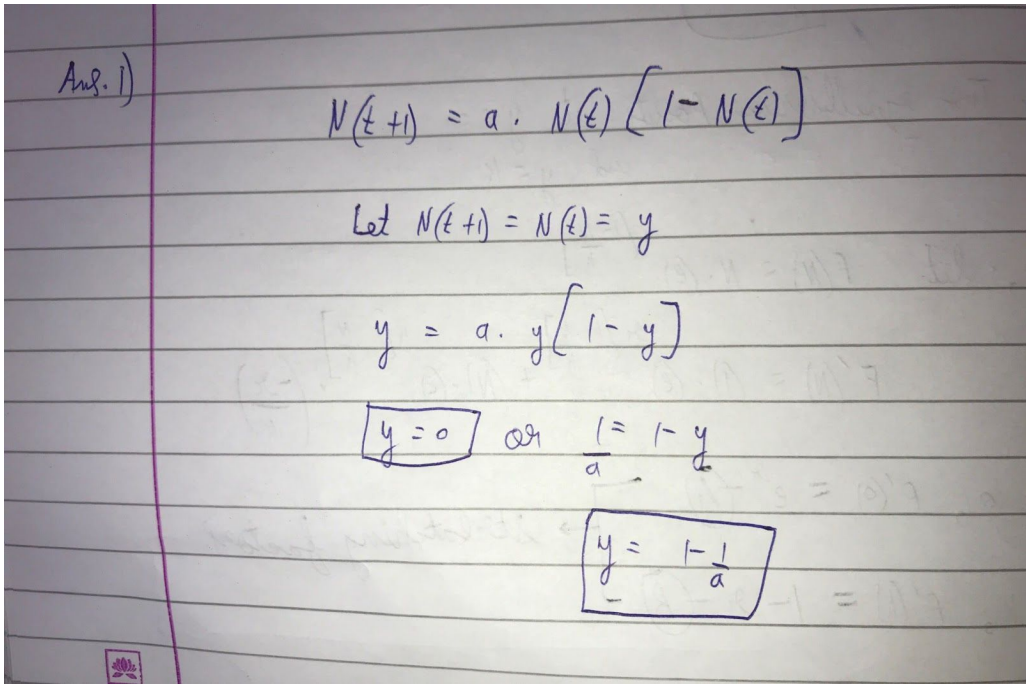
Assignment 1

Answer 1.



∴ Two eq^{n} points, $y = 0$

$$y = 1 - \frac{1}{a}$$

Now, let $F(N) = a \cdot N \cdot [1 - N]$

$$F'(N) = a - 2aN$$

At $F'(0) = a$ → Stretching factors.

At $F'\left[1 - \frac{1}{a}\right] = a - 2a\left(\frac{a-1}{a}\right) = a - 2a + 2 = 2 - a$

(A) **Stable Equilliberium.**

↓

$0 < \lambda < 1$

For $\lambda = a$

$0 < a < 1$ //

For $\lambda = 2 - a$

$0 < 2 - a < 1$

$-2 < -a < -1$

$2 > a > 1$ //

(B) **Stable oscillations.**

↓

$-1 < \lambda < 0$

For $\lambda = a$

$-1 < a < 0$ //

We will not consider this case, because in this a is negative.

For $\lambda = 2 - a$

$-1 < 2 - a < 0$

$-3 < -a < -2$

$3 > a > 2$ //

Date :
Page No. :

(c) Unstable oscillations.

$-1 > \lambda$

For $\lambda = a$

$-1 > a$ //

As "a" is negative, we will not consider this case.

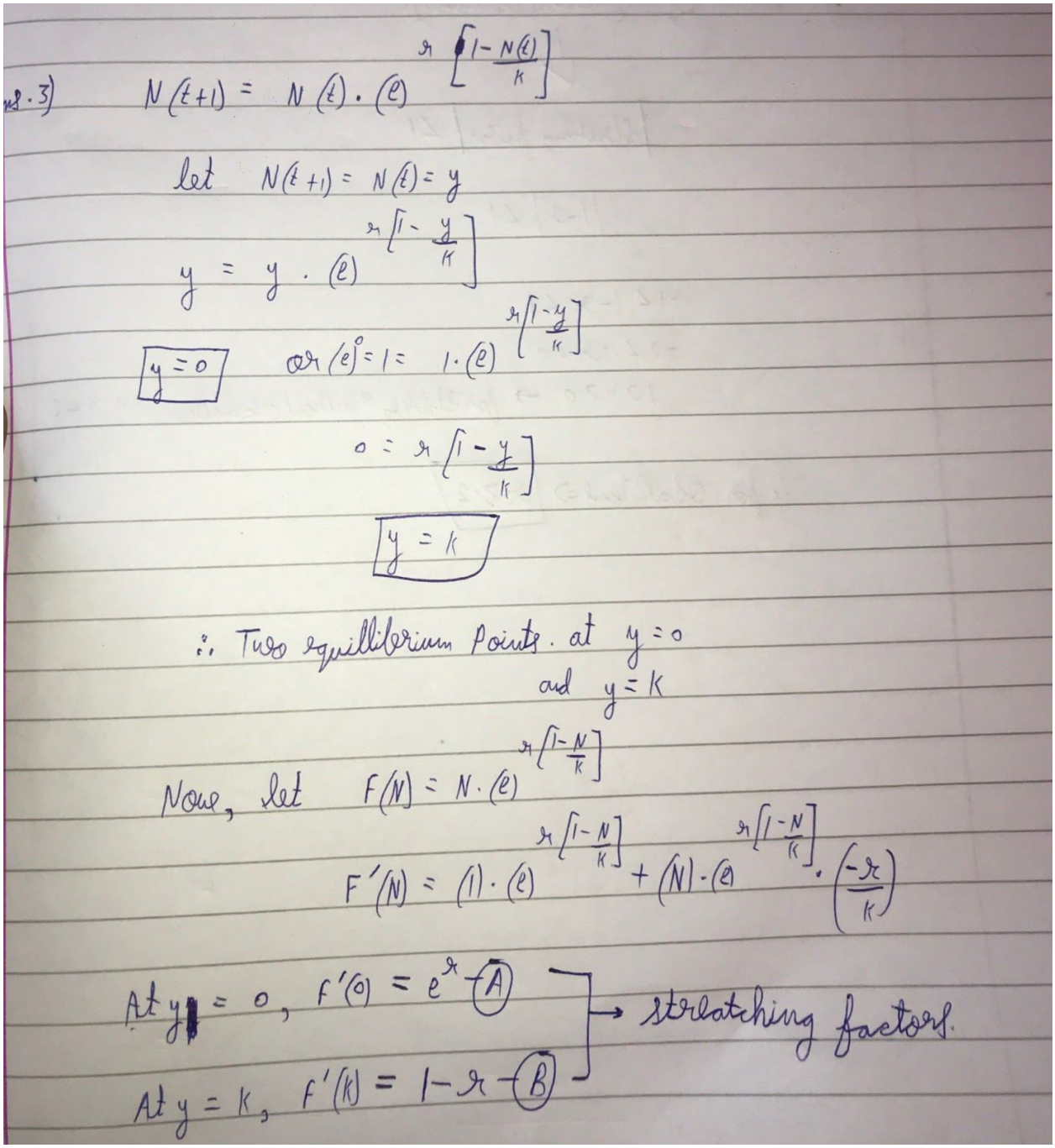
For $\lambda = 2 - a$

$-1 > 2 - a$

$-3 > -a$

$3 < a$ //

Answer 3.


Q.3) $N(t+1) = N(t) \cdot (e)^{r\left[1-\frac{N(t)}{K}\right]}$
let $N(t+1) = N(t) = y$
$y = y \cdot (e)^{r\left[1-\frac{y}{K}\right]}$
$y = 0$ or $(e)^0 = 1 = 1 \cdot (e)^{r\left[1-\frac{y}{K}\right]}$
$0 = r\left[1-\frac{y}{K}\right]$
$y = K$
∴ Two equilibrium Points. at $y = 0$ and $y = K$
Now, let $F(N) = N \cdot (e)^{r\left[1-\frac{N}{K}\right]}$
$F'(N) = (1) \cdot (e)^{r\left[1-\frac{N}{K}\right]} + (N) \cdot (e)^{r\left[1-\frac{N}{K}\right]} \cdot \left(\frac{-r}{K}\right)$
At $y = 0$, $F'(0) = e^r$ (A)
At $y = K$, $F'(K) = 1 - r$ (B)
→ stretching factors.

~~for oscillations to exhibit, stretching~~

For stretching factor = $1-r$

for stable Eq^{n}. [without oscillation].

$\Downarrow$

$|\text{Stretching factor}| < 1$

$|1-r| < 1$

$-1 < 1-r < 1$

$-2 < -r < 0$

$2 > r > 0$ → for stable Eq^{n} without oscillations, $0 < r < 1$

$\therefore$ for oscillations $\Rightarrow \boxed{r \geq 2}$

Answer 4

Ans. 4)

$$N(t) = \frac{K}{1 + \left[\frac{K - N(0)}{N(0)}\right] e^{-rt}} \quad \text{--- (1)}$$

manipulating (1)

$$\left[\frac{K - N(0)}{N(0)}\right] e^{-rt} = \frac{K}{N(t)} - 1 \quad \text{--- (2)}$$

Put $t \to t+1$ in (1)

$$N(t+1) = \frac{K}{1 + \frac{\left[\frac{K - N(0)}{N(0)}\right] (e)^{-rt}}{e^{r}}}$$

$$N(t+1) = \frac{K \cdot (e)^{r}}{(e)^{r} + \left[\frac{K - N(0)}{N(0)}\right] (e)^{-rt}}$$

$$N(t+1) = \frac{K \cdot (e)^{r}}{(e)^{r} + \left[\frac{K}{N(t)} - 1\right]} \qquad \text{[from (2)]}$$

$$N(t+1) = \frac{K}{1 + (e)^{-r}\left[\frac{K}{N(t)} - 1\right]} = \frac{K}{1 - \left[1 - \frac{K}{N(t)}\right](e)^{-r}}$$

Answer 2.

```
# Harshit Rai
# 2017152

import numpy as np
import matplotlib.pyplot as plt

# Given function
def func_gen(a, Nt):
    return a*Nt*(1-Nt)

# For first part
def part_a(a, start, gra=None):
    gra.set_title(f"$a={a:.1f}, x_0={start:.1f}, Part A$")
    x=0
    graph = np.linspace(0,1,200)
    t = func_gen(a,graph)
    gra.plot(graph, t, 'k', lw=1)
    gra.plot([0,1], [0,1],'k', lw=1)

    x_p_1=0
    x=start
    for k in range(200):
        x_p_1 = func_gen(a,x)
        gra.plot([x,x], [x, x_p_1], 'k', lw=1)
        gra.plot([x, x_p_1], [x_p_1, x_p_1], 'k', lw=1)
        x = x_p_1

# For third part
def part_c(start, gra=None):
    gra.set_title(f"$x_0={start:.1f},Part C$")
    x=start
```

```
    a = np.linspace(1.4,4,260)
    for i in range(160):
        x_p_1 = func_gen(a,x)
        if (i>=100):
            gra.plot(a, x, 'k', lw=1)
        x=x_p_1

# For second part
def b_part(a, start, gra=None):
    gra.set_title(f"$a={a:.1f}, x_0={start:.1f}, Part B$")
    x=0
    arre2 = [];
    arre = np.linspace(0,1, 100)
    val=0
    x=start
    for k in range(100):
        x_p_1 = func_gen(a,x)
        arre2.append(x_p_1)
        x = x_p_1
    gra.plot(arre, arre2, 'k', lw=1)

# Plotting Graphs
fig_1, (gra1) = plt.subplots(1, 1, figsize=(6, 6))
fig_2, (gra2) = plt.subplots(1, 1, figsize=(6, 6))
fig_3, (gra3) = plt.subplots(1, 1, figsize=(6, 6))
fig_a, (grgra) = plt.subplots(1, 1, figsize=(6, 6))
fig_4, (gra4) = plt.subplots(1, 1, figsize=(6, 6))
fig_4, (gra5) = plt.subplots(1, 1, figsize=(6, 6))
fig_4, (gra6) = plt.subplots(1, 1, figsize=(6, 6))
fig_b, (gray) = plt.subplots(1, 1, figsize=(6, 6))
fig_5, (gra7) = plt.subplots(1, 1, figsize=(6, 6))

gra1.set_ylim(0,1)
```

```
gra1.set_xlim(0,1)
grgra.set_ylim(0,1)
grgra.set_xlim(0,1)
gray.set_xlim(0,1)
gray.set_ylim(0,1)
gra2.set_ylim(0,1)
gra2.set_xlim(0,1)
gra3.set_ylim(0,1)
gra3.set_xlim(0,1)
gra4.set_ylim(0,1)
gra4.set_xlim(0,1)
gra5.set_ylim(0,1)
gra5.set_xlim(0,1)
gra6.set_ylim(0,1)

part_a(0.1, .1, gra=gra1)
part_a(1.356, .1, gra=gra2)
part_a(2.9, .1, gra=gra3)
part_a(4.1, .1, gra=grgra)
b_part(0.1, .1, gra=gra4)
b_part(1.356, .1, gra=gra5)
b_part(2.9, .1, gra=gra6)
b_part(4.1, .1, gra=gray)
part_c(.1, gra=gra7)

plt.show()
```