ELL100: INTRODUCTION TO ELECTRICAL ENGG.

Forced Response and Phasor Analysis

Course Instructors:
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Forced Response

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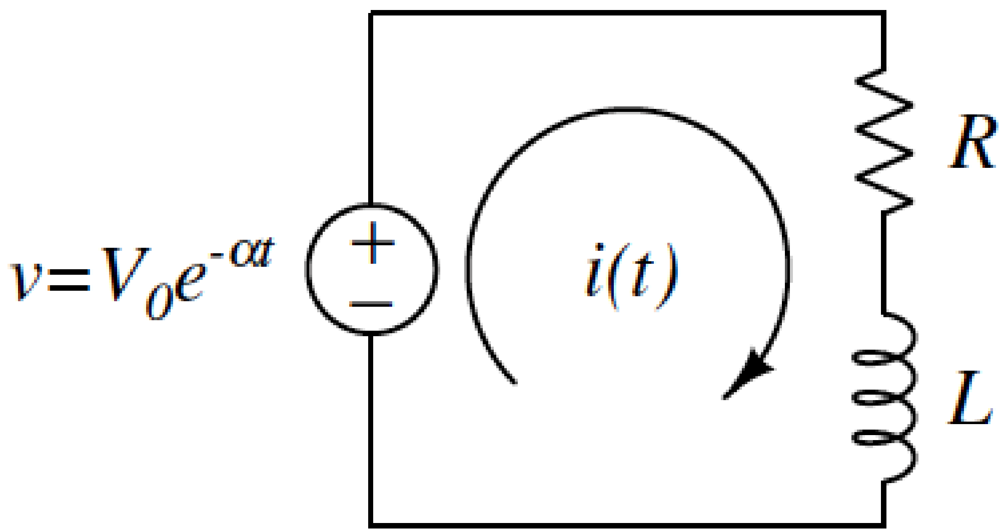
- Types of input worth exploring.
  - Decaying exponential signal (s= negative real)
  - DC (s=0)
  - AC (s=imaginary)

Impedance (Recap)

- For Resistor $Z_R = \frac{v}{i} = \frac{iR}{i} = R \ \Omega$
- For Inductor $Z_L = \frac{v}{i} = \frac{L\frac{di}{dt}}{i} = \frac{Lsi}{i} = sL \ \Omega$
- For Capacitor $Z_C = \frac{v}{i} = \frac{v}{C\frac{dv}{dt}} = \frac{v}{Csv} = \frac{1}{sC} \ \Omega$

Important Pointers

- The differential equation reads $Ri + L\frac{di}{dt} = V_0 e^{-\alpha t}$
- Equation is no longer homogenous



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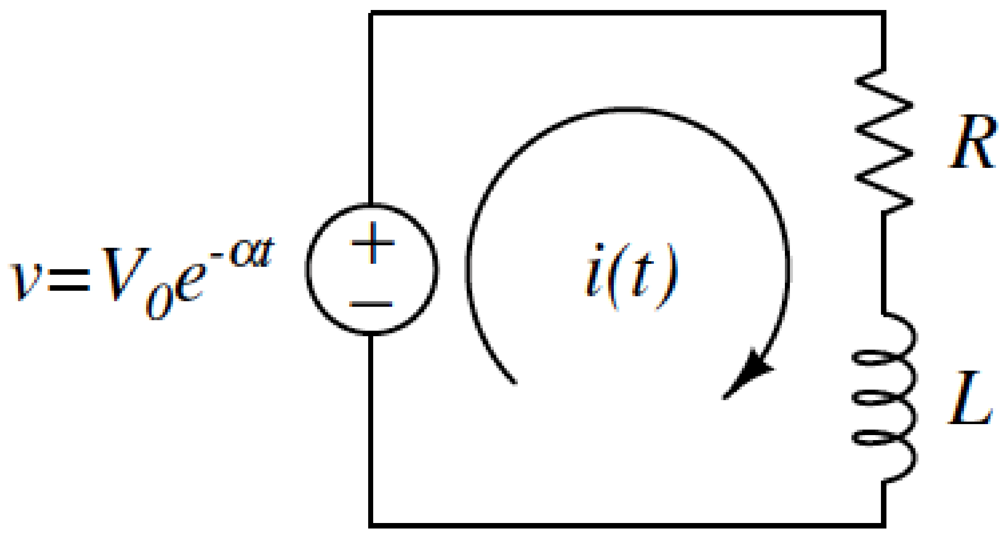

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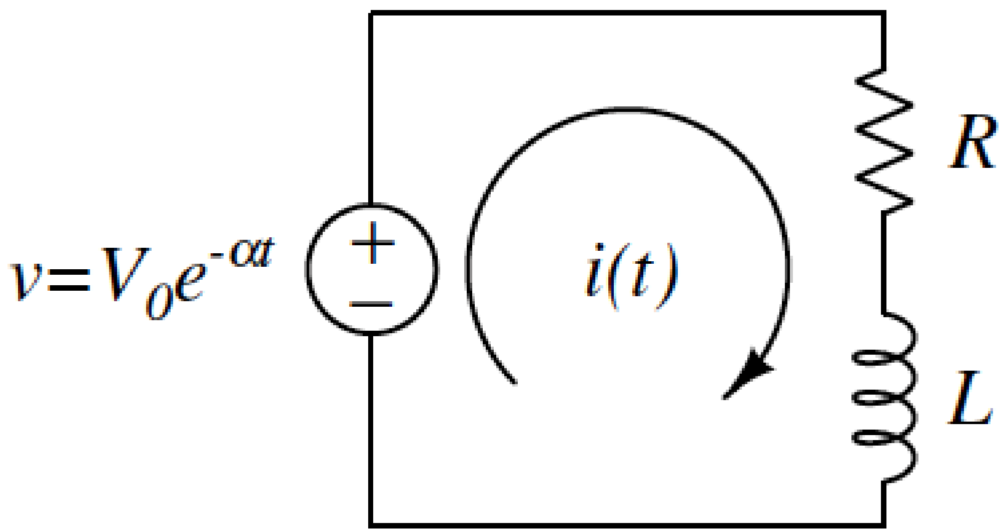
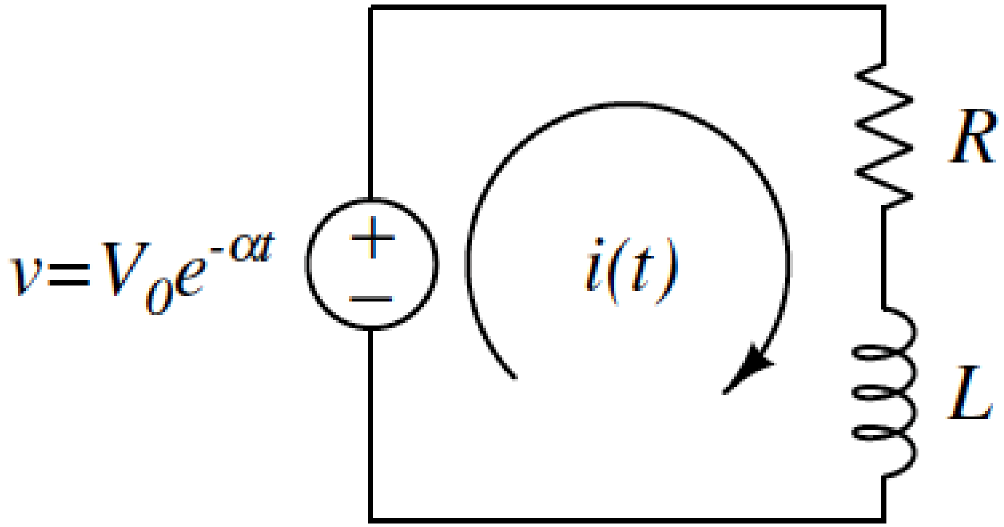
- Equation is no longer homogenous
- For forced response calculation initial conditions are **assumed** as zero.
- The forced response would be composed only of components present in the input ($e^{-\alpha t}$ here)



Forced Response - Exponentials

- Using the template solution $i(t) = Ae^{-\alpha t}$

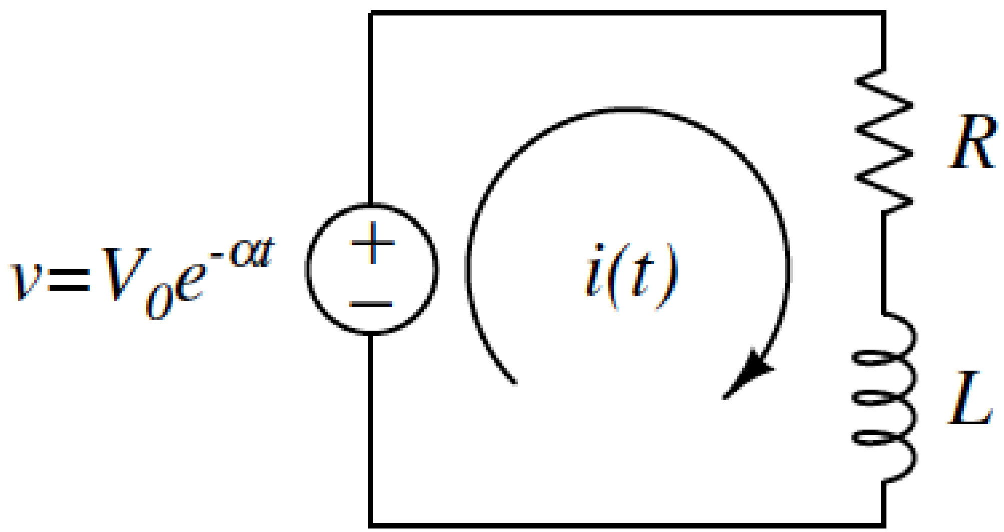


$$Ri + L\frac{di}{dt} = V_0 e^{-\alpha t}$$

Forced Response - Exponentials

- Using the template solution $i(t) = Ae^{-\alpha t}$
- We have

$$-L\alpha Ae^{-\alpha t} + RAe^{-\alpha t} = V_0 e^{-\alpha t}$$
$$A = \frac{V_0}{R - \alpha L}$$



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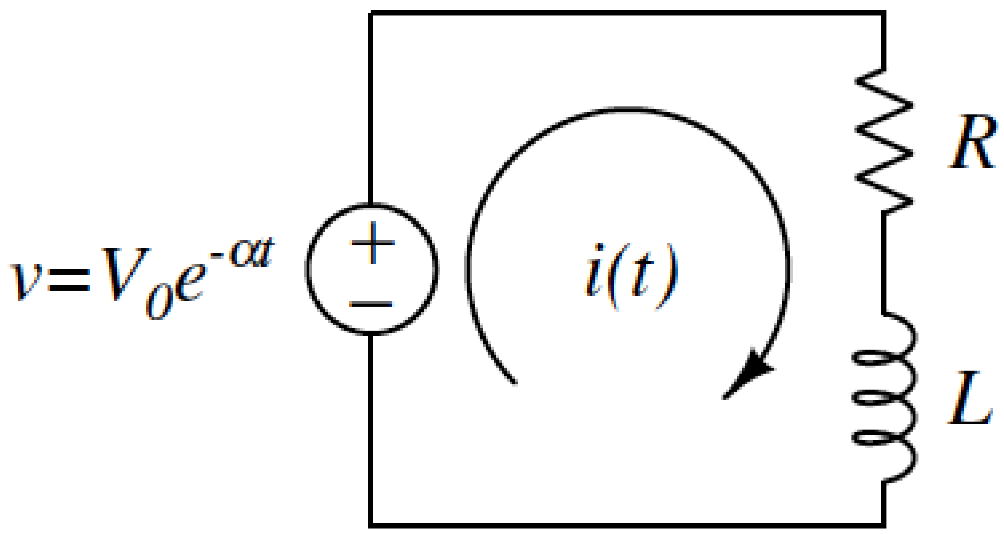
$$A = \frac{V_0}{R-\alpha L}$$

$$\begin{aligned} i(t) &= \frac{V_0}{R-\alpha L} e^{-\alpha t} \\ v_R(t) &= \frac{RV_0}{R-\alpha L} e^{-\alpha t} \\ v_L(t) &= -\frac{\alpha L V_0}{R-\alpha L} e^{-\alpha t} \end{aligned}$$



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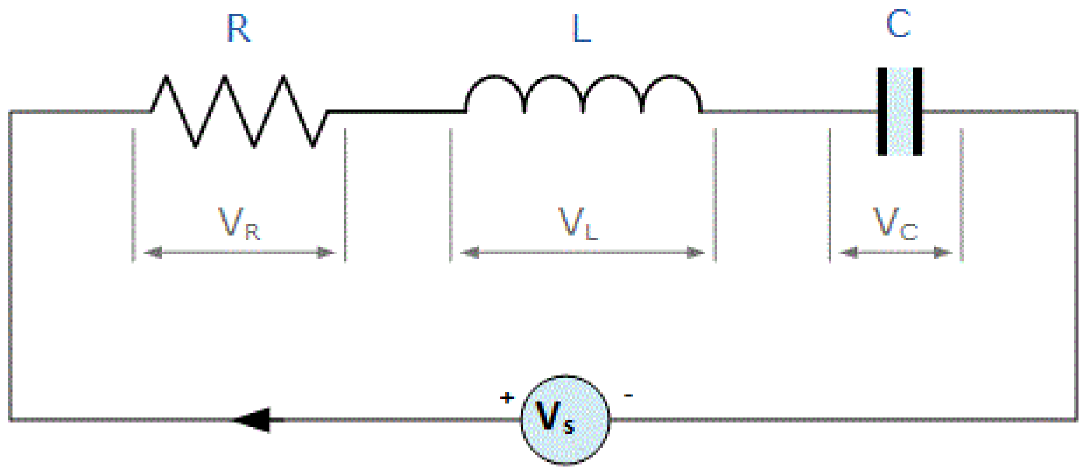
Example 2

- An R-L-C circuit is powered by a source $V_s(t) = 5e^{-2t}$

With R = 4 Ω, L = 1 H, C = 1/3 F

- None of the components are energized till t=0.



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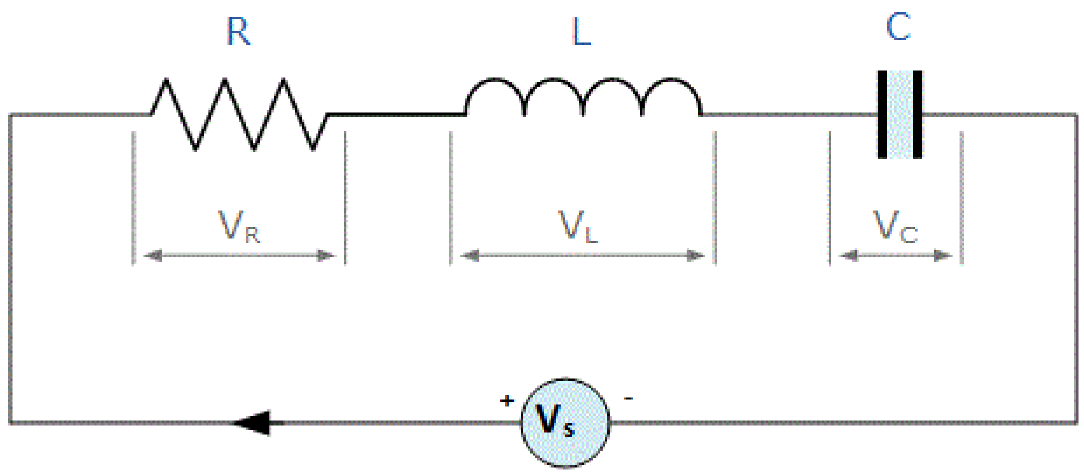
- None of the components are energized till t=0.
- What is (i) the current in the circuit, (ii) voltages across the three components?
- The impedance function is

$$Z(\alpha) = \frac{\alpha^2 LC + \alpha RC + 1}{\alpha C}$$
$$\alpha = -2$$



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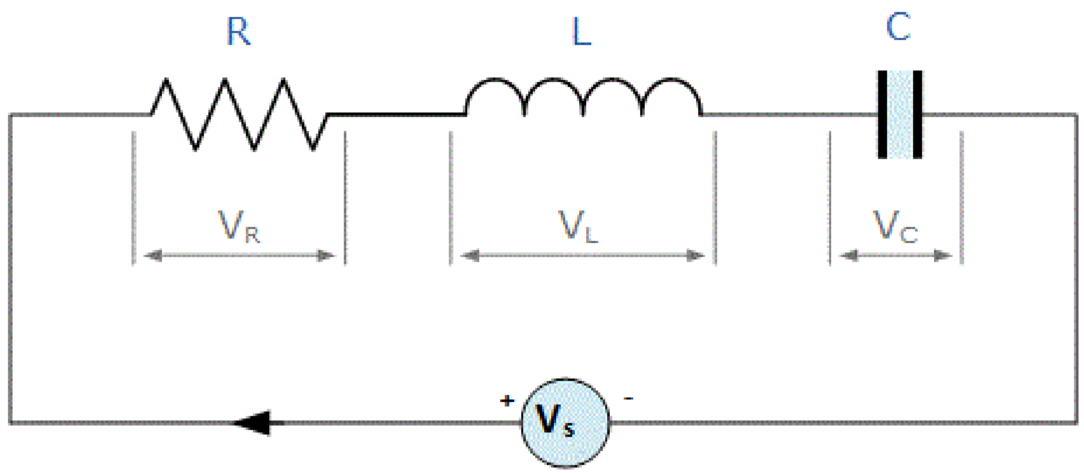

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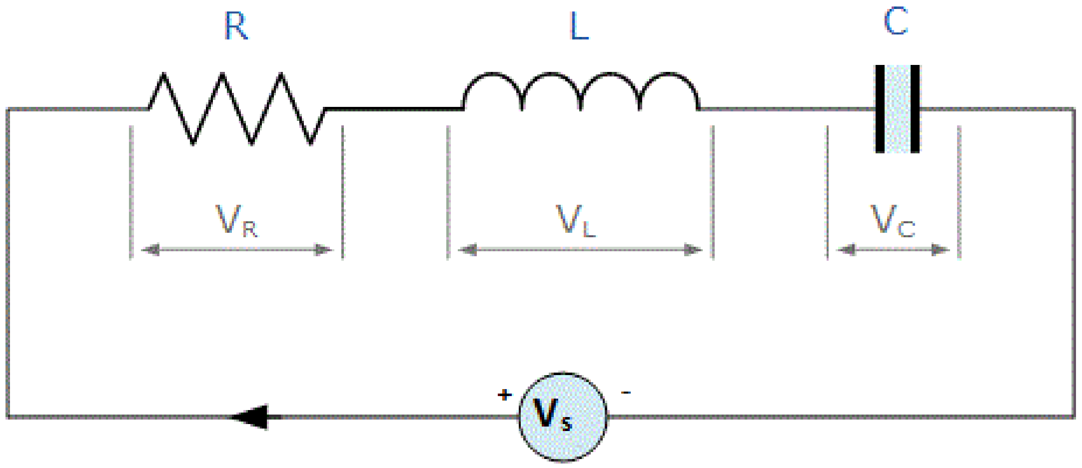
$$\alpha = -2$$

$$Z(-2) = 0.5$$

$$i(t) = \frac{V}{Z} = \frac{5e^{-2t}}{0.5}$$



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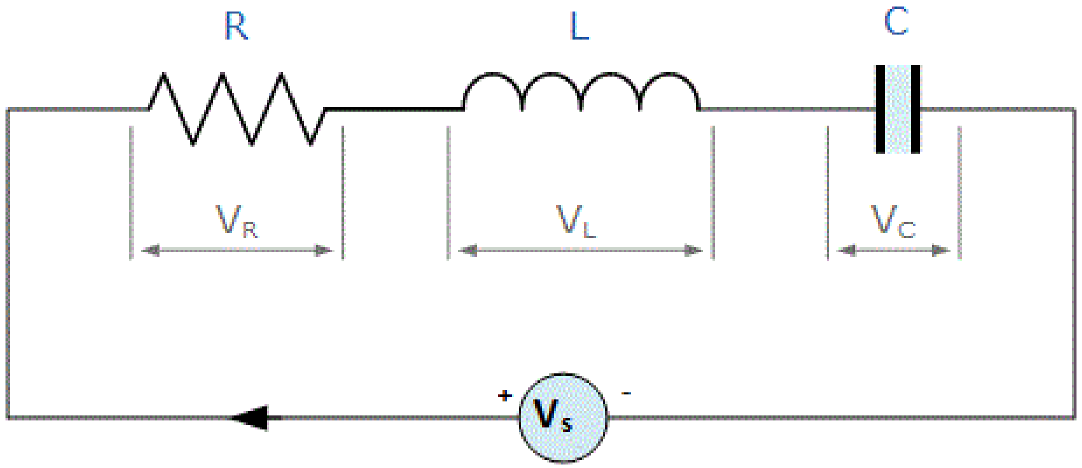

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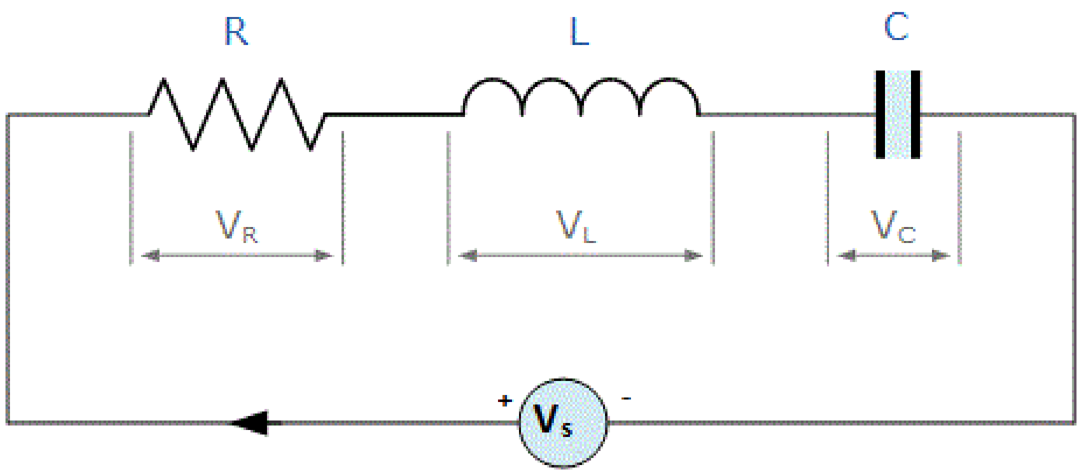
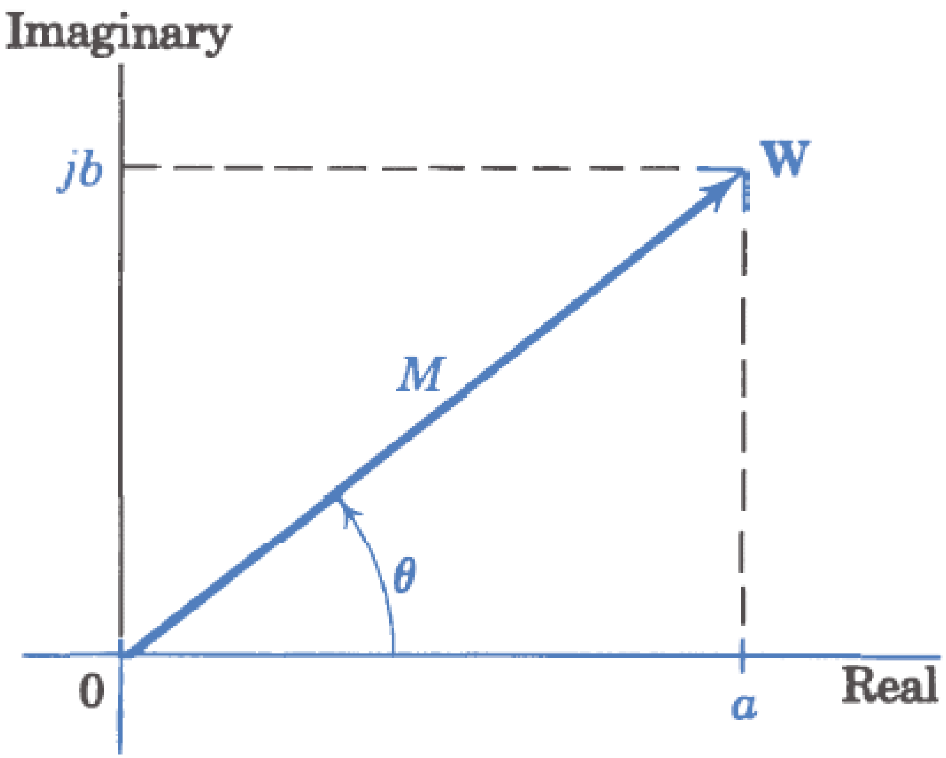
$$i(t) = \frac{V}{Z} = \frac{5e^{-2t}}{0.5} = 10e^{-2t}$$

$$V_R = iR = 40e^{-2t}$$
$$V_L = \alpha L i = -20e^{-2t}$$
$$V_C = \frac{i}{\alpha C} = -15e^{-2t}$$



Phasors – Representation

- Sine/Cosine value can be visualized as imaginary/real part of a complex number (vector).



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$$W = Me^{j\theta} = M\angle\theta$$

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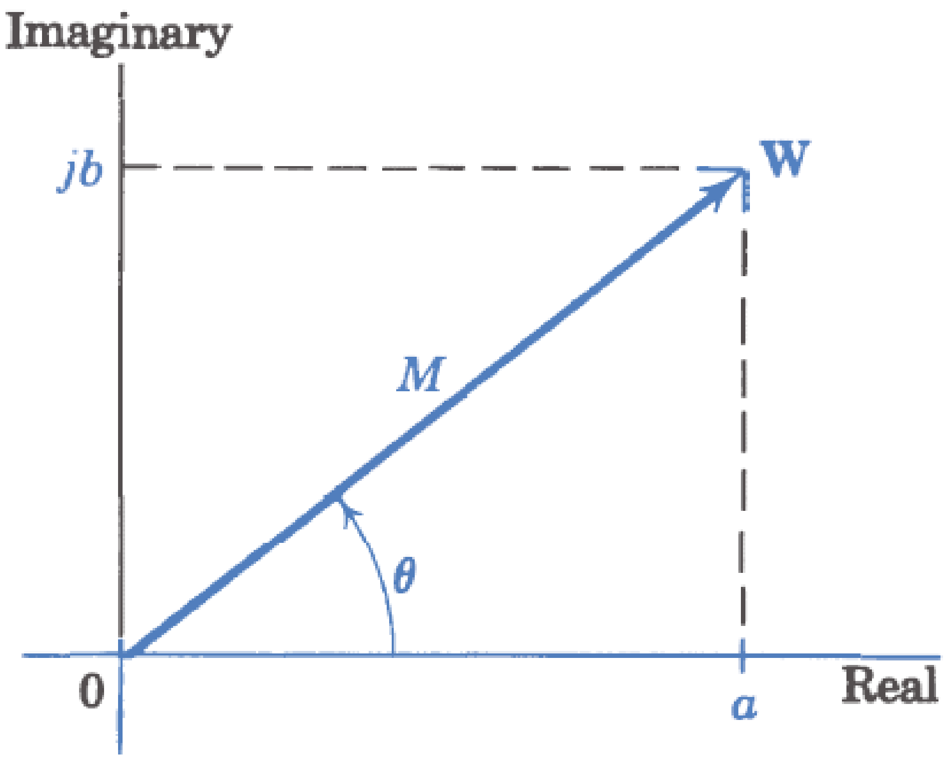

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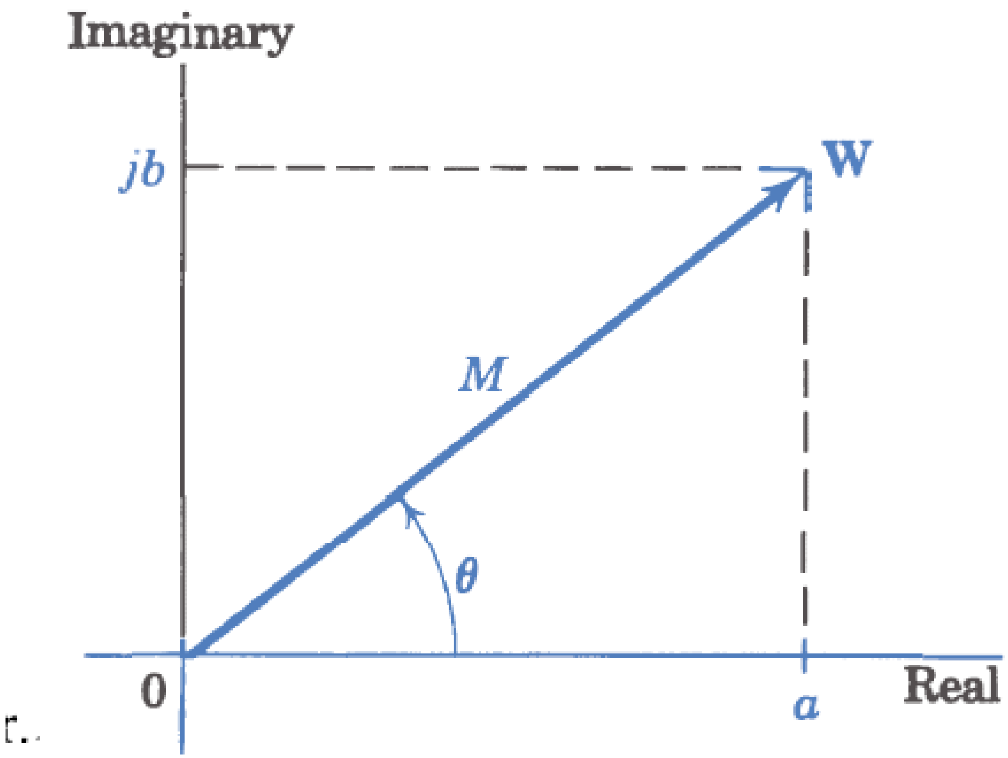

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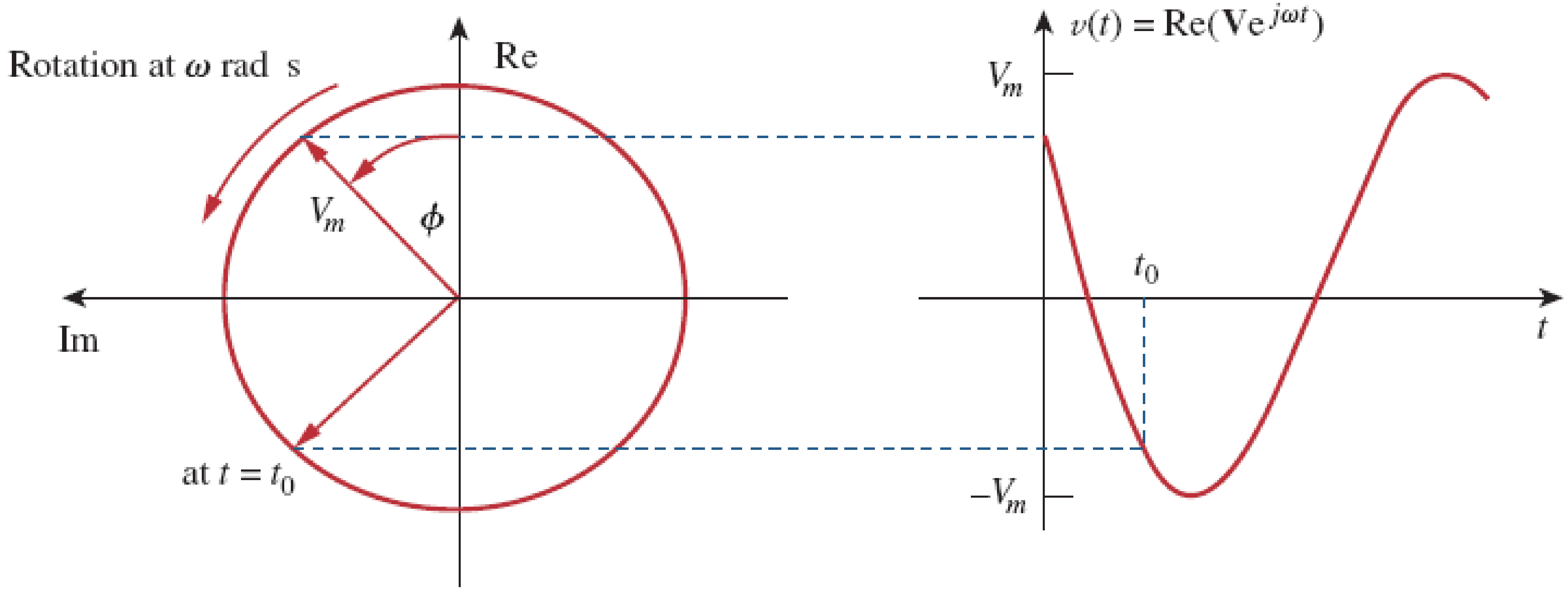
- This representation is called as a phasor
- A convention of either the real or imaginary part can be adopted.
- Here, we use real part.

Phasors – Computations

- Certain arithmetic operations involving phasors :
- Addition/Subtraction : $z_1 \pm z_2 = (x_1 \pm x_2) + j(y_1 \pm y_2)$
- Multiplication : $z_1 z_2 = r_1 r_2 \angle(\phi_1 + \phi_2)$
- Division : $\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle(\phi_1 - \phi_2)$ Reciprocal : $\frac{1}{z_2} = \frac{1}{r_2} \angle(-\phi_2)$
- Square root : $\sqrt{z} = \sqrt{r} \angle(\phi/2)$

Phasor Diagrams

- When a linear circuit is excited by a sinusoidal source of frequency ω (a rotating phasor)
- Voltages/currents in the circuit are also sinusoids of frequency ω, but with some phase difference



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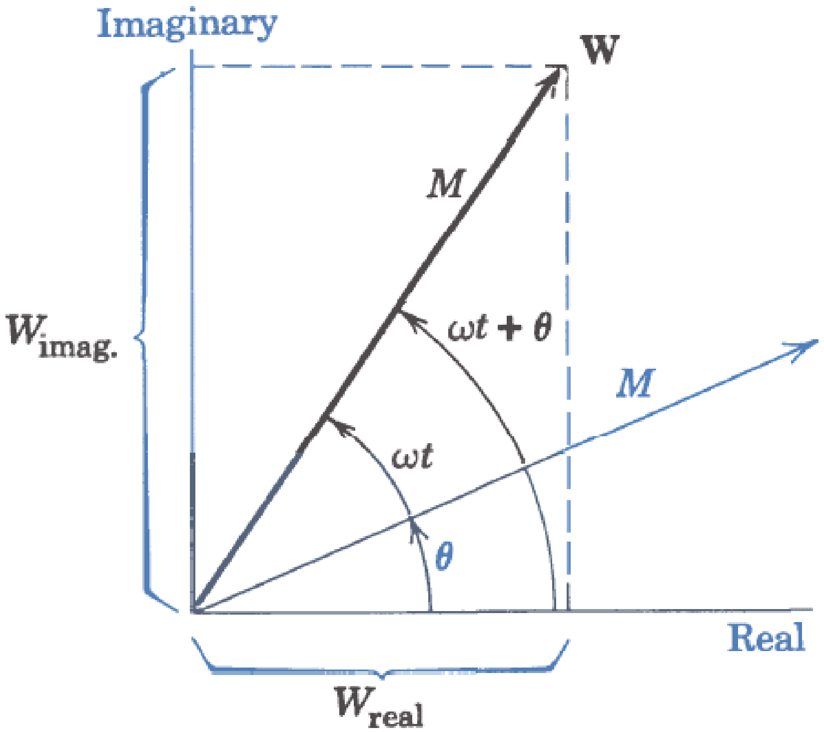

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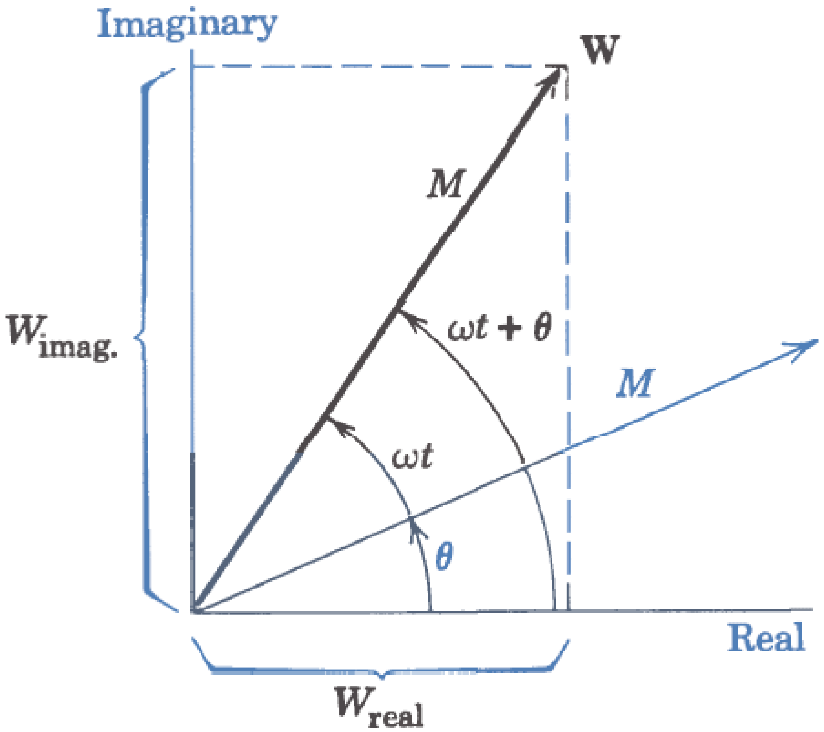

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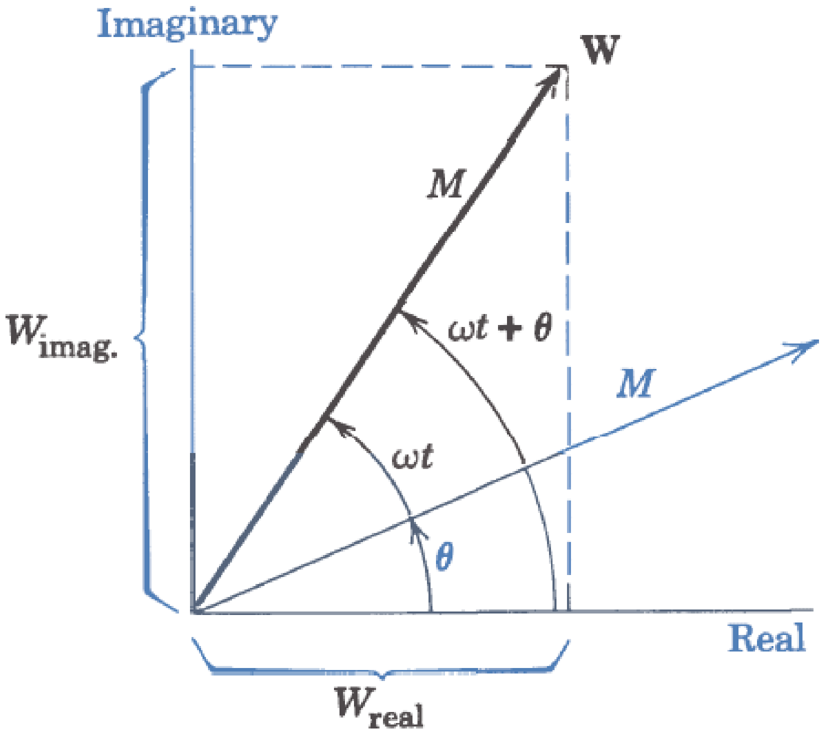

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$$v(t) = V_m \cos(\omega t + \theta)$$

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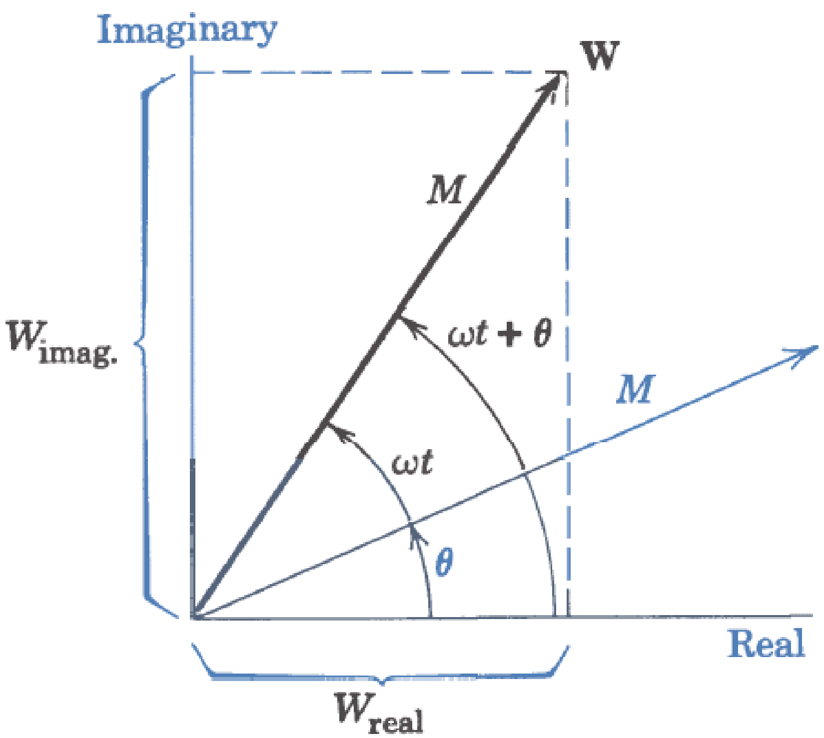

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$$
\begin{aligned}
v(t) &= V_m \cos(\omega t + \theta) \\
&= \sqrt{2} V \cos(\omega t + \theta) \quad \text{RMS Value}
\end{aligned}
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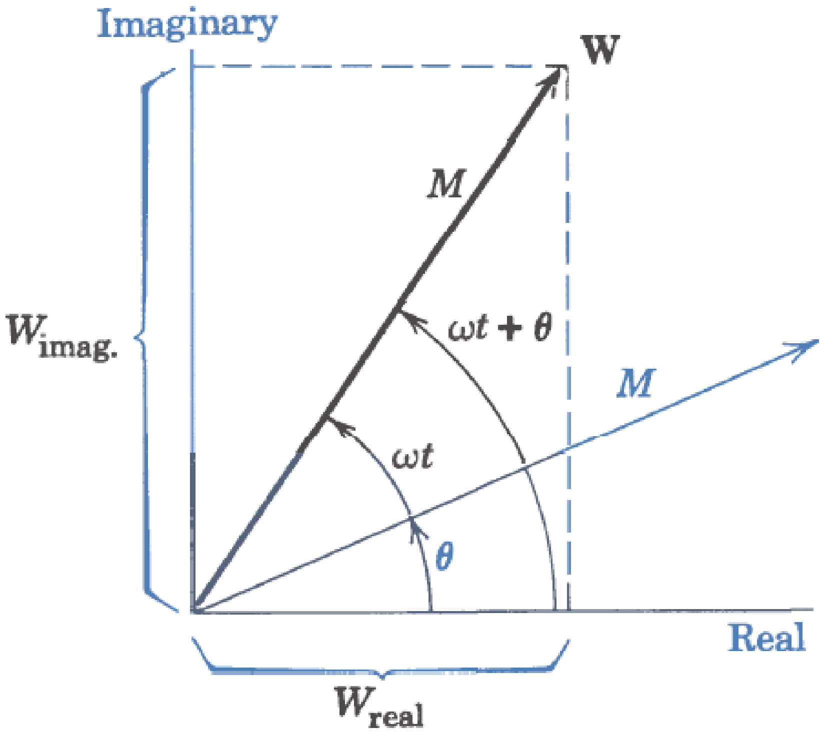

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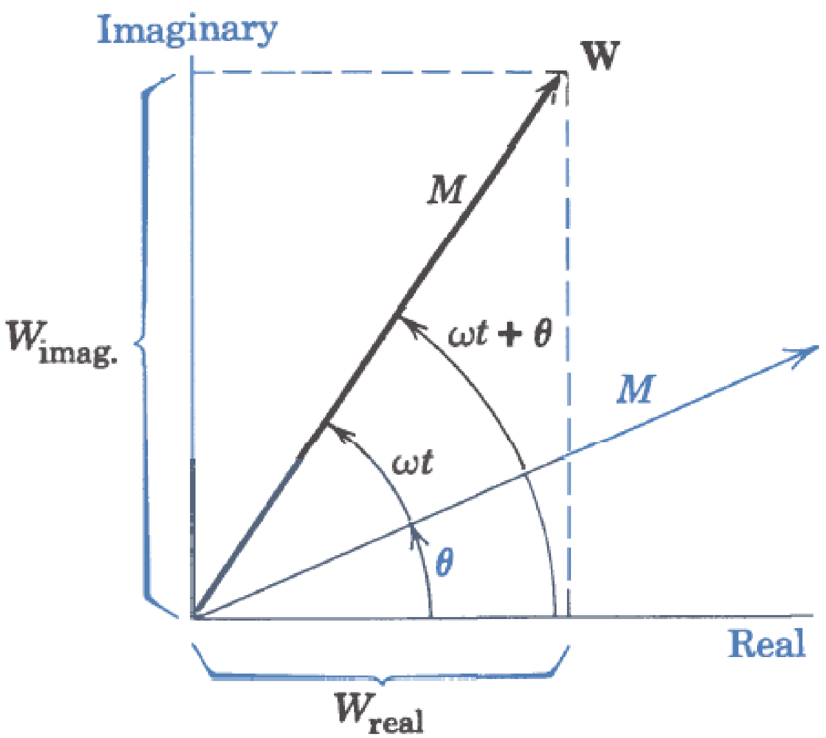

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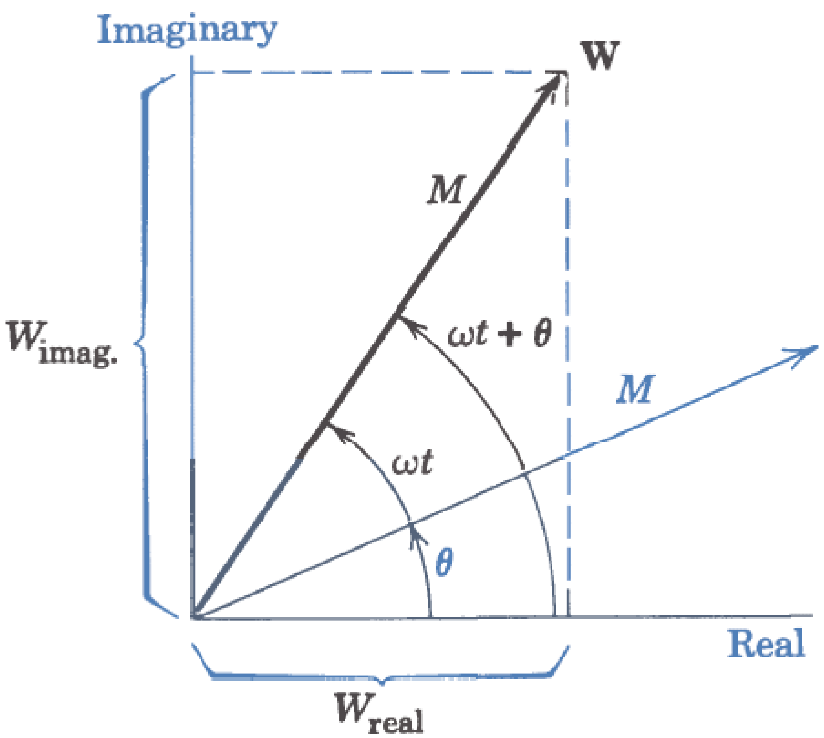

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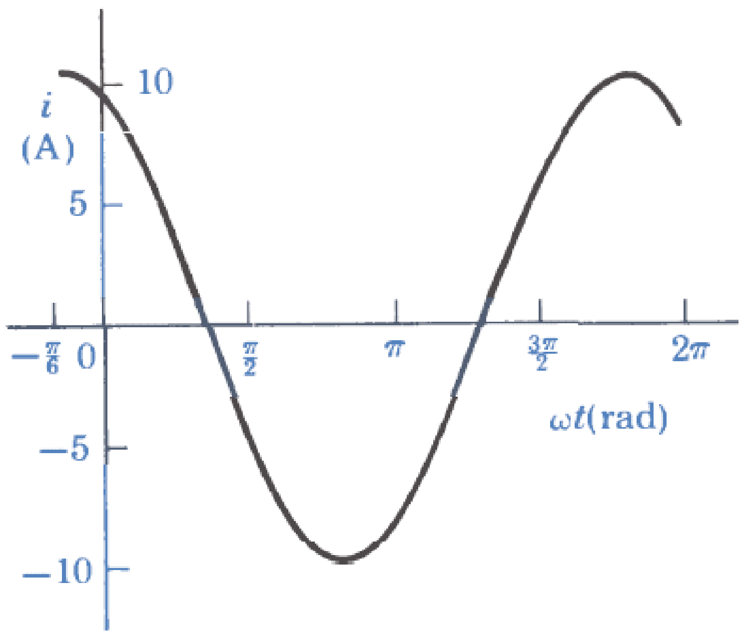
All essential information of magnitude and phase difference is retained. The common 'rotation' is ignored.

Phasor Diagram Example

Write the eq. of the current shown in fig. as a function of time and represent the current by a phasor.



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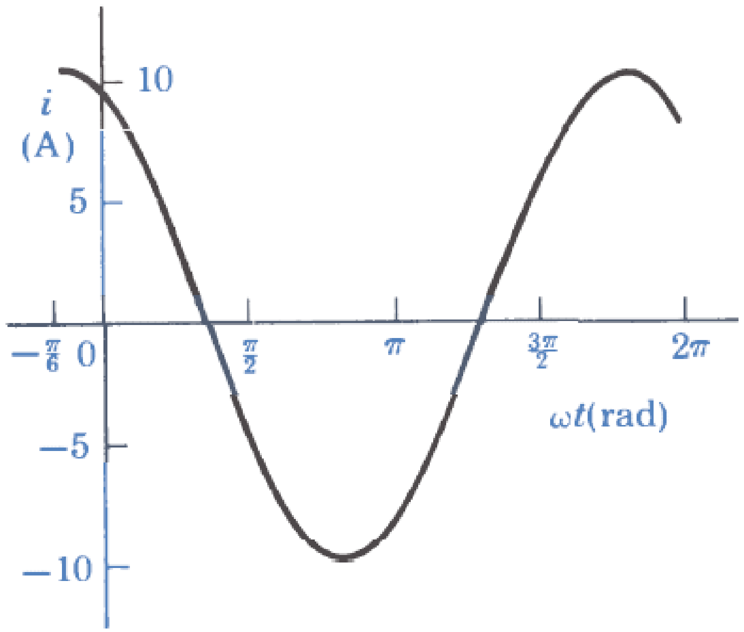
Write the eq. of the current shown in fig. as a function of time and represent the current by a phasor.

The current reaches a positive maximum of 10 A at $\pi/6$ rad or 30^0 before $\omega t = 0$; therefore , $i = 10\cos(\omega t + \pi/6)\ A$



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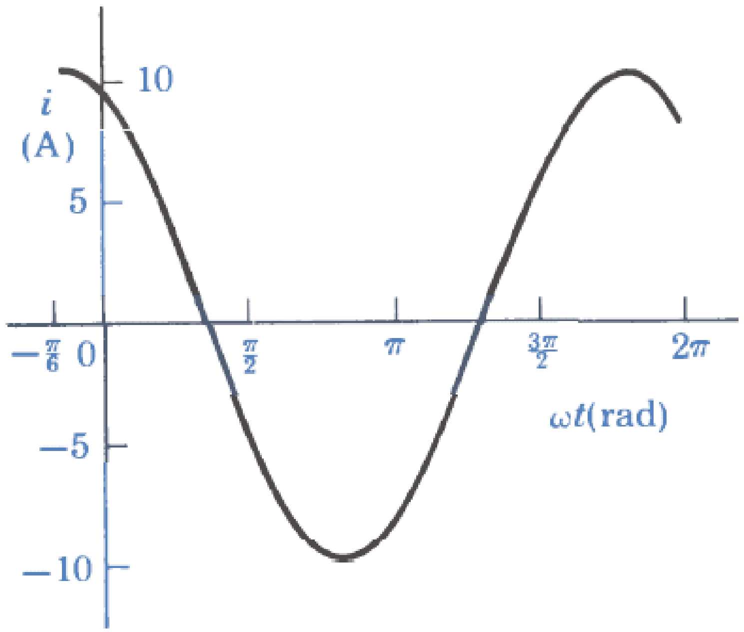

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or

$$i(t) = Re\{10\ e^{j(\omega t + \pi/6)}\}$$
$$= Re\{7.07\ e^{j(\pi/6)}\sqrt{2}e^{j\omega t}\}\ A$$

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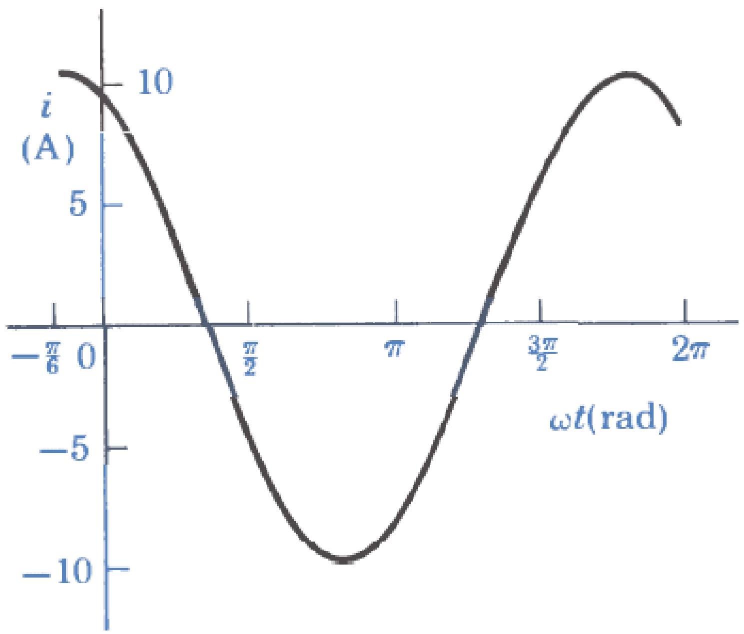

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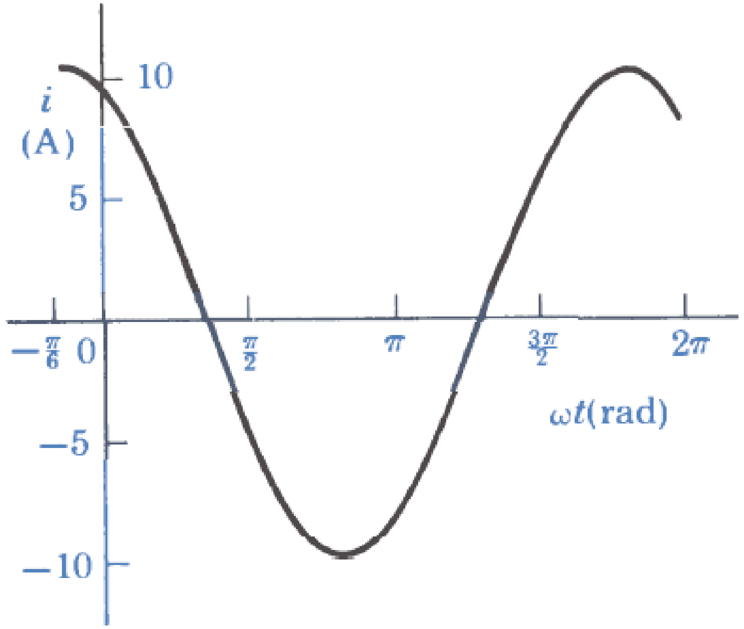

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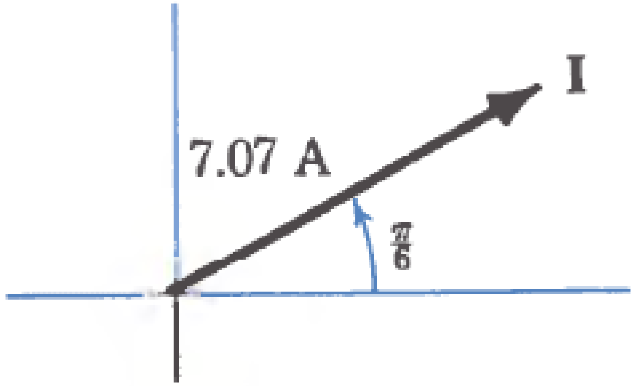
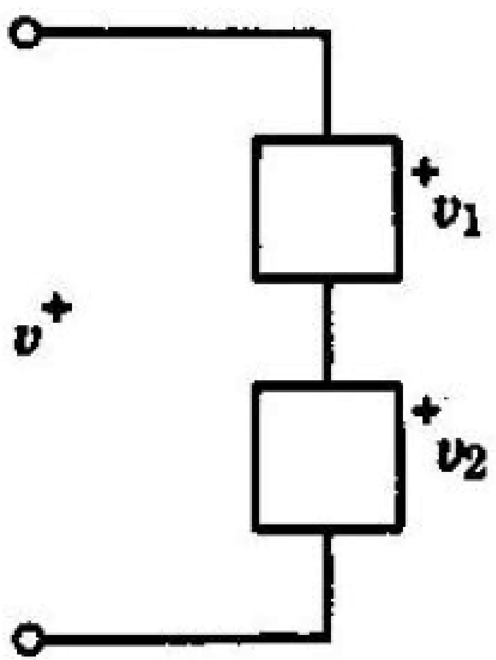


Figure: Phasor

Phasor Diagram Example 2

Given
$v_1 = 150\sqrt{2}\cos(377t - \pi/6)\ V$
and $V_2 = 200\angle{+60^o}$ V, find
$v = v_1 + v_2$



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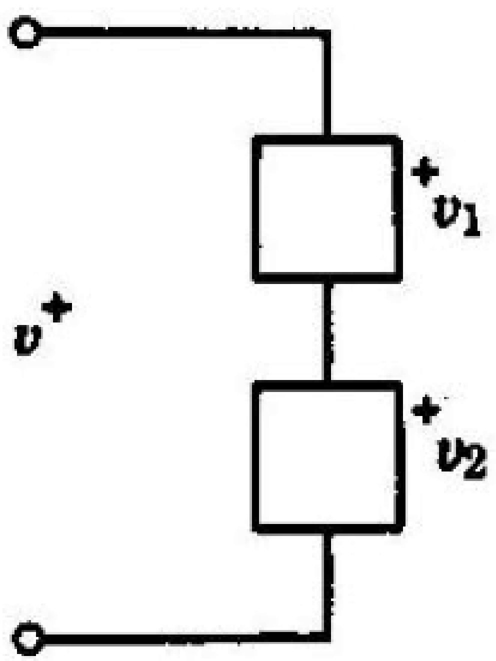
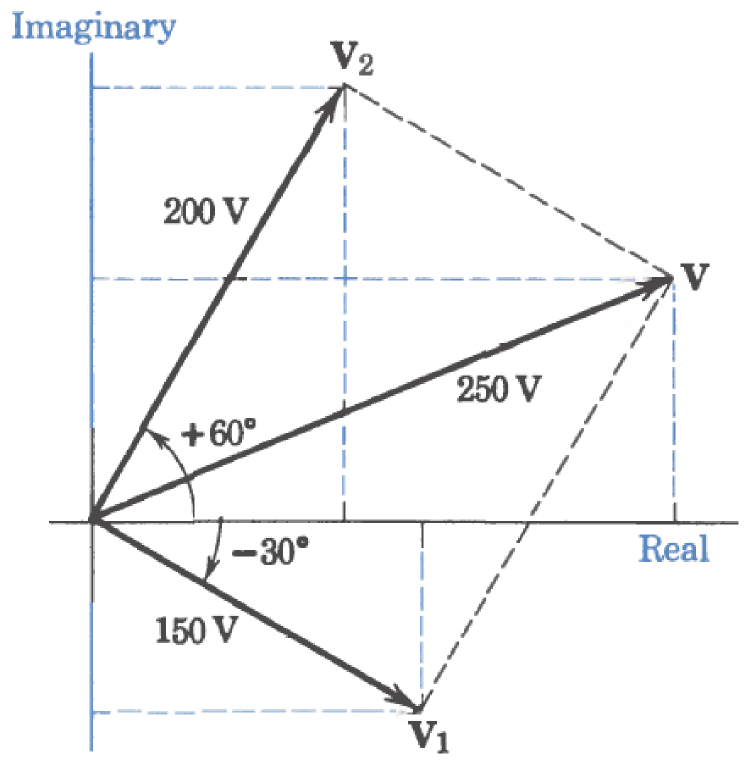


Phasor notation:
$V_1 = 150\angle - 30^o$ V



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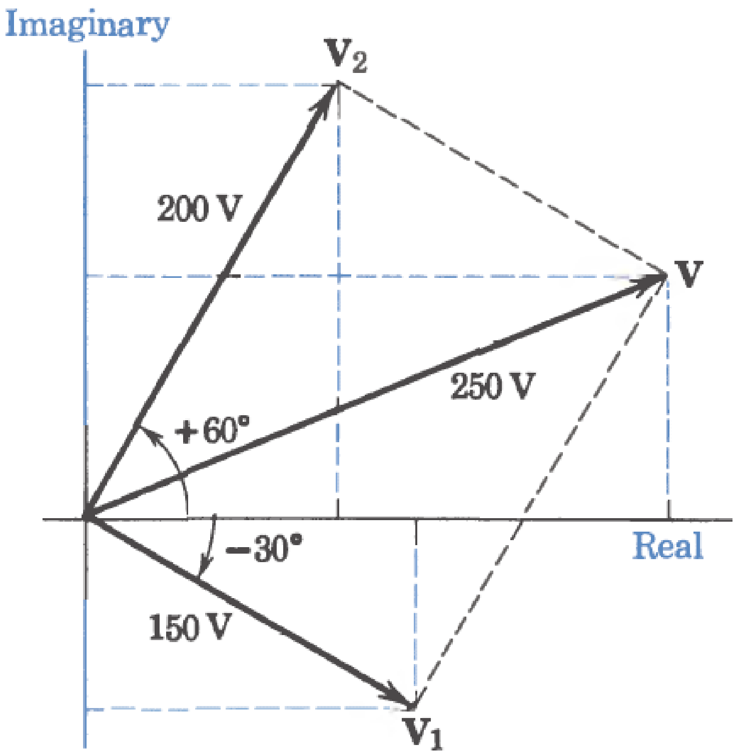

$$V_1 = 150\cos(-30^o) + j150\sin(-30^o)$$
$$= 130 - j75 \text{ V}$$
$$V_2 = 200\cos 60^o + j200\sin 60^o$$
$$= 100 + j173\text{V}$$
$$V = V_1 + V_2 = 230 + j98$$
$$= 250\angle 23.1^o \text{ V}$$

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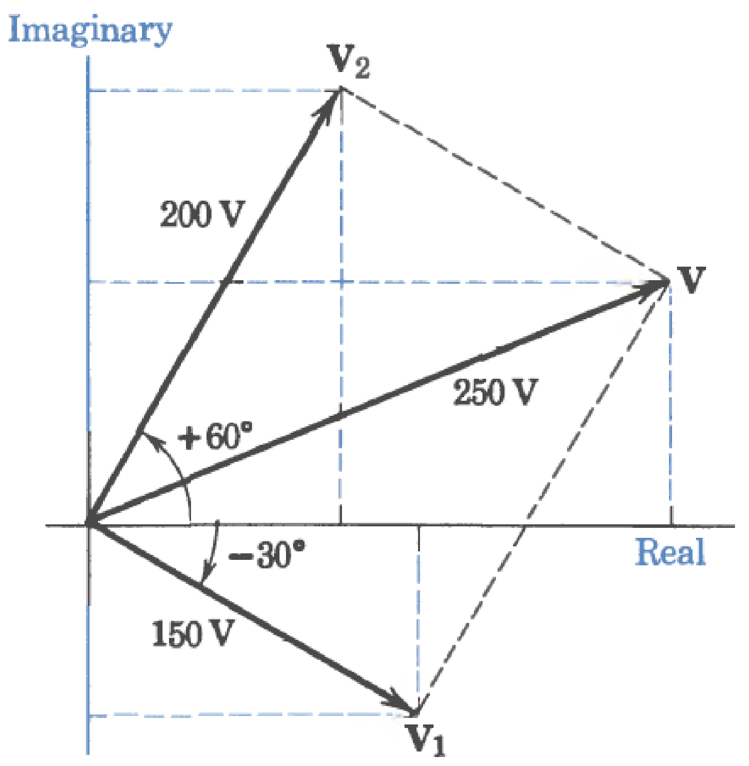

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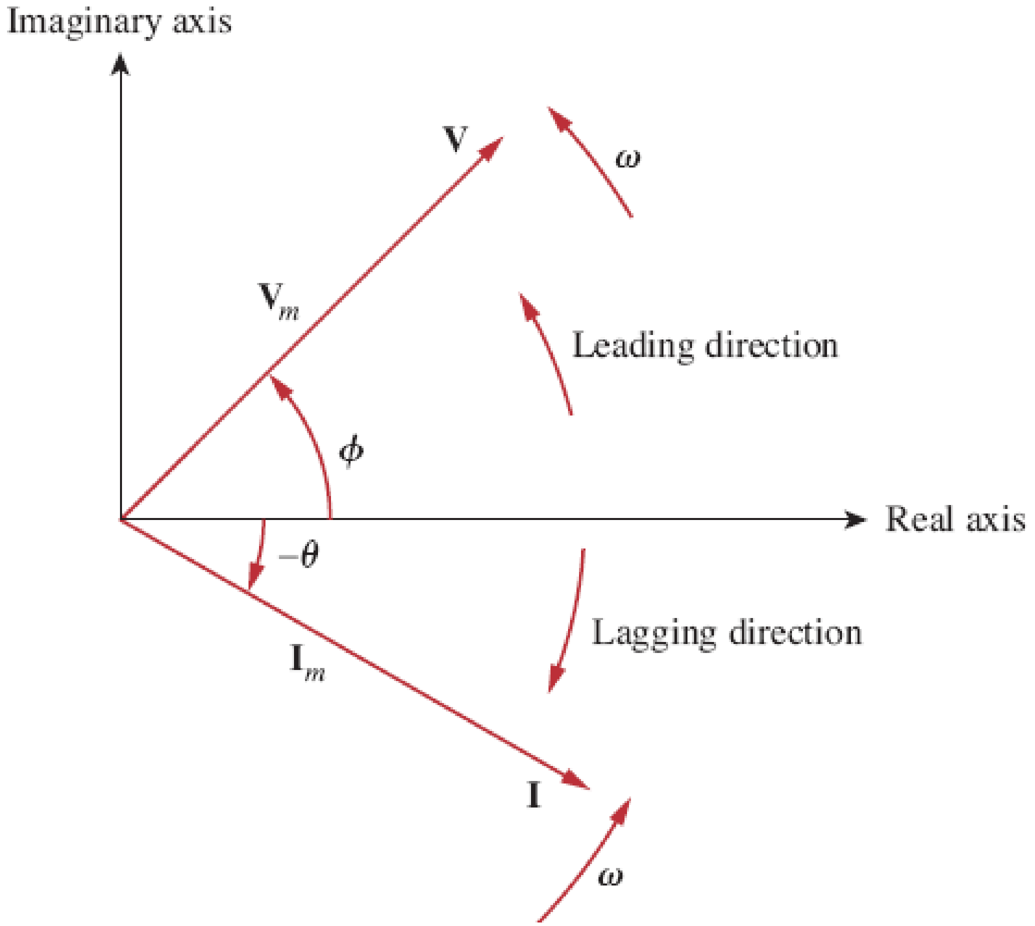
After inverse transformation on V,

$v = v_1 + v_2 =$

$250\sqrt{2}\cos(377t + 23.1^o)$V

Phasor Diagram : Leading/Lagging

- Phasors may have positive/negative phase difference with the reference signal.



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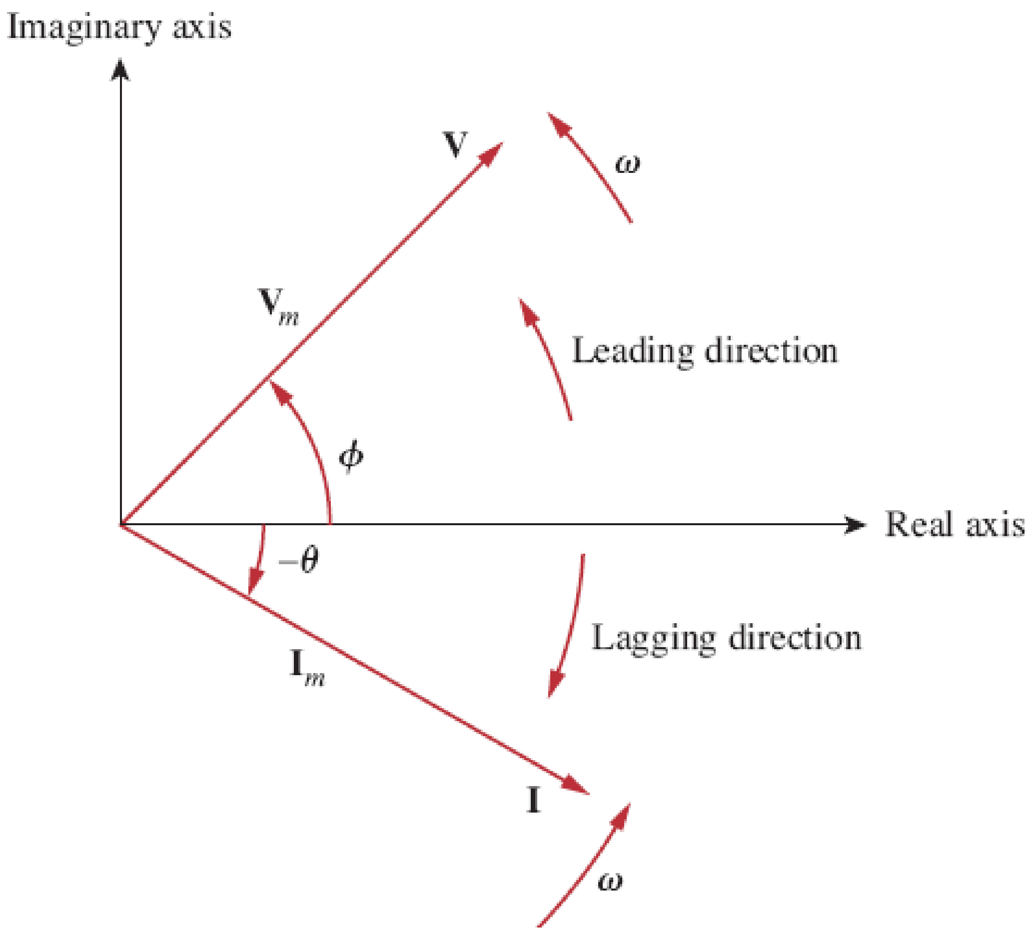
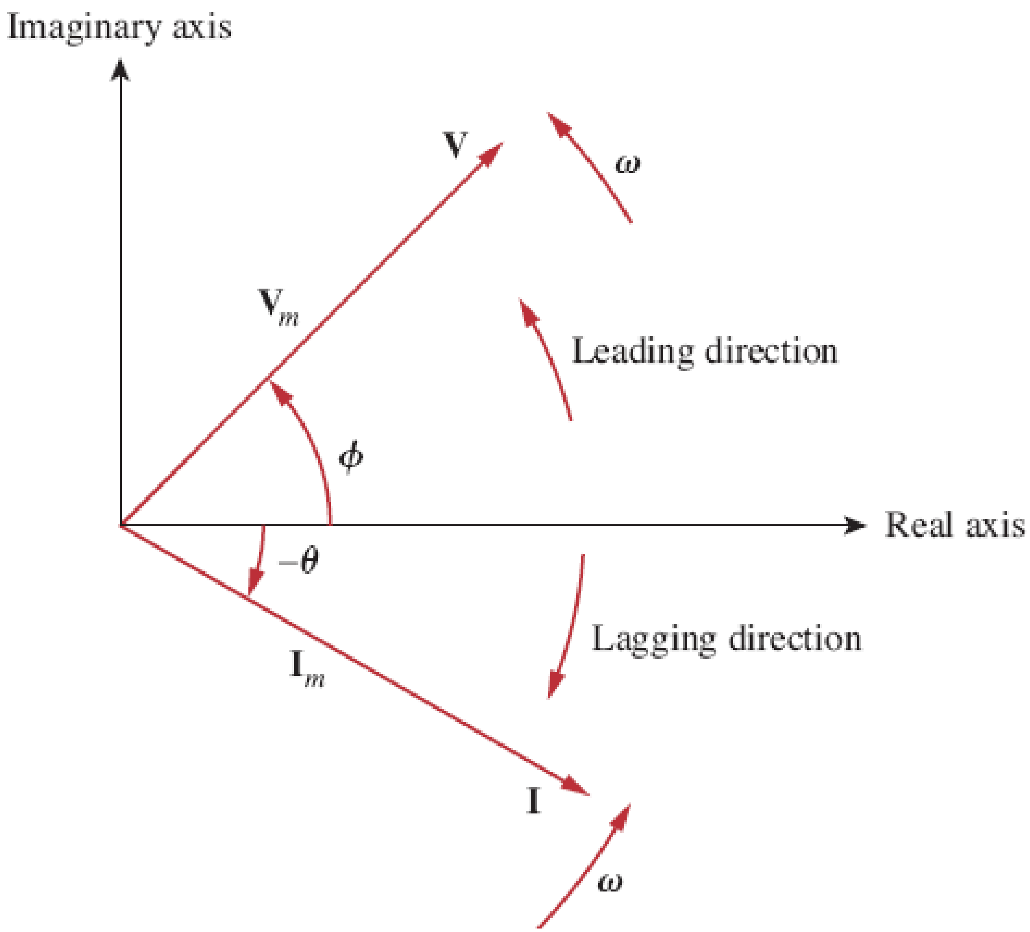


- Positive phase difference wrt reference is termed as **leading**
- Negative phase difference wrt reference is termed as **lagging**

Phasor Diagram : Leading/Lagging

- Phasors may have positive/negative phase difference with the reference signal.



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Reference Signal : Signal common to various circuit parts.

- Current in series-connected circuits
- Voltage in parallel connected circuit (power supply)

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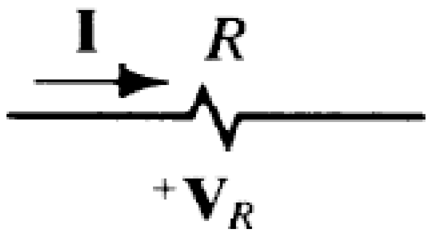
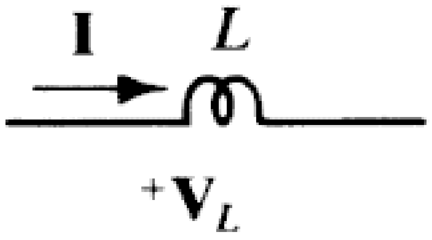
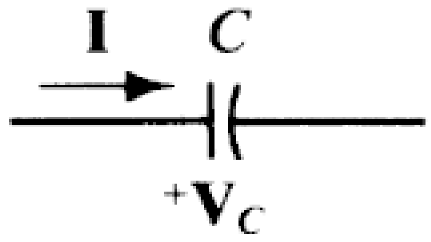
Impedance and Phasors







$$\mathbf{V}_R = R\mathbf{I} \qquad \mathbf{V}_L = j\omega L\mathbf{I} \qquad \mathbf{V}_C = \frac{1}{j\omega C}\mathbf{I}$$

$$\mathbf{I}_R = \frac{1}{R}\mathbf{V}_R \qquad \mathbf{I}_L = \frac{1}{j\omega L}\mathbf{V}_L \qquad \mathbf{I}_C = j\omega C\mathbf{V}_C$$

V in phase with I

V leads I by 90 deg

V lags I by 90 deg

Admittance

The reciprocal of impedance is admittance, SI unit: siemens or mho

$$Y = \frac{I}{V} = \frac{1}{Z} Siemens$$

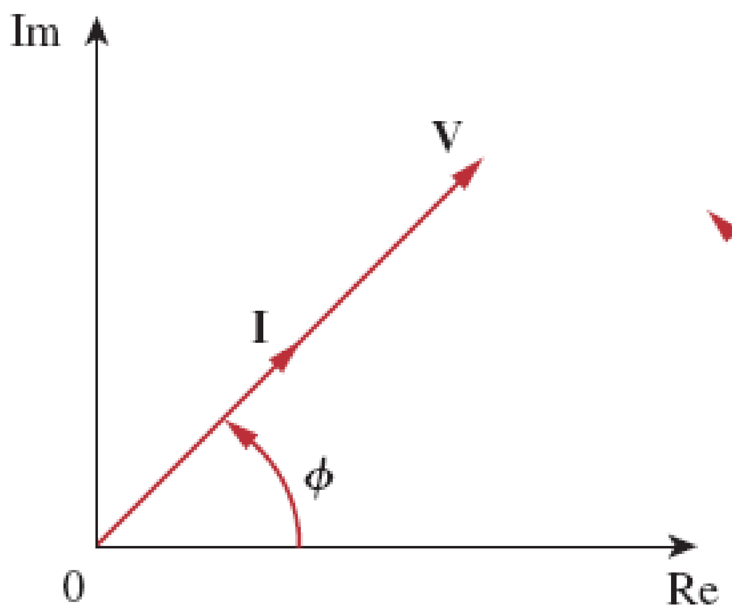
$$Y_R = \frac{1}{R\angle 0^o} = G\angle 0^o$$

$$Y_L = \frac{1}{j\omega L} = \frac{1}{\omega L}\angle -90^o$$

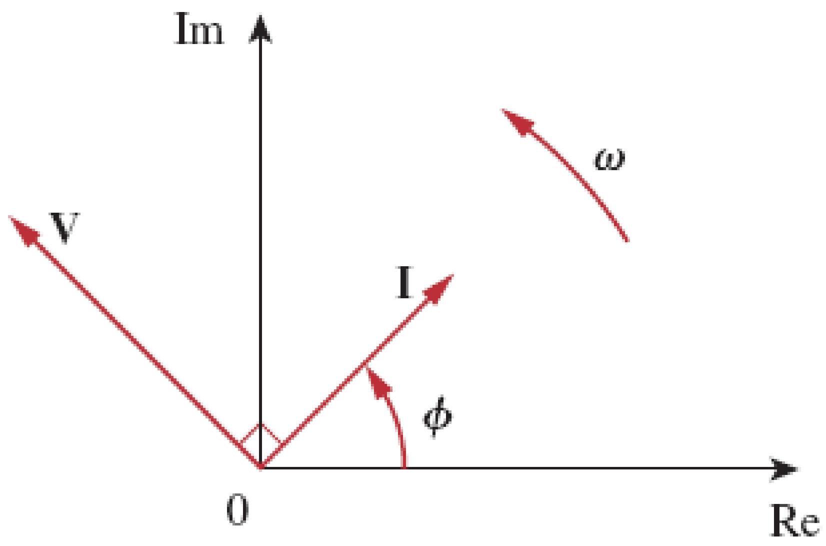
$$Y_C = j\omega C = \omega C\angle 90^o$$

Phasors for R-L-C elements

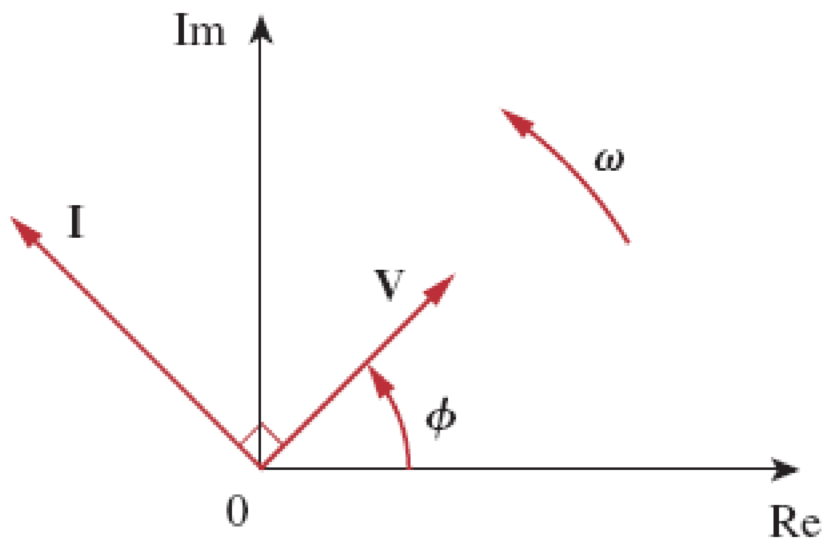
The typical phasors for R-L-C elements



Resistor



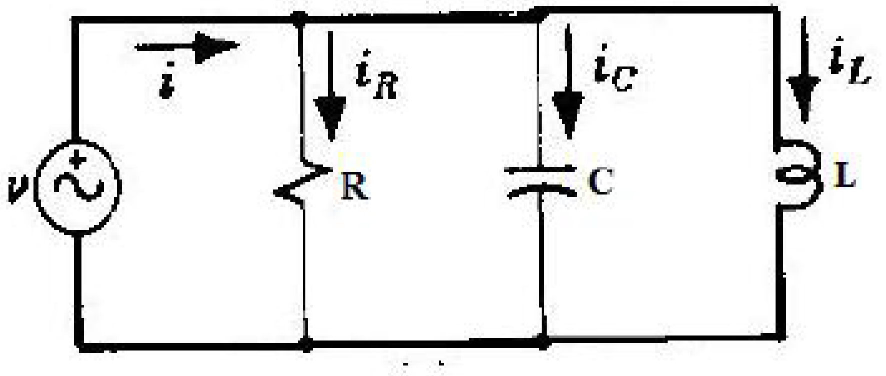
Inductor



Capacitor

Example

Voltage
$v = 120\sqrt(2)\cos(1000t + 90^o)$ V
is applied to the circuit where
$R = 15\Omega$, $C = 83.3\mu F$, and
$L = 30mH$. Find i(t)



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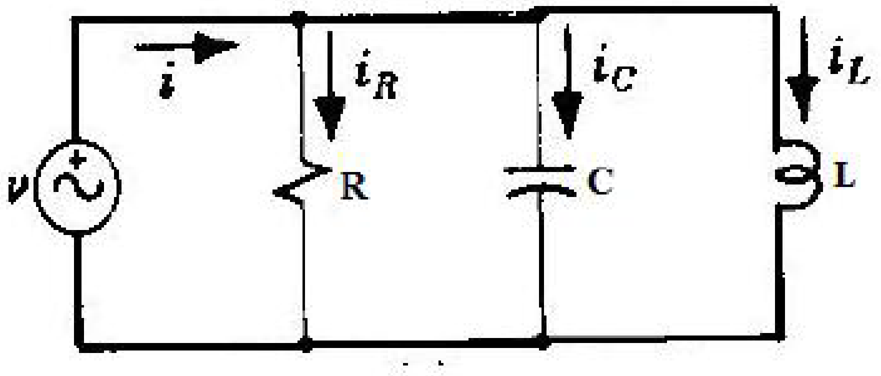

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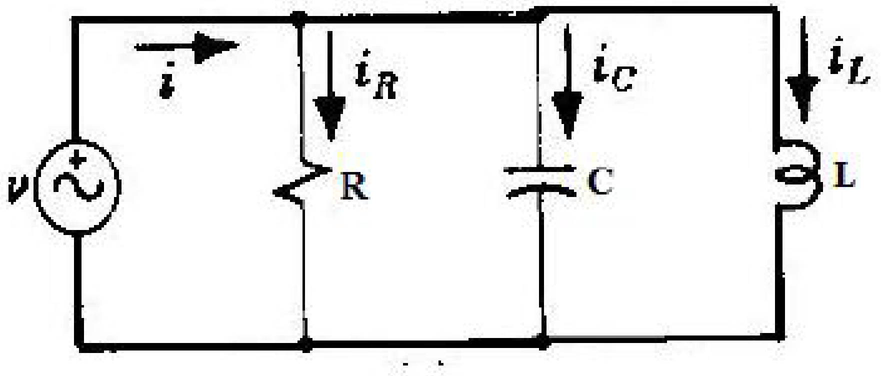

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$I_R = \frac{1}{R}V = \frac{1}{15}120\angle 90^o = 8\angle 90^o$
$= 0 + j8$ A

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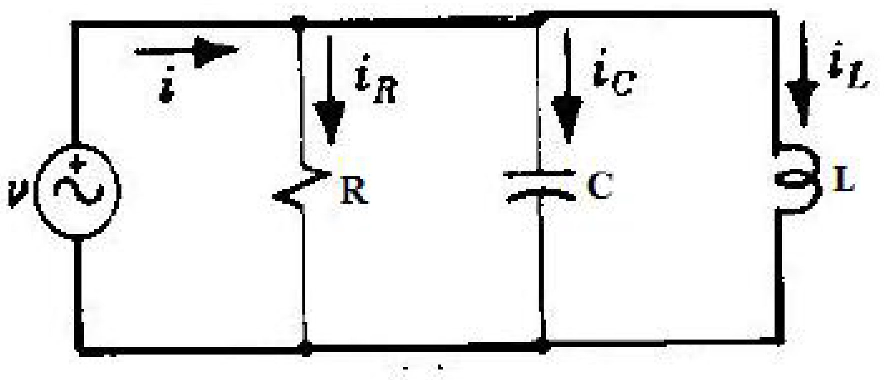

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$I_C = j\omega C\ V$
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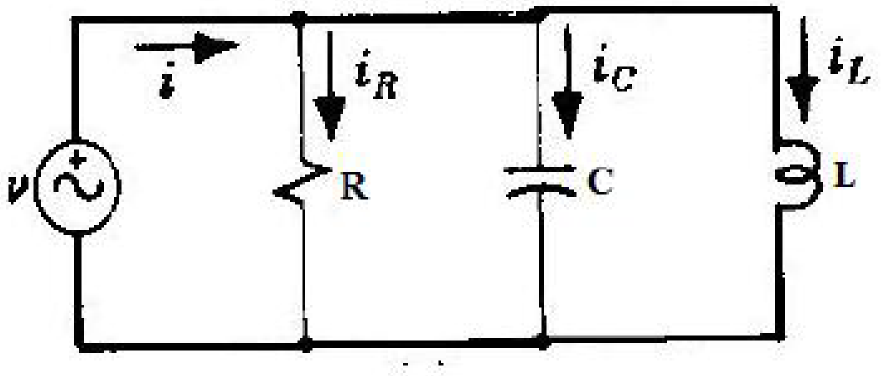

$\mathbb{C} = M_1 * M_2\angle\phi_1 + \phi_2$

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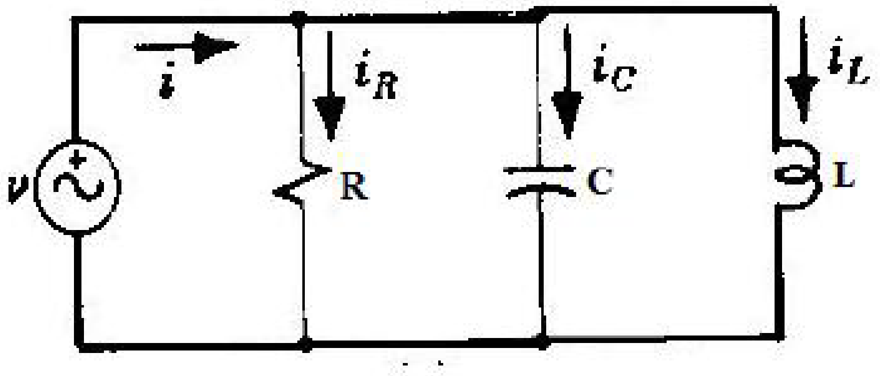

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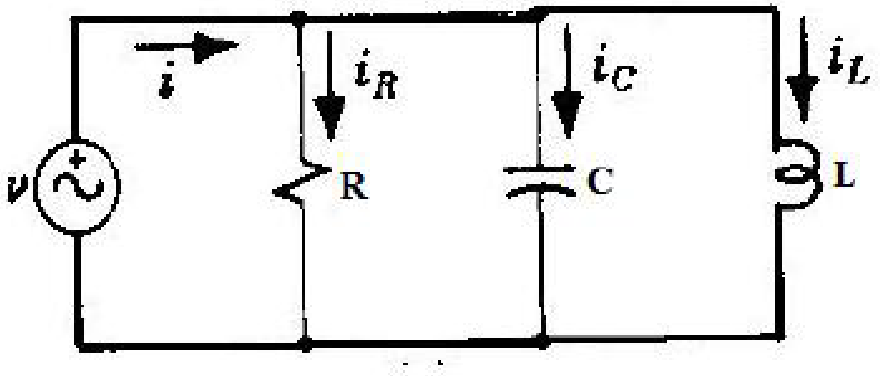

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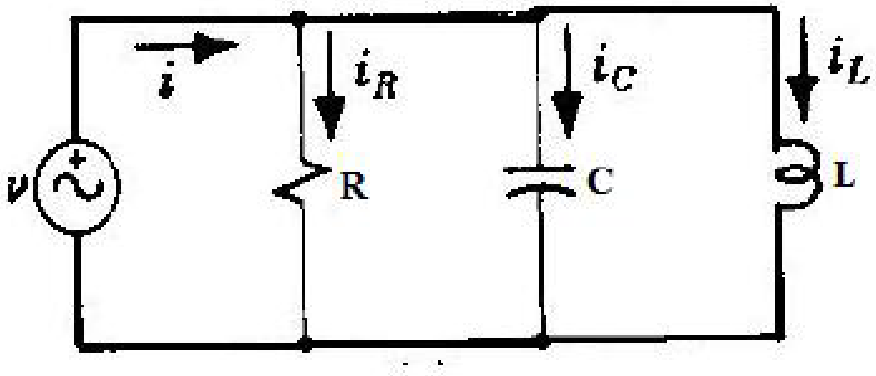

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$$\boxed{\mathbb{C} = M_1 * M_2 \angle \phi_1 + \phi_2}$$

$$\boxed{\mathbb{C} = \frac{M_1}{M_2}\angle \phi_1 - \phi_2}$$

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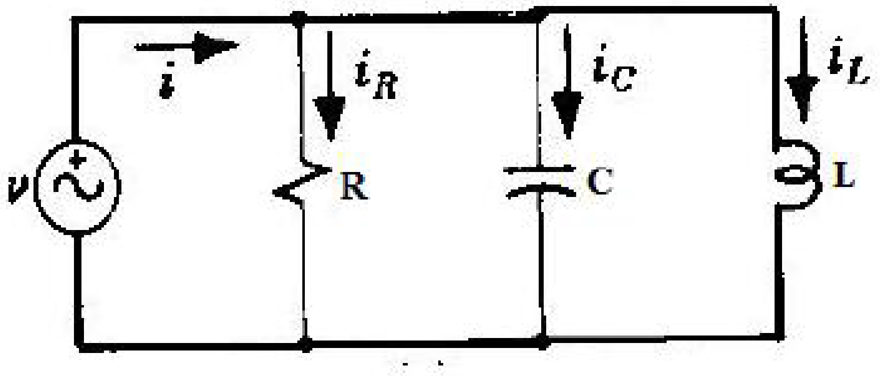

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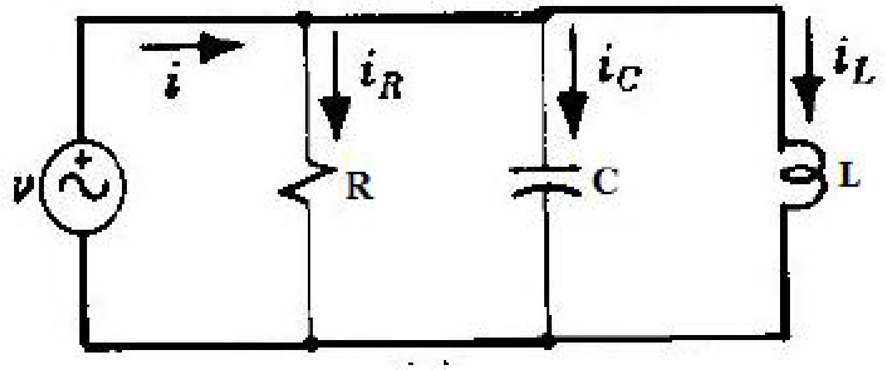

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By Kirchhoff's current law, $\sum I = 0$
or
$I = I_R + I_C + I_L = (0-10+4)+j(8+0+0)$
$= -6 + j8 = 10\angle 127^o$ A

$i(t) = 10\sqrt(2)\cos(1000t + 127^o)$ A

Example

Voltage
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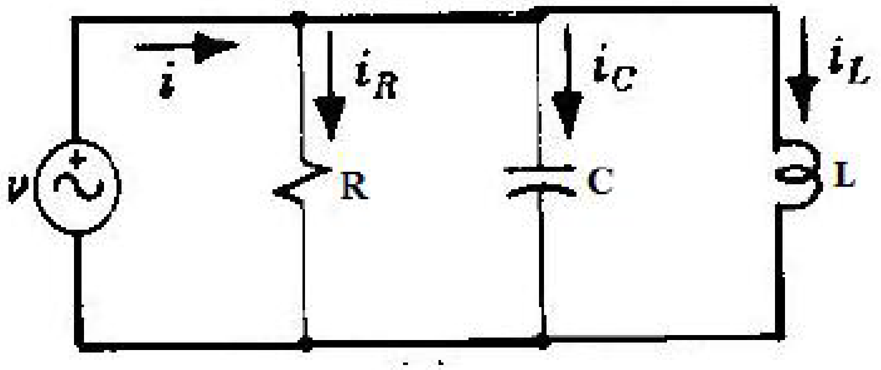

$$\mathbb{C} = M_1 * M_2 \angle \phi_1 + \phi_2$$

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$V = 120\angle 90^o$ V

$I_R = \frac{1}{R} V = \frac{1}{15} 120\angle 90^o = 8\angle 90^o$
$= 0 + j8$ A **In Phase**

$I_C = j\omega C\ V$
$= (0.0833\angle 90^o)(120\angle 90^o)$
$= 10\angle 180^o = -10 + j0$ A **Leading**

$I_L = \frac{1}{j\omega L} V = \frac{120\angle 90^o}{30\angle 90^o}$
$= 4\angle 0^o = 4 + j0$ A **Lagging**

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$i(t) = 10\sqrt(2)\cos(1000t + 127^o)$ A **Leading**

Example – Impedance as Complex No.

Voltage $v = 12\sqrt{(2)}\cos 5000t$ V is applied to the circuit. Find the individual and combined impedances and the current i(t)

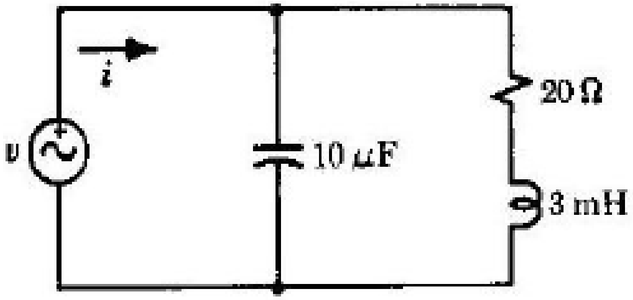
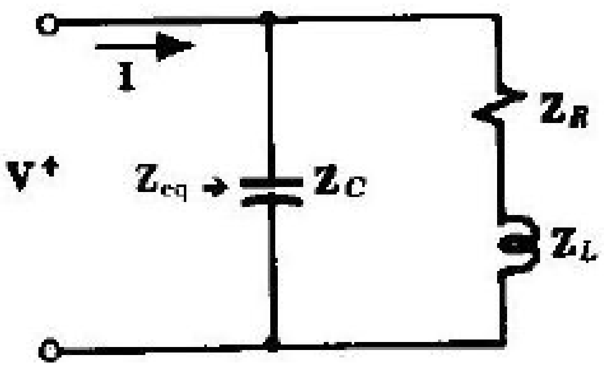


Figure: Circuit Elements



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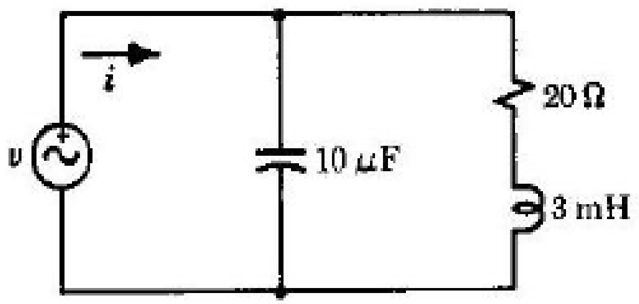

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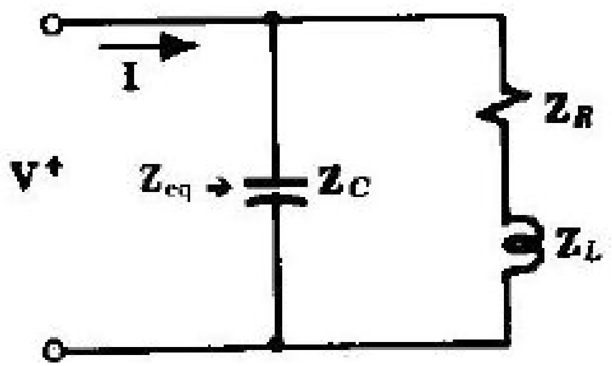

Solution:

$\mathcal{Z}_R = R = 20 = 20 + j0 = 20\angle 0°\Omega$

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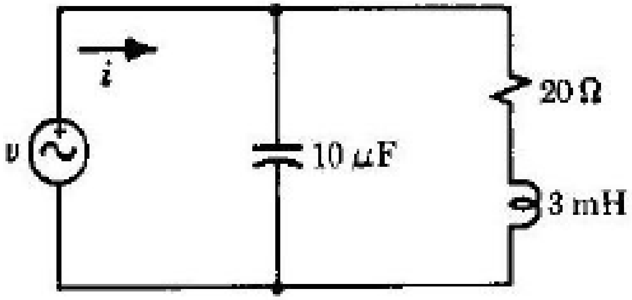

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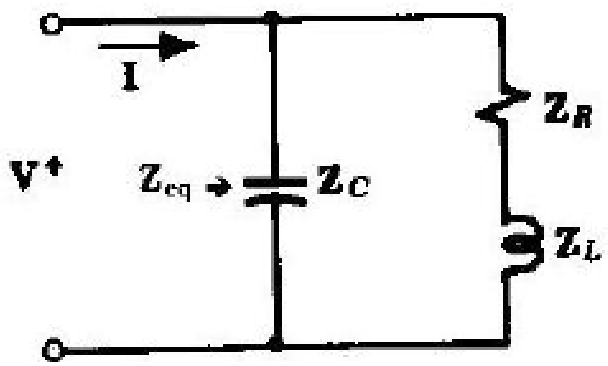

Solution:

$\mathcal{Z}_R = R = 20 = 20 + j0 = 20\angle 0^\circ \Omega$

$\mathcal{Z}_L = j\omega L = j5000 \times 0.003 = j15\Omega$
$= 15\angle 90^\circ \Omega$

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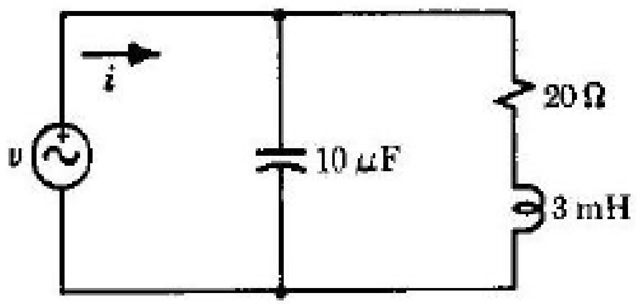

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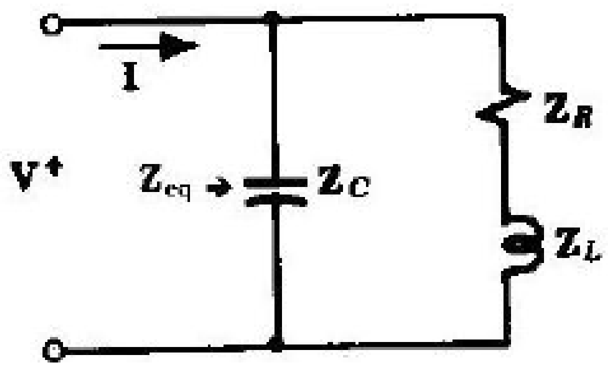

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$\mathcal{Z}_C = -j\frac{1}{\omega C} = \frac{-j}{5000\times 10^{-5}}$
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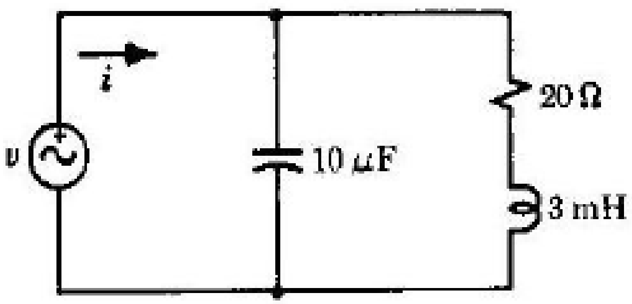

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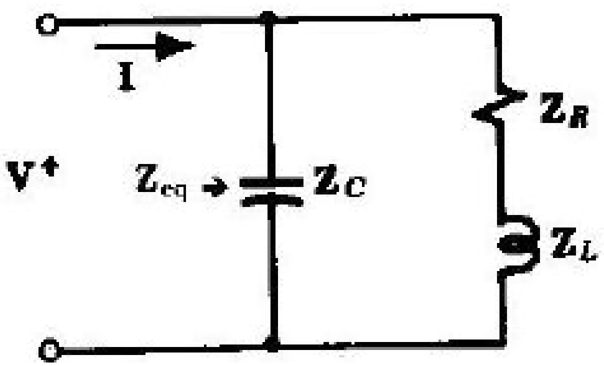

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$\mathcal{Z}_{RL} = \mathcal{Z}_R + \mathcal{Z}_L = 20 + j15 = 25\angle 37^o \Omega$

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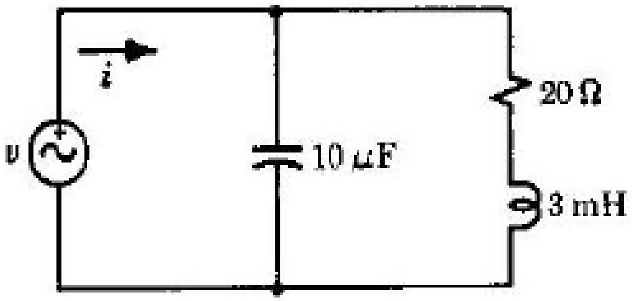

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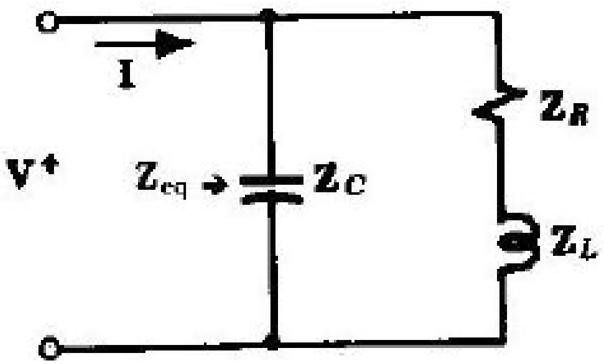

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$= 15\angle 90^o\Omega$

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$= -j20 = 20\angle - 90^o\Omega$

$\mathcal{Z}_{RL} = \mathcal{Z}_R + \mathcal{Z}_L = 20 + j15 = 25\angle 37^o\Omega$

$\mathcal{Z}_{eq} = \frac{\mathcal{Z}_{RL}.\mathcal{Z}_C}{\mathcal{Z}_{RL}+\mathcal{Z}_C} = \frac{25\angle 37^o.20\angle -90^o}{20+j15-j20}$
$= \frac{500\angle -53^o}{20-j5} = 24.3\angle - 39^o$
$= 18.9 - j15.3\Omega$

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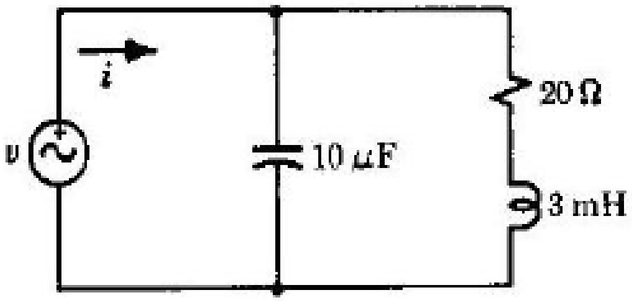

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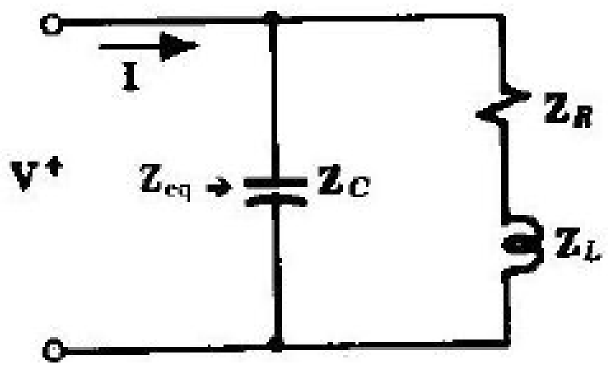

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$$= \frac{500\angle -53^o}{20 - j5} = 24.3\angle -39^o$$
$$= 18.9 - j15.3\Omega$$

$$I = \frac{V}{\mathcal{Z}} = \frac{12\angle 0^o}{24.3\angle -39^o} = 0.49\angle 39^o \text{ A}$$
$$i(t) = 0.49\sqrt{(2)}\cos(5000t + 39^o) \text{ A}$$

General Procedure

- Transform the time functions into phasors and element values into impedances/admittances.
- Combine impedances/admittances to simplify circuit.
- Draw the phasor diagram of the phasors of interest
- Compute the desired result in phasor form and transform to time function (if required)