# Data Structures

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## Assignment -1

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(1) $t_{n=10} \Rightarrow 5\,sec$

$t_{n=5} \Rightarrow ?$

$Kn^2 = t$

$K \times 100 = 5$

$K = 1/20$

$t_{n=50} = \frac{1}{20} \times 50 \times 50$

$= 125\,sec$

(2) $T_n = n^3$ $\quad T_B = 2n^2$

break pt

$n^3 = 2n^2$

$n = 2$

(3) $f(n) = n2^n$ $\quad g(n) = n^n$

Applying limit rule

$\lim_{n\to\infty} \frac{f(n)}{g(n)} \to \lim_{n\to\infty} \frac{n2^n}{4^n} \to \lim_{n\to\infty} \frac{n2^n}{2^{2n}}$

Applying L'Hospital $\Rightarrow \lim_{n\to\infty} \frac{1}{2^n \log 2}$

Hence $f(n)$ is $o(g(n))$

(4) let polynm $f^n$ $P(n)$ and $\log(n)$ $f^n$, using limit rule $\lim_{n\to\infty} \frac{\log(n)}{(n)} \left(\frac{\infty}{\infty}\right)$

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applying L'hospital rule 11912039

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$$\Rightarrow \lim_{n \to \infty} \frac{1}{n(\ln)}$$

$\to 0$

thus login grow slower than all fun.

let 2 log funⁿ $\frac{\log_a n}{\log_b (n+1)}$ and

Applying limit rule

$$\lim_{n \to \infty} \frac{\log_a n}{\log_b (n+1)} = \lim_{n \to \infty} \frac{\log_a n}{\log_b + \log(1+1/n)}$$

$$= \lim_{n \to \infty} \frac{\log_a n}{\log_b n}$$

$$= \lim_{n \to \infty} \frac{\ln b}{\ln a}$$

hence all log fun grow rate i.e. constn

| Q(4) Average case (Θ) | Worst case (O) |
| --- | --- |
| * performs avg. no. of steps on input data of n Elements | perform max no. of step on input data of n elements |
| * Averaged across all possible input Eg:- $\Theta(n \log(n))$ Quicksort | Input is arbitrary size & output is upperbound Eg:- $O(n^2)$ Quick sort |

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Q(b)

| Worst case (O) | Asymptotically Bounded ($\Theta$) |
| --- | --- |
| It is arbitrary fun. O/p is upper bound. | Growth of a remaining time to within const-factors below. |

Q

(a) $f(n) = n^4 + \log n + n$, $g(n)$ $n^4$

$$\lim_{n\to\infty} f(n) = \lim_{n\to\infty} \frac{n^4 + \log + n}{n^4}$$

$$= \lim_{n\to\infty} \frac{4n^3 + 1/n}{4n^3}$$

$$= 1.\text{cons}$$

hence $f(n)$ is $O(g(n))$

Q(7)(a) $k=1$

$k=2$

$k=3$

$|$ $O(n-1)$

$\to \Theta O(n)$

$k = n-1$

(b) for $i=1$ $j = 2, 3, \ldots n$ $(n-1)$

$i=2$ $j = 3, n \ldots (n-2)$

$i = n-1$ $(n-1)$

$(n-1) + (n-2) \ldots 3 + 2 + 1$

$$= \frac{n((n+1))}{2}$$

$$= O(n)$$

8 Quadratic fun $f(n) = n^2$

$n \times n = n^2$

$1 + 2 + 3 \ldots (n-1) + n = \frac{n(n-1)}{2}$

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Q(9) $T_A = 100^n$

$T_B = n^4$

using limit rule, $\lim \frac{n^4}{100^n}$ $\frac{\infty}{\infty}$

applying L hospital $= \frac{4n^3}{100^n \log_e 100} = 0$

hence two grow faster when $n = 120$

10 $f(n) = n \log n$ $g_n = \log(n!)$

$= \log(1 \times 2 \times 3 \cdots (n-1)n)$

$= \log\left(\frac{n}{2}^{n/2}\right)$

$= \frac{n}{2} \log n/2$

$= \frac{n}{2}(\log n - \log 2) \simeq \Omega\, n \log n$

hence $f(n) \in \Omega\, g(n)$

11 (a) $2^{n+} + 4^{n+1}$

for $\Theta$ $c_1 g(n) < f(n) \leq c_2 g(n)$

$2^{n+1} + 2^{2n+2}$

$= 2^{2n}\left(4 + \frac{1}{2^{n+1}}\right)$

Highest order Hence $2^{2n} \Rightarrow \Theta\,(4^n)$

(b) $(n^2+6)^8$

highest order $= (n^2)^8$

$= n^{16}$

$\Rightarrow \Theta\,(n^{16})$

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$T_A = n^2$

$T_B = n+1$

Breakpoint.

$n^2 = n+1$

$(n-2)(n+1) = 0$

$n = 2$.