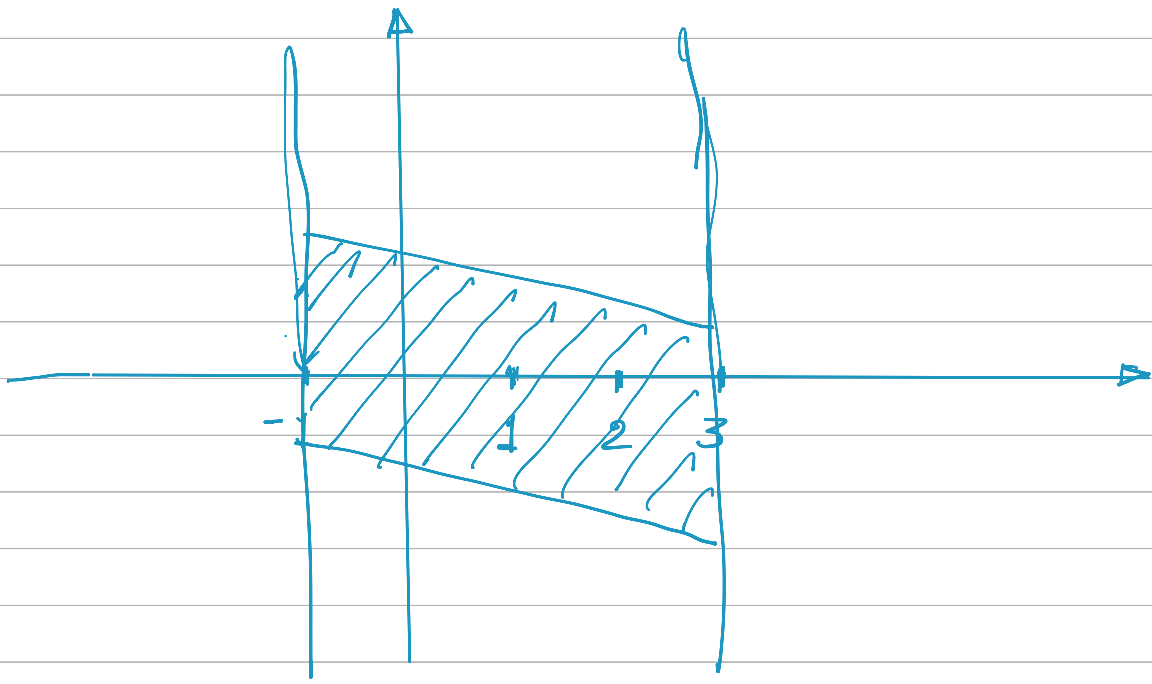
Lecture 2 Tue 25 Jul 2023.

$\max\ (-x+2y)$ LP — Linear programming

s.t. $-1 \le x \le 3$

$-5 \le 4y + x \le 7$



$\max/\min\ c^T x = \langle c, x \rangle.$

s. to $x \in S = \{x \in \mathbb{R}^n : Ax \le b,\ x \ge 0\}$

S: feasible set

$x \ge 0$: non negativity restriction

$Ax \le b$, $x \ge 0$ (consistent) $\rightarrow$ $S \ne \phi$

A: $m \times n$, x: $n \times 1$

$S \ne \phi$ $\rightarrow$ (singleton) unique solⁿ of LP. / more than one element.

inconsistent $\Rightarrow$ $S = \phi$ = (LP) is infeasible

$z = -x + 2y$

let $z = c \Rightarrow -x + 2y = c$ → change (parameter).

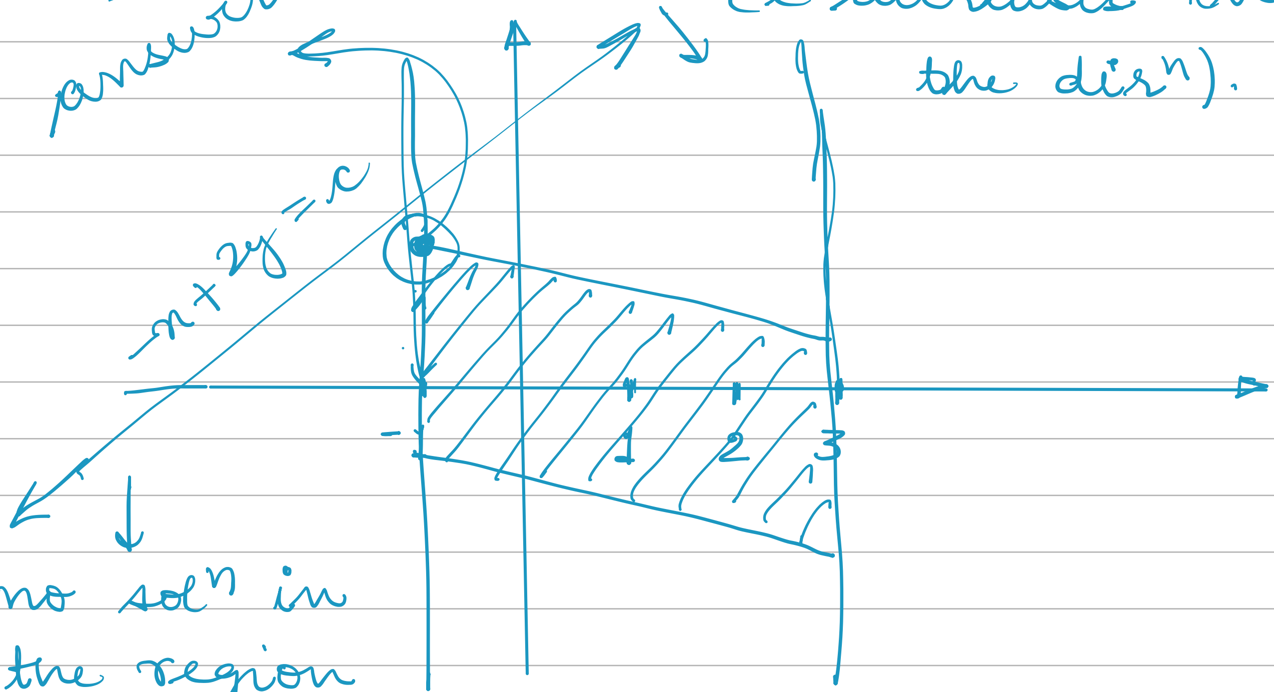
level curve.

let $c = 6$

$-x + 2y = 6$

$(-1, 2)$ answer

(c decreases in the dir^n).



no sol^n in the region

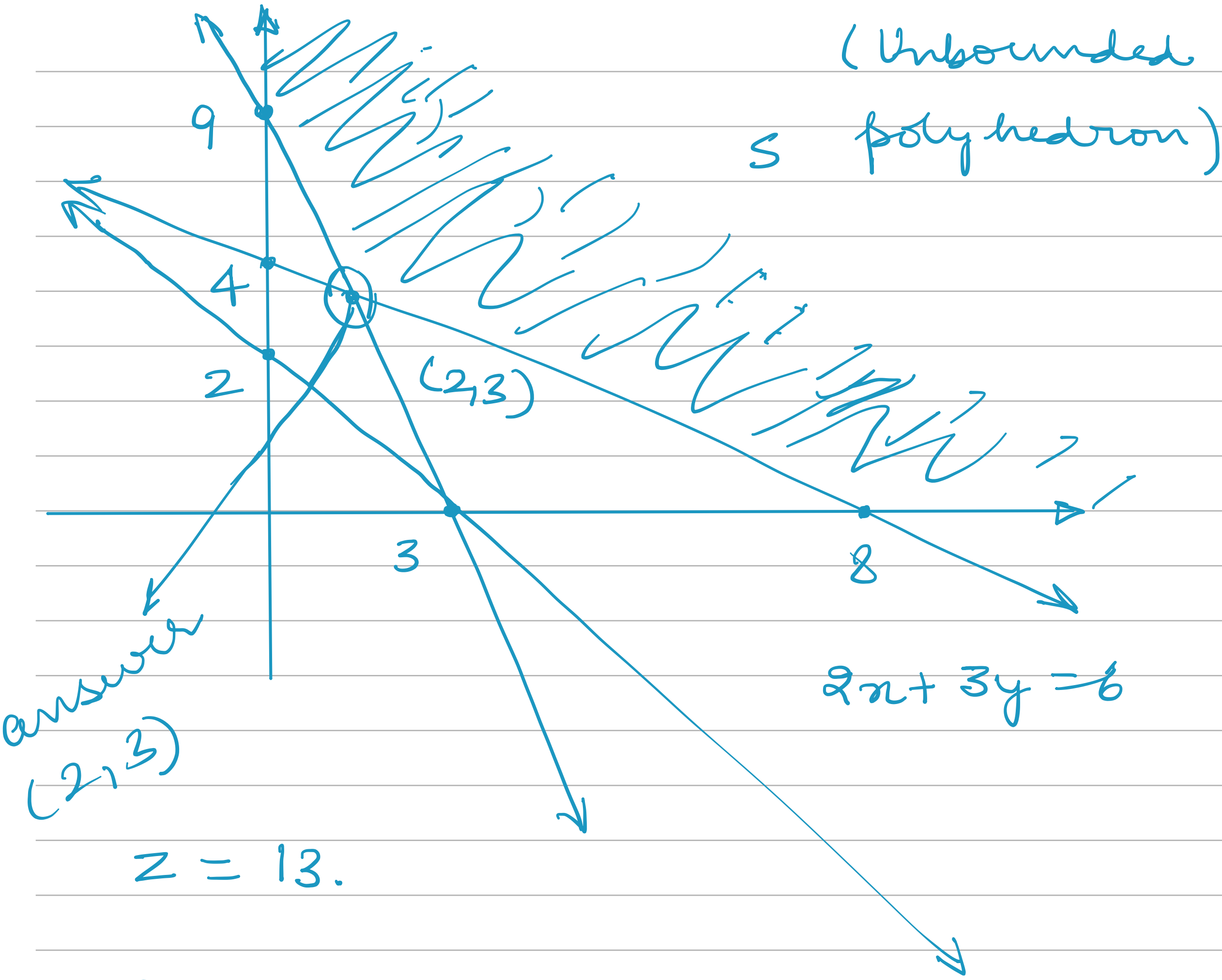
$-(-1) + 2(2) = 5$

$$\min\ (2x + 3y)$$

s.t. $\quad x + 2y \geq 8$

$\quad 3x + y \geq 8$

$\quad x, y \geq 0$



$Z = 13.$

Cases

① $S = \phi$ problem is infeasible

② $S \neq \phi$ solⁿ exist

$\exists$ seq. $\{x_k\} \in S$ $\qquad x_k \in \mathbb{R}^n.$

s.t. $f(x_k) = z_k \longrightarrow +\infty$ (max)

OR

$-\infty$ (min).

$\downarrow$

S cannot be bounded.

③ $S \neq \phi \rightarrow$ solⁿ exist

(S May be bounded)

The LP is feasible bounded &

hence solvable.

(1) unique sol^n

(2) multiple sol^n.

eg. $\max(-x + 2y)$

$$1 \le x \le 3.$$

$$4y + x \le 7$$

$$2x - y \ge 7$$