Robot Controls Final project(ENPM667)

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This is the report the solution to
the double pendulum and cart problem.

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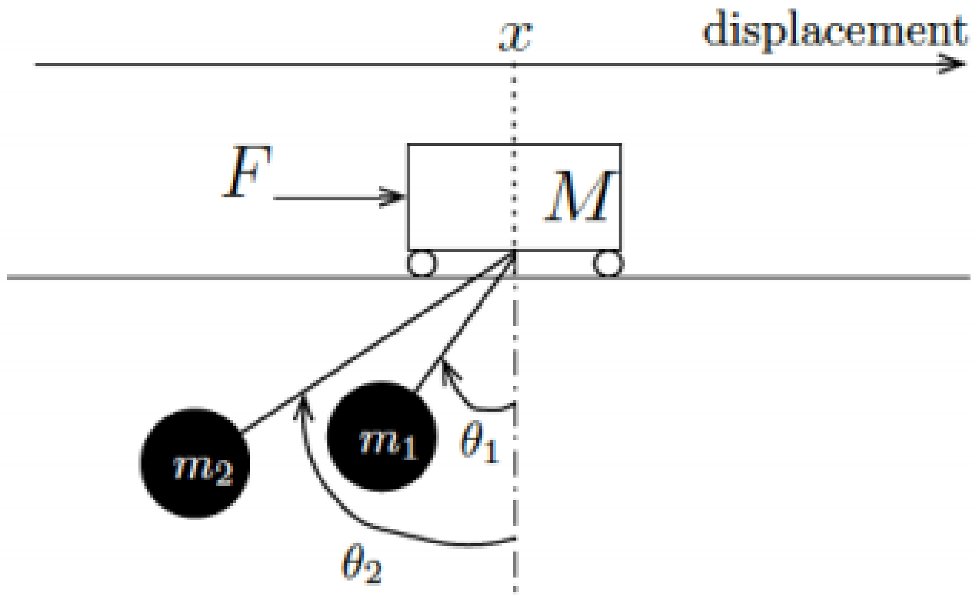
1 Problem

1.1 Question

Consider a crane that moves along an one-dimensional track. It behaves as a frictionless cart with mass M actuated by an external force F that constitutes the input of the system. There are two loads suspended from cables attached to the crane. The loads have mass m1 and m2, and the lengths of the cables are l1 and l2, respectively. The following figure depicts the crane and associated variables used throughout this project.



A) (25 points) Obtain the equations of motion for the system and the corresponding nonlinear state-space representation.

B) (25 points) Obtain the linearized system around the equilibrium point specified by x = 0 and $\theta 1$ = 0 $\theta 2$ = 0. Write the state-space representation of the linearized system.

C) (25 points) Obtain conditions on M, m1, m2, l1, l2 for which the linearized system is controllable.

D) (25 points) Choose M = 1000Kg, m1 = m2 = 100Kg, l1 = 20m and l2 = 10m. Check that the system is controllable and obtain an LQR controller. Simulate the resulting response to initial conditions when the controller is applied to the linearized system and also to the original nonlinear system. Adjust the parameters of the LQR cost until you obtain a suitable response. Use Lyapunov's indirect method to certify stability (locally or globally) of the closed-loop system.

Second Component (100 points): Consider the parameters selected in C) above.

E) Suppose that you can select the following output vectors: x(t), ($\theta_1(t)$, $\theta_1(t)$), (x(t), $\theta_1(t)$) or (x(t), $\theta_1(t)$, $\theta_2(t)$). Determine for which output vectors the linearized system is observable.

F) Obtain your "best" Luenberger observer for each one of the output vectors for which the system is observable and simulate its response to initial conditions and unit step input. The simulation should be done for the observer applied to both the linearized system and the original nonlinear system.

G) Design an output feedback controller for your choice of the "smallest" output vector. Use the LQG method and apply the resulting output feedback controller to the original nonlinear system. Obtain your best design and illustrate its performance in simulation. How would you reconfigure your controller to asymptotically track a constant reference on x ? Will your design reject constant force disturbances applied on the cart ?

2 Solution

2.1 Part A

To convert the problem into state space we first find the kinetic and potential energy which will give us Euler-Lagrangian equation

$$\begin{aligned}K = \frac{1}{2}(M\dot{x}^2) + \frac{1}{2}(m_1(\frac{d}{dt}(x - l_1 sin\theta_1))^2) + \frac{1}{2}(m_1(\frac{d}{dt}(l_1 cos\theta_1))^2) \\ +\frac{1}{2}(m_2(\frac{d}{dt}(x - l_2 sin\theta_2))^2) + \frac{1}{2}(m_2(\frac{d}{dt}(l_2 cos\theta_2))^2)\end{aligned} \tag{1}$$

$$\begin{aligned}K = \frac{1}{2}(M\dot{x}^2) + \frac{1}{2}(m_1(\dot{x}^2 + l_1^2\dot{\theta_1}^2 - 2l_1\dot{x}cos(\theta_1)\dot{\theta_1}) \\ +\frac{1}{2}(m_2(\dot{x}^2 + l_2^2\dot{\theta_2}^2 - 2l_2\dot{x}cos(\theta_2)\dot{\theta_2})\end{aligned} \tag{2}$$

$$P = m_1 g l_1(1 - cos(\theta_1)) + m_2 g l_2(1 - cos(\theta_2)) \tag{3}$$

The lagrangian interms of the kinetic and potential energy is given as

$$L = K - P \tag{4}$$

Therefore we have

$$\begin{aligned}L = \frac{1}{2}(M\dot{x}^2) + \frac{1}{2}(m_1(\dot{x}^2 + l_1^2\theta_1^2 - 2l_1\dot{x}cos(\theta_1)\dot{\theta_1}) + \frac{1}{2}(m_2(\dot{x}^2 + l_2^2\theta_2^2 - 2l_2\dot{x}cos(\theta_2)\dot{\theta_2}) \\ -m_1 g l_1(1 - cos(\theta_1)) - m_2 g l_2(1 - cos(\theta_2))\end{aligned} \tag{5}$$

The generalized coordinates are x, θ_1 and θ_2. We have the equations for the Lagrangian (L) given in equation A as follows

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F \tag{6}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0 \tag{7}$$

We compute the Lagrange's equation for the first generalized coordinate x

$$\frac{\partial L}{\partial x} = 0 \tag{8}$$

$$\frac{\partial L}{\partial \dot{x}} = m_1\dot{x} - m_1 l_1 \cos\theta_1\dot{\theta_1} + m_2\dot{x} - m_2 l_2 \cos\theta_2\dot{\theta_2} + M\dot{x} \tag{9}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m_1\ddot{x} + m_1 l_1 \sin\theta_1\dot{\theta}_1^2 - m_1 l_1 \cos\theta_1\ddot{\theta_1} + m_2\ddot{x} + m_2 l_2 \sin\theta_2\dot{\theta}_2^2 - m_2 l_2 \cos\theta_2\ddot{\theta_2} + M\ddot{x} \tag{10}$$

Substituting (8), (9) and (10) in (6) and separating out $\ddot{x}$ we get

$$\ddot{x} = \frac{F + m_1 l_1(\cos\theta_1\ddot{\theta_1} - \sin\theta_1\dot{\theta}_1^2) + m_2 l_2(\cos\theta_2\ddot{\theta_2} - \sin\theta_2\dot{\theta}_2^2)}{M + m_1 + m_2} \tag{11}$$

We compute the Lagrange's equation for the second generalized coordinate θ_1

$$\frac{\partial L}{\partial \theta_1} = m_1 l_1 \dot{x}\dot{\theta_1}\sin\theta_1 - m_1 g l_1 \sin\theta_1 \tag{12}$$

$$\frac{\partial L}{\partial \dot{\theta_1}} = \dot{\theta_1} m_1 l_1^2 - m_1 l_1 \dot{x}\cos\theta_1 \tag{13}$$

Taking the time derivative we get

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta_1}}\right) = m_1 l_1 \dot{x}\sin\theta_1\dot{\theta_1} + m_1 l_1^2\ddot{\theta_1} - m_1 l_1 \ddot{x}\cos\theta_1 \tag{14}$$

Substituting (12), (13) and (14) in (7) and separating out $\ddot{\theta_1}$ we get

$$\ddot{\theta_1} = \frac{\ddot{x}\cos\theta_1 - g\sin\theta_1}{l_1} \tag{15}$$

Similarly for θ_2, we compute the Lagrange's equation as follows

$$\frac{\partial L}{\partial \theta_2} = m_2 l_2 \dot{x}\sin\theta_2\dot{\theta_2} - m_2 g l_2 \sin\theta_2 \tag{16}$$

$$\frac{\partial L}{\partial \dot{\theta_2}} = \dot{\theta}_2 m_2 l_2^2 - m_2 l_2 \dot{x} \cos\theta_2 \tag{17}$$

Taking the time derivative we get

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta_2}}\right) = m_2 l_2 \dot{x} \sin\theta_2 \dot{\theta_2} + m_2 l_2^2 \ddot{\theta_2} - m_2 l_2 \ddot{x} \cos\theta_2 \tag{18}$$

Substituting (16), (17) and (18) in (7) and separating out $\ddot{\theta_2}$ we get

$$\ddot{\theta_2} = \frac{\ddot{x}\cos\theta_2 - g\sin\theta_2}{l_2} \tag{19}$$

Our state vector can be represented as

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \\ \theta_1 \\ \dot{\theta_1} \\ \theta_2 \\ \dot{\theta_2} \end{bmatrix} \tag{20}$$

where $x_1 = x$, $x_2 = \dot{x}$, $x_3 = \theta_1$, $x_4 = \dot{\theta_1}$, $x_5 = \theta_2$ and $x_6 = \dot{\theta_2}$

The corresponding non-linear state space representation is given by

$$\dot{\mathbf{X}} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta_1} \\ \ddot{\theta_1} \\ \dot{\theta_2} \\ \ddot{\theta_2} \end{bmatrix} \tag{21}$$

where $\ddot{x}$, $\ddot{\theta_1}$ and $\ddot{\theta_2}$ are given by eq (11), (15) and (19) respectively.

Also note we have here we have $[F_1 F_2 F_3 F_4 F_5 F_6]$ as functions of other state variables. Hence following is state space of non-linear system.

2.2 Part B

In the previous section, we have obtained the non-linear state space representation given by

$$\dot{\mathbf{X}} = f(X, U) \tag{22}$$

where $\mathbf{x}$ is the state vector and $\mathbf{u}$ is the vector of inputs. We now obtain the linearized system ($\dot{\mathbf{X}} = AX + BU$) around the equilibrium point specified by $x = 0$, $\theta_1 = \theta_2 = 0$. At equilibrium point given by,

$$X_{eqbm} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{23}$$

the system is characterized by setting the state derivative to zero. This can be represented as a Taylor series expansion of $f(\cdot\,,\cdot)$ about nominal state x_e and input u_e given by

$$\frac{d}{dt}(x_e + \partial x) = f(x_e, u_e) + \left.\frac{\partial f_i}{\partial x}\right|_0 \partial x + \left.\frac{\partial f_i}{\partial u}\right|_0 \partial u \tag{24}$$

Ignoring the higher order terms in the expansion due to small variations, and writing for all the states we have

$$\frac{d}{dt}(\partial x_i) = \left.\frac{\partial f_i}{\partial x}\right|_0 + \left.\frac{\partial f_i}{\partial u}\right|_0 \tag{25}$$

where $\left.\frac{\partial f_i}{\partial x}\right|_0$ gives us the A matrix shown below

$$\mathbf{A} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} & \frac{\partial f_1}{\partial x_5} & \frac{\partial f_1}{\partial x_6} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} & \frac{\partial f_2}{\partial x_5} & \frac{\partial f_2}{\partial x_6} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_4} & \frac{\partial f_3}{\partial x_5} & \frac{\partial f_3}{\partial x_6} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_2} & \frac{\partial f_4}{\partial x_3} & \frac{\partial f_4}{\partial x_4} & \frac{\partial f_4}{\partial x_5} & \frac{\partial f_4}{\partial x_6} \\ \frac{\partial f_5}{\partial x_1} & \frac{\partial f_5}{\partial x_2} & \frac{\partial f_5}{\partial x_3} & \frac{\partial f_5}{\partial x_4} & \frac{\partial f_5}{\partial x_5} & \frac{\partial f_5}{\partial x_6} \\ \frac{\partial f_6}{\partial x_1} & \frac{\partial f_6}{\partial x_2} & \frac{\partial f_6}{\partial x_3} & \frac{\partial f_6}{\partial x_4} & \frac{\partial f_6}{\partial x_5} & \frac{\partial f_6}{\partial x_6} \end{bmatrix} \tag{26}$$

and $\left.\frac{\partial f_i}{\partial u}\right|_0$ gives us the B matrix as

$$\mathbf{B} = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \\ \frac{\partial f_3}{\partial u} \\ \frac{\partial f_4}{\partial u} \\ \frac{\partial f_5}{\partial u} \\ \frac{\partial f_6}{\partial u} \end{bmatrix} \tag{27}$$

Computing $\mathbf{A}$ and $\mathbf{B}$ at the equilibrium point we get

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{gm_1}{M} & 0 & -\frac{gm_2}{M} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -(\frac{g}{l_1} + \frac{gm_1}{Ml_1}) & 0 & -\frac{gm_2}{Ml_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{gm_1}{Ml_2} & 0 & -(\frac{g}{l_2} + \frac{gm_2}{Ml_2}) & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{1}{Ml_1} \\ 0 \\ \frac{1}{Ml_2} \end{bmatrix} \tag{28}$$

We have three outputs of our system ie. x, θ_1 and θ_2. We have the **C** matrix as follows

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{29}$$

So, we can write this in linear state space equation as

$$\dot{\mathbf{X}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{gm_1}{M} & 0 & -\frac{gm_2}{M} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -(\frac{g}{l_1} + \frac{gm_1}{Ml_1}) & 0 & -\frac{gm_2}{Ml_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{gm_1}{Ml_2} & 0 & -(\frac{g}{l_2} + \frac{gm_2}{Ml_2}) & 0 \end{bmatrix} \mathbf{X} + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{1}{Ml_1} \\ 0 \\ \frac{1}{Ml_2} \end{bmatrix} \mathbf{U} \tag{30}$$

$$\mathbf{Y} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{X} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \mathbf{U} \tag{31}$$

2.3 Part C

For the linear system to be controllable we check the rank of the controllability matrix which is given by

$$C = \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix} \tag{32}$$

To be controllable, $rank(C)$ should be equal to 6 ie. full rank. In other words, the matrix should not be singular and value of determinant should not be equal to zero. The C matrix in our case is as follows

$$C = \begin{bmatrix} 0 & \frac{1}{M} & 0 & \cdots \\ \frac{1}{M} & 0 & \frac{-gm_1}{M^2 l_1} \frac{-gm_2}{M^2 l_2} & \cdots \\ 0 & \frac{1}{Ml_1} & 0 & \cdots \\ \frac{1}{Ml_1} & 0 & -\frac{g+\frac{(gm_1)}{M}}{Ml_1^2} - \frac{gm_2}{M^2 l_1 l_2} & \cdots \\ 0 & \frac{1}{Ml_2} & 0 & \cdots \\ \frac{1}{Ml_2} & 0 & -\frac{g+\frac{(gm_2)}{M}}{Ml_2^2} - \frac{gm_1}{M^2 l_1 l_2} & \cdots \end{bmatrix} \tag{33}$$

we have only shown the first three columns and the code for the calculating the full C matrix in Matlab is attached. If we calculate the determinant of the C matrix we get the following

$$Det(C) = -\frac{g^6 (l_1 - l_2)^2}{M^6 l_1^6 l_2^6} \tag{34}$$

we see that the determinant is equal to zero when l_1 equals l_2. Thus the linearized system is controllable if $l_1 \neq l_2$.

2.4 Part D

Now we are given values of parameters as follows:-

$M = 1000Kg$

$m_1 = m_2 = 100Kg$

$l_1 = 20m$

$l_2 = 10m$

So our linear state space from part B we have.

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{gm_1}{M} & 0 & -\frac{gm_2}{M} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -(\frac{g}{l_1} + \frac{gm_1}{Ml_1}) & 0 & -\frac{gm_2}{Ml_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{gm_1}{Ml_2} & 0 & -(\frac{g}{l_2} + \frac{gm_2}{Ml_2}) & 0 \end{bmatrix} \tag{35}$$

Substituting values we get.

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -981/1000 & 0 & -981/1000 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -10791/20000 & 0 & -981/20000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -981/10000 & 0 & -10791/10000 & 0 \end{bmatrix} \tag{36}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ \frac{1}{1000} \\ 0 \\ \frac{1}{20000} \\ 0 \\ \frac{1}{10000} \end{bmatrix}$$

One method of computing

$Q = C^T C$

But in our case. as the values of Cart mass, length of string of pendulum and pendulum masses are very high so we need to choose Q matrix in order of 10^6 or else system does not decay. R=0.001.

$$\mathbf{Q} = \begin{bmatrix} 100000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1000000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10000000 & \\ 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix} \tag{37}$$

procedure to choose Q in by going on increasing zeros to the cost of important state. So with this conditions we designed LQR controller which has values of K as

K= 1.0e+04 * $\begin{bmatrix} 1.000 & 0.9495 & 2.1169 & -3.2256 & 1.1278 & -2.9447 \end{bmatrix}$

By this values we get the following output

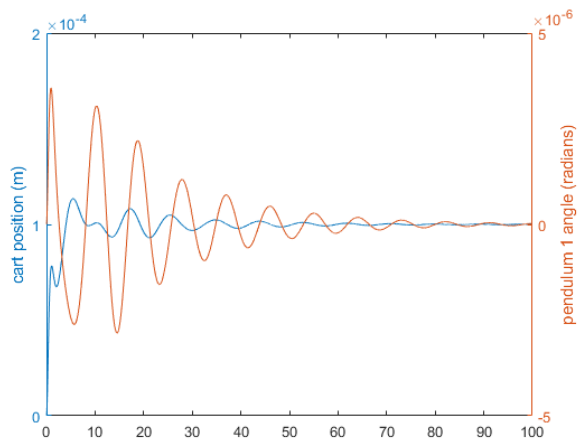


Figure 1: Graph with LQR controller of θ_1 , X in step input

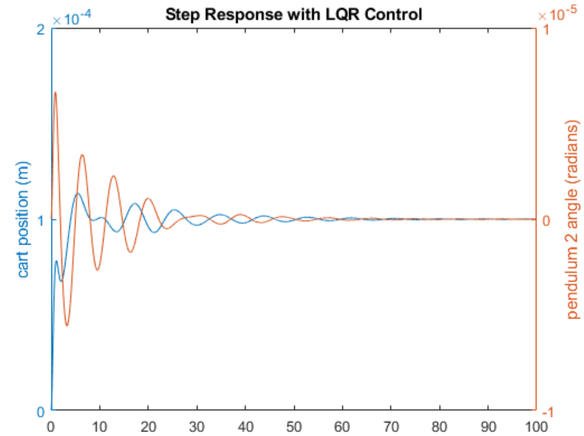


Figure 2: Graph with LQR controller of θ_2 , X in step input

Results from the following graphs.

- As Value of Length of pendulum 1 is greater than length of pendulum 2 so it take more time for the system to come in equilibrium.
- Error of Pendulum increases to higher value and die down fast than cart because cart has 10 times more mass so has higher inertia to oppose to change in state of system to stability.
- Similar results can also be seen in case if we give system an initial condition on x= $\begin{bmatrix} 2 & 0 & 0.3 & 0 & 0.3 & 0 \end{bmatrix}^T$

So following is the output we get.

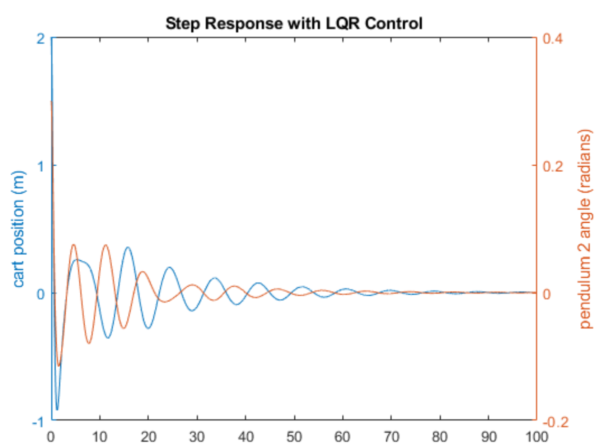
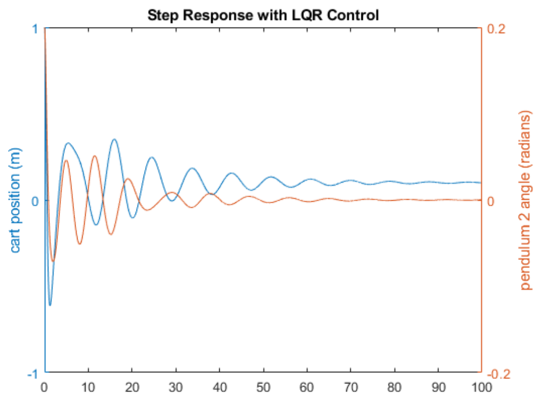
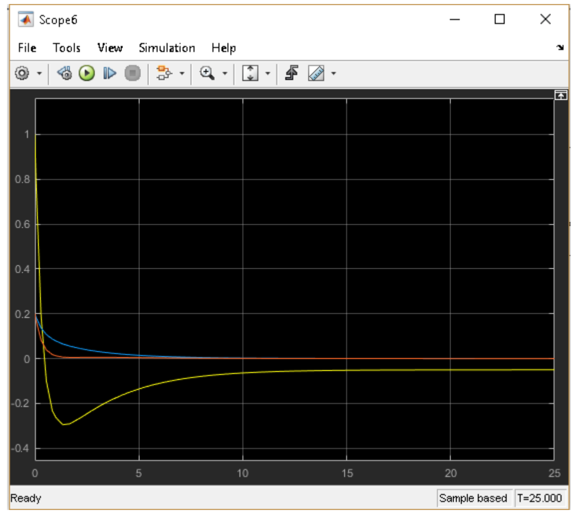


Figure 3: Graph with LQR controller of θ_2 , X with initial condition



(a) Linear system



(b) Nonlinear system

Figure 5: Graph with LQR controller with initial condition of 1000 N step input

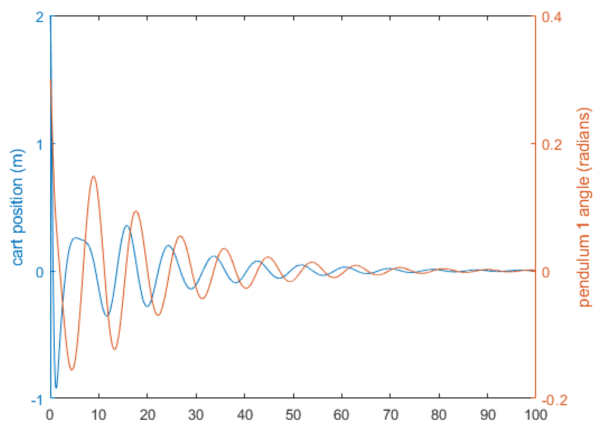


Figure 4: Graph with LQR controller of θ_2 , X with initial condition

Hence, we have a good LQR contoller which dies down quickly.

Now lets apply linear controller to Non-linear system.

And we see the following output.

For this case we give input step response as 1000N of force. Now we can compare the output of linear and non-linear system.

Lyapunov's indirect method test

Eigen values of after LQR controller implementation.

Eigen Values =

$$\begin{bmatrix} -2.27476483568212 + 2.32665936966114i \\ -2.27476483568212 - 2.32665936966114i \\ -0.141362748764850 + 0.955789076908457i \\ -0.141362748764850 - 0.955789076908457i \\ -0.0528182311971062 + 0.695729234667867i \\ -0.0528182311971062 - 0.695729234667867i \end{bmatrix}$$

So from this we can infer from eigen values that real parts of all Eigen values is negative. So linearized system is stable. And with Lyapunov's indirect method we can tell that the non linear system we are working with is locally stable near equlilbrium point.

2.5 Part E

To determine if a system is observable, we determine the rank of the Observability matrix. If it has full rank, then the system is said to be observable. The observability matrix denoted by O is given by

$$\mathbf{O} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} \tag{38}$$

We are given different combinations for which we want to determine if the system is observable or not. The cases are analyzed one by one

Case 1: In this case we select the output vector $x(t)$. Rank of observability is 6 ie. full rank, so system **is** observable with just displacement x(t) data.

Case 2: In this case we select the output vector $(\theta_1(t)\ ,\ \theta_2(t))$. Rank of the observability matrix is 4 so system **is not** observable with just $(\theta_1(t)\ ,\ \theta_2(t))$ of system.

Case 3: In this case we select the output vector $(x(t), \theta_2(t))$. Rank of the observability matrix is 6 ie. full rank so the system **is** observable with just $(x(t), \theta_2(t))$ data.

Case 4: In this case we select the output vector $(x(t)), \theta_1(t), \theta_2(t))$. Rank of observability matrix is 6 ie. full rank so system **is** observable with $(x(t)), \theta 1(t), \theta 2(t))$ data.

2.6 Part F

Case 1: In this case we select the output vector $x(t)$.

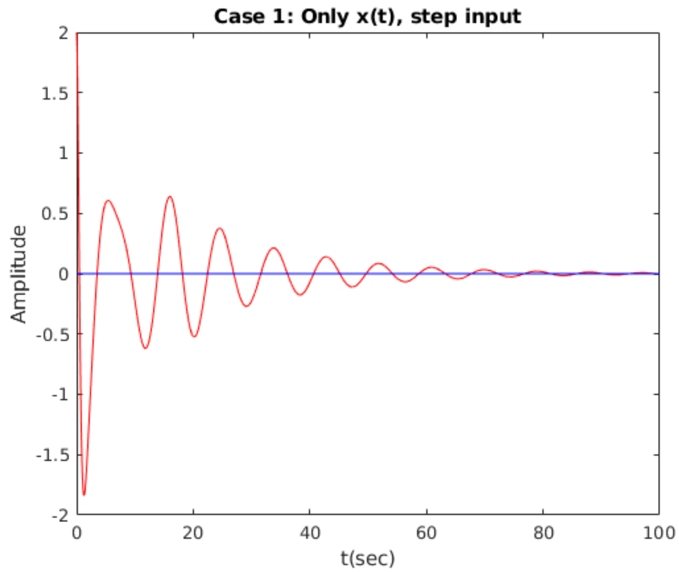


Figure 6: Response for output vector x(t) with initial state

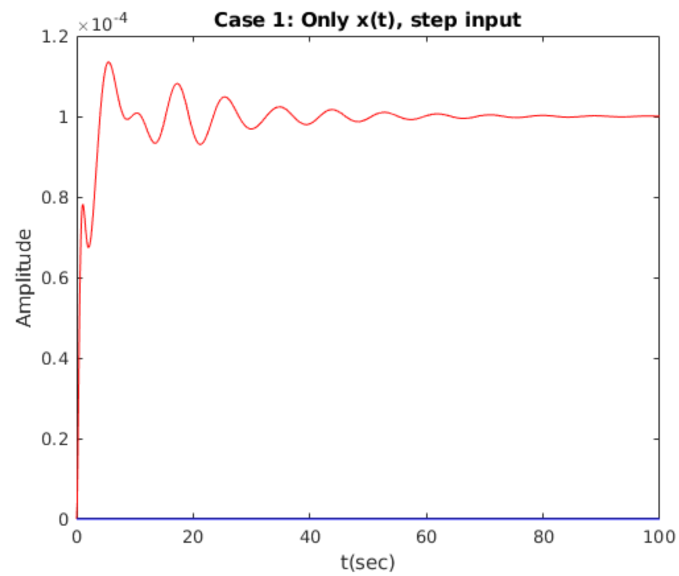


Figure 7: Response for output vector x(t) with step input

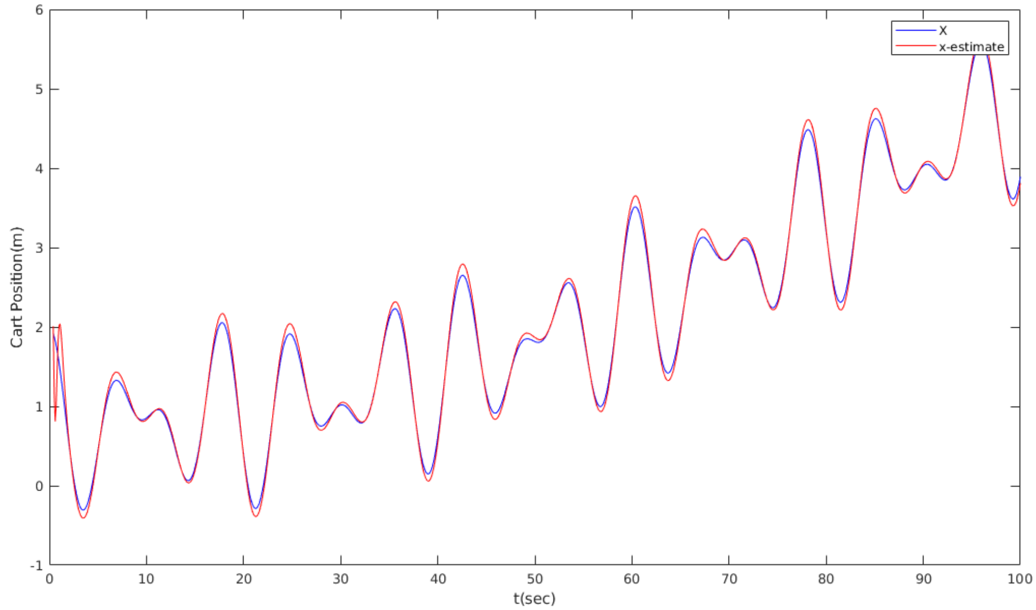


Figure 8: Observer tracking for output vector x(t)

Output of Non-linear system

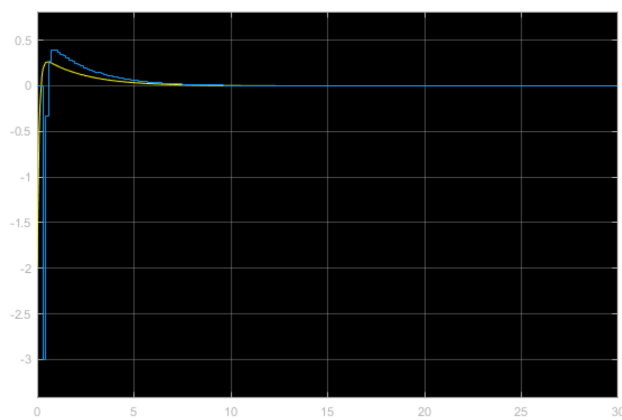


Figure 9: Observer tracking for output of nonlinear vector x(t)

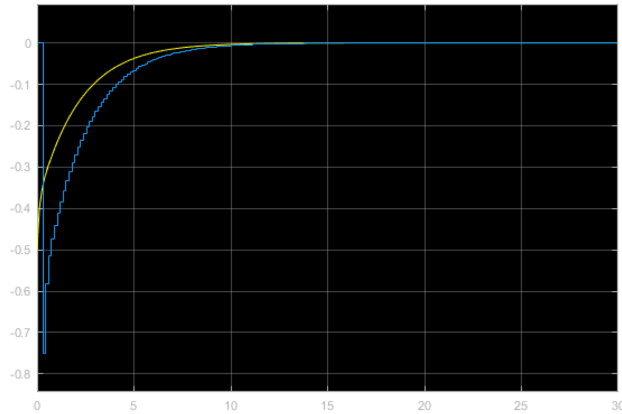


Figure 10: Observer tracking for output of nonlinear vector θ_1(t)

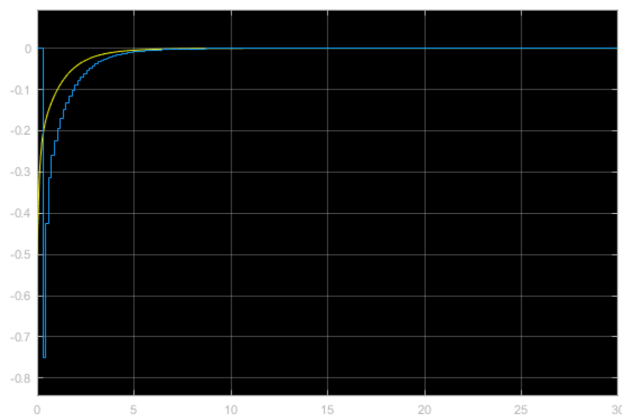


Figure 11: Observer tracking for output of nonlinear vector θ_2(t)

Case 3: In this case we select the output vector $(x(t), \theta_2(t))$.

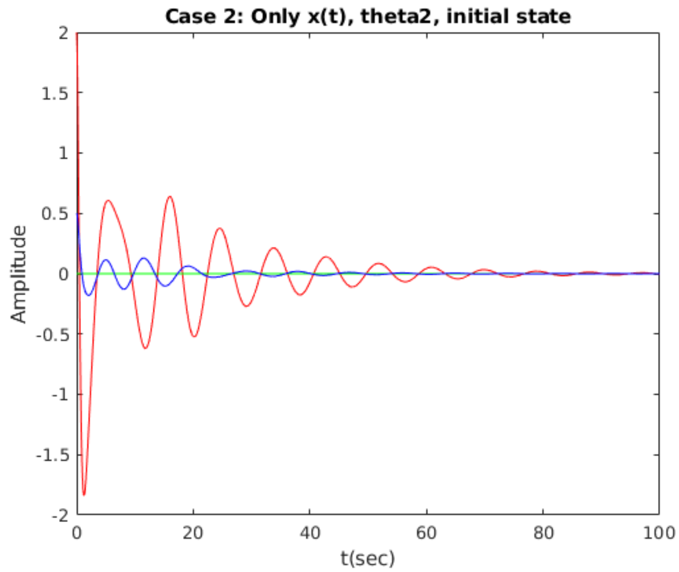


Figure 12: Response for output vector x(t) and θ_2 with initial state

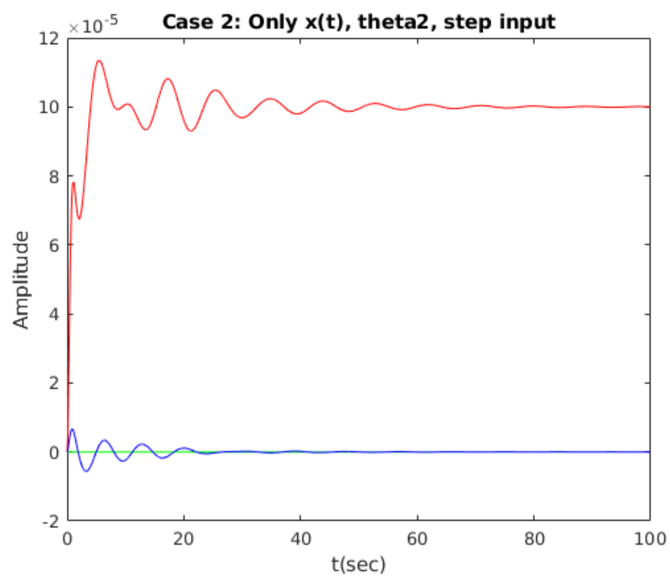


Figure 13: Response for output vector x(t) and θ_2 with step input

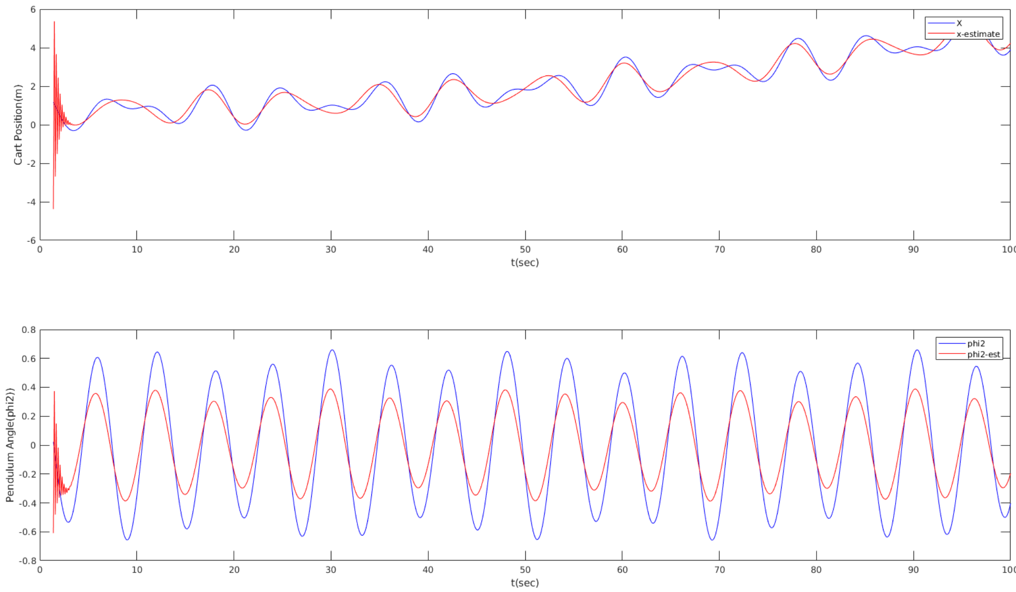


Figure 14: Observer tracking for output vector x(t) and θ_2

Output of Non-linear system

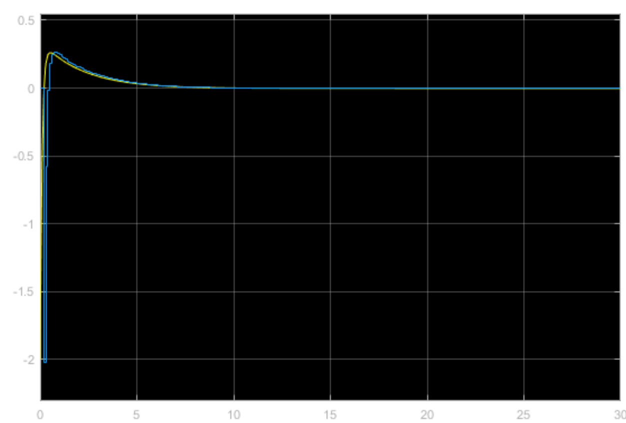


Figure 15: Observer tracking for output of nonlinear vector x(t)

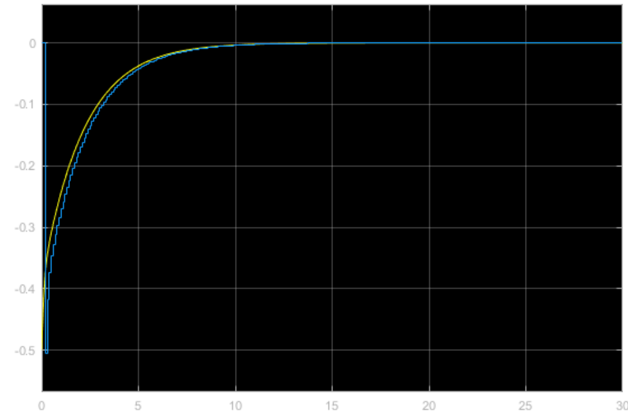


Figure 16: Observer tracking for output of nonlinear vector θ_1(t)

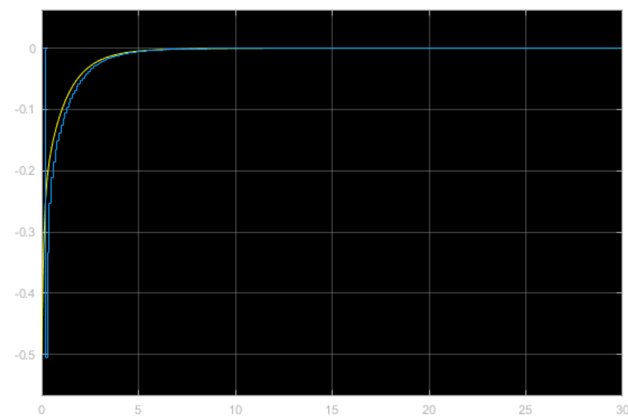


Figure 17: Observer tracking for output of nonlinear vector θ_2(t)

Case 4: In this case we select the output vector $(x(t)), \theta_1(t), \theta_2(t))$.

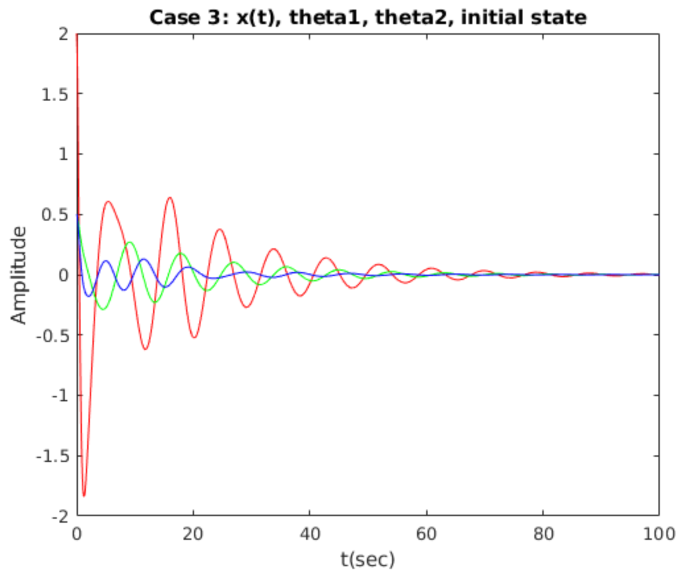


Figure 18: Response for output vector x(t), θ_1 and θ_2 with initial state

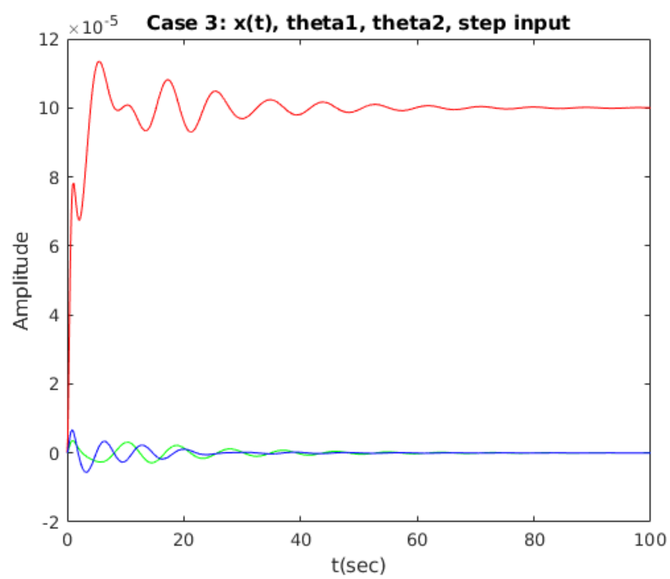


Figure 19: Response for output vector x(t), θ_1 and θ_2 with step input

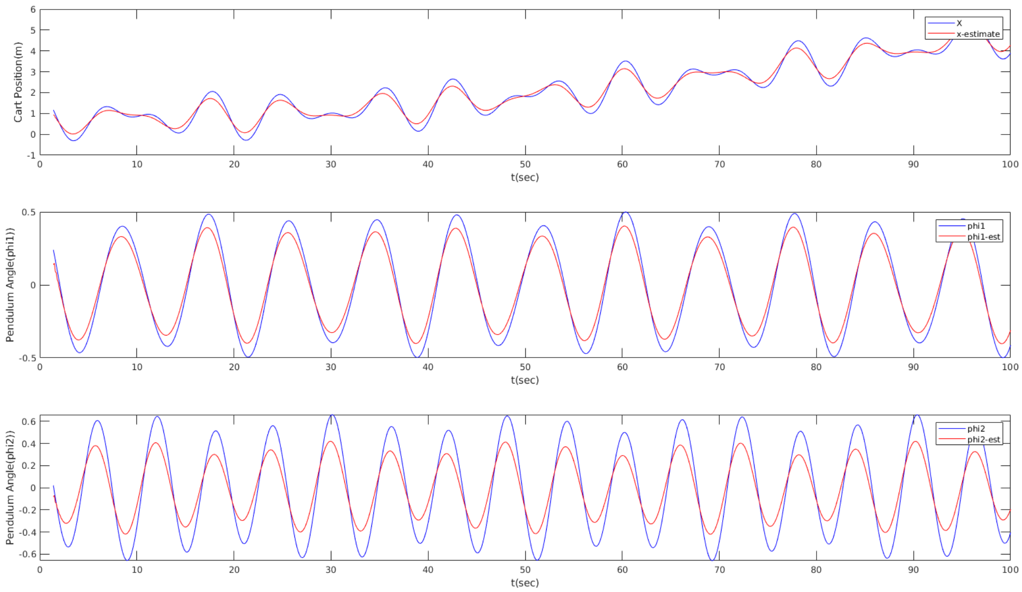


Figure 20: Observer tracking for output vector x(t), θ_1 and θ_2

Output of Non-linear system

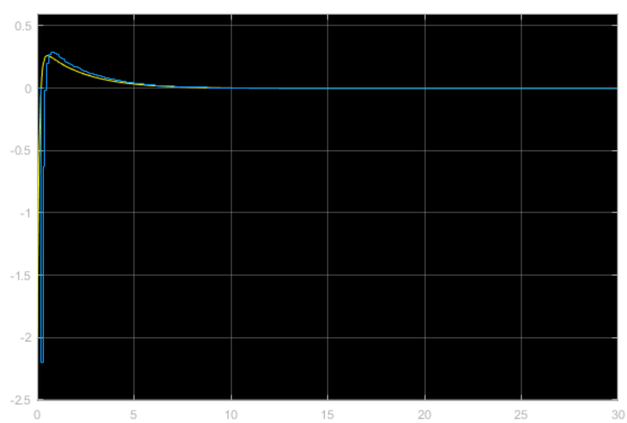


Figure 21: Observer tracking for output of nonlinear vector x(t)

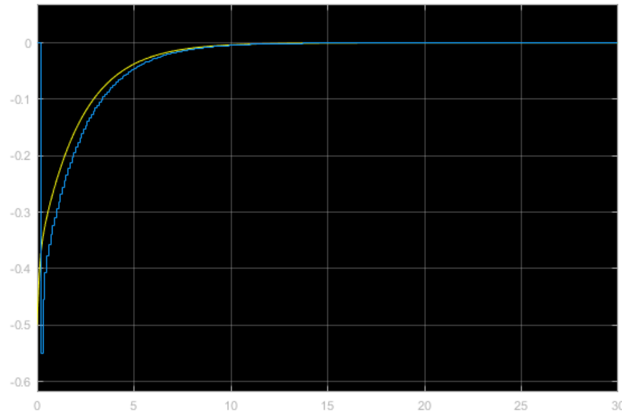


Figure 22: Observer tracking for output of nonlinear vector θ_1(t)

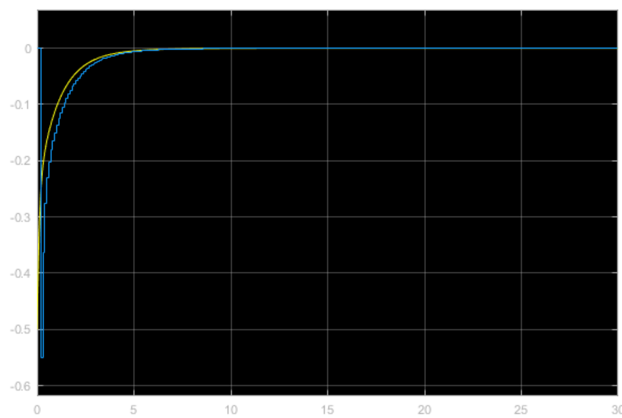


Figure 23: Observer tracking for output of nonlinear vector θ_2(t)

2.7 Part G

In control theory, the linear–quadratic–Gaussian (LQG) control problem is one of the most fundamental optimal control problems. The continuous time linear dynamic system can be represented as

$$\dot{\mathbf{x}}(t) = A(t)\mathbf{x}(t) + B(t)\mathbf{u}(t) + \mathbf{v}(t), \tag{39}$$

$$\mathbf{y}(t) = C(t)\mathbf{x}(t) + \mathbf{w}(t) \tag{40}$$

where $w(t)$ is the measurement noise and $v(t)$ is the process noise.

Reference Tracking To track a constant reference x, we can use LQR with an integral component to it.

Non-linear Graph with LQG

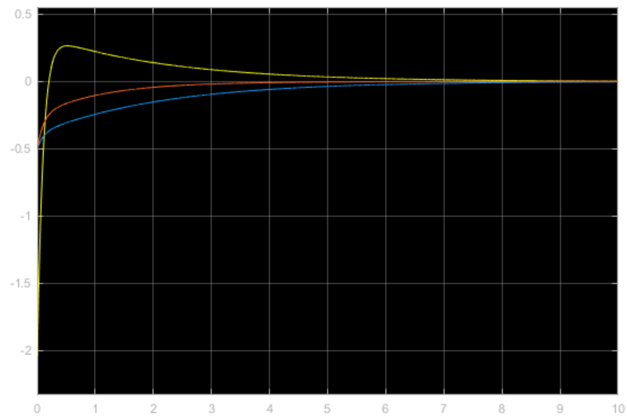


Figure 24: Observer tracking for output with LQG controller

2.8 Simulink Files

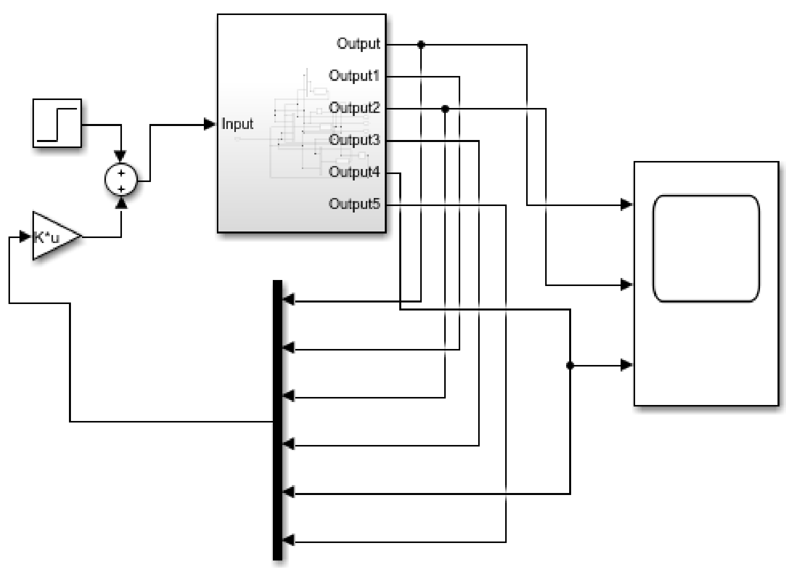


Figure 25: LQR for non linear System block Diagram

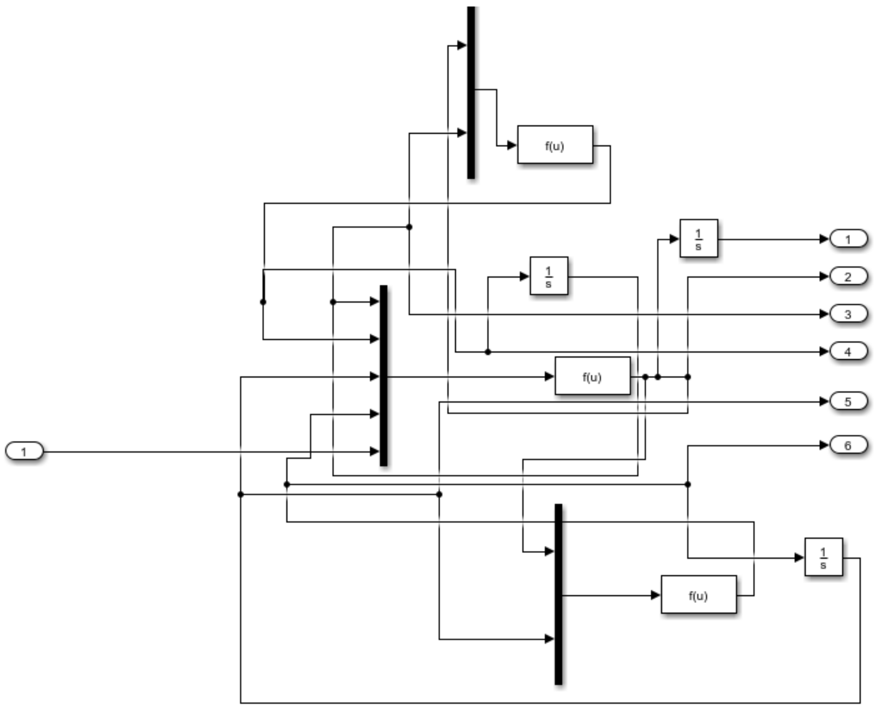


Figure 26: Internal block of LQR for non linear System block Diagram

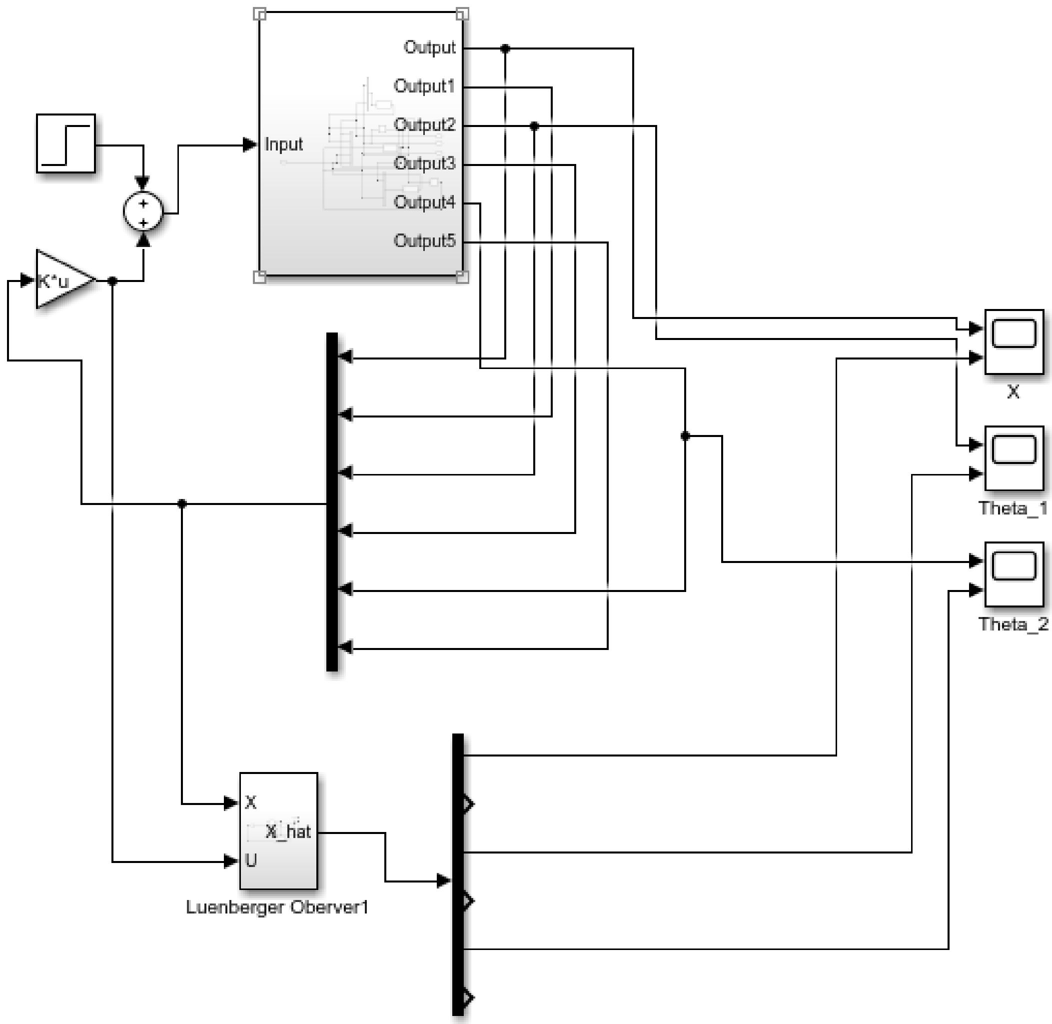


Figure 27: Observer tracking with Luenberger Observer block Diagram

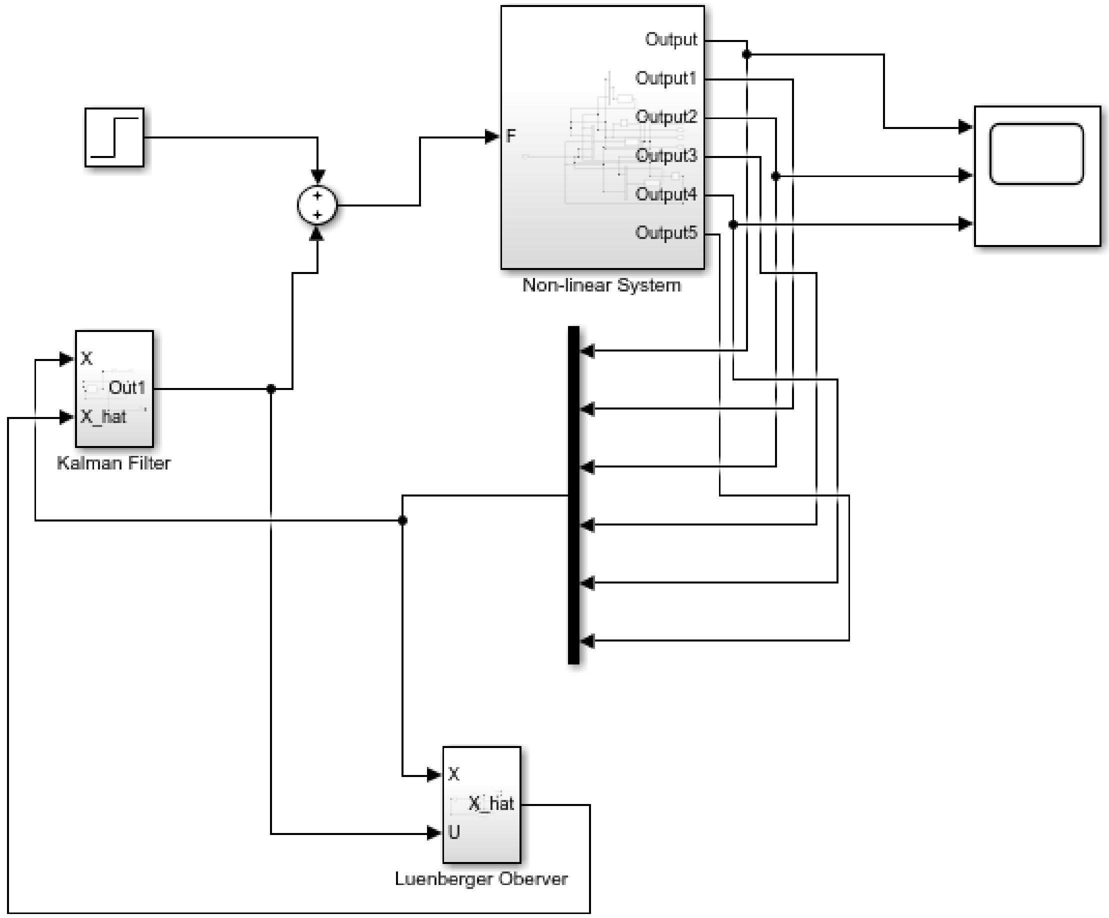


Figure 28: LQG controller block Diagram