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Pg-301

3 (a) ~~$f_{b}(x)$~~.

first generation.

$$f(x) = x$$

$$4x - 9x^2 = x$$

$x = 0$ ← we wont take trivial solution

$$x = \frac{3}{9} = \frac{1}{3} = x^{(1)}$$

$$\boxed{x^{(1)} = \frac{1}{3}}$$

for stability of ~~period~~ point ①.

$$\left| f'(x) \right|_{x=x^{(1)}} < 1$$

$$f'(x) = 4 - 18x$$

$$f'(x)\Big|_{x=x^{(1)}} = 4 - 18\left(\frac{1}{3}\right)$$

$$\left| f'(x) \right| = \left| 4 - 6 \right|$$

$$= 2 > 1$$

So, $x^{(1)}$ is unstable here.

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Pg → 3.2

(3) Map of dynamical system

$$f(x) = 4x - 9x^2$$

(b) $f^{(2)}(x) = f(f(x)) = x$

$$(4x - 9x^2)(4 - 9(4 - 9x^2)) = x$$

$$16 - 16\times 9\,x + 81\times 4\,x^2 - 36\,x + 16\times 9\,x^2 - 9^3 x^3 - 1 = 0$$

$x = \frac{4-1}{9} = \frac{1}{3}$ will also root of above equation.

$$x^{(1)} + x^{(2)} + x^{(3)} = \frac{2(4)}{9} = \frac{8}{9}$$

$$x^{(2)} + x^{(3)} = \frac{4+1}{9} = 5/9$$

$$x^{(1)}\,x^{(2)}\,x^{(3)} = \frac{4^2-1}{9^3} = \frac{15}{729}$$

$$x^{(2)}\,x^{(3)} = \frac{5}{81}$$

$x^{(2)}$ & $x^{(3)}$ are root of

$$x^2 - \left(\frac{5}{9}\right)x + \left(\frac{5}{81}\right) = 0.$$

$$x = \frac{5 \pm \sqrt{5}}{18}$$

$x = \frac{5+\sqrt{15}}{18}$, $\frac{5-\sqrt{5}}{18}$, $\frac{1}{3}$ are only fixed points for stability.

$$\frac{d}{dx} f^2(x) = \frac{d}{dx}\left(f(f(x))\right)$$

$$= f'(f(x))\, f'(x)$$

and we know that $f(x) = x$ at fixed points

So,

$$\frac{d}{dx} f^2(x) = f'(x) \cdot f'(x)$$

$$= \left(f'(x)\right)^2$$

$$f'(x) = \frac{d}{dx}\left(4x - 9x^2\right)$$

$$= 4 - 18x$$

$$\frac{d}{dx} f^2(x) = (4 - 18x)^2$$

at $x = \frac{5 + \sqrt{85}}{18}$, $\frac{d}{dx} f^2(x) = \left(4 - \frac{\cancel{18}(5 + \sqrt{85})}{\cancel{18}}\right)^2$

$$= (-1 - \sqrt{85})^2 > 1$$

Not stable.

$x = \frac{5 - \sqrt{85}}{18}$, $\frac{d}{dx} f^2(x) = \left(4 - \frac{18(5 - \sqrt{85})}{18}\right)^2$

$$= (-1 + \sqrt{5})^2 > 1 \rightarrow \text{Not stable}$$

$x = \frac{1}{3}$, $\frac{d}{dx} f^2(x) \Rightarrow \left(4 - \frac{18}{3}\right)^2 \Rightarrow 4 > 1$

Not stable

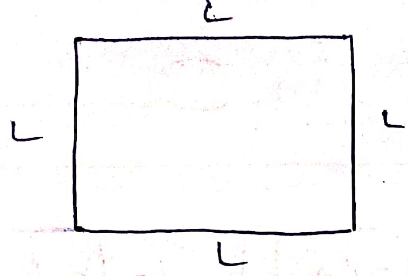
(2) <u>Fractals :-</u>

fractals are sets which appears to have complex structure and it can splits into several parts and each part is similiar to the original fractal (basically a reduced form of original fractal). Each part has some statistical appearance as the whole curve. fractals can be of infinite sets.

<u>fractals dimension</u> → It is a ratio that tells about index of complexity of fractals patterns as they change with the scale at which it is measured.

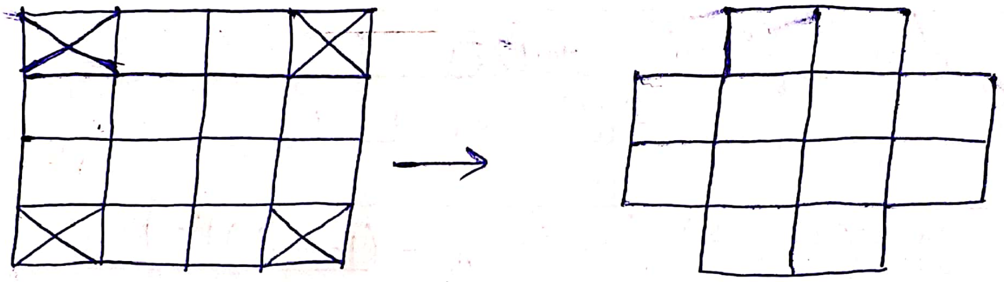
P.T.O

(a) Take square, divide in 16 part, remove the corner one.



$N = 1$
$E = L$

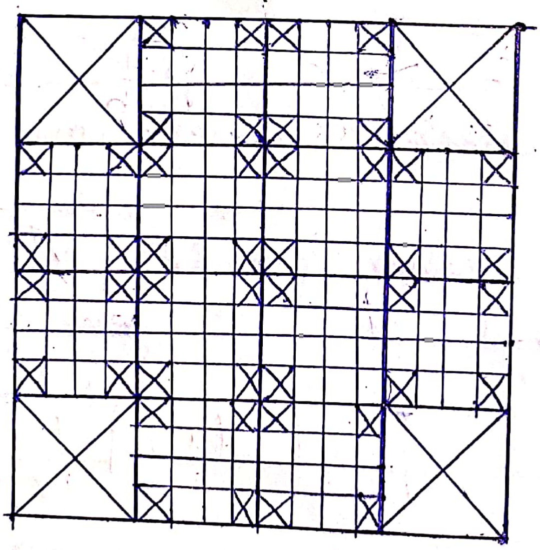
Divide in equal part and then remove the corner one



$N = 12$
$E = \frac{L}{4}$

2nd step.

At 3rd step we have after removing all corners 12 new squares are received



$N = 12^2$

$E = \frac{L}{4^2}$

Hence we have seen that from each of 12 squares obtained from them we again get 12 new squares.

for n step, $N = 12^n$
$E = \frac{L}{4^n}$

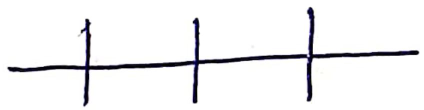
fractral dimension is

$$= \lim_{E \to 0} \left(\frac{\log N}{\log\left(\frac{L}{E}\right)} \right)$$

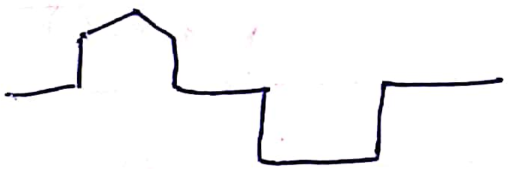
$$= \frac{\log 12^n}{\log 4^n} \Rightarrow \frac{\log 12}{\log 4} \approx \boxed{1.8}$$

$$\boxed{d_c = 1.8}$$

(b) $N = 1$, $E = L$

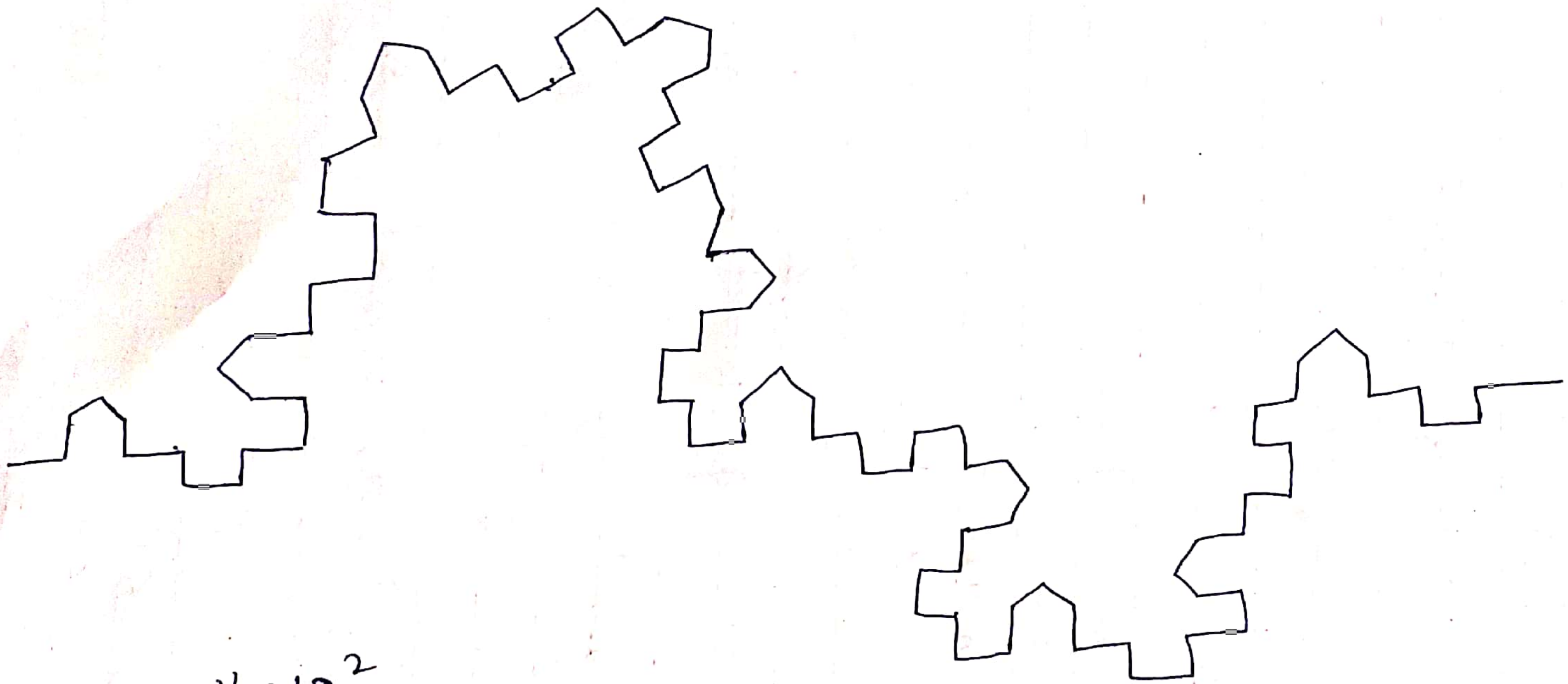


divide into 5 parts & construct.



$N = 10$, $E = 4/5$

P.T.O



$N = 10^2$

$\epsilon = \frac{L}{5^2}$

So, ~~we get~~ after n steps, $N = 10^n$
$\epsilon = L/5^n$

$$d_c = \lim_{\epsilon \to 0} \left(\frac{\log N}{\log L/\epsilon} \right) = \frac{\log 10^n}{\log 5^n} = \frac{\log 10}{\log 5} \simeq 1.43$$

$$\boxed{d_c = 1.43}$$

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Q (1)

(a) If $x^{(1)}$, $x^{(2)}$ are fixed points of first generation map $f_b(x)$, then they will be the fixed points of higher generation i.e $f_b^{(2)}(x)$, $f_b^{(3)}(x)$ — $f_b^{(n)}x$.

eg logistic map equation
$$x_{n+1} = r x_n (1 - x_n)$$
for first generation.
$$r x_n (1 - x_n) = x_n \quad , x_n \neq 0$$
$$r(1-x) = 1.$$

for second generation.
$$r^2 x_n (1-x_n)\left[1 - (r x_n (1-x_n))\right] = x_n, \quad x_n \neq 0.$$
we will see that $x = 1 - \frac{1}{r}$ will statisfies this equation.

further for third generation.
$x = 1 - \frac{1}{r}$ will also satisfies this
So this will be a fixed point.

Hence proved that any fixed point of previous generation will be fixed point of further generation.

fig → 1.2.

(b)

As proved in part (a) that any fixed point of previous generation will be fixed point of further generation.

fixed point of first generation → $x^{(1)}, x^{(2)}$

Exclusive fixed point of second generation → $x^{(3)}, x^{(4)}$

So, fixed point of second will be $x^{(1)}, x^{(2)}, x^{(3)}, x^{(4)}$

fixed point of $F_b(x) \Rightarrow x^{(1)}, x^{(2)}$

One attractor system

$$x_{n+1} = b x_n (1 - x_n^2)$$

Range of b for which period one attractor

$$|f_b'(x^{(1)})| < 1 \qquad x^{(1)} = \sqrt{1 - \frac{1}{b}}$$

$$f_b'(x) = b(1 - 3x^2)$$

$$f_b'(x^{(1)}) = b\left(1 - 3\left(\sqrt{1 - \frac{1}{b}}\right)^2\right)$$

$$f_b'(x^{(1)}) = b\left(1 - 3\left(1 - \frac{1}{b}\right)\right)$$

$$= -2b + 3$$

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$$\left| f_b'(x^{(1)}) \right| < 1 \Rightarrow |-2b+3| < 1$$

$$2 > b > 1$$

So, $\boxed{1 < b < 2}$

for $b \in (1,2)$ period one attractor would be there

for period two attractor system.
On solving
we get $x^{(1)} = \sqrt{1 - \frac{1}{b}}$

So, $\boxed{1 < b < 2}$ for stability of $x^{(1)}$

& now for $x^{(2)}$ & $x^{(3)}$

$$f_b^{(2)'}(x)\Big|_{x^{(2)}} = \left[f_b(f_b(x)) \right]'\Big|_{x = x^{(2)}}$$

$$= \left[f_b'(f_b(x)) \cdot f_b'(x) \right]\Big|_{x = x^{(2)}}$$

$$f_b(x)\Big|_{x = x^{(2)}} = x^{(3)}$$

$$f_b^{(2)'}(x)\Big|_{x^{(2)}} = f_b'(x^{(2)})\, f_b'(x^{(3)})$$

for stability of $x^{(2)}$ & $x^{(3)}$

$$\boxed{\left| f_b'(x^{(2)})\, f_b'(x^{(3)}) \right| < 1}$$

(c) $x^{(3)}, x^{(4)} \rightarrow$ fixed point of second generation

Yes they will be fixed point of $f_b^{(3)}, f_b^{(4)}$ & $f_b^{(5)}$. As proved in part (a) ~~the~~ Every fixed point of previous generation will be ~~the~~ fixed point of further one.

Logistic map is designed in this way that higher order generation have fixed point that lower order have.

Logistic map,

$$x_{n+1} = bx_n(1-x_n)$$

Hence proved.