MANIPAL INSTITUTE OF TECHNOLOGY

(Manipal University, Manipal-576 104)

FIRST SEM. M.TECH.(CSE/CSIS) DEGREE EXAMINATION

Nov/Dec 2014

SUB: COMPUTATIONAL METHODS AND STOCHASTIC PROCESSES

MAT 511

Time: 3hrs Max Marks: 50

- Answer any **FIVE** full questions.
- Subquestions A and B are of 3 marks each, subquestion C is of 4 marks.

Q.1A: Find the coefficient of correlation, regression equations of y on x and x on y for the following data.

X	1	3	4	6	8
Y	1	12	24	10	5

Q.1B: Find the dimension of vector space of 2×2 symmetric matrices over the field of real numbers.

Q.1C: Show that the stochastic process given by $X(t) = Acos(\omega t + \theta)$ is wide sense stationary, where A, ω are constants and θ is uniformly distributed on the interval $[0, 2\pi]$.

(3+3+4=10)

Q.2A: A particle performs a random walk with absorbing barriers, say, at 0 and 4. Whenever it is at any position r $(0 < r < 4)$, it moves to $r+1$ with probability q, $p+q=1$. But as soon as it reaches 0 or 4 it remains there itself. Let X_n be the position of the particle after n moves and the different states of X_n be the different positions of the particle. Prove that $\{X_n\}$ is a markov chain. Sketch the transition graph of the Markov chain and form the transition matrix.

Q.2B: Let A and B are uncorrelated random variables each with mean 0 and variance 1. Test whether the stochastic process given by $X(t) = A\cos(0.75t) + B\sin(0.75t)$ is covariance stationary.

Q.2C: Let $\{X_n, n \geq 0\}$ be a Markov chain with three states 0, 1, 2 and with transition matrix given by

$$\begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

Find the following probabilities assuming that the initial distribution is equally likely for the three states 0, 1 and 2.
(i) $P(X_1 = 1 \mid X_0 = 2)$; (ii) $P(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2)$; (iii) $P(X_2 = 1, X_0 = 0)$.

(3+3+4=10)

Q.3A: Three persons A, B, C are throwing a ball to each other. A always throws the ball to B and B always to C. However, C is just as likely to throw the ball to B as to A. Find the transition matrix and classify the states.

Q.3B: Illustrate the notion of Prufer sequence with an example.

Q.3C: Find the co-variance and the coefficient of correlation of the Poisson process.

(3+3+4=10)

Q.4A: A computer program while adding numbers rounds each number off to the nearest integer. Suppose that all rounding errors are independent and are uniformly distributed over $(-0.5, 0.5)$. How many numbers may be added together in order that the magnitude of the total error is less than 10 with probability 0.90?

Q.4B: A year selected at random is observed to have 53 sundays. What is the probability that the year is not a leap year?

Q.4C: Consider a communication system which transmits the two digits 0 and 1 through several stages. Let $X_n, n \geq 1$ be the digit leaving the n^{th} stage of system and X_0 be the digit entering the first stage. At each stage there is a constant probability q that the digit which enters will be transmitted unchanged and probability p otherwise; such that $p + q = 1$. Assuming $P(X_0 = 0) = 0.75$ and $P(X_0 = 1) = 0.25$, find $P(X_m = 0, X_0 = 0)$.

(3+3+4=10)

Q.5A: Find the stationary (invariant) probability distribution for Markov chain with two states 1, 2 and transition matrix given by

$$\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Q.5B: With step size $h = \frac{1}{3}$ solve the Poisson's equation

$$u_{xx} + u_{yy} = 0,$$

$0 \leq x \leq 1, 0 \leq y \leq 1, u(x, 1) = u(0, y) = 0, u(1, y) = 9(y - y^2); u(x, 0) = 9(x - x^2)$.

Q.5C: Derive the Chapman-Kolmogorov equation.

(3+3+4=10)

Q.6A: Consider the stochastic process $\{X(t), t \in T\}$ whose probability distribution under a certain condition is given by

$$P(X(t) = n) = \frac{(0.5t)^{n-1}}{(1+0.5t)^{n+1}}, \qquad n = 1, 2, 3, ...$$

$$P(X(t) = 0) = \frac{0.5t}{1+0.5t}.$$

Find $E[X(t)]$ and $V[X(t)]$.

Q.6B: Solve by Crank Nicolson's method $u_t = u_{xx}$, $0 < x < 1, t > 0, u(0,t) = u(1,t) = 0, u(x,0) = 100(x - x^2)$. Compute u for one time step by taking $h = 0.25$ and $\lambda = 1$.

Q.6C: Use the Simplex method to solve the following linear programming problem.

Maximize $Z = 4x_1 + 3x_2$ subject to
$2x_1 + x_2 \leq 1000$, $x_1 + x_2 \leq 800$, $x_1 \leq 400$, $x_2 \leq 700$,
$x_1 \geq 0, x_2 \geq 0$.

(3+3+4=10)