SET 2 A :

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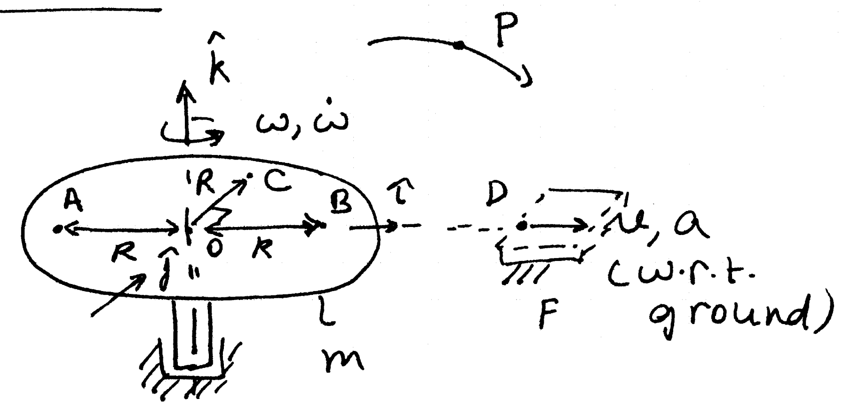

$\vec{AP} = \vec{AB} + \vec{BP}$

$\Rightarrow \vec{r}_{PA} = \vec{r}_{BA} + \vec{r}_{PB}$

a)

$\vec{v}_{P/m}$ (observer at A) $= \frac{d}{dt}(\vec{r}_{PA})/_m$

$\vec{v}_{P/m}$ (observer at B) $= \frac{d}{dt}(\vec{r}_{PB})/_m$.

$$\frac{d}{dt}(\vec{r}_{PA})/_m = \frac{d}{dt}(\vec{r}_{PB} + \vec{r}_{BA})/_m = \frac{d}{dt}(\vec{r}_{PB})/_m + 0$$

since $\vec{AB}$ is a fixed vector w.r.t. the platform

Hence the velocity of P seen by observer at A and the observer at B, with respect to the platform, are the same!

By the same argument

$$\frac{d^2}{dt^2}(\vec{r}_{PA})/_m = \frac{d^2(\vec{r}_{PB})}{dt^2}/_m$$

Hence the accelerations measured by the two observers A and B, with respect to the platform, are the SAME.

b) $\vec{v}_{D/F} = v\hat{\imath}$; $\vec{a}_{D/F} = a\hat{\imath}$.

If we choose A as the origin for m :

$\vec{r}_{DA} = (d+R)\hat{\imath}$. ; $\vec{v}_{A/F} = \vec{v}_{O/F} + \omega\hat{k} \times (-R\hat{\imath}) = -\omega R\hat{\jmath}$

$$\vec{v}_{D/F} = \vec{v}_{D/m} + \vec{v}_{A/F} + \omega\hat{k} \times \vec{r}_{DA} = \vec{v}_{D/m} - \omega R\hat{\jmath} + \omega(R+d)\hat{\jmath}$$

$\therefore \vec{v}_{D/m} = \vec{v}_{D/F} - \omega d \hat{j} = u \hat{i} - \omega d \hat{j}$. —— I

If we choose B as the origin for M:

$\vec{r}_{DB} = (d-R)\hat{i}$; $\vec{v}_{B/F} = \vec{v}_{O/F} + \omega \hat{k} \times (R\hat{i}) = \omega R \hat{j}$.

$\therefore \vec{v}_{D/F} = \vec{v}_{D/m} + \vec{v}_{B/F} + \omega \hat{k} \times \vec{r}_{DB} = \vec{v}_{D/m} + \omega d \hat{j}$

$\therefore \vec{v}_{D/m} = v \hat{i} - \omega d \hat{j}$ —— II

We see from I and II that we get the same result for $\vec{v}_{D/m}$, irrespective of the choice of origin.

IIIly

$\vec{a}_{D/F} = \vec{a}_{D/m} + \vec{a}_{A/F} + 2\omega \hat{k} \times \vec{v}_{D/m} + \dot{\omega} \hat{k} \times \vec{r}_{DA} - \omega^2 \vec{r}_{DA}$

$\vec{a}_{A/F} = -\dot{\omega} R \hat{j} + \omega^2 R \hat{i}$.

$$\therefore \vec{a}_{D/m} = a\hat{i} + \dot{\omega} R \hat{j} - \omega^2 R \hat{i} - 2\omega(u\hat{j} + \omega d \hat{i}) - \dot{\omega}(d+R)\hat{j} + \omega^2(R+d)\hat{i}$$
$$= a\hat{i} - 2\omega u \hat{j} - \dot{\omega} d \hat{j} - \omega^2 d \hat{i}$$
$$= (a - \omega^2 d)\hat{i} - (2\omega v + \dot{\omega} d)\hat{j}. \quad \text{—— III}$$

It can easily be verified that choosing B as the origin for m gives the same expression for $\vec{a}_{D/m}$ as III.

c) $\vec{v}_{BA/m} = \vec{v}_{B/m} - \vec{v}_{A/m} = 0$ since both A and B are fixed in m.

d) C is fixed in 'm' $\Rightarrow$ Any observer on m would see C at rest $\Rightarrow$ $\vec{a}_{C/m} = 0$; $\vec{v}_{C/m} = 0$.

Suppose OA, AB and cockpit represent frame 1, 2 and 3.

we have $\vec{\omega}_{1|F} = 0.4\hat{j}$, $\dot{\vec{\omega}}_{1|F} = -0.5\hat{j}$ [Because both work in opposite direction]

$\vec{\omega}_{2|1} = 0.2\hat{k}$, $\dot{\vec{\omega}}_{2|1} = 0.1\hat{k}$

$\vec{\omega}_{3|2} = 2\hat{i}$ $\dot{\vec{\omega}}_{3|2} = -0.3\hat{i}$

Using the composition of angular acceleration, we obtain

$$\vec{\omega}_{2|F} = \vec{\omega}_{2|1} + \vec{\omega}_{1|F} = 0.4\hat{j} + 0.2\hat{k}$$

$$\dot{\vec{\omega}}_{2|F}\big|_F = \dot{\vec{\omega}}_{1|F}\big|_F + \dot{\vec{\omega}}_{2|1}\big|_F = \dot{\vec{\omega}}_{1|F}\big|_F + \dot{\vec{\omega}}_{2|1}\big|_1 + \vec{\omega}_{1|F} \times \vec{\omega}_{2|1}$$

$$= -0.5\hat{j} + 0.1\hat{k} + 0.4\hat{j} \times 0.2\hat{k} = 0.08\hat{i} - 0.5\hat{j} + 0.1\hat{k}$$

$$\vec{\omega}_{3|F} = \vec{\omega}_{3|2} + \vec{\omega}_{2|F} = 2\hat{i} + 0.4\hat{j} + 0.2\hat{k}$$

$$\dot{\vec{\omega}}_{3|F} = \dot{\vec{\omega}}_{3|2}\big|_F + \dot{\vec{\omega}}_{2|F}\big|_F = \dot{\vec{\omega}}_{3|2}\big|_2 + \dot{\vec{\omega}}_{2|F} \times \vec{\omega}_{3|2} + \dot{\vec{\omega}}_{2|F}$$

$$= -0.3\hat{i} + (0.08\hat{i} - 0.5\hat{j} + 0.1\hat{k}) + (0.4\hat{j} + 0.2\hat{k}) \times 2\hat{i}$$

$$= -0.3\hat{i} + (0.08\hat{i} - 0.5\hat{j} + 0.1\hat{k}) + (-0.8\hat{k} + 0.4\hat{j})$$

$$= -0.22\hat{i} - 0.1\hat{j} - 0.7\hat{k}$$

Now we need to calculate acceleration of the point P w.r.t frame F.

First we calculate the acceleration of A w.r.to Frame F.

$$\vec{a}_{A|F} = \vec{a}_{O|F} + \vec{a}_{A|m} + 2\vec{\omega}_{m|F} \times \vec{v}_{A|m} + \dot{\vec{\omega}}_{m|F} \times \vec{OA} + \vec{\omega}_{m|F} \times (\vec{\omega}_{m|F} \times \vec{OA})$$

Here $m =$ frame 1.

[OA is a rigid body $\therefore \vec{a}_{O|F} = 0$, $\vec{a}_{A|m} = 0$, $\vec{v}_{A|m} = 0$

$$\vec{a}_{A|F} = \dot{\vec{\omega}}_{1|F} \times \vec{OA} + \vec{\omega}_{1|F} \times (\vec{\omega}_{1|F} \times \vec{OA})$$

$$= -0.5\hat{j} \times \hat{i} + 0.4\hat{j} \times (0.4\hat{j} \times \hat{i})$$

$$= 0.5\hat{k} + 0.4\hat{j} \times (-0.4\hat{k}) = 0.5\hat{k} - 0.16\hat{i}$$

Now, we calculate the acceleration at B w.r.t frame F

$$\vec{a}_{B|F} = \vec{a}_{A|F} + \vec{a}_{B|2} + 2\vec{\omega}_{2|F} \times \vec{v}_{B|2} + \dot{\vec{\omega}}_{2|F} \times \vec{AB} + \vec{\omega}_{2|F} \times (\vec{\omega}_{2|F} \times \vec{AB})$$

$$= (-0.16\hat{i} + 0.5\hat{k}) + 0.1\hat{i} + 2(0.4\hat{j} + 0.2\hat{k}) \times 0.2\hat{i} + (0.08\hat{i} - 0.5\hat{j} + 0.1\hat{k}) \times 4\hat{i}$$

$$+ (0.4\hat{j} + 0.2\hat{k}) \times [(0.4\hat{j} + 0.2\hat{k}) \times 4\hat{i}]$$

$$= -0.06\hat{i} + 0.5\hat{k} + (-0.16\hat{k} + 0.08\hat{j}) + (2\hat{k} + 0.4\hat{j}) + (0.4\hat{j} + 0.2\hat{k}) \times (-1.6\hat{k} + 0.8\hat{j})$$

$$= -0.06\hat{i} + 0.5\hat{k} + (-0.16\hat{k} + 0.08\hat{j}) + (2\hat{k} + 0.4\hat{j}) + (-0.64\hat{i} - 0.16\hat{i})$$

$$= -0.86\hat{i} + 0.48\hat{j} + 2.34\hat{k}$$

$\therefore$ The resulting acceleration at P w.r.t frame F

$$\vec{a}_{P|F} = \vec{a}_{B|F} + \vec{a}_{P|3} + 2\vec{\omega}_{3|F} \times \vec{v}_{P|3} + \dot{\vec{\omega}}_{3|F} \times \vec{BP} + \vec{\omega}_{3|F} \times (\vec{\omega}_{3|F} \times \vec{BP})$$

where $\vec{BP} = (1.5\hat{i} + 0.8\hat{j})$

$$\vec{\dot{\omega}}_{3|F} \times \vec{BP} = (-0.22\hat{i} - 0.1\hat{j} - 0.7\hat{k}) \times (1.5\hat{i} + 0.8\hat{j})$$

$$= (0.56\hat{i} - 1.05\hat{j} - 0.026\hat{k})$$

$$\vec{\omega}_{3|F} \times (\vec{\omega}_{3|F} \times \vec{BP}) = (2\hat{i} + 0.4\hat{j} + 0.2\hat{k}) \times [(2\hat{i} + 0.4\hat{j} + 0.2\hat{k}) \times (1.5\hat{i} + 0.8\hat{j})]$$

$$= 0.34\hat{i} - 2.032\hat{j} + 0.664\hat{k}$$

$$\therefore \vec{a}_{P|F} = (-0.86\hat{i} + 0.48\hat{j} + 2.34\hat{k}) + (0.56\hat{i} - 1.05\hat{j} - 0.026\hat{k})$$

$$+ (0.34\hat{i} - 2.032\hat{j} + 0.664\hat{k})$$

$$= (0.04\hat{i} - 2.602\hat{j} + 2.978\hat{k})\ m/s^2$$

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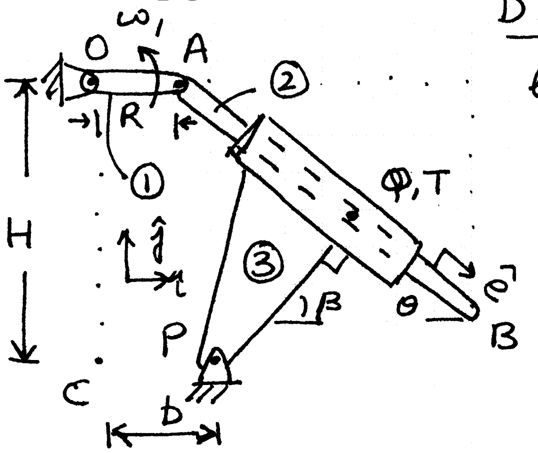


Data:

$l(AB) = L = 20\,cm$; $H = 14\,cm$

$b = 6\,cm$; $R = 4\,cm$; $\omega_1 = -2\,rad/s$

$\cos\theta = 4/5 = 0.8$; $\sin\theta = 0.6$

let: $l(A\varphi) = l_1$; $l(PT) = l_2$

$\beta = 90° - \theta \Rightarrow \cos\beta = 0.6$; $\sin\beta = 0.8$

$\hat{e} = \cos\theta\,\hat{i} - \sin\theta\,\hat{j}$.

We can easily determine l_1 and l_2 using:

a) $H = l_1\sin\theta + l_2\sin\beta$; b) $R + l_1\cos\theta = b + l_2\cos\beta$. $\Rightarrow l_1 = l_2 = 10\,cm$

Let $\vec{\omega}_1$, $\vec{\omega}_2$ and $\vec{\omega}_3$ be the angular velocities of the 3 bodies w.r.t. the ground frame, respectively and let the angular accelerations be $\dot{\vec{\omega}}_1$, $\dot{\vec{\omega}}_2$ and $\dot{\vec{\omega}}_3$.

$\vec{\omega}_1 = \omega_1\hat{k} = -2\,rad/s\,\hat{k}$; $\vec{\omega}_2 = \omega_2\hat{k}$ and $\vec{\omega}_3 = \omega_3\hat{k}$

(Since the motion is in the x-y plane all the $\vec{\omega}$ and $\dot{\vec{\omega}}$ vectors are along $\hat{k}$.)

Similarly $\dot{\vec{\omega}}_1 = 0$ (given) ; $\dot{\vec{\omega}}_2 = \dot{\omega}_2\hat{k}$; $\dot{\vec{\omega}}_3 = \dot{\omega}_3\hat{k}$.

[To obtain $\vec{\omega}_2$ and $\vec{\omega}_3$ we need to set up equations for the velocity at some point in 2 different ways and equate them.].

O and A lie on body (1) ; $\vec{V}_{O/G} \equiv \vec{V}_O = 0$.

$\vec{V}_A = \vec{V}_O + \omega_1\hat{k}\times\vec{r}_{AO} = \omega_1 R\,\hat{j}$.

A and φ lie on body (2)

$\vec{V}_\varphi = \vec{V}_A + \omega_2\hat{k}\times\vec{r}_{\varphi A} = \omega_1 R\hat{j} + \omega_2\hat{k}\times(l_1\,\hat{e})$

$= \omega_1 R\hat{j} + \omega_2 l_1\cos\theta\,\hat{j} + \omega_2 l_1\sin\theta\,\hat{i}$.

P and T lie on body (3) ; $\vec{V}_P = 0$

$\therefore \vec{V}_T = 0 + \omega_3\hat{k}\times\vec{r}_{TP} = -\omega_3 l_2\sin\beta\,\hat{i} + \omega_3 l_2\cos\beta\,\hat{j}$.

Now, T and Q are coincident but body 2 slides in body 3

$\Rightarrow \vec{V}_T = \vec{V}_Q + \dot{s}\hat{e}$ where $\dot{s}$ is the sliding speed.

$$\therefore\ -\omega_3 l_2 \sin\beta\,\hat{\imath} + \omega_3 l_2 \cos\beta\,\hat{\jmath} = (\omega_1 R + \omega_2 l_1 \cos\theta)\hat{\jmath} + \omega_2 l_1 \sin\theta\,\hat{\imath} + \dot{s}(\cos\theta\,\hat{\imath} - \sin\theta\,\hat{\jmath}) \quad \text{—(I)}$$

This is a vector equation $\Rightarrow$ we can generate 2 scalar equations however, we have 3 scalar unknowns. ω_2, ω_3 and $\dot{s}$.

BUT, since body 2 can only have a sliding (translating) motion relative to body 3 there can be no relative rotation between body 2 and body 3

$\Rightarrow \vec{\omega}_2 = \vec{\omega}_3$; $\dot{\vec{\omega}}_2 = \dot{\vec{\omega}}_3$!

$\Rightarrow$ (I) reduces to :

$$-\omega_2 l_2 \sin\beta\,\hat{\imath} + \omega_2 l_2 \cos\beta\,\hat{\jmath} = (\omega_2 l_1 \sin\theta + \dot{s}\cos\theta)\hat{\imath} + (\omega_1 R + \omega_2 l_1 \cos\theta - \dot{s}\sin\theta)\hat{\jmath}.$$

$\rightarrow$ solving we have

$\dot{s} = -1.75\, l_2 \omega_2$; $\omega_2 = -\omega_1 R/(1.25 l_2) = 0.64$ rad/s

$$\vec{V}_B = \vec{V}_A + \omega_2 \hat{k} \times \vec{r}_{BA} = (\omega_1 R + \omega_2 L\cos\theta)\hat{\jmath} + \omega_2 L \sin\theta\,\hat{\imath}.$$

Similarly for the accelerations:

A and O lie on body 1

$$\Rightarrow \vec{a}_A = \vec{a}_O + \dot{\omega}_1 \hat{k} \times \vec{r}_{AO} - \omega_1^2 \vec{r}_{AO} = -\omega_1^2 R\,\hat{\imath}$$

A and Q lie on body 2

$$\Rightarrow \vec{a}_Q = \vec{a}_A + \dot{\omega}_2 \hat{k} \times \vec{r}_{QA} - \omega_2^2 \vec{r}_{QA}$$
$$= -\omega_1^2 R\,\hat{\imath} + \dot{\omega}_2 l_1 (\cos\theta\,\hat{\jmath} + \sin\theta\,\hat{\imath}) - \omega_2^2 l_1(\cos\theta\,\hat{\imath} - \sin\theta\,\hat{\jmath})$$

P and T lie on body 3

$$\Rightarrow \vec{a}_T = \underbrace{\cancel{\vec{a}_P}}_{0} + \dot{\omega}_2 l_2 (\cos\beta\,\hat{\jmath} - \sin\beta\,\hat{\imath}) - \omega_2^2 l_2 (\cos\beta\,\hat{\imath} + \sin\beta\,\hat{\jmath})$$

Again $\vec{a}_T = \vec{a}_Q + \ddot{s}\,\hat{e}$

$$\therefore -(\omega_2^2 l_2 \cos\beta + \dot{\omega}_2 l_2 \sin\beta)\hat{i} + (\dot{\omega}_2 l_2 \cos\beta - \omega_2^2 l_2 \sin\beta)\hat{j}$$

$$= -(\omega_1^2 R + \omega_2^2 l_1 \cos\theta - \dot{\omega}_2 l_1 \sin\theta)\hat{i}$$

$$+ (\dot{\omega}_2 l_1 \cos\theta + \omega_2^2 l_1 \sin\theta)\hat{j} + \ddot{s}(\cos\theta\,\hat{i} - \sin\theta\,\hat{j})$$

This equation (vector) provides 2 scalar equations that can be solved to yield $\dot{\omega}_2$ and $\ddot{s}$.

$\dot{\omega}_2 = 1.98$ rad/s^2

$$\vec{a}_B = \vec{a}_A + \dot{\omega}_2 k \times \vec{r}_{BA} - \omega_2^2 \vec{r}_{BA}$$

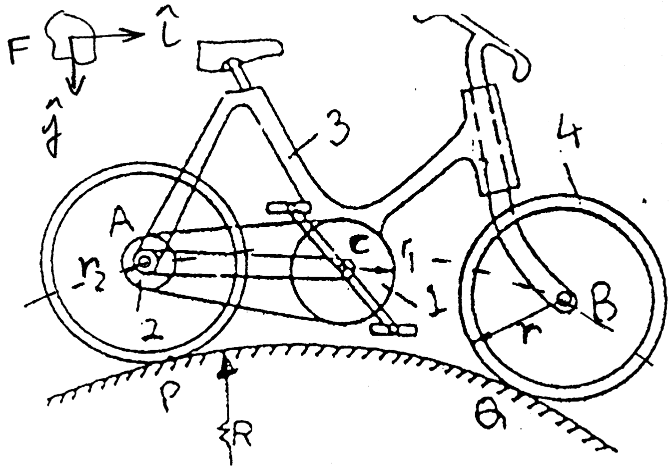
$$= -\omega_1^2 R\,\hat{i} + \dot{\omega}_2 L(\cos\theta\,\hat{j} + \sin\theta\,\hat{i}) - \omega_2^2 L(\cos\theta\,\hat{i} - \sin\theta\,\hat{j})$$

• ——— x ——— y

Body 1 Front sprocket attached to the pedals

Body 2 Rear wheel with rear sprocket

Body 3 Bicycle frame



Motion in the x-y plane

$\Rightarrow$ All $\vec{\omega}$ vectors are along $\hat{k}$.

$\vec{\omega}_1 = \omega \hat{k}$.

Since the chain is inextensible we can say that ~~the velocity~~ the two ends of the straight portion of the chain must have the same tangential velocity component.

If the cycle were ~~two~~ moving on a level road the ~~velocities~~ centres of the two sprockets would have the same velocity and we could simply write

$$\omega r_1 = \omega_2 r_2 \quad \Rightarrow \quad \omega_2 = \omega r_1 / r_2 .$$

However, since the cycle is going over a cylindrical mound, the sprocket centres have different velocities

$\Rightarrow$ This must be taken into account when calculating the velocity of the chain end using the inextensible criterion. $\rightarrow$ The algebra here may be quite tedious. There is another view wherein the algebra is simpler.

Consider the reference frame of the cycle (body 3). In this frame the centres of the two sprockets are at rest.

The inextensibility criterion then simply becomes

$$\omega_{1/3}\, r_1 = \omega_{2/3}\, r_2$$

$$\Rightarrow (\omega_1 - \omega_3)\, r_1 = (\omega_2 - \omega_3)\, r_2 . \qquad — (I)$$

To solve this we need an equation relating ω_3 with either ω_1 or ω_2.
Consider the centre of the wheel A. Let its speed relative to the ground be $\dot{s}_A$.

i.e. $\dot{s}_A = \omega_2 r$ since there is no slip at P.

The path taken by A is a circle with radius $R+r$.

$$\therefore \omega_3 = \dot{\phi}_A = \frac{\dot{s}_A}{R+r} \Rightarrow \omega_3 = \frac{\omega_2 r}{R+r} \qquad — II.$$

Substituting II in I we have

$$\left(\omega_1 - \frac{\omega_2 r}{R+r}\right) r_1 = \left(\omega_2 - \frac{\omega_2 r}{R+r}\right) r_2 \qquad (\omega_1 = \omega)$$

Solving :

$$\omega_2 = \frac{\omega\, r_1\, (R+r)}{(r_2(R+r) + r(r_1 - r_2))}$$