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Q1) $f(x,y) = \dfrac{x \sin(x^2+2y^2)}{x^2+y^2}$

for $f(x,y)$ to be continuous at $(0,0)$.

$\lim_{(x,y)\to(0,0)} f(x,y)$ should exist and be equal to $f(0,0)$

i.e. ~~Let $f(0,0)$ be defined and equal to~~

Along the x-axis $y=0$

$\lim_{(x,0)\to(0,0)} \dfrac{x\sin(x^2)}{x^2} \longrightarrow 0$ as $x \to 0$

There fore if $f(x,y)$ is continuous at $(0,0)$

$f(0,0)$ will be equal to 0.

~~Now for $\lim_{(x,y)\to(0,0)}$ for $x^2 y$~~

$\therefore$ There should exist an ε $\forall$ δ, where

$|f(x,y) - f(0,0)| < \varepsilon$ $\wedge$

$\sqrt{(x-0)^2+(y-0)^2} < \delta$

$\therefore$ $|f(x,y)| < \varepsilon$ (since $f(0,0)$ has to be 0 for continuity)

$\left|\dfrac{x\sin(x^2+2y^2)}{x^2+y^2}\right| \leq \left|\dfrac{x}{x^2+y^2}\right|$ $(|\sin(x)| \leq 1)$

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Now for $\left|\frac{x}{x^2+y^2}\right|$

Along x axis $y=0$

$$\lim_{x\to 0}\left|\frac{x}{x^2+0^2}\right| \to \infty$$

Along y axis $x=0$

$$\lim_{y\to 0}\left|\frac{0}{0+y^2}\right| = 0$$

so this limit does not exist.

hence, It is not possible to define $f(0,0)$ so that $f(x,y)$ is continuous

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Q 2) for $f(x, y)$ to be differentiable at $(0,0)$

$$\lim_{\rho \to 0} \frac{\Delta f - df}{\rho} = 0 \quad \text{for } \rho = \sqrt{h^2 + k^2}$$

where $x = (0+h)$ $y = (0+k)$

$$\left.\frac{\partial f}{\partial x}\right|_{(x,y)=(0,0)} = 0$$

(clearly since $y = 0$)

similarly

$$\left.\frac{\partial f}{\partial y}\right|_{(x,y)=(0,0)} = 0$$

$\therefore$ $df = 0$

$$\lim_{(h,k) \to (0,0)} \frac{hk(h^2 + k^4)}{(k^4 + h^2)\sqrt{h^2 + k^4}}$$

$h = \rho \cos\theta$ $k = \rho \sin\theta$

$$\lim_{\rho \to 0} \frac{\rho^4 (\cos^2\theta + \rho^2 \sin^2\theta)}{\rho^3 (\rho^2 \cos^2\theta + \sin^2\theta)}$$

$$= \lim_{\rho \to 0} \frac{\rho (\cos^2\theta + \rho^2 \sin^2\theta)}{(\rho^2 \cos^2\theta + \sin^2\theta)}$$

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for $\theta = 0$ $\sin\theta = 0$ $\cos\theta = 1$

$$\lim_{\rho \to 0} \frac{\rho}{\rho^2} = \lim_{\rho \to 0} \frac{1}{\rho} \neq 0$$

Hence $f(x, y)$ is not differentiable at $(0, 0)$

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Q3) $P_2 = f(0,0) + h f_x(0,0) + k f_y(0,0) + \frac{h^2 f_{xx}(0,0) + 2hk f_{xy}(0,0) + k^2 f_{yy}(0,0)}{\lfloor 2}$ where $x = 0 + h$, $y = 0 + k$

$f(0,0) = 2$

$f_x(x,y) = 2e^{x+y} - y\cos(xy)$

$f_x(0,0) = 2$

$f_y(x,y) = 2e^{x+y} - x\cos(xy)$

$f_y(0,0) = 2$

$f_{xx}(x,y) = 2e^{x+y} + y^2\sin(xy) \cancel{- y\cos(xy)}$

$f_{xx}(0,0) = 2 \cancel{-1}$

$f_{yy}(x,y) = 2e^{x+y} + x^2\sin(xy) \cancel{-\cos(xy)}$

$f_{yy}(0,0) = 2 \cancel{-1}$

$f_{xy}(x,y) = 2e^{x+y} + xy\sin(xy) - \cos xy$

$f_{xy}(0,0) = 2 - 1 = 1$

$P_2 = 2 + 2h + 2k + \frac{h^2 + 4hk + k^2}{2}$

$= 2 + 2x + 2y + \frac{2x^2 + 2xy + 2y^2}{2}$

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$$|f(x,y) - P_2(x,y)| = \left|\frac{f'''(c\,x, cy)}{\lfloor 3}\right|$$

for some $c \in (0,1)$

$f_{xxx} = 2xe^{x+y} + y^3\cos(xy) + \cancel{2y\sin(xy)} + y\sin(xy)$

$f_{yyy} = 2e^{x+y} + x^3\cos(xy) + \cancel{2x\sin(xy)} + x\sin(xy)$

$f_{xxx}\big|_{(0,0)} = 2 = f_{yyy}\big|_{(0,0)}$

$f_{xxy} = 2e^{x+y} + 2y\sin(xy) + y^2x\cos(xy)$

$f_{x,x,y}\big|_{(0,0)} = 2$

similarly

$f_{yyx}\big|_{(0,0)} = 2$

$$\frac{f'''(x,y)}{\lfloor 3} = \frac{2(x^3 + 3x^2y + 3y^2x + y^3)}{\lfloor 3} < \frac{\cancel{2} \times 8 \times 0.001}{\cancel{2} \times 3}$$

for $|x|, |y| < 0.1$

~~max value $|f'''(x,y)|$ will be when~~

$$= \frac{0.008}{3} = \frac{8}{3000}$$

Error $|f(x,y) - P_2(x,y)| < \frac{8}{3000}$ for $|x|, |y| < 0.1$

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Q4) We have find the minimum value of
$$\sqrt{(x-0)^2+(y-0)^2+(z-0)^2}$$
which is same as minimum value of

$$x^2+y^2+z^2$$

let $f(x,y,z) = x^2+y^2+z^2$

Subject to the constraint $z^2 = xy+4$

let $g(x,y,z) = z^2 - xy - 4 = 0$ — (4)

By Lagrange multiplier for ~~minimum~~ points of extremum

$$\nabla f(x,y,z) = \lambda \nabla g(x,y,z)$$

$$(2x, 2y, 2z) = \lambda(-y, -x, 2z)$$

$2x = -\lambda y$ — (1)

$2y = -\lambda x$ — (2)

$2z = \lambda 2z$ — (3)

By (3) either $z=0$ or $\lambda = 1$

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Case (1)

Let $\lambda = 1$

By (1) and (2)

we get $x = 0$ & $y = 0$

By putting in $g(x, y, z) = 0$

we get $z^2 = 4$ or $z = \pm 2$

case (2) Let $z = 0$

we get $xy = -4$ by (4)

By (1) and (2) we get $\lambda = -2$

$|xy| = 4 < \sqrt{x^2 + y^2 + 0^2}$ always

$\therefore$ this case will not be the solution

$\therefore$ the point on the surface closest to origin is $(0, 0, \pm 2) = (0, 0, 2)$ or $(0, 0, -2)$