

Math for ML - Ram 10 of 2018

Random Walks

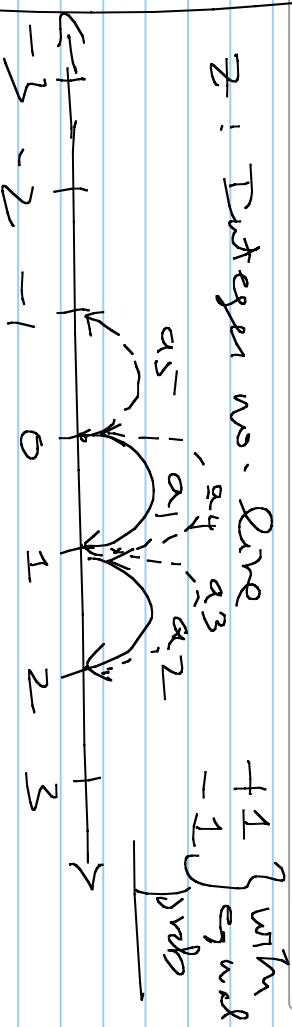
- Stochastic or random process
- Describes a path:

Succession of random

steps on some mathematical space

→ e.g. real line,

- integer no. line (\mathbb{Z})
- lattice (2-d, d-dim)
- graph (directed / undirected)



step
 \downarrow
 $0 \rightarrow +1 \quad +1 \quad -1 \quad -1 \quad \boxed{-1}$

e.g. path traced by a molecule in liquid or gas

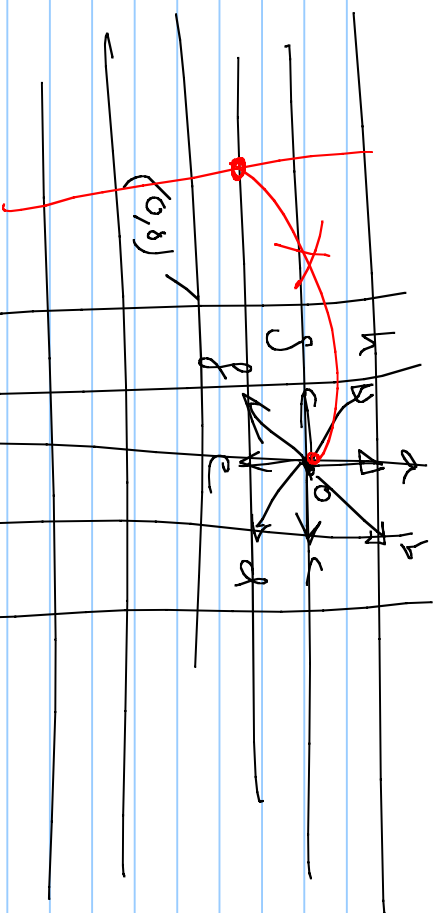
- Search path of a foraging animal
- Price of a fluctuating stock

Random Walk on Regular Lattice

Each step: Jump to 'another' site according to some probability distribution.

Simple Random Walk

Lattice path is formed by "jumps" from current lattice point to 'ONLY' neighbors per site.



$0 \rightarrow (a, b, c, \dots, h)$

\rightarrow with different probabilities

$p(0 \rightarrow a), p(0 \rightarrow b), \dots, p(0 \rightarrow h)$

Simple Symmetric RW

- on a locally finite lattice
- Prob of the location (point) jumping to each one of its immediate neighbors' are same

(IN) - In d-dim integer lattice

Qn \mathbb{Z}^d : Hypercube lattice

- Assignment: a) How many IN's are there?

b) What are their prob?

Simple Bordered Symmetric RW

- State space is limited to finite dim

- Transition probabilities depend on the location of the state

(Margin, Corner States Don't move)



RW on Graphs*

Defn: A sequence of vertices generated from a start vertex by selecting an edge, traversing the edge to a new vertex, and repeating

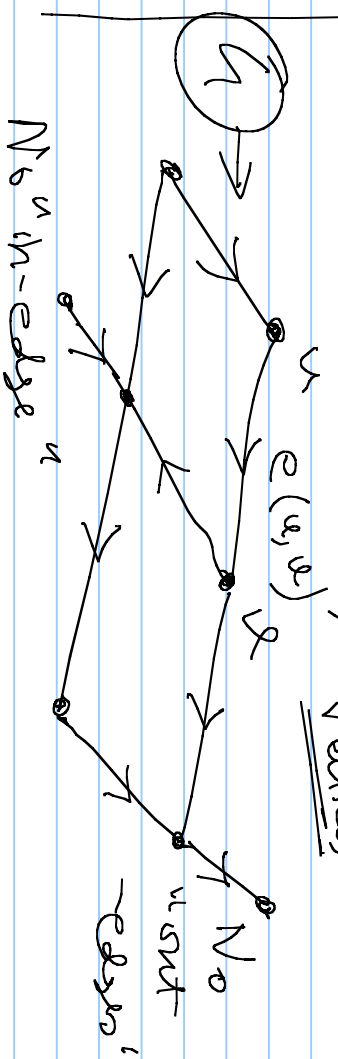
the process "1"

* Directed Graphs

$$G = (V, E)$$

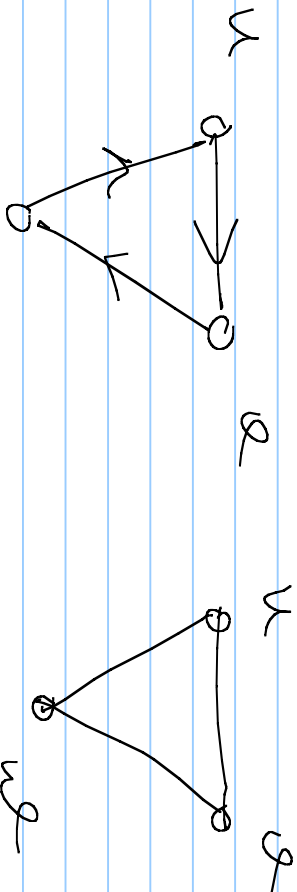
$V \rightarrow$ Set of vertices (nodes or points)
 $E \rightarrow$ Set of edges

where (u, v) is an edge of E
if vertex " u " is pointing to / connected to vertex " v ". \rightarrow ordered pair of vertices



Undirected Graph

edges have no orientation



w

Directed

Undirected

edge (u, v)

$= \text{edge } (v, u)$

→ Not an "ordered pair"
but an "unordered pair"

Directed Graph

An ordered pair of vertices

(u, v) is called "strongly

connected" if a directed

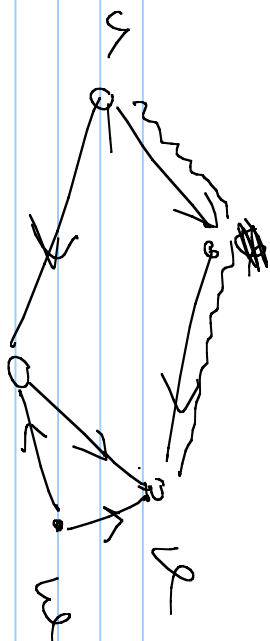
path leads from u to v

Definition: The pair of vertices (u, v)

~~are~~ is called "weakly connected"

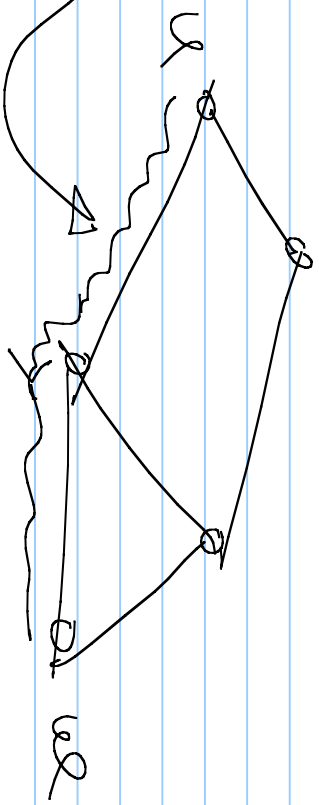
If an undirected path leads from u to v after replacing all of its "directed" edges with undirected

edges.



(u, v) : Strongly connected

(u, w) : weakly connected



A "strongly connected graph"
is a directed graph
in which every "ordered pair"
of vertices in the graph
is strongly connected.