APL Formula List

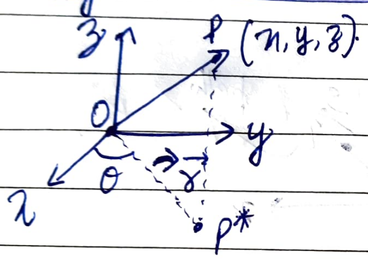
1) $\vec{v} = \frac{d\vec{r}}{dt} = \dot{\vec{r}}$; $a = \frac{d^2\vec{r}}{dt} = \ddot{\vec{r}}$

2) If $y = y(x)$,

then $\dot{y} = \frac{dy}{dx}\dot{x}$

$\ddot{y} = \frac{dy}{dx}\ddot{x} + \left(\frac{d^2y}{dx^2}\dot{x}\right)\dot{x}$

3) Cylindrical Polar co-ordinate



(a) $(\hat{e}_r, \hat{e}_\theta, \hat{e}_z)$ form a right-handed triad.

$\hat{e}_r$ → unit radial vector

$\hat{e}_\theta$ → Transverse/ Circumferential/ Azimuthal unit vector

$\hat{e}_z$ → axial unit vector

(b) $\hat{e}_r = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$\hat{e}_\theta = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$

$\frac{d\hat{e}_r}{dt} = \dot{\theta}\,\hat{e}_\theta$

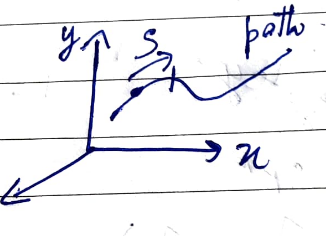
$\frac{d\hat{e}_\theta}{dt} = -\dot{\theta}\,\hat{e}_r$

(c) $\vec{V}_r = \dot{r}$, $\vec{V}_\theta = r\dot{\theta}$, $\vec{V}_z = \dot{z}$,
where $OP^* = r\hat{e}_r$ & $P^*P = z\hat{e}_z$.

(d) $a_r = (\ddot{r} - r\dot{\theta}^2)$
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
$a_z = \ddot{z}$

(e) A circular path of radius R is equivalent to cylindrical co-ordinate system with $z = 0$.

4) Path co-ordinates



(a) $\hat{e}_t = \frac{d\vec{r}}{ds}$,
$\hat{e}_n \rightarrow$ unit vector in direction of $\frac{d\hat{e}_t}{ds}$.
$\hat{e}_t \times \hat{e}_n = \hat{e}_b \rightarrow$ binormal unit vector.

(b) $\frac{d\hat{e}_t}{ds} = \frac{1}{\rho}\hat{e}_n$; $\rho \rightarrow$ radius of curvature.

(c) $\vec{v} = \dot{s}\hat{e}_t$; $\vec{a} = \ddot{s}\hat{e}_t + \frac{\dot{s}^2}{\rho}\hat{e}_n$

(d) $\frac{1}{\rho} = \frac{\left|\frac{d\vec{r}}{ds} \times \frac{d^2\vec{r}}{ds^2}\right|}{\left|\frac{d\vec{r}}{ds}\right|^3}$, where s is any parameter.

When s is time,

$$\frac{1}{\rho} = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3}$$

5) $\hat{i}, \hat{j}, \hat{k} \longrightarrow$ unit vectors in (m)
$\hat{I}, \hat{J}, \hat{K} \longrightarrow$ unit vectors in (F)

$$\vec{\omega}_{m/F} = \frac{1}{2}\left\{ \hat{i} \times \left.\frac{d\hat{i}}{dt}\right|_F + \hat{j} \times \left.\frac{d\hat{j}}{dt}\right|_F + \hat{k} \times \left.\frac{d\hat{k}}{dt}\right|_F \right\}$$

6) $$\left.\frac{d\vec{A}}{dt}\right|_F = \left.\frac{dA}{dt}\right|_m + \vec{\omega}_{m/F} \times \vec{A}$$

7) When P is a fixed point,

$$\boxed{\begin{gathered} \vec{v}_{P/F} = \vec{v}_{A/F} + \vec{v}_{P/m} + \vec{\omega} \times \vec{r}_{P/A}, \\ \text{where } \vec{\omega} = \vec{\omega}_{m/F} \\ A \text{ is the origin.} \end{gathered}}$$

8) $$\boxed{\begin{aligned} \vec{a}_{P/XYZ} &= \vec{a}_{A/XYZ} + \vec{a}_{P/xyz} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/A}) \\ &\quad + \dot{\vec{\omega}} \times \vec{r}_{P/A} + \underbrace{2(\vec{\omega} \times \vec{V}_{P/m})}_{\text{Coriolis acceleration}} \end{aligned}}$$

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9) When A and P are both on the rigid body.

$$\vec{v}_{P/G} = \vec{v}_A + \underbrace{\vec{\omega} \times \vec{r}_{PA}}_{\vec{v}_{P/A}}$$

$$\vec{a}_{P/XYZ} = \vec{a}_{A/XYZ} + \underbrace{\dot{\vec{\omega}} \times \vec{r}_{P/A}}_{\text{tangential acceleration}} + \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/A})}_{\text{centripetal acceleration}}.$$

10) $$\dot{\omega}_{31} = \dot{\omega}_{32} + \dot{\omega}_{21} + \omega_{32} \times \omega_{21}.$$