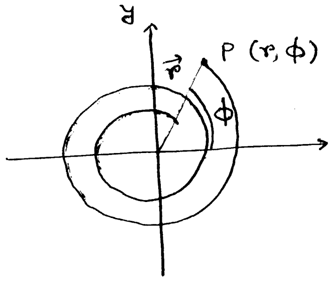
Q 1)

Solution:-



The path of the logarithmic spiral is given by the equation $r = c e^{\phi}$, where $\dot{\phi} = \frac{b}{r^2}$.

We know that in polar coordinate, the velocity of the particle at a point P is obtained as
$$\vec{v} = v_r \hat{e}_r + v_\phi \hat{e}_\phi, \quad \text{where}$$
$v_r = \dot{r}$, $v_\phi = r\dot{\phi}$, and $\hat{e}_r$, $\hat{e}_\phi$ are unit vectors along radial and cross-radial directions.

$$\therefore \; v_r = \dot{r} \Rightarrow v_r = c e^{\phi} \dot{\phi} = c e^{\phi} \frac{b}{r^2} = r \frac{b}{r^2} = \frac{b}{r}$$

$$\Rightarrow v_r = \frac{b}{r}$$

Similarly, $$v_\phi = r\dot{\phi} = r \frac{b}{r^2} = \frac{b}{r} = \frac{b}{c e^{\phi}}$$

$\therefore$ velocity of the particle is ~~given by~~

$$\vec{v} = \frac{b}{r}\hat{e}_r + \frac{b}{r}\hat{e}_\phi = \frac{b}{r}(\hat{e}_r + \hat{e}_\phi)$$

Again we know that the acceleration of the particle at a point P can be obtained as

$$\vec{a} = a_r \hat{e}_r + a_\phi \hat{e}_\phi, \quad \text{where}$$

$$a_r = \ddot{r} - r\dot{\phi}^2, \quad a_\phi = \frac{1}{r}\frac{d}{dt}(r^2\dot{\phi})$$

Now, $$a_r = -\frac{b^2}{r^3} - r\frac{b^2}{r^4} = -\frac{2b^2}{r^3}$$

Similarly, $$a_\phi = \frac{1}{r}\frac{d}{dt}(r^2\dot{\phi}) = \frac{r^2}{r}\ddot{\phi} + 2\dot{r}\dot{\phi}$$

$$= r\left(-\frac{2b^2}{r^4}\right) + 2\frac{b}{r}\frac{b}{r^2} = 0$$

We have $\dot{r} = \frac{b}{r}$

$\Rightarrow \ddot{r} = -\frac{b}{r^2}\dot{r}$

$= -\frac{b^2}{r^3}$

$\dot{\phi} = \frac{b}{r^2}$

$\Rightarrow \ddot{\phi} = -\frac{2b}{r^3}\dot{r}$

$= -\frac{2b^2}{r^4}$

$\therefore$ The resulting acceleration is

$$\vec{a} = -\frac{2b^2}{r^3}\hat{e}_r$$

The radius of curvature ρ of the path can be obtained as

$$\frac{1}{\rho} = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3}$$

Now, $$\vec{v} \times \vec{a} = -\frac{2b^2}{r^4} \cdot b \, (\hat{e}_r + \hat{e}_\phi) \times \hat{e}_r = \frac{2b^3}{r^4}\hat{e}_z,$$

where $\hat{e}_z$ is the normal to the x-y plane.

$$\therefore \; \frac{1}{\rho} = \frac{2b^3/r^4}{2^{3/2}\, b^3/r^3} = \frac{1}{\sqrt{2}\, r} \Rightarrow \boxed{\rho = \sqrt{2}\, r}$$

[Here the Constant b is +ve].

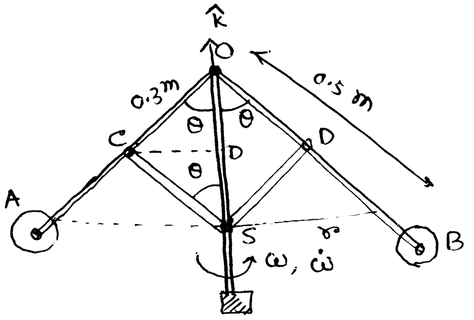
why is the radius of curvature independent of b?

2)

Solution:-

we have $v = 2$ m/s, The speed of the sleeve

$a = -0.1$ m/s^2, the acceleration of the sleeve, which is negative because, the speed is decreasing.



As the arms OA and OB are rotating with respect to the OS axis, then we can use cylindrical co-ordinate system. So at any instant, the position B has cylindrical co-ordinat (r, ϕ, z), where $\dot{\phi} = \omega = 2$ rad/s, $\dot{\omega} = 0.4$ rad/s^2

and $\theta = 40^\circ$.

The velocity of the ball B is given by

$$\vec{v}_B = \dot{r}\,\hat{e}_r + r\dot{\phi}\,\hat{e}_\phi + \dot{z}\,\hat{e}_z = \dot{r}\,\hat{e}_r + r\omega\,\hat{e}_\phi + \dot{z}\,\hat{e}_z$$

Here $r = OB \sin\theta = 0.5 \sin\theta$

$z = -OB\cos\theta = -0.5\cos\theta$

$$\Rightarrow \dot{z} = 0.5\sin\theta\,\dot{\theta} \Rightarrow \ddot{z} = 0.5\cos\theta\,\dot{\theta}^2 + 0.5\sin\theta\,\ddot{\theta}$$

$$\dot{r} = 0.5\cos\theta\,\dot{\theta} \Rightarrow \ddot{r} = 0.5\cos\theta\,\ddot{\theta} - 0.5\sin\theta\,\dot{\theta}^2$$

Now, we need to determine $\dot{\theta}$?

Initially θ was 0°. $\therefore$ Initial distance of OS $= 0.6$ m.

If the arms are rotated, the sleeve moves up with velocity 2 m/s.

$\therefore$ OD $= 0.3\cos\theta$

$\therefore$ New distance of OS $= 0.6\cos\theta$

$\therefore$ The resulting distance is moved by the sleeve S is $(0.6 - 0.6\cos\theta)$

$$\therefore v = \frac{d}{dt}\left[0.6(1-\cos\theta)\right]$$

$$\Rightarrow 0.6\sin\theta\,\dot{\theta} = 2$$

$$\Rightarrow \dot{\theta} = \frac{2}{0.6\sin\theta} = 5.18575 \text{ m/s}$$

Similarly $$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 0.6\sin\theta\,\ddot{\theta} + 0.6\cos\theta\,\dot{\theta}^2 = -0.1$$

$$\Rightarrow \ddot{\theta} = -32.3079 \text{ m/s}^2$$

$$\therefore \dot{r} = v_r = 0.5\cos\theta\,\dot{\theta} = 1.98626 \text{ m/s}$$

$$r\dot{\phi} = v_\phi = r\dot{\phi} = 0.5\sin\theta\,\omega = 0.642788 \text{ m/s}$$

$$\dot{z} = v_z = 0.5\sin\theta\,\dot{\theta} = 1.66667 \text{ m/s}$$

$\because$ The resulting velocity is

$$\vec{v} = 1.9863\,\hat{e}_r + 0.643\,\hat{e}_\phi + 1.667\,\hat{e}_z$$

The acceleration of the ball B is given by

$$\vec{a}_B = (\ddot{r} - r\dot{\phi}^2)\,\hat{e}_r + \frac{1}{r}\frac{d}{dt}(r^2\dot{\phi})\,\hat{e}_\phi + \ddot{z}\,\hat{e}_z$$

$$\therefore\ a_r = \ddot{r} - r\dot{\phi}^2 = 0.5\cos\theta\,\ddot{\theta} - 0.5\sin\theta\,\dot{\theta}^2 - r\omega^2$$

$$= -22.3031\ m/s^2$$

$$a_\phi = \frac{1}{r}\frac{d}{dt}(r^2\dot{\phi}) = 2\dot{r}\dot{\phi} + r\ddot{\phi} = 8.0736\ m/s^2$$

$$a_z = \ddot{z} = -0.0833242\ m/s^2$$

$\therefore$ The resulting acceleration is $\vec{a}_B = -22.3031\,\hat{e}_r + 8.0736\,\hat{e}_\phi - 0.0833242\,\hat{e}_z$

$\therefore$ Hence the force exerted by the ball B, on the shaft is

$$\vec{F} = m\vec{a}_B = (-4.46062\,\hat{e}_r + 1.61472\,\hat{e}_\phi - 0.0166648\,\hat{e}_z)$$

Q3) $y_p = 0$ at $t = 0$; and both slots are rest at $t = 0$.

$$\Rightarrow \cancel{y_p}\ \ddot{y}_p = a_0 \Rightarrow \dot{y}_p = \dot{y}_p(0) + a_0 t = a_0 t$$

$$\Rightarrow y_p = y_p(0) + \tfrac{1}{2} a_0 t^2 = \tfrac{1}{2} a_0 t^2 .$$

$x_p = c\, y_p^2$ (Since the pin is common to both slots).

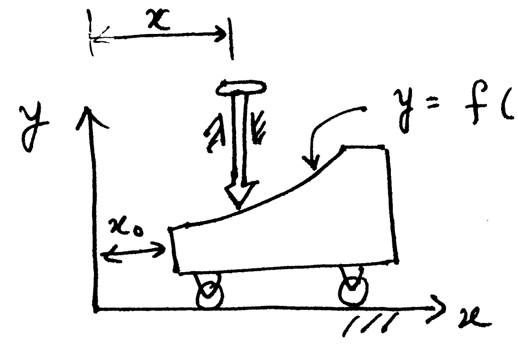
$$\therefore x_p = \tfrac{1}{4} c a_0^2 t^4. \Rightarrow \dot{x}_p = c a_0^2 t^3 \; ; \; \ddot{x}_p = 3 c a_0^2 t^2$$

$$\therefore \vec{r}_p = \tfrac{1}{4} c a_0^2 t^4 \,\hat{i} + \tfrac{1}{2} a_0 t^2 \,\hat{j}$$

$$\vec{v}_p = c a_0^2 t^3 \,\hat{i} + a_0 t \,\hat{j}$$

$$\vec{a}_p = 3 c a_0^2 t^2 \,\hat{i} + a_0 \,\hat{j} .$$

Q4) Keeping the coordinate axes fixed as indicated consider the cam after time t



At $t = 0 \rightarrow x_0 = 0, \; \dot{x}_0 = u, \; \ddot{x}_0 = a$.

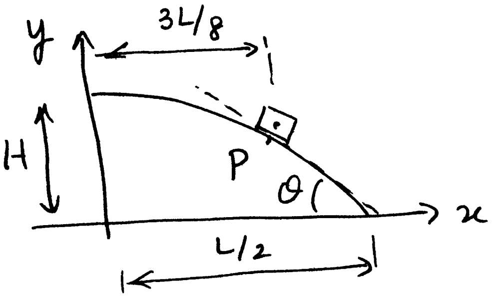
In this coordinate system the follower moves only along y. We need $\dot{y}\big|_{t=0}$ and $\ddot{y}\big|_{t=0}$.

$$\dot{y} = f' \frac{d}{dt}(x - x_0) = -\frac{dx_0}{dt} f'$$

$$\ddot{y} = -\frac{d^2 x_0}{dt^2} f' + f'' \left(\frac{dx_0}{dt}\right)^2$$

$$\dot{y}\big|_{t=0} = -u\, f'(x) \; ; \; \ddot{y}\big|_{t=0} = -a\, f'(x) + u^2 f''(x) .$$

Q 5)



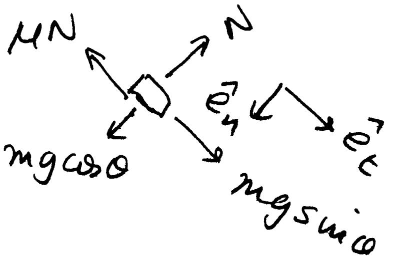
$$y = H\cos(\pi x / L)$$

$$x_p = 3L/8 \Rightarrow y_p = H\cos(3\pi/8) = 0.383\,H.$$

$$-\tan\theta = \left.\frac{dy}{dx}\right|_P = \left.-\frac{H\pi}{L}\sin(\pi x/L)\right|_{\frac{3L}{8}} = -0.726$$

$$\therefore \sin\theta = 0.59\,; \quad \cos\theta = 0.81.$$

$$\left.\frac{d^2y}{dx^2}\right|_P = \left.-\frac{H\pi^2}{L^2}\cos(\pi x/L)\right|_{\frac{3L}{8}} = -\frac{0.945}{L} \Rightarrow \rho_p = 1597.5\,m.$$



Forces acting on the mass.

$$a_p = \ddot{s}\,\hat{e}_t + \frac{\dot{s}^2}{\rho}\hat{e}_n$$

$$\dot{s} = 40\,m/s. \qquad \mu = 0.2$$

$$\hat{e}_n : \quad mg\cos\theta - N = m\frac{\dot{s}^2}{\rho} \Rightarrow N = m\left(g\cos\theta - \frac{\dot{s}^2}{\rho_p}\right)$$

$$\hat{e}_t : \quad mg\sin\theta - \mu N = m\ddot{s}$$

$$\Rightarrow \ddot{s} = g\sin\theta - \mu\left(g\cos\theta - \dot{s}^2/\rho_p\right)$$

$$= 4.4\,m/s^2.$$