ELL100: INTRODUCTION TO ELECTRICAL ENGG.

Boolean Logic

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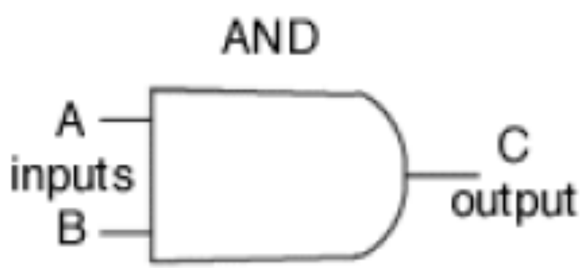
Textbook: Moris Mano's 'Digital Design':

Chapter 2 (Boolean Algebra and Logic Gates)

Boolean Algebra

- Branch of algebra in which variable values are: True (1) or False (0)
- Basic set of TRUE/FALSE operations and rules
- Formulated by George Boole in 19th century
  - Two binary operators:
    - AND (conjunction): Output is TRUE ‘only if’ E JE

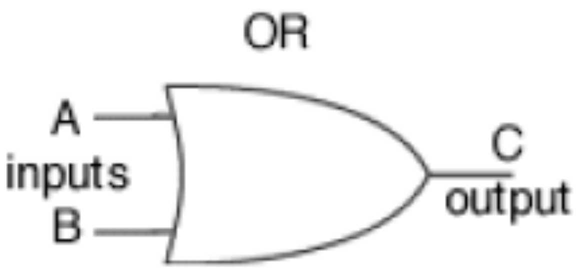


$$C = A \cdot B$$

A	B	C
0	0	0
1	0	0
0	1	0
1	1	1

Boolean Algebra

- Two binary operators:
  - AND (conjunction): Output is TRUE ‘only if’ BOTH inputs are TRUE
  - OR (disjunction): Output is TRUE ‘if’ ANY input is TRUE

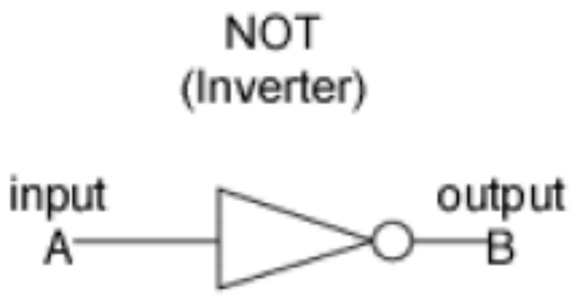


$$C = A+B$$

A	B	C
0	0	0
1	0	1
0	1	1
1	1	1

Boolean Algebra

- Two binary operators: AND (conjunction), OR (disjunction)
- One unary operator: NOT (Negation)



$$B = \overline{A}$$

$$B = A'$$

A	B
0	1
1	0

Boolean Algebra: Basic Properties

- Both the binary operations satisfy: If A,B are 1/0
  - Closure (If A, B are 1/0 → A+B, A.B are also 1/0)
  - Commutativity
    - $A + B = B + A,$ $A.B = B.A$
  - Associativity
    - $(A + B) + C = A + (B + C),$ $A.(B.C) = (A.B).C$
  - Distributivity
    - $A.(B + C) = A.B + A.C$ $A + (B.C) = (A + B).(A + C)$
  - Idempotence
    - $A + A = A$ $A.A = A$
  - Identity
    - $A + 1 = 1$ $A.0 = 0$

Boolean Algebra: Basic Properties

- Both the binary operations satisfy: If A,B are 1/0
  - Closure
  - Commutativity
  - Associativity
  - Distributivity
  - Idempotence
  - Absorption ($A + A.B = A.(A + B) = A$)
  - Involution ($\bar{\bar{A}} = A$ → NOT(NOT(A))=A)
  - Complement ($A + \bar{A} = 1,\ A.\bar{A} = 0$)

$$\begin{aligned} A + A.B &= (A.1) + (A.B) \\ &= A.(B + 1) = A \end{aligned}$$

$$\begin{aligned} A.(A + B) &= (A + 0).(A + B) \\ &= A + (B.0) = A \end{aligned}$$

Boolean Algebra : Basic Properties

- Both the binary operations satisfy: If A,B are 1/0
  - Closure
  - Commutativity
  - Associativity
  - Distributivity
  - Idempotence
  - Absorption
  - Involution
  - Complement

De Morgan's Laws:

$$\overline{A+B} = \bar{A}.\bar{B}$$

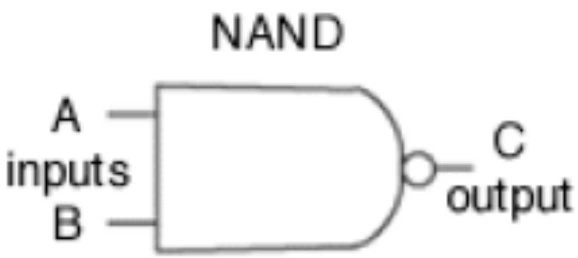
$$\overline{A.B} = \bar{A} + \bar{B}$$

Universal Gates

- **Universal Gates:** Logic Gates that can implement any Boolean function, without the use of any other type of gates.
- They are more easier to fabricate and are the *actual* ‘basic’ gates in digital electronics

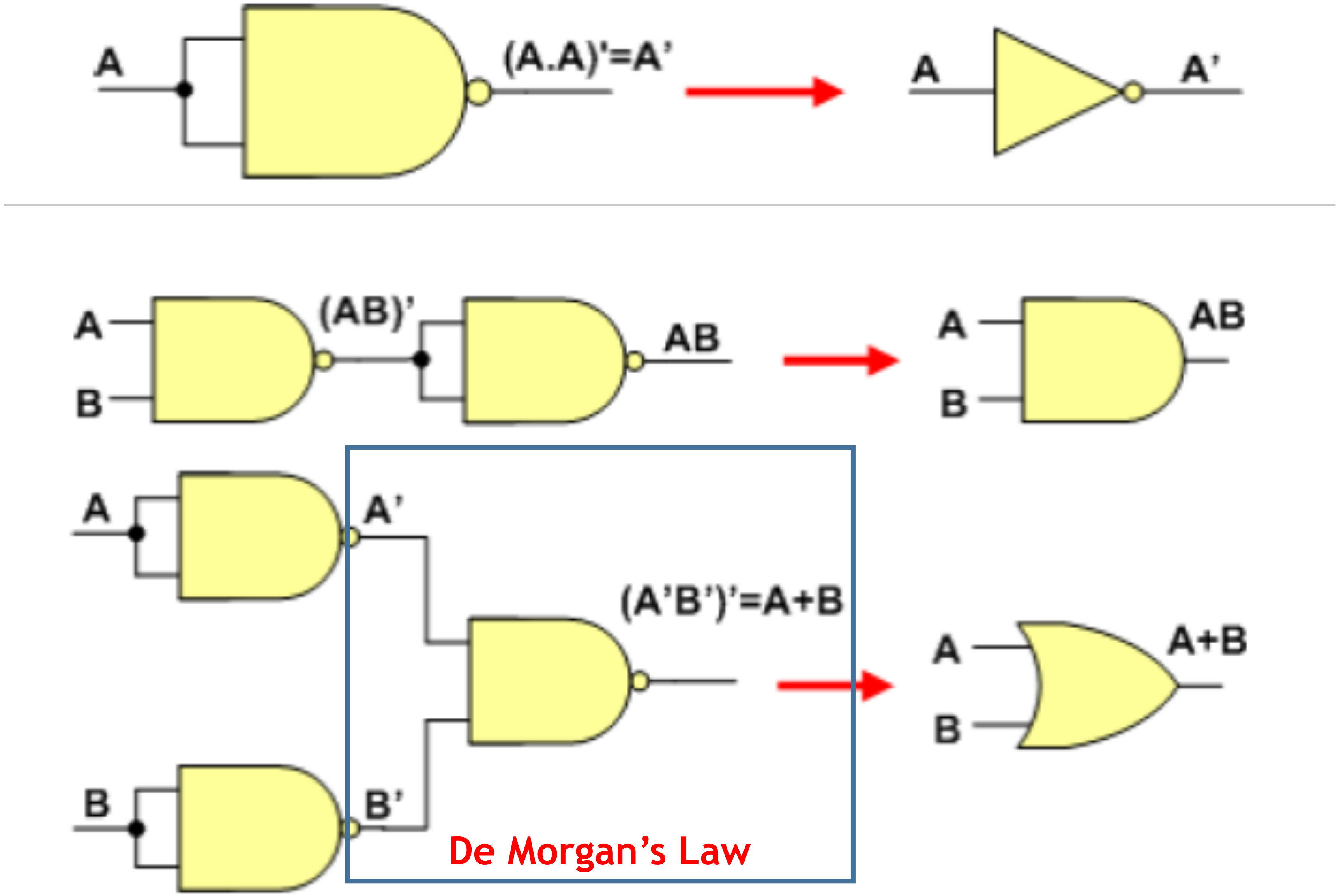
Universal Gates

- **Universal Gates:** Logic Gates that can implement any Boolean function, without the use of any other type of gates.
- **NAND Gate**: $\overline{AB}$ (NOT-AND)



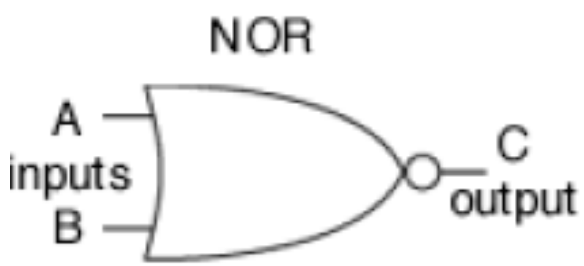
A	B	C
0	0	1
1	0	1
0	1	1
1	1	0

NAND Implementation of NOT/AND/OR



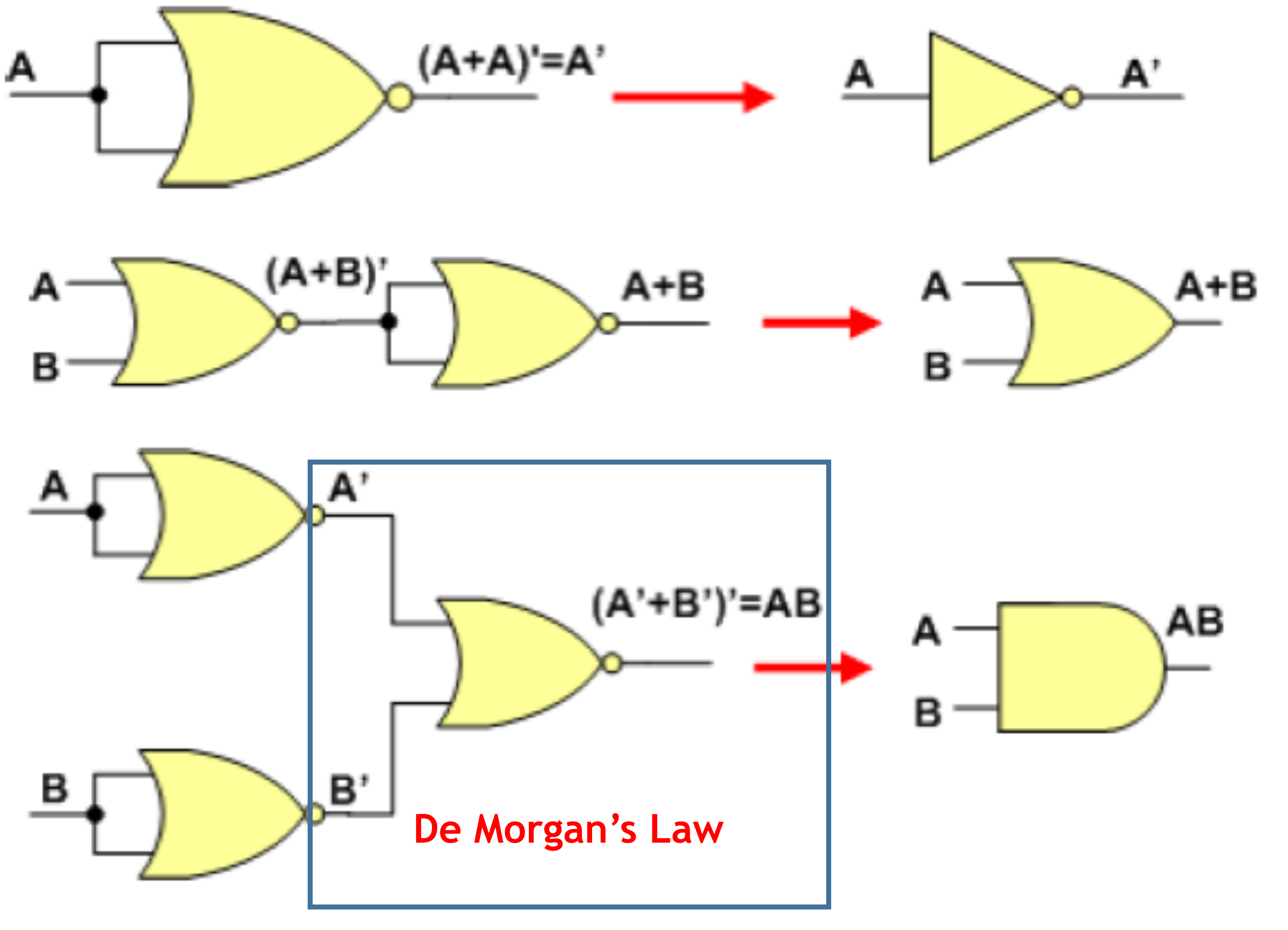
Universal Gates

- **Universal Gates:** Logic Gates that can implement any Boolean function, without the use of any other type of gates.
- **NOR Gate** : $\overline{A + B}$ (NOT-OR)



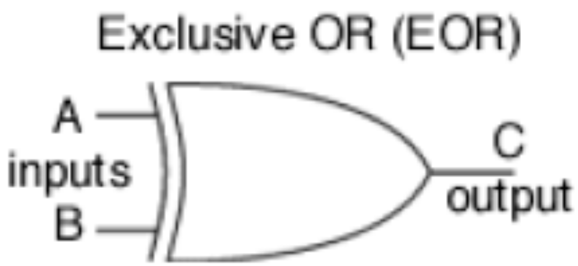
A	B	C
0	0	1
1	0	0
0	1	0
1	1	0

NOR Implementation of NOT/OR/AND



Other Commonly Used Gates

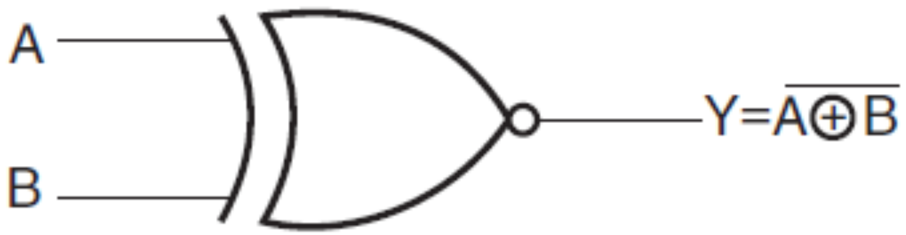
- **Exclusive OR (XOR):** $\mathrm{A} \oplus B = \bar{A}B + A\bar{B}$
  - Also called Exclusive Disjunction
  - Output is TRUE if **odd** number of inputs are TRUE
  - Becomes "one and only one" in case of 2 inputs



A	B	C
0	0	0
1	0	1
0	1	1
1	1	0

Other Commonly Used Gates

- **Exclusive NOR (XNOR) :** $A \odot B = \bar{A}\bar{B} + AB = \overline{A \oplus B}$
  - Output is TRUE if **even** number of inputs are TRUE
  - Becomes 'Equivalence' in case of two inputs



$$Y = (\overline{A \oplus B}) = (A.B + \overline{A}.\overline{B})$$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1