Lecture 12 Sat 19th Aug 2023

Result : local minima $\Rightarrow$ Global minima
" maxima $\Rightarrow$ " maxima

Convex func^n

$f: X \subseteq \mathbb{R}^n \to \mathbb{R}$, where
$X \subseteq \mathbb{R}^n$ is a convex set.
Then f is called a
convex function on X if

$$f((1-\lambda)x + \lambda y) \leq (1-\lambda) f(x) + \lambda f(y) \quad \forall \lambda \in [0,1]$$
$$\forall x, y \in X.$$

$f: X \subseteq \mathbb{R}^n \to \mathbb{R}$ is concave if

$-f$ is a convex function.

$$\left\{ \min_{x \in X} f(x) \right.$$

X is convex set in $\mathbb{R}^n$ & $f: X \to \mathbb{R}$ is a convex fn

local minima of problem (P) is the global minima of (P).

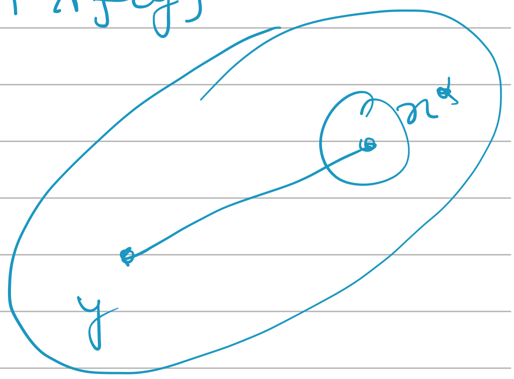
Let $x^* \in X$ be a local minima of (P).

$\Rightarrow \exists \ \varepsilon > 0$ such that $f(x^*) \leq f(x)$
$\forall x \in B_\varepsilon(x^*) \cap X$.

Choose $y \in X - B_\varepsilon(x^*)$. $\forall \lambda \in [0,1]$

$(1-\lambda) x^* + \lambda y \in X$ ($\because X$ is a convex set given)

$$f((1-\lambda)x^* + \lambda y) \le (1-\lambda) f(x^*) + \lambda f(y)$$



$\lambda \to 0 \quad (1-\lambda)x^* + \lambda y \to x^*$

Choose $0 < \lambda < \dfrac{\varepsilon}{\|y - x^*\|}$

$\Rightarrow (1-\lambda)x^* + \lambda y \in B_\varepsilon(x^*)$

$$f((1-\lambda)x^* + \lambda y) \ge f(x^*)$$

$$\Rightarrow (1-\lambda) f(x^*) + \lambda f(y) \ge f(x^*)$$

$$\Rightarrow f(y) \ge f(x^*)$$

$$\therefore f(x^*) \le f(x) \quad \forall x \in X$$

$\Rightarrow x^*$ is global minima of P.

Suppose, $\max_{x \in X} f(x)$

X is convex set in $\mathbb{R}^n$

and, $f : X \to \mathbb{R}$ is a concave func^n on X.

For concave,

$$f((1-\lambda)x + \lambda y) \ge (1-\lambda) f(x) + \lambda f(y)$$

$\forall \lambda \in [0,1]$

$\forall x, y \in X$.

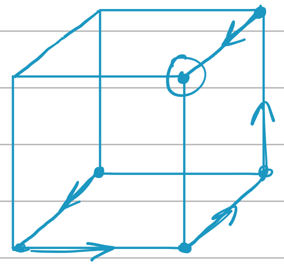
linear func^n $\to$ Both convex and concave func^n

$\downarrow$

local minima / maxima

is global " / "

* Simplex Method (always convergent,



$\max z = c^T x$

Set, $A_{m\times n} x_{n\times 1} \; (\leq \; = \; \geq) \; b_{m\times 1}$

$m << n$ $\quad$ $\text{rank}(A) = m$

$x \geq 0$ (or unrestricted)

① $b \geq 0$

$3x - 4y \leq -10$

$\Rightarrow -3x + 4y \geq 10$

② $\forall\, x \geq 0$

Suppose

$3x - 4y \geq 10$

$x \geq 0, \; y \in \mathbb{R}$

$y = u - v$ (difference of 2 variables) $\quad u \geq 0, \; v \geq 0$

$3x - 4u + 4v \geq 10$

$x, u, v \geq 0$

③ The system of constraints must contain identity matrix when all constraints are converted into eq^n form.