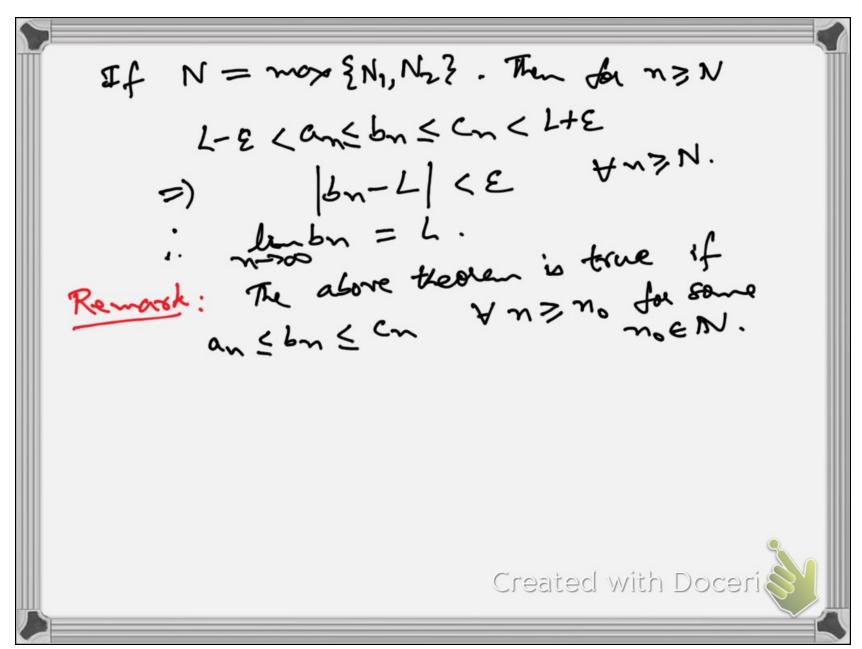
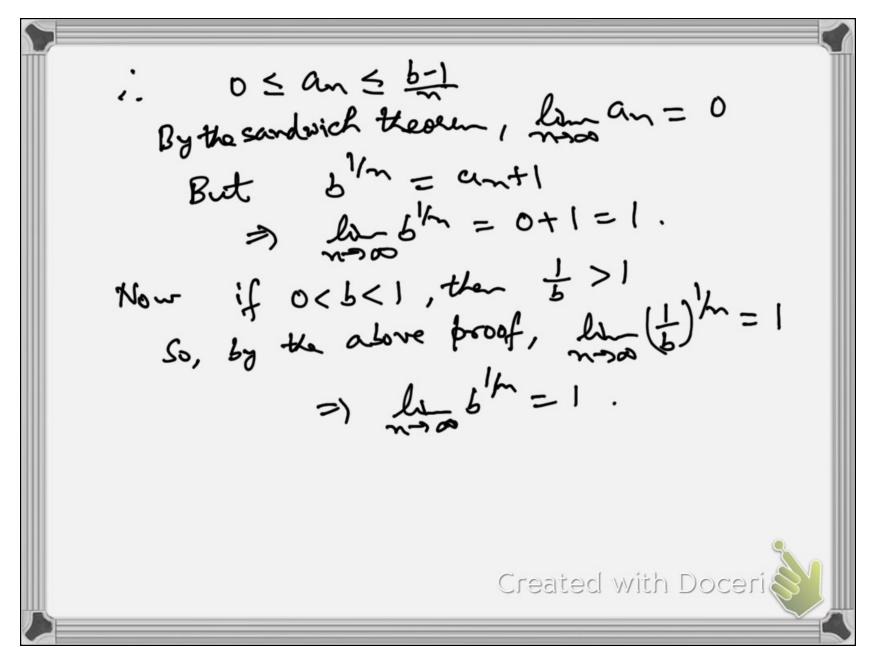
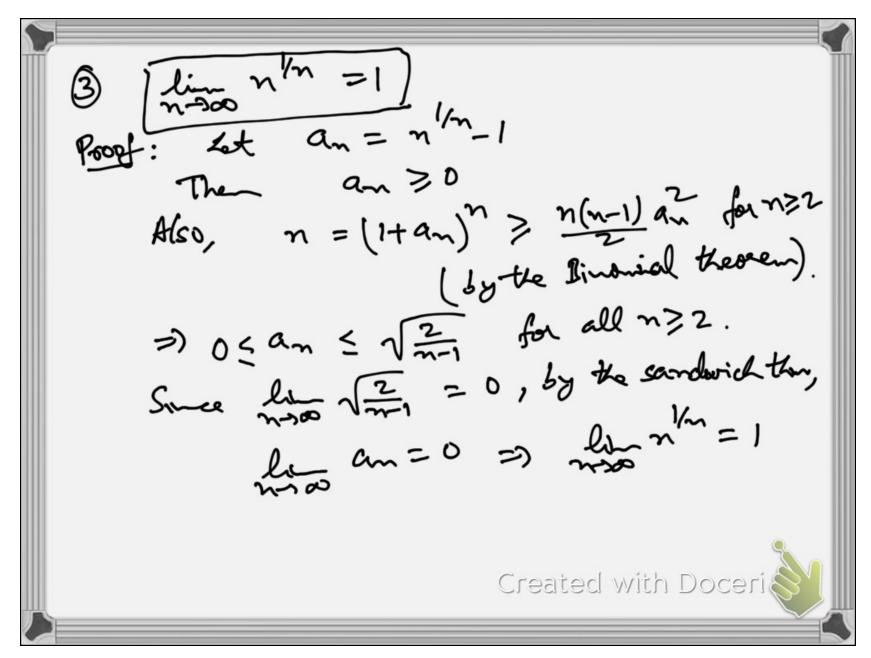
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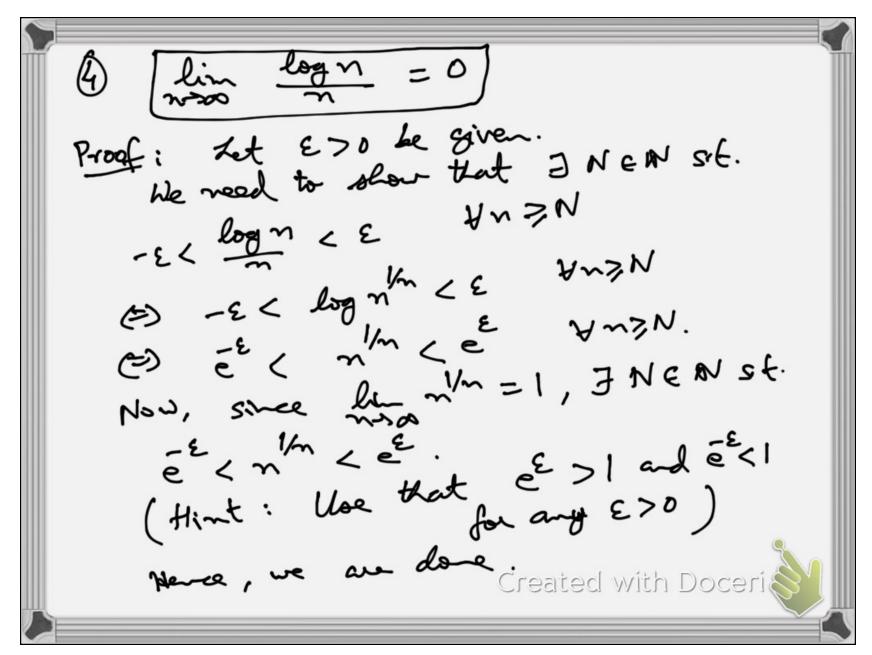
Lecture -3 MTL-100 Sequences (continued) Sandwich Theorem (or Squeeze Theorem): Let (an), (bn) and (cn) be three sequences such that an  $\leq$  bn  $\leq$  cn for all  $n \in \mathbb{N}$ . If himan = L & lin on = L, then limbn=h. Proof: Let EDD be given. since liman = L, JN, EN st. 1an-L/< E Y n7, N1 Shilarly, since land C=L, 7 NZEN sit ~ > N2 => L-E < Cn < L+E - (ii) Created with Doceria

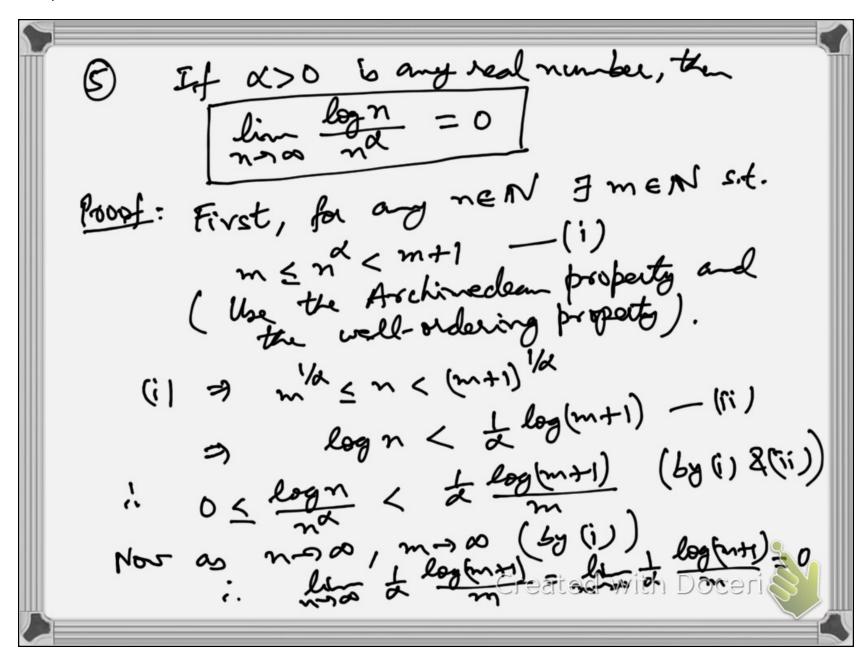


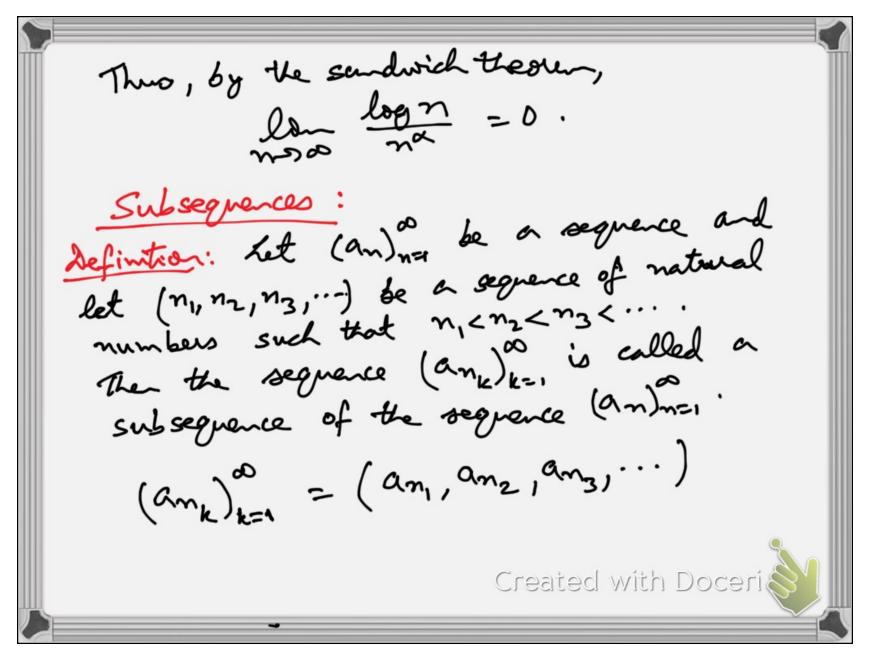
## Some applications of Sandwich Theorem Pf: We know that -1 < sin(n) < 1 4men i) If 6>0, then lime b/m = 1 PE: First assume 6>1. At an = b/m-1 > 0 Then b = (1+an) = 1+nan (by )

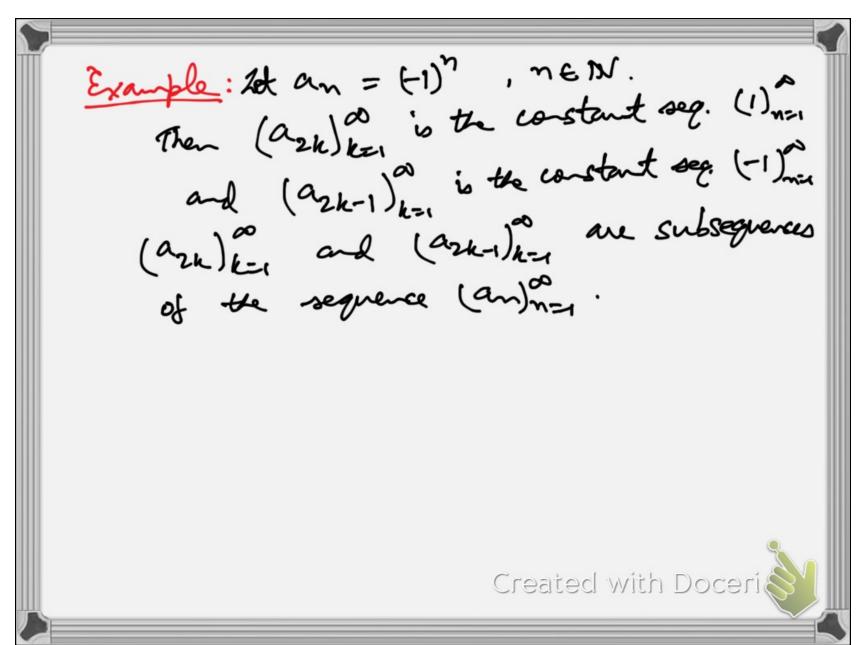




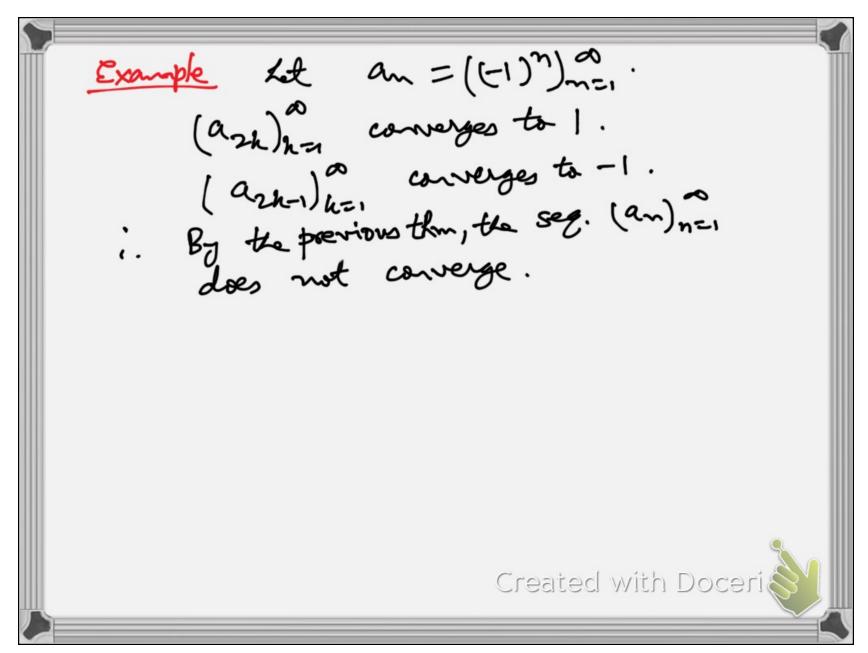


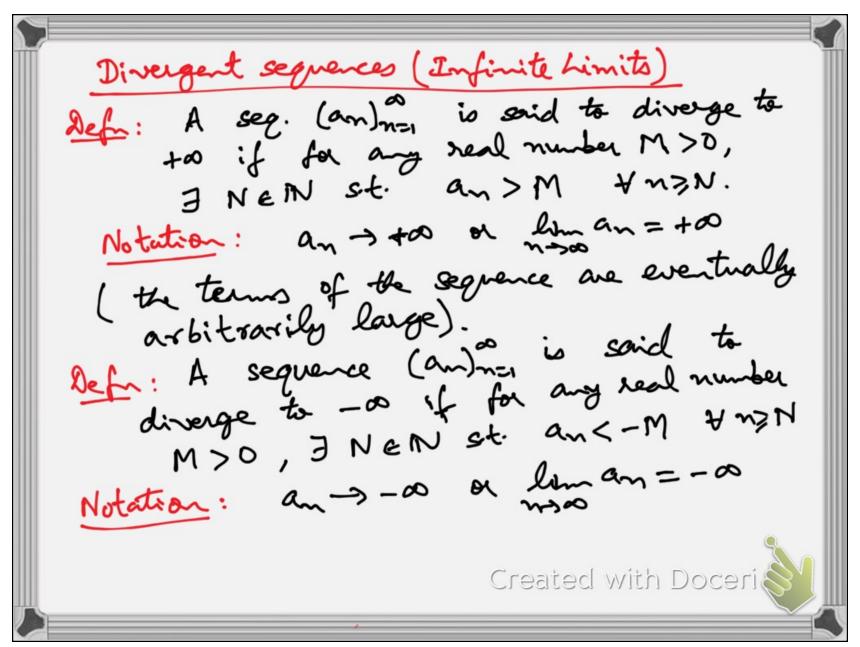


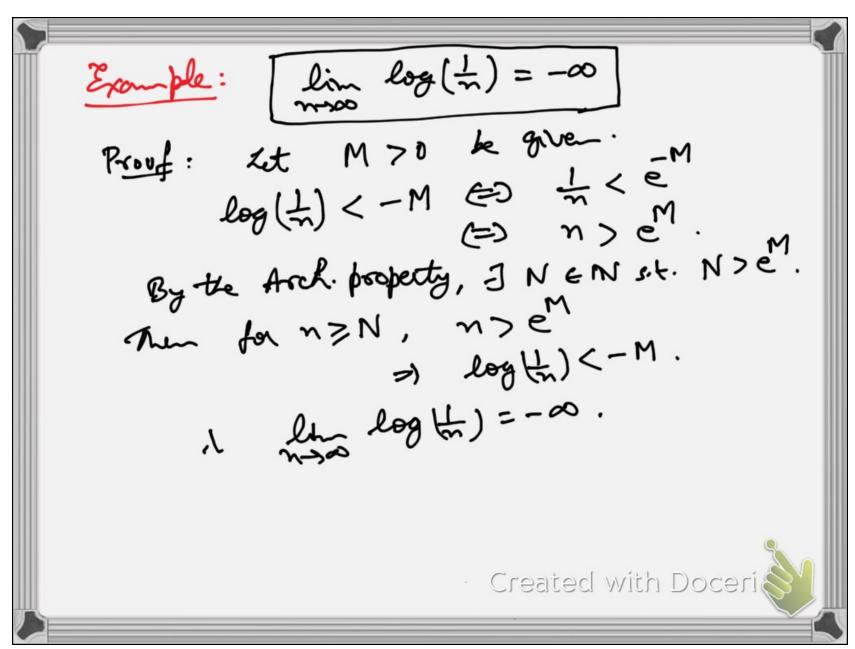




Theorem: If a segmence converge to Lither all its subsequences also converge to L. Proof: Let (an) he a sequence converging to Land let (anh) k=1 be a subsequence. Let E>0 be given. Since him an=L, 3 NEW st |an-L|<E YngN Now note that MEZK YKEN. So if K > N, then Mh > k > N i. (an, -L/< & (by (i)) lan ank = L. Created with Doceria







Theorem: 1) If him an = +00 and him bn = +00, then len (arthr)=to & line (antr)=to

(2) If his an = -o and len bn = -o, the him (anthon) = -a and him (anton) = ta. 3 If him an = +0 and him bn = LER then line (anton) = +00 him  $(a_n b_n) = \begin{cases} +\infty & \text{if } L > 0 \\ -\infty & \text{if } L < 0 \end{cases}$ now (if L=0, not conclusive). Proof: Left as an exercise.

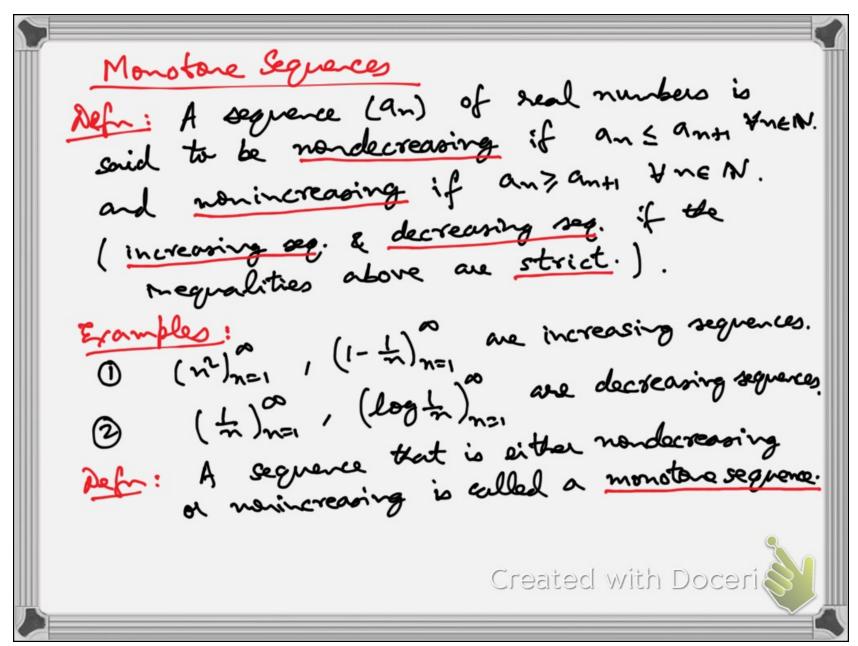
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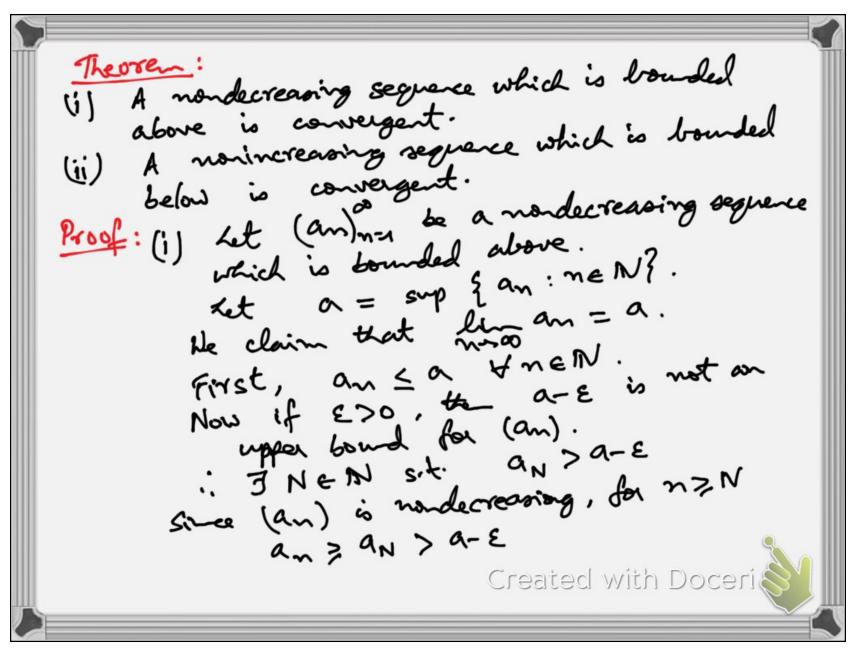
Example: 
$$\lim_{n\to\infty} (\sqrt{n}+\sqrt{n}) = ?$$
 $\sqrt{n}+\sqrt{n} = (\sqrt{n}+\sqrt{n}) (\sqrt{n}+\sqrt{n})$ 
 $= \frac{1}{\sqrt{n}+\sqrt{n}} = 0$ 

(If  $\lim_{n\to\infty} a_n = \pm \infty$ , the  $\lim_{n\to\infty} a_n = 0$ )

. Difference of two diverging sequences may converge.

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.. mon > a-E<am < a<a+E (ii) Created with Doceric Lec-3.pdf Page 19 of 20

Example: If O<b<1, then lim b" = 0 Let an = b , no in. Then  $a_{n+1} = b^{n+1} = b \cdot b^n < b^n = a_n$  then

Then  $a_{n+1} = b^n = b \cdot b^n < b^n = a_n$  then  $(a_n)_{n=1}^{\infty}$  is a decreasing sequence. b>0,  $a_n = b^n > 0$   $y_n$ . Created with Doceric

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