Assignment 06 - Parameter Estimation.

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Question:

1. Let (X1, X2, ...) be a random sample of size n taken from a Normal Population with parameters: mean = $\theta 1$ and variance = $\theta 2$. Find the Maximum Likelihood Estimates of these two parameters.
2. Let X_1, X_2, ---- X_n be a random sample from B(m, θ) distribution, where $\theta \in \Theta = (0,1)$ is unknown and 'm' is a known positive integer. Compute value of θ using the M.L.E.

Soln. 1. Mean = $\theta 1$, Variance = $\theta 2$ (Normal distribution).

∴ The function is :-

$$L(\theta 1, \theta 2 \mid x_1, x_2 --- x_n) = \pi_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{\frac{-(x-\theta_2)^2}{2\theta_2}}$$

take log on both side :-

$$\ln L(\theta 1, \theta 2 \mid x_1, x_2 --- x_n) = -\frac{n}{2} \ln(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i \theta_i)^2$$

Diff. w.r.t. $\theta 1$ & $\theta 2$ & then equal to 0.

for θ_1 :-

$$\frac{\partial}{\partial\theta_1} \ln L(\theta_1, \theta_2 \mid x_1, x_2 --- x_n) = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\frac{1}{\theta 2} \sum_{i=1}^{n} (x_i - \hat{\theta_i}) = 0$$

$$\sum_{i=1}^{n} (x_i - \hat{\theta}_1) = 0$$

$$\hat{\theta_1} = \frac{1}{n} \sum_{i=1}^{n} x_i$$ ∴ MLE for $\theta 1$ is sample mean.

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* For $\theta 2$:-

$$\frac{\partial}{\partial \theta_2} \ln L(\theta 1, \theta 2 | x_1, x_2 \ldots x_n) = \frac{-n}{2\theta 2} + \frac{1}{2\theta_2} \cdot \sum_{i=1}^{n} (xi - \theta_1)^2$$

equating to 0

$$\frac{-n}{2\theta_2} + \frac{1}{2\hat{\theta}_2^2} \sum_{i=1}^{n} (xi - \hat{\theta}_1)^2$$

$$\hat{\theta_2} = \frac{1}{n} \sum_{i=1}^{n} (xi - \hat{\theta}_1)^2$$

$\therefore$ MLE for $\theta 2$ is sample variance.

Solⁿ 2:- Bernoulli distribution :-

PMF :- $^nC_x \cdot p^x \cdot (1-p)^{n-x}$

$P(x_1) = {}^nC_x \cdot p^{x_1} \cdot (1-p)^{n-x_1}$

$P(x_2) = {}^nC_x \cdot p^{x_2} \cdot (1-p)^{n-x_2}$

⋮

$P(x_n) = {}^nC_x \cdot p^{x_n} (1-p)^{n-x_n}$

* Joint Probability :-

$$L(x_1, x_2, \ldots x_n) = ({}^nC_x)^n \cdot p^{\Sigma xi} \cdot (1-p)^{n^2 - \Sigma xi}$$

taking log on both side.

$$\ln (L) = n \cdot \ln [{}^nC_x] + \Sigma xi \ln p + (n^2 - \Sigma xi) \cdot \ln(1-p)$$

$$\frac{dL}{dp} = \frac{\Sigma xi}{p} - \frac{(n^2 - \Sigma xi)}{(1-p)} = 0$$

$$= \frac{\Sigma xi (1-p) - (n^2 - \Sigma xi) \cdot P}{P(1-P)} = 0$$

$$= \Sigma xi - \Sigma xi \times P - n^2 P + \Sigma xi \cdot P = 0$$

$$= P = \frac{\Sigma xi}{n^2}$$