COL 352 Introduction to Automata and Theory of Computation

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Introduction, Logistics, Motivation etc.

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  3. If we can solve a problem in theory, can it still be solvable in practice?

Automata and Theory of Computation

Dictionary definition

noun (plural automata)

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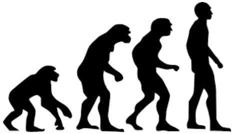

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- Move towards the most versatile*
- Prove their power and limitations
- Determine the relationships between models

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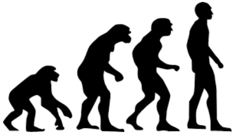

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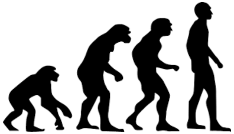

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Main Goal: Learn to formally reason about computation!

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- Math is good for you! ☺

"Computer Science is no more about computers than astronomy is about telescopes"

Edgar Dijkstra *(Turing award 1972)*

Logistics

Quizzes	30%
Minors	35%
Major	35%

Attendance: 75% (to qualify for audit pass and remajor).
Audit Pass: 40% total with at least 50% in Major+Minors.

Teaching Assistants: Sourav Bansal (cs5180421), Suraj Patni (csy217550), Sanaa Siddiqui (csy227516), Surbhi Rajput (csy227519), Harsh Singh Chauhan (mcs222052)

Forums: Piazza (Q & A), Moodle (?), MS Teams (Announcements).

Textbooks:

- Automata and Computability - Dexter Kozen.
- Introduction to Theory of Computation - Michael Sipser.

Logistics contd.

Web: https://sites.google.com/view/nikhilbalaji/toc2023

Prerequisites: COL 202, COL 351 or equivalent (if you don't satisfy these, email me!)

- Proof techniques - construction, contradiction, induction, counting techniques.
- Sets and set operations, relations Functions – injections, surjections, and bijections, cardinality.
- Countability and uncountability, diagonalization.
- Equivalence relations, partial orders

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Let's get started!

Modelling computation

Modeling a computing device: an automaton

- Input
- Processing
- Output

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Example:

- Consider a bulb with a switch.
- Its input are: on/off.
- Behavior during the day is: “on off on off”
- When light is off, I can turn it on and vice versa.

Inputs

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- Domain of inputs/actions $\Sigma = \{\text{on}, \text{off}\}$.
- Σ is called the Alphabet.
- Elements of the alphabet are Letters.
- Inputs or behaviors are strings or sequences of letters, called Words.
- Example: "on off on off on off" is a word in Σ^*.

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- $\Sigma^+ = \Sigma^* \setminus \Sigma^0$

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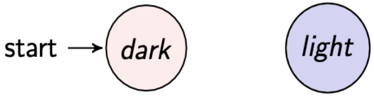

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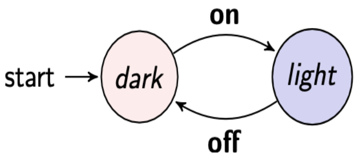
- States: current information which allows to process the next letter/input.
- Transitions: read an input and move from a state to another



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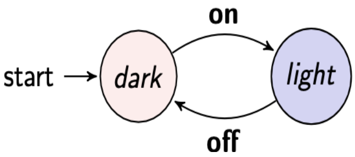

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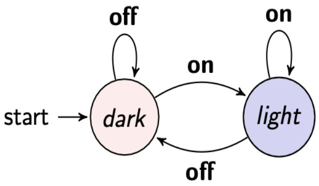

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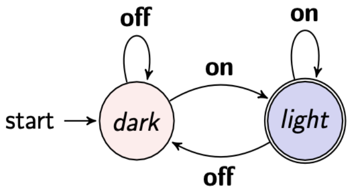
- "on off on"
- "off off"
- "on (off on)*"

Outputs



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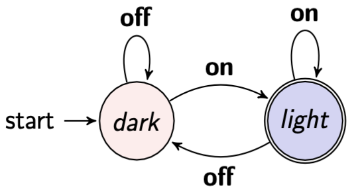

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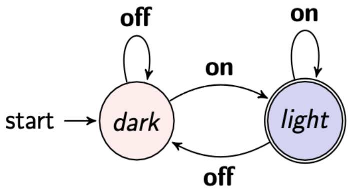
- What are the words accepted by above automaton?

Language of an automaton



All words accepted by an automaton form its language.

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Exercise:

- Give an example of a finite and infinite language.
- Give an example of a language that contains any language over Σ.

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We will go back-and-forth between languages and problems.