PYL101
(Electromagnetic Waves and Quantum Mechanics)
Tutorial Sheet 3 (L5-L6)

(1) Consider the two states $|\psi> = i\,|\phi_1> + 3i\,|\phi_2> - |\phi_3>$ and $|\chi> = |\phi_1> - i|\phi_2> + 5i|\phi_3>$, where $|\phi_1>, |\phi_2>$ and $|\phi_3>$ are orthonormal. Then calculate:

$<\psi|\psi>$, $<\chi|\chi>$, $<\psi|\chi>$, $<\chi|\psi>$ and $<\psi+\chi|\psi+\chi>$.

(2) Find the constant α so that the states $|\psi> = \alpha|\phi_1> + 5|\phi_2>$ and $|\chi> = 3\alpha|\phi_1> - 4|\phi_2>$ are orthogonal. $|\phi_1>$ and $|\phi_2>$ are orthonormal wave functions.

(3) Consider a state which is given in terms of three orthonormal vectors $|\phi_1>$, $|\phi_2>$ and $|\phi_3>$

$$|\psi> = \frac{1}{\sqrt{15}}|\phi_1> + \frac{1}{\sqrt{3}}|\phi_2> + \frac{1}{\sqrt{5}}|\phi_3>,$$

where $|\phi_n>$ are eigenstates to an operator $\hat{B}$ which satisfies the relation $\hat{B}|\phi_n> = (3n^2-1)|\phi_n>$, where n = 1, 2, 3. Then

(a) Find the norm of $|\psi>$. $= <\psi|\psi>$

(b) Find the expectation value of $\hat{B}$ with respect to $|\psi>$

(4) Show that the operator $|\psi><\psi|$ is a projection operator only when $|\psi>$ is normalized.

(5) Check whether the operators $\hat{x}$, d/dx and $i\,d/dx$ are Hermitian operators.

(6) Consider a system whose Hamiltonian is given by $\hat{H} = \alpha(|\phi_1><\phi_2| + |\phi_2><\phi_1|)$, α is a real number having the dimensions of energy and $|\phi_1>$, $|\phi_2>$ are normalized eigenstates of a Hermitian operator $\hat{A}$ that has no degenerate eigenvalues.

(a) Check whether $|\phi_1>$ and $|\phi_2>$ are eigenstates of $\hat{H}$

(b) Calculate the commutators $[\hat{H}, |\phi_1><\phi_1|]$ and $[\hat{H}, |\phi_2><\phi_2|]$

(7) Consider an operator $\widehat{D_x}$ to be $\frac{\partial}{\partial x}$ and the wave function of the system to be $\psi(x) = A\sin(\frac{n\pi x}{a})$, then calculate

(a) $\widehat{D_x}\,\psi(x)$ and $\widehat{D_x^2}\psi(x)$

(b) Which one of these forms an eigenvalue problem and what is the corresponding eigenvalue.

(8) If the function $e^{-\alpha x^2}$ represents an eigenfunction of the operator $\hat{A} = \left(\frac{d^2}{dx^2} - Bx^2\right)$, then find the value of B.

(9) The state of a system at $t = 0$ is given by $|\psi(0)> = \frac{1}{\sqrt{3}}|\phi_1> + A|\phi_2> + \frac{1}{\sqrt{6}}|\phi_3>$, where $|\phi_1>, |\phi_2>$ and $|\phi_3>$ are orthonormal wave functions and A is a real constant.

(a) Find A so that $|\psi(0)>$ is normalized.

(b) Write down the state of the system $|\psi(t)>$ at any later time t. Given: E_1, E_2 and E_3

are the energies corresponding to $|\phi_1>$, $|\phi_2>$ and $|\phi_3>$, respectively.

(10) If $\psi(x) = A\ exp(-x^4)$ is the eigenfunction of one-dimensional Hamiltonian with eigenvalue $E = 0$, then calculate the potential $V(x)$ (in units where $\hbar = 2m = 1$).