Lecture on 03/12/2020

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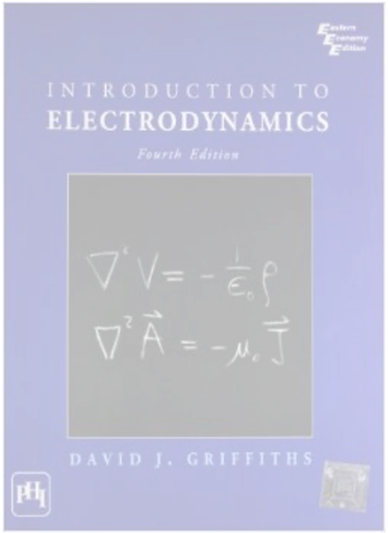
PYL101 course:

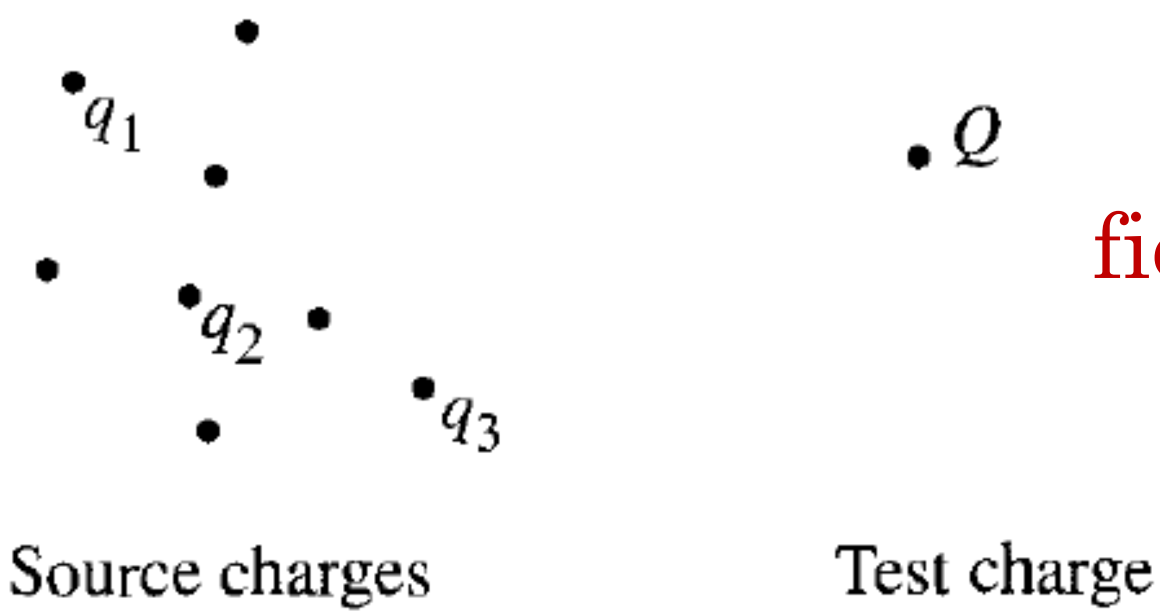
Electromagnetics & Quantum Mechanics

- *Next few classes, we will discuss the following topics:*
- *Magnetostatics (ch5)*
- *Magnetic fields in matter (ch6)*
- *Electrodynamics (ch7)*
- *Continuity equation and Poynting's theorem (ch8)*



Recap and start with magnetostatics...

As, you have already learned about electrostatics, let's start by recalling the basic problem of classical electrodynamics:, force "source" charges (rest) exert on Q, principle of superposition, find the force of a single charge and vector sum…



Stationary charge produces electric field that is constant in time; hence the term electrostatic

Now we move to another topic, which is magnetostatics….i.e., let's understand what happens when charges are in motion, how to find the force between them? In next classes, we will discuss……

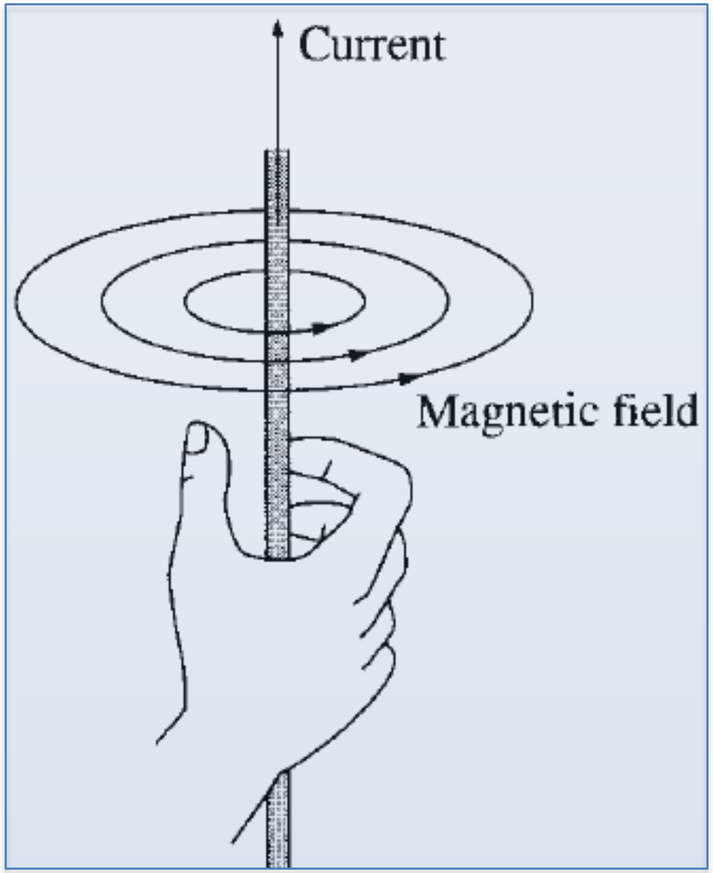
Magnetic fields:

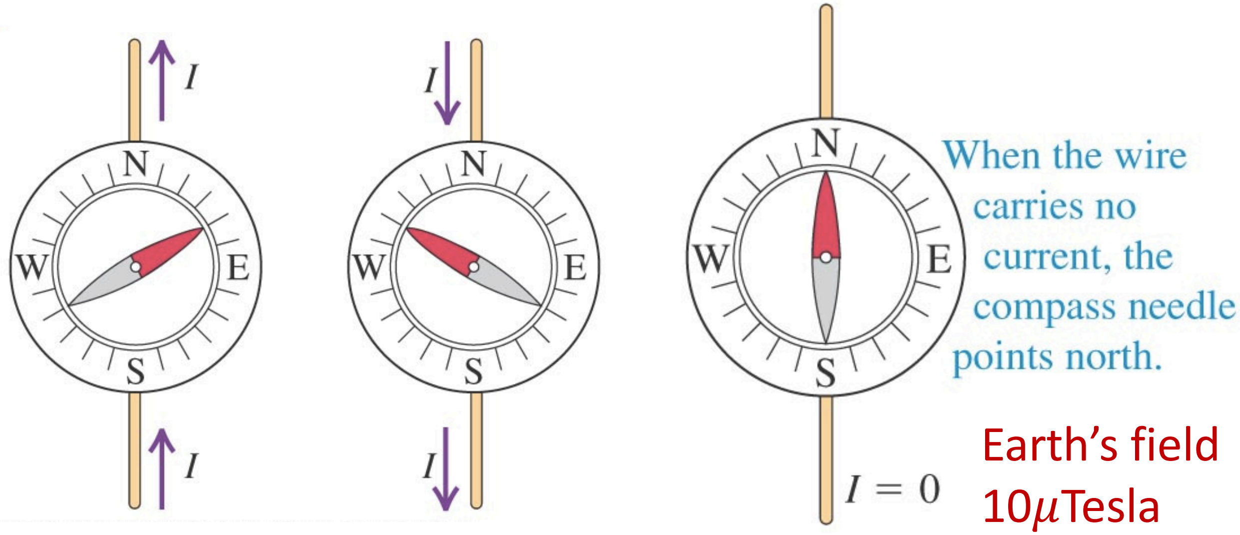
Steady current in a wire generates magnetic field that is constant in time,….

…..the theory is called magnetostatics…..

How do we know that there is a magnetic field around the wire and the field line direction?

let's bring a small compass close to the wire, you will see that field circles around the wire….right hand rule…



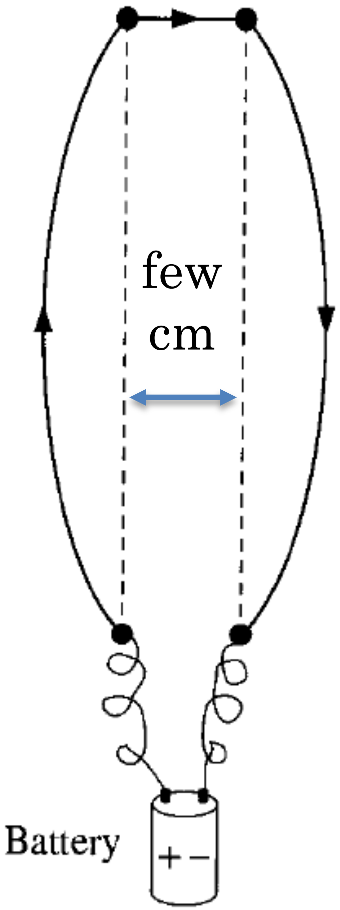


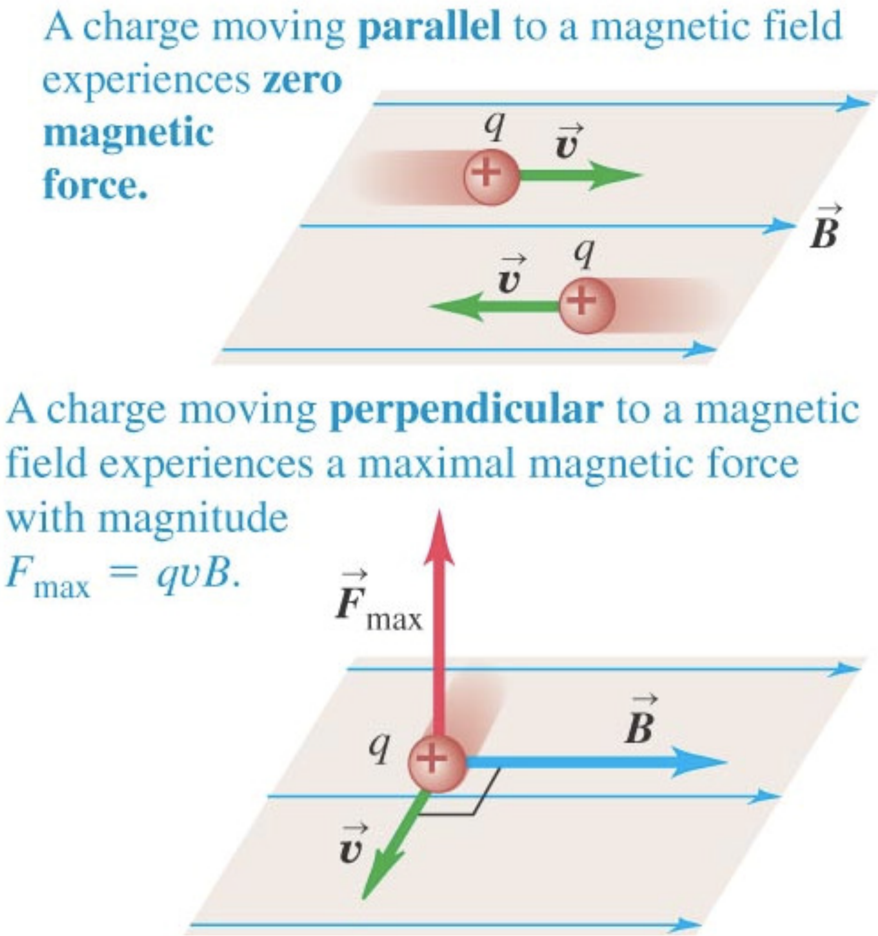
Earth's field

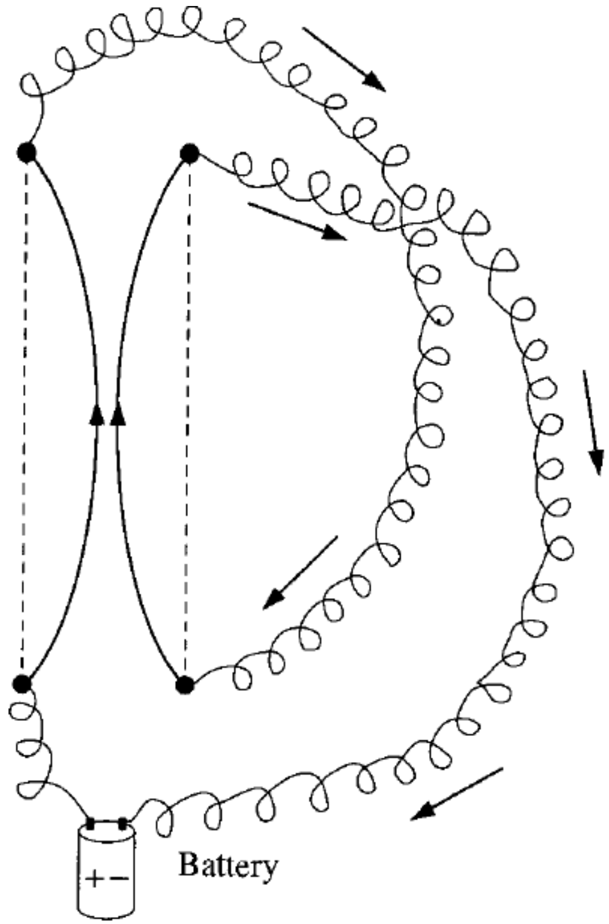
10μTesla

Magnetic fields and magnetic force:

The magnetic field, which is a vector, exerts a force on any other nearby moving charge…





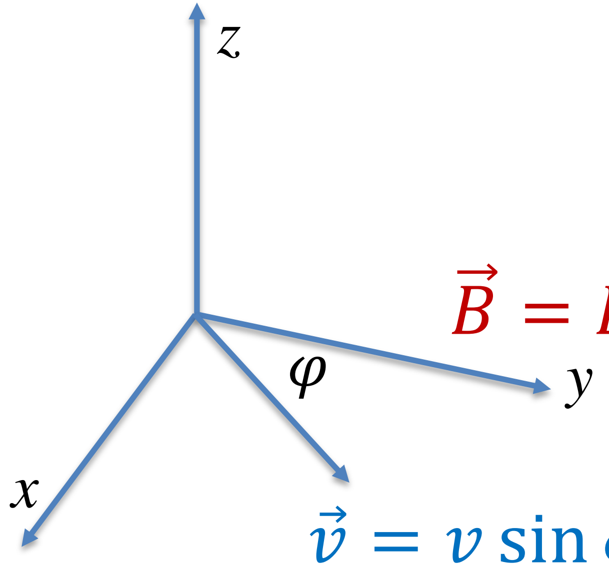


magnitude $F_m = qvB\sin\varphi$ or $\vec{F}_m = q\vec{v} \times \vec{B}$

this is known as Lorentz force law….

How to find magnetic force and direction?:

Let's take a coordinate system,



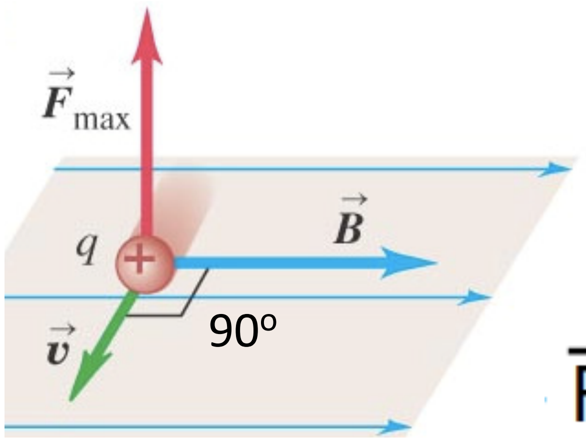
$\vec{B} = B\,\hat{j}$

$\vec{v} = v\sin\varphi\,\hat{\imath} + v\cos\varphi\;\hat{j}$

$$\overrightarrow{F_m} = q\left(\vec{v}\times\vec{B}\right)$$
$$= q(v\sin\varphi\,\hat{\imath} + v\cos\varphi\,\hat{j})\times B\,\hat{j}$$
$$= qvB\sin\varphi\,(\hat{\imath}\times\hat{j})$$
$$= qvB\sin\varphi\,\hat{k}$$

Force on +q charge is in $\hat{k}$ direction…

We can understand the force direction by right hand rule



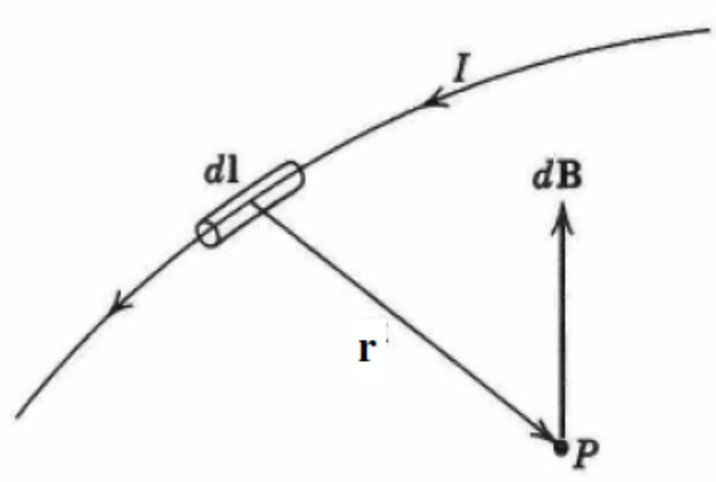
One example: (where v and B are $\perp$)

q=1μC = 10^{-6} C, v=10 m/s, B=10mT

$F_{max} = qvB = 10^{-6} \times 10 \times 10 \times 10^{-3} = 10^{-7}$ N

$\overrightarrow{F_m}$ is always perpendicular to $\vec{B}$ and $\vec{v}$.

What is the magnetic field produced by a current (steady) carrying conductor?: named as the Biot-Savart law:



Quantitative rule for computing the magnetic field from any electric current

$$d\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$$

with $\mu_0 = 4\pi\times10^{-7}\,\mathrm{T\cdot m/A}$

the permeability of free space

$$\vec{\mathbf{B}} = \int_{\text{wire}} d\vec{\mathbf{B}}$$

Let's understand few things about electric (E) and magnetic (B) fields:

1) Both E and B are long range and vector quantity
2) Both decreases as $1/r^2$
3) Both obey principle of superposition
4) E is produced by a scaler charge
5) B is produced by current element
6) E is along the line joining the charge and point P
7) B is perpendicular to the plan of $Id\vec{l}\times\vec{r}$
8) B depends on the angle between $\vec{r}$ and $Id\vec{l}$

$$\varepsilon_0\mu_0 = \frac{1}{c^2}$$

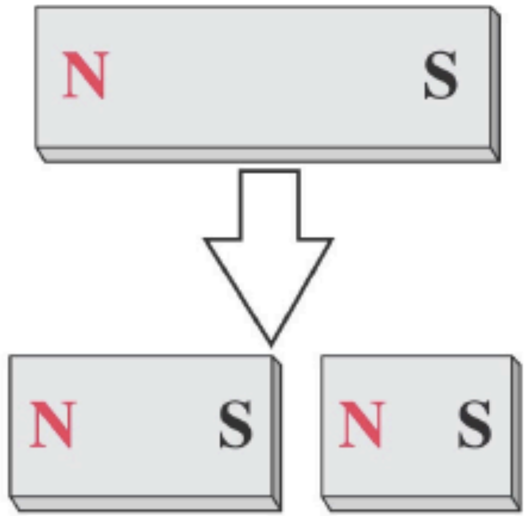
will discuss in Maxwell's equations

Magnetic poles versus electric charges:

- We observed monopoles in electricity.
- (+) or (-) charge alone was stable, and field lines could be drawn around it.

However, magnets cannot exist as monopoles.

If you break a bar magnet between N and S poles, you get two smaller magnets, each with its own N and S pole.



Magnetic poles always come in pairs and cannot be isolated…

Can magnetic forces do work?:

Let's assume that charge q moves an amount dl which can be written as vdt

Then the work done is

$$dW_m = Fm.dl = q(v \times B).vdt = 0$$

This is because $v \times B \; is \; \perp$ to v, so $(v \times B).v = 0$

Magnetic forces cannot speed up or slow down a particle, but can alter the direction…

This means work done by magnetic forces is zero…

It may appear to you strange, but you will understand this when we will discuss about the work done on the magnetic dipoles…

Magnetic fields in matter:

Processes which create magnetic fields in an atom are:

– Nuclear spin. Some nuclei, such as a hydrogen atom, have a net spin, which creates a magnetic field.

– Electron spin. An electron has two intrinsic spin states, which we call up and down

– Electron orbital motion. There is a magnetic field due to the electron moving around the nucleus.

• Each of these magnetic fields interact with one another as well as with external magnetic fields.

However, some of these interactions are strong and others are negligible…

Magnetic fields in matter:

- All magnetic phenomena are attributed to motion of electric charges (current)…….

- electrons in most atoms exist in pairs (spin ↑↓), orbiting around nuclei, and spinning about their axes, each causes a magnetic field & gets cancelled in pair

- Interestingly, many materials have unpaired electrons, so they have certain magnetic field..

- Magnetic field is always connected to current loops..

- These loops are so small, act like **magnetic dipoles**..

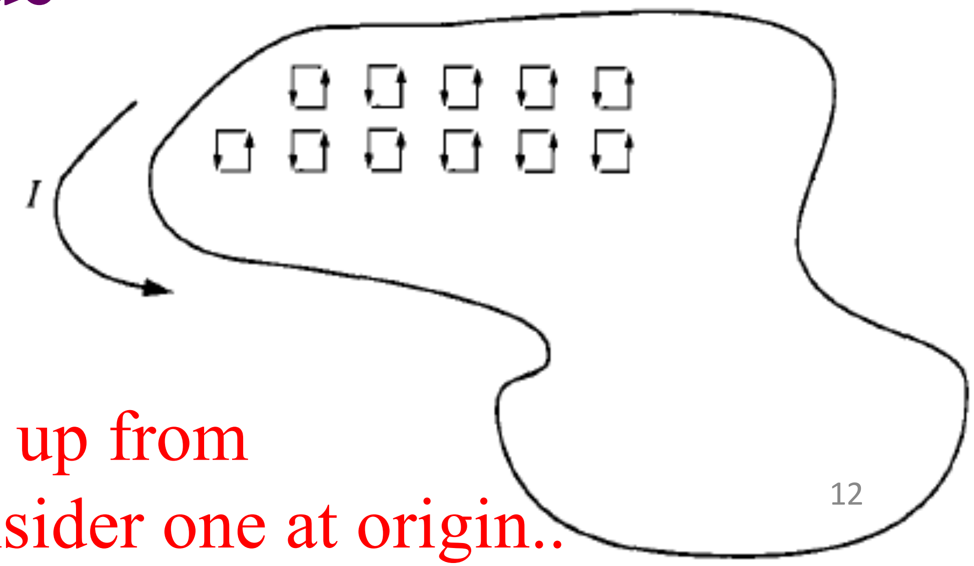
- So, each atom of these elements acts like small magnet..

Magnetization:

- Apply electric field **E**: ⇨ Electric polarization
- Apply magnetic field: ⇨ Magnetization (i.e. alignment of magnetic dipoles..)
- Unlike Electric Polarization (usually in the same direction as **E**), magnetization can be…
- parallel to applied field ⇨ Para-magnets
- opposite to applied field ⇨ Dia-magnets
- magnetization retained after B is removed ⇨ Ferro-magnets

Torques and forces on magnetic diploes:

- A magnetic dipole experiences a torque in a magnetic field.....like electric dipole does…..
- Take a current loop, which can be divided into tiny rectangular (or any shape) loops of large number, with internal sides cancelling..
- Let us calculate the torque on a rectangular current loop in a uniform magnetic field **B.**

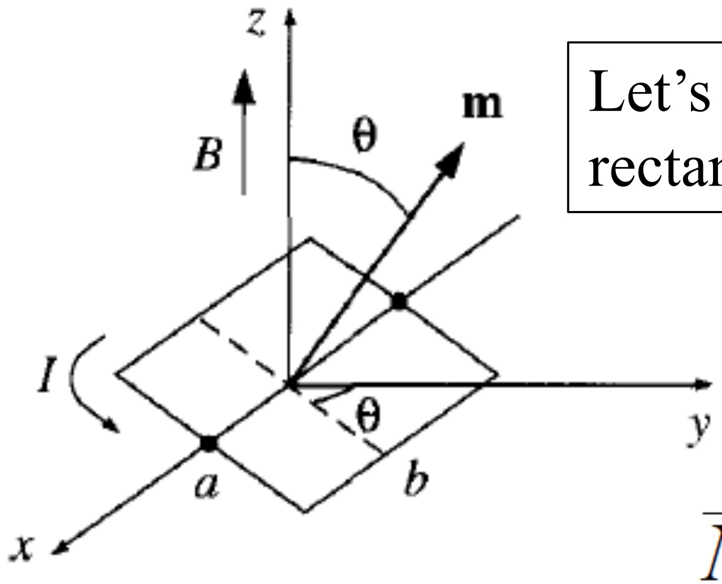


As, any current loop could be built up from infinitesimal rectangles….let's consider one at origin..

Torques and forces on magnetic diploes:

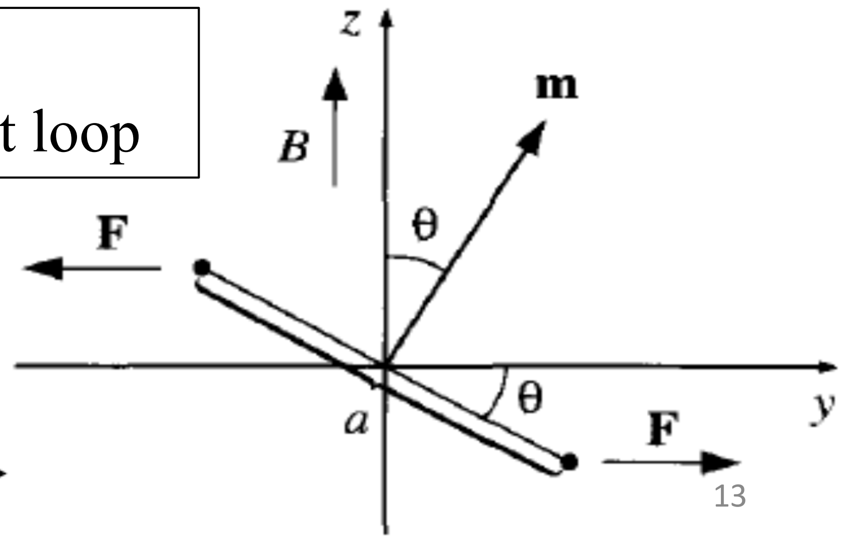
- ✧ The forces on the two sloping sides cancel (they tend to *stretch* the loop, but they don't *rotate* it).

- ✧ The forces on the "horizontal" sides are likewise equal and opposite (so the net *force* on the loop is zero), but they do generate a torque because **the forces are not collinear**



Let's consider a rectangular current loop

$$\overrightarrow{N} = \overrightarrow{a} \times \overrightarrow{F}$$



Torques and forces on magnetic diploes:

$$\mathbf{N} = aF\sin\theta\,\hat{\mathbf{x}}$$ torque along the x-direction

- And, since d**F** = Id**l** x **B**, the magnitude of the force is $F = I\,b\,B$, as the side b of the loop give the torque…we can write…

$$\mathbf{N} = IabB\sin\theta\,\hat{\mathbf{x}}$$
$$= mB\sin\theta\,\hat{\mathbf{x}}$$

$$\Rightarrow \mathbf{N} = \mathbf{m} \times \mathbf{B}$$

where, $m = Iab$ = dipole moment of the loop (area=ab)

The above equation gives the exact torque on any localized current distribution, in uniform B..

Torques and forces on magnetic diploes:

$$\mathbf{N} = \mathbf{m} \times \mathbf{B}$$

- ❖ The torque aligns the dipoles parallel to the field.

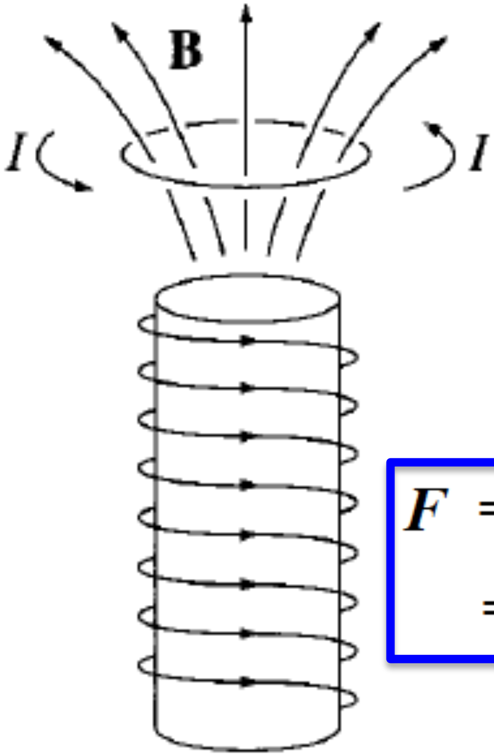
This mechanism is responsible for paramagnetism. **m** must be nonzero, occurs in atoms/molecules with odd no. of electrons (i.e. unpaired electrons)……..

- ❖ In a uniform field, the net force on a current loop is zero…..(as constant B outside integral)

- ❖ And, the line integral of *dl* is equal to zero around any closed loop……..

$$\mathbf{F} = I \oint (d\mathbf{l} \times \mathbf{B}) = I \left(\oint d\mathbf{l} \right) \times \mathbf{B} = 0$$

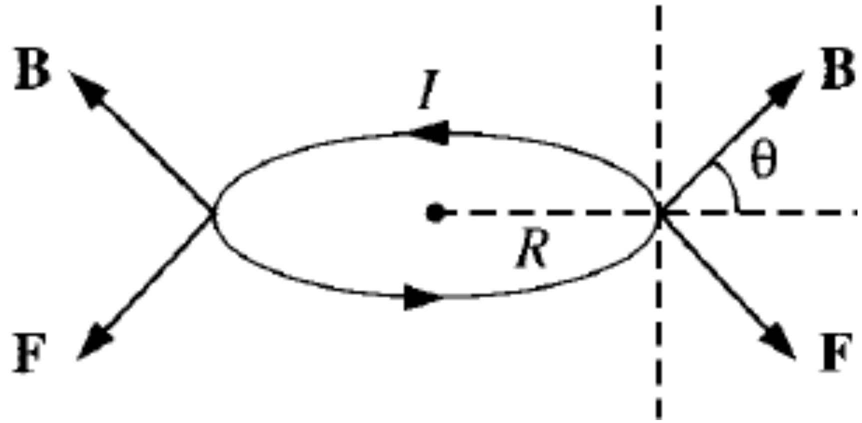
What happens when the field is non-uniform?

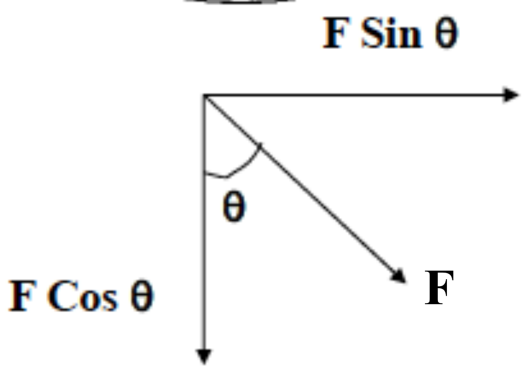


- take a circular wire of radius R, carrying current I, suspended in the fringing field above a solenoid..

Linear charge density

$$\boxed{\begin{aligned} F &= Q\,v\,B \quad ; \quad Q = 2\pi R\,\lambda \\ &= 2\pi R\,\lambda\,v\,B = 2\pi R\,I\,B \end{aligned}}$$





- B has a radial component……...
- Net downward magnetic force...

$$\boxed{\mathbf{F} = I\oint (d\mathbf{l}\times\mathbf{B}) = 2\pi I R B\cos\theta.}$$

It can be shown that the force on a small loop, with dipole moment **m**, in a field **B**, is: $\boxed{\mathbf{F} = \nabla(\mathbf{m}\cdot\mathbf{B})}$

Effect of magnetic field on atomic orbits:

✧ Electrons not only ***spin****;* they also *revolve* around the nucleus in ***orbits*** (like a tiny current loop)…..

- $\mathbf{m_s}$-magnetic dipole moment from **spin motion**
- $\mathbf{m_o}$-magnetic dipole moment from **orbital motion**

Effect of magnetic field **B**:

- $\mathbf{m_s}$ gets tilted (due to the torque) with **B.**

  (Para-magnetism) [$\mathbf{m_s}$ gets parallel to **B**]

- $\mathbf{m_o}$ cannot be tilted by **B**, but it leads to change in speed, leading to change in $\mathbf{m_o}$, and is ***opposite*** to **B**.

  (Diamagnetism) [$\mathbf{m_o}$ antiparallel to **B**]

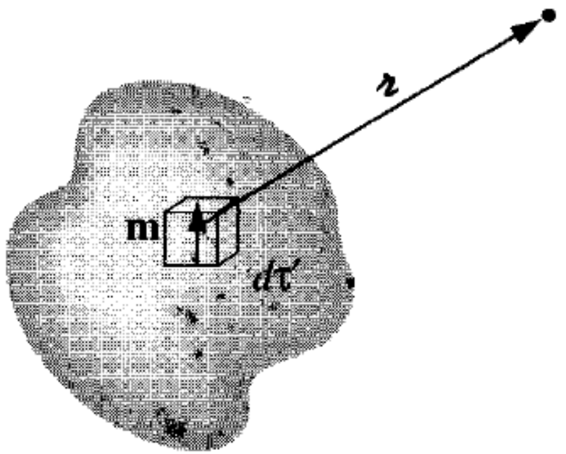
Magnetization:

- In the presence of magnetic field, matter gets magnetized……….
- Para-magnetism: dipoles of unpaired e^- spins experience torque to line them up parallel to **B**...
- Dia-magnetism: e^- orbital speed altered to change dipole moment opposite to **B**…….

Magnetization (M):

M = magnetic dipole moment per unit volume [Similar to electric polarization **P**]…...

The Field of a Magnetized Object: bound currents



Vector potential of a single dipole moment **m** is:

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\boldsymbol{\imath}}}{\imath^2}$$

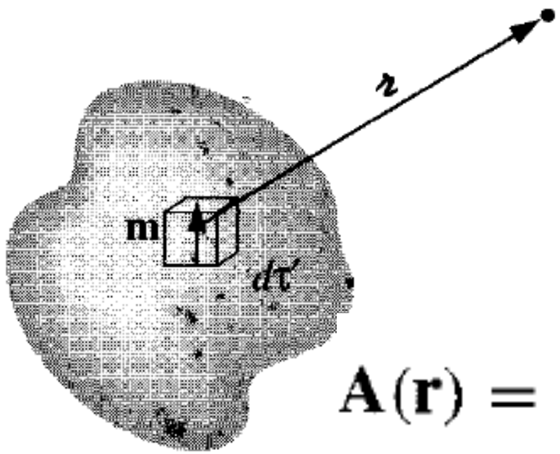
Each volume element dτ carries a dipole moment **m**=**M**dτ, so the total vector potential is:

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{M}(\mathbf{r}')\times\hat{\boldsymbol{\imath}}}{\imath^2}\,d\tau'$$

$$\boxed{\nabla'\frac{1}{\imath} = \frac{\hat{\boldsymbol{\imath}}}{\imath^2}}$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\left[\mathbf{M}(\mathbf{r}')\times\left(\nabla'\frac{1}{\imath}\right)\right]d\tau$$

The Field of a Magnetized Object: bound currents



using product rule…..

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\left\{\int \frac{1}{\mathscr{r}}[\nabla' \times \mathbf{M}(\mathbf{r}')]\,d\tau' - \int \nabla' \times \left[\frac{\mathbf{M}(\mathbf{r}')}{\mathscr{r}}\right] d\tau'\right\}$$

using surface integral….

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{1}{\mathscr{r}}[\nabla' \times \mathbf{M}(\mathbf{r}')]\,d\tau' + \frac{\mu_0}{4\pi}\oint \frac{1}{\mathscr{r}}[\mathbf{M}(\mathbf{r}') \times d\mathbf{a}']$$

first term like potential of volume current….

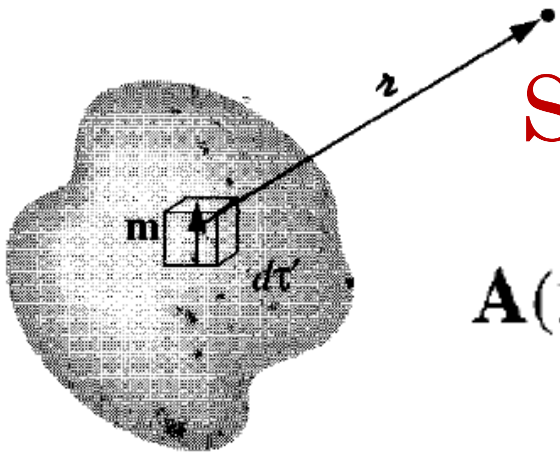
$$\mathbf{J}_b = \nabla \times \mathbf{M}$$

second term like potential of a surface current

$$\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}}$$

$\hat{n}$ is normal unit vector

The Field of a Magnetized Object: bound currents



So, we can write the eq. like below:

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_{\mathcal{V}} \frac{\mathbf{J}_b(\mathbf{r}')}{\imath}\, d\tau' + \frac{\mu_0}{4\pi}\oint_{S} \frac{\mathbf{K}_b(\mathbf{r}')}{\imath}\, da'$$

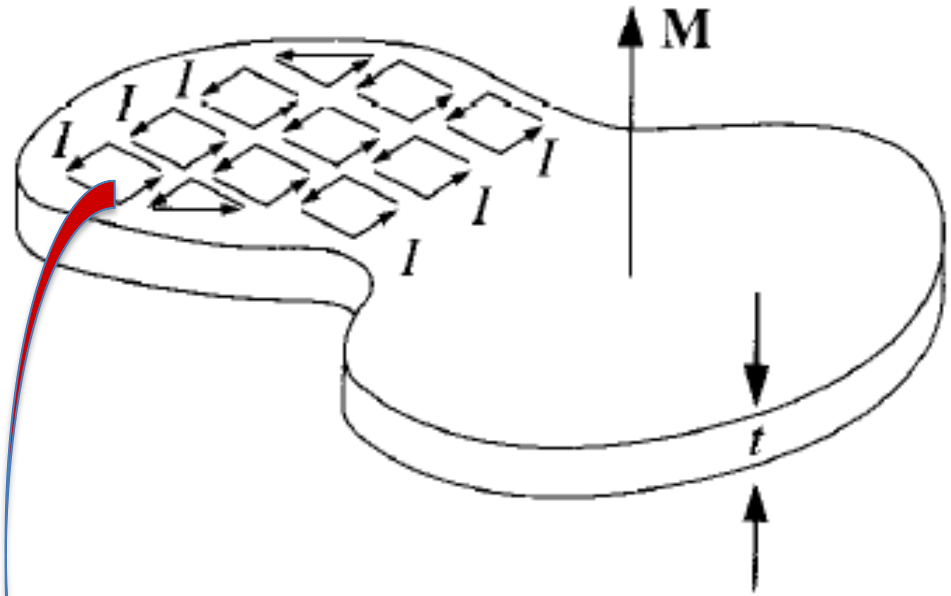
What does this mean?

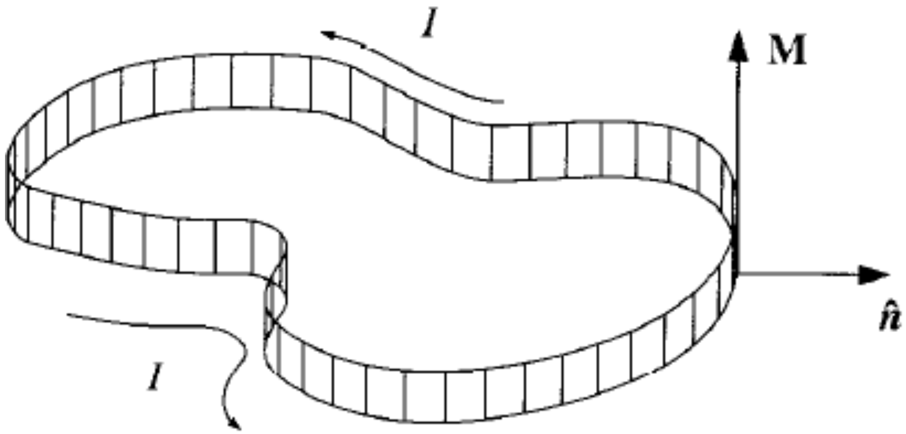
The potential and hence the field of a magnetized object is the same as would be produced by a volume current throughout the material plus a surface current on the boundary.

So, instead of integrating the contributions of all small dipoles, we can first determine these bound currents, and then find the field they produce….

Physical interpretation of bound currents:

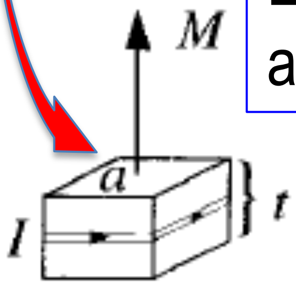
✧ Consider a slab of uniformly magnetized material





Equivalent to a simple ribbon of current I flowing around the boundary

✧ Tiny current loops are like dipoles



Each tiny loop has an area “a” and thickness “t”

$$\therefore\ \vec{m} = \vec{M}at$$ In terms of Magnetization

$$m = Ia$$ In terms of circulating current

$$\Rightarrow Ia = Mat \Rightarrow K_b = \frac{I}{t} = M$$

(surface current)

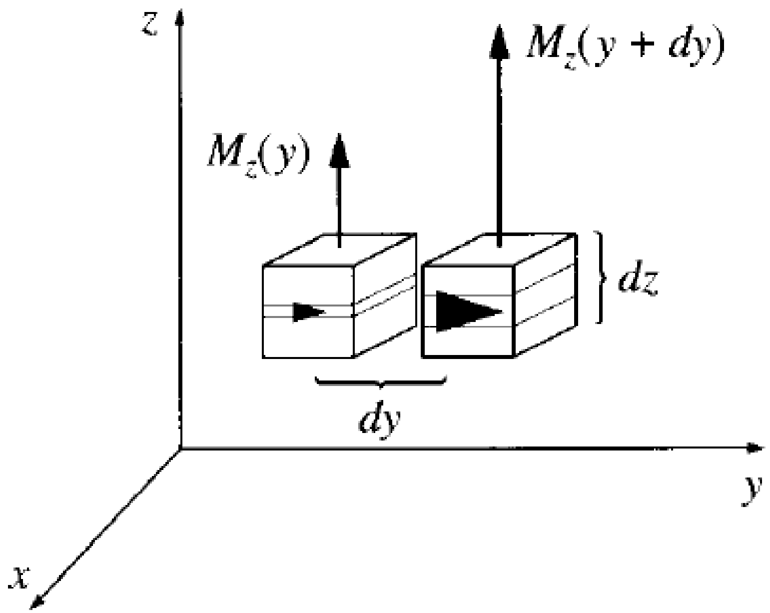
Let $\hat{n}$ is the outward-drawn unit vector then

(bound surface current) $\vec{K}_b = \vec{M} \times \hat{n}$

Physical interpretation of bound currents:

- ✧ It is a peculiar kind of current, as not a single charge makes the whole trip, each charge moves only in a tiny little loop within a single atom…..
- ✧ The net effect is a macroscopic current flowing over the surface of the magnetized object……...
- ✧ This is called "bound" surface current.
- ✧ Every charge is attached to a particular atom, but it is a perfectly genuine current…
- ✧ It produces a magnetic field in the same way as any other current does……..

What if the Magnetization is non-uniform?:



The internal currents no longer cancel.

Consider two adjacent chunks of magnetized materials:

Thick arrow — greater magnetization at that point.

—there is a difference in magnetization in the y– direction:

When they join there is a net current (at the interface between the two current loops) in the *x*-direction.

Which can be written as,

$$I_x = [M_z(y+dy) - M_z(y)]dz = \frac{\partial M_z}{\partial y} dydz$$

$$\boxed{I = Mt}$$

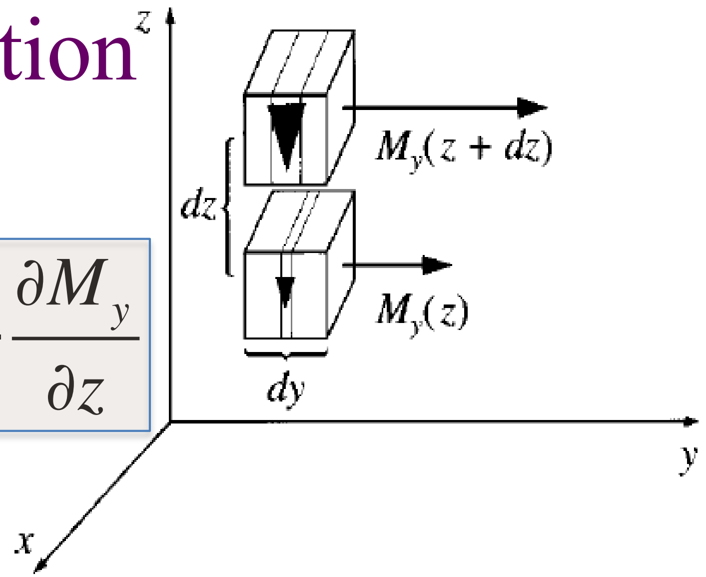
The corresponding volume current density is

$$\boxed{(J_b)_x = \frac{\partial M_z}{\partial y}}$$

Non-uniform Magnetization

Similarly, any change in magnetization in y-direction would produce:

$$I_x = [M_y(z+dz) - M_y(z)]dy = \frac{\partial M_y}{\partial z} dydz \Rightarrow \boxed{(J_b)_x = -\frac{\partial M_y}{\partial z}}$$



(-ve because, when they join, excess current flows in opposite direction)

✧ So, the net current density in *x*-direction:

$$(J_b)_x = \frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z}$$ (*x*-component of a curl)

Extending this to 3D, we get the expected result:

$$\vec{J}_b = \vec{\nabla} \times \vec{M}$$

and, this should follow conservation law..

$$\nabla \cdot \mathbf{J}_b = 0.$$

Summary: Bound currents

The effect of magnetization is to establish bound currents

$$\vec{J}_b = \vec{\nabla} \times \vec{M} \qquad \text{within the material}$$

and

$$\vec{K}_b = \vec{M} \times \hat{n} \qquad \text{on the surface}$$

The field due to magnetization of the medium is just the field produced by these bound currents.

Total Field = field due to bound currents + field due to everything else (or free current)

Total current: $\vec{J} = \vec{J}_b + \vec{J}_f$

$\vec{J}_b$= due to magnetization results from the conspiracy of many aligned atomic dipoles.

$\vec{J}_f$ = due to supply of current or transport of charge.