ELL101: INTRODUCTION TO ELECTRICAL ENG.

Network Theorems
(Thevenin's and Norton's Theorems)

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Introduction

- One of the main uses of Thevenin's and Norton's theorems is the replacement of a large part of a circuit, often a complicated and uninteresting part, with a very simple equivalent

- The new, simpler circuit enables us to make rapid calculations of the voltage, current, and power which the original circuit is able to deliver to a load

Introduction(contd.)

- It also helps us to choose the best value of this load resistance

- In a transistor power amplifier, for example, the Thévenin or Norton equivalent enables us to determine the maximum power that can be taken from the amplifier and delivered to the speakers

Thevenin's Theorem

- Thevenin's theorem tells us that it is possible to replace everything except the load resistor(R_L) with an independent voltage source(V_{TH}) in series with a resistor(R_{TH})

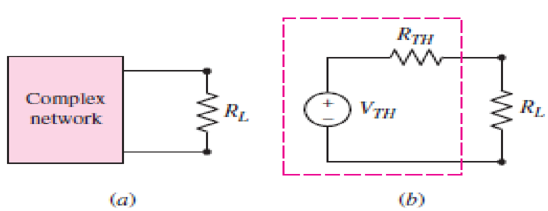


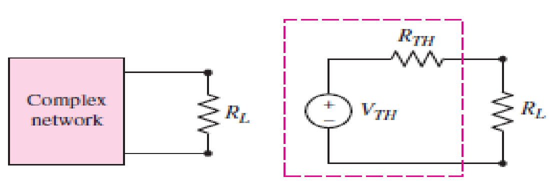
Fig. 1 (a) A complex network including a load resistor R_L. (b) A Thevenin equivalent network connected to the load resistor R_L.

Procedure To Determine Thevenin's Equivalent Circuit

- Make the Load resistor R_L as open circuit
- Now, define a voltage V_{TH} or V_{oc} (Thevenin's voltage) across the open circuited terminals of R_L and determine V_{TH} or V_{oc}
- V_{oc} is the voltage that could be measured by a voltmeter at the output
- To determine R_{TH} (Thevenin's resistance or the equivalent resistance seen from the open circuited terminals of R_L), we proceed as follows:
- Deactivate all the independent sources, i.e., short circuit the voltage source and open circuit the current source

Contd.

- Measure the resistance at across the open circuited terminals of R_L
- Finally, we can obtain the Thevenin's equivalent circuit as shown in Fig. 1(b)



(a) (b)

Problem 1(Independent source)

- Find V_X by first finding V_{TH} and R_{TH} to the left of A-B

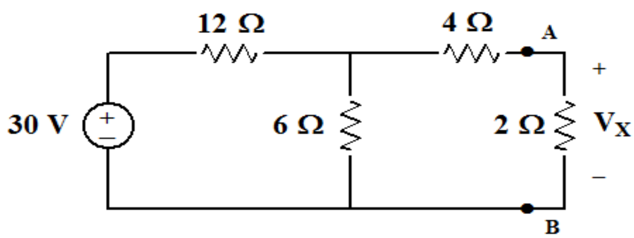


Fig. 2: Circuit for Problem 1

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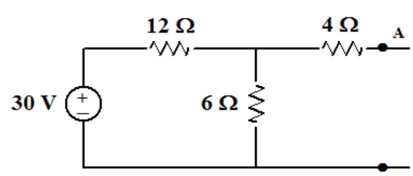
Solution

- We first open circuit the terminals of A-B to obtain V_{TH} as shown below:



- Using voltage division rule, we get:

$$V_{AB} = V_{TH} = \frac{(30)(6)}{6+12} = 10V$$

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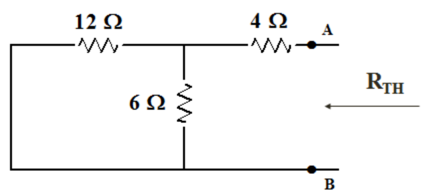
Solution(contd.)

- To get R_{TH}, we deactivate all the independent sources as described in the procedure as follows:



$$R_{TH} = 12||6 + 4 = 8\Omega$$

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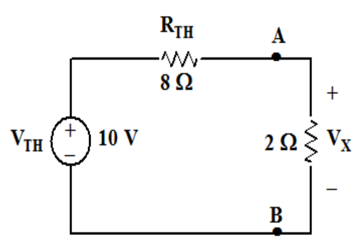
Solution(contd.)

- Now, the Thevenin's equivalent circuit can be obtained as:



Using voltage division rule, we get:

$$\therefore V_X = \frac{(10)(2)}{8+2} = 2V$$

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Problem 2(Scenario of Dependent Source)

- Determine the Thevenin equivalent of the circuit shown below:

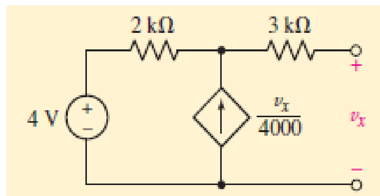


Fig.3: A given network whose Thevenin equivalent is desired

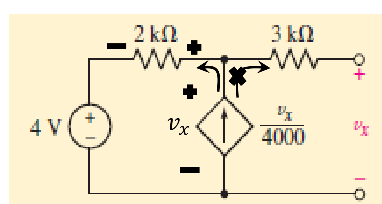
Solution

- To find V_{TH}, we note that $v_x = V_{TH}$ and that the dependent source current must pass through the $2k\Omega$ resistor, since no current can flow through the $3k\Omega$ resistor.
- Using KVL around the outer loop:

$$4 + 2 \times 10^3 \left(\frac{v_x}{4000}\right) = v_x$$

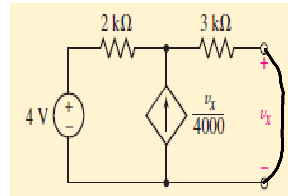
and

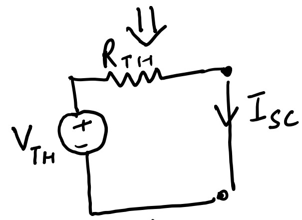
$$V_{TH} = v_x = 8V$$



Solution(contd.)

- The dependent source prevents us from determining directly R_{TH}
- Therefore, we short circuit the output terminals of the given circuit and determine the short-circuited current (I_{sc})
- Upon short-circuiting the output terminals in figure, it is apparent that $v_x = 0$ and therefore the dependent current source becomes inactive





$$\Rightarrow I_{sc} = \frac{V_{TH}}{R_{TH}}$$

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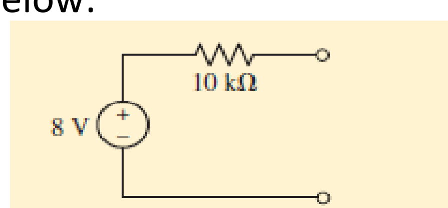
Solution(contd.)

- Hence, $I_{sc} = \frac{4}{5\times10^3} = 0.8\text{mA}$
- Thus, $R_{TH} = \frac{V_{TH}}{I_{sc}} = \frac{8}{0.8\times10^{-3}} = 10k\Omega$

- Therefore, the thevenin's equivalent circuit is as shown below:



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Norton's Theorem

- Using Norton's theorem, we obtain an equivalent composed of an independent current source(I_N) in parallel with a resistor(R_N)

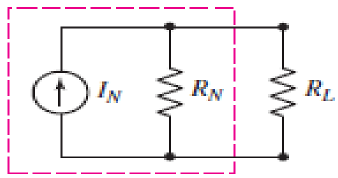

Fig.4: A Norton equivalent network connected to the load resistor R_L

Procedure To Determine Norton's Equivalent Circuit

- Make the Load resistor R_L as short circuit
- Now, define a current I_N or I_{sc} (Norton's current) across the short-circuited terminals of R_L and determine I_N or I_{sc}
- The procedure for determining R_N(Norton's equivalent resistance) is same as that of R_{TH}
- Finally, we can obtain the Norton's equivalent circuit as shown in Fig. 4

Problem 3 (Independent source)

- Find the Norton's equivalent circuit for the network faced by the $1k\Omega$ load resistor

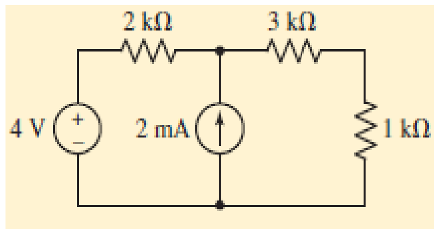

Fig. 5: Circuit for Problem 3

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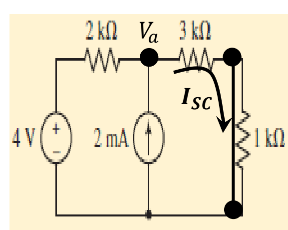
Solution

- We first define the node voltage (V_a)at the upper terminal of 2mA current source
- Now, we short circuit the $1k\Omega$ load resistor and determine the short-circuited current I_N or I_{sc} using nodal analysis as follows:

$$\frac{V_a - 4}{2k\Omega} - 2\text{mA} + \frac{V_a - 0}{3k\Omega} = 0$$

$$\Longrightarrow V_a = 4.8V$$

$$\therefore I_N = \frac{V_a}{3k} = 1.6\text{mA}$$



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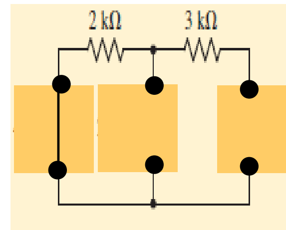
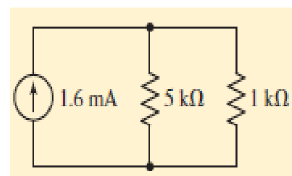
Solution(contd.)

- Now, to determine R_N, the $1\,k\Omega$ load resistor is open circuited
- Moreover, we deactivate all the independent sources, i.e., voltage source is short-circuited and current source is open-circuited
- We get: $R_N = 2k\Omega + 3k\Omega = 5k\Omega$ and the Norton's equivalent circuit is as shown below:





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Problem 4(Scenario of Dependent Source)

- For the circuit shown below, find the Norton's equivalent circuit to the left of terminals A-B

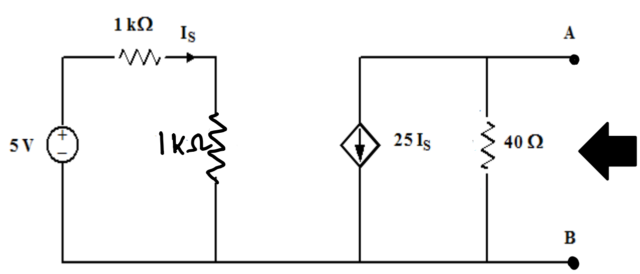

Fig. 6 Circuit for Problem 4

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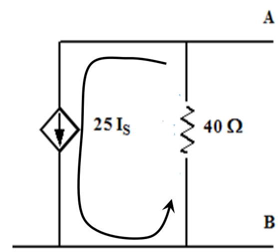
Solution

- In case of a dependent source, R_N is obtained as:

$$R_N = \frac{V_{OC}}{I_{SC}} = \frac{V_{AB}}{I_{SC}}$$

- Where $V_{OC} = V_{AB}$ is the open-circuited voltage across the load resistor
- We first find V_{AB}:

$$V_{AB} = (-25I_s)(40)$$
$$= -1000I_s$$



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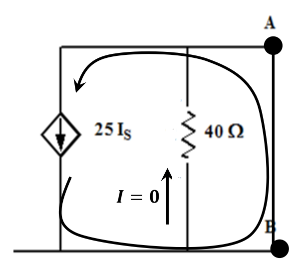
Solution(contd.)

- To find I_{sc}, the output terminals are short-circuited and due to this the entire current of $25I_s$ flows through the short-circuited arm
- Therefore, $I_{sc} = -25\ I_s$
- Hence, $R_N = \frac{V_{AB}}{I_{sc}} = \frac{-1000I_s}{-25\ I_s} = 40\Omega$



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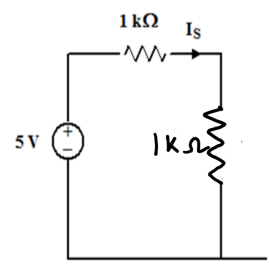
Solution(contd.)

- Note that we have obtained V_{AB} in terms of I_s,i.e., $V_{AB} = -1000I_s$
- Now, I_s can be obtained using mesh analysis as follows:

$$-5 + 1000I_s + 1000I_s = 0$$

$$\Longrightarrow I_s = 2.5\text{mA}$$

- Thus $I_{sc} = -25I_s = -62.5\text{mA}$



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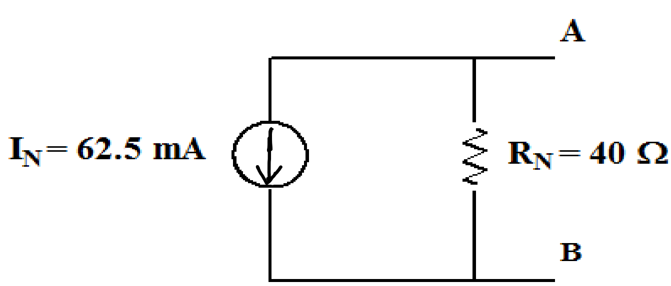
Solution(contd.)

- The Norton's equivalent circuit is as shown below:



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Problem 5 (Another Scenario of Dependent Source)

- For the circuit shown below, find the Norton's equivalent circuit

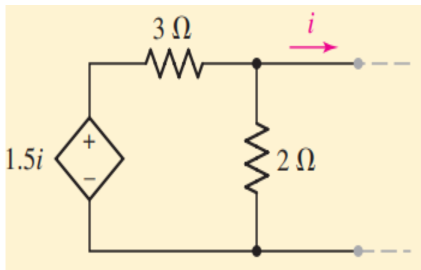
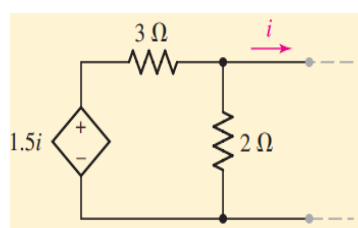


Fig. 7 Circuit for Problem 5

Solution

- Dependent voltage source depends upon the current i
- i=0 for short circuit or open circuit
- So the dependent voltage source of 1.5i value is inactive
- It is not possible to determine the value of I_{SC} and V_{OC}

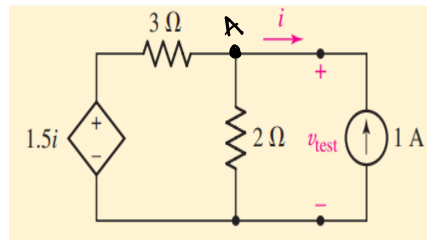


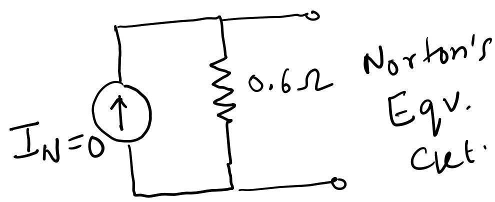
Solution(contd.)

- Let us apply a 1A current source at the O/P nodes
- Then $R_N = \frac{v_{test}}{1}$
- As $i = -1$ A
- Apply nodal analysis at A

$\frac{v_{test}-1.5(-1)}{3} + \frac{v_{test}}{2} = 1$

- $v_{test} = 0.6$V
- R_N =0.6 ohm





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