1. A particle is in the ground state of an infinite square well potential given by

$$V(x) = \begin{cases} 0 & \text{for } -a \leq x \leq a \\ \infty & \text{otherwise} \end{cases}$$

What is the probability to find the particle in the interval between –a/2 and a/2?

2. Consider the box potential

$$V(x) = \begin{cases} 0, & 0 < x < a, \\ \infty, & \text{elsewhere.} \end{cases}.$$

(a) Estimate the energies of the ground state as well as those of the first and the second excited states for (i) an electron enclosed in a box of size $a = 10^{-10}$ m (express your answer in electron volts; you may use these values: $\hbar c = 200\,\text{MeV fm}$, $m_e c^2 = 0.5\,\text{MeV}$); (ii) a 1 g metallic sphere which is moving in a box of size $a = 10\,\text{cm}$ (express your answer in joules).

(b) Discuss the importance of the quantum effects for both of these two systems.

(c) Use the uncertainty principle to estimate the velocities of the electron and the metallic sphere.

3. If the wave function of a particle trapped in space between x = 0 and x = L is given by , where A is a constant, for which value(s) of x will the probability of finding the particle be the maximum?

4. A particle of mass $m$ is in the state

Where $A$ and $a$ are positive real constants.

(a) Find $A$.

(b) For what potential energy function $V(x)$ does $\Psi$ satisfy the Schrödinger equation?

(c) Calculate the expectation values of $x$, $x^2$, $p$, and $p^2$.

5. At $t = 0$ a particle is represented by the wave function

$$\Psi(x,0) = \begin{cases} Ax/a, & \text{if } 0 \le x \le a \\ A(b-x)/(b-a), & \text{if } a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

where $a$ and $b$ are constants, and $A$ is a normalization constant.

(a) Normalize $\Psi(x,0)$.

(b) Sketch $\Psi(x,0)$ as a function of $x$.

(c) Where is the particle most likely to be found at $t = 0$?

(d) What is the probability of finding the particle to the left of the point $a$?

6. A proton is confined in an infinite square well of width 10 fm. (The nuclear potential that binds protons and neutrons in the nucleus of an atom is often approximated by an infinite square well potential.)

- Calculate the energy and wavelength of the photon emitted when the proton undergoes a transition from the first excited state ($n = 2$) to the ground state ($n = 1$).
- In what region of the electromagnetic spectrum does this wavelength belong?

7. An electron is confined to a 1 micron thin layer of silicon. Assuming that the semiconductor can be adequately described by a one-dimensional quantum well with infinite walls, calculate the lowest possible energy within the material in units of electron volt. If the energy is interpreted as the kinetic energy of the electron, what is the corresponding electron velocity? (The effective mass of electrons in silicon is $m^* = 0.26\ m_0$, where $m_0 = 9.11 \times 10^{-31}$ kg is the free electron rest mass).

8. A particle in a 1-D box has a minimum allowed energy of 2.5 eV.

(a) What is the next higher energy it can have? And the next higher after that? Does it have a maximum allowed energy?

(b) If the particle is an electron, how wide is the box?

(c) The fact that particles in a 1-D box have a minimum energy is not completely unrelated to the uncertainty principle. Find the minimum momentum of a particle, with mass m, trapped in a 1-D box of size L. How does this compare with the momentum uncertainty required by the uncertainty principle, if we assume $\Delta x = L$ ?

9. An electron is traded in a one-dimensional potential well of length  m. Find the longest wavelength photons emitted by the electron as it changes energy levels in the well.

10. An important property of the eigenfunctions of a system is that they are orthogonal to one
another, which means that

$$\int_{-\infty}^{+\infty} \psi_n \psi_m \, dV = 0 \qquad n \neq m$$

Verify this relationship for the eigenfunctions of a particle in a one-dimensional box

11. Draw probability density of the particle in three different regions i.e., x<0, 0 to a and x>a for two cases: