ELL100: INTRODUCTION TO ELECTRICAL ENGG.

Complete Response - II

Course Instructors:
Manav Bhatnagar, Subashish Dutta, Debanjan Bhaumik, Harshan Jagadeesh
Department of Electrical Engineering, IITD

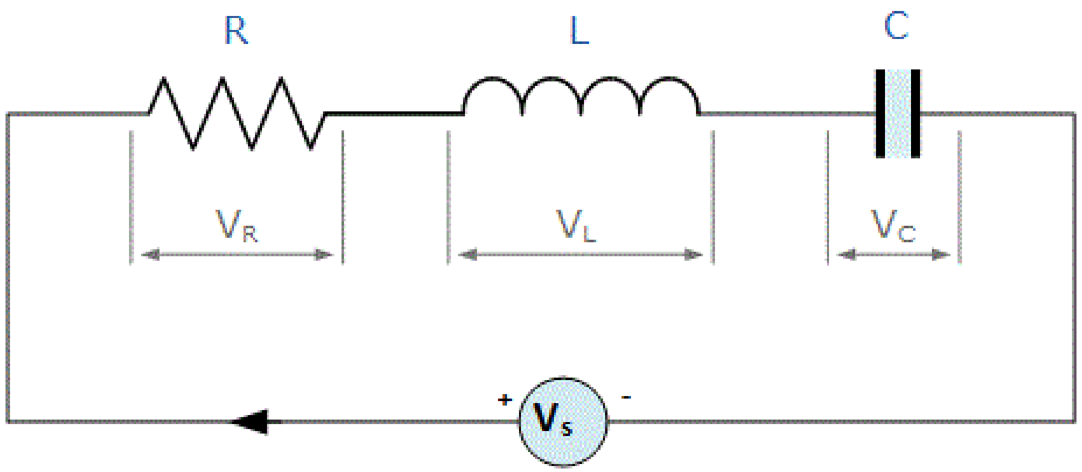
Complete Response

- If source (excitation) is exponential/sinusoidal :
  - Forced response – composed of exponentials present in source signal
  - Natural response – composed of exponentials depending on circuit components.
- EXCEPT when there are common modes
- For other excitations, ONLY

Complete Response = Natural Response + Forced Response

Example

$V_s(t) = 5e^{-t}$ with R = 4 Ω, L = 1 H, C = 1/3 F, i(0)=1 A, di/dt(0) = 3 A/s. What is i(t) for t >=0?



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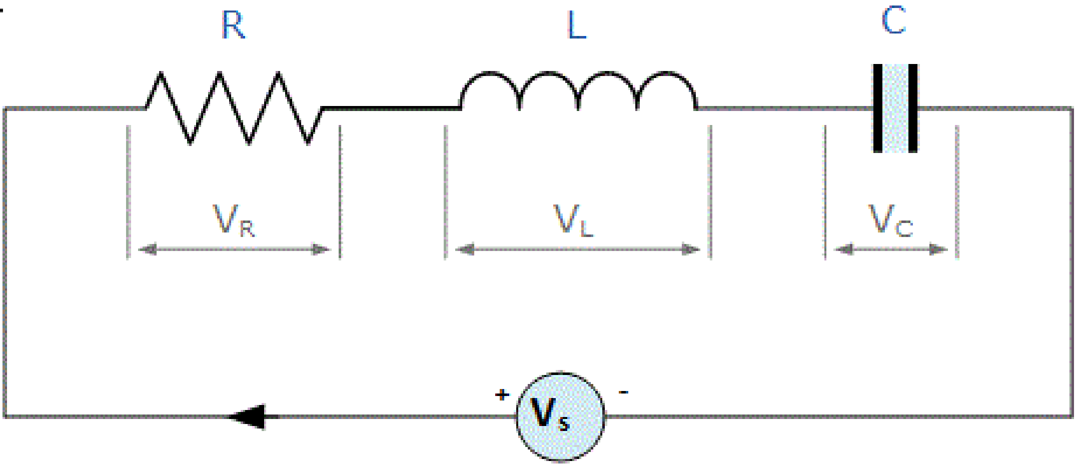
- The impedance function is

$$Z(\alpha) = \frac{\alpha^2 LC + \alpha RC + 1}{\alpha C} = \frac{\alpha^2 + 4\alpha + 3}{\alpha}$$
$$\alpha = -1$$
$$Z(\alpha) = 0$$



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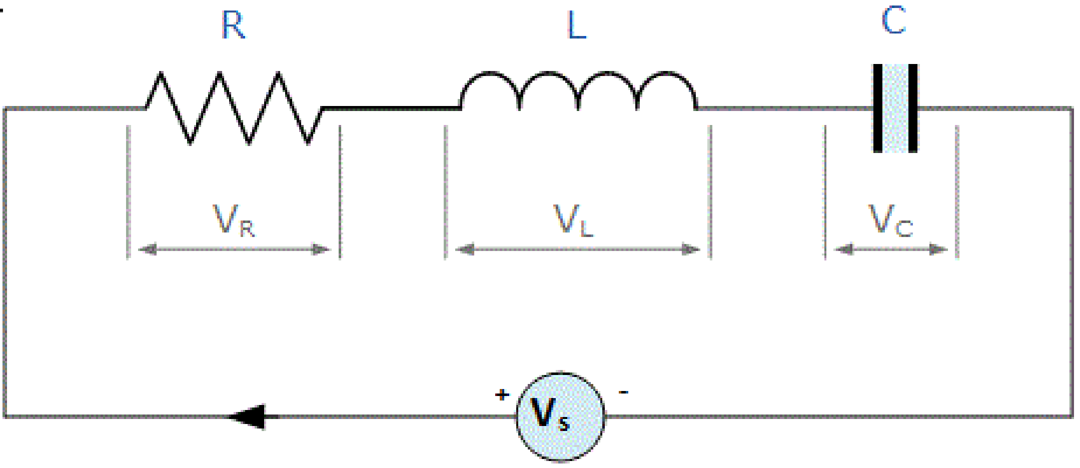
$$i(t) = \frac{V_s(t)}{Z(\alpha)} = ??$$



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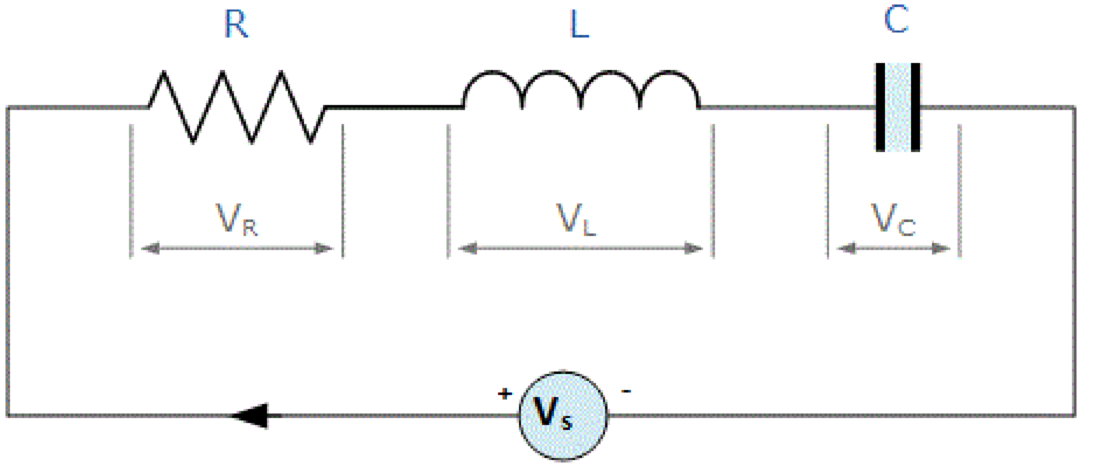

- We cannot determine i(t) using Impedance function.

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Solution to our example

The complete response will be $(A_1 + A_2 t)e^{-t} + A_3 e^{-3t}$

Particular Solution and Initial Conditions

- $V_s(t) = 5e^{-t}$ with R = 4 Ω, L = 1 H, C = 1/3 F, i(0)=1 A, di/dt(0) = 3 A/s.

- $$i(t) = (A_1 + A_2 t)e^{-t} + A_3 e^{-3t}$$
  $$\frac{di}{dt} = (-A_1 + A_2 - A_2 t)e^{-t} - 3A_3 e^{-3t}$$
  $$\frac{d^2 i}{dt^2} = (A_1 - 2A_2 + A_2 t)e^{-t} + 9A_3 e^{-3t}$$

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$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = \frac{d}{dt}V_s$$

$$\begin{aligned} -5e^{-t} &= \left((A_1 + A_2 t)e^{-t} + A_3 e^{-3t}\right) \\ &\quad +4\left((-A_1 + A_2 - A_2 t)e^{-t} - 3A_3 e^{-3t}\right) \\ &\quad +3\left((A_1 - 2A_2 + A_2 t)e^{-t} + 9e^{-3t}\right) \\ &= -2A_2 e^{-t} \quad \Longrightarrow \boxed{A_2 = 2.5} \end{aligned}$$

$$\boxed{\begin{array}{l} A_1 = 1.75,\ \ A_2 = 2.5,\ \ A_3 = -0.75 \\ i(t) = (1.75 + 2.5t)e^{-t} - 0.75e^{-3t} \end{array}}$$

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   For each e^{at}, find the maximum 'n' such that $t^n e^{at}$ appears.

   Make (n+1) copies of the exponent in the set of exponents for source.

   Eg: $(3t^2 + 2)e^{-t} + te^{-2t} + 2e^{-4t} \implies \{-1, -1, -1, -2, -2, -4\}$

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In our example problem, the polynomial is

$$s^2 + 4s + 3 = 0 \qquad\qquad \Longrightarrow \quad e^{-t},\ e^{-3t}$$

Thus, set corresponding to natural response is $\{-1, -3\}$

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