Decision Surface

DR. ANIL SINGH PARIHAR

Basis of a vector space:

Orthogonal basis:

$\bar{q}_1, \bar{q}_2, \bar{q}_n$

orthogonal Matrix

$$Q = \begin{bmatrix} | & | & & | \\ \bar{q}_1 & \bar{q}_2 & \cdots & \bar{q}_n \\ | & | & & | \end{bmatrix}$$

Orthonormal vectors: $\bar{q}_i^T \bar{q}_j = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$ $\quad \|q_i\| = 1$ for all i

Dot product: $\bar{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$ & $\bar{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$ $\quad \bar{a}.\bar{b} = a_1b_1 + a_2b_2 \cdots + a_nb_n$

$$\bar{a}^T.\bar{b} = \begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = a_1b_1 + a_2b_2 + \cdots + a_nb_n$$

$$a^Tb = b^Ta$$

Orthogonal basis:

$$Q = \begin{bmatrix} | & | & & | \\ \bar{q}_1 & \bar{q}_2 & \cdots & \bar{q}_n \\ | & | & & | \end{bmatrix}$$

$$Q^T Q = \begin{bmatrix} - & q_1^T & - \\ - & q_2^T & - \\ - & q_n^T & - \end{bmatrix} \begin{bmatrix} | & | & & | \\ \bar{q}_1 & \bar{q}_2 & \cdots & \bar{q}_n \\ | & | & & | \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & & 1 \end{bmatrix}}_{I}$$

$$Q^T Q = I \quad \text{--- (1)}$$

If Q is Square matrix, then $Q^{-1} Q = I$ --- (2)

$$\Rightarrow \boxed{Q^T = Q^{-1}}$$

Eigenvalues & Eigenvectors:

Sys. $f(\ )$ ← x input → $y = f(x)$ output

$$\begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = [\ ]$$

$A \to$ Matrix

$\bar{x} \to$ vector

In general $\underbrace{A\bar{x}}$ direction different from $\bar{x}$

But $A\bar{x} \parallel \bar{x} \to$ Eigenvector

$$A\bar{x} = \lambda \bar{x}$$

$\vec{x} \to$ Eigen vector

$\lambda \to$ Eigenvalue.

Note: if A is singular ie $\det(A) = 0$ then $\lambda = 0$

Eigenvalues & Eigenvectors:

Solution: $A\bar{x} = \lambda\bar{x}$

$\Rightarrow \quad (A - \lambda I)\bar{x} = 0$

if $\bar{x} \neq 0$, $\underline{A - \lambda I}$ must be Singular Matrix

$$\det(A - \lambda I) = 0$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots \\ a_{21} & & \\ & & \cdots \end{bmatrix}_{n \times n}$$

① Sum of λ's = Sum of diagonal element $= a_{11} + a_{22} + \cdots + a_{nn}$

② for $n \times n$ Matrix, there will be 'n' Eigenvalues

③ If matrix is Symmetric then eigenvalues will be Real

Matrix multiplication:

S

$$\begin{bmatrix} | & | & & | \\ c_1 & c_2 & \cdots & c_n \\ | & | & & | \end{bmatrix} \begin{bmatrix} | \\ b_1 \\ | \end{bmatrix} = \begin{bmatrix} | \\ Cb_1 \\ | \end{bmatrix}$$

C

$$\begin{bmatrix} | & | & & | \\ c_1 & c_2 & \cdots & c_n \\ | & | & & | \end{bmatrix} \begin{bmatrix} | & | & & | \\ b_1 & b_2 & \cdots & b_m \\ | & | & & | \end{bmatrix} = \begin{bmatrix} | & | & & | \\ Cb_1 & Cb_2 & \cdots & Cb_m \\ | & | & & | \end{bmatrix}$$

Diagonalization:

Suppose we have n linearly independent eigenvectors of A

$\bar{x}_1, \bar{x}_2, \ldots \bar{x}_n$ & eigenvalues $\lambda_1, \lambda_2, \ldots \lambda_n$

$$S = \begin{bmatrix} | & | & & | \\ \bar{x}_1 & \bar{x}_2 & \cdots & \bar{x}_n \\ | & | & & | \end{bmatrix} \qquad A \to \text{Original Matrix}$$

$$AS = A\begin{bmatrix} | & | & & | \\ \bar{x}_1 & \bar{x}_2 & \cdots & \bar{x}_n \\ | & | & & | \end{bmatrix} = \begin{bmatrix} | & | & & | \\ A\bar{x}_1 & A\bar{x}_2 & \cdots & A\bar{x}_n \\ | & | & & | \end{bmatrix} = \begin{bmatrix} | & | & & | \\ \lambda_1\bar{x}_1 & \lambda_2\bar{x}_2 & \cdots & \lambda_n\bar{x}_n \\ | & | & & | \end{bmatrix}$$

$$\underline{AS} = \begin{bmatrix} | & | & & | \\ \lambda_1\bar{x}_1 & \lambda_2\bar{x}_2 & \cdots & \lambda_n\bar{x}_n \\ | & | & & | \end{bmatrix} = \underbrace{\begin{bmatrix} | & | & & | \\ \bar{x}_1 & \bar{x}_2 & \cdots & \bar{x}_n \\ | & | & & | \end{bmatrix}}_{S} \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ 0 & \vdots & & \vdots \\ 0 & 0 & & \lambda_n \end{bmatrix} = \underline{S\Lambda}$$

$$\boxed{AS = S\Lambda}$$

Diagonalization:

$$AS = S\Lambda \quad \text{— } (1)$$

$[A]_{n\times n}$ Symmetric matrix

$$S^{-1}AS = S^{-1}S\Lambda = I\Lambda = \Lambda$$

$[S]_{n\times n}$ formed by eigenvect.

$$\boxed{S^{-1}AS = \Lambda}$$

$[\Lambda] \rightarrow$ Diagonal

$$ASS^{-1} = AI = A = S\Lambda S^{-1}$$

$$\boxed{A = S\Lambda S^{-1}}$$

Diagonalization:

Symmetric Matrix: $A^T = A$

① The eigenvalues are real

② The eigenvectors can be chosen to be perpendicular (othogonal eigenvectors)

$$A = S \Lambda S^{-1}$$

$$S = Q \ni Q^{-1} = Q^T$$

for symmetric $A = Q \Lambda Q^{-1}$

$$\boxed{A = Q \Lambda Q^T}$$

Norms:

A norm on $\mathbb{R}^n$ is a real-valued function

$$\|\cdot\| : \mathbb{R}^n \longrightarrow \mathbb{R}$$

which obeys

- $\|\bar{x}\| \geq 0 \quad \forall x \in \mathbb{R}^n$ & $\|\bar{x}\| = 0$ iff $\bar{x} = \bar{0}$
- $\|\alpha \bar{x}\| = |\alpha| \, \|\bar{x}\|$
- $\|\bar{x} + \bar{y}\| \leq \|\bar{x}\| + \|\bar{y}\|$

Norms: (popular norms) : $\bar{x} \in \mathbb{R}^n$

- L_2 or Euclidean norm

$$\|\bar{x}\|_2 = \left[\sum_{i=1}^{n} (x_i)^2\right]^{1/2}$$

- L_1 norm

$$\|\bar{x}\|_1 = \sum_{i=1}^{n} |x_i|$$

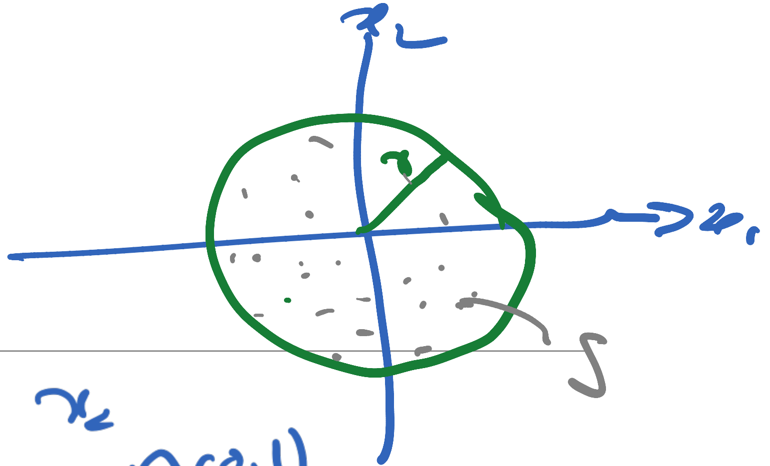
- L_∞ norm

$$\|\bar{x}\|_\infty = \max_{i=1,2,\ldots n} |x_i|$$

Norm:

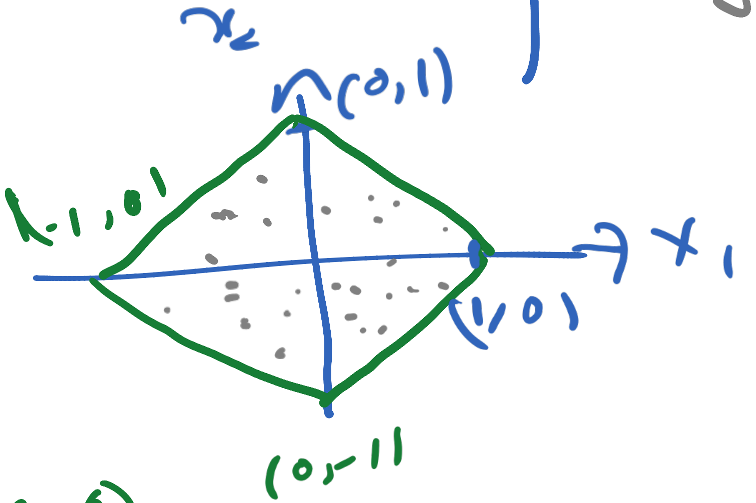
① $S = \{\bar{x} \in \mathbb{R}^2 : \|\bar{x}\|_2 \le r\}$

$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

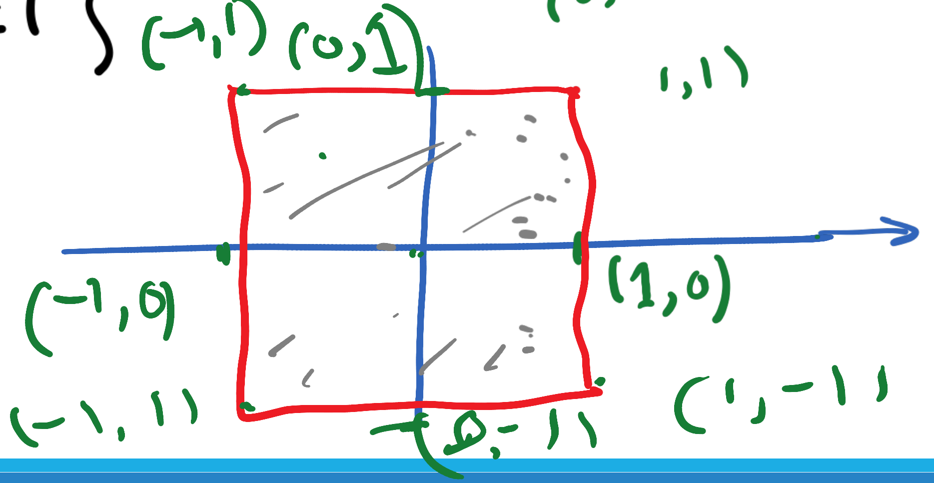


② $S = \{\bar{x} \in \mathbb{R}^2 : \|\bar{x}\|_1 \le 1\}$

$x_1 + x_2 = 1 \Rightarrow x_2 = 1 - x_1$



③ $S = \{\bar{x} \in \mathbb{R}^2 : \|\bar{x}\|_\infty \le 1\}$



Norm:

In general; L_p norm $\quad 1 \le p < \infty$

$$\|\bar{X}\|_p = \left[\sum_{i=1}^{n} |x_i|^p\right]^{1/p}$$

Inner Product:

$$\bar{x}, \bar{y} \in \mathbb{R}^n \quad , \& \quad \bar{x} \neq 0 , \; \bar{y} \neq 0$$

$$\bar{x} \cdot \bar{y} = \bar{x}^T \bar{y} = \sum_{i=1}^{n} x_i y_i = \|\bar{x}\| \cdot \|\bar{y}\| \cos\theta$$

$\theta \rightarrow$ angle between $\bar{x}$, & $\bar{y}$

Note

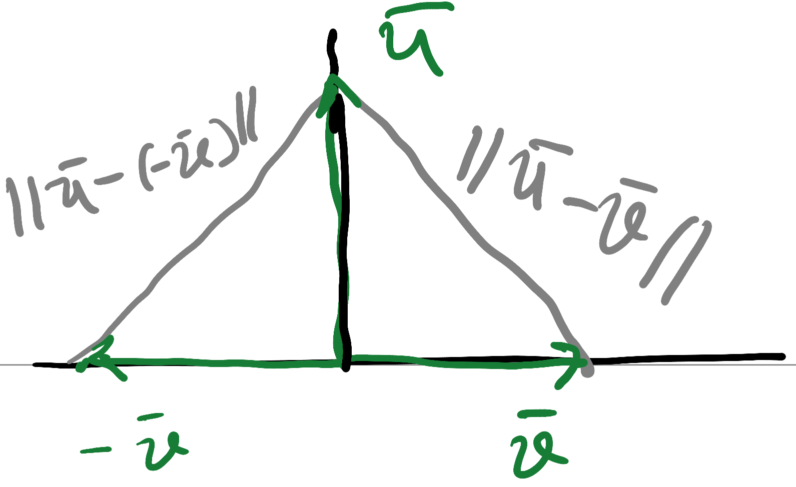
① $\bar{x}^T \bar{x} = \|\bar{x}\|^2$

② $\bar{x}^T \bar{y} = \bar{y}^T \bar{x}$

③ $\|\bar{x} \cdot \bar{y}\| \leq \|\bar{x}\| \; \|\bar{y}\|$

Orthogonality:

$\bar{u}$ & $\bar{v} \in \mathbb{R}^n$

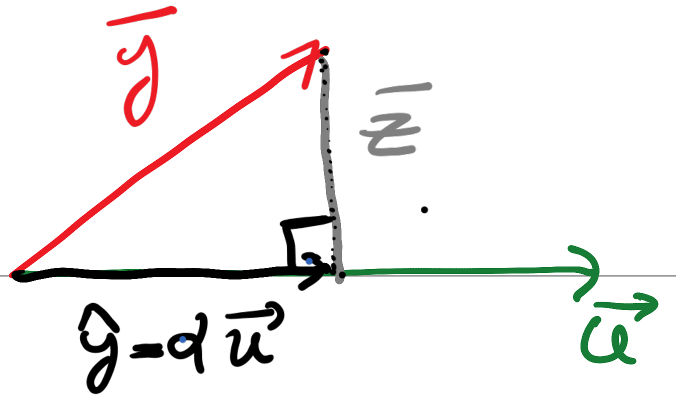


$\bar{u}$ & $\bar{v} \neq 0$

if $\bar{u}^T\bar{v} = 0 \Rightarrow \bar{u}$ & $\bar{v}$ are $\perp$

Orthogonality:

Orthogonal Projection:



$\|\bar{u}\| = 1$

$u^T u = \|u\|_2^2 = 1$

$\bar{y} = \hat{y} + \bar{z}$ $\qquad \hat{y} = \alpha \bar{u}$

$= \alpha \bar{u} + z \quad \Rightarrow \bar{z} = \bar{y} - \alpha \bar{u}$

$\hat{y} \cdot \bar{z} = 0$ or $\bar{u} \cdot \bar{z} = 0$

$\bar{u} \cdot (\bar{y} - \alpha \bar{u}) = 0 \; ; \quad \bar{u}^T (\bar{y} - \alpha \bar{u}) = 0$

$\Rightarrow \alpha = \dfrac{u^T y}{u^T u} = \dfrac{y^T u}{u^T u} \Rightarrow \underline{\underline{\alpha = u^T y}}$

Change of Basis:

$$\hat{y} = \alpha \bar{u}$$

$$\hat{y} = u u^T \bar{y} \qquad \text{if } u \text{ ~~is~~ un}$$

$$\hat{y} = \bar{u} \frac{\bar{y}^T \bar{u}}{\bar{u}^T \bar{u}} = \frac{\bar{u} u^T}{\bar{u}^T \bar{u}} y$$

Change of Basis:

B basis for vector subspace (k-dimensional) of $\mathbb{R}^n$

$$B = \{\bar{v}_1, \bar{v}_2, \ldots \bar{v}_k\}$$

Let vector $a \in \mathbb{R}^n$

$$\bar{a} = c_1 \bar{v}_1 + c_2 \bar{v}_2 + \ldots + c_k \bar{v}_k = \underbrace{\begin{bmatrix} | & | & & | \\ \bar{v}_1 & \bar{v}_2 & \cdots & \bar{v}_k \\ | & | & & | \end{bmatrix}}_{C_{n \times k}} \underbrace{\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_k \end{bmatrix}}_{k \times 1}$$

$$[\bar{a}]_B = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_k \end{bmatrix}_{k \times 1}$$

$$\bar{a} = C_{n \times k} [\bar{a}]_B$$

change of Basis Matrix

Random Variables:

Example:

$\bar{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\bar{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $B = \{\bar{v}_1, \bar{v}_2\}$

$\bar{a} \in \mathbb{R}^3$ $\quad [\bar{a}]_B = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$ $\quad$ find $\bar{a} = ?$

$$\bar{a} = \underbrace{\begin{bmatrix} v_1 & v_2 \end{bmatrix}}_{C} [a]_B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ -4 \end{bmatrix} = 7\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - 4\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$[\bar{a}]_{\mathcal{E}} = \begin{bmatrix} 3 \\ 14 \\ 17 \end{bmatrix} \qquad [\bar{a}]_B = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$$

Change of basis:

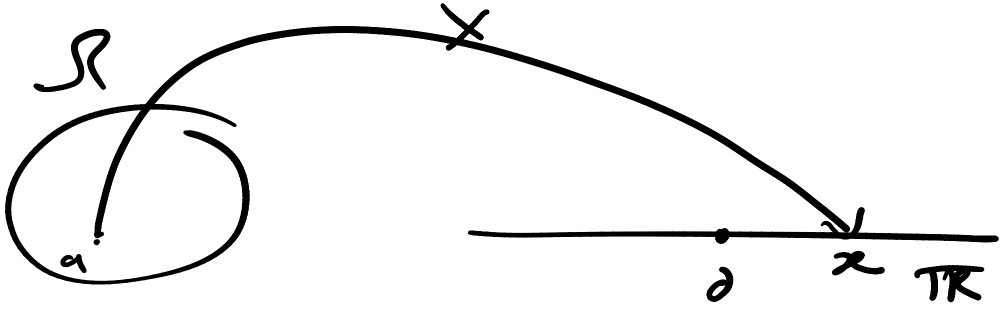
$\bar{d} = \begin{bmatrix} 8 \\ -6 \\ 2 \end{bmatrix}$ in $\mathbb{R}^3$, $\bar{d}$ is in a subspace $B = \{v_1, v_2\}$ as well

Find $[d]_B = ?$

Random Variables:

Scalar

$$X : \Omega \longrightarrow \mathbb{R}$$

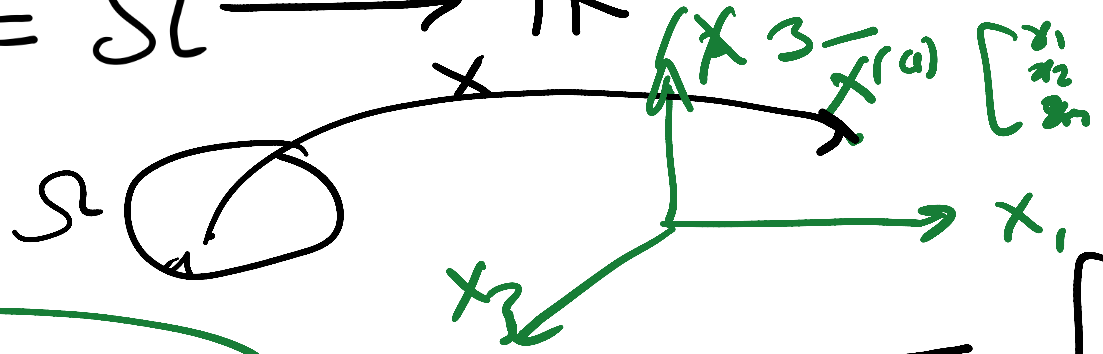


$$\Omega = \{H, T\} \qquad X = \begin{cases} 0 & \text{if } H \\ 1 & \text{if } T \end{cases}$$

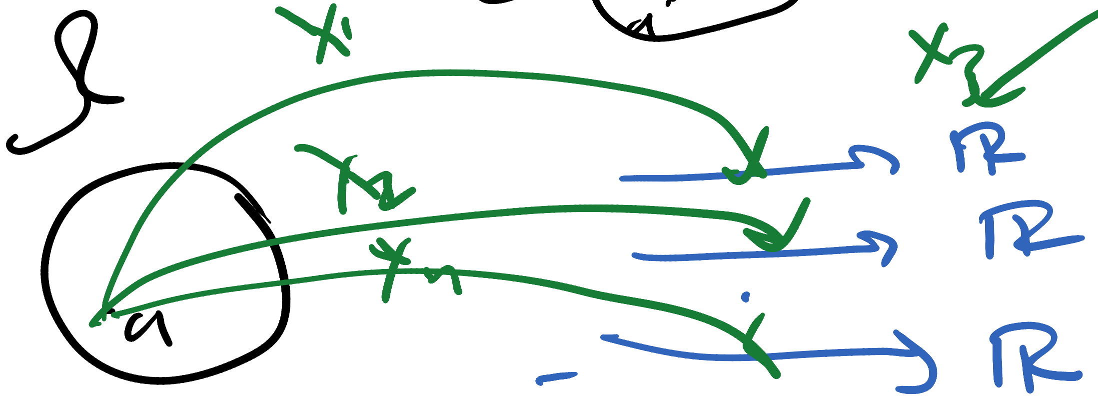
Random vector

$$\bar{X} = \Omega \longrightarrow \mathbb{R}^n$$

(1)



(2)



$$\bar{X} = \begin{bmatrix} X_1(a) \\ X_2(a) \\ \vdots \\ X_n(a) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Random Vectors:

Expectation

Scalar (1) $X \in \{x_1, x_2, \ldots x_m\}$

$$E[X] = \frac{1}{m}\sum_{i=1}^{m} x_i = \frac{x_1 + x_2 + \cdots + x_m}{m}$$

$$E[\bar{X} + \bar{Y}] = E[\bar{X}] + E[\bar{Y}]$$

$$E[\alpha \bar{X}] = \alpha E[\bar{X}]$$

Random Variables:

Random Vector

$$\bar{X} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}$$

n - dimensional

$$E[\bar{X}] = \begin{bmatrix} E[X_1] \\ E[X_2] \\ \vdots \\ E[X_n] \end{bmatrix}$$

$$E[\bar{X}_1 + \bar{X}_2] = E[\bar{X}_1] + E[\bar{X}_2]$$

$$E[a\bar{X}] = aE[\bar{X}]$$

$$E[\tilde{A}\bar{X}] = \tilde{A}E[\bar{X}]$$

$\tilde{A} \rightarrow$ Constant Matrix

Random Variables: Variance & Covariance

Scalars

$$\sigma^2(X) = \frac{\sum_{i=1}^{m} (x_i - \mu)^2}{m} = \frac{\sum_{i=1}^{m} (x_i - \mu)(x_i - \mu)}{m}$$

$$\bar{\mu} = \frac{\sum_{i=1}^{m} (x_i)}{m}$$

$$Var(X) = E[(X - \mu)^2] \quad ; \quad \mu = E[X]$$

$$= E[(X - \mu)(X - \mu)]$$

Random Variables: Variance & Covariance

$X \qquad X \in \{x_1, x_2 \ldots x_m\}$

$Y \qquad Y \to \{y_1, y_2, \ldots y_m\}$

$\mu_X = E[X]$

$\mu_Y = E[Y]$

$$\mathrm{cov}(X,Y) = \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_X)(y_i - \mu_Y)$$

$$\mathrm{Var}(X) = \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_X)(x_i - \mu_X)$$

$$= \mathrm{Cov}(X, X)$$

Note: $\mathrm{Cov}(X,Y) = \mathrm{Cov}(Y,X)$

Random Variables: Variance & Covariance

Vectors

$$\bar{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

$$Var(\bar{X}) = \begin{bmatrix} Var(x_1) & cov(x_1, x_2) & \cdots & cov(x_1, x_n) \\ cov(x_2, x_1) & \cdots & & \\ \vdots & & \ddots & \\ cov(x_n, x_1) & & \cdots & Var(x_n) \end{bmatrix}$$

m samples of random vector $\bar{X}$

i.e. $\{\bar{X}^{(1)}, \bar{X}^{(2)}, \ldots\ldots, X^{(m)}\}$

$$Var(\bar{X}) = \frac{1}{(m-1)}(XX^T)$$

A, B, C

$Var(A), Var(B), Var(C)$

$cov(A, B), cov(B, C)$

$cov(A, C)$

$$M = \begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} Var(A) & \underline{Cov(A,B)} & \underline{Cov(A,C)} \\ \underline{Cov(B,A)} & Var(B) & \underline{Cov(B,C)} \\ Cov(C,A) & \underline{Cov(C,B)} & Var(C) \end{bmatrix} \end{array}$$

symmetric

$=$

Random Variables: Variance & Covariance

$$Var(\bar{X}) = E[(\bar{X} - \bar{u})(\bar{X} - \bar{u})^T]$$

$$= \frac{1}{m-1} \bar{X} \bar{X}^T$$

$$\bar{X}^{(1)} = \begin{bmatrix} X_1^{(1)} \\ X_2 \\ \vdots \\ X_n^{(1)} \end{bmatrix}, \ldots, \bar{X}_n^{(m)} = \begin{bmatrix} X_1^{(m)} \\ X_2^{(m)} \\ \vdots \\ X_n^{(m)} \end{bmatrix}$$

$$\bar{u} = \begin{bmatrix} \frac{X_1^{(1)} + X_1^{(2)} + \cdots + X_1^{(m)}}{m} \\ \vdots \end{bmatrix}$$

$$\bar{X} = \begin{bmatrix} | & & | \\ \bar{X}^{(1)} & \cdots & X^{(m)} \\ | & & | \end{bmatrix}$$

S.N	angle	o/p	..	..
1	. .	. .	-	.
2	.	- -	—	.
i	-	—	..	
⋮				

If row represents an vector (entry)

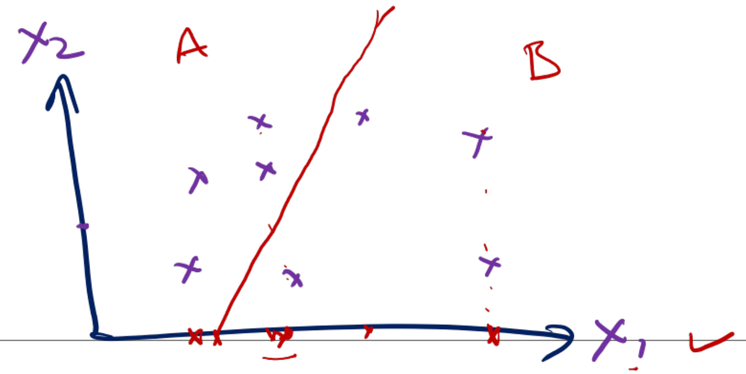
$X = X^T$ in above eqn

$$Var(X) = \frac{1}{m-1} X^T X$$

Random Variables: Variance & Covariance

Random Variables: Variance & Covariance

Principal Component Analysis:



Basis

$$\bar{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \in \mathbb{R}^3$$

standard Basis of $\mathbb{R}^3$ $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \begin{matrix} c_1 = 1 \\ c_2 = 2 \\ c_3 = 3 \end{matrix}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Principal Component Analysis:

Another Basis $\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}0\\0\\3\end{bmatrix}$

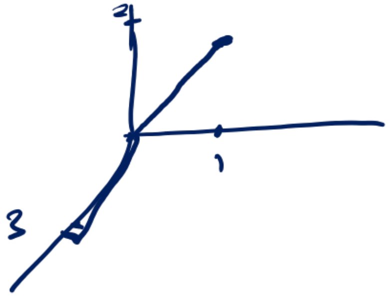
$[a]_B = \begin{pmatrix}1\\1\end{pmatrix}$

$$\begin{bmatrix}1\\2\\3\end{bmatrix} = x_1\begin{bmatrix}1\\2\\0\end{bmatrix} + x_2\begin{bmatrix}0\\0\\3\end{bmatrix}$$

$x_1 = 1$

$x_2 = 1$

$\begin{bmatrix}1\\2\\0\end{bmatrix}$ & $\begin{bmatrix}0\\0\\3\end{bmatrix}$

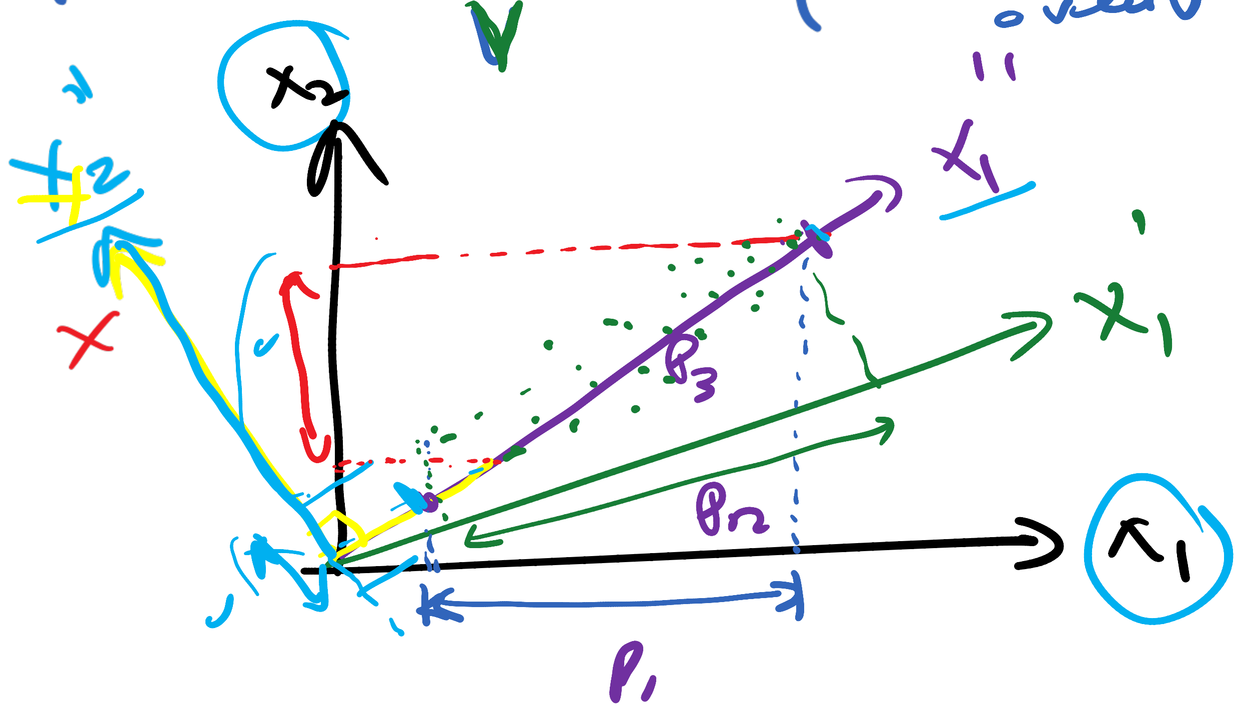


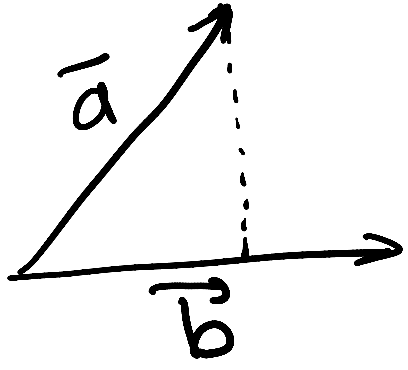
$$\begin{bmatrix}1\\2\\3\end{bmatrix} = c_1\begin{bmatrix}1\\PC_1\\1\end{bmatrix} + c_2\begin{bmatrix}1\\PC_2\\1\end{bmatrix} + \cdots$$

Principal Component Analysis

Maximize the variance of Projection of the data on the principal (basis) vectors

1st PC : max var (Proj of D over V)





$$\frac{a^T b}{b^T b}$$

Principal Component Analysis

m points. (data given).

$$x^{(i)} = \begin{bmatrix} x_1^{(i)} \\ x_2^{(i)} \\ \vdots \\ x_m^{(i)} \end{bmatrix}_{n\times 1}$$

$$X = \begin{bmatrix} | & | & & | \\ \bar{x}^{(1)} & \bar{x}^{(2)} & \cdots & \bar{x}^{(m)} \\ | & | & & | \end{bmatrix}_{n\times m}$$

let cons[i]d a new base

$$V = \begin{bmatrix} | & | & & | \\ \bar{v}_1 & \bar{v}_2 & \cdots & \bar{v}_n \\ | & | & & | \end{bmatrix}$$

$$X = VY$$

$$Y = \begin{bmatrix} | & | & & | \\ y^{(1)} & y^{(2)} & \cdots & y^{(m)} \\ | & | & & | \end{bmatrix}$$

$$\bar{x}^{(i)} = V \bar{y}^{(i)}$$

$$\Rightarrow \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix} = \underbrace{\begin{bmatrix} | & | & & | \\ v_1 & v_2 & \cdots & v_n \\ | & | & & | \end{bmatrix}}_{n\times n} \begin{bmatrix} y^{(1)} & y^{(2)} & \cdots & y^{(m)} \end{bmatrix}$$

Principal Component Analysis

$$\underset{n\times 1}{\begin{bmatrix} \bar{x}_1 \end{bmatrix}} = \underset{n\times p}{\begin{bmatrix} u_1 & \cdots & u_p \end{bmatrix}} \underset{p\times 1}{\begin{bmatrix} \bar{y}_1 \end{bmatrix}}$$

$$p < n$$

$$X = UY$$

$$\Rightarrow \underline{Y = U^{-1} \underline{\underline{X}}}$$

Principal Component Analysis

Decorrelation : data X

$$\Sigma_x = E[(\bar{X} - \bar{\mu}_x)(\bar{X} - \bar{\mu}_x)^T]$$

if data is mean adjusted $\bar{\mu}_x = 0$

$$\Sigma_x = E[\bar{X}\bar{X}^T] = \frac{1}{n-1}[XX^T]$$

Σ_x is not diagonal matrix.

$$\underset{\text{correl}}{X} \longrightarrow \underset{\text{decorrelated}}{Y}$$

Principal Component Analysis

$$E[YY^T] = \frac{1}{n-1} YY^T = D \rightarrow \text{diagonal Matx.}$$

$$X \rightarrow \Sigma_X \rightarrow \text{eigen vectors } (e_1, e_2 \ldots e_n) \; \Phi$$

$$\Phi = \underbrace{\begin{bmatrix} | & | & & | \\ e_1 & e_2 & \cdots & e_n \\ | & | & & | \end{bmatrix}}_{} \rightarrow \text{change of base vec.}$$

$$\Rightarrow \quad \Phi^{-1} = \Phi^T$$

$$X = \Phi Y \Rightarrow Y = \Phi^{-1} X = \Phi^T X$$

Principal Component Analysis

$$Y = \phi^T x$$

$$E[YY^T] = E[\phi^T x (\phi^T x)^T]$$

$$= E[\phi^T \underbrace{x x^T} \phi]$$

$$= \frac{1}{n-1} \phi^T x x^T \phi = \phi^T \frac{x x^T}{n-1} \phi$$

$$= \phi^T \Sigma_x \phi = \Delta$$

Principal Component Analysis