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Q1) A and B are invertible i.e. $|A| \neq 0$ & $|B| \neq 0$ & also $|A^{-1}| = \frac{1}{|A|}$ and $|B^{-1}| = \frac{1}{|B|}$

$$A^{-1} B^{-1} A B = cI \quad \text{(given)}$$

taking determinant both sides

$$|A^{-1} B^{-1} A B| = |cI|$$

$$|A^{-1}||A||B^{-1}||B| = c^{n}|I| \quad (|\lambda M|_{n\times n} = \lambda^n \times |M|) \text{ (proved ahead)}$$

& Now $|A^{-1}||A| = 1$ & $|B^{-1}||B| = 1$

and $|I| = 1$

$\therefore$

$$c^{n} \times 1 = 1 \times 1$$

$$\boxed{c^{n} = 1}$$

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Q2) Let Matrix $A = \begin{bmatrix} 2a+4b & 2a+5b & 2a+6b \\ 2a+5b & 2a+6b & 2a+7b \\ 2a+6b & 2a+7b & 2a+8b \end{bmatrix}$

$$|A| = \begin{vmatrix} 2a+4b & 2a+5b & 2a+6b \\ 2a+5b & 2a+6b & 2a+7b \\ 2a+6b & 2a+7b & 2a+8b \end{vmatrix}$$

→ Now using Elementary ~~Row Transformation~~ Operation on matrices (which does not change the value of determinant)

→ Adding or subtracting scalar multiple of one row to another ~~does not change the~~ is an elementary operation

Now, $R_2 \longrightarrow R_2 - \left(\frac{R_1}{2} + \frac{R_3}{2}\right)$ gives

$$\begin{vmatrix} 2a+4b & 2a+5b & 2a+6b \\ 0 & 0 & 0 \\ 2a+6b & 2a+7b & 2a+8b \end{vmatrix}$$

The 2^{nd} Row of this matrix is a zero row, so the value of determinant $= 0$

Hence $|A| = 0$

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Q3) $W = \{(x, y, z) \in \mathbb{R}^3 \mid ax + by + cz = d\}$

Let $u, v \in W$, such that

$u = (x_1, y_1, z_1)$ and $v = (x_2, y_2, z_2)$

Now we know

$$ax_1 + by_1 + cz_1 = d \quad \text{--- (1)}$$

and

$$ax_2 + by_2 + cz_2 = d \quad \text{--- (2)}$$

Now for W to be a subspace of $\mathbb{R}^3(F)$

$\alpha u + \beta v \in W$ for all $\alpha, \beta \in F$

$\Rightarrow \alpha(x_1, y_1, z_1) + \beta(x_2, y_2, z_2)$

$\Rightarrow (\alpha x_1 + \beta x_1, \alpha y_1 + \beta y_2, \alpha z_1 + \beta z_2)$

For this to belond to W

we have

$a(\alpha x_1 + \beta x_2) + b(\alpha y_2 + \beta y_2) + c(\alpha z_1 + \beta z_2) = d$

$= \alpha(ax_1 + by_1 + cz_1) + \beta(ax_2 + by_2 + cz_2) = d$

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From (1) and (2) we know

$$\alpha d + \beta d = d$$

$$d(\alpha + \beta) = d$$

$$d(\alpha + \beta - 1) = 0$$

Either $d = 0$ or $\alpha + \beta = 1$

We know $\alpha + \beta$ will not always be 1 as they can be any value in F

$\therefore$ $\boxed{d = 0}$

Hence Proved

$$A \times \operatorname{adj} A = |A| I = \begin{vmatrix} |A| & 0 & 0 & \cdots & n \\ 0 & |A| & 0 & \cdots & n \\ \vdots & 0 & |A| & & \\ \vdots & & & |A| & \\ n & & & & |A| \cdots n \end{vmatrix}$$

taking determinant both sides: we get

$$|A \times \operatorname{adj} A| = ||A| I| = \begin{vmatrix} |A| & 0 & 0 & 0 \\ 0 & |A| & 0 & 0 \\ 0 & 0 & |A| & 0 \\ 0 & \cdot & \cdot & |A| \cdots n \end{vmatrix}$$

Let $|A| = \lambda$

$$|\lambda I| = \lambda^n$$ (by taking out λ from each row)