

COL215L: Digital Logic & System Design

Lecture 8: Combinational Circuits (Cont.)



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Challenges with K-Map

- K-Map
 - A graphical method and thus not suitable for large no. of variables
 - Not suitable for programming
- Quine-Mccluskey (QM) method
 - Does not suffer from these disadvantages

QM Method

- $y = f(a,b,c,d,e) = \Sigma m(0,6,7,8,14,20,22,24,26,30,31)$
- Step-1: Order by number of 1's in the binary representation



No. of 1's	Minterms
0	0
1	8
2	6, 20, 24
3	7, 14, 22, 26
4	30
5	31

QM Method (Cont.)

$$a'b'c'd'e' + a'b'c'd'e' = a'b'c'd'e' + a'b'c'd'e'$$

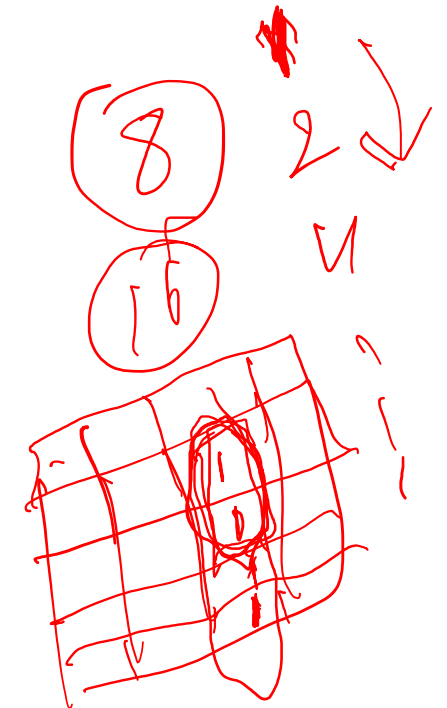
$$= a'b'c'd'e' + a'b'c'd'e'$$

- Step-2: Combine minterms considering following conditions
 - The integer values of the two minterms differ by 2^k for some $k \geq 0$.
 - The minterm with a larger integer value has one more 1's than the minterm with a smaller integer value.
- The combination of the minterms is called an implicant.

$$a'b'c'd'e' + a'b'c'd'e' = a'b'c'd'e'$$

$$(a' + a) \cdot c'd'e' = c'd'e'$$

0	0	(0,8)	
1	8	(8,24)	
2	6, 20, 24	(6,7), (6,14), (6,22), (20,22), (24,26)	(6,14,22,30), (6,22,14,30)
3	7, 14, 22, 26	(14,30), (22,30), (26,30)	
4	30	(30,31)	
5	31		



QM Method (Cont.)

- Step-3: Identify prime implicants
 - A prime implicant is not fully contained in any other implicant, i.e., no other implicant contains all the minterms contained in this implicant.

0	0	(0,8) ✓	
1	8	(8,24) ✓	
2	6, 20, 24	(6,7), ✓ (6,14), (6,22), (20,22), ✓ (24,26) ✓	(6,14,22,30), ✓ (6,22,14,30)
3	7, 14, 22, 26	(14,30), (22,30), (26,30) ✓	
4	30	(30,31) ✓	
5	31		

~~$c'd'e'$~~
 ~~$(8,24) c'd'e'$~~
 ~~$(6,22) a'b'cde'$~~
 ~~$a'b'cde'$~~
 $(12,28)$
 $a'b'c'd'e'$

QM Method (Cont.)

- Step-4: Prepare a cover table

	(0,8)	(6,7)	(6,14,22,30)	(8,24)	(20,22)	(24,26)	(26,30)	(30,31)
0	✓							
6		✓	✓					
7		✓						
8	✓			✓				
14			✓					
20					✓			
22			✓		✓			
24				✓		✓		
26						✓	✓	
30			✓				✓	✓
31								✓

$$y = a'c'd'e' + \dots$$

Essential Prime Implicants

- Step-5: Identify essential prime implicants
 - Essential prime implicants are those which cover minterm(s) not covered by any other prime implicant.
- Any minimal function have to include the essential prime implicants
- There can be some non-essential prime implicants

QM Method: Steps

1. Order minterms by number of 1's
2. Generate implicants by combination of minterms/implicants
3. Identify prime implicants
4. Prepare a cover table
5. Identify essential/other prime implicants covering all minterms
6. Express the function as a sum of identified prime implicants