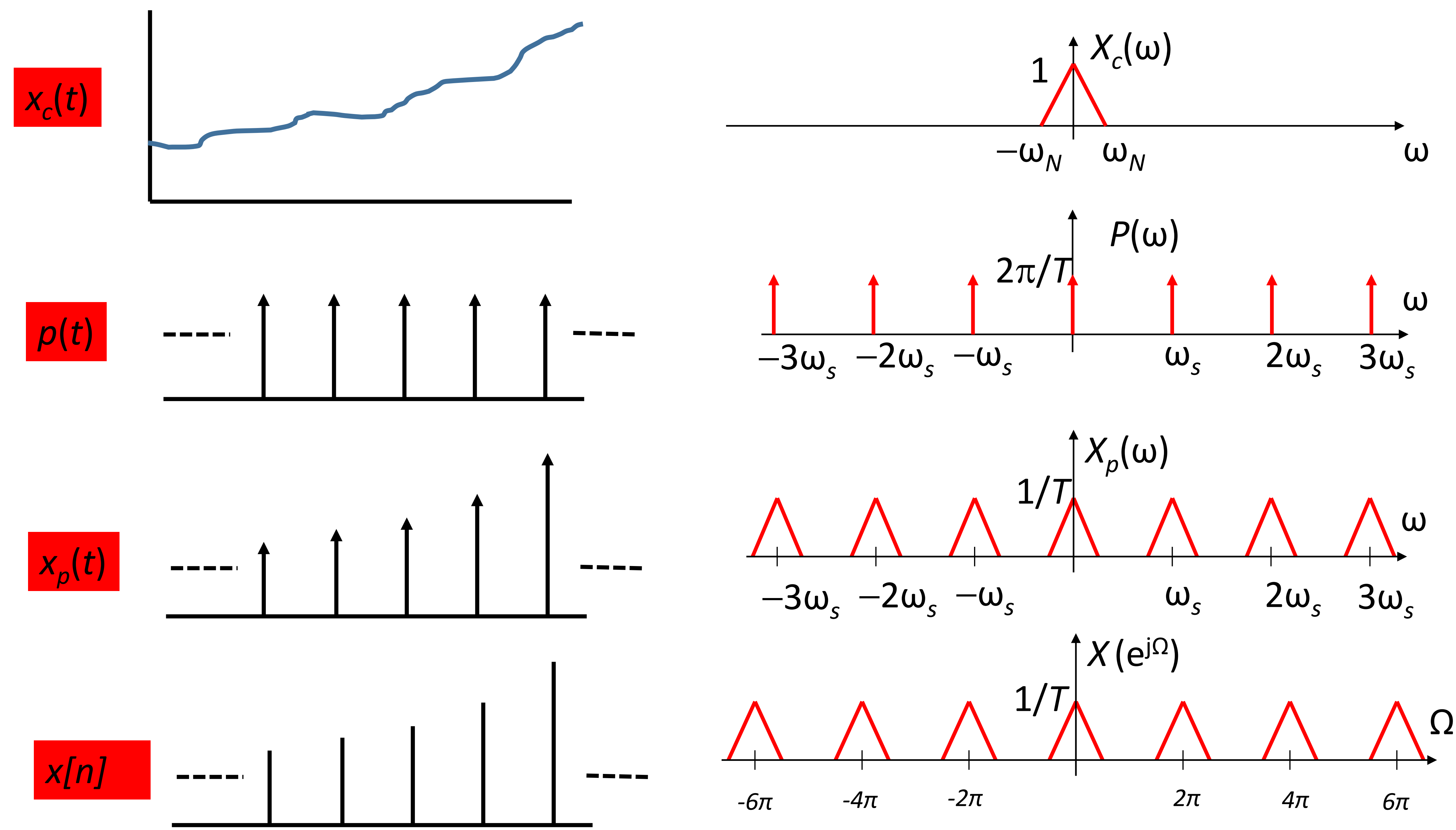
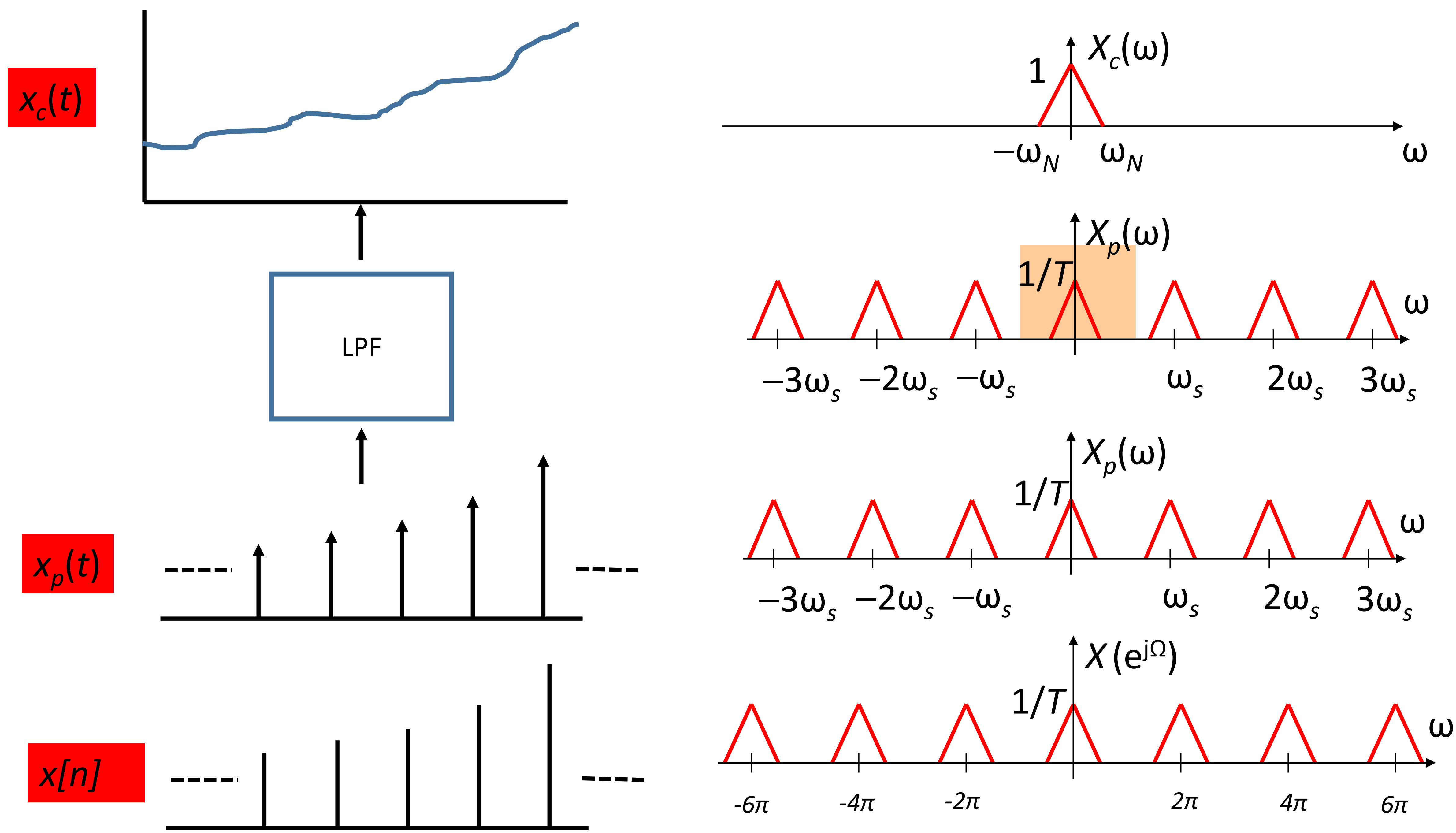
Interpolation

Lecture 32

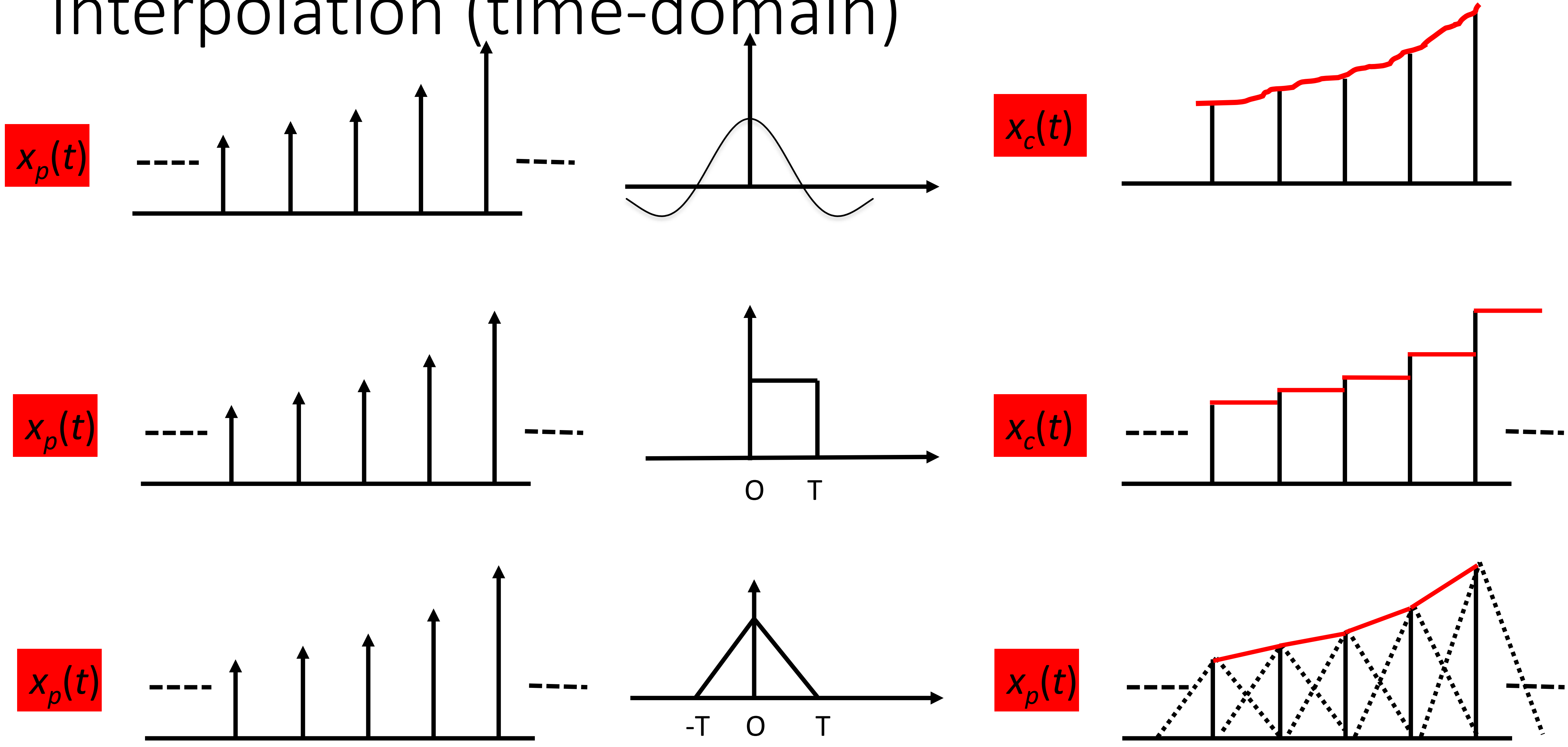


$x_c(t)$
$p(t)$
$x_p(t)$
$x[n]$
$X_c(\omega)$
1
$-\omega_N$
ω_N
ω
$P(\omega)$
$2\pi/T$
ω
$-3\omega_s$
$-2\omega_s$
$-\omega_s$
ω_s
$2\omega_s$
$3\omega_s$
$X_p(\omega)$
$1/T$
ω
$-3\omega_s$
$-2\omega_s$
$-\omega_s$
ω_s
$2\omega_s$
$3\omega_s$
$X(e^{j\Omega})$
$1/T$
Ω
-6π
-4π
-2π
2π
4π
6π

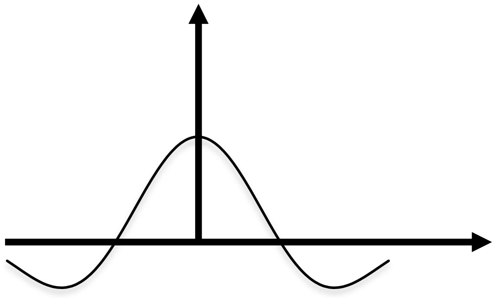


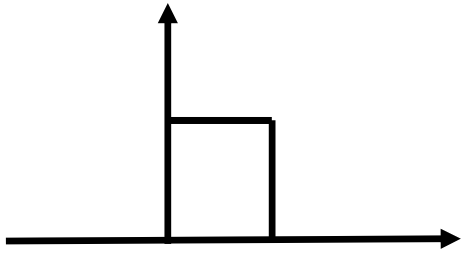
$x_c(t)$
LPF
$x_p(t)$
$x[n]$
$X_c(\omega)$
1
$-\omega_N$
ω_N
ω
$X_p(\omega)$
1/T
$-3\omega_s$ $-2\omega_s$ $-\omega_s$ ω_s $2\omega_s$ $3\omega_s$
ω
$X_p(\omega)$
1/T
$-3\omega_s$ $-2\omega_s$ $-\omega_s$ ω_s $2\omega_s$ $3\omega_s$
ω
$X(e^{j\Omega})$
1/T
ω
-6π -4π -2π 2π 4π 6π

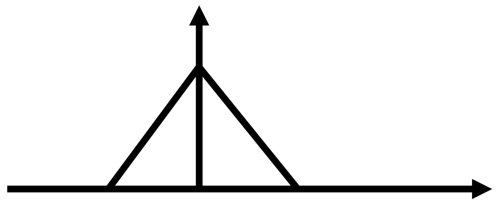
Interpolation (time-domain)

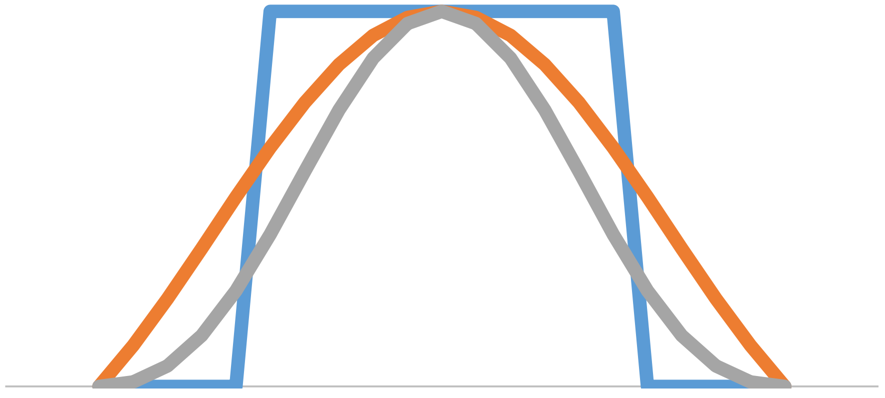


Time domain vs. Frequency domain









Sampling period = 2 times

Sampling period = 4 times

Sampling period = 8 times

Effect of Interpolation

Effect of Interpolation

Anti-aliased

Linear

ZOH

Effect of Interpolation

Effect of Interpolation

Effect of Interpolation

ZOH

Linear

Anti-aliased

Linear

ZOH

Anti-aliased

Linear

Music

- Music (linear interpolation)

- Music (ZOH)

Music

- Music (linear interpolation)

- Music (ZOH)

Music

- Music (linear interpolation)

- Music (ZOH)

Laplace Transforms

Pierre-Simon Laplace (1749-1827)

1. He was an astronomer
2. He has significant contributions in calculus, Bayesian estimations, black holes, Laplace equation and Laplace transforms .
3. Central limit theorem and absurd theories like rule of succession

$$\Pr(\textit{sun will rise tomorrow}) = \frac{d+1}{d+2}$$

4. He is known as the “Newton of France.”
5. D’Alembert interaction.

Pierre-Simon Laplace

Pierre-Simon Laplace (1749–1827).

Continuous-Time Fourier Transform

- Representing signals as linear combination of basic signals $e^{j\omega t}$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$$

Synthesis equation

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

Analysis equation

LTI systems

- Impulse response h(t)

$$e^{j\omega t} \longrightarrow \boxed{\text{LTI}} \longrightarrow H(\omega)e^{j\omega t}$$

LTI systems

- Impulse response h(t)

$$e^{st} \longrightarrow \boxed{\text{LTI}}$$

$$s = \sigma + j\omega$$

LTI systems

- Impulse response h(t)

$$e^{st} \longrightarrow \boxed{\text{LTI}} \longrightarrow \int_{-\infty}^{\infty} h(\tau)e^{s(t-\tau)}d\tau$$

$$s = \sigma + j\omega$$

LTI systems

- Impulse response h(t)

$$e^{st} \longrightarrow \boxed{\text{LTI}} \longrightarrow \int_{-\infty}^{\infty} h(\tau) e^{s(t-\tau)} d\tau$$

$$s = \sigma + j\omega$$

LTI systems

- Impulse response h(t)

$$e^{st}$$

$$e^{st}\int_{-\infty}^{\infty} h(\tau)e^{-s\tau}d\tau$$

$$s = \sigma + j\omega$$

$$\boxed{H(s) = \int_{-\infty}^{\infty} h(\tau)e^{-s\tau}d\tau}$$

LTI systems

- Impulse response h(t)

$$e^{st} \longrightarrow \boxed{\text{LTI}} \longrightarrow e^{st}\,H(s)$$

$$s = \sigma + j\omega$$

$$\boxed{H(s) = \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau}$$

Laplace Transform

Bilateral

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

$$x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s)$$

Connection between Laplace and Fourier Transform

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$s = \sigma + j\omega \qquad X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

$$X(j\omega) = X(s)\big|_{s=j\omega} = \mathcal{F}\{x(t)\} = X(\omega)$$

$$\boxed{X(j\omega) = \mathcal{F}\{x(t)\}}$$

New notation