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CON101: Assignment 4b
Optimum Payoff Strategy

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1 Strategy

We are given that X_1 is uniform in $\{1, 4, 16, 64\}$, X_2 is uniform in $\{1, 3, 9, 27\}$ and X_3 is uniform in $\{1, 2, 4, 8\}$. The expected values of the random variables taken from each of the boxes is $E(X_1) = 21.25$, $E(X_2) = 10$ and $E(X_3) = 3.75$. The strategy we should follow to maximize our expected payoff is as follows:

1. First, pick the box X_1.
2. Note that if we reject X_1 we will be forced to pick both X_2 and X_3. Thus we should only reject X_1 if $X_1 < \min(E(X_2), E(X_3)) = 3.75$.
3. In case we reject X_1 then the only choice as stated above is to choose both X_2 and X_3.
4. If we accept X_1, then the strategy for choosing X_2 is similar. If $X_2 < 3.75$, which is the expected value from the 3rd box, we reject it and accept whatever we get from the third box. Else we choose X_2.

2 Expected Payoff

As described in the above strategy, we can compute the payoff as follows:

- If $X_1 < 3.75$, which occurs with probability $\frac{1}{4}$, then the expected sum is $E(X_2) + E(X_3) = 13.75$.
- On the other hand, if $X_1 > 3.75$ which occurs with probability $\frac{3}{4}$, $E(X_1/X_1 > 3.75) = 28$. Now two subcases:
  - For X_2, if $X_2 < 3.75$ which can occur with probability $\frac{1}{2}$ then we reject it. Then the expected sum is $28 + 3.75 = 31.75$.
  - Whereas, if $X_2 > 3.75$ which can occur with probability $\frac{1}{2}$, we accept it. In that case $E(X_2/X_2 > 3.75) = 18$. The expected sum is 46.

Therefore, the expected value of the sum (say X) can be computed as:

$$E(X) = \frac{1}{4} * 13.75 + \frac{3}{4} * \left(\frac{1}{2} * 31.75 + \frac{1}{2} * 46\right) = 32.59$$