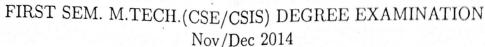
MANIPAL INSTITUTE OF TECHNOLOGY

(Manipal University, Manipal-576 104)



SUB: COMPUTATIONAL METHODS AND STOCHASTIC PROCESSES
MAT 511

Time: 3hrs

प्रज्ञानं ब्रह

Max Marks: 50

- Answer any FIVE full questions.
- Subquestions A and B are of 3 marks each, subquestion C is of 4 marks.

Q.1A: Find the coefficient of correlation, regression equations of y on x and x on y for the following data.

X	1	3	4	6	8
Y	1	12	24	10	5

- Q.1B: Find the dimension of vector space of 2×2 symmetric matrices over the field of real numbers.
- **Q.1C:** Show that the stochastic process given by $X(t) = A\cos(\omega t + \theta)$ is wide sense stationary, where A, ω are constants and θ is uniformly distributed on the interval $[0, 2\pi]$.

(3+3+4=10)

Q.2A: A particle performs a random walk with absorbing barriers, say, at 0 and 4. Whenever it is at any position r (0 < r < 4), it moves to r + 1 with probability q, p + q = 1. But as soon as it reaches 0 or 4 it remains there itself. Let X_n be the position of the particle after n moves and the different states of X_n be the different positions of the particle. Prove that $\{X_n\}$ is a markov chain. Sketch the transition graph of the Markov chain and form the transition matrix.

- Q.2B: Let A and B are uncorrelated random variables each with mean 0 and variance 1. Test whether the stochastic process given by $X(t) = A\cos(0.75t) + B\sin(0.75t)$ is covariance stationary.
- Q.2C: Let $\{X_n, n \geq 0\}$ be a Markov chain with three states 0, 1, 2 and with transition matrix given by

$$\begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

Find the following probabilities assuming that the initial distribution is equally likely for the three states 0, 1 and 2.

(i)
$$P(X_1 = 1 \mid X_0 = 2)$$
; (ii) $P(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2)$; (iii) $P(X_2 = 1, X_0 = 0)$.

$$(3+3+4=10)$$

- Q.3A: Three persons A, B, C are throwing a ball to each other. A always throws the ball to B and B always to C. However, C is just as likely to throw the ball to B as to A. Find the transition matrix and classify the states.
 - Q.3B: Illustrate the notion of Prufer sequence with an example.
- Q.3C: Find the co-variance and the coefficient of correlation of the Poisson process.

$$(3+3+4=10)$$

- Q.4A: A computer program while adding numbers rounds each number off to the nearest integer. Suppose that all rounding errors are independent and are uniformly distributed over (-0.5, 0.5). How many numbers may be added together in order that the magnitude of the total error is less than 10 with probability 0.90?
- Q.4B: A year selected at random is observed to have 53 sundays. What is the probability that the year is not a leap year?
- Q.4C: Consider a communication system which transmits the two digits 0 and 1 through several stages. Let $X_n, n \ge 1$ be the digit leaving the n^{th} stage

of system and X_0 be the digit entering the first stage. At each stage there is a constant probability q that the digit which enters will be transmitted unchanged and probability p otherwise; such that p + q = 1. Assuming $P(X_0 = 0) = 0.75$ and $P(X_0 = 1) = 0.25$, find $P(X_m = 0, X_0 = 0)$.

$$(3+3+4=10)$$

Q.5A: Find the stationary (invariant) probability distribution for Markov chain with two states 1, 2 and transition matrix given by

$$\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Q.5B: With step size $h = \frac{1}{3}$ solve the Poisson's equation

$$u_{xx} + u_{yy} = 0,$$

$$0 \le x \le 1, 0 \le y \le 1, u(x, 1) = u(0, y) = 0, u(1, y) = 9(y - y^2); u(x, 0) = 9(x - x^2).$$

Q.5C: Derive the Chapman-Kolmogorov equation.

$$(3+3+4=10)$$

Q.6A: Consider the stochastic process $\{X(t), t \in T\}$ whose probability distribution under a certain condition is given by

$$P(X(t) = n) = \frac{(0.5t)^{n-1}}{(1+0.5t)^{n+1}}, \quad n = 1, 2, 3, \dots$$

$$P(X(t) = 0) = \frac{0.5t}{1 + 0.5t}$$

Find E[X(t)] and V[X(t)].

Q.6B: Solve by Crank Nicolson's method $u_t = u_{xx}$, 0 < x < 1, t > 0, u(0,t) = u(1,t) = 0, $u(x,0) = 100(x-x^2)$. Compute u for one time step by taking h = 0.25 and $\lambda = 1$.

Q.6C: Use the Simplex method to solve the following linear programming problem.

Maximize $Z = 4x_1 + 3x_2$ subject to $2x_1 + x_2 \le 1000$, $x_1 + x_2 \le 800$, $x_1 \le 400$, $x_2 \le 700$, $x_1 \ge 0$, $x_2 \ge 0$.

(3+3+4=10)