Tutorial - 4

1.1 4-bit 2's complement numbers $\rightarrow (-8, 7)$

8-bit " " $\rightarrow (-2^7, 2^7 - 1)$

$(-128, 127)$

16-bit " " $\rightarrow (-2^{15}, 2^{15} - 1)$

$(-32768, 32767)$

1.2 Let's first think about the cases when the overflow affects the results. If we write down the truth ~~of~~ table corresponding to the addition of the last bit x_{n-1} & y_{n-1} given the carry-in c_{n-1} resulting in the sum s_{n-1} and carry-out c_n.

x_{n-1}	y_{n-1}	c_{n-1}	c_n	s_{n-1}
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Suspicious cases

$c_n \oplus c_{n-1} = 1$

Now recall that the MSB indicates the sign of the number. Hence, adding two positive numbers should result in a +ve number, while adding two -ve numbers should result in a -ve number. With these underlying conditions, there are two suspicious cases where the overflow results into unexpected results.

Now for the three cases of 4-bit, 8-bit & 16-bit numbers, it is straightforward to verify with some examples.

(2.1) We have.

$X = x_7 x_6 x_5 x_4 x_3 x_2 x_1 x_0$

$Y = y_7 y_6 y_5 y_4 y_3 y_2 y_1 y_0$

$C = c_8 c_7 c_6 c_5 c_4 c_3 c_2 c_1 c_0$

Since $c_i = g_{i-1} + p_{i-1} \cdot c_{i-1}$, ~~the~~ given the inputs X and Y, c_i will take the value of c_{i-1} after time Δt which is also equivalent to the delay corresponding an AND gate followed by an OR gate.

Now since ~~out~~ we want to make sure that c_8 toggles after every Δt, we can have

$C = 010101010$

If ~~this~~ we would ensure that the carry value propagate, we can allow c_8 to get the value of c_7 after Δt, c_6 after $2\Delta t$, c_5 after $3\Delta t$ and so on.

Hence, the set of selected inputs should ensure that

(1) $C = 010101010$

(2) the carry values propagate, but ~~the~~ are not generated.

To satisfy the first condition, we ~~need to~~ can have identical X and Y ~~as identi~~ with alternate bits as 0 and 1.

$X = 01010101$

$Y = 01010101$

Now at $T=0$, we have to set each x_i and y_i, such that

$g_i = x_i \cdot y_i = 0$ & $p_i = x_i + y_i = 1$

Therefore, to satisfy second condition, we can have one of x_i and y_i as 1 and we set the other as 0.

~~That is,~~

For example,

$$X = 0\,0\,0\,0\,0\,0\,0\,0$$
$$Y = 1\,1\,1\,1\,1\,1\,1\,1$$

Now, c_8 will toggle after every Δt.

Time		c_8	c_7	c_6	c_5	c_4	c_3	c_2	c_1	c_0
$T = 0$	$C =$	0	1	0	1	0	1	0	1	0
Δt	~~C =~~	1	0	1	0	1	0	1	0	0
$2\Delta t$	C	0	1	0	1	0	1	0	0	0
$3\Delta t$		1	0	1	0	1	0	0	0	0
$4\Delta t$		0	1	0	1	0	0	0	0	0
$5\Delta t$		1	0	1	0	0	0	0	0	0
$6\Delta t$		0	1	0	0	0	0	0	0	0
$7\Delta t$		1	0	0	0	0	0	0	0	0
$8\Delta t$		0	0	0	0	0	0	0	0	0