Harshit Sahu | BSC CS Hons | 20211467 | Practical- 8

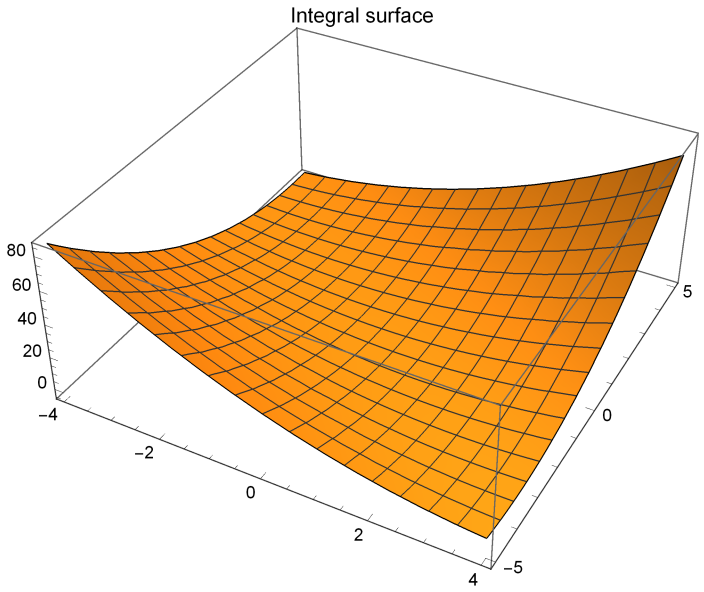
Plot the integral surface of a given first order PDE with the initial data

Problem 1: Obtain the solution of the linear equation u[(x,y),x]-u[(x,y),y]=1 with
the Cauchy data u(x,0) = x*x.
Plot the integral surface with in the range {x,-4,4} and {y,-5,5}.
Solution :

```
pde = D[u[x, y], x] - D[u[x, y], y] == 1
DSolve[{D[u[x, y], x] - D[u[x, y], y] == 1, u[x, 0] == (x * x)}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5}, PlotLabel → "Integral surface "]
```

$$-u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1$$

$$\{\{u[x, y] \to x^2 - y + 2xy + y^2\}\}$$



DSolve: "Equation or list of equations expected instead of \!\(\(−□\)\\\\\\ *SuperscriptBox[\\\"u\\\", TagBox[\\n RowBox[{\\\"(\\\", \\n RowBox[{\\\"0\\\", \\\",\\\", \\\"1\\\"}], \\\")\\\"}],\\n Derivative],\\nMultilineFunction->None][x, y] + *SuperscriptBox[\\\"u\\\", TagBox[\\n RowBox[{\\\"(\\\", \\n RowBox[{\\\"1\\\", \\\",\\\", \\\"0\\\"}], \\\")\\\"}],\\n Derivative],\\nMultilineFunction->None][x, y]\) in the first argument \!\({\(−□\)\\\\\\ *SuperscriptBox[\\\"u\\\", TagBox[\\n RowBox[{\\\"(\\\", \\n RowBox[{\\\"0\\\", \\\",\\\", \\\"1\\\"}], \\\")\\\"}],\\n Derivative],\\nMultilineFunction->None][x, y] + *SuperscriptBox[\\\"u\\\", TagBox[\\n RowBox[{\\\"(\\\", \\n RowBox[{\\\"1\\\", \\\",\\\", \\\"0\\\"}], \\\")\\\"}],\\n Derivative],\\nMultilineFunction->None][x, y], □\\\\\\ *SuperscriptBox[\(x\), \(2\)]\\\\\\ u[x, 0]}\)."

```
DSolve[{−□ u^(0,1)[x, y] + u^(1,0)[x, y], □ x^2 u[x, 0]}, u[x, y], {x, y}]
```

Problem 2: Find the solution of the equation y*u[(x,y),x]-2*x*y*u[(x,y),y]=2*x*u[x,y] with the Cauchy data u(0,y) = y*y*y. Plot the integral surface with in the range {x,-7,7} and {y,-5,5}.

Solution

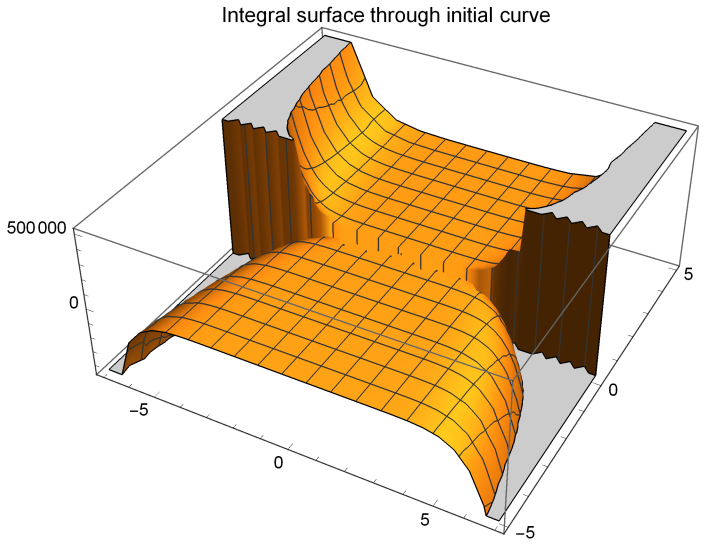
Plot3D: Options expected (instead of PlotLabel → Integral surface) beyond position 3 in Plot3D[u[x, y] /. %, {x, −4, 4}, {y, −5, 5}, PlotLabel → Integral surface]. An option must be a rule or a list of rules.

```
Plot3D[u[x, y] /. %, {x, −4, 4}, {y, −5, 5}, PlotLabel → Integral surface ]
```

```
pde = y * D[u[x, y], x] - 2 * x * y * D[u[x, y], y] == 2 * x * u[x, y]
sol3 = DSolve[{pde, u[0, y] == y * y * y}, u[x, y], {x, y}]
Plot3D[u[x, y] /. sol3, {x, -7, 7}, {y, -5, 5},
 PlotLabel → "Integral surface through initial curve"]
```

$$-2\,x\,y\,u^{(0,1)}[x, y] + y\,u^{(1,0)}[x, y] == 2\,x\,u[x, y]$$

$$\left\{\left\{u[x, y] \to \frac{\left(x^2 + y\right)^4}{y}\right\}\right\}$$



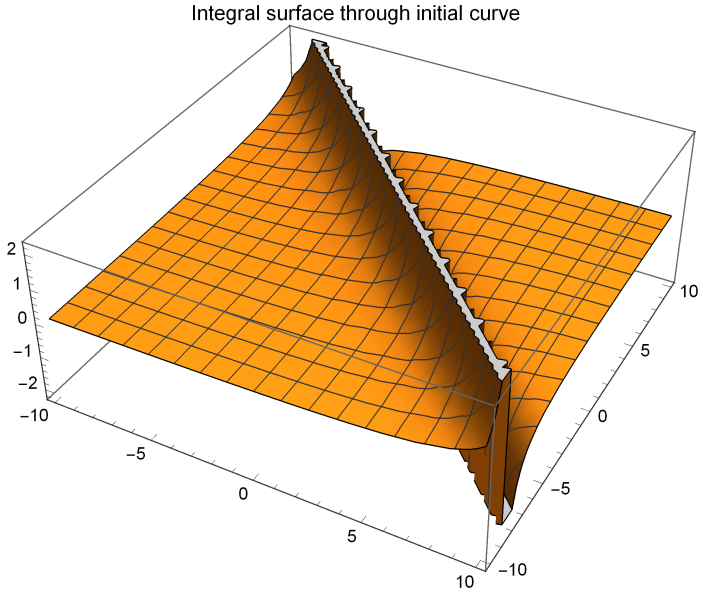
Problem 3 : Determine the integral surfaces of the equation u[(x,y),x]+u[(x,y),y]=u[x,y]*u[x,y] , with the data x+y=0,u=1. Plot the integral surface with in the range {x,10,10} and {y,-10,10}.

Solution :

```
Eqn = D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y]
DSolve[{D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y], u[x, -x] == 1}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -10, 10}, {y, -10, 10},
 PlotLabel → "Integral surface through initial curve "]
```

$$u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == u[x, y]^2$$

$$\left\{\left\{u[x, y] \to -\frac{2}{-2 + x + y}\right\}\right\}$$

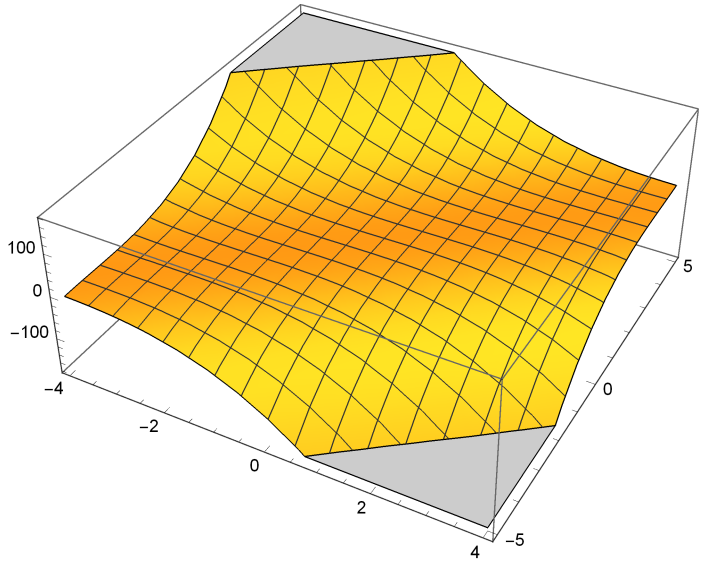


Problem 4 : Obtain the solution of the linear equation u[(x,y),x]+u[(x,y),y]=1 with
the Cauchy data u(x,2x)=x*x*x.
Plot the integral surface with in the range {x,-4,4} and {y,-5,5}.
Solution :

```
D[u[x, y], x] + D[u[x, y], y] == 1
DSolve[{D[u[x, y], x] + D[u[x, y], y] == 1, u[x, 2 x] == x * x * x}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5}]
```

$$u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1$$

$$\left\{\left\{u[x, y] \to 2 x - x^3 - y + 3 x^2 y - 3 x y^2 + y^3\right\}\right\}$$



Problem 5 : Obtain the solution of the linear equation u(x+y)*u[(x,y),x]+u(x□□y)*u[(x,y),y]=x*x+y*y with the Cauchy data u(x,2x)=0. Plot the integral surface with in the range {x,-4,4} and {y,-5,5}.

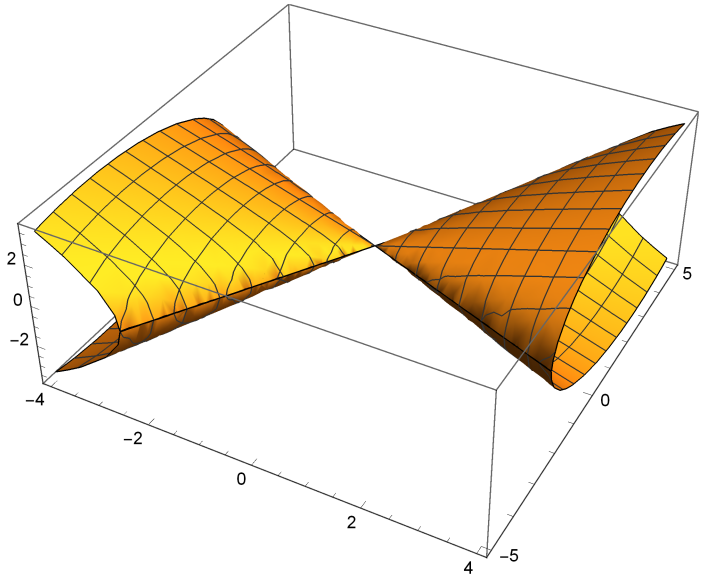
Solution :

```
eqn = u[x, y] * (x + y) * D[u[x, y], x] + u[x, y] * (x - y) * D[u[x, y], y] == x * x + y * y
DSolve[{u[x, y] * (x + y) * D[u[x, y], x] + u[x, y] * (x - y) * D[u[x, y], y] == (x * x) + (y * y),
  u[x, 2 x] == 0}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5}]
```

$$(x - y)\, u[x, y]\, u^{(0,1)}[x, y] + (x + y)\, u[x, y]\, u^{(1,0)}[x, y] == x^2 + y^2$$

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

$$\left\{\left\{u[x, y] \to -\sqrt{\frac{2}{7}}\ \sqrt{2 x^2 + 3 x y - 2 y^2}\right\}, \left\{u[x, y] \to \sqrt{\frac{2}{7}}\ \sqrt{2 x^2 + 3 x y - 2 y^2}\right\},\right.$$
$$\left.\left\{u[x, y] \to -\sqrt{\frac{2}{7}}\ \sqrt{2 x^2 + 3 x y - 2 y^2}\right\}, \left\{u[x, y] \to \sqrt{\frac{2}{7}}\ \sqrt{2 x^2 + 3 x y - 2 y^2}\right\}\right\}$$



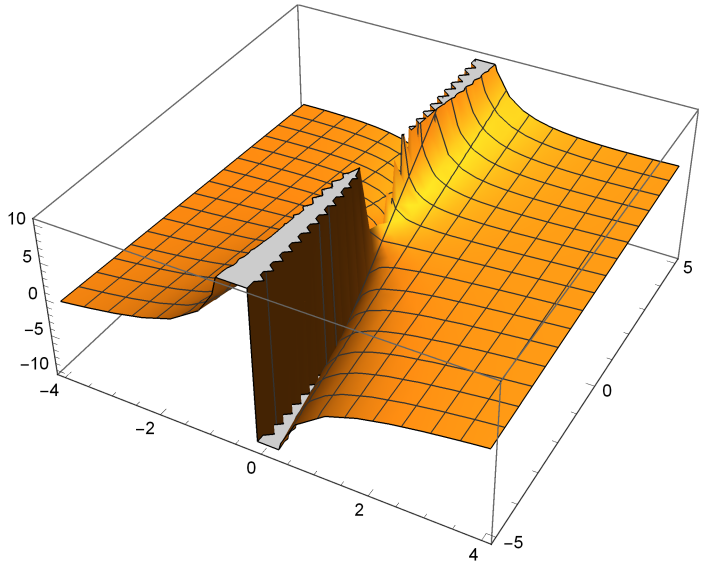
Problem 6 : Obtain the solution of the linear equation u[(x,y),x]+u[x,y]*u[(x,y),y]=1 with the Cauchy data u(0,y)=4*y. Plot the integral surface with in the range {x,-4,4} and {y,-5,5}.

Solution

```
D[u[x, y], x] + u[x, y] * D[u[x, y], y] == 1
DSolve[{D[u[x, y], x] + u[x, y] * D[u[x, y], y] == 1, u[0, y] == 4 * y}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5}]
```

$$u[x, y]\, u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1$$

$$\left\{\left\{u[x, y] \to \frac{x + 2\,x^2 + 4\,y}{1 + 4\,x}\right\}\right\}$$



Problem 7 : Obtain the solution of the linear equation u[(x,y),x]+y*u[(x,y),y]=0 with the Cauchy data u(0,y)=4*y.

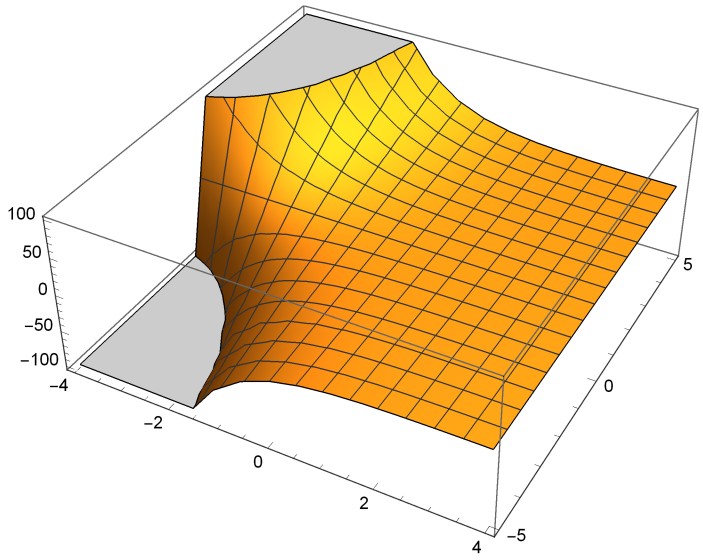
Plot the integral surface with in the range {x,-4,4} and {y,-5,5}.

Solution :

```
D[u[x, y], x] + y * D[u[x, y], y] == 0
DSolve[{D[u[x, y], x] + y * D[u[x, y], y] == 0, u[0, y] == 4 * y}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5}]
```

$$y\,u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 0$$

$$\{\{u[x, y] \to 4\,e^{-x}\,y\}\}$$



Problem 8 : Obtain the solution of the linear equation u[(x,y),x]+2*u[(x,y),y]=0 with the Cauchy data u(0,y)=Exp[-y*y].

Plot the integral surface with in the range {x,-4,4} and {y,-5,5}.

Solution :

```
D[u[x, y], x] + 2 * D[u[x, y], y] == 0
DSolve[{D[u[x, y], x] + 2 * D[u[x, y], y] == 0, u[0, y] == Exp[-y * y]}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -4, 4}, {y, -5, 5}]
```

$$2\,u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 0$$

$$\left\{\left\{u[x, y] \to e^{-(-2x+y)^2}\right\}\right\}$$