Advanced Topics in statistical signal Processing

TECHNISCHE UNIVERSITÄT DARMSTADT

Mobile Adaptive Networks

Signal Processing Group

Signal Processing Group
Technische Universität Darmstadt
Supervisor: Sahar Khawatmi

Overview

Motivation

TECHNISCHE UNIVERSITÄT DARMSTADT

Many biological systems exhibit sophisticated levels of adaptation and coordination, which result in remarkable and observable forms of collective motion and self-organization.

Motivation

Mobile adaptive networks exhibits these qualities:

- Robust
- Can react in real time to changes in the statistical properties of data
- Can adjust the network topology

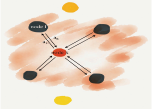


Figure: Distributed Solution

Disadvantages of centralised solution:

- Single point of failure
- Too much information exchange between node and fusion center
- it is not easily scalable

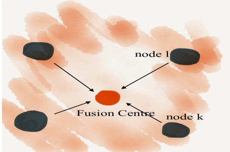


Figure: Centralised Solution

Diffusion Strategies

- ATC-Adapt Then Combine
- CTA-Combine Then Adapt

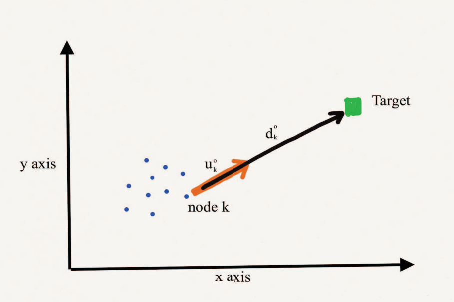
Data Model

- Measurement Model

$$d_k^o(i) = u_{k,i}^o(w^o - x_{k,i})$$
$$u_k^o = \frac{(w^o - x_k)^T}{\|w^o - x_k\|}$$
$$\mathbf{d}_k(i) = \mathbf{u}_{k,i}w^o + \mathbf{n}_k(i)$$

- Cost Function

$$J^{\text{glob}}(w) = \sum_{k=1}^{N} E|\mathbf{d}_k(i) - \mathbf{u}_{k,i}w|^2$$



Data Model

The noisy location of the target is denoted by $q_{k,i}$

$$
\begin{aligned}
q_{k,i} &= x_{k,i} + d_k(i)u_{k,i}^T \\
&= w^o + \eta_{k,i}
\end{aligned}
$$

where the vector noise term is given by:

$$\eta_{k,i} = n_k^d(i)u_{k,i}^T + d_k(i)n_{k,i}^{uT} + n_k^d(i)n_{k,i}^{uT}$$

We assume that $\eta_{k,i}$ is zero mean white random process with covariance matrix $C_{k,i}$ and let $\sigma_k^2(i) = \mathrm{Tr}(C_{k,i})$ denote the trace of $C_{k,i}$.

$$C_{k,i} = \kappa \|w - \mathbf{x}_{k,i}\|^2 I_M$$

ATC diffusion algorithm

TECHNISCHE UNIVERSITÄT DARMSTADT

ATC diffusion algorithm: Adapt-then-Combine diffusion algorithm

1) location $x_{k,i}$, $\{d_k(i), u_{k,i}, v_{k,i}, \sigma_k^2(i)\}$

2) Find $q_{k,i} = x_{k,i} + d_k(i)u_{k,i}^T$

3)

$$\varphi_{k,i} = w_{k,i-1} + \mu_k \sum_{l \in \mathcal{N}_{k,i}} c_{l,k}^w (q_{l,i} - w_{k,i-1})$$

$$\phi_{k,i} = v_{k,i-1}^g + \nu_k \sum_{l \in \mathcal{N}_{k,i}} c_{l,k}^v (v_{l,i} - v_{k,i-1}^g)$$

ATC diffusion algorithm

4)

$$w_{k,i} = \sum_{l \in \mathcal{N}_{k,i}} a^{w}_{l,k} \varphi_{l,i}$$

$$v^{g}_{k,i} = \sum_{l \in \mathcal{N}_{k,i}} a^{v}_{l,k} \phi_{l,i}$$

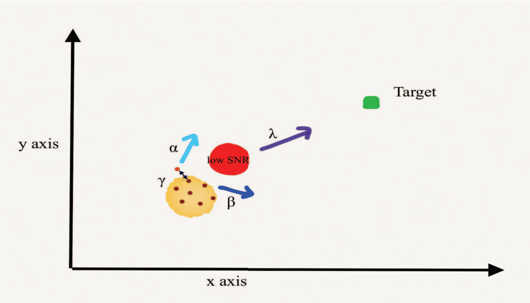
5)

$$v_{k,i+1} = \lambda \cdot h(w_{k,i} - x_{k,i}) + \alpha \frac{g_{k,i}}{\|g_{k,i}\|} + \beta v^{g}_{k,i} + \gamma \delta_{k,i}$$

$$x_{k,i+1} = x_{k,i} + \Delta t \cdot v_{k,i+1}$$

ATC diffusion algorithm

$$v_{k,i+1} = \lambda \cdot h\left(w_{k,i} - x_{k,i}\right) + \alpha \frac{g_{k,i}}{\left\| g_{k,i} \right\|} + \beta v^{g}_{k,i} + \gamma \delta_{k,i}$$



ATC diffusion algorithm

TECHNISCHE UNIVERSITÄT DARMSTADT

Assign every node k two sets of non-negative real coefficients $c_{k,l}$ and $a_{l,k}$

$$\sum_{l=1}^{N} c_{k,l} = \sum_{l=1}^{N} a_{l,k} = 1, \quad c_{l,k} = a_{l,k} = 0 \quad \text{if} \quad l \notin \mathcal{N}_k.$$

$$h(w - x_k) = \begin{cases} w - x_k, & \text{if} \quad \|w - x_k\| \leq s \\ s \cdot \frac{w - x_k}{\|w - x_k\|}, & \text{otherwise} \end{cases}$$

$$\delta_{k,i} = \sum_{l \in \mathcal{N}_k \setminus \{k\}} (\|x_{l,i} - x_{k,i}\| - r) \frac{x_{l,i} - x_{k,i}}{\|x_{l,i} - x_{k,i}\|}$$

$$g_{k,i} = - \sum_{l \in \mathcal{N}_k \setminus \{k\}} [\sigma_i^2(l) - \sigma_k^2(i)] \frac{x_{l,i} - x_{k,i}}{\|x_{l,i} - x_{k,i}\|}$$

Performance Analysis

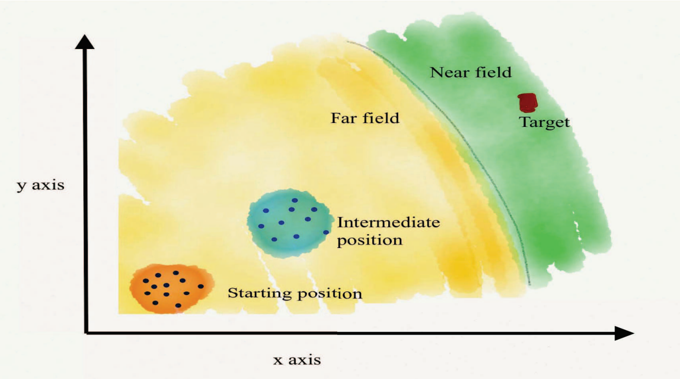


Figure: Near-field and far-field

Performance Analysis

$$0 < \mu_k < \frac{2}{\lambda_{max}(R_u)}$$

$R_u = E[\mathbf{u}^T \mathbf{u}]$ Covariance matrix

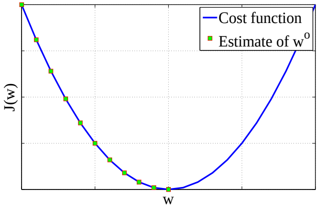


Figure: Cost function

Performance Analysis

Mean-Square-Deviation:

$$\mathrm{MSD}_w \triangleq \lim_{i\to\infty} \frac{1}{N} \sum_{k=1}^{N} E\|w - \mathbf{w}_{k,i}\|^2$$

Mean-Square-Error:

$$\mathrm{MSE}_v \triangleq \frac{1}{N} \sum_{k=1}^{N} E\|\mathbf{v}_{\hat{i}}^g - \mathbf{v}_{k,\hat{i}}^g\|^2$$

Mean-Square-Disagreement:

$$D_v \triangleq \frac{1}{N} \sum_{k=1}^{N} E\|\mathbf{v}_{\hat{i}}^g - \mathbf{v}_{k,\hat{i}}\|^2$$

Performance Analysis

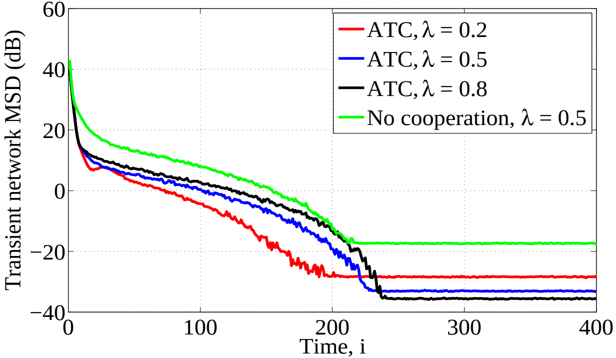


Figure: Transient network MSD for estimating the target location, w^o

Performance Analysis

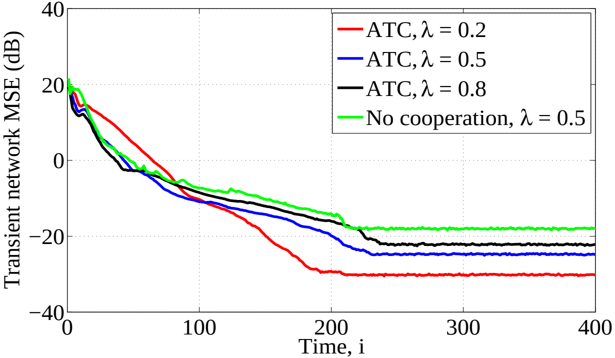


Figure: Transient network MSE for estimating the velocity of the center gravity in the far-field

Performance Analysis

TECHNISCHE UNIVERSITÄT DARMSTADT

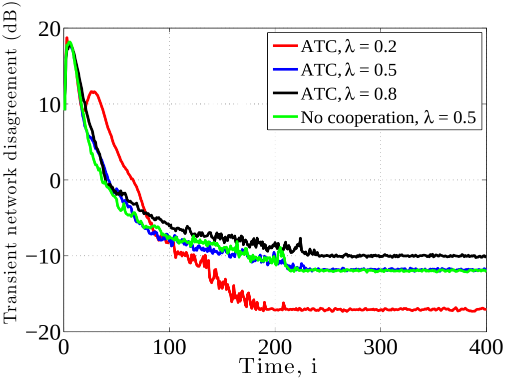


Figure: Transient network mean-square disagreement of velocities in the far-field

Performance Analysis

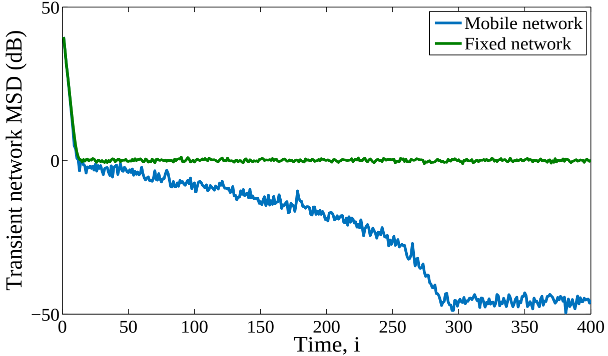


Figure: Transient network MSD for estimating the target location w^o

Simulation

Number of nodes N	50
Dimension	2
C-Matrix	Identity Matrix
A-Matrix	Uniform distribution
μ	0.5
Max-Neighbours	10
α	0.5
γ	0.5
κ	0.0005
β	0.5
λ	0.5

Table: Simulation Parameters

Simulation results

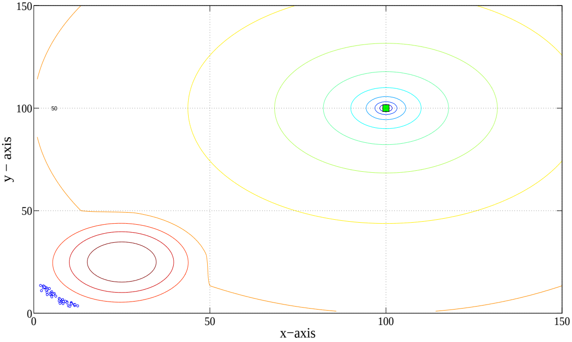


Figure: Maneuvers of mobile networks at $t = 25$ sec

Simulation results

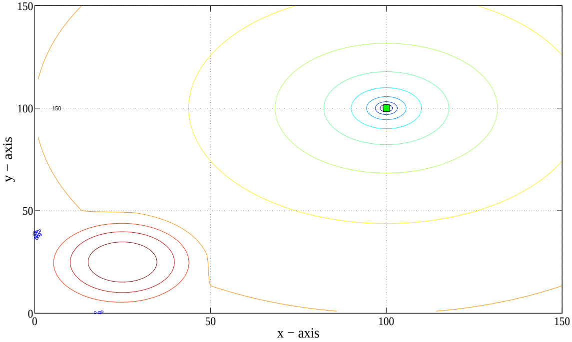


Figure: Maneuvers of mobile networks at $t = 75$ sec

Simulation results

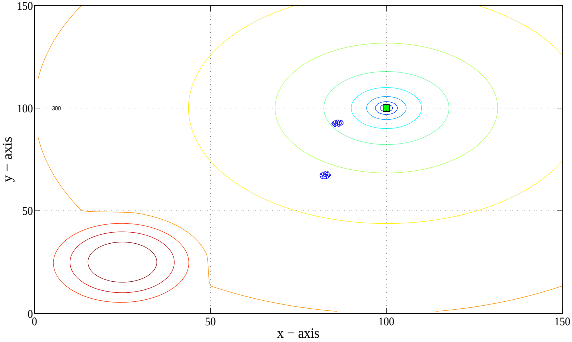


Figure: Maneuvers of mobile networks at $t = 150$ sec

Simulation results

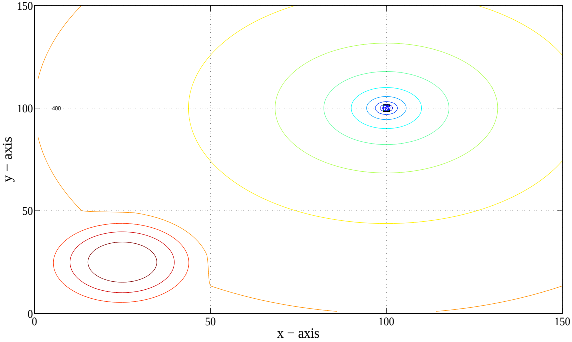


Figure: Maneuvers of mobile networks at $t = 200$ sec

Simulation results

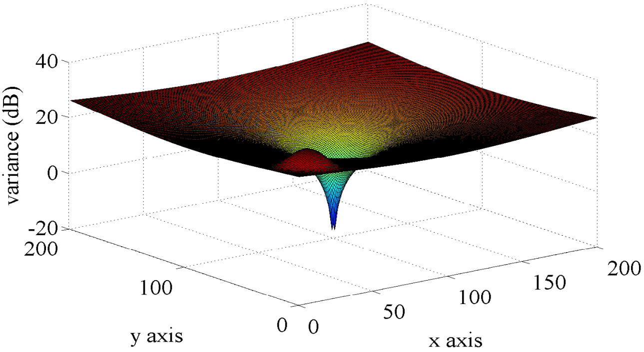
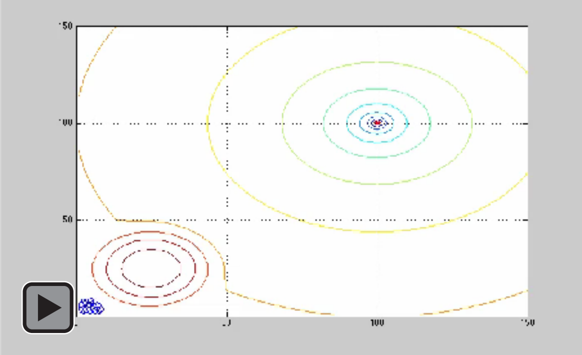


Figure: Noise variance over the plane.

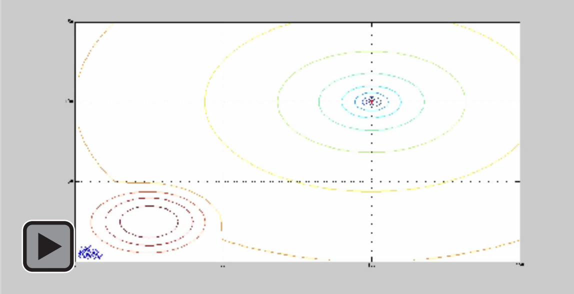
Simulation results

Maneuvers of mobile networks in $\mathbb{R}^2$ over time.



Simulation results

Maneuvers of mobile networks in $\mathbb{R}^2$ over time.



Conclusion

- Strategies involve two diffusion steps
  - Estimation of Target
  - Tracking the centre of the mass of the network
- Analysis of mean-square performance of the diffusion scheme
- Simulation of Algorithm to emulate coherent motion

References

1. S. Y. Tu and A. H. Sayed,"Mobile Adaptive Networks", *IEEE journal of selected topics in signal processing*, vol. 5, no. 4, pp. 649-664, August 2011.
2. A. H. Sayed, S. Y. Tu, J. Chen, X. Zhao, and Z. J. Towfic, "Diffusion Strategies for Adaptation and Learning over Networks", *IEEE signal processing magazine*, pp. 155-171, May 2013.
3. F. S. Cattivelli and A. H. Sayed, Diffusion LMS Strategies for Distributed Estimation, *IEEE Transactions on Signal Processing*, vol. 58, no. 3, pp. 10351048, Mar. 2010.

Thanks for your attention!

Any questions?