Lecture 4
Signals and Systems (ELL205)

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Outline

- Different kinds of Signals
  - Continuous-time vs. Discrete-time signals
  - Energy vs. Power signals
- Signal transformations
  - Flipping
  - Scaling
  - Shifting
- Further classifications of Signals
  - Even vs. Odd signals
  - Periodic vs. Aperiodic signals
- Basic Signals
  - Exponential

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Transformation of DT signal

$$x[n] = \{-5,3,\underset{\substack{\uparrow \\ n=0}}{2},1,4,5\}$$

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$$x[n+2] = \{-5,3,2,1,\underset{\substack{\uparrow \\ n\,=\,0}}{4},5\}$$

Transformation of DT signal

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$$x[n-2] = \{\underset{\substack{\uparrow \\ n=0}}{-5},3,2,1,4,5\}$$

$$x[-n] = \{5,4,1,\underset{\substack{\uparrow \\ n=0}}{2},3,-5\}$$

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$$y[n] = x[2n]$$

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$$y[-1] = x[-2] = -5$$

$$y[0] = x[0] = 2$$

$$y[1] = x[2] = 4$$

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Leads to decimation of samples (samples are permanently lost)

Transformation of DT signal

$$x[n] = \{-5,3,\underset{\substack{\uparrow \\ n=0}}{2},1,4,5\}$$

$$y[n] = \begin{cases} x\left[\dfrac{n}{2}\right] & n \text{ is even} \\ 0 & n \text{ is odd} \end{cases}$$

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$$y[n] = \begin{cases} x\left[\frac{n}{p}\right] & n = mp, \text{where } m \epsilon I \\ 0 & n \neq mp, \text{where } m \epsilon I \end{cases}$$

If $p > 1$ & $an\ I$, then insert $p - 1$ zeros between samples.

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Even vs. Odd signals

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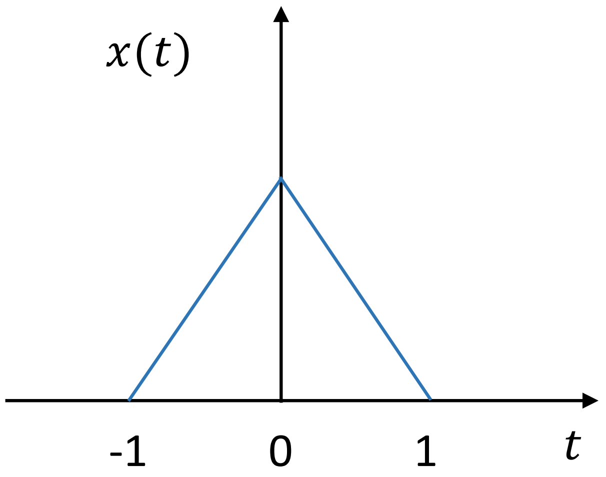
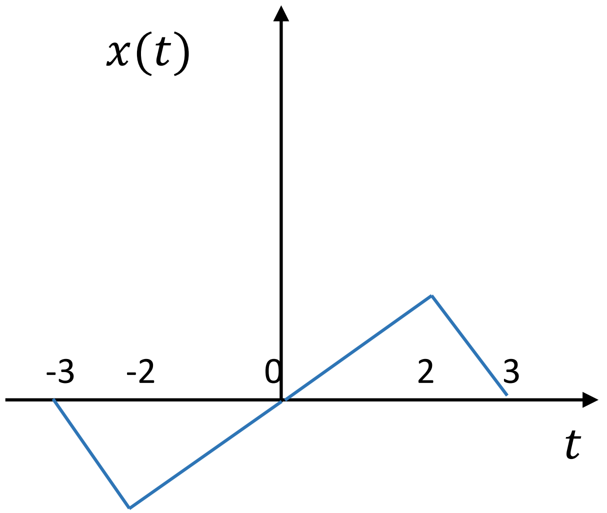


Even vs. Odd signals

- $x(t) = -x(-t)$ Odd signal



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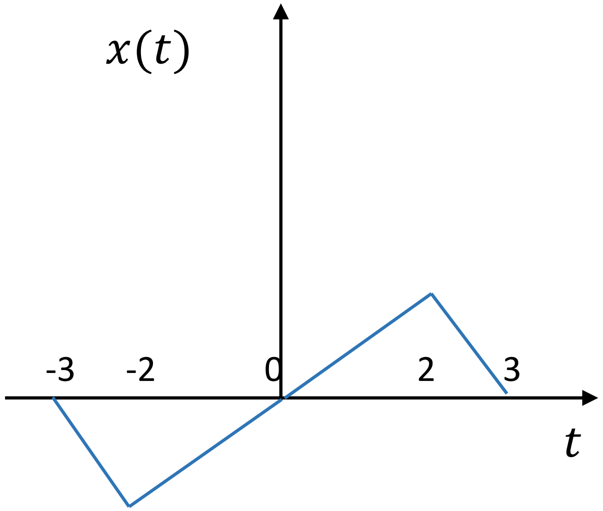

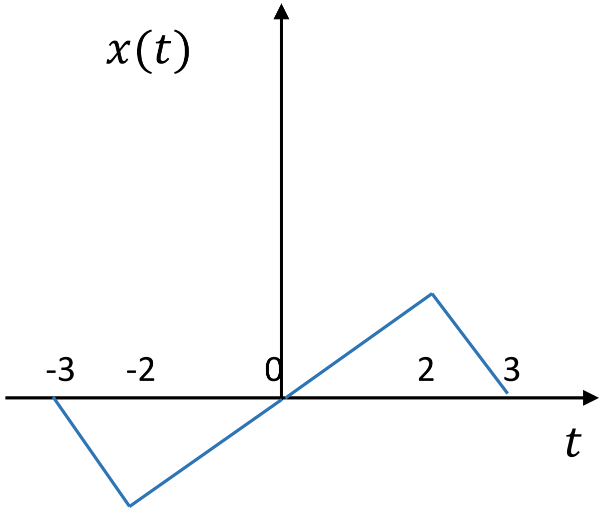

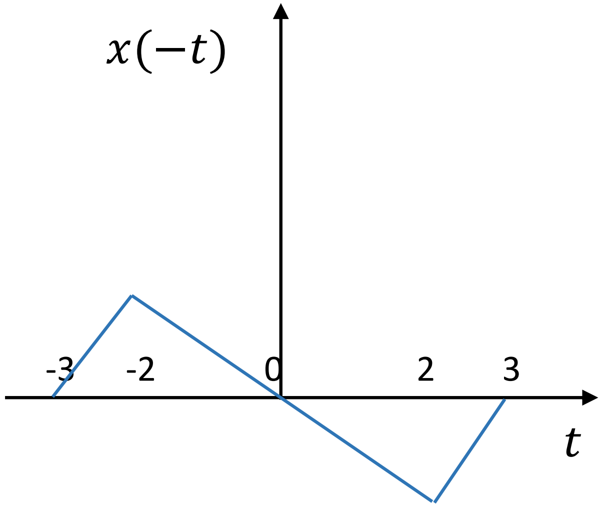

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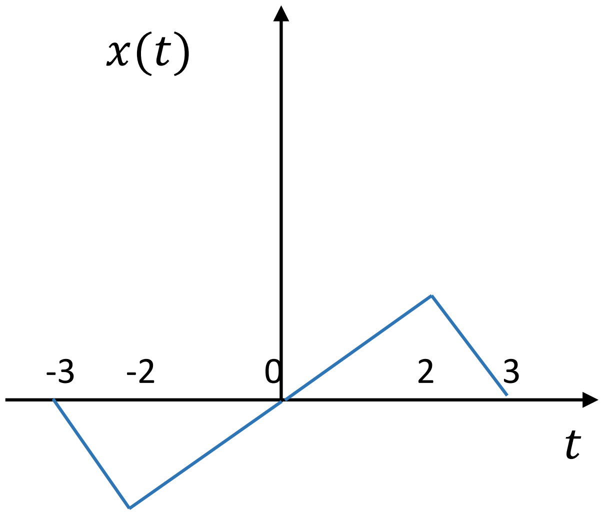
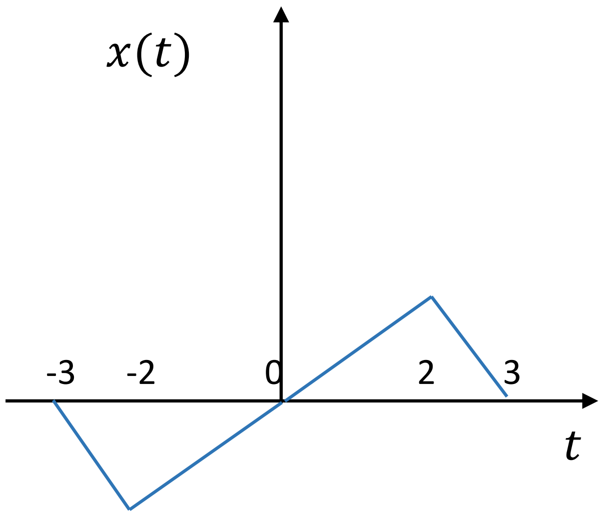
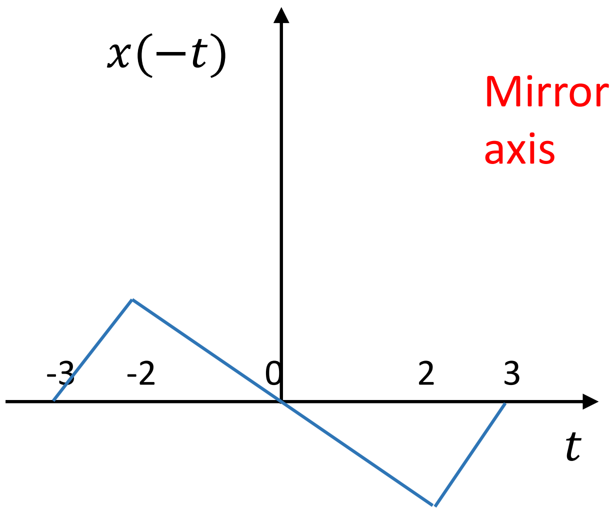


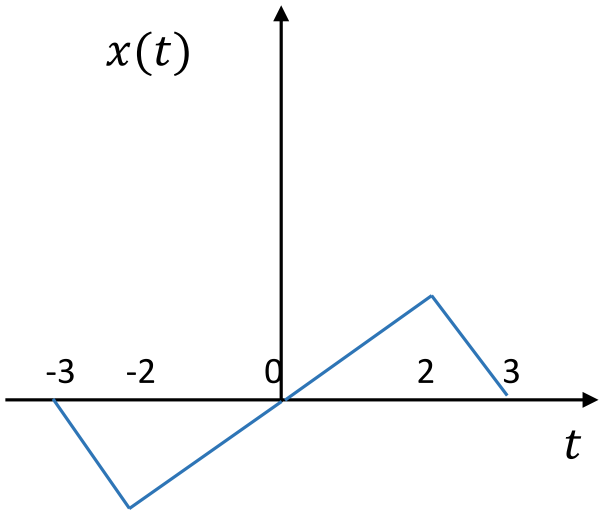




Mirror Image along x axis

Even vs. Odd signals

- $x(t) = -x(-t)$ Odd signal



For odd signal

$$x(0) = -x(0)$$

$$x(0) = 0$$

Even vs. Odd signals

- $x(t) = x_e(t) + x_o(t)$ {Signal decomposition}

Even vs. Odd signals

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- $x(-t) = x_e(-t) + x_o(-t)$
- $x(-t) = x_e(t) - x_o(t)$

Even vs. Odd signals

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Even vs. Odd signals

- $x(t) = x_e(t) + x_o(t)$ {Signal decomposition}
- $x(-t) = x_e(t) - x_o(t)$
- Adding (and subtracting) these two equations,
- $x_e(t) = \frac{x(t)+x(-t)}{2}$
- $x_0(t) = \frac{x(t)-x(-t)}{2}$

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Periodic vs. Aperiodic signals

- $x(t) = x(t \pm T)$ for all values of t and where T is a positive real number, then $x(t)$ is a periodic signal.

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  - Fundamental period T_o is the minimum period T for which this equation is satisfied.

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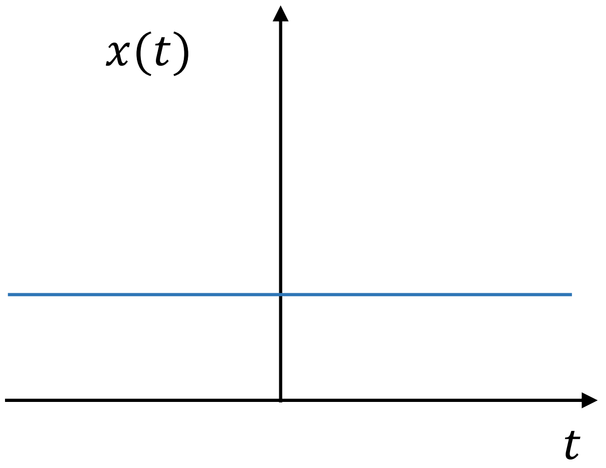
  - $x(t) = x(t \pm T) = x(t \pm 2T) = x(t \pm 3T)$ …
  - Fundamental period T_o is the minimum period T for which this equation is satisfied.
- Similarly, $x[n] = x[n \pm N]$ for all values of n and where N is a positive integer, then $x[n]$ is a periodic signal.
  - Fundamental period N_o is the minimum period N for which this equation is satisfied.

Periodic vs. Aperiodic signals

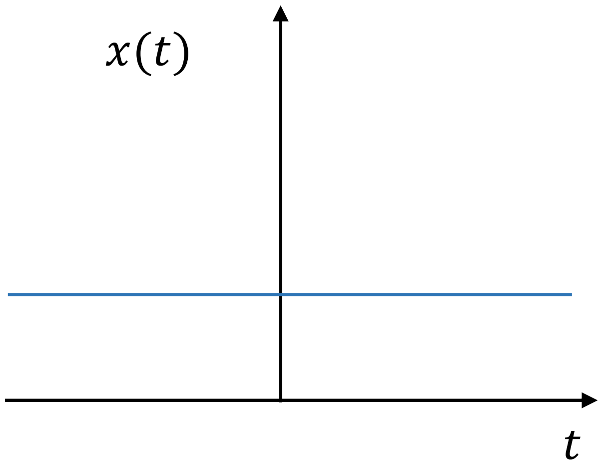
- Fundamental period of $x(t)$?



a) Undefined	**b) 0**
c) 1	**d) ∞**

Periodic vs. Aperiodic signals

- Fundamental period of $x(t)$?



a) Undefined	b) 0
c) 1	d) ∞

Periodic vs. Aperiodic signals

- Fundamental period of cos$(t/6)$?

a) 6π	b) 12π
c) 6	d) 12

Periodic vs. Aperiodic signals

- Fundamental period of cos$(t/6)$?

$$\frac{t+T}{6} = \frac{t}{6} + 2m\pi, \text{ where } m \text{ is some integer}$$

$$\frac{T}{6} = 2m\pi, \text{ where } m \text{ is some integer}$$

$$T = 12m\pi, \text{ where } m \text{ is some integer}$$

$$T_0 = 12\pi$$

a) 6π	b) 12π
c) 6	d) 12

Periodic vs. Aperiodic signals

- Fundamental period of $\cos[n/6]$?

a) 6π	b) 12π
c) 6	d) None of these

Periodic vs. Aperiodic signals

- Fundamental period of $\cos[n/6]$?

$$\frac{n+N}{6} = \frac{n}{6} + 2m\pi, \text{ where } m \text{ is some integer}$$

$$\frac{N}{6} = 2m\pi, \text{ where } m \text{ is some integer}$$

$$N = 12m\pi, \text{ where } m \text{ is some integer}$$

Thus, aperiodic

a) 6π	b) 12π
c) 6	d) None of these

Periodic vs. Aperiodic signals

- Fundamental period of $\cos[\Omega_0 n]$?

$\Omega_0 N = 2m\pi$, where m is some integer

$\frac{N}{m} = \frac{2\pi}{\Omega_0}$, where m is some integer

$\frac{N}{m}$ is rational.

If $\frac{2\pi}{\Omega_0}$ has to be rational, then Ω_0 is some rational multiple of π.

Thus, for $\boldsymbol{cos[\Omega_0 n]}$ to be periodic, $\boldsymbol{\Omega_0}$ is some rational multiple of $\boldsymbol{\pi}$.

Periodic vs. Aperiodic signals

- Fundamental period of $\cos[\Omega_0 n]$, if Ω_0 is some rational multiple of π?

$\Omega_0 N = 2m\pi$, where m is some integer

$N = \frac{2\pi m}{\Omega_0}$, where m is some integer

For N_0, we choose smallest value of m, for which N is an integer.

Periodic vs. Aperiodic signals

- Fundamental period of $\cos[5\pi n/31]$?

$$\frac{N}{m} = \frac{2\pi}{5\pi/31} \qquad \frac{N}{m} = \frac{62}{5} \qquad N_0 = 62$$

Its fundamental frequency is $\frac{2\pi}{62} = \frac{\Omega_0}{5}$

Periodic vs. Aperiodic signals

$\cos[\Omega_0 n]$ **vs.**

- May not be periodic
- If periodic, $N_0 = \frac{2\pi}{\Omega_0} m$
- Fundamental (angular) frequency, Ω_0 / m

$\cos(\omega_0 t)$

- Always periodic
- $T_0 = \frac{2\pi}{\omega_0}$
- Fundamental (angular) frequency, ω_0

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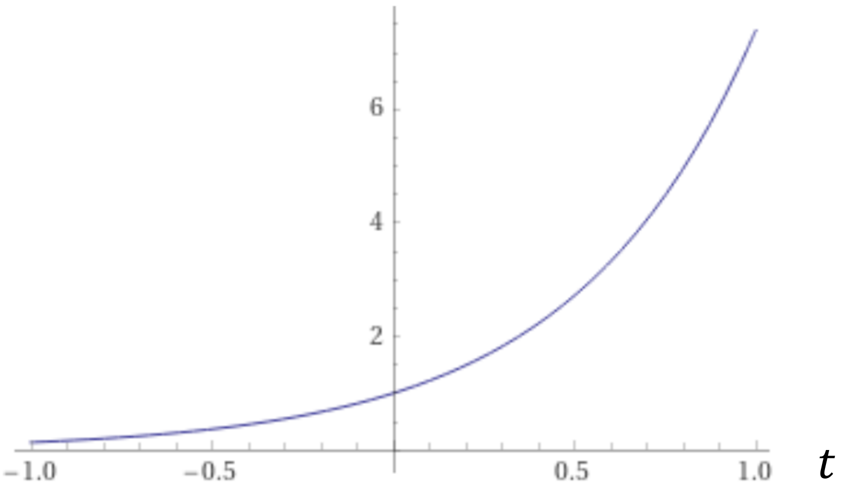
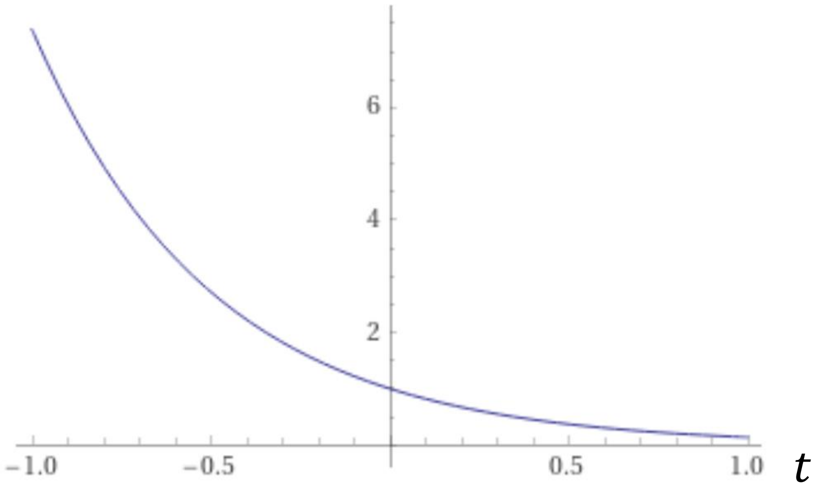
$\alpha > 0$



$\alpha < 0$



Exponential

$Ce^{\alpha t}$

C is 1, and α purely imaginary

$\alpha = j\omega_0$

$j = \sqrt{-1}$

$Ce^{\alpha t} = e^{j\omega_0 t}$

Exponential

$Ce^{\alpha t}$

C is 1, and α purely imaginary $\alpha = j\omega_0$ $j = \sqrt{-1}$

$Ce^{\alpha t} = e^{j\omega_0 t} = cos\omega_0 t + jsin\omega_0 t$ Euler's theorem

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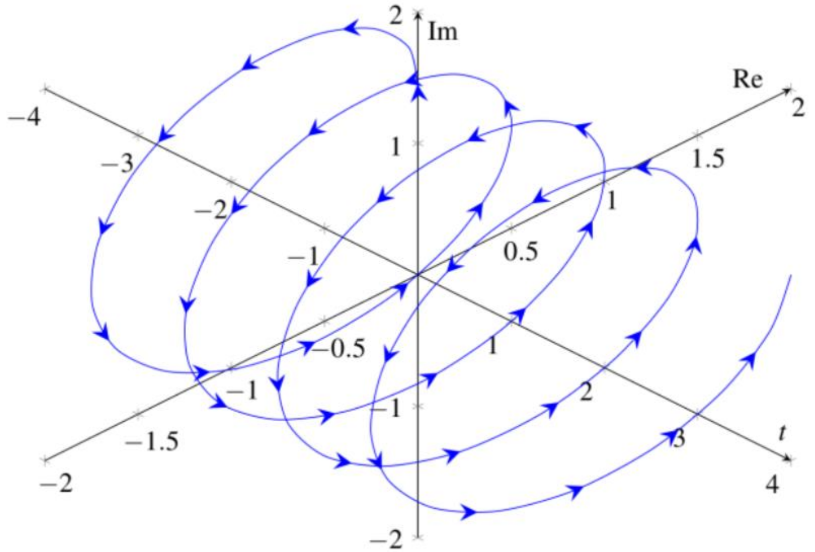

Exponential

$e^{j\omega_0 t}$ is a periodic signal in t

$$e^{j\omega_0(t+T)} = e^{j\omega_0 t}\underbrace{e^{j2\pi m}}_{1 \text{ (for } m \text{ an integer)}}$$

$$\mathrm{T} = \frac{2\pi m}{\omega_0} \qquad \mathrm{T}_0 = \frac{2\pi}{\omega_0}$$

Exponential

$e^{j\omega_0 t}$ is a periodic signal in t

$x_1(t) = e^{j\omega_0 t}$ $\quad x_2(t) = e^{j2\omega_0 t}$ … $\quad x_n(t) = e^{jn\omega_0 t}$

$n\omega_0$ is a harmonic of ω_0

A natural signal which contains "harmonics" of frequencies is a music signal.