Mini Project

M&I – ENG 686X – Multi-Criteria Decision Making

By Team #4

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IV. Executive Summary

Electricity generation and resources are becoming increasingly important due to climate change and environmental issues, resulting in many research projects. Many states in the US are trying to minimize their CO_2 emissions to address climate change. This report will deal with the data presented in Table 1 to minimize CO_2 emission damages, electricity generation costs, and local pollution by considering three input factors while maintaining the fixed per capita demand of 9 kWh.

Table 1: Data for the three alternative inputs.

	Local pollution mg/kWh	CO2/kWh	Cents/kWh
Imports	0	0	75
Coal+CC	100	0.9	20
NG	45	2	10

Throughout the report, P_i represents the local pollution of input factor i per kWh, where i: 1, 2, and 3 for imports, coal plus carbon capture, and natural gas, respectively. CO_i is the CO_2 emissions from input i per kWh, and C_i is the cost of generating 1 kWh using i. Moreover, in terms of Decision variables, let X_i represent the production quantity in kWh using input i.

In each question, we will tackle specific requirements and try to find an optimal solution based on the assigned objectives or given portfolio sets. We face uncertainty from expert opinions regarding the exponent of damage cost function from CO_2 emissions, presented in Table 2. We will be dealing with the views of Expert 1 all through questions I to IV.

Table 2: Probabilities assigned to expert opinions regarding the exponent of damages.

	Exp = 1.5	2	3
Expert 1	0.3	0.4	0.3
Expert 2	0.4	0.45	0.15
Expert 3	0.15	0.25	0.6

We show that when the policymaker strictly prefers to minimize the net cost, i.e., he neglects local pollution, coal plus carbon capture appropriates almost all the inputs for electricity generation. On the other hand, when the decision-maker solely prefers to minimize local pollution, imports are used to satisfy all of the electricity demand. Meanwhile, for more balanced objectives, i.e., when the decision-maker has different, albeit non-zero, levels of preference for both net cost and local pollution, natural gas can play a small role as an input factor; however, even in these more balanced cases, coal plus carbon capture and imports are the driving inputs.

The rest of the report is structured as follows. Section V will discuss the extreme instance when the decision-maker only cares about the net cost of electricity generation. Section VI discusses the tradeoff between net cost and local pollution and tries to develop a non-dominated set of alternative portfolios with two separate methods. In Section VII, we will construct and use an additive value function to compare a finite set of available portfolios. Stochastic dominance is assessed among a limited number of available portfolios in Section VIII. Finally, in Section IX, we conclude.

V. Question I

In this question, we will ignore local pollution. As an objective, we will minimize the net cost of CO_2 emissions and electricity generation while maintaining a per capita demand of 9 kWh.

Our objective function will be as follows:

$$Min \sum_{1}^{3} X_i C_i + E[CO]$$

Observe that the net cost is equivalent to the cost of damages (the expected cost of CO_2 emissions using the expert's assigned probabilities) plus the cost of generating electricity. Also, note that the expected damage cost of CO_2 emissions is calculated based on the total amount of CO_2 emissions (i.e., $\sum_1^3 x_i CO_i$) and the probabilities assigned by Expert 1. In mathematical terms:

$$E[CO] = Pr_{1.5}(\sum_{1}^{3} X_i CO_i)^{1.5} + Pr_2(\sum_{1}^{3} X_i CO_i)^2 + Pr_3(\sum_{1}^{3} X_i CO_i)^3$$

Table 3: Question I results.

Decision Variables		
X_1: Imports	X_2: Coal + CC	X_3: NG
0.418747743	**8.581253264**	**0**

Total CO_2 emissions	**7.723128**

Cost Of Damages using Expert 1 opinions	
Exponent = 1.5	21.46297
Exponent = 2	59.64671
Exponent = 3	460.6591
Expected CO_2 cost	**168.4953**

Energy Cost	**203.0311**
Net Cost	**371.5265**

This model has one constraint, which is:

$$S.t \sum_{1}^{3} X_i = 9$$

This means the total electricity produced from the three inputs should always equal 9.

Using the Excel Solver to find the optimal solution to the above nonlinear optimization problem, we found an optimal objective value of 371.53 cents per day per capita.

Table 3 shows the results for the three decision variables. In the optimal solution, the portfolio is such that we produce 0.419 kWh using imports and 8.58 kWh using coal plus CC.

VI. Question II

In this question, we will include the local pollution to what we already had in Question I.

Here, we will present two different methods. We assign weights to local pollution and net cost objectives in the first method. In the second one, we turn the objective function assigned to local pollution into a constraint with specific values at each iteration.

The first method to produce the non-dominated set is the weighted method. Table 4 shows the solution using equal weights of 0.5 and 0.5:

Table 4: Optimal solution for minimizing the net cost and local pollution with 50-50 weights.

Decision Variables (for 50-50 weights)		
X_1: Imports	X_2: Coal + CC	X_3: NG
7.604106243	**0**	**1.395895**

Local Pollution	**62.81526**
Net Cost	**595.3118**
Objective Value	**329.0635**

The net cost is calculated in a way utterly similar to what we had in question I. However, the objective value differs since we must also integrate local pollution using the weights w and 1-w.

Thus, the objective function is as follows:

$$Min \;\; w\left(\sum_{1}^{3} X_i C_i + E[CO]\right) + (1-w)\left(\sum_{1}^{3} X_i P_i\right)$$

Where w is the weight of the net cost and (1-w) is the weight of the local pollution. Note that the constraint remains the same as before; the daily demand is fixed at 9 kWh. The values of w are varied from 0 to 1 by an increment of 0.1. This resulted in the Pareto chart shown in Figure 1.

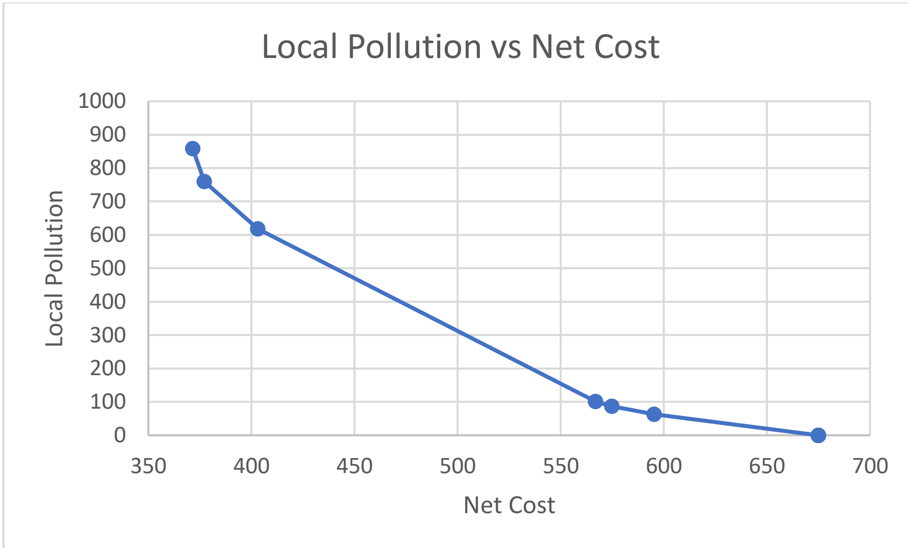

Figure 1: Pareto chart of the non-dominated portfolio set using the weighting method.

Now, we will discuss the second method, i.e., the constraint method. In this case, we omit the relevant objective function and add it as a new constraint to the model that sets the total local pollution equal to a specific value. This value ranges from 0 to 900 as we have 9 kWh of daily demand, and this will lead to a minimum of 0 mg of local pollution in case of using only the imports input and a maximum of 900 mg if we use the input that results in the highest local pollution, i.e., coal plus CC with 100 mg of pollution per kWh.

The added constraint is as follows:

$$S.t \sum_{1}^{3} X_i P_i = Constant$$

Where we vary the constant from 0 to 900 by increments of 50.

This will result in the Pareto chart for a non-dominated set of portfolios presented in Figure 2; note that, for comparison, the constraint method results are shown along those of the weighting method.

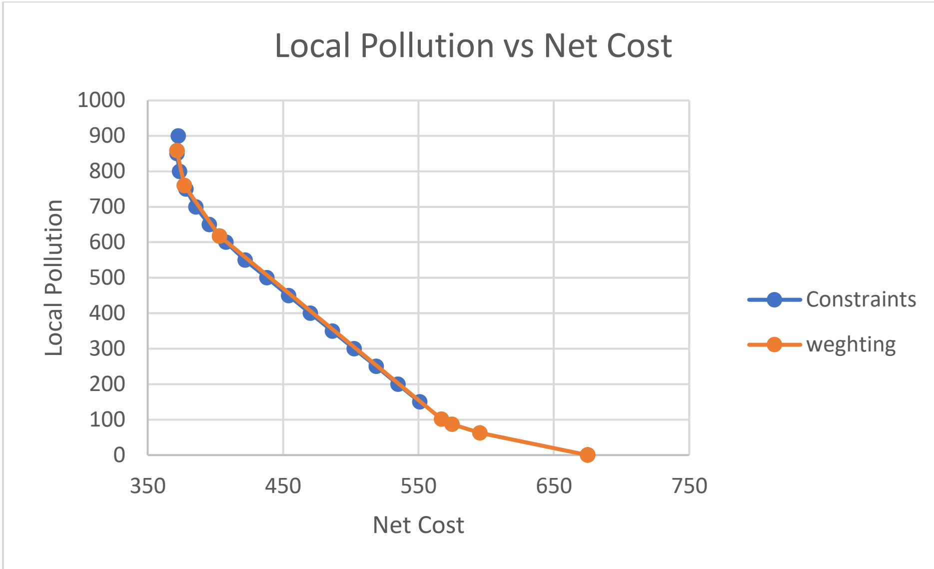

Figure 2: Pareto chart of the non-dominated portfolio set using weighting and the constraint methods.

As one can see, both methods lead to the same results—a higher preference for minimizing local pollution results in a high net cost. One can observe the decreasing characteristic of the slope at the extremes of the tradeoff; that is, e.g., changing the limit of local pollution from 400 to 200 results in a cost reduction of around 100 cents, whereas the same magnitude of change could result in a much lower net cost reduction (about 15 cents) at the lower end of the tradeoff.

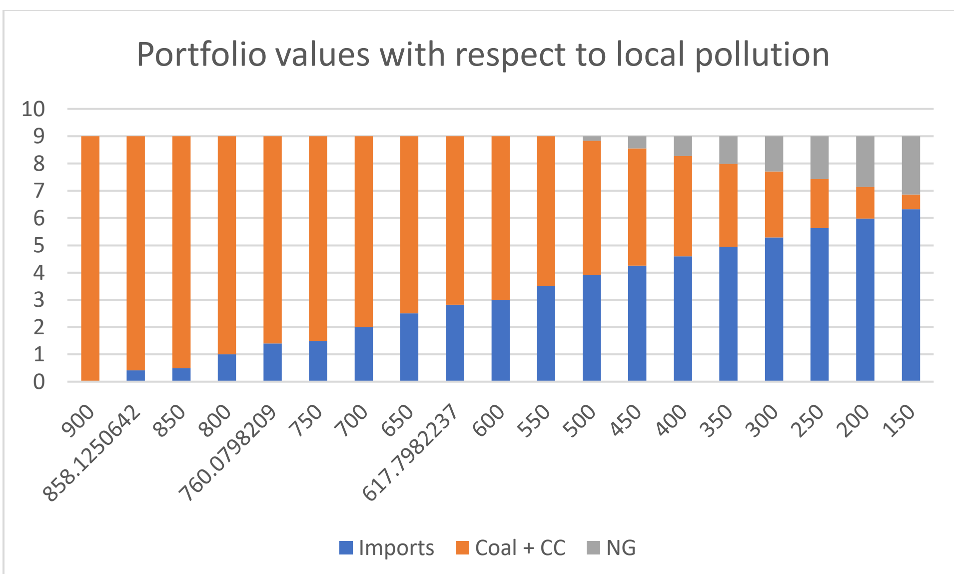

Figure 3: Portfolio values variation with respect to the local pollution.

When local pollution is preferred, more imports are used to generate electricity to achieve 0 levels of local pollution. On the other hand, being less sensitive to local pollution means that coal with carbon capture will be the preferred input since it results in the lowest net cost among the alternative inputs.

A better representation of both methods in one plot is Figure 4. This graph is handy for the decision-maker/policymaker since it offers the results interactively and clarifies the relationship between local pollution and net cost. If one wants to minimize both in a balanced manner, one possible scenario is the 13th portfolio, where the relevant curves intercept.

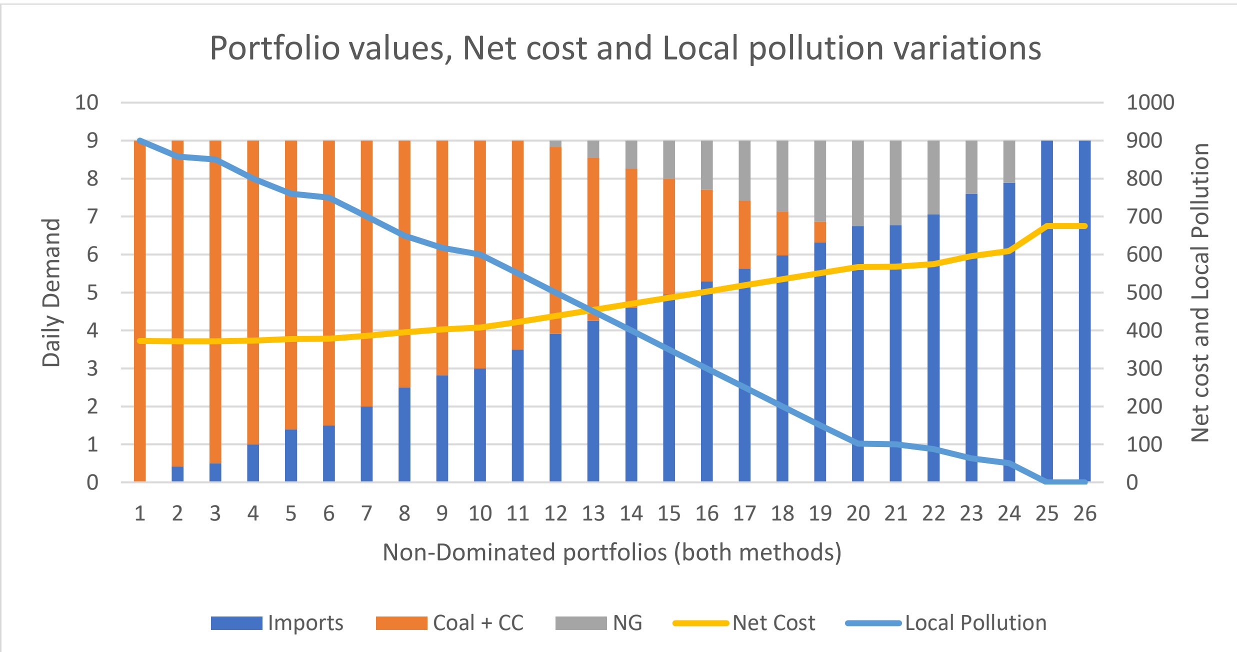

Figure 4: The Non dominated portfolio values along with the net cost and local pollution.

VII. Question III

Here, we use an additive value function with weights of 0.7 and 0.3 assigned to net cost and local pollution, respectively. Additionally, note that the maximum and minimum resultant net cost and local pollution amounts are used to construct the value functions and derive the total value outcomes; these maximum and minimum amounts are across all three exponents. The results are shown in Table 5.

Table 5: Results of local pollution and net cost for the given portfolios.

Import	Coal + CC	NG	Total CO2	Expected CO2 Cost	Energy Cost	Net Cost	Local Pollution
0.4	8.6	0	7.74	169.5285	202	371.53	860
2	7	0	6.3	95.63396	290	385.63	700
3.7	5.3	0	4.77	44.78591	383.5	428.29	530
2.9	4.1	2	7.69	166.4789	319.5	485.98	500
4.9	2.1	2	5.89	79.46617	429.5	508.97	300
5.9	1.1	2	4.99	50.57953	484.5	535.08	200
6.6	0	2.4	4.8	45.54848	519	564.55	108
8.1	0	0.9	1.8	3.770086	616.5	620.27	40.5
9	0	0	0	0	675	675	0
6.7	0	2.3	4.6	40.62457	525.5	566.12	103.5

We use the following formulas to normalize the values:

$$V_{NC} = (X_{NC\,max} - X_{NC}) / (X_{NC\,max} - X_{NC\,min})$$

$$V_{LP} = (X_{LP\,max} - X_{LP}) / (X_{LP\,max} - X_{LP\,min})$$

V_{NC} and V_{LP} refer to the value function of the expected net cost and that of the local pollution, respectively. The X values of the different parameters are found in Table 5.

After calculating the two value functions based on the given portfolios, we will use the given weights to calculate the outcome of the additive value function, as shown in Table 6. Note that these values are found using a maximum net cost across all exponents, which is found to be 675, and a minimum net cost of 371.528, again, across all exponents, whereas, for local pollution, the maximum used is 860, and the minimum is 0.

Table 6: Value results for the given portfolios.

Import	Coal + CC	NG	Value
0.4	8.6	0	0.700
2	7	0	0.723
3.7	5.3	0	0.684
2.9	4.1	2	0.562
4.9	2.1	2	0.578
5.9	1.1	2	0.553
6.6	0	2.4	0.517
8.1	0	0.9	0.412
9	0	0	0.300
6.7	0	2.3	0.515

The optimal portfolio has the highest value. Accordingly, the optimal solution here is to produce two units of electricity from imports and the rest from coal plus CC, i.e., without using natural gas at all, which results in a maximum value of 0.723.

VIII. Question IV

In this question, we use the net costs and resultant values for each probability scenario instead of the expected amounts. In this case, we have three portfolios, and the scenario-based net cost and the total local pollution are calculated for each.

Table 7: Value results for the three portfolios.

					Net Cost				Value Functions		
Import	Coal + CC	NG	Total CO2	Energy Cost	Exp = 1.5	Exp = 2	Exp = 3	Local Pollution	Exp = 1.5	Exp = 2	Exp = 3
2.9	4.1	2	7.69	319.5	340.825	378.63	774.256	500	0.7	0.63893	0
6.6	0	2.4	4.8	519	529.516	542.04	629.592	108	0.6511	0.63096	0.489566
8.1	0	0.9	1.8	616.5	618.915	619.74	622.332	40.5	0.5508	0.54954	0.545361

Note that the values in Table 7 are calculated using the same weights and additive value function as those in Question 3.

The cumulative probability chart is presented in Figure 5 to check for stochastic dominance. It is observable that there is no first-order stochastic dominance. However, we have two second-order stochastic dominance, where portfolio 2 is SOSD portfolio 1, and portfolio 3 is SOSD portfolio 1.

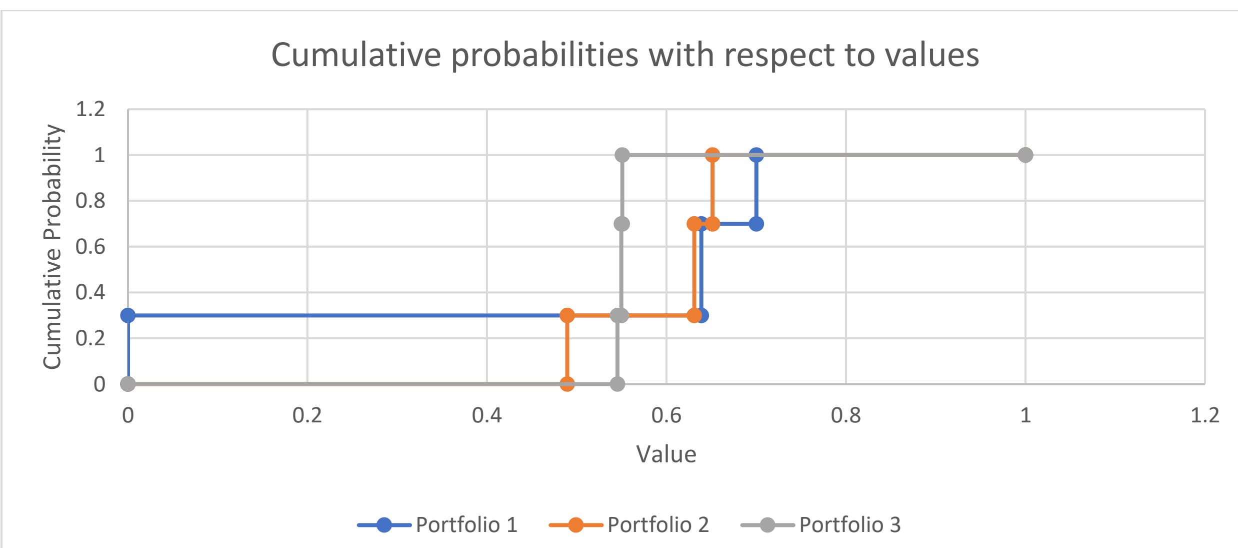

Figure 5: Stochastic dominance results.

IX. Conclusion

In this report, we assessed the problem of finding the optimal portfolio of input factors among three alternatives of imports, coal plus carbon capture, and natural gas for electricity generation to minimize the net cost of CO2 emissions damages, electricity generation costs, and local pollution. We started with a simple case where the decision-maker only cared about the net cost as their objective and found coal plus CC to be the primary source of electricity, with a minimal amount sourced through imports and no natural gas.

Then, we considered the tradeoff that comes into play when considering both net cost and local pollution. We approximated a non-dominated set of alternative portfolios based on two methods, i.e., the weighting method and the constraint method, and found coal plus CC and imports to be the significant input factors interplaying in the tradeoff as the preferences of the decision-maker change. We also considered applying an additive value function to the problem to compare a finite set of alternatives and come up with one single best portfolio; yet again, it heavily depends on the decision-maker's preferences.

Finally, we discussed stochastic dominance for a limited number of available portfolios and found two second-order stochastic dominance (SOSD) occasions and no first-order stochastic dominance (FOSD). In terms of methodology, it is clear that the approximation methods to develop a non-dominated set of portfolios provide the policymaker with much more insight; however, these methods require coming up with a specific objective function or something similar to enable the development of a set of alternatives and their comparison. On the other hand, although the stochastic dominance method results in much less insight, it allows the decision-maker to rule out some other options or compare a few known ones. As such, the stochastic dominance method can still provide much-needed support and guidance to the policymaker in complicated situations with a lack of data.