{matrix} x_1 & x_4 \\ \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \end{matrix}$$

$x_1 = 0$, $x_4 = 4$ (degenerate)

$x_2 = x_3 = 0$

$$\begin{matrix} x_2 & x_3 \\ \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \end{matrix} =$$

$x_2 = 4$

$x_3 = -4$ (Not possible)

$x_1 = x_4 = 0$

$$\begin{matrix} x_2 & x_4 \\ \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \end{matrix}$$

$x_1 = x_3 = 0 = x_2$ (degenerate)

$x_4 = 4$

$$\begin{matrix} x_3 & x_4 \\ \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \end{matrix}$$

$x_1 = x_2 = 0 = x_3$ (degenerate)

$x_4 = 4$