Harshit Mawandia

2020 CS 10348

GRP – 4

Q1)

a) Photo electric effect:

A certain threshold frequency is required for the current to be generated. Also there is no time lag between illumination and release of electrons. Current depends on intensity of the photons

b) ~~Quantum~~ Mechanical ~~effects on carrier distri~~

b) Width of depletion region in the p-n junction:

The depletion layer that has formed around the p-n junction is calculated quantum mechanically. ~~For example~~ The p-n product deviates by orders of magnitude from results from different doping concentration.

c) Quantum Tunelling:

The wavefunction can propagate even though a potential barrier higher than the total energy. Therefore, some electrons ~~were~~ not having enough energy may also cross the potential barrier to reach conduction band.

0.5

d) $E = h\nu$

Energy of photons are integral multiple of $h\nu$ where ν is the frequency of the wave. These packets of energy are called 'photons'.

Q 2) From uncertainty principle

For ground state of H atom:

$$\Delta x \cdot \Delta p \sim \hbar$$

Maximum uncertainty in position r :-

$$\Delta x \to r$$

$$r \Delta p \sim \hbar$$

$$\Delta p \sim \frac{\hbar}{r}$$

Min value of momentum cannot be less than uncertainty

$$p \sim \frac{\hbar}{r}$$

Now electron proton energy -

$$E(r) = \frac{p^2}{2m_e} - \frac{e^2}{4\pi\epsilon_0 r} = \frac{\hbar^2}{2m_e r^2} - \frac{e^2}{4\pi\epsilon_0 r} \quad \text{— (1)}$$

At the ground state, energy must be minimum

$$\frac{dE(r)}{dr} = 0 \quad \text{at } r = r_0 \Rightarrow \frac{\hbar^2}{m_e r^3} + \frac{e^2}{4\pi\epsilon_0 r^2} = 0$$

$$\frac{\hbar^2}{m_e} \times \frac{4\pi\epsilon_0}{e^2} = r$$

After solving this $r_0 = 0.53$ nm

Now energy at this $r_0 = -13.6$ eV

From eqn 1

$$p = \left(\left(E(r) + \frac{e^2}{4\pi\epsilon_0 r} \right) \times 2m_e \right)^{1/2}$$

a)

$\lambda_{de\,Broglie} = \frac{h}{p}$

$r_2 = 2 r_1$ (given)

$E_2 = E_1/4$ (given)

$\therefore p_1 = \left(\left(E_1 + \frac{e^2}{4\pi\epsilon_0 r_1} \right) \times 2me \right)^{1/2}$

$\therefore p_2 = \left(\left(\frac{E_1}{4} + \frac{e^2}{2 \times 4\pi\epsilon_0 r_1} \right) \times 2me \right)^{1/2}$ ✓

$\frac{\lambda_2}{\lambda_1} = \frac{p_1}{p_2} = \left(\frac{E_1 + \frac{e^2}{4\pi\epsilon_0 r_1}}{\frac{E_1}{4} + \frac{e^2}{2 \times 4\pi\epsilon_0 r_1}} \right)^{1/2}$

Now $r_1 = \frac{\hbar^2}{me} \times \frac{4\pi\epsilon_0}{e^2}$

$E_1 = \frac{e^4 me}{32\hbar^2 \times \pi^2 \epsilon_0^2} - \frac{e^2}{4\pi\epsilon_0 \hbar^2 \times 4\pi\epsilon_0} \times me$

$= \frac{-e^4 me}{32\hbar^2 \times \pi^2 \epsilon_0^2} = \boxed{\frac{-e^2}{8\pi\epsilon_0 r_1}}$ ✓ 6

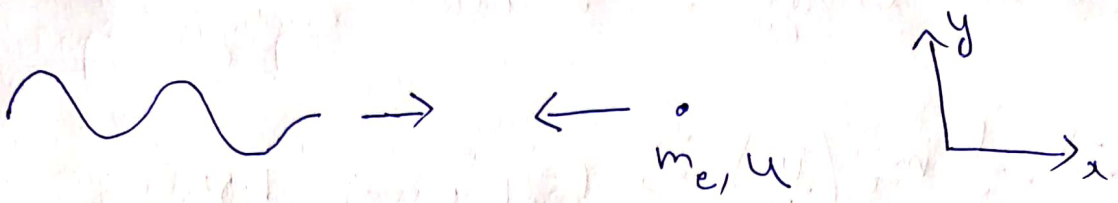
$\frac{\lambda_2}{\lambda_1} = \left(\frac{\frac{-e^2}{8\pi\epsilon_0 r_1} + \frac{e^2}{4\pi\epsilon_0 r_1}}{\frac{-e^2}{4 \times 8\pi\epsilon_0 r_1} + \frac{e^2}{8\pi\epsilon_0 r_1}} \right)^{1/2} = \left(\frac{\frac{-1}{2} + 1}{\frac{-1}{8} + \frac{1}{2}} \right)^{1/2}$

nice

$= \left(\frac{1/2}{3/8} \right)^{1/2} = (4/3)^{1/2}$

Q3)

a)



Since before and after collision it is in negative x direction and the photon was also moving in +ve x direction.

Let the momentum of photon be $\vec{p} = p_{photon}\,\hat{i}$

where p is +ve

By momentum conservation:

$$-m_e u\,\hat{i} + p_{photon}\,\hat{i} = p_{final}\,\hat{i} - m_e\frac{u}{2}\,\hat{i}$$

$$p_{final} = \left(-m_e\frac{u}{2} + p_{photon}\right)\hat{i}$$

since $p_{phon} << m_e\frac{u}{2}$ It's not given

direction will be both along negative x axis. direction possible

b) Energy of photon:

Initial energy = $h\nu_i$ / E_{photon} $+ \frac{1}{2}m_e u_0^2$ E_{e-} should be relativistic

Since collision is elastic

energy lost = 0

Final energy = Initial energy

$$\Rightarrow \frac{1}{2} m_e \left(\frac{u}{2}\right)^2 + E_{photon} = h\nu_i + \frac{1}{2} m_e u^2$$

$$E_{photon} = h\nu_i + \frac{3}{8} m_e u^2$$

Scanned with CamScanner

Q4) a) Since ϕ is a quantum mechanical wave function it must be normalised

$\therefore \langle \phi | \phi \rangle = 1$

Since ψ_i are orthonormal $\psi_i^* \psi_j = 0 \quad \forall \; i \neq j$

$\therefore \langle \phi | \phi \rangle = A^2 \langle \psi_1 | \psi_1 \rangle + \frac{1}{5} \langle \psi_2 | \psi_2 \rangle + \frac{1}{7} \langle \psi_3 | \psi_3 \rangle = 1$

$\Rightarrow A^2 + \frac{1}{5} + \frac{1}{7} = 1 \qquad (\langle \psi_i | \psi_i \rangle = 1$ (they are normalised))

$\Rightarrow A^2 = 1 - \frac{12}{35} = \frac{23}{35}$

$\Rightarrow A = \sqrt{\frac{23}{35}}$ ✓ 2

b) $\langle \hat{O} \rangle = \langle \phi | \hat{O} | \phi \rangle$

$\hat{O} | \phi \rangle = 2 \sqrt{\frac{23}{35}} \psi_1 + \frac{5}{\sqrt{5}} \psi_2 + i \times \frac{10}{\sqrt{7}} \psi_3$ ✓ 1

$\langle \hat{O} \rangle = \langle \phi | \hat{O} | \phi \rangle$ (since $\langle \phi | \phi \rangle = 1$)

$= 2 \times \frac{23}{35} + \frac{5}{5} + \frac{10}{7}$

$= \frac{46}{35} + \frac{35}{35} + \frac{50}{35}$ ✓

$= \frac{131}{35} \approx 3.742$ 2

c) $\langle \psi_2 | \hat{O} | \psi_2 \rangle = \langle \psi_2 | 5 \psi_2 \rangle$

$= 5 \langle \psi_2 | \psi_2 \rangle$

$= 5$

2

Q5) Particle is confined to move between $-\frac{a}{2} \leq x \leq \frac{a}{2}$

By 1-D Shrodinger equation

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} E\psi(x) = 0$$

gives $\psi(x) = A\cos kx + B\sin kx$

where $k = \sqrt{\frac{2mE}{\hbar^2}}$

a) The boundary conditions are

$\psi\left(\frac{a}{2}\right) = 0$ & $\psi\left(-\frac{a}{2}\right) = 0$ ✓ (1)

which gives $A\cos\left(\frac{ak}{2}\right) = 0$

$B\sin\left(\frac{ak}{2}\right) = 0$

b) $\psi_n(x) = \sqrt{\frac{2a}{a}}\cos\left(\frac{n\pi x}{a}\right)$ $n = 1, 3, 5$

and $\sqrt{\frac{2}{a}}$ is obtained after normalisation

$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$ ✓ (1) $n = 2, 4, 6$

∴ Ground state function $(n=1)$

$= \sqrt{\frac{2}{a}}\cos\left(\frac{\pi x}{a}\right)$ (+1)

1st excited state wave function $(n=2)$ is

$\sqrt{\frac{2}{a}}\sin\left(\frac{2\pi x}{a}\right)$ ✓

c) $$P = \int_{-a/4}^{a/4} \frac{2}{a} \sin\left(\frac{2\pi x}{a}\right) \sin\left(\frac{2\pi x}{a}\right) dx$$ ✓ (1)

$$= \frac{2}{a} \int_{-a/4}^{a/4} \frac{1}{2} \left(1 - \cos\left(\frac{4\pi x}{a}\right)\right) dx$$

$$= \frac{1}{a} \left[x - \frac{a}{4\pi} \sin\left(\frac{4\pi x}{a}\right)\right]_{-a/4}^{a/4}$$

$$= \frac{1}{2} \times \frac{a}{2} = \frac{1}{2} \text{ Ans}$$ ✓ +2

d) $$k = \sqrt{\frac{2mE}{\hbar^2}} \Rightarrow E = \frac{k^2\hbar^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

$$\therefore \Delta E = E_2 - E_1$$ (1)

$$= \frac{3}{2} \frac{\pi^2\hbar^2}{ma^2} = \frac{3}{8} \frac{\pi^2\hbar^2}{m_e a^2}$$ ✓

$$= 0.45 \times 10^{-19} \text{ J}$$

$$= 0.28 \text{ eV}$$ ✓ +2

6) a) For a We know that

$$\frac{p^2}{2m} = E$$

for particles having $p = 2\hbar k$

$$E = \frac{2\hbar^2 k^2}{m}$$

By time evaluation of wave function:

$$\psi(x,t) = \psi(x,0)\, e^{-\frac{i}{\hbar}\frac{2\hbar^2 k^2}{m}t}$$

$$\psi(x,t) = \sqrt{\frac{\alpha}{\pi}}\, e^{\frac{-\alpha x^2}{2}}\, e^{-\frac{i}{\hbar}\frac{2\hbar^2 k^2}{m}t}$$

Probability of function is independent of time

as $\psi^* \psi = \sqrt{\frac{\alpha}{\pi}}\, e^{\frac{-\alpha x^2}{2}}\, e^{\frac{-i}{\hbar}\frac{2\hbar^2 k^2}{m}t} \cdot \sqrt{\frac{\alpha}{\pi}}\, e^{\frac{-\alpha x^2}{2}}\, e^{\frac{i}{\hbar}\frac{2\hbar^2 k^2}{m}t}$

probability density $= \frac{\alpha}{\pi} e^{-\alpha x^2}$

b) $\langle E \rangle = \frac{\langle \psi | \hat{H} | \psi \rangle}{\langle \psi | \psi \rangle}$

$$= \frac{\int_{-\infty}^{\infty} \psi^* H \psi \, dx}{\int_{-\infty}^{\infty} \psi^* \psi \, dx} = \frac{\int_{-\infty}^{\infty} \sqrt{\frac{\alpha}{\pi}} e^{\frac{-\alpha x^2}{2}} \cdot \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} e^{\frac{-\alpha x^2}{2}} dx}{\int_{-\infty}^{\infty} \sqrt{\frac{\alpha}{\pi}} e^{\frac{-\alpha x^2}{2}} \sqrt{\frac{\alpha}{\pi}} e^{\frac{-\alpha x^2}{2}} dx}$$

$$= \int_{-\infty}^{\infty} \frac{\alpha}{\pi} e^{-\frac{\alpha x^2}{2}} \cdot -\frac{\hbar^2}{2m} \left(-\alpha + \alpha x^2\right) e^{\frac{-\alpha x^2}{2}} dx$$

$$\int_{-\infty}^{\infty} \frac{\alpha}{\pi} e^{-\alpha x^2} dx$$

$$= \frac{\int_{-\infty}^{\infty} e^{-\alpha x^2} \left(\frac{-\hbar^2}{2m}\right)\left(\alpha x^2 - \alpha\right) dx}{\int_{-\infty}^{\infty} e^{-\alpha x^2} dx}$$

$$= -\frac{\hbar^2}{2m} \frac{\left(\frac{\alpha}{2}\sqrt{\frac{\pi}{\alpha}} - \alpha\sqrt{\frac{\pi}{\alpha}}\right)}{\sqrt{\pi/\alpha}} = \frac{\hbar^2}{2m} \cdot \frac{\alpha}{2}$$

$$\langle E \rangle = \frac{\hbar^2 \alpha}{4m}$$

For value of α we can normalise ψ

$$\langle \psi | \psi \rangle = 1 = \int_{-\infty}^{\infty} \frac{\alpha}{\pi} e^{-\alpha x^2} dx$$

$$\Rightarrow \frac{\alpha}{\pi} \times \sqrt{\pi/\alpha} = 1$$

$$\Rightarrow \alpha = \pi$$

$$\therefore \langle E \rangle = \frac{\hbar^2 \pi}{4m}$$

Q 7)

a) i) $V_0 \to \infty$ for $x < 0$

$\therefore \psi(x) = 0$ for $x < 0$

For $x > 0$ for bound state soln

$-V_0 < E < 0$ is given by

$$\frac{d^2(\psi_1)}{dx^2} + k_1^2 \psi(x) = 0 \quad (0 < x < a) \quad - (1)$$

and $$\frac{d^2}{dx^2} \psi_2(x) - k_2^2 \psi_2(x) = 0 \quad (a < x) \quad - (2)$$

where $k_n^2 = \frac{2m(V_0 + E)}{\hbar^2}$ & $k_2^2 = \frac{-2mE}{\hbar^2}$

The soln of eq (1) is of the type

$$\psi_1(x) = A \sin k_1 x + B \cos k_1 x$$

Boundary condition

$$\psi_1(0) = 0 \Rightarrow B = 0$$

$$\psi_1(x) = A \sin k_1 x$$

for ψ_2

$D = 0$ since $e^{k_2 x}$ is unbounded

$$\psi_2 = C e^{-k_2 x}$$

$$\psi_{(x)} = \begin{cases} 0 & 0 > x \\ A \sin k_1 x & 0 < x < a \\ C e^{-k_2 x} & a < x \end{cases}$$

$$\psi_1(a) = \psi_2(a)$$

$$A \sin k_1 a = C e^{-k_2 a} \quad \text{— (3)}$$

$$\psi_1'(a) = \psi_2'(a)$$

$$A k_1 \cos k_1 a = -C k_2 e^{-k_2 a} \quad \text{— (4)}$$

$\frac{(4)}{(3)} \Rightarrow$ $k_1 \cot k_1 a = -k_2 a$

Also, $(k_1 a)^2 + (k_2 a)^2 = r_0^2$

$$r_0 = \frac{\sqrt{2mV}\, a}{\hbar}$$

x2

4) For case II $0 < E < V_2$

similarly we get

$$\psi_1(x) = A e^{k_1 x} \qquad (x < -a)$$

$$\psi_3(x) = D e^{-k_1 x} \qquad (x > a)$$

also $k_2 \cot(k_2 a) = -k_1$

and $k_2 \tan(k_2 a) = k_1$

+1

These eqn can be solved graphically

$-\alpha_n \cot \alpha_n = \sqrt{R^2 - \alpha_n^2}$ (for odd cases)

$\alpha_n \tan \alpha_n = \sqrt{R^2 - \alpha_n^2}$ (for even cases)

where $R^2 = \dfrac{2m a^2 V_0}{\hbar^2}$

We always get atleast 1 bound state no matter what for $V_0 > 0$