Harshit Marwandia 2020CS10348 Group-4

Q1) Let $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

Eigenvalues of A = roots of det$(\lambda I - A) = 0$

$$\begin{vmatrix} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda-1 & 1 \\ 0 & 0 & 1 & \lambda-1 \end{vmatrix} = 0$$

~~$\Rightarrow \lambda^2 (\lambda-1)^2 = 0$ $\lambda = 0$ or $\lambda = 1$~~

~~Eigenvalues of A = {0, 1}~~

$$\Rightarrow \lambda^2 \times ((\lambda-1)^2 - 1) = 0$$

$= \lambda = 0$ or $(\lambda-1)^2 = 1$

$\lambda - 1 = \pm 1$

$\lambda = 2, 0$

Eigen values of A = {0, 2}

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Finding the corresponding eigenspace :

$V_0 \Rightarrow$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0x \\ 0y \\ 0z \\ 0w \end{bmatrix}$$

$\Rightarrow$ $y = 0$

$0 = 0$

$z + w = 0$

$z + w = 0$

$V_0 = \{ (x, 0, z, -z) \mid (x, z) \in \mathbb{R}^2 \}$

It has dim = 2 as it has 2 arbitrary parameters (x, z)

basis of V_0 are $\rightarrow \{ (0, 0, 1, -1), (1, 0, 0, 0) \}$

$V_2 \Rightarrow$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \\ 2z \\ 2w \end{bmatrix}$$

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$y = 2x \Rightarrow x = y/2 = 0$

$0 = 2y \Rightarrow y = 0$

$z + w = 2z \Rightarrow w = z$

$z + w = 2w \Rightarrow w = z$

$V_2 = \{ (0, 0, z, z) \mid z \in \mathbb{R} \}$

$\downarrow$

It has dimension 1 and basis $\Rightarrow (0, 0, 1, 1)$

$\Rightarrow$ Now ~~sum~~ sum of dimension of eigenspaces of eigenvalues $= 2 + 1 = 3 \neq 2 = \dim$ of A

$\downarrow$

row reduced form

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$(R_4 \rightarrow R_4 - R_3)$

Hence the matrix is not diagonalizable

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Ans 2) Given,

$$t^2x'' + 5tx' + 4x = 0$$

Substitution $\Rightarrow$ $t = e^s$

$t = e^s$

$\frac{dt}{ds} = e^s$

$\frac{d^2t}{ds^2} = e^s$

$\frac{dx}{ds} = \frac{dx}{dt} \times \frac{dt}{ds}$

$\Rightarrow x' e^s \rightarrow (1)$

$\frac{d^2x}{ds^2} = \frac{d}{ds}\left(\frac{dx}{dt} \times \frac{dt}{ds}\right)$

$\Rightarrow \frac{d^2x}{dt^2}\left(\frac{dt}{ds}\right)^2 + x' \frac{d^2t}{ds^2}$

$x''_{(s)} = x'' e^{2s} + x' e^s \rightarrow (2)$

$\Rightarrow x'' t^2 + x'$

$t^2x'' \Rightarrow x''_{(s)} - x'_{(s)} \rightarrow (3)$

$x'_{(s)} \Rightarrow x' e^s \Rightarrow x't \rightarrow (4)$

Using eq (3) & (4) and substituting in parent eq^n. (As $t = e^s$)

$x''_{(s)} - x'_{(s)} + 5(x'_s) + 4x = 0$

$x''_{(s)} + 4x'_{(s)} + 4x = 0$

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putting $x = e^{ms}$

$x'_{(s)} = m e^{ms}$

$x''(s) = m^2 e^{ms}$

$m^2 e^{ms} + 4me^{ms} + 4e^{ms} = 0$

$(m^2 + 4m + 4) e^{ms} = 0$

$\hookrightarrow \neq 0$

$m^2 + 4m + 4 = 0$

$(m+2)^2 = 0$

$m = -2$

So $x(s) \Rightarrow e^{-2s} \Rightarrow$ solution

let $x_2(s) = v(s) e^{-2s}$

$x_2'(s) = v' e^{-2s} - 2ve^{-2s}$

$x_2''(s) = v'' e^{-2s} - 2v'e^{-2s}$
$- 2v' e^{-2s} + 4ve^{-2s}$

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So :- $x_2''(s) + 4(x_2'(s)) + 4x_2 = 0$

$v''e^{-2s} - 2v'e^{-2s} - 2v'e^{-2s} + 4ve^{-2s}$
$+ 4(v'e^{-2s} - 2ve^{-2s}) + 4ve^{-2s} = 0$

$v''e^{-2s} = 0$

$v'' = 0$

so $v' = c$

$\Rightarrow V = s$

so general soln $x = (ks + c)e^{-2s}$

$(ks + c)e^{-2s}$

now $t = e^s$

$(k\ln t + c)t^{-2}$

$x(1) = 0 \Rightarrow c = 0$

$x'(1) = 1 \Rightarrow K = 1$

$\Rightarrow$ Ans $= \frac{t\ln t}{t^2}$

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