MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.

(a) Governments issue bonds instead of printing more money to avoid inflation and maintain economic stability by borrowing from the public rather than increasing the money supply.

(b) The long-term part of a yield curve might flatten if investors expect slower economic growth or anticipate future interest rate cuts by the central bank.

(c) Quantitative easing is a monetary policy where central banks purchase long-term securities to inject liquidity into the economy, and the US Fed employed this during the COVID-19 pandemic to lower interest rates and stabilize financial markets.

ISIN	**Coupon (%)**	**Maturity Date**	**Issue Date**
CA135087K528	1.25	2025-03-01	10/11/2019
CA135087K940	0.50	2025-09-01	04/03/2020
CA135087L518	0.25	2026-03-01	10/09/2020
CA135087L930	1.00	2026-09-01	04/16/2021
CA135087M847	1.25	2027-03-01	10/15/2021
CA135087N837	2.75	2027-09-01	05/13/2022
CA135087P576	3.50	2028-03-01	10/21/2022
CA135087Q491	3.25	2028-09-01	04/21/2023
CA135087Q988	4.00	2029-03-01	10/13/2023
CA135087R895	3.50	2029-09-01	04/08/2024

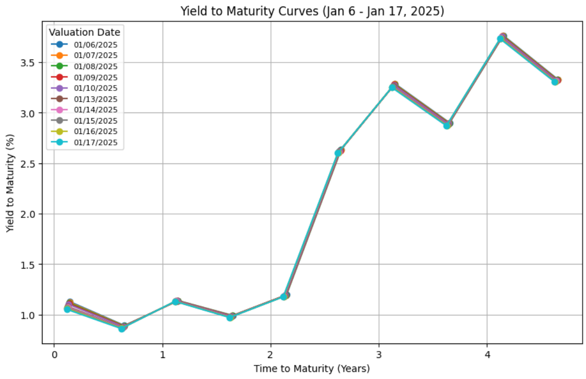
2. I chose these 10 bonds because their maturity dates are evenly spaced in March and September between 2025 and 2029, which makes them ideal for building the yield curve. This spacing avoids any big gaps and keeps things consistent, making it easier to connect the dots when calculating yields. I also tried to pick bonds with earlier issue dates when there were overlaps in maturity, just to keep the data as clean as possible. Overall, this set gives a nice balance and helps capture the term structure clearly without overcomplicating things

3. In the context of stochastic processes, the eigenvectors of the covariance matrix represent the main patterns or trends in how the processes vary together. The corresponding eigenvalues indicate how much of the total variability across all the processes is explained by each trend. A larger eigenvalue means that its associated eigenvector captures a more significant part of the overall variability.

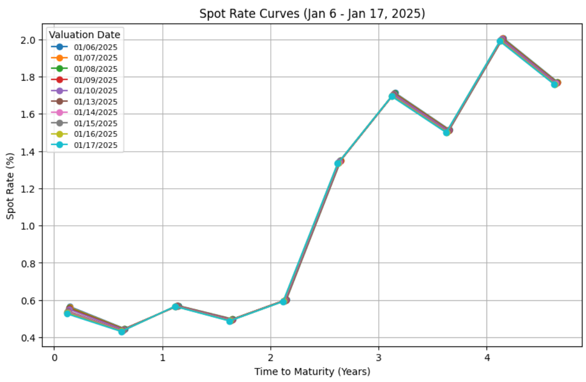
Empirical Questions - 75 points

4.

(a) The Yield to Maturity (YTM) for each bond was calculated by solving the bond pricing equation: $P = \sum_{t=1}^{n} \frac{C}{(1+r)^t} + \frac{F}{(1+r)^n}$ where P is the dirty price, C is the coupon payment, F is the face value, n is the total periods, and r is the unknown YTM. Using the bond's dirty price, coupon rate, and maturity date, numerical methods (via SymPy's `nsolve`) were applied to iteratively solve for r over different valuation dates. To address intermediate maturities or missing points, linear interpolation was applied. Linear interpolation was chosen for its simplicity and computational efficiency. Moreover, the data points are relatively close together so the interpolation will be a relatively good approximation of the missing data. The resulting graph shows the YTM curves for the selected bonds across multiple valuation dates.

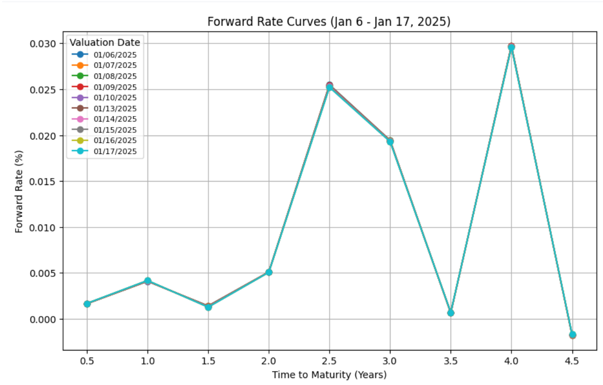


(b) To derive the spot rate curve, we use the bootstrapping method, starting with the bond of the shortest maturity. For each bond, we first compute its cash flows, represented as $C = \frac{\text{coupon rate}}{2} \cdot$ face value, repeated for all periods, with the final payment including the face value. We then calculate the dirty price P as the sum of discounted cash flows. For the first bond, the spot rate r_1 is calculated using $r_1 = \left(\frac{F+C}{P}\right)^{1/T} - 1$, where F is the face value, C is the coupon payment, P is the dirty price, and T is the maturity in years. For subsequent bonds, we discount prior cash flows using previously computed spot rates: Discounted Cash Flows $= \sum_{t=1}^{T-1} \frac{C}{(1+r_t/2)^{2t}}$. The residual is calculated as Residual $= P -$ Discounted Cash Flows, and the new spot rate r_T is derived as $r_T = \left(\frac{\text{Residual}}{\text{Final Payment}}\right)^{1/T} - 1$. This process iterates over all bonds, appending each



r_T to the spot rate curve.

(c) To calculate forward rates from spot rates, we use the formula $F_{t,t+n} = \left(\frac{(1+S_{t+n}/2)^{2(t+n)}}{(1+S_t/2)^{2t}}\right)^{\frac{1}{2n}} - 1$, where S_t and S_{t+n} are the spot rates for maturities t and $t+n$ (in years), respectively, and $n = t + n - t$. We initialize an empty list F to store the forward rates. For each consecutive pair of spot rates (S_t, S_{t+n}), we compute the corresponding forward rate using the formula and append the result to F. This process continues for all pairs of consecutive maturities. The final



list F contains the forward rates.

5. Covariance Matrix for Selected YTMs

$$
\begin{array}{c}
 \\ X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5
\end{array}
\begin{bmatrix}
X_1 & X_2 & X_3 & X_4 & X_5 \\
4.37 \times 10^{-6} & 2.03 \times 10^{-6} & 8.51 \times 10^{-7} & 1.29 \times 10^{-6} & 1.10 \times 10^{-6} \\
2.03 \times 10^{-6} & 9.48 \times 10^{-7} & 3.97 \times 10^{-7} & 5.99 \times 10^{-7} & 5.11 \times 10^{-7} \\
8.51 \times 10^{-7} & 3.97 \times 10^{-7} & 1.66 \times 10^{-7} & 2.51 \times 10^{-7} & 2.14 \times 10^{-7} \\
1.29 \times 10^{-6} & 5.99 \times 10^{-7} & 2.51 \times 10^{-7} & 3.79 \times 10^{-7} & 3.23 \times 10^{-7} \\
1.10 \times 10^{-6} & 5.11 \times 10^{-7} & 2.14 \times 10^{-7} & 3.23 \times 10^{-7} & 2.75 \times 10^{-7}
\end{bmatrix}
$$

Covariance Matrix for Selected Forward Rates

$$
\begin{array}{c}
 \\ X_1 \\ X_2 \\ X_3 \\ X_4
\end{array}
\begin{bmatrix}
X_1 & X_2 & X_3 & X_4 \\
1.88 \times 10^{-6} & 1.89 \times 10^{-6} & -9.04 \times 10^{-6} & -6.02 \times 10^{-7} \\
1.89 \times 10^{-6} & 1.90 \times 10^{-6} & -9.10 \times 10^{-6} & -6.06 \times 10^{-7} \\
-9.04 \times 10^{-6} & -9.10 \times 10^{-6} & 4.37 \times 10^{-5} & 2.90 \times 10^{-6} \\
-6.02 \times 10^{-7} & -6.06 \times 10^{-7} & 2.90 \times 10^{-6} & 1.93 \times 10^{-7}
\end{bmatrix}
$$

6. YTMs:

Eigenvalue	Eigenvector (components)
$4.36993334 \times 10^{-6}$	$[-0.8438, -0.3929, -0.1644, -0.2486, -0.2118]$
$2.03482442 \times 10^{-6}$	$[0.5255, -0.4687, -0.3403, -0.4559, -0.4250]$
$8.51445319 \times 10^{-7}$	$[0.1088, -0.7745, 0.3859, 0.2325, 0.4305]$
$1.28762064 \times 10^{-6}$	$[0.0055, -0.1606, -0.3808, 0.8216, -0.3926]$
$1.09703912 \times 10^{-6}$	$[-0.0007, -0.0022, -0.7089, -0.1176, 0.6954]$

Forward Rates:

Eigenvalue	Eigenvector (components)
$4.76333285 \times 10^{-5}$	$[-0.1982, -0.1996, 0.9575, 0.0636]$
$1.57888675 \times 10^{-8}$	$[0.6708, 0.6645, 0.2880, -0.1599]$
$1.78757222 \times 10^{-14}$	$[-0.2381, 0.0001, 0.0152, -0.9711]$
$1.51534133 \times 10^{-20}$	$[-0.6738, 0.7202, -0.0004, 0.1653]$

The largest eigenvalue of the covariance matrix represents the direction with the most significant variation in the data, while its corresponding eigenvector indicates how different components contribute to this variation. For the YTM covariance matrix, this highlights which maturities or rates dominate the overall changes in yields, showing systemic trends across the time series. Similarly, for the forward rate covariance matrix, the largest eigenvalue and its eigenvector reveal the most influential patterns in forward rate movements, reflecting collective shifts across maturities. These results provide insight into the primary sources of variation, emphasizing dominant factors affecting yields or forward rates over time.

References and GitHub Link to Code

Github Project Link

References:

1. Investopedia - Differences Between Bonds: Yield Rate and It's Coupon Rate
2. Investopedia - YTM
3. Clean to Dirty Price
4. Eigevalues and Eigenvectors in Quant Finance