AKTU

B.Tech I-Year

Electrical Engg.

ONE SHOT Revision

Unit-2

Steady State Analysis of Single Phase AC Circuits

Avinash Sir

Fundamentals of Electrical Engineering

Unit-2 : Steady State Analysis of Single Phase AC Circuits

3-ϕ Circuit

AKTU Syllabus

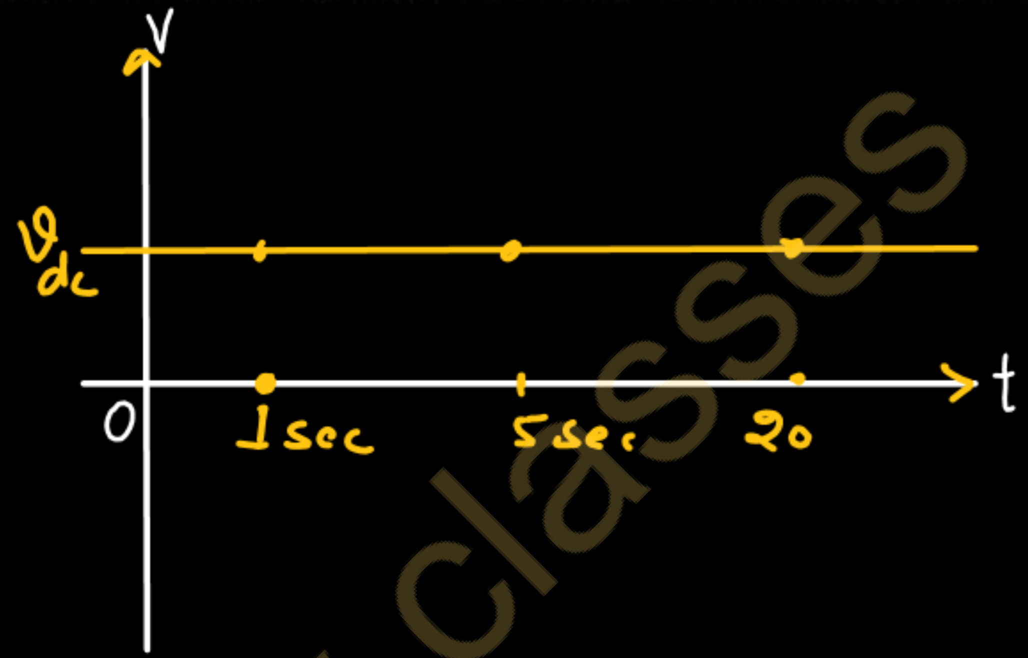
Representation of Sinusoidal waveforms – Average and effective values, Form and peak factors.

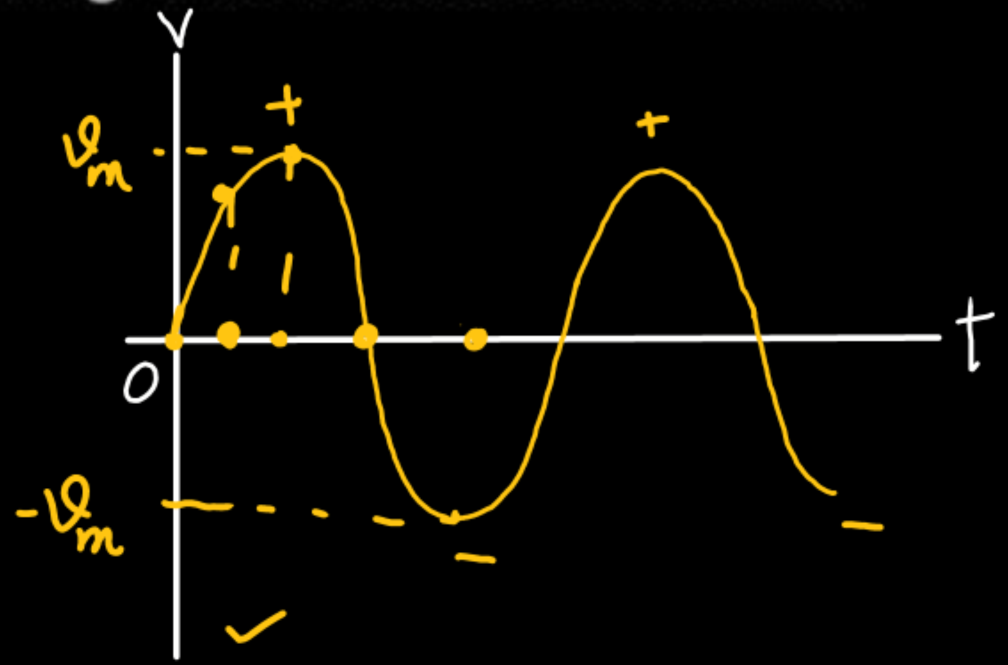
Analysis of single phase AC Circuits consisting R-L-C combination (Series and Parallel) Apparent, active & reactive power, Power factor. Concept of Resonance in series & parallel circuits, bandwidth and quality factor.

Three phase balanced circuits, voltage and current relations in star and delta connections.

Types of Electrical Supply

Direct Current (D.C.) Supply:- A DC power supply is one that provides a constant magnitude and direction with respect to time.





Alternating Current (A.C.) Supply: An alternating supply is one whose magnitude and direction changes periodically with time.

Advantage of AC over DC

1. *Varying voltage level of AC by using transformer.*
2. *More Economical then DC*
3. *Generation of high voltage AC is much easier & Cheaper than DC*
4. *AC can be easily converted to DC.*
5. *AC machine are more simple in construction and requires less maintenance*

Alternating Quantity

Equation of Alternating Quantity (AKTU-2002-03)

For a pure sine wave $v = v_m \sin \omega t$ → angular frequency $(\omega) = 2\pi f$

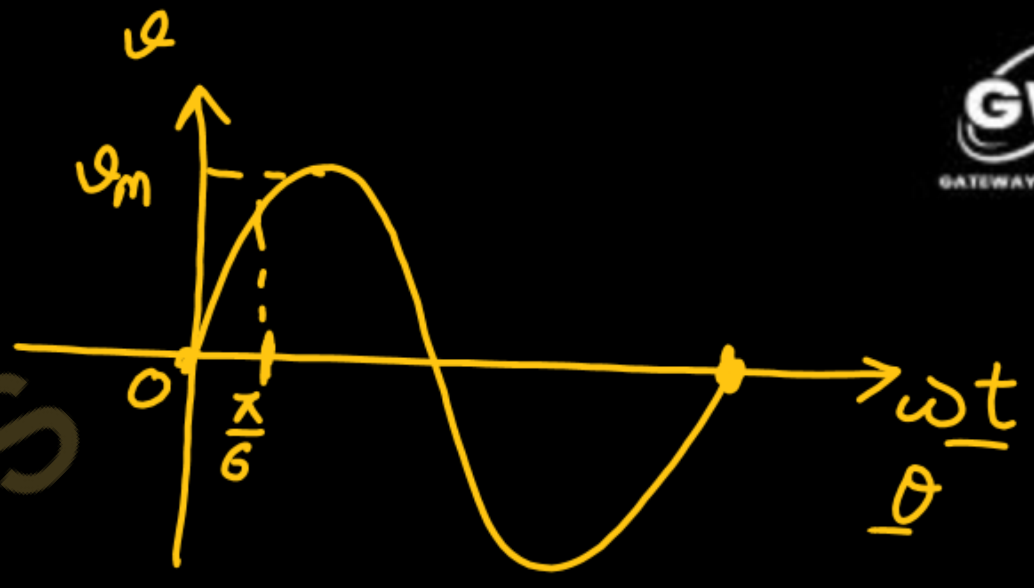
Instantaneous Value ← $v = v_m \sin \theta$

Maximum Value

$I = I_m \sin \omega t$

$e = E_m \sin \omega t$

$\phi = \phi_m \sin \omega t$



1 Cycle

Time period (T)

$$\boxed{f = \frac{1}{T}}$$

Important Terms related to Alternating Quantity -

Instantaneous Value:- Value of alternating quantity at any instant of time.

Waveform Cycle:- Each repetition of positive and negative instantaneous values of the AC Quantity.

angular frequency $(\omega) = 2\pi f$

Time period:- Time taken to complete one cycle.

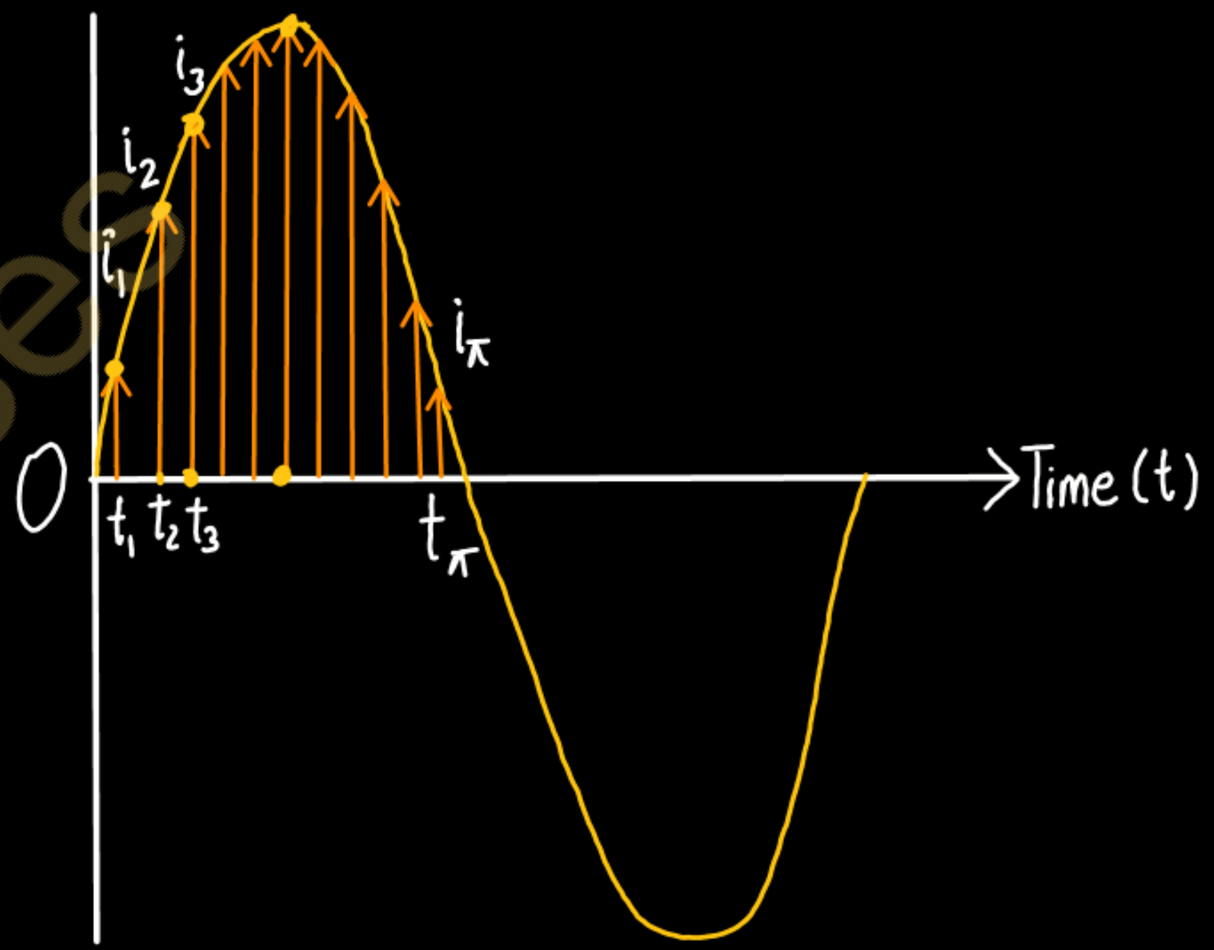
Frequency:- Reciprocal of Time period or no of cycle in a second.

Average Value (Mean Value)

The average of all the instantaneous values of an alternating voltage and currents over one complete cycle is called **Average Value.**

Or

(The Average Value (also known as Mean Value) of an Alternating Current (AC) is expressed by that Direct Current (DC) which transfers across any circuit the same amount of charge as is transferred by that Alternating Current (AC) during the same time.)

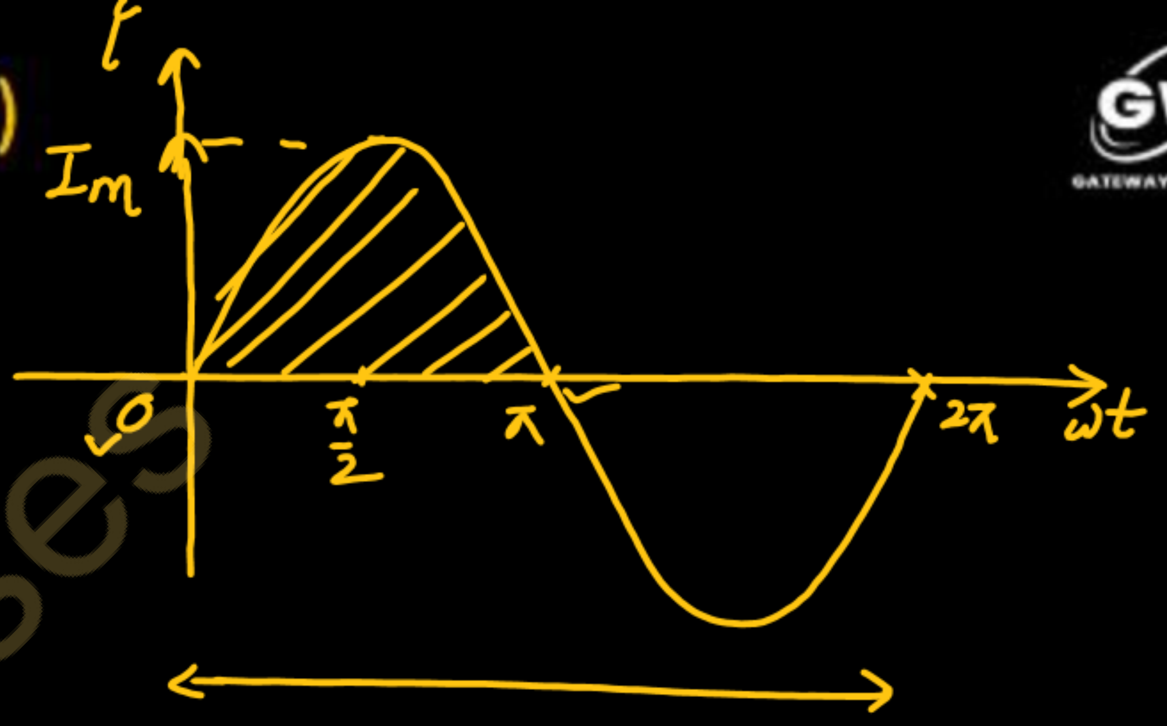


Average Value by Graphical Method:-

$$Average\ value = \frac{Sum\ of\ all\ instantaneous\ values\ over\ one\ cycle}{Number\ of\ instants}$$

$$I_{avg} = \frac{i_1 + i_2 + i_3 \dots\dots + i_n}{n}$$

Average Value (Mean Value)



Average Value by Analytical Method:-

$$I_{avg} = \frac{\textit{Area under the curve for half cycle}}{\textit{Length of base over half cycle}}$$

Imp

Ques. Determine Average value of a sine wave?

The Equation of given Sine wave

$i = I_m \sin\omega t$

$$I_{avg} = \frac{\int_0^{\pi} i \cdot d\omega t}{\pi} = \frac{\int_0^{\pi} I_m \sin\omega t}{\pi} = \frac{I_m}{\pi}\int_0^{\pi} \sin\omega t \cdot d\omega t$$

$$= \frac{I_m}{\pi}\left[-\cos\omega t\right]_0^{\pi} = \frac{I_m}{\pi}\left[-\cancel{\cos\pi}^{\,-1} + \cancel{\cos 0}^{\,+1}\right] = \boxed{\frac{2 I_m}{\pi} = I_{avg}}$$

Root Mean Square (R.M.S.) Value/ Effective Value

$$\sqrt{\frac{i_1^2+i_2^2+i_3^2+\cdots}{n}}$$

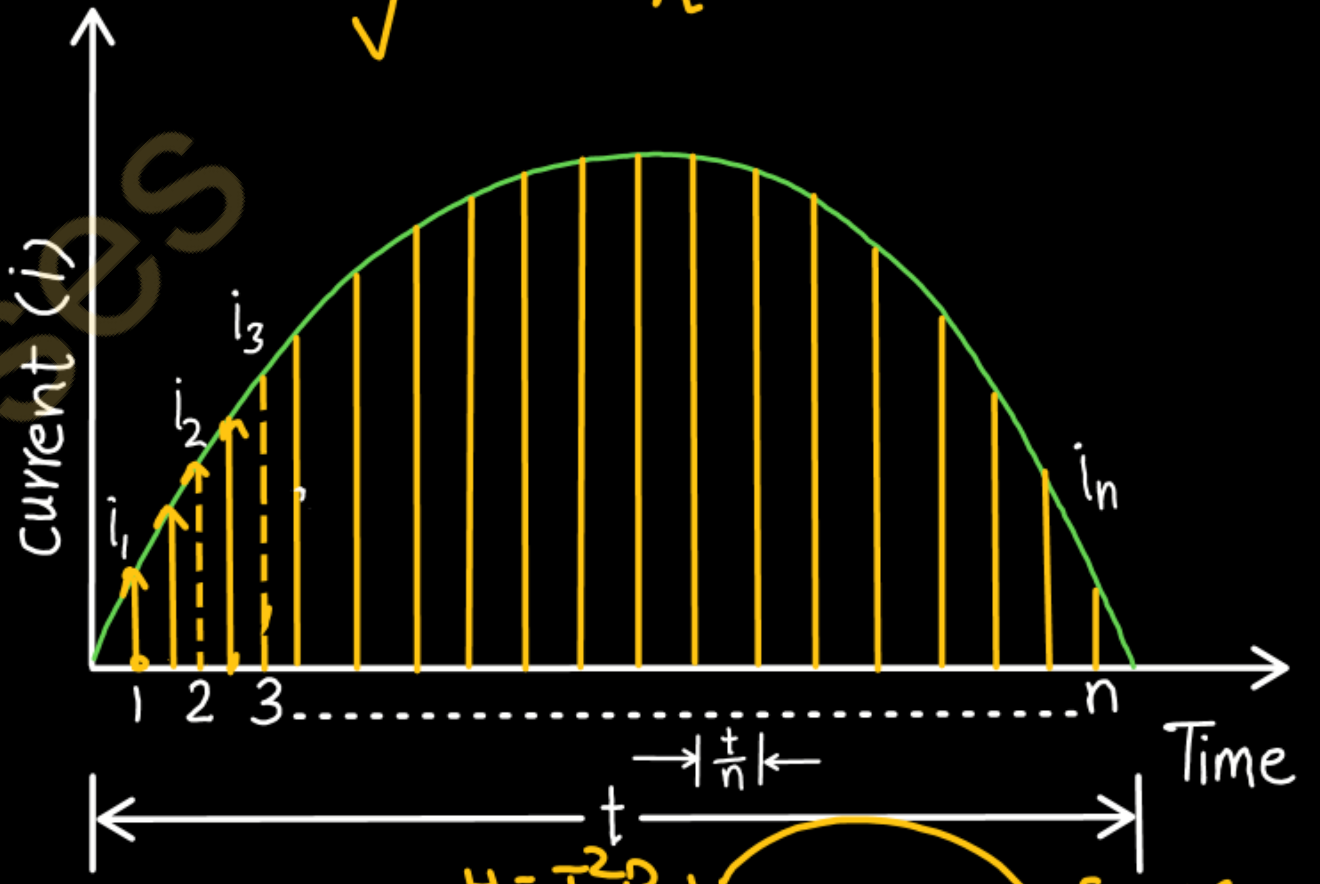
Definition: The RMS value of AC current is equal to that amount of DC current which produces the same heating effect flowing through the same resistance for the same time.

For example, if 3A (RMS) AC current is flowing through a circuit, And it will produce the same amount of heat (energy) as will be produced by 3A DC current.

"Graphical Method"

$$RMS\ value\ of\ alternating\ current\ (I) = \sqrt{mean\ value\ of\ i^2}$$

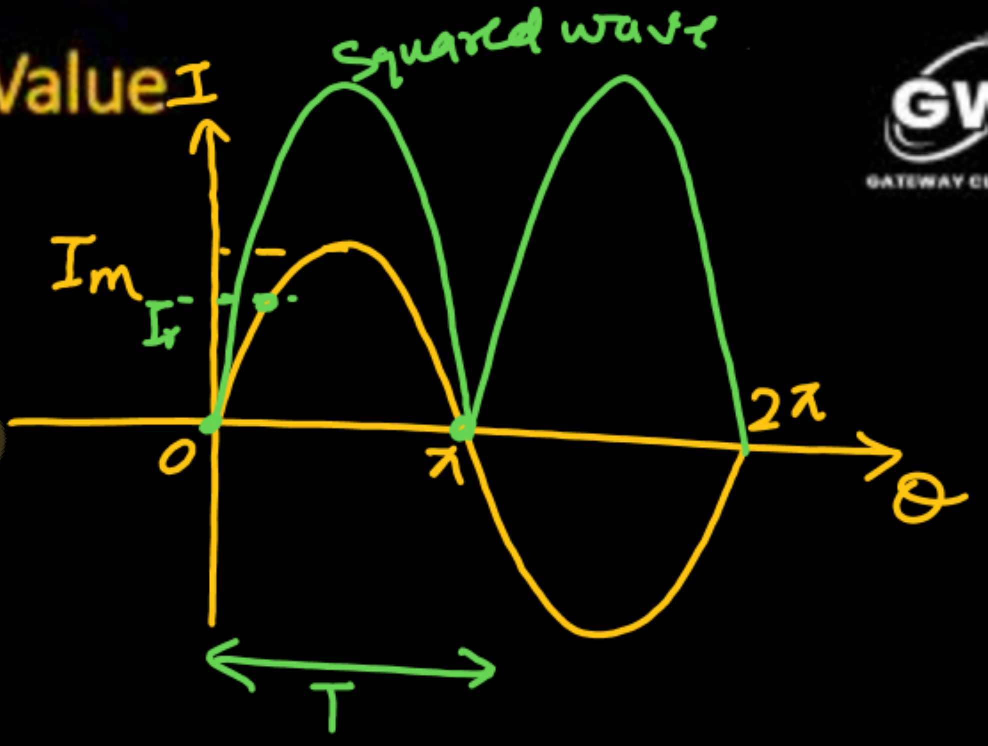
$$= \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \cdots \ldots \ldots i_n^2}{n}}$$



$H = I^2 R.t$

same

V_{ac} = R.M.S Value



Analytical Method:-

$$I_{RMS} = \sqrt{\frac{\text{Area of half cycle squared wave}}{\text{base length of half cycle}}}$$

Ques. Determine RMS value of a sine wave? (Same for full wave rectifier output)

Solⁿ:→ $I = I_m \sin\theta$

$$I_{rms} = \sqrt{\frac{\int_0^\pi i^2\, d\theta}{\pi}}$$

$$I_{rms}^2 = \frac{1}{\pi}\int_0^\pi I_m^2 \sin^2\theta\, d\theta = \frac{I_m^2}{\pi}\int_0^\pi \left(\frac{1-\cos 2\theta}{2}\right) d\theta = \frac{I_m^2}{2\pi}\int_0^\pi (1-\cos 2\theta)\, d\theta$$

$$= \frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^\pi = \frac{I_m^2}{2\pi}\left[\pi - \cancel{\frac{\sin 2\pi}{2}}^{\,0} - 0 + \cancel{\frac{\sin 0}{2}}^{\,0}\right] = \frac{I_m^2}{2\cancel{\pi}}\cdot\cancel{\pi}$$

$$I_{rms}^2 = \frac{I_m^2}{2}$$

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}} = 0.707 I_m$$

Form Factor & Peak Factor

Form Factor:- The ratio of the root mean square value to the average value of an alternating quantity (current or voltage) is called **Form Factor.**

$$Form\ Factor = \frac{I_{r.m.s}}{I_{av}}\ or\ \frac{E_{r.m.s}}{E_{av}}$$

For a sine wave, $= \dfrac{\cancel{I_m}/\sqrt{2}}{\dfrac{2\cancel{I_m}}{\pi}} = 1.11$

Peak Factor/ Crest Factor :- Peak Factor is defined as the ratio of maximum value to the R.M.S value of an alternating quantity.

K_p = Peak value / RMS value

For a sine Wave

$= \dfrac{\cancel{I_m}}{\dfrac{\cancel{I_m}}{\sqrt{2}}} = \sqrt{2} = 1.414$

Numerical

Ques. Determine Form Factor and Peak Factor for Half wave rectified Signal (V_m = 10volt). **(AKTU- 2003-04, 2020-21, 2023-24)**

Imp

Sol^n

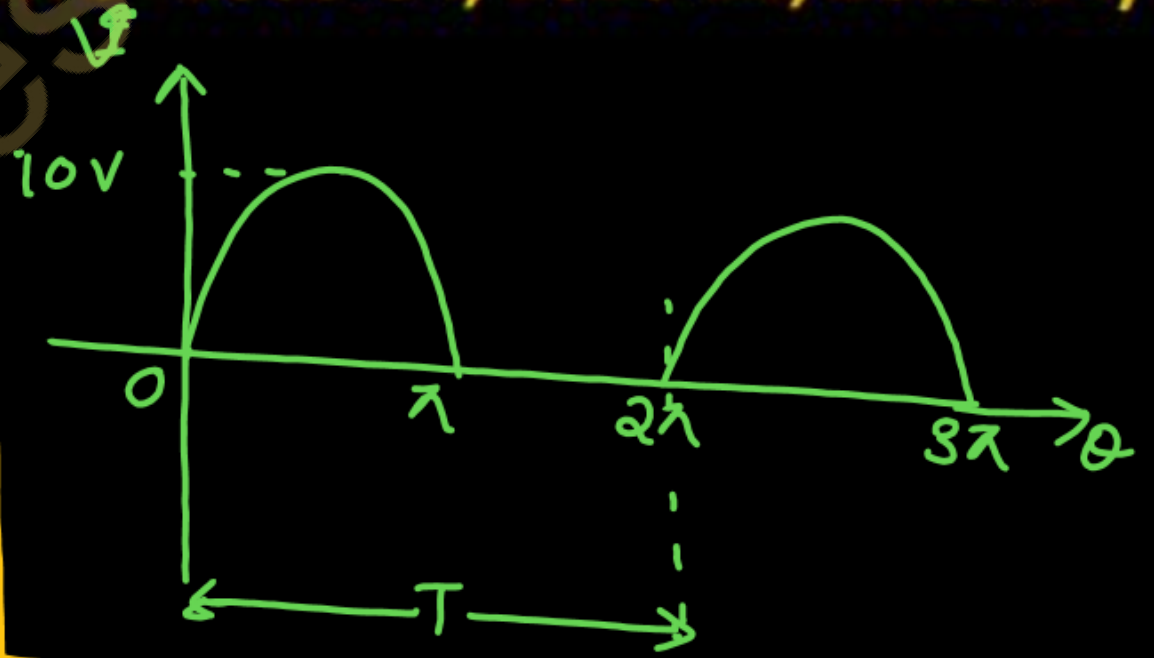
$$v = v_m \sin\theta \quad (\text{from } 0 \text{ to } \pi)$$
$$= 0 \quad (\text{from } \pi \text{ to } 2\pi)$$

$$v_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v\, d\theta$$
$$= \frac{1}{2\pi}\left[\int_0^{\pi} v_m \sin\theta\, d\theta + \int_{\pi}^{2\pi} \cancel{0}\, d\theta\right]$$
$$= \frac{v_m}{2\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{v_m}{2\pi}\left[-\cos\pi + \cos 0\right]$$

$$\boxed{v_{avg} = \frac{v_m}{\pi} = \frac{10}{\pi}}$$

$$V_{rms}^2 = \frac{\int_0^{2\pi} v^2\, d\theta}{2\pi}$$
$$= \frac{1}{2\pi}\left[\int_0^{\pi} v_m^2 \sin^2\theta\, d\theta + \int_{\pi}^{2\pi} 0\right]$$
$$= \frac{v_m^2}{2\pi}\left[\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta\right]$$
$$= \frac{v_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$
$$= \frac{v_m^2}{4\pi}\left[\pi - \frac{\sin 2\pi}{2} - 0 + \frac{\sin 0}{2}\right]$$
$$= \frac{v_m^2}{4}$$

$$V_{rms} = \frac{v_m}{2} = \frac{10}{2} = 5\,V$$



$$\text{Form factor} = \frac{\text{Rms Value}}{\text{Avg}} = \frac{5}{10/\pi} = \frac{\pi}{2}$$

$$\text{Peak factor} = \frac{\text{Max}}{\text{Rms}} = \frac{10}{5} = 2$$

Concept of Phasor

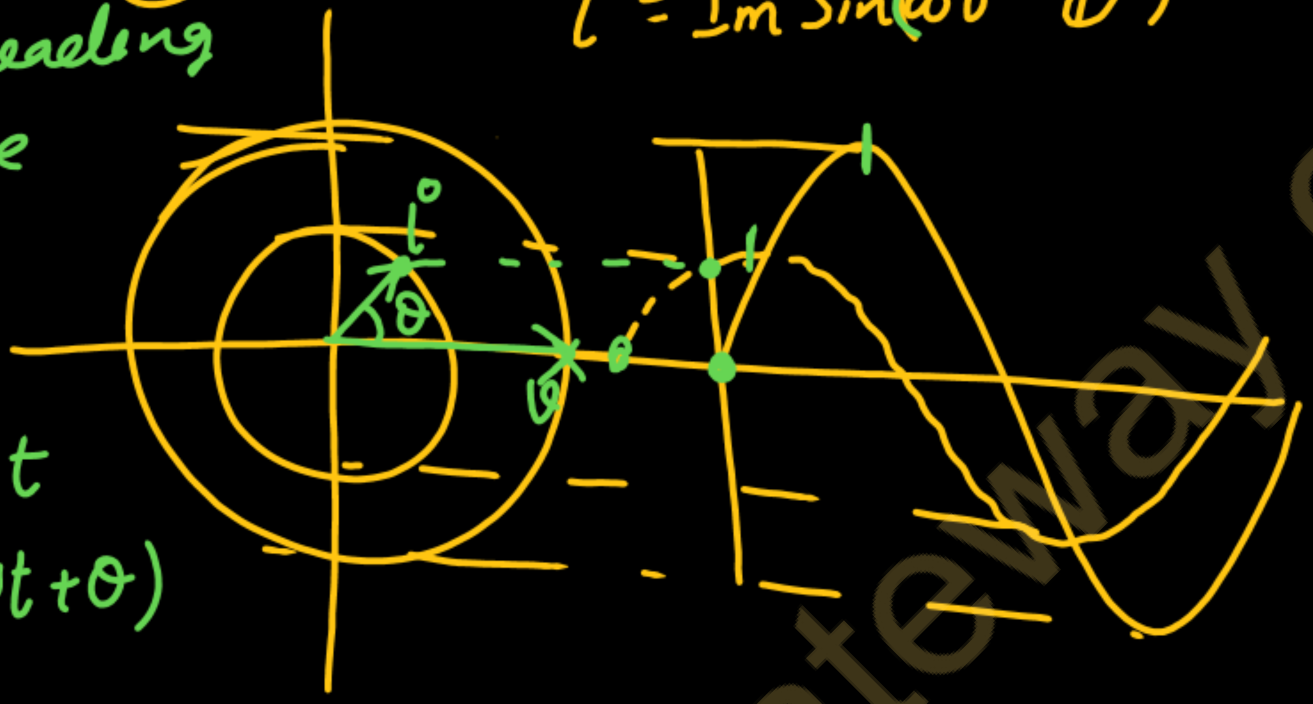
It gives information about any A.C quantity that how much it is from ref. Position

(3)

$v = V_m \sin \omega t$

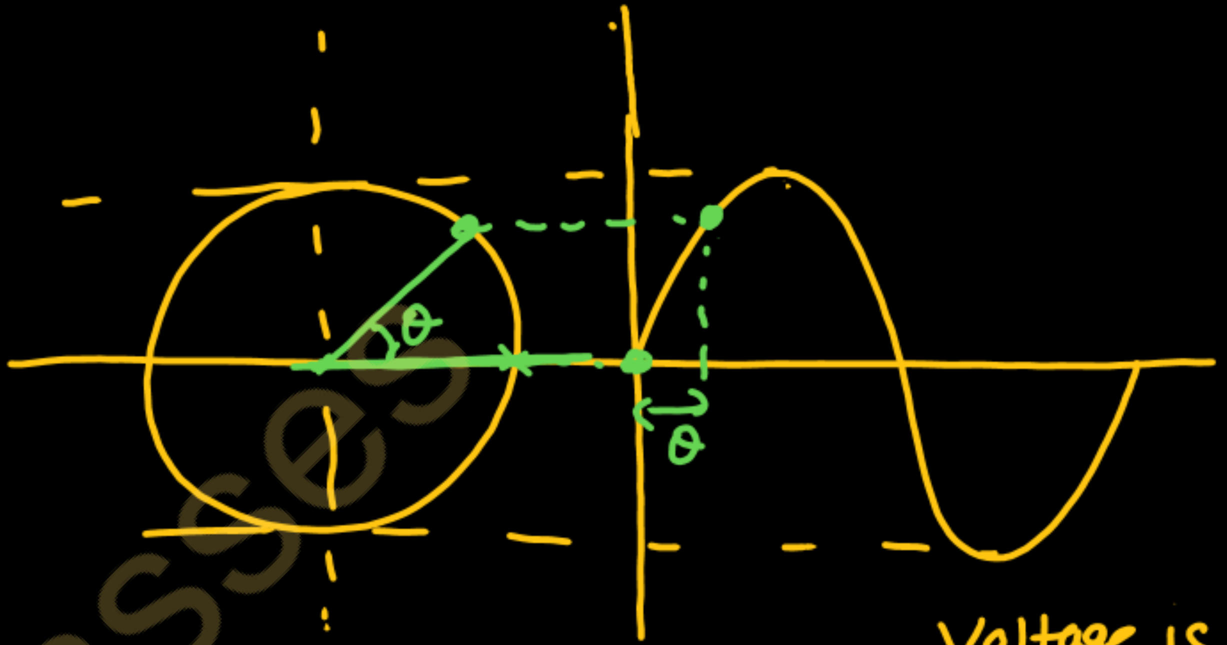
$i = I_m \sin(\omega t - \theta)$

Current is leading by θ degree



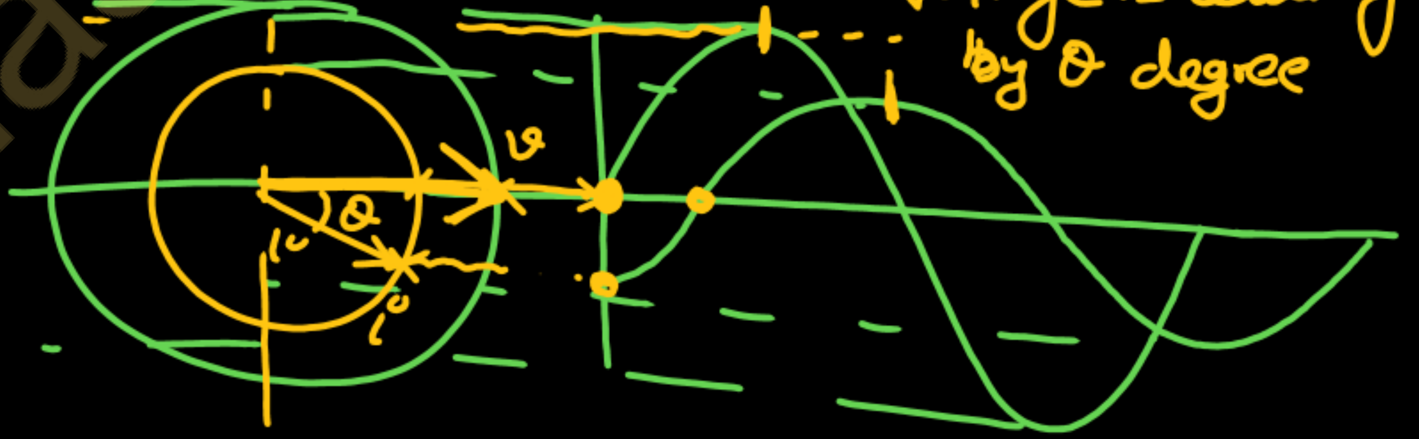
$v = V_m \sin \omega t$

$i = I_m \sin(\omega t + \theta)$



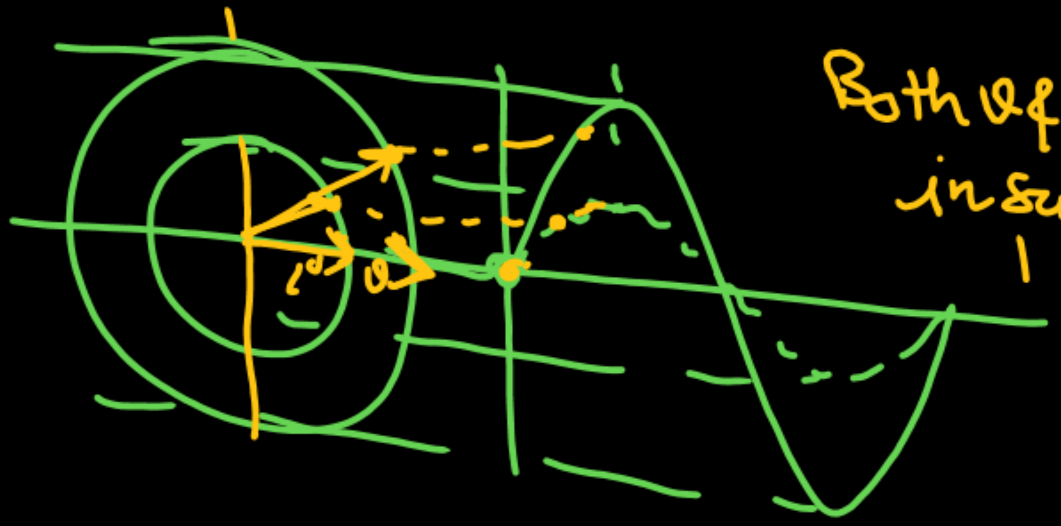
(1)

Voltage is leading by θ degree



(2)

Both v & i are in same



A.C. Through Pure Resistive Circuit

In given Circuit

$$\boxed{v = V_m \sin\omega t \quad \text{—— (1)}}$$

$$i = \frac{v}{R} = \frac{V_m \sin\omega t}{R}$$

$$\boxed{i = I_m \sin\omega t \quad \text{—— (2)}}$$

where $I_m = \frac{V_m}{R}$

Here Phase difference is 0

bcoz voltage & Current are in Same Phase

Instantaneous power

$$P = V.I = V_m \sin\omega t \, I_m \sin\omega t$$

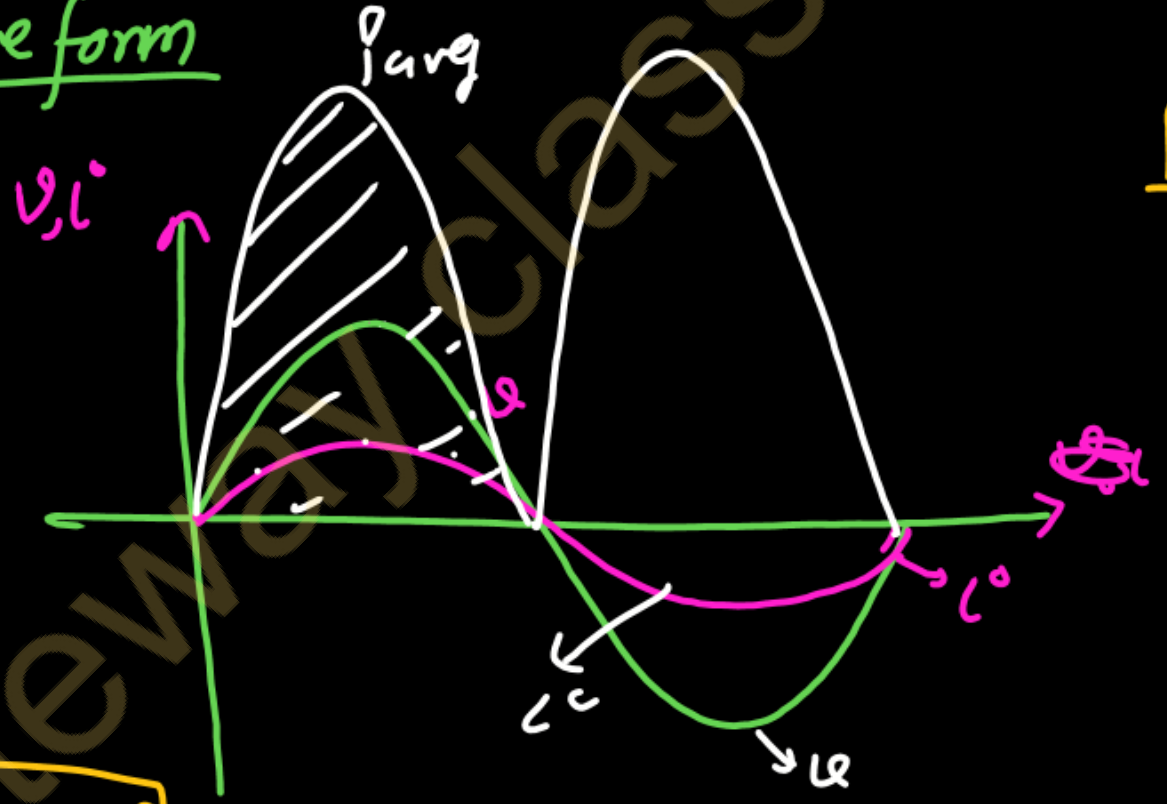
$$= \frac{V_m I_m}{2} 2\sin^2\omega t = \frac{V_m I_m}{2} \cancel{2} \left[\frac{1 - \cos 2\omega t}{\cancel{2}}\right]$$

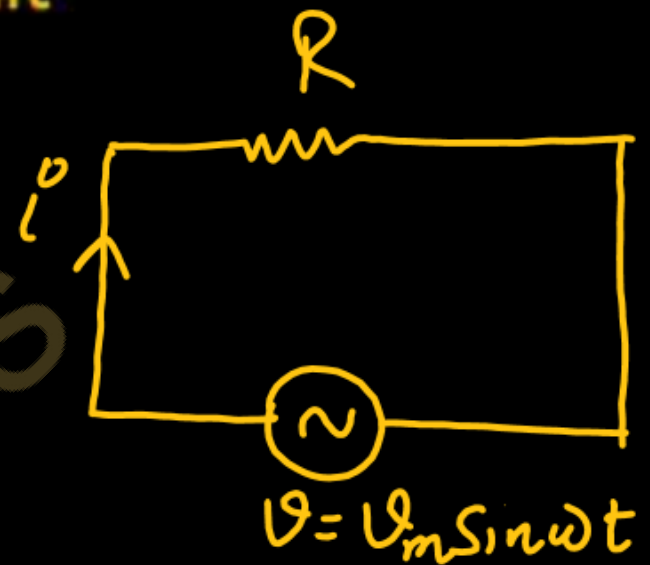
$$P = \frac{V_m I_m}{2} - \underbrace{\frac{V_m I_m}{2}\cos 2\omega t}_{\text{Avg. value} = 0}$$

Average Power

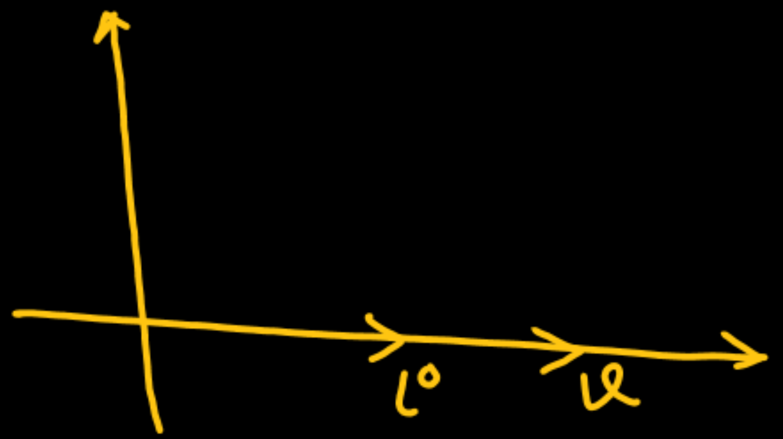
$$P_{avg} = \frac{V_m \cdot I_m}{2} - 0 = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = V_{rms} \cdot I_{rms}$$

Wave form



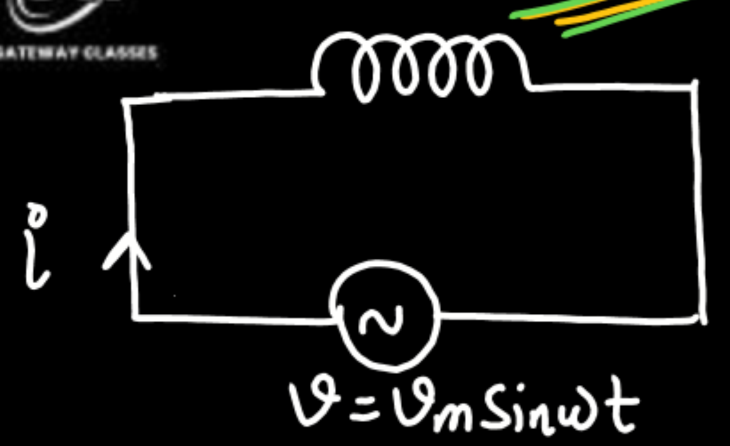


Phasor



A.C. Through Pure Inductive Circuit

v·Imp



Given

$v = V_m \sin\omega t$ —(1)

Emf in inductor

$e = -L\dfrac{di}{dt}$

$v = -e = L\dfrac{di}{dt}$

$\dfrac{di}{dt} = \dfrac{v}{L} = \dfrac{V_m \sin\omega t}{L}$

$i = \dfrac{V_m}{L}\displaystyle\int \sin\omega t \cdot dt$

$i = \dfrac{V_m}{L}\left(-\dfrac{\cos\omega t}{\omega}\right)$ —(2)

$i = \dfrac{V_m}{\omega L}\sin\left(\omega t - \dfrac{\pi}{2}\right)$

$\boxed{i = I_m \sin\left(\omega t - \dfrac{\pi}{2}\right)}$ —(2)

$i = -I_m \cos\omega t$ —(3)

where

$I_m = \dfrac{V_m}{\omega L}$

$\boxed{\dfrac{V_m}{I_m} = \omega L = X_L}$ —(4)

It is known as inductive reactance.

Power

$P = vI$

$= V_m \sin\omega t \, I_m \cos\omega t$

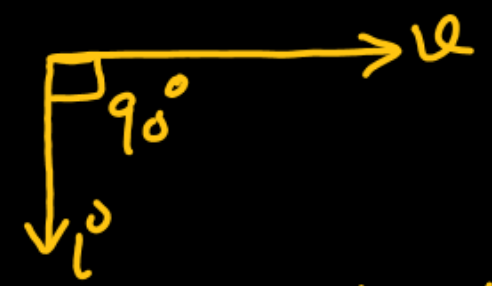
$= \dfrac{V_m I_m}{2} 2\sin\omega t \cdot \cos\omega t$

$P = -\dfrac{V_m I_m}{2}\sin 2\omega t \rightarrow 0$

Average Power

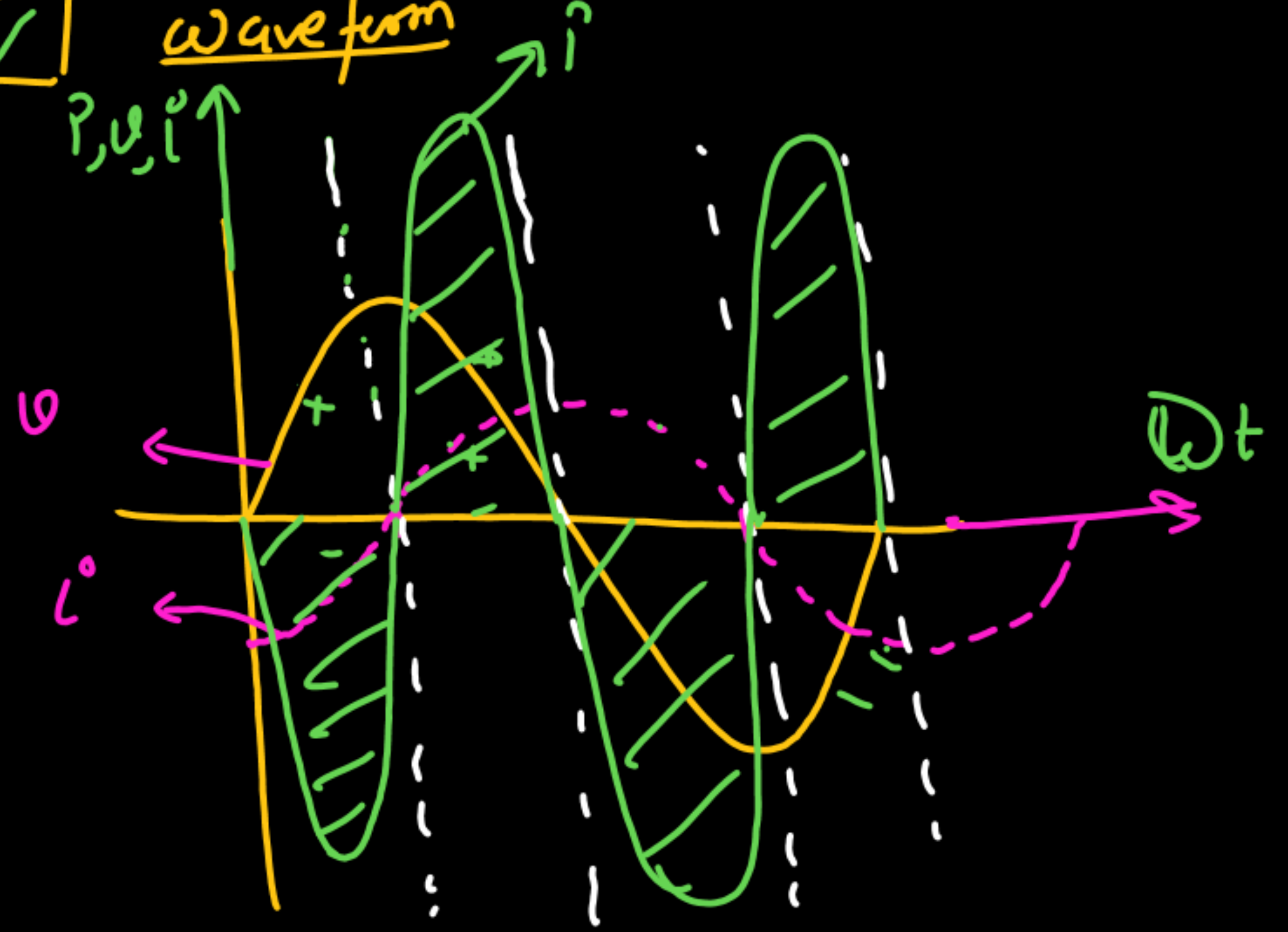
$\boxed{P_{avg} = 0}$

Phasor



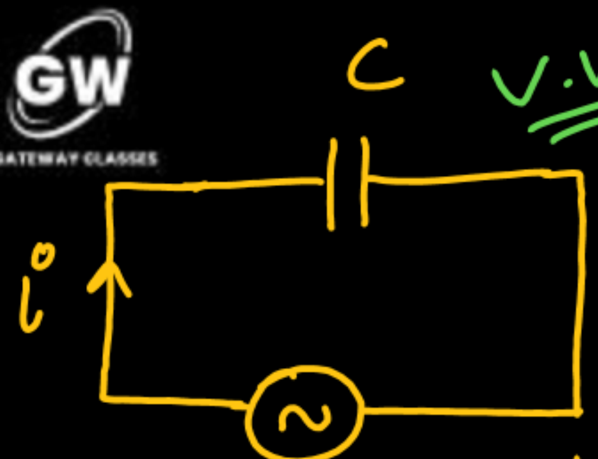
Current is lagging behind the voltage by 90°.

waveform



A.C. Through Pure Capacitive Circuit

V.V.Imp



$v = v_m \sin\omega t$

$$\boxed{v = v_m \sin\omega t} \quad \text{—(1)}$$

Charge

$q = C v$

$q = C v_m \sin\omega t$

and $i = \frac{dq}{dt} = \frac{d}{dt}[C v_m \sin\omega t]$

$i = \underbrace{C v_m \omega}\cos\omega t$

$i = I_m \cos\omega t$ —(2)

$$\boxed{i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)} \quad \text{—(3)}$$

where

$I_m = C v_m \omega$

$$\boxed{\frac{v_m}{I_m} = \frac{1}{\omega C} = X_C}$$

This the resistance offered by the capacitor. Known as capacitive reactance)

Power

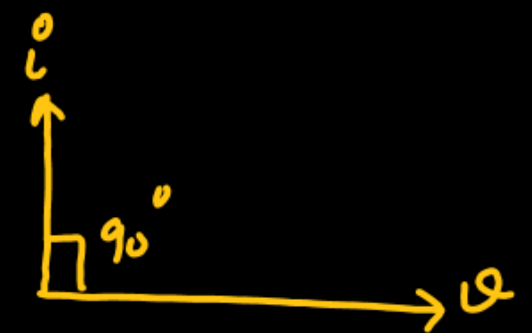
$P = v.I = v_m \sin\omega t . I_m \cos\omega t$

$P = \frac{v_m I_m}{2} \sin 2\omega t \rightarrow 0$

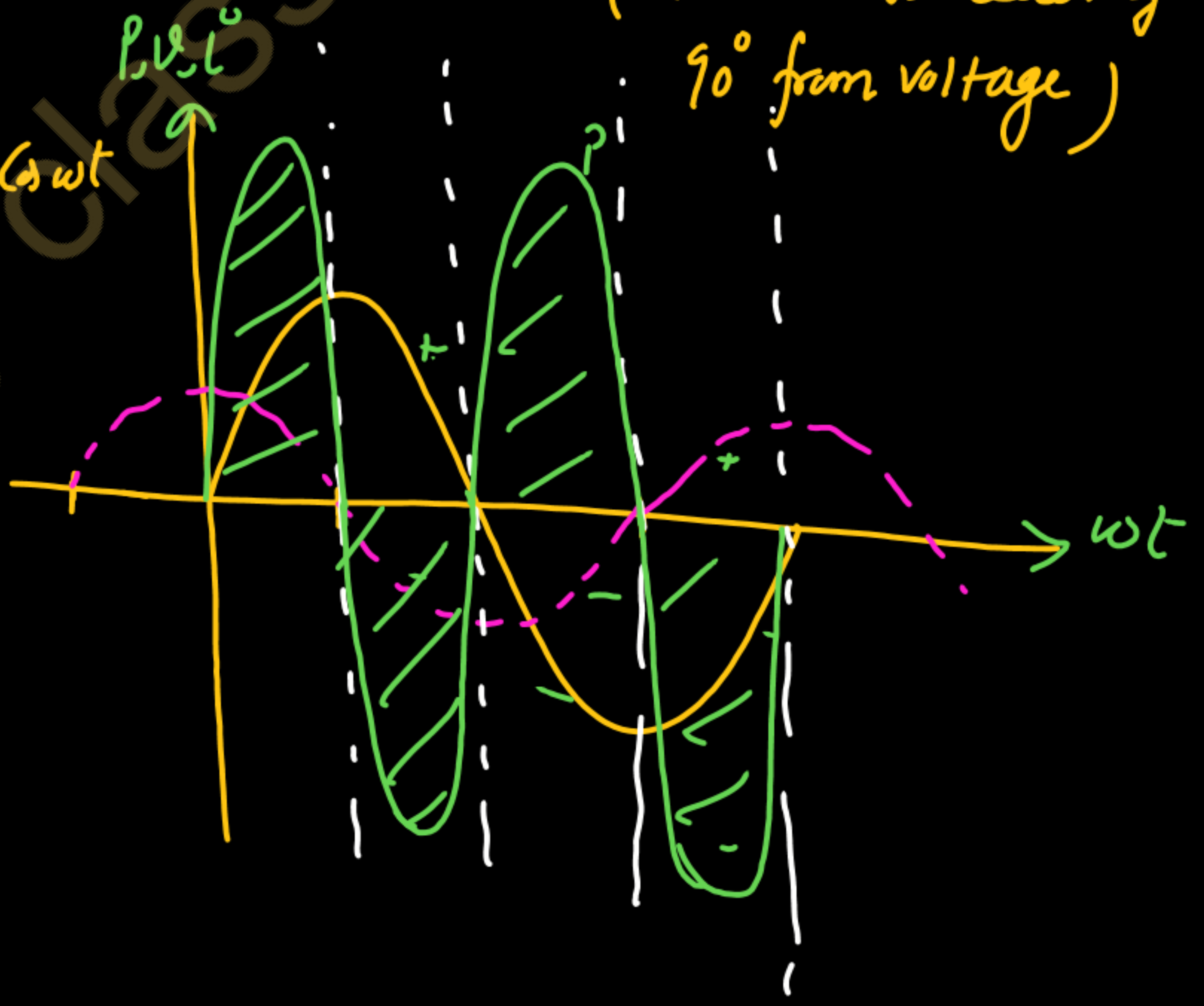
Average Power

$$\boxed{P_{avg} = 0}$$

Phasor



((Current is leading by 90° from voltage)



Concept of Impedance

Resistance $\xrightarrow{\text{Reciprocal}}$ Conductance

Reactance (X) $\longrightarrow$ Susceptance

Impedence (Z) $\longrightarrow$ Admittance

(R & X)

Impedance

$$\boxed{Z = R + jX_L} \Rightarrow \begin{cases} \sqrt{R^2 + X_L^2} = |Z| \\ \angle Z = \tan^{-1}\left(\frac{X_L}{R}\right) \end{cases}$$

$$Z = R - jX_C$$

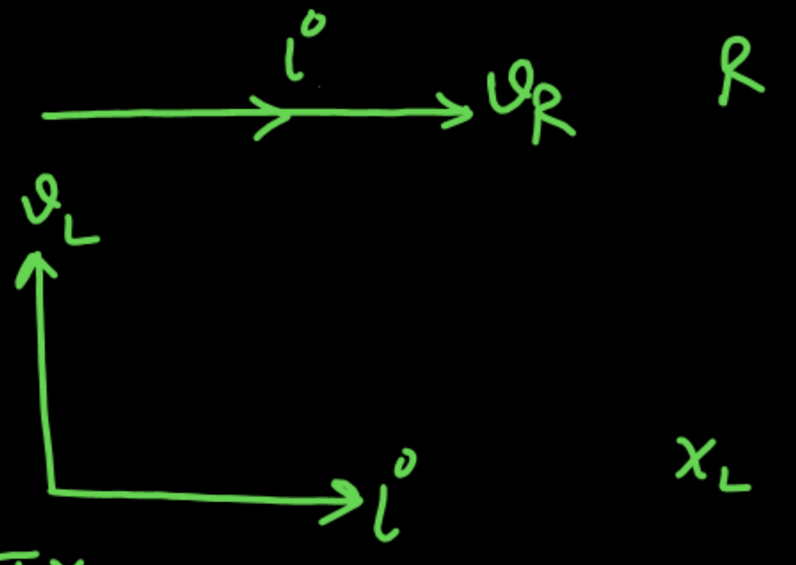
$$Z = R + jX_L$$
$$= Z\angle\theta$$

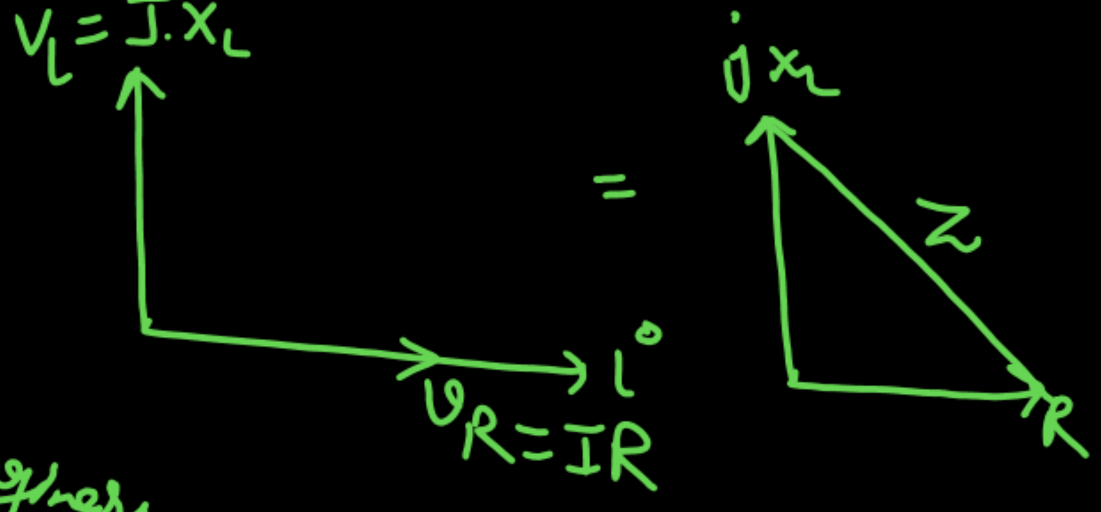
Polar $\rightleftarrows$ Rectangular

$r\angle\theta$ $\rightleftarrows$ $a + ib$

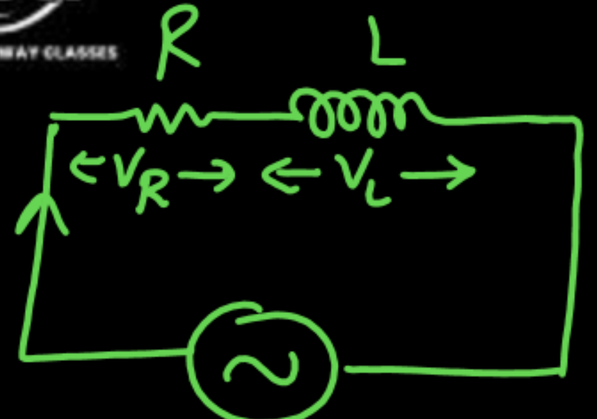
r → magnitude, θ → angle

a → Real, b → Imaginary





A.C. Through Series R-L Circuit (AKTU-2022-23)



$v = V_m \sin\omega t$ — (1)

$\vec{V} \equiv \vec{V_R} + \vec{V_L}$

$I\vec{Z} = I\vec{R} + I\vec{X_L}$

$\vec{Z} = \vec{R} + \vec{X_L}$

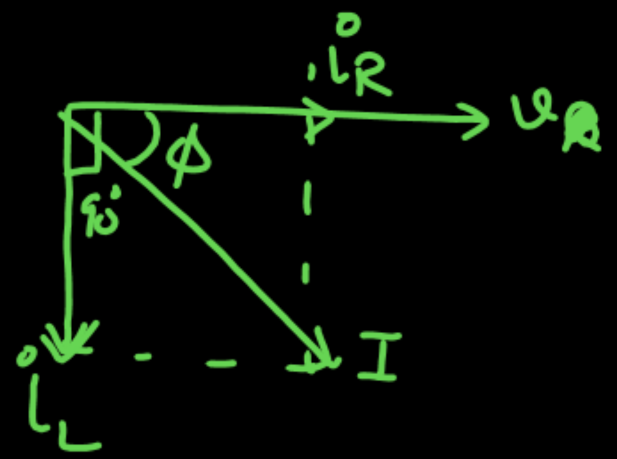
$|Z| = \sqrt{R^2 + X_L^2}$

$\angle = \tan^{-1}\left(\frac{X_L}{R}\right)$

} Impedance

$v = V_m \sin\omega t$ — (1)

$i = I_m \sin(\omega t - \phi)$ — (2)



Power $= P = VI$

$P = V_m \sin\omega t \; I_m \sin(\omega t - \phi)$

$= \frac{V_m I_m}{2} \, 2\sin\omega t \cdot \sin(\omega t - \phi)$

$= \frac{V_m I_m}{2}\left[\cos(\omega t - \omega t + \phi) - \cos(\omega t + \omega t - \phi)\right]$

$P = \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t - \phi)\right]$

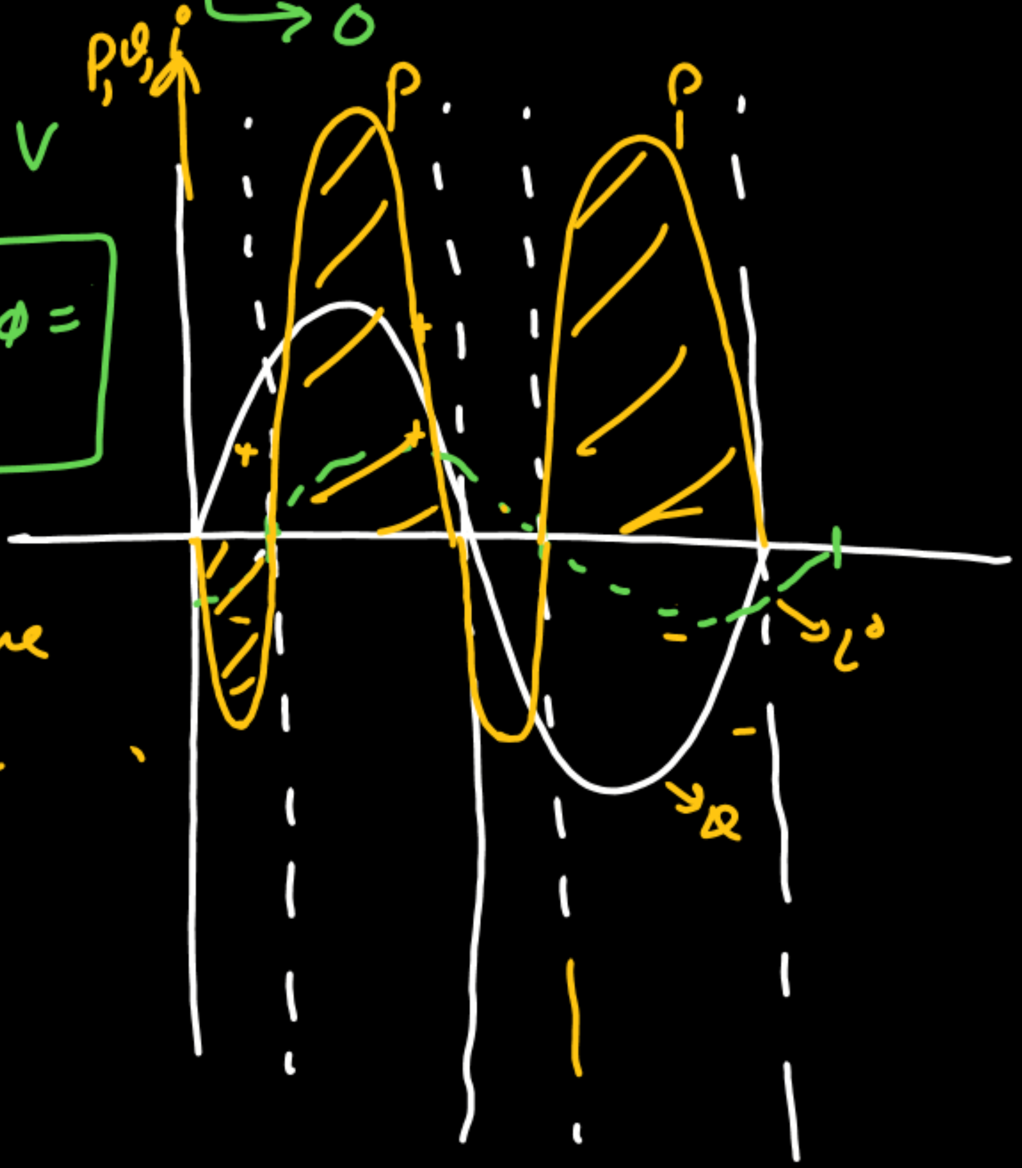
$P = \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)$

constant; $\cos(2\omega t - \phi) \to 0$

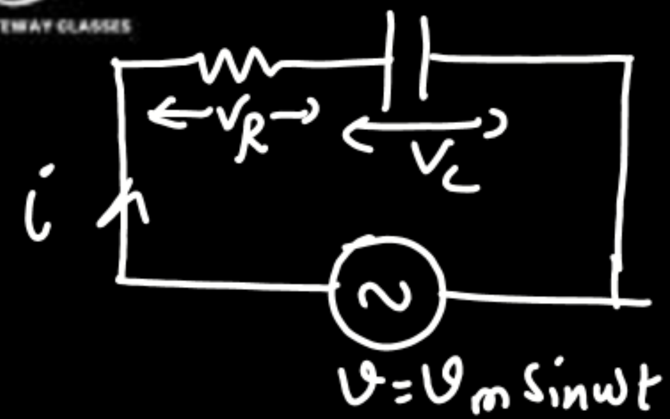
$P_{avg} = \frac{V_m I_m}{2}\cos\phi = V$

$$\boxed{P_{avg} = V_{rms} I_{rms} \cos\phi = VI\cos\phi}$$

This power is due to the Presence of Resistance.



A.C. Through Series R-C Circuit



$\vec{v} = \vec{v}_R + \vec{V}_C$

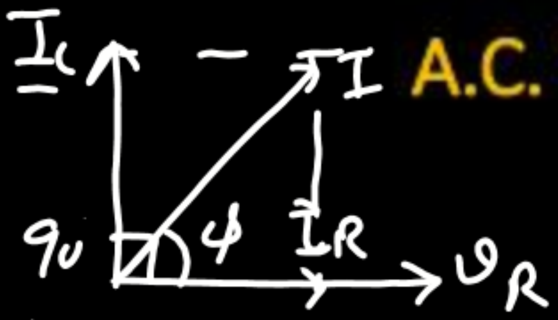
$\vec{IZ} = \vec{IR} + \vec{IX_C}$

$|Z| = \sqrt{R^2 + X_C^2}$

$\angle = \tan^{-1}\left(\frac{-X_C}{R}\right)$

$v = v_m \sin\omega t$

$i = I_m \sin(\omega t + \phi)$



Power

$P = vI$

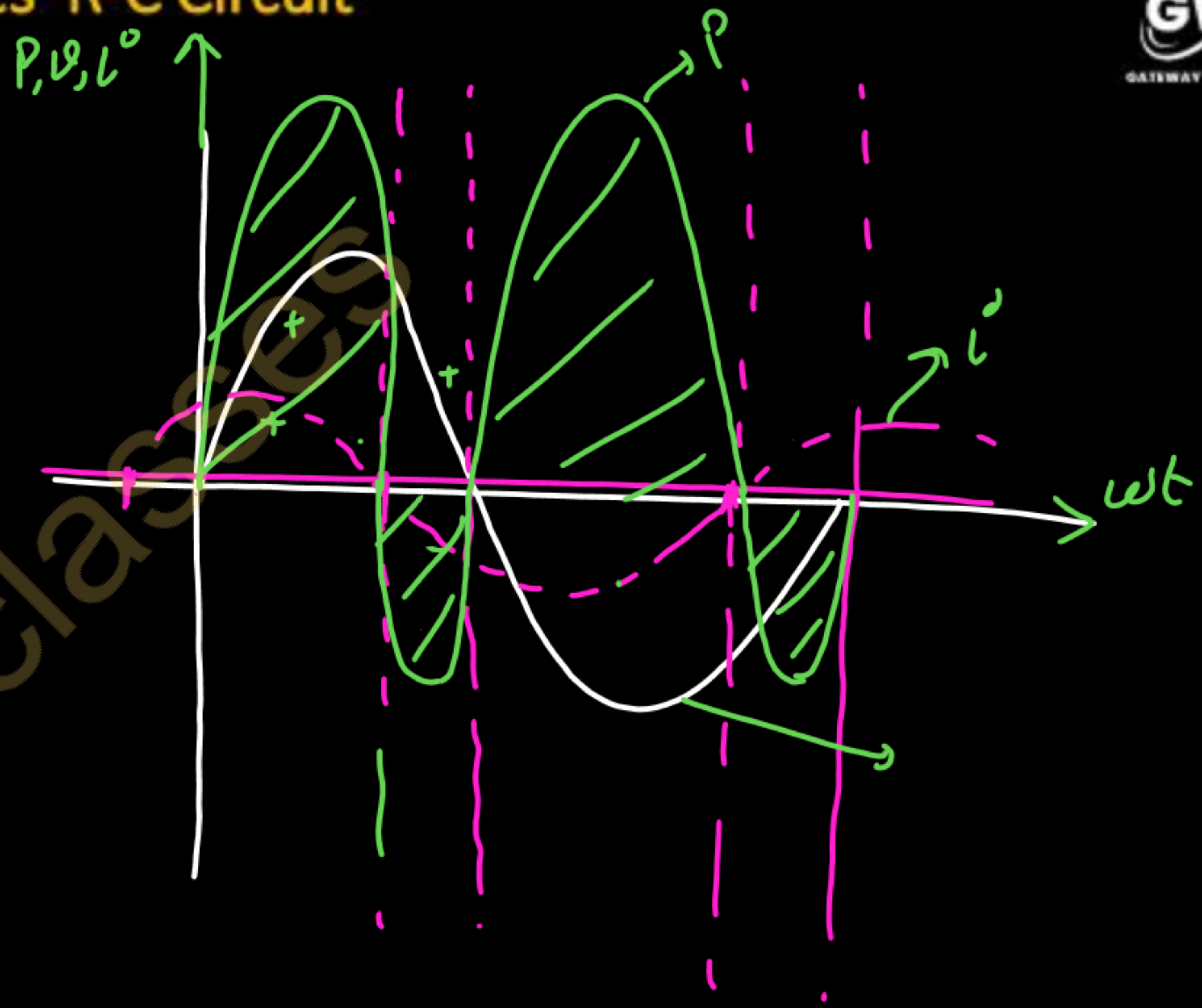
$= v_m \sin\omega t \cdot I_m \sin(\omega t + \phi)$

$= \frac{v_m I_m}{2}\left[\cos\phi - \underbrace{\cos(2\omega t + \phi)}_{0}\right]$

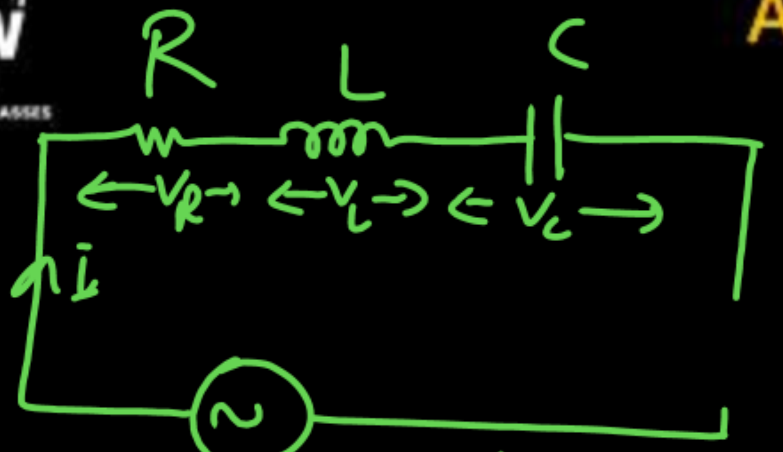
Average Power

$P_{avg} = \frac{v_m I_m}{2}\cos\phi$

$\boxed{P_{avg} = vI\cos\phi}$



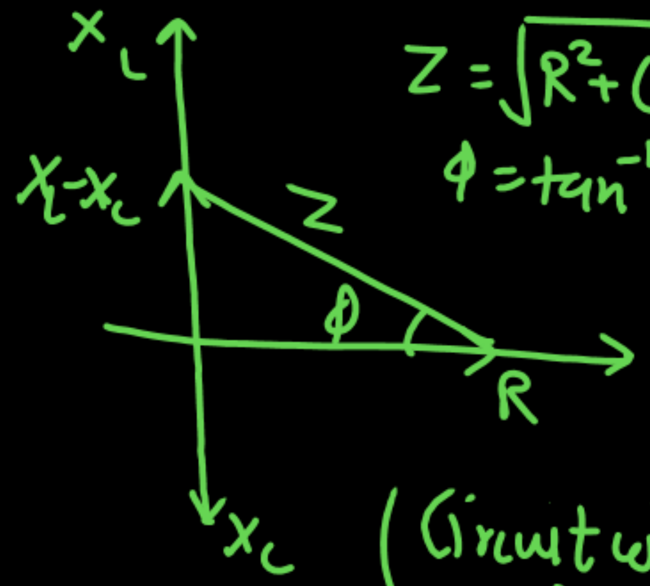
A.C. Through Series R-L-C Circuit (AKTU 2022-23)



$v = V_m \sin \omega t$

$$Z = R + j(X_L - X_C)$$

(i) when $X_L > X_C$



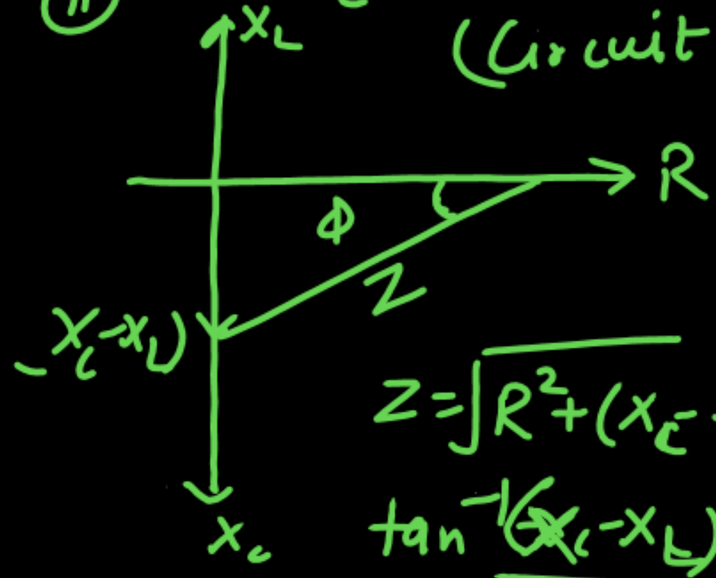
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

(Circuit will be inductive)

(ii) when $X_C > X_L$

(Circuit will be capacitive)



$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

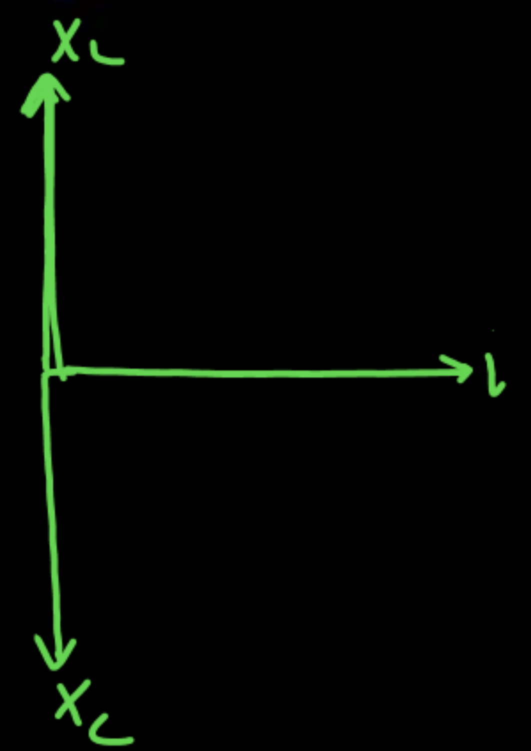
$$\tan^{-1}\frac{(X_C - X_L)}{R} = \phi$$

(iii) when $X_L = X_C$

$$\boxed{X_L - X_C = 0}$$

$$\boxed{Z = R}$$

Resistive



Concept of Power

Concept of Power:-

Apparent Power:- Apparent Power (S) is Product of rms value of Voltage (V) and current (I) .

Unit- ~~KW~~ KVA

$\boxed{S = VI}$

Active/True Power :- True Power (P) is Product of applied Voltage (V) and active component of current (I).

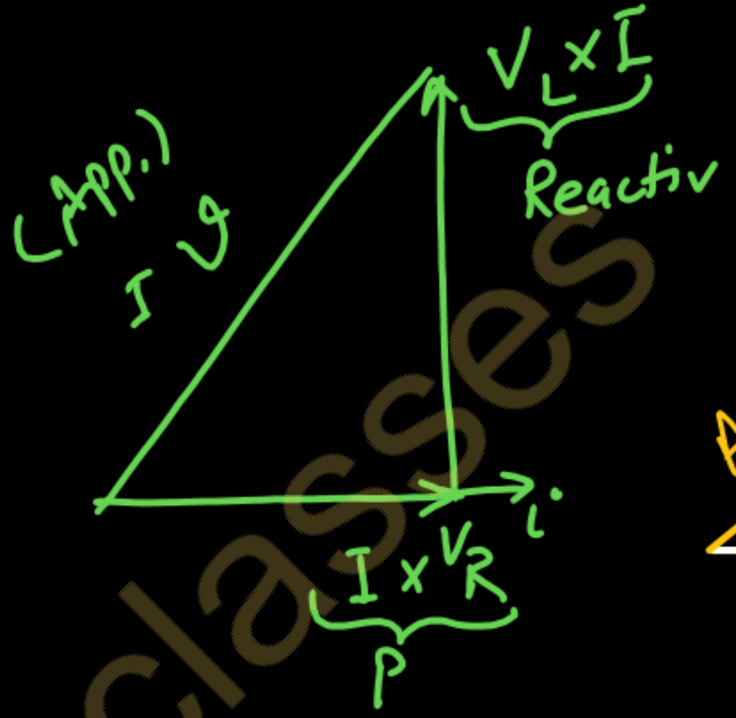
Unit- ~~KVA~~ KW

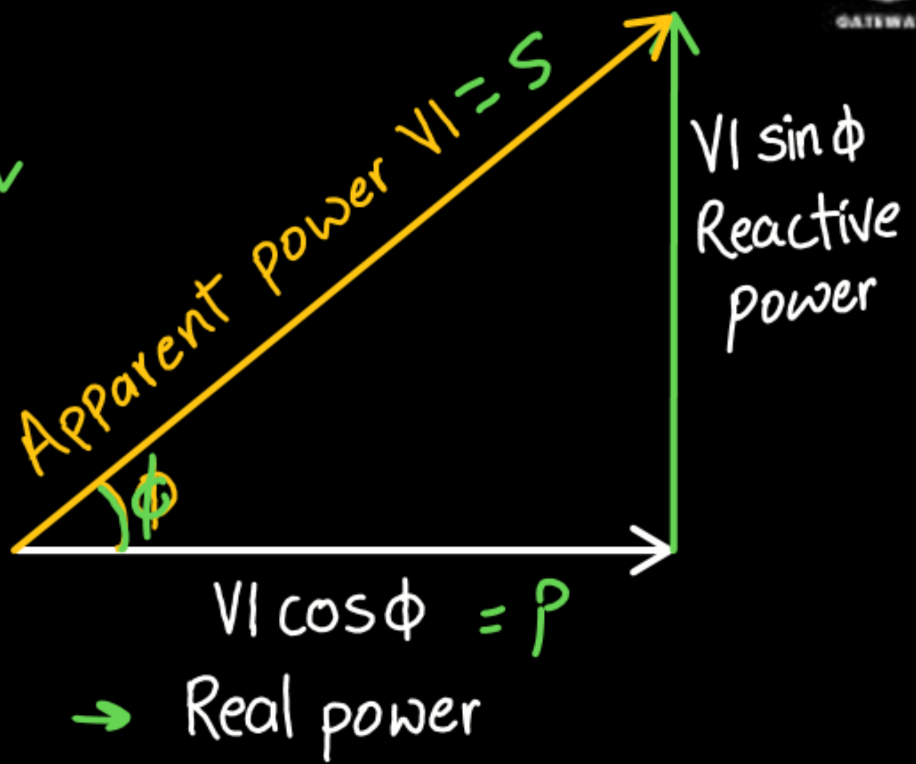
$P = VI \cos\phi = S \cos\phi$

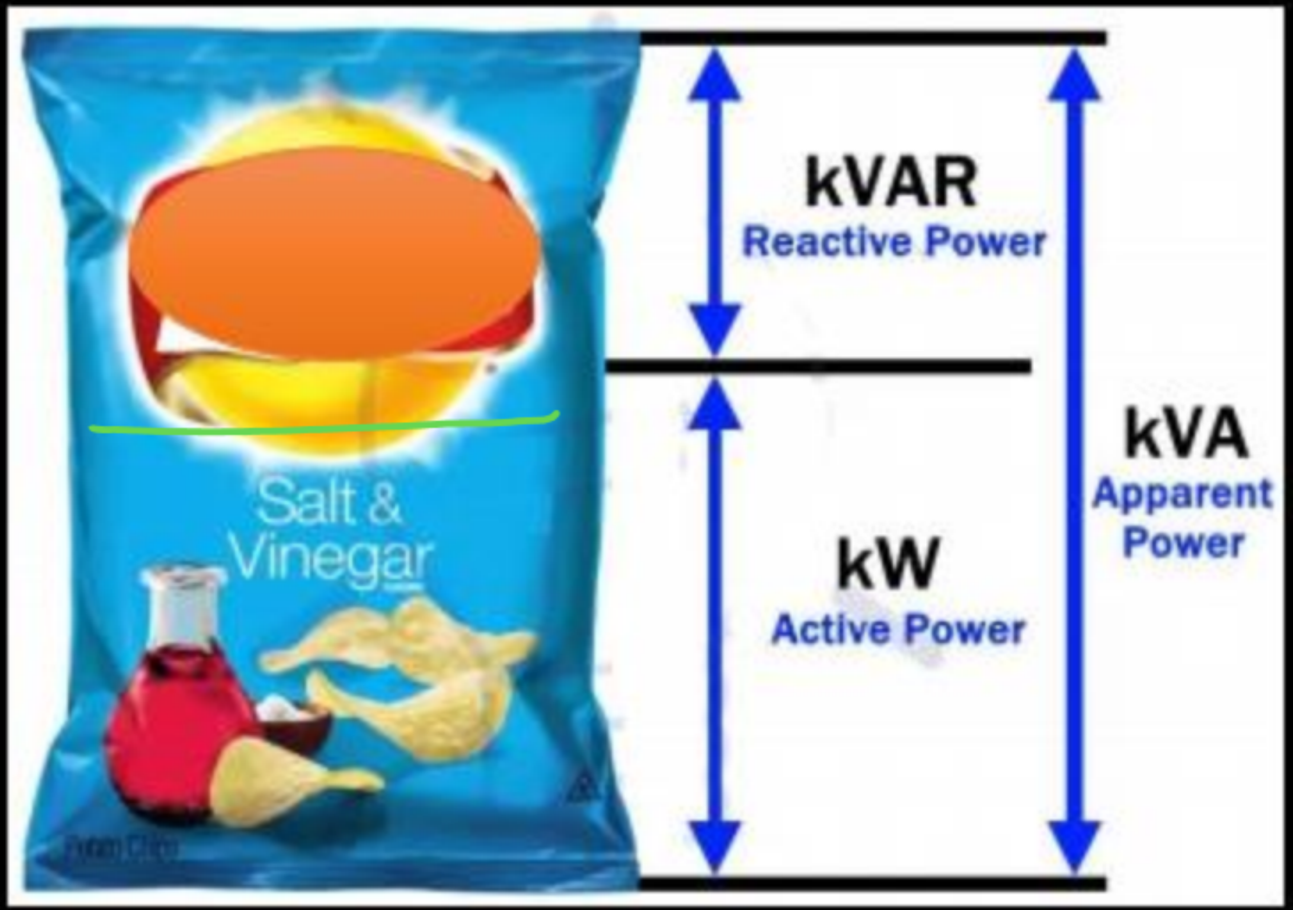
Reactive Power :- Reactive power (Q) is Product of applied Voltage (V) and reactive component of current (I).

Unit- KVAR

$\boxed{Q = VI \sin\phi = S \sin\phi}$







Power Factor

$\cos\phi = \frac{R}{Z} = \frac{VI\cos\phi}{VI} = \frac{P}{S}$

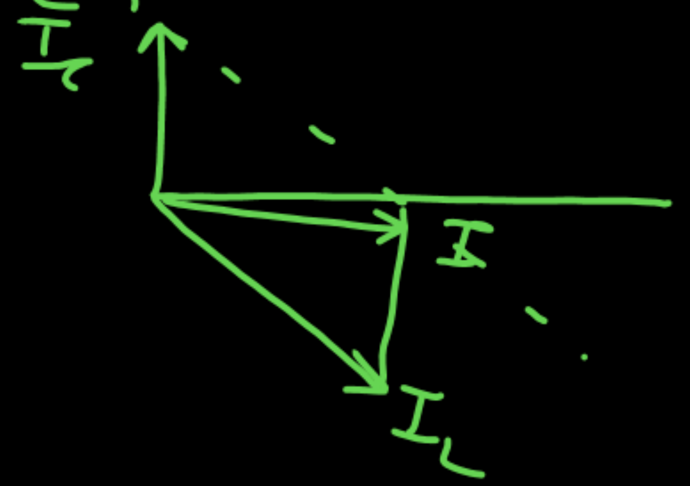
Power factor is the

- Ratio between true power and apparent power is called the for this circuit.
- Ratio of resistance to impedance.
- Cosine of phase angle between voltage and current.

Induction motor

pf = 0.6 → 1

S ← { P ← 60% Power - use; Q ← 40% " = waste }



Cause of Low Power Factor.

- All AC motor transformers has low power factor.
- Most of the loads has low power factors/
- Industrial heating furnace , induction furnace has low power factor.

Improvement of Power Factor.

- Using Phase advancers
- Using Capacitor Bank, Synchronous Condenser

Effects of low power factor

- a) Large Copper Losses b) Large kVA rating c) Poor Voltage Regulation.

Ques1 A 120 V, 60 Watt Lamp is to be operated on 220 V , 50 Hz supply mains, In order that lamp should operate on correct voltage Calculate value of (i) Non inductive resistance (ii) Pure Inductance (iii) Pure Capacitance **AKTU**

Sol^n (i) Lamp $P = 60W$, $V = 120V$

$I = \frac{P}{V} = \frac{60}{120} = 0.5A$

The value of 'R'

$R = \frac{V}{I} = \frac{100}{0.5} = 200\Omega$

(ii)

$\vec{V} = \vec{V_R} + \vec{V_L}$

$V_L = \sqrt{V^2 - V_R^2} = \sqrt{220^2 - 120^2}$

$V_L = 184.39V$

$X_L = \frac{V_L}{I_L} = \frac{184.39}{0.5} = 368.78\Omega$

$X_L = 2\pi f L$

$L = \frac{X_L}{2\pi f} = \frac{368.78}{2\pi \times 50} = 1.17H$

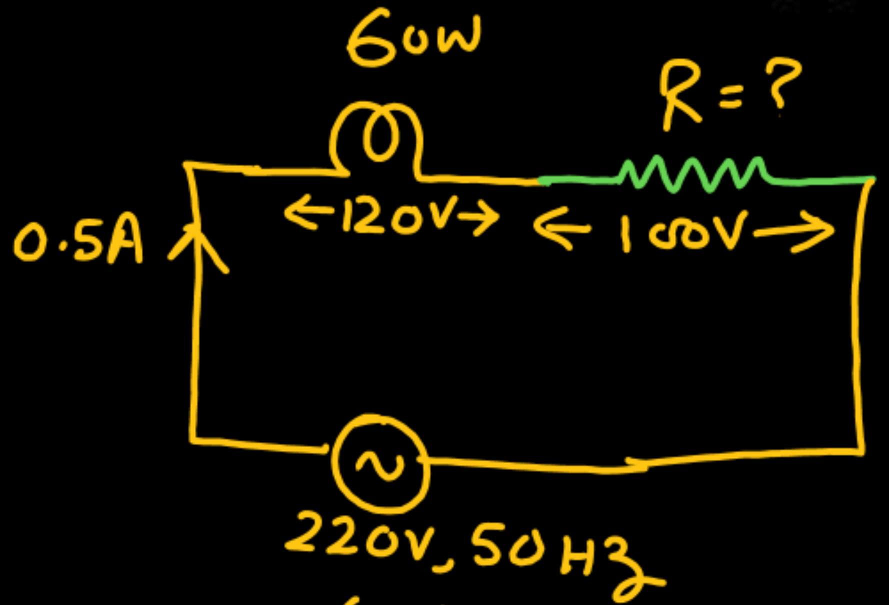
(iii)

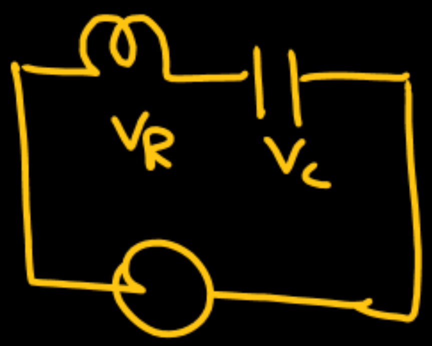
$V_C = \sqrt{V^2 - V_R^2}$

$= 184.39V$

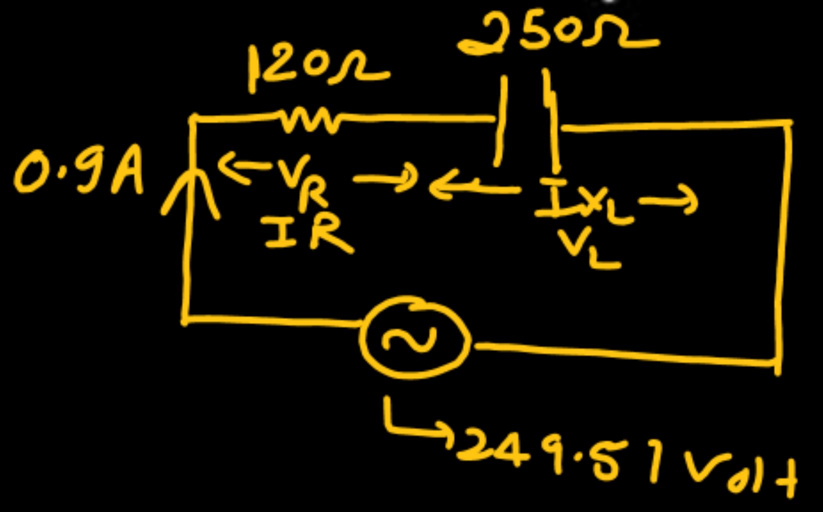
$X_C = \frac{V_C}{I_C} = 368.78\Omega$

$X_C = \frac{1}{2\pi f C} \Rightarrow C = \frac{1}{2\pi f X_C} = 8.63\mu f$





Ques2 A resistance of 120 ohms and a capacitive reactance of 250 ohms are connected in series across a A.C. voltage source. If a current of 0.9 A is flowing in the circuit find out (i) power factor (ii) supply voltage (iii) voltage across resistance and capacitance (iv) Active and reactive power.



Sol^n (i) Power factor $= \frac{R}{Z} = \frac{120}{\sqrt{120^2+250^2}} = 0.43$

(ii) Supply Voltage $V = I.Z = 0.9 \cdot \sqrt{120^2+250^2} = 249.57$

(iii) $V_R = IR = 0.9 \times 120 = 108$ Volt

$V_L = IX_L = 0.9 \times 250 = 225$ volt

$\cos\phi = 0.43$

$\phi = 64.53$

$\sin\phi = 0.90$

(iv) $P = VI\cos\phi = 249.57 \times 0.9 \times 0.43 = 96.58\ W$

$Q = VI\sin\phi = 249.57 \times 0.9 \times 0.90 = 202.15\ W$

Numerical Examples

Ques3 A series RLC circuit is composed of 10 ohm resistance , 0.1H inductance and 50 microfarad capacitance. A voltage $\underbrace{v(t) = \underbrace{141.4}_{v_m} \cos(\underbrace{100\pi}_{\omega} t)}$ V is impressed upon the circuit . **Determine** (i) The expression for instantaneous current, (ii) The voltage drops V_R, V_L, V_C across resistor capacitor and inductor (iii) Draw the phasor diagram using all the voltage relation. **AKTU- 2014-15**

Sol^n

$\omega = 100\pi$, $2\pi f = 100\pi$, $v_m = 141.4$ Volt

$f = 50$ Hz

$R = 10\Omega$

$X_L = 2\pi f L = 100\pi \times 0.1 = 10\pi\ \Omega = 31.4\ \Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{100\pi \times 50 \times 10^{-6}} = 63.66\ \Omega$

$X_C > X_L \rightarrow$ Circuit will be capacitive

$Z = \sqrt{R^2 + (X_L - X_C)^2} = 33.77\ \Omega$

So $I_m = \frac{v_m}{Z} = 4.18$ Amp

$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

$\phi = 72.77^\circ$

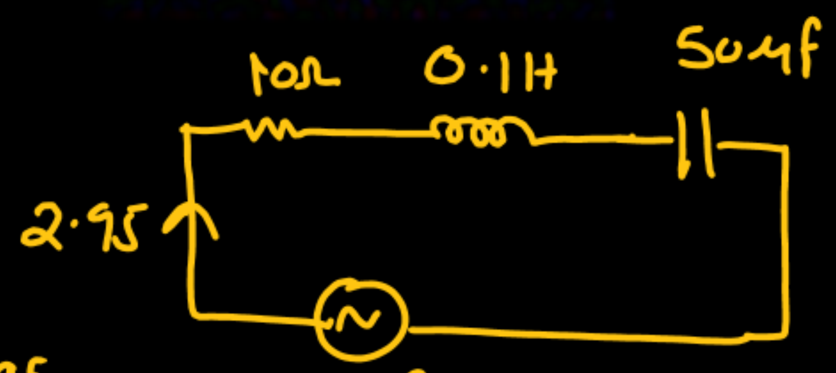
$i = 4.18 \cos(100\pi t + 72.77^\circ)$

(ii) $I_m = 4.18$, $I_{rms} = \frac{I_m}{\sqrt{2}} = 2.95$

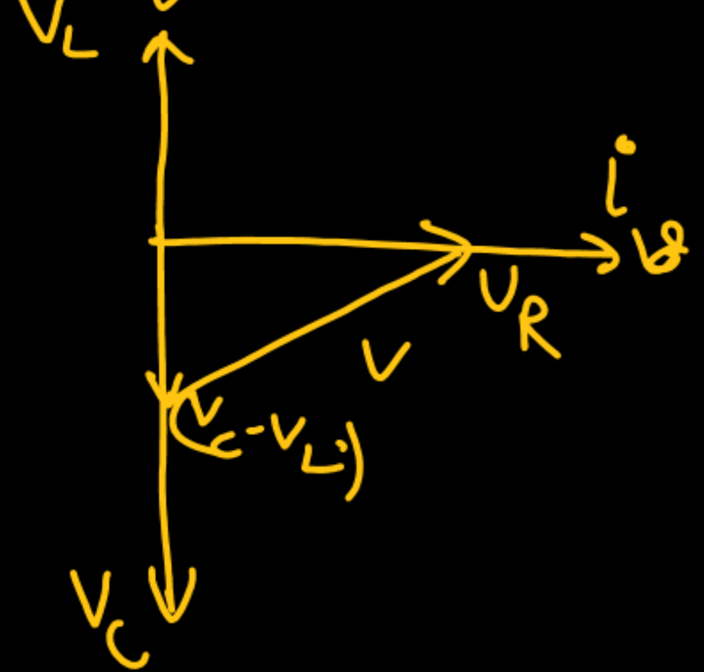
$V_R = IR = 2.95 \times 10 = 29.5$ Volt

$V_L = I X_L = 2.95 \times 31.4 = 92.8$ Volt

$V_C = I X_C = 2.95 \times 63.66 = 187.97$ Volt



$v =$



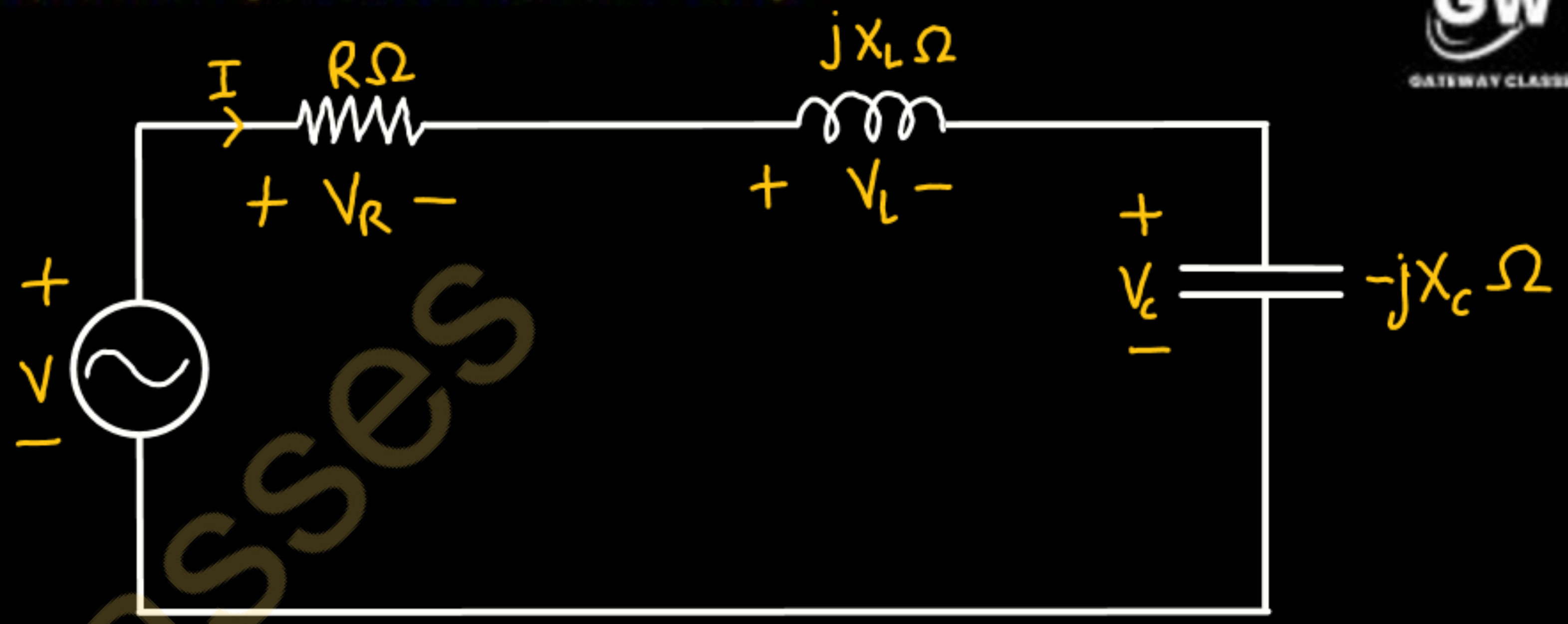
Resonance in Series RLC circuit (AKTU-2022-23)

V.V.Imp

Condition of Resonance in Series RLC circuit:-

- Power Factor is unity
  → Circuit will be resistive.
- Volatge and current are in same phase
- A series resonant circuit has the capability to draw heavy current and power from the mains; it is also called acceptor circuit.

2 marks



Parameters at Resonance Frequency:

(i) Resonance Frequency

$$X_L = X_C$$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

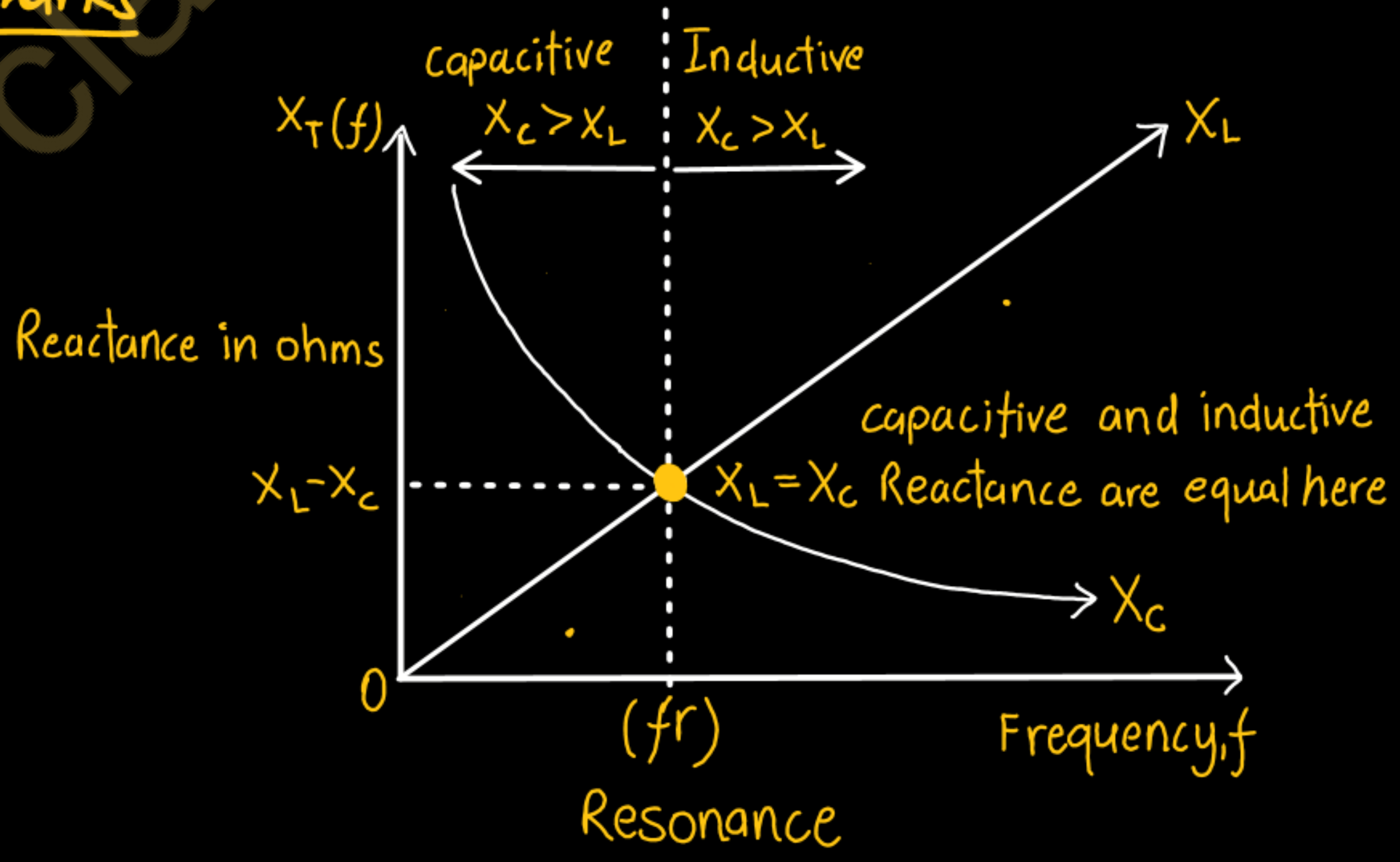
$$f_0^2 = \frac{1}{4\pi^2 LC}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

$X_L = 2\pi f L$

$X_L \propto f$

$X_C \propto \frac{1}{f}$



Resonance in Series RLC circuit

Parameters at Resonance Frequency:

(ii) Impedance at Resonance: At resonance, the total impedance of series RLC circuit is equal to resistance i.e $\boxed{Z = R}$, impedance has only real part but no imaginary part and this impedance at resonant frequency is called dynamic impedance.

$$\boxed{Z_{min} = R}$$

(iii) Current at Resonance: if Z is minimum then I will be maxi

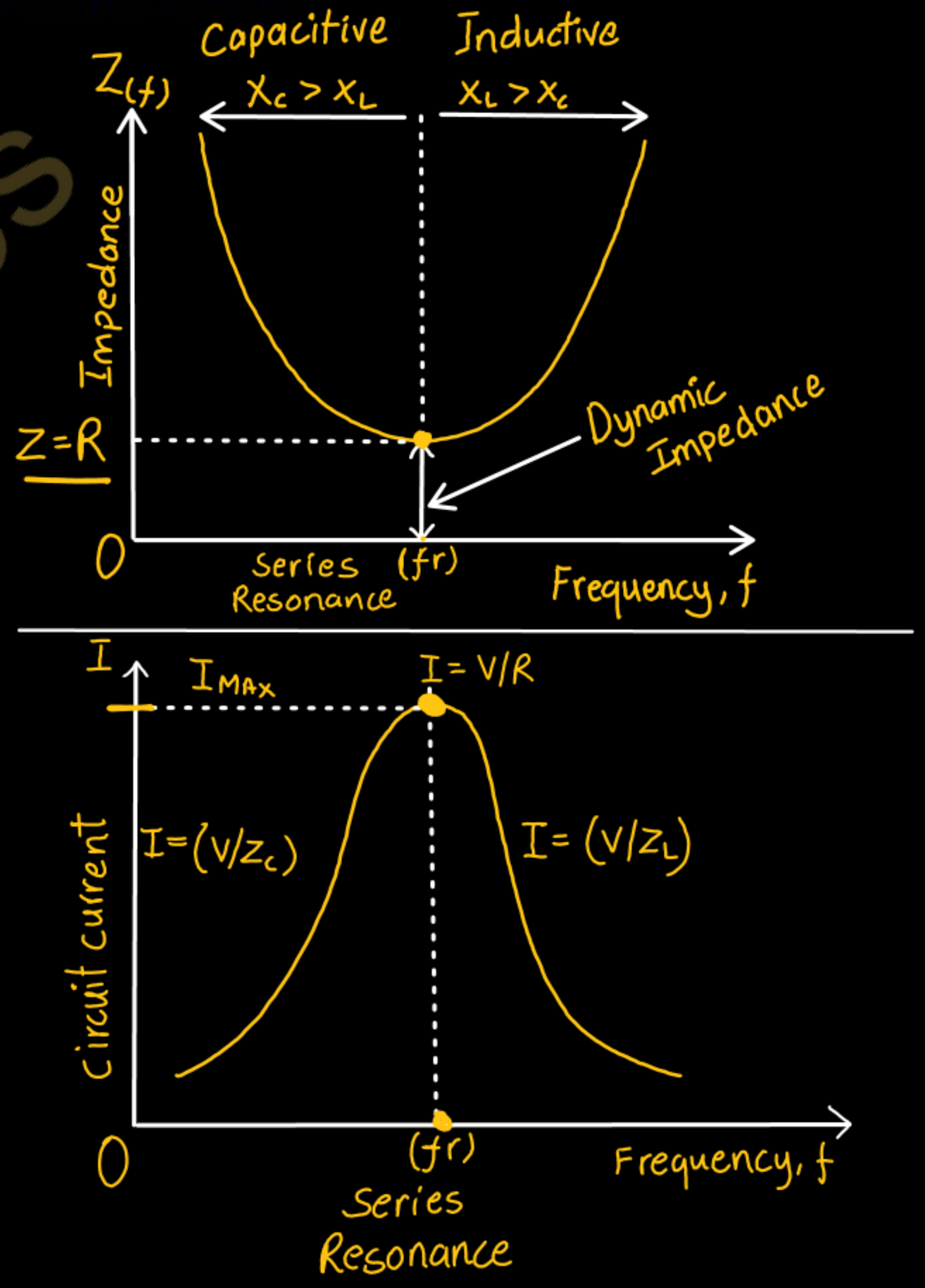
$$\uparrow I = \frac{V}{Z}\downarrow$$

(iv) Quality Factor:- The ratio of a voltage developed across the inductance or Capacitance at resonance to the applied voltage

$$Q = \frac{V_L \text{ or } V_c}{V} = \frac{\cancel{I}\cdot \omega L}{\cancel{I} R} = \boxed{\frac{\omega L}{R} = Q} \boxed{= \frac{1}{\omega C R}}$$

$$Q = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} = \boxed{\frac{1}{R}\sqrt{\frac{L}{C}} = Q}$$

Resonance Curve:

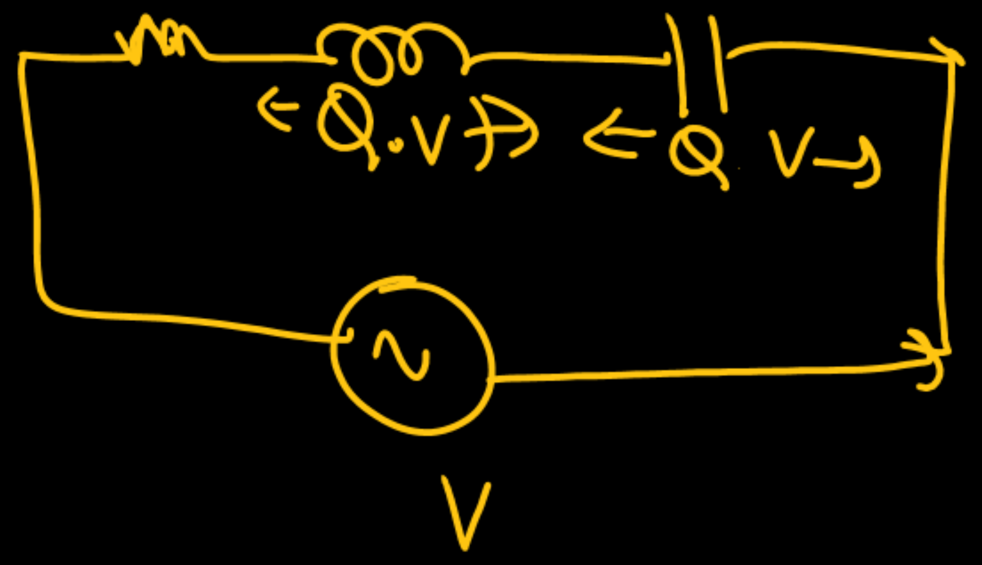


Resonance in Series RLC circuit

- Series resonance RLC circuit is called as **voltage magnification** circuit, because the magnitude of voltage across the inductor and the capacitor is equal to Q times the input sinusoidal voltage V.

Application of Series RLC Resonant Circuit

- Since **resonance in series RLC circuit** occurs at particular frequency, so it is used for filtering and tuning purpose as it does not allow unwanted oscillations that would otherwise cause signal distortion, noise and damage to circuit to pass through it.



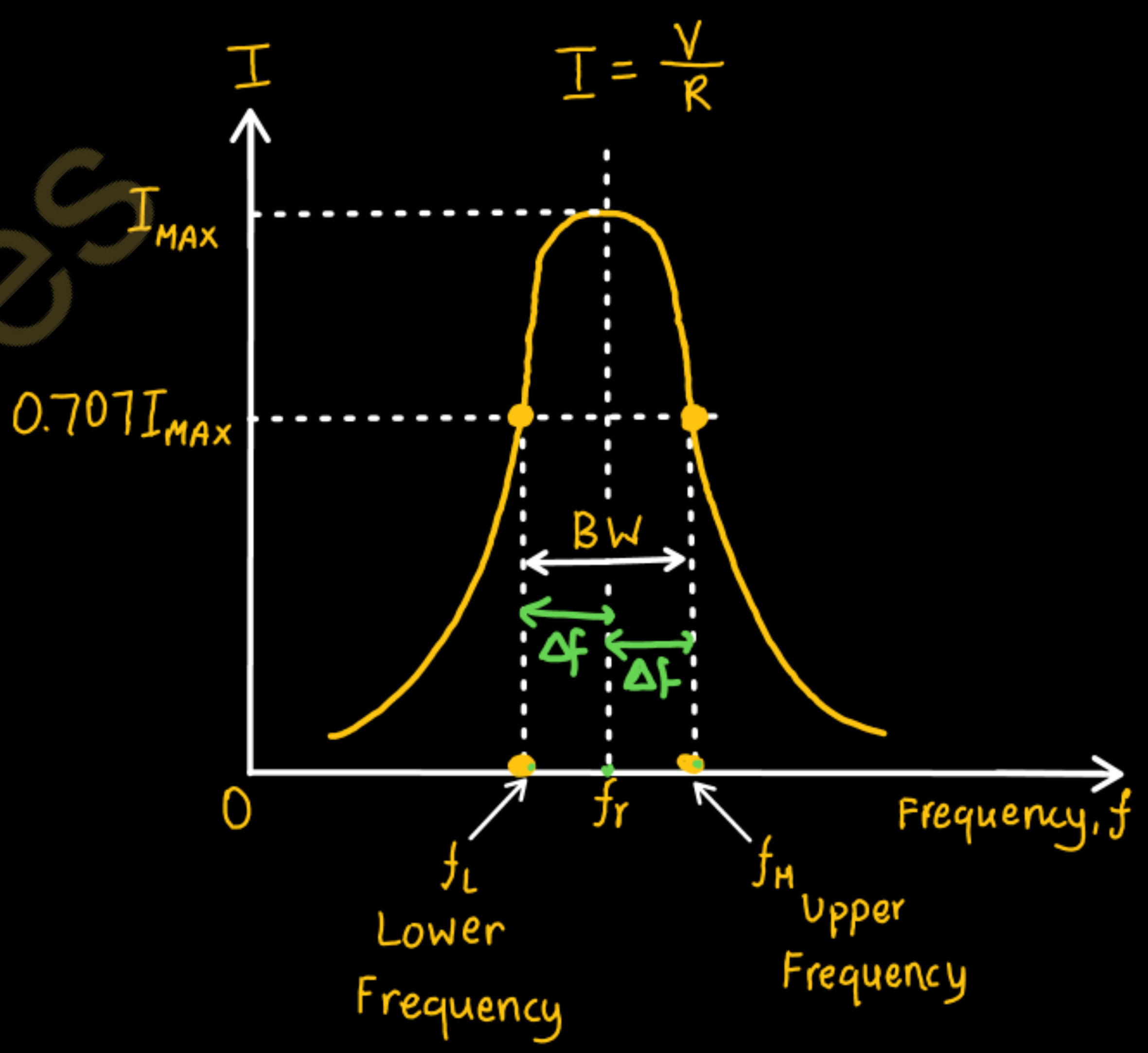
Bandwidth of Series RLC Circuit at resonance (AKTU-2023-24)

Imp

The bandwidth of any system is the range of frequencies for which the current or output voltage is equal to 70.7% of its value at the resonant frequency, and it is denoted by BW.

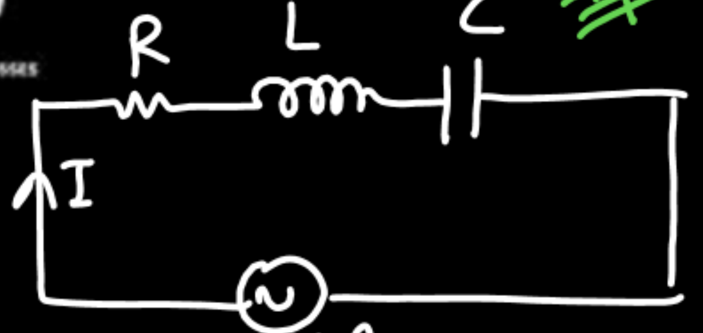
Frequency f_1 is the frequency at which the current is 0.707 times the current at resonant value, and it is called the **lower cut-off frequency**.

The frequency f_2 is the frequency at which the current is 0.707 times the current at resonant value (i.e. maximum value), and is called the **upper cut-off frequency**. The bandwidth, or BW, is defined as the frequency difference between f_2 and f_1.



Bandwidth of Series RLC Circuit at resonance

Imp



$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2+(X_L-X_C)^2}} \quad \text{— (1)}$$

At resonance
Current will be maximum

$$I_m = \frac{V}{R} \quad \text{— (2)}$$

and $\frac{P_m}{2} = \frac{I_m^2}{2} R$

$$= \left(\frac{I_m}{\sqrt{2}}\right)^2 \cdot R$$

$$\boxed{\frac{P_m}{2} = I_{rms}^2 R}$$

So these points are also Known as half Power Points

$$I = \frac{I_m}{\sqrt{2}} = \frac{V}{R\sqrt{2}} \quad \text{— (3)}$$

from Eqn (1) & (3)

$$\frac{\cancel{V}}{R\sqrt{2}} = \frac{\cancel{V}}{\sqrt{R^2+(X_L-X_C)^2}}$$

$$2R^2 = R^2+(X_L-X_C)^2$$

$$(X_L-X_C)^2 = R^2$$

$$X_L - X_C = \pm R$$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \quad \text{— (4)}$$

$$\omega_2 L - \frac{1}{\omega_2 C} = +R \quad \text{— (5)}$$

Add Eqn 4 & 5

$$L(\omega_1+\omega_2) - \frac{1}{C}\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right) = 0$$

$$L(\omega_1+\omega_2) - \frac{1}{C}\left(\frac{\omega_1+\omega_2}{\omega_1\omega_2}\right) = 0$$

$$L - \frac{1}{C\omega_1\omega_2} = 0$$

$$L = \frac{1}{C\omega_1\omega_2}$$

$$\omega_1\omega_2 = \frac{1}{LC} \quad \text{— (6)}$$

$$f_1 f_2 = \frac{1}{4\pi^2 LC}$$

$$f_1 f_2 = f_r^2$$

$$\boxed{f_r = \sqrt{f_1 f_2}} \quad \text{— (7)}$$

Subtract Eqn (4) from (5)

$$L(\omega_2-\omega_1) - \frac{1}{C}\left[\frac{1}{\omega_2}-\frac{1}{\omega_1}\right] = 2R$$

$$L(\omega_2-\omega_1) - \frac{1}{C}\left(\frac{\omega_1-\omega_2}{\omega_1\omega_2}\right) = 2R$$

$$(\omega_2-\omega_1)\left[L+\frac{1}{C\omega_1\omega_2}\right] = 2R$$

$$(\omega_2-\omega_1)\left[L+\frac{1}{\cancel{C}\times\frac{1}{L\cancel{C}}}\right] = 2R$$

$$\cancel{2}L(\omega_2-\omega_1) = \cancel{2}R$$

$$(\omega_2-\omega_1) = R/L$$

$$\boxed{(f_2-f_1) = \frac{R}{2\pi L} = B.W} \quad \text{— (8)}$$

Now $\Delta f = \frac{B.W}{2} = \frac{R}{4\pi L}$

$$f_2 = f_r + \Delta f = f_r + \frac{R}{4\pi L} \quad \text{— (9)}$$

$$f_1 = f_r - \Delta f = f_r - \frac{R}{4\pi L} \quad \text{— (10)}$$

Numerical Examples

Ques4 A Series RLC circuit consisting a resistance of 20 ohm , inductance of 0.2 H and Capacitance of 150µF is connected across 230V, 50 Hz source. Calculate (i) Impedance (ii) Current (III) Power Factor (iv) Frequency Required to Make power factor Unity (v) Quality Factor **AKTU- 2012-13**

Sol: $R = 20\Omega$

$X_L = 2\pi f L = 100\pi \times 0.2 = 62.83\Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{100\pi \, 150\times10^{-6}} = 21.22\Omega$

(i) $Z = \sqrt{R^2 + (X_L - X_C)^2} = 46.16\Omega$

(ii) $I = \frac{V}{Z} = 4.98\ A$

(iii) $\cos\phi = \frac{R}{Z} = \frac{20}{46.16} = 0.43$

(iv)

(iv) $f_r = \frac{1}{2\pi\sqrt{LC}}$

$= \frac{1}{2\pi\sqrt{0.2\times150\times10^{-6}}}$

$\boxed{f_r = 29.05}$ Hz.

(v) $Q = \frac{\omega L}{R} = \frac{2\pi 29.05}{20} = 9.128$

Resonance in Parallel RLC circuit

Condition of Resonance in Parallel RLC circuit:-

Parallel Resonance means when the circuit current is in phase with the applied voltage of an AC circuit containing an inductor and a capacitor connected together in parallel.

Parameters at Resonance Frequency:

(i) Resonance Frequency

$$\vec{I_r} = \vec{I_L} + \vec{I_C}$$

$$I_C = I_L \sin\phi_L$$

$$\sin\phi_L = \frac{X_L}{Z_L}, \quad I_C = \frac{V}{X_C}, \quad I_L = \frac{V}{Z_L}$$

$$\frac{\cancel{V}}{X_C} = \frac{\cancel{V}}{Z_L}\cdot\frac{X_L}{Z_L}$$

$$\boxed{Z_L^2 = X_L X_C}$$

$$\boxed{Z_L^2 = \frac{L}{C}}$$

$$(R^2 + X_L^2) = \frac{L}{C}$$

$$X_L^2 = \frac{L}{C} - R^2$$

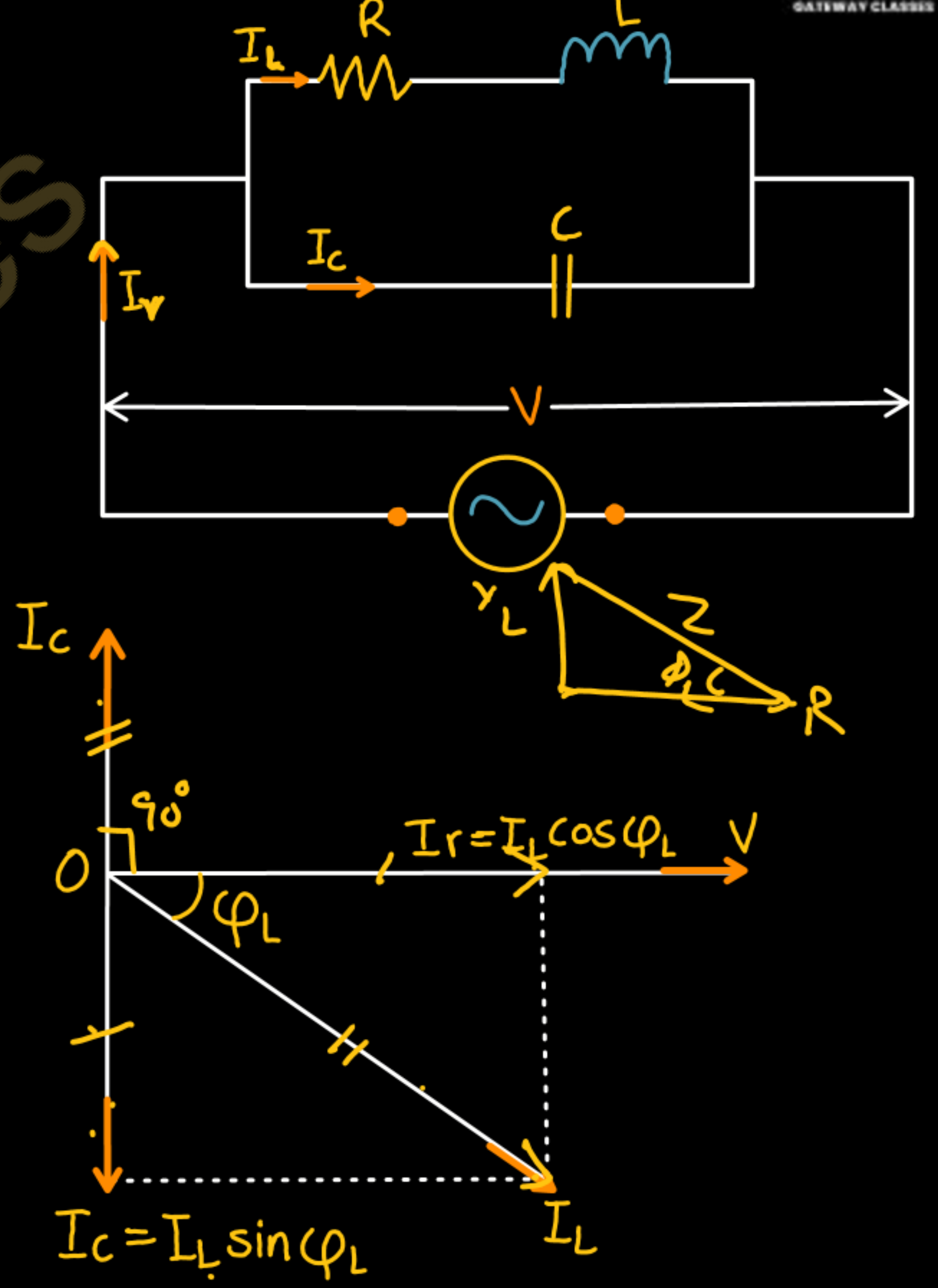
$$(2\pi f L)^2 = \frac{L}{C} - R^2$$

$$f^2 = \frac{1}{4\pi^2 L^2}\left(\frac{L}{C} - R^2\right)$$

$$f_0^2 = \frac{1}{4\pi^2}\left[\frac{1}{LC} - \frac{R^2}{L^2}\right]$$

$$\boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

$\rightarrow$ neglect



Resonance in Parallel RLC circuit

(ii) Impedance at Parallel Resonance:

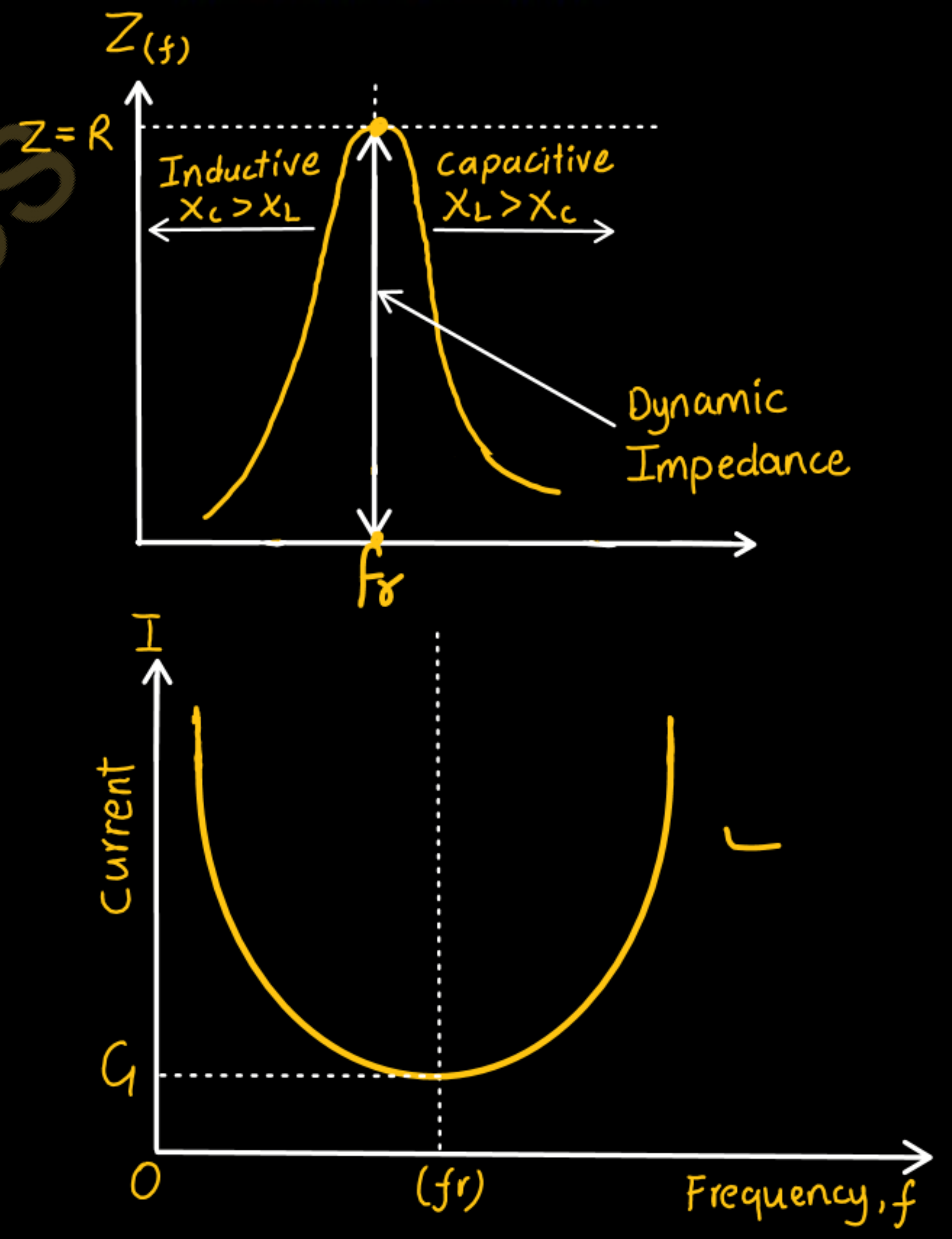
At parallel resonance line current $I_r = I_L \cos\varphi$ or

$$\frac{\cancel{V}}{Z_r} = \frac{\cancel{V}}{Z_L}\cdot\frac{R}{Z_L}$$

$$Z_r = \frac{Z_L^2}{R} = \frac{L}{RC}$$

$$\boxed{Z_r = \frac{L}{RC}} \rightarrow \text{maximum}$$

Resonance Curve:



- The circuit impedance is purely resistive because there is no frequency term present in it. Circuit impedance Z_r will be in Ohms.
- The value of **Zr will be very high** because the ratio L/C is very large at parallel resonance.
- The value of ***circuit current, Ir = V/Zr is very small*** because the value of Zr is very high.
- parallel resonant circuit can draw a very small current and power from the mains, therefore, it is also called as ***Rejector Circuit.***

Quality Factor at Parallel Resonance

- The ratio of a Current across the inductance or Capacitance at resonance to the Current at resonance.
- Parallel resonance RLC circuit is called as **Current magnification** circuit.

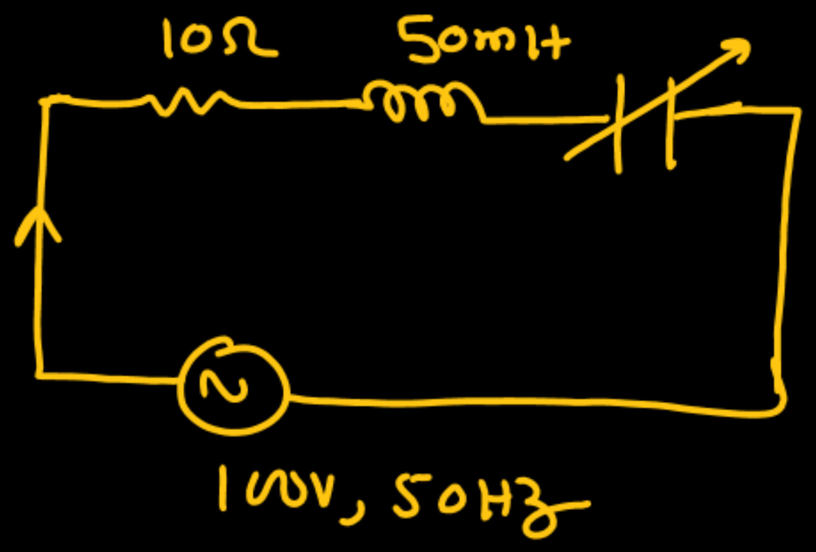
$$Q = \frac{I_{L-C}}{I} = \frac{\cancel{V}/X_L}{\cancel{V}/Z_r} = \frac{Z_r}{X_L}$$

$$\boxed{Q = \frac{L/R\cancel{C}}{1/\omega\cancel{C}} = \frac{\omega L}{R}}$$

Numerical Examples

GW GATEWAY CLASSES

Ques5 A Series circuit consists of a resistance of 10 ohm and inductance of 50mH and a variable capacitance in series across a 100V , 50 Hz supply. Calculate (i) Value of capacitance to produce resonance (ii) Voltage across the capacitance (iii) Q- Factor. **AKTU 2018-19**



(1) $X_L = 2\pi f L$

$= 2\pi \times 50 \times 10^{-3}$

$X_L = 15.70\ \Omega$

for Resonance

$X_L = X_C = 15.70$

$C = \frac{1}{2\pi X_C} = \frac{1}{2\pi \times 15.70}$

$\boxed{C = 0.01 f}$

$V_C = I X_C$

$I = \frac{V}{R} = \frac{100}{10} = 10\,A$

$V_C = 10 \times 15.70$

$\boxed{V_C = 157.0\ \text{Volt}}$

$Q = \frac{\omega L}{R} = \frac{2\pi \times 50 \times 50 \times 10^{-3}}{10} = 1.57$

Ques.6 Three impedances of (70.7+j70.7)ohm, (120+j160)ohm and (120+j90) ohm are connected in parallel across a 250 V supply. Determine (i) admittance of the circuit (ii) supply Current and (iii) power factor

AKTU- 2021-22

$Z_1 = (70.7 + j\,70.7)$

$Z_2 = (120 + j160)$

$Z_3 = (120 + j90)$

$Y = [(70.7 + 70.7j)^{-1} + (120 + 160j)^{-1} + (120 + 90j)^{-1}]$

$Y = [0.015 - 0.015j]\ \mho$

(ii) $I = \frac{V}{Z} = V.Y = (3.85 - 3.76j)$ Volt

$= 5.38 \angle -44.37^\circ$ V Ans

$\boxed{S}$

$\boxed{=}$

$(a+ib)\ \boxed{S}\ \boxed{+}$

$r\angle\theta$

P.f $= \cos\phi$

$= 0.714$ (Lagging)

$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$

$Y = \frac{1}{Z} = (Z_1^{-1} + Z_2^{-1} + Z_3^{-1})$

$Z = (Z_1^{-1} + Z_2^{-1} + Z_3^{-1})^{-1}$

Numerical Examples

Ques.7 Two coils having resistance 5 Ω and 10 Ω and inductances 0.04 H and 0.05 H respectively are connected in parallel across a 200 V, 50 Hz supply. Calculate: i. Conductance, susceptance and admittance of each coil) ii. Total current drawn by the circuit and its power factor. iii. Power absorbed by the circuit. (AKTU)

$X_{L_1} = 2\pi f L_1 = 100\pi \cdot 04 = 12.56\,\Omega$

$Z_1 = R_1 + jX_{L_1} = 5 + 12.56j$

$Z_2 = R_2 + jX_{L_2} = 10 + 15.70j$

Coil ①

$G = \frac{1}{R} = \frac{1}{5} = 0.2$

Susceptance $= \frac{1}{X_L} = \frac{1}{12.56}$

Admittance $= \frac{1}{Z} = \frac{1}{(5+12.56j)}$

Coil-2

$G_2 = \frac{1}{10} = .01\ S$

$X_2 = \frac{1}{15.70} =$

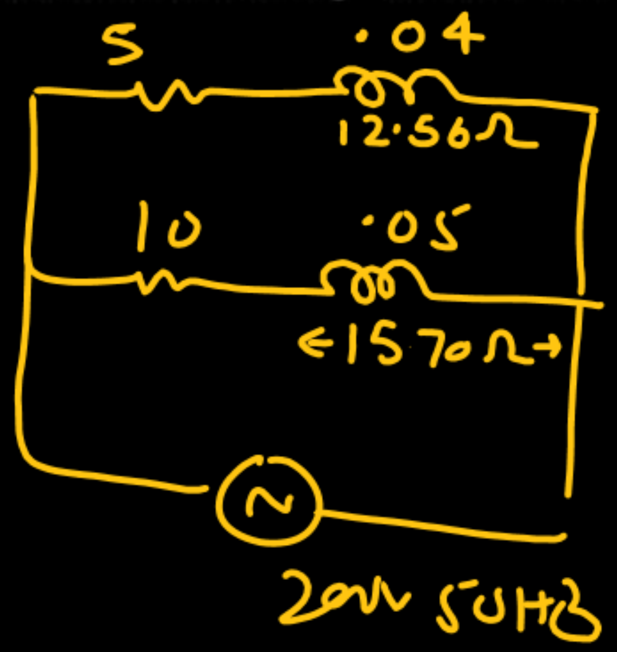
$Y_2 = \frac{1}{Z_2} = \frac{1}{(10+15.70j)}$

② $Z = \left(Z_1^{-1} + Z_2^{-1}\right)^{-1}$

$Z = 3.47 + 7.05j$

$I = \frac{V}{Z} = \frac{200}{Z} = 25.42\angle -63.78^\circ$

$P.f = \cos 63.75 = 0.44$ lagg

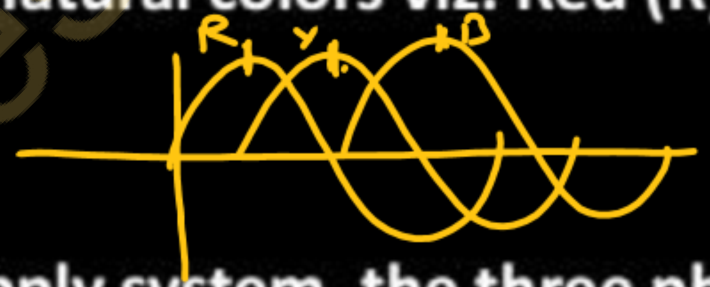


$P = 250 \times 25.42 \times 0.44$

$\boxed{P = 2796.2 \text{ watt}}$

Important Definitions

- **Phase Sequence** – The *phase sequence* is defined as the order in which the emf in three phase or coils of an alternator attains the positive maximum value. It is determined by the direction of rotation of alternator.
- **Naming the Phases** – The name of the three phases are given by the natural colors viz. Red (R), Yellow (Y) and Blue (B). Thus, the phase sequence being *RYB*.

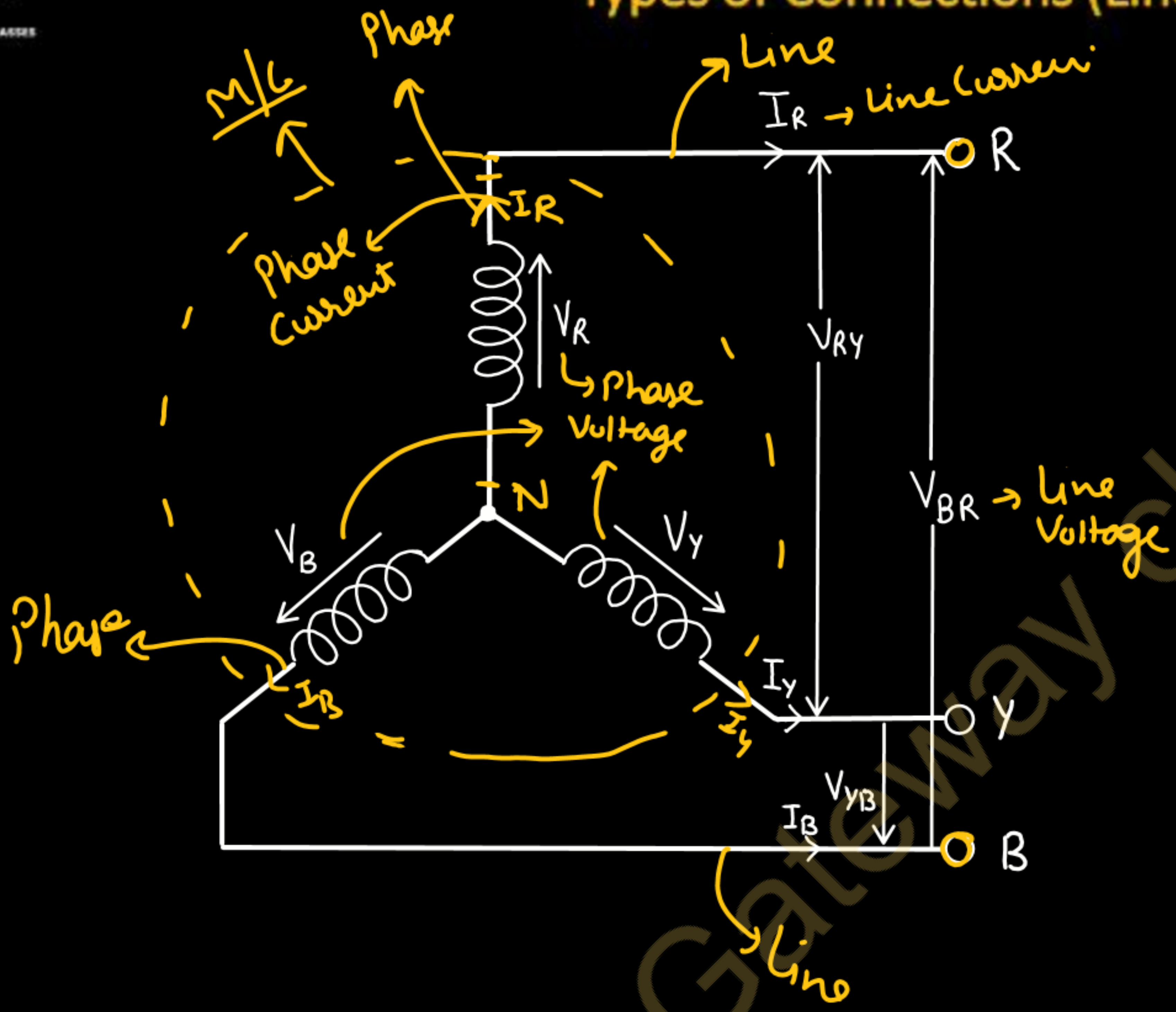


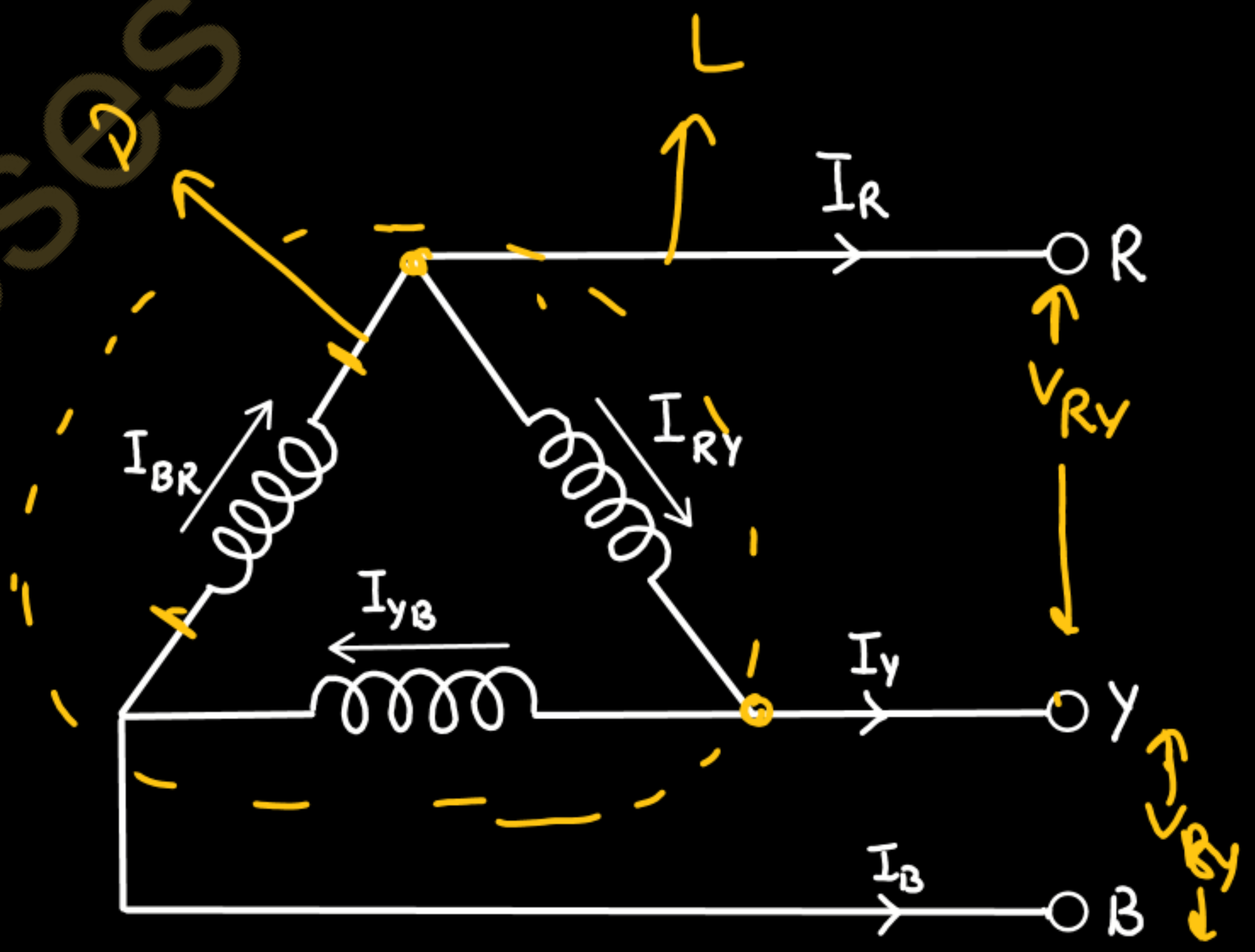
- **Balanced Three-Phase Supply System** – In a balanced three-phase supply system, the three phase voltages are equal in magnitude and ~~frequency~~ and having a phase diff of 120°

Advantages of Three Phase Supply

- The three phase power has a constant magnitude while the power in single phase system is the function of supply frequency i.e. pulsates from zero to maximum value at twice the supply frequency.
- In the 3-phase system, a rotating magnetic field can be created in stationary windings.
- For the same rating, three phase machines are smaller and simpler in construction
- To transmit the same amount of power over certain distance at a given voltage, the 3-ϕ system requires less (or 3/4) of the weight of copper than that required by the 1-ϕ system.
- The voltage regulation of three phase transmission lines is better than that of 1-phase line.
- A single phase load can by supplied by a three phase system but, the converse is not true.

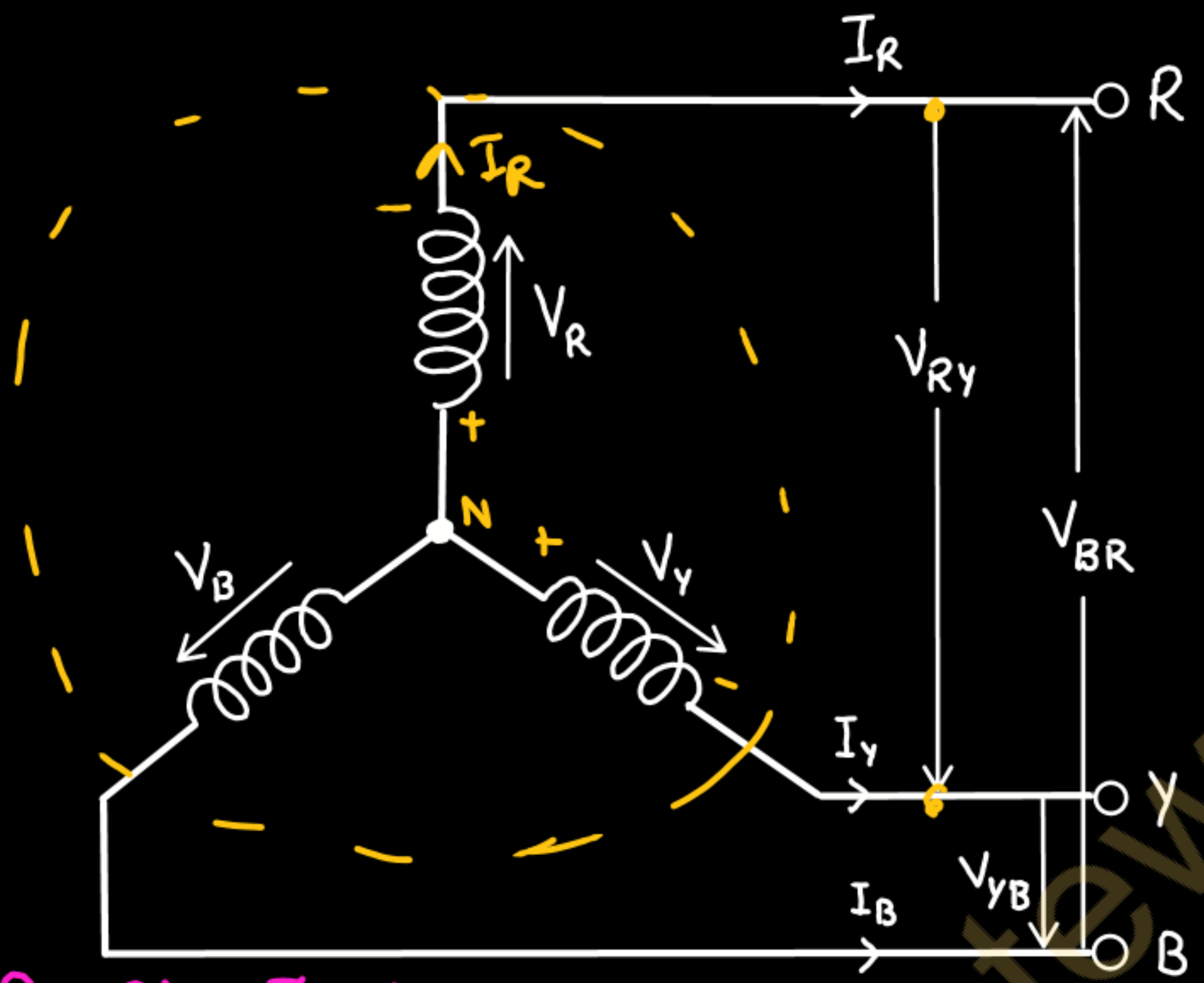
Types of Connections (Line and Phase Values)





Relation Between Line and phase Values in Star Connection (AKTU 2022-23)

v v Imp



from diagram

$\boxed{I_{Ph} = I_L}$

Also,

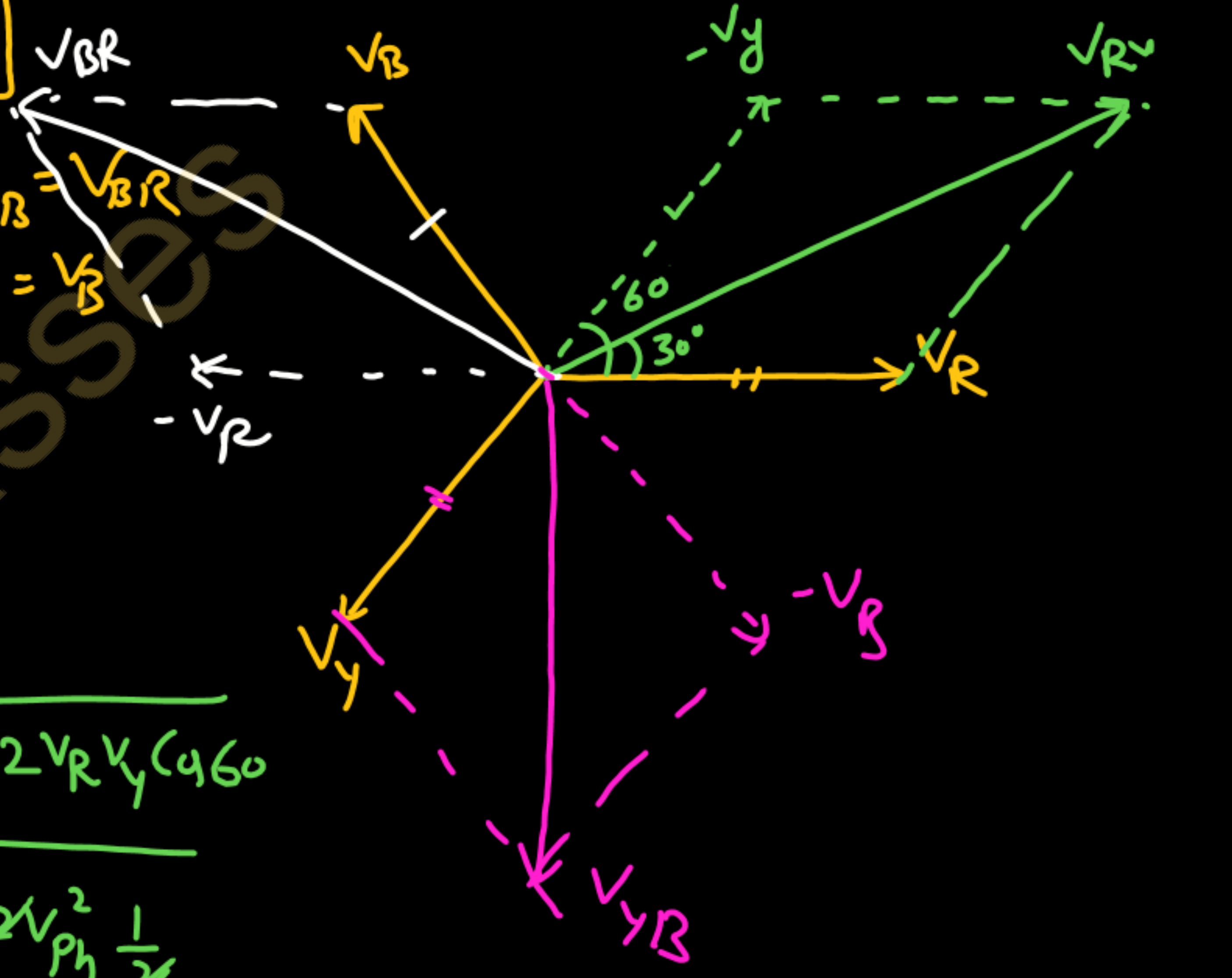
$V_L = V_{RY} = V_{YB} = V_{BR}$

$V_{Ph} = V_R = V_Y = V_B$

$V_{RY} = V_R - V_Y$

$V_{BR} = V_B - V_R$

$V_{YB} = V_Y - V_B$



$V_{RY} = \sqrt{V_R^2 + V_Y^2 + 2V_RV_Y\cos 60}$

$V_L = \sqrt{V_{Ph}^2 + V_{Ph}^2 + 2V_{Ph}^2 \frac{1}{2}}$

$V_L = \sqrt{3V_{Ph}^2}$

$\boxed{V_L = \sqrt{3}V_{Ph}}$

$P = 3V_{Ph}I_{Ph}\cos\phi$

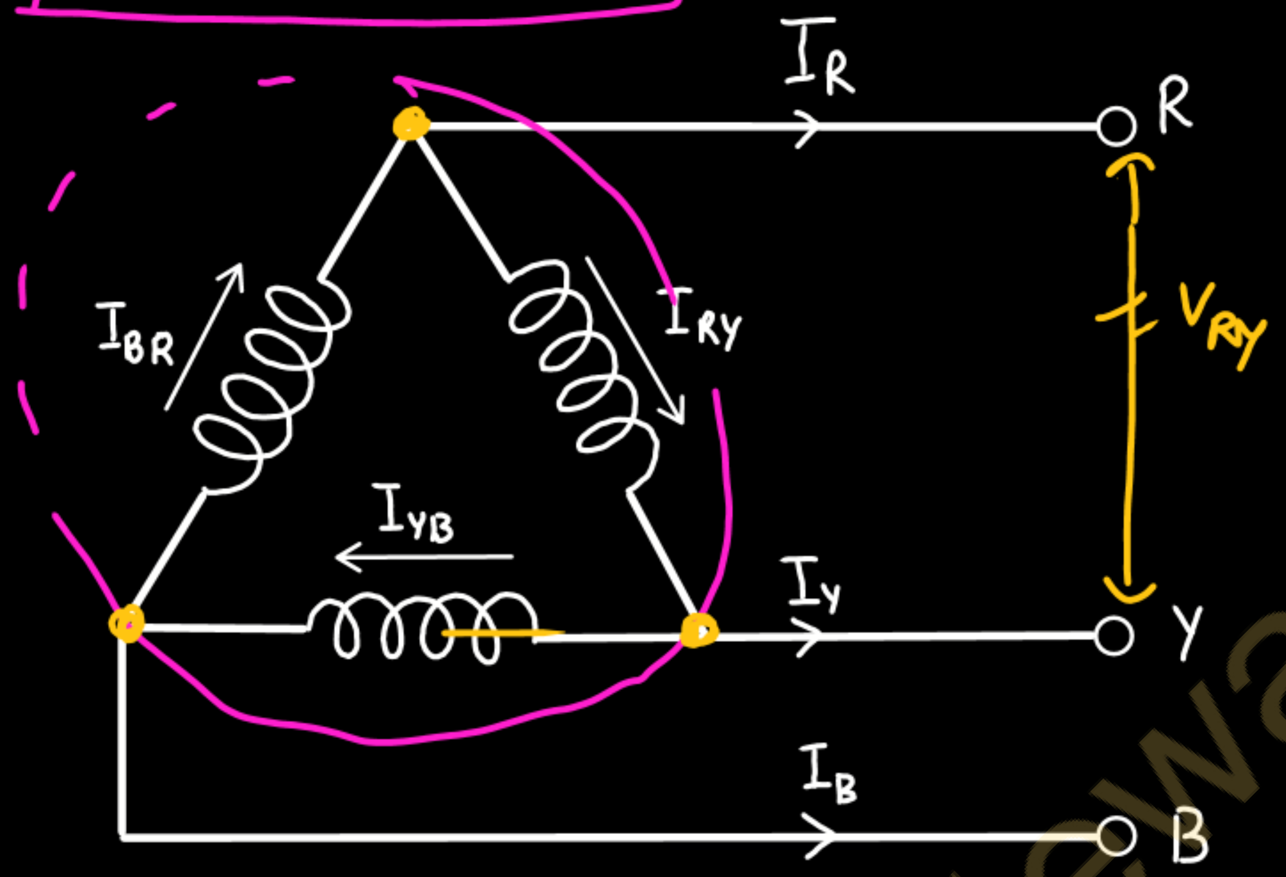
$P_{3\phi} = 3\frac{V_L}{\sqrt{3}} I_L \cos\phi = \sqrt{3} V_L I_L \cos\phi$

Relation Between Line and phase Values in Delta Connection

$P = 3 V_{ph} I_{ph} \cos\phi$

$= 3 \times V_L \cdot \frac{I_L}{\sqrt{3}} \cos\phi$

$\boxed{P = \sqrt{3} V I_L \cos\phi}$



$Q = \sqrt{3} V_L I_L \sin\phi$

$S = \sqrt{3} V_L I_L$

from diagram

$\boxed{V_L = V_{ph}}$

from Current Relation

$I_{BR} = I_R + I_{RY}$

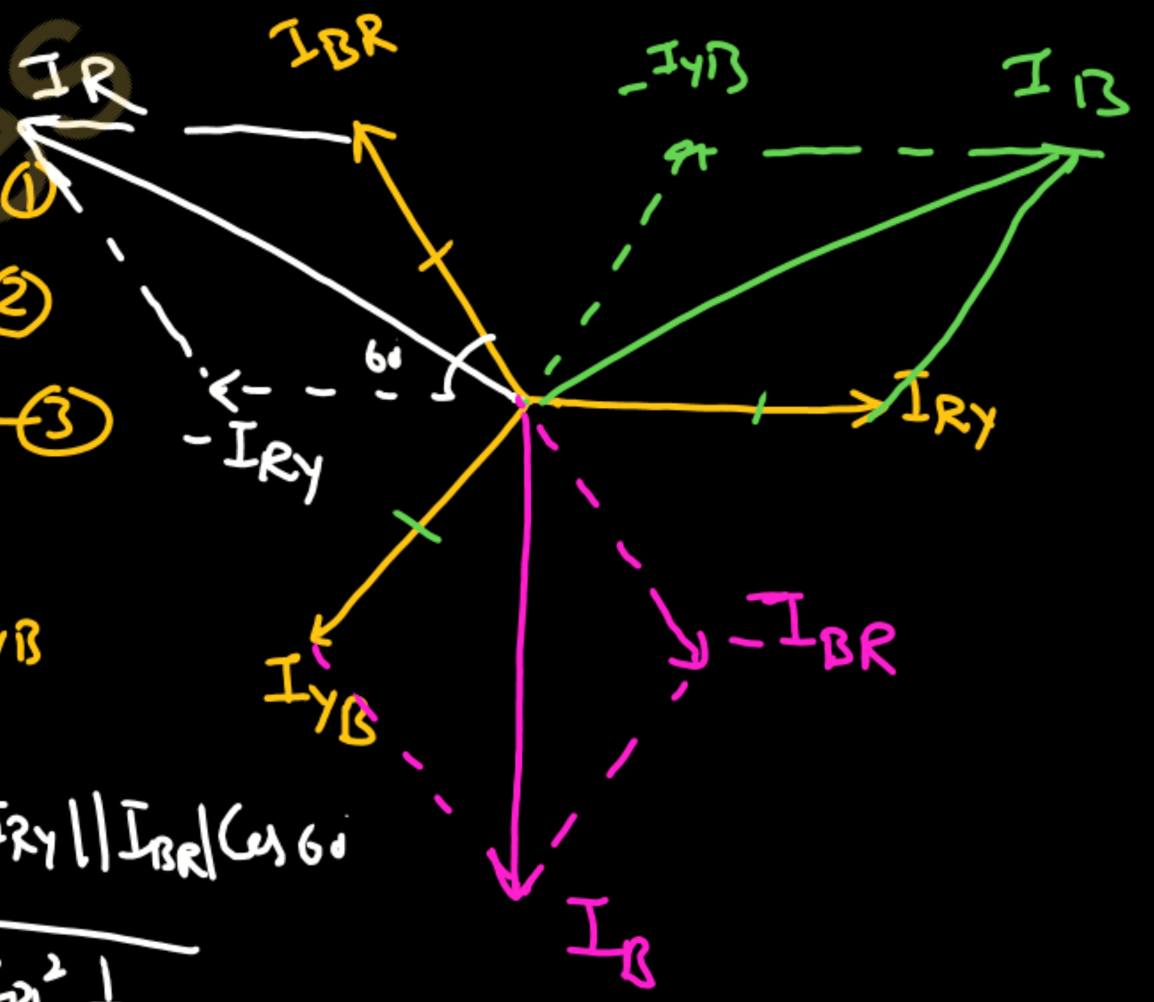
$I_R = I_{BR} - I_{RY}$ — (1)

$I_Y = I_{RY} - I_{YB}$ — (2)

$I_B = I_{YB} - I_{BR}$ — (3)

$I_L = I_R = I_Y = I_B$

$I_{ph} = I_{BR} = I_{RY} = I_{YB}$



$I_R = \sqrt{|I_{BR}|^2 + |I_{RY}|^2 + 2|I_{RY}||I_{BR}|\cos 60}$

$I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + \cancel{2} I_{ph}^2 \frac{1}{\cancel{2}}}$

$\boxed{I_L = \sqrt{3} I_{ph}}$

Power Relations in Star and Delta connection

Numerical Examples

GW GATEWAY CLASSES

Ques8 A balanced star connected load of (8 + j6) ohm per phase is connected to a balanced 3- phase, 400 V supply. Find the line current, phase current, power factor, power and total volt-amperes. **AKTU- 2021-22**

Soln

$Z_{ph} = (8+6j)\,\Omega$

$V_L = 400\,V$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231$ Volt

$$\boxed{I_L = I_{ph} = \frac{V_{ph}}{Z_{ph}} = 23.09\angle{-36.86^\circ}}$$

$pf = \cos 36.86 = 0.8$ lag

$P = 3 V_{ph} I_{ph} \cos\phi$

$= 3 \times 231 \times 23.09 \times 0.8$

$= 12.8$ KW

$S = 3 V_{ph} I_{ph}$

$$\boxed{S = 16\ KW}$$

Numerical Examples

Ques9 A balanced 3-Phase star connected load takes 30KW at a leading current of 48A from a 3-φ source of 500V, 50Hz. Find the circuit parameters per phase. **AKTU- 2021-22**

$P = 30KW$, $I_L = 48A$

$V_L = 500V$

$I_L = I_{ph} = 48A$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{500}{\sqrt{3}} = 288.67V$

$Z_{ph} = \frac{V_{ph}}{I_{ph}} = 6.01\Omega$

$P = \sqrt{3} V_L I_L \cos\phi$

$\cos\phi = \frac{P}{\sqrt{3} V_L I_L} = \frac{30\times10^3}{\sqrt{3}\times500\times48} = 0.72$

$\phi = 43.80$ leading

$Z = |Z|\angle\theta$

$Z = 6.01\angle -43.80$

$Z = 4.33 - 4.159j$

$Z = R - jX_C$

$R = 4.33\Omega$

$X_C = 4.159\Omega$

$(-) \Rightarrow L \quad Z = \frac{V}{I\angle-\theta}$

$= \angle\theta$

Ques.10 A balanced delta connected load of (12 + j 9) Ω/ phase is connected to 3- phase 400 V supply. Calculate line current, power factor and power drawn by it. **AKTU- 2014-15**

$Z_{ph} = (12 + 9j)$

$V_L = V_{ph} = 400\ V$

$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{(12+9j)}$

$I_{ph} = 26.67 \angle -36.86\ A$

$I_L = \sqrt{3} I_{ph} = 80 \angle -36.86\ A$

$P.f = \cos 36.86 = 0.8$ lag

$P = 3 V_{ph} I_{ph} \cos\phi$

$\boxed{P = 25.6\ KW}$

Ques.11 Derive the relation between line and phase voltages in a 3-φ, star-connected circuit.

A balanced star-connected load of (3+j4) Ω/phase is connected to a 3-φ, 400 V supply. Calculate the line current, power factor, active and reactive power drawn from the supply.

Similar to Ques 8

AKTU- 2023-24

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Link in Description

Thank You

Q ① $3-\phi \rightarrow$ Relation $\Delta - Y$

② L, & C $\rightarrow$ Power Relation
Resonance – series

③ half wave – R.ms & Average volt
Sine wave