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Method of Variation of Parameters

QUESTION 1 : Solve second order differential equation y''[x] + y[x] = tan[x] by method of variation of parameter

Solution :

Step – 1 : Find complementary function

In[92]:=
```
eqn := y''[x] + y[x];
f[x_] := Tan[x];
P = DSolve[eqn == 0, y[x], x]
```

Out[94]=
```
{{y[x] → C[1] Cos[x] + C[2] Sin[x]}}
```

Step – 2 Consider fundamental solution function u (x) and v (x)

In[95]:=
```
u[x_] := Cos[x];
v[x_] := Sin[x];
```

Step – 3 Find Wronskian W = `({u[x], v[x]}, {u'[x], v'[x]})`

In[97]:=
```
w = Simplify[Det[{{u[x], v[x]}, {u'[x], v'[x]}}]]
```

Out[97]=
```
1
```

Step – 4 Find g[x] = (–v[x] f[x]) / w and h[x] = (u[x] f[x]) / w

In[98]:=
```
g[x_] := (-v[x] * f[x]) / w
h[x] := (u[x] * f[x]) / w
```

Step – 5 Find G = Integrate[g[x], x] and H = Integrate[h[x], x]

In[100]:=
```
G = Integrate[g[x], x]
H = Simplify[Integrate[h[x], x]]
```

Out[100]= $\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Sin}[x]$

Out[101]= $-\text{Cos}[x]$

Step – 6 Find PI = u[x] G + v[x] H

In[102]:=
```
PI = u[x] G + v[x] H
```

Out[102]= $-\text{Cos}[x]\ \text{Sin}[x] + \text{Cos}[x]\ \left(\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Sin}[x]\right)$

QUESTION 2 : Solve second order differential equation y''[x] – 2 y'[x] = e^x Sin[x] by method of variation of parameter

Step – 1 : Find complementary function

In[77]:=
```
eqn := y''[x] - 2 y'[x];
f[x_] := e^x * Sin[x];
P = DSolve[eqn == 0, y[x], x]
```

$\{\{y[x] \to \frac{1}{2} e^{2x} C[1] + C[2]\}\}$

Step – 2 Consider fundamental solution function u (x) and v (x)

```
u[x_] := 1 / 2 Exp[2 x]
v[x_] := 1
```

Step – 3 Find Wronskian W = ({u[x], v[x]}, {u'[x], v'[x]})

In[82]:=
```
w = Simplify[Det[{{u[x], v[x]}, {u'[x], v'[x]}}]]
```

$-e^{2x}$

Step – 4 Find g[x] = (–v[x] f[x]) / w and h[x] = (u[x] f[x]) / w

```
g[x_] := (-v[x] × f[x]) / w
h[x_] := (u[x] × f[x]) / w
```

Step – 5 Find G = Integrate[g[x], x] and H = Integrate[h[x], x]

In[85]:=
```
G = Integrate[g[x], x]
H = Simplify[Integrate[h[x], x]]
```

Out[85]=
$$\frac{e^x \, e^{-2x} \left(-\text{Cos}[x] + \left(-2 + \text{Log}[e]\right) \text{Sin}[x]\right)}{5 - 4\,\text{Log}[e] + \text{Log}[e]^2}$$

$$\frac{e^x \left(\text{Cos}[x] - \text{Log}[e]\,\text{Sin}[x]\right)}{2\left(1 + \text{Log}[e]^2\right)}$$

Step – 6 Find PI = u[x] G + v[x] H

In[87]:=
```
PI = u[x] G + v[x] H
```

Out[87]=
$$\frac{e^x \left(-\text{Cos}[x] + \left(-2 + \text{Log}[e]\right) \text{Sin}[x]\right)}{2\left(5 - 4\,\text{Log}[e] + \text{Log}[e]^2\right)} + \frac{e^x \left(\text{Cos}[x] - \text{Log}[e]\,\text{Sin}[x]\right)}{2\left(1 + \text{Log}[e]^2\right)}$$