ELL101: INTRODUCTION TO ELECTRICAL ENG.

Miscellaneous Theorems

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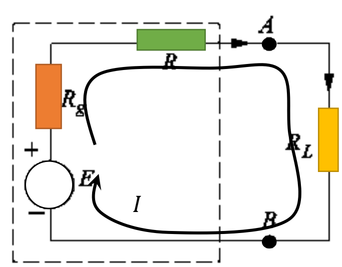
Maximum Power Transfer Theorem

Statement--A resistive load will abstract maximum power from a network when the load resistance is equal to the resistance of the network as viewed from the output terminals, with all energy sources removed leaving behind their internal resistances.



Let $R_i = R_g + R$, then maximum power will be absorbed by the load if $R_L = R_i$

Proof: Circuit current: $I = \frac{E}{R_i + R_L}$

Power consumed by the load is $P_L = I^2 R_L = \left(\frac{E}{R_i + R_L}\right)^2 R_L$

By setting $\frac{dP_L}{dR_L} = 0$ it can be shown that $R_L = R_i$

$$\boxed{P_{L\,\max} = \frac{E^2 R_L}{4 R_L^2} = \frac{E^2}{4 R_L} = \frac{E^2}{4 R_i}}$$

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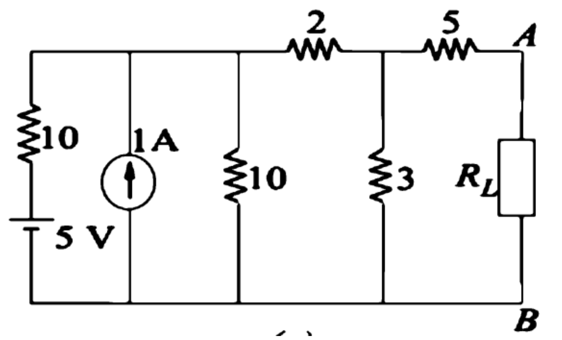
Example

In the circuit, calculate the maximum power transferred to the load.



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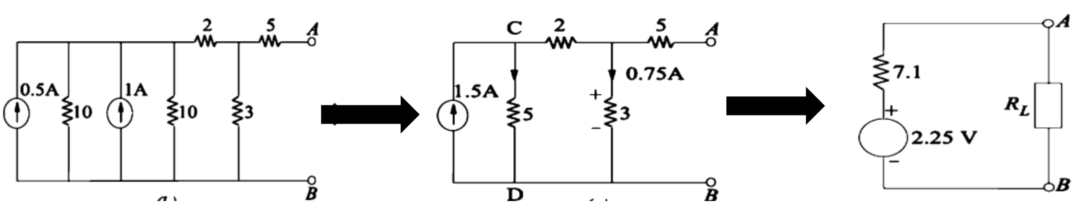
Solution



- First we find the Norton equivalent for the 5V voltage source which is a current source of 0.5A in parallel with 10 ohm resistance.
- Both current source have the same direction, total current source is 1.5A.
- Two 10ohm resistance are in parallel and get reduce to 5ohm equivalent resistance.
- Open circuiting the A and B nodes will result no current in 5ohm resistance connected to node A.
- So open ckt voltage at A and B will be the voltage drop across 3ohm resistance.
- From current division rule, 3ohm resistance carries a current of 0.75A, hence there is 2.25V drop at 3 ohm resistance, which is also V_{TH}, i.e., the voltage measured at A and B by the voltmeter.
- After open ckting the current source, total resistance between node A and B will be 7.1 ohm.
- For maximum power transfer, the value of load resistance should be 7.1ohm.
- Maximum power will be 178 mW.

$$\boxed{P_{L\,max} = \frac{E^2 R_L}{4 R_L^2} = \frac{E^2}{4 R_L} = \frac{E^2}{4 R_i}}$$

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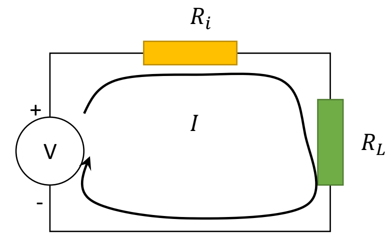
Power Transfer Efficiency



- Total power $P_T = P_i + P_L$
- P_i is the power consumed by the internal resistance R_i
- P_L is the power delivered to the load
- Power Transfer Efficiency

$$\eta = \frac{P_L}{P_i + P_L} = \frac{I^2 R_L}{I^2 R_i + I^2 R_L} = \frac{1}{1 + R_i/R_L}$$

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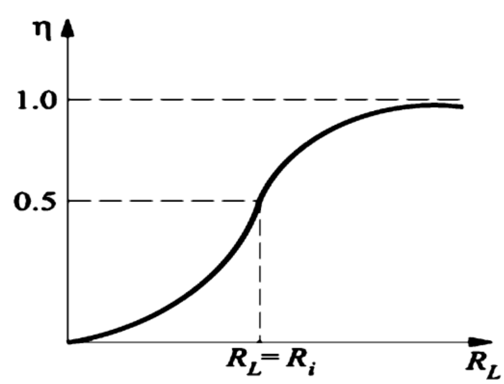
Power Transfer Efficiency

- The behavior of η w.r.t. R_L:

$$\eta = \frac{1}{1 + R_i/R_L}$$

- The maximum value of η is unity when $R_L \rightarrow \infty$
- η=50% for $R_i = R_L$
- It means that under maximum power transfer conditions, the power transfer efficiency is only 50%.



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Reciprocity Theorem

- Statement--In any linear bilateral network, if a source of e.m.f. E in any branch produces a current I in any other branch, then the same e.m.f. E acting in the second branch would produce the same current I in the first branch.
- In other words, it simply means that E and I are mutually transferrable.
- It also means that interchange of an ideal voltage sources and an ideal ammeter in any network will not change the ammeter reading.
- Same is the case with the interchange of an ideal current source and an ideal voltmeter.

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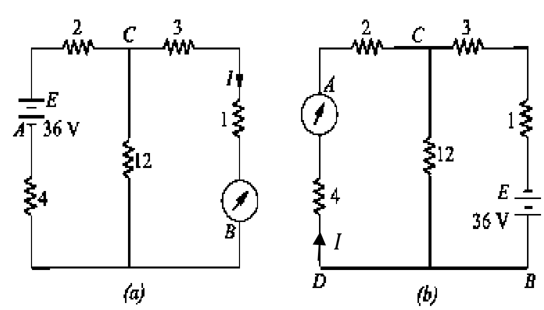
Example

- In Fig. a 36V battery is at node A and ammeter is at node B. Find the value of current I, measured by the ammeter.
- In Fig. b, the places of the battery and ammeter are exchanged. Find the current measured by the ammeter, i.e., I in this case.



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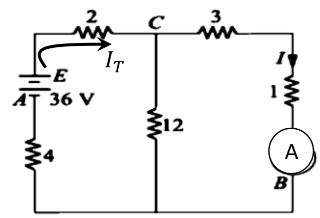
Solution

- Note that ideal ammeter has zero internal resistance.
- So total resistance faced by the battery will be
$$R = 2 + ((3+1)||12) + 4 = 9ohm$$
- So total current flowing out of the battery
$$I_T = \frac{36}{9} = 4A$$
- From figure, applying current division rule
$$I = \frac{12}{(12+3+1)} I_T = 3A$$



(a)

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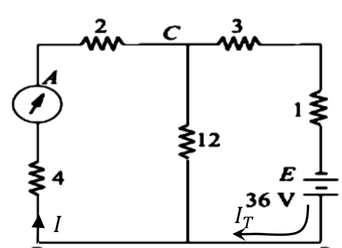
Solution

- Total resistance faced by the battery
$$R = 1 + 3 + ((2+4)||12) = 8ohm$$
- So total current flowing out of the battery
$$I_T = \frac{36}{8} = 4.5A$$
- From figure, applying current division rule
$$I = \frac{12}{(12+4+2)} I_T = 3A$$



(b)

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Millman's Theorem

- The Millman's Theorem states that – when a number of voltage sources (V_1, V_2, V_3……… V_n) are in parallel having internal resistance (R_1, R_2, R_3…………R_n) respectively, the arrangement can replace by a single equivalent voltage source V in series with an equivalent series resistance R.
- This theorem is also called as PARALLEL GENERATOR THEOREM.
- It is used to simplify circuits that have:
  - Several parallel-connected branches containing a voltage source and series resistance
  - Current source and parallel resistance
  - Combination of both

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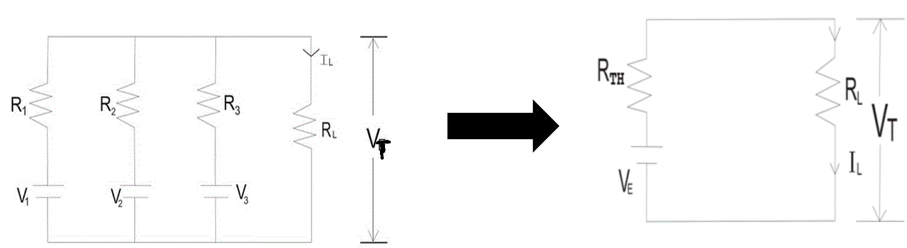
Circuit Consisting Only Voltage Sources



$$V_E = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \qquad R_{TH} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \qquad I_L = \frac{V_{TH}}{R_L + R_{TH}} \qquad V_T = I_L \times R_L$$

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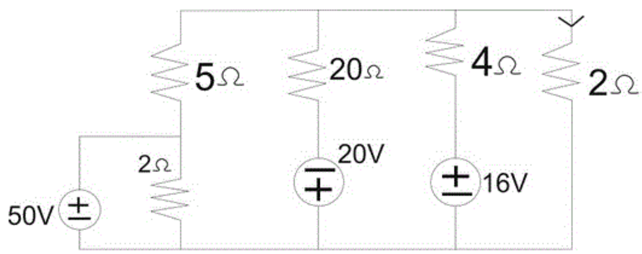
Example

Find out the voltage across 2 ohm resistance and current through the 2 ohm resistance.



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Solution

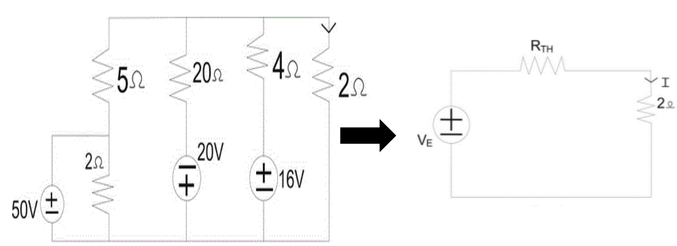
- Apply Millman's Theorem

$$V_E = \frac{\frac{50}{5} - \frac{20}{20} + \frac{16}{4}}{\frac{1}{5} + \frac{1}{20} + \frac{1}{4}} = 26V$$

- Eqv. Resistance

$$R_{TH} = \frac{1}{\frac{1}{5} + \frac{1}{20} + \frac{1}{4}} = 2\Omega$$

- Note that 2ohm resistance in parallel with 50V source is ineffective for the calculations



$$I_{2\Omega} = \frac{26}{2+2} = 6.5A$$

$$V_L = I_{2\Omega} \times 2 = 6.5 \times 2 = 13V$$

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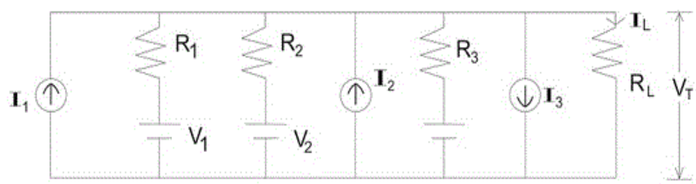
Circuit is Consisting Mixture of Voltage and Current Source



Millman's Theorem is also helpful to reduce a mixture of voltage and current source connected in parallel to a single equivalent voltage or current source.

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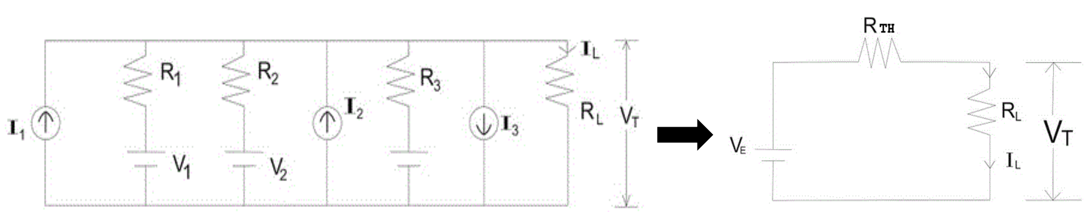
Solution



$$V_E = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + I_1 + I_2 - I_3}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{\sum \frac{V}{R} + \sum I}{\sum \frac{1}{R}}$$

$$R_{TH} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$I_L = \frac{V_E}{R_L + R_{TH}}$$

$$V_T = I_L \times R_L$$

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