Lecture 13
Signals and Systems (ELL205)

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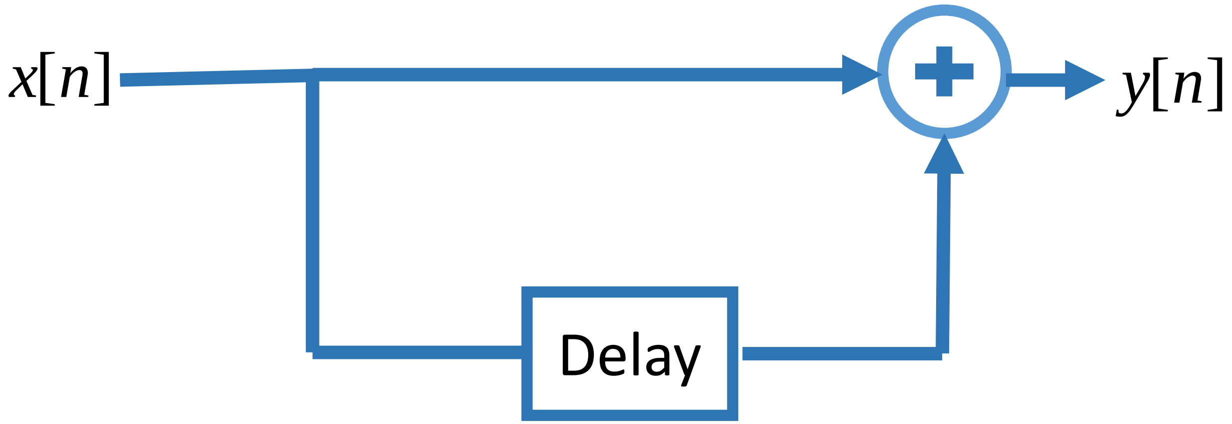
Outline of the lecture

- System designing

Basic DT system



Basic characteristics:

Linear (if delay starts at rest)

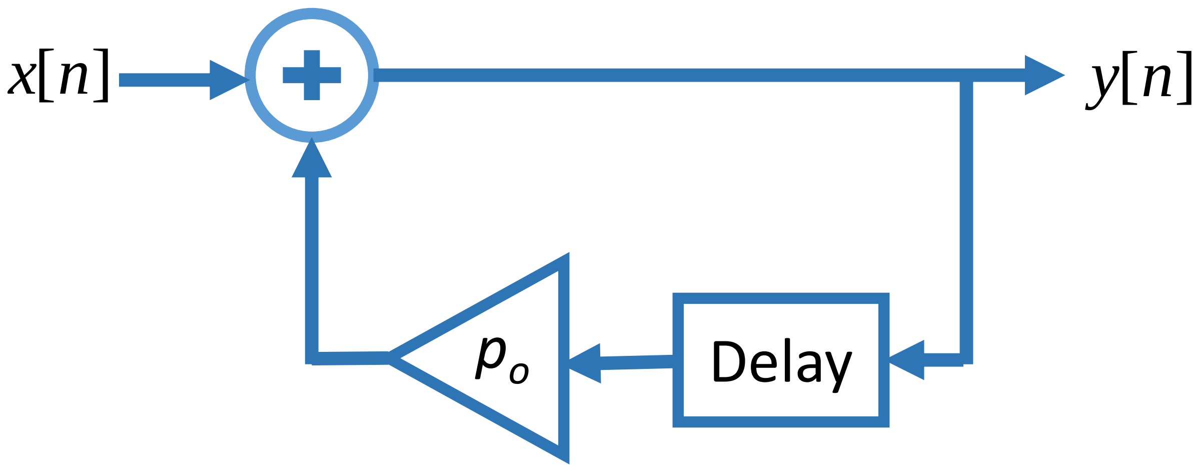
Time-Invariant

Causal

Recipe system

FIR system

Basic DT system



Basic characteristics:

Linear

Time-Invariant

Causal

Constraint/feedback system

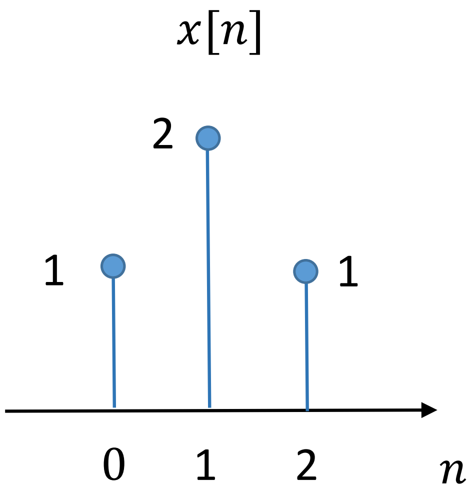
IIR system

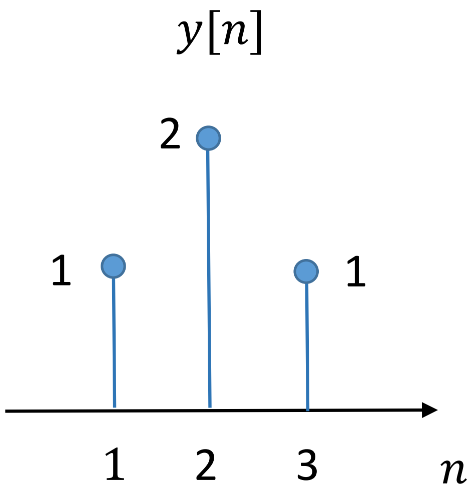
Different approaches

1) Graphical method
2) Step-by-step method
3) Guess method
4) Polynomial approach

Polynomials

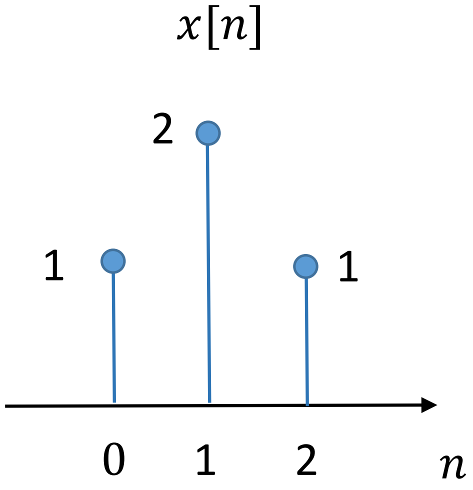
$$x[n] \longrightarrow \boxed{\text{Delay}} \longrightarrow y[n]$$

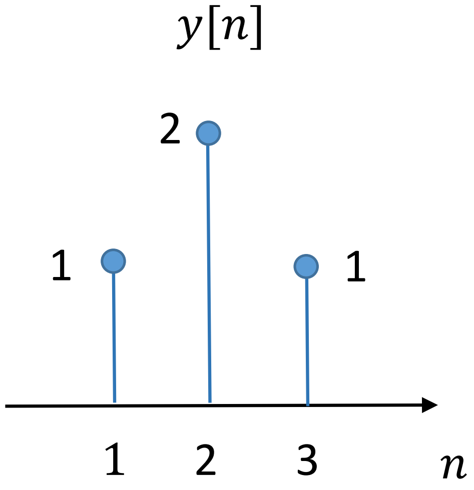




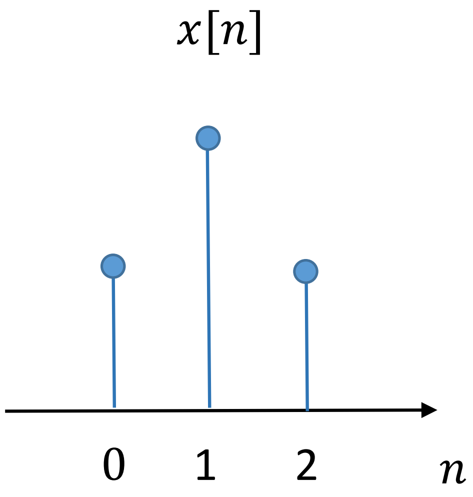
Polynomials

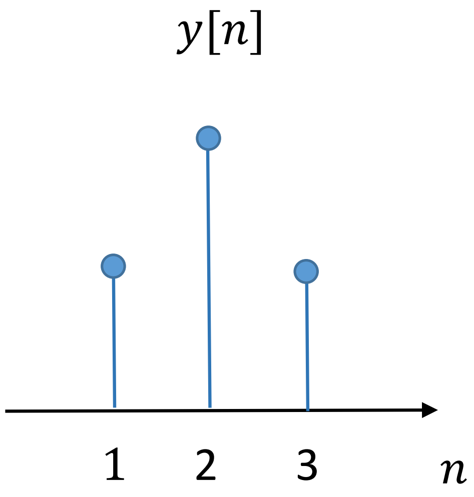
$$Y = RX$$





Polynomials



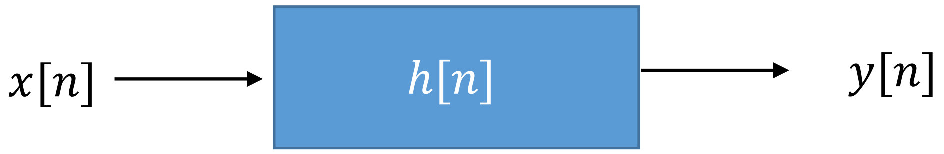


$$Y = RX$$

$$X = 1 + 2R + R^2$$

$$Y = R + 2R^2 + R^3$$

Why does polynomial method works?



Polynomials

$X = x_o + x_1 z + x_2 z^2$ $\quad$ $H = h_o + h_1 z + h_2 z^2$

$$
\begin{aligned}
&Y = HX \\
&= x_0 h_0 + (x_0 h_1 + x_1 h_0) z \\
&+ (x_0 h_2 + x_1 h_1 + x_2 h_0) z^2 + (x_1 h_2 + x_2 h_1) z^3 \\
&+ (x_2 h_2) z^4
\end{aligned}
$$

Why does polynomial method works?

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

Polynomials

$$X = x_o + x_1 z + x_2 z^2 \qquad H = h_o + h_1 z + h_2 z^2$$

$$\begin{aligned} Y &= HX \\ &= x_0 h_0 + (x_0 h_1 + x_1 h_0) z \\ &+ (x_0 h_2 + x_1 h_1 + x_2 h_0) z^2 + (x_1 h_2 + x_2 h_1) z^3 \\ &+ (x_2 h_2) z^4 \end{aligned}$$

Convolution

$$y[n] = (x * h)[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=0}^{2} x[k]h[n-k]$$

$$y[0] = \sum_{k=0}^{2} x[k]h[n-k] = x[0]h[0]$$

Why does polynomial method works?

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Polynomials

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Convolution

$$y[n] = (x * h)[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=0}^{2} x[k]h[n-k]$$

$$y[1] = \sum_{k=0}^{2} x[k]h[1-k] = x[0]h[1] + x[1]h[0]$$

Why does polynomial method works?

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

Polynomials

$$X = x_o + x_1 z + x_2 z^2 \qquad H = h_o + h_1 z + h_2 z^2$$

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Convolution

$$y[n] = (x * h)[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=0}^{2} x[k]h[n-k]$$

$$y[2] = \sum_{k=0}^{2} x[k]h[2-k] = x[0]h[2] + x[1]h[1] + x[2]h[0]$$

Why does polynomial method works?

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

Polynomials

$$X = x_o + x_1 z + x_2 z^2 \qquad H = h_o + h_1 z + h_2 z^2$$

Convolution

$$y[n] = (x * h)[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=0}^{2} x[k]h[n-k]$$

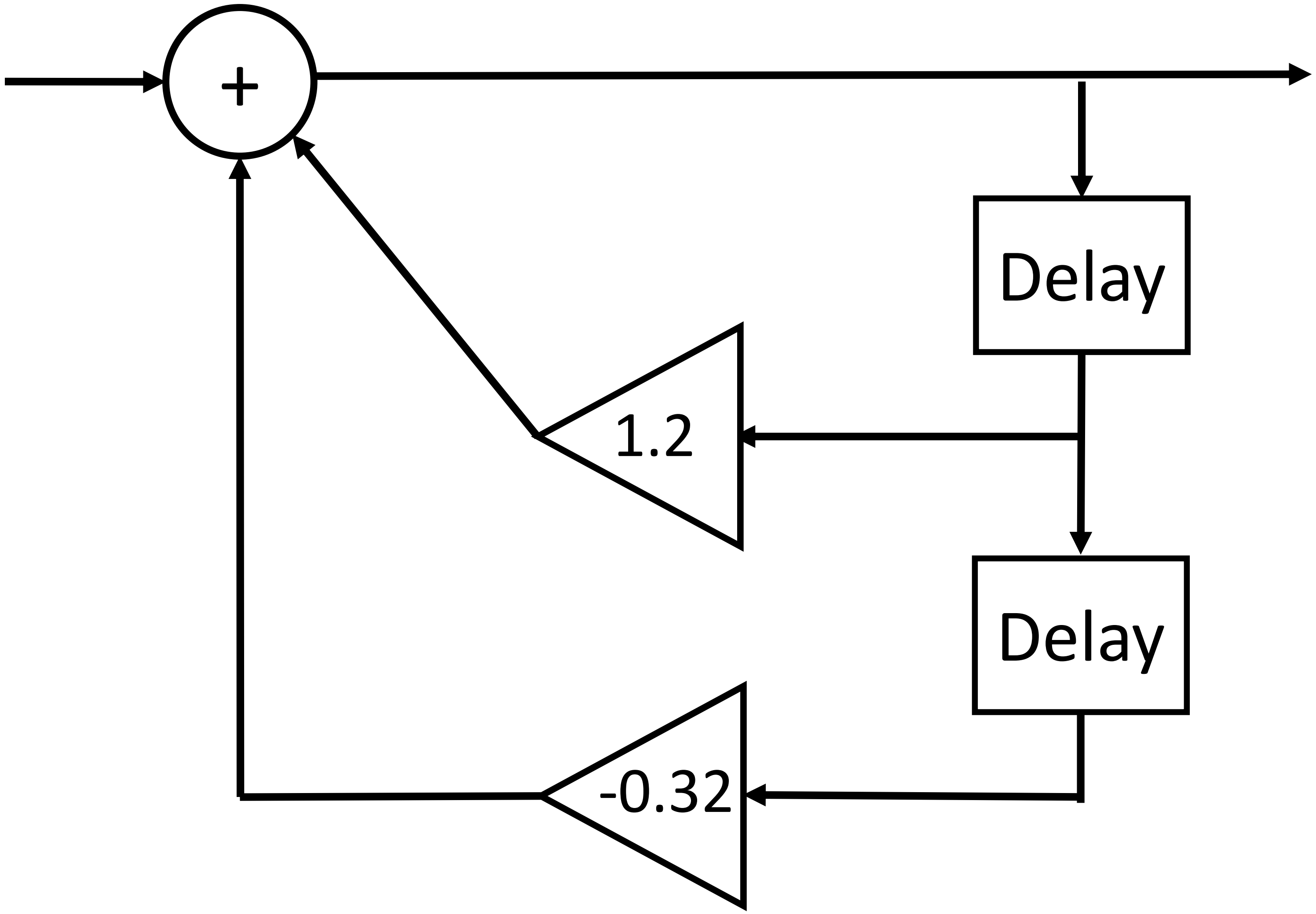
Polynomials are:

a) commutative: $HX = XH$
b) Associative: $(HX)Z = H(XZ)$
c) Distributive: $H(X+Y) = HX + HY$

Convolution is:

a) Commutative
b) Associative
c) Distributive

Representing in elementary form



Representing in elementary form

$$Y = X + 1.2RY - 0.32R^2Y$$

$$Y(1 - 1.2R + 0.32R^2) = X$$

$$\frac{Y}{X} = \frac{1}{(1 - 1.2R + 0.32R^2)}$$

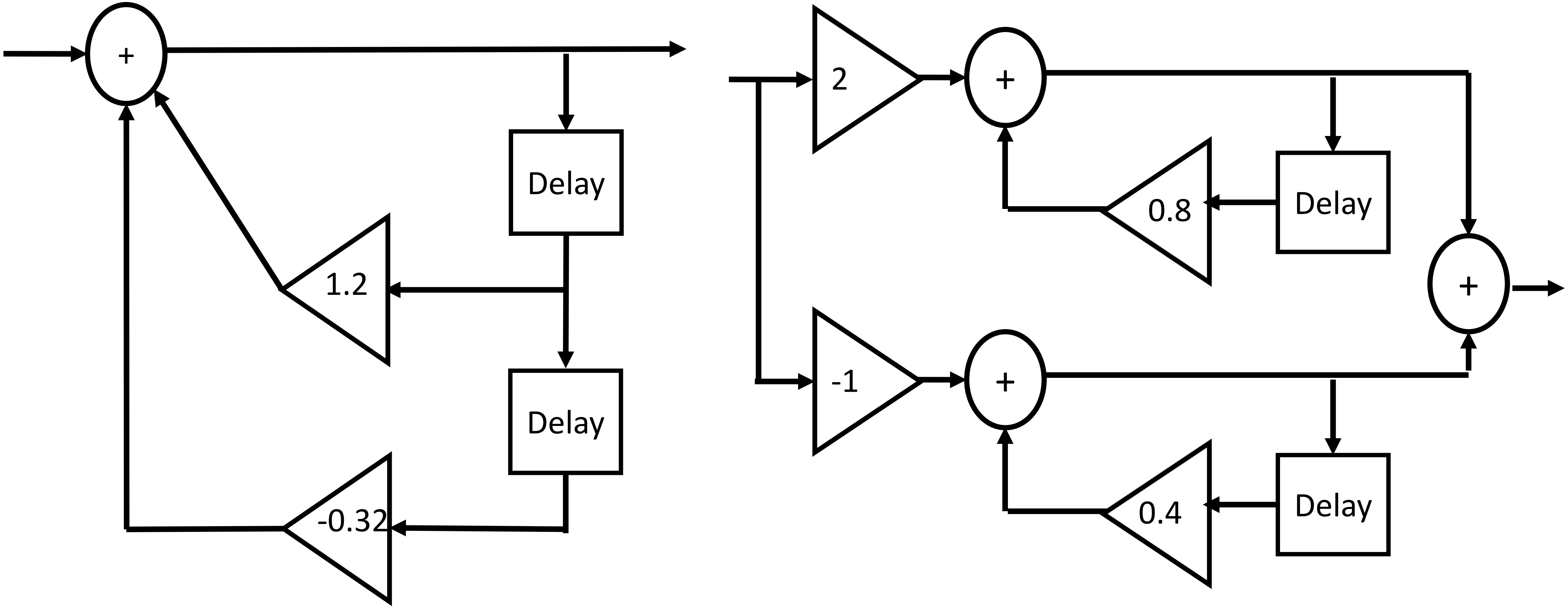
$$\frac{Y}{X} = \frac{1}{(1 - 0.8R)(1 - 0.4R)}$$

Representing in elementary form

$$\frac{Y}{X} = \frac{1}{(1 - 0.8R)(1 - 0.4R)}$$

$$\frac{Y}{X} = \frac{2}{1 - 0.8R} - \frac{1}{1 - 0.4R}$$

Representing in elementary form



Find the impulse response of the system:

$$H(R) = \frac{1}{(1-R)^2}$$

1) $\boldsymbol{h[n] = (n+1)u[n]}$	**2)** $\boldsymbol{h[n] = nu[n]}$
3) $\boldsymbol{h[n]} = \delta[\boldsymbol{n}] + \delta[\boldsymbol{n+1}]$	4) $\boldsymbol{h[n]} = \delta[\boldsymbol{n}] - \delta[\boldsymbol{n+1}]$

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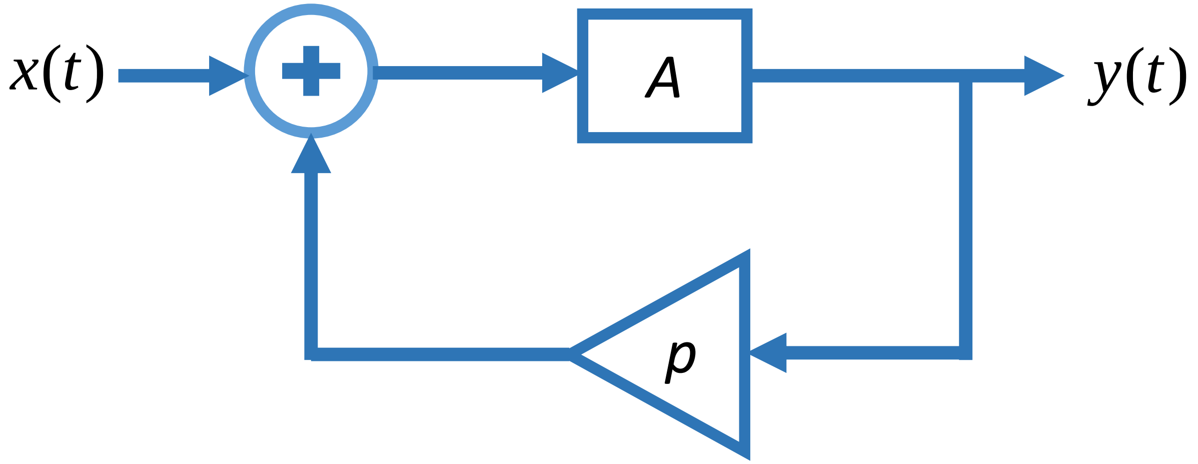
CT system (Graphical)

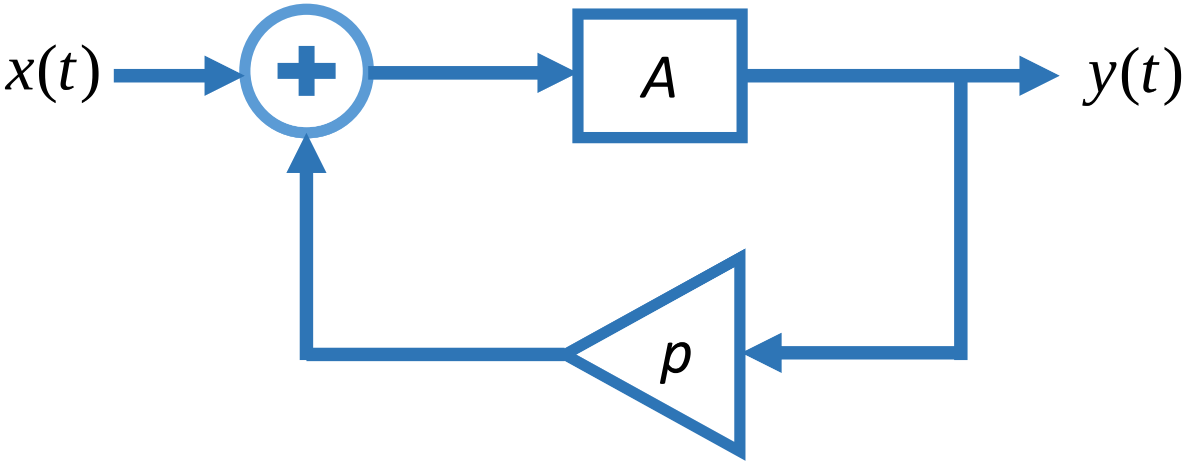
Basic CT system



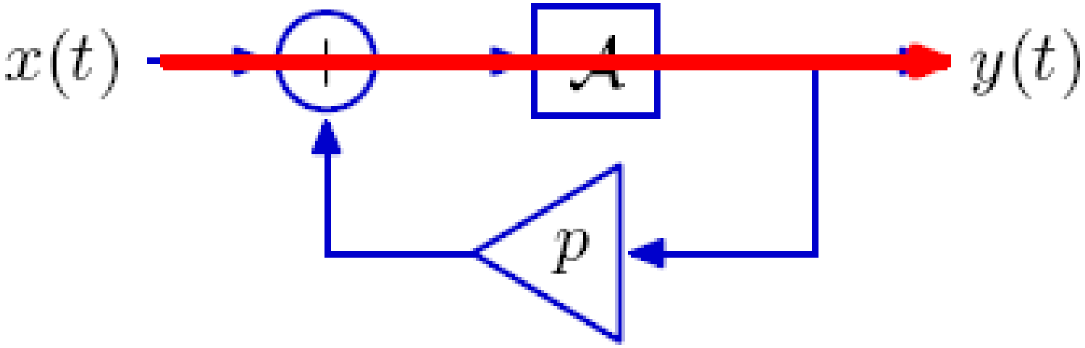
$$h(t) = e^{pt}u(t)$$

CT system (Graphical)

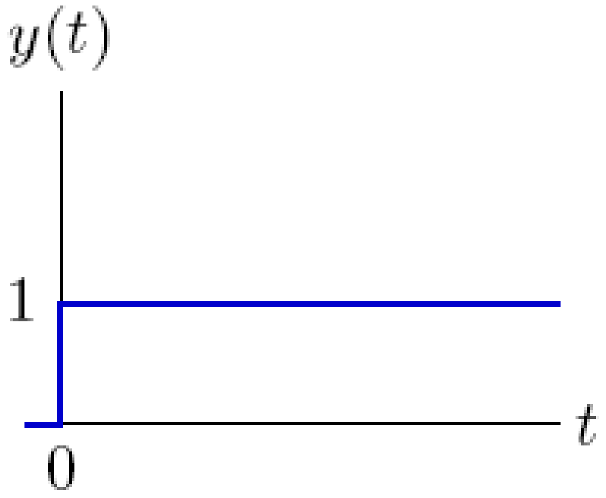
Basic CT system



$$h(t) = e^{pt}u(t)$$



h(t)=1



CT system (Graphical)

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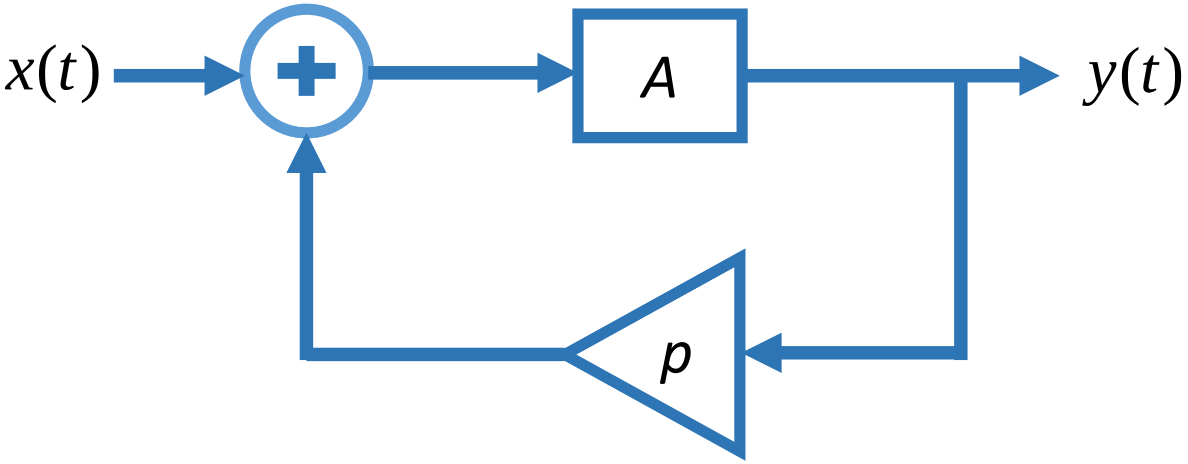
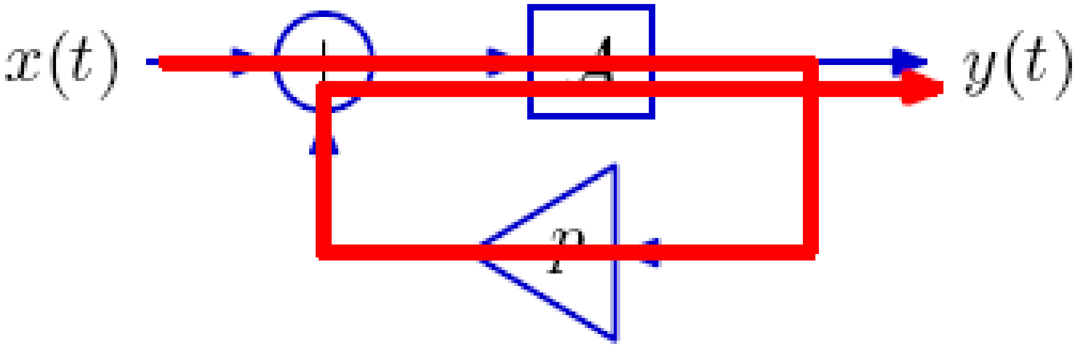
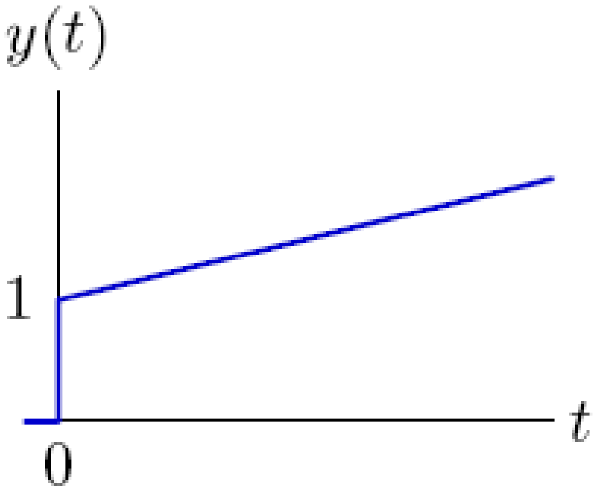


$$h(t) = e^{pt}u(t)$$



$h(t)$=1+pt



CT system (Graphical)

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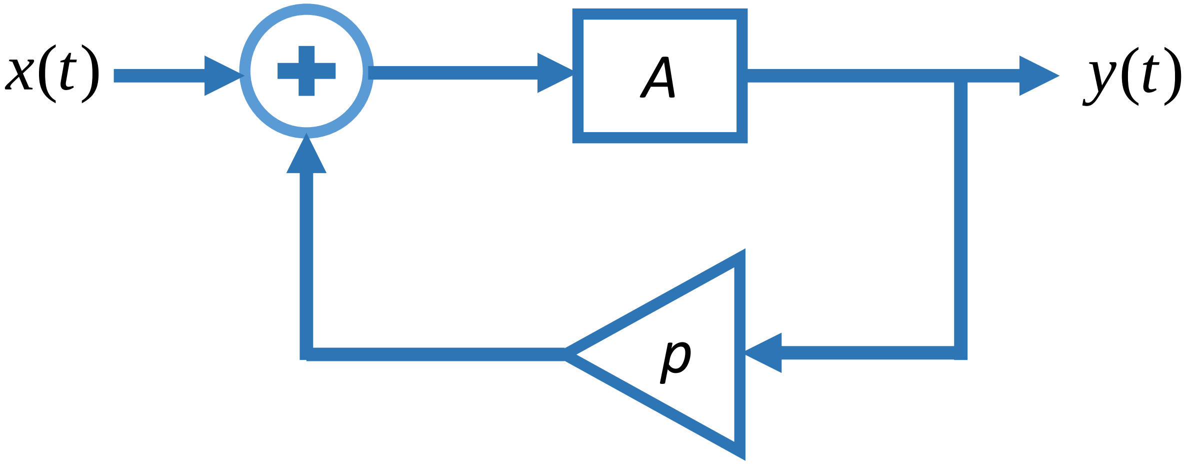
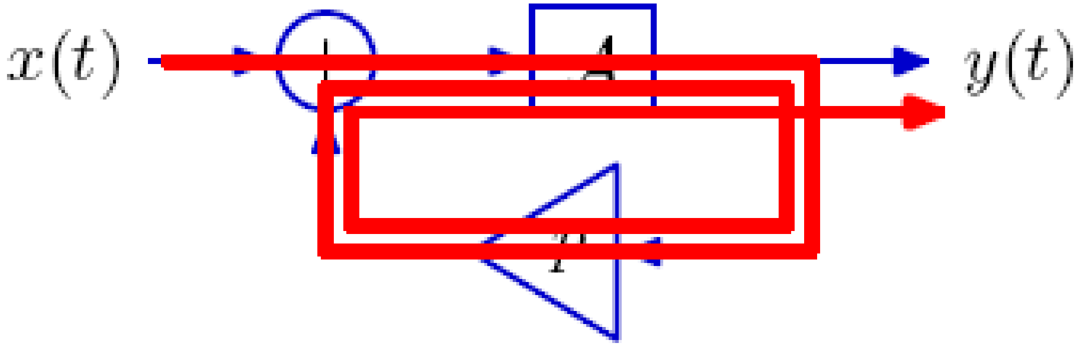
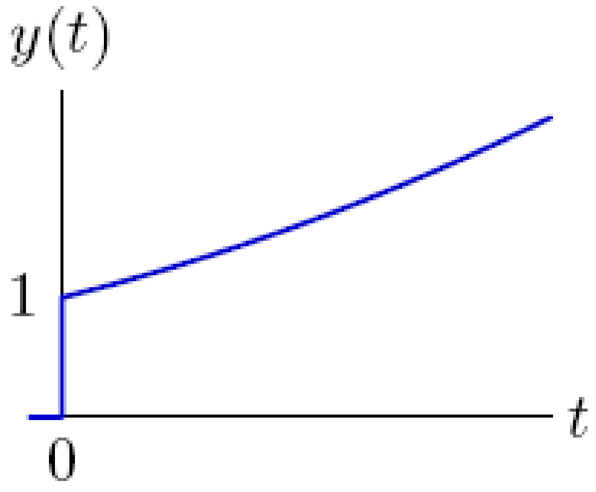


$$h(t) = e^{pt}u(t)$$

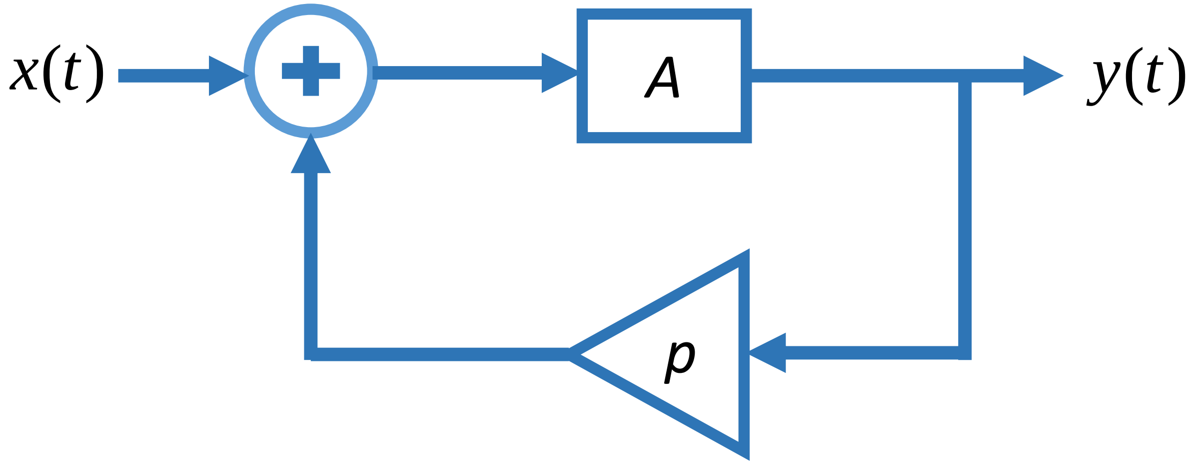


$h(t) = 1 + pt + \frac{1}{2}p^2t^2$

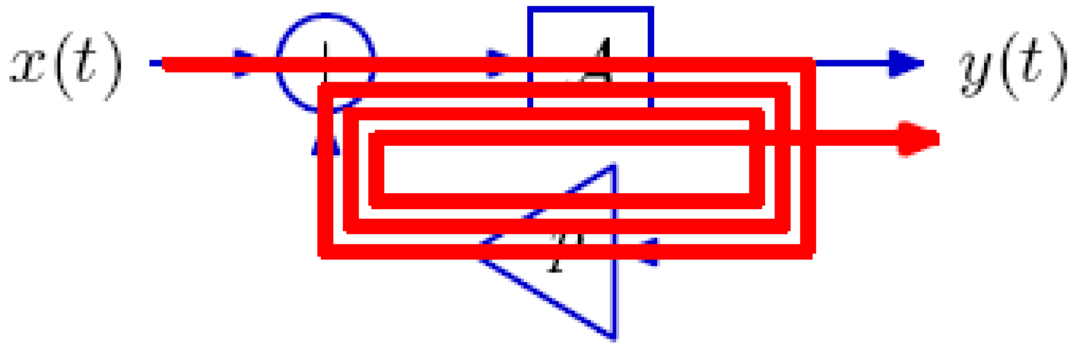


CT system (Graphical)

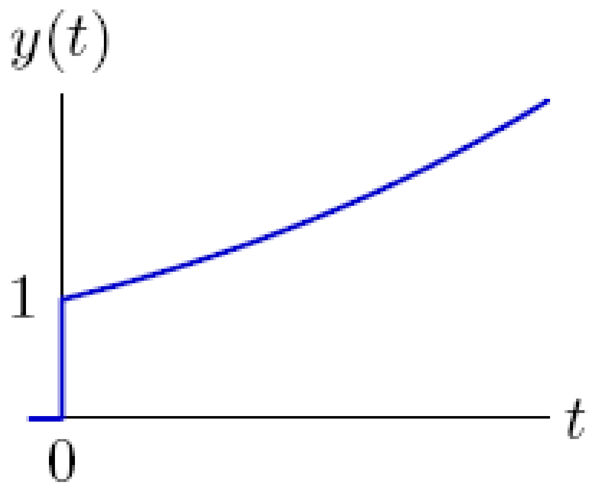
Basic CT system



$$h(t) = e^{pt}u(t)$$

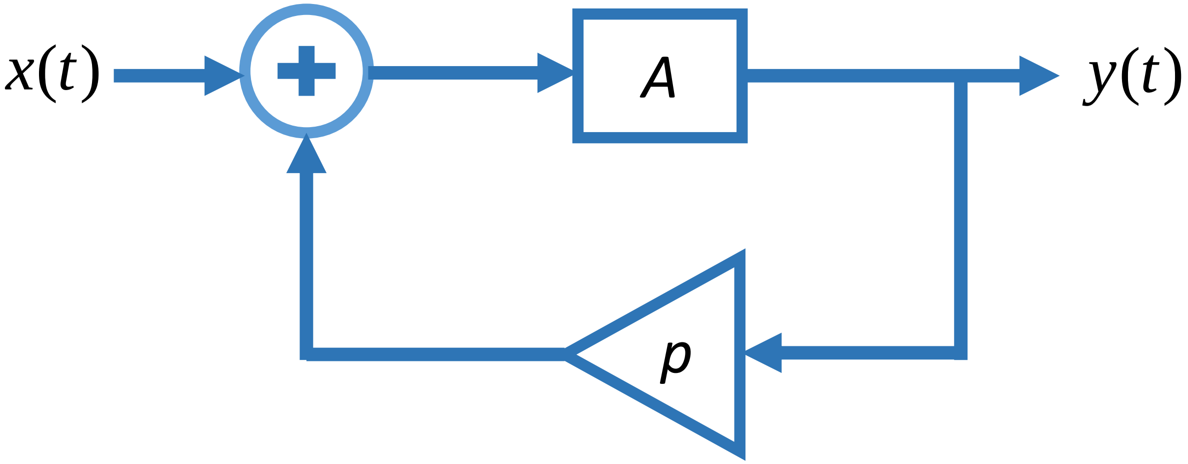


$$h(t) = 1 + pt + \frac{1}{2}p^2t^2 + \frac{1}{6}p^3t^3$$



CT system (Graphical)

Basic CT system



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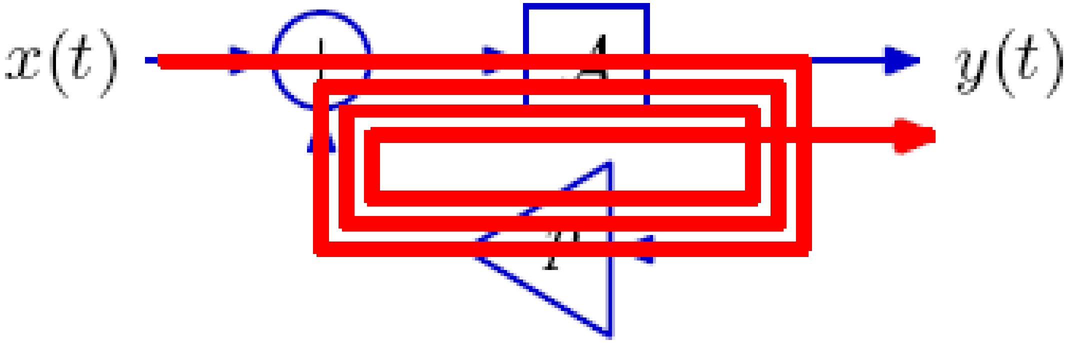
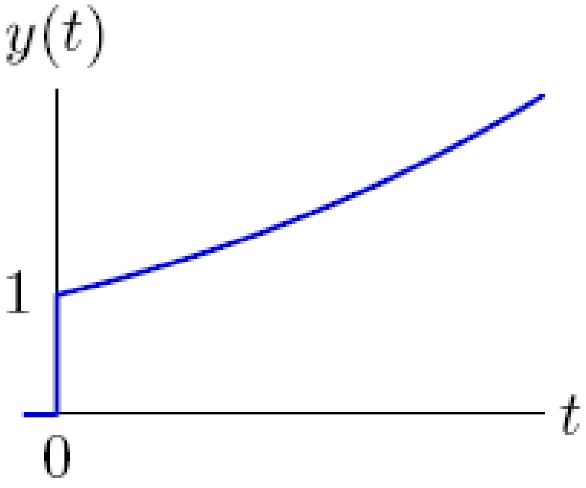
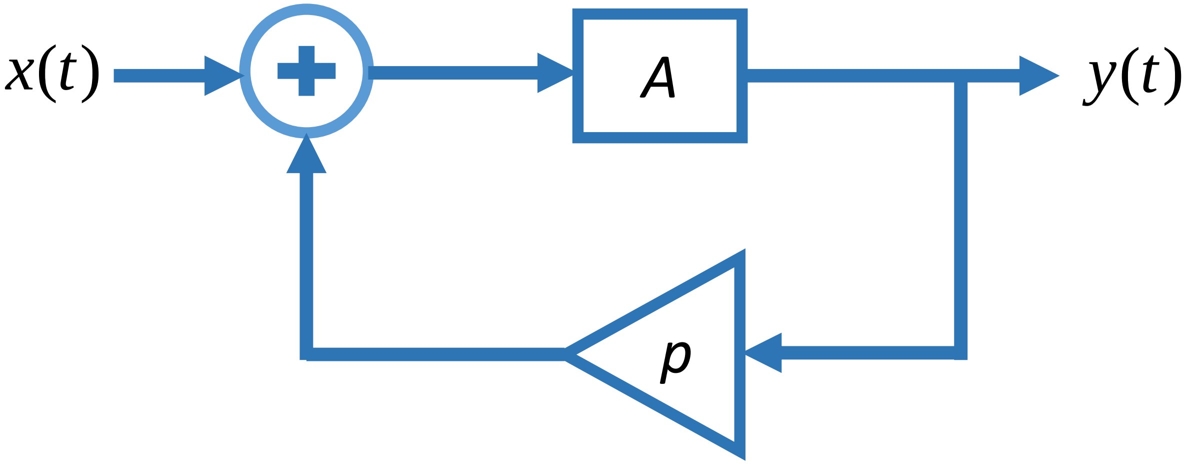


$$h(t) = \left(1 + pt + \frac{1}{2}p^2t^2 + \frac{1}{6}p^3t^3 + \right.$$



CT system (Graphical)

Basic CT system



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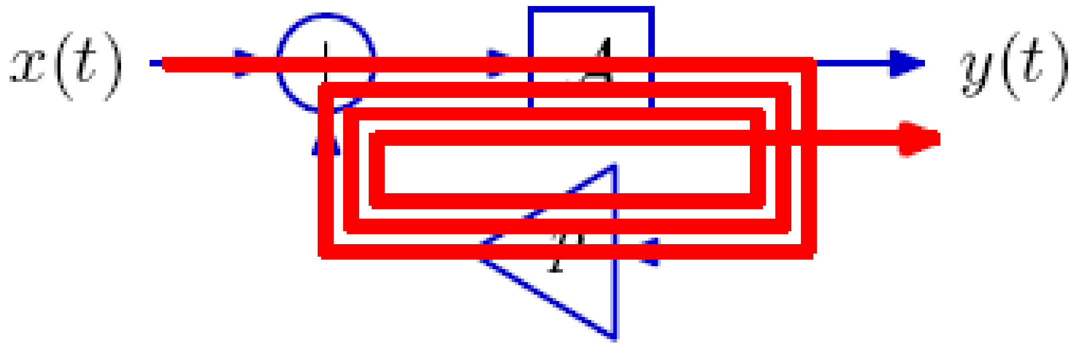
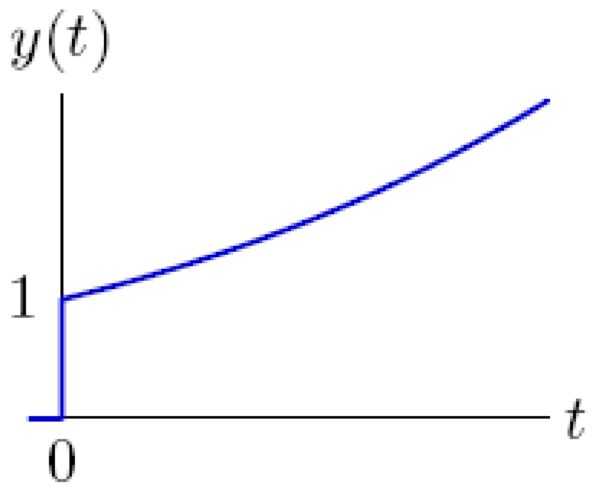
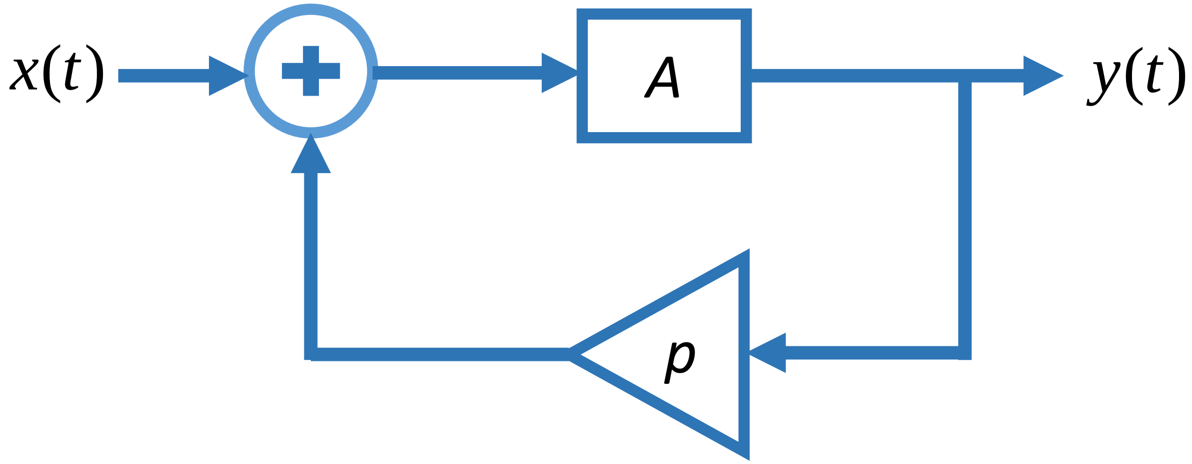


$h(t)=e^{pt}u(t)$



CT system (Guess)

Basic CT system



$$h(t) = e^{pt}u(t)$$

$$\dot{y}(t) = x(t) + py(t)$$

By Guess

$$y(t) = Ce^{st}u(t)$$

Substituting

$$Ce^{st}\delta(t) + sCe^{st}u(t) = \delta(t) + pCe^{st}u(t)$$

Comparing

$$C = 1 \;\&\; s = p$$

$$y(t) = e^{pt}u(t)$$