Machine Learning II – Assignment 2

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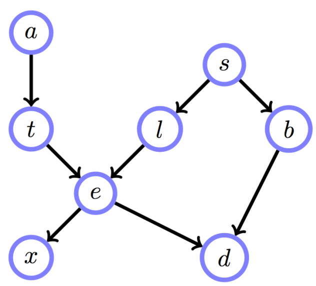
This homework is due November 24, 2014 at 20:00.
Please read the instructions carefully!

1 Conditional Independence (8 points)



The chest clinic network above concerns the diagnosis of lung disease (tuberculosis, lung cancer, or both, or neither). In this model a visit to Asia is assumed to increase the probability of lung cancer. We have the following binary variables.

x = Positive X-ray
d = Dyspnea (Shortness of breath)
e = Either Tuberculosis or Lung Cancer
t = Tuberculosis
l = Lung cancer
b = Bronchitis
a = Visited Asia
s = Smoker

1. Are the following independence statements implied by the graph? (And how do you conclude this?)
   (a) tuberculosis $\perp\!\!\!\perp$ smoking | shortness of breath
   (b) lung cancer $\perp\!\!\!\perp$ bronchitis | smoking

(c) visit to Asia $\perp\!\!\!\perp$ smoking | lung cancer

(d) visit to Asia $\perp\!\!\!\perp$ smoking | lung cancer, shortness of breath

2. Calculate by hand the values for $p(d)$ and $p(d|s = 1)$.
   The CPT is:

$$
\begin{array}{lcllcl}
p(a = 1) & = & 0.01, & p(s = 1) & = & 0.5 \\
p(t = 1 \mid a = 1) & = & 0.05, & p(t = 1 \mid a = 0) & = & 0.01 \\
p(l = 1 \mid s = 1) & = & 0.1, & p(l = 1 \mid s = 0) & = & 0.01 \\
p(b = 1 \mid s = 1) & = & 0.6, & p(b = 1 \mid s = 0) & = & 0.3 \\
p(x = 1 \mid e = 1) & = & 0.98, & p(x = 1 \mid e = 0) & = & 0.05 \\
p(d = 1 \mid e = 1, b = 1) & = & 0.9, & p(d = 1 \mid e = 1, b = 0) & = & 0.7 \\
p(d = 1 \mid e = 0, b = 1) & = & 0.8, & p(d = 1 \mid e = 0, b = 0) & = & 0.1
\end{array}
$$

   and

$$
p(e = 1 \mid t, l) = \begin{cases} 0 & t = 0 \wedge l = 0 \\ 1 & \text{otherwise} \end{cases}
$$

2 Belief Network (4 points)

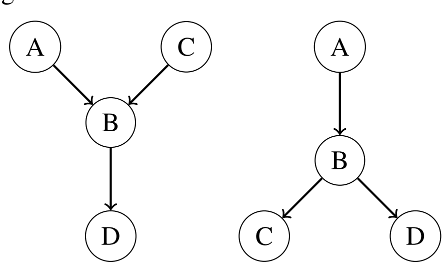
Assume that the day of the week that females are born on, x, is independent of the day of the week, y, on which males are born. However, assume that personality is dependent on the day of the week you're born on. If a represents the female personality type and b the male personality type, then $a \top\!\!\!\top x$ and $b \top\!\!\!\top y$, but $a \perp\!\!\!\perp b$. Whether or not a male and a female are married, m, depends strongly on their personality types, $m \top\!\!\!\top \{a, b\}$, but is independent of x and y if we know a and b.

1. Draw a belief network that can represent this setting.

2. What can we say about the (graphical) dependency between the days of the week that John and Jane are born on, given that they are not married?

3 Markov Equivalence (4 points)

Consider the following two belief networks:



For each of them say whether there is another Markov equivalent belief network. If yes, draw a Markov equivalent belief network. If no, give reason why there can't be such a network.

4 Markov Networks (4 points)

Consider the pairwise Markov network

$$p(x) = \phi(x_1, x_2)\phi(x_2, x_3)\phi(x_3, x_4)\phi(x_4, x_1). \tag{1}$$

Express in terms of ϕ the following

$$p(x_1 \mid x_2, x_4), \quad p(x_2 \mid x_1, x_3), \quad p(x_3 \mid x_2, x_4), \quad p(x_4 \mid x_1, x_3). \tag{2}$$