Lecture on 07/12/2020

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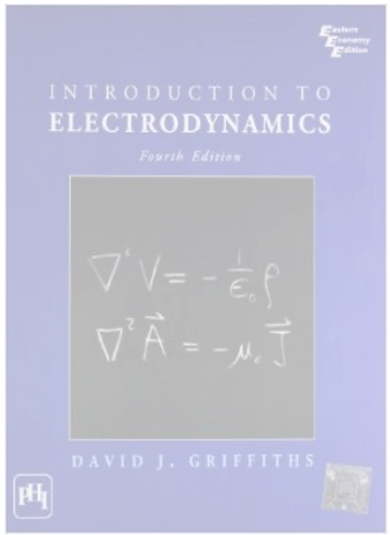
PYL101 course:

Electromagnetics & Quantum Mechanics

- *Next few classes, we will discuss the following topics:*
- *Magnetostatics (ch5)*
- *Magnetic fields in matter (ch6)*
- *Electrodynamics (ch7)*
- *Continuity equation and Poynting's theorem (ch8)*



Ch.7: Electrodynamics:

Ohm's law:

As we know that in order to have a current in wire, we have to push on the charges.

How fast these charges move, in response to a given push, depends on the nature of the material.

For most substances, the current density (J) is proportional to the force per unit charge (f),

so we can write $J = \sigma f$

Here σ is the proportionality factor (not to be confused with surface charge) i.e., an empirical constant that varies from one material to another; it's called the conductivity of the medium….

Ch.7: Electrodynamics:

Ohm's law:

Usually reciprocal of σ is called resistivity $\rho = 1/\sigma$

It is important to note here that these values are in ohm-meters and for 1 atm, 20°C,.. changes with P and T.

Material	Resistivity	Material	Resistivity
Conductors:		*Semiconductors:*	
Silver	1.59×10^{-8}	Sea water	0.2
Copper	1.68×10^{-8}	Germanium	0.46
Gold	2.21×10^{-8}	Diamond	2.7
Aluminum	2.65×10^{-8}	Silicon	2500
Iron	9.61×10^{-8}	*Insulators:*	
Mercury	9.61×10^{-7}	Water (pure)	8.3×10^{3}
Nichrome	1.08×10^{-6}	Glass	$10^{9} - 10^{14}$
Manganese	1.44×10^{-6}	Rubber	$10^{13} - 10^{15}$
Graphite	1.6×10^{-5}	Teflon	$10^{22} - 10^{24}$

In principle, the force that drives the charges to produce the current could be anything-chemical, gravitational, etc., but our purposes, it's usually an electromagnetic force that does the job.

In this case, we can write, J= $\sigma(E + v \times B)$

Normally, the velocity of the charges is sufficiently small that the second term can be ignored…so we can write J= σE, called the Ohm's law

Ch.7: Electrodynamics:

In practice, metals are such good conductors that the E field required to drive current in them is negligible.

Thus we normally treat the connecting wires in electric circuits (for example) as equi-potentials.
However, resistors are made from poorly conducting materials.....

the total current flowing from one electrode to the other is proportional to the potential difference between them:

$V = I\,R$ (R is measured in ohms (Ω): an ohm is a volt per ampere)

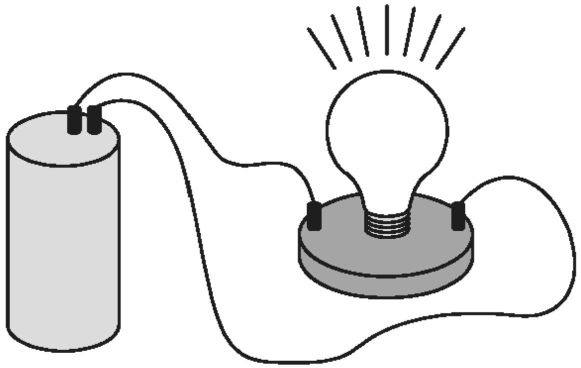
This, of course, is the more familiar version of Ohm's law.

The constant of proportionality R is called the resistance; it's a function of the geometry of the arrangement and the conductivity of the medium between the electrodes....

Ch.7: Electrodynamics: electromotive force…

Now we will discuss about emf, let's connect a bulb with the battery to make a electric circuit…

Now you can ask, why the current is the same all the way around the loop?, how much time it takes to reach the current at the light bulb? whether all the charges start moving at the same time?



In this context, let's discuss the forces involved in driving current around a circuit, which are the source $\mathbf{f}_s$, from a battery and other one is electrostatic force, which serves to smooth out the flow and communicate the effect of the source to distant parts of the circuit….

This means we can write $\mathbf{f}=\mathbf{f}_s + \mathbf{E}$

How about the physical source responsible for $\mathbf{f}_s$…..?

Ch.7: Electrodynamics: electromotive force…

- In a battery, it is a chemical force,
- In a piezoelectric crystal, it is mechanical pressure that is converted into an electrical pulse…
- In a thermocouple, it is a temperature gradient…
- In a photoelectric cell, it is light…

Whatever the mechanism, its net effect is determined by the line integral of **f** around the circuit…

$$\varepsilon = \oint \mathbf{f}.\, d\mathbf{l} = \oint \mathbf{f}_s.\, d\mathbf{l}$$

Here, $\mathbf{f}_s$ only, because $\oint \mathbf{E}.\, d\mathbf{l} = 0$ for electrostatic field…

Here, ε is called the electromotive force, or *emf*….

If we take an ideal source of *emf*, like resistance less battery…the net force on charges will be zero (negligible) and σ is ∞…

Ch.7: Electrodynamics: electromotive force…

So, E= -$\mathbf{f}_s$, and the potential different between the terminals (*a* & *b*) can be written as…

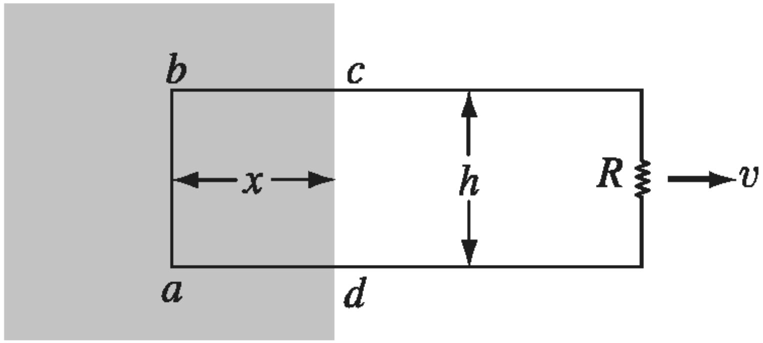
$$V = -\int_a^b \mathbf{E}.\,d\mathbf{l} = \int_a^b \mathbf{f}_s.\,d\mathbf{l} = \oint \mathbf{f}_s.\,d\mathbf{l} = \varepsilon$$

The role of a battery is to establish and maintain a voltage different equal to the emf, for example,

a 6 V battery holds the +ve terminal 6 V above the –ve terminal….

The resulting electrostatic field drives current around the circuit…

Now, let's what is <u>motional *emf*</u>? It arises when we move a wire through a magnetic field…



In this Fig., the shaded region represents a uniform magnetic field pointing into the page…. and R is the resistor through which we want to drive the current…

Ch.7: Electrodynamics: motional emf...

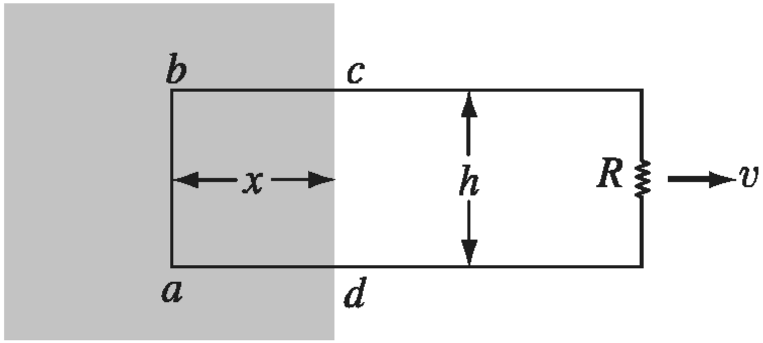
Let's try by moving the entire loop to the right with speed v, the charges in segment *ab* experience magnetic force, whose vertical component is qvB drives the current around the loop in the clockwise direction…

The *emf* can be written as $\varepsilon = \oint \mathbf{f}_{\mathrm{mag}} . d\mathbf{l} = vBh$

The *bc* and *ad* segments contribute nothing as the F is ⊥ to the wire

Note that above integral is carried out at one instant of time, thus, d**l** for *ab* segment points straight up, even the loop is moving right..

What we learn, we lean that although the F_{mag} is responsible for establishing *emf*, it is not doing any work…



As we know that magnetic forces never do work…

Who is then supplying the energy that heats the resistor…?

Ch.7: Electrodynamics: motional emf …

Answer: the person who is pulling the loop to right….
Now let's understand it further….

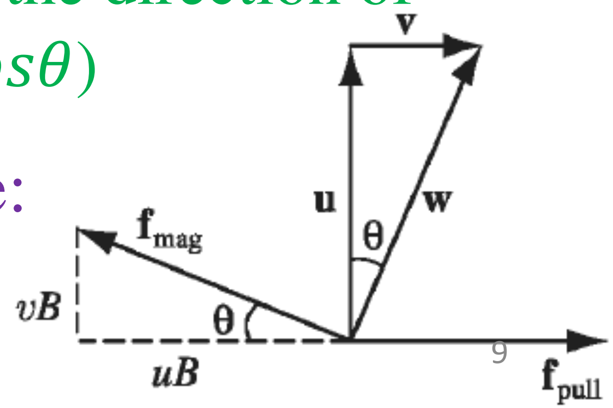
When the current is flowing in the wire, the free charges in *ab* segment will have a vertical velocity (u) and, also a horizontal velocity v they have from the motion of the loop…
Accordingly, in this configuration, the magnetic force will have a component quB to the left…

In order to counteract this force, the person pulling the wire must exert a force per unit charge $f_{pull} = uB$ to the right….

Due to both these effects, the particle is moving in the direction of resultant velocity w & the distance it goes is $(h/cos\theta)$

So, we can write the work done per unit charge:

$$\int \mathbf{f}_{\text{pull}} \cdot d\mathbf{l} = (uB)\left(\frac{h}{\cos\theta}\right)\sin\theta = vBh = \mathcal{E}$$



Ch.7: Electrodynamics: motional emf ...

$$\int \mathbf{f}_{\text{pull}} \cdot d\mathbf{l} = (uB)\left(\frac{h}{\cos\theta}\right)\sin\theta = vBh = \mathcal{E}$$

Though we have taken integrals along different paths and different forces are involved, we learn that the work done/charge is exactly equal to the *emf*….

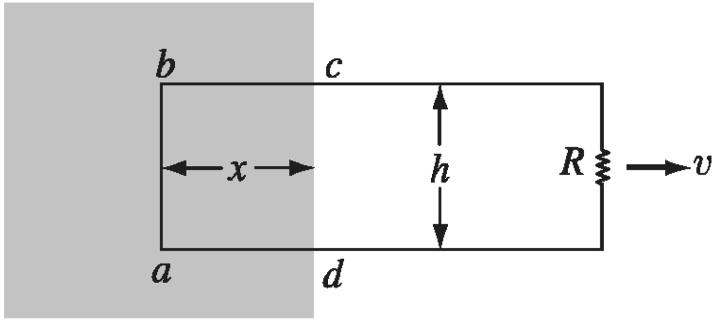
In order to calculate the emf, we integrate around the loop at one instant, but to calculate the work done we follow a charge in its journey around the loop…

Note that $\mathbf{f}_{\text{pull}}$ contributes nothing to the emf, because it is ⊥ to the wire..., whereas $\mathbf{f}_{\text{mag}}$ contributes nothing to work because it is ⊥ to the motion of the charge…

Now let's define ϕ as the flux of B through the loop… we can write… $\phi = \int B.\,da$ and for the rectangular loop in that figure… $\phi = Bhx$ and as the loop moves to right, the flux decreases….

Ch.7: Electrodynamics: motional emf ...

We can write… $\frac{d\phi}{dt} = Bh\frac{dx}{dt} = -Bhv$ (as d*x*/dt is –ve)



Interestingly, this is precisely the *emf*, i.e., the generated *emf* in the loop is minus the rate of change of flux through the loop…

And it can be written as…

$$\varepsilon = -\frac{d\phi}{dt}$$

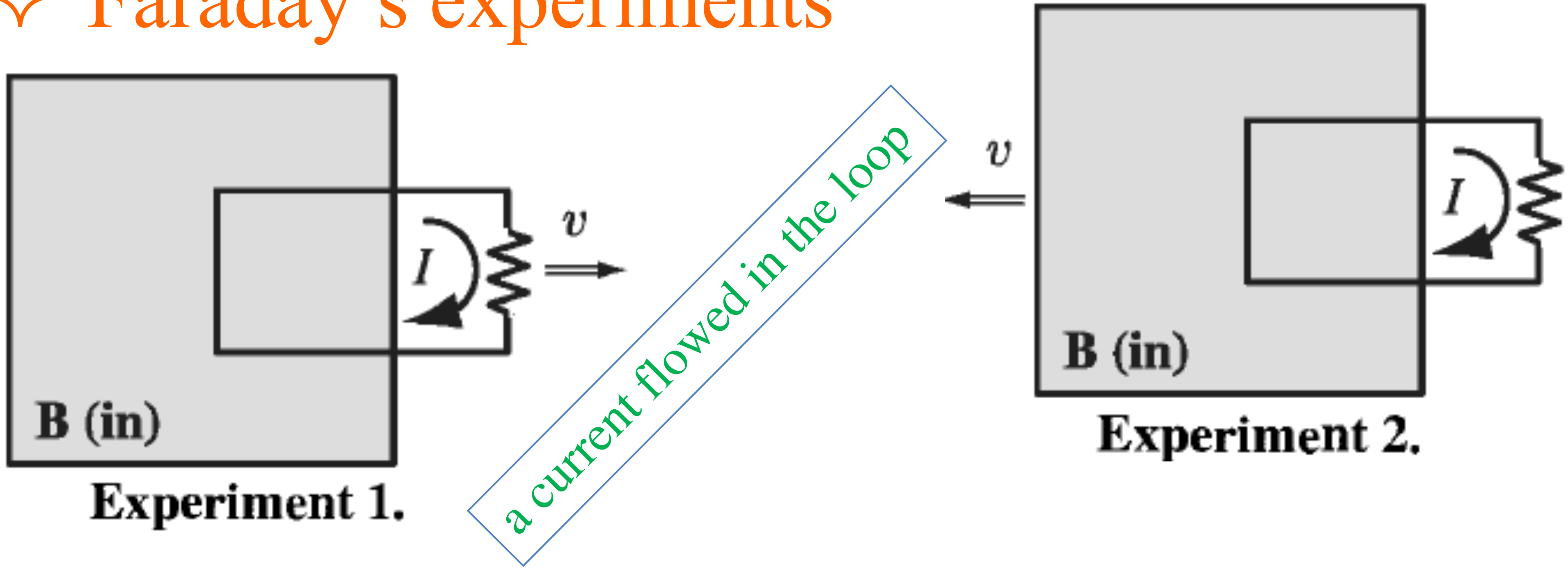
The above equation is the flux rule of motional *emf*….

Also, this can be applied to nonrectangular loops moving in arbitrary directions through nonuniform magnetic fields….

In the context of above discussion, Faraday performed a series of experiments in 1831…..

Ch.7: Electromagnetic induction:

✧ Faraday's experiments



✧ Pulled a loop of wire *w.r.t.* a magnetic field...

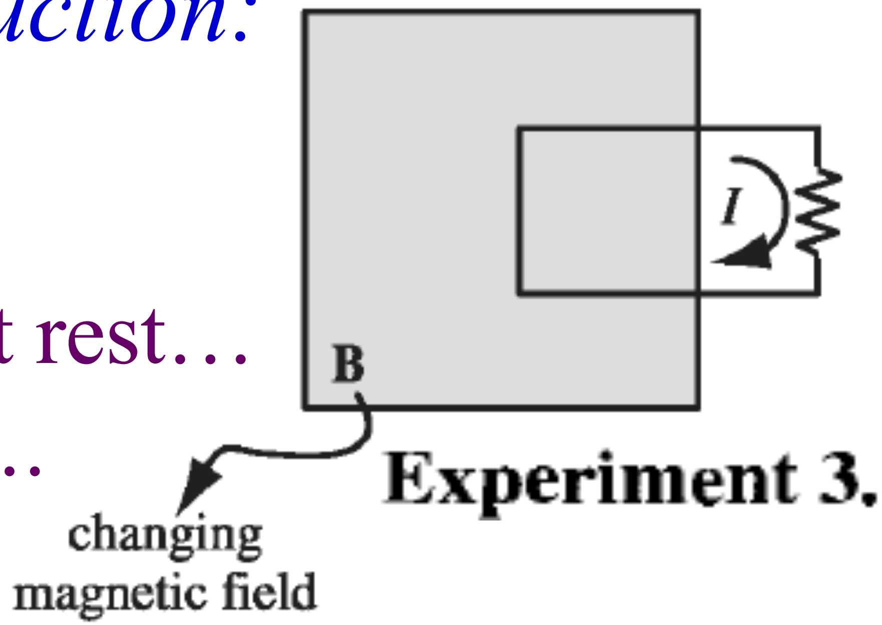
✧ Moved the magnet, holding the loop still…..

✧ All that really matters is the relative motion…………..**motional *emf***……..

Ch.7: Electromagnetic induction:

✧ Faraday's experiments

✧ Both the loop & magnet at rest… strength of the field changed….



Observations:

…….and, current is observed in all the cases….
If the loop moves, it's F_{mag} sets up the *emf*, but if the loop is stationary, the F cannot be magnetic (charges at rest, no F_{mag})….

In this case, what is responsible? What sort of field that exerts a force on charges at rest?; well, we know electric fields do… But then where this electric field is coming from? This means….

A changing magnetic field induces an electric field….

Ch.7: Faraday's Law:

- Changing magnetic field induces an electric field:

Electromotive Force (*emf*) $\varepsilon = \oint \vec{E} \cdot d\vec{l} = -\dfrac{d\phi}{dt}$

where, the magnetic flux $\phi = \int \vec{B} \cdot d\vec{a}$

$$\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$$

Faraday's law in integral form

by using Stoke's theorem: $\int_s (\vec{\nabla} \times \vec{v}) \cdot d\vec{a} = \oint_p \vec{v} \cdot d\vec{l}$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Faraday's law in differential form

The minus sign in Faraday's law indicates that a changing magnetic flux will induce an electric field and current such that the magnetic field induces by the current leads to a flux change in the opposite direction. (called Lenz's Law.)……..

Ch.7: Electrostatics & Magnetostatics:

Electrostatics

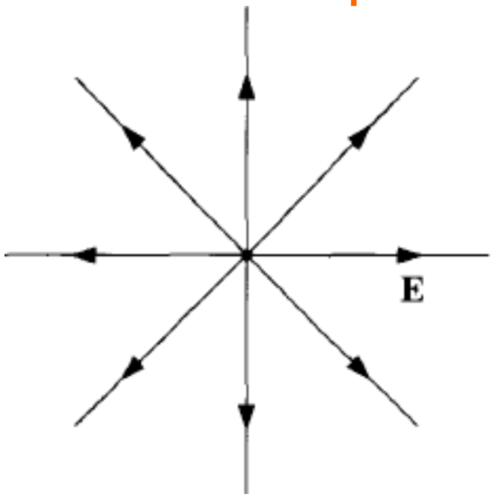
$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho, \text{ (Gauss's law)}$$

$$\nabla \times \mathbf{E} = 0,$$

Div of **E** is nonzero
Curl of **E** is zero

for the static case (constant B)

Electric field diverges away from a +ve point charge



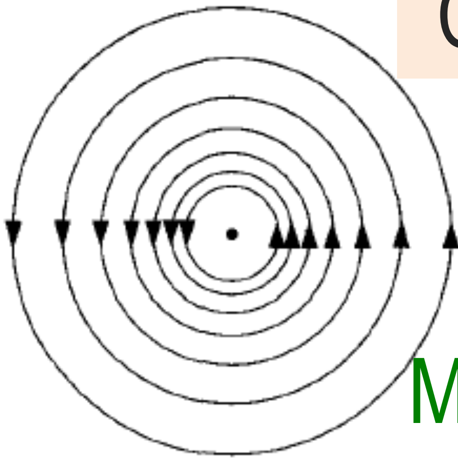
Magnetostatics

$$\nabla \cdot \mathbf{B} = 0,$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J},$$

Ampere's law

Div of B is zero
Curl of B is nonzero



Magnetic field curls around a current

Ch.7: Electrostatics & Magnetostatics:

- ✧ Divergence of **E** is nonzero means, they originate on +ve charges and end on –ve charges.

- ✧ Divergence of **B** is zero means, they do not begin or end anywhere. They either form closed loops or extend to infinity.

i.e., there exist no magnetic analog of electric point charge.

- ✧ Thus, $\nabla . B = 0$ means magnetic charges or *magnetic monopoles* do not exist.

Electrodynamics before Maxwell:

✧ Summary of the laws regarding Div and Curl of electric and magnetic fields:

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\varepsilon_0}\rho$$

Gauss's law

$$\vec{\nabla} \cdot \vec{B} = 0$$

No name

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Faraday's law

$$\vec{\nabla} \times \vec{B} = \mu_0 J$$

Ampere's law

However, there is one inconsistency in these formulas.

What are the problems?

As we know from vector analysis, that the divergence of curl is always zero.

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{v}) = 0$$

Let's apply to Faraday's law:

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\cdot\left(-\frac{\partial\vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\vec{\nabla}\cdot\vec{B}\right)$$

In this case, both LHS and RHS are zero and hence is satisfied.

Now, let's apply to Ampere's law:

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \mu_0\left(\vec{\nabla}\cdot\vec{J}\right)$$

✧ LHS is zero, but RHS is zero only for steady currents; otherwise not.

for steady currents ⇨ $\nabla\cdot\mathbf{J} = -\frac{\partial\rho}{\partial t} = 0$

But for non steady currents, there is inconsistency.

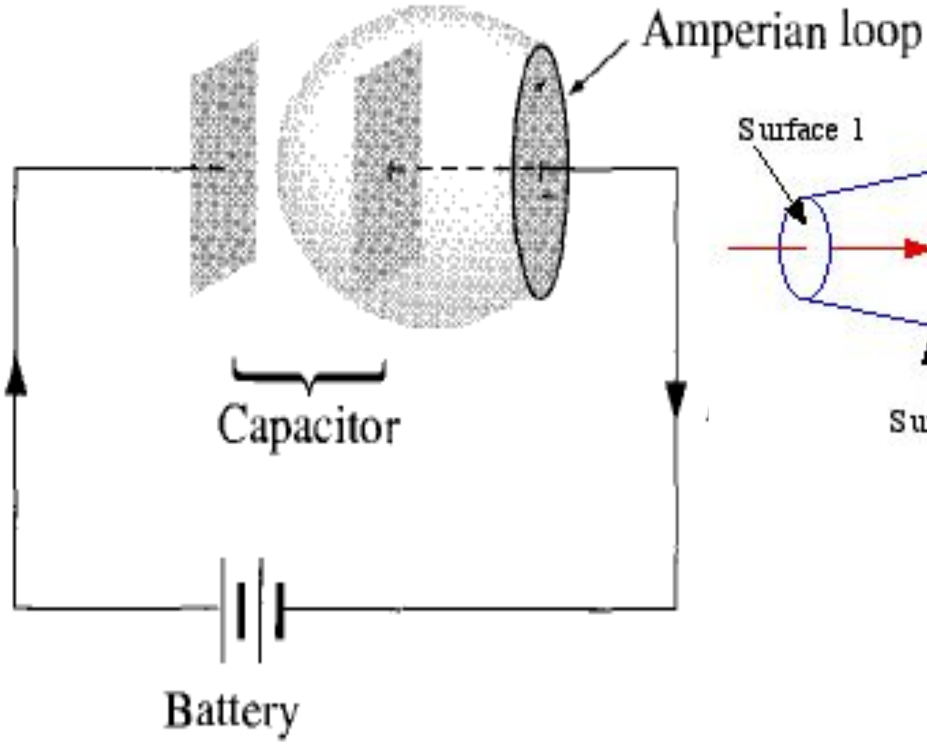
i.e., Ampere's law does not work beyond magnetostatics, and fails for non-steady currents. How can we prove this?

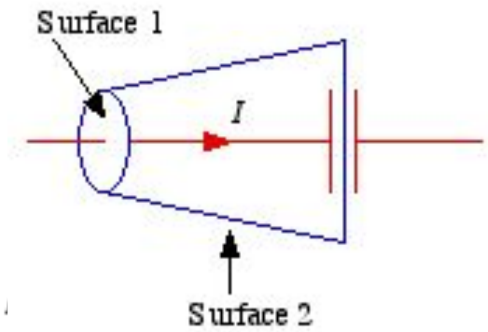
Let's see an example: showing Ampere's law fails

Consider the process of charging up a capacitor and apply Ampere's law

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$$

to the Amperian loop.





where, I_{enc} is the total current passing through the loop

Depending on the surface selected to define the loop,

(plane loop) $I_{\text{enc}} = I$ or $I_{\text{enc}} = 0$ (balloon shaped loop)

Conflict arises since charge is 'piling up' somewhere.

⇒ for non steady current, the "current enclosed by loop" is an ill-defined notion. (as it depends on what surface you choose)..

Let's see how Maxwell fixed Ampere's Law?

We should make $\text{RHS} = 0$ in $\Rightarrow \vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \mu_0\left(\vec{\nabla}\cdot\vec{J}\right)$

The problem term is $\vec{\nabla}\cdot\vec{J}$

What we have to do.......

is use the continuity equation, which describes local charge conservation, and apply the Gauss' law:...

$$\nabla\cdot\mathbf{J} = -\frac{\partial\rho}{\partial t} = -\frac{\partial}{\partial t}(\epsilon_0\nabla\cdot\mathbf{E}) = -\nabla\cdot\left(\epsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)$$

Thus the solution may be to add the term $\epsilon_0\frac{\partial\mathbf{E}}{\partial t}$ to $\mathbf{J}$ in Ampere's law, so as to kill the extra divergence, to get:

$$\nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t}$$

Maxwell's corrected Ampere's Law

$$\vec{\nabla} \times \vec{B} = \mu_0 J + \mu_o \varepsilon_o \frac{\partial \vec{E}}{\partial t}$$

- ✧ This modification changes nothing, as far as magnetostatics is concerned: when **E** is constant, we still have $\vec{\nabla} \times \vec{B} = \mu_0 J$.
- ✧ However, it plays a crucial role in the propagation of Electromagnetic Waves. (will be discussed later…..)
- ✧ Maxwell called this extra term, Displacement Current

$$\mathbf{J}_d \equiv \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

- ✧ It's a misleading name, since it has nothing to do with current, except that it adds to **J** in Ampere's law.
- ✧ Apart from curing the defect in Ampere's law, Maxwell's term <u>implies that a changing **E** induces a magnetic field</u>…..

Let's discuss again, the process of charging capacitor:

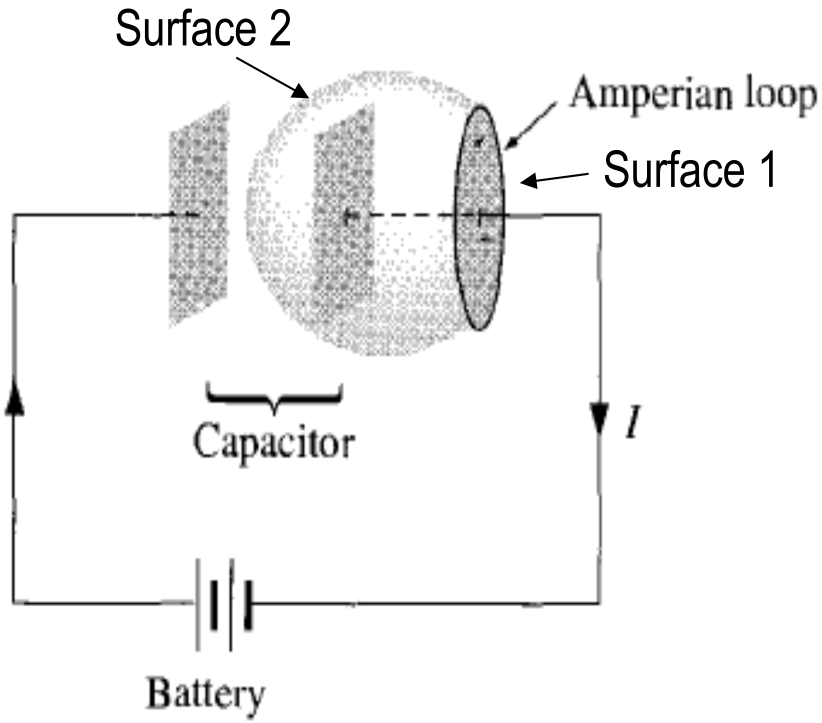
Electric field between the plates of capacitor is, $E = \frac{1}{\epsilon_0}\sigma = \frac{1}{\epsilon_0}\frac{Q}{A}$

Then, we can get.. $\frac{\partial E}{\partial t} = \frac{1}{\epsilon_0 A}\frac{dQ}{dt} = \frac{1}{\epsilon_0 A} I$ between the plates..

Now, $\vec{\nabla} \times \vec{B} = \mu_0 J + \mu_o \varepsilon_o \frac{\partial \vec{E}}{\partial t}$

In integral form:

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc} + \mu_0 \epsilon_0 \int \left(\frac{\partial \mathbf{E}}{\partial t} \right) \cdot d\mathbf{a}$$



For flat surface 1: $E = 0$ and $I_{enc} = I$.

For balloon-shaped surface 2: $I_{enc} = 0$, but $\int (\partial \mathbf{E}/\partial t) \cdot d\mathbf{a} = I/\epsilon_0$.

Hence the same answer, in both cases…means problem solved…..

Now we have Maxwell's equations:

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_0}\rho$$

Gauss's law

$$\vec{\nabla}\cdot\vec{B} = 0$$

No name

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

Faraday's law:
changing magnetic field
induces an electric field.

$$\vec{\nabla}\times\vec{B} = \mu_0 J + \mu_o\varepsilon_o\frac{\partial\vec{E}}{\partial t}$$

Ampere's law after
Maxwell's correction:
changing electric field
induces a magnetic field.

E produced by either charges (ρ) or changing **B** ($\frac{\partial\vec{B}}{\partial t}$)

B produced by either current (***J***) or changing electric field ($\frac{\partial\vec{E}}{\partial t}$)