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MANIPAL INSTITUTE OF TECHNOLOGY

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FIRST SEM. M.TECH.(CSE/CSIS) DEGREE EXAMINATION
Nov/Dec 2014
SUB: COMPUTATIONAL METHODS AND STOCHASTIC PROCESSES
MAT 511

Time: 3hrs | Max Marks: 50

- Answer any **FIVE** full questions.
- Subquestions A and B are of 3 marks each, subquestion C is of 4 marks.

Q.1A: Find the regression equations of y on x and x on y for the following data.

X	1	3	4	6	8
Y	1	2	24	12	5

Q.1B: A number X is selected from $1, 2, ..., n$. Find a lower bound for

$$P[\mid X - E(X) \mid < \sqrt{n^2 - 1}].$$

Q.1C: Consider a communication system which transmits the two digits 0 and 1 through several stages. Let $X_n, n \geq 1$ be the digit leaving the n^{th} stage of system and X_0 be the digit entering the first stage. At each stage there is a constant probability q that the digit which enters will be transmitted unchanged and probability p otherwise; such that $p + q = 1$. Assuming $P(X_0 = 0) = 0.75$ and $P(X_0 = 1) = 0.25$, find $P(X_m = 1, X_0 = 1)$.

(3+3+4=10)

Q.2A: Suppose that the probability of a dry day following a rainy day is $\frac{1}{4}$ and that the probability of a rainy day following a dry day is $\frac{1}{2}$. Form a transition matrix. If December 1 is a dry day, find the probability that December 4 is a dry day.

Q.2B: Let A and B are uncorrelated random variables each with mean 0 and variance 1. Test whether the stochastic process given by $X(t) = A\cos(0.25t) + B\sin(0.25t)$ is covariance stationary.

Q.2C: Let $\{X_n, n \geq 0\}$ be a Markov chain with three states 0, 1, 2 and with transition matrix given by

$$\begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

Find the following probabilities assuming that the initial distribution is equally likely for the three states 0, 1 and 2.

(i) $P(X_1 = 1 \mid X_0 = 2)$; (ii) $P(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2)$; (iii) $P(X_2 = 1, X_0 = 0)$.

(3+3+4=10)

Q.3A: Classify the states of the Markov chain with four states 1, 2, 3, 4 and transition matrix given by

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{2} \end{bmatrix}$$

Q.3B: Prove that the number of different labelled trees with n vertices is n^{n-2} and the number of different rooted labelled trees with n vertices is n^{n-1}, where $n \geq 2$.

Q.3C: Find the mean and variance of the Poisson process. A machine goes out of order whenever a component part fails. The failure of this part is in accordance with a poisson process with mean rate of 1 per week. Suppose that there are 5 spare parts of the component in an inventory and that the supply is not due in next 10 weeks. Find the probability that the machine will not be out of order in the next 10 weeks.

(3+3+4=10)

Q.4A: Let (X, Y) be uniformly distributed over $\{(x, y) \mid 0 < x < y < 1\}$. Find the coefficient of correlation ρ_{xy}.

Q.4B: If X, Y are two integer valued random variables having probability function $f(x, y) = k(2x + y)$ where $0 \le x \le 2$, $0 \le y \le 3$. Find k, $E(X)$, $E(Y)$, $E(Y/X = 2)$.

Q.4C: Consider the stochastic process $\{X(t), t \in T\}$ whose probability distribution under a certain condition is given by

$$P(X(t) = n) = \frac{(0.5t)^{n-1}}{(1 + 0.5t)^{n+1}}, \quad n = 1, 2, 3, \ldots$$

$$P(X(t) = 0) = \frac{0.5t}{1 + 0.5t}$$

Find $E[X(t)]$ and $V[X(t)]$.

(3+3+4=10)

Q.5A: Use graph theoretic approach to find the stationary (invariant) probability distribution for Markov chain with three states 1, 2, 3 and transition matrix given by

$$\begin{bmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Q.5B: With step size $h = \frac{1}{3}$ solve the Poisson's equation

$$u_{xx} + u_{yy} = -81xy,$$

$0 < x < 1, 0 < y < 1, u(0, y) = u(x, 0) = 0, u(1, y) = u(x, 1) = 100.$

Q.5C: Define random telegraph process. Show that the random telegraph process is a wide sense stationary stochastic process.

(3+3+4=10)

Q.6A: The scores of a random sample of 45 people who took TOEFL exam had a mean of 540 and a standard deviation of 50. Construct a 95 percent confidence interval for μ.

Q.6B: Explain the importance of matrix diagonalization. Diagonalize the following matrix.

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 0 \\ 2 & -4 & 2 \end{pmatrix}$$

Q.6C: Use the Simplex method to solve the following linear programming problem.

Maximize $Z = 3x_1 + 5x_2 + 4x_3$ subject to
$2x_1 + 3x_2 \leq 8$, $2x_2 + 3x_3 \leq 10$, $3x_1 + 2x_2 + 4x_3 \leq 15$,
$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$.

(3+3+4=10)