The giant virtual image of the photographer, who is standing closer than one focal length from a multi-element telescope mirror in Tucson, Arizona. He's wearing a hat and has his right hand raised. (Joseph Shaw)

5.5 Prisms

Prisms play many different roles in Optics; there are prism combinations that serve as beamsplitters (p. 138), polarizing devices (see Section 8.4.3), and interferometers. Despite this diversity, the vast majority of applications make use of only one of two main prism functions. First, a prism can serve as a dispersive device, as it does in a variety of spectrum analyzers (Fig. 5.66). As such it is capable of separating, to some extent, the constituent frequency components in a polychromatic light beam. Recall that the term *dispersion* was introduced earlier (p. 78) in connection with the frequency dependence of the index of refraction, $n(\omega)$, for dielectrics. In fact, the prism provides a highly useful means of measuring $n(\omega)$ over a wide range of frequencies and for a variety of materials (including gases and liquids).

Its second and more common function is to effect a change in the orientation of an image or in the direction of propagation of a beam. Prisms are incorporated into many optical instruments, often simply to fold the system into a confined space. There are inversion prisms, reversion prisms, and prisms that deviate a beam without inversion or reversion—and all of this without dispersion.

5.5.1 Dispersing Prisms

Prisms come in many sizes and shapes and perform a variety of functions (see photo). Let's first consider the group known as **dispersing prisms**. Typically, a ray entering a dispersing prism, as in Fig. 5.66, will emerge having been deflected from its original direction by an angle δ known as the **angular deviation**. At the first refraction the ray is deviated through an angle $(\theta_{i1} - \theta_{t1})$, and at the second refraction it is further deflected through $(\theta_{t2} - \theta_{i2})$. The total deviation is then

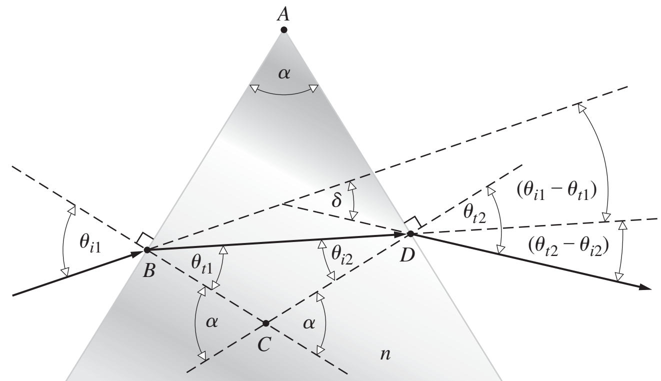


Figure 5.66 Geometry of a dispersing prism.

$$\delta = (\theta_{i1} - \theta_{t1}) + (\theta_{t2} - \theta_{i2})$$

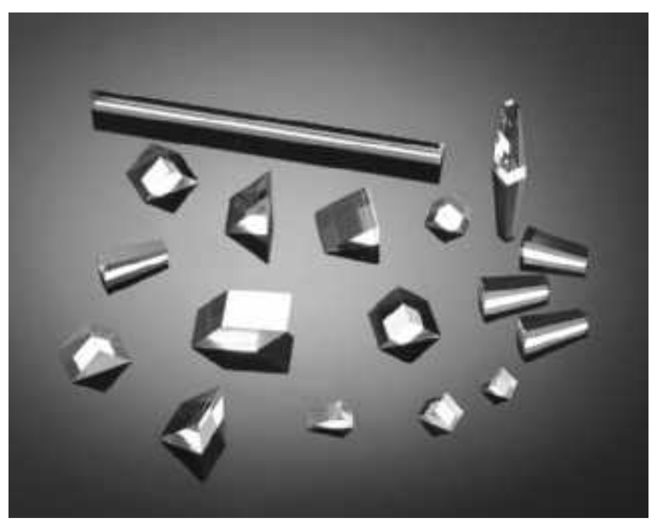
Since the polygon $ABCD$ contains two right angles, $\measuredangle BCD$ must be the supplement of the **apex angle** α. As the exterior angle to triangle BCD, α is also the sum of the alternate interior angles, that is,

$$\alpha = \theta_{t1} + \theta_{i2} \tag{5.51}$$

Thus

$$\delta = \theta_{i1} + \theta_{t2} - \alpha \tag{5.52}$$

We would like to write δ as a function of both the angle-of-incidence for the ray (i.e., θ_{i1}) and the prism angle α; these presumably would be known. If the prism index is n and it's



A selection of various prisms. (Perkins Precision Developments) immersed in air ($n_a \approx 1$), it follows from Snell's Law that

$$\theta_{t2} = \sin^{-1}(n \sin\theta_{i2}) = \sin^{-1}[n \sin(\alpha - \theta_{t1})]$$

Upon expanding this expression, replacing $\cos\theta_{t1}$ by $(1 - \sin^2\theta_{t1})^{1/2}$, and using Snell's Law we have

$$\theta_{t2} = \sin^{-1}[(\sin\alpha)(n^2 - \sin^2\theta_{i1})^{1/2} - \sin\theta_{i1}\cos\alpha]$$

The deviation is then

$$\delta = \theta_{i1} + \sin^{-1}[(\sin\alpha)(n^2 - \sin^2\theta_{i1})^{1/2} - \sin\theta_{i1}\cos\alpha] - \alpha \tag{5.53}$$

Apparently, δ increases with n, which is itself a function of frequency, so we might designate the deviation as $\delta(\nu)$ or $\delta(\lambda)$. For most transparent dielectrics of practical concern, $n(\lambda)$ decreases as the wavelength increases across the visible [refer back to Fig. 3.41 for a plot of $n(\lambda)$ versus λ for various glasses]. Clearly, then, $\delta(\lambda)$ will be less for red light than it is for blue.

Missionary reports from Asia in the early 1600s indicated that prisms were well known and highly valued in China because of their ability to generate color. A number of scientists of the era, particularly Marci, Grimaldi, and Boyle, had made some observations using prisms, but it remained for the great Sir Isaac Newton to perform the first definitive studies of dispersion. On February 6, 1672, Newton presented a classic paper to the Royal Society titled "A New Theory about Light and Colours." He had concluded that white light consisted of a mixture of various colors and that the process of refraction was color-dependent.

Returning to Eq. (5.53), it's evident that the deviation suffered by a monochromatic beam on traversing a given prism (i.e., n and α are fixed) is a function only of the incident angle at the first face, θ_{i1}. A plot of the results of Eq. (5.53) as applied to a typical glass prism is shown in Fig. 5.67. The smallest value of δ is known as the **minimum deviation**, δ_m, and it is of particular interest for practical reasons. The value of δ_m can be determined analytically by differentiating Eq. (5.53) and then setting $d\delta/d\theta_{i1} = 0$, but a more indirect route will certainly be simpler. Differentiating Eq. (5.52) and setting it equal to zero yields

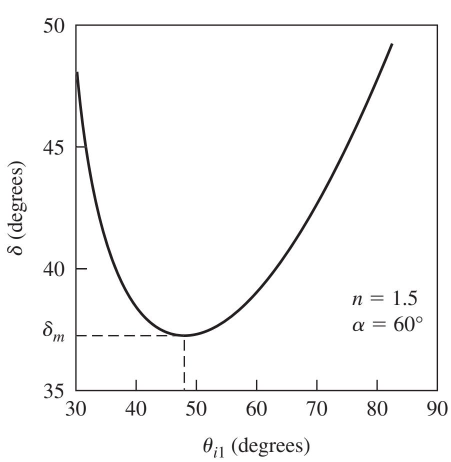


Figure 5.67 Deviation versus incident angle.

$$\frac{d\delta}{d\theta_{i1}} = 1 + \frac{d\theta_{t2}}{d\theta_{i1}} = 0$$

or $d\theta_{t2}/d\theta_{i1} = -1$. Taking the derivative of Snell's Law at each interface, we get

$$\cos\theta_{i1}\, d\theta_{i1} = n \cos\theta_{t1}\, d\theta_{t1}$$

and

$$\cos\theta_{t2}\, d\theta_{t2} = n \cos\theta_{i2}\, d\theta_{i2}$$

Note as well, on differentiating Eq. (5.51), that $d\theta_{t1} = -d\theta_{i2}$, since $d\alpha = 0$. Dividing the last two equations and substituting for the derivatives leads to

$$\frac{\cos\theta_{i1}}{\cos\theta_{t2}} = \frac{\cos\theta_{t1}}{\cos\theta_{i2}}$$

Making use of Snell's Law once again, we can rewrite this as

$$\frac{1 - \sin^2\theta_{i1}}{1 - \sin^2\theta_{t2}} = \frac{n^2 - \sin^2\theta_{i1}}{n^2 - \sin^2\theta_{t2}}$$

The value of θ_{i1} for which this is true is the one for which $d\delta/d\theta_{i1} = 0$. Inasmuch as $n \neq 1$, it follows that

$$\theta_{i1} = \theta_{t2}$$

and therefore

$$\theta_{t1} = \theta_{i2}$$

This means that *the ray for which the deviation is a minimum traverses the prism symmetrically, that is, parallel to its base*. Incidentally, there is a lovely argument for why θ_{i1} must equal θ_{t2}, which is neither as mathematical nor as tedious as the one we have evolved. In brief, suppose a ray undergoes a minimum deviation and $\theta_{i1} \neq \theta_{t2}$. Then if we reverse the ray, it will retrace the same path, so δ must be unchanged (i.e., $\delta = \delta_m$). But this implies that there are two different incident angles for which the deviation is a minimum, and this we know is not true—ergo $\theta_{i1} = \theta_{t2}$.

In the case when $\delta = \delta_m$, it follows from Eqs. (5.51) and (5.52) that $\theta_{i1} = (\delta_m + \alpha)/2$ and $\theta_{t1} = \alpha/2$, whereupon Snell's Law at the first interface leads to

$$n = \frac{\sin[(\delta_m + \alpha)/2]}{\sin\alpha/2} \tag{5.54}$$

This equation forms the basis of one of the most accurate techniques for determining the refractive index of a transparent substance. Effectively, one fashions a prism out of the material in question, and then, measuring α and $\delta_m(\lambda)$, $n(\lambda)$ is computed employing Eq. (5.54) at each wavelength of interest. Hollow

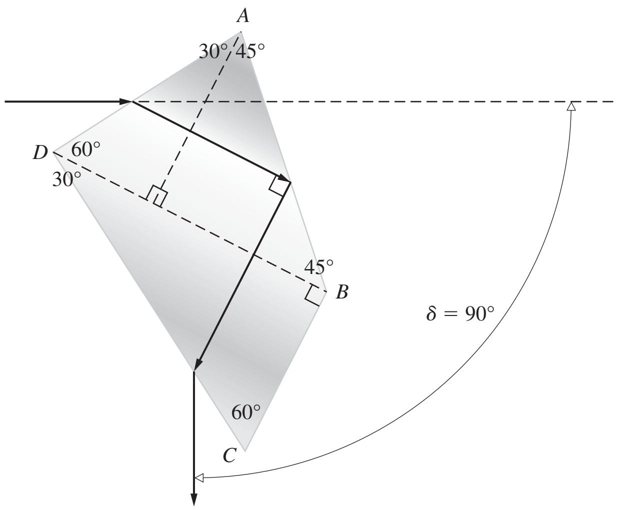


Figure 5.68 The Pellin–Broca prism.

prisms whose sides are fabricated of plane-parallel glass can be filled with liquids or gases under high pressure; the glass plates will not result in any deviation of their own.

Figures 5.68 and 5.69 show two examples of **constant-deviation dispersing prisms**, which are important primarily in spectroscopy. The **Pellin–Broca prism** is probably the most common of the group. Albeit a single block of glass, it can be envisaged as consisting of two 30°–60°–90° prisms and one 45°–45°–90° prism. Suppose that in the position shown a single monochromatic ray of wavelength λ traverses the component prism *DAE* symmetrically, thereafter to be reflected at 45° from face *AB*. The ray will then traverse prism *CDB* symmetrically, having experienced a total deviation of 90°. The ray can be thought of as having passed through an ordinary 60° prism (*DAE* combined with *CDB*) at minimum deviation. All other wavelengths present in the beam will emerge at other angles. If the prism is now rotated slightly about an axis normal to the paper, the incoming beam will have a new incident angle. A different wavelength component, say λ_2, will now undergo a minimum deviation, which is again 90°—hence the name *constant deviation*. With a prism of this sort, one can conveniently set up the light source and viewing system at a fixed angle (here 90°), and then simply rotate the prism to look at a particular wavelength. The device can be calibrated so that the prism-rotating dial reads directly in wavelength.

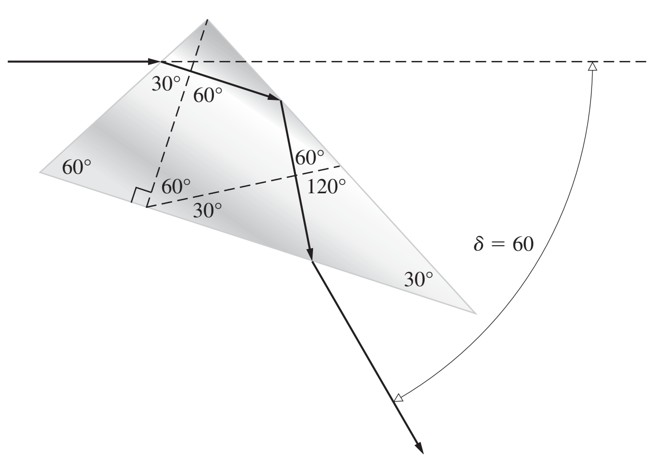


Figure 5.69 The Abbe prism.

5.5.2 Reflecting Prisms

We now examine **reflecting prisms**, in which dispersion is not desirable. In this case, the beam is introduced in such a way that at least one internal reflection takes place, for the specific purpose of changing either the direction of propagation or the orientation of the image, or both.

Let's first establish that it is actually possible to have such an internal reflection without dispersion. Is δ independent of λ? The prism in Fig. 5.70 is assumed to have as its profile an isosceles triangle—this happens to be a rather common configuration in any event. The ray refracted at the first interface is later reflected from face *FG*. As we saw earlier (Section 4.7), this will occur when the internal incident angle is greater than the critical angle θ_c, defined by

$$\sin\theta_c = n_{ti} \qquad [4.69]$$

For a glass–air interface, this requires that θ_i be greater than roughly 42°. To avoid any difficulties at smaller angles, let's further suppose that the base of our hypothetical prism is silvered as well—certain prisms do in fact require silvered faces. The angle of deviation between the incoming and outgoing rays is

$$\delta = 180° - \measuredangle BED \qquad (5.55)$$

From the polygon *ABED* it follows that

$$\alpha + \measuredangle ADE + \measuredangle BED + \measuredangle ABE = 360°$$

Moreover, at the two refracting surfaces

$$\measuredangle ABE = 90° + \theta_{i1}$$

and

$$\measuredangle ADE = 90° + \theta_{t2}$$

Substituting for $\measuredangle BED$ in Eq. (5.55) leads to

$$\delta = \theta_{i1} + \theta_{t2} + \alpha \qquad (5.56)$$

Since the ray at point-*C* has equal angles-of-incidence and reflection, $\measuredangle BCF = \measuredangle DCG$. Thus, because the prism is isosceles, $\measuredangle BFC = \measuredangle DGC$, and triangles *FBC* and *DGC* are similar. It follows that $\measuredangle FBC = \measuredangle CDG$, and therefore $\theta_{t1} = \theta_{i2}$. From Snell's Law we know that this is equivalent to $\theta_{i1} = \theta_{t2}$, whereupon the deviation becomes

$$\delta = 2\theta_{i1} + \alpha \qquad (5.57)$$

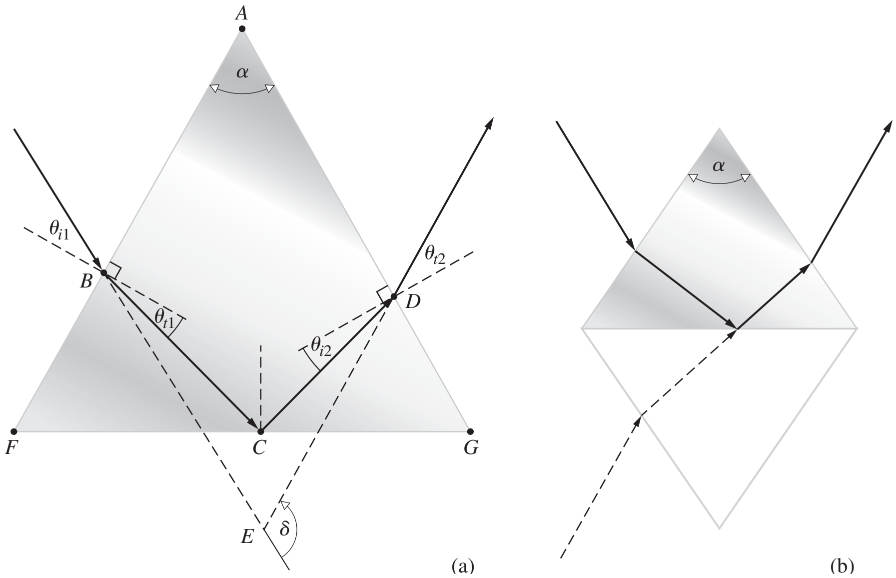


Figure 5.70 Geometry of a reflecting prism.

which is certainly independent of both λ and n. The reflection will occur without any color preferences, and the prism is said to be **achromatic**. Unfolding the prism, that is, drawing its image in the reflecting surface *FG*, as in Fig. 5.70*b*, we see that it is equivalent in a sense to a parallelepiped or thick planar plate. The image of the incident ray emerges parallel to itself, regardless of wavelength.

A few of the many widely used reflecting prisms are shown in the next several figures. These are often made from BSC-2 or C-1 glass (see Table 6.2). For the most part, the illustrations are self-explanatory, so the descriptive commentary will be brief.

The **right-angle prism** (Fig. 5.71) deviates rays normal to the incident face by 90°. Notice that the top and bottom of the image have been interchanged; that is, the arrow has been flipped over, but the right and left sides have not. It is therefore an inversion system with the top face acting like a plane mirror. (To see this, imagine that the arrow and lollypop are vectors and take their cross-product. The resultant, arrow × lollypop, was initially in the propagation direction but is reversed by the prism.)

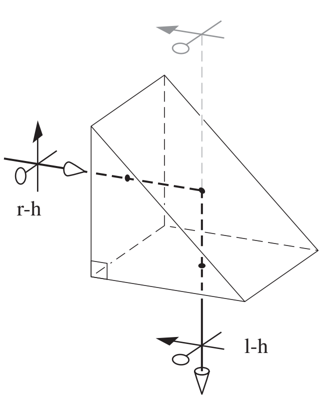


Figure 5.71 The right-angle prism.

The **Porro prism** (Fig. 5.72) is physically the same as the right-angle prism but is used in a different orientation. After two reflections, the beam is deviated by 180°. Thus, if it enters right-handed, it leaves right-handed.

The **Dove prism** (Fig. 5.73) is a truncated version (to reduce size and weight) of the right-angle prism, used almost exclusively in collimated light. It has the interesting property (Problem 5.92) of rotating the image twice as fast as it is itself rotated about the longitudinal axis.

The **Amici prism** (Fig. 5.74) is essentially a truncated right-angle prism with a roof section added on to the hypotenuse face. In its most common use, it has the effect of splitting the image down the middle and interchanging the right and left portions.* These prisms are expensive, because the 90° roof

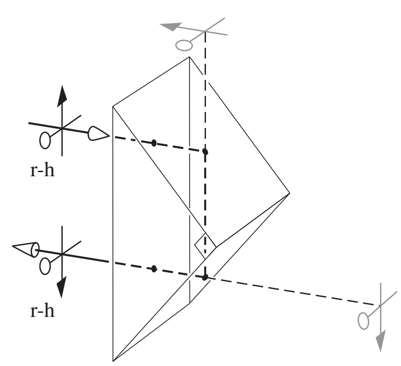


Figure 5.72 The Porro prism.

*You can see how it actually works by placing two plane mirrors at right angles and looking directly into the combination. If you wink your *right* eye, the image will wink its *right* eye. Incidentally, if your eyes are equally strong, you will see two seams (images of the line where the mirrors meet), one running down the middle of each eye, with your nose presumably between them. If one eye is stronger, there will be only one seam, down the middle of that eye. If you close it, the seam will jump over to the other eye. This must be tried to be appreciated.

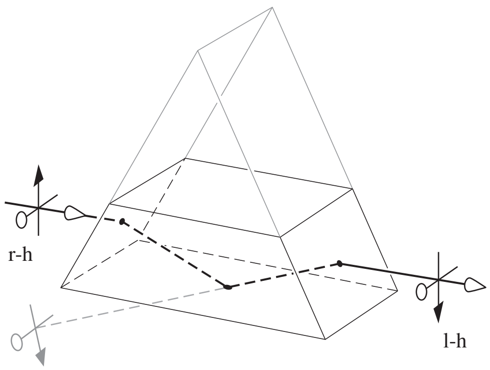


Figure 5.73 The Dove prism.

angle must be held to roughly 3 or 4 seconds of arc, or a troublesome double image will result. They are often used in simple telescope systems to correct for the reversion introduced by the lenses.

The **rhomboid prism** (Fig. 5.75) displaces the line-of-sight without producing any angular deviation or changes in the orientation of the image.

The **penta prism** (Fig. 5.76) will deviate the beam by 90° without affecting the orientation of the image. Note that two of its surfaces must be silvered. These prisms are often used as end reflectors in small range finders.

The **Leman–Springer prism** (Fig. 5.77) also has a 90° roof. Here the line-of-sight is displaced without being deviated, but the emerging image is right-handed and rotated through 180°. The prism can therefore serve to erect images in telescope systems, such as gun sights and the like.

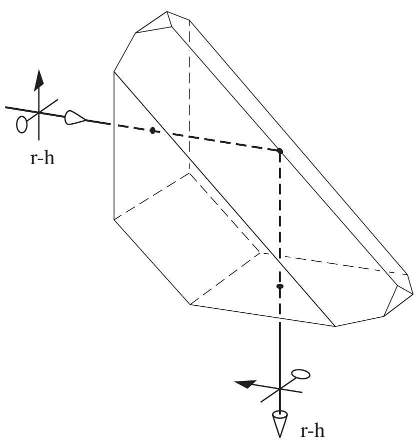


Figure 5.74 The Amici prism.

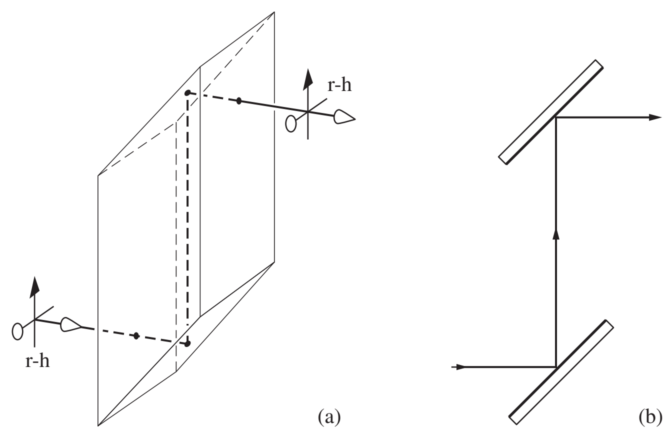


Figure 5.75 The rhomboid prism and its mirror equivalent.

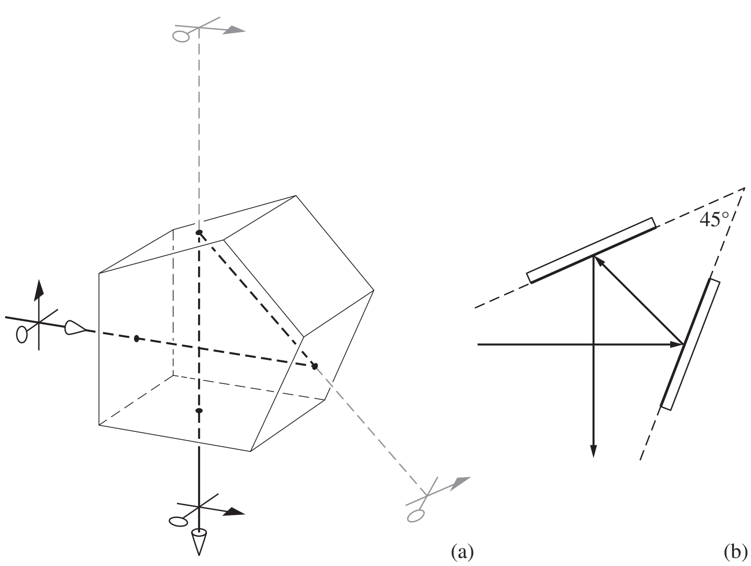


Figure 5.76 The penta prism and its mirror equivalent.

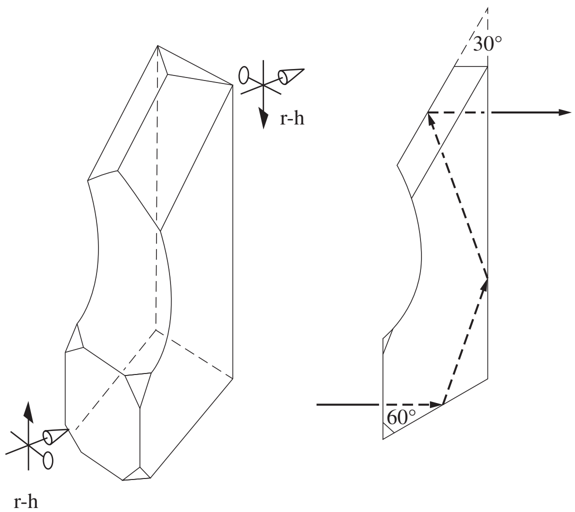


Figure 5.77 The Leman–Springer prism.

Many more reflecting prisms perform specific functions. For example, if one cuts a cube so that the piece removed has three mutually perpendicular faces, it is called a **corner-cube prism**. It has the property of being retrodirective; that is, it will reflect all incoming rays back along their original directions. One hundred of these prisms are sitting in an 18-inch square array 240 000 miles from here, having been placed on the Moon during the Apollo 11 flight.*

The most common erecting system consists of two Porro prisms, as illustrated in Fig. 5.78. These are relatively easy to manufacture and are shown here with rounded corners to

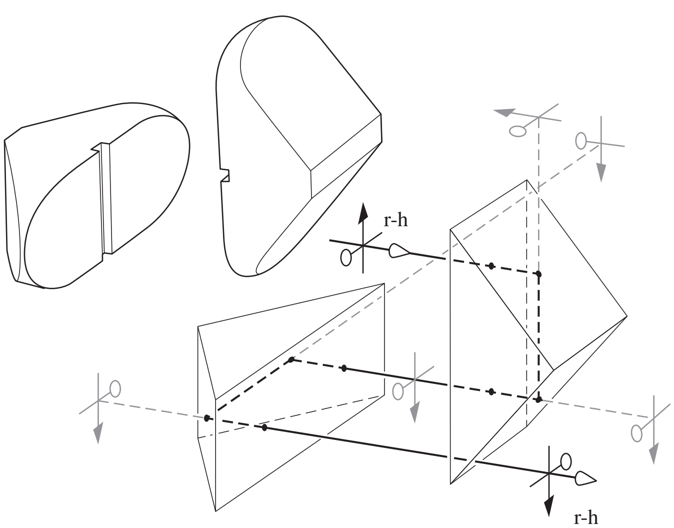


Figure 5.78 The double Porro prism.

*J. E. Foller and E. J. Wampler, "The Lunar Laser Reflector," *Sci. Am.*, March 1970, p. 38.

reduce weight and size. Since there are four reflections, the exiting image will be right-handed. A small slot is often cut in the hypotenuse face to obstruct rays that are internally reflected at glancing angles. Finding these slots after dismantling the family's binoculars is often an inexplicable surprise.

5.6 Fiberoptics

The concept of channeling light within a long, narrow dielectric (via total internal reflection) has been around for quite a while. John Tyndall (1870) showed that light could be contained within and guided along a thin stream of water. Soon after that, glass "light pipes" and, later, threads of fused quartz were used to further demonstrate the effect. But it wasn't until the early 1950s that serious work was done to transport images along bundles of short glass fibers.

After the advent of the laser (1960), there was an immediate appreciation of the potential benefits of sending information from one place to another using light, as opposed to electric currents or even microwaves. At those high optical frequencies (of the order of 10^{15} Hz), one hundred thousand times more information can be carried than with microwaves. Theoretically, that's the equivalent of sending tens of millions of television programs all at once on a beam of light. It wasn't long (1966) before the possibility of coupling lasers with fiberoptics for long-distance communications was pointed out. Thus began a tremendous technological transformation that's still roaring along today.

In 1970 researchers at the Corning Glass Works produced a silica fiber with a signal-power transmission of better than 1% over a distance of 1 km (i.e., an attenuation of 20 dB/km), which was comparable to existing copper electrical systems. During the next two decades, the transmission rose to about 96% over 1 km (i.e., an attenuation of only 0.16 dB/km).

Because of its low-loss transmission, high-information-carrying capacity, small size and weight, immunity to electromagnetic interference, unparalleled signal security, and the abundant availability of the required raw materials (i.e., ordinary sand), ultrapure glass fibers have become the premier communications medium.

As long as the diameter of these fibers is large compared with the wavelength of the radiant energy, the inherent wave nature of the propagation is of little importance, and the process obeys the familiar laws of Geometrical Optics. On the other hand, if the diameter is of the order of λ, the transmission closely resembles the manner in which microwaves advance along waveguides. Some of the propagation modes are evident in the photomicrographic end views of fibers shown in Fig. 5.79. Here the wave nature of light must be reckoned with, and this behavior resides in the domain of Physical Optics. Although optical waveguides, particularly of