

## Mathematics of Learning – Worksheet 6

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- The exercise sheets will be uploaded every Monday. Solution sketches will be uploaded one week later.
  - You can hand in your own solutions via StudOn and we correct them - this is not mandatory. Please hand in in small groups of 2-3 students.
  - For questions, please use the forum on StudOn since other students may have similar questions. If you have a more personal question about the exercises please send an email to ehsan.waiezi@fau.de or lars.weidner@fau.de respectively.
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### Exercise 1 [Regularization].

The generalized Tikhonov regularization is formulated as

$$RSS(\beta, \lambda) = \sum_{n=1}^N (y_n - x_n^T \beta)^2 + \lambda \sum_{i=1}^{M+1} \sum_{j=1}^{M+1} q_{ij} \beta_i \beta_j$$

for  $\lambda \geq 0$  and with  $q_{ij} = q_{ji}$ .

1. Write  $RSS(\beta, \lambda)$  in matrix notation.
2. Let  $\lambda \geq 0$  be fixed. Compute the minimizer  $\hat{\beta}$  assuming that the regularization term is convex. (What does that mean in the matrix notation?). State a condition under which it is unique.

### Exercise 2 [Examples].

Let  $x = (1, 2, 3, 4, 5)^T$  and  $y = (4, 2, 5, 7, 2)^T$ .

1. Calculate  $\beta_0, \beta_1, \beta_2, \beta_3 \in \mathbb{R}$  such that  $\sum_{i=1}^5 (\beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i^3 - y_i)^2$  is minimal.
2. Calculate  $\beta_0, \beta_1, \beta_2, \beta_3 \in \mathbb{R}$  such that  $\sum_{i=1}^5 (\beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i^3 - y_i)^2 + 2 \cdot \|\beta\|^2$  is minimal.
3. Calculate  $\beta_0, \beta_1, \beta_2, \beta_3 \in \mathbb{R}$  with  $\|\beta\|_2 \leq 1$  such that  $\sum_{i=1}^5 (\beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i^3 - y_i)^2$  is minimal. (You need knowledge about constrained optimization (KKT-conditions) for this exercise. This will be explained later in the semester.)