

Harshit Sahu | BSc (Hons) Computer Science | 2021 | 4 | 4 | Practical 7

Find the Characteristics
for the first order PDE and
Plotting them

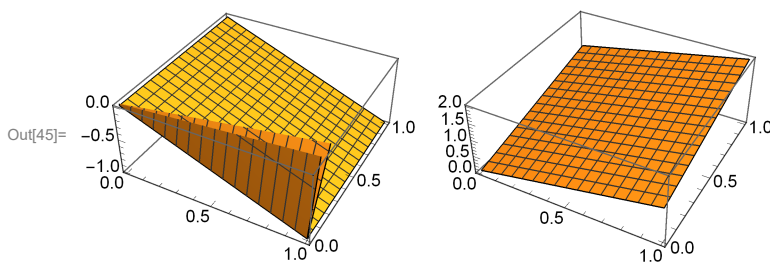
Example I :

Find the Characteristics of the equation $(u - y)u_x + y u_y = x + y$ and plot them.
Solution:

The characteristics system is $dx/(u - y) = dy/y = du/(x + y)$ using (i) + (ii) + (iii),
we have $v = (u + x)/y = c_1$ is a first integral. using (i) + (ii) = (iii),
we have $w = (x + y)^2 - u \cdot u = c_2$ is a second first integral.

```
In[37]:= f0 = Plot3D[-x, {x, 0, 1}, {y, 0, 1}, PlotPoints -> 10];  
f1 = Plot3D[5 y - x, {x, 0, 1}, {y, 0, 1}, PlotPoints -> 10];  
f2 = Plot3D[10 y - x, {x, 0, 1}, {y, 0, 1}, PlotPoints -> 10];  
g1 = Show[f0, f1, f2];  
h0 = Plot3D[x + y, {x, 0, 1}, {y, 0, 1}, PlotPoints -> 10];  
h1 = Plot3D[Sqrt[(x + y)^2 + 5], {x, 0, 1}, {y, 0, 1}, PlotPoints -> 10];  
h2 = Plot3D[Sqrt[(x + y)^2 + 10], {x, 0, 1}, {y, 0, 1}, PlotPoints -> 10];  
g2 = Show[h0, h1, h2];  
Show[GraphicsArray[{g1, g2}]]
```

... GraphicsArray: GraphicsArray is obsolete. Switching to GraphicsGrid.



Example 2 :

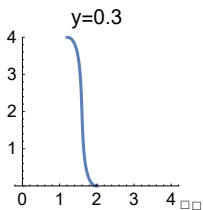
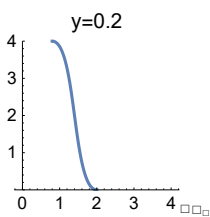
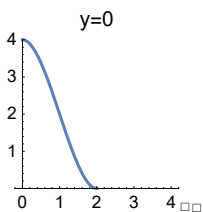
The solution of the equation $u[(x, y), y] + u[x, y] * u[(x, y), x] = 0$ can be interpreted as a vector field on the $x -$ axis varying with time y . Find the integral satisfying the initial condition $u(s, 0) = h(s)$, where h is a given function.

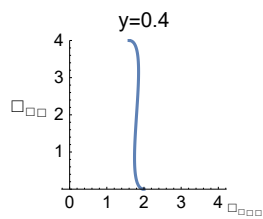
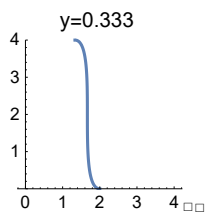
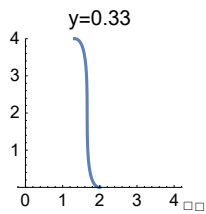
Solution:

We plot the curves

{Ct: $x = s + t(s^3 - 3s^2 + 4)$, $u = s^3 - 3s^2 + 4$ }

```
In[61]:= u[s_] := s^3 - 3 s^2 + 4;
x[s_, t_] := s + t * u[s];
h0 = ParametricPlot[{x[s, 0], u[s]}, {s, 0, 2}, PlotRange -> {0, 4}, PlotLabel -> "y=0"]
h1 = ParametricPlot[{x[s, 0.2], u[s]}, {s, 0, 2}, PlotRange -> {0, 4}, PlotLabel -> "y=0.2"]
h2 = ParametricPlot[{x[s, 0.3], u[s]}, {s, 0, 2}, PlotRange -> {0, 4}, PlotLabel -> "y=0.3"]
h3 = ParametricPlot[{x[s, 0.33], u[s]}, {s, 0, 2}, PlotRange -> {0, 4}, PlotLabel -> "y=0.33"]
h4 = ParametricPlot[{x[s, 0.333], u[s]}, {s, 0, 2}, PlotRange -> {0, 4}, PlotLabel -> "y=0.333 "}]
h5 = ParametricPlot[{x[s, 0.4], u[s]}, {s, 0, 2}, PlotRange -> {0, 4}, PlotLabel -> "y=0.4"]
Show[GraphicsArray[{h0, h1, h2}, {h3, h4, h5}].FrameTicks -> None, Frame -> False]
```





GraphicsArray: GraphicsArray is obsolete. Switching to GraphicsGrid.

Show: No graphical objects to show.

Out[69]= Show[
y=0 y=0.2 y=0.3
y=0.33=0.333 y=0.4
].FrameTicks → None, Frame → False]