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COURSE :
BSc (Hons) Computer Science
Practical – 6

SOLUTION OF CAUCHY PROBLEM FOR FIRST ORDER PDE

QUESTION 1 :

Obtain the solution of the linear equation u[(x, y), x] – u[(x, y), y] = 1 with the Cauchy data u (x, 0) = x * x

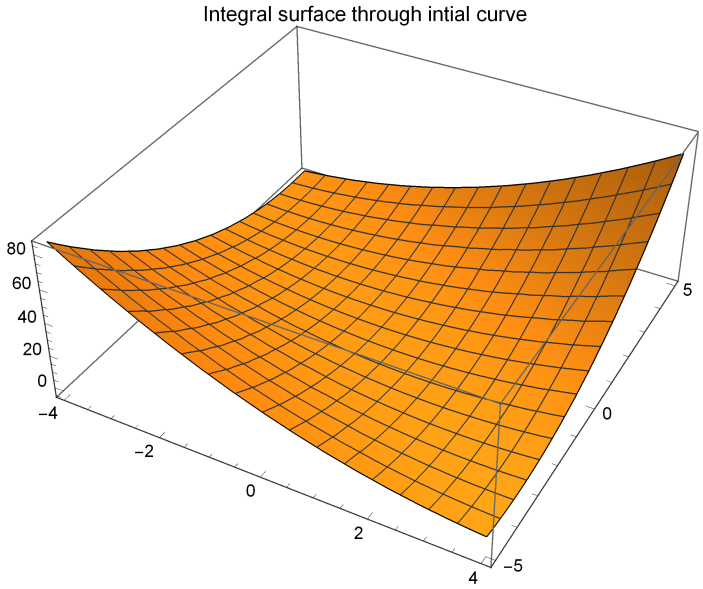
```
pde = D[u[x, y], x] - D[u[x, y], y] == 1
```

$$-u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1$$

```
sol = DSolve[{pde, u[x, 0] == x * x}, u[x, y], {x, y}]
```

$$\{\{u[x, y] \to x^2 - y + 2\,x\,y + y^2\}\}$$

```
Plot3D[u[x, y] /. sol, {x, -4, 4}, {y, -5, 5},
 PlotLabel → "Integral surface through intial curve"]
```



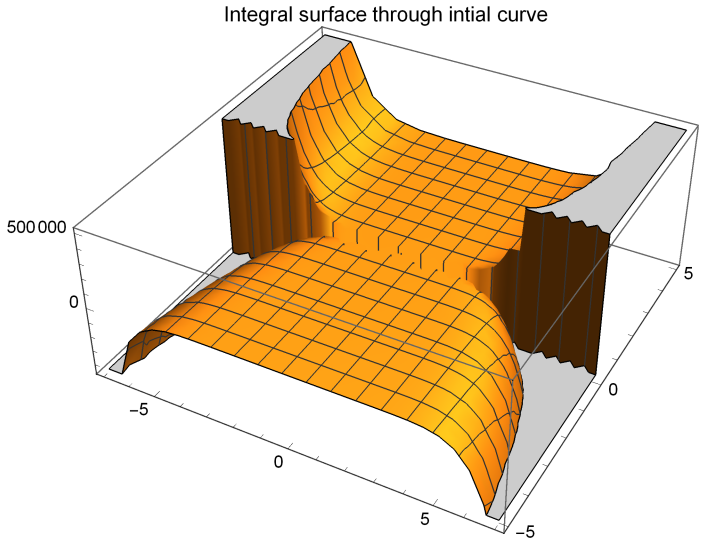
QUESTION 2 : Obtain the solution of the linear equation y * u[(x, y), x] – 2 * x * u[x, y] with the Cauchy data u (0, y) = y * y * y

SOLUTION :

```
pde = y * D[u[x, y], x] - 2 * x * y * D[u[x, y], y] == 2 * x * u[x, y]
sol3 = DSolve[{pde, u[0, y] == y * y * y}, u[x, y], {x, y}]
Plot3D[u[x, y] /. sol3, {x, -7, 7}, {y, -5, 5},
 PlotLabel → "Integral surface through intial curve"]
```

$$-2\,x\,y\,u^{(0,1)}[x, y] + y\,u^{(1,0)}[x, y] == 2\,x\,u[x, y]$$

$$\left\{\left\{u[x, y] \to \frac{\left(x^2 + y\right)^4}{y}\right\}\right\}$$



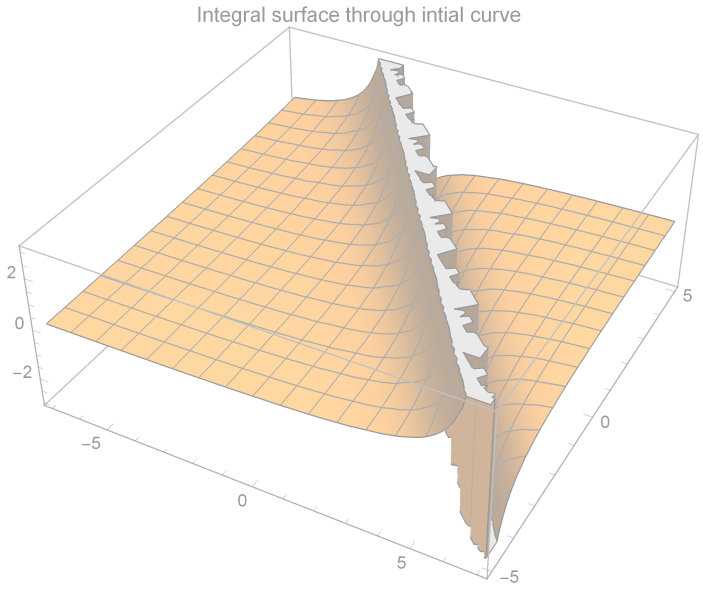
QUESTION 3 : Determine the integral surfaces of the equation u[(x, y), x] + u[(x, y), y] = u[x, y] * u[x, y], (a) with the data x + y = 0, u = 1. (b) with the data u (x, 0) = tanh (x).

```
Eqn = D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y]
sol4 =
 DSolve[{D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y], u[x, -x] == 1}, u[x, y], {x, y}]
Plot3D[u[x, y] /. sol4, {x, -7, 7}, {y, -5, 5},
  PlotLabel → "Integral surface through intial curve"]
```

Out[1]= $u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == u[x, y]^2$

Out[2]= $\left\{\left\{u[x, y] \to -\frac{2}{-2 + x + y}\right\}\right\}$

Out[3]=



(b)

In[5]:=
```
D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y]
```

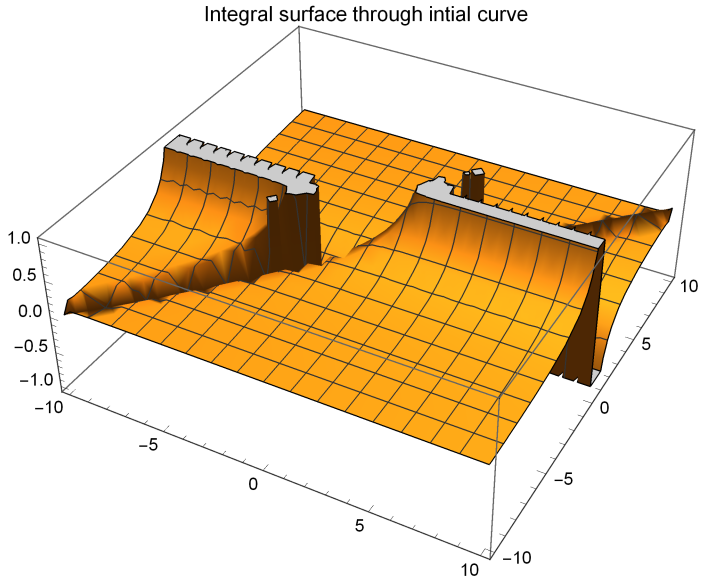
Out[5]= $u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == u[x, y]^2$

In[7]:=
```
sol5 = DSolve[
  {D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y], u[x, 0] == Tanh[x]}, u[x, y], {x, y}]
```

Out[7]= $\left\{\left\{u[x, y] \to \frac{1}{-y + \text{Coth}[x - y]}\right\}\right\}$

In[8]:=
```
Plot3D[u[x, y] /. sol5, {x, -10, 10}, {y, -10, 10},
 PlotLabel → "Integral surface through intial curve"]
```

Out[8]=



QUESTION 4 : Obtain the solution of the linear equation

u[(x, y), x] + u[(x, y), y] = 1 with the Cauchy data u (x, 2 x) = x * x * x

In[9]:=
```
D[u[x, y], x] + D[u[x, y], y] == 1
```

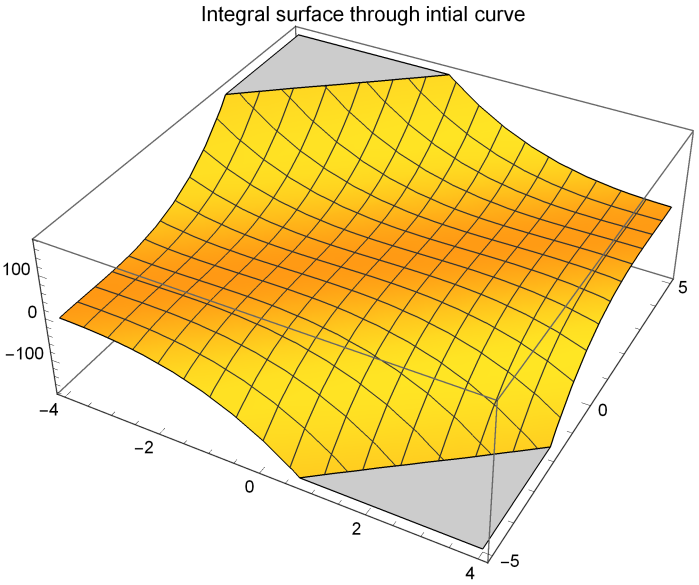
Out[9]=
$$u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1$$

In[11]:=
```
sol6 = DSolve[{D[u[x, y], x] + D[u[x, y], y] == 1, u[x, 2 x] == x * x * x}, u[x, y], {x, y}]
```

Out[11]=
$$\left\{\left\{u[x, y] \to 2\,x - x^3 - y + 3\,x^2\,y - 3\,x\,y^2 + y^3\right\}\right\}$$

In[12]:=
```
Plot3D[u[x, y] /. sol6, {x, -4, 4}, {y, -5, 5},
 PlotLabel → "Integral surface through intial curve"]
```

Out[12]=



QUESTION 5 : Obtain the solution of the linear equation

u (x + y) * u[(x, y), x] + u (x − y) * u[(x, y), y] =

x * x + y * y with the Cauchy data

u (x, 2 x) = 0

SOLUTION :

In[14]:=
```
u[x, y] * (x + y) * D[u[x, y], x] + u[x, y] * (x - y) * D[u[x, y], y] == x * x + y * y
```

Out[14]=
$$(x-y)\,u[x,y]\,u^{(0,1)}[x,y]+(x+y)\,u[x,y]\,u^{(1,0)}[x,y]==x^2+y^2$$

In[16]:=
```
DSolve[{u[x, y] * (x + y) * D[u[x, y], x] + u[x, y] * (x - y) * D[u[x, y], y] == (x * x) + (y * y),
  u[x, 2 x] == 0}, u[x, y], {x, y}]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[16]=
$$\left\{\left\{u[x,y]\to-\sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\},\left\{u[x,y]\to\sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\},\right.$$
$$\left.\left\{u[x,y]\to-\sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\},\left\{u[x,y]\to\sqrt{\frac{2}{7}}\sqrt{2x^2+3xy-2y^2}\right\}\right\}$$