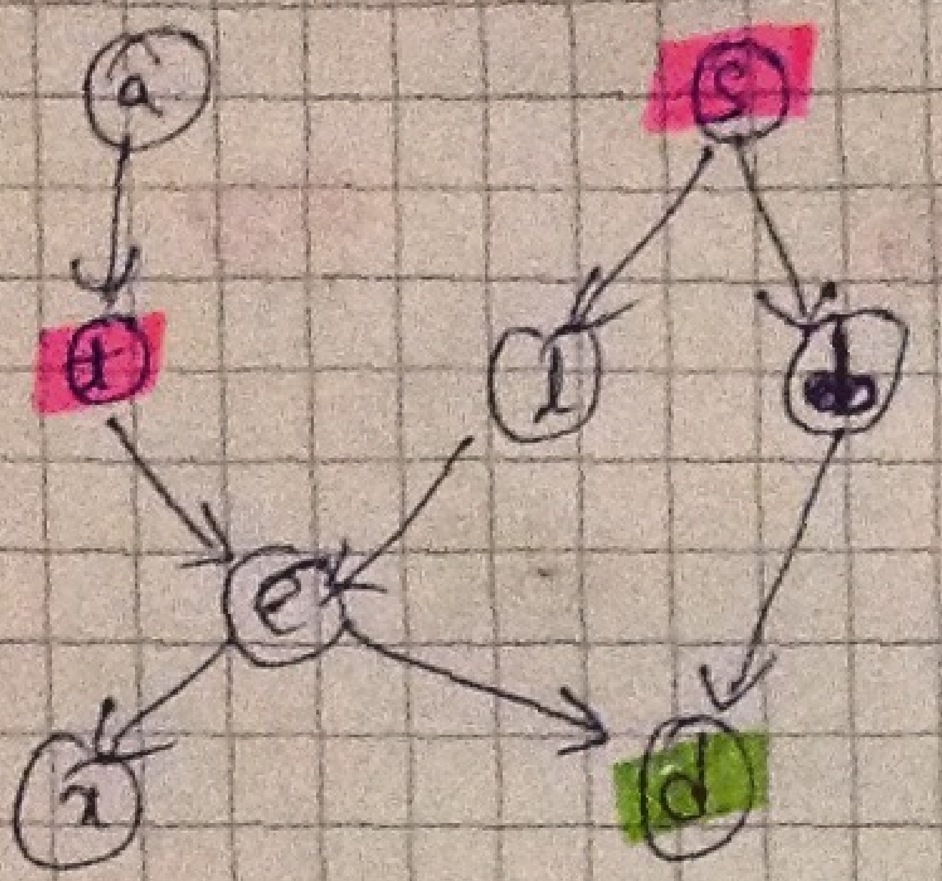
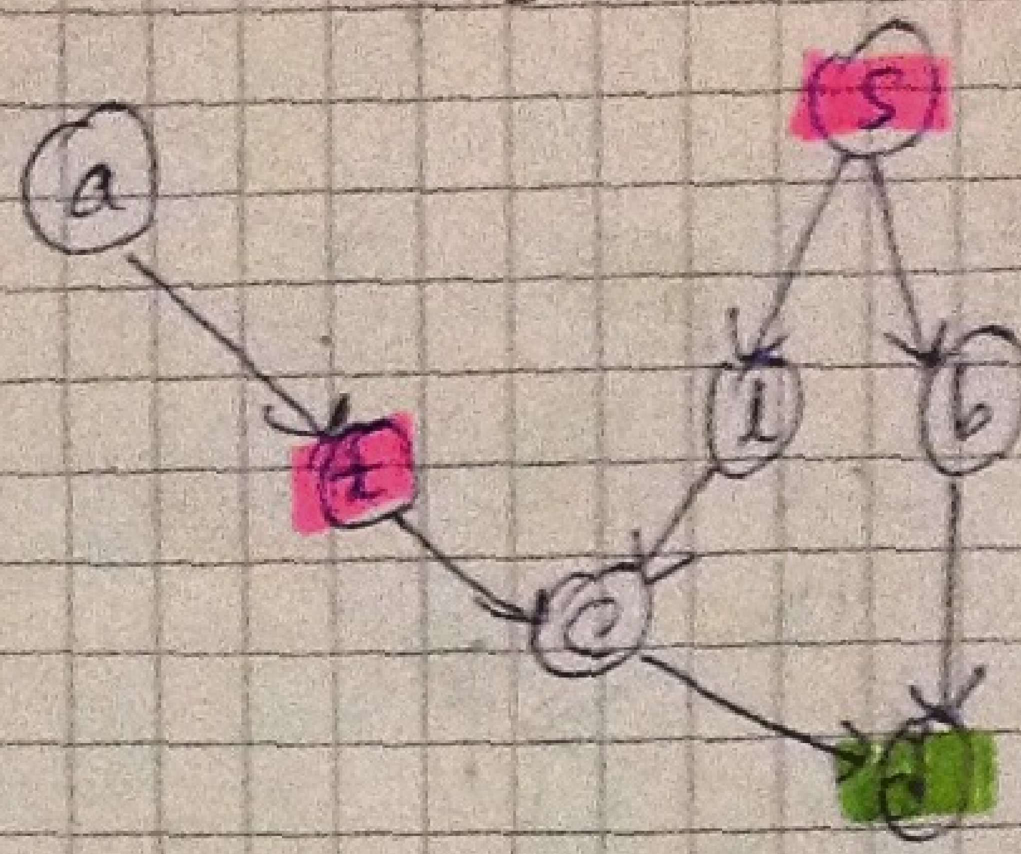
Problem 1 Conditional Independence

a) tuberculosis ⫫ smoking | shortness of breath $(t \perp\!\!\!\perp s \mid d)$

Ancestral graph

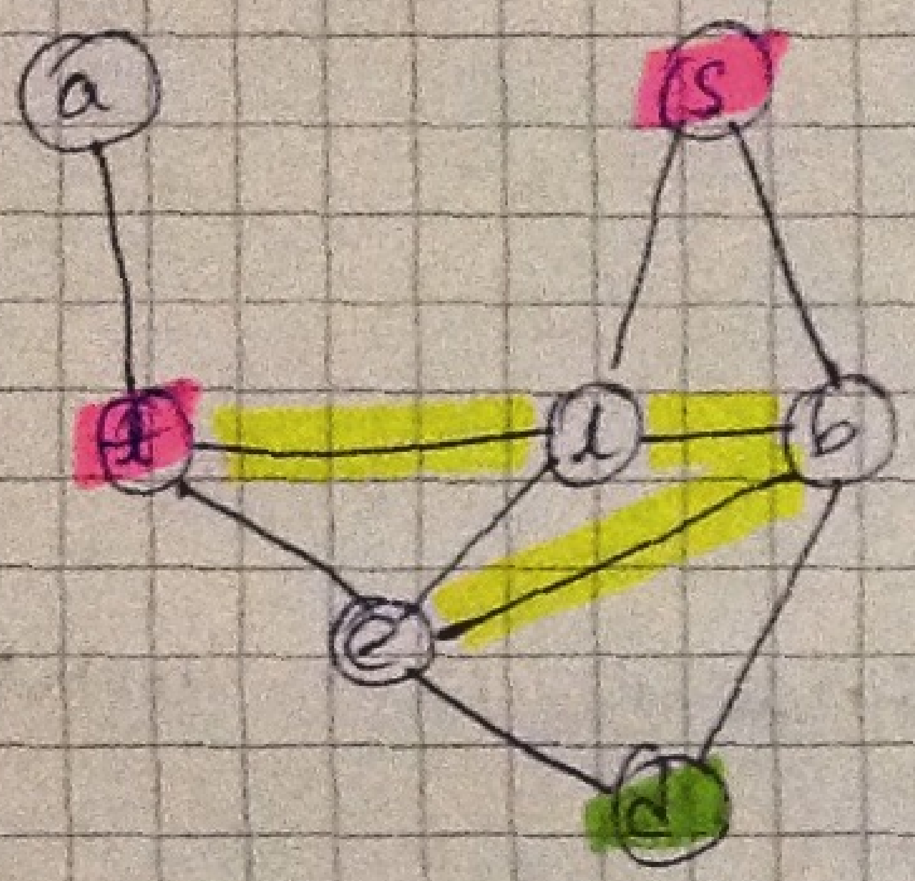


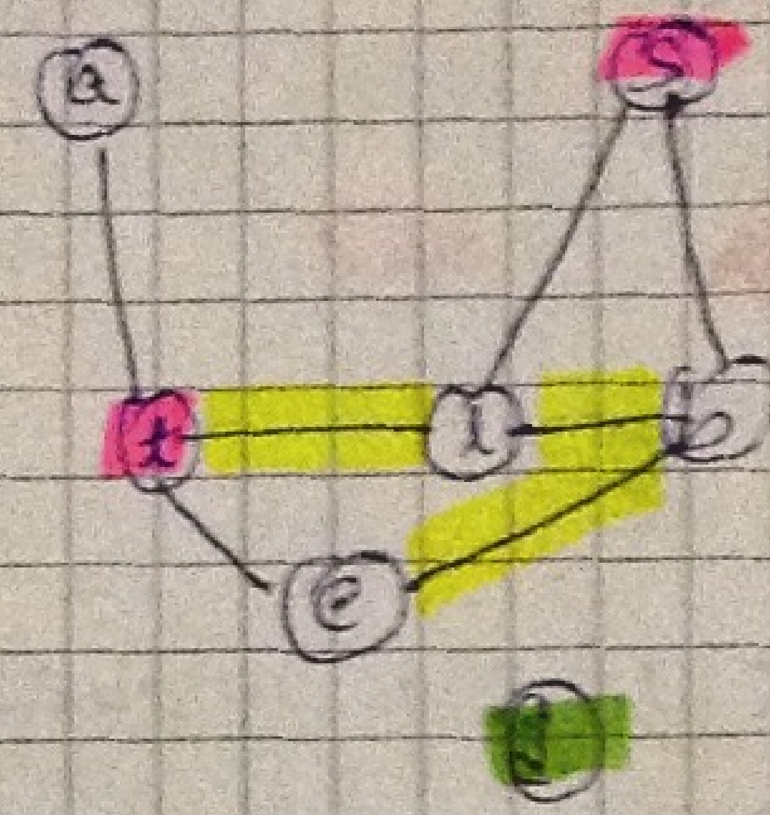


Moralization

Seperation

Text

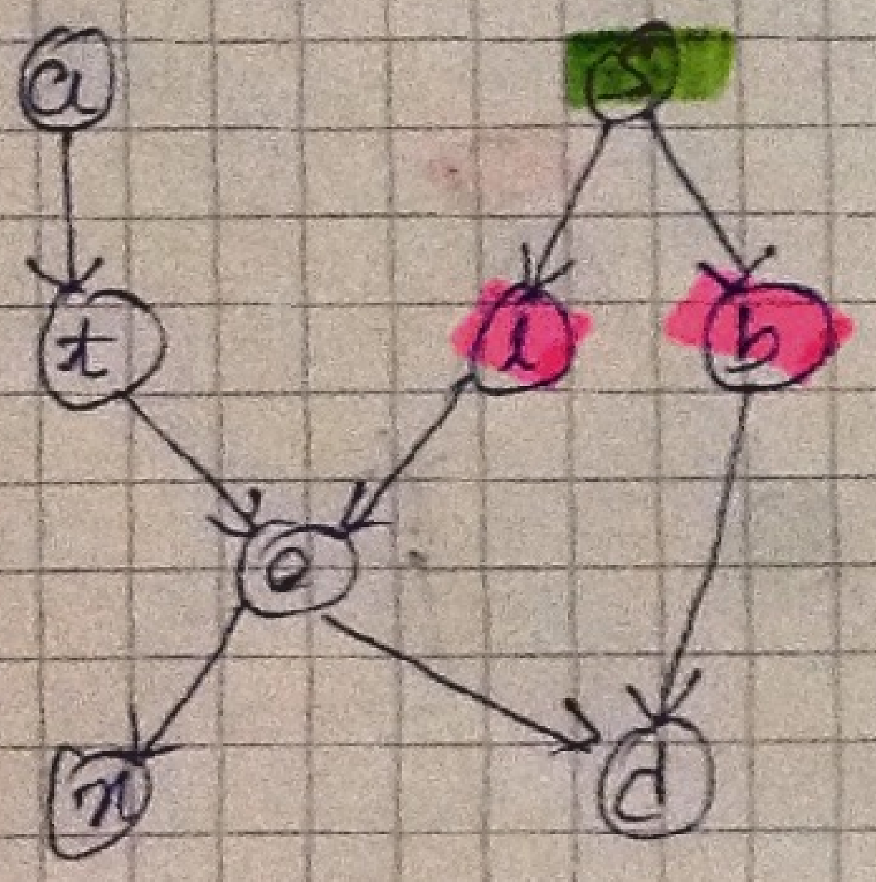


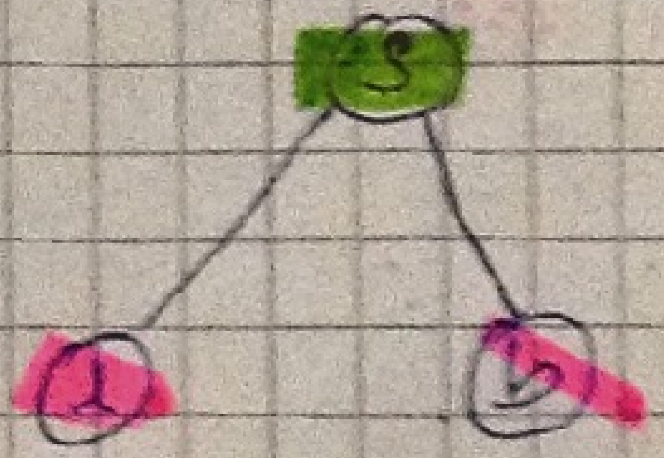


$(t \perp\!\!\!\perp s \mid d)$ $\boxed{\text{NOT implied}}$ because a path from t to s exists

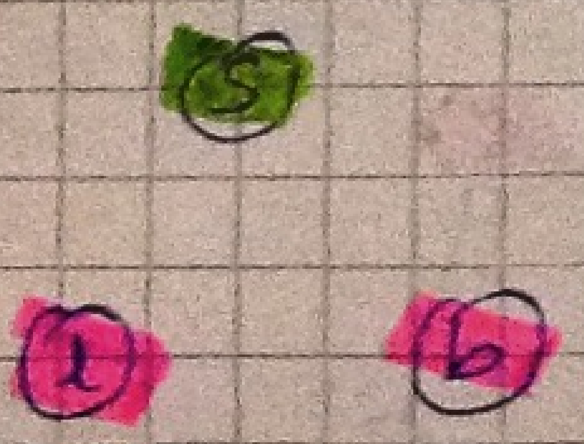
b) lungcancer ⫫ bronchitis | smoking $l \perp\!\!\!\perp b \mid s$

Ancestral graph & moralization





seperation



$l \perp\!\!\!\perp b \mid s$ is implied in graph because no path links l & b

Group 8

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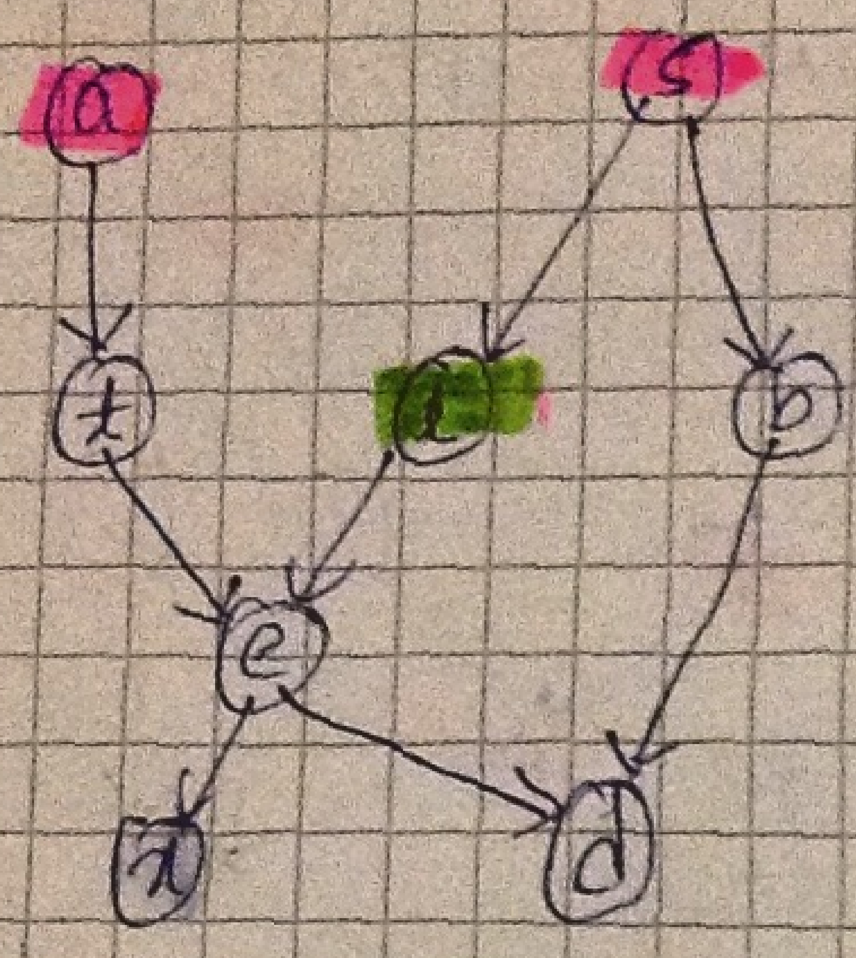
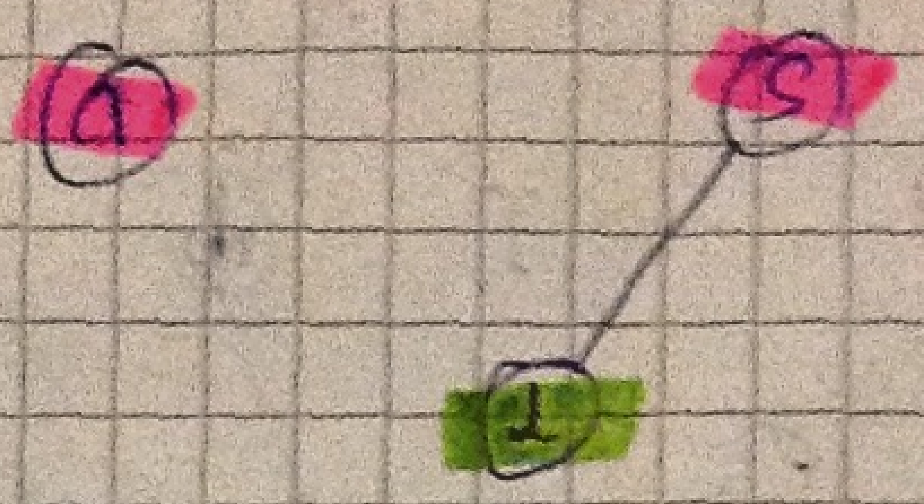
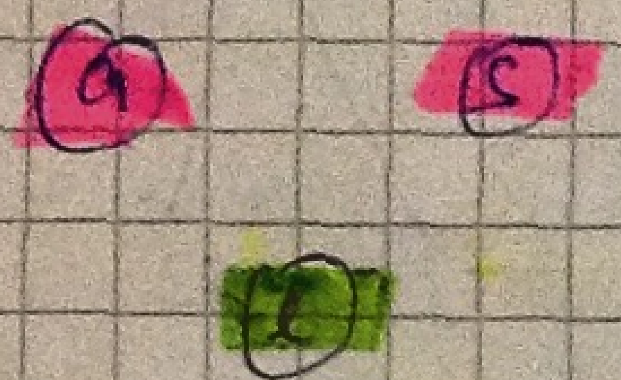
(c) Visit to asia $\perp\!\!\!\perp$ smoking | Lung cancer $a \perp\!\!\!\perp s \mid l$



Ancestral graph & Moralization



Seperation

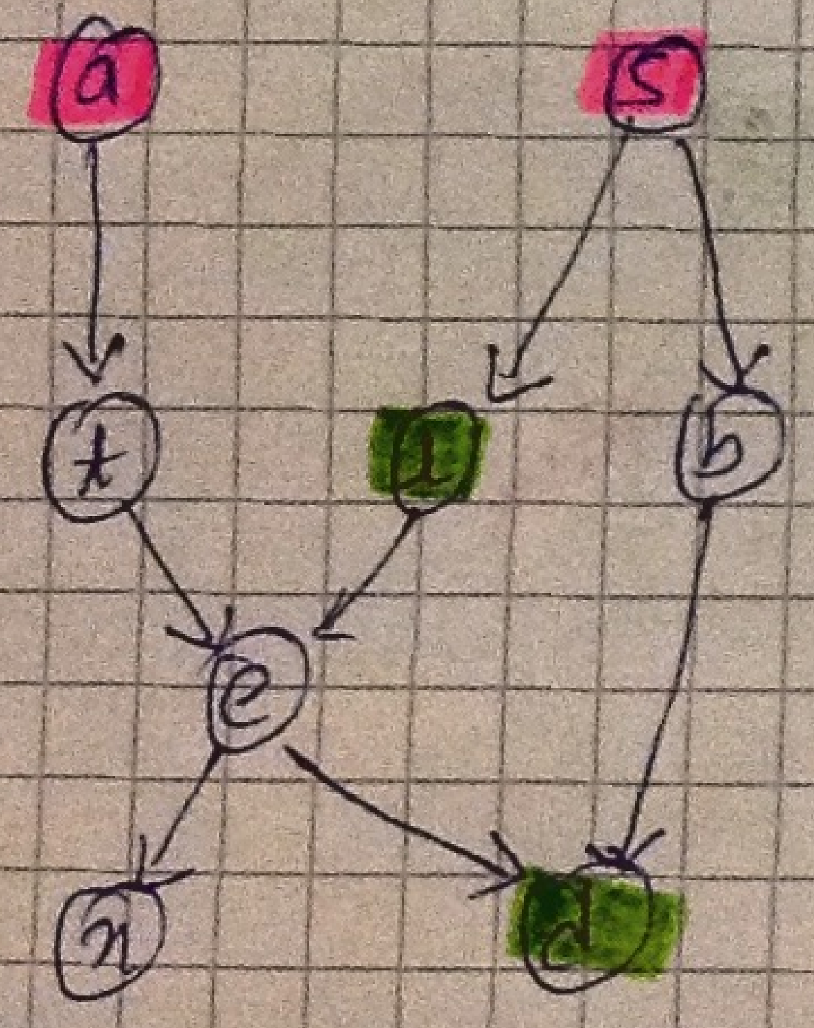


$(a \perp\!\!\!\perp s \mid l)$ is implied in graph since no path links

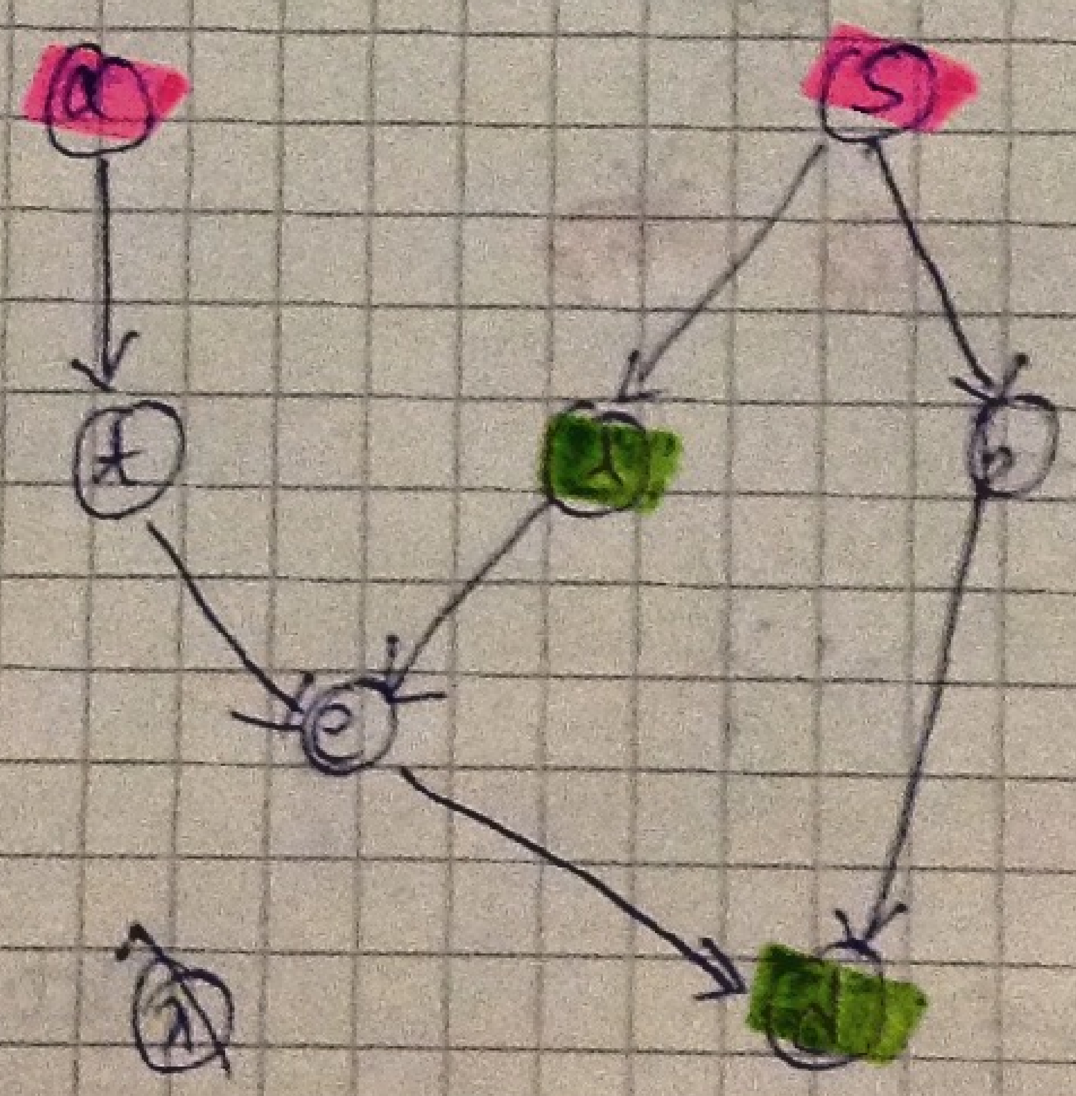
a to s

(d) Visited asia $\perp\!\!\!\perp$ smoking | lungcancer, shortness of breath

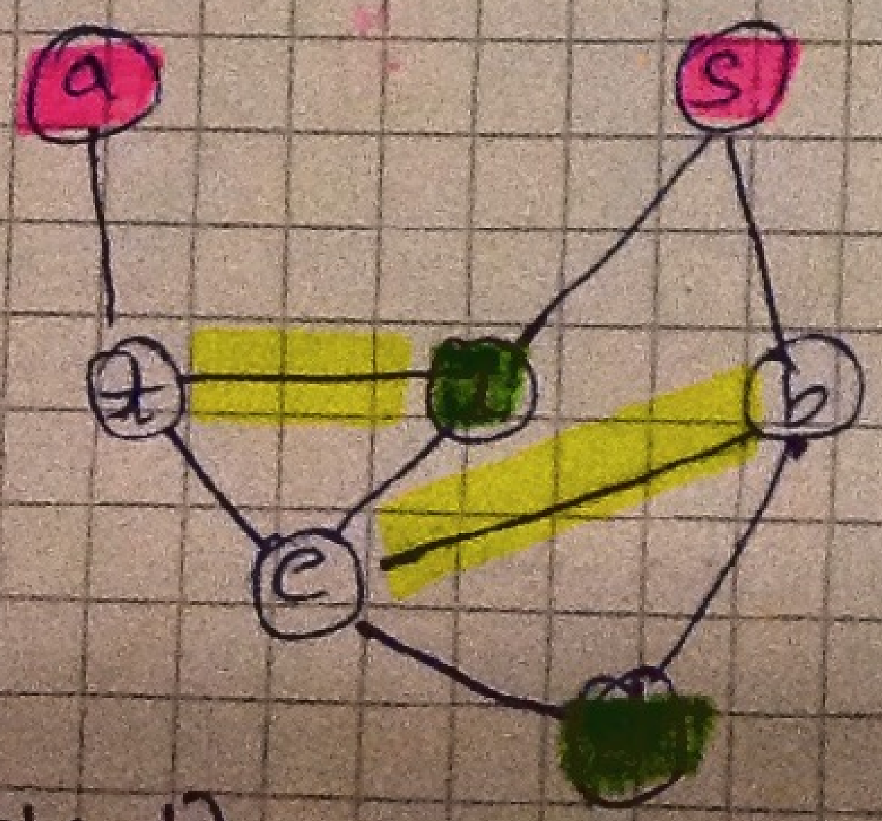
$a \perp\!\!\!\perp s \mid l, d$



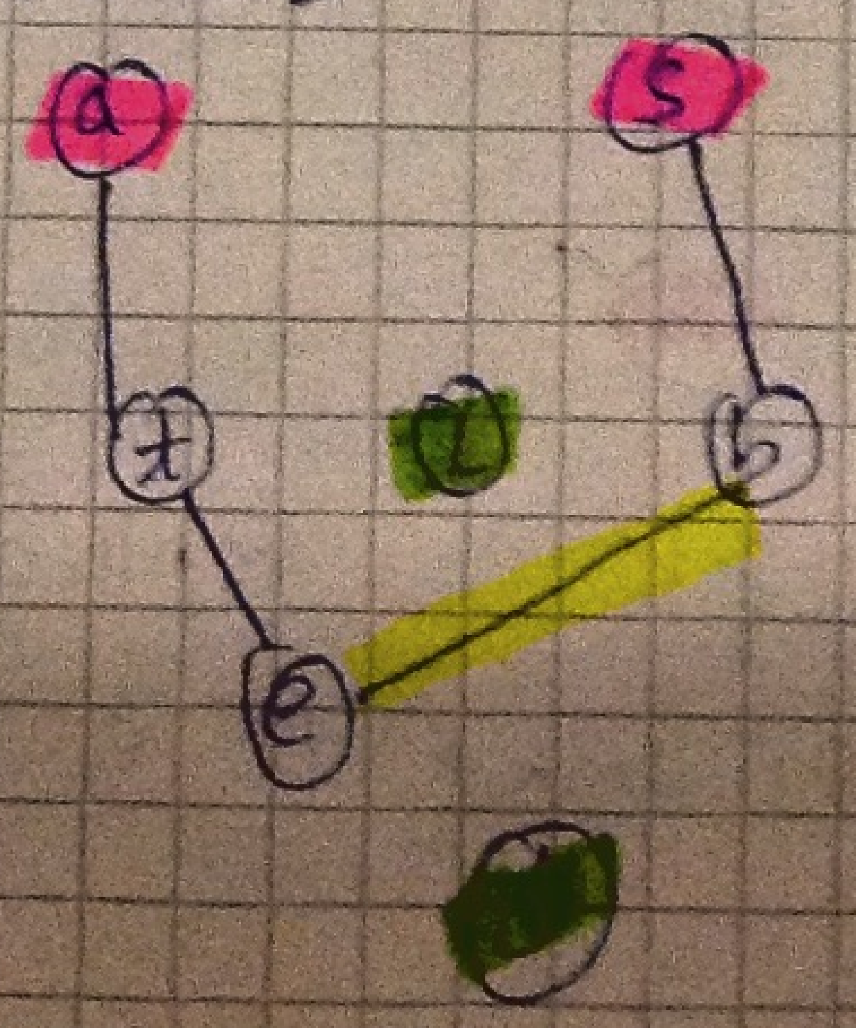
Ancestral graph



Moralization



Seperation



$(a \perp\!\!\!\perp s \mid l, d)$ <u>NOT</u> implied since path link exists from a $\rightarrow$ s

Problem 1 - section 2

$p(x,d,e,t,l,b,a,x)$

$p(a)\,p(t|a)\,p(e|t,l)\,p(x|e)\,p(d|e,b)\,p(l|s)\,p(s)\,p(b|s)$

$$p(d)=\sum_{\substack{\text{all}\\\text{except}\\d}} p(x,d,e,t,l,b,a,s)$$

$$=\underbrace{\sum_a p(a)\,p(t|a)}_{\gamma(a)}\ \underbrace{\sum_s p(b|s)\,p(l|s)\,p(s)}_{\gamma(s)}\ \underbrace{\sum_{b,e} p(d|e,b)}_{\gamma(b,e)}\ \underbrace{\sum_e p(x|e)}_{\gamma(e)}\ \underbrace{\sum_{l,t} p(e|t,l)}_{\gamma(l,t)}$$

$\gamma(a)_{t=1} = 0.1\times0.05+\ 0.99\times0.01 = 0.014$

$\gamma(s)_{l=1,b=1} = 0.5\times0.1\times0.6+0.5\times0.01\times0.3 = 0.0315$

$\gamma(b,e) = 0.1+0.8+0.7+0.9 = 2.5$

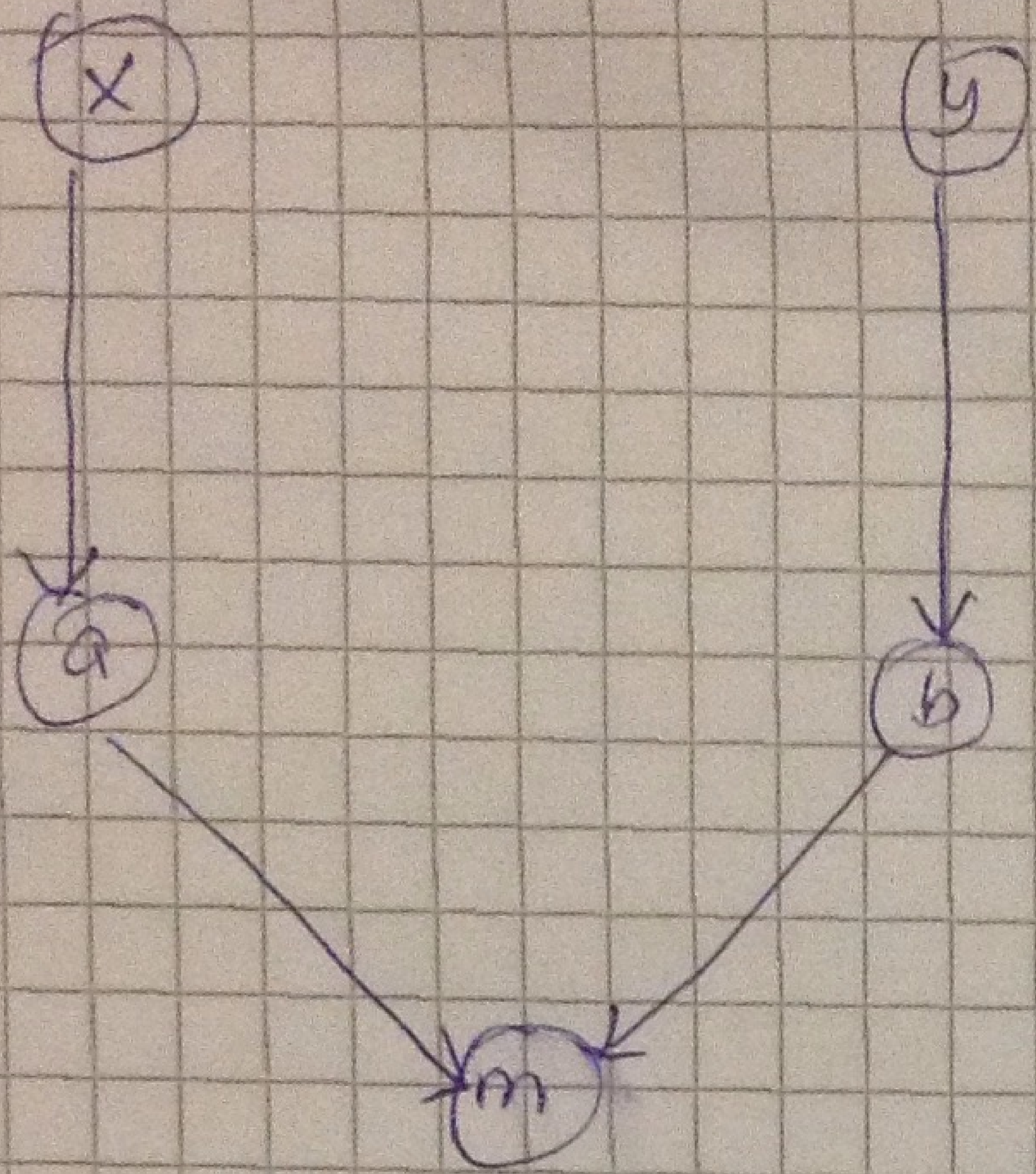
$\gamma(t,l) = 1+1+1 = 3$

$\gamma(e) = 0.98+0.05 = 1.03$

~~γ(a) = 0.1×0.5 + 0.99×0.01 + 0.5×0.05 +~~

Problem 2 Belief Network

1)



$(x \perp\!\!\!\perp y)$

$(a \not\perp\!\!\!\perp x)$ $(b \not\perp\!\!\!\perp y)$ $(a \perp\!\!\!\perp b)$

$(m \not\perp\!\!\!\perp \{a,b\})$ $(m \perp\!\!\!\perp x \mid a,b)$

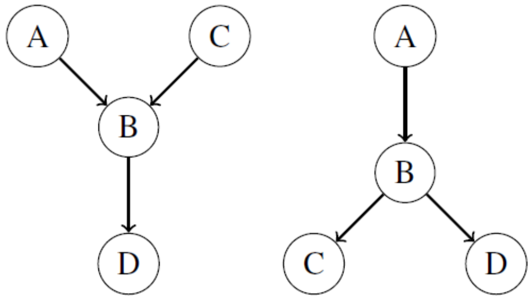
$(m \perp\!\!\!\perp y \mid a,b)$

2). days of week x, y

conditioned on m (not married)

x & y are graphically dependent on m

Problem 3.



A B

Figure 1. Belief networks A and B.

For the Markov equivalent we have to check the same skeleton and the same set of immoralities.

We can say, that we cannot find another equivalent DAG for the A graph (Figure 1), because we have an immorality (nodes A, B and C).

We can find another equivalent network for the belief network B (Figure 1), because it has no immoralities. And equivalent network is a very similar network, but arrow between A and B has direction from B to A (No immoralities and the same skeleton)

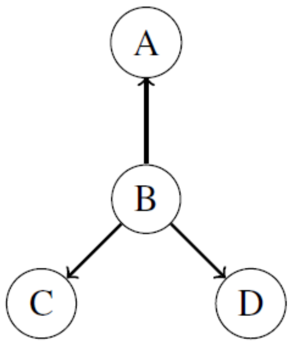
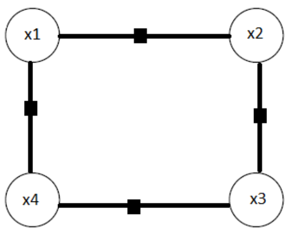


Figure 2. Markov equivalent belief networks for network B from figure 1.

<u>Problem 4</u>



$$p(x_1 \mid x_2, x_4) = \frac{\varphi(x_1, x_2)\varphi(x_2, x_3)\varphi(x_3, x_4)\varphi(x_4, x_1)}{\sum_1 \varphi(x_1, x_2)\varphi(x_2, x_3)\varphi(x_3, x_4)\varphi(x_4, x_1)} =$$

$$\frac{\varphi(x_1, x_2)\varphi(x_4, x_1)}{\sum_1 \varphi(x_1, x_2)\varphi(x_4, x_1)};$$

$$p(x_2 \mid x_1, x_3) = \frac{\varphi(x_1, x_2)\varphi(x_2, x_3)\varphi(x_3, x_4)\varphi(x_4, x_1)}{\sum_2 \varphi(x_1, x_2)\varphi(x_2, x_3)\varphi(x_3, x_4)\varphi(x_4, x_1)} =$$

$$\frac{\varphi(x_1, x_2)\varphi(x_2, x_3)}{\sum_2 \varphi(x_1, x_2)\varphi(x_2, x_3)};$$

$$p(x_3 \mid x_2, x_4) = \frac{\varphi(x_1, x_2)\varphi(x_2, x_3)\varphi(x_3, x_4)\varphi(x_4, x_1)}{\sum_3 \varphi(x_1, x_2)\varphi(x_2, x_3)\varphi(x_3, x_4)\varphi(x_4, x_1)} =$$

$$\frac{\varphi(x_2, x_3)\varphi(x_3, x_4)}{\sum_3 \varphi(x_2, x_3)\varphi(x_3, x_4)};$$

$$p(x_4 \mid x_1, x_3) = \frac{\varphi(x_1, x_2)\varphi(x_2, x_3)\varphi(x_3, x_4)\varphi(x_4, x_1)}{\sum_4 \varphi(x_1, x_2)\varphi(x_2, x_3)\varphi(x_3, x_4)\varphi(x_4, x_1)} =$$

$$\frac{\varphi(x_3, x_4)\varphi(x_4, x_1)}{\sum_4 \varphi(x_3, x_4)\varphi(x_4, x_1)};$$