TARUN GAUR, 2019CE10314

ESL 300 - Minor

Sol.ⁿ 3 Map of dynamical system is:

(a) $f(x) = 4x - 9x^2$

(a) To find fixed points,

$$f(x) = x$$

$$\therefore 4x - 9x^2 = x$$

$$\therefore 3x - 9x^2 = 0$$

$$3x(1-3x) = 0$$

$x^{(0)} = 0$ is trivial &

$1 - 3x = 0 \Rightarrow x^{(1)} = \frac{1}{3}$ is other fixed point.

Now, $f'(x) = 4 - 18x$

For stability of $x^{(1)}$, $|f'(x^{(1)})| < 1$

$$f'(x^{(1)}) = 4 - 18\left(\frac{1}{3}\right)$$

$$= -2$$

$\therefore |f'(x^{(1)})| > 1 \Rightarrow$ unstable.

Hence, $x^{(0)} = 0$ is trivial & $x^{(1)} = \frac{1}{3}$ is fixed point of first generation & is unstable.

sol^n ③ (b) 2^{nd} generation map is,

$$f^{(2)}(x) = f(f(x))$$
$$= 4[f(x)] - 9[f(x)]^2$$
$$= 4[4x - 9x^2] - 9[4x - 9x^2]^2$$

For fixed points, $f(f(x)) = x$

$$\therefore (4x - 9x^2)[4 - 9(4x - 9x^2)] = x$$

$$\therefore x(4 - 9x)[4 - 9(4x - 9x^2)] - x = 0$$

$$\therefore x[(4 - 9x)(4 - 9(4x - 9x^2)) - 1] = 0$$

$x^{(0)} = 0$ is trivial solution.

$$(4 - 9x)(4 - 36x + 81x^2) - 1 = 0$$

$$\therefore 16 - 144x + 324x^2 - 36x + 324x^2 - 729x^3 - 1 = 0$$

$$\Rightarrow 729x^3 - 648x^2 + 180x - 15 = 0$$

$$\Rightarrow x^3 - \frac{648}{729}x^2 + \frac{180}{729}x - \frac{15}{729} = 0$$

$x^{(1)} = \frac{1}{3}$ is a solution of this & is a fixed point of 2^{nd} gene. map.

$$\therefore \left(x - \frac{1}{3}\right)\left(x^2 - \frac{5x}{9} + \frac{5}{81}\right) = 0$$

To find fixed points, $x^{(2)}$ & $x^{(3)}$, solve quadratic eq^n,

$$x = \frac{\frac{5}{9} \pm \sqrt{\frac{25}{81} - \frac{20}{81}}}{2}$$

$$\therefore x = \frac{\frac{5}{9} \pm \frac{\sqrt{5}}{9}}{2} = \frac{5 \pm \sqrt{5}}{18}$$

$$\therefore x^{(2)} = \frac{5 + \sqrt{5}}{18} \simeq 0.402$$

$$\therefore x^{(3)} = \frac{5 - \sqrt{5}}{18} \simeq 0.153$$

For stability,

$x^{(1)} = 0.333$

$x^{(2)} = 0.402$

$x^{(3)} = 0.153$

$$\frac{d}{dx} f^{(2)}(x) = \frac{d}{dx} f(f(x))$$

$$= \frac{d}{dx}\left[4(4x-9x^2) - 9(4x-9x^2)^2\right]$$

$$= -2916x^3 + 1944x^2 - 360x + 16$$

$\therefore \frac{d}{dx} f^{(2)}(x^{(1)}) = \cancel{4.01}\ 4.01 \Rightarrow \therefore \left|\frac{d}{dx} f^2(x^{(1)})\right| > 1 \rightarrow$ unstable

$\therefore \frac{d}{dx} f^{(2)}(x^{(2)}) = -4 \Rightarrow \left|\frac{d}{dx} f^2(x^{(2)})\right| > 1 \rightarrow$ unstable

$\therefore \frac{d}{dx} f^{(2)}(x^{(3)}) = -4.02 \Rightarrow \left|\frac{d}{dx} f^2(x^{(3)})\right| > 1 \longrightarrow$ unstable

Hence, $x^{(1)} = 0.333$; $x^{(2)} = 0.402$ & $x^{(3)} = 0.153$ are fixed points of 2nd generation & all are unstable

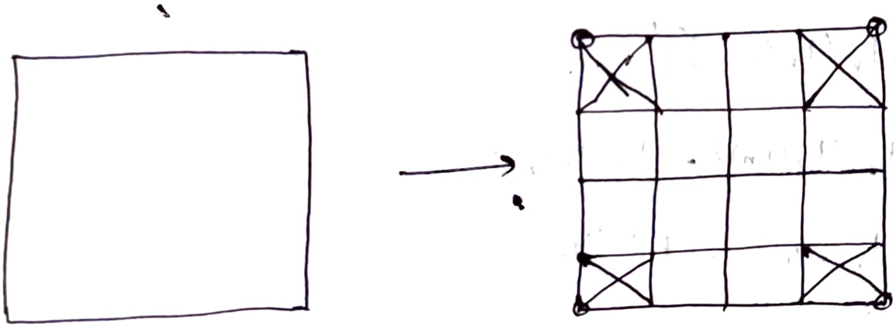
Sol.ⁿ (2)

Fractal: A curve or a figure, parts of which has the same ~~statiscal~~ statistical character as the whole curve / figure. These are sets that appear to have same detailed complex structure, no matter at what scale you examine them. Theoretically, true fractals are infinite sets & have self-similarity across different scales, so that same quality of structure is observed.

Fractal dimension: It is a ratio which provides a mathematical index of complexity, describing how the information in the fractal pattern changes with measuring scale.

Eg. → ~~kou~~ koch (As in koch snowflake)

a) Taking a square, dividing it into 16 equal parts & after removing all corners, we get 12 new squares.

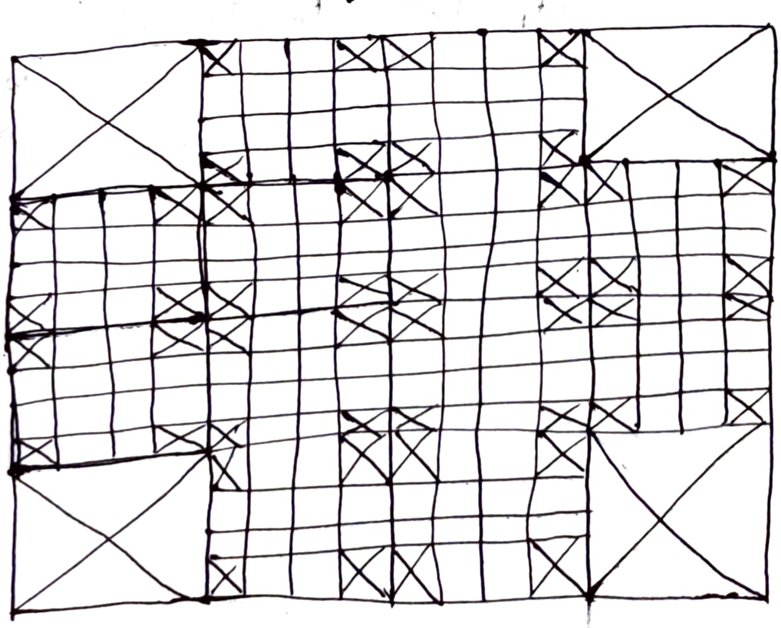


(1)

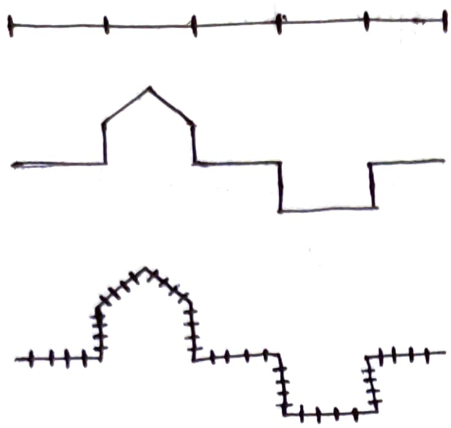
From each of these 12 new squares, we get 12 more new squares.

Hence,

$$\text{Fractal dimension} = \frac{\log 12}{\log 4} = 1.79$$



Sol^n 2 (b)

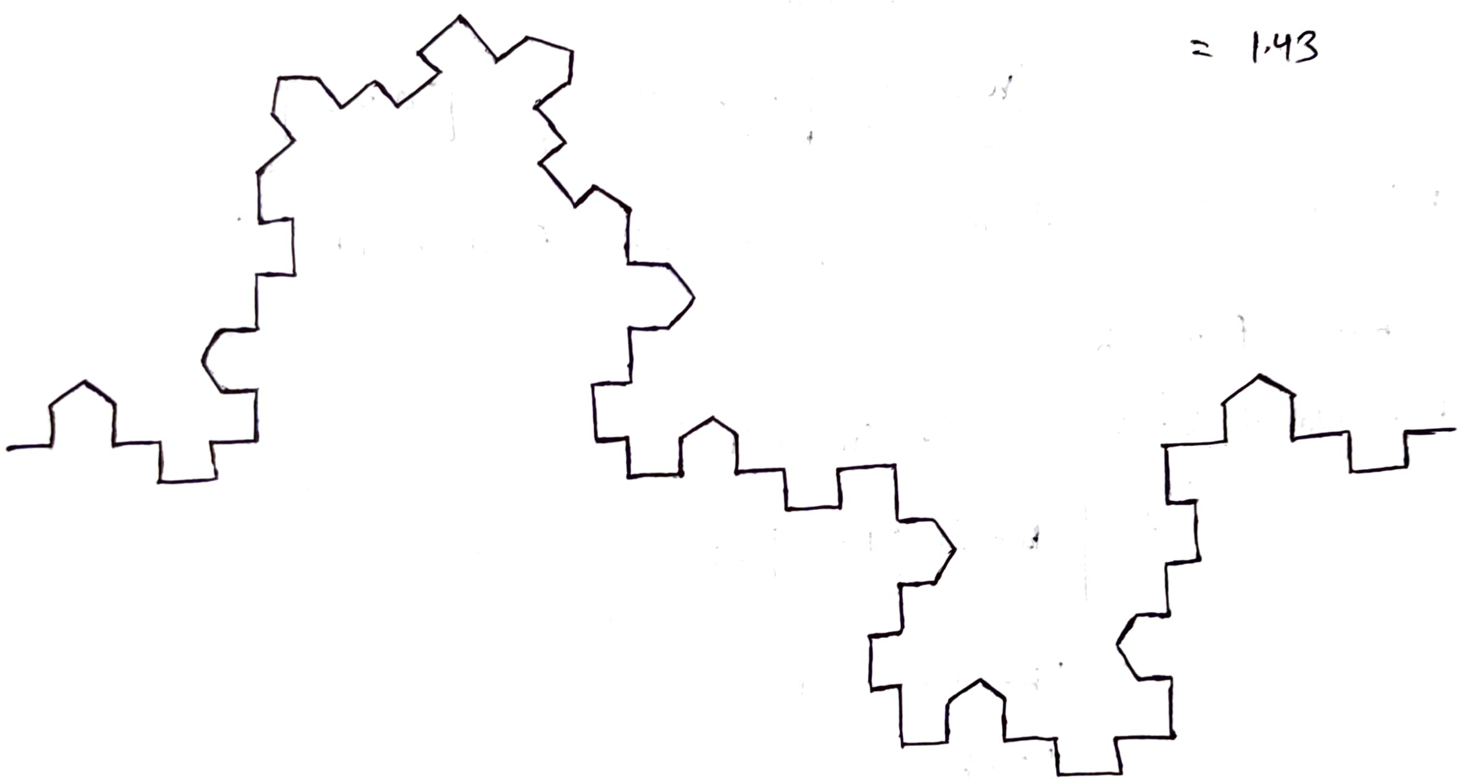


For 1 line, 10 parts / lines are formed.

→ taking a line & dividing it into 5 equals parts, then performing given operations

$$\text{Fractal dimension} = \frac{\log(10)}{\log(5)}$$

$$= 1.43$$



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Sol^n 1

(a) Yes they are fixed points of higher generation also.

Like, $F_b^2(x) = f(f(x))$

As in Q. 3, $x = \frac{1}{3} \rightarrow$ fix pnt of 1st gen is also fix. pnt of 2nd gen

$F_b^n(x) = f(f \ldots n \text{ times } (x))$

For 1st gen, fix points $\Rightarrow f(x) = x$ — 1 for $x^{(1)}, x^{(2)} \rightarrow$ 1st gen fix points

2nd gen, " " $\Rightarrow f(f(x)) = x$ — 2, 1 & 2 combine leads to $x^{(1)}$ & $x^{(2)}$ being end pnt of 2nd gen

Similarly n^{th} gen, $f_b^n(x) = f(f \ldots n \text{ times } (x)) = x$, — n

From eq^n 1, 2 ... n, we see $x^{(1)}$ & $x^{(2)}$ are fix points of all higher generations

(b) $x^{(1)}, x^{(2)} \rightarrow$ fix points of 1st gen

$x^{(3)}, x^{(4)} \rightarrow$ " " 2nd gen

$\therefore$ $x^{(1)}$ & $x^{(2)}$ are also fix. pnt of 2nd gen. [proved in (a)]

If slope $f_b^{(2)}(x)$ is >1 at $x^{(1)}$, then we get additional fix. pnts. which are not of $f_b'(x)$.

→ value of b @ which $x^{(1)}$ becomes unstable, stable cycle period 2 is generated.

→ For period 1 attractor system, |propagator| < 1

$|f_b'(x)| < 1$

" " " " , |proparagator| > 1 & < 2.

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(c) yes they are. proof is same as (a).

$f_b^2(n) = f(f(n)) = n;$ — ① n^3, n^4 are fix pnts

$f_b^3(n) \equiv f(f(f(n))) = n,$ — ② ① + ② gives

n^3, n^4 to be fix end

∴ similarly for high gen.,

$(n^{(3)})$ $(n^{(4)})$ are fixed points.