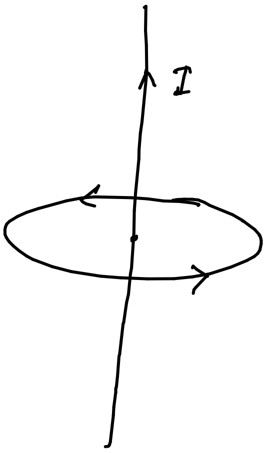
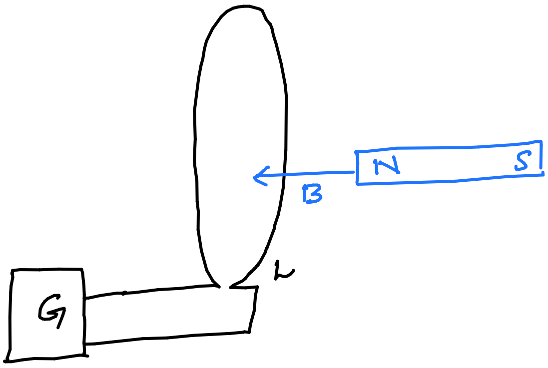




- A changing magnetic produces a current in the loop L. However, a steady magnetic field does not produce any current in the loop.

↓

"Electromagnetic induction"

↓

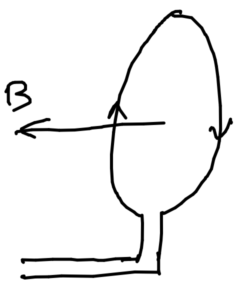
"induced current"

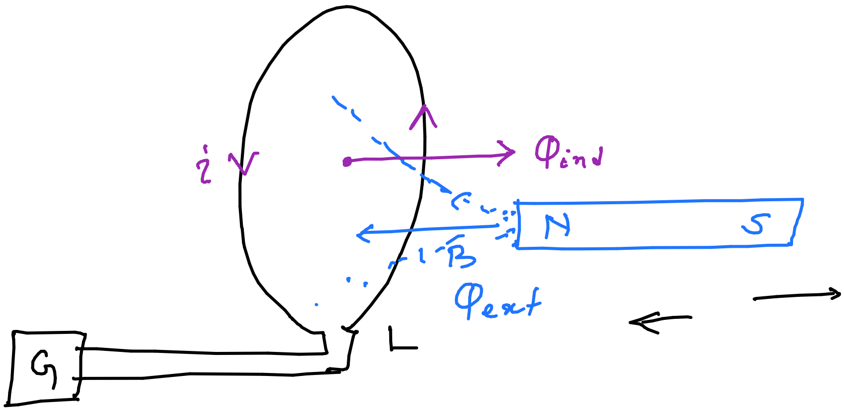
↓

induced voltage or
- induced electromotive force (emf)
- back emf

→ Lenz's law:

The flow of current in the loop will be in a direction s.t. the magnetic field produced by the induced current opposes the source magnetic field.





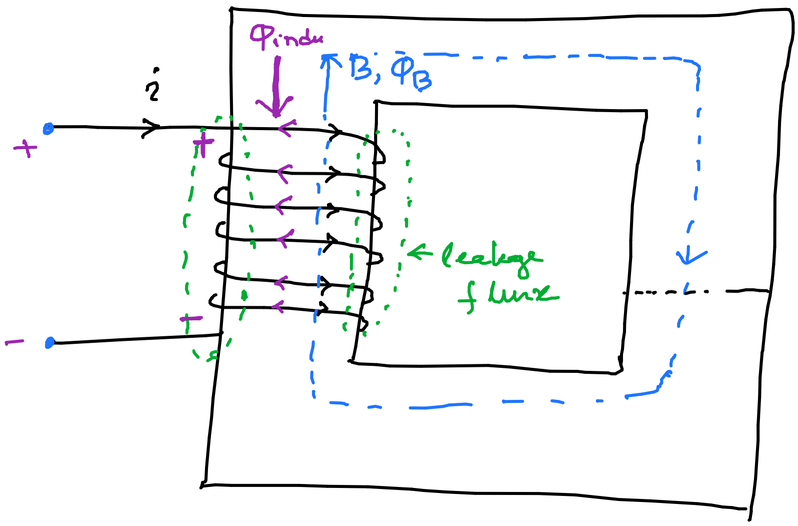
- Faraday law:

Emf induced in the loop

$$E_{emf} = -\frac{d\varphi_{ext}}{dt} \quad \text{(single turn coil)}$$

$$\boxed{E_{emf} = -N\frac{d\varphi_{ext}}{dt}} \qquad \varphi_B = \varphi_{ext}$$

N : number of turns in the coil.



Assumptions:

- B & H are linear
- There is no leakage flux ✓

A_c

$$✓ \quad \varphi_B = \frac{F}{\mathcal{R}_{total}} = \frac{Ni}{\mathcal{R}_c} \qquad \boxed{B = \mu_c H} ✓$$

↑ Assumption

- Varying current → φ_B varying with time.

-

A time varying mmf $\Rightarrow$ time varying $\varphi_B = \varphi_{ext}$

$\Downarrow$

Back emf ($\varphi_{induced}$)

$\varphi_{induced}$ is exactly opposite to φ_{ext}.

- Flux linkage $\psi = N\varphi_B$ Since $\varphi_B \to \varphi(t)$

$\psi(t) \to \psi(t)$

- Back emf $E_{emf} = -\dfrac{d\psi}{dt}$

$$\psi = N\varphi_B$$

$$= N\frac{\mathcal{F}}{\mathcal{R}_{total}}$$

$$\mathcal{R}_{total} = \mathcal{R}_c = \frac{l_c}{\mu_c A_c}$$

$$= N\frac{Ni}{l_c/\mu_c A_c}$$

$$= \frac{\mu_c N^2 A_c}{l_c} i$$

inductance of the coil.

L

$$\boxed{\psi = Li}$$

Under the assumption that μ_c is const.

$\Downarrow$

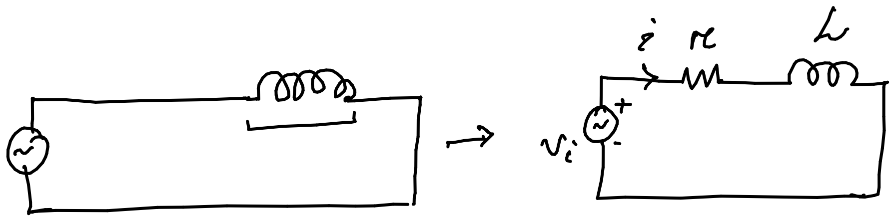
L is also const.

$$\boxed{L = \frac{\psi}{i}}$$

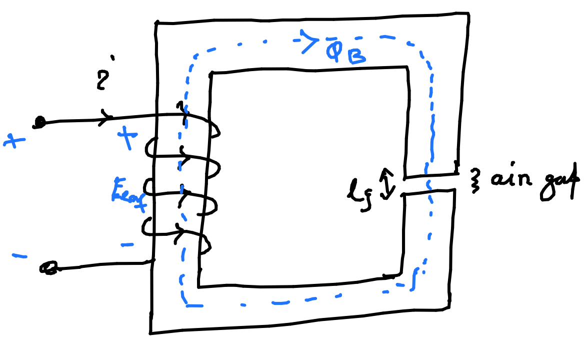
$$\boxed{L = \frac{N^2}{\mathcal{R}_{total}}}$$

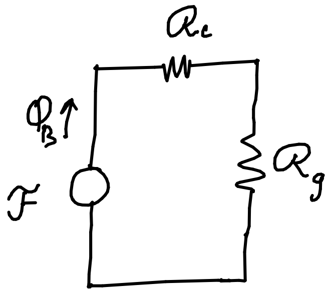
- Back emf $E_{emf} = -\frac{d\psi}{dt}$

$$\boxed{E_{emf} = -L\frac{di}{dt}}$$



$$v_i = ri + E_{emf}$$
$$= ri + L\frac{di}{dt}$$





$$\mathcal{F} = \phi_B(R_c + R_g)$$

$$\Rightarrow \phi_B = \frac{\mathcal{F}}{R_c + R_g}$$

$R_{total} = R_c + R_g$

$$R_c = \frac{l_c}{\mu_c A_c} \qquad R_g = \frac{l_g}{\mu_0 A_g}$$

$\mu_c >> \mu_0$

$\downarrow$

R_c can be neglected in comparison to R_g

Flux linkage $\psi = N\phi_B$

$$= N\frac{\mathcal{F}}{R_{total}}$$

$$= \frac{N^2}{R_c + R_g} i$$

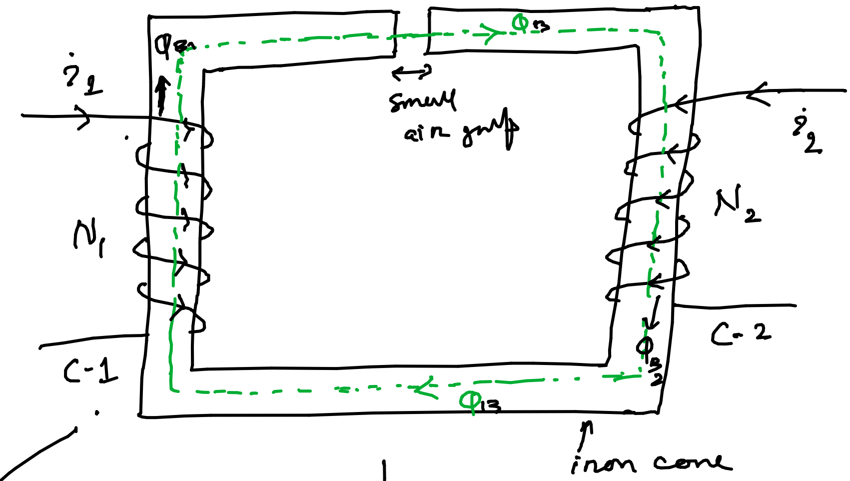
$\leftarrow$ inductance of coil L

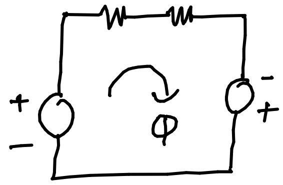
Neglecting R_c

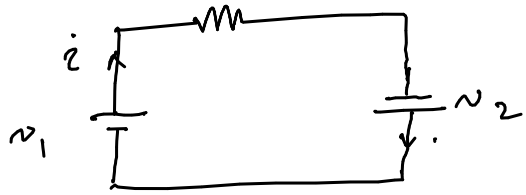
inductance is primarily due to air gap

$$L = \frac{N^2}{R_g} = \frac{\mu_0 N^2 A_g}{l_g}$$

Fig-1



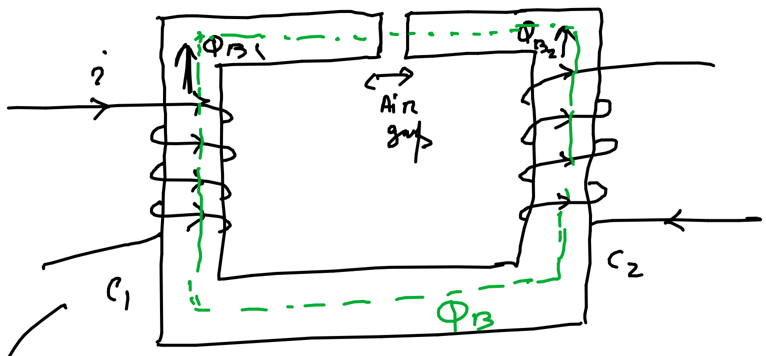


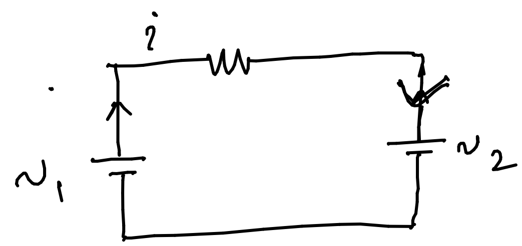


$$v_1 - iR + v_2 = 0$$

$$\Rightarrow \frac{v_1 + v_2}{R} = i$$

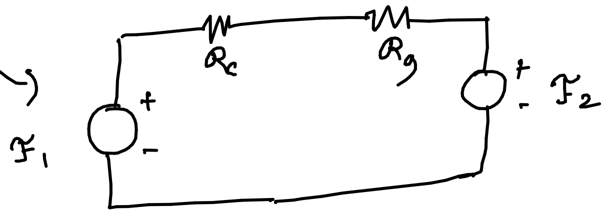
$$\Phi_B = \frac{\mathcal{F}_1 + \mathcal{F}_2}{\mathcal{R}_{total}}$$





$$v_1 - iR - v_2 = 0$$

$$\Rightarrow \frac{v_1 - v_2}{R} = i$$



$$\Phi_B = \frac{\mathcal{F}_1 - \mathcal{F}_2}{\mathcal{R}_{total}}$$

Consider - Fig - 1

$\mu_c >> \mu_0$

$$\mathcal{F}_t = \mathcal{F}_1 + \mathcal{F}_2$$

$$\varphi_B = \frac{\mathcal{F}_1 + \mathcal{F}_2}{\mathcal{R}_{total}} = \frac{\mathcal{F}_1 + \mathcal{F}_2}{\mathcal{R}_c + \mathcal{R}_g}$$

Since $\mu_c >> \mu_0$, neglect $\mathcal{R}_c$

$$\boxed{\varphi_B = (N_1 i_1 + N_2 i_2)\left(\frac{\mu_0 A_g}{l_g}\right)}$$

The resultant core flux due to the mmf produced by two coils.

→ The flux linkage in Coil-1 (C-1)

(Assume that the leakage flux are neglected)

⇓

$$\psi_1 = N_1 \varphi = N_1 \left[N_1 i_1 \left(\frac{\mu_0 A_g}{l_g}\right) + N_2 i_2 \left(\frac{\mu_0 A_g}{l_g}\right) \right]$$

Core Area $A_g = A_c$

$$= N_1^2 i_1 \left(\frac{\mu_0 A_g}{l_g}\right) + N_1 N_2 i_2 \left(\frac{\mu_0 A_g}{l_g}\right)$$

$$= L_{11} i_1 + L_{12} i$$

where

$$L_1 = \frac{\mu_0 N_1^2 A_g}{l_g} \qquad L_{12} = \frac{\mu_0 N_1 N_2 A_g}{l_g}$$

↓ self inductance of coil-1

↓ Mutual inductance between coil-1 & coil-2

$L_{11} i_1 \rightarrow$ Flux linkage of coil-1 due to its own mmf

$L_{12} i_2 \rightarrow$ Flux linkage of coil-1 due the flux produced by coil-2 (mmf)

Similarly

$\rightarrow$ The flux linkage in coil-2

$$\Psi_2 = N_2 \phi = N_2 \left[\quad \cdots \quad \right]$$

$$= \underbrace{N_1 N_2 \left(\frac{\mu_0 A_g}{l_g} \right)}_{L_{12}} i_1 + \underbrace{N_2^2 \left(\frac{\mu_0 A_g}{l_g} \right)}_{L_{22}} i_2$$

Mutual inductance between Coil-1 & Coil-2

Self-inductance of coil-2

$$\boxed{L_{12} = \frac{\mu_0 N_1 N_2 A_g}{l_g} \qquad L_{22} = \frac{\mu_0 N_2^2 A_g}{l_g}}$$