Performance comparison of different digital Modulation schemes:-

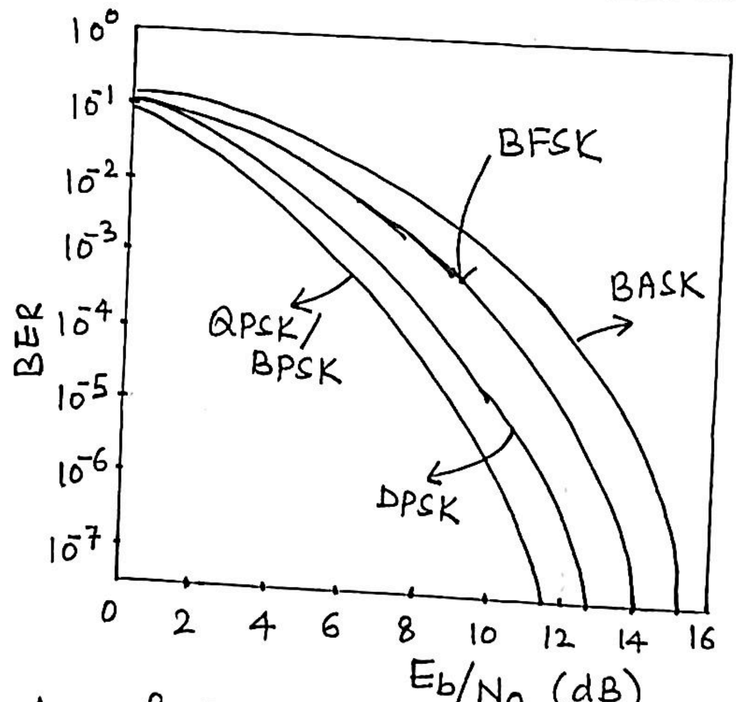
Modulation scheme	Euclidean Distance (d_{12})	Probability of error, P_e: $Q(x)$	Probability of error, P_e: $\text{erfc}(x)$	Transmission BW, B_T: Min^m	Transmission BW, B_T: Max^m
BASK	$\sqrt{E_b}$	$Q\left(\sqrt{\frac{E_b}{2N_0}}\right)$	$\frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{4N_0}}\right)$	R_b	$2R_b$
BFSK	$\sqrt{2E_b}$	$Q\left(\sqrt{\frac{E_b}{N_0}}\right)$	$\frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$	$3R_b$	$4R_b$ $(2R_b+f_1-f_2)$
BPSK	$2\sqrt{E_b}$	$Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$	$\frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$	R_b	$2R_b$
QPSK		$Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$	$\frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$	R_b	R_b
DPSK		$\frac{1}{2}\exp\left[-\frac{E_b}{N_0}\right]$		R_b	$2R_b$

$$Q(x) = \frac{1}{2}\text{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-z^2}\,dz$$

In, general

$$\boxed{P_e = Q\left(\sqrt{\frac{d_{12}^2}{2N_0}}\right)}$$



Note: (1) For the same value of E_b/N_0 in all modulation schemes, BER is least in BPSK and QPSK.

$$\underbrace{\text{erfc}(\uparrow)}_{\text{Argument}\uparrow} \longrightarrow \text{value}\downarrow \quad \Rightarrow \text{BER}\downarrow$$

(2) Euclidean distance $\uparrow\ \Rightarrow$ BER $\downarrow$

$\Rightarrow$ Among BASK, BFSK, and BPSK $\Rightarrow$ BPSK is most reliable.

- Channel bandwidth and transmitted power are two primary communication resources and have to be used as efficiently as possible.

⇒ Power utilization efficiency (energy efficiency): Measured by the required E_b/N_0 to achieve a certain P_e.

⇒ Spectrum utilization efficiency (bandwidth efficiency): Measured by the achievable data rate per unit bandwidth

$$\boxed{\rho = \frac{R_b}{B_T}}$$

- It is always desired to maximize bandwidth efficiency at to minimal required E_b/N_0.

To see the ultimate power-bandwidth tradeoff, we use Shannon's channel capacity theorem (1948).

- Channel capacity is the theoretical upper bound for the maximum rate at which information could be transmitted without error.

- For a bandlimited AWGN channel, the max^m rate achievable is given by -

$$R_b \leq C = B_T \log_2(1 + SNR) = B_T \log_2\left(1 + \frac{P_S}{N_0 B_T}\right)$$

$$\frac{E_b}{N_0} = \frac{P_S T_b}{N_0} = \frac{P_S}{R_b N_0} = \frac{P_S B_T}{R_b N_0 B_T} = SNR\, \frac{B_T}{R_b}$$

$$\boxed{\frac{E_b}{N_0} = \frac{B_T}{R_b}\left(2^{R_b/B_T} - 1\right)}$$

As $\frac{R_b}{B_T} \to 0$

$$\frac{E_b}{N_0} = \ln 2 = 0.693 = -1.59\,dB$$

⇒ This value is called the Shannon Limit. Received E_b/N_0 must be > -1.6 dB to ensure reliable communications.

⑥ Binary data are transmitted at a rate of 10^6 bits per second over a radio link. Assuming channel noise is AWGN with zero mean and PSD at the receiver input is 10^{-10} W/Hz, find the average carrier power required to maintain an average probability of error $P_e \leq 10^{-4}$ for coherent BFSK. Determine the minimum channel bandwidth required.

<u>Ans:</u> $R_b = 10^6$ bps, $\frac{N_0}{2} = 10^{-10} \Rightarrow N_0 = 2 \times 10^{-10}$ W/Hz

$$P_e \leq 10^{-4}$$

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$10^{-4} = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2 \times 2 \times 10^{-10}}}\right)$$

$$E_b = 3.025 \times 10^{-9} \text{ Joules}$$

$$P_s = \frac{E_b}{T_b} = E_b R_b = 3.025 \text{ mW}$$

$$P_s \geq 3.025 \text{ mW}$$

carrier power

$$(B_T)_{min} = 3R_b = 3 \times 10^6 = 3 \text{ MHz.}$$

(7) An FSK system transmits binary data at a rate of 10^6 bps. Assuming channel noise AWGN with zero mean and PSD 2×10^{-20} watts/Hz, determine the average probability of error. Assume coherent detection, and amplitude of received sinusoidal signal for both symbol 1 and 0 to be $1.2\mu V$.

Ans:. $R_b = 10^6$ bps , $\frac{N_0}{2} = 2\times10^{-20} \Rightarrow N_0 = 4\times10^{-20}$

$A_c = 1.2\ \mu V$

$$E_b = P_s T_b = \frac{A_c^2}{2}\times\frac{1}{R_b} = 7.2\times10^{-19} \text{ Joule}$$

$$P_e = \frac{1}{2}\,erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$= \frac{1}{2}\,erfc\left(\sqrt{\frac{7.2\times10^{-19}}{2\times4\times10^{-20}}}\right)$$

$$P_e = \frac{1}{2}\,erfc(3)$$

$$\boxed{P_e = 1\times10^{-5}}$$

$$\boxed{erf(x) + erfc(x) = 1}$$

$erf(x)$: error function; $erfc(x)$: complementary error funⁿ.

(8) A binary data is transmitted over an AWGN channel using BPSK at a rate of 1 Mbps. It is desired to have average probability of error $P_e \le 10^{-4}$. Noise power spectral density is 10^{-12} W/Hz. Find the average carrier power to maintain this P_e. Also, $erfc(3.5) = 0.0002$ given.

Ans:. $P_e \le 10^{-4}$, $\frac{N_0}{2} = 10^{-12} \Rightarrow \boxed{2\times10^{-12} = N_0}$

$R_b = 1$ Mbps

$$P_s = E_b/T_b$$

$$E_b = P_s T_b = \frac{P_s}{R_b} = P_s\times10^{-6}$$

$$\frac{E_b}{N_0} = \frac{P_s\times10^{-6}}{2\times10^{-12}}$$

$$P_e = \frac{1}{2}\,erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

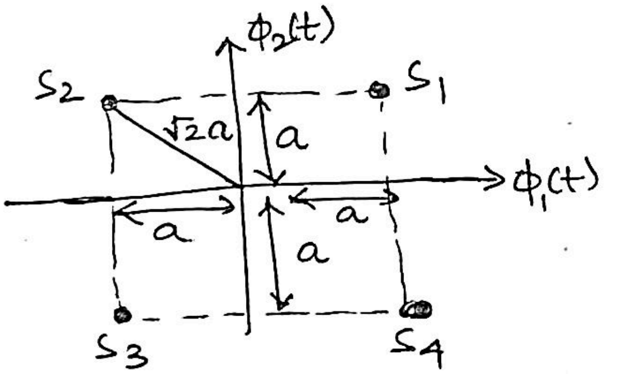
$$10^{-4} = \frac{1}{2}\,erfc\left(\sqrt{\frac{P_s\times10^{-6}}{2\times10^{-12}}}\right)$$

$$erfc^{-1}(0.0002) = \sqrt{5\times10^{5}P_s} \Rightarrow \boxed{P_s = 24.5\ \mu \text{Watts}}$$

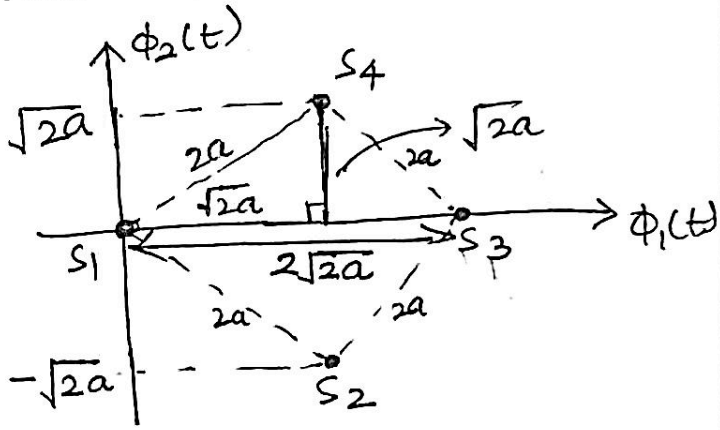
⑨ In a binary communication channel, the symbol 0 is transmitted with $p(0) = 0.6$, and the symbol 1 is transmitted with $P(1) = 0.4$. Determine, the error probability of the channel if conditional probability of detecting symbol 0 is 10^{-4} and that of symbol 1 is 10^{-6}.

Ans:-
$$P_e = P(e/0)\,P(0) + P(e/1)\,P(1)$$
$$= 10^{-4} \times 0.6 + 10^{-6} \times 0.4$$
$$= 0.604 \times 10^{-4}$$

⑩ Two signal constellations are shown below. Compare the two modulation schemes in terms of their BER as well as energy efficiency performances.



Constellation A



Constellation B

Solution:- The min^m distance between signal points which are adjacent in the constellation decides the error probability for both the signal constellations. So Here, $d_{min} = 2a$ for both the signal constellations. So both modulation schemes will perform identically in terms of BER.

Const. A $\Rightarrow d_{min} = 2a$

AVNB = 2

Const B. $d_{min} = 2a$

AVNB = 2

$$\boxed{BER_A = BER_B}$$

For energy efficiency.

Const. A. $\Rightarrow$ the average energy

$$E_{avg} = \frac{E_{s1} + E_{s2} + E_{s3} + E_{s4}}{4} = \frac{1}{4}\left[(\sqrt{2}a)^2 \times 4\right]$$

$$= 2a^2$$

$$E_{s1} = E_{s2} = E_{s3} = E_{s4} = (\sqrt{2}a)^2$$

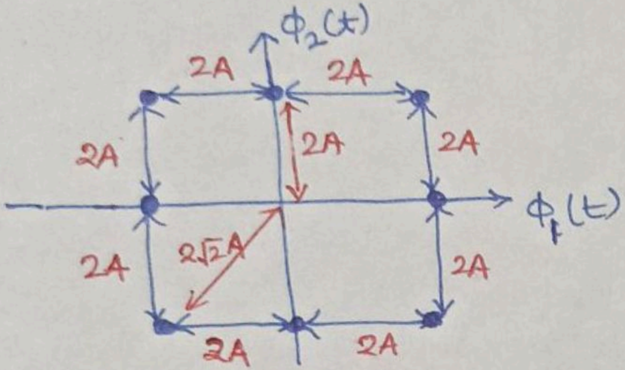
Note:. The signal vector amplitude on the signal space is a measure of $\sqrt{E_b}$, where E_b is its energy.

Const. B.

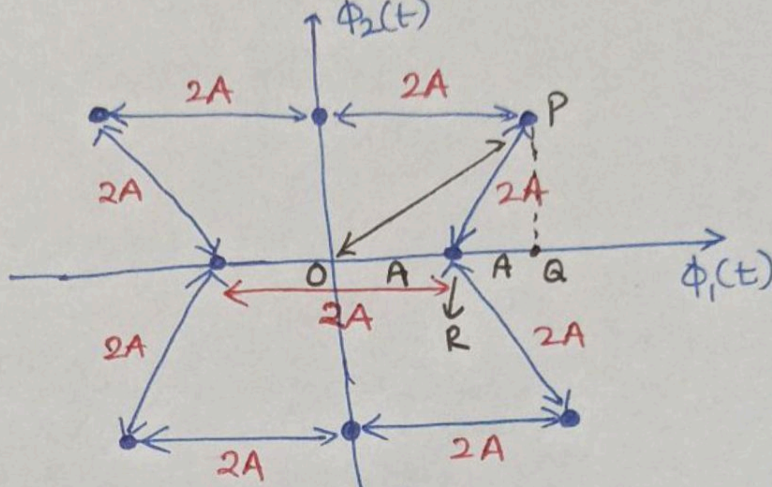
$$E_{avg} = \frac{0 + (2a)^2 + (2\sqrt{2}a)^2 + (2a)^2}{4} = 4a^2$$

Conclusion :- Constellation A is more energy efficient. It can give the same BER performance as constellation B, by transmitting half the energy.

(12) Two sets of signal constellation for some modulation schemes are shown in the figure below. Minimum distance between adjacent points is 2A. Assume that the signal points are equally probable, which constellate is more power efficient? Explain the nature of modulation shown by these two constellation.



A



B

Ans:-

Constellation A

$$E_{avg} = \frac{1}{8}\left[4(2A)^2 + 4(2\sqrt{2}A)^2\right]$$

$$= 6A^2$$

$d_{min} = 2A$

ANNB = 2

$$\boxed{BER_A = BER_B}$$

$E_b = P_s \times T_b$

$P_s = \frac{E_b}{T_b} \Rightarrow P_{avg} = \frac{E_{avg}}{T_b}$

$$\boxed{P_{avg} = \frac{6A^2}{T_b}}$$

Constellation B.

$\triangle PQR$

$$PQ = \sqrt{(2A)^2 - (A)^2}$$

$$= \sqrt{4A^2 - A^2} = \sqrt{3}A$$

$\triangle OPQ$

$$OQ^2 + PQ^2 = OP^2$$

$$OP = \sqrt{(\sqrt{3}A)^2 + (2A)^2}$$

$$= \sqrt{7}A$$

$$E_{avg} = \frac{1}{8}\left[4(\sqrt{7}A)^2 + 2(A)^2 + 2(\sqrt{3}A)^2\right]$$

$$= 4.5A^2$$

AVNB = 2

$d_{min} = 2A$

$$\boxed{P_{avg} = \frac{4.5A^2}{T_b}}$$

$\Rightarrow$ Thus the second constellation is more power efficient.